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January 1, 1967

ADVANCED ELECTRO-OPTICAL SIGNAL PROCESSING TECHNIQUES
QUARTERLY PROGRESS REPORT P-9/321
October 1, 1966 to December 31, 1966

Prepared for
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Arlington, Virginia 22209

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ABSTRACT

This report summarizes the initial theoretical and experimental studies of the electro-optical signal processor for the synthetic spectrum radar.

The present digital radar processing is briefly reviewed. The proposed electro-optical processor technique is presented. This processor operating in real-time is capable of coherently phase-referencing and continuously recording the multi-channel radar output on photographic film.

An experimental, single-channel, CW-signal recorder system is described. Preliminary experimental results are included which verify the electro-optical recorder theory and show that the recorder output light intensity distribution corresponds to the input signal.

The investigation of wide band solid light modulators for optical signal processors is continued. The theory of optimal transducer depth to minimize internal refractive effects is verified experimentally for the compression mode. Additional experimental data on the cross coupling between 0.5 mm spaced transducers shows no evidence of cross coupling (to within 20 db) for compression mode transducers. Wider separations may be necessary if shear mode transducers are employed.
AUTHORIZATION

The research described in this report was performed at the Electronics Research Laboratories of Columbia University. This report was prepared by A. Aimette, M. Arm, M. King and J. Minkoff.

This project is directed by the Advanced Research Projects Agency of the Department of Defense and is administered by the Air Force Office of Scientific Research under Contract AF 49(638)-1478.

Submitted by: Approved by:

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I. INTRODUCTION AND SUMMARY

The investigations of electro-optical signal processing techniques at these Laboratories have produced a technology which can now be employed to solve difficult, real-time, high-data rate signal processing problems arising in Ballistic Missile Defense systems. In particular the purpose of this program is the application of devices which combine multi-dimensional, coherent optical configurations with real-time spatial and temporal light modulators to provide signal processing capabilities consistent with advanced BMD radar system requirements.

The objectives of this program are twofold:

1. to investigate new techniques and extend the present electro-optical systems in order to further enhance signal processing capabilities consistent with the anticipated requirements of BMD radar systems and

2. to perfect an electro-optical signal processor which can be demonstrated in the field on currently operating experimental radar systems.

To meet the first of these objectives the basic studies with solid media light modulators have been continued. These studies are necessary to evaluate design parameters such as bandwidth, beam-broadening, cross-coupling, and voltage drive levels for wideband, multiple channel light modulators. During this research period experimental verification for the optimum compressional transducer lengths theory was obtained.
at 100 MHz for normal light incidence. The results agreed very well with the theory. Since small transducers permit larger capacity systems, an investigation was performed to detect any cross coupling between closely spaced transducers. For 70-MHz compressional transducers spaced 0.5 mm, no cross coupling was noted. This investigation will now be extended to the case of non-normal light incidence.

The major effort during this past report period has been spent in investigating and developing an electro-optical processor for Space Object Identification compatible with the Westinghouse Synthetic Spectrum Radar. At present the signal processing for this system requires an analog-to-digital conversion, storage, and recording technique to convert the multi-channel radar signals into a form suitable for processing via a digital computer. This technique is expensive, slow, and, prevents real-time data reduction.

The optical processor system under investigation would coherently record and phase reference the multi-channel radar signal output on photographic film. A second optical system subsequently processes the recorded signals to form a two-dimensional image of the target scattering centers. This scheme eliminates the use of conversion equipment, computer processing and may lead to on-line and real-time data processing if advanced photorecording techniques are employed.

The initial emphasis of this program has been concentrated upon demonstrating the feasibility of the coherent recorder system. During this past period studies were made to determine the processor component requirements.
A mathematical model for signals encountered at all points of interest within the radar system and the processor was derived.\(^1\) A theoretical analysis of the electrooptical recorder was performed to compare alternate modes of operation and to provide a reference point for the experimental program.\(^2\) A single-channel recorder system was designed and constructed. Preliminary test results for a CW input signal show that the light intensity distributions obtained (see Sec. II) agree quite well with the theory.\(^3\)

A second optical system for evaluating photographic film characteristics is presently under development and will shortly be operational. A film development system has been obtained and is being installed. During this next period film recordings using the single-channel system will be made for various types of films and several modes of processor operation.

\(^1\) For numbered references, see Sec. IV.
II. ELECTRO-OPTICAL SIGNAL PROCESSORS FOR A SPACE OBJECT IDENTIFICATION (SOI) RADAR

A. INTRODUCTION

One technique useful for identifying unknown orbiting space objects is to measure their size and shape. To perform this measurement radar systems which coherently transmit and receive signals with very large time-bandwidth products are employed. These signals are then processed to extract two-dimensional information about the size and shape of the reflecting object. The Westinghouse Corporation has developed such a system in the form of their Synthetic Spectrum (S.S.) Radar.\(^4,5\)

In the S.S. radar, a high bandwidth signal is used to provide the required range resolution. For extended targets the received echoes consist of several distinct pulses reflected from particular target scattering centers. Therefore, for each transmitted pulse, the received signals can be processed to construct a "range profile" of the object. The second dimension is obtained by measuring the relative rates of change of the backscattering centers as the object rotates in space relative to the radar site. This measurement is performed by coherently integrating the signals over many transmissions, computing the Doppler frequency shift and then computing the distribution of the reflecting centers in the "transverse" range dimension.
B. THE PRESENT DATA PROCESSING TECHNIQUE

The Synthetic Spectrum Radar receiver separates the received signals into 100 parallel (simultaneous) range channels, each channel corresponding to one range cell. The range profile data, therefore, exists in analog form as the amplitude and phase of 100 parallel, simultaneous output signals.

To obtain the "range profile" and measure the "transverse range" from these 100 signals, it is necessary to establish a phase reference extracted directly from the range profile data or from refined orbit data. The gross Doppler variations caused by overall target translation or other signal phase perturbation occurring during the coherent integration time are removed by subtracting the phase of the most prominent range profile signal from all the signals in a single range profile. Hence for each transmission, the prominent signal is used as both a range and phase reference and coherent integration can be performed as long as the 'prominent point' signal exists.

At present, the 100 analog signals are digitized, stored, commutated, and recorded on magnetic tape. These tapes are then fed to a high-speed digital computer which performs the necessary operations of prominent point selection, profile alignment, phase subtraction, and coherent integration on a pulse-to-pulse basis. This procedure is slow, expensive and necessarily prohibits real-time or on-line operation.

The electro-optical processor now under investigation for use with the S.S. Radar eliminates the digital computer.

and offers the possibility of real-time operation, directly from the 100-analog channels. Without commutation this produces the aligned range profile data on photographic film in a format which permits subsequent coherent integration in an optical film processor. This may permit real-time recording of the data in analog form and hence facilitates an on-line estimate of the target size and shape.

C. THE ELECTRO-OPTIC S. S. RADAR PROCESSOR

The processor, shown schematically in Fig. 1, employs a Schlieren-type coherent optical configuration with a 100-channel spatially-multiplexed Debye-Sears light modulator cell, each channel corresponding to one signal channel of the S. S. Radar output. The plane-collimated light passes through the cell and emerges spatially-phased modulated by the input signals. The integrating lens, spatial filter and imaging lens form a moving image of the 100 signals in the light modulator onto the moving film. To record the image on the film the image is essentially frozen in time by amplitude modulating the light from a signal coherently derived from one channel of the radar output. If this channel is also the 'prominent point' or reference signal it may be shown that all the signal channels recorded on the film are thus automatically phase referenced to the prominent point channel. Note that only the signals corresponding to the range of interest are recorded since the laser is pulsed by a signal derived from the radar range tracking gate.

The film is advanced at a slow rate relative to the allowable exposure time of 4 μsec. A schematic diagram of the film format produced is shown in Fig. 2. The
B. THE PRESENT DATA PROCESSING TECHNIQUE

The Synthetic Spectrum Radar receiver separates the received signals into 100 parallel (simultaneous) range channels, each channel corresponding to one range cell. The range profile data, therefore, exists in analog form as the amplitude and phase of 100 parallel, simultaneous output signals.

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At present, the 100 analog signals are digitized, stored, commutated, and recorded on magnetic tape. These tapes are then fed to a high-speed digital computer which performs the necessary operations of prominent point selection, profile alignment, phase subtraction, and coherent integration on a pulse-to-pulse basis. This procedure is slow, expensive, and necessarily prohibits real-time or on-line operation.

The electro-optical processor now under investigation for use with the S.S. Radar eliminates the digital computer and offers the possibility of real-time operation, directly from the 100-analog channels. Without commutation this produces the aligned range profile data on photographic film in a format which permits subsequent coherent integration in an optical film processor. This may permit real-time recording of the data in analog form and hence facilitates an on-line estimate of the target size and shape.

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The film is advanced at a slow rate relative to the allowable exposure time of 1 μsec. A schematic diagram of the film format produced is shown in Fig. 2. The
FIG. 1  SCHEMATIC DIAGRAM-ELECTRO-OPTICAL SIGNAL PROCESSOR
FOR THE WESTINGHOUSE S.S. RADAR
FIG. 2
FILM FORMAT—ELECTRO-OPTICAL SIGNAL PROCESSOR FOR THE WESTINGHOUSE S.S. RADAR

100 CHANNELS OF RADAR I.F. OUTPUT WITHOUT PHASE ALIGNMENT IN REAL TIME

SEGMENT OF ELECTRICAL INPUT

TRACKING CHANNEL

RADAR PULSE PERIOD

4 µsec

RECORDED PULSE WIDTH

RECORDED PULSE PERIOD

100 CHANNELS OF RADAR I.F. OUTPUT WITH PHASE ALIGNMENT

SEGMENT OF PHOTOGRAPHIC FILM RECORD
rows in the diagram correspond to the spatially-multiplexed light modulator channels and the columns to the successive pulses. Therefore, range information is recorded in corresponding rows and each column represents a range profile. The incremental phase changes between successive range profiles (successive rows) contain the Doppler information. The film output from this processor can now be placed in a second optical system where coherent Doppler filtering can be done between successive echoes (columns) to yield the transverse-range distance between scattering centers.

D. ELECTRO-OPTICAL PROCESSOR - ANALYTICAL STUDIES

The technical report entitled "Signal Processing for the Synthetic Spectrum Radar"\(^1\) contains the results of a detailed analytical study of the radar. Beginning with a mathematical model of the transmitted signal, an expression relating the radar return to the target dynamics is obtained. A step-by-step analysis of the effects of the processing of the return within the radar then yields expressions for the IF output signals in all range channels as a function of target dynamics. Finally, the report shows that by phase alignment and coherent integration of the IF output signals it is possible to obtain a two-dimensional radar image of the target. Formulas which relate the target dimensions to the signals achieved after coherent integration are derived.

The technical report entitled "Multi-Channel Electro-Optical Recorder for the Synthetic Spectrum Radar"\(^2\) is an exposition of the theory of operation of the recorder we expect to build for the Synthetic Spectrum Radar. Starting
with a simple recording system that can handle a CW carrier input, the exposition gradually increases in complexity until the performance of the final recorder is explained. The properties of the photographic film are accounted for, as well as those of the electro-optical components, when the final expressions for film transmittance as a function of electrical input signal are derived. An appendix evaluates three alternate modes of operation of the recorder, and derives the recording-system transfer functions for all three cases.

E. PRELIMINARY EXPERIMENTAL PROGRAM - ELECTRO-OPTICAL RECORDER SYSTEM

1. Single Channel CW Electro-Optical Recorder System

To establish the engineering feasibility and to determine the component specifications of the electro-optic recorder system, the preliminary test system (Fig. 3) was constructed.\(^3\)

The CW light beam from the gas laser is amplitude modulated in passing through a retardation-type modulator (Spectra Physics 320). A single-channel 20-MHz Debye-Sears light modulator spatially phase modulates the amplitude modulated plane light. Both the amplitude (temporal) and the phase (spatial) light modulators are driven from a common CW oscillator. The image of the CW input signal in the spatial modulator is formed in focal plane of the imaging lens. This light distribution is scanned by a 1-micron slit and photomultiplier system and displayed on the y axis of an x-y plotter, together with a horizontal position voltage from the slit scanner drive system. Hence the output display
FIG. 3 SCHEMATIC DIAGRAM—PRELIMINARY OPTICAL SIGNAL RECORDER TEST SYSTEM
is a signal whose spatial frequency and amplitude is a function of the electrical input signal to the spatial light modulator and whose relative phase is determined by the phase difference between the temporal (amplitude) and the spatial light modulators.

2. Experimental Results

The spatial light modulator shown in Fig. 3 can be operated in several alternate modes. The results obtained using the so-called "Bragg mode" are shown in Figs. 4 and 5. In this mode of operation the undiffracted or zero order fringe and one of the first order fringes are allowed to pass through the spatial filter and the acoustic beam is tilted to the Bragg angle with respect to the incident light. All the incident light therefore is contained in these two fringes. This lowers the input light power required by the system.

Figure 4 shows the light intensity distribution in the image focal plane of the system in Fig. 3 as a function of the spatial modulator transducer voltage. Only a small number of cycles in the image is shown and the scale is approximately 300 μ/in.

Note that the AC signal shown is the time stationary image of the acoustic signal traveling inside the spatial light modulator (a recorded image). These AC signals ride on a DC bias level proportional to the peak intensity of the zero order. For no input signal the image is merely a constant DC voltage. As the input signal $V_m$ increases the AC component spatially amplitude modulates the DC signal. The contrast or percentage modulation is proportional
FIG. 4 ELECTRO-OPTIC SIGNAL RECORDER IMAGE PLANE LIGHT INTENSITY DISTRIBUTION FOR BRAGG MODE, $B_{01}$, OPERATION
FIG. 5  MEASURED CONTRAST IN ELECTRO-OPTIC RECORDER BRAGG MODE
to $\sin kV_m$, and the intensity of the first and zero orders is proportional to $\cos^2 kV_m$ and $\sin^2 kV_m^2$, respectively. Figure 4 shows the individually measured values of the first and zero order.

Figure 5 shows the contrast as a function of the transducer drive voltage from Fig. 4. The upper curve is derived from the measurements of the zero and first orders individually and the lower curve is measured by taking the ratio of the peak AC signal to the DC bias level. These agree quite well; the difference is most probably due to modulation distortions on the input signal. At low input voltages the AC signal recorded is linearly proportional to the input voltage.

These results verify the recorder theory and show that the light intensity distributions in the image plane correspond to the CW electrical input signals to the spatial light modulator with the predicted contrast. The next step is to record these signals on photographic film.

A second optical system has been designed and is almost operational. This system will be used to investigate the characteristics of various types of photographic film, particularly under the short exposure, low intensity conditions requisite for the radar signal recordings. A developing system for the film has been purchased and is nearly operational. During the next period photographic recordings of the signals will be made and evaluated for the zero Doppler condition.
III. WIDE-BAND FUSED-SILICA LIGHT MODULATORS

It has been shown that the development of a wide-band solid light modulator is necessary in order to meet the requirements for electro-optical processing of array antennas. The initial objectives of this study are the theoretical and experimental determination of the basic operating characteristics of fused-silica light modulators in order to establish design parameters. Some experimental results of these basic investigations have already been reported and additional experimental results are presented below. In particular:

1. The theory of the optimal transducer depth which minimizes the effect of internal refraction is verified experimentally for the compression mode.

2. Additional experimental data on cross coupling between very closely spaced transducers is obtained. (For preliminary work see Ref. 8). Transducer spacings in these experiments are approximately 0.5 mm. No evidence of cross coupling (to within 20 db) is observed for the compression mode but results indicate that wider separations may be necessary if shear transducers are employed.

Thus far, all theoretical and experimental considerations have been based on the assumption of normal light incidence. It is possible, however, that the voltage requirements, which have been shown to result in considerable transducer heating, may be reduced by employing a configuration in which the light is incident at the Bragg angle. It is expected that future research efforts in this program will consider this mode of operation.
A. EXPERIMENTAL VERIFICATION OF OPTIMAL TRANSDUCER DEPTH

It has been shown\textsuperscript{6} that, for conditions of normal diffraction,\textsuperscript{9} because of internal refraction considerations, the optimal transducer depth for normal light incidence is given by:

\[ L_{\text{opt}} = \frac{n_o S f_i}{\lambda} \]  

where:

- \( L \) = transducer depth
- \( n_o \) = equilibrium value of refractive index
- \( S \) = sonic velocity
- \( f_i \) = input frequency
- \( \lambda \) = light wavelength

That is, for any given value of peak input voltage, \( V_m \) (assuming that Willard's criterion\textsuperscript{9} and the assumptions of Rao and Murty\textsuperscript{10} are not violated), this value of transducer depth theoretically maximizes the effective phase modulation, \( \overline{\psi}_m \), which is defined here as:

\[ \overline{\psi}_m = \frac{\psi_m}{\gamma} \]  

where:

\[ \psi_m = 2 \pi \overline{n} \]  

and \( \overline{n} \) = maximum perturbation of refractive index,

\[ \gamma = \frac{\pi L f_i^2}{\lambda n_o S^2} \]  

and \( \sin \gamma \) is the attenuation factor due to internal refraction\textsuperscript{6,10} for normal light incidence.

The experimental verification of this relationship (Eq. (1)) utilized the arrangement shown in Fig. 6. In this case a fused-silica light modulator with a compression transducer was used, the amplitude of the input signal was held at a fixed value, and the input frequency was 100 MHz. The peak first-order intensity was then measured as a function of transducer depth, \( L \), which was varied from 11 mm to 5 mm in steps of 1 mm. It was found that the peak first-order light intensity was a maximum for \( L = 8 \) mm. This result agrees with the theory which has been developed since the value of \( L_{\text{opt}} \) for the compression mode at 100 MHz in fused silica is approximately 8 mm.\textsuperscript{6}

The results are presented, normalized to \( \overline{\psi}_m (8) \), in Fig. 7.

From Eqs. 2, 3, and 4 it is seen that the effective normalized phase modulation is:

\[ \frac{\overline{\psi}_m (L)}{\overline{\psi}_m (8)} = \sin k_2 \frac{k L_2}{k L_1} \]  

where \( k_1 \) and \( k_2 \) are constants, and since from Eqs. 1 and 4

\[ \gamma \bigg|_{L = L_{\text{opt}}} = \frac{\pi}{2} \]
A. EXPERIMENTAL VERIFICATION OF OPTIMAL TRANSDUCER DEPTH

It has been shown⁹ that, for conditions of normal diffraction, because of internal refraction considerations, the optimal transducer depth for normal light incidence is given by:

\[ L_{\text{opt}} = \frac{n_0^2 \gamma}{\lambda f_1^2} \]  

(1)

where:

- \( L = \) transducer depth
- \( n_0 = \) equilibrium value of refractive index
- \( S = \) sonic velocity
- \( f_1 = \) input frequency
- \( \lambda = \) light wavelength

That is, for any given value of peak input voltage, \( V_m \) (assuming that Willard’s criterion⁹ and the assumptions of Rao and Murty¹⁰ are not violated), this value of transducer depth theoretically maximizes the effective phase modulation, \( \bar{\gamma} \), which is defined here as:

\[ \bar{\gamma} = \frac{\gamma}{y} \]  

(2)

where:

\[ \gamma = \frac{2 \pi n_0^2}{\lambda} \]  

(3)

and \( n = \) maximum perturbation of refractive index.

The experimental verification of this relationship (Eq. (1)) utilized the arrangement shown in Fig. 6. In this case a fused-silica light modulator with a compression transducer was used, the amplitude of the input signal was held at a fixed value, and the input frequency was 100 MHz. The peak first-order intensity was then measured as a function of transducer depth, \( L \), which was varied from 11 mm to 5 mm in steps of 1 mm. It was found that the peak first-order light intensity was a maximum for \( L = 8 \) mm. This result agrees with the theory which has been developed since the value of \( L_{\text{opt}} \) for the compression mode at 100 MHz in fused silica is approximately 8 mm.⁸

The results are presented, normalized to \( \bar{\gamma}_m(8) \), in Fig. 7. From Eqs. 2, 3, and 4 it is seen that the effective normalized phase modulation is:

\[ \frac{\bar{\gamma}_m(L)}{\bar{\gamma}_m(8)} = \frac{k_1 L \sin k_1 L}{k_2 \sin k_2 L} \]  

(5)

where \( k_1 \) and \( k_2 \) are constants, and since from Eqs. 1 and 4

\[ \gamma = \frac{\pi}{2} \]

\[ L = L_{\text{opt}} \]
FIG. 6 SYSTEM FOR MEASURING DIFFRACTED LIGHT INTENSITY
FIG. 7. NORMALIZED EFFECTIVE PHASE MODULATION vs TRANSUDER DEPTH

$\sin \frac{\pi L}{16}$

○ MEASURED VALUES
then:

\[ \gamma = \frac{k_2 L}{2} \approx \frac{\pi}{16}, \]

\[ L = 8 \]

therefore

\[ \frac{\psi_m(L)}{\psi_m(8)} \approx \sin \frac{\pi L}{16}. \] (6)

This curve is included in Fig. 7.

These results provide experimental verification, for the compression mode, of Eq. (1). It is seen that this value of transducer depth will minimize the input voltage requirements. Note that this theory has been developed under the assumption of normal light incidence. Future research will consider light incident at the Bragg angle which may result in other specifications for this design parameter.

**B. EFFECTS OF ELECTROMECHANICAL CROSS COUPLING**

It has previously been reported\(^8\) that, for both shear and compression modes, no cross-coupling was observed between transducers which were spaced by 2 mm. Additional experimental data concerning cross-coupling between shear and compression transducers with spacings of the order of 0.5 mm is now presented.

In this experiment, for each mode, a single electrode 7 mm wide was excited by a low voltage signal and a Schlieren photograph was taken of the resulting acoustic wave (Figs. 8a and 9a). The frequency for both modes was 70 MHz. The electrode was then divided in half by a scratch, which ef-
FIG. 8  SHEAR-MODE TRANSDUCER: SCHLIEREN PHOTOGRAPHIC-STUDY OF CROSS COUPLING.
FIG. 9  COMPRESSION-MODE TRANSDUCER: SCHLIEREN PHOTOGRAPHIC-STUDY OF CROSS COUPLING.
fectively produced two 3.5-mm electrodes separated by less than 0.5 mm, and a photograph was taken (using the same exposure as before) of the beams produced by exciting one of the halves by a signal 10 times as large (Figs. 8b) and 9b). In the case of the shear transducer, the excited electrode happened to be located equidistantly between the unexcited 3.5 mm half and a second electrode which was in place from a previous experiment.

It is seen that, for the compression mode, no spurious signal is visible which verifies that very closely spaced compression transducers will be isolated by at least 20 db. For the shear mode, however, this might not be the case. Although it may not be readily apparent from the reproduction of Fig. 8b, the original photograph for the case of the shear transducer showed faint traces at the locations of the passive electrodes on both sides of the active region which could represent a spurious excitation arising from the extremely narrow separations in this experiment. These results therefore seem to favor the use of the compression mode for a multi-channel spatially-multiplexed light-modulator configuration.
IV. REFERENCES

References 1, 2, 3, 6, 7 and 8 were prepared at the Electronics Research Laboratories, Columbia University, New York, New York 10027.


This report summarizes the initial theoretical and experimental studies of the electro-optical signal processor for the synthetic spectrum radar.

The present digital radar processing is briefly reviewed. The proposed electro-optical processor technique is presented. This processor operating in real-time is capable of coherently phase-referencing and continuously recording the multi-channel radar output on photographic film.

An experimental, single-channel, CW-signal recorder system is described. Preliminary experimental results are included which verify the electro-optical recorder theory and show that the recorder output light intensity distribution corresponds to the input signal.

The investigation of wide band solid light modulators for optical signal processors is continued. The theory of optimal transducer depth to minimize internal refractive effects is verified experimentally for the compression mode. Additional experimental data on the cross coupling between 0.5 mm spaced transducers shows no evidence of cross coupling (to within 20 db) for compression mode transducers. Wider separations may be necessary if shear mode transducers are employed.