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TURBULENCE, TOROIDAL CIRCULATION AND DISPERSION OF FALLOUT PARTICLES FROM THE RISING NUCLEAR CLOUD

by
I. O. Huebsch

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ABSTRACT

The entraining-parcel model of the rise and expansion of the nuclear cloud is revised to allow for (1) production of turbulent kinetic energy from kinetic energy of rise during the momentum-conserving, inelastic-collision entrainment, so that total kinetic energy is conserved in entrainment. This production of turbulent energy is in addition to that due to eddy viscosity; (2) dissipation of turbulent energy to heat.

The resulting nuclear-cloud model is represented as an energy cycle between enthalpy and energy of rise, turbulent energy, and potential energy.

Calculations of toroidal circulation and particle motion in the nuclear cloud are discussed, using the cloud model and turbulent similarity theory. If the toroidal circulation is to be represented by a vortex ring superimposed on the parcel-model cloud, the vortex ring can be considered the largest eddy in the turbulent spectrum, containing a fixed fraction of the turbulent energy. This gives an estimate of the circulation without recourse to, but in general agreement with, estimates based on measurements of nuclear cloud films. Alternately, circulation may be calculated by an adaptation of Kelvin's theorem. This method assumes neither a particular vortex form nor steady-state flow, and is therefore more consistent with actual cloud conditions.

Attempts have been made to calculate the effect of toroidal circulation on particle dispersion from the cloud using laminar flow methods. Here, this dispersion is, instead, represented as due to turbulent diffusion, using the calculated dissipation rate as the governing parameter. Diffusion coefficients and concentration gradients are derived from turbulent similarity theory. The resulting dispersion rate is shown to be small compared with that due to gravitational fallout rate indicating that dispersion induced by circulation or turbulence can be ignored.

SUMMARY PAGE

The Problem

A vortex ring or so-called toroidal circulation is often observed in the intensely turbulent cloud rising from a nuclear explosion. Particles of different sizes in this nuclear cloud may follow the circulatory motion of the fluid to different extents, and some of them may be centrifuged out of the cloud. The circulation, then, affects the dispersion of particles from the cloud and the distribution of radioactive fallout in the air and on the ground.

Calculation of the fluid motion and the particle trajectories by classical hydrodynamic methods presents both theoretical and computational difficulties, due largely to the turbulent character of the flow in and around the cloud. Calculations of cloud motion can be considerably simplified by using a parcel method, which gives average values of cloud velocity, size and other variables as functions of time. An entraining-parcel cloud model has previously been developed at this laboratory.

The objectives of the present work were:

(1) to revise the cloud model to take into account dissipation of turbulent energy to heat, and conservation of kinetic plus turbulent energy in entrainment.

(2) to discuss attempts to calculate toroidal circulation in the nuclear cloud and its effects on particle dispersion.

(3) to investigate alternate methods of modeling non-gravitational particle dispersion, using turbulence theory.

Findings

(1) In the revised cloud model, total energy is conserved. The model implies a nuclear cloud energy cycle.

(2) If toroidal circulation is to be represented by a classical vortex ring superimposed on a parcel-model cloud, then the vortex parameters may be estimated from the cloud model consistent with turbulenc theory. The calculated values of circulation are in general agreement with the few, scattered values based on measurements of nuclear cloud films. Alternately, the rate of change of circulation may be calculated by a free adaptation of Kelvin's theorem, and the rate integrated to give circulation. This method does not require assuming a particular vortex structure and steady-state flow. It is therefore more consistent with physical conditions in the nuclear cloud.

(3) Non-gravitational dispersion of particles can be represented as due to turbulent diffusion instead of laminar vortex flow. Using turbulent diffusion coefficients derived from small-scale-turbulence theory, and the turbulent energy dissipation rate given by the cloud model, it is shown that turbulent-diffusive dispersion of particles is unimportant compared to gravitational fallout.

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1. INTRODUCTION

1.1 Background

One of the striking features of nuclear explosions is the vortex ring and/or "toroidal circulation" in the rising, intensely turbulent nuclear cloud. It has been suggested that particles of different sizes in the cloud follow the motion of the fluid circulation to different extents, so that the circulation influences the dispersion of particles from the cloud (by a sort of centrifuging process) and thus the distribution of radioactive fallout in the air and on the ground. Calculation of the fluid motion and of the particle trajectories by classical hydrodynamic methods presents both theoretical and computational difficulties, not yet overcome. These difficulties arise largely from the turbulent character of the flow in and around the cloud.

1.2 Objectives

The purposes of this report are

1. To present a revision of a previously published[1] entraining-parcel model of the atomic cloud, allowing, as before, for transfer of kinetic energy of cloud rise to turbulence by eddy-viscosity, and also for

(a) additional transfer of kinetic energy of rise to turbulence by the inelastic-collision, momentum-conserving entrainment process, so that total energy is conserved

(b) transfer of energy from turbulence to heat.

2. To discuss the energy cycle of the atomic cloud which is implicit in this model.

3. To discuss attempts to superimpose a classical vortex motion on a parcel-method cloud, and to show how such attempts can at least be made consistent with the energy balance of the cloud, and how cloud circulation can be estimated without use of the superimposed vortex.

4. To show how the particle-dispersive effect attributed to fluid circulation can be represented as due to turbulent diffusion consistent with the parcel method, and to estimate the relative importance of diffusive and gravitational dispersion of particles from the cloud.

1.3 Approach

The principal tool used in the present study is the revised entraining-parcel model of the nuclear cloud. This model was originally developed for water-surface bursts, but is also applicable to air bursts, since, in both cases, the mass fraction of condensed matter is small. The model can further be applied to land-surface bursts provided the mass and size distribution of condensed matter at start of cloud rise are specified.

The present development of the original model is largely inspired by turbulence theory, especially by the concept of the energy cascade and the theory of local similarity, or similarity of small eddies.[2] Therefore, the relevant parts of turbulence theory are reviewed in this report. (Sec. 3)

2. MODELS OF THE NUCLEAR CLOUD

2.1 The Formation of the Nuclear Cloud

In a nuclear explosion, typically, from 1 KT to 10 MT of energy (10^{12} to 10^{16} calories) is released in less than 1 microsecond (μ sec.) in a space 1 meter in diameter.

Since the energy release per unit mass is thousands of times that of a chemical explosion, the resulting "fireball" consisting of vaporized bomb materials is initially at a temperature on the order of 10^7K instead of 3000 - 5000 K as for a chemical explosion, and a pressure of several million atmospheres.

The total energy, W, of a nuclear explosion in the air, near sea level, is partitioned thus:[3]

5% = initial nuclear radiation (radiation during the first minute after the explosion)
10% = residual nuclear radiation (radiation after the first minute after the explosion)
50% = blast and/or shock
35% = thermal radiation

All of this energy is eventually degraded to heat.

The diameter of the fireball increases by radiative and shock heating of the surrounding air, while the pressure in the fireball drops by expansion and radiative cooling. Within a few seconds (about 1 second for W = 20 KT) pressure has dropped to one atmosphere and temperature has dropped to about 3000 to 4500 K. The fireball then starts rising and can be called the nuclear cloud.

At start of cloud rise, the fireball contains about $\frac{1}{3}$ of the total energy of the explosion. This $\frac{1}{3}$ value need not be the same as the 35% thermal energy mentioned above, since some of the $\frac{1}{3}$ is shock heating and some of the 35% was radiated beyond the fireball.

The fraction, f, of the total energy contained in the fireball at start of rise, fW, increases slightly with yield but the subsequent cloud history is not sensitive to variations in assumed initial cloud energy, fW. Taylor[4] calculated $f > \frac{1}{3}$ as the basis of shock heating plus adiabatic

expansion to atmospheric pressure, and suggested that the effects of neglecting both energy loss by radiation and the increased specific heat of air at high temperature tend to cancel each other.

The maximum radius of the fireball before it starts to rise and becomes the atomic cloud is proportional[3] to $W^{2/5}$. The exponent 2/5 is close to 1/3 and the fireball size is approximately as if 1/3 of the total explosion energy were used to heat air at constant pressure to 3000K. The calculated fireball volume (and thus radius) at start of rise is nearly independent of assumed fireball temperature.

(Proof: using the perfect gas law, and the equality of cloud and ambient pressure

$$p = \rho RT = \rho_e RT_e; \; fW = mc_p (T-T_e) \text{ so that}$$

$$V = \frac{m}{\rho} = \frac{R}{p} \cdot \frac{fWT}{c_p(T-T_e)} \simeq \frac{fRW}{c_p p}$$

Here the subscript e denotes initial environment conditions, p is pressure, ρ is density, R is the universal gas constant, T is temperature, V is cloud volume, m is cloud mass, and c_p is specific heat at constant pressure.)

At start of rise, the cloud may contain some refractory particulate matter; more and larger particles form in the rising, cooling cloud by condensation and coagulation. The dispersion of these particles from the cloud may be affected by conditions inside the cloud, including vortex flow or turbulence. The following section considers calculation of such conditions; i.e. cloud models.

2.2 Cloud Calculations

Calculations of the behavior of the nuclear cloud have been made by a number of investigators using different approaches, often patterned after studies of cumulus clouds or of the rise of hot gases from fires and factory chimneys.

Possible models of cloud behavior can be divided into those using (1) local methods and (2) parcel methods.

2.2.1 Local Methods

In a "local" method, the flow conditions in the cloud and surrounding atmosphere are described by a set of partial differential equations (Navier-Stokes equations) giving local values of velocity, temperature, etc., as functions of time. For numerical computation, the partial differential equations are replaced by a set of ordinary differential equations at each point of a grid or mesh of points covering the flow region. Some beginnings using this treatment have been made[5,6] for developing cumulus clouds, making the assumption that temperature differences between cloud and atmosphere are small relative to absolute temperature, and using an eddy viscosity in the Navier-Stokes equations. The values of velocity, etc., computed represent time averages, not the instantaneous values which are subject to turbulent fluctuations.

Eddy-viscosity is needed in the equations used for computation because molecular viscosity (ordinary viscosity) operates to convert the energy of only the smallest eddies of a turbulent flow into heat. It can be shown (see for instance, Section 3.1.5) that the ratio of the size of the smallest eddies, λ_o, to the overall flow dimensions, ℓ, for one-dimensional flow, is

$$\frac{\lambda_o}{\ell} \quad Re^{-3/4}$$

where Re is the Reynolds number of the flow. Such a flow could be pictured as containing a row of ℓ/λ_o such eddies, (on which are superimposed larger eddies) and so having $Re^{3/4}$ degrees of freedom. Then the mesh for a two-dimensional calculation must have $Re^{3/2}$ points, so that numerical computation is out of the question when Re is large. The eddy viscosity is used to blur together small eddies, and thus reduce the number of degrees of freedom of the flow problem, and the required number of mesh points, to where numerical computation is possible.

No local-method computation has yet been carried out for nuclear clouds. Even for cumulus clouds, the original two-dimensional array of mesh points (in an axisymmetric finite-difference calculation) is rapidly distorted. For nuclear clouds, this distortion would be accompanied by extreme temperature gradients, so that heat conduction could not be neglected. Presumably, an eddy conductivity would be required as well as eddy viscosity. Altogether, one is inclined to question the use of eddy transport coefficients to calculate mean local conditions at the extremely high Reynolds numbers ($\sim 10^{10}$) of these intensely turbulent flows. In any case, the attempt has not yet been made.

2.2.2 Parcel Methods

In a parcel method, the cloud is treated as a whole, as if all parts of it had the same properties. Temperature, velocity, etc., are represented by average values for the whole cloud. Pressure is taken as equal to ambient pressure at the altitude of the center of the cloud. Thus a parcel method is a simple, cheap substitute for a local method. But it is far more amenable to numerical computation. And with a few enlightened assumptions, a parcel method can give a wide variety of information, including an estimate of particle dispersion by toroidal circulation, represented as turbulent diffusion. See Sec. 5.

2.3 The TR-741 Atomic Cloud Model, Revised

An atomic cloud model, using an entraining parcel method has been developed at this Laboratory[1]. This model, (referred to as the TR-741 model) was particularly designed for sea-water-surface nuclear explosions. The model treats the cloud gas as a mixture of air and water vapor and allows for the release of latent heat by water-vapor condensation. The key assumptions in this model are:

1. The effective rate of flow of ambient air into the cloud (entrainment) per unit surface area is equal to the product of a constant, λ, a characteristic velocity, v, and the ratio of cloud density to ambient density, ρ/ρ_e. Some previous cloud studies have used the same entrainment rule except for omitting the density ratio.
2. Cloud rise is retarded by both entrainment and an apparent eddy-viscous force, directly proportional to the same characteristic velocity and inversely proportional to the density ratio. (This is formally equivalent to a drag coefficient.)
3. The kinetic energy of rise corresponding to the momentum lost by eddy viscosity is converted into kinetic energy of turbulence, which remains in the cloud.
4. The characteristic velocity, v, is the greater of absolute rate of cloud rise, u, and average velocity of turbulence, $\sqrt{2E}$ *. Thus entrainment does not necessarily end when cloud rise ends, but continues as a turbulent diffusion.
5. In accelerating from rest to its maximum velocity, the cloud must set in motion a volume of ambient air equal to one half the initial cloud volume. Because of this "virtual mass", the acceleration is always less than twice that of gravity.

No provision was made in this model for transformation of turbulent energy into heat. Not only does such transformation actually take place, but also it may affect the formation of fallout particles through the mechanism of turbulent coagulation.[7] The rate of this transformation, the so-called "dissipation rate", ε, is found both theoretically and experimentally to be proportional to the cube of a large-scale turbulent velocity divided by a large-scale length, ℓ. (see Section 3, Turbulence) Therefore, we now add one assumption to the cloud model.

6. Turbulent energy is dissipated at a rate proportional to the cube of the average velocity of turbulence divided by a char-

* The symbol E_k was used in TR-741 for the quantity called E in the present report.

acteristic length. (See Revisions, Appendix A) The average velocity of turbulence is defined as $\sqrt{2E}$ where E is the turbulent energy per unit mass. The characteristic length used is the vertical radius of the cloud.

It was not previously noted that in entrainment of stationary ambient air with conservation of momentum, kinetic energy is lost, i.e., entrainment is an inelastic-collision process. The proof is as follows:

The rate of change of cloud momentum per unit mass, u, (i.e., velocity) with time, t, due to entrainment, is

$$\frac{du}{dt} = -u \frac{1}{m} \frac{dm}{dt} \qquad (2.3.1)$$

The corresponding change in kinetic energy per unit mass is

$$u \frac{du}{dt} = \frac{d(u^2/2)}{dt} = -2\left(\frac{u^2}{2}\right) \frac{1}{m} \frac{dm}{dt} \qquad (2.3.2)$$

On the other hand, if total kinetic energy is conserved, then for a small change in velocity and mass

$\frac{1}{2} mu^2 = \frac{1}{2} (m+dm) (u+du)^2$, and neglecting 2nd-order terms

$$\frac{d(u^2/2)}{dt} = -\frac{u^2}{2} \frac{1}{m} \frac{dm}{dt} \qquad (2.3.3)$$

Entrainment, then, results in a loss of total kinetic energy, at a rate, per unit mass, of $\frac{u^2}{2} \frac{1}{m} \frac{dm}{dt}$. The law of conservation of total energy and the concept of turbulent energy already used suggest the following assumption:

7. The kinetic energy of rise lost in the inelastic-collision, momentum-conserving entrainment process, remains in the cloud as turbulent energy.

A simplified version of the essential equations is given below, neglecting the effects of the 2-gas mixture, latent heat release, initial virtual mass and condensed-water mass fraction of the cloud. These effects are considered in the full set of equations used in computation. (See TR-741 and Appendix A)

<u>Mass:</u> $$\frac{1}{m} \frac{dm}{dt} = \frac{S}{V} \lambda v \qquad (2.3.4)$$

where m = mass

t = time

S = cloud surface area

V = cloud volume

λ = dimensionless entrainment constant

v = characteristic velocity of entrainment

<u>Momentum:</u> $$\frac{du}{dt} = \left(\frac{T}{T_e} - 1\right) g - \left(\frac{2k_2 v}{\ell} \frac{T}{T_e} + \frac{1}{m}\frac{dm}{dt}\right) u \qquad (2.3.5)$$

where u = rate of rise

T = cloud temperature

T_e = environment temperature

g = acceleration of gravity

k_2 = dimensionless drag or eddy viscosity constant

ℓ = a characteristic length of the cloud

<u>Temperature:</u> $$\frac{dT}{dt} = -\frac{g}{c_p}\frac{T}{T_e} u - (T - T_e)\frac{1}{m}\frac{dm}{dt} + \frac{\varepsilon}{c_p} \qquad (2.3.6)$$

where c_p = specific heat of air

<u>Turbulent kinetic energy density:</u>

$$\frac{dE}{dt} = 2k_2 \frac{T}{T_e}\frac{v}{\ell} u^2 + \frac{u^2}{2}\frac{1}{m}\frac{dm}{dt} - E\frac{1}{m}\frac{dm}{dt} - \varepsilon \qquad (2.3.7)$$

where E = turbulent energy per unit mass

The term $\frac{u^2}{2}\frac{1}{m}\frac{dm}{dt}$ compensates for the loss of kinetic energy of rise in entrainment (assumption 7). Thus, some turbulent energy is produced even in the absence of eddy viscosity, i.e. even if $k_2 = 0$.

Environment temperature, T_e, is a specified function of height, z, which is given by

$$u = \frac{dz}{dt} \qquad (2.3.8)$$

Dissipation rate is given by

$$\varepsilon = k_3 \, (2E)^{3/2} / \ell \qquad (2.3.9)$$

where k_3 is a dimensionless constant.

Cloud form is a sphere (initially tangent to sea level, say, in the case of a surface burst) until the top (not the center, as in TR-741) reaches the tropopause, and is a horizontally expanding spheroid thereafter. The vertical radius of the spheroid is fixed as the sphere's radius at the tropopause. The cloud volume and surface area can then be calculated from the perfect gas law using the assumption of pressure equilibrium. The model actually uses a differential equation for volume, so that the gas law is available as a cross-check.

The characteristic length, ℓ, is taken as the (vertical) radius of the cloud. The characteristic velocity, v, is taken as max $(|u|, \sqrt{2E})$.

The equations have been programmed for machine computation, and the computations give predictions for rate of rise, cloud size, final cloud height and the late horizontal expansion of high-yield clouds, in general agreement with observations of nuclear clouds. Late horizontal expansion had not previously been predicted by a cloud model. Detailed revisions to the TR-741 cloud model are given in Appendix A. Appendix F gives the corresponding revised computer program. Tables 2.1 and 2.2 are sample computer output for the same input parameters as Tables 3.1 and 3.2 of TR-741 respectively, except for the dimensionless parameters λ, k_2, and k_3. A modified tropical atmosphere[8] was again used. The two tables are for a high yield (5 MT) and low yield (20 KT) explosion, respectively. Appendix E gives a glossary of printout symbols. All quantities are in mks units except explosion energy, W, which is in kilotons. For values of physical constants and initial conditions used in numerical solution of the cloud equations, see Appendix C of TR-741. For a discussion of appropriate values of the dimensionless parameters λ, k_2 and k_3, see Appendix D.

TABLE 2.1

COMPUTER OUTPUT FOR A HIGH-YIELD (5 MT) BURST

DST= 0.06250 K2= 0.1000000 LAMBDA= 0.200000 C1=0. C2= 0. W= 0.500E+04 F= 0.33000000 PHI= 0.5000

TE0=300.000000 CHANGE= 30.000 DST2= 5.0000 B0= 1091.00 B1= 0.1328 B2= 0. DST1= 1.0 K=2

A1= 0.00850 A2=-0.00440 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000. Z3= 52000. P0= 101300.

ZT= 16500. D0= 1910.0 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0.[illegible]5000 PRINT= 900.0 RK3= 1.0

RLH = 78747066 62585450 4642 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
0.	0.	0.3329	3000.0	1535.	1500.	0.	1.51E+10	0.	300.0	1.30E+0	920666443.	85159.	29728.	0.000	1.64E-02
1.0	15.7	0.3316	2994.9	1536.	1508.	2.	1.52E+10	-0.	290.2	1.30E+0	920766486.	85080.	29625.	5.935	6.98E-04
2.0	31.2	0.3277	2979.8	1539.	1531.	26.	1.55E+10	-0.	290.0	1.32E+0	921068739.	84846.	29318.	46.605	4.35E-02
3.0	46.6	0.3215	2954.6	1543.	1570.	130.	1.54E+10	-0.	289.8	1.34E+0	921579419.	84458.	28818.	152.286	4.75E-01
4.0	61.5	0.3131	2919.9	1550.	1624.	395.	1.56E+10	-0.	289.4	1.36E+0	922308804.	83921.	28138.	345.165	2.51E+00
5.0	76.0	0.3029	2875.8	1559.	1693.	919.	1.59E+10	-0.	289.0	1.40E+0	923270934.	83242.	27297.	636.788	8.85E+00
6.0	89.9	0.2911	2822.8	1569.	1776.	1795.	1.62E+10	-0.	288.5	1.45E+0	924483226.	82429.	26317.	1027.097	2.40E+01
7.0	103.2	0.2782	2761.7	1580.	1873.	3100.	1.65E+10	-0.	287.8	1.50E+0	925965703.	81490.	25222.	1505.035	5.41E+01
8.0	115.8	0.2646	2693.1	1594.	1982.	4880.	1.70E+10	-0.	287.1	1.56E+0	927739890.	80437.	24039.	2050.496	1.06E+02
9.0	127.6	0.2504	2617.8	1608.	2104.	7144.	1.74E+10	-0.	286.3	1.64E+0	929827141.	79279.	22789.	2638.192	1.86E+02
10.0	138.5	0.2360	2536.7	1624.	2237.	9868.	1.79E+10	-0.	285.5	1.72E+0	932251337.	78028.	21498.	3278.922	2.99E+02
11.0	148.4	0.2211	2446.9	1641.	2381.	13137.	1.85E+10	-0.	284.5	1.82E+0	935152427.	76698.	20145.	4133.430	4.54E+02
12.0	156.8	0.2055	2347.3	1659.	2534.	16927.	1.91E+10	-0.	283.5	1.94E+0	938649505.	75303.	18731.	4958.195	6.57E+02
13.0	163.7	0.1897	2239.3	1679.	2694.	21050.	1.98E+10	-0.	282.5	2.08E+0	942775257.	73860.	17288.	5669.712	9.00E+02
14.0	169.1	0.1740	2124.9	1701.	2860.	25259.	2.06E+10	-0.	281.4	2.24E+0	947505032.	72385.	15849.	6200.063	1.17E+03
15.0	173.2	0.1588	2006.8	1723.	3032.	29293.	2.14E+10	-0.	280.3	2.43E+0	952731407.	70894.	14445.	6512.907	1.44E+03
16.0	175.9	0.1444	1887.5	1747.	3206.	32925.	2.23E+10	-0.	279.2	2.65E+0	958244556.	69398.	13100.	6602.019	1.69E+03
17.0	177.5	0.1310	1769.6	1771.	3383.	35989.	2.33E+10	-0.	278.0	2.90E+0	963727306.	67910.	11836.	6493.357	1.91E+03
18.0	178.3	0.1187	1655.3	1795.	3561.	38396.	2.42E+10	-0.	276.9	3.18E+0	968774876.	66439.	10668.	6229.168	2.07E+03
19.0	178.2	0.1076	1546.3	1820.	3739.	40130.	2.53E+10	-0.	275.7	3.49E+0	972941399.	64991.	9599.	5857.297	2.19E+03
20.0	177.5	0.0975	1443.7	1846.	3917.	41226.	2.64E+10	-0.	274.5	3.83E+0	975806233.	63571.	8632.	5422.436	2.24E+03
21.0	176.4	0.0885	1348.2	1872.	4094.	41758.	2.75E+10	-0.	273.4	4.20E+0	977045467.	62184.	7762.	4962.054	2.26E+03
22.0	174.9	0.0805	1260.1	1899.	4270.	41814.	2.87E+10	-0.	272.2	4.61E+0	976408611.	60831.	6984.	4501.530	2.23E+03
23.0	173.2	0.0734	1179.3	1926.	4444.	41487.	2.99E+10	-0.	271.1	5.05E+0	974146036.	59515.	6292.	4059.913	2.17E+03
24.0	171.4	0.0670	1105.7	1954.	4616.	40865.	3.13E+10	-0.	270.0	5.52E+0	970200536.	58234.	5677.	3648.044	2.09E+03
25.0	169.4	0.0614	1038.7	1982.	4787.	40024.	3.26E+10	-0.	268.9	6.01E+0	964968588.	56990.	5132.	3271.330	2.00E+03
26.0	167.4	0.0564	977.9	2011.	4955.	39030.	3.41E+10	-0.	267.8	6.55E+0	958843406.	55782.	4649.	2931.428	1.90E+03
27.0	165.3	0.0520	922.9	2040.	5122.	37934.	3.56E+10	-0.	266.7	7.11E+0	952235340.	54609.	4220.	2627.905	1.79E+03
28.0	163.3	0.0480	873.0	2070.	5286.	36780.	3.71E+10	-0.	265.6	7.70E+0	945521942.	53470.	3839.	2357.992	1.69E+03
29.0	161.3	0.0445	827.7	2100.	5448.	35598.	3.88E+10	-0.	264.6	8.32E+0	939014398.	52364.	3500.	2119.176	1.58E+03
30.0	159.3	0.0413	786.7	2130.	5608.	34413.	4.05E+10	-0.	263.5	8.96E+0	932941749.	51289.	3199.	1908.343	1.48E+03
35.0	150.0	0.0295	630.8	2289.	6381.	28898.	5.02E+10	-0.	258.5	1.26E+1	011984554.	46367.	2106.	1184.826	1.06E+03
40.0	141.9	0.0222	530.3	2456.	7111.	24455.	6.20E+10	-0.	253.8	1.69E+10	3755431.	42066.	1454.	772.278	7.71E+02
45.0	135.1	0.0174	462.2	2629.	7803.	21030.	7.62E+10	-0.	249.3	2.17E+10	1140176.	38292.	1044.	534.735	5.74E+02
50.0	129.1	0.0140	413.7	2808.	8463.	18392.	9.28E+10	-0.	245.0	2.70E+10	357006.	34953.	773.	388.709	4.40E+02
55.0	124.0	0.0116	377.8	2991.	9095.	16331.	1.12E+11	-0.	240.9	3.28E+10	118496.	31979.	588.	293.787	3.45E+02
60.0	119.4	0.0098	350.1	3177.	9704.	14693.	1.34E+11	-0.	236.9	3.89E+10	42023.	29316.	457.	229.131	2.77E+02
65.0	115.4	0.0084	328.2	3366.	10291.	13367.	1.60E+11	-0.	233.1	4.53E+10	15910.	26920.	361.	183.322	2.27E+02
70.0	111.8	0.0073	310.3	3558.	10859.	12276.	1.89E+11	-0.	229.4	5.21E+10	6389.	24754.	2[illegible]9.	149.790	1.89E+02
75.0	108.6	0.0065	295.3	3751.	11409.	11364.	2.21E+11	-0.	225.8	5.91E+10	2702.	22791.	235.	124.500	1.60E+02
80.0	105.6	0.0058	282.6	3947.	11945.	10591.	2.58E+11	-0.	222.4	6.63E+10	1194.	21005.	193.	104.971	1.37E+02
85.0	102.9	0.0052	271.5	4144.	12466.	9928.	2.98E+11	-0.	219.0	7.38E+10	548.	19378.	160.	89.573	1.18E+02
SWITCH TO ELLIPSE, R=RH															
90.0	100.3	0.0047	261.8	4448.	12974.	9349.	3.43E+11	-0.	215.7	8.14E+10	260.	17892.	134.	80.780	1.08E+02
95.0	97.7	0.0043	253.1	4761.	13469.	8834.	3.94E+11	-0.	212.5	8.91E+10	127.	16534.	113.	73.106	9.92E+01
100.0	95.1	0.0039	245.4	5085.	13950.	8369.	4.49E+11	-0.	209.3	9.71E+10	63.	15293.	96.	66.352	9.14E+01
95.5	97.4	0.0042	252.3	4793.	13517.	8785.	3.99E+11	-0.	212.1	8.99E+10	118.	16405.	111.	72.363	9.83E+01
96.0	97.1	0.0042	251.5	4825.	13566.	8737.	4.04E+11	-0.	211.8	9.07E+10	110.	16277.	109.	71.658	9.75E+01
96.5	96.9	0.0042	250.7	4857.	13614.	8690.	4.10E+11	-0.	211.5	9.15E+10	103.	16150.	108.	70.963	9.67E+01
SWITCH TO WET															
101.5	94.6	0.0028	245.8	5211.	14093.	8241.	4.71E+11	0.0011	208.4	9.95E+10	66.	14940.	66.	65.393	8.94E+01

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
106.5	92.6	0.0017	240.6	5570.	14561.	7841.	5.39E+11	0.0018	205.4	1.08E+11	41.	13830.	39.	60.741	8.29E+01
111.5	91.0	0.0011	235.3	5937.	15020.	7481.	6.12E+11	0.0022	202.4	1.16E+11	24.	12807.	22.	56.689	7.73E+01
116.5	89.4	0.0006	229.8	6311.	15470.	7155.	6.91E+11	0.0025	199.4	1.24E+11	14.	11862.	12.	53.029	7.23E+01
121.5	87.8	0.0003	224.4	6695.	15913.	6859.	7.78E+11	0.0025	196.6	1.33E+11	7.	10990.	6.	49.633	6.78E+01
126.5	86.2	0.0002	219.0	7090.	16348.	6585.	8.73E+11	0.0025	193.7	1.41E+11	4.	10185.	3.	46.429	6.38E+01
131.5	84.3	0.0001	213.9	7498.	16775.	6330.	9.76E+11	0.0024	194.0	1.50E+11	2.	9446.	2.	42.553	6.01E+01
136.5	81.4	0.0001	209.4	7916.	17190.	6080.	1.09E+12	0.0023	195.8	1.59E+11	1.	8784.	1.	37.614	5.66E+01
141.5	77.5	0.0000	205.4	8342.	17587.	5827.	1.21E+12	0.0022	197.5	1.68E+11	1.	8198.	0.	32.506	5.31E+01
146.5	73.0	0.0000	202.0	8773.	17964.	5569.	1.34E+12	0.0021	199.2	1.77E+11	0.	7683.	0.	27.461	4.96E+01
151.5	67.9	0.0000	199.1	9205.	18316.	5306.	1.47E+12	0.0020	200.7	1.86E+11	0.	7235.	0.	22.689	4.62E+01
156.5	62.4	0.0000	196.7	9636.	18642.	5038.	1.61E+12	0.0019	202.2	1.95E+11	0.	6846.	0.	18.329	4.27E+01
161.5	56.7	0.0000	194.7	10061.	18940.	4770.	1.76E+12	0.0019	203.5	2.04E+11	0.	6511.	0.	14.461	3.93E+01
166.5	50.3	0.0000	193.1	10477.	19208.	4503.	1.91E+12	0.0018	204.7	2.14E+11	0.	6225.	0.	11.119	3.61E+01
171.5	44.8	0.0000	191.8	10882.	19447.	4241.	2.06E+12	0.0017	205.7	2.23E+11	0.	5982.	0.	8.303	3.30E+01
176.5	38.8	0.0000	190.9	11273.	19656.	3986.	2.21E+12	0.0016	206.6	2.32E+11	0.	5779.	0.	5.990	3.01E+01
181.5	32.9	0.0000	190.2	11648.	19836.	3742.	2.36E+12	0.0016	207.4	2.42E+11	0.	5610.	0.	4.142	2.73E+01
186.5	27.1	0.0000	189.8	12003.	19986.	3509.	2.50E+12	0.0015	208.1	2.51E+11	0.	5474.	0.	2.710	2.48E+01
191.5	21.5	0.0000	189.6	12339.	20107.	3289.	2.64E+12	0.0015	208.6	2.60E+11	0.	5366.	0.	1.644	2.25E+01
196.5	16.1	0.0000	189.6	12653.	20201.	3083.	2.78E+12	0.0014	209.0	2.69E+11	0.	5284.	0.	0.890	2.04E+01
201.5	10.9	0.0000	189.8	12945.	20269.	2891.	2.91E+12	0.0014	209.3	2.79E+11	0.	5226.	0.	0.398	1.86E+01
206.5	6.0	0.0000	190.2	13214.	20311.	2715.	3.03E+12	0.0013	209.5	2.88E+11	0.	5190.	0.	0.118	1.69E+01
211.5	1.5	0.0000	190.7	13460.	20329.	2552.	3.15E+12	0.0013	209.6	2.97E+11	0.	5174.	0.	0.007	1.54E+01
216.5	-2.8	0.0000	191.4	13684.	20326.	2404.	3.25E+12	0.0012	209.6	3.06E+11	0.	5177.	0.	0.024	1.41E+01
221.5	-6.7	0.0000	192.1	13885.	20302.	2270.	3.35E+12	0.0012	209.5	3.15E+11	0.	5198.	0.	0.134	1.29E+01
226.5	-10.3	0.0000	193.0	14066.	20259.	2147.	3.43E+12	0.0012	209.3	3.24E+11	0.	5234.	0.	0.307	1.19E+01
231.5	-13.4	0.0000	194.0	14226.	20200.	2037.	3.51E+12	0.0011	209.0	3.33E+11	0.	5285.	0.	0.514	1.10E+01
236.5	-16.2	0.0000	195.0	14367.	20126.	1937.	3.58E+12	0.0011	208.7	3.42E+11	0.	5349.	0.	0.735	1.02E+01
241.5	-18.5	0.0000	196.1	14492.	20039.	1847.	3.65E+12	0.0011	208.3	3.51E+11	0.	5426.	0.	0.949	9.48E+00
246.5	-20.5	0.0000	197.2	14601.	19941.	1766.	3.70E+12	0.0010	207.9	3.60E+11	0.	5514.	0.	1.141	8.86E+00
251.5	-22.0	0.0000	198.4	14697.	19835.	1692.	3.75E+12	0.0010	207.4	3.69E+11	0.	5611.	0.	1.301	8.31E+00
256.5	-23.1	0.0000	199.6	14782.	19721.	1625.	3.79E+12	0.0010	206.9	3.78E+11	0.	5717.	0.	1.419	7.82E+00
261.5	-23.9	0.0000	200.7	14857.	19604.	1563.	3.83E+12	0.0010	206.4	3.87E+11	0.	5829.	0.	1.491	7.38E+00
266.5	-24.2	0.0000	201.9	14924.	19484.	1505.	3.87E+12	0.0009	205.9	3.96E+11	0.	5946.	0.	1.515	6.98E+00
271.5	-24.1	0.0000	203.0	14986.	19363.	1452.	3.90E+12	0.0009	205.3	4.05E+11	1.	6067.	0.	1.492	6.61E+00
276.5	-23.7	0.0000	204.1	15044.	19243.	1402.	3.93E+12	0.0009	204.8	4.14E+11	1.	6189.	0.	1.426	6.27E+00
281.5	-22.9	0.0000	205.1	15101.	19127.	1354.	3.96E+12	0.0009	204.3	4.23E+11	1.	6310.	0.	1.322	5.95E+00
286.5	-21.8	0.0001	206.1	15158.	19015.	1308.	3.99E+12	0.0008	203.8	4.33E+11	1.	6430.	1.	1.188	5.65E+00
291.5	-20.5	0.0001	207.0	15216.	18909.	1263.	4.02E+12	0.0008	203.3	4.42E+11	1.	6545.	1.	1.033	5.36E+00
296.	-18.8	0.0001	207.8	15278.	18811.	1220.	4.05E+12	0.0008	202.9	4.51E+11	1.	6654.	1.	0.867	5.09E+00
301.5	-17.0	0.0001	208.5	15344.	18721.	1178.	4.09E+12	0.0008	202.5	4.60E+11	1.	6755.	1.	0.698	4.83E+00
306.5	-15.0	0.0001	209.1	15416.	18641.	1137.	4.13E+12	0.0007	202.2	4.70E+11	1.	6847.	1.	0.536	4.58E+00
311.5	-12.9	0.0001	209.6	15495.	18571.	1097.	4.17E+12	0.0007	201.9	4.79E+11	1.	6928.	1.	0.388	4.34E+00
316.5	-10.6	0.0001	210.0	15582.	18512.	1058.	4.21E+12	0.0007	201.6	4.88E+11	1.	6998.	1.	0.260	4.11E+00
321.5	-8.3	0.0001	210.2	15677.	18465.	1020.	4.27E+12	0.0007	201.4	4.98E+11	1.	7054.	1.	0.157	3.89E+00
326.5	-6.0	0.0001	210.4	15782.	18429.	984.	4.32E+12	0.0007	201.2	5.07E+11	1.	7097.	1.	0.079	3.69E+00
331.5	-3.6	0.0001	210.5	15896.	18405.	949.	4.39E+12	0.0006	201.1	5.16E+11	1.	7126.	1.	0.029	3.49E+00
336.5	-1.3	0.0001	210.4	16020.	18393.	915.	4.46E+12	0.0006	201.1	5.26E+11	1.	7141.	1.	0.004	3.31E+00
341.5	0.9	0.0001	210.3	16154.	18392.	884.	4.53E+12	0.0006	201.1	5.35E+11	1.	7142.	1.	0.002	3.14E+00
346.5	3.0	0.0001	210.1	16298.	18402.	853.	4.61E+12	0.0006	201.1	5.44E+11	1.	7130.	1.	0.018	2.98E+00
351.5	4.9	0.0001	209.7	16451.	18421.	825.	4.70E+12	0.0006	201.2	5.53E+11	1.	7106.	1.	0.050	2.83E+00
356.5	6.8	0.0001	209.4	16614.	18451.	799.	4.79E+12	0.0006	201.3	5.63E+11	1.	7071.	1.	0.091	2.69E+00
361.5	8.4	0.0001	208.9	16785.	18489.	774.	4.89E+12	0.0006	201.5	5.72E+11	1.	7026.	1.	0.138	2.57E+00
366.5	9.8	0.0001	208.4	16964.	18534.	750.	5.00E+12	0.0006	201.7	5.81E+11	1.	6972.	1.	0.186	2.46E+00
371.5	11.1	0.0001	207.9	17151.	18587.	729.	5.11E+12	0.0006	201.9	5.90E+11	1.	6910.	1.	0.232	2.35E+00
376.5	12.1	0.0001	207.3	17344.	18645.	708.	5.22E+12	0.0006	202.2	5.99E+11	1.	6843.	1.	0.271	2.25E+00
381.5	12.8	0.0001	206.7	17542.	18707.	689.	5.34E+12	0.0006	202.5	6.09E+11	1.	6771.	1.	0.302	2.16E+00
386.5	13.4	0.0001	206.0	17745.	18773.	671.	5.47E+12	0.0006	202.7	6.18E+11	1.	6697.	1.	0.323	2.08E+00
391.5	13.7	0.0000	205.4	17952.	18841.	654.	5.59E+12	0.0006	203.0	6.27E+11	1.	6621.	0.	0.333	2.00E+00
396.5	13.8	0.0000	204.8	18160.	18910.	638.	5.73E+12	0.0006	203.4	6.36E+11	1.	6544.	0.	0.332	1.93E+00
401.5	13.7	0.0000	204.2	18370.	18979.	623.	5.86E+12	0.0005	203.7	6.45E+11	1.	6469.	0.	0.322	1.86E+00
406.5	13.4	0.0000	203.6	18579.	19046.	608.	5.99E+12	0.0005	204.0	6.55E+11	1.	6396.	0.	0.302	1.79E+00
411.5	12.9	0.0000	203.0	18787.	19112.	593.	6.13E+12	0.0005	204.2	6.64E+11	1.	6325.	0.	0.276	1.73E+00
416.5	12.2	0.0000	202.5	18992.	19175.	579.	6.26E+12	0.0005	204.5	6.73E+11	0.	6259.	0.	0.244	1.67E+00

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
421.5	11.0	0.0000	202.1	19153.	19235.	565.	6.40E+12	0.0005	204.8	6.82E+11	0.	6198.	0.	0.208	1.61E+00
426.5	10.4	0.0000	201.6	19390.	19289.	552.	6.53E+12	0.0005	205.0	6.91E+11	0.	6142.	0.	0.171	1.55E+00
431.5	9.3	0.0000	201.3	19581.	19339.	538.	6.66E+12	0.0005	205.2	7.00E+11	0.	6091.	0.	0.135	1.49E+00
436.5	8.1	0.0000	201.0	19765.	19382.	525.	6.78E+12	0.0005	205.4	7.10E+11	0.	6047.	0.	0.101	1.44E+00
441.5	6.8	0.0000	200.7	19941.	19420.	512.	6.90E+12	0.0005	205.6	7.19E+11	0.	6010.	0.	0.071	1.39E+00
446.5	5.5	0.0000	200.5	20110.	19451.	500.	7.02E+12	0.0005	205.7	7.28E+11	0.	5979.	0.	0.045	1.33E+00
451.5	4.1	0.0000	200.4	20269.	19475.	487.	7.13E+12	0.0005	205.8	7.37E+11	0.	5955.	0.	0.025	1.28E+00
456.5	2.7	0.0000	200.3	20420.	19492.	475.	7.24E+12	0.0005	205.9	7.46E+11	0.	5938.	0.	0.011	1.24E+00
461.5	1.3	0.0000	200.3	20561.	19502.	464.	7.34E+12	0.0005	206.0	7.55E+11	0.	5928.	0.	0.003	1.19E+00
466.5	-0.0	0.0000	200.3	20692.	19505.	452.	7.43E+12	0.0005	206.0	7.64E+11	0.	5925.	0.	0.000	1.15E+00
471.5	-1.4	0.0000	200.4	20814.	19502.	441.	7.52E+12	0.0005	206.0	7.73E+11	0.	5928.	0.	0.003	1.11E+00
476.5	-2.6	0.0000	200.6	20926.	19492.	431.	7.60E+12	0.0005	205.9	7.83E+11	0.	5938.	0.	0.009	1.07E+00
481.5	-3.8	0.0000	200.8	21030.	19476.	421.	7.68E+12	0.0005	205.8	7.92E+11	0.	5954.	0.	0.020	1.03E+00
486.5	-4.9	0.0000	201.0	21124.	19454.	411.	7.75E+12	0.0004	205.7	8.01E+11	0.	5976.	0.	0.032	9.96E-01
491.5	-5.9	0.0000	201.3	21211.	19427.	402.	7.81E+12	0.0004	205.6	8.10E+11	0.	6003.	0.	0.046	9.63E-01
496.5	-6.8	0.0000	201.6	21290.	19395.	393.	7.87E+12	0.0004	205.5	8.19E+11	0.	6034.	0.	0.061	9.32E-01
501.5	-7.5	0.0000	202.0	21362.	19359.	385.	7.92E+12	0.0004	205.3	8.28E+11	0.	6070.	0.	0.074	9.03E-01
506.5	-8.1	0.0000	202.4	21429.	19320.	377.	7.97E+12	0.0004	205.2	8.37E+11	0.	6110.	0.	0.086	8.75E-01
511.5	-8.6	0.0000	202.7	21490.	19278.	370.	8.02E+12	0.0004	205.0	8.46E+11	0.	6153.	0.	0.096	8.49E-01
516.5	-8.9	0.0000	203.1	21547.	19234.	363.	8.06E+12	0.0004	204.8	8.55E+11	1.	6198.	0.	0.103	8.25E-01
521.5	-9.1	0.0000	203.5	21601.	19189.	356.	8.10E+12	0.0004	204.6	8.64E+11	1.	6245.	0.	0.106	8.01E-01
526.5	-9.2	0.0000	203.9	21653.	19144.	349.	8.14E+12	0.0004	204.4	8.73E+11	1.	6293.	0.	0.107	7.79E-01
531.5	-9.1	0.0000	204.3	21704.	19098.	343.	8.18E+12	0.0004	204.2	8.82E+11	1.	6341.	0.	0.104	7.57E-01
536.5	-8.8	0.0000	204.7	21754.	19053.	336.	8.22E+12	0.0004	204.0	8.91E+11	1.	6388.	0.	0.098	7.37E-01
541.5	-8.4	0.0000	205.1	21806.	19010.	330.	8.25E+12	0.0004	203.8	9.01E+11	1.	6435.	0.	0.089	7.16E-01
546.5	-8.0	0.0000	205.4	21859.	18969.	324.	8.29E+12	0.0004	203.6	9.10E+11	1.	6479.	0.	0.078	6.97E-01
551.5	-7.4	0.0000	205.8	21914.	18931.	318.	8.34E+12	0.0004	203.4	9.19E+11	1.	6521.	1.	0.067	6.78E-01
556.5	-6.7	0.0001	206.0	21973.	18896.	312.	8.38E+12	0.0004	203.3	9.28E+11	1.	6559.	1.	0.054	6.59E-01
561.5	-5.9	0.0001	206.3	22035.	18864.	306.	8.43E+12	0.0004	203.2	9.37E+11	1.	6594.	1.	0.042	6.40E-01
566.5	-5.0	0.0001	206.5	22103.	18837.	301.	8.48E+12	0.0003	203.0	9.46E+11	1.	6624.	1.	0.030	6.22E-01
571.5	-4.1	0.0001	206.7	22175.	18814.	295.	8.54E+12	0.0003	202.9	9.55E+11	1.	6650.	1.	0.020	6.05E-01
576.5	-3.1	0.0001	206.8	22254.	18796.	289.	8.60E+12	0.0003	202.9	9.64E+11	1.	6670.	1.	0.012	5.88E-01
581.5	-2.1	0.0001	206.9	22338.	18783.	284.	8.66E+12	0.0003	202.8	9.73E+11	1.	6685.	1.	0.005	5.71E-01
586.5	-1.1	0.0001	206.9	22428.	18775.	278.	8.73E+12	0.0003	202.8	9.82E+11	1.	6694.	1.	0.002	5.55E-01
591.5	-0.2	0.0001	206.9	22525.	18772.	273.	8.81E+12	0.0003	202.7	9.91E+11	1.	6698.	1.	0.000	5.39E-01
596.5	0.8	0.0001	206.8	22628.	18773.	268.	8.89E+12	0.0003	202.8	1.00E+12	1.	6696.	1.	0.001	5.24E-01
601.5	1.8	0.0001	206.8	22738.	18780.	263.	8.98E+12	0.0003	202.8	1.01E+12	1.	6689.	1.	0.003	5.10E-01
606.5	2.6	0.0001	206.6	22854.	18791.	258.	9.07E+12	0.0003	202.8	1.02E+12	1.	6676.	1.	0.008	4.96E-01
611.5	3.5	0.0001	206.5	22975.	18806.	254.	9.16E+12	0.0003	202.9	1.03E+12	1.	6659.	1.	0.013	4.82E-01
616.5	4.2	0.0001	206.3	23102.	18825.	249.	9.27E+12	0.0003	203.0	1.04E+12	1.	6638.	1.	0.019	4.70E-01
621.5	4.9	0.0001	206.0	23235.	18848.	245.	9.37E+12	0.0003	203.1	1.05E+12	1.	6612.	1.	0.026	4.58E-01
626.5	5.5	0.0000	205.8	23371.	18874.	241.	9.48E+12	0.0003	203.2	1.05E+12	1.	6583.	1.	0.032	4.46E-01
631.5	5.9	0.0000	205.5	23512.	18903.	237.	9.60E+12	0.0003	203.3	1.06E+12	1.	6552.	1.	0.038	4.36E-01
636.5	6.3	0.0000	205.2	23656.	18933.	233.	9.72E+12	0.0003	203.5	1.07E+12	1.	6518.	0.	0.042	4.25E-01
641.5	6.6	0.0000	204.9	23803.	18966.	230.	9.84E+12	0.0003	203.6	1.08E+12	1.	6483.	0.	0.045	4.15E-01
646.5	6.8	0.0000	204.6	23952.	18999.	226.	9.96E+12	0.0003	203.7	1.09E+12	1.	6447.	0.	0.047	4.06E-01
651.5	6.8	0.0000	204.3	24101.	19033.	223.	1.01E+13	0.0003	203.9	1.10E+12	1.	6410.	0.	0.047	3.97E-01
656.5	6.8	0.0000	204.0	24251.	19067.	219.	1.02E+13	0.0003	204.0	1.11E+12	1.	6373.	0.	0.046	3.88E-01
661.5	6.6	0.0000	203.8	24401.	19101.	216.	1.03E+13	0.0003	204.2	1.12E+12	1.	6338.	0.	0.044	3.79E-01
666.5	6.4	0.0000	203.5	24549.	19133.	213.	1.05E+13	0.0003	204.3	1.13E+12	1.	6303.	0.	0.040	3.71E-01
671.5	6.0	0.0000	203.2	24695.	19164.	210.	1.06E+13	0.0003	204.5	1.14E+12	1.	6271.	0.	0.036	3.62E-01
676.5	5.6	0.0000	203.0	24839.	19193.	206.	1.07E+13	0.0003	204.6	1.14E+12	1.	6241.	0.	0.030	3.54E-01
681.5	5.1	0.0000	202.8	24979.	19220.	203.	1.08E+13	0.0003	204.7	1.15E+12	0.	6213.	0.	0.025	3.46E-01
686.5	4.5	0.0000	202.6	25115.	19244.	200.	1.10E+13	0.0003	204.8	1.16E+12	0.	6188.	0.	0.020	3.38E-01
691.5	3.9	0.0000	202.4	25246.	19265.	197.	1.11E+13	0.0003	204.9	1.17E+12	0.	6166.	0.	0.014	3.31E-01
696.5	3.2	0.0000	202.3	25372.	19283.	194.	1.12E+13	0.0003	205.0	1.18E+12	0.	6148.	0.	0.010	3.23E-01
701.5	2.5	0.0000	202.2	25492.	19297.	191.	1.13E+13	0.0003	205.1	1.19E+12	0.	6133.	0.	0.006	3.16E-01
706.5	1.8	0.0000	202.1	25607.	19308.	188.	1.14E+13	0.0003	205.1	1.20E+12	0.	6122.	0.	0.003	3.08E-01
711.5	1.0	0.0000	202.1	25715.	19315.	185.	1.15E+13	0.0003	205.1	1.21E+12	0.	6115.	0.	0.001	3.01E-01
716.5	0.3	0.0000	202.1	25817.	19318.	182.	1.16E+13	0.0003	205.2	1.22E+12	0.	6112.	0.	0.000	2.94E-01
721.5	-0.5	0.0000	202.1	25913.	19318.	180.	1.17E+13	0.0003	205.1	1.23E+12	0.	6112.	0.	0.000	2.88E-01
726.5	-1.2	0.0000	202.2	26002.	19314.	177.	1.17E+13	0.0003	205.1	1.23E+12	0.	6116.	0.	0.001	2.81E-01
731.5	-1.9	0.0000	202.2	26085.	19306.	174.	1.18E+13	0.0003	205.1	1.24E+12	0.	6124.	0.	0.003	2.75E-01

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
736.5	-2.5	0.0000	202.4	26163.	19295.	172.	1.19E+13	0.0003	205.0	1.25E+12	0.	6135.	0.	0.006	2.69E-01
741.5	-3.1	0.0000	202.5	26234.	19281.	169.	1.19E+13	0.0003	205.0	1.26E+12	0.	6150.	0.	0.008	2.63E-01
746.5	-3.6	0.0000	202.7	26300.	19264.	167.	1.20E+13	0.0003	204.9	1.27E+12	0.	6167.	0.	0.012	2.58E-01
751.5	-4.1	0.0000	202.8	26362.	19245.	165.	1.21E+13	0.0003	204.8	1.28E+12	0.	6187.	0.	0.015	2.52E-01
756.5	-4.5	0.0000	203.0	26419.	19223.	162.	1.21E+13	0.0003	204.7	1.29E+12	1.	6209.	0.	0.018	2.47E-01
761.5	-4.8	0.0000	203.3	26473.	19200.	160.	1.22E+13	0.0003	204.6	1.30E+12	1.	6234.	0.	0.020	2.42E-01
766.5	-5.1	0.0000	203.5	26523.	19175.	158.	1.22E+13	0.0003	204.5	1.31E+12	1.	6259.	0.	0.022	2.37E-01
771.5	-5.2	0.0000	203.7	26571.	19149.	156.	1.23E+13	0.0003	204.4	1.32E+12	1.	6287.	0.	0.023	2.33E-01
776.5	-5.3	0.0000	203.9	26618.	19123.	154.	1.23E+13	0.0002	204.3	1.32E+12	1.	6314.	0.	0.024	2.28E-01
781.5	-5.3	0.0000	204.2	26663.	19096.	152.	1.23E+13	0.0002	204.2	1.33E+12	1.	6343.	0.	0.024	2.24E-01
786.5	-5.2	0.0000	204.4	26709.	19070.	150.	1.24E+13	0.0002	204.1	1.34E+12	1.	6371.	0.	0.023	2.20E-01
791.5	-5.1	0.0000	204.6	26754.	19044.	148.	1.24E+13	0.0002	203.9	1.35E+12	1.	6398.	0.	0.021	2.16E-01
796.5	-4.8	0.0000	204.8	26801.	19019.	146.	1.25E+13	0.0002	203.8	1.36E+12	1.	6425.	0.	0.019	2.12E-01
801.5	-4.5	0.0000	205.0	26849.	18996.	145.	1.25E+13	0.0002	203.7	1.37E+12	1.	6450.	0.	0.017	2.08E-01
806.5	-4.1	0.0000	205.2	26900.	18974.	143.	1.26E+13	0.0002	203.6	1.38E+12	1.	6473.	0.	0.014	2.04E-01
811.5	-3.7	0.0000	205.4	26953.	18955.	141.	1.26E+13	0.0002	203.6	1.39E+12	1.	6494.	0.	0.011	2.00E-01
816.5	-3.2	0.0000	205.5	27010.	18938.	139.	1.27E+13	0.0002	203.5	1.40E+12	1.	6513.	1.	0.008	1.96E-01
821.5	-2.7	0.0000	205.6	27070.	18923.	137.	1.27E+13	0.0002	203.4	1.40E+12	1.	6529.	1.	0.006	1.92E-01
826.5	-2.1	0.0000	205.7	27135.	18911.	136.	1.28E+13	0.0002	203.4	1.41E+12	1.	6542.	1.	0.004	1.89E-01
831.5	-1.5	0.0000	205.8	27204.	18902.	134.	1.28E+13	0.0002	203.3	1.42E+12	1.	6553.	1.	0.002	1.85E-01
836.5	-0.9	0.0000	205.8	27278.	18896.	132.	1.29E+13	0.0002	203.3	1.43E+12	1.	6559.	1.	0.001	1.81E-01
841.5	-0.3	0.0000	205.8	27356.	18893.	130.	1.30E+13	0.0002	203.3	1.44E+12	1.	6563.	1.	0.000	1.78E-01
846.5	0.3	0.0000	205.8	27440.	18893.	129.	1.31E+13	0.0002	203.3	1.45E+12	1.	6563.	1.	0.000	1.74E-01
851.5	0.9	0.0000	205.8	27528.	18895.	127.	1.32E+13	0.0002	203.3	1.46E+12	1.	6560.	1.	0.001	1.71E-01
856.5	1.4	0.0000	205.7	27621.	18901.	125.	1.32E+13	0.0002	203.3	1.47E+12	1.	6553.	1.	0.002	1.68E-01
861.5	2.0	0.0000	205.6	27718.	18910.	124.	1.33E+13	0.0002	203.4	1.48E+12	1.	6544.	1.	0.003	1.65E-01
866.5	2.5	0.0000	205.5	27820.	18921.	122.	1.34E+13	0.0002	203.4	1.48E+12	1.	6532.	1.	0.005	1.62E-01
871.5	2.9	0.0000	205.4	27926.	18934.	121.	1.35E+13	0.0002	203.5	1.49E+12	1.	6517.	0.	0.006	1.59E-01
876.5	3.3	0.0000	205.2	28036.	18950.	119.	1.36E+13	0.0002	203.5	1.50E+12	1.	6500.	0.	0.008	1.56E-01
881.5	3.6	0.0000	205.1	28148.	18967.	118.	1.38E+13	0.0002	203.6	1.51E+12	1.	6481.	0.	0.010	1.53E-01
886.5	3.9	0.0000	204.9	28264.	18986.	117.	1.39E+13	0.0002	203.7	1.52E+12	1.	6461.	0.	0.011	1.51E-01
891.5	4.1	0.0000	204.7	28382.	19006.	115.	1.40E+13	0.0002	203.8	1.53E+12	1.	6439.	0.	0.013	1.48E-01
896.5	4.3	0.0000	204.5	28501.	19027.	114.	1.41E+13	0.0002	203.9	1.54E+12	1.	6417.	0.	0.013	1.46E-01
901.5	4.3	0.0000	204.3	28621.	19048.	113.	1.42E+13	0.0002	204.0	1.55E+12	1.	6393.	0.	0.014	1.43E-01

TABLE 2.2

COMPUTER OUTPUT FOR A LOW-YIELD (20 KT) BURST

DST= 0.06250 K2= 0.1000000 LAMBDA= 0.200000 C1=0. C2= 0. W= 0.200E+02 F= 0.33000000 PHI= 0.5000

TE0=300.000000 CHANGE= 30.000 DST2= 5.0000 B0= 1091.00 B1= 0.1328 B2= 0. DST1= 1.0 K=2

A1= 0.00650 A2=-0.00440 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000. Z3= 52000. P0= 101300.

ZT= 16500. D0= 1910.0 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0.175000 PRINT= 900.0 RK3= 1.0

RLH = 78747066 62585450 4642 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
0.	0.	0.3329	3000.0	230.	0.	0.	5.09E+07	0.	300.0	5.20E+06	20666443.	101300.	35362.	0.	0.
1.0	15.0	0.3249	2969.8	231.	8.	9.	5.13E+07	-0.	300.0	5.30E+06	21269015.	101213.	34776.	32.731	6.11E-02
2.0	29.2	0.3029	2881.1	232.	30.	135.	5.25E+07	-0.	299.8	5.61E+06	23152142.	100959.	33108.	234.025	3.34E+00
3.0	42.1	0.2715	2739.4	235.	65.	577.	5.44E+07	-0.	299.6	6.15E+06	26531000.	100549.	30597.	651.738	2.92E+01
4.0	53.1	0.2362	2554.3	239.	113.	1446.	5.69E+07	-0.	299.3	6.95E+06	31709914.	100004.	27563.	1191.960	1.14E+02
5.0	61.4	0.1979	2316.9	243.	171.	2765.	6.01E+07	-0.	298.9	8.15E+06	39777197.	99350.	24022.	1937.416	2.96E+02
6.0	65.9	0.1596	2030.9	248.	235.	4234.	6.41E+07	-0.	298.5	10.00E+06	51640887.	98627.	20174.	2350.551	5.49E+02
7.0	67.4	0.1263	1734.6	254.	302.	5314.	6.87E+07	-0.	298.0	1.26E+07	65317057.	97875.	16549.	2275.832	7.55E+02
8.0	66.6	0.1001	1464.3	260.	369.	5774.	7.39E+07	-0.	297.6	1.62E+07	75339596.	97123.	13486.	1897.372	8.34E+02
9.0	64.7	0.0805	1238.1	267.	434.	5727.	7.97E+07	-0.	297.2	2.07E+07	76040003.	96392.	11065.	1458.026	8.04E+02
10.0	62.3	0.0661	1058.1	274.	498.	5389.	8.61E+07	-0.	296.8	2.61E+07	66645561.	95689.	9214.	1084.236	7.15E+02
11.0	59.8	0.0556	917.9	281.	559.	4934.	9.32E+07	-0.	296.4	3.25E+07	51595798.	95018.	7821.	802.729	6.10E+02
12.0	57.4	0.0478	809.4	289.	618.	4460.	1.01E+08	-0.	296.0	3.99E+07	36308084.	94379.	6745.	600.191	5.10E+02
13.0	55.0	0.0419	725.1	297.	674.	4016.	1.10E+08	-0.	295.6	4.82E+07	23942442.	93768.	5925.	456.027	4.24E+02
14.0	52.9	0.0373	658.9	305.	728.	3619.	1.19E+08	-0.	295.3	5.73E+07	15200098.	93186.	5286.	352.838	3.53E+02
15.0	50.9	0.0338	606.4	314.	780.	3272.	1.29E+08	-0.	294.9	6.73E+07	9493096.	92628.	4780.	278.011	2.95E+02
16.0	49.1	0.0309	564.3	322.	830.	2971.	1.40E+08	-0.	294.6	7.82E+07	5926415.	92093.	4373.	222.871	2.49E+02
17.0	47.4	0.0287	530.0	331.	878.	2711.	1.52E+08	-0.	294.3	8.98E+07	3739527.	91580.	4042.	181.543	2.11E+02
18.0	45.8	0.0268	501.8	340.	924.	2486.	1.64E+08	-0.	294.0	1.02E+08	2402195.	91086.	3767.	150.045	1.81E+02
19.0	44.4	0.0252	478.4	349.	969.	2290.	1.77E+08	-0.	293.7	1.15E+08	1577690.	90610.	3537.	125.653	1.56E+02
20.0	43.1	0.0239	458.8	357.	1013.	2120.	1.91E+08	-0.	293.4	1.29E+08	1061688.	90150.	3340.	106.521	1.35E+02
21.0	41.9	0.0227	442.1	366.	1056.	1971.	2.06E+08	-0.	293.1	1.43E+08	732546.	89705.	3164.	91.249	1.18E+02
22.0	40.7	0.0217	427.8	375.	1097.	1840.	2.21E+08	-0.	292.9	1.58E+08	518114.	89274.	3011.	78.914	1.04E+02
23.0	39.7	0.0208	415.5	384.	1137.	1724.	2.37E+08	-0.	292.6	1.74E+08	375313.	88857.	2878.	68.836	9.23E+01
24.0	38.7	0.0200	404.9	393.	1176.	1621.	2.53E+08	-0.	292.4	1.90E+08	278105.	88451.	2761.	60.514	8.23E+01
25.0	37.8	0.0193	395.5	401.	1215.	1528.	2.71E+08	-0.	292.1	2.07E+08	210503.	88056.	2657.	53.573	7.37E+01
26.0	36.9	0.0187	387.3	410.	1252.	1445.	2.88E+08	-0.	291.9	2.25E+08	162514.	87672.	2565.	47.733	6.64E+01
27.0	36.1	0.0182	380.0	418.	1289.	1371.	3.07E+08	-0.	291.6	2.43E+08	127778.	87298.	2481.	42.777	6.00E+01
28.0	35.4	0.0177	373.5	427.	1324.	1303.	3.26E+08	-0.	291.4	2.61E+08	102172.	86933.	2406.	38.539	5.45E+01
29.0	34.7	0.0172	367.7	435.	1359.	1241.	3.45E+08	-0.	291.2	2.80E+08	82968.	86577.	2337.	34.890	4.97E+01
30.0	34.0	0.0168	362.5	444.	1394.	1185.	3.66E+08	-0.	290.9	3.00E+08	68335.	86229.	2274.	31.727	4.55E+01
35.0	31.1	0.0152	342.8	484.	1556.	965.	4.75E+08	-0.	289.9	4.04E+08	30829.	84596.	2026.	20.964	3.07E+01
40.0	28.9	0.0141	329.8	522.	1706.	813.	5.97E+08	-0.	288.9	5.19E+08	17205.	83115.	1848.	14.756	2.20E+01
45.0	27.0	0.0133	320.8	559.	1845.	702.	7.32E+08	-0.	288.0	6.44E+08	11050.	81753.	1712.	10.934	1.65E+01
50.0	25.5	0.0126	314.0	594.	1977.	618.	8.79E+08	-0.	287.2	7.78E+08	7809.	80491.	1602.	8.416	1.28E+01
55.0	24.2	0.0120	308.8	628.	2100.	552.	1.04E+09	-0.	286.3	9.20E+08	5902.	79312.	1503.	6.678	1.02E+01
60.0	23.0	0.0115	304.7	660.	2218.	499.	1.21E+09	-0.	285.6	1.07E+09	4682.	78204.	1418.	5.419	8.34E+00
65.0	22.0	0.0110	301.3	692.	2331.	455.	1.39E+09	-0.	284.8	1.23E+09	3851.	77158.	1346.	4.479	6.94E+00
70.0	21.1	0.0106	298.4	722.	2439.	418.	1.58E+09	-0.	284.1	1.39E+09	3256.	76166.	1282.	3.757	5.85E+00
75.0	20.3	0.0103	296.0	751.	2542.	386.	1.78E+09	-0.	283.5	1.56E+09	2812.	75223.	1225.	3.192	5.00E+00
80.0	19.6	0.0100	293.9	780.	2642.	359.	1.99E+09	-0.	282.8	1.74E+09	2470.	74325.	1174.	2.740	4.32E+00
85.0	18.9	0.0097	292.0	808.	2738.	336.	2.21E+09	-0.	282.2	1.92E+09	2200.	73467.	1128.	2.373	3.77E+00
90.0	18.3	0.0094	290.3	835.	2831.	315.	2.44E+09	-0.	281.6	2.11E+09	1981.	72645.	1087.	2.071	3.31E+00
95.0	17.7	0.0092	288.8	861.	2921.	296.	2.67E+09	-0.	281.0	2.30E+09	1800.	71857.	1048.	1.819	2.93E+00
100.0	17.1	0.0090	287.5	887.	3008.	280.	2.92E+09	-0.	280.5	2.50E+09	1649.	71101.	1012.	1.608	2.61E+00
105.0	16.6	0.0087	286.2	912.	3092.	265.	3.17E+09	-0.	279.9	2.70E+09	1520.	70374.	976.	1.430	2.34E+00
110.0	16.1	0.0085	285.1	936.	3174.	251.	3.44E+09	-0.	279.4	2.90E+09	1409.	69673.	943.	1.277	2.11E+00
115.0	15.7	0.0083	284.0	960.	3253.	239.	3.71E+09	-0.	278.9	3.12E+09	1313.	68998.	912.	1.144	1.91E+00
120.0	15.3	0.0081	283.0	984.	3331.	228.	3.99E+09	-0.	278.4	3.33E+09	1228.	68347.	883.	1.029	1.73E+00
125.0	14.8	0.0080	282.1	1007.	3406.	218.	4.27E+09	-0.	277.9	3.55E+09	1153.	67719.	857.	0.927	1.58E+00
130.0	14.4	0.0078	281.2	1029.	3479.	208.	4.57E+09	-0.	277.4	3.77E+09	1087.	67112.	832.	0.838	1.44E+00
135.0	14.0	0.0076	280.4	1052.	3550.	199.	4.87E+09	-0.	276.9	4.00E+09	1027.	66526.	808.	0.759	1.32E+00

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
140.0	13.7	0.0075	279.6	1073.	3620.	191.	5.18E+09	-0.	276.5	4.23E+09	973.	65960.	786.	0.689	1.22E+00
145.0	13.3	0.0073	278.8	1095.	3687.	183.	5.49E+09	-0.	276.0	4.46E+09	925.	65412.	765.	0.627	1.12E+00
150.0	13.0	0.0072	278.1	1116.	3753.	176.	5.82E+09	-0.	275.6	4.69E+09	880.	64883.	745.	0.571	1.04E+00
155.0	12.6	0.0071	277.5	1136.	3817.	169.	6.15E+09	-0.	275.2	4.93E+09	840.	64371.	727.	0.520	9.59E-01
160.0	12.3	0.0070	276.8	1157.	3879.	163.	6.48E+09	-0.	274.8	5.17E+09	803.	63875.	709.	0.475	8.90E-01
165.0	12.0	0.0069	276.2	1176.	3939.	157.	6.82E+09	-0.	274.4	5.42E+09	769.	63396.	692.	0.434	8.27E-01
170.0	11.6	0.0067	275.6	1196.	3998.	151.	7.17E+09	-0.	274.0	5.66E+09	738.	62932.	676.	0.396	7.70E-01
175.0	11.3	0.0066	275.1	1215.	4056.	146.	7.52E+09	-0.	273.6	5.91E+09	709.	62483.	660.	0.363	7.17E-01
180.0	11.0	0.0065	274.5	1234.	4112.	141.	7.88E+09	-0.	273.3	6.16E+09	682.	62049.	645.	0.333	6.70E-01
185.0	10.7	0.0064	274.0	1253.	4166.	136.	8.24E+09	-0.	272.9	6.41E+09	657.	61628.	630.	0.305	6.26E-01
190.0	10.4	0.0063	273.5	1271.	4219.	131.	8.61E+09	-0.	272.6	6.67E+09	634.	61221.	616.	0.279	5.86E-01
195.0	10.2	0.0062	273.0	1289.	4270.	127.	8.98E+09	-0.	272.2	6.92E+09	612.	60828.	603.	0.256	5.49E-01
200.0	9.9	0.0061	272.6	1307.	4320.	123.	9.36E+09	-0.	271.9	7.18E+09	592.	60447.	590.	0.235	5.15E-01
205.0	9.6	0.0060	272.1	1325.	4369.	119.	9.74E+09	-0.	271.6	7.44E+09	573.	60078.	578.	0.215	4.84E-01
200.5	9.8	0.0061	272.5	1309.	4325.	122.	9.39E+09	-0.	271.9	7.21E+09	590.	60409.	589.	0.233	5.12E-01
201.0	9.3	0.0061	272.5	1311.	4330.	122.	9.43E+09	-0.	271.9	7.23E+09	588.	60372.	588.	0.230	5.09E-01
201.5	9.8	0.0061	272.4	1312.	4335.	122.	9.47E+09	-0.	271.8	7.26E+09	586.	60335.	586.	0.228	5.06E-01
SWITCH TO WET															
206.5	9.5	0.0060	272.1	1330.	4383.	118.	9.85E+09	0.0000	271.5	7.52E+09	572.	59970.	570.	0.210	4.75E-01
211.5	9.3	0.0058	271.8	1347.	4430.	114.	1.02E+10	0.0001	271.2	7.78E+09	559.	59617.	555.	0.193	4.47E-01
216.5	9.0	0.0057	271.5	1364.	4476.	110.	1.06E+10	0.0001	270.9	8.04E+09	546.	59274.	541.	0.178	4.21E-01
221.5	8.8	0.0056	271.2	1381.	4521.	107.	1.10E+10	0.0002	270.6	8.30E+09	534.	58942.	527.	0.165	3.97E-01
226.5	8.6	0.0055	270.9	1398.	4564.	104.	1.14E+10	0.0002	270.3	8.57E+09	522.	58619.	514.	0.153	3.74E-01
231.5	8.4	0.0054	270.6	1414.	4607.	101.	1.18E+10	0.0002	270.1	8.83E+09	511.	58305.	502.	0.142	3.53E-01
236.5	8.2	0.0053	270.3	1430.	4648.	98.	1.22E+10	0.0002	269.8	9.10E+09	500.	58000.	490.	0.132	3.34E-01
241.5	8.0	0.0052	270.0	1446.	4689.	95.	1.27E+10	0.0003	269.5	9.37E+09	490.	57702.	479.	0.123	3.16E-01
246.5	7.9	0.0051	269.7	1461.	4729.	92.	1.31E+10	0.0003	269.3	9.64E+09	480.	57412.	468.	0.115	3.00E-01
251.5	7.7	0.0050	269.5	1477.	4768.	90.	1.35E+10	0.0003	269.0	9.91E+09	471.	57129.	458.	0.108	2.84E-01
256.5	7.6	0.0049	269.2	1492.	4806.	87.	1.39E+10	0.0004	268.8	1.02E+10	462.	56853.	448.	0.101	2.70E-01
261.5	7.4	0.0048	269.0	1507.	4843.	85.	1.43E+10	0.0004	268.5	1.04E+10	453.	56583.	438.	0.095	2.56E-01
266.5	7.3	0.0048	268.7	1522.	4880.	83.	1.48E+10	0.0004	268.3	1.07E+10	444.	56319.	429.	0.090	2.44E-01
271.5	7.1	0.0047	268.5	1536.	4916.	80.	1.52E+10	0.0004	268.0	1.10E+10	436.	56061.	420.	0.085	2.32E-01
276.5	7.0	0.0046	268.2	1551.	4951.	78.	1.56E+10	0.0004	267.8	1.13E+10	428.	55809.	412.	0.080	2.21E-01
281.5	6.9	0.0045	268.0	1565.	4986.	76.	1.61E+10	0.0005	267.6	1.15E+10	421.	55561.	404.	0.075	2.11E-01
286.5	6.8	0.0045	267.7	1579.	5020.	75.	1.65E+10	0.0005	267.4	1.18E+10	413.	55319.	396.	0.071	2.02E-01
291.5	6.7	0.0044	267.5	1593.	5054.	73.	1.69E+10	0.0005	267.1	1.21E+10	406.	55081.	388.	0.068	1.93E-01
296.5	6.6	0.0043	267.3	1607.	5087.	71.	1.74E+10	0.0005	266.9	1.24E+10	399.	54848.	380.	0.064	1.84E-01
301.5	6.5	0.0043	267.1	1621.	5120.	69.	1.78E+10	0.0005	266.7	1.26E+10	392.	54620.	373.	0.061	1.76E-01
306.5	6.4	0.0042	266.8	1634.	5152.	68.	1.83E+10	0.0005	266.5	1.29E+10	386.	54395.	366.	0.058	1.69E-01
311.5	6.3	0.0041	266.6	1647.	5184.	66.	1.87E+10	0.0006	266.3	1.32E+10	379.	54175.	359.	0.055	1.62E-01
316.5	6.2	0.0041	266.4	1660.	5215.	65.	1.92E+10	0.0006	266.1	1.35E+10	373.	53959.	353.	0.053	1.55E-01
321.5	6.1	0.0040	266.2	1674.	5246.	63.	1.96E+10	0.0006	265.9	1.37E+10	367.	53746.	346.	0.050	1.49E-01
326.5	6.0	0.0040	266.0	1686.	5276.	62.	2.01E+10	0.0006	265.7	1.40E+10	361.	53537.	340.	0.048	1.43E-01
331.5	5.9	0.0039	265.8	1699.	5306.	61.	2.06E+10	0.0006	265.5	1.43E+10	356.	53333.	334.	0.046	1.37E-01
336.5	5.8	0.0039	265.6	1712.	5335.	59.	2.10E+10	0.0006	265.3	1.46E+10	350.	53131.	328.	0.043	1.32E-01
341.5	5.8	0.0038	265.4	1724.	5364.	58.	2.15E+10	0.0006	265.1	1.48E+10	345.	52933.	323.	0.041	1.27E-01
346.5	5.7	0.0038	265.2	1737.	5393.	57.	2.19E+10	0.0006	264.9	1.51E+10	339.	52739.	317.	0.040	1.22E-01
351.5	5.6	0.0037	265.0	1749.	5421.	56.	2.24E+10	0.0007	264.8	1.54E+10	334.	52548.	312.	0.038	1.18E-01
356.5	5.5	0.0037	264.8	1761.	5449.	55.	2.29E+10	0.0007	264.6	1.57E+10	330.	52360.	307.	0.036	1.14E-01
361.5	5.4	0.0036	264.6	1773.	5476.	54.	2.34E+10	0.0007	264.4	1.60E+10	325.	52176.	[illegible]	0.034	1.09E-01
366.5	5.3	0.0036	264.4	1785.	5503.	53.	2.38E+10	0.0007	264.2	1.62E+10	320.	51994.	297.	0.033	1.06E-01
371.5	5.3	0.0035	264.2	1797.	5529.	52.	2.43E+10	0.0007	264.1	1.65E+10	316.	51816.	292.	0.031	1.02E-01
376.5	5.2	0.0035	264.1	1809.	5556.	51.	2.48E+10	0.0007	263.9	1.68E+10	311.	51641.	288.	0.030	9.84E-02
381.5	5.1	0.0034	263.9	1820.	5581.	50.	2.53E+10	0.0007	263.7	1.71E+10	307.	51469.	283.	0.029	9.50E-02
386.5	5.0	0.0034	263.7	1832.	5607.	49.	2.58E+10	0.0007	263.6	1.74E+10	303.	51300.	279.	0.027	9.18E-02
391.5	5.0	0.0034	263.5	1843.	5632.	48.	2.62E+10	0.0007	263.4	1.76E+10	299.	51134.	275.	0.026	8.88E-02
396.5	4.9	0.0033	263.4	1855.	5657.	47.	2.67E+10	0.0007	263.2	1.79E+10	295.	50970.	271.	0.025	8.58E-02
401.5	4.8	0.0033	263.2	1866.	5681.	46.	2.72E+10	0.0007	263.1	1.82E+10	291.	50810.	267.	0.024	8.30E-02
406.5	4.8	0.0032	263.0	1877.	5705.	45.	2.77E+10	0.0007	262.9	1.85E+10	287.	50652.	263.	0.023	8.04E-02
411.5	4.7	0.0032	262.9	1888.	5728.	44.	2.82E+10	0.0008	262.8	1.88E+10	283.	50497.	259.	0.022	7.78E-02
416.5	4.6	0.0032	262.7	1899.	5752.	44.	2.87E+10	0.0008	262.6	1.91E+10	280.	50345.	256.	0.021	7.54E-02
421.5	4.5	0.0031	262.6	1910.	5775.	43.	2.92E+10	0.0008	262.5	1.93E+10	276.	50195.	252.	0.020	7.30E-02

ST	U	X	T	R	Z	EK	V	WT	TE	M	ES	P	PW	ED	EPS
426.5	4.5	0.0031	262.4	1920.	5797.	42.	2.97E+10	0.0008	262.3	1.96E+10	273.	50048.	249.	0.019	7.08E-02
431.5	4.4	0.0031	262.3	1931.	5819.	42.	3.02E+10	0.0008	262.2	1.99E+10	270.	49904.	245.	0.018	6.86E-02
436.5	4.3	0.0030	262.1	1942.	5841.	41.	3.07E+10	0.0008	262.0	2.02E+10	267.	49762.	242.	0.017	6.65E-02
441.5	4.3	0.0030	262.0	1952.	5862.	40.	3.12E+10	0.0008	261.9	2.05E+10	263.	49623.	239.	0.017	6.45E-02
446.5	4.2	0.0030	261.8	1962.	5884.	39.	3.17E+10	0.0008	261.8	2.07E+10	260.	49487.	236.	0.016	6.26E-02
451.5	4.1	0.0029	261.7	1973.	5904.	39.	3.21E+10	0.0008	261.6	2.10E+10	257.	49353.	233.	0.015	6.08E-02
456.5	4.0	0.0029	261.5	1983.	5925.	38.	3.26E+10	0.0008	261.5	2.13E+10	255.	49222.	230.	0.014	5.90E-02
461.5	4.0	0.0029	261.4	1993.	5945.	38.	3.32E+10	0.0008	261.4	2.16E+10	252.	49093.	227.	0.014	5.73E-02
466.5	3.9	0.0029	261.3	2003.	5965.	37.	3.37E+10	0.0008	261.2	2.19E+10	249.	48967.	224.	0.013	5.57E-02
471.5	3.8	0.0028	261.1	2013.	5984.	36.	3.42E+10	0.0008	261.1	2.21E+10	246.	48843.	222.	0.012	5.41E-02
476.5	3.8	0.0028	261.0	2023.	6003.	36.	3.47E+10	0.0008	261.0	2.24E+10	244.	48721.	219.	0.012	5.26E-02
481.5	3.7	0.0028	260.9	2032.	6022.	35.	3.52E+10	0.0008	260.9	2.27E+10	241.	48602.	216.	0.011	5.11E-02
486.5	3.6	0.0028	260.7	2042.	6040.	35.	3.57E+10	0.0008	260.7	2.30E+10	239.	48486.	214.	0.011	4.97E-02
491.5	3.6	0.0027	260.6	2052.	6058.	34.	3.62E+10	0.0008	260.6	2.33E+10	236.	48372.	212.	0.010	4.83E-02
496.5	3.5	0.0027	260.5	2061.	6076.	34.	3.67E+10	0.0008	260.5	2.36E+10	234.	48260.	209.	0.010	4.70E-02
501.5	3.4	0.0027	260.4	2070.	6093.	33.	3.72E+10	0.0008	260.4	2.38E+10	232.	48150.	207.	0.009	4.57E-02
506.5	3.4	0.0027	260.2	2080.	6110.	33.	3.77E+10	0.0008	260.3	2.41E+10	229.	48043.	205.	0.009	4.45E-02
511.5	3.3	0.0026	260.1	2089.	6127.	32.	3.82E+10	0.0008	260.2	2.44E+10	227.	47938.	202.	0.008	4.33E-02
516.5	3.2	0.0026	260.0	2098.	6143.	32.	3.87E+10	0.0008	260.1	2.47E+10	225.	47836.	200.	0.008	4.22E-02
521.5	3.2	0.0026	259.9	2107.	6159.	31.	3.92E+10	0.0008	260.0	2.50E+10	223.	47736.	198.	0.007	4.10E-02
526.5	3.1	0.0026	259.8	2116.	6174.	31.	3.97E+10	0.0008	259.9	2.52E+10	221.	47638.	196.	0.007	4.00E-02
531.5	3.0	0.0025	259.7	2125.	6190.	30.	4.02E+10	0.0008	259.8	2.55E+10	219.	47543.	194.	0.007	3.89E-02
536.5	2.9	0.0025	259.6	2134.	6204.	30.	4.07E+10	0.0008	259.7	2.58E+10	217.	47450.	193.	0.006	3.79E-02
541.5	2.9	0.0025	259.5	2143.	6219.	29.	4.12E+10	0.0008	259.6	2.61E+10	215.	47360.	191.	0.006	3.70E-02
546.5	2.8	0.0025	259.4	2152.	6233.	29.	4.17E+10	0.0008	259.5	2.64E+10	213.	47272.	189.	0.006	3.60E-02
551.5	2.7	0.0025	259.3	2160.	6247.	29.	4.22E+10	0.0008	259.4	2.66E+10	212.	47186.	187.	0.005	3.51E-02
556.5	2.6	0.0025	259.2	2169.	6260.	28.	4.27E+10	0.0008	259.3	2.69E+10	210.	47103.	186.	0.005	3.42E-02
561.5	2.5	0.0024	259.1	2177.	6273.	28.	4.32E+10	0.0008	259.2	2.72E+10	209.	47023.	184.	0.005	3.34E-02
566.5	2.5	0.0024	259.0	2186.	6286.	27.	4.37E+10	0.0008	259.1	2.75E+10	207.	46945.	183.	0.004	3.25E-02
571.5	2.4	0.0024	258.9	2194.	6298.	27.	4.42E+10	0.0008	259.1	2.78E+10	205.	46869.	181.	0.004	3.17E-02
576.5	2.3	0.0024	258.8	2202.	6310.	27.	4.47E+10	0.0008	259.0	2.80E+10	204.	46795.	180.	0.004	3.10E-02
581.5	2.3	0.0024	258.7	2210.	6322.	26.	4.52E+10	0.0008	258.9	2.83E+10	203.	46725.	178.	0.003	3.02E-02
586.5	2.2	0.0024	258.6	2218.	6333.	26.	4.57E+10	0.0008	258.8	2.86E+10	201.	46656.	177.	0.003	2.95E-02
591.5	2.1	0.0023	258.6	2226.	6343.	26.	4.62E+10	0.0008	258.8	2.89E+10	200.	46590.	176.	0.003	2.88E-02
596.5	2.0	0.0023	258.5	2234.	6354.	25.	4.67E+10	0.0008	258.7	2.92E+10	199.	46527.	174.	0.003	2.81E-02
601.5	1.9	0.0023	258.4	2242.	6364.	25.	4.72E+10	0.0008	258.6	2.94E+10	198.	46466.	173.	0.002	2.74E-02
606.5	1.9	0.0023	258.3	2250.	6373.	25.	4.77E+10	0.0008	258.6	2.97E+10	196.	46408.	172.	0.002	2.67E-02
611.5	1.8	0.0023	258.3	2257.	6382.	24.	4.82E+10	0.0008	258.5	3.00E+10	195.	46352.	171.	0.002	2.61E-02
616.5	1.7	0.0023	258.2	2265.	6391.	24.	4.87E+10	0.0008	258.5	3.03E+10	194.	46298.	170.	0.002	2.55E-02
621.5	1.6	0.0023	258.1	2273.	6399.	24.	4.92E+10	0.0008	258.4	3.05E+10	193.	46247.	169.	0.002	2.49E-02
626.5	1.5	0.0023	258.1	2280.	6407.	23.	4.97E+10	0.0008	258.4	3.08E+10	192.	46199.	168.	0.001	2.43E-02
631.5	1.5	0.0023	258.0	2287.	6415.	23.	5.01E+10	0.0008	258.3	3.11E+10	191.	46153.	167.	0.001	2.38E-02
636.5	1.4	0.0022	258.0	2295.	6422.	23.	5.06E+10	0.0008	258.3	3.14E+10	191.	46110.	166.	0.001	2.32E-02
641.5	1.3	0.0022	257.9	2302.	6428.	22.	5.11E+10	0.0008	258.2	3.17E+10	190.	46069.	166.	0.001	2.27E-02
646.5	1.2	0.0022	257.9	2309.	6435.	22.	5.16E+10	0.0008	258.2	3.19E+10	189.	46031.	165.	0.001	2.22E-02
651.5	1.1	0.0022	257.8	2316.	6441.	22.	5.20E+10	0.0008	258.1	3.22E+10	188.	45995.	164.	0.001	2.17E-02
656.5	1.0	0.0022	257.8	2323.	6446.	21.	5.25E+10	0.0007	258.1	3.25E+10	188.	45962.	164.	0.001	2.12E-02
661.5	1.0	0.0022	257.7	2330.	6451.	21.	5.30E+10	0.0007	258.1	3.28E+10	187.	45932.	163.	0.001	2.07E-02
666.5	0.9	0.0022	257.7	2337.	6456.	21.	5.35E+10	0.0007	258.0	3.30E+10	186.	45904.	162.	0.000	2.03E-02
671.5	0.8	0.0022	257.7	2344.	6460.	21.	5.39E+10	0.0007	258.0	3.33E+10	186.	45878.	162.	0.000	1.98E-02
676.5	0.7	0.0022	257.6	2350.	6464.	20.	5.44E+10	0.0007	258.0	3.36E+10	185.	45855.	161.	0.000	1.94E-02
681.5	0.6	0.0022	257.6	2357.	6467.	20.	5.49E+10	0.0007	258.0	3.38E+10	185.	45835.	161.	0.000	1.90E-02
686.5	0.5	0.0022	257.6	2364.	6470.	20.	5.53E+10	0.0007	257.9	3.41E+10	184.	45818.	161.	0.000	1.86E-02
691.5	0.5	0.0022	257.6	2370.	6472.	20.	5.58E+10	0.0007	257.9	3.44E+10	184.	45803.	160.	0.000	1.82E-02
696.5	0.4	0.0022	257.5	2376.	6474.	19.	5.62E+10	0.0007	257.9	3.47E+10	184.	45790.	160.	0.000	1.78E-02
701.5	0.3	0.0022	257.5	2383.	6476.	19.	5.67E+10	0.0007	257.9	3.49E+10	183.	45780.	160.	0.000	1.74E-02
706.5	0.2	0.0022	257.5	2389.	6477.	19.	5.71E+10	0.0007	257.9	3.52E+10	183.	45773.	159.	0.000	1.70E-02
711.5	0.1	0.0022	257.5	2395.	6478.	19.	5.76E+10	0.0007	257.9	3.55E+10	183.	45768.	159.	0.000	1.67E-02
716.5	0.0	0.0022	257.5	2401.	6478.	18.	5.80E+10	0.0007	257.9	3.58E+10	183.	45766.	159.	0.000	1.64E-02

2.4 The Energy Cycle of the Nuclear Cloud

If the momentum equation (2.3.5) is multiplied by u, the temperature equation (2.3.6) by c_p and the height equation (2.3.8) by g, then with the turbulent-energy equation (2.3.7), they form a set of four energy equations whose terms have the indicated meanings.

Kinetic energy, KE

$$\frac{du^2/2}{dt} =$$

gain from H	loss to PE i.e. work done lifting cloud	eddy-viscous loss to turbulence	inelastic-collision loss to turbulence	dilution by entrainment

$$= \frac{T}{T_e} gu \qquad -gu \qquad - 2k_2 \frac{v}{\ell} \frac{T}{T_e} u^2 \qquad - \frac{u^2}{2} \frac{1}{m} \frac{dm}{dt} \qquad - \frac{u^2}{2} \frac{1}{m} \frac{dm}{dt} \qquad (2.4.1)$$

Enthalpy, H

	loss to KE	gain from TE i.e. dissipation, viscous heating	dilution by entrainment

$$c_p \frac{dT}{dt} = \frac{dH}{dt} = - \frac{T}{T_e} gu \quad + k_3 \frac{(2E)^{3/2}}{\ell} \qquad -c_p(T-T_e) \frac{1}{m} \frac{dm}{dt} \qquad (2.4.2)$$

Turbulent energy, TE

gain from KE by eddy viscosity	gain from KE by inelastic-collision entrainment	dissipation loss to H	dilution by entrainment

$$\frac{dE}{dt} = 2k_2 \frac{v}{\ell} \frac{T}{T_e} u^2 \quad + \frac{u^2}{2} \frac{1}{m} \frac{dm}{dt} \qquad -k_3 \frac{(2E)^{3/2}}{\ell} \qquad -E \frac{1}{m} \frac{dm}{dt} \qquad (2.4.3)$$

Potential energy, PE

gain from KE

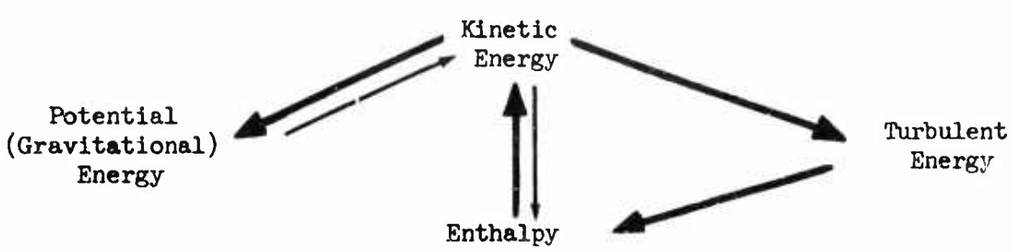
$$g \frac{dz}{dt} = gu \qquad (2.4.4)$$

Kinetic energy here means energy of directed motion of the cloud, $u^2/2$, turbulent energy, E, is actually kinetic energy of randomly moving lumps of fluid within the cloud. See Sec. 3.1.1. Adding the four equations gives an equation for total energy per unit mass

$$\frac{d}{dt}\left(u^2/2 + c_pT + E + gz\right) = -\frac{1}{m}\frac{dm}{dt}\left(u^2/2 + c_p(T-T_e) + E\right) \qquad (2.4.5)$$

The dimensionless constants k_2 and k_3 do not appear in the total energy equation; they only affect the rate of transfer between different forms of energy.

The present cloud model, then, is based on conservation of mass and energy. The equations for velocity (or momentum or kinetic energy), temperature (or enthalpy), turbulent energy and height can be considered merely rules for energy transformation. These rules imply an energy cycle which may be diagrammed as follows. The heavier arrows indicate the usual direction of transfer (when $T>T_e$ and $u>0$)



The energy flow through turbulence is always in the same direction; this part of the cycle is intrinsically irreversible. The novelty in this model is the use of turbulence as a "delay line" between kinetic energy of rise and heat. Although momentum is lost by drag or eddy viscosity, kinetic energy is not lost but rather transformed.

Because of entrainment, the cloud cannot be treated as a closed system, with constant total energy per unit mass, except under special atmospheric conditions. To examine change in total cloud energy (not per unit mass) all mass should be referred to the same datum, even though the mass was entrained at different altitudes and temperatures (at different potential energies and enthalpies). As a datum, we use sea level, $z = 0$, with the corresponding ambient temperature, $T_e = T_{eo}$.

Multiplying Eq. (2.4.5) by m and rearranging terms, the rate of change of total energy relative to this datum is

$$\frac{d}{dt}\left[m(u^2/2 + c_p(T-T_{eo}) + E + gz)\right] = \frac{dm}{dt}\left[(T_e + gz/c_p)-T_{eo}\right]c_p \tag{2.4.6}$$

Now $T_e + gz/c_p$ is the potential temperature of the atmosphere at altitude z, (the temperature air at the given altitude would have if compressed adiabatically to sea-level pressure), so that total cloud energy, referred to sea level, increases with altitude in proportion to the excess of potential temperature over sea-level temperature. We can now sketch a total-energy cycle for the cloud, referred to the sea-level datum. (See Fig. 2.1)

In an adiabatic atmosphere (one in which potential temperature does not vary with altitude), we have

$$T_e = T_{eo} - gz/c_p$$

so that, substituting in Eq. (2.4.6),

$$\frac{d}{dt}\left[m(u^2/2 + E + c_p(T-T_{eo}) + gz)\right] = 0 \qquad (2.4.7$$

i.e., total energy, referred to sea level, is constant.

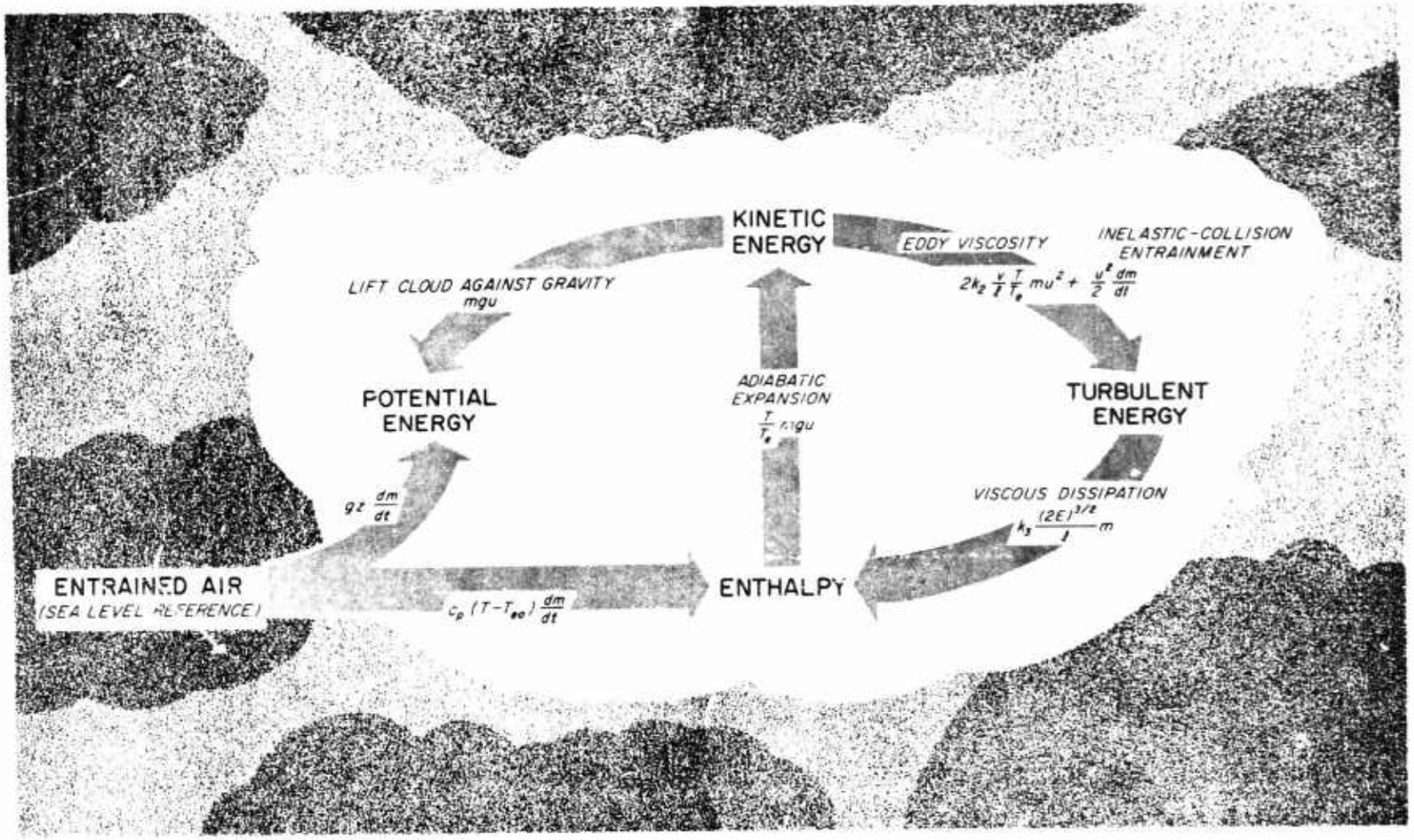


Fig. 2.1 TOTAL ENERGY CYCLE FOR THE RISING NUCLEAR CLOUD

3. TURBULENCE IN THE NUCLEAR CLOUD

3.1 Theoretical Background

Turbulent motion of a fluid has by definition a fluctuating, random nature in time and space. This nature has suggested Fourier analysis of velocity components (and other fluctuating properties) into a spectrum of motions of various length scales, or "eddy sizes". We shall be concerned mostly with small eddies.*

3.1.1 The Energy Cascade

The kinetic energy of a large-scale, intensely turbulent flow is supposed to be transmitted from the over-all flow to the largest eddies (turbulent motions) and through a cascade of successively smaller eddies* to an approximate minimum eddy size, and finally, to be dissipated by viscosity to heat (molecular motions). This idea dates back at least to L. F. Richardson, c.1920. The cascade picture suggests that eddies sufficiently far along the cascade may be independent of large-scale flow conditions, and leads to the equilibrium hypothesis. (See next section)

3.1.2 The Equilibrium Hypothesis

During the transfer of energy to smaller and smaller eddies, the direction of motion becomes random and the concentration of energy in particular eddy sizes (wave lengths) is smoothed out by transfer to neighboring sizes. Information on eddy orientation is lost during energy transmission, so to speak. Thus, small-scale turbulence is homogeneous and isotropic even though the largest eddies (such as the vortex ring of a nuclear cloud) are not. The motions at different small length

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* 'Small eddy' is used. . .as a concise term for a Fourier component belonging to a small length scale, or large wave number.[2]

scales are similar in the sense that they differ only by scale factors which are independent of large-scale flow conditions. Eddies of these sizes are said to be in the equilibrium range. This is "Kolmogorov's hypothesis"*: "the small scale components of turbulence are approximately in statistical equilibrium."[2] Equivalent names for this hypothesis are: theory of "local similarity", of "universal equilibrium", or of "similarity of small eddies". Kolmogorov's hypothesis applies only to very high Reynolds-number flows ($Re > 10^6$) such as a rising nuclear cloud.

While sufficiently small eddies are independent of large-scale flow, all except very small eddies are independent of fluid viscosity. This latter independence leads to the principle of Reynolds-number similarity (Ref. 10, Sec. 5.4):

> "Geometrically similar flows are similar at all sufficiently high Reynolds numbers. The aspects of the motion that are excluded from the similarity are quite simply those that occur at low Reynolds numbers, i.e., those in which viscous forces are comparable with inertial and pressure forces."

This principle is applied in the cloud model, for instance, in the choice of characteristic length involved in momentum and energy transfer. This length is scaled to cloud size, independent of fluid viscosity. It is used again in developing a theory of turbulent-diffusive dispersion of particles from the cloud. (Sec. 5) Note that the principle of Reynolds-number similarity is a statement about large-scale conditions, while that of local similarity is a statement about small-scale conditions.

The equilibrium range of the eddy spectrum is independent of overall flow dimensions and thus of energy input by large eddies at one end of the spectrum. Furthermore, for extremely large Reynolds numbers[11] as in nuclear cloud turbulence, the larger-eddy part of this equilibrium range, called the "inertial sub-range"[2] is also independent of viscous

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* For translations of Kolmogorov's original papers, see Ref. 9.

dissipation by the smallest eddies (the "dissipation range") at the other end of the spectrum. The existence of the "inertial range" or "sub-range" is sometimes[11] called "Kolmogorov's second hypothesis". Dimensional analysis, applied to the inertial and dissipation ranges, predicts the velocity, energy, etc., at each wave length, λ, or wave number, k, (reciprocal of wave length). See Sec. 3.1.3.

For a quasi-steady fluid flow, the rate of energy dissipation per unit mass, ε, by the smallest eddies, is equal to the rate of energy transmission through the spectrum from the large eddies, (energy-input) to the smallest, dissipative eddies. That is, if λ is an eddy size much smaller than the maximum flow dimensions, and somewhat larger than the minimum eddy size, λ_o, then the rate of energy transmission past λ is equal to ε. (The size range of λ might be about $10\lambda_o < \lambda \leq \ell/10$, but see Sec. 3.1.4) A quasi-steady fluid flow is one in which the response time of the energy spectrum at a given small-eddy wave number is small compared with the time scale of changes in the overall flow.

The dissipation rate, ε, is determined by large-scale flow conditions, although it is the smallest eddies which actually dissipate kinetic energy to heat. At this largest scale of the flow, the Reynolds number is extremely large and viscosity is of negligible importance. Then, using dimensional analysis, ε must have the form

$$\varepsilon \sim u^3/\ell$$

where u and ℓ are a large-scale velocity and length, respectively, since these are the only large-scale quantities out of which a dissipation rate (power per unit mass, or length^2/time^3) can be formed. Here and later, the symbol $\sim$ means "proportional to and of the order of magnitude of". This type of dimensional reasoning, which is used later in this report, is especially popular with Russian authors (Levich,[13] Landau and Lifshitz[14]).

Writing $\varepsilon \sim u^3/\ell \sim \ell u(u/\ell)^2$

it is seen that ℓu has the dimensions of kinematic viscosity, and u/ℓ

has the dimensions of shear. Just as molecular kinematic viscosity is proportional to the product of molecular speed and mean free path, an eddy viscosity can be defined:

$$\nu_{turb} \sim u\ell$$

Then we can write

$$\varepsilon \sim \nu_{turb}\,(u/\ell)^2$$

in analogy to the dissipation rate in laminar viscous flow, whose form is $\nu\left(\frac{\partial u}{\partial y}\right)^2$ where y is a coordinate perpendicular to u.

3.1.3 Small-Scale Turbulence Parameters

The smallest eddy size, λ_o, the "Kolmogorov microscale", can depend only on (molecular) kinematic viscosity and on ε, so that again by dimensional analysis,

$$\lambda_o \sim (\nu^3/\varepsilon)^{\frac{1}{4}}$$

(The constant of proportionality in these relations is usually of order unity.) Thus, the microscale is only weakly dependent (to the $-\frac{1}{4}$ power) on dissipation rate. It is typically of order 1 mm in severe atmospheric storms, where $\varepsilon \sim u^3/\ell \sim 10^3 cm^2/sec^3$ or exceptionally, $\varepsilon \sim 10^4$ (See for instance, Ref. 15). Even if $\varepsilon \sim 10^6$ or 10^7, as seems possible in a megaton nuclear cloud, (see Table 2.1) ν_o is not much less -- say .3 mm = 300 μ, since at the high temperature and/or low density of nuclear clouds, the value of ν will be greater than at standard temperature and pressure. (See Sec. 3.2.3). Further predictions of characteristic quantities in the inertial and dissipation ranges are readily derived (as in Refs. 7 and 13) and are listed here (Table 3.1).

The most quoted of these predictions concerns the form of the energy spectrum in the inertial range. If v_λ is the velocity at length scale λ in this range, then its form must be $v_\lambda \sim (\varepsilon\lambda)^{1/3}$. The corresponding kinetic energy per unit mass is $\sim v_\lambda^{\,2} \sim (\varepsilon\lambda)^{2/3}$. Then the energy, $\mathcal{E}$, in a spectral interval $d\lambda$ is $\mathcal{E}(\lambda)d\lambda \sim \frac{(\varepsilon\lambda)^{2/3}}{\lambda}\,d\lambda = \varepsilon^{2/3}\lambda^{-1/3}d\lambda$. In terms of wave number, $k \sim 1/\lambda$, this gives "Kolmogorov's law":

TABLE 3.1 SMALL SCALE TURBULENCE PARAMETERS

PARAMETER \ RANGE	INERTIAL	MICROSCALE	VISCOUS
Length, λ	$\lambda > \lambda_o$	$\lambda_o = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$	$\lambda < \lambda_o$
Velocity, v_λ	$(\varepsilon\lambda)^{1/3} = v_o\left(\frac{\lambda}{\lambda_o}\right)^{1/3}$	$v_o = (\varepsilon\nu)^{1/4}$	$\frac{\lambda}{t_o} = \lambda\frac{v_o}{\lambda_o}$
Time, $t_\lambda = \frac{\lambda}{v_\lambda}$	$\left(\frac{\lambda^2}{\varepsilon}\right)^{1/3} = t_o\left(\frac{\lambda}{\lambda_o}\right)^{2/3}$	$t_o = \left(\frac{\nu}{\varepsilon}\right)^{1/2} = \frac{\lambda_o}{v_o}$	t_o
Acceleration, $a_\lambda = \frac{v_\lambda}{t_\lambda}$	$\left(\frac{\varepsilon^2}{\lambda}\right)^{1/3} = \frac{v_o}{t_o}\left(\frac{\lambda_o}{\lambda}\right)^{1/3}$	$a_o = \frac{v_o}{t_o} = \left(\frac{\varepsilon^3}{\nu}\right)^{1/4} = \left(\frac{\varepsilon^2}{\lambda_o}\right)^{1/3}$	$\frac{v_\lambda}{t_o} = \frac{\lambda}{t_o^2} = \frac{\lambda}{\lambda_o}\frac{v_o}{t_o}$
Shear Rate, $\frac{v_\lambda}{\lambda} = \frac{1}{t_\lambda}$	$\left(\frac{\varepsilon}{\lambda^2}\right)^{1/3} = \frac{1}{t_o}\left(\frac{\lambda_o}{\lambda}\right)^{2/3}$	$\frac{1}{t_o} = \frac{v_o}{\lambda_o} = \left(\frac{\varepsilon}{\nu}\right)^{1/2}$	$\frac{1}{t_o}$
Energy Per Unit Wave Length,	SAME AS ACCELERATION		

TABLE 3.1 SMALL SCALE TURBULENCE PARAMETERS (cont.)

PARAMETER \ RANGE	INERTIAL	MICROSCALE	VISCOUS
Energy Per Unit Wave Number, $\mathcal{E}(k)$	$\varepsilon^{2/3} k^{-5/3}$	$\mathcal{E}(1/\lambda_o) = (\varepsilon\nu^5)^{1/4} = \frac{\nu^2}{\lambda_o} = (\varepsilon^2\lambda_o^5)^{1/3}$	$\frac{\lambda^3 v_o^2}{\lambda_o^2} = \frac{\nu^2}{\lambda_o}\left(\frac{\lambda}{\lambda_o}\right)^3 = \mathcal{E}(1/\lambda_o)\left(\frac{\lambda}{\lambda_o}\right)^3$
Spectral Reynolds Number $Re_\lambda = \frac{\lambda v_\lambda}{\nu}$	$\frac{\lambda^{4/3}\varepsilon^{1/3}}{\nu} = \left(\frac{\lambda}{\lambda_o}\right)^{4/3} > 1$	1	$\left(\frac{\lambda}{\lambda_o}\right)^2 < 1$

$$\mathcal{E}(k)dk \sim K\varepsilon^{2/3}\, k^{-5/3} dk$$

where K is a universal constant. Experimental[16] and theoretical[12] estimates indicate the coefficient K is of order unity.

3.1.4 Approximations to the Energy Spectrum

It has been suggested[12] that for very high Reynolds numbers, the inertial energy-distribution range, i.e., the -5/3 law, can be extended over the entire spectrum, from the largest wave lengths, ℓ, to the microscale, λ_o.

3.1.5 The Degrees of Freedom of a Turbulent Flow

Associated with any eddy size, λ, there is a "spectral Reynolds number" $Re_\lambda = \frac{\lambda v_\lambda}{\nu}$.

From Table 3.1, substituting $\varepsilon \sim u^3/\ell$,

$$Re_\lambda \sim \frac{\varepsilon^{1/3}\lambda^{4/3}}{\nu} \sim \frac{u\ell}{\nu}\left(\frac{\lambda^{4/3}}{\ell^{4/3}}\right) = Re\left(\frac{\lambda}{\ell}\right)^{4/3}$$

where Re refers to the overall flow. Since for $\lambda = \lambda_o$, $Re_\lambda = 1$, the ratio of size of largest to smallest eddies is of order $Re^{3/4}$.

Taking the largest eddies as of nearly the same size as the overall flow, there are then $Re^{3/4}$ independent microscale eddies, or degrees of freedom. In a three-dimensional flow, there are then $Re^{9/4}$ independent eddies, or degrees of freedom. This suggests the problems associated with numerical computation of turbulent flows. (See Sec. 2.2.1).

3.2 Estimates of Nuclear Cloud Turbulence

In this discussion, reference is made to the revised TR-741 cloud model for numerical estimates of characteristics of explosion clouds. (See Tables 2.1 and 2.2 of the present report. They are revised versions of Tables 3.1 and 3.2 of Ref. 1, recomputed with the changes in the model mentioned in Sec. 2 and specified in Appendix A.)

3.2.1 Scales and Reynolds numbers

As a large-scale characteristic length for calculating Reynolds number we use cloud diameter. The size of the largest eddies (such as the vortex-ring cross section), used as a length scale for energy dissipation rate, $\varepsilon \sim u^3/\ell$, is less than this diameter, but we assume that ℓ is a constant fraction of cloud diameter. For the corresponding characteristic velocity, u, we use the "velocity of turbulence", $\sqrt{2E}$.

In the early part of cloud rise (up to about 60 sec. for 5 MT), the cloud is much hotter than the environment. This leads to different values for "external" and "internal" Reynolds numbers.

$$Re_{ext} = \frac{u\ell\rho_e}{\mu_e} \quad ; \qquad Re_{int} = \frac{u\ell\rho}{\mu}$$

Here, u and ℓ are appropriate velocity and length scales. Since, for a perfect gas

$$\frac{\rho}{\rho_e} \simeq \frac{T_e}{T} \quad \text{and} \quad \frac{\mu}{\mu_e} \simeq \left(\frac{T}{T_e}\right)^{1/2} , \quad \text{then} \quad \frac{Re_{ext}}{Re_{int}} = \left(\frac{T}{T_e}\right)^{3/2} .$$

It is Re_{int} that is of interest in cloud turbulence. Although calculated turbulence velocities are high -- of the same order as rate of rise -- the Mach number of turbulence is (even for a 5 MT shot) never more than 1/3 because of the higher speed of sound in the hot cloud than in the cold environment. Thus, compressible-flow effects can be neglected.

3.2.2 The Turbulent Spectrum of the Cloud

Since nuclear-cloud Reynolds numbers are extremely high (for a 5 MT cloud, $Re \sim 10^{10}$ from 30 to 600 sec. after burst) one can certainly assume homogeneity and isotropy of small eddies. Homogeneity is also consistent with the "parcel" cloud method. Because of the high Reynolds number, we can also, with some confidence, extend the inertial range to nearly the entire spectrum from the microscale to almost the largest eddies. (Sec 3.1.4)

Because the nuclear cloud is not in a steady state, there is difficulty in defining the dissipation rate, ε. The rate of "loss" (dilution) of energy per unit mass due to entrainment may be larger than viscous loss (i.e., $\frac{E}{m}\frac{dm}{dt} > \varepsilon$), and the flux of energy from its source in the large eddies to the viscous sink is actually neither independent of time nor of eddy size. Nevertheless, we propose to neglect any distorting effect of entrainment on the spectrum and estimate turbulence parameters quasi-statically at each time of interest on the basis of E and ℓ. That is, we treat the turbulence as "quasi-steady" as defined in Sec. 3.1.2. This treatment is justified if the characteristic time of change of E, say $\left[\frac{1}{E}\left|\frac{dE}{dt}\right|\right]^{-1}$ is much greater than the "local transfer time"[12] of energy flow through the spectrum, $\tau(k) = \left[k^3\mathcal{E}(k)\right]^{-1/2}$.

There are several possible choices for $\frac{dE}{dt}$:

(1) the total derivative (net rate of change) of E. This cannot be used at some early times, (e.g., near 22 sec. in Table 2.1) where it is nearly zero because increase of E by generation of turbulence and decrease by dissipation and entrainment nearly cancel;

(2) the generation rate, $2k_2 \frac{T}{T_e}\frac{v}{\ell}u^2 + \frac{1}{m}\frac{dm}{dt}\frac{u^2}{2}$

(3) the dissipation rate, $\varepsilon = k_3 \frac{(2E)^{3/2}}{\ell}$

The characteristic time $\left(\frac{1}{E}\frac{dE}{dt}\right)^{-1}$, calculated using each of these choices for $\frac{dE}{dt}$, is given in Table 3.2 and compared with the local transfer time, for the Table 2.1 cloud computation. For the local transfer time, τ, we use a characteristic time based on similarity theory (Table 3.1). $\tau = (\varepsilon k^2)^{-1/3} = (\lambda^2/\varepsilon)^{1/3}$. This form for τ can be obtained by substituting Kolmogorov's law (Sec 3.1.3) in $\tau(k) = \left[k^3\mathcal{E}(k)\right]^{-1/2}$.

From Table 3.2, it is evident that for typical inertial-range wavelengths, τ is much smaller than $\left[\frac{1}{E}\frac{dE}{dt}\right]^{-1}$ so that the equilibrium treatment of small-scale turbulence is justified for the nuclear cloud. We might say that if the local transfer time is small compared with the characteristic time, turbulent energy is flowing through the eddy spectrum like an incompressible fluid.

TABLE 3.2

CHARACTERISTIC TIME AND LOCAL TRANSFER TIME FOR A HIGH-YIELD (5 MT) BURST

Time, T (sec)	Turbulent Energy, E (m^2/sec^2)	Rate, $\frac{dE}{dt}$ (m^2/sec^3) given by			Characteristic Time, (sec) $\left[\frac{1}{E}\frac{dE}{dt}\right]^{-1}$, based on			Local Transfer Time (sec) $\tau = (\lambda^2/\varepsilon)^{\frac{1}{3}}$, for λ(cm) =			
		(a) Generation Rate	(b) Dissipation Rate	(c) Total Rate of Change of E	(a)	(b)	(c)	10^{-2}	10^{-1}	1.	10.
25	4.0×10^{4}	4.6×10^{3}	2.0×10^{3}	9.2×10^{2}	8.7	2.0×10^{1}	4.3×10^{1}	3.7×10^{-3}	1.7×10^{-2}	8.0×10^{-2}	3.7×10^{-1}
60	1.5×10^{4}	4.8×10^{2}	2.8×10^{2}	3.0×10^{2}	3.3×10^{1}	5.2×10^{1}	4.9×10^{1}	7.1×10^{-3}	3.3×10^{-2}	1.5×10^{-1}	7.1×10^{-1}
120	6.9×10^{3}	1.0×10^{2}	6.8×10^{1}	5.7×10^{1}	6.9×10^{1}	1.0×10^{2}	1.2×10^{2}	1.1×10^{-2}	5.3×10^{-2}	2.5×10^{-1}	1.1
300	1.2×10^{3}	1.3	4.8	8.3	9.1×10^{2}	2.5×10^{2}	1.4×10^{2}	2.8×10^{-2}	1.3×10^{-1}	5.9×10^{-1}	2.8

NOTE: Generation Rate is $2k_2 \frac{T}{T_e} \frac{v}{\ell} u^2 + \frac{1}{m}\frac{dm}{dt}\frac{u^2}{2}$.

Dissipation Rate is $\varepsilon = k_3 \frac{(2E)^{3/2}}{\ell}$.

Source of Data: Table 2.1

4. VORTEX MOTION IN THE CLOUD

Pictures of nuclear clouds often show a vortex ring of hot incandescent gas within the rising cloud (see for instance, Figure 2.07b of Ref. 3), and also, even at later times when the gas is no longer incandescent, a general toroidal circulation of the cloud boundary. These phenomena appear more clearly in air bursts than surface blasts. Surface material in and around the cloud may diminish their visibility or actually interfere with their occurrence, say by mass loading of the circulating gas. We will generally refer to these phenomena as "toroidal circulation", restricting "vortex ring" to its classical hydrodynamic meaning. The reasons for interest in the circulation are (1) that it may affect cloud rise and expansion and especially (2) its effect on particle dispersion from the cloud.

Attempts to calculate circulation and dispersion in terms of laminar vortex flow are considered in this section. In the next section, we suggest an alternate turbulent-diffusion treatment of dispersion.

To calculate the flow in and around the cloud from first principles requires a "local" method (section 2.2) but, as noted, numerical solution of the equations of fluid motion for the nuclear cloud is not within the state of the art of machine computation. Is it either possible or desirable to use any of the classical theory of vortex rings (see Lamb[17], Chapter VII) to describe the atomic cloud? The conditions under which the classical theory was developed--incompressible steady flow, homogeneous medium, conservation of vorticity, etc.--are not valid here. The cloud starts rising from rest and input of turbulent energy (or generation of vorticity) continues during the entire period of rise. The streamlines of a vortex ring flow are closed, a condition which cannot be reconciled with the mass entrainment characteristic of atomic clouds. The nuclear cloud is better described as a "thermal"[18] (a mass of heated, buoyant air) than as a vortex ring. Thermals

"mix with the surroundings as they advance, and they grow bigger thereby. After a time, therefore, most of the fluid of which a thermal is composed was originally motionless in the surroundings and most of the momentum and vorticity it possesses will have been generated by the buoyancy forces since its birth."

"The difference between a vortex ring and a thermal is that in the former case all the vorticity and vertical momentum have been supplied by an impulse.... whereas a thermal is ideally a configuration of vorticity and momentum produced entirely by the action of the buoyancy forces. The vorticity and vertical momentum produced in the latter manner increase in proportion to the time for which the buoyancy has operated, and will therefore ultimately exceed any impressed upon the fluid at the beginning. A vortex ring composed of buoyant fluid will therefore ultimately become a thermal and the laminar flow will break down so that mixing with the fluid into which it is advancing can take place."[18]

From the preceding quotation, one might expect that at least early in cloud rise the vortex-ring treatment could be applicable. This is also suggested by Turner's[19] remark on the rise of buoyant vortex rings, that early in their rise,

"the ring diameter is increasing faster than the ... material can spread as the result of diffusion."

In our cloud model, this early rise would be the time when rate of cloud rise exceeds average velocity of turbulence; $|u| > \sqrt{2E}$. (see assumption 4 of the cloud model, Sec. 2.3) But we find by computation (Tables 2.1 and 2.2) that rate of cloud rise is nearly always less than average

velocity of turbulence: $|u| \leq \sqrt{2E}$ except during the first few seconds of cloud rise, so that in Turner's terms, cloud expansion is nearly always governed by turbulent diffusion.

4.1 Late Horizontal Cloud Expansion

It has been suggested that classical vortex theory be applied to the late horizontal expansion of megaton clouds, using the method of images. The cloud would be represented by a vortex ring approaching the tropopause from below while an image ring approached from above. The image ring induces an expansion of the approaching cloud vortex ring. This is mathematically equivalent to treating the tropopause as a rigid boundary.

The physical objections to this treatment would be the same as those to any classical vortex treatment as given above. Furthermore, the tropopause is by no means impenetrable, and the height of the boundary plane would have to be specified as a function of yield and actual tropopause height.

On the other hand, (Section 2.3, assumption 4) the present model does predict the late expansion of megaton clouds, in effect as a turbulent diffusion without invoking a rigid or yield-dependent tropopause.

4.2 The Pasted Vortex

Atomic cloud vortex models proposed to date consist of an assumed classical vortex flow superimposed, or "pasted", on a parcel method cloud, without regard even to conservation of energy, much less to whether the resulting "pseudo-local" velocities satisfy the time-averaged Navier-Stokes equations. Since the streamlines thus drawn in the cloud are intended to represent average flow directions in the intensely turbulent cloud, the time of averaging must be long compared with cloud rise. The time of averaging then, should be just shorter than the periods of the largest eddies, i.e. of the vortex ring.

Actually, the local velocities can represent only one solution to the equation of motion, that for steady motion in an infinite homogeneous, non-viscous, incompressible medium; the vortex, however, is being superimposed on radically different conditions.

If the pasted vortex is to be used, it is necessary to specify first the vortex form and then its numerical parameters. The two extremes of vortex structure[17] are Hill's spherical vortex, in which the entire moving spherical fluid body is the vortex, and a thin ring vortex, which constitutes only a small part of the traveling fluid volume. The use of Hill's vortex has been suggested for cumulus cloud elements.[20] Observations of nuclear clouds indicate that the thin ring is more appropriate than the spherical vortex, (and that early in cloud rise, the ring cross section radius is not more than 1/10 of the cloud radius).

Given the general ring-vortex form, its numerical parameters can be obtained either empirically or theoretically. The flow is completely determined by specifying ring radius, vortex-core cross-section radius, and circulation (line integral of velocity around a closed circuit through the ring). If cross-section radius is very small, i.e., a thin-ring vortex, then local velocities outside the core are completely determined by ring radius and circulation. Alternatively, a number of local values of velocity can be used to determine the vortex parameters and the flow structure.

4.2.1 Vortex Parameters From Cloud Films

In the empirical method, vortex parameters are estimated from motion pictures of atomic clouds. The motion of cumulus-like lobes or projections on the rising cloud is supposed to correspond to the local velocities around the vortex ring. This method has been well described by Norment.[21] He notes that the theoretical rate of vortex translation given by vortex parameters calculated from the observed local velocities does not generally agree with observed rate of cloud

rise. Presumably vortex parameters must be somehow scaled or interpolated between yields and times for which films give information.

The shot from which most of the film data has been obtained, shot Grable of Operation Upshot-Knothole, was an air burst.[3] To apply the resulting calculated vortex parameters to fallout particle trajectories, it must then further be assumed that the presence in the cloud of considerable amounts of condensed material from a surface burst does not significantly alter the fluid circulation.

4.2.2 Specification of Vortex Parameters by the Cloud Model.

The vortex ring is certainly the largest, most permanent eddy in the turbulent atomic cloud. In fact, the vortex ring might be pictured as the flywheel maintaining the cascade of energy to smaller eddies in the turbulent spectrum (see, for instance, Hinze,[11] sec. 3.5) even after the end of cloud rise, when no more energy is supplied to turbulence. This energy supply picture is compatible with the late horizontal cloud expansion. Now we propose that the transfer of energy from mean motion to turbulence begins with the generation of the vortex, and that the vortex ring be allocated a fixed fraction of the total turbulent energy density E, say 0.2 (Ref. 11) or 0.25 (Ref. 2). This proposal is suggested by Hinze's statement:[11]

> "In the fully developed state it is not the largest eddies that will have the maximum kinetic energy, but the eddies in a higher-wave number range. The range of the energy spectrum where the eddies make the main contribution to the total kinetic energy of turbulence will be called the range of the energy-containing eddies.... Though the more permanent largest eddies contain much less energy than the energy-containing eddies, their energy is by no means negligibly small and may still amount to as much as 20 per cent of the total kinetic energy."

We can now outline a procedure for specifying vortex parameters using the cloud model. Our three input parameters will be rate of cloud rise, u, vertical cloud radius, r, and rotational (vortex) energy, 0.2mE, where m is cloud mass.

The total energy of a thin-ring vortex flow is[17]

$$T = \frac{\kappa^2 \rho \bar{\omega}_o}{2} \left(\ell n \frac{8\bar{\omega}_o}{a} - \frac{7}{4}\right) \tag{4.1}$$

Here we use Lamb's[17] notation: κ is circulation, $\bar{\omega}_o$ is the radius of the vortex ring, a is the cross-section (vortex core) radius.

This energy consists, (we assume) of kinetic energy of rise of the cloud, $\frac{1}{2} mu^2$, and rotational energy, 0.2mE. Then the energy equation becomes

$$\frac{1}{2} mu^2 + 0.2mE = \frac{\kappa^2 \rho \bar{\omega}_o}{2} \left(\ell n \frac{8\bar{\omega}_o}{a} - \frac{7}{4}\right) \tag{4.2}$$

The velocity of translation of the vortex is[17]

$$u = \frac{\kappa}{4\pi\bar{\omega}_o} \left(\ell n \frac{8\bar{\omega}_o}{a} - \frac{1}{4}\right) \tag{4.3}$$

The radius of the cloud is related to the radius of the vortex ring by the fact that the stream function[17] equals zero at the boundary of the "cloud", that is, at the boundary of the fluid which is carried along with the vortex ring. This relation gives a third equation (Lamb,[17] sec. 161, Equation 11) so that κ, a, and $\bar{\omega}_o$ can be found. The algebra of the solution is involved and only numerical solutions for κ, a, and $\bar{\omega}_o$ can be obtained explicitly. In any case, for a thin ring the stream function can be given in terms of κ, $\bar{\omega}_o$ and u, without using a, which does not affect velocities outside the core.

For purposes of illustration, we take a simplified case. Set $\bar{\omega}_o = 3/5\ r$ as indicated by Figure 3 of Reference 21. Treat the cloud as spherical, so that $\rho = m/(\frac{4}{3}\pi r^3)$ and drop the second term in paren-

thesis in Equations (4.2) and (4.3) since $\omega_o \gg a$. Then, dividing the energy equation (4.2) by the velocity equation (4.3) and substituting the given values of ω_o and ρ,

$$\kappa = 1.85r \frac{u^2/2 + 0.2E}{u} \tag{4.4}$$

But near the end of cloud rise, $u^2/2 > 0.2E$ so that approximately

$$\kappa = ru \tag{4.5}$$

This result[19] can be obtained directly by dimensional analysis: if κ depends only on r and u, it must have the form of Eq. (4.5).

Values of circulation calculated from Eqs. (4.4) and (4.5) for a 20 KT burst (Table 2.2) are rather lower at early times (5 to 10 sec) than comparable values estimated from cloud films.[21,22] Now, at these times, ambient air is several times as dense as the cloud, and circulation is changing rapidly, so that the use of classical vortex theory is on especially shaky ground. However, it is reasonable that if circulation is due to the drag of the ambient air on the cloud, it should be proportional to the density ratio $\rho_e/\rho = T/T_e$, like the eddy-viscous momentum loss rate in the momentum equation (2.3.5). An appropriate formula would then be

$$\kappa = ruT/T_e \tag{4.6}$$

No theoretical consistency with Eqs. (4.1) - (4.3) can be claimed for Eq. (4.6): the latter is offered as an intuitive value for circulation in a situation where attempts to use formulas from classical vortex theory seem particularly remote from reality.

Table 4.1 gives values of κ calculated from Eqs. (4.4), (4.5), and (4.6), for several times of interest using Tables 2.1 and 2.2 (for 5 MT and 20 KT bursts, respectively) to supply values of r, u, and T/T_e. The resulting values of κ are of the same order of magnitude as those estimated from cloud films[21,22] assuming a ring vortex form. These values from films are widely scattered, some being smaller and others larger

TABLE 4.1

VORTEX CIRCULATION CALCULATED FROM CLOUD MODEL

Time (sec.)	Radius r(m)	Rate of Rise u(m/sec)	Turbulent Energy Density $E(m^2/sec^2)$	Temperature Ratio $T/T_e = \rho_e/\rho$	Circulation, $\kappa(m^2/sec)$ Eq (4.4) $1.85 \frac{r}{u} (\frac{u^2}{2} + 0.2E)$	Eq (4.5) ru	Eq (4.6) ruT/T_e
W = 5 MT (from Table 2.1)							
20	1.8×10^3	1.8×10^2	4.1×10^4	5.3	4.5×10^5	3.2×10^5	1.7×10^6
30	2.1	1.6	3.4	3.0	4.8	3.3	9.9×10^5
60	3.2	1.2	1.5	1.5	5.0	3.8	5.7
80	3.9	1.1	1.1	1.3	5.3	4.1	5.3
W = 20 KT (from Table 2.2)							
5	2.4×10^2	61.	2.8×10^3	7.7	1.8×10^4	1.5×10^4	1.2×10^5
10	2.7	62.	5.4	3.6	2.5	1.7	6.1×10^4
15	3.1	51.	3.3	2.1	2.2	1.6	3.4
30	4.4	34.	1.2	1.2	2.0	1.5	1.8

than those in Table 4.1. They are for somewhat different yields and times than those in the table. Our conclusion is that if one wishes to superimpose an assumed vortex ring on a parcel-method cloud model, one may as well derive vortex parameters from the model itself as from film observations. Not only is laborious film measurement avoided, but, at least for Eqs. (4.4) and (4.5), the vortex velocity and energy are now compatible with those of the cloud.

4.3 Use of Kelvin's Theorem to Estimate Cloud Circulation

The equations for the classical vortex ring are derived assuming steady incompressible flow. Under such circumstances, circulation is constant, hence it is illogical to claim that the rate of change of circulation can be obtained by differentiating Eq. (4.4) or (4.5). An expression for rate of change of circulation* $\frac{d\Gamma}{dt}$, can be obtained more consistently from a consequence of Kelvin's equation,[23]

$$\frac{d\Gamma}{dt} = \frac{d}{dt} \oint \vec{v} \cdot d\vec{x} = \oint \vec{a} \cdot d\vec{x} \tag{4.7}$$

where $\vec{v}$ is velocity, $\vec{a}$ is acceleration, and $d\vec{x}$ is a line element. The integral is taken around a circuit moving with the fluid, i.e., consisting of the same fluid particles.

Writing the Euler equation of motion as

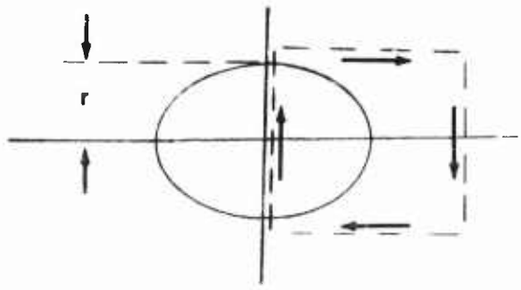
$$\vec{a} = -\frac{1}{\rho} \nabla p$$

the rate of change of circulation around the circuit is[23]

$$\frac{d\Gamma}{dt} = -\oint \frac{1}{\rho}\, dp \tag{4.8}$$

* The symbol Γ is used here for circulation to conform with Ref. 23, while in Sec. 4.2.2 the symbol κ was used to conform with Ref. 17.

To calculate cloud circulation, we take a rectangular circuit in a vertical plane through the cloud axis. As indicated in the following diagram, the circuit extends vertically from below the bottom to above the top of the cloud, and horizontally from the axis to well out in the ambient air.



Now dp is zero along the horizontal parts of the circuit. Furthermore, the parts of the upward vertical path outside the cloud are exactly cancelled by the corresponding parts of the downward path at the same altitudes so that the only possible non-zero contribution to the line integral is from the remaining vertical parts, i.e., between the top and bottom of the cloud, a distance $\Delta z = 2r$. On each of these two vertical parts, take an average value of specific volume, namely, $1/\rho$ inside and $1/\rho_e$ outside the cloud, and using the hydrostatic law, replace dp by $-\rho_e g dz$. On the upward path, the change in p is then $-\rho_e g\Delta z = -2\rho_e gr$ and similarly on the downward path (pressures inside and outside being equal) so that

$$\frac{d\Gamma}{dt} = -\left[-\frac{2r}{\rho}\rho_e g + \frac{2r}{\rho_e}\rho_e g\right] = 2rg\left(\frac{\rho_e}{\rho} - 1\right) \tag{4.9}$$

Note that the cloud need not be spherical; it is only its vertical extent, 2r, that matters.

The foregoing derivation makes no allowance for decrease of circulation due to (1) eddy viscosity or (2) entrainment. Inclusion of these two effects violates, respectively, the (1) ideal-fluid and (2) closed-streamline assumptions of Kelvin's theorem. Nevertheless, we offer a way to allow for these effects on circulation. Substitute in Eq. (4.7) the value of acceleration given by the momentum equation (2.3.5) and integrate around the same circuit as before. The entrainment rate is taken at zero on the part of the circuit outside the cloud. The resulting rate of change of circulation is

$$\frac{d\Gamma}{dt} = 2r\left[\left(\frac{\rho_e}{\rho} - 1\right) g - \left(\frac{\rho_e}{\rho} - 1\right)\frac{2k_2 v}{\ell} u - \frac{1}{m}\frac{dm}{dt} u\right] \qquad (4.10)$$

The first term in the brackets on the right side of this equation is the ideal-fluid term, corresponding to Eq. (4.9). The second and third terms represent the effect of eddy-viscosity and entrainment respectively. In this heuristic derivation, it seems a worse abuse of hydrodynamic theory to introduce entrainment across streamlines than eddy viscosity. Figure 4.1 gives numerical values of circulation from a hand integration of Eq. 4.10 with and without the entrainment term, using data from Tables 2.1 and 2.2. The integration was in 1-second steps to 5 sec., then 5-second steps to 60 sec., and 10-second steps thereafter.

It is seen in Fig. 4.1 that circulation attains nearly its maximum value early in cloud rise, at about the time of maximum rate of rise. The time of rapid increase in circulation can be associated with the formation of a vortex ring from the initial spherical cloud. This picture is reinforced if the entrainment term is omitted - no entrained mass crosses the surface of the initial cloud even though this surface is deformed from sphere to toroid.

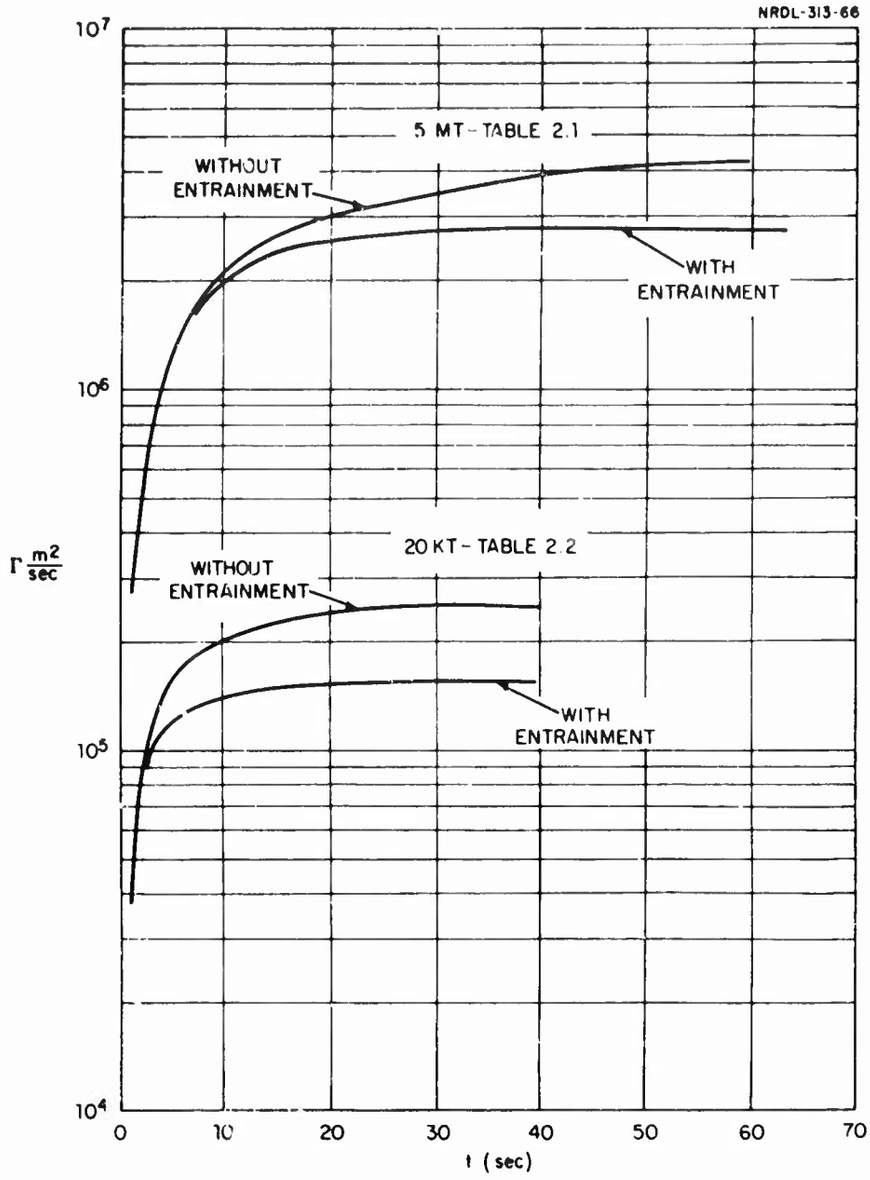


Fig. 4.1 CIRCULATION VS TIME

No direct experimental confirmation of these theoretical values of circulation is available. Film measurements can give only peripheral velocities, not velocities around the part of the circuit inside the cloud. To obtain a "measured" circulation, a specific vortex form (the ring vortex) is assumed[21,22] giving a mathematical relation between peripheral velocity and circulation. An indirect confirmation of a good choice of superimposed vortex would be given by agreement between the corresponding "measured" circulation and the theoretical values given by Eq. (4.10).

4.4 Centrifugal Throwout From the Vortex Flow

The effect of "toroidal circulation" in the cloud on particle motion may be determined by calculating first the fluid motion and then the particle trajectories through the circulating fluid (provided the presence of particles does not substantially perturb the flow). For sufficiently small particles, the flow past the particles can be considered viscous (Stokes' flow); for larger particles, empirical expressions for drag force must be used. In both cases, of course, turbulence is completely neglected if the trajectories are calculated from a laminar vortex flow. (Stokes' law in itself is not incompatible with turbulence, provided we look only at the motion of a small particle within an eddy.[24,7]) Even if small particles are centrifugal distribution effect will be blurred by atmospheric diffusion long before they reach the ground. It is, therefore, larger particles, say of diameter of order 1000μ, for which toroidal circulation could significantly affect surface fallout distribution. For such particles, empirical expressions for drag force must be used. Calculations of particle trajectories have been carried out by Norment[21,22].

5. TURBULENT DIFFUSION OF PARTICLES

If particles escape from thenuclear cloud by centrifugal force as well as gravity, due to "centrifugal throwout" from the toroidal circulation of the fluid, then the concentrat on of these particles, immediately after leaving the cloud, would vary with distance from the axis of the rising cloud. Their distribution, projected on the ground, would increase with distance from the projected cloud center, while if particles escaped under gravity alone, the distribution would be nearly uniform over the projected cloud area. The intense turbulence of the cloud suggests treating any non-gravitational dispersion of particles as a turbulent diffusion. How do particles "really" get out of the cloud? Perhaps the question is meaningless. We propose to use the language of turbulence, as more appropriate than that of laminar flow, to model non-gravitational dispersion.

5.1 Geometrical Effect of Turbulent Diffusion

Suppose turbulent diffusion of particles is uniform over the spheroidal cloud surface (a sphere before the cloud top has reached the tropopause; an oblate spheroid thereafter). Only radial diffusion causes dispersion of particles from the cloud. Dispersion of particles out through the upper surface of the cloud can be neglected since particles so dispersed are either recaptured by the rising cloud or fall back into the cloud after end of rise. Therefore, only dispersion through the lower surface need be considered.

If particle flux density, I_o, is uniform over the cloud surface, then its projection, I, on a horizontal plane varies with radial distance, x, for the projected cloud center as the ratio of arc length on a cross section through the vertical axis, ds, to projected arc length, dx. If y is the vertical coordinate, then $ds = \sqrt{(dx)^2 + (dy)^2}$ so that for a spheroid whose vertical cross section has the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have

$$\frac{ds}{dx} = \frac{\sqrt{1-(x^2/a^2)(1-b^2/a^2)}}{1-x^2/a^2}$$

Since the projected flux density is proportional to $\frac{ds}{dx}$, it becomes infinite below the periphery of the cloud, $x = a$, (neglecting diffusion in the atmosphere, wind, etc) A more realistic approach is to divide the projection into say five concentric rings whose mid-radii are x/a = 0.1, 0.3, 0.5, 0.7, 0.9. The corresponding relative projected flux densities $\frac{ds}{dx}$ are, for a sphere, 1.01, 1.05, 1.15, 1.40, 2.20 and for a 2/1 ellipsoid (such as the 5 MT cloud in Table w.1 at 140 seconds) 1.00, 1.01, 1.04, 1.11, 1.44. Thus, the geometrical model predicts non-uniform, radially increasing projected particle flux density. This is the same effect attributed to toroidal circulation.

The total projected flux is equal to the flux density integrated over rings of area $2\pi\ x\ dx$, and equal to the total flux through the lower half of the cloud. For a spherical cloud of radius, $R = a = b$,* the demonstration is particularly simple:

$$\text{Total projected flux} = \int_0^R I(x) \bullet 2\pi\ x\ dx = I_c \int_0^R \frac{ds}{dx} \bullet 2\pi\ x\ dx =$$

$$= 2\pi I_o \int_0^R \frac{x}{\sqrt{1-x^2/R^2}}\ dx = 2\pi R^2 I_o = \text{total flux}$$

5.2 Turbulent Diffusion Coefficients

Turbulent diffusion coefficients as a function of particle size, have been determined in a study[7] of relative motion and coagulation of particles in a turbulent gas.

The derivation of the diffusion coefficients and other particle parameters uses the similarity theory of turbulence and was suggested by a remark of V. G Levich[13] that for any particle size there is a minimum eddy size which just contains the particle during a turbulent fluctuation. (Exception: small particles below a certain size are contained

* The symbol R is used for cloud radius in this section to distinguish it from particle radius, r.

by all, even microscale eddies). Mean particle speed relative to fluid is a maximum for this eddy size, since turbulent velocity increases with eddy size (see Table 3.1).

By analogy with the molecular diffusion coefficient in kinetic theory, the turbulent diffusion coefficient is defined as the product of the mean free path and the speed, relative to the fluid, of the particle. The table of diffusion coefficients and other parameters[7] is reproduced here (Table 5.1). As indicated in the table, there are four flow regimes, which apply to particles of increasing size, reading from left to right. The mean speed of theparticle relative to the fluid in the first, microscale regime is equivalent to that given by Levich[13].

In each of the other three regimes, the speed, q, is given by

$$q^3 \sim \frac{r\varepsilon\rho_p}{C_D\rho}$$

where ε is the dissipation rate, ρ_p is density of the particulate material, and drag coefficient C_D is a specified function of Reynolds number, $Re = \frac{2rq}{\nu}$. This equation for q was derived by Levich for the case C_D = constant, and derived in Ref. 7 without this restriction.

In these three regimes, the forms $C_D = 24/Re$ (Stokes' law), $C_D = 24\ Re^{-\frac{3}{4}}$ (empirical) and C_D = constant were used, respectively.

The relaxation time, τ, in each of the four regimes is given by $\tau \sim \frac{r\rho_p}{C_D q\rho}$ where Stokes' law is used for C_D in both of the first two regimes.

The mean free path is $q\tau$, which equals the minimum eddy size containing the particle for all but the smallest particles. The turbulent-diffusion coefficient is $q^2\tau$. For the smallest particles, contained by all eddies, the turbulent-diffusion coefficient is less than the molecular-diffusion coefficient, taken as nearly equal to kinematic viscosity.

TABLE 5.1 RELATIVE VELOCITY PARAMETERS AS A FUNCTION OF PARTICLE SIZE

PARTICLE RADIUS, r	r	$<$	$\lambda_o\left(\frac{\rho_a}{\rho}\right)^{1/2}$	$<$	r	$<$	$\lambda_o\left(\frac{\rho_a}{\rho}\right)^{1/4}$	$<$	r	$<$	$\sim 20\lambda_o\left(\frac{\rho_a}{\rho}\right)^{1/4}$	$<$	r
FLOW REGIME	STOKES' LAW Microscale				Stokes - Inertial				EMPIRICAL FORMS				CONSTANT DRAG COEFFICIENT
MEAN RELATIVE SPEED, q	$v_o\left(\frac{r}{\lambda_o}\right)^2\frac{\rho}{\rho_a}$	$<$	$v_o = (\varepsilon\nu)^{1/4}$	$<$	$v_o\frac{r}{\lambda_o}\left(\frac{\rho}{\rho_a}\right)^{1/2}$	$<$	$v_o\left(\frac{r}{\rho_a}\right)^{1/2}$	$<$	$v_o\left(\frac{r}{\lambda_o}\right)^{3/4}\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	$\sim 3v_o\left(\frac{\rho}{\rho_a}\right)^{1/4}$	$<$	$v_o\left(\frac{r}{\lambda_o}\right)^{1/2}\left(\frac{\rho}{\rho_a}\right)^{1/2} = (\varepsilon r\frac{\rho}{\rho_a})^{1/2}$
MINIMUM CONTAINING EDDY SIZE	λ_o	$=$	$\lambda_o = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$	$<$	$\lambda_o\left(\frac{r}{\lambda_o}\right)^2\left(\frac{\rho}{\rho_a}\right)^{3/2}$	$<$	$\lambda_o\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	$\lambda_o\left(\frac{r}{\lambda_o}\right)^{3/2}\left(\frac{\rho}{\rho_a}\right)^{3/2}$	$<$	$\sim 25\lambda_o\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	$\lambda_o\left(\frac{r}{\lambda_o}\right)\frac{\rho}{\rho_a} = r\frac{\rho}{\rho_a}$
RELAXATION TIME	$t_o\left(\frac{r}{\lambda_o}\right)^2\frac{\rho}{\rho_a}$	$<$	$t_o = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$	$<$	$t_o\left(\frac{r}{\lambda_o}\right)^2\frac{\rho}{\rho_a}$	$<$	$t_o\left(\frac{\rho}{\rho_a}\right)^{1/2}$	$<$	$t_o\left(\frac{r}{\lambda_o}\right)^{3/4}\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	$\sim 10t_o\left(\frac{\rho}{\rho_a}\right)^{1/2}$	$<$	$t_o\left(\frac{r}{\lambda_o}\right)^{3/2}\left(\frac{\rho}{\rho_a}\right)^{3/2} = \left(\frac{r}{\varepsilon}\right)^{1/2}\left(\frac{\rho}{\rho_a}\right)^{3/2}$
TURBULENT DIFFUSION COEFFICIENT	$\nu\left(\frac{r}{\lambda_o}\right)^2\left(\frac{\rho}{\rho_a}\right)^3$	$<$	$v_o\lambda_o = \nu$	$<$	$\nu\left(\frac{r}{\lambda_o}\right)^4\left(\frac{\rho}{\rho_a}\right)^2$	$<$	$\nu\frac{\rho}{\rho_a}$	$<$	$\nu\left(\frac{r}{\lambda_o}\right)^{3/4}\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	$\sim 100\nu\frac{\rho}{\rho_a}$	$<$	$\nu\left(\frac{r}{\lambda_o}\right)^{3/2}\left(\frac{\rho}{\rho_a}\right)^{3/2} = (r^{4}\varepsilon)^{1/2}\left(\frac{\rho}{\rho_a}\right)^{3/2}$
REYNOLDS NUMBER, $Re \sim qr/\nu$	$\left(\frac{r}{\lambda_o}\right)^2\frac{\rho}{\rho_a}$	$<$	$\left(\frac{\rho_a}{\rho}\right)^{1/2}$	$<$	$\left(\frac{r}{\lambda_o}\right)^2\left(\frac{\rho}{\rho_a}\right)^{1/2}$	$<$	1	$<$	$\left(\frac{r}{\lambda_o}\right)^{3/4}\left(\frac{\rho}{\rho_a}\right)^{3/4}$	$<$	200	$<$	$\left(\frac{r}{\lambda_o}\right)^{3/2}\left(\frac{\rho}{\rho_a}\right)^{3/2}$

NOTES

- Numerical Constants Are Omitted Or Order-Of-Magnitude
- $C_D \sim Re^{-3/4}$ Is Used As A Representative Empirical Form
- $Re = 200$ Is Given As An Empirical, Approximate Bound
- ρ Is Particle Density, ρ_a Is Gas Density
- Parameter Value In Shaded Columns Correspond To Limits Of Each Particle Size Range
- Parameter Value Between Shaded Columns Are Functions Of Particle Radius In The Corresponding Size Range

Note that with increasing particle size, i.e., moving from left to right in Table 5.1, kinematic viscosity becomes ever less important, appearing raised to smaller absolute powers when the microscales λ_o, v_o, t_o (Table 3.1) are expressed in terms of ν and ε. Finally, in the extreme right column, ν does not appear at all, except in the Reynolds number. The expressions in this column, then, are pure inertial-range forms: they apply to interaction of particles with eddies so much larger than the microscale that the interaction parameters depend only on the dissipation rate, for a given value of the ratio of particle density to fluid density.

Since there is a maximum eddy size of order ℓ, somewhat less than cloud radius R (Sec. 3.2.1), there is also a maximum particle size for which containing eddies exist. The maximum relative speed for any particle size cannot be greater than the large-scale turbulent velocity $\sqrt{2E}$. Since q is one component of relative speed, then the maximum speed is

$$\sqrt{3}q \leq \sqrt{2E}$$

The relative speed for large particles given in Table 5.1 is $q = (\varepsilon r \rho_p/\rho)^{\frac{1}{3}}$, and ε is defined by Eq. (2.3.9), where for a spherical cloud $\ell = R$. Substituting these values of q and ε in the inequality, we find

$$r \leq R \frac{1}{3^{3/2} k_3} \frac{\rho}{\rho_p}$$

It is found empirically (App. D) that $k_3 \sim .175$, so that the maximum contained particle size is at most

$$r \leq R \frac{\rho}{\rho_p}$$

For still larger particles, the definition of a turbulent diffusion coefficient does not make sense, since such particles cannot travel several mean free paths within the dimensions of the cloud. At any rate

their effective diffusion coefficient cannot be greater than that formed from large-scale speed and length, $\sqrt{2E}\,R$.

5.3 Turbulent-Diffusive Flux of Particles

The flux of particles through a unit area of cloud surface equals the product of the diffusion coefficient and the concentration gradient (as for any transport process). The gradient has the units of concentration per unit length. This length is taken as proportional to the cloud radius (vertical radius when the cloud is an ellipsoid). This is the same characteristic length used in the cloud-model equations, Section 2.3. That is, the principle of Reynolds-number similarity, (Sec. 3.1) is again used here: since we are dealing with a turbulent process, the length is independent of fluid, (molecular) viscosity, or diffusivity. This principle may not apply to the smallest particles mentioned above, governed by molecular diffusion, but their diffusion rate, and falling rate, are extremely small.

The flux through a unit area of cloud surface, then, is

$$I_o = k_4 D \frac{n}{\ell} \tag{5.3.1}$$

where n and D are the concentration and diffusion coefficient of particles of the given size, and k_4 is a dimensionless constant. The value of k_4 is not specified, but by analogy with the other turbulent transfer coefficients involving ℓ in the cloud equations, Section 2.3, we estimate that

$$.10 \leq k_4 \leq .25.$$

The flux through the lower half of the cloud is then

$$\Phi_t = \frac{S}{2} k_4 D \frac{n}{\ell} \tag{5.3.2}$$

where S is cloud surface area.

5.4 Gravitational vs Turbulent-Diffusive Dispersion of Particles

The total gravitational flux equals the product of the horizontal projected cloud area, the particle concentration (assumed uniform in the cloud), and the particle falling rate, p.

$$\Phi_g = \pi R^2 np \tag{5.4.1}$$

For a spherical cloud, $S = 4\pi R^2$ and, substituting this in Eq. (5.3.2)

$$\frac{\Phi_t}{\Phi_g} = k_4 \frac{2D}{Rp} \tag{5.4.2}$$

So the larger the cloud the less important becomes particle dispersion due to turbulent particle diffusion, relative to gravitational dispersion. The turbulent-diffusion coefficient for a given particle size increases with turbulent-energy dissipation rate, and, therefore, with yield and finally with radius, R, but such dependence does not significantly alter the relative importance of turbulent and gravitational dispersion established above.

5.4.1 Large Particles

Consider now particles of radius, r, so large that their drag coefficient, C_D, in gravitational fall is constant. Then

$$p = \left(\frac{8}{3}\frac{rg}{C_D}\frac{\rho_p}{\rho}\right)^{\frac{1}{2}} \tag{5.4.3}$$

where ρ_p and ρ are particle and fluid density, and $(\rho_p-\rho)/\rho \sim \rho_p/\rho$. Such particles are so large that they are also in a constant-drag-coefficient regime with respect to intensely turbulent motion, (i.e., at high dissipation rates). Then (see Table 5.1) the particle Reynolds number is over 200 and the diffusion coefficient is

$$D \sim \varepsilon^{1/3}\, r^{4/3} \left(\frac{\rho_p}{\rho}\right)^{4/3} \tag{5.4.4}$$

Thus, taking $1.5C_D \sim 1$, (as is approximately valid for such Reynolds numbers) and substituting Eqs. (5.4.3) and (5.4.4) in Eq. (5.4.2)

$$\frac{\Phi_t}{\Phi_g} \sim k_4 \left(r\,\frac{\rho_p}{\rho}\right)^{5/6} \frac{\varepsilon^{1/3}}{Rg^{1/2}} \tag{5.4.5}$$

The diffusion coefficient, Eq. (5.4.4), and therefore the turbulent-to-gravitational flux ratio, Eq. (5.4.5), are only valid for contained particles of sizes up to $r \sim R\rho/\rho_p$ (See Sec. 5.2). Substituting the maximum diffusion coefficient, $\sqrt{2E}\,R$, and Eq. (5.4.3) in Eq. (5.4.2), the flux ratio for oversized, non-contained particles is at most about

$$\frac{\Phi_t}{\Phi_g} \sim k_4 \left(\frac{2E}{gr}\,\frac{\rho}{\rho_p}\right)^{1/2} \tag{5.4.6}$$

Comparing Eqs. (5.4.5) and (5.4.6), it is seen that the flux ratio increases with increasing r in the first equation and decreases with increasing r in the second. It is therefore a maximum for the just-contained particles. In terms of toroidal circulation (reverting to the language of laminar flow) it would be these just-contained particles for which centrifugal throwout is most important. Smaller particles tend to follow fluid streamlines and larger ones, to fall like stones.

For particles of the just-contained size, $r \sim R\rho/\rho_p$, both flux ratios, Eqs. (5.4.5) and (5.4.6), reduce to

$$\left.\frac{\Phi_t}{\Phi_g}\right)_{max} \sim \frac{\varepsilon^{1/3}}{R^{1/6}\, g^{1/2}} \sim \left(\frac{\varepsilon^2}{gR}\right)^{1/2}$$

where we have again used the definition of ε, Eq. (2.3.9), and taken the turbulent velocity $\sqrt{2E}$ as approximately equal to rate of cloud rise, u. But the Froude number[25] of the cloud, or ratio of inertial to gravitational forces, is

$$Fr = u^2/gR$$

and these forces are nearly in balance during the "steady" rise of the cloud, i.e., $Fr \sim 1$ as is roughly confirmed by the first page of Table 2.1. Therefore the maximum flux ratio for any particle size, is order unity

$$\left.\frac{\Phi_t}{\Phi_g}\right)_{max} \sim Fr \sim 1 \tag{5.4.7}$$

Furthermore, fallout particles as large as $R\rho/\rho_p$, hardly exist. Taking $\rho/\rho_p \sim 10^{-4}$, then even for a yield as low as 20 KT, this size is $r \sim 2.5$ cm, taking $R \sim 250$ m (Table 2.2).

5.4.2 Small Particles

The largest particles to whose motion Stokes' law can be applied are of radius $r \sim \lambda_o(\rho/\rho_p)^{\frac{1}{4}}$. These particles are at the upper limit of the "Stokes-inertial" size range[7].

From page 1 of Table 2.1, the viscosity of air[26], μ, and the definition of kinematic viscosity, $\nu = \mu/\rho$, the microscale $\lambda_o = (\nu^3/\varepsilon)^{\frac{1}{4}}$ is of order 300 microns, and the limiting size $r \sim \lambda_o(\rho/\rho_p)^{\frac{1}{4}}$ is about 30 microns. The diffusion coefficient at this upper limit is

$$D = \nu \frac{\rho_p}{\rho} \tag{5.4.8}$$

while the falling rate for such particles is given by Stokes' law,

$$p = \frac{2}{9}\frac{r^2 g}{\nu}\frac{\rho_p}{\rho} \tag{5.4.9}$$

where again $(\rho_p - \rho)/\rho \sim \rho_p/\rho$. Then the flux ratio, substituting $r \sim \lambda_o(\rho/\rho_p)^{1/4}$, and Eqs. (5.4.8) and (5.4.9) in Eq. (5.4.2) is

$$\frac{\Phi_t}{\Phi_g} \sim 9k_4 \frac{\nu^2}{Rg\lambda_o^2} \left(\frac{\rho_p}{\rho}\right)^{1/2} \tag{5.4.10}$$

Now we use the fact that (Sec. 3.1.5) $\lambda_o \sim \ell\, Re^{-3/4}$ and for the large-scale length, ℓ, take cloud radius, R. Using also the definition of Reynolds number $Re \sim uR/\nu$, Eq. (5.4.10) becomes

$$\frac{\Phi_t}{\Phi_g} \sim 9k_4 \frac{u^2}{RgRe^{1/2}} \left(\frac{\rho_p}{\rho}\right)^{1/2} \tag{5.4.11}$$

As in Sec. (5.4.1), we substitute the value of the Froude number, $u^2/gR = Fr \sim 1$ in Eq. (5.4.11) giving for the ratio of turbulent-diffusive to gravitational flux of small particles,

$$\frac{\Phi_t}{\Phi_g} \sim 9 \frac{k_4}{Re^{1/2}} \left(\frac{\rho_p}{\rho}\right)^{1/2} \tag{5.4.12}$$

Now even for a low yield cloud (say 20 KT) $Re \sim 10^7$, while $(\rho_p/\rho) \sim 10^4$, so that $\Phi_t/\Phi_g \sim 10^{-3/2}$, i.e., this ratio is small. To be sure, for these small particles gravitational dispersion is itself quite slow.

6. RESULTS AND CONCLUSIONS

6.1 Results

The TR-741 cloud model has been revised to take into account the decay of turbulent energy to heat. The rate of this decay per unit mass, the turbulent-energy dissipation rate, is the governing parameter for small-scale turbulent motions. The model was shown to imply a nuclear cloud energy cycle.

The problem of toroidal circulation in the nuclear cloud has been discussed. It is shown that if this circulation is to be represented by a classical vortex-ring flow superimposed on the parcel-model cloud, then the vortex parameters, in particular the circulation, may be estimated from the cloud model by taking vortex rotational energy as a given fraction of the turbulent energy, consistent with conservation of energy and with turbulence theory. The resulting values of circulation are of the same order of magnitude as those calculated from films of nuclear clouds. Alternately, cloud circulation as a function of time may be calculated directly from the model by adapting Kelvin's theorem. This approach is more consistent with nuclear cloud conditions since it does not require an assumed steady state vortex form.

It is proposed that the centrifugal throwout of particles by the vortex flow be represented by a turbulent-diffusive dispersion, using a previously derived theory of turbulent motions. For both large and small particles, the rate of this turbulent-diffusive dispersion is shown to be small compared with the rate of gravitational fallout.

6.2 Conclusions

The energy dissipation rate and small-scale turbulence parameters can be calculated from the cloud model using turbulent similarity theory. The model and turbulence theory can be used to specify vortex parameters, if a steady state vortex form is to be superimposed on the cloud. The model can also be used to calculate cloud circulation as a function of

time without use of the superimposed vortex.

The turbulent-diffusive flux of particles from the rising nuclear cloud is small relative to gravitational flux, and can be ignored for practical purposes well within the accuracy obtainable from current treatments of cloud rise and expansion and fallout-particle deposition.

APPENDIX A

REVISIONS TO THE TR-741 CLOUD MODEL

A.1 Dissipation of Turbulent Kinetic Energy to Heat and Production of Turbulent Kinetic Energy by Inelastic-collision Entrainment

Define the turbulent energy dissipation rate per unit mass as

$$\varepsilon = k_3 \frac{(2E)^{3/2}}{\ell}$$

where k_3 is a dimensionless constant

E is the turbulent energy per unit mass

ℓ is a characteristic length

This definition is consistent with turbulence theory, (Sec. 3.1)

The following changes are required in the equations of TR-741.

Turbulent kinetic energy density, Equation (3.7) of TR-741:

(1) Subtract ε from right side (corresponding to assumption 6 of Sec. 2.3., dissipation of turbulent kinetic energy)

(2) Add $\frac{u^2}{2}\frac{1}{m}\frac{dm}{dt}$ to right side (corresponding to assumption 7 of Sec 2.3., conservation of total kinetic energy in entrainment)

(3) Multiply first term on right side by 2 (this corrects an error).

The revised Eq. (3.7) reads

$$\frac{dE}{dt} = 2k_2 \frac{T^*}{T_e^*} \beta \frac{u^2 v}{\ell} + \frac{u^2}{2}\frac{1}{m}\frac{dm}{dt} - \varepsilon - E\frac{1}{m}\frac{dm}{dt}$$

Temperature, Equation (3.4D) of TR-741, "dry" (unsaturated) cloud: Add ε/c_p to right side, to obtain

$$\frac{dT}{dt} = -\frac{g}{c_p(T)}\frac{T^*}{T_e^*}u - \frac{T_e\int^{T} c_{pa}(T)dT}{c_p(T)}\frac{1}{m}\frac{dm}{dt} + \frac{\varepsilon}{c_p(T)}$$

Temperature, Equation (3.4W) of TR-741 "wet" (unsaturated) cloud

Add $\dfrac{\varepsilon/c_p}{1 + \dfrac{L^2 \varepsilon x}{c_p R_a T^2}}$ to right side to obtain:

$$\frac{dT}{dt} = - \frac{\dfrac{g}{c_p} \dfrac{T^*}{T_e^*} \left(1 + \dfrac{xL}{R_a T}\right) u + \left[(x - x_e) \dfrac{L}{c_p} + (T - T_e)\right] \dfrac{1}{m} \dfrac{dm}{dt} - \dfrac{\varepsilon}{c_p}}{1 + \dfrac{L^2 \epsilon x}{c_p R_a T^2}}$$

The denominator allows for the increase in saturation vapor pressure with temperature; some of the dissipated energy is absorbed as heat of vaporization.

A.2 Cloud Form

A certain altitude z_T, the nominal height of the tropopause, is designated such that the vertical cloud radius remains constant after the top of the cloud reaches the tropopause: $r + z = z_T$. The corresponding condition in TR-741 applied to the center of the cloud: $z = z_T$. Thus, in the modified treatment, the cloud is a sphere until $r + z = z_T$, and a horizontally expanding oblate spheroid with vertical radius $r\ (z_T)$ thereafter.

The theoretical justification is that the stability of the stratosphere restricts the upward turbulent diffusion of the cloud top. The practical reason considers viscous dissipation, i.e., transfer of energy from turbulence to heat (Sec. 2.3., assumption 6); such energy transfer leaves less turbulent energy available for late horizontal expansion of megaton clouds than in the TR-741 model. The cloud-form modification compensates for the effect of assumption 6 on late horizontal expansion and on vertical oscillations, by lessening dilution of turbulent energy by entrainment and starting exclusively horizontal expansion earlier. The new computed cloud dimensions and velocities (Table 2.1) are now about the same as those in TR-741 (Table 3.1), which are in general agreement with atomic cloud observations.

A.3 Characteristic Velocity

It was thought at one time that the mean and turbulent velocities should be added to give the characteristic velocity

$$v = \sqrt{u^2 + 2E}$$

This form has been discarded in favor of $v = \max(|u|, 2E)$ which gives better agreement with observations of nuclear cloud heights and dimensions, and is consistent with the rise of buoyant vortex rings. (Section 4.1)

A.4 Environmental Conditions

(Appendix C.2 of TR-741)

Relative humidity is now given by a table of values, one for each 1000-meter layer of the atmosphere up to 28,000 meters, and is taken as zero above this altitude. (In Appendix C.2 of TR-741, humidity was given by a quadratic polynomial function of altitude.)

A.5 Revision of Volume Equation

Volume, Eqs. (3.5D) and (3.5W) of TR-741

To calculate change in cloud volume, V, the gas law, $p = \frac{m}{V} R_a q(x) T$, is used in the differential form

$$\frac{dV}{V} = \frac{dT}{T} + \frac{dq(x)}{q(x)} - \frac{dp}{p}$$

Substituting

$$dq = \left[(1/\epsilon - 1)/(1 + x)^2\right] dx$$

and using the gas law and the hydrostatic law,

$$\frac{dV}{dt} = V\left[\frac{g}{R_a T_e^*} u + \frac{1}{T}\frac{dT}{dt} + \frac{1/\epsilon - 1}{q(x)(1+x)^2}\frac{dx}{dt} + \frac{1}{m}\frac{dm}{dt}\right]$$

This equation replaces both volume equations of TR-741. Change in q(x) is now taken into account during the "wet" as well as the "dry" period. The substitution of the values of $\frac{dT}{dt}$ given by Eqs. (3.4) (see Sec. A.1) is not made explicitly as it is unnecessary for computation and makes the equation more complicated in appearance.

APPENDIX B

ERRATA IN TR-741

B.1 Typographical errors

These errata are typographical only. They do not affect derived equations or the computer program.

page		
15	lines 10-11	For "a unit volume of cloud" Read "a volume V_1 of cloud"
	Eq. (2.3)	Multiply left side and first term on right side by V_1
	Eq. (2.4)	Multiply left side and first term on right side by V_1
	Eq. (2.4)	Multiply right side by V_1
16	Eq. (2.2)	Delete dt from right side
	Eq. (2.8)	For E_k read 2 E_k
17	Para.2, line 4	For "environment" read "entrainment"
	(1), line 2	For dz read dz_1
18	Eq. (2.9)	For $\frac{dm}{dt} = \rho_e A \frac{dz_1}{dz}$ read $\frac{dm}{dt} = \rho_e A \frac{dz_1}{dt}$
19	Eq. (2.11)	Delete λ
	line 5	For ℓ read λ
	Eq. (2.12)	Delete λ
27	last line	For ρ_e/ρ_e read ρ/ρ_e
30	Eq. for $c_p(T)$	Multiply $c_{pw}(T)$ by x
32	Sec. 3.5.1 Eq. for $dV)_m/V$	Insert minus sign before each $dT)_m$
	Eq. for $dV)_m$	Multiply right side by dz
35	Sec. 3.7 Eq. (3.7)	Insert ℓ in denominator under u^2v. For E_K read E_k

57	lines 15-16	For Eq. (3.6), Sec.(3.6) Read Eq. (3.7), Sec.(3.7)
73	App. C.3 Eq. for m_{wo}	Replace ϕ by $(1-\phi)$
81	line 2	For "the column" read "ten column"

B.2 Errors affecting the cloud model

page		
19	Eq. (2.12)	For k_2 read $2k_2$
35	Eq. (3.7)	For k_2 read $2k_2$
36	Eq. (3.7A)	For k_2 read $2k_2$

The corresponding correction in the computer program is

87 Statement 555. Multiply right side by 2.0.

For the revised Eq. (3.7), see App. A.1. For the revised computer program, see App. F.

APPENDIX C

SYMBOLS USED IN THE REPORT

C.1 A Note on Notation

This report uses hydrodynamics, thermodynamics and meteorology. These fields use the same symbols for different quantities; consequently, any notation used must violate some usage. For example, in meteorology x and w are used for ratios of water vapor and condensed water mass to dry air mass, respectively. But in hydrodynamics, the velocity components u, v, w correspond to the coordinates x, y, z. Since z is the usual symbol for the vertical coordinate, as in $dp = -\rho_e g\, dz$, inconsistency cannot be avoided.

C.2 List of Symbols used in the present report

a	horizontal semi-axis of cloud, or acceleration, or vortex cross-section radius
b	vertical semi-axis of cloud
C_D	drag coefficient
c_p	specific heat of gas at constant pressure
D	turbulent diffusion coefficient
E	turbulent kinetic energy per unit mass
$\mathcal{E}(k)$	turbulent kinetic energy per unit mass per unit wave number
ε	energy dissipation rate per unit mass
Fr	Froude number
f	fraction of explosion energy, W, contained in fireball at start of rise
g	acceleration of gravity
H	enthalpy
I_o	turbulent-diffusive particle flux density
I	horizontal projection of I_o
k_2	empirical constant (in eddy viscosity)
k_3	empirical constant (in dissipation rate)

k_4	empirical constant in turbulent diffusion
k	Boltzmann's constant; or wave number $1/\lambda$
k^*	$1/\lambda_o$
L	latent heat of evaporation of water
ℓ	characteristic large-scale length
M	mean molecular weight of air
m	mass of cloud
n	concentration of particles of a given size
p	pressure, or particle falling rate
$q(x)$	ratio of virtual to actual temperature, $\frac{1 + x/\epsilon}{1 + x}$
R^*	universal gas constant
R_a	gas constant of air $= \frac{R^*}{M_a}$
R	weighted mean value of gas constant $= R_a \frac{1 + x/\epsilon}{1 + x}$;
Re	Reynolds number
r	radius of cloud, or particle radius
S	surface area of cloud
s	curvilinear coordinate
T	temperature, or vortex energy
T^*	virtual temperature, i.e., $Tq(x)$
t	time
u	vertical velocity, or large-scale velocity
V	volume of cloud
v	characteristic velocity, $v = \max(\lvert u \rvert, \sqrt{2E})$; or turbulent velocity
W	total explosion energy (kilotons)
y	length coordinate
z	vertical coordinate

β	ratio of gas density to total density of cloud $= \frac{1 + x}{1 + x + w}$
Γ	circulation
ϵ	ratio of molecular weights of water and air = 18/29
ε	energy dissipation rate per unit mass
κ	vortex circulation
λ	empirical constant (in entrainment rate) or eddy size
μ	viscosity
ν	kinematic viscosity (μ/ρ_a)
ρ	density
τ	relaxation time of accelerated particle, or local transfer time of turbulent energy
$\bar{\omega}_o$	radial coordinate of vortex ring
Φ	total particle flux

SUBSCRIPTS

a	air (dry air)
e	ambient (environment) conditions
g	gravitational
h	horizontal (radius of cloud)
o	Kolmogorov microscale values; or initial values
p	particle, or constant pressure
t	turbulent
λ	values for eddies of size λ

APPENDIX D

DIMENSIONLESS PARAMETERS USED IN THE CLOUD MODEL

The dimensionless parameters λ, k_2 and k_3 appearing in the cloud model (Sec. 2.3) are empirical, although intuitive arguments can be made to derive some particular values (e.g., $\lambda = .25$, see pp. 17-18 of Ref. 1).

D.1 Sensitivity of cloud variables to the dimensionless parameters

The maximum rate of cloud rise depends mainly on k_2.

The maximum height of cloud center depends mainly on λ.

The late horizontal radius of the cloud (if it reaches the tropopause) depends almost entirely on k_3.

D.2 Suggested numerical values of the parameters

Values of the parameters found to give good agreement of the cloud model with observed cloud heights and diameters as functions of time are:

$\lambda = 0.2$. This is the value given by G. I. Taylor[4] in the earliest discussion of nuclear cloud rise. In Taylor's report, however, the entrainment equation corresponding to Eq. (2.3.4) had the additional factor T/T_e on the right side. See discussion in Sec. 2.6.1 of Ref. 1. Sensitivity of λ: 0.25 is too large; 0.15 is too small.

$k_2 = 0.1$ For a sphere, a given value of k_2 corresponds to a value for the drag coefficient of $C_D = 16\ k_2/3$. See Eqs. (2.13) and (2.15) of Ref. 1. Thus, $k_2 = 0.1$ is very nearly equivalent to $C_D = 0.5$. Sensitivity of k_2: 0.125 is too large; 0.075 is too small.

$k_3 = 0.175$. Sensitivity of k_3: 0.25 is too large, 0.125 is too small.

APPENDIX E

GLOSSARY OF COMPUTER PRINTOUT SYMBOLS

Symbol in Printout	Symbol in Text*	Comment
Output Symbols, in order of printout		
ST	t	
U	u	
X	x	
T	T	
R	r	After SWITCH TO ELLIPSE R refers to horizontal radius, vertical radius remaining fixed
Z	z	
EK	E	
V	V	
WT	w	
TE	T_e	
M	m	
ES	e_s	Saturation water vapor pressure at T. Values of ES printed for T>373 have no physical significance
P	p	
PW	$p \frac{x/\epsilon}{1 + x/\epsilon}$	partial pressure of water vapor in cloud
ED	$k_2 \frac{u^2}{l} v \frac{T^*}{T_e^*}$	rate of loss of kinetic energy of rise due to eddy viscosity, per unit mass
EPS	ε	turbulent energy dissipation rate per unit mass

*Some of these symbols appear only in Ref. 1, not in the present report.

Symbol in Printout	Symbol in Text	Comment
Parameter Symbols, in order of printout		
DST		Runge-Kutta step size between t = 0 and t = 1
K2	k_2	
LAMBDA	λ	
W	w	
F	f	
PHI	ϕ	
TEO	T_{eo}	Sea-level temperature
CHANGE		Value of ST after which Runge-Kutta step size changes to DST2
DST2		Runge-Kutta step size when ST > CHANGE
B0, B1. B2		Coefficients of quadratic polynomial in T for c_{pa}.
DST1		Runge-Kutta step size for 1 < ST < CHANGE
K		Option in program. K = 1, 3 select the "alternate" equations, (See TR-741). K = 1, 2 select the characteristic length $\ell = r$. K = 3, 4 select $\ell = 100$. This last value of ℓ is in a dummy equation in which some other value of ℓ, such as a Mach-number correction, can be inserted.
A1, A2, A3, A4	$\alpha_1, \alpha_2, \alpha_3, \alpha_4$	Lapse rates in the 4 layers of the atmosphere
Z1, Z2, Z3	z_1, z_2, z_3	Dividing altitudes of the 4-layer atmosphere
PO	p_o	Sea level pressure
ZT		Altitude such that if R + Z > ZT (SWITCH TO ELLIPSE) cloud vertical radius remains constant (nominal height of tropopause)
D0, D1, D2		Coefficients of quadratic polynomial in T for c_{pw}.
TF		Freezing point used to select value of L (after SWITCH TO WET)

Symbol in Printout	Symbol in Text	Comment
C3	k_3	
PRINT		End time of computation
RK3		If RK3 = 1, the momentum equation contains the initial virtual mass factor. If RK3 = 0 this factor is omitted.
RLH		Percent relative humidity in 1000-meter layers of the atmosphere, in ascending order.

APPENDIX F

COMPUTER PROGRAM FOR THE CLOUD MODEL

The following FORTRAN II computer program for the cloud model is the same as that given in TR-741, except that

(1) it incorporates the modifications given in Appendix A,

(2) the Runge-Kutta integrator, SUBROUTINE RKGILL, is now written in FORTRAN II like the rest of the program, instead of FAP, as in TR-741.

The program computes cloud center height, rate of rise, temperature, turbulent energy dissipation rate, etc., as functions of time. It gives a complete numerical solution of the set of differential equations given in TR-741 with the modifications given in App. A of the present report.

The program was originally written by David Hutchinson of CEIR, Inc. and revised by J. H. Crawford* and D. C. Galant** of this Laboratory.

Punching the Input Data Cards

All the input data required by the program is read from a set of six cards; see the table below. The input data words on cards 1-5 of an input card set are punched as decimal numbers, seven words in the first consecutive ten column fields per card. Card 3 has an eighth data word which is punched as an integer in column 71. Card 6 of an input card set has a different format which is specified in the table.

The program expects to process consecutive input data card sets. It halts computation and calls EXIT when it reads a card 1 of an input data card set which has a negative decimal number punched in the third word, i.e., data word named T. When consecutive input data card sets are read by the program, only card 1 and those data words on the other cards which differ from the preceding input set need be punched. The

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program tests each input data word; and if it has not been punched, replaces it with the corresponding data word of the previous input data set. This means the initial input data card set and card 1 of each set must be punched completely, and the program interprets (internally) un unpunched input data words as "negative" zeros. The six data cards are punched as follows:

Cols	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Card 1	U	RK3	T	PO	Z	EK	DST	
Card 2	K2	λ	C1	C2	W	F	ϕ	
Card 3	TEO	CHANGE	DST2	BO	B1	B2	DST1	K (must be in col 71)
Card 4	A1	A2	A3	A4	Z1	Z2	Z3	
Card 5	ZT	DO	D1	D2	TF	K3	PRINT	

Card 6 Columns 1-8, 11-18, etc., up to 61-68. Two digit values of percent relative humidity for 1000m layers of the atmosphere up to 28000m. Zero values may be omitted.

Parameters C1 and C2 are not used.

```
*     FORTRAN   CLOUDRISE
C     PROG. 3100. CEIR. J. CRAWFORD (CLOUD RISE)
CLOUDRISE PROGRAM MODIFIED BY D C GALANT 25 APRIL 1965
C     MODIFIED 7/16/65 TO PRINT THE ENERGY DISSIPATION RATE DC GALANT
C     MODIFIED FOR FORTRAN INTEGRATOR RKGILL  26 MAY 1965
C      THIS IS THE MAIN ROUTINE
      DIMENSION PAR(37),A(37),DVBL(8),VBL(8),RKG(8)
      COMMON A, DWT,DRM,DU,DX,DT,DV,DZ,DEK,SMALLT,DVBL,TE,P,P3,P1,P2,N,
     1     TE1,TE2,TE3,ED,NZT,RZT,RK3,QI  , VBL
      COMMON EPS
      EQUIVALENCE (VBL(1),WT),(VBL(2),RM),(VBL(3),U,A(1)),(VBL(4),X,A(2)
     1 ),(VBL(5),T,A(3)),(VBL(6),V,A(4)),(VBL(7),Z,A(5)),(VBL(8),EK,A(6)
     2 ),(A(7),DST),(A(8),RK2),(A(9),RL),(A(10),C1),(A(11),C2),(A(12),W)
     3 ,(A(13),F),(A(14),PHI),(A(15),TEO),(A(16),CHANGE),(A(17),DST2),
     4 (A(18),B0),(A(19),B1),(A(20),B2),(A(21),DST1),(A(22),K),(A(23),A1
     5 ),(A(24),A2),(A(25),A3),(A(26),A4),(A(27),Z1),(A(28),Z2),(A(29),
     6 Z3),(A(30),P0),(A(31),ZT),(A(32),D0),(A(33),D1),(A(34),D2),
     7 (A(35),TF),(A(36),C3),(A(37),PRINT),(DVBL(1),DWT),(DVBL(2),DRM),
     8 (DVBL(3),DU),(DVBL(4),DX),(DVBL(5),DT),(DVBL(6),DV),(DVBL(7),DZ),
     9  (DVBL(8),DEK)
      DIMENSION RLH (28)
      DIMENSION IRLH (28)
      COMMON RLH
      PRINT 9999
9999  FORMAT(1H1)
C     INPUT THE INITIAL CONDITIONS FOLLOWED BY DST AND PARAMETERS
    1 READ 19, (PAR (I), I = 1, 7)
   19 FORMAT (7F10.0)
C                    IF T=PAR(3) IS NEGATIVE IT IS A SIGNAL TO STOP
      IF(PAR(3))18,4,4
18    PRINT 9999
      CALL EXIT
    4 READ 1900, (PAR (I), I =  8, 29), (PAR (I), I = 31, 37)
 1900 FORMAT (7F10.0/7F10.0,I1/7F10.0/7F10.0)
      DO 3 I = 1, 37
C     IF FIELD IS BLANK USE LAST PARAMETER VALUE INPUT
      IF(PAR(I))2,33,2
B  33  D=PAR(I)+201400000000
       IF(D) 3,2,2
2     A(I) = PAR(I)
3     CONTINUE
       RK3=X
      P0 = V
C                    INPUT THE RELATIVE HUMIDITY. 704 AND 7094 PROGRAMS.
      READ 1901, (PAR (I), I = 1, 28)
 1901 FORMAT (7(4F2.0, 2X))
      DO 1905 I = 1, 28
      IF (PAR (I)) 1904, 1903, 1904
B1903 IF (PAR (I) + 201400000000) 1905, 1904, 1904
 1904 RLH (I) = PAR (I)
 1905 CONTINUE
C                                    PRINT THE PARAMETERS
      PRINT 17, (A(I), I = 7, 37), RK3
17    FORMAT(5H1DST=F8.5,5H  K2=F10.7,9H  LAMBDA=F10.6,5H  C1=F 9.7,5H
     2C2=F11.7,4H  W=E10.3,4H  F=F12.8,6H  PHI=F 7.4/
     3 7H0  TE0=F10.6,9H  CHANGE=F7.3,7H  DST2=F10.4,5H  B0=F11.2 ,
```

```
     4 5H  B1=F11.4,5H  B2=E12.4,7H  DST1=F7.1,4H  K=I1/6H0  A1=F8.5,
     55H  A2=F8.5,5H  A3=F8.5,5H  A4=F8.5,5H  Z1=F8.0,5H  Z2=F8.0,5H  Z3
     6=F8.0,5H  P0=F9.0/6H0  ZT=F8.0,5H  D0=F8.1,5H  D1=F8.5,5H  D2=F10.
     78,5H  TF=F8.1,5H  C3=F8.6,8H  PRINT=F6.1,6H  RK3=F4.1/1H0)
      DO 1999 I = 1, 28
 1999 IRLH (I) = RLH (I) + 0.5
      PRINT 1910, (IRLH (I), I = 1, 28)
 1910 FORMAT (7H RLH =       7(4I2, 2X))
      SMALLT = 0.
       DWT=0.
       DRM=0.
      DU=0.
      DX=0.
      DT=0.
      DV=0.
      DZ=0.
      DEK=0.
      T0=T
       TE=TE0
       TE1=TE0-A1*Z1
       TE2=TE1-A2*(Z2-Z1)
       TE3=TE2-A3*(Z3-Z2)
       XX=9.8/287.
       P1=P0*(TE1/TE0)**(XX/A1)
       P2=P1*(TE2/TE1)**(XX/A2)
       P3=P2*(TE3/TE2)**(XX/A3)
      RMA0=PHI*F*W*4.18E12/(B0*(T0-TE0)+B1/2.*  (T0*T0-TE0*TE0)
     2      +B2/3. *(T0*T0*T0 - TE0*TE0*TE0))
      RMW0=(1.-PHI)*F*W*4.18E12/( D0  *(T0-TE0)+ D1/2.*(T0*T0-TE0*TE0)+
     2    D2/3.   *(T0*T0*T0-TE0*TE0*TE0)+2500000.)
C     SET INITIAL COND. FOR X
      X = RMW0/RMA0
C                                          SET INITIAL FOR V
       P=P0*((TE0-A1*Z )/TE0)**(XX/A1)
      V=(RMA0+RMW0)*287.*T0*(1.+29.*X/18.)/ (P*(1. X))
C     SET INITIAL COND. FOR R
       R=(3.*V/12.5663706)**.333333333
       RM=RMA0+RMW0
      QI=.5*RM*T*(18.+29.*X)*(1.+XE)/(TE*(18.+29.*XE)*(1.+X))
       N=1
       WT=0.
       RZT=-1.0
       PRINT  99
99    FORMAT(2X,2HST,4X,1HU,5X,1HX,7X,1HT,6X,1HR,7X,1HZ,6X,2HEK,4X,1HV,
     1          8X,2HWT,4X,2HTE,5X,1HM,11X,2HES,7X,1HP,5X,2HPW,5X,2HED,
     2          4X,3HEPS)
C                                     PRINT INITIAL CONDITIONS
       GO TO 35
8      PU=U
       PX=X
       PT=T
       PZ=Z
       PEK=EK
       PV=V
       PRM=RM
C                                  TAKE A RUNGE-KUTTA STEP
```

```
      CALL RKGILL(VBL, DVBL, RKG, DST, 8)
      SMALLT = SMALLT + DST
C                                TAKE SMALL STEPS INITIALLY
       IF(SMALLT   -1.0)8,87,88
87     DST=DST1
88     IF(RZT)888,89,89
89     R=SQRTF(3.*V/(RZT*12.5663706))
      EPS = C3*(2.*EK)**1.5/RZT
       GO TO 35
888    R=(3.*V/12.5663706)**.333333333
      EPS = C3*(2.*EK)**1.5/R
35    PW=P*X*29./(18. +29.*X)
      ES=611.* (T/273.)**(-5.13)*EXPF((25.*(T-273.))/T)
11    PRINT 16,
     1      SMALLT,U,X,T,R,Z,EK,V,WT,TE,RM,ES,P,PW,ED,EPS
16    FORMAT(F6.1,F6.1,F7.4,F7.1,F8.0,F7.0,F7.0,1PE9.2,0PF7.4,F6.1,
     1           1PE9.2,0PF9.0,F8.0,F7.0,F8.3,1PE9.2)
C                 N=1, DRY MODE  N=2, WET MODE   N=3, SMALL STEP DRY MODE
       IF (N-2) 150, 154, 1531
150    IF (ES-PW) 152,152,151
152    SAVE=DS1
       DST=0.5
C                      RESTORE VARIABLE VALUES AT START OF LAST STEP
      SMALLT = SMALLT-SAVE
       U=PU
       X=PX
       T=PT
       Z=PZ
       EK=PEK
       V=PV
       RM=PRM
       N=3
C                          NOW TAKE SMALL STEPS UNTIL ES LESS THAN PW
       GO TO 8
1531   IF (ES-PW)41,41,8
41     DST=SAVE
       N=2
       PRINT  77
77     FORMAT(14H0SWITCH TO WET)
       GO TO 151
154    IF(WT+.00000001) 153,153,151
153    N=1
       WT=0.
       DWT=0.0
       PRINT  66
66     FORMAT (14H0SWITCH TO DRY)
C                     IF RZT IS POSITIVE WE ARE IN ELLIPSOIDAL MODE
151    IF(RZT)50,1511,1511
C                                SWITCH TO ELLIPSOIDAL IF Z LARGER THAN ZT
CHANGED 25 APRIL 1965      TOP OF CLOUD AT TROPOPAUSE   DCG
50    IF(Z + R - ZT)  1511, 51, 51
51     RZT=R
       PRINT 52
52     FORMAT(24H SWITCH TO ELLIPSE, R=RH)
1511   IF(SMALLT   -CHANGE) 14, 15,14
15     DST=DST2
```

```
14     CONTINUE
        IF(ABSF(T)-10.) 1,20,20
20      IF(R-1.) 1,21,21
21      IF (SMALLT    -10.)22,210,210
210    IF(ABSF(DU)*DST-.1)211, 22, 22
211     IF(ABSF(U)-.1) 1,22,22
22      IF (SMALLT    -PRINT)13, 13, 1
13     IF(Z-10000.*W**.25)8,8,1
       END(0,0,0,0,0)
```

```
      SUBROUTINE RKGILL
     1     ( VBL, DVBL, RKG, H, NNN)
C     FORTRAN RUNGE-KUTTA-GILL INTEGRATOR FOR CLOUDRISE EQUATIONS
C     DC GALLANT 27 MAY 1965   158 DEC STORAGE
      DIMENSION VBL(99), DVBL(99), RKG(99)
C
COMMENCE THE INTEGRATION OF THE CLOUDRISE EQUATIONS
C
      CALL DERIV
      DO 2 J = 1, NNN
      VBL(J) = VBL(J) + .5*H*DVBL(J)
2     RKG(J) = DVBL(J)
      CALL DERIV
      DO 4 J = 1, NNN
      VBL(J) = VBL(J) + .29289322*H*(DVBL(J) - RKG(J))
4     RKG(J) = .58578644*DVBL(J) + .12132034*RKG(J)
      CALL DERIV
      DO 6 J = 1, NNN
      VBL(J) = VBL(J) + 1.7071068*H*(DVBL(J)-RKG(J))
6     RKG(J) = 3.41421356*DVBL(J)- 4.1213203*RKG(J)
      CALL DERIV
      DO 8 J = 1, NNN
8     VBL(J) = VBL(J) + .16666667*H*(DVBL(J) - 2.*RKG(J))
      RETURN
      END ( 0,1,0,0,0)
```

```
*     FORTRAN
       SUBROUTINE DERIV
C     VERSION OF DERIV TO BE USED WITH RKGILL
C     DERIV WITH CHANGES OF 11/1/65
      DIMENSION PAR(37),A(37),DVBL(8),VBL(8),RKG(8)
      COMMON A, DWT,DRM,DU,DX,DT,DV,DZ,DEK,SMALLT,DVBL,TE,P,P3,P1,P2,N,
     1     TE1,TE2,TE3,ED,NZT,RZT,RK3,QI , VBL
      COMMON EPS
      EQUIVALENCE (VBL(1),WT),(VBL(2),RM),(VBL(3),U,A(1)),(VBL(4),X,A(2)
     1 ),(VBL(5),T,A(3)),(VBL(6),V,A(4)),(VBL(7),Z,A(5)),(VBL(8),EK,A(6)
     2 ),(A(7),DST),(A(8),RK2),(A(9),RL),(A(10),C1),(A(11),C2),(A(12),W)
     3 ,(A(13),F),(A(14),PHI),(A(15),TE0),(A(16),CHANGE),(A(17),DST2),
     4 (A(18),B0),(A(19),B1),(A(20),B2),(A(21),DST1),(A(22),K),(A(23),A1
     5 ),(A(24),A2),(A(25),A3),(A(26),A4),(A(27),Z1),(A(28),Z2),(A(29),
     6 Z3),(A(30),P0),(A(31),ZT),(A(32),D0),(A(33),D1),(A(34),D2),
     7 (A(35),TF),(A(36),C3),(A(37),PRINT),(DVBL(1),DWT),(DVBL(2),DRM),
     8 (DVBL(3),DU),(DVBL(4),DX),(DVBL(5),DT),(DVBL(6),DV),(DVBL(7),DZ),
     9  (DVBL(8),DEK)
      DIMENSION RLH (28)
      COMMON RLH
      DZ = U
C                                  COMPUTE TE AND P
       XX=9.8/287.
       IF(Z-Z1)80,80,81
80     TE=TE0-A1*Z
       P=P0*(TE/TE0)**(XX/A1)
       GO TO 89
81     IF(Z-Z2)82,82,83
82     TE=TE1-A2*(Z-Z1)
       P=P1*(TE/TE1)**(XX/A2)
       GO TO 89
83     IF(Z-Z3)84,84,85
84     TE=TE2-A3*(Z-Z2)
       P=P2*(TE/TE2)**(XX/A3)
       GO TO 89
85     TE=TE3-A4*(Z-Z3)
       P=P3*(TE/TE3)**(XX/A4)
89     CONTINUE
C                    FIND THE RELATIVE HUMIDITY. (704 AND 7094)        JC
   62 L = XMIN1F (Z / 1000.0 + 1.00000, 28.5)
   36 XE = 109.98 * RLH (L)
     1                          *(TE/273.)**(-5.13)*EXPF((25.*(TE-273.))/TE
     2)/(P*29.)
38     CPAI=B0*(T-TF)+B1/2.*(T*T-TF*TE)+B2/3.* (T*T*T-TE*TE*TE)
       CPW=D0 + D1*T + D2*T*T
       CP=(B0+B1*T+B2* T*T  + X*CPW)/(X 1.)
       QXE=( 1.+XE)/(1.+29.*XE/18.)
      QX=(1.+29.*X/18.)/(1.+X)
      QT=T/TE
       IF(RZT)35,70,70
35     R=(3.*V/12.5663706)**.333333333
       SV=3./R
      RLL = R
       GO TO 49
70     R=SQRTF(3.*V/(RZT*12.5663706))
       ECC=SQRTF(R*R-RZT*RZT )/R + 1.0E 15
```

```
      SV=3.1415926*(2.*R*R+RZT*RZT* LOGF((1.+ECC)/(1.-ECC))/ECC)/V
      RLL = RZT
49     J=K
      EPS = C3 * (2. * EK) ** 1.5 / RLL
      Q7 = MAX1F (      ABSF (U),       SQRTF (2.0 * EK))
      GO TO (50,50,51,51),J
   50 IF (RZT) 53, 54, 54
   53 R2 = R
      GO TO 52
   54 R2 =  RZT
      GO TO 52
51    R2=100.
52    GO TO (60,61,60,61),J
60    DRM   =SV*Q7*(1.+29.*X/18.)*T * RL    *RM*QXE/((1.+X+WT)*TE)
      QQ=1.0
      GO TO 621
   61 DRM     = SV * RM * Q7* RL
      QQ = QT*QX*QXE*(1.+X)/(1.+X+WT)
621   DU   =(9.8*( QT*QX * (1.+X)/(1.+X+WT)*QXE - 1.)
     2                      -(QQ*Q7      *2.*RK2/R2 + DRM    /RM)*U)*
     3 (RM+QI*(1.-RK3))/(RM+QI)
      M=N
      GO TO (100,101,100),M
100   DX   =-(1.+X)*(X-XE)*DRM    /(RM*(1.+XE))
      DT   =-QX*QT*9.8*U/CP * QXE                  CPAI*DRM    /(CP*RM)
     1      + EPS / CP
C                                        RK2 IS K2, RL IS LAMBDA, RM IS M
      DV = V*(DRM/RM+DT/T+9.8*U/(287.*QXE*TE)+(29./18.-1.)*DX
     1                /(QX*(1.+X)**2))
      GO TO 555
C  THIS IS THE WET PART
101   Q1 = 1. + X*29./18.
      IF(T-TF)102,103,103
102   CL=2.83E6
      GO TO 104
103   CL=2.5E6
104   Q2 = CL*X/( 287.*T )
      Q3 = 18.*Q2/( T*29. )
      Q4 = 1. + Q2
      Q5 = 1.+ CL*Q3/CP
      Q6 = CL*(X-XE)/CP + T-TE
      DT   = (-QX*QT*9.8*Q4*U/CP *QXE    Q6*DRM    /RM)/Q5
     1      + EPS /(CP*Q5)
      DX    = Q1*(Q3*DT     + 9.8*X*U/(287.*TE)*QXE)
      DV = V*(DRM/RM+DT/T+9.8*U/(287.*QXE*TE)+(29./18.-1.)*DX
     1                /(QX*(1.+X)**2))
      DWT   =-( 1.+X+WT)*(WT+X-XE)*DRM    /((1.+XE)*RM) - DX
555   ED=RK2*  U*U*Q7 *QQ/R2  *2.0
      DEK = ED - ( EK - U*U/2.) * DRM/RM
     1      - EPS
      RETURN
      END (0,0,0,0,0)
```

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13. ABSTRACT
The entraining-parcel model of the rise and expansion of the nuclear cloud is revised to allow for (1) production of turbulent kinetic energy from kinetic energy of rise during the momentum-conserving, inelastic-collision entrainment, so that total kinetic energy is conserved in entrainment. This production of turbulent energy is in addition to that due to eddy viscosity; (2) dissipation of turbulent energy to heat.

The resulting nuclear-cloud model is represented as an energy cycle between enthalpy and energy of rise, turbulent energy, and potential energy.

Calculations of toroidal circulation and particle motion in the nuclear cloud are discussed, using the cloud model and turbulent similarity theory. If the toroidal circulation is to be represented by a vortex ring superimposed on the parcel-model cloud, the vortex ring can be considered the largest eddy in the turbulent spectrum, containing a fixed fraction of the turbulent energy. This gives an estimate of the circulation without recourse to, but in general agreement with, estimates based on measurements of nuclear cloud films. Alternately, circulation may be calcuated by an adptation of Kelvin's theore . This method assumes neither a particular vortex form nor steady-state flow, and is therefore more consistent with actual cloud conditions.

Attempts have been made to calculate the effect of toroidal circulation on particle dispersion from the cloud using laminar flow methods. Here, this dispersion is, instead, represented as due to turbulent diffusion, using the
(Abstract continued on another page)

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calculated dissipation rate as the governing parameter. Diffusion coefficients and concentration gradients are derived from turbulent similarity theory. The resulting dispersion rate is shown to be small compared with that due to gravitational fallout rate indicating that dispersion induced by circulation or turbulence can be ignored.

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