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UNSTEADY HEAT TRANSFER ANALYSIS FOR ANY AMMUNITION, GUN, AND FIRING SCHEDULE

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UNSTEADY HEAT TRANSFER ANALYSIS FOR ANY AMMUNITION, GUN, AND FIRING SCHEDULE

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A self contained program has been developed to meet the objective: (1) Generation of thermochemical properties for any assigned propellant, (2) Analysis of transient, inviscid, compressible flow through the gun barrel, (3) Analysis of unsteady, viscous, compressible flow with arbitrary pressure gradients on the bore surface, (4) Analysis of transient heat diffusion through single or multi-layer gun tube, (5) Analysis of unsteady free convection and radiation around gun tube, and (6) Control of sequence of procedures due to the nature of unknown and transient boundary conditions and also due to coupling between problems. The investigations in these areas revealed: Carbon monoxide is more than 40%, supersonic velocity at the base of the projectile for a constant diameter tube even though the gas temperature is high and also impossible for steady state flows, period in which LaGrangian assumption may be approximate enough, extension of the state of the art in unsteady viscous flows, need for new approaches without the use of analogy between momentum and energy transfer, need for inclusion of non-Fourier effects in classical Fourier heat conduction models, and order of magnitude for free convection and radiation. It is obvious that there is strong coupling between unsteady viscous flow and unsteady heat diffusion. For hypervelocity systems, there is a possibility of interaction between inviscid and viscous flows as in hypersonic flow. When all these programs are arranged for a digital computer, the overall program can be big like NASA's NASTRAN program. When these two programs are utilized, then only the resulting gun would be realistic.

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INTRODUCTION

Rapid fire automatic weapons permit discharge of hundreds of projectiles per minute. As a result, the surface temperatures of the gun tube easily reach 1500°F at the inner surface, and 1000°F at the outer surface in a short period of time, even though both surfaces were initially at room temperature. There is only one approach apparent that can fulfill the objective stated in the title other than cut and try techniques. Any experimental approach has limited value due to moving action of the projectile and the presence of large transient gradients at the bore surface. If the instantaneous effects are not considered, structural failure may result because of excessive thermal stresses near the surface of the solid where structural strength was already reduced by the high temperature.

What is the state of the art in the development of weapon or weapon systems? It is, in the author's opinion, a cut and try procedure. That is, in other words, a capability does not exist to generate the pertinent information regarding the behavior of the weapon subjected to any ammunition and firing schedule without preliminary design, prototype development, tests, and analyses. The next logical question is: Is it possible to formulate a program to obtain future weapon characteristics before actual development? The answer may be stated as "yes". NASA's NASTRAN program is quite familiar to the structural analysis community and also to the designers. If a program can be developed to deal with the influences of fluid dynamic aspects of weapon systems, this program in addition to NASTRAN can make it possible to develop future weapon systems without cut and try techniques. So, the rest of this paper will deal with the propellant gases, in particular, its influence on unsteady heat transfer analysis for any ammunition, gun, and firing schedule.

As the projectile moves ahead because of the high pressure gases created by burning propellant, the propellant gas will be set into motion starting from rest. Since the governing equations of fluid dynamics for many problems of interest are a system of nonlinear partial differential equations and are also dominated by real gas and nonequilibrium effects, no general solutions exist that allow arbitrary initial and boundary conditions. Therefore, examination of the flow field and subdivision of the overall problem by consideration of dominant features only seems appropriate. The objective is to establish a capability to perform overall heat transfer analysis for any given dimensions of the weapon and for specified propellant characteristics. Towards this goal, the propellant gas convective heat transfer problem is divided into several subproblems [1]: I - Generation of thermochemical properties for any chosen propellant; II - Analysis of

transient, inviscid, compressible flow through the gun barrel; ~~III~~ - Analysis of unsteady, viscous, compressible flow with arbitrary pressure gradients on the bore surface; ~~IV~~ - Analysis of transient heat diffusion through single or multi-layer gun tubes; V - Analysis of unsteady free convection and radiation around the gun tube; and ~~VI~~ - Control of a sequence of procedures due to the nature of unknown and transient boundary conditions and also due to coupling between the problems. Let us review each and every one of these problems in the following sections.

I THERMOCHEMISTRY

The generation of thermochemical properties for any propellant under consideration is essential to the development of a capability to perform overall heat transfer analysis and thereafter stress analysis for any given dimensions of the weapon and for any specified propellant characteristics. The thermochemistry of propellants involves determination of chemical composition of propellant gases either by finite-rate chemistry or by chemical equilibrium chemistry and the derivation of propellant gas properties from the composition [2]. The formulation of a finite-rate chemistry model involves the following events [3]:

- Selection of only important species, such as a dozen. The proper selection may depend upon the actual experiments.

- Selection of only important reactions. Of course, the number of species have to be matched with the number of reactions. Again, the proper selection may depend upon the experience, intuition, and observations.

- Determination of forward and backward reaction rates - sometimes, this amounts to either conducting special experiments or estimating them from kinetic theory of gases and empirical models.

- Formation of nonlinear ordinary differential equations for rate of change of species - one for each specie.

- Linearization (implies no significant change of species concentrations within a time step) by Taylor's series expansion.

- Development of special solution procedures to solve a large system of linearized, nonhomogeneous, first order ordinary differential equations, such as, combination of subdomain method, i.e., particular case of method of weighted residuals and matrix solution procedure.

• Postulation of "quasi-global" mechanism to bridge the gap between the original fuel and the equilibrium products of combustion - This step is essential because the detailed reaction mechanism becomes extremely complex even for hydrocarbon fuels, such as, kerosene (C_9H_{20}). Of course, this must be compatible with burning rate and ignition delay period.

It is clear that the above events up to a certain extent are hypothetical and also require prior knowledge. The feasibility of such a formulation is definitely questionable for any new (would be) propellants. It is important to look into the structure of propellants and the available information on such propellants. The constituents of typical propellants are as follows:

Physical	Chemical	IMR	M18
Nitrocellulose	$C_6\ H_{7.365}\ O_{10.27}\ N_{2.635}$	.8984	.7960
Nitroglycerin	$C_3\ H_5\ O_9\ N_3$	-	.0995
Dibutylphthalate	$C_{16}\ H_{22}\ O_4$	-	.0895
Dinitrotoluene	$C_7\ H_6\ O_4\ N_2$	.0718	-
Diphenylamine	$C_{12}\ H_{11}\ N$	.0062	.0099
Ethyl Alcohol (Residual)	$C_2\ H_6\ O$	.0053	.0049
Potassium Sulfate	$K_2\ SO_4$	.0089	-
Water (Residual)	H_2O	.0089	-

Since the important constituents are quite complex in their structure and the chain of reactions leading to lower compounds is unknown, it is impossible to consider finite-rate chemistry at the present time. However, the chemical equilibrium chemistry can be considered with possible hundreds of species and reactions in order to obtain the thermochemical properties of typical propellant gases.

The information on chemical equilibrium compositions of a chemical system permits one to calculate theoretical thermodynamic and transport properties for the system. Therefore, the first step is to find chemical equilibrium composition for any assigned thermodynamic state. Chemical equilibrium is usually described by either of two equivalent formulations, i.e., equilibrium constants or minimization of free energy. However, the minimization of free energy is somewhat better and, therefore, it is utilized.

The condition for equilibrium may be stated in terms of any of several thermodynamic functions such as the minimization of the Gibbs

free energy or Helmholtz free energy or the maximization of entropy. If one uses temperature and pressure to characterize a thermodynamic state, the Gibbs free energy is most easily minimized in as much as temperature and pressure are its natural variables. Similarly, the Helmholtz free energy is most easily minimized if the thermodynamic state is characterized by temperature and volume or density. The computer program based on these formulations is utilized to predict the composition for any propellant and for any assigned thermodynamic state.

The theory for deriving transport properties of gases from intermolecular potentials which determine the forces between any two molecules in the gas is available [4]. However, the determination of intermolecular potentials between complex molecules is almost impossible. Measurement and tabulation of transport properties such as thermal conductivity, λ, and dynamic viscosity, μ, for all gas mixtures and under all circumstances is not feasible. Thus, a means of predicting the transport properties from known data is necessary. Since computed composition of gases may not be that accurate either, the transport properties are computed by summing the products of individual mole fractions and corresponding transport properties. The individual transport properties at the desired tremperature, T, are obtained by the use of modified Sutherland's relation.

Some of the highlights of typical propellant gases are as follows:

- About 73 products of combustion are possible for IMR. However, the important species are: $CO = 0.43$, $H_2 = 0.12$, $H_2O = 0.21$, $N_2 = 0.12$, $CO_2 = 0.12$

- About 48 (less than 73, due to absence of potassium and sulphur elements) products of combustion are possible for M18. The important species are: $CO = 0.41$, $H_2 = 0.19$, $H_2O = 0.16$, $N_2 = 0.1$, $CO_2 = 0.08$.

- The gases were found to be highly toxic. The resultant gases may be due to the availability of less oxidants in those propellants. The consequences of incomplete combustion are muzzle flash, smoke, and fire in addition to low efficiency. However, there may be other effects that may be favorable from another view point. The ideal propellant gas should possess low burning, high impetus, and low molecular weight characteristics from the thermodynamic point of view. However, the impetus is directly proportional to the flame temperature and inversely proportional to the molecular weight. Also, the elements nitrogen and oxygen are heavier than carbon and hydrogen. Therefore, the ideal thermodynamic properties may be obtained at the expense of less and

less oxidants in the formation of propellants and also by promoting incomplete combustion.

• The major component of present day propellants is nitrocellulose, $C_6H_{7.365}O_{10.27}N_{2.365}$ (molecular weight = 274.575, oxygen = 59.85 per cent). Such propellants are expected to yield about 42 per cent of carbon monoxide. With the use of other major elements, such as, nitramine-HMX ($C_4H_8N_8O_8$) being investigated by other elements of DOD, such as the Air Force, the carbon monoxide is expected to be more than 50 per cent of propellant gases. The oxygen content of this propellant (molecular weight, 296) is only 43.24 per cent. However, this type of propellant showed higher burning rates and a larger spike in the pressure-time trace (a safety hazard), and thus contributed to lower ballistic efficiency. If the burning rates can be controlled to yield approximately constant pressures in a chamber, many of the weapon problems that exist today can be solved.

• The thermodynamic properties including adiabatic flame temperature and composition are determined for various pressures. The variations with pressure is interpreted as due to variations in composition with pressure. The increase in pressure causes increase in molecular weight, adiabatic flame temperature and specific heats, but a net decrease in specific heats ratio.

• The transport properties of propellant gases are much more important than the thermodynamic properties as far as forced convection is concerned. The convective heat transfer coefficient is directly proportional to the thermal conductivity of gases. Since the mean free path of gas molecules is inversely proportional to the pressure, and number of molecules per unit volume is directly proportional to the pressure, the thermal conductivity (of gases) which is a product of the two is not a strong function of pressure. However, the thermal conductivity depends strongly on the temperature of gases. The thermal conductivity increases slightly faster than the dynamic viscosity, but increases many fold (more than three times, 500 - 2000°K range) with increase in temperature. However, the Prandtl number is nearly independent of temperature. It is to be noted that the variation of specific heat with temperature is included in the calculation of the Prandtl number.

II CORE FLOW

Since the composition and thermodynamic properties are established in the above section for any chosen propellant, it is now logical to define the transient inviscid compressible flow through the central section of the barrel. Almost all predictions of the behavior

of propellant systems are based on an admixture of intuition, experience and the use of interior ballistic equations. These equations assume space-mean averages for the thermodynamic variables and include dynamic effects only through the inclusion of various correction factors determined more or less, empirically. It is also common to assume either uniform density or a linear velocity gradient in the flow field between the breech and the bullet at any time during weapon firing.

The core flow model, while necessarily a gross over-simplification of the physical system, would permit at least a study of the distribution of thermodynamic variables and provide more accurate interior ballistics data than the data obtained by conventional methods to aid in the design of muzzle brakes, noise and flash suppressors, and also gas ports for automatic actuating mechanisms.

A model involving unsteady, one-dimensional motion would apparently be the most that could be tolerated in complexity. Moreover, this model does not warrant any more complexity due to the limited knowledge in solid propellant particle movements and in associated burning models. The gas is assumed to be inert and also without viscosity and heat conduction effects. This is in compliance with Prandtl's idea of separating the flow into core flow and boundary layer flow. For convenience, the mass and energy are assumed to be distributed continuously throughout the gas flow, although such an assumption is unnecessary to apply the chosen method of approach for the solution to the problem. The governing equations are as follows:

Continuity:

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \rho u \frac{\partial}{\partial x} \ln A = Q \left(1 - \frac{\rho}{\delta}\right)$$

Momentum:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{u}{\rho} Q - \frac{2f}{D} u^2$$

Energy:

$$\rho \frac{\partial}{\partial t} \left(e + \frac{u^2}{2}\right) + \rho u \frac{\partial}{\partial x} \left(e + \frac{u^2}{2}\right) + Q \left(e + \frac{u^2}{2}\right) = WQ - \frac{1}{A} \frac{\partial}{\partial x} (pAu)$$

State:

$$p \left(\frac{1}{\rho} - \eta\right) = RT \quad \text{or} \quad p \left(\frac{1}{\rho} - \eta\right)^{\gamma} = C\, e^{s/C_v}$$

The wall friction does not appear directly in the energy equation since its action is that of converting mean kinetic energy to thermal energy. The above governing equations of unsteady, inviscid, and compressible flow are hyperbolic in nature. In principle, one can solve them by the well-known method of characteristics. Along certain (characteristic) directions, the coupled partial differential equations can be reduced to a set of simultaneous ordinary differential (characteristic) equations. The solution of such a set of equations is further complicated by an unknown nonhomogeneous, and unsteady (moving bullet) boundary condition and time-dependent propellant burning model.

The governing equations can be conveniently written with u, a (speed of sound), and s (entropy) as dependent variables instead of u, ρ (density), and e (internal energy). If the definition of perfect differentials is included as additional equations, one can obtain any partial derivative of these dependent variables by Cramer's rule of determinants. Since the characteristic directions can be defined as the curves along which these derivatives are discontinuous, one can obtain the following sets of equations by forcing respectively the denominator and the numerator of any one of those partial derivatives to zero.

Characteristic directions:

$$\frac{dx}{dt} = x + a \qquad \frac{dx}{dt} = x - a \qquad \frac{dx}{dt} = u$$

Governing equations along characteristic directions:

$$\frac{Du}{Dt} + \frac{2(1-\rho\eta)}{\gamma-1+2\rho\eta}\frac{Da}{Dt} = \frac{a}{C_p}\frac{(1-\rho\eta)(1-2\rho\eta)}{\gamma-1+2\rho\eta}\frac{Ds}{Dt} + aQ(1-\rho\eta)(W + \frac{u^2}{2} - C_pT)/C_p$$

$$+ aQ(1/\rho-1/\delta) - uQ/\rho - 2fu^2/D$$

$$\frac{Du}{Dt} - \frac{2(1-\rho\eta)}{\gamma-1+2\rho\eta}\frac{Da}{Dt} = -\frac{a}{C_p}\frac{(1-\rho\eta)(1-2\rho\eta)}{\gamma-1+2\rho\eta}\frac{Ds}{Dt} - aQ(1-\rho\eta)(W + \frac{u^2}{2} - C_pT)/C_p$$

$$- aQ(1/\rho-1/\delta) - uQ/\rho - 2fu^2/D$$

$$\frac{Ds}{Dt} = \frac{Q}{\rho T}\left[W + \frac{u^2}{2} - e - p(\frac{1}{\rho} - \eta)\right]$$

The following information is assumed in order to complete the formulation:

Propellant burning rate (r):

$$r = kp^n, \quad \omega = \sigma\int_0^{\hat{\sigma}} r\,dt, \quad Q = \delta r\sigma/\omega$$

Initial and Boundary Conditions:

The gases are at rest at the breech. The gases at the base of the projectile are allowed to move with the projectile velocity because of a continuity requirement. Even though, the continuity requirement is assumed, this is still an unknown boundary condition. However, a relation can be formulated by consideration of a dynamic balance for the moving projectile.

$$\text{i.e.,} \quad m\frac{dV}{dt} = pA-F$$

Two numerical examples are generated, one of which is applicable to small arms and the other to an artillery weapon. Further details are available elsewhere [5]. The highlights are as follows:

- As the bullet moves ahead because of the high pressure gases, it issues rarefaction waves which will travel toward the breech. The reflected waves from the breech will interact with the oncoming rarefaction waves and thus form a network similar to a coordinate grid system. The path of these waves may be called characteristic curves (Figure 1 of reference 5). The applicable ordinary differential equations along these curves can be solved by finite difference techniques.

- The density distribution at various times is shown in Figure 5 of reference 5. The pressure distribution is similar to the density distribution. The uniform density between bullet and breech is difficult to justify.

- The velocity distribution is shown in Figure 7 of reference 5. For the first portion of bullet travel (approximately 20 per cent), the linear velocity distribution is not true.

- In general, the assumption of a linear velocity gradient is better than the assumption of uniform density. Note that the uniform density assumption implies a linear velocity gradient due to the equation of continuity.

- The velocity of the gas at any location increases with time and decreases significantly even before the bullet leaves the muzzle.

- Since this solution procedure is not significantly more difficult than the current practice of solving only ordinary differential equations by consideration of space mean averages, it is recommended to consider at least the model presented here in any interior ballistics studies.

III BOUNDARY LAYERS

The separation of gas flow into core flow and boundary layer flow and procedures to solve independently can be justified for thin boundary layers, i.e., for low viscosity and large Reynolds number flows. Otherwise, one may be able to solve either simultaneously or one after the other by some iteration scheme. Under any circumstances, one should have a capability of handling laminar and turbulent cases of unsteady, viscous, compressible flow with arbitrary pressure gradients on a bore surface.

The rate of heat transfer from the hot propellant gases to the barrel is controlled by the development of the boundary layers. The flow in the gun barrel boundary layers could be laminar, transitional, or turbulent in nature. The type of boundary layer at a particular cross section at any instant need not be the same as at another instant. Since the flow has to start from rest and must also satisfy zero boundary layer thickness at the bullet base because of the scraping action of bullet, laminar flow always exists in some parts of the gun barrel boundary layers.

Flow in a laminar boundary layer will eventually become unstable as the Reynolds number is increased. The boundary layer thickness, skin friction, and heat transfer increases many fold for turbulent flow over laminar flow. The mechanics of turbulence or more common eddy viscosity is the dominating mechanism for such increases. The boundary layer flow can be turbulent somewhere in the middle of the flow between the breech and the bullet base. A transitional regime should exist between the laminar and turbulent regimes. However, because of limited knowledge about transitional regimes, the flow will be assumed to change suddenly from laminar to turbulent at a time and a place determined by a well-known laminar-turbulent transition criteria. Therefore, the unsteady boundary layer analysis is needed for laminar and turbulent boundary layers.

The state of the art in unsteady boundary layers [6] and turbulent models [7,8] is quite limited. The literature cannot be reviewed here due to lack of space. However, Yalamanchili [9,10] reviewed it in some detail in the past. It is concluded that no other investigator has solved an unsteady compressible laminar, and turbulent boundary layer problems with arbitrary pressure gradients and arbitrary free stream conditions. The governing boundary layer equations with eddy viscosity, ϵ_m, formulation in dimensionless form are as follows:

Continuity: $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

X-Momentum:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \rho \frac{\partial u_w}{\partial t} + \frac{1}{Re} \frac{\partial}{\partial y} \left[\frac{\partial u}{\partial y} (\mu + \rho \varepsilon_m)\right]$$

Energy:

$$\rho\left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial}{\partial y} \left[\frac{\partial T}{\partial y} \left(\frac{\mu}{Pr} + \frac{\rho \varepsilon_m}{Pr_t}\right)\right]$$
$$+ \frac{1}{Re} \left(\frac{\partial u}{\partial y}\right)^2 \left(\mu + \frac{\rho \varepsilon_m}{Pr_t}\right) + \frac{1}{Re} \mu \frac{\partial}{\partial y} \left[\frac{\partial u}{\partial y} \frac{\rho \varepsilon_m}{Pr_t} (1 - Pr_t)\right]$$

Thermodynamic State: $p\,(1/\rho - \eta) = RT$

$$Re = \frac{U^* L^* \rho^*}{\mu^*} \qquad Pr = \frac{\mu c_p}{k} \qquad Pr_t = \varepsilon_m / \varepsilon_h$$

Inner layer:

$$\varepsilon_m = Re(\kappa_m y)^2 \left(1 - e^{-y/A}\right)^2 \frac{\partial u}{\partial y}, \quad A = \frac{26}{\sqrt{Re}} \nu \left(\rho/\tau_w\right)^{1/2} \left(1 - 11.8\, p^+\right)^{-1/2}$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}, \quad p^+ = - \sqrt{Re} \left(\frac{\partial p}{\partial x}\right) \frac{\nu}{\rho} \left(\frac{\rho}{\tau_w}\right)^{1/2}; \quad k_m = 0.4$$

Outer layer:

$$\varepsilon_m = 0.0168\, Re \left| \int_0^\infty (u_i - u)\, dy \right| \Big/ \left[1 + 5.5 \left(\frac{y}{\delta}\right)^6\right]$$

Boundary Conditions:

$y = o$: $u = u_w(t)$, $v = o$, $T = T_w(x,t)$

$y = \infty$: $u = u_i(x,t)$, $T = T_i(x,t)$, $p = p_i(x,t)$

$x = x_u$: $u = u_u(y,t)$, $T = T_u(y,t)$

The governing equations are a system of nonlinear parabolic partial differential equations with three independent variables. No general solutions exist which allow arbitrary initial and boundary conditions. Various solution procedures are attempted because of complexity and importance. One approach involves the use of method of weighted residuals (method of moments) and the method of characteristics in order to reduce only continuity and momentum equations to ordinary differential equations and numerical integration in the end [1]. Thus, the use of existing analogy between momentum and energy transfer for steady flows is required to predict the convective heat transfer. To check the validity of analogy for unsteady cases, another approach

*Referenced quantities

involves the use of some techniques on continuity, momentum and energy equations, but with some more assumptions. If the analogy, such as Colburn, is valid for unsteady boundary layers with arbitrary pressure gradients, the convective heat transfer coefficients computed by two approaches for any example should be almost the same. However, these are not. The results based on the analogy underestimate as much as 100 per cent for small times, however, the same analogy overestimate the convective heat transfer coefficients up to 50 per cent for large times. It is surprising that the Colburn analogy predict almost identical results as obtained without the use of an analogy at one of the times considered. This time is about one-half of the projectile travel time inside the tube.

Since more assumptions are involved in the above approaches than desirable, other solution procedures are attempted. The transverse coordinate is modified to absorb the compressibility effect. The stream function is introduced to satisfy the continuity and thus to eliminate one of the dependent variables. In the analytical approach [11], the method of weighted residuals is used to reduce by one the number of independent variables. No way presently seems to be available to select the functions systematically for all problems. However, the functional in solution form for the dependent variables is chosen on the basis of the asymptotic solution of steady differential equations for large values of the space-like coordinate. The error functions consequently occur for forced convective flows. The weighting function (Galerkin) is chosen on the basis of least error for wall shear. All integrations across the boundary layer are performed analytically. The Method of Lines is used to eliminate another independent variable. Finally, numerical solutions are obtained by a fourth order Runge-Kutta integration scheme. Since no real test case exists for such a general solution procedure, the Rayleigh Blasius incompressible flow on a plate was considered [9,10]. The results are in satisfactory agreement with Hall's transient results and Blasius steady state results with even one term in solution form. Another example, the shock induced compressible boundary layer problem is investigated because of its similarity to bullet induced boundary layer problem. Again, the results are satisfactory.

The above procedure may be tedious for turbulent boundary layers and also for arbitrary initial and boundary conditions. Therefore, a second-order accurate numerical scheme is being developed for the analysis of laminar and turbulent boundary layers. The scheme has many desirable features, theoretically and computationally. For example, the Richardson's extrapolation scheme is adaptable and thus possible to increase the accuracy by another two orders of magnitude. However,

the preliminary results indicate drastic increase in computational times over the previously discussed techniques and also quite sensitive to the input of initial and boundary conditions. But, this is a valuable tool to qualify analytical models, to study the structure of unsteady boundary layers, and also to determine accurately the highly unsteady convective heat fluxes.

IV HEAT DIFFUSION

In addition to various investigators all over the world, Yalamanchili and Chu [12,13,14,15] investigated finite element, finite difference, and weighted residuals methods for transient two-dimensional heat diffusion problems. The governing equations, initial and boundary conditions, stability and oscillation characteristics, and applications are available in those references.

V FREE CONVECTION AND RADIATION

The outer surface temperature of the gun tube may change from -60°F to 1500°F because of the high rate of fire. Since wind velocities help to cool the gun tube faster than free convection and radiation, thermal design of a gun tube should not include wind velocities. The computation of the product of the Grashof and the Prandtl numbers for typical gun barrel conditions (10^6) indicate that the flow is in the laminar regime (10^3 to 10^9). The contributions of free convection and radiation are almost equal where the outer surface temperature of the gun tube is 400°F. The free convective contribution dominates over radiation for temperatures much less than 400°F and vice versa. The governing equations of unsteady free convection and radiation are a system of nonlinear partial differential equations of parabolic type with three independent variables and with variable linear and nonlinear (radiation) boundary conditions. Yalamanchili [16] discussed the solution procedure by explicit finite differences and also results. The conclusions are as follows: The results are stable even with 40 per cent larger time step sizes than obtained by stability criteria. The thermal boundary layer in the upper half of the cylinder is much thicker than in the lower half of the cylinder. The thermal boundary layer thickness increases not only with time but also with position for large times. The velocity boundary layer thickness also grows with time but not as fast as the growth of the thermal boundary layer thickness. The dimensionless heat transfer is fairly uniform for small times and changes up to 50 per cent over its maximum at that time.

COMBINED ANALYSIS

The solutions obtained for the individual fictitious problems do not represent the solutions for the real gun tube problems because of continuous change in boundary conditions. Compatible boundary conditions must be introduced at interfaces between the problems. It is

convenient to generate ahead the thermochemical properties for various pressures for any chosen propellant and later input in the form of tables for problems II and III. To solve core flow problem, one may require the location of boundaries, in particular, the interface between problems II and III. Since the location of this interface is unknown and also the thickness of boundary layers, in general, is small, one can reasonably assume that the interface is the bore surface for problem II. However, one should include the information at the interface such as the heat loss to the gun tube and skin friction. These quantities will depend on the development of boundary layers and also the temperature of the bore surface. Sometimes, the assumption that the outer edge of the boundary layer extends to the centerline of the tube because of negligible viscous and heat conduction effects in core flow is convenient.

Even though the boundaries are well defined for a chosen gun for heat diffusion, the information at the boundaries such as heat-in due to forced convection and heat-out due to free convection and radiation are lacking because these in turn are dependent upon problem IV. It seems that there is strong coupling between boundary layer (forced convection) problem and heat diffusion problem because of rapid change in conditions at the interface. The combined unsteady forced convective and heat conductive problem will be solved once analytical capability in unsteady boundary layers is well established. The classical heat diffusion equation possesses several pathological anomalies, the most important one being an infinite speed of heat wave or thermal shock propagation. The use of the Fourier model is definitely questionable for heat pulse problems, plasma torch, etc., where high heat fluxes may be present. Therefore, a non-Fourier model is being investigated because of the possibility that the bore surface temperature and its gradient could be even higher than the classical results. The heat transfer rates due to frictional heating between rotating band and rifling at the rubbing contact are very high. Probably, these could be much higher than the propellant gas convective heat transfer rates. Similarly, the forced convective heat transfer will be substantially higher for hypervelocity systems because of higher gas velocities and also large temperature differences between the gas and the tube. The effect of non-Fourier effects could be significant at least in such circumstances. Finally, it should not be forgotten about the possibility of interaction between core flow and boundary layer flow for hypervelocity systems such as in hypersonic flow [17]. It is also important to recognize the existence of shock ahead of the projectile for hypervelocity systems and consider its effects, such as resistance induced on projectiles (core flow) and also shock induced compressible boundary layers. All the models proposed are quite rigorous, but fail to forecast why the overall objective can not be fulfilled.

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