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TECHNICAL REPORT ECOM-01821-4

**INVESTIGATION OF HIGH-POWER  
BEAM PLASMA INTERACTIONS**

**4th QUARTERLY REPORT**

*By*

*SHELDON GRUBER*

**MARCH 1967**

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**SPERRY RAND RESEARCH CENTER**

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INVESTIGATIONS OF HIGH-POWER  
BEAM PLASMA INTERACTIONS

4th QUARTERLY REPORT

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Prepared By

Sheldon Gruber

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for

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ABSTRACT

This research is directed toward the investigation of high-power beam plasma interactions, with specific investigation of the transverse velocity beam modes called for.

Experimental results have been obtained for the propagation characteristics of the transverse plasma wave modes. They are qualitatively consistent with a traveling backward wave propagating toward the axis in a non-uniform plasma column whose density decreases toward the outside. The observed wavenumber is about twice that predicted by the theoretical dispersion relation based on a Maxwellian electron velocity distribution function. The theory has been reformulated to accommodate arbitrary distribution functions. The two-beam experiment has been completely set up in a form which prevents interaction in the varying perpendicular velocity region.

This research is part of PROJECT DEFENDER, sponsored by the Advanced Research Project Agency, Department of Defense, and administered by the U. S. Army Electronics Command under Contract No. DA 28-043-AMC-01821(E).

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## I. PURPOSE

This investigation has as its purpose the theoretical and experimental investigation of new (i.e., not space charge) modes of beam plasma interaction. In particular, it includes the investigation of the feasibility of these new modes as an improved means of generation and amplification of microwaves.

## II. INTRODUCTION AND STATEMENT OF PROBLEM

It is apparent from recent developments in the linear theory of plasma waves (of which electron beam waves are a subgroup) that the wave interactions used thus far in devices for microwave generation and amplification represent only a small fraction of those which are possible and which should be considered. Performance and design limitations of existing devices are due to the characteristics of the particular waves used, and they may well be extended or removed if different waves are employed.

The Sperry Rand Research Center (SRRC) has contracted to conduct a comprehensive theoretical and experimental study of particular plasma waves (including electron beam waves) which are candidates for application to high-power microwave generators or amplifiers and which have not as yet been adequately investigated.

The work being undertaken is an extension of research which has been in progress at SRRC. As a result of company sponsored investigations performed during the past three years, an important set of beam and plasma waves - the so-called electrostatic, cyclotron-harmonic waves - have been identified. These waves merit further study because they remove the plasma density, magnetic field, and parallel phase velocity restrictions inherent in the wave modes used in existing devices. Their dispersion relation has been formulated and solved for many interesting cases, including growing wave interactions.

In particular the research program includes: measurement of propagation characteristics for comparison with existing linear dispersion theory; coordinated theoretical and experimental study of the effect of finite geometry, velocity spread, and density and temperature gradients on linear propagation characteristics and wave impedance; a primarily experimental study of non-linear amplitude limiting and spurious frequency generation; and a study of the noise properties of the amplification medium. Special emphasis will be given to a search for practical methods of efficiently coupling these waves to conventional transmission lines.

The program will also include extension of the range of solutions to linear plasma and beam wave dispersion relations in a search for additional wave modes of potential usefulness in high-power microwave devices. For while the past theoretical program at SRRC has been extensive, there remain many possible relative orientations of beam velocity, wave velocity, rf electric field and dc magnetic field vectors, wide ranges of parameters, and many beam and plasma velocity distributions of potential interest which have not yet been considered.

### III. TECHNICAL BACKGROUND

#### A. LINEAR THEORY OF WAVES IN A UNIFORM PLASMA OR BEAM

The non-relativistic dispersion relation for high-frequency electrostatic waves in an infinite uniform electron medium neutralized by massive ions is:

$$\frac{\omega_p^2}{k_\perp^2 + k_\parallel^2} \sum_{n=-\infty}^{\infty} \int_0^\infty v_\perp dv_\perp \int_{-\infty}^{\infty} dv_\parallel \frac{2\pi J_n^2(k_\perp v_\perp / \Omega)}{\omega - k_\parallel v_\parallel - n\Omega} \left( \frac{n\Omega}{v_\perp} \frac{\partial}{\partial v_\perp} + k_\parallel \frac{\partial}{\partial v_\parallel} \right) f_0(v_\perp, v_\parallel) = -1 \quad (1)$$

where

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m}$$

$k_\parallel$  = component of propagation vector along the static magnetic field,  $B_0$ .

$k_\perp$  = component of propagation vector across the static magnetic field.

$f_0(v_\perp, v_\parallel)$  = the normalized electron velocity distribution, where  $v_\perp$  is the velocity across the field and  $v_\parallel$  is the velocity along the field.

$$\Omega = \frac{eB_0}{m}$$

and  $J_n(k_\perp v_\perp / \Omega)$  is the Bessel function of the first kind and order  $n$  with argument  $k_\perp v_\perp / \Omega$ .

The importance of the distribution function in determining the behavior of the waves which may be supported in the medium is striking. To begin with, if we consider an electron beam with velocity parallel to the magnetic field, then

$$f_0(v_\perp, v_\parallel) = \frac{1}{2\pi v_\perp} \delta(v_\perp) \delta(v_\parallel - v_{0\parallel})$$

and the dispersion relation is

$$k_{\parallel}^2 \frac{\omega_p^2}{(\omega - k_{\parallel} v_{0\parallel})^2} + k_{\perp}^2 \frac{\omega_p^2}{(\omega - k_{\parallel} v_{0\parallel})^2 - \Omega^2} = k_{\perp}^2 + k_{\parallel}^2 \quad (2)$$

This equation, originally discussed by Gould and Trivelpiece,<sup>1</sup> describes the fast and slow space charge waves as well as the fast and slow fundamental cyclotron waves. The case of a finite beam diameter has also been discussed in detail.<sup>2</sup> The general conclusion drawn from the simple case of a uniform density finite diameter beam is that the geometry merely restricts the set of  $k$  values ( $k_{\perp}, k_{\parallel}$ ) which can be used to satisfy (2) but that the infinite medium dispersion equation must still be satisfied. The dispersion diagram for this "cold" electron beam is given in Fig. 1. It will be noted that the slow "negative energy" waves used for oscillators or amplifiers have a parallel phase velocity less than the beam velocity.

When electron motion about the lines of magnetic field is taken into account, an infinite set of waves is found.<sup>3,4</sup> In addition to modified space charge waves, two waves exist for each harmonic of the electron cyclotron frequency. One of each such pair of waves is found to have negative energy,<sup>5</sup> and can thus be used for growing wave interaction, as is the slow space charge wave in conventional microwave tubes. The dispersion curve shown in Fig. 2 is for a beam of monoenergetic, spiraling electrons, whose velocity distribution is given by

$$f_0(v_{\perp}, v_{\parallel}) = \frac{1}{2\pi v_{0\perp}} \delta(v_{\perp} - v_{0\perp}) \delta(v_{\parallel} - v_{0\parallel})$$

and whose dispersion relation is

$$k^2 = k_{\perp}^2 + k_{\parallel}^2 = \omega_p^2 \sum_{n=-\infty}^{\infty} \left[ \frac{k_{\parallel}^2 J_n^2(k_{\perp} v_{0\perp} / \Omega)}{(\omega - k_{\parallel} v_{0\parallel} - n\Omega)^2} + \frac{k_{\perp}^2 \left( J_{n-1}^2(k_{\perp} v_{0\perp} / \Omega) - J_{n+1}^2(k_{\perp} v_{0\perp} / \Omega) \right)}{2n(\omega - k_{\parallel} v_{0\parallel} - n\Omega)} \right] \quad (3)$$

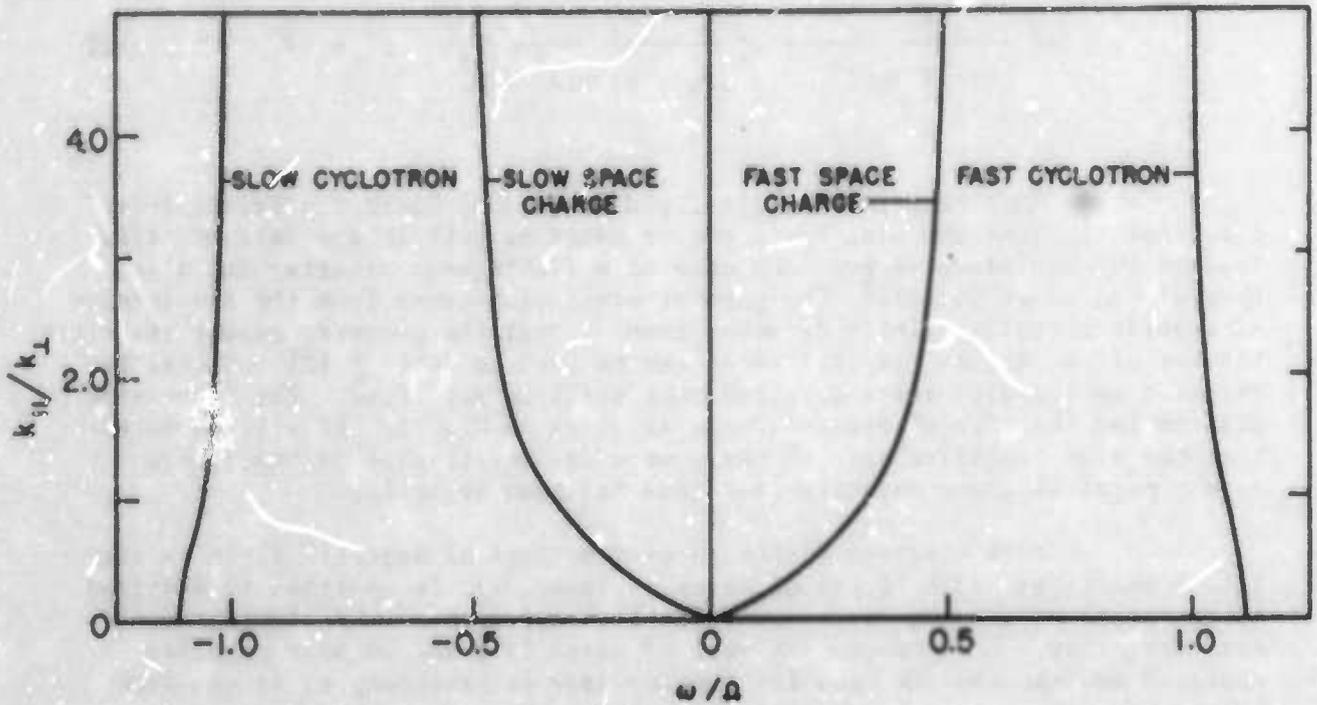


FIG. 1 Dispersion diagram of so-called cold electron beam where electrons move only along the magnetic field with a single speed (in this case zero velocity). The beam plasma frequency divided by the cyclotron frequency  $\omega_p/\Omega = 0.5$  for the example chosen.

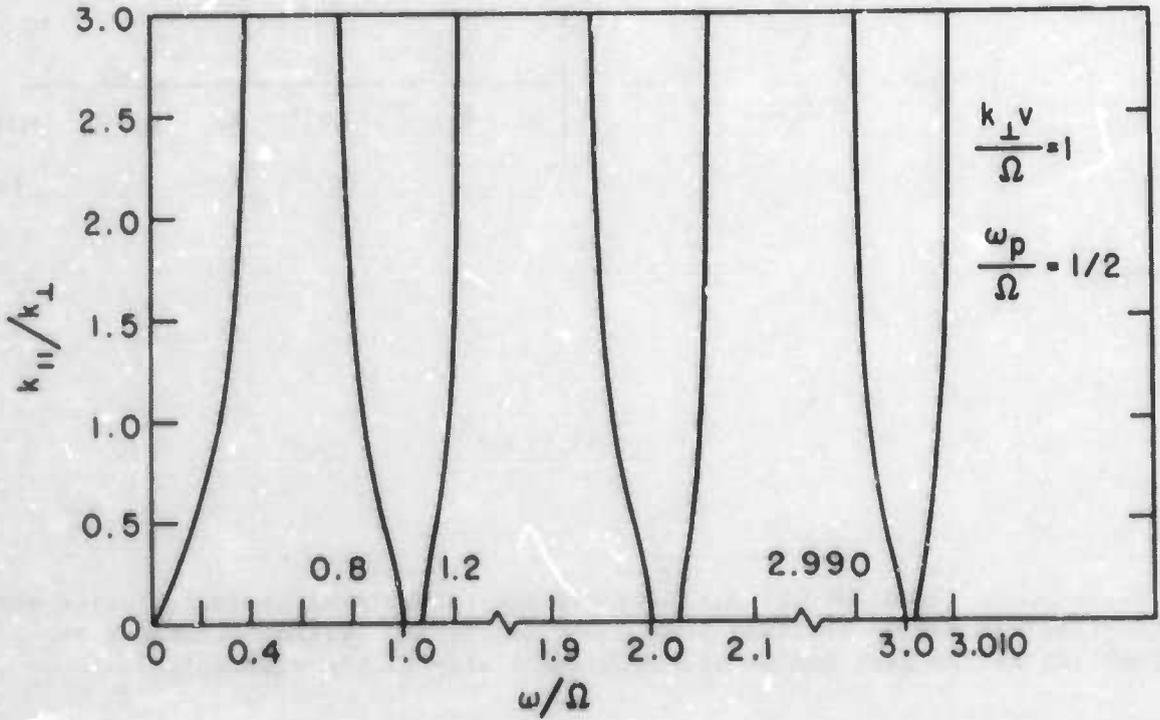


FIG. 2 Dispersion diagram for transverse beam modes on a mono-energetic beam of spiraling electrons. Only positive frequencies are shown for convenience. The dispersion relation is symmetric about both axes. Here again  $\omega_p/\Omega = 0.5$  but  $v_{0\perp} = \Omega/k_{\perp}$  (fixed  $k_{\perp}$ ).

If we consider an electron beam whose distribution function is Maxwellian across the field and having a single velocity along the field, then the results of using this in Eq. (1) yields

$$k^2 = k_{\perp}^2 + k_{\parallel}^2 = \omega_p^2 \sum_{n=-\infty}^{\infty} I_n(\lambda) e^{-\lambda} \left[ \frac{k_{\parallel}^2 v_{\perp}^2}{(\omega - k_{\parallel} v_{0\parallel} - n\Omega)^2} + \frac{n\Omega}{(\omega - k_{\parallel} v_{0\parallel} - n\Omega)} \right] \quad (4)$$

where

$$\lambda = (k_{\perp} v_{\perp} / \Omega)^2 .$$

Equation (3) exhibits certain interesting characteristics which in principle may be utilized in a power generation system. Consider the function in the last sum on the right-hand side of the equation.

$$J_{n-1}^2(\rho) - J_{n+1}^2(\rho) = J_n(\rho) \frac{d}{d\rho} J_n(\rho) .$$

where

$$\rho = k_{\perp} v_{\perp} / \Omega .$$

This function becomes negative whenever  $J_n(\rho)$  and its derivative are of opposite sign. It is possible, for a sufficiently dense beam, to have instability over critical perpendicular velocity ranges for which  $J_n(\rho) d/d\rho J_n(\rho)$  is negative. Too much velocity spread in the perpendicular direction can eliminate these unstable regions, however, since the Maxwellian velocity distribution beam does not exhibit this characteristic. Our theoretical investigation will include detailed calculations of the effect both of perpendicular velocity spread (we will employ a shifted Maxwellian distribution with variable velocity spread) and of axial velocity spread, a spread which leads to the so-called "collisionless cyclotron damping."

Interaction between the transverse velocity, negative energy wave on the beam near the cyclotron harmonic and a circuit (or beam or plasma) positive energy wave leads to wave growth. This is dramatically illustrated in Fig. 3, where we present the negative and positive energy waves on a single electron beam. As the beam electron density increases, the positive energy wave originating at zero frequency for  $k_{\parallel} = 0$  (the fast space charge wave) couples with the negative-energy transverse velocity wave at the cyclotron frequency, and an instability results in growing wave solutions.<sup>6</sup> The growth rate and frequency spectrum of these waves are presented in Fig. 4 for several harmonics of the electron cyclotron frequency.

The interaction of a monoenergetic beam excited in the transverse velocity mode with a plasma whose electrons have a Maxwellian velocity distribution has been considered under somewhat restricted conditions by us. We have found wave growth in the region where the axially-traveling beam electrons see the cyclotron harmonic frequencies after the approximate doppler shift. This interaction occurs if the plasma appears to be lossy (resistive instability) or slightly reactive (reactive instability). In Fig. 5 we show the results of a calculation of the reactive instability.

The effect of boundaries in a finite beam of uniform electron density is subtly complicated by the non-zero orbits of the electrons. Those electrons traveling on field lines within a Larmor radius of the outer edge of the beam penetrate through the beam boundary and, hence, through what would be a region of radial field discontinuity. These electrons may interact more strongly with harmonics of the cyclotron motion than electrons nearer the axis.<sup>7</sup>

## B. THE EFFECT OF GRADIENTS

The importance of density and temperature gradients in beams or plasmas is well recognized. Because of theoretical difficulties, few attempts toward adequate solutions have been made. Recently, Nickel, Parker and Gould<sup>8</sup> and others investigated the effect of plasma gradients upon electrostatic waves propagating across a plasma column in order to explain the so-called Tonks-Dattner resonances which occur with no magnetic field. Buchsbaum and Hasegawa<sup>9</sup> and Schmitt, Meltz and Freyheit<sup>10</sup> have considered wave propagation across a radial density gradient in a magnetized plasma. In all cases, it is assumed that the change in density across a Larmor orbit is either so small that the gradient slightly perturbs the wave-equation or so large that the zero magnetic field condition is valid.

Emission<sup>11</sup> and absorption<sup>9,10</sup> measurements of a plasma column immersed in a magnetic field have shown very interesting fine structure when the frequency of observation is in the vicinity of twice the electron cyclotron frequency (and higher harmonics as well). The theory of Buchsbaum and Hasegawa is that waves can propagate within the high-density core of the plasma out toward the walls of the discharge tube until the wave frequency corresponds to the local hybrid frequency ( $\omega_{\text{hybrid}} = \sqrt{\omega_p^2 + \Omega_e^2}$ ), as long as the wave frequency is less than the second harmonic of the cyclotron frequency.

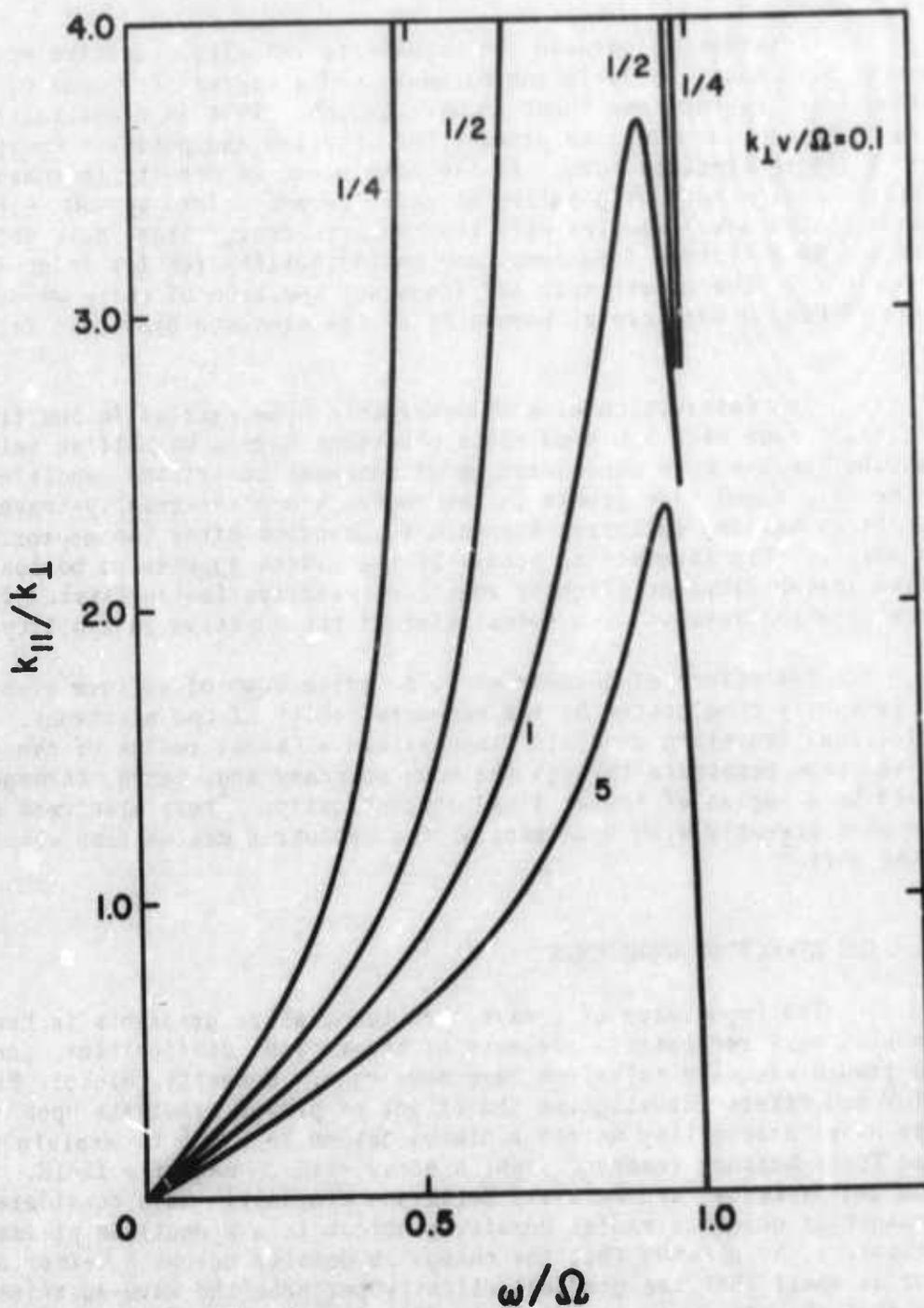


FIG. 3 Dispersion relation for mono-energetic, fixed perpendicular energy beam waves of frequency less than the first cyclotron harmonic. Each curve is for different beam density expressed in terms of the ratio  $\omega_p^2/\Omega^2$  all having  $k_{\perp}v_{0\perp} = 0.1\Omega$ . As density increases, the wave originating near zero frequency couples with the negative energy wave below the cyclotron frequency, and wave growth ensues.

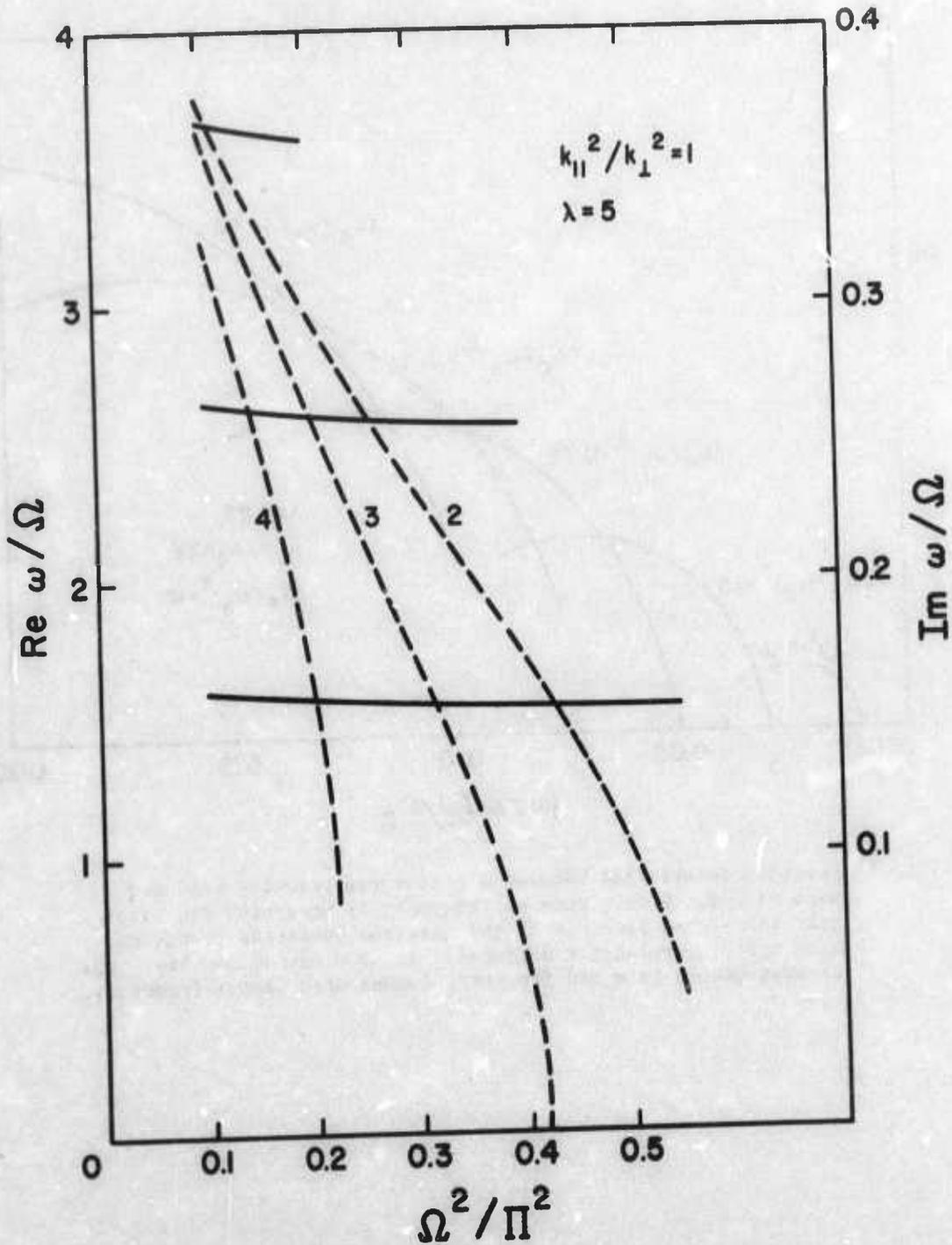


FIG. 4 Growth rate and frequency of growing waves associated with the interaction of the positive energy beam wave with the negative energy beam wave on the same beam as a function of the ratio of cyclotron to beam plasma frequency. Growth curves are shown by the dashed lines.

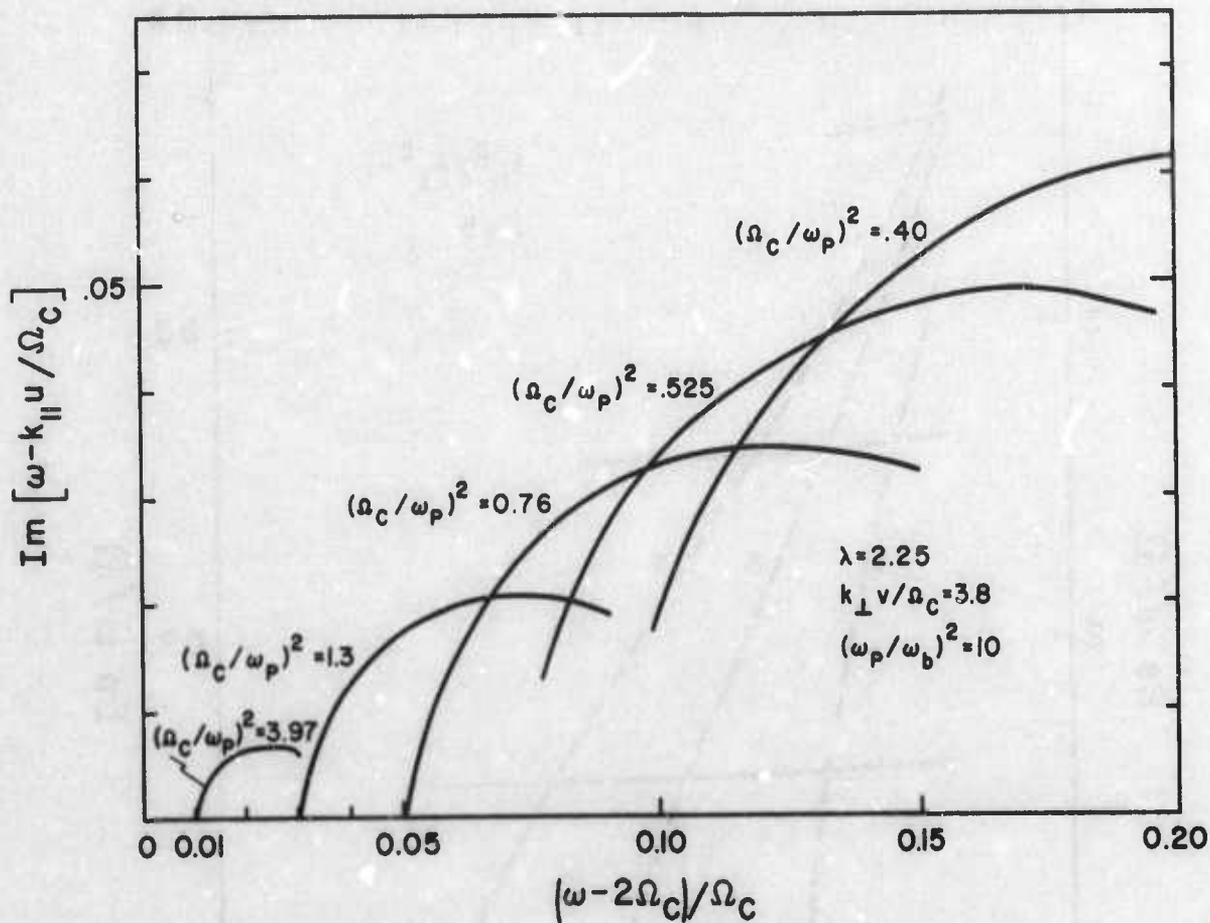


FIG. 5 Reactive interaction between a transverse velocity beam and a warm-plasma. Growth rate vs frequency is presented for waves near the second harmonic of the electron cyclotron frequency. Each curve represents a different plasma electron density, thus showing growth rate and frequency depend upon plasma frequency.

At the hybrid frequency the waves become evanescent on the outside and are reflected toward the interior, thus setting up a standing wave or radial resonant condition. The importance of radial density gradients is stressed by their analysis.

The combination of the linearized Boltzmann equation with Poisson's equation leads to the following differential equation for slab geometry:

$$\frac{d^2}{dx^2} [g(x)E(x)] + \frac{1}{\lambda^2} \left[ \frac{\omega_{po}^2}{\omega^2 - \Omega^2} - \frac{1}{g(x)} \right] g(x)E(x) = 0 \quad (5)$$

where

$$\lambda^2 = \frac{3eT/m \omega_{po}^2}{(\omega^2 - \Omega^2)(4\Omega^2 - \omega^2)}$$

$T$  is the electron temperature (assumed to be uniform),  $g(x)$  is the normalized electron density profile, and  $\omega_{po}$  is the peak electron plasma frequency.

If the medium is uniform,  $g = 1$  and  $d^2/dx^2 \rightarrow -k^2$ , so that the dispersion relation is

$$(4\Omega^2 - \omega^2)(\Omega^2 + \omega_{po}^2 - \omega^2) = k_{\perp}^2 \left( \frac{3eT}{m} \right) \omega_{po}^2 \quad (6)$$

from which we can verify that

$k$  is real where  $\omega^2 < \Omega^2 + \omega_{po}^2$  and  $2\Omega > \omega$  .

$k$  is real where  $\omega^2 > \omega_{po}^2 + \Omega^2$  and  $2\Omega < \omega$  .

Case 2 has been experimentally and theoretically treated by Schmitt, Meltz and Freyheit.<sup>10</sup>

The solution of Eq. (5) can be explicitly given for a density profile

$$g(x) = \frac{1}{1 + \gamma \left(\frac{x}{l}\right)^2}$$

and is

$$E(x) = \left[1 + \gamma \left(\frac{x}{l}\right)^2\right] \left[ D_\nu \left(\frac{x}{c}\right) \pm D_\nu \left(\frac{-x}{c}\right) \right]$$

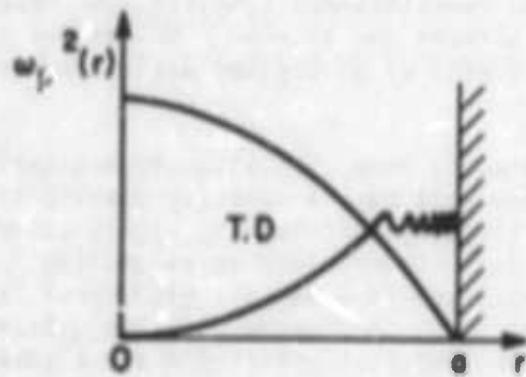
where

$$c = \left[ \frac{\frac{3eT}{m} \omega_{p0}^2 l^2}{4\gamma(\omega^2 - \Omega^2)(4\Omega^2 - \omega^2)} \right]^{1/4}$$

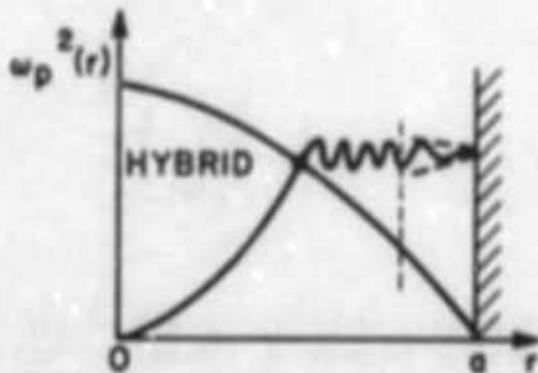
$$\nu = \frac{1}{2} \left[ \left\{ \frac{(4\Omega^2 - \omega^2) l^2}{3\gamma \omega_{p0}^2 \frac{eT}{m} (\omega^2 - \Omega^2)} \right\}^{1/2} (\Omega^2 + \omega_{p0}^2 - \omega^2) - 1 \right]$$

and the D functions are parabolic cylinder functions. These functions oscillate in space in the manner of a radial standing wave, showing that physically the wave propagating out from the core is continuously reflected from the density gradient. Buchsbaum and Hasegawa's work has been extended by us to include cylindrical geometry, and the same essential feature of the standing wave pattern is found. In Fig. 6 we illustrate the nature of the solutions associated with waves propagating across a density gradient both with and without a static axial magnetic field.

The solution given above for the non-uniform plasma is valid only in the region where  $\omega \approx 2\Omega$  and is a result of an expansion to first order in the quantity  $(L_r d/dx)$ , which is the ratio of Larmor orbit ( $L_r^2 = eT/m\Omega^2$ ) to gradient scale length. In order to consider waves in the

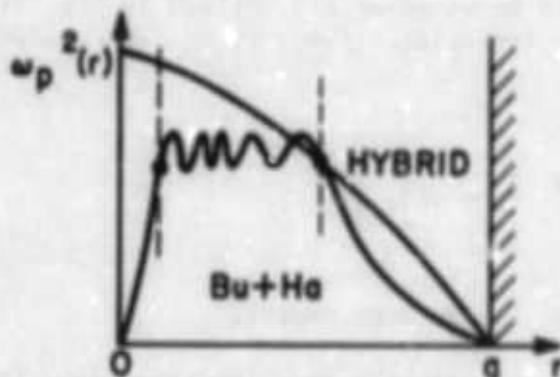


$$\Omega = 0$$



$$p < \frac{\omega_c}{\Omega} < p+1$$

$$(p \geq 2)$$



$$\frac{\omega_c}{\Omega} > 1$$

FIG. 6 Electric potential associated with the radial wave resonances on a plasma column. Three cases are indicated: Tunks-Dattner (TD) resonances with no magnetic field, the Buchsbaum-Hasegawa core resonances (B-H) in a magnetic field and external resonances in a magnetic field.

vicinity of the third harmonic, terms to second order in  $L_T d/dx$  are required and so on. It is obvious that computational complications increase with higher harmonic number if such a technique is used. We propose to consider the extension of this method as well as attempting different attacks on the problem.

While a plasma will probably have only slightly non-uniform electron temperature, an electron beam may well have a velocity distribution (it would be incorrect to consider it a temperature) which is highly inhomogeneous as a result of generation and injection methods. The terms arising from inhomogeneous beam electron velocity distribution (and density gradients) in the Boltzmann equation are from the term  $v_0 \nabla_T f_0$ , where  $f_0 = n(r)g_{\perp}(r, v_{\perp})g_{\parallel}(r, v_{\parallel})$ . That is, we assume that the density and velocity variations are separable. For example, we could consider a local Maxwellian velocity distribution in the  $\perp$  direction.

$$g_{\perp}(r, v_{\perp}) = \frac{1}{\sqrt{2\pi v_{10}^2(r)}} \exp\left(-\frac{v_{\perp}^2}{2v_{10}^2(r)}\right).$$

The consequences of such a distribution (or, for that matter, of any temperature gradient) upon the cyclotron harmonic beam waves are not evident, but approaching the problem via a perturbation technique allows the insights obtained in the uniform analysis to be extended and applied to the very difficult case of spatial temperature variation. That is, we can consider

$$v_{10}^2(r) = v_{10}^2 \left[ 1 + \gamma \left(\frac{r}{l}\right)^2 \right]$$

where  $\gamma$  is a small number and  $l$  is the beam radius.

We are exploring, as one possible coupling mechanism, the non-uniform plasma resonances discussed in the previous section. These resonances set up the high-order radial field variations required to excite transverse velocity beam waves. The resonances themselves may be excited by electrodes which are located entirely outside the beam-plasma region. (See Fig. 8.)

Many experiments related to this aspect have been conducted. From these experiments, it appears that the core resonances in a plasma are strongly excited by an external circuit. The depth of the absorption is well illustrated in Fig. 7, which shows oscilloscope traces of resonant dips in reflected power as viewed on a strip line excited at a frequency of 400 Mc/sec. Each trace is for the indicated ratio of wave frequency to cyclotron frequency;

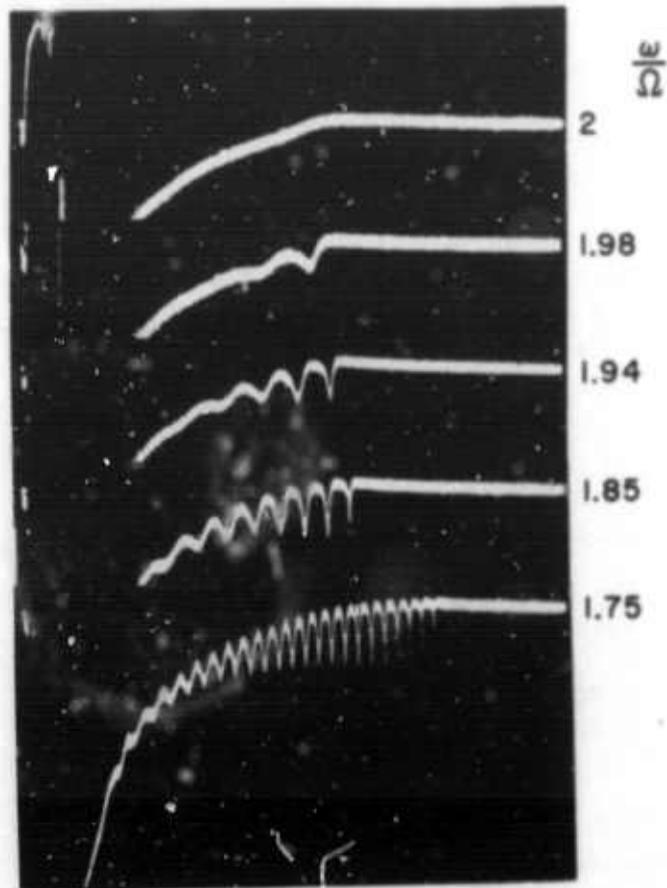


FIG. 7 Experimentally observed radial electron plasma wave resonances in the core of a cylindrical plasma column for different magnetic field strengths, observed in reflection in a neon afterglow plasma, 0.02 torr,  $f = 400$  Mc/sec, time scale 0.2 msec/div. Note depth of resonant structure.

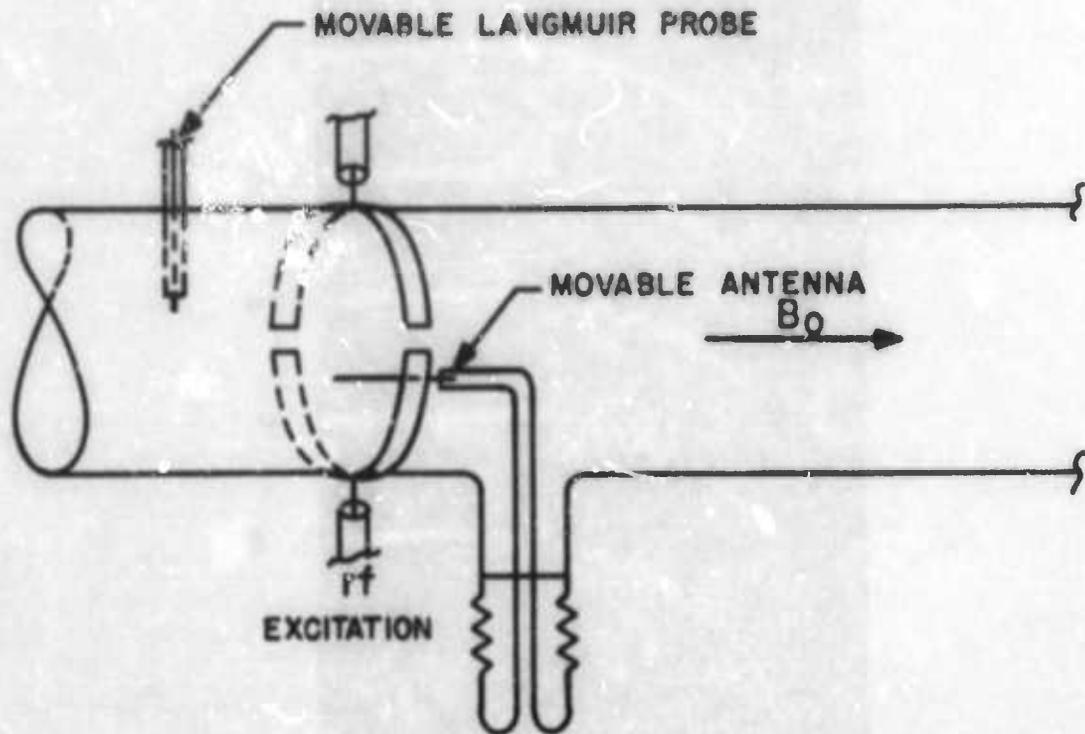


FIG. 8 Experimental apparatus showing method of excitation as well as method of internal probing of radial resonances on a plasma column in magnetic field. The rf excitation at 1175 Mc/sec is made on a capacitive type system employing striplines fed out of phase. A simple coaxial cable with the center conductor bared for one inch serves as the probe inside the dc discharge.

all traces are in the vicinity of the second harmonic of the cyclotron frequency, as predicted by the dispersion relation of Bernstein<sup>12</sup> from longitudinal waves propagating across the magnetic field.

#### IV. WORK PERFORMED DURING REPORT PERIOD

##### A. Coupling Techniques

##### 1. Theory

The experimental results of the electrostatic waves propagating in the plasma component of the beam-plasma system have shown consistently that the waves do not obey the dispersion relation predicted by Bernstein [12]. For a given frequency the waves are observed to have a wave number about twice that which is predicted for a Maxwellian plasma. This results in a marked change in the plasma slab impedance at a given frequency than that which is predicted for a plasma with a Maxwellian velocity distribution (see report ECOM-01821-3). In order to determine whether a non-Maxwellian plasma could account for the experimental dispersion relation an attempt to determine the theoretical  $\omega, k$  relation for general distribution functions is being programmed.

The equation governing the electrostatic waves propagating across the magnetic field is

$$1 + \frac{\pi}{k_{\perp}^2} \int_0^{\infty} \frac{2\pi J_n^2(k_{\perp} v_{\perp} / \Omega)}{(\frac{\omega}{\Omega} - n)} \frac{\partial f(v_{\perp})}{\partial v_{\perp}} dv_{\perp} = 0 \quad (7)$$

which is obtained by setting  $k_{\parallel} = 0$  in Eq. (1) and integrating over the axial velocity,  $v_{\parallel}$ .

Initially, two distributions will be considered, the exponential

$$f_a(v_{\perp}) = \frac{3}{2\pi} \frac{m}{kT} e^{-\sqrt{3m/kT} v_{\perp}} \quad (8a)$$

and the Druyvestyn

$$f_b(v_{\perp}) = \frac{1}{n} \left( \frac{m}{kT} \right) e^{-((v_{\perp}^2/4\pi)(m/kT)^2)} \quad (8b)$$

Both distributions are normalized and have second moments equal to the second moment of the Maxwellian distribution so that

$$\overline{v^2} = 2v_t^2 = \frac{2kT}{m}$$

making the energy equal in all cases.

In a low pressure discharge (pressure  $\sim 10^{-3}$  Torr) it is expected that the distribution is more like Eq. (8a) whereas Eq. (8b) is the theoretical velocity distribution for a high pressure dc discharge.

## 2. Experiment

In order to verify that the experimental technique previously employed had been valid and that the experimentally determined values for wavelength were not off by a factor of two as a result of the measurement technique the wave probing experiments were repeated using the set up shown in Fig. 9.

The signal driving the plasma oscillations is split in two by the 3 dB directional coupler. The impedance of the probe in the plasma is tuned by a short-circuited line and the probe signal coupled out by a 10 dB coupler. This signal is phase shifted by means of a motor-driven delay line and combined with the reference signal in a 10 dB coupler. This combined signal is detected and recorded as a function of position on an xy recorder. An attenuator pad was added to the reference to test the circuit operation. Since the crystal detector operates as a square-law device, its low frequency output contains the terms

$$S_o(x) = S_p^2(x) + S_R + 2S_p(x) S_R \cos \varphi(x) \quad (9)$$

where:  $S_p(x)$  is the probe signal and is a function of position,  $x$ , within the plasma;

$S_R$  is the amplitude of the reference signal and is independent of probe position;

and  $\varphi(x)$  is the phase difference between the reference and probe channel which, in general, will be a function of position ( $x$ ).

If a standing wave were present in the plasma the phase  $\varphi(x)$  would be independent of position, and  $S_p(x)$  would vary spatially in a sinusoidal like manner whereas for traveling wave,  $\cos \varphi(x)$  would vary in a sinusoidal manner and  $S_p(x)$  would vary slowly (as a result of the plasma inhomogeneity).

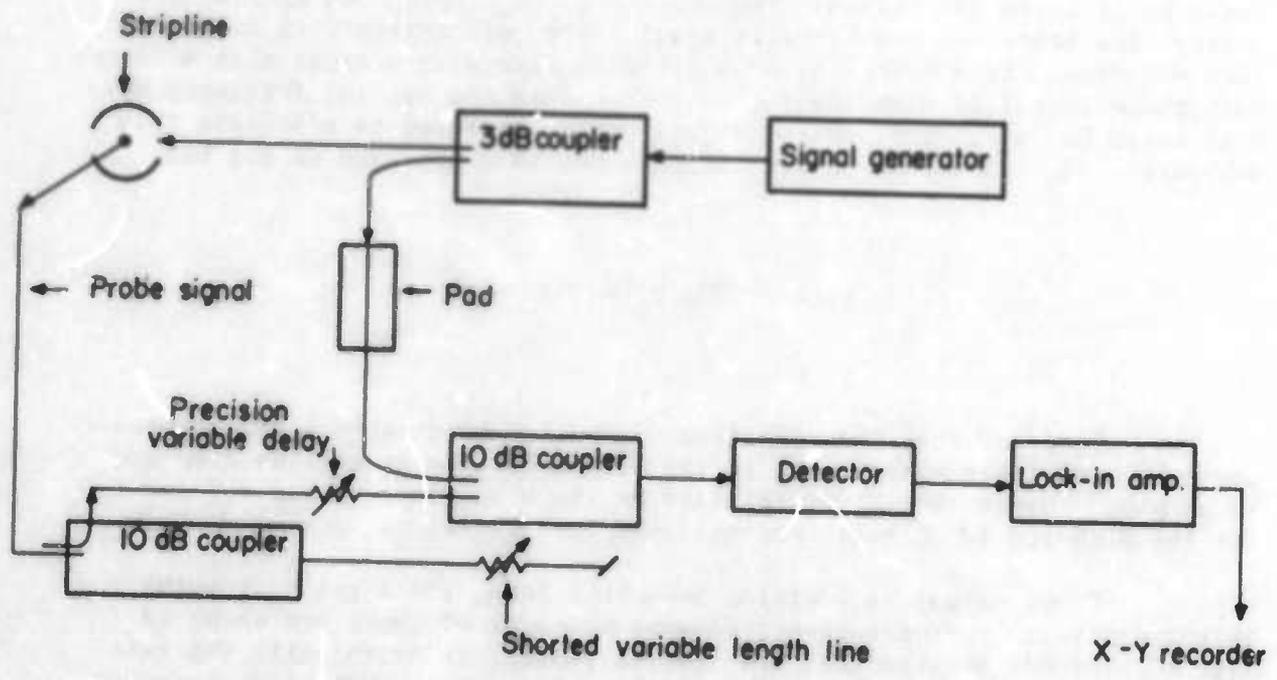


FIG. 9 Experimental set up for the detection of the phase velocity of the electrostatic waves propagating across the plasma column.

Consider the case in question. If  $S_R = 0$  and only a traveling wave were present then the output,  $S_D^2(x)$ , would be independent of position which is contrary to previous experiments. (See report ECOM-01821(E)-1.) If only a standing wave were present then the spatial oscillations measured would be at twice the spatial frequency  $2k_{\perp}$  ( $k_{\perp}$  being the spatial frequency) and hence the measurements should have been interpreted differently than was done. If a traveling or a standing wave were present with a larger background signal of slow spatial variation then the spatial frequency measured would be  $k_{\perp}$ . The present experiment is designed to eliminate this ambiguity.  $S_R$  is set to be very much larger than  $S_p$  and so one has

$$S_o(x) \approx 2S_R S_p(x) \cos \varphi(x) \quad (10)$$

In order that the condition  $S_R \gg S_p$  be checked a set of measurements were taken with no pad in the reference line as well as 3 dB and 10 dB pads. Since the output relative to the 0 dB pad case was 0.7 and 0.3 for the 3 dB and 10 dB cases respectively, the conditions were satisfied.

Probe output vs position were then taken for a set of lengths in the precision variable line. Several examples of these are shown in Fig. 10. It can be seen that the spatial pattern is essentially the same but that the pattern moves outward as the delay in the probe line increases. This is a positive indication of a wave whose phase velocity is directed toward the plasma axis. (The strip lines were reversed and the measurements repeated and again inward propagation was found so that it was determined that the unbalanced strip line did not lead to wave propagation.)

In Fig. 11 we plot the position of the maxima and minima of the set of data taken at 485 MHz (in the neighborhood of the third harmonic of the cyclotron frequency, 175 MHz) as a function of the time delay introduced in the probe arm. The slope of these curves gives the local phase velocity which is plotted as a function of position in Fig. 12.

The behavior of the phase velocity across the column is in qualitative agreement with the motion of a wave propagating through a non-uniform plasma column whose density decreases toward the outside.

#### B. Two-Beam Experiment

The two-beam experiment was remounted after a cathode failure. The physical structure has been changed as well. The "linear" beam (the beam not having perpendicular velocity superimposed) has been formed by a hollow beam system which was obtained from a commercial backward-wave oscillator. The perpendicular velocity beam is as before. The experimental arrangement is shown in Fig. 13. It can be seen that the hollow beam is collected before it arrives in the region where the perpendicular energy is imparted to the solid beam. This was a measure taken to avoid interaction in a region of varying perpendicular velocity.

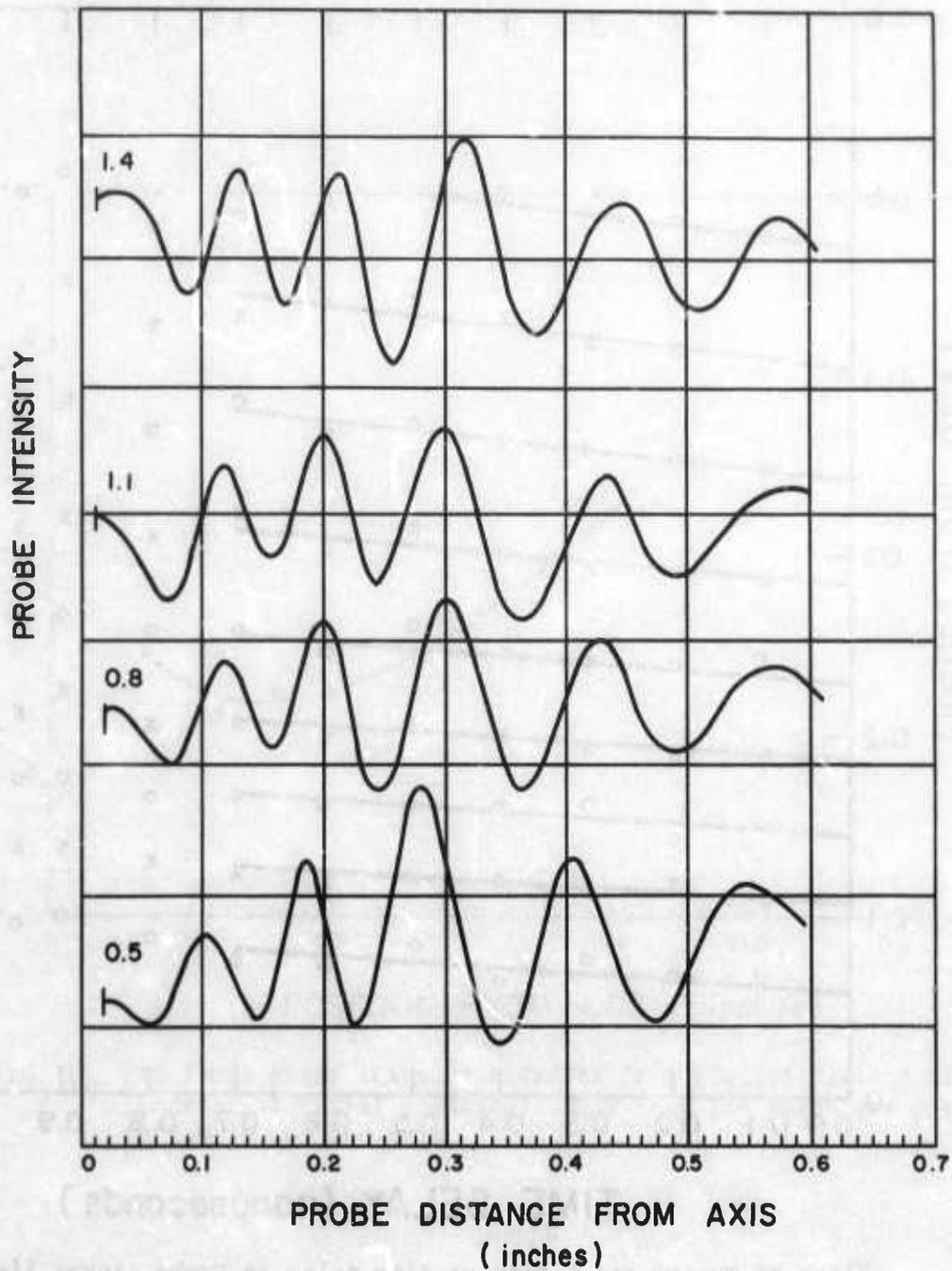


FIG. 10 Probe signal vs probe distance from axis for several delay times in probe signal line. The time delays are indicated in nanoseconds on the curves. It can be seen that the peaks move outward indicating that the wave is moving inward. The curves were taken in a 10 mA Hg discharge at 485 MHz with the cyclotron frequency at 175 MHz and are displaced from one another for clarity.

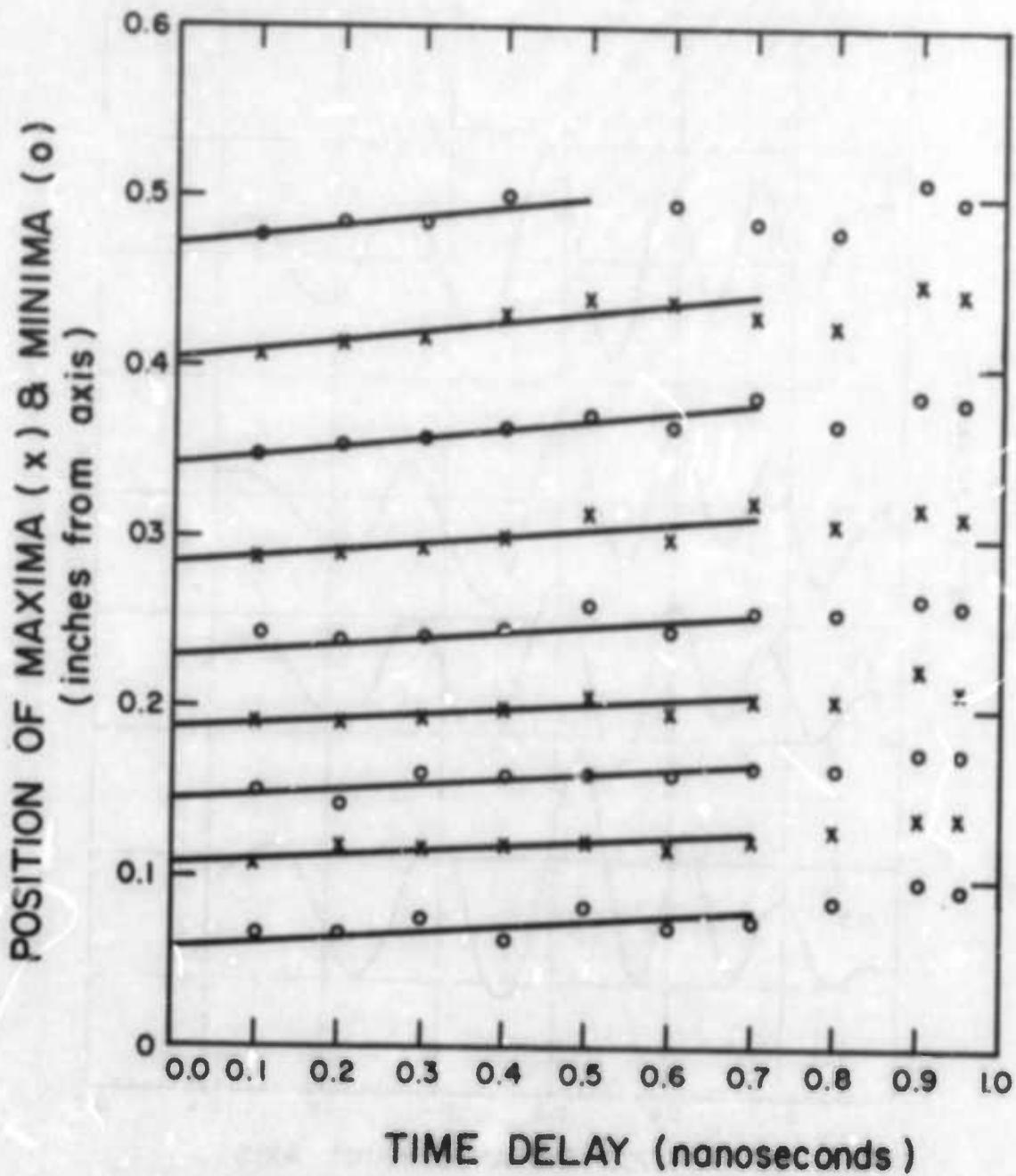


FIG. 11 Plots of maxima and minima vs time delay in probe signal line. The slopes of these curves determine the local phase velocity. The data were obtained under the same conditions indicated in Fig. 10.

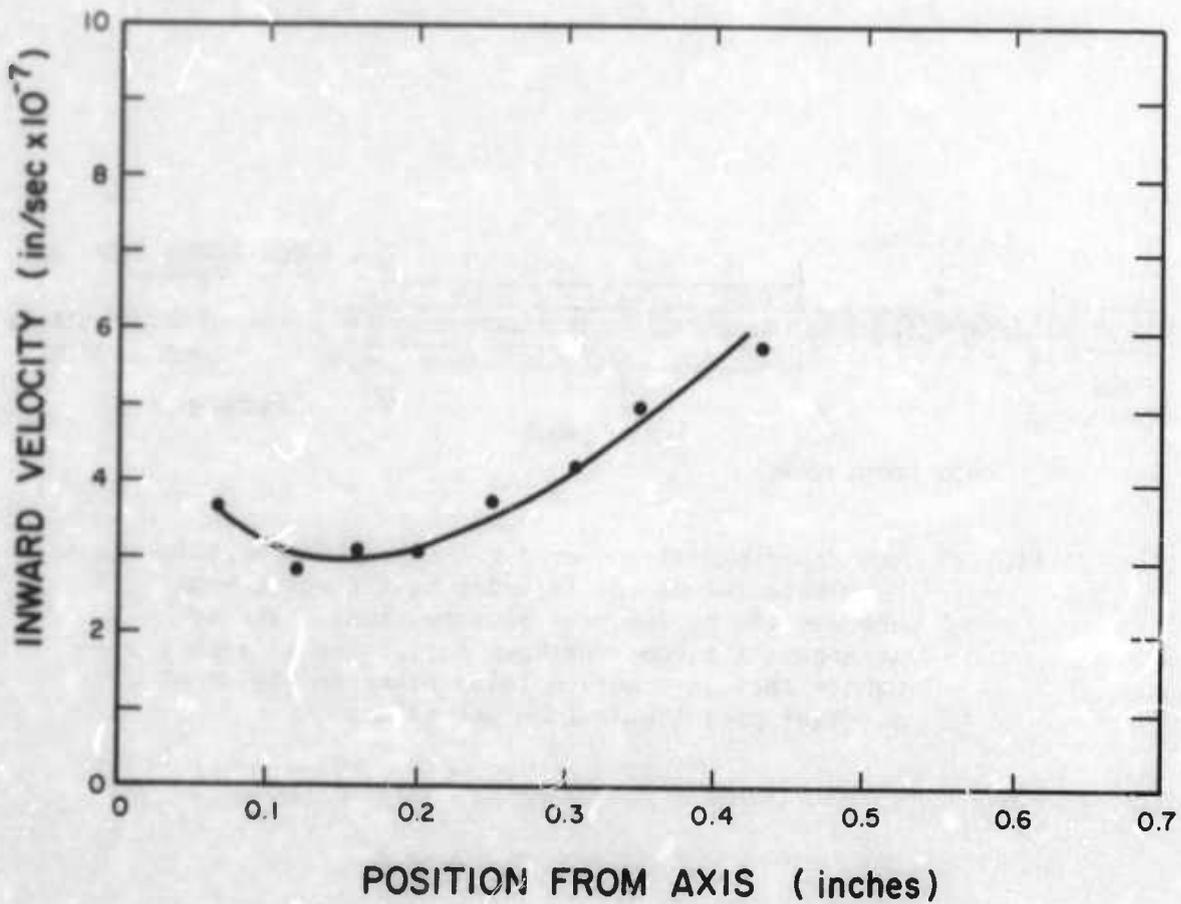


FIG. 12 The local phase velocity obtained from Fig. 11 plotted as a function of radial position. Note that the waves are propagating toward the axis.

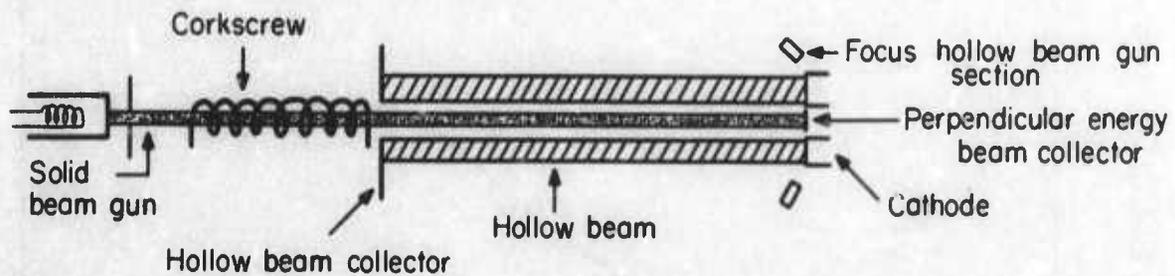


FIG. 13 New experimental set up for the two-beam experiment. Perpendicular energy is added to the solid beam whereas the hollow beam receives none. The new arrangement allows for beam collection in such a manner that interaction takes place in region of constant predictable beam parameters.

The first attempt resulted in a heater failure and the system is readied for a new trial.

## V. CONCLUSIONS

The plasma standing waves are now experimentally determined to be a result of the spatial superposition of a long wavelength field with the propagating short wavelength electrostatic waves under the discharge conditions encountered in these experiments with a low pressure dc discharge. The discrepancy between the measured wavelengths and those predicted by Bernstein dispersion relation are thought to be a result of the non-Maxwellian velocity distribution of the plasma electrons and a computation of this is in progress.

## VI. FUTURE PLANS

The non-Maxwellian dispersion relation will be calculated and the modulated beam introduced into the plasma so that the effort in the study of the beam plasma interaction will be sped up.

SUPPLEMENT TO REPORT NO. 4 ECOM-1821(E)-4

Services under Contract No. DA 28-043-AMC-01821(E)

A. IDENTIFICATION OF PERSONNEL

<u>Name</u>	<u>Title</u>	<u>Estimated Man-Hours</u>
Warren D. McBee	Department Head	50
Sheldon Gruber	Research Staff Member (Principal Investigator)	290
Claude D. Lustig	Research Staff Member	50

Biographies of each of these key technical personnel are attached.

B. PUBLICATIONS, LECTURES, REPORTS & CONFERENCES

- 10/17/66, S. Gruber attended a Seminar by Ronald Parker at M.I.T., "Some Results on Ion Cyclotron Wave Excitation and Propagation."
- 10/14/66, S. Gruber attended American Physical Society Meeting in Boston, Mass. and gave the following paper, "Experimental Evidence of Cyclotron Harmonic Wave Instabilities due to Anisotropic Velocity Distribution."
- 11/18/66, S. Gruber gave a seminar at Columbia University entitled "Longitudinal Wave Properties of a Magnetized Inhomogeneous Plasma Near the Second Electron Cyclotron Harmonic."
- 12/6/66, Col. Benjamin I. Hill, Irving Reingold and William Saxton visited S. Gruber.
- 12/9/66, Drs. Gruber and Lustig attended a seminar at M.I.T. by Professor Wright entitled, "Plasma Relaxation Rates as Determined from the Observed Time Dependent Electron Velocity Distribution."

## LITERATURE CITED

1. R. W. Gould and A. W. Trivelpiece, A New Mode of Wave Propagation on Electron Beams, Proceedings of the Symposium on Electronic Waveguides, (Polytechnic Press, Brooklyn, New York, 1958).
2. L. D. and P. Chorney, Propagation in Ion Loaded Waveguides, ibid.
3. P. L. Auer and S. Gruber, Bull. Am. Phys. Soc. II 9, 565 (1964).
4. S. Gruber, Bull. Am. Phys. Soc. II 10, 221 (1965).
5. A. Bers and S. Gruber, Appl. Phys. Letters 6, 27 (1965).
6. S. Gruber, M. W. Klein and P. L. Auer, Phys. Fluids 8, 1504 (1965).
7. A. Simon and M. N. Rosenbluth, Phys. Fluids 6, 1566 (1963).
8. J. Nickel, J. Parker and R. W. Gould, Phys. Fluids 7, 1489 (1964).
9. S. J. Buchsbaum and A. Hasegawa, Phys. Rev. Letters 12, 605 (1964).
10. H. J. Schmitt, G. Meltz and P. J. Freyheit, Phys. Rev. 139, A1432 (1965).
11. K. Mitani, H. Kubo and S. Tanaka, J. Phys. Soc. (Japan) 19, 221 (1964).
12. J. B. Bernstein, Phys. Rev. 109, 10 (1958).
13. W. P. Allis and J. C. deAlmeida Azevedo (private communication).
14. R. C. Wingerson, T. H. Dupree and D. J. Rose, Phys. Fluids 7, 1475 (1964).
15. L. M. Lidsky, Phys. Fluids 7, 1484 (1964).
16. A. Hasegawa, Phys. Fluids 8, 761 (1965).
17. S. J. Buchsbau, and A. Hasegawa, Phys. Rev. 143, 303 (1966).
18. E. G. Harris, J. Nucl. Energy, Pt. C, 2, 138 (1961).
19. R. C. Wingerson, T. H. Dupree and D. J. Rose, Phys. Fluids 7, 1485 (1964).
20. B. J. Maxum and A. W. Trivelpiece, J. Appl. Phys. 36, 481 (1965).
21. C. Etievant and M. Perulli, Compt. Rendu. Acad. Sci. 256, 5000 (1963).

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13. ABSTRACT <p>This research is directed toward the investigation of high-power beam plasma interactions, with specific investigation of the transverse velocity beam modes called for.</p> <p>Experimental results have been obtained for the propagation characteristics of the transverse plasma wave models. They are qualitatively consistent with a traveling backward wave propagating toward the axis in a non-uniform plasma column whose density decreases toward the outside. The observed wavenumber is about twice that predicted by the theoretical dispersion relation based on a Maxwellian electron velocity distribution function. The theory has been reformulated to accommodate arbitrary distribution functions. The two-beam experiment has been completely set up in a form which prevents interaction in the varying perpendicular velocity region.</p> <p>This research is part of PROJECT DEFENDER, sponsored by the Advanced Research Project Agency, Department of Defense, and administered by the U. S. Army Electronics Command under Contract No. DA 28-043-AMC-01821(E).</p>			

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