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**A NOTE ON TENSILE TESTING AT  
HIGH STRAIN RATES**

*T. NICHOLAS*

TECHNICAL REPORT AFML-TR-66-341

OCTOBER 1966

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# A NOTE ON TENSILE TESTING AT HIGH STRAIN RATES

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## FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals." The research work was conducted in the AF Materials Laboratory, Research and Technology Division, Wright-Patterson Air Force Base, Ohio, by Dr. T. Nicholas of AFML.

This report covers work performed from July 1966 to August 1966.

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This technical report has been reviewed and is approved.



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## ABSTRACT

The response of an elastic uniaxial tension specimen to a constant velocity applied at one end is presented. The application to the problem of testing specimens with high rate testing machines where the velocity of the ram approaches the dilatational wave velocity of the material is discussed. The concept of high strain rates is shown to break down under high loading rates.

## INTRODUCTION

The last several years have seen an increased interest in the properties of materials at high rates of strain. These properties have been investigated by various techniques including wave propagation and impact studies as well as by the conventional uniaxial tension test. The advent of high speed testing machines has made it possible to conduct tests at relatively high rates of strain, in some cases approaching the dilatational wave velocity of the material for some soft materials.

It is the purpose of this note to show the theoretical response of an elastic material to a constant cross-head velocity applied at one end of a uniaxial tension specimen.

## THEORY

Consider a prismatic bar of length  $l$  which is fixed at one end as shown in Fig. 1. The other end is given a uniform velocity  $v_0$  at time  $t=0$ . If  $u(x,t)$  denotes the displacement of any point in the  $x$ -direction, the one dimensional equation of motion neglecting rotational inertia is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (1)$$

where  $c = \sqrt{E/\rho}$  (2)

is the dilatational wave velocity,  $E$  is Young's modulus and  $\rho$  the mass density of the material. The bar is initially at rest, i.e.

$$\begin{aligned} u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned} \quad (3)$$

and the boundary conditions are

$$u(0,t) = 0$$

$$u(l,t) = \begin{cases} 0 & t < 0 \\ v_0 t & t \geq 0 \end{cases} \quad (4)$$

Taking the Laplace transform of (1) and incorporating the initial conditions (3) we get<sup>2</sup>

$$s^2 \bar{u}(x,s) = c^2 \frac{d^2 \bar{u}(x,s)}{dx^2} \quad (5)$$

where  $\bar{u}$  is the Laplace transform of  $u$  with respect to the parameter  $s$ . The solution to (5) is

$$\bar{u} = c_1 \sinh \frac{sx}{c} + c_2 \cosh \frac{sx}{a} \quad (6)$$

with the associated transformed boundary conditions

$$\bar{u}(0,s) = 0 \quad (7)$$

$$\bar{u}(l,s) = \frac{v_0}{s^2}$$

Substitution of (7) into (6) gives the solution

$$\bar{u}(x,s) = \frac{v_0}{s^2} \frac{\sinh(\frac{3x}{c})}{\sinh(\frac{sl}{c})} \quad (8)$$

which can be rewritten for solution purposes as

$$\bar{u}(x,s) = \frac{v_0}{2s^2} [e^{sx/c} - e^{-sx/c}] \operatorname{csch}(\frac{sl}{c}) \quad (9)$$

To invert this transform it is noted from the table of transforms of Roberts and Kaufman<sup>3</sup> that

$$L^{-1} \left[ \frac{1}{s^2} \operatorname{csch} \left( \frac{S \ell}{c} \right) \right] = \begin{cases} 0 & 0 < t < \frac{\ell}{c} \\ 2n \left( t - \frac{\ell n}{c} \right) & (2n-1) \frac{\ell}{c} < t < (2n+1) \frac{\ell}{c} \end{cases} \quad (10)$$

If the dimensionless variable  $\tau = t \frac{c}{\ell}$  is introduced, a function  $f(\tau)$  can be defined from (10) as

$$f(\tau) = \begin{cases} 0 & 0 < \tau < 1 \\ 2n(\tau - n) & (2n-1) < \tau < (2n+1) \end{cases} \quad (11)$$

A graph of this function is shown in Fig. 2. The term  $e^{-sx/c}$  in the transformed solution (9) has the property of shifting the inverse transform (10) to the right by an amount  $(x/c)$  along the time axis, or by  $(x/\ell)$  along the  $\tau$  axis. The function  $e^{sx/c}$  has a similar property of shifting the solution to the left by the same amount, provided the solution is zero in the region  $0 < t < x/c$ . The displacement can then be written for any point as

$$u(x, \tau) = \frac{v_0 \ell}{2c} \left[ f \left( \tau + \frac{x}{\ell} \right) - f \left( \tau - \frac{x}{\ell} \right) \right] \quad (12)$$

Introducing the following dimensionless variables,

$$\begin{aligned} \xi &= x/\ell \\ u^* &= u/\ell \\ v^* &= v_0/c \end{aligned} \quad (13)$$

the dimensionless displacement becomes

$$u^*(\xi, \tau) = \frac{v^*}{2} [f(\tau + \xi) - f(\tau - \xi)] \quad (14)$$

where  $f(\tau)$  is defined by equation (11). The displacement at any interior point  $\xi$  of the bar is shown in Fig. 3, plotted as  $u^*/v^*$  against  $\tau$ . The dashed curve is the displacement of the end in motion,  $\xi=1$ , which is  $u^* = v^* \tau$  or  $u = v_0 t$ .

The strain at any point is easily derived from  $\epsilon_x = \partial u / \partial x$  or in terms of dimensionless variables,

$$\epsilon_x = \frac{\partial u^*}{\partial \xi} \quad (15)$$

From the definition of  $f(\tau)$  in (11) and (14), we find

$$\epsilon_x = v^*(n_1 + r_2) \quad (16)$$

where  $n_1$  and  $n_2$  are defined in the relations

$$(2n_1 - 1) < \tau + \xi < (2n_1 + 1) \quad (17)$$

$$(2n_2 - 1) < \tau - \xi < (2n_2 + 1)$$

The strain function is shown in Fig. 4. It is to be noted that the average value of the strain is the same at any point in the bar, i.e.  $\epsilon_{av} = vt/\ell$ .

This average strain is what is observed in ordinary slow speed tests of uniform cross-head or ram velocity as on an Instron tester.

The stress is calculated from  $\sigma = \epsilon E$  and is plotted for both ends of the bar in Fig. 5. The stress at  $\xi=1$  is what the machine applies; the stress at  $\xi=0$  is what would be observed with a load cell at the fixed end.

#### DISCUSSION

From the diagrams of stress and strain along the bar several conclusions may be drawn concerning the use of constant cross-head or ram velocity machines for determining stress strain properties at very high loading rates. First, the machine must be capable of applying a step type loading as shown in Fig. 5 if the velocity of the ram is to remain constant during the test. Secondly, the observed stress at the fixed end (see Fig. 5) does not give a stress-strain curve for the material when plotted against average strain because of the discrete jumps in the response. This phenomenon has been observed in the high speed testing of

rubbers by Dannis<sup>4</sup>. Finally, the concept of constant strain rate is no longer valid at very high loading rates because the problem now becomes one of a wave propagating through the material.

The constant cross-head velocity experiment thus becomes an impact or wave propagation test when the loading rates approach the dilatational wave velocity of the material. This, along with the mechanical difficulties of constructing such a machine, limits the range of usefulness of these high rate universal testing machines. Above certain loading rates, which depend on the stiffness of the material, wave propagation techniques are necessary for the determination of material properties. However, the data obtained from constant loading rate tests may be used for determining material properties if interpreted properly in conjunction with the equations and diagrams presented here.

It must also be noted here that the "strain rates" which a material undergoes during the propagation of a wave are average values of step type functions (cf. Fig. 4).

The strain rates that actually occur are infinite for a zero time interval and are then zero for a finite time interval. The concept of material response as a function of strain rate should be carefully reviewed in conjunction with this observation. An analysis of what is really happening can only lead one to conclude that claims by experimentalists of achieving "strain rates" in the laboratory of  $10^4$  and  $10^5$   $\text{sec}^{-1}$  are really meaningless statements.

REFERENCES

1. Timoshenko, S., Vibration Problems in Engineering, 3rd ed., D. Van Nostrand, New York, 1955.
2. Churchill, R. V., Operational Mathematics, 2nd ed., McGraw Hill, New York, 1958.
3. Roberts, G. E. and Kaufman, H., Table of Laplace Transforms, W. B. Saunders Co., Philadelphia, 1966, p. 279.
4. Dannis, M. L., "Stress-Strain Testing of Rubbers at High Rates of Elongation", in High Speed Testing, Vol. 3, Interscience, New York, 1962.

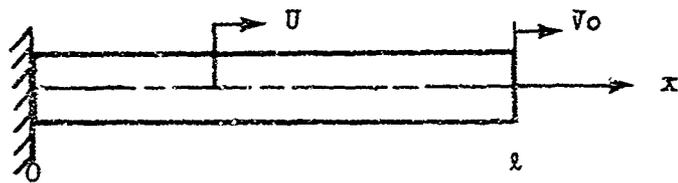


Fig. 1 Uniaxial tension test

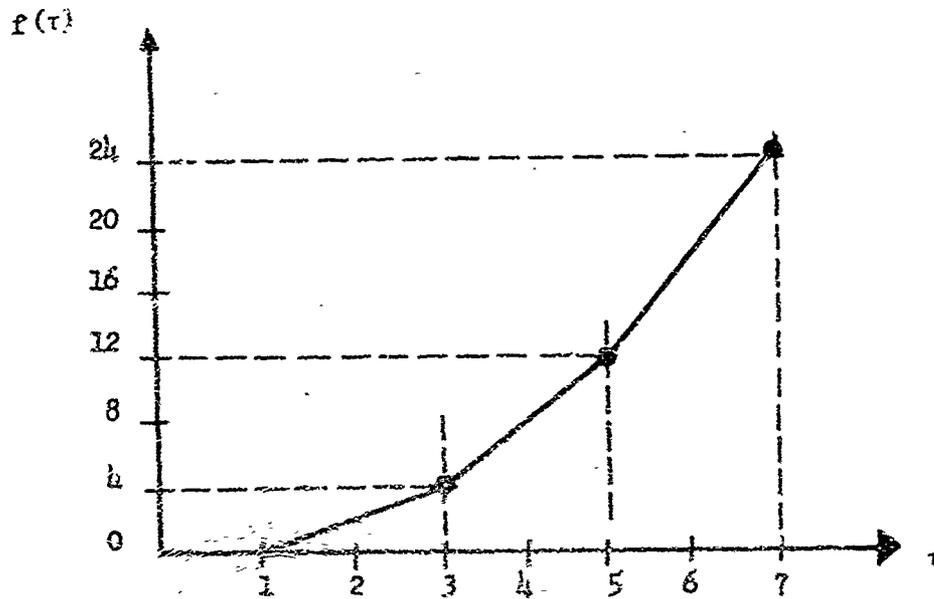


Fig 2 Graph of the Function  $f(\tau)$

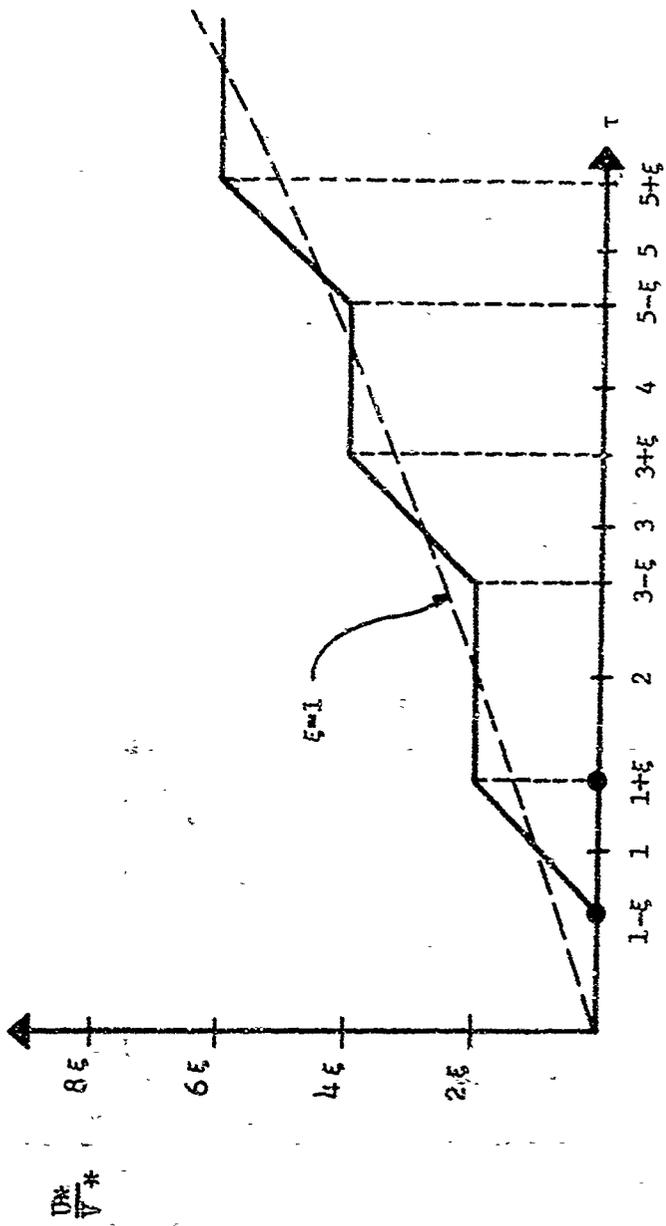


Fig 3 Displacement - time curve

$$\frac{\epsilon^*}{V^*}$$

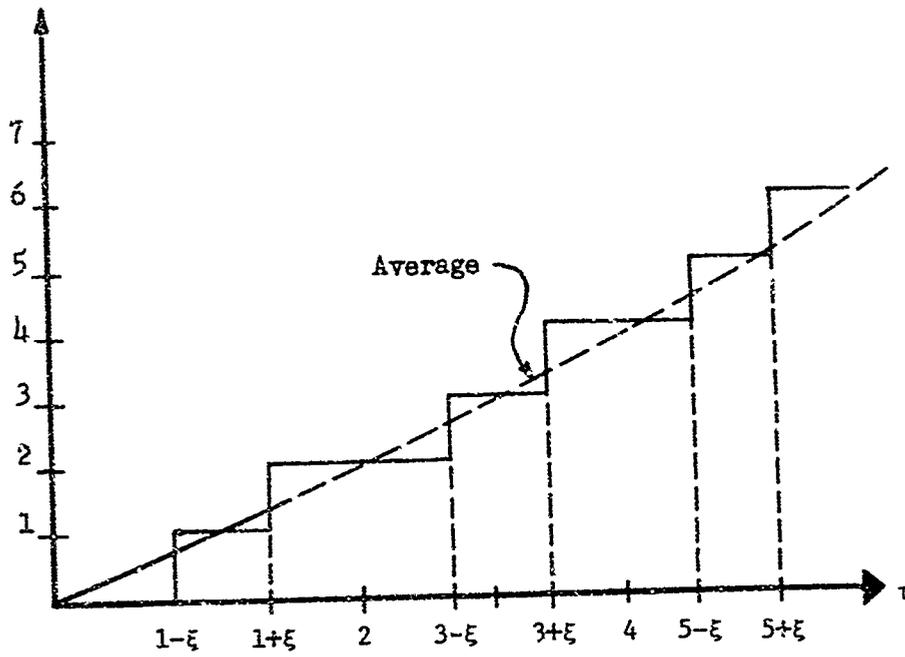


Fig 4 Strain - time curve

$$\frac{\sigma}{EV^*}$$

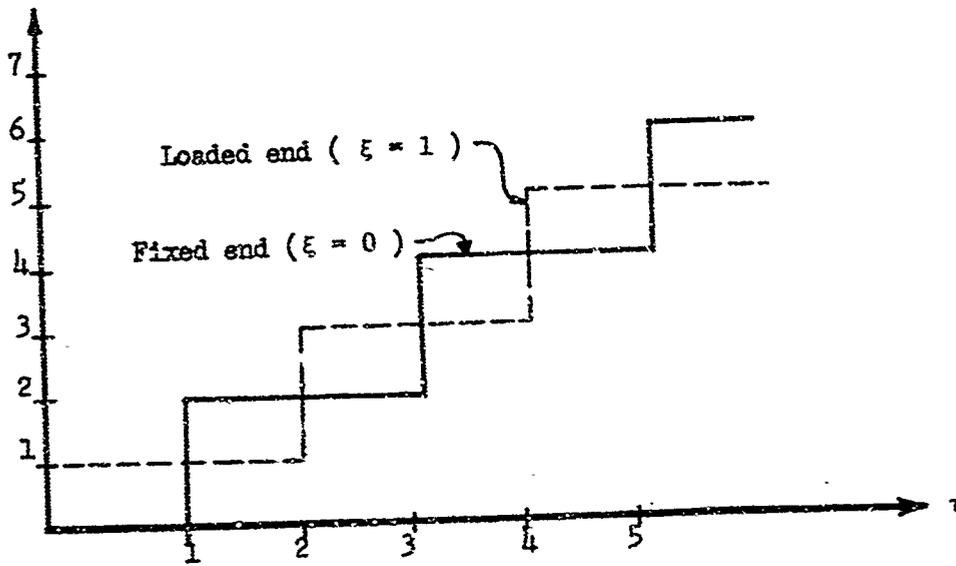


Fig 5 Stress at both ends of bar

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