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PRINCIPAL COMPONENT ANALYSIS OF SEISMIC DATA and DIRECTION OF THE PRINCIPAL COMPONENT FOR SEISMIC RECORD

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PRINCIPAL COMPONENT ANALYSIS OF SEISMIC DATA and DIRECTION OF THE PRINCIPAL COMPONENT FOR SEISMIC RECORD

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ABSTRACT

This report consists of two technical notes prepared by S.C. Choi, of the Measurement Analysis Corporation. The first is MAC Technical Note 409-16. In this report principal component theory is developed. It is noted that the main use of principal components is in the reduction of random variables to a small number of linear combinations of random variables. A summary of statistical interpretation of the principal component is given. An example of principal component analysis is given for LASA noise data. Results are given concerning the proportion of the total variance (power) from 10 seismometers as explained by the first four principal components. At 0.2 cps approximately 30% of the total variance is accounted for by first principal component. In the summary it is concluded that it seems quite worthwhile to investigate the applications of the principal component to seismic noise study.

The second report is MAC 409-21. In the first report it was mentioned that the direction of noise source might be determined by examination of the principal component. This report pursues this argument.

The seismic signal data used are from LONGSHOT and a geographically nearby earthquake recorded at a LASA subarray. The phase shifts of the first principal component at 0.2 cps corrected for LASA instrument response prove to be interesting. From these phase shifts it appears that the general direction of the main noise source can be estimated. Computed examples for LONGSHOT and the nearby earthquake are given to verify this claim.

PRINCIPAL COMPONENT ANALYSIS OF SEISMIC DATA

S. C. Choi

1. INTRODUCTION

The purpose of this report is to describe some interpretations and applications of the principal component to seismic noise records. The main use of the principal component is in the reduction of random variables to a small number of linear combinations. It can be shown that the sum of the variances of all principal components is the sum of the variances of the original variables. See Reference 1. Thus, if there exist principal components with large variances which account for most of the variability, the dimensionality of the problem might be reduced by attention only to these principal components.

Let $\Sigma(\omega)$ be the k x k spectral density matrix at frequency ω of a zero-mean multiple time series $x(t)$. $\Sigma(\omega)$ is the Hermitian non-negative definite matrix. Further, let $Z(\omega)$ be the Fourier transform of $x(t)$. Then there exists a p-component column vector $\beta'(\omega)$ such that

$$\beta'(\omega)\,\beta(\omega) = 1 \tag{1}$$

and the variance of $\beta'(\omega)\, Z(\omega)$ is

$$E[\beta'(\omega)\, Z(\omega)]^2 = E[\beta'(\omega)\, Z(\omega)\, Z(\omega)\, \beta(\omega)]$$

$$= \beta'(\omega)\, \Sigma(\omega)\, \beta(\omega) \tag{2}$$

Henceforth, ω will be omitted from the notation for simplicity. It will be understood that the results apply independently to each frequency value ω. To determine the normalized linear combination $\beta' Z$ with a maximum variance, it is necessary to find a vector β satisfying $\beta'\beta = 1$ which maximizes the variance $\beta'\Sigma\beta$. Let

$$T_1 = \beta'\Sigma\beta - \lambda(\beta'\beta - 1) \tag{3}$$

where λ is a Lagrange multiplier. The vector of partial derivatives is

$$\frac{\partial T_1}{\partial \beta} = 2\Sigma\beta - 2\lambda\beta \tag{4}$$

Setting Eq. (4) equal to zero, one obtains

$$(\Sigma - \lambda I)\,\beta = 0 \tag{5}$$

and Z must satisfy

$$\left|\Sigma - \lambda I\right| = 0 \tag{6}$$

Since Eq. (5) is a polynomial equation of degree k, it has k roots. Let these be $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$. If Eq. (5) is multiplied by β', then

$$\beta' \Sigma \beta = \lambda \beta' \beta = \lambda$$

Note that $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the eigenvalues of Σ. This shows that the variance of $\beta' \Sigma$, given by Eq. (2) is simply λ. Let β_1 be normalized solution of $(\Sigma - \lambda_1 I)\beta = 0$. Then $C_1 = \beta_1' Z$ is a normalized linear combination with maximum variance with the variance equal to λ_1. C_1 is called the first principal component.

The second principal component C_2 is defined as a normalized combination that has a maximum variance of all linear combinations uncorrelated with C_1. Lack of correlation is specified by the condition

$$E(\beta' Z P_1) = E(\beta' Z Z' \beta_1)$$

$$= \beta' \Sigma \beta_1 = \lambda_1 \beta' \beta_1 = 0 \tag{7}$$

Thus, one must maximize

$$T_2 = \beta' \Sigma \beta - \lambda(\beta' \beta - 1) - 2v\beta' \Sigma \beta_1 = 0 \qquad (8)$$

where λ and v are Lagrange multipliers. Let β_2 be the normalized solution of Eq. (8). Then $C_2 = \beta_2' Z$ is the second principal component with the variance λ_2.

The remaining $k - 2$ principal components are similarly defined.

2. THE PRINCIPAL COMPONENTS IN SEISMIC RECORDS

Suppose that there are n sample records each obtained from an array of k seismometers. Undoubtedly there exists some linear relation between the k seismometers. Principal component analysis is designed to explain observed relations among k records in terms of simpler relations. The simplification consists of creating a smaller number of hypothetical variables called principal components. The principal components might be interpreted physically as representing underlying independent noise sources or possibly noise vibration modes.

In Section 1, eigenvectors and eigenvalues denoted by β and λ refer to population values. These statistics are estimated by corresponding samples values $\hat{\beta}$ and $\hat{\lambda}$ derived from the sample spectral density matrix $\hat{\Sigma}$. For simplicity of notation, the $(\hat{\ })$ notation will be omitted henceforth. In the previous section it was found that the jth principal component C_j is the normalized linear combination of variable Z that has a maximum variance, but is uncorrelated with the 1st to the j - 1st principal components. The variance of C_j is given by the jth largest eigenvalue λ_j.

Suppose that there are k seismometers. Let $Z_i(\omega)$ be the Fourier transform of record at the ith seismometer. Then the jth principal component C_j has the form

$$C_j = \sum_{i=1}^{k} \beta_{ij} Z_i \quad , \qquad j = 1, 2, \ldots, k \tag{9}$$

where β_{ij} is the ith component of the jth eigenvector associated with eigenvalue λ_j. Note that λ_j is a real value since the sample spectral density matrix Σ is a Hermitian. The proportion P_j of the variance of all k seismic recorders explained by j linear combinations $C_1, C_2, \ldots, C_j$ is clearly

$$P_j = \sum_{i=1}^{j} \lambda_i \Big/ \sum_{i=1}^{k} \lambda_i \tag{10}$$

For example, if $P_j = 0.99$, then 99% of the variances of all records is explained by $C_1, C_2, \ldots, C_j$. In this case, one would only investigate these j linear functions. In addition, these functions are uncorrelated.

After extracting j eigenvalues the possibility exists that all the remaining k - j eigenvalues may be the same, especially when they are small. If this is the case then there is no reason to find the remaining principal components since all the remaining principal components are identical and they have the same variance. Therefore, it is of interest to test the following hypothesis:

$$\begin{aligned} H_0 &: \lambda_{j+1} = \lambda_{j+2} = \ldots = \lambda_k \\ H_1 &: \lambda_i > \lambda_m \quad , \quad k \geq i > m \geq j+1 \end{aligned} \tag{11}$$

Reference 4 suggests the following statistic to test the above hypothesis.

$$\chi^2_{n'} = n'[-\log|\Sigma| + \log(\lambda_1 \cdot \lambda_2 \ldots \lambda_j) = (k - j)\log\lambda] \tag{12}$$

where $\quad n' = n - j - (2k - 2j + 1 + \frac{2}{k - j})/6$

$$\lambda = (\text{trace } \Sigma - \lambda_1 - \lambda_2 - \ldots - \lambda_j)/(k - j)$$

It can be shown $\chi^2_{n'}$ given by Eq. (12) has an approximate chi-square distribution with n' degree-of-freedom.

Now, solving the k linear equations from (9), one can express Z_i in terms of C's, i.e.,

$$Z_i = a_{i1} C_1 + a_{i2} C_2 + \ldots + a_{ik} C_k \tag{13}$$

$$i = 1, 2, \ldots, k$$

It can be shown that the jth coefficient vector of the component C_j is given by $\sqrt{\lambda_j}\beta_j$, i.e.,

$$\begin{pmatrix} a_{1j} \\ a_{2j} \\ \cdot \\ \cdot \\ \cdot \\ a_{kj} \end{pmatrix} = \sqrt{\lambda_j} \begin{pmatrix} \beta_{1j} \\ \beta_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{kj} \end{pmatrix} \tag{14}$$

where λ_j and β_{ij} are defined as before. Therefore, the coefficients $(a_{i1}, a_{i2}, \ldots, a_{ik})$ of Eq. (13) are given by:

$$(a_{i1}, a_{i2}, \ldots, a_{ik}) = (\beta_{i1}\sqrt{\lambda_1}, \beta_{i2}\sqrt{\lambda_2}, \ldots, \beta_{ik}\sqrt{\lambda_k}) \tag{15}$$

where β_{ij} is the ith component of the jth eigenvector. The component of the coefficient vector $(a_{i1}, a_{i2}, \ldots, a_{ik})$ is sometimes called the factor loadings.

In Eq. (15), suppose that $a_{ij+1} = a_{ij+2} = \ldots = a_{ik} = 0$. Then

$$Z_i = a_{i1} C_1 + a_{i2} C_2 + \ldots + a_{ij} C_j \tag{16}$$

and one may infer that Z_i is governed by j uncorrelated components. In particular, the quantity

$$H = a_{i1}^2 + a_{i2}^2 + \ldots + a_{ij}^2 \tag{17}$$

is called the communality of a variable Z_i, and it is an index of the contribution of the underlying common components to the total unit variance of the variable. In particular, a_{im}^2 indicates the contribution of the component C_m to the communality of Z_i. Since communality of a variable Z_i is the amount of the variance of the variable accounted for by the

common components together, this will be less than the whole variance of the ith seismometer. Thus, a residue may remain which is uniquely accounted for by a specific error component. Table 1 presents the complete component matrix where S_i^2 denotes the variance due to specific and error components.

Table 1. Contribution of Components to Total Variance

Seismometers	Common Components				Specific and Error Components			
	1	2	...	j	1	2	...	k
1	a_{11}^2	a_{12}^2	...	a_{1j}^2	S_1^2			
2	a_{21}^2	a_{22}^2	...	a_{2j}^2		S_2^2		
.	.	.		.			.	
.	.	.		.			.	
.	.	.		.			.	
k	a_{k1}^2	a_{k2}^2	...	a_{kj}^2				S_k^2

As a summary the following statistical interpretation of the principal component can be given:

a. The sum of the variances of all principal components is identical to the sum of the variances of the original variables.

b. Of all linear functions of the variables, the first principal component accounts for largest variance of the sum of the original variances. The second component has a maximum variance of all linear combinations uncorrelated with the first component. The remaining components are analogously defined.

c. The first principal component is the linear function of the variables which has least variance due to error of measurement. Among all linear functions of variables which are uncorrelated with the first component, the second component has least variance resulting from such errors, and so on for the other components.

d. Of all linear functions of variables, the first component has the greatest mean-square correlation with the variables; the second component the next mean-square correlation with the variables, and so on for the remaining components.

3. EXAMPLE

An example of the principal component analysis given below which is performed on LASA noise data available at the Earth Science Division of Teledyne, Inc. Refer to Reference 2.

The first principal component of the records of seismogram 5507 at f = 0.20 cps is computed as

$$C_1 = (0.2630 - 0.0668i)\, Z_1 + (0.2774 - 0.0565i)\, Z_2 + (0.3008 - 0.0696i)\, Z_3$$
$$+ (0.1753 - 0.0690i)\, Z_4 + (0.2502 - 0.0814i)\, Z_5 + (0.1024 - 0.0991i)\, Z_6$$
$$+ (0.2596 - 0.0684i)\, Z_7 + (0.2424 - 0.0473i)\, Z_8 + (0.2596 - 0.0162i)\, Z_9 + Z_{10} \tag{18}$$

where Z_i is the Fourier transform of the noise record at the ith seismometer. In polar form Eq. (15) can be written as

$$C_1 = 0.271\ell^{-14.3i}\, Z_1 + 0.283\ell^{-11.5i}\, Z_2 + 0.309\ell^{-13.0i}\, Z_3$$
$$+ 0.188\ell^{-21.4i}\, Z_4 + 0.263\ell^{-18.0i}\, Z_5 + 0.142\ell^{-44.1i}\, Z_6 \quad . \tag{19}$$
$$+ 0.268\ell^{-14.7i}\, Z_7 + 0.247\ell^{-11.1i}\, Z_8 + 0.258\ell^{-3.6i}\, Z_9 + Z_{10}$$

Equation (19) expresses the first principal component in terms of the gain and phase of the coefficients for the first 9 seismometers relative to the 10th seismometer. The magnitudes of gain and phase factors would be expected to lead to interpretations regarding the makeup of the mass field when considered relative to the principal components in other frequency bands.

Using Eq. (15) it is possible to express the seismic record of each seismometer in terms of the principal components. For the above example they turn out as follows:

$$Z_1 = (.2630 - .0668i)(1.071 \times 10^{-4}) C_1 + (.0301 - .0935i)(1.552 \times 10^{-5}) C_{10}$$
$$= .290 \times 10^{-4} e^{-14.3i} C_1 + .152 \times 10^{-5} e^{-72.2i} C_{10}$$

$$Z_2 = (.2774 - .0565i)(1.071 \times 10^{-4}) C_1 + (-.0254 - .0123i)(1.552 \times 10^{-5}) C_{10}$$
$$= .303 \times 10^{-4} e^{-11.5i} C_1 + .043 \times 10^{-5} e^{-154.2i} C_{10}$$

$$Z_3 = (.3008 - .0696i)(1.071 \times 10^{-4}) C_1 + (-.4499 + .2292i)(1.552 \times 10^{-5}) C_{10}$$
$$= .331 \times 10^{-4} e^{-13.0i} C_1 + .843 \times 10^{-5} e^{-207.0i} C_{10}$$

$$Z_4 = (.1753 - .0690i)(1.071 \times 10^{-4}) C_1 + (.3414 - .0156i)(1.552 \times 10^{-5}) C_{10}$$
$$= .201 \times 10^{-4} e^{-21.4i} C_1 + .531 \times 10^{-5} e^{-2.6i} C_{10}$$

$$Z_5 = (.2502 - .0814i)(1.071 \times 10^{-4}) C_1 + (.2279 - .1040i)(1.552 \times 10^{-5}) C_{10}$$
$$= .282 \times 10^{-4} e^{-18.0i} C_1 + .388 \times 10^{-5} e^{-24.5i} C_{10}$$

$$Z_6 = (.1024 - .0991i)(1.071 \times 10^{-4})\, C_1 + 1\,(1.552 \times 10^{-5})\, C_{10}$$
$$= .152 \times 10^{-4}\, e^{-44.1i}\, C_1 + 1.552 \times 10^{-5}\, C_{10}$$

$$Z_7 = (.2596 - .0684i)(1.071 \times 10^{-4})\, C_1 + (.3685 - .0574i)(1.552 \times 10^{-5})\, C_{10}$$
$$= .268 \times 10^{-4}\, e^{-14.7i}\, C_1 + .579 \times 10^{-5}\, e^{-8.9i}\, C_{10}$$

$$Z_8 = (.2424 - .0473i)(1.071 \times 10^{-4})\, C_1 + (-.0310 - .1200i)(1.552 \times 10^{-5})\, C_{10}$$
$$= .247 \times 10^{-4}\, e^{-11.1i}\, C_1 + .192 \times 10^{-5}\, e^{-104.4i}\, C_{10}$$

$$Z_9 = (.2596 - .0162i)(1.071 \times 10^{-4})\, C_1 + (-.4112 - .1878i)(1.552 \times 10^{-5})\, C_{10}$$
$$= .258 \times 10^{-4}\, e^{-3.6i}\, C_1 + .702 \times 10^{-5}\, e^{-155.4i}\, C_{10}$$

$$Z_{10} = 1\,(1.071 \times 10^{-4})\, C_1 + (-.4465 - .1495i)(1.552 \times 10^{-5})\, C_{10}$$
$$= 1.071 \times 10^{-4}\, C_1 + .731 \times 10^{-5}\, e^{-161.5i}\, C_{10}$$

Note that approximately 93 percent of noise source at each seismometer, 1 through 10, are explained by the respective equations in the above.

From the above equations it is suspected that there are two underlying power sources from two directions. In order to make a definite statement about the number and directions of the noise sources, it is necessary to make a further empirical study with data of which one knows the information ahead of time.

The variance contributed by C_1 is given by the largest eigenvalue which is

$$\lambda_1 = 1.0709 \times 10^{-4}$$

Since the total variance due to all principal components is 1.3166×10^{-4}, the percentage of variance accounted for by the 1st principal component C_1 is

$$\frac{1.0709}{1.3166} \times 100 = 81.34\%$$

Therefore, approximately 80% of the variation (power) in the data from all 10 seismometers can be accounted for by investigation of the single linear combination C_1. This tends to imply that there exists one major underlying noise component in the low frequency range.

Proceeding in the above way, one can obtain the following tables which illustrate the contributions due to the first four principal components of seismograms 5507, 5508, and 5509 at 0.20 cps.

Table 2. Proportions of Variance

Components / Seismograms	C_1	C_2	C_3	C_4
5507	81.34	11.78	4.65	0.98
5508	86.39	7.50	3.78	1.28
5509	84.69	9.14	4.25	0.77

Table 3. Cumulative Proportions of Variance

Components / Seismograms	C_1	C_2	C_3	C_4
5507	81.34	93.12	97.77	98.75
5508	86.39	93.89	97.67	98.93
5509	84.69	93.83	98.08	98.85

It will be observed that in all three seismograms, the first component accounts for over 80% of the total variance in the 10 seismic measurements. If one is interested in studying the conditions that lead to variations of 10 seismic records at 0.20 cps, one can look for variations in conditions that lead to variations of the first principal component, for example, C_1 given in Eq. (18) in the case of seismogram 5507. If one wants to account for 90% of the total variances (or power) then one should study the first two components.

Table 4 shows the proportions of variance and cumulative proportions of the first two components for seismogram 5507 at 0.20, 0.60, 1.00, 1.40, and 1.80 cps.

Table 4. Proportions of Variance at Different Frequencies

Component cps	C_1	C_2	Total
0.20	81.34	11.78	93.12
0.60	69.00	9.68	78.68
1.00	62.49	9.78	72.27
1.40	50.26	15.54	65.80
1.80	63.97	13.30	76.27

Table 4 indicates that in higher frequencies, the first two principal components account for lesser amounts of the sum of the variance than in a lower frequency. This is consistent for all other seismograms. The above result is quite similar to the coherence study made on 10 seismometers (Reference 3). It has been reported that the coherence between records at higher frequencies is in general less than that at lower frequencies. It is clear that if there are very high coherences between records, then the first component would account for a large portion of the sum of the variances. This indicates that the noise fields are more local in higher frequency bands.

4. SUMMARY

The principal component may prove to be a useful tool to analyze the seismic data. It seems quite worthwhile to investigate the applications of the principal component to the seismic noise study. The following two fields of study are particularly worth undertaking with the vast amount of data at UED.

First, it is clearly suspected that if the underlying power source is from one or two directions and powerful, then the first couple principal components will account for a major portion of the total variance both at lower and higher frequencies. Therefore, like the multiple coherence function, the principal component may prove to be a useful tool to determine if the underlying power source is from one or two directions and powerful as in the case of a bomb explosion or earthquake.

Secondly, the direction of bomb explosions or earthquakes might be empirically determined by examination of the principal component. It appears quite feasible to determine the approximate direction of the seismic noise source by studying the gain and the phase of each seismometer in polar form of the principal component. See Eq. (15) for example.

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DIRECTION OF THE PRINCIPAL COMPONENT FOR SEISMIC RECORD

1. INTRODUCTION

Some potential applications of principal component analysis to seismic array data were discussed in MAC Technical Note 409-16. In that note it was mentioned that the direction of noise source might be determined by examination of the principal component. This note is intended to pursue this argument.

Suppose that there are n seismometers. Let $Z_i(\omega)$ be the Fourier transform of record $X(t)$ at frequency ω at the ith seismometer. If p $(p \leq n)$ principal components are denoted by $C_1(\omega)$, $C_2(\omega)$, ..., $C_p(\omega)$ then one can express $Z_i(\omega)$ as

$$Z_i(\omega) = a_{i1}(\omega)\, C_1(\omega) + a_{i2}(\omega)\, C_2(\omega) + \ldots + a_{ip}(\omega)\, C_p(\omega),$$

$$i = 1, 2, \ldots, n \qquad (1)$$

Henceforth, ω may be omitted from the notation for simplicity. Each $Z_i(\omega)$ may be interpreted as an output variable with m uncorrelated inputs $C_1, C_2, \ldots, C_p$ in a constant parameter linear system. The quantity $a_{im}\, C_m$ is the part of the output $Z_i(\omega)$ that is produced by the mth input component. See Figure 1.

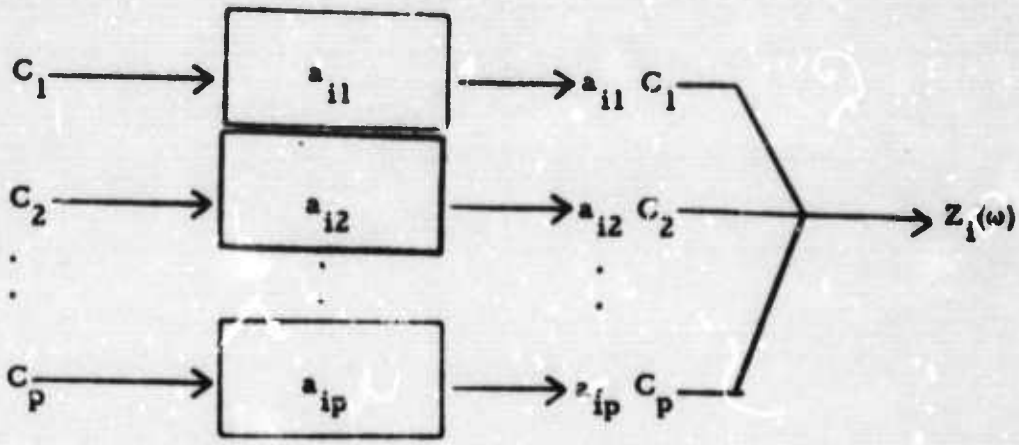

Figure 1. Components as Inputs

The coefficient a_{ij} can be interpreted as the transfer function which is associated with the input C.

2. APPLICATIONS OF PRINCIPAL COMPONENTS

First, consider the coherence function between $Z_i(\omega)$ and the mth principal component C_m denoted by $\gamma^2_{C_m i}$.

$$\gamma^2_{C_m i}(\omega) = \frac{\left| S_{C_m i}(\omega) \right|^2}{S_{C_m}(\omega)\, S_i(\omega)} \tag{2}$$

$$= \frac{\left| E(C_m\, Z_i^*) \right|^2}{E(C_m\, C_m^*)\, E(Z_i\, Z_i^*)}$$

$$= \frac{\left| a_{im}\, a_{im}^* \right| \left| \mathrm{Var}\,(C_m) \right|^2}{\mathrm{Var}\,(C_m) \sum_{k=1}^{p} \left| a_{ik}\, a_{ik}^* \right| \mathrm{Var}\,(C_k)}$$

$$= \frac{1}{1 + \left[\dfrac{\sum_{k \neq m}^{p} \left| a_{ik}\, a_{ik}^* \right| \lambda_k}{\left| a_{im}\, a_{im}^* \right| \lambda_m^2} \right]} \tag{3}$$

where λ_k is the kth largest eigenvalue of the $n \times n$ spectral matrix at frequency ω of a zero-mean multiple time series $X(t)$. The coherence function given by Eq. (3) can be interpreted as the proportion of power in $X(t)$ accounted by the kth component. It seems that this is a much more sensible application of the coherence function than that of previous study when the "output" is the record from a more or less arbitrary selected seismometer. What would happen if the selected record has bad data? For example, the multiple coherence function could be near zero even if records at all other seismometers have high coherence.

In previous reports (see Reference 2 for example) a central seismometer was always chosen as the "representative" of a subarray. Statistically, it appears that the best representative is one which minimizes the residual variance in predicting its record by the best linear regression on records of other seismometers. The residual variance σ^2 in predicting Z_i by a linear regression on

$$L(Z_i) = b_1 Z_1 + \ldots + b_{i-1} Z_{i-1} + b_{i+1} Z_{i+1} + \ldots + b_n Z_n \quad (4)$$

is

$$\sigma_i^2 = \mathrm{Var}\,(Z_i) - \frac{\mathrm{Cov}^2\,[Z_i,\ L(Z_i)]}{\mathrm{Var}\,[L(Z_i)]} \quad (5)$$

Therefore, the kth seismometer can be selected as the "best representative" where

$$\sigma_k^2 = \min_{i=1,n} (\sigma_i^2) \quad (6)$$

It can be shown (see Reference 3) that the vector $(b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n)$ of Eq. (4) which minimizes σ_i^2 given by Eq. (5) (for given index i) is the eigenvector corresponding to the largest root of the following equation.

$$\left| \Sigma_{21} \Sigma_{12} \Sigma_{22}^{-1} - \lambda I \right| = 0 \tag{7}$$

where the n x n spectral matrix Σ is partioned as follows

$$= \left[\begin{array}{c|c} \sigma_i^2 & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array} \right] \tag{8}$$

Of course, the second alternative is to use the first principal component itself or the seismometer with the highest coherence with the first component as the best representative. Empirical studies have indicated that approximately 70% of total variance is accounted for by the first principal component. The following table illustrates this statement. The data are from Longshot (LS) and an earthquake (EQ) recorded by a LASA subarray.

Table 1. Percentage of Total Variance Accounted for by the First Two Principal Components.

	ω	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
LS	C_1	70.4	69.7	91.9	76.9	85.6	64.5	56.7	79.4	88.3	72.8
	C_2	11.0	12.1	2.7	8.0	6.1	16.8	23.2	8.0	4.6	9.9
EQ	C_1	71.6	54.3	15.4	62.1	75.1	54.1	62.6	71.9	80.2	71.2
	C_2	10.2	14.0	12.6	18.0	10.2	22.4	10.5	9.9	7.3	9.3

Note that the first and second components account for 75.6 and 10.2 percent of total variance for the long shot data and 66.9 and 12.4 percent for earthquake data respectively in the above example.

In order to find the seismometer whose record has the highest coherence with the first principal component C_1, Eq. (3) can be used. From Eq. (3) it can be seen that the coherence function $\gamma^2_{C_1 i}$, between C_1 and the ith seismometer is a monotonic increasing function of the gain factor of the seismometer. Therefore, kth seismometer has the highest coherence with C_1 at the frequency ω if

$$\gamma^2_{C_1 k}(\omega) \geq \gamma^2_{C_1 i}(\omega) \quad \text{for all } i \tag{9}$$

For the particular example summarized in the Appendix, the seismometer with the maximum coherence with C_1 at each frequency ω is summarized in Table 2.

Table 2. Seismometers with the Maximum Coherences

ω	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
LS	81	81	26	81	83	22	85	54	10	52
EQ	83	10	81	26	85	85	83	56	10	22

Now, consider the first principal component denoted by $C_1(\omega)$. One can express $Z_i(\omega)$ as follows in terms of $C_1(\omega)$.

$$Z_i(\omega) = a_{i1}(\omega)\, C_1(\omega) + e_i(\omega) \qquad i = 1, 2, \ldots, m \tag{10}$$

where

$$e_i = \sum_{j=2}^{m} a_{ij}(\omega)\, C_j(\omega)$$

Equation (4) can be written as

$$\left|Z_i(\omega)\right| e^{j\phi_i(\omega)} = \left|a_{i1}(\omega)\right| \left|C_1(\omega)\right| e^{j[\phi_i'(\omega) + \phi(\omega)]} + e_i(\omega) \tag{11}$$

where $\phi_i(\omega)$, $\phi_i'(\omega)$ and $\phi(\omega)$ denote the associated phase shifts of Z_i, a_{i1} and C_1 respectively. One may suppose that the first component is the principal input and $Z_i(\omega)$ is an output at the given frequency ω. Then the ratio of the output amplitude to the input amplitude is equal to $\left|a_{i1}(\omega)\right|$ and the phase shift of the output $Z_i(\omega)$ from the input $C_1(\omega)$ is given by $\phi_i'(\omega)$. Therefore, the relative phase shifts of the outputs $Z_1(\omega)$, $Z_2(\omega)$, ..., $Z_n(\omega)$ from the first principal component are given by $\phi_1'(\omega)$, $\phi_2'(\omega)$, ..., $\phi_n'(\omega)$. Examination of these phase shifts by ordering them will possibly indicate the general direction of the first principal axis of an ellipsoid. This may be considered as the direction of main noise source because the axis has the greatest sum of all coherences with all seismometer records.

As an illustration consider a three-dimensional space with $\phi_1'(\omega) = \phi_2'(\omega) \neq \phi_3'(\omega)$. Then the principal axis of an ellipsoid is parallel to the line joining Z_1 and Z_2 and it may be illustrated as in Figure 2.

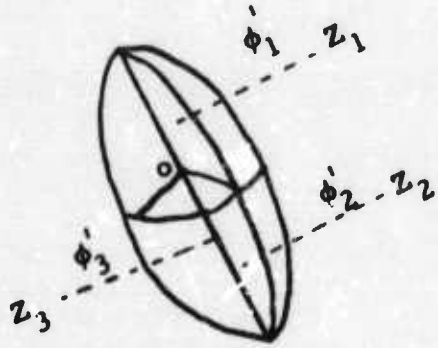

Figure 2. Principal Axis of Ellipsoid

In Figure 2, it is interesting to observe that a large eigenvalue means that in the direction of the principal axis the quadratic surface comes near to the center. The smaller the eigenvalue the greater the distance from the surface point to the center. This can be seen as follows.

Let $\sum$ be the $n \times n$ spectral density matrix. Then

$$\sum \beta = \lambda \beta \tag{12}$$

or

$$\beta' \sum \beta = \lambda \beta' \beta = \lambda \sum_{i=1}^{n} b_i^2 = 1 \tag{13}$$

where b_i denotes the ith element of a eigenvector β. It follows from Eq. (13)

$$\lambda = \frac{1}{\sum_{i=1}^{n} b_i^2} \tag{14}$$

Thus λ is the reciprocal of the square of distance from the center.

The application of the second and remaining principal components in terms of the above argument is analogous. Suppose that one has $C_j(\omega)$ in terms of $Z_i(\omega)$'s, that is,

$$c_j(\omega) = b_{j1} Z_1(\omega) + b_{j2} Z_2(\omega) + \ldots + b_{jn} Z_n(\omega) \tag{15}$$

Then it can be shown (see MAC Technical Note 409-16) that a_{1i}, a_{2i}, ..., a_{ni} are simply obtained by multiplying b_{i1}, b_{i2}, ..., b_{in} by λ_i. Therefore, the relative magnitudes of the phase shifts of Z_1, Z_2, ..., Z_n are conveniently obtained by the phase factors of b_{j1}, b_{j2}, ..., b_{jn} of Eq. (15).

The following examples illustrate the preceding discussion. Data are obtained from LASA available at the ESD of Teledyne, Inc. A complete set of coefficients of the first principal component data from Longshot and a geographically nearby earthquake is given in the Appendix.

<u>Example 1</u>

The first principal component of an earthquake record at the frequency $\omega = 0.20$ cps is computed by the computer program COMPNT as follows:

$$\begin{aligned} C_1 = {} & .365e^{-5j} Z_1 + .425e^{23.4j} Z_2 + .273e^{-59.1j} Z_3 \\ & + .329e^{35.0j} Z_4 + .391e^{-9.8j} Z_5 + .281e^{41.6j} Z_6 \\ & + .310e^{-14.1j} Z_7 + .268e^{2.9j} Z_8 + .295e^{12.9j} Z_9 + .130e^{3.9j} Z_{10} \end{aligned} \tag{16}$$

When 10 seismometers are ordered according to the corresponding phase angles of the coefficients in Eq. (16) one obtains the following sequence.

(52 83 81 22 24 26 10 56 54 85)

From Eq. (4) and Figure 3, one can infer that a projected direction of the first principal component is parallel to the plane connecting three seismometers 52, 83 and 81; that is, it has north-west direction. Since the above data is the earthquake record from Alaska the direction seems agreeable. Note that how regularly the phase angles change as the distance from the principal axis changes.

Example 2

Data from the Longshot test explosion was processed by the program COMPNT. The first component at the frequency $\omega = 0.20$ is given by

$$\begin{aligned} C_1 = {} & .358e^{-4.0j} Z_1 + .452e^{30.2j} Z_2 + .258e^{-84.2j} Z_3 \\ & + .267e^{20.1j} Z_4 + .354e^{-15.3j} Z_5 + .236e^{37.6j} Z_6 \qquad (17) \\ & + .252e^{-25.2j} Z_7 + .212e^{-6.6j} Z_8 + .343e^{2.15j} Z_9 + .349e^{-2.9j} Z_{10} \end{aligned}$$

The 10 seismometers are agin ordered algebraically by the phase angles of the corresponding coefficients in Eq. (17).

(52 81 83 22 24 10 26 56 54 85)

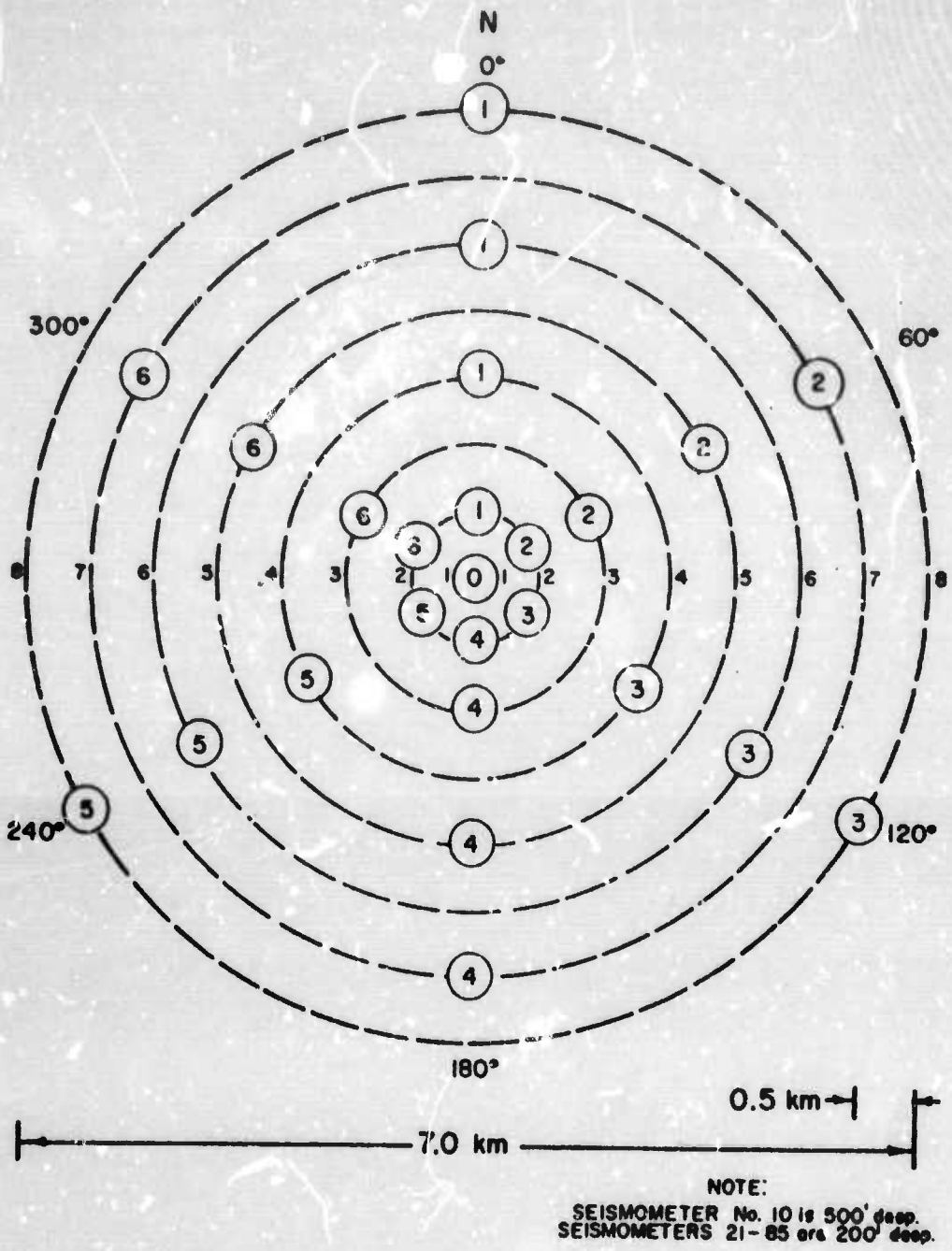

Figure 3. LASA Subarray

Immediately, one can infer that the direction of the principal component is roughly the same as that of the result in Example 1. It is again noted that the phase shift of each seismogram varies regularly as the distance from the principal axis.

Example 3

The same analysis by the computer program yields the following result for a random noise record obtained by LASA

(24 22 26 81 85 10 54 56 83 52)

Although in all three examples the first principal component accounts for approximately 75% of the total variance, there exists no regularity of phase angles such as observed in Examples 1 and 2. See Figure 3.

3. CONCLUSIONS

Some potential applications of principal component analysis to seismic array data have been discussed. It appears that the phase shift of $Z_i(.20)$ from the first principal component $C_1(.20)$ can be used in estimating the general direction of main noise source. The gain factor may be useful to select the most representative seismometer. An alternative way of selecting an optimum representative is to search for one which minimizes the residual variance in predicting its record by the best linear regression on records of other seismometers. Some of these arguments are based on intuition and fragmentary empirical results. It is quite desirable to verify and extend these by theoretical or extensive empirical study.

Closely related to principal component analysis is canonical analysis. Briefly, the first canonical coherence function is the maximum coherence between all possible combinations of the first array with those of the second array. It seems that canonical analysis is a logical approach to studying the coherency between two different subarrays.

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APPENDIX

LONG-SHOT

Seismometer Number	ω = 0.20		ω = 0.40		ω = 0.60		ω = 0.80		ω = 1.00	
	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle
10	2.36	37.6	2.65	-3.4	2.45	-16.6	2.97	-3.0	3.51	7.6
81	4.52	30.2	4.18	-2.0	3.56	-68.6	4.00	-74.9	3.06	53.5
85	3.49	-2.9	3.67	2.2	3.00	23.4	3.65	45.4	0.60	48.5
83	2.58	-84.2	3.41	-1.2	3.03	-5.4	2.88	-9.8	3.87	2.4
56	2.12	-6.6	2.48	-39.5	3.55	-2.6	2.74	-4.7	3.62	-4.8
52	3.58	-4.0	4.18	21.5	2.65	0.2	2.01	53.2	3.58	6.9
54	3.43	2.1	2.39	16.2	3.09	1.5	3.36	-5.3	3.84	6.8
26	2.52	-25.2	2.45	-4.2	3.96	16.7	3.87	43.5	3.28	-81.4
22	2.67	20.1	3.07	10.0	3.51	33.4	3.04	56.7	2.32	83.4
24	3.54	-15.3	2.37	17.8	2.43	31.7	2.55	-1.2	2.48	44.0

ω = frequency in cps.

LONG-SHOT (cont'd)

Seismometer Number	ω = 1.20		ω = 1.40		ω = 1.60		ω = 1.80		ω = 2.00	
	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle
10	3.63	-8.3	3.55	52.7	3.30	65.3	4.03	-18.7	2.32	-14.7
81	3.14	-1.1	2.55	-1.7	3.49	15.3	3.19	10.6	2.62	5.9
85	2.76	67.9	4.56	27.5	2.31	-38.5	2.10	-4.9	1.89	4.2
83	2.72	-3.1	2.37	17.5	1.86	56.8	2.84	81.9	2.55	84.4
56	2.79	10.2	3.34	-9.7	3.71	11.2	3.10	78.8	3.36	7.1
52	3.76	-53.8	3.52	7.9	1.27	40.8	3.76	5.3	3.98	-22.6
54	2.44	80.8	3.57	14.0	4.12	-3.3	3.03	26.6	3.47	69.3
26	2.60	-63.6	2.70	-68.2	3.11	-5.9	3.53	-24.3	3.98	-24.5
22	4.01	81.6	2.91	85.2	3.24	-56.2	1.39	36.7	3.83	-27.2
24	3.36	3.1	1.54	-54.5	3.95	21.5	3.71	-13.0	2.83	16.9

ω = frequency in cps.

EARTHQUAKE

Seismometer Number	ω = 0.20		ω = 0.40		ω = 0.60		ω = 0.80		ω = 1.00	
	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle
10	3.10	-14.1	4.07	1.1	2.39	16.6	3.38	43.0	4.10	-60.8
81	2.73	-59.1	3.35	25.0	4.92	-37.2	1.15	79.4	0.21	32.7
85	1.30	3.9	3.72	17.4	2.48	4.8	3.27	31.4	5.69	49.7
83	4.25	23.4	1.86	11.5	2.62	2.9	2.12	-16.9	0.71	6.0
56	2.68	2.9	3.00	5.9	3.09	34.8	0.63	35.8	3.79	10.2
52	3.65	-0.5	3.76	-0.8	2.78	23.3	3.18	-20.6	0.35	-41.6
54	2.95	12.9	2.56	4.5	3.63	-1.3	2.64	-8.5	3.35	-58.8
26	2.81	41.6	2.93	-0.4	3.05	6.9	7.01	-67.3	0.52	86.9
22	3.28	35.0	3.10	-9.8	3.37	-4.9	1.14	-11.8	3.03	-12.4
24	3.91	-9.8	2.64	32.9	2.43	-4.7	2.06	31.5	3.38	-58.8

ω = frequency in cps.

EARTHQUAKE (cont'd)

Seismometer Number	ω = 1.20		ω = 1.40		ω = 1.60		ω = 1.80		ω = 2.00	
	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle	Gain Factor x 10^{-1}	Phase Angle
10	4.01	-75.4	3.01	-36.0	3.54	60.2	4.83	56.1	0.46	-77.7
81	2.06	77.1	3.02	-29.0	2.66	-45.1	3.12	-76.9	2.71	-75.9
85	6.32	6.2	2.77	-57.9	3.99	-50.3	3.54	-28.1	2.97	28.7
83	0.97	85.5	4.96	11.2	3.44	69.9	0.72	-89.8	0.72	78.1
56	3.56	22.5	1.17	-16.8	5.39	50.6	4.58	-89.7	0.28	-57.0
52	1.01	-34.0	1.19	65.7	0.60	72.5	3.25	88.3	4.50	-15.7
54	2.79	-73.7	3.63	-16.2	0.76	86.5	2.92	4.9	3.98	-79.7
26	1.04	52.5	3.86	35.4	3.85	58.1	0.71	59.0	3.83	-45.7
22	2.81	6.3	0.84	-12.0	2.62	-21.4	3.63	-22.1	4.61	52.0
24	2.75	-60.9	4.23	26.7	0.95	15.1	0.30	-68.6	3.32	-84.9

ω = frequency in cps.

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13. ABSTRACT

The first part of this report develops the theory of principal components as applied to noise backgrounds measured at seismic arrays. It is noted that the main use of principal components is the reduction of random variables. A summary of statistical interpretation of the principal component is given. An example of principal component analysis is given for LASA noise data. Results are given concerning the proportion of the total variance (power) from 10 seismometers as explained by the first four principal components. At 0.2 cps approximately 80% of the total variance is accounted for by first principal component. In the summary it is concluded that it seems quite worthwhile to investigate the applications of the principal component to seismic noise study.

In the second part the seismic signal data used are from LONGSHOT and a geographically nearby earthquake recorded at a LASA subarray. The phase shifts of the first principal component at 0.2 cps corrected for LASA instrument response prove to be interesting. From these phase shifts its appears that the general direction of the main noise source can be estimated. Computed examples for LONGSHOT and the nearby earthquake are given to verify this claim.

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