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**AUTHORITY**
USAF ltr, 25 Jan 1972
ENERGY FLUCTUATIONS IN SEISMIC NOISE

7 October 1966

Prepared For
AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C.

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SEISMIC DATA LABORATORY
TELEDYNE, INC.

Under
Project VELA UNIFORM

Sponsored By
ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Test Detection Office
ARPA Order No. 624
ENERGY FLUCTUATIONS IN SEISMIC NOISE

SEISMIC DATA LABORATORY REPORT NO. 167

AFTAC Project No.: VELA T/6702
Project Title: Seismic Data Laboratory
ARPA Order No.: 624
ARPA Program Code No.: 581C

Name of Contractor: EARTH SCIENCES DIVISION
TELEDYNE INDUSTRIES, INC.

Contract No.: AF 33(657)-15919
Date of Contract: 18 February 1966
Amount of Contract: $ 1,842,884
Contract Expiration Date: 17 February 1967
Project Manager: William C. Dean
(703) 836-7644

P. O. Box 334, Alexandria, Virginia

AVAILABILITY

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This research was supported by the Advanced Research Projects Agency, Nuclear Test Detection Office, under Project VELA-UNIFORM and accomplished under the technical direction of the Air Force Technical Applications Center under Contract AF 33(657)-15919.

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ABSTRACT

Recent observations of seismic noise have indicated a large standing wave or isotropic component composed of the admixture of many propagation modes. Statistical theories such as equipartition of energy have been reasonably successful for deriving the excitation of the propagation modes. Since such models are classically described by waves from a zero-mean Gaussian population, we test this hypothesis by observing if the amplitude envelope is described by a Boltzmann or exponential probability distribution. The results are affirmative in that, on comparing a large number of observations of seismic and thermal vacuum tube noise, we cannot distinguish between the two sets of data in fitting exponential distributions. The same holds for both narrow-band and broad-band measurements of the noise.
1. **INTRODUCTION**

In recent years, statistical methods have been successfully used to predict the change in ambient seismic noise power sensed by burying a seismometer beneath the surface of the earth. In abandoned oil explorations wells, noise measurements were carried out by the Geotechnical Corporation for AFTAC at twelve different locations and by Shell Development Company for the Air Force Cambridge Research Laboratory at one deepwell, approximately 4½ kilometers in depth. In order to explain the observed noise attenuation and the change in coherency between vertically separated seismometers, it was necessary to describe the noise vibration by several propagation modes. This was pointed out from observations of noise power spectra by Archambeau, et al., (1964), Douze (1964), Sax and Hartenberger (1964) and Seriff, et al., (1965). A simple statistical model for the wave motion is a Gaussian distributed ensemble of standing waves trapped between the walls of a large arbitrary enclosure. At any point in the medium, the state of a unit mass particle excited by the system of waves can be described by the energy changing sequentially in time. The classical model for obtaining the power spectral density is to multiply the density of eigenfrequencies in each propagation mode times the average energy per normal mode. For a layered elastic half-space, the spectral value of the normal mode density was derived for dispersive surface waves by Sax (1965). The average energy per normal mode is also a spectral quantity which is constant for an enclosure bounded by rigid insulated walls but in general is an unknown quantity. For a geological region, it depends on the equilibrium between the
absorbed incident radiation and the average radiative dissipation at the walls of the enclosure. Even though the dissipation mechanism is unknown, at a given frequency the average energy per mode can be taken constant for all of the propagation modes and their relative excitation can thus be obtained (Rosenbaum 1964, Sax and Hartenberger 1964, or Sax 1965). The results obtained in predicting the noise attenuation in a deepwell has been in most cases within several db of that observed and appears to be within the accuracy of the observations and given parameters describing the layering.

The purpose of this experiment is to tentatively assume that the seismic noise is Gaussian and to try to reject the hypothesis by measuring the relative frequency of occurrence of specific particle energies sequentially observed on samples of seismic noise. As control for the experiment, the same frequency of occurrence observations are made of a thermal noise sample recorded on tape from a Gaussian noise generator. The energy envelope of Gaussian noise is theoretically described by a Boltzman or exponential probability function, thus the fit of a straight line to the log of the relative frequency of occurrence is a test of the Gaussian hypothesis. Two tests are ised; one is the chi-square test of the deviation from the least squares straight line; the other more powerful test compares the variance of the seismic noise and that of the Gaussian noise generator.

There is scant background of such tests on seismic data. Galbraith (1963) performed chi-square tests on samples of seismic noise. Recently, very similar experiments have been
conducted to test the Gaussian and log normal distribution of radio noise, for example, Aiya (1962).

2. **SEISMIC DATA**

Three hours of seismic noise was selected which appeared to be typical of the normal ambient background. The noise was measured by a Geotech model #11167 seismometer, Shappee (1964), located at a depth of 7,452 feet in a deepwell near Apache, Oklahoma (Geographic Coordinates - 34°49'59.0" N and 98°26'09.0" W). The stratigraphic and velocity profiles as well as the data are described in a research report by the Geotechnical Corporation (1964); the near surface layering is described as high-velocity limestones of approximately 6 km/sec overlying an igneous basement complex of velocities in the neighborhood of 5 km/sec.

3. **ANALOG DATA PROCESSING**

The purpose of the analog data processing is to compute the specific energy (mean squared particle velocity amplitude), and to divide the energy by magnitude into a number of class intervals and determine the relative frequency of occurrence of states in each class interval. The description of the analog equipment used to process the data are shown on Figure 1 and Figure 2.

The analog circuit is shown in block diagram form in Figure 1. The circuit consists of two Khron-Hite low frequency bandpass filters (Model 330), one narrow-band (hi-Q) filter, an energy envelope former, and the energy envelope distribution
Figure 1. Analog Block Diagram

Figure 2. Distribution Analyzer (Typical)
analyzer. All but the Khron-Hite filters are modeled on the general purpose analog computer (an EAI 231R).

The Khron-Hite filters have a transfer function

\[
\frac{s^4 f_2^4}{\left[(s^2 + 2 A s f_1 + f_1^2)^2\right] \left[(s^2 + 2 A s f_2 + f_2^2)\right]}
\]

where \( A = 0.6 \), and \( f_1 \) and \( f_2 \) are the low and high cut-off frequencies, respectively.

For each data frequency analyzed, the high and low cutoffs are set so that the effective bandwidth is several times that desired in the analysis. The Khron-Hite filters are used to isolate the frequency band preventing power leakage from other frequencies. The hi-Q filters have a transfer function

\[
\frac{s^\omega}{Q} \frac{s^\omega}{Q} \frac{s^\omega}{Q} \frac{s^\omega}{Q}
\]

where \( \omega_o = 2\pi f_o \) and along with the Khron-Hite filters the \( Q \) is computed to maintain a constant bandwidth about each center frequency, \( f_o \). The combination of Khron-Hite and hi-Q isolate each frequency band by at least 52 db/octave.

The envelope former consists of a pair of all pass circuits having the transfer functions
\[
\frac{s^2 - \frac{\omega_1}{Q} s + \omega_1^2}{s^2 + \frac{\omega_1}{Q} s + \omega_1^2}
\]

and

\[
\frac{s^2 - \frac{\omega_2}{Q} s + \omega_2^2}{s^2 + \frac{\omega_2}{Q} s + \omega_2^2}
\]

respectively. Where

\[\omega_1 = \frac{2\pi \text{fo}}{1.835}, \quad \omega_2 = 2\pi(1.835) \text{fo} \quad \text{and} \quad Q = 0.322\]

The outputs of these circuits have the same amplitude spectrum as the input and have a quadrature (90°) phase relationship to each other in a band of about two octaves around the center frequency, \(\text{fo}\). The squared envelope of the signal amplitude can be formed by taking the sum of the squares of the two outputs.

The distribution analyzer circuit consists of a ladder of \(N\) (in this case \(N = 20\)) identical, precision outputs feeding into \(N\) analog integrators. A pair of typical sections of the ladder are shown in Figure 2. A detailed description of the design and operation of the distribution analyzer is given in Appendix 1.
4. DIGITAL DATA PROCESSING

The relative frequency of occurrence values for each class interval are printed by the analog computer for each half hour of data along with the system gain, frequency, and effective bandwidth of the filter. Six, one half hour samples are taken to cover a three hour sample of seismic noise. The information is punched on cards as input data for the digital computer. The relative energy at the center of each class interval is computed by dividing by the instrument gain. The geometric mean of the six samples is computed for each class interval with a listing of the mean, variance, and confidence limits.

The logarithm of the mean frequency of occurrence of each class is taken as a dependent variable, and the relative energy of the class interval as an independent variable. The inverse of the variance of the mean (from six samples for each class) is used to construct a set of normal weights. This data is put into the least squares polynomial program to obtain the best fit in the form, \( Y = A - BX \). The slope \( B \) is measured along with its standard deviation, the per cent error (standard deviation divided by the mean), chi-square, and the variance of the fit to the data. Each set of seismic data run at each frequency is accompanied for control by a similar run of noise data taken from a thermal noise generator. The noise generator is vacuum tube noise played back at one sixtieth of the recording speed to place its nearly white portion in the seismic noise band. All of the above parameters are also computed for the noise generator for comparison with
seismic noise.

5. RESULTS

After initially hypothesizing that the seismic noise is Gaussian, we will attempt to reject the hypothesis on the analysis of our three hour sample of seismic noise. If the noise is Gaussian, the probability distribution of the energy envelope is Boltzman or exponential. After plotting the relative frequency of occurrence of each class interval on semi-log paper we compare visually the distribution for seismic noise and a known Gaussian sample, observing the quality of the linear trend on each set of data. Further, we fit a straight line by least squares to the frequency of occurrence versus energy of each class interval. For the least squared derivation of the slope through the points we obtain the percentage accuracy based on the ratio of the standard deviation to the mean, the chi-square of the deviations from the straight line fit to the data, and the variance of the fit. The first quantity is a measure of the precision with which the power spectral density can be computed. The latter two quantities can be used in statistical hypothesis tests wherein we require a 99% probability that the distribution observed is Gaussian.

As shown on Figure 3 for seismic noise, the relative frequency of occurrence is given by the vertical axis and the class interval in order of increasing energy is shown on the horizontal axis. On each plot, the center frequency and effective bandwidth of the filter is labeled. On Figure 4, the
Figure 3. Relative Frequency of Occurrence (Ordinate) Versus Energy Level (Abscissa)
Figure 4. Relative Frequency of Occurrence (Ordinate) Versus Energy Level (Abscissa)
same information is shown for the Gaussian noise generator. Comparing Figure 3 with Figure 4, the scatter from a linear trend is the same overall for both sets of data, seismic and Gaussian generator.

All of the plots except the last are based on filters with an effective bandwidth of 0.05 cps. The last two filters are held constant at 1.0 cps, and the bandwidth is taken as 0.5 cps for one case and 1.0 cps for the other case. These were computed for the purpose of evaluating broadband data as well as the narrow band data of the other examples. Since a linear filter is basically a linear sum of samples, then by the central limit theorem, a non-Gaussian input occurring at time intervals much less than the filter memory can be converted to a Gaussian output by the filter. Thus, the interpretation that the input of a filter is Gaussian must be qualified as applying to independent states sampled independently at a time interval greater than the pulse width of the filter which is about 20 seconds for the .05 cps filter, 2 seconds for the .5 cps filter and one second for the 1.0 cps filter. Thus, we can show that the Gaussian interpretation is as good for independent samples separated by only one second as it is for separations in excess of 20 seconds.

On Tables 1 and 2, for seismic noise and thermal noise respectively, we summarize the information obtained from a least squares analysis of the data. After chi-square we have a column labeled "Chi-Square Test." The word "Gaussian" placed in this Column means that there is a 99% or better probability that the sample is from a Gaussian population.
## Table 1. Fit of Exponential Distribution to Energy Envelope of Seismic Noise

<table>
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<tr>
<th>FREQUENCY</th>
<th>BANDWIDTH</th>
<th>MEAN SLOPE</th>
<th>% ERROR</th>
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<th>CHI-SQ TEST</th>
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<td>3.5 %</td>
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### Table 2. Fit of Exponential Distribution to Energy Envelope of Gaussian Noise Generator

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<th>CHI-SQ TEST</th>
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<td>1.75</td>
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"no," implies rejection on the basis of less than a 99% probability that the sample is from a Gaussian population. The column labeled "F" shows the statistic F computed as the ratio of the largest to smallest variance of the seismic noise and noise generator. Then we have another column labeled "F Test." Again, Gaussian implies 99% or more probability that the deviations of the seismic and thermal noise generator are from the same random population.

Although this experiment was not designed to compute the power spectral density of noise, the percentage errors computed for the least squares slope suggest that the method can be used to compute noise power with a precision of approximately 2%. Since the exponential distribution is of the form \( \frac{\text{EXP} \left( \frac{E}{E} \right)}{E} \), the inverse of the slope can be used to compute \( \bar{E} \), the mean squared amplitude of the noise. The power spectral density computed in this way, is shown, both for the seismic noise and the vacuum tube noise generator, on Figure 5, as well as for five minute samples based on sample mean calculations using the Blackman-Tukey method. The general similarity of the two results verifies that the computer indeed was operating on the seismic noise sample and thermal noise sample as stated and not errors produced by our system.

6. **CONCLUSION**

Based on the chi-square test, it is more than 99% probable that samples at all of the frequencies are from a zero-mean Gaussian population. The hypothesis that the class deviation of the seismic noise sample and the thermal noise
Figure 5. Comparison of Power Spectra from Fitting Exponential Distribution Versus Sample Variance Calculation
sample are from the same random population is rejected only for the sample at 1.4 cps, suggesting that a non-Gaussian component may be included with the noise in this band. If we accept the Gaussian hypothesis, theoretical interest in the power spectral density of the noise may bear ultimately on dissipation mechanisms and may possibly lead to analysis of geological structure, with the real changes in the power spectrum in a region depending primarily on changes in structure. Of more practical interest is that modern literature on detection and filtering is most meaningful in the context of seismic signals added to Gaussian noise.
REFERENCES


APPENDIX I

Operation of the section of the distribution analyzer is as follows:

1) When the input voltage \( e_{in} \) is smaller than the \( i^{th} \) bias voltage \( B_i \), amplifier 1 (Figure 2) has a net negative input causing the output to tend to go positive (because of polarity reversal by the operational amplifier). The positive output of amplifier #1 is, however, held to a very small value by the forward conduction of the zener diode, CR1. This voltage and similarly that voltage from amplifier #5 being small, amplifier #2 has a net positive input due to the 5V bias. This causes the output of amplifier #2 to tend to go negative, but it is held to a very small value by the forward conduction of CR2. This small negative voltage out of amplifier #2 is nulled out by the positive bias voltage \( E_i \), giving the integrating amplifier #3 a net zero input in this state.

2) When \( e_{in} \) exceeds \( B_i \), amplifier #1 has a net positive input, and its output goes negative to the zener voltage of CR1 (approx. 10v). This gives amplifier #2 a net negative input and causes its output to go sharply to the zener voltage of CR2 (also approx. 10v). The potentiometer \( P_i \) is adjusted so that the net voltage into integrating amplifier #3, \( (e_{z(CR2)} \times P_i - E_i) \), is precisely 10.0 volts in this state.
3) When $e_{in}$ exceeds $B_{i+1}$, amplifier #4 is turned "on" (i.e., its output goes negative to the zener voltage just as that of amplifier #1 is inverted by amplifier #5 to a positive 10v which summed into amplifier #2 gives it a net positive input which essentially returns amplifiers #2 and #3 to the condition described in (1) above where the net input to the integrator is zero.) Thus, the input to the $i^{th}$ integrator is zero except when the signal value (in this case the energy envelope) is between the values $B_i$ and $B_{i+1}$, and is a constant (10.0v) when the signal is between $B_i$ and $B_{i+1}$. Therefore, the output of the integrator is a measure of the amount of time that the signal remained in the range $B_i$ to $B_{i+1}$.

By selecting the proper number of sections in the ladder, and the proper values of $B_i$, it is possible to analyze the probability distribution in any detail desired, within the accuracy of the equipment used.
ENERGY FLUCTUATIONS IN SEISMIC NOISE

Recent observations of seismic noise has indicated a large standing wave or isotropic component composed of the admixture of many propagation modes. Statistical theories such as equipartition of energy have been reasonably successful for deriving the excitation of the propagation modes. Since such models are classically described by waves from a zero-mean Gaussian population, we test this hypothesis by observing if the amplitude envelope is described by a Boltzman or exponential probability distribution. The results are affirmative in that, on comparing a large number of observations of seismic and thermal vacuum tube noise, we cannot distinguish between the two sets of data in fitting exponential distributions. The same holds for both narrow-band and broad-band measurements of the noise.
Deepwell Measurements of Seismic Noise
Gaussian Noise
Thermal Noise