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THEORETICAL INVESTIGATION OF ROCK AND SUPPORT INTERACTION - DEVELOP MORE RATIONAL DESIGN METHODS AND DEVELOP NEW TYPES OF SUPPORT AND MINING SYSTEMS

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Bureau of Mines

Prepared for:

Advanced Research Projects Agency

January 1974

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER P89-2A	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Theoretical Investigation of Rock and Support Interaction - Develop More Rational Design Methods and Develop New Types of Support and Mining Systems - Part 1		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report Feb. 1971 - June 1972
		6. PERFORMING ORG. REPORT NUMBER P89-2A
7. AUTHOR(s) William J. Karwoski		8. CONTRACT OR GRANT NUMBER(s) ———
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Bureau of Mines, Spokane Mining Research Center, North 1430 Washington Street Spokane, Washington 99201		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62701D, 1579 2B32, F53405
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209		12. REPORT DATE January 1974
		13. NUMBER OF PAGES 92
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this document is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Finite Element Computer Codes
Geotechnical Problems
Rock Mass Behavior
Roof Support

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The finite element method was applied to several practical problems in geotechnical engineering. An underground power station was investigated with a finite element computer code having features of elastic, elastic-plastic, and no-tension modeling of rock properties. This study showed that large wall displacements in the power plant were caused by movements of a large rock block in the rock mass.

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20. ABSTRACT (Continued)

Another study was made of roof deflection due to increasing roof spac in an underground mine. From this study it was possible to determine the in-situ modulus of the rock mass. This modulus can then be used to design openings in the in-situ material.

A third study was conducted on stresses in a steel liner used in underground support. The results showed that increasing the number of blocking points reduced the bending moments in the support.

Detailed descriptions of these studies and information on obtaining copies of technical reports and other materials generated under these contracts are given in the Appendices.

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UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF MINES

SPOKANE MINING RESEARCH CENTER
SPOKANE, WASHINGTON

FINAL TECHNICAL REPORT

Bureau of Mines In-House Research

Theoretical Investigation of Rock and Support Interaction - Part 1

Sponsored by

Advanced Research Projects Agency (ARPA)
ARPA Order No. 1579, Amendment No. 2
Program Code No. 1F10

92

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Form Approved
Budget Bureau No.: 22-R0293

Final Technical Report

ARPA Order No.: 1579 Amendment 2

Effective Date: Feb. 1, 1971

Program Code: 1F10

Expiration Date: June 30, 1972

Originating Agency: U.S. Bureau of Mines
Spokane Mining Research Center
1430 N. Washington St.
Spokane, Washington 99201

Amount Funded: $57,642

Principal Investigator: W. J. Karwoski
Telephone No.: 509/456-2523

Title: Theoretical Investigation of Rock and Support Interaction - Develop More Rational Design Methods and Develop New Types of Support and Mining Systems - Part 1

Sponsored by:

Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, Virginia 22209

TECHNICAL REPORT SUMMARY

Objective

The purpose of this study is to mathematically model the load-transfer characteristics between rock and selected support systems, and to investigate the response to varying rock-behavioral characteristics.

General Approach and Technical Results

Three geotechnical engineering of commercial value were investigated. Detailed reports are given in Appendices A, B, and C. Roof and wall movements in an underground power plant were calculated and composed with field data. In-situ rock mass properties were determined from measured roof deflections, and axial and bending moments in a steel support were calculated to determine the applicability of two-dimensional finite element computer codes to that type of problem. With the experience gained from these studies a computer code was developed which consolidated all the existing and proven finite element technology for two- and three-dimensional computer codes into one "user oriented" computer code.

Technical Problems

In setting up and executing these problems, it was determined that modeling of the structural features of the rock using finite-element computer codes has out-stripped the availability of rock property data.

DOD Implications

None.

Implications for Further Research

Additional research should be made in the field to determine in-situ conditions of rock mass behavior and the state of stress. This information can then be used in existing finite element computer codes.

Special Comments

None.

TABLE OF CONTENTS

FIGURES

THEORETICAL INVESTIGATION OF ROCK AND SUPPORT INTERACTION

The purpose of this study is to mathematically model the load-transfer characteristics between rock and selected support systems, and to investigate the response of these structures to varying rock-behavioral characteristics. The schedule and tasks required to accomplish this mission is shown in figure 1. Tasks undertaken include: (1) determine the state-of-the-art of rock--support interaction models; (2) determine the availability of engineering parameters; (3) develop a weighting process for the parameters; (4) obtain and modify computer codes; and (5) perform parameters study. Through contract monitoring (task 6), computer codes were developed to model rock mass and ground support systems. This technology was then applied to problems of commercial value.

Manpower shortages, employment freeze, and the reduction-in-force prevented full accomplishment of the tasks delineated.

Tasks 1 through 4 were completed. With respect to tasks 1 through 4, the following conclusions were reached.

Modeling of the structural features of rock using finite-element computer codes has out-stripped the availability of rock property data. Because of the complexity of rock mass, it is doubtful that detailed finite-element calculations will ever be practicable for routine design of underground openings. Generalized models incorporating only the salient features of the rock mass could be used for phenominological studies. Finite-element techniques can be helpful in computing the state of stress for structures whose behavior is unsatisfactory when known displacements serve as a check on the values of physical properties chosen. Definite need exists for artificial support property data; and capability of 3-D models must be improved and extended to increase the scope of practical problems that can be analyzed.

A study also was conducted on circular and square openings in layered rock. The magnitude of the field stress used was that expected in deep mining in the Coeur d'Alene mining district as no agreement on the magnitude and direction of the stress field could be found in literature. The physical properties, chosen to give an in situ overall elastic modulus of 2.5×10^6, and the allowable compression and tension values were based on physical property tests made at the University of Idaho. Trend of the results was as expected; that is with stiffer material there is less displacement; there is a preferred orientation of the opening with respect to the stress field; and the steeper the Mohr's envelope, the less plastic deformation. Significance of this study is that under similar conditions of stress and rock properties, rock displacements in the district can be 18 inches while the elastic-plastic calculations give only 3 inches--some phenomena other than plastic flow is the primary cause of the rock movement in field situations. Two manuscripts were written describing the finite element analysis of rock movement in an underground powerplant and roof movements due to increasing roof span. These documents are in Appendix A and Appendix B. Task 5 (to begin in FY73) was incorporated into a statement of work change and included as a modification to Ohio State University Contract No. H0210017[1/]. The results of the task 5 parametric investigation showed that:

The load transfer to the structural support system is dependent upon the tendency of the rock to deform. Higher modulus implies less deformation for the same rock load and consequently less load transferred to the steel member.

Higher values, tending toward .5, of Poisson's ratio are associated with redistribution of stress in the rock. This stress redistribution is reflected in a more uniform stress distribution in the steel support.

1/ Contract H0210017 "Stresses, Deformations, and Progressive Failure on Non-Homogeneous Rock" with Ohio State University was awarded on February 1, 1971. The contract was later amended to include the modification mentioned (Task 5). Information on obtaining reports and magnetic tape containing this computer program is shown on page 55.

Sparce blocking increases bending moments in the support structure. The total load transferred to the support structure decreases as the amount of blocking between the steel set and rock increases. The wood blocking acts much like a "back packing" material.

A complete manuscript describing the details of this work is given in Appendix C.

As a consequence of this work, it was recommended and accepted that a contract be funded which would consolidate all the existing and proven finite element technology for two- and three-dimensional computer codes into one "user oriented" computer code. This contract was funded (ARPA contract H0220035) and completed. Appendix D is a descriptive summary of the computer code capability. Subsequently, an era of computer code application is anticipated. From application, we hope to discover the deficiencies within the state-of-the-art and identify where fundamental research is needed and make appropriate recommendations for future research.

Manuscripts:

"Finite Element Analysis of Excavations in Rock," by Chin-Yung Chang, Keshavan Nair, and William J. Karwoski. Oral presentation and subsequent publication in the proceedings of Corps of Engineers, Waterways Experiment Station Symposium on "Applications of the Finite-Element Method to Problems in Geotechnical Engineering", held May 1-4 in Vicksburg, Mississippi.

"Roof Movement Due to Increasing Roof Span in an Underground Opening," by William J. Karwoski, Ranbir S. Sandhu, R. Singh.

Oral presentation and subsequent publication of abstract in the proceedings of the 14th. Annual Rock Mechanics Symposium, held June 12-14, in University Park, Pennsylvania.

"A Parametric Study of Stresses in Steel Support for a Tunnel," by Ranbir S. Sandhu, and William J. Karwoski. Oral presentation and subsequent publication in proceedings of the 15th Annual Rock Mechanics Symposium, held September 17-19, at the South Dakota School of Mines and Technology, Rapid City, South Dakota.

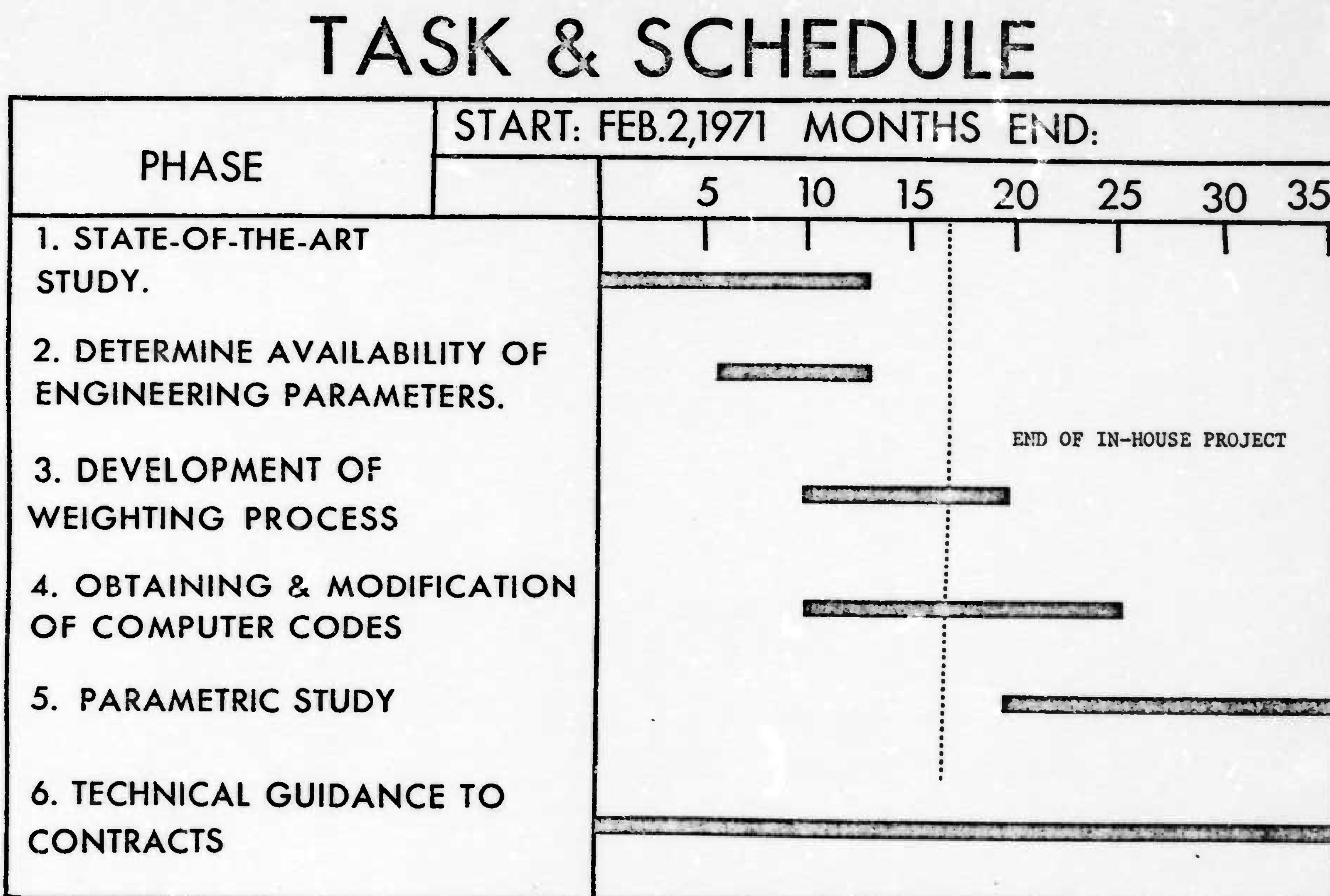

FIGURE 1. - Task and Schedule.

APPENDIX A

The manuscript of "Finite Element Analysis of Excavating in Rock" presented by the Symposium on Applications of The Finite Element Method in Geotechnical Engineering held at the U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, MS, May 1-4, 1972.

This presentation included results of research performed under contract H0210046 "A Theoretical Method for Evaluating Stability of Openings in Rock" with Woodward-Lundgren & Assoc., Oakland, Calif., Dr. Keshavan Nair, Principal Investigator. Technical reports from this contract and the extension of this work (contract H0220038 with the same organization) are available from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, Va. 22151. Following is the information necessary for ordering these reports.

	AD No.	Price
Contract H0210046 (first year effort)		$3.00[1]
Final Technical Report	AD-740 341	[1]
Contract H0220038 (second year effort)		
Final Technical Report	AD-773 861/OGI	$5.75[1]

[1] Price shown is for paper copy, microfiche copies are also available at $1.45 per report.

APPENDIX A

FINITE ELEMENT ANALYSIS OF EXCAVATIONS IN ROCK

By

Chin-Yung Chang, Research Engineer, Woodward-Lundgren & Associates, Oakland, California

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William J. Karwoski, Mining Engineer, Bureau of Mines, Spokane, Washington

Prepared for

Symposium on Applications of the Finite Element Method

to Problems in Geotechnical Engineering

Department of the Army
Waterways Experiment Station
Corps of Engineers

Vicksburg, Mississippi
1-3 May 1972

FINITE ELEMENT ANALYSIS OF EXCAVATIONS IN ROCK

By

Chin-Yung Chang[1], Keshavan Nair[2] and William J. Karwoski[3]

SUMMARY

A general finite element computer program for the analysis of plane problems which incorporates no tension, joint perturbation and elasto-plastic behavior was developed. The results of analyses were compared with actual performance for the Morrow Point Powerplant excavation and a laboratory model study of an opening in a rock-like material.

The comparison indicated that the developed analytical techniques had the capability of providing a good qualitative prediction of performance. Quantitative prediction was also considered to be acceptable in view of the input information and idealization.

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mass due to yielding have been presented by Reyes and Deere (1966). Preliminary analysis has indicated that these new methods of analysis using finite element techniques have the potential of predicting performance with improved accuracy. There has been very limited verification of these techniques on the basis of comparison with measured field performance. Without field verification, the use of these new techniques in the design of excavations in rock will remain limited. Therefore, to develop a theoretically sound method for designing excavations in rock, which will be used in practice, an essential step is to verify the ability of the available methods of stress analysis in predicting the behavior of rock masses.

The study reported herein can be divided into two phases: (1) Development of a general analytical procedure (i.e., a finite element computer program with wide capabilities) for determining the mechanical state in a rock mass, and (2) Analysis of case histories to compare predicted and observed values.

DEVELOPMENT OF A GENERAL COMPUTER PROGRAM

In order to develop a single general computer program, it was necessary to develop a consistent computational technique to model the different aspects of rock behavior. On the basis of the review of the available techniques, it was concluded that the "initial stress" (stress transfer) technique presented by Zienkiewicz and his co-workers would provide a consistent approach in the development of a finite element computer program for time-independent plane problems which included the following capabilities: (1) No Tension Analysis, (2) Joint Perturbation Analysis, and (3) Elasto-Plastic Analysis. The major reason for selecting this approach was the computational advantage that results from using the initial elastic stiffness at every stage of the solution process. The essential concepts used to include the above listed rock characteristics are discussed in the subsequent paragraphs.

I. No Tension Analysis

The basic concept used in the combined computer program for the no tension analysis is similar to that developed by Zienkiewicz, et al (1968). The major steps in the analysis used can be summarized as follows:

1. Assign initial stresses to the rock mass, and calculate the boundary loads required on the cavity face to simulate the creation of the opening and other structural loads applied to the system.

2. Analyze the problem as a linear elastic problem. Add the induced changes in stress to the initial stresses and compute the principal stresses.

3. Determine those elements in which tension exists. If the material is assumed incapable of sustaining any tension, or if the tensile stress exceeds the tensile strength, the excess tensile stresses have to be eliminated. This is done by applying nodal point forces calculated to eliminate tensile stresses.

4. The elastic analysis is repeated for the calculated equivalent nodal point forces; stresses are determined in the elements. The check for tensile stresses is repeated.

5. If, at the end of step (4), principal tensile stresses are still present, steps (3) and (4) are repeated until there is no appreciable difference in magnitude and distribution of stresses with further iterations.

In Step (3) when the linear elastic solution indicates that the rock is subjected to tensile stresses greater than the tensile strength, the rock is assumed to be fractured and incapable of transferring stresses between two orthogonal directions. To include this effect, a correction has to be made on the stress before the stress transfer process is performed. In the case where both principal tensile stresses σ_x' and σ_y' are to be transferred, the 'corrected' stresses are given by (Chang and Nair 1972)

$$\begin{aligned} \sigma_x &= (1 + \nu^2 + \ldots)\,\sigma_x' + \nu\,(1 + \nu^2 + \ldots)\,\sigma_y' \\ \sigma_y &= (1 + \nu^2 + \ldots)\,\sigma_y' + \nu\,(1 + \nu^2 + \ldots)\,\sigma_x' \end{aligned} \tag{1}$$

II. Joint Perturbation Analysis

The stiffness matrix of one-dimensional joints is formulated according to the procedure developed by Goodman, Taylor, and Brekke (1968). The one-dimensional joint with its local coordinate system and the sign convention of its normal and shear stresses is illustrated in Figure 1.

The shear strength of a joint may be expressed by

$$\tau_f = c + \sigma_N \tan \phi_e \tag{2}$$

where

τ_f = shear strength of the joint
c = cohesion along the joint
σ_N = normal stress across the joint
ϕ_e = effective friction angle of the joint surface (Patton 1966)

If the normal stress across the joint is tensile, it is assumed that the joint is incapable of resisting any shear stress, i.e., it has no strength. A flow diagram illustrating the stress transfer technique for the joint perturbation analysis is shown in Figure 2.

III. Elasto-Plastic Analysis

In developing an elasto-plastic analysis, it is necessary to define a yield function and the stress-strain relations before

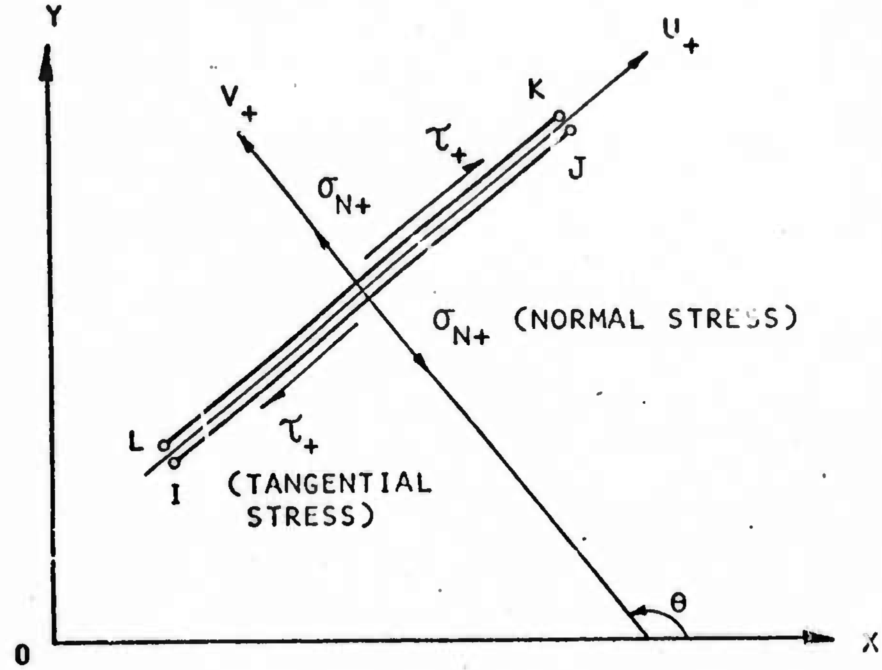


FIG. 1 - LINKAGE OR "JOINT" ELEMENT WITH ITS LOCAL COORDINATE SYSTEM (After Goodman, et al. 1968)

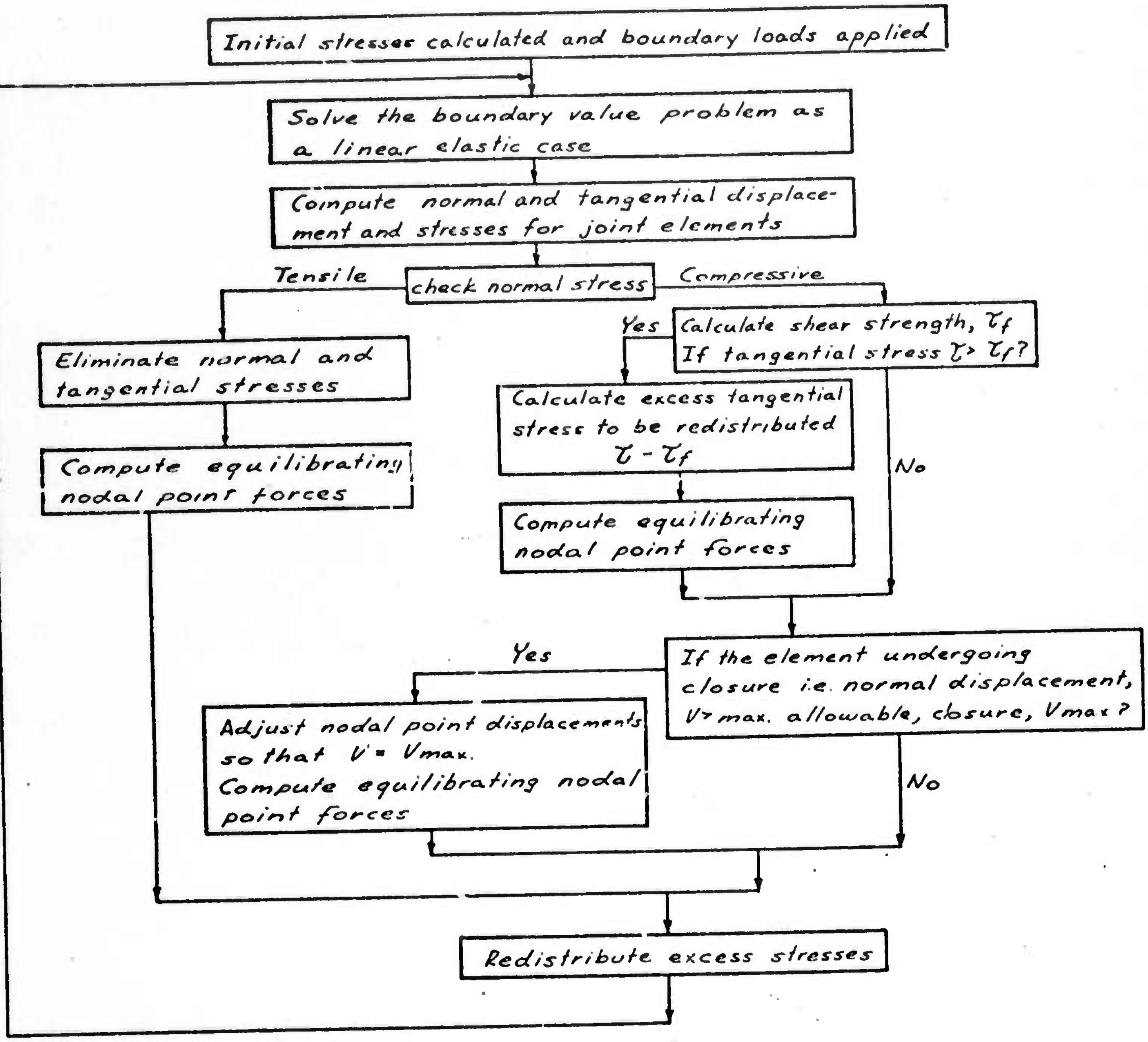


FIG. 2 -"STRESS TRANSFER" TECHNIQUE FOR JOINT PERTURBATION ANALYSIS

and after yield. Prior to yield, it is assumed that linear elastic stress-strain relations are applicable.

Yield Function

The yield function utilized is a generalization of the Mohr-Coulomb hypothesis suggested by Drucker and Prager (1952). The yield function is represented by the following equation:

$$f = \alpha J_1 + \sqrt{J_2} = k \tag{3}$$

where:

f and k = material constants

J_1 = first stress invarient

J_2 = second invarient of stress deviation.

α = material constant

The yield surface expressed by Equation (3) for $\alpha>0$ is a right circular cone with its axis equally inclined to the coordinate axis.

In the case of plane strain, Drucker and Prager (1952) have shown that

$$\alpha = \frac{\tan \phi}{(9 + 12 \tan^2 \phi)^{\frac{1}{2}}} \tag{4}$$

and

$$k = \frac{3c}{(9 + 12 \tan^2 \phi)^{\frac{1}{2}}} \tag{5}$$

where:

c = the cohesion of the material

ϕ = angle of internal friction of the material.

Stress-Strain Relations

During an infinitesimal increment of stress, changes in strain are assumed to be composed of elastic and plastic parts if the element is in yield, i.e.

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon\}_e + \{\Delta\varepsilon\}_p \tag{6}$$

The elastic strain increments are related to stress increments by the generalized Hooke's law as

$$\{\Delta\sigma\} = [D]\,\{\Delta\varepsilon\}_e \tag{7}$$

[D]is the linear isotropic elastic coefficient matrix.

Utilizing the Drucker Prager criteria (equation 3), Reyes (1966) developed elasto-plastic stress-strain relations. These relations can be expressed as follows:

$$\{\Delta\sigma\} = [D]_{e.p.}\,\{\Delta\varepsilon\}_p \tag{8}$$

where the elements of $[D]_{e.p.}$ are defined as:

$$\begin{aligned}
D_{11} &= 2G(1 - h_2 - 2h_1\,\sigma_x - h_3\,\sigma_x{}^2) \\
D_{22} &= 2G(1 - h_2 - 2h_1\,\sigma_y - h_3\,\sigma_y{}^2) \\
D_{33} &= 2G(1/2 - h_3\,\tau_{xy}{}^2) \\
D_{12} &= D_{21} = -2G\left[h_2 + h_1\,(\sigma_x + \sigma_y) + h_3\,\sigma_x\,\sigma_y\right] \\
D_{13} &= D_{31} = -2G(h_1\,\tau_{xy} + h_3\,\sigma_x\,\tau_{xy}) \\
D_{23} &= D_{32} = -2G(h_1\,\tau_{xy} + h_3\,\sigma_y\,\tau_{xy})
\end{aligned} \tag{9}$$

$$h_1 = \frac{\frac{3K\alpha}{2G} - \frac{J_1}{6J_2^{\frac{1}{2}}}}{J_2^{\frac{1}{2}}\,(1 + 9\alpha^2\,\frac{K}{G})}$$

$$h_2 = \frac{\left[\alpha - \frac{J_1}{6J_2^{\frac{1}{2}}}\right]\left[\frac{3K\alpha}{G} - \frac{J_1}{3J_2^{\frac{1}{2}}}\right]}{(1 + 9\alpha^2\,\frac{K}{G})} - \frac{3\nu Kk}{EJ_2^{\frac{1}{2}}\,(1 + 9\alpha^2\,\frac{K}{G})} \qquad (10)$$

$$h_3 = \frac{1}{2J_2(1 + 9\alpha^2\,\frac{K}{G})}$$

$$G\ (\text{shear modulus}) = \frac{E}{2(1 + \nu)}$$

$$K\ (\text{bulk modulus}) = \frac{E}{3(1 - 2\nu)} \qquad (11)$$

Since the elasto-plastic stress-strain relation is a function of the stress state, it is necessary to keep a record of the axial stress σ_z. In the elastic range under the plane strain conditions, the following relationship exists between the change in axial stress ($\Delta\sigma_z$) and the changes in stress in the x - y plane.

$$\Delta\sigma_z = \nu(\Delta\sigma_1 + \Delta\sigma_2) \qquad (12)$$

where $\Delta\sigma_1$ and $\Delta\sigma_2$ are changes in stress in two principal stress directions in x - y plane. In the plastic range, Drucker and Prager (1952) have shown that the change in axial stress is given by:

$$\Delta\sigma_z = \frac{1}{2}\,(\Delta\sigma_1 + \Delta\sigma_2) - \frac{1}{2}\,(\Delta\sigma_1 - \Delta\sigma_2)\ \text{Sin}\ \phi \qquad (13)$$

To conduct an incremental elasto-plastic analysis, the load is applied in a series of increments. A flow diagram illustrating the computational procedure using the stress transfer technique is shown in Figure 3.

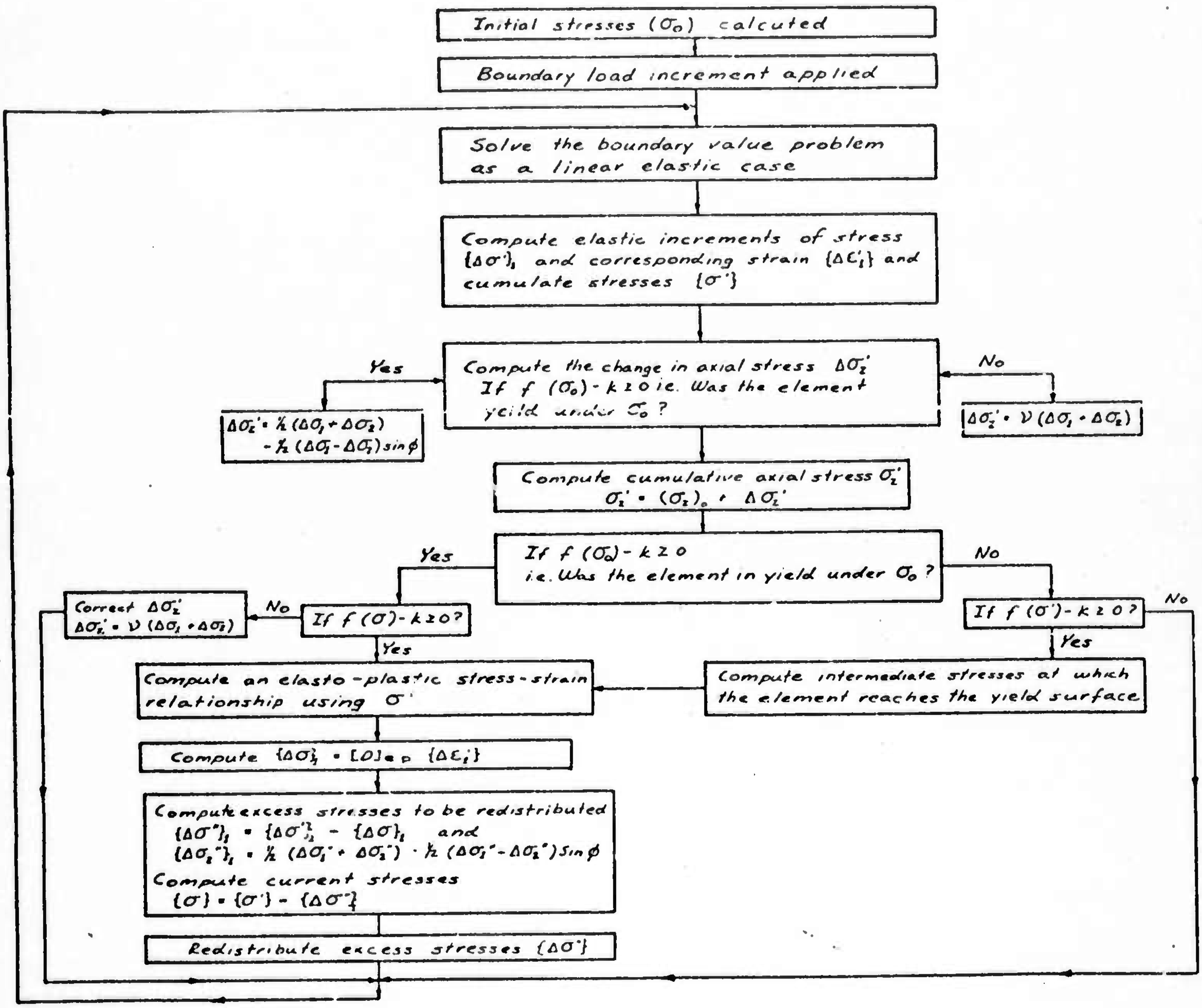


FIG. 3 – "STRESS TRANSFER" TECHNIQUE FOR ELASTO-PLASTIC ANALYSIS

A macro flow chart indicating the organization of the complete program is presented in Figure 4.

ILLUSTRATIVE EXAMPLES

The following three problems were solved to verify the program and illustrate its use:

I. Elasto-plastic analysis of a thick-walled circular tube with the Von Mises yield criterion. A closed form solution is available for this case for verification, Prager and Hodge (1951).

II. Elasto-plastic analysis of a circular opening with the generalized Mohr-Coulomb yield criterion. The results are compared with those obtained by Reyes (1966) and Baker, et al (1969).

III. Combined no tension, joint perturbation and elasto-plastic analysis of a rectangular underground opening to demonstrate the usage of the combined computer program.

The results of these analyses have been compared with closed form solutions and the solutions obtained by other researchers. Typical results are presented in the Appendix.

ANALYSIS OF LABORATORY MODEL

Description of Model Study

Heuer and Hendron (1971) conducted a series of model tests under plane strain conditions on unlined openings in a rock-like material. The geometry of the model block which consisted of two halves assembled together is shown in Figure 5a. The behavior of the model was monitored by internal foil strain gage rosettes installed on the block interface at various radii. Circumferential strain gages were also installed on the wall of the opening. Two sets of diametrical extensometers were also used to measure the

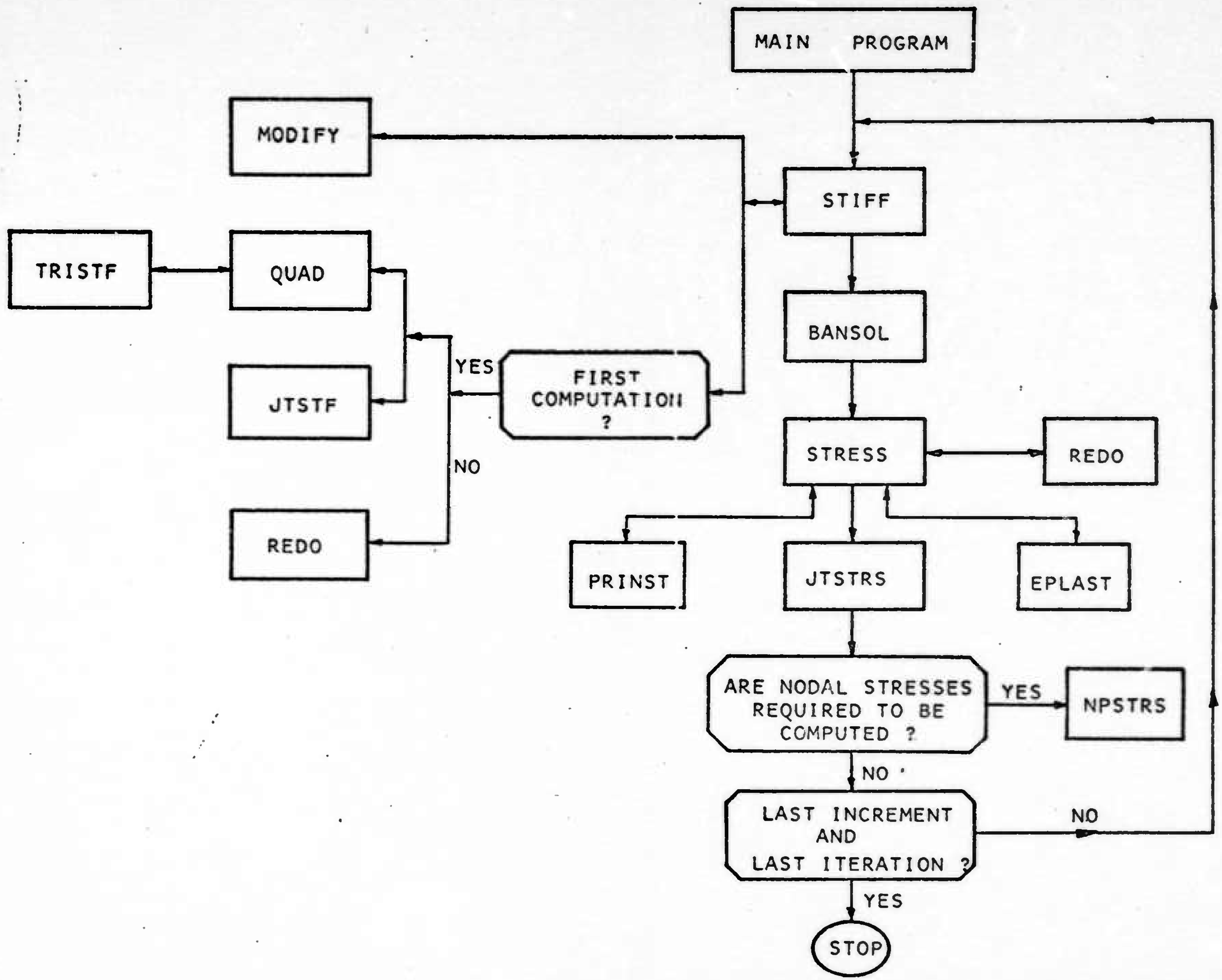
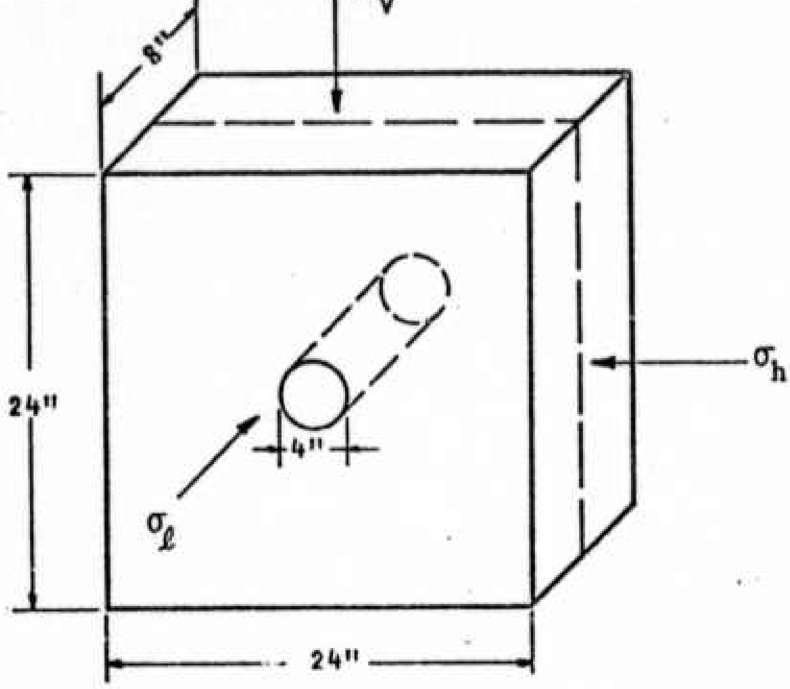


FIG. 4 - SIMPLIFIED FLOW DIAGRAM SHOWING SEQUENCE OF OPERATION OF ALL SUBROUTINES

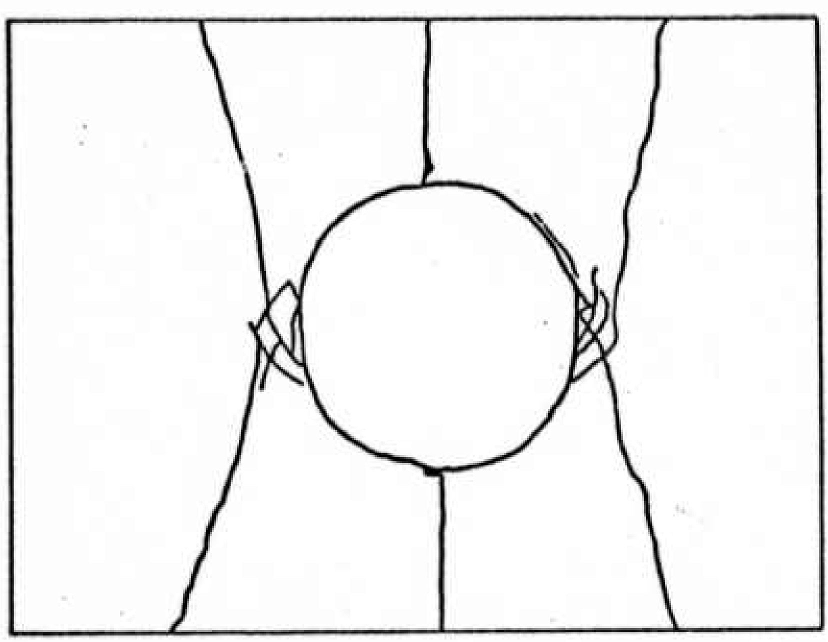
closure of the opening. The loads applied to the model consisted of horizontal and vertical stresses applied at the boundary as shown in Figure 5a. Tests were conducted under three ratios of horizontal to vertical stress, (σ_h/σ_v), 1.0, .25 and .75. Only the tests with $\sigma_h/\sigma_v = .25$ are analyzed in this study.

Based on the results obtained from triaxial tests on the material utilized for constructing the model (Heuer and Hendron, 1971) the material constants, (α, k) used in Equation (3) for the yield function were calculated to be 0.262 and 189 psi, respectively. The tensile strength of the material was determined to be 35 psi as compared with the unconfined compressive strength of 600 psi. The axial stress-strain curves and the value of Poisson's ratio were found to be functions of the principal stress difference. The stress-strain curves indicate that a linear approximation is valid up to a principal stress difference $(\sigma_1 - \sigma_3)$ of 600 psi. A linear isotropic elastic characterization was utilized for the analysis.

During testing at $\sigma_h/\sigma_v = 1/4$, fractured wedges occurred at the springlines when the applied load was incremented from $\sigma_v = 690$ psi to $\sigma_v = 765$ psi, Figure 5b. As the stress level was increased, more fractures formed and the wedges moved into the opening. Another significant set of fractures were formed extending away from the opening back into the model. These fractures appear to be extensions of the fractures which formed the wedges at the springline, Figure 5b. At the crown and invert, high circumferential tensile strains were measured at applied stress levels above $\sigma_v = 400$ psi. Heuer and Hendron (1971) hypothesized that tension cracks might have formed during testing. However, no cracks were visible after testing. It was assumed that the cracks were open during the test and then closed upon removal of load.



(a) Geometry



(b) Fractures Developed in Model Test

FIG. 5 - GEOMETRY OF MODEL BLOCK AND FRACTURES DEVELOPED IN MODEL TEST ($\sigma_h/\sigma_v = 1/4$) (AFTER HEUER AND HENDRON, 1971)

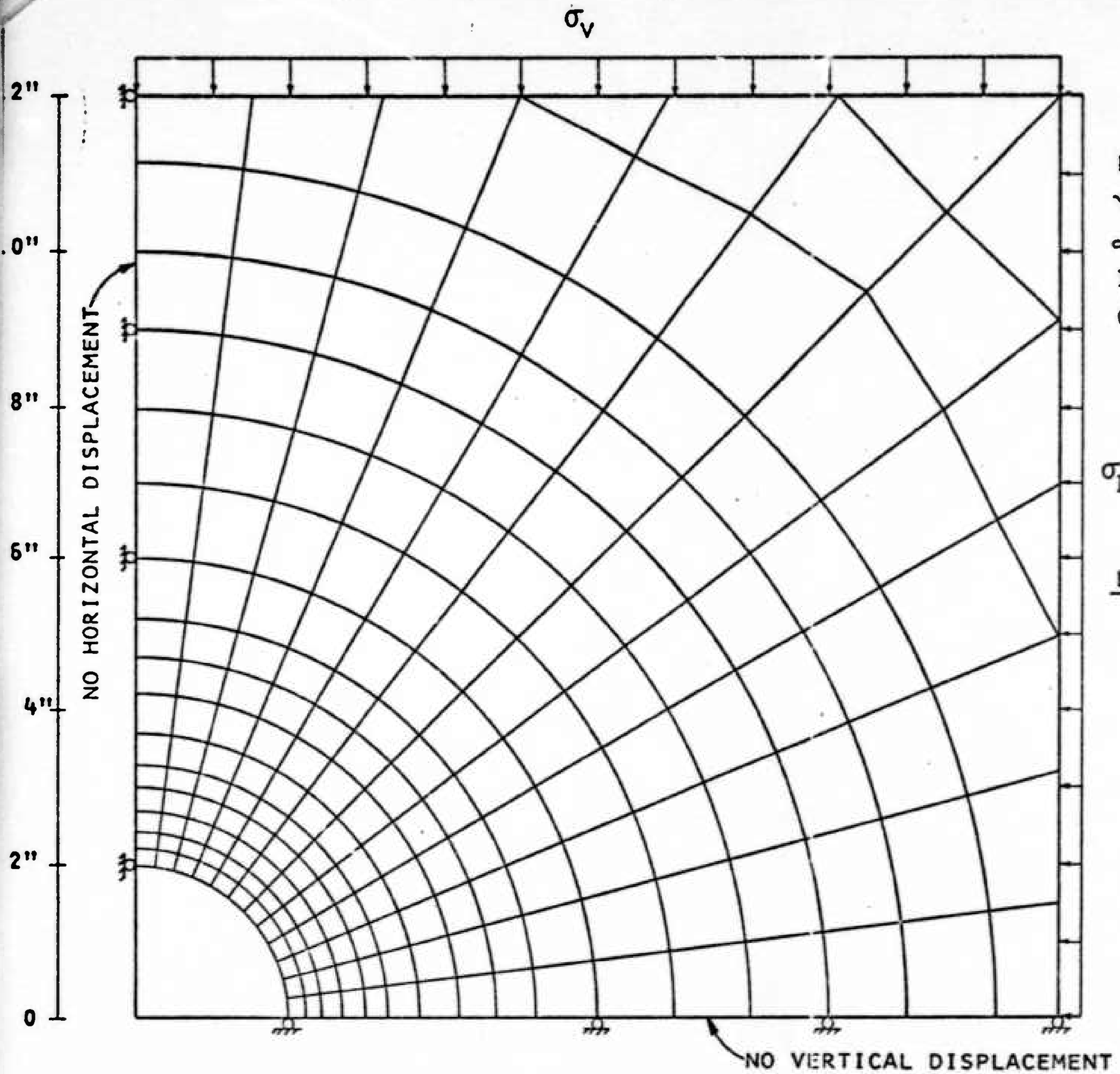
Analysis of Model Study

Idealization of the Model - Because of the symmetry of the model with respect to its axes, as shown in Figure 5a, it was only necessary to analyze a quadrant of the cross-section. The finite element idealization and the elastic parameters used in the analysis are shown in Figure 6.

The incremental analysis was performed to simulate the actual loading path. The incremental loads applied are shown in Figure 6. Two cases were analyzed under the assumption that the material behaved as an elasto-plastic material and had a limited tensile strength of 35 psi. The first case (Case 1) assumed that the block acted as a continuum. The second case (Case 2) assumed that a fractured wedge formed at the springline during testing and moved into the opening.

Presentation and Discussion of Results

Figure 7 illustrates the development of plastic and tensile regions at three levels of the axial stress 400, 600, and 800 psi. The observed fractures are also shown in Figure 7. It may be noted in Figure 7 that the development of the plastic region calculated at σ_v = 600 psi is similar to the shear-fractured wedges developed at the springline. At σ_v = 800 psi, a large plastic region developed back into the model. The location of the actual shear fracture is shown to be along the edge of the plastic region. It was observed in the model test that a region of tensile stress developed at the crown and invert. However, there was little failure in this zone indicating that the tensile stresses were below the tensile strength of the material. An elasto-plastic analysis indicates that a small zone of tension cracks would develop if the material exhibits elasto-plastic behavior. The results of the elasto-plastic analysis appear to agree with observed behavior in the model test.



$E = 833{,}000$ PSI

$\nu = 0.14$

$\alpha = 0.262$

$k = 189$ PSI

$\sigma_h/\sigma_v = 1/4$

LOAD INCREMENT NO.	$\Delta\sigma_v$ PSI	$\Delta\sigma_h$ PSI
1	200	50
2	100	25
3	100	25
4	100	25
5	100	25
6	100	25
8	100	25

FIG. 6 - FINITE ELEMENT IDEALIZATION OF MODEL BLOCK

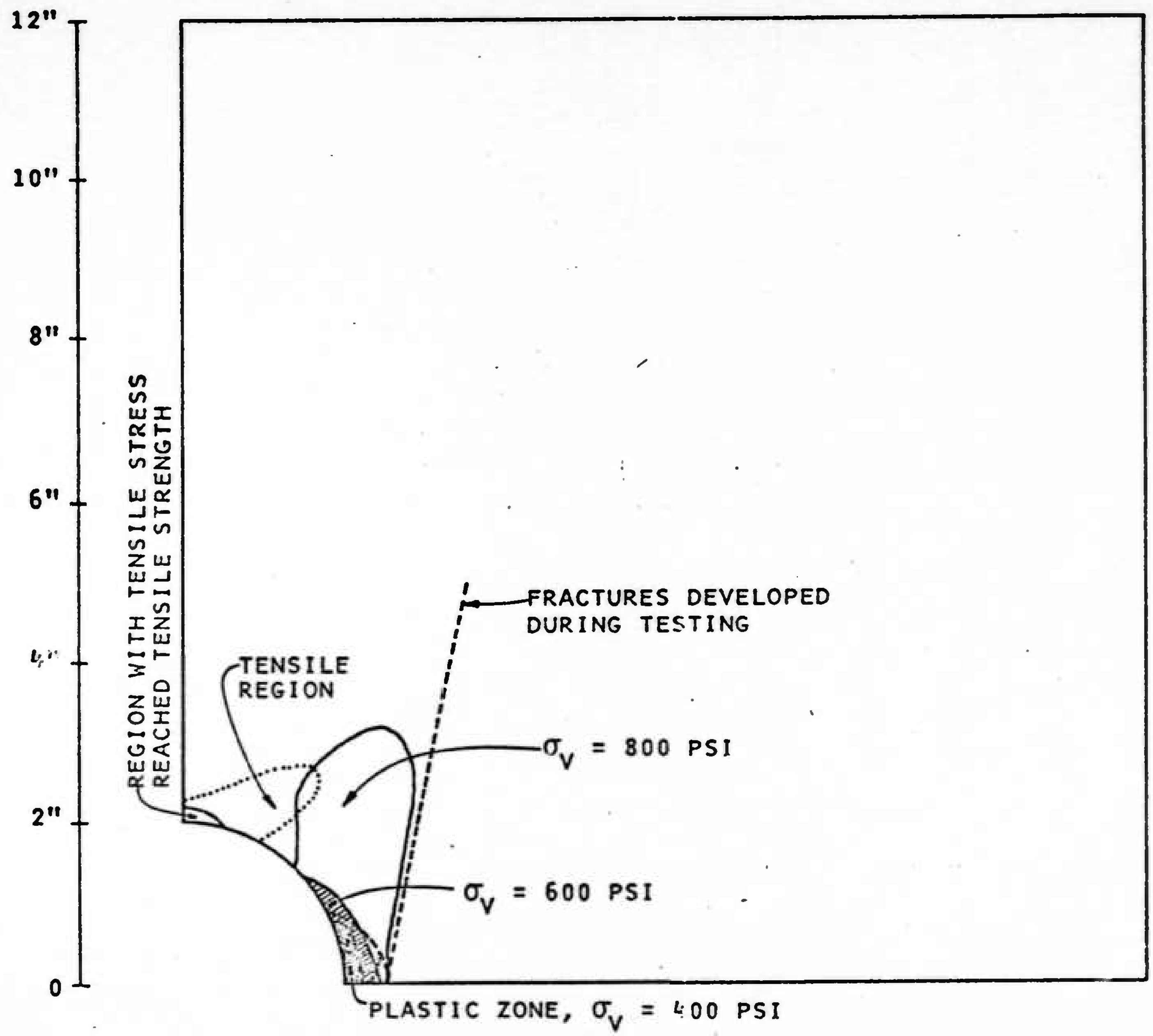


FIG. 7 - RESULTS OF ANALYSIS OF MODEL TESTS, σ_h/σ_v = 1-1/4, SHOWING DEVELOPMENT OF PLASTIC AND TENSILE REGIONS, CASE 1

In order to simulate the propagation of fractures and their effects on the behavior of the model, an analysis was performed (Case 2) after removing the wedge that formed under a vertical stress of 600 psi. Under these conditions, the plastic zone extended behind the wedge and propagated into the model block as shown in Figure 8. The direction of the propagation of the plastic zone and the actual propagation of the fracture as the loads increased were similar. The extend of the plastic zone as the vertical stress was increased to 800 psi is also shown in Figure 8. Also shown is the fracture which developed during the test as illustrated in Figure 5b. It may be noted that the shear fracture observed lies in the middle of the plastic zone indicating the analysis can provide a good indication with respect to the development of the critical zone in the vicinity of the opening.

If it is recognized that the yield function can also represent a limiting stress state for fracture to occur, then the results of the elasto-plastic analysis are in good agreement with observed performance.

In order to compare with the strains measured in the model tests, element strains were computed from the nodal point displacements. These strains were assumed to occur at the centroid of the element. Computed vertical and horizontal strains along the line ($\theta = 86.25^{\circ}$) close to the horizontal axis ($\theta = 90^{\circ}$) for Cases 1 and 2 are shown in Figures 9 and 10, respectively. For the purpose of comparison, the measured tangential and radial strains along the horizontal axis are also plotted in Figures 9 and 10.

Because of the lack of reliability of the test data obtained from the electrical strain gages as evidenced by the relatively large scatter in the measurements, Heuer and Hendron (1971) concluded that the strain measurements would only give a qualitative indication of the response of the model. Comparison of the measured

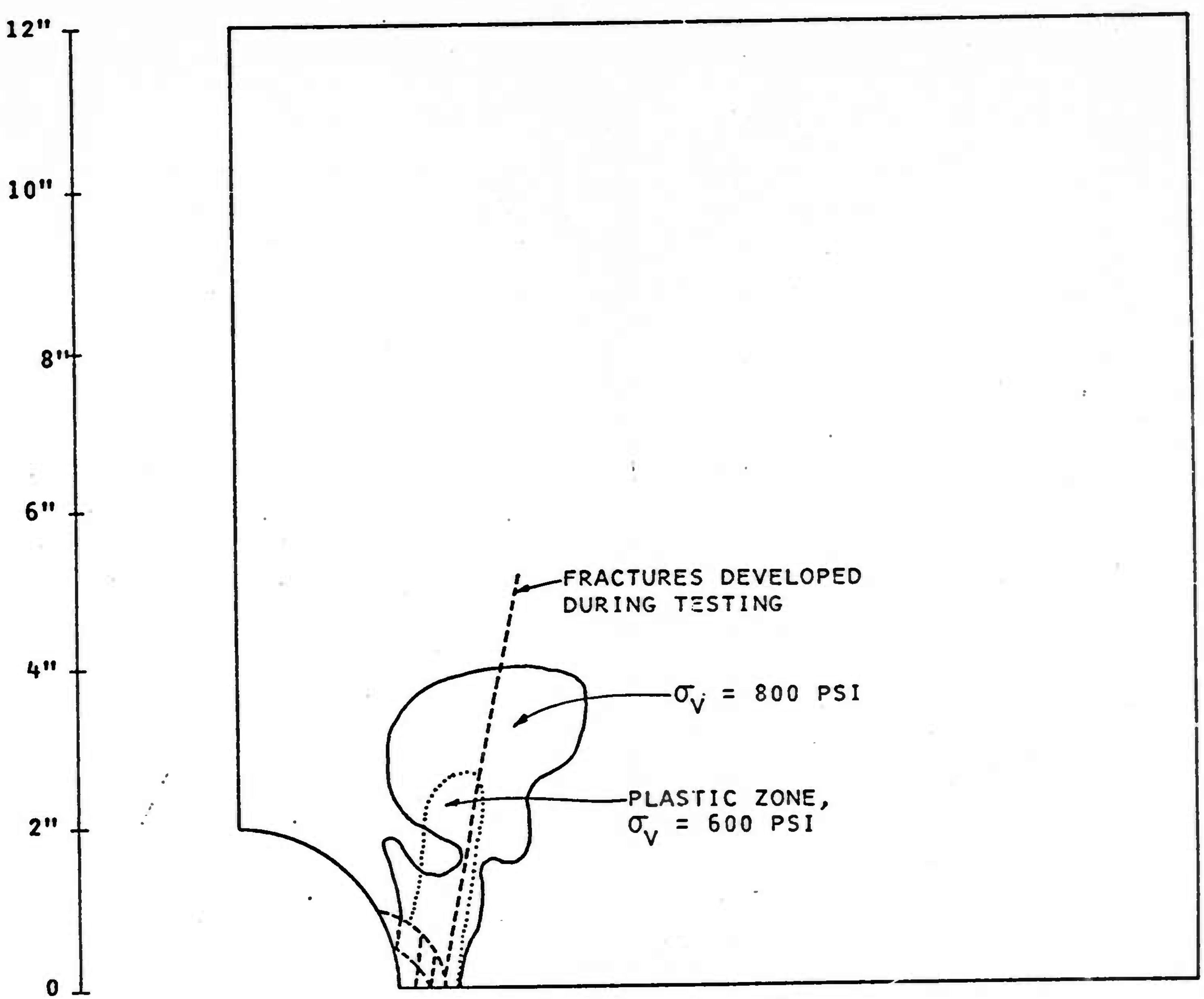


FIG. 8 - RESULTS OF ANALYSIS OF MODEL TEST, $\sigma_h/\sigma_v = 1/4$, SHOWING DEVELOPMENT OF PLASTIC REGIONS, CASE 2

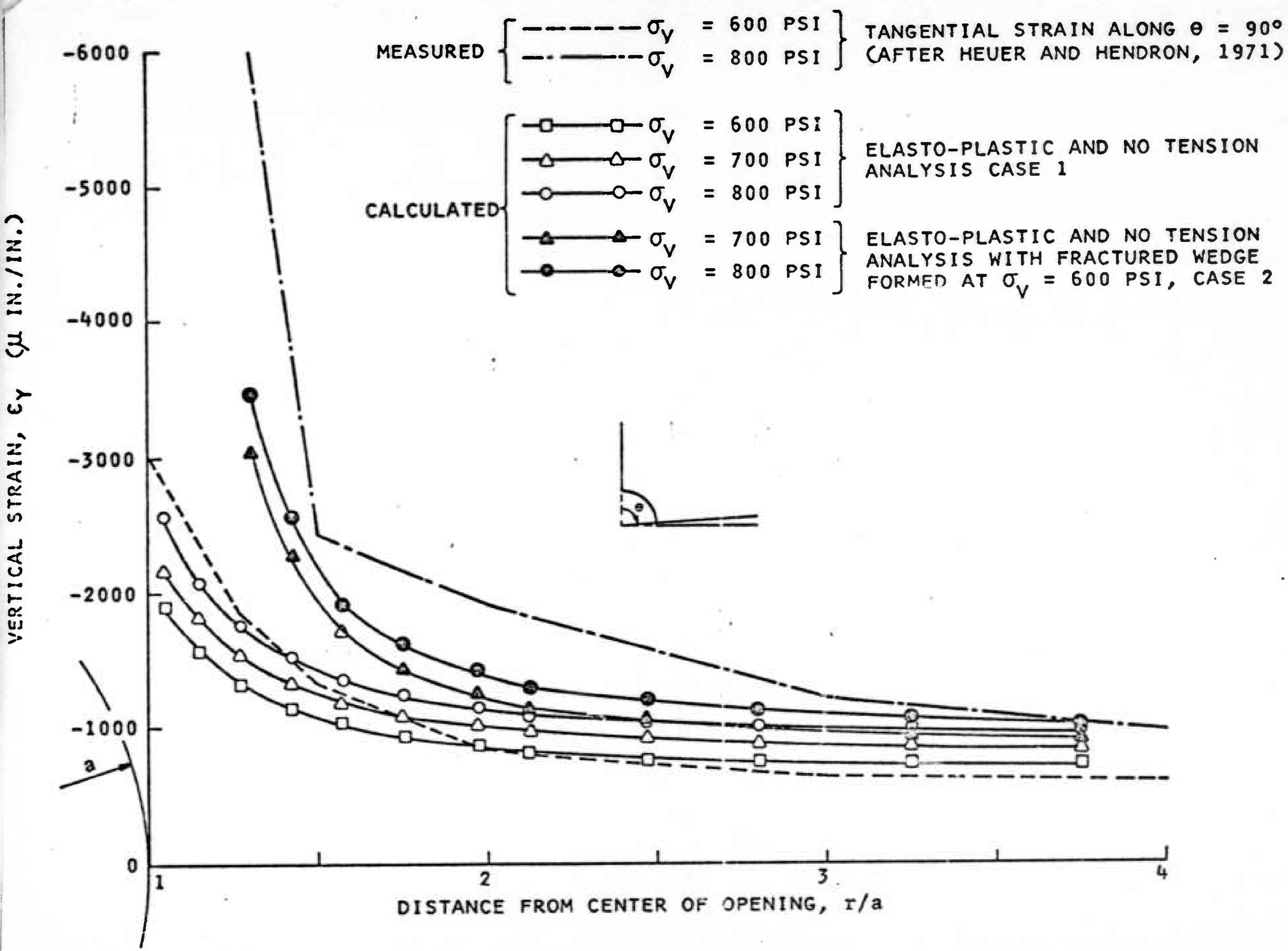


FIG. 9 - DISTRIBUTION OF VERTICAL STRAINS ALONG $\theta = 86.25^{\circ}$, $\sigma_h/\sigma_V = 1/4$, ANALYSIS OF MODEL TESTS

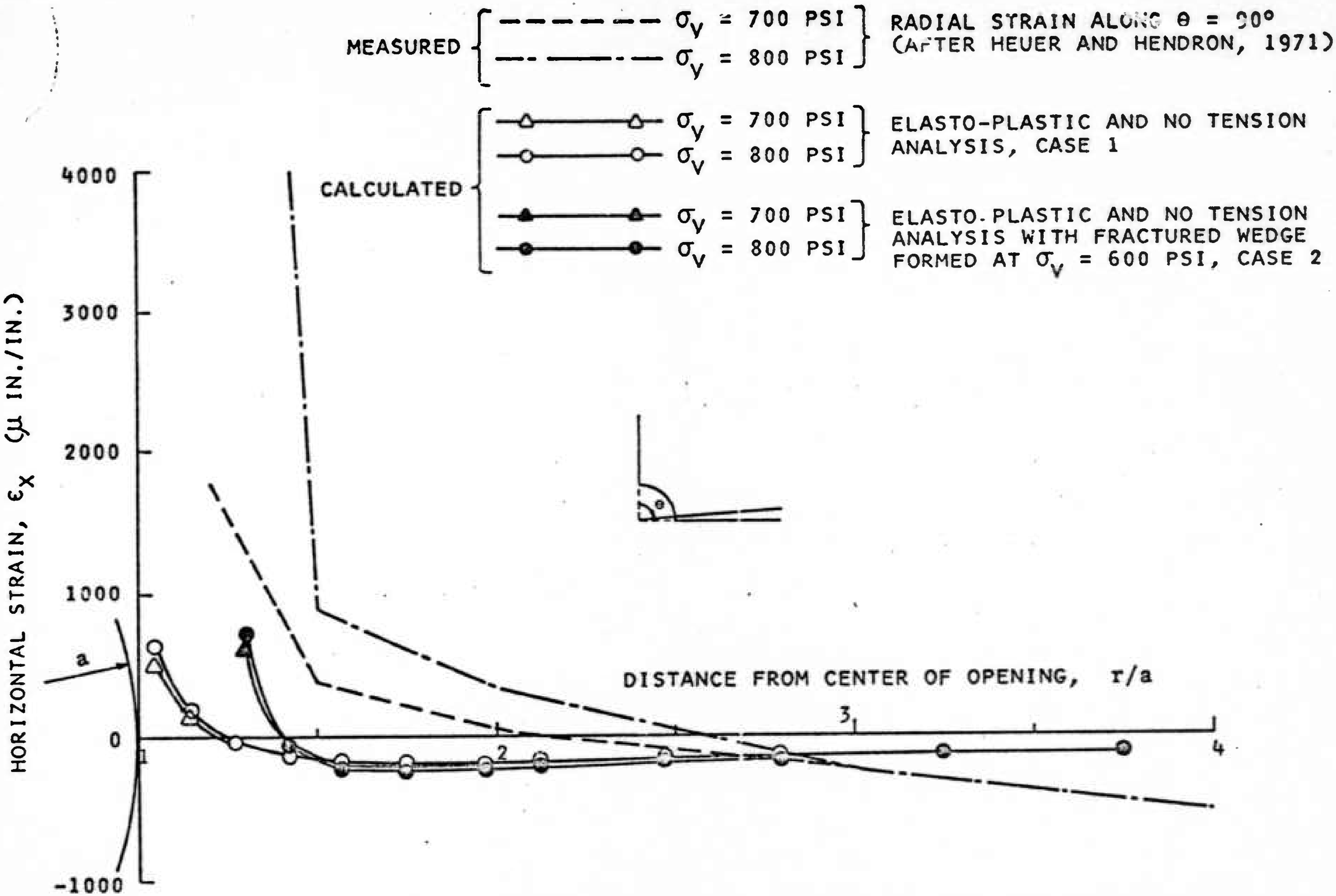


FIG. 10 - DISTRIBUTION OF HORIZONTAL STRAINS ALONG $\theta = 86.25^{\circ}$, $h/v = 1/4$, ANALYSIS OF MODEL TESTS

and calculated behavior shows that, at the low stress levels, they are in good agreement. However, as the stress level increased and the fractured wedges formed, the strains increased to a level much greater than that predicted from the Case 1 analysis in which the model block was assumed to be a continuum. The results of the Case 2 analysis indicated a strain pattern and magnitude in approximate agreement with measured values. This indicates that the formation of fractured wedges at the springline has a great effect on the behavior of the model block. Discrepancies between the measured strains and the strains computed by Case 2 analysis at the higher stress levels may, in large part, be attributed to the formation of the second set of fractures extending into the model block away from the opening.

Analysis of the model tests indicates that the elasto-plastic analysis may provide valuable information for evaluating the critical zone in an intact rock surrounding an opening. Although the analysis cannot simulate accurately the behavior of the model after the formation of shear fractures surrounding the opening, some indication can be obtained with regard to propagation of fractured zones.

Analysis of a Field Case History

The Morrow Point Underground Powerplant was constructed by the U.S. Bureau of Reclamation on the Gunnison River some 20 miles east of Montrose, Colorado (Dodd, 1967 and Brown, et al. 1971). The powerplant chamber is 206 ft long and 57 ft wide with a height ranging from 65 to 134 ft and is located about 400 ft below the ground surface. The plan of the powerplant and other adjacent structures is shown in Figure 11. A cross-section of the powerplant chamber along A-A'(line 4 + 12 ft) is shown in Figure 12. It may be noted from Figures 11 and 12 that the powerplant chamber is situated behind a steep valley rock wall. The crown of the chamber on the river side (b line wall, Figure 12) is about 200 ft behind the ground surface. Details with regard to the geology of the rocks are given by the Bureau of Reclamation (1965).

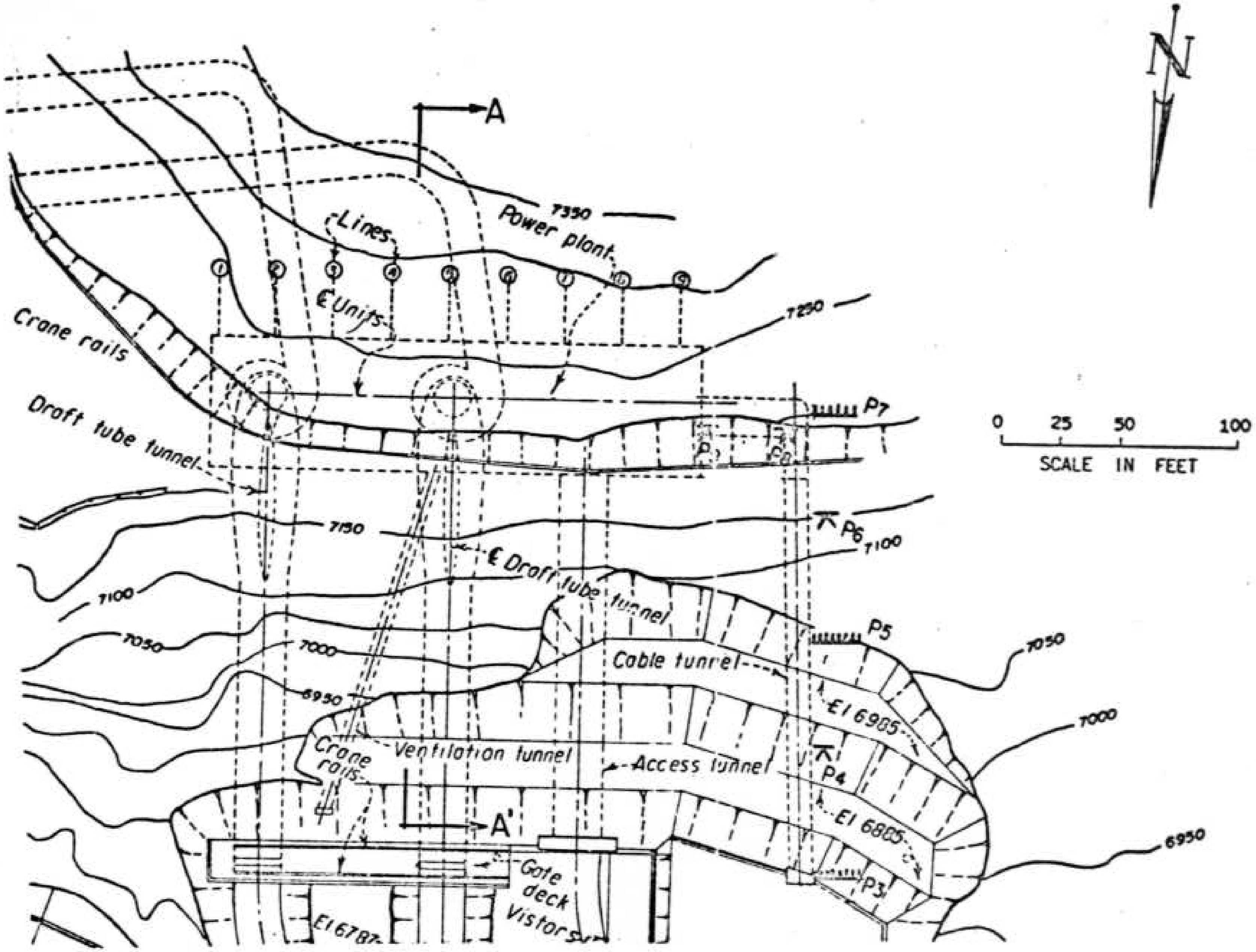


FIG. 11 - PLAN AND LOCATION OF MORROW POINT POWERPLANT (AFTER DODD, 1967)

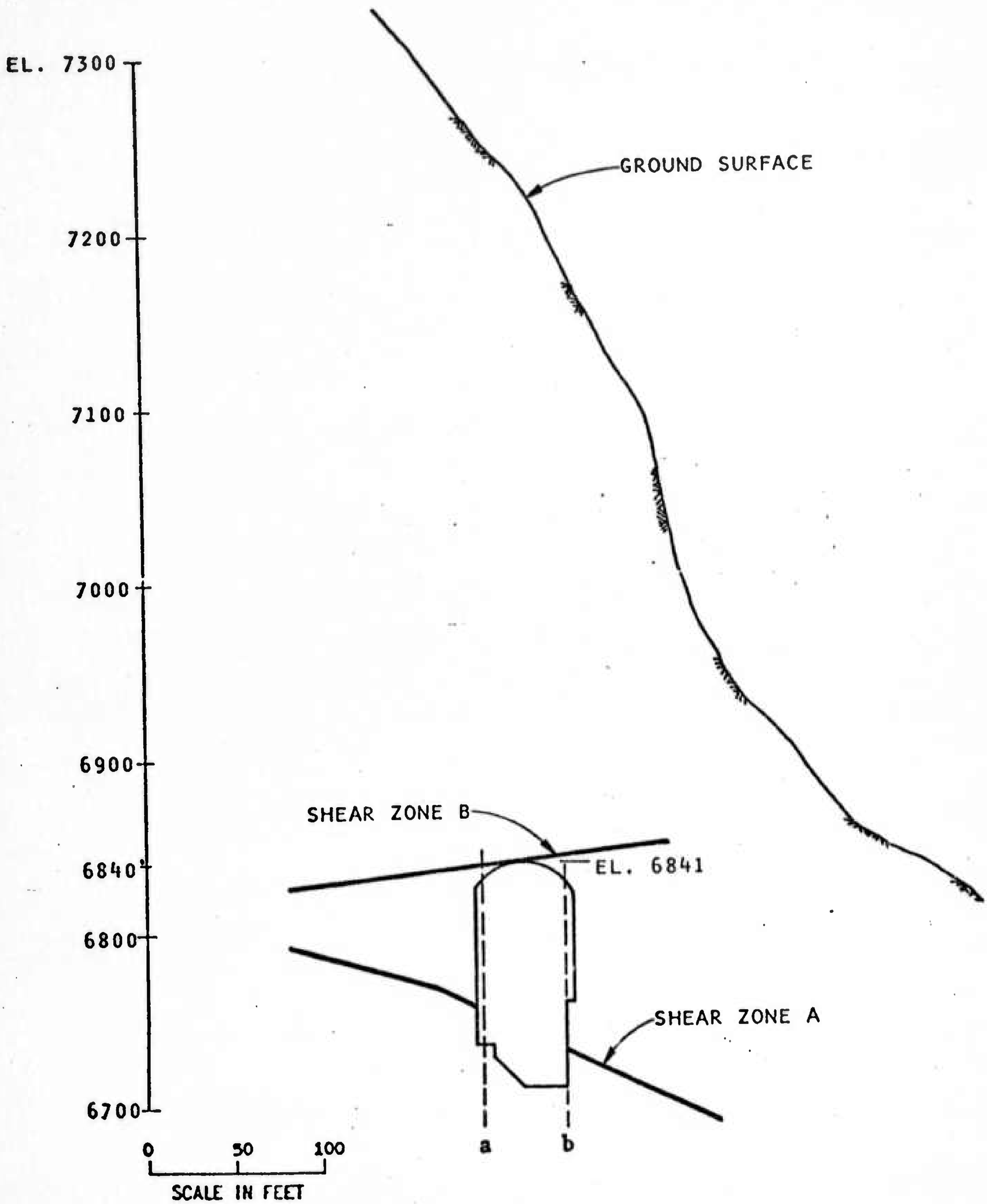


FIG. 12 - CROSS-SECTION A-A' (LINE 4 + 12 FT) OF THE POWERPLANT CHAMBER, MORROW POINT POWERPLANT

There are two distinct shear zones intersecting the powerplant area. The lower zone (shear zone A) strikes N 40° W, dips 32°E, and consists of a one to five ft zone of fault gouge and fractured rock. The upper zone (shear zone B) has a strike of N 20° E, a dip of 23° E and consists of a one to three ft zone of fault gouge and fractured rock. The orientations of the three major joint sets that intersect the chambers are: strike N 63° W and dip 82° SW; strike N 36° E, and dip 80° W; strike N-S and dip 43° E.

Significant Features of the Behavior of the Powerplant Excavation

The crown of the chamber was first excavated during a three-month period from May of 1965 through July of 1965. The excavation of the remaining chamber continued from August of 1965 through March of 1966. When the excavation reached the El. 6793 bench, 48 ft below the crown, some initial movement of the a-line rock wall occurred. When the powerplant was excavated to El. 6748, 93 ft below the crown, an accelerated inward movement of 1.5 inches was measured at line 4 + 12 ft. As the excavation proceeded, the a-line rock wall continuously moved inward but at a diminishing rate. Significant rock movement ceased by the end of March, 1966 (Brown, Morgan and Dodd, 1971). The maximum observed inward movement of the a-line wall at line 4 + 12 ft was 2.5 inches. Figure 13 shows the history of the observed rock movements at line 4 + 12 ft reported by Brown, et al (1971).

Brown, et al. (1971) hypothesized that the anomalous rock behavior along the a-line rock wall might involve a mass movement of a rock wedge shown in Figure 14. It is hypothesized that shear zones A and B intersect at an average distance of 130 ft behind the a-line wall, forming a rock wedge. In addition, two planes of failure must exist for the wedge to move into the opening. One is the upper shear failure plane with an apparent dip of 17° intersecting the chamber near the springline of the rock arch. The other is essentially vertical and

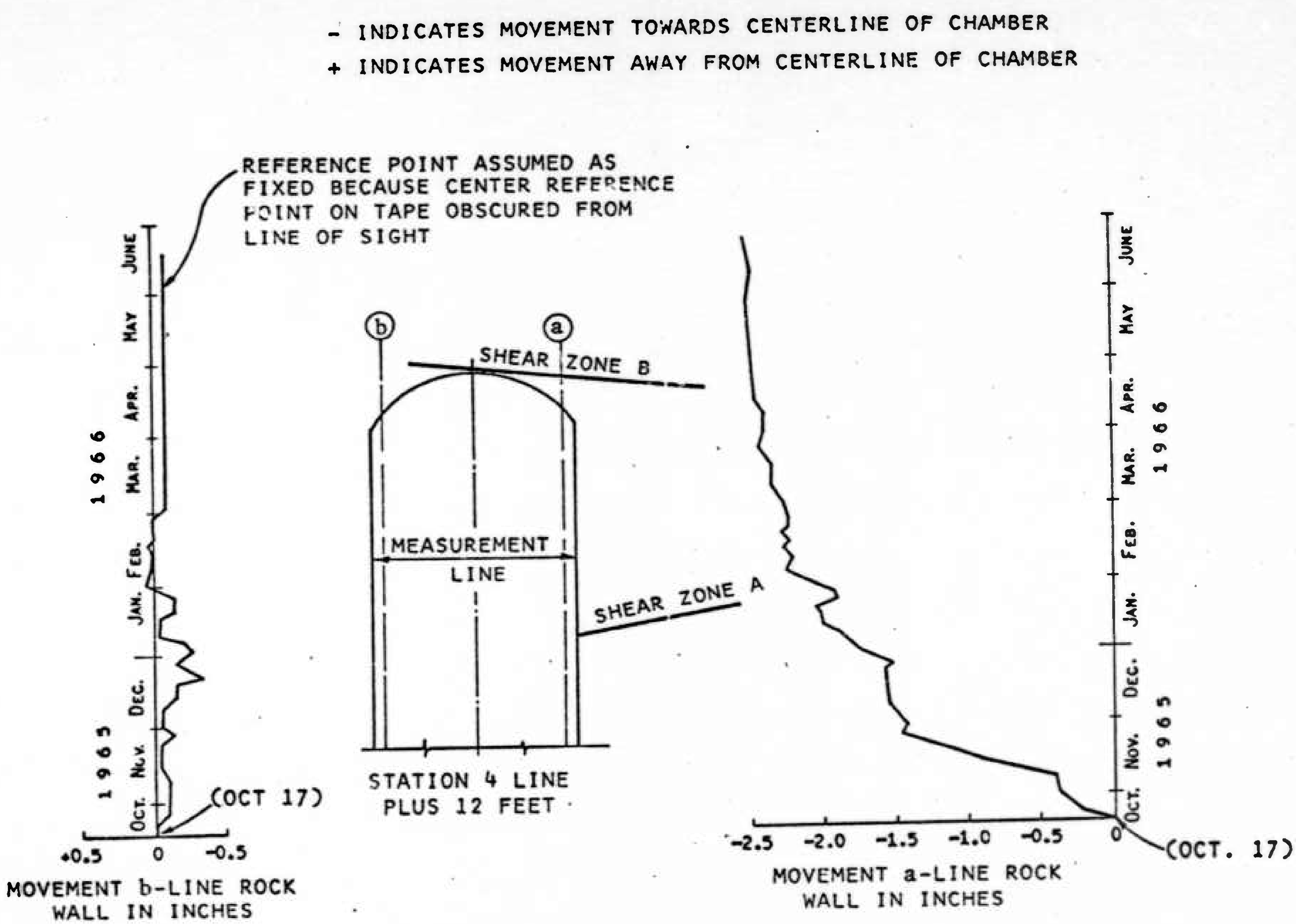


FIG. 13 - HISTORY OF ROCK MOVEMENT AT MORROW POINT POWERPLANT
(AFTER BROWN, ET AL, 1971)

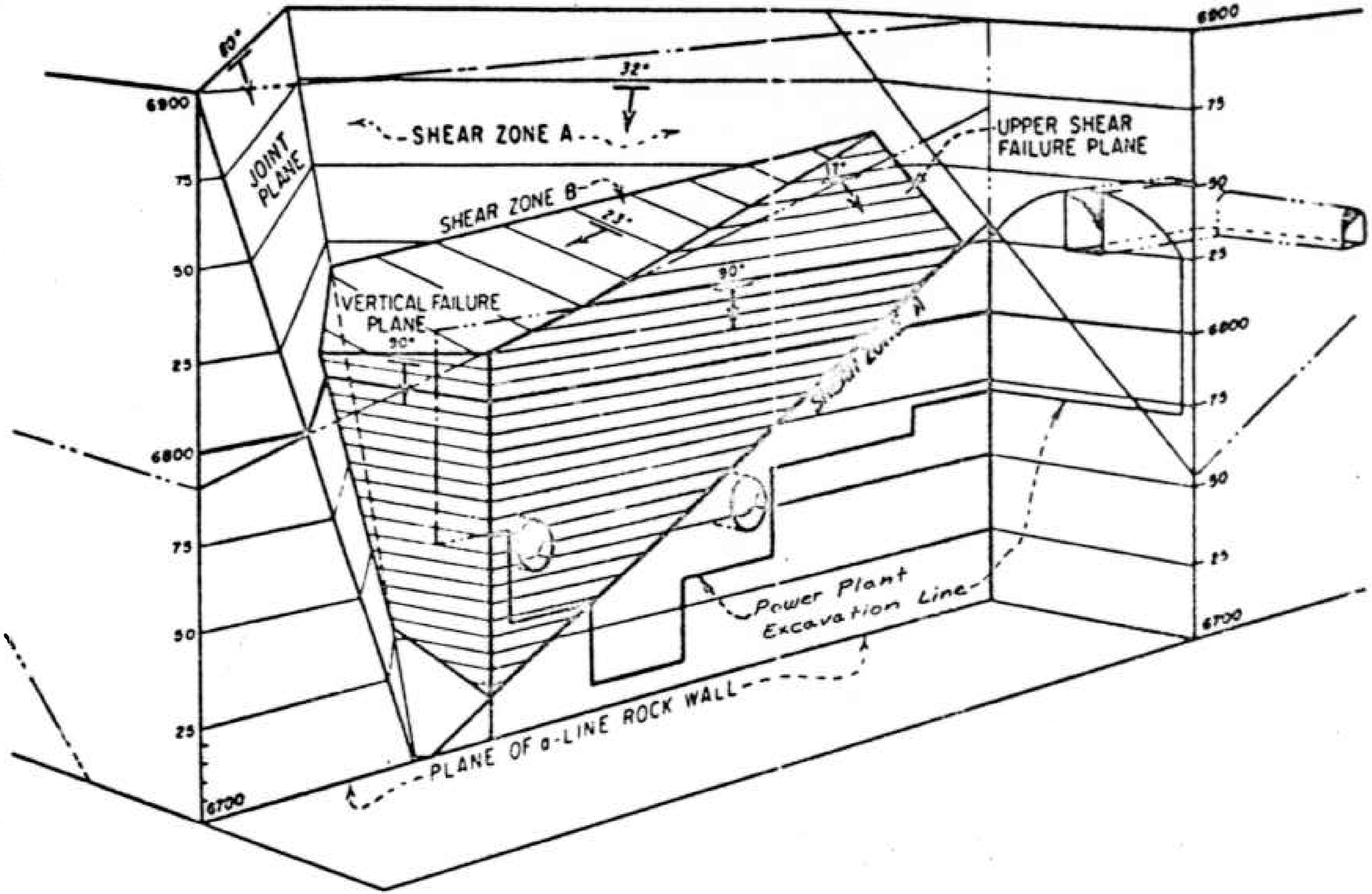


FIG. 14 - THREE-DIMENSION VIEW OF ROCK WEDGE, MORROW POINT POWERPLANT
(AFTER BROWN, ET AL. 1971)

intersects the chamber between line 2 + 15.5 ft and the east end wall of the chamber. A cross-section through the chamber at line 4 + 12 ft showing the rock wedge is presented in Figure 15.

Idealization of the Powerplant Excavation

a. Finite Element Idealization - As discussed previously the observed maximum rock movements occurred at line 4 + 12 ft. Therefore a cross-section through the opening at this location (Figure 12) was chosen for analysis. The essential features in the idealization are the steep sloping valley ground surface, shear zones A and B and two other incipient failure planes. The steep sloping ground surface makes the estimation of the initial in-situ stresses difficult. As described previously, both the rock wedge and the movements associated with it are three-dimensional in nature. However, for the purposes of analysis, it was necessary to idealize it as a two-dimensional plane strain problem. Shear zones A, B and the other two incipient failure planes are shown as distinct discontinuities extending in the direction perpendicular to the cross-section. These discontinuities are idealized as Goodman's one-dimensional joints. Its deformability is approximated by normal and shear stiffness.

b. Material Properties - Extensive testing programs were conducted by the Bureau of Reclamation (1965) to determine rock properties at the Morrow Point Dam Site. These included direct shearing and sliding friction both in the field and laboratory, triaxial testings and field jacking tests of foundation rock. The strength and elastic properties of the rock above and below shear zone A used in the analysis are summarized in Table 1. As described previously, both shear zones A and B were idealized by one-dimensional joint elements. The properties of these shear zones are approximated by the normal and shear joint stiffnesses (Goodman, 1969) which are functions of normal and tangential deformability of the shear zones and their geometries.

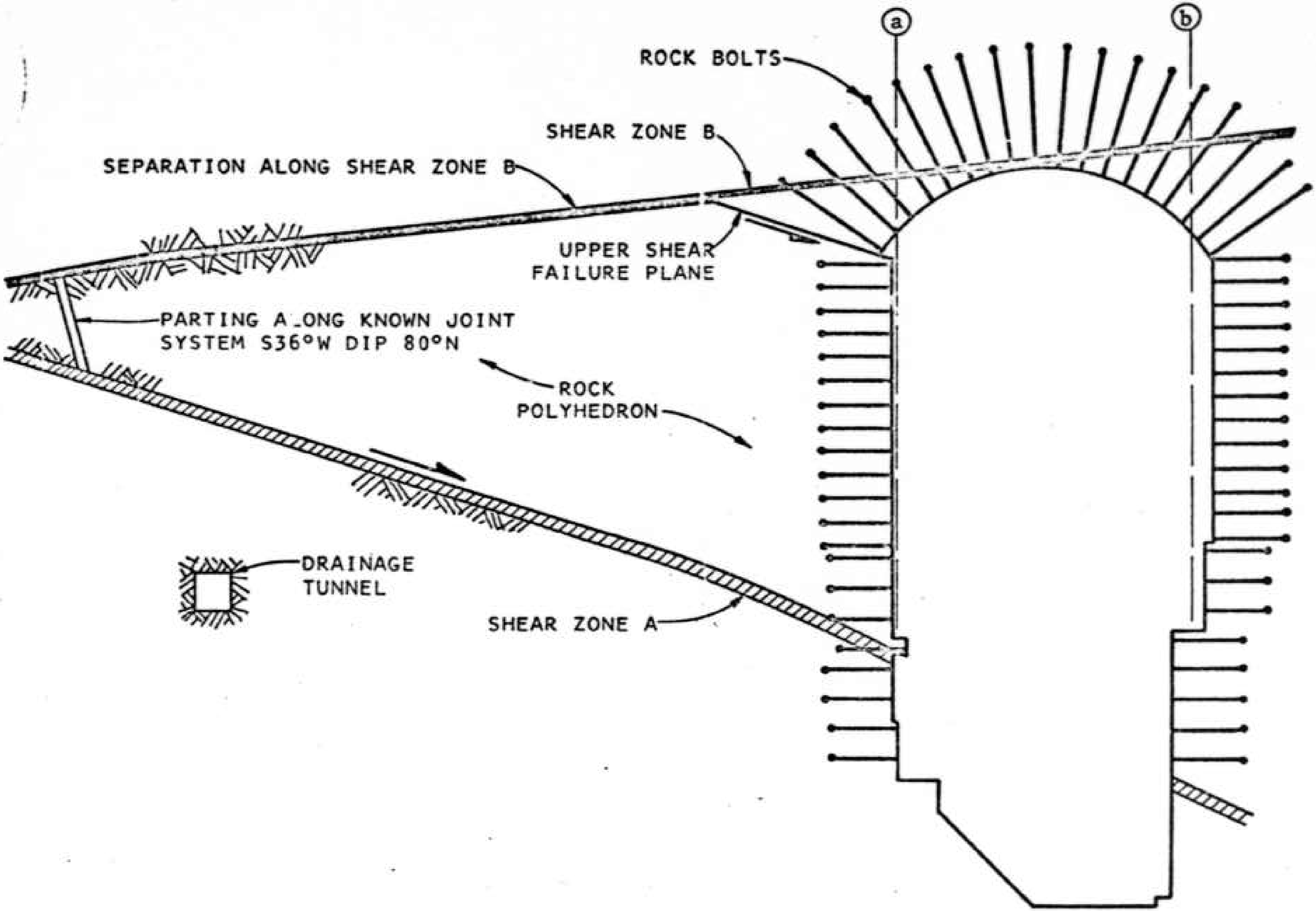


FIG. 15 - SCHEMATIC DIAGRAM OF A-LINE ROCK WEDGE MOVEMENT
(AFTER BROWN, ET AL. 1971)

TABLE 1 - SUMMARY OF MATERIAL PROPERTIES USED IN ANALYSES, MORROW POINT POWERPLANT

Case / Material	1G Gravity Loading Without Opening	1 Simulation of Powerplant Excavation	2G Gravity Loading Without Opening	2 Simulation of Powerplant Excavation
1 Rock above Shear Zone A	E = 1.8 x 10^8 psf ν = 0.49 (assumed) γ = 171 pcf	E = 1.8 x 10^8 psf ν = 0.02 c = 115,000 psf ϕ_r = 36°	E = 1.8 x 10^8 psf ν = 0.49 (assumed) γ = 171 pcf	E = 1.8 x 10^8 psf ν = 0.05 c = 115,000 psf ϕ_r = 36°
2 Rock below Shear Zone A	E = 2.52 x 10^8 psf ν = 0.49 (assumed γ = 171 pcf	E = 2.52 x 10^8 psf ν = 0.05 c = 288,000 psf ϕ_r = 55°	E = 2.52 x 10^8 psf ν = 0.49 (assumed) γ = 171 pcf	E = 2.52 x 10^8 psf ν = 0.10 c = 288,000 psf ϕ_r = 55°
3 Shear Zone A Shear Zone B Two Incipient Failure Planes	K_N = 2 x 10^6 pcf K_S = 1 x 10^6 pcf (assumed)	K_N = 2 x 10^6 pcf K_S = 1 x 10^6 pcf c = 0 ϕ_r = 25° (assumed)	K_N = 1 x 10^6 pcf K_S = 1 x 10^5 pcf	K_N = 1 x 10^6 pcf K_S = 1 x 10^5 pcf c = 0 ϕ_r = 25° (assumed)
4 Part of Shear Zone B Reinforced by Rock Bolts	-----	K_N = 2 x 10^6 pcf K_S = 1 x 10^7 pcf c = 100,000 psf ϕ_r = 45° (assumed)	-----	K_N = 1 x 10^6 pcf K_S = 1 x 10^7 pcf c = 100,000 psf ϕ_r = 45° (assumed)

No data were available for the normal and shear stiffnesses of the shear zones with thickness varying from 1 to 5 ft. Therefore, two sets of joint stiffness were utilized to study the influence of the deformability of the shear zones on the behavior of the powerplant excavation. The strength parameters assumed for the shear zones are presented in Table 1.

Because of the limited data with regard to the magnitude of the in-situ stresses and the difficulties associated with estimating their values in the rock mass, the initial state of stress used in the analysis was obtained by a gravity turn-on analysis of the rock mass without the opening. These analyses were conducted to account for the effects of the steep valley wall located in the vicinity of the powerplant excavation. For these analyses, the higher values of the Poisson's ratio (=0.49) was assumed for the rock to simulate the likelihood of a high horizontal stress.

The values of the initial stresses which have to be applied at excavated boundary to simulate the excavation were obtained by a method similar to that suggested by Clough and Duncan (1969). The nodal point stresses on the excavated boundary were estimated from the stresses in the surrounding elements.

Analysis Procedures

As indicated previously two cases using high and low normal and shear joint stiffnesses for the shear zones were analyzed. For each case, the initial stresses for each element were first obtained by performing a gravity turn-on-analysis. The nodal stresses were then determined along the excavated surface and applied at the excavated boundary to simulate the excavation. Very small values of the elastic constants were assigned to those elements in the opening to simulate the cavity.

Presentation and Discussion of Results

The results of the analyses are presented in Figures 16 and 17. Figure 16 illustrates the horizontal displacements for points along the face of the powerplant chamber, and Figure 17 shows the vertical displacements. The observed movements on a- and b-line walls are also shown in Figures 16 and 17. For the case with high joint stiffnesses, the horizontal inward movement at El. 6793 was calculated to be 0.44 in. on the a-line wall and 0.41 in. on the b-line wall. When the low joint stiffnesses were used, the calculated inward movements increased to 1.35 in. on the a-line wall and 0.62 in. on the b-line wall.

The results indicate that the analysis provided a good qualitative estimate of rock movements at the Morrow Point Powerplant excavation. The computed deformations are of the same order of magnitude as those observed. The difference between the computed and observed deformation is significant in terms of percentages but is considered satisfactory in terms of numerical values when it is recognized that the following approximations were made in the analysis (1) The rock wedge and the movements associated with it are three-dimensional in nature, Figure 14. A two-dimensional plane strain analysis would tend to underestimate the movements. (2) The initial state of stress was estimated in the analyses. It is quite possible that a higher initial state of stress, especially higher horizontal stresses might exist in the rock mass which would cause higher induced stresses and movements. (3) No data were available on the properties of the shear zone present in the rock mass. It was necessary to assume their values in the analysis.

It should also be recognized that the powerplant was excavated in stages whereas the analysis assumed that the excavation was created instantaneously. For an ideal elastic material the assumption would not result in any error. However, in those

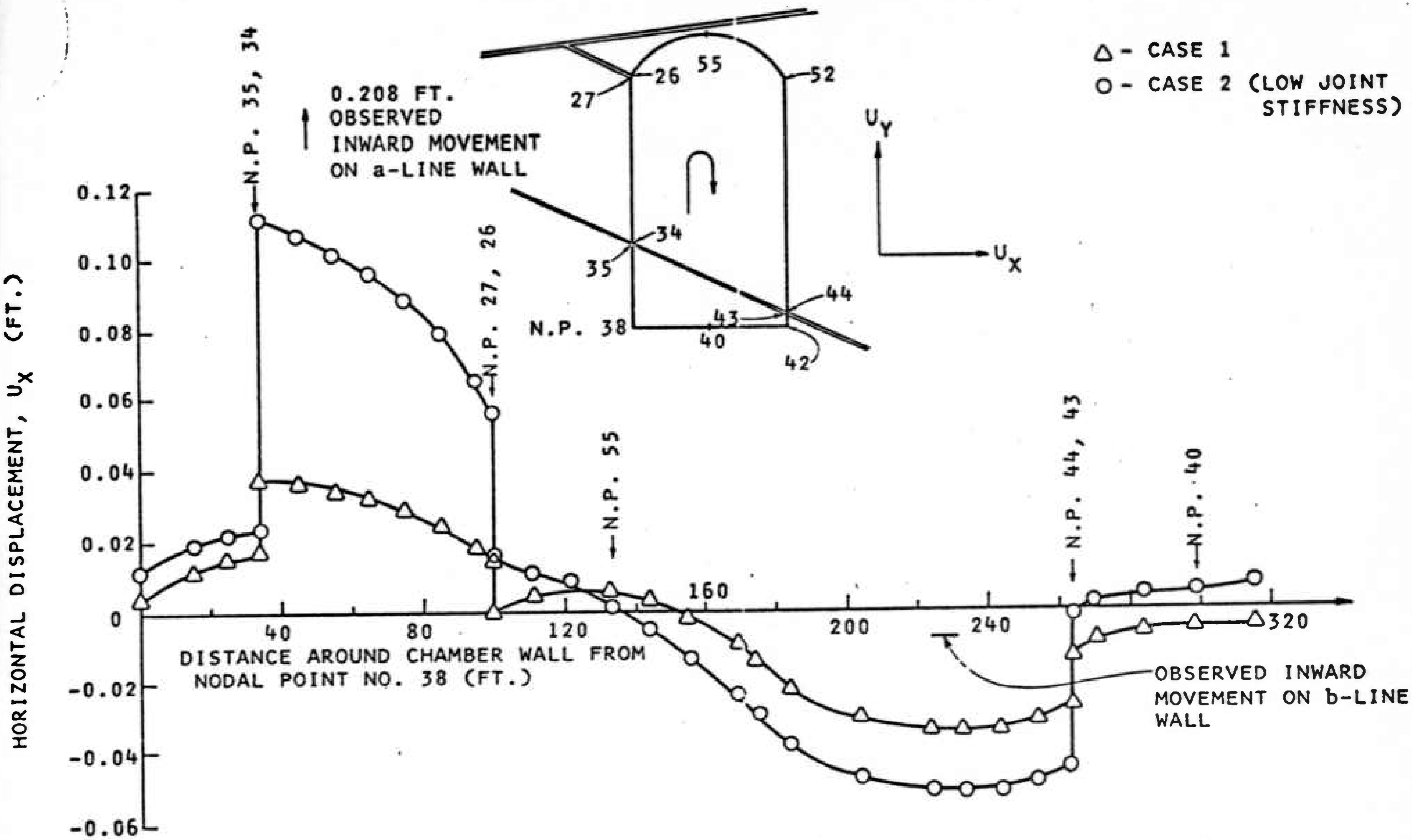


FIG. 16 - HORIZONTAL DISPLACEMENTS ALONG FACE OF POWERPLANT CHAMBER, MORROW POINT POWERPLANT

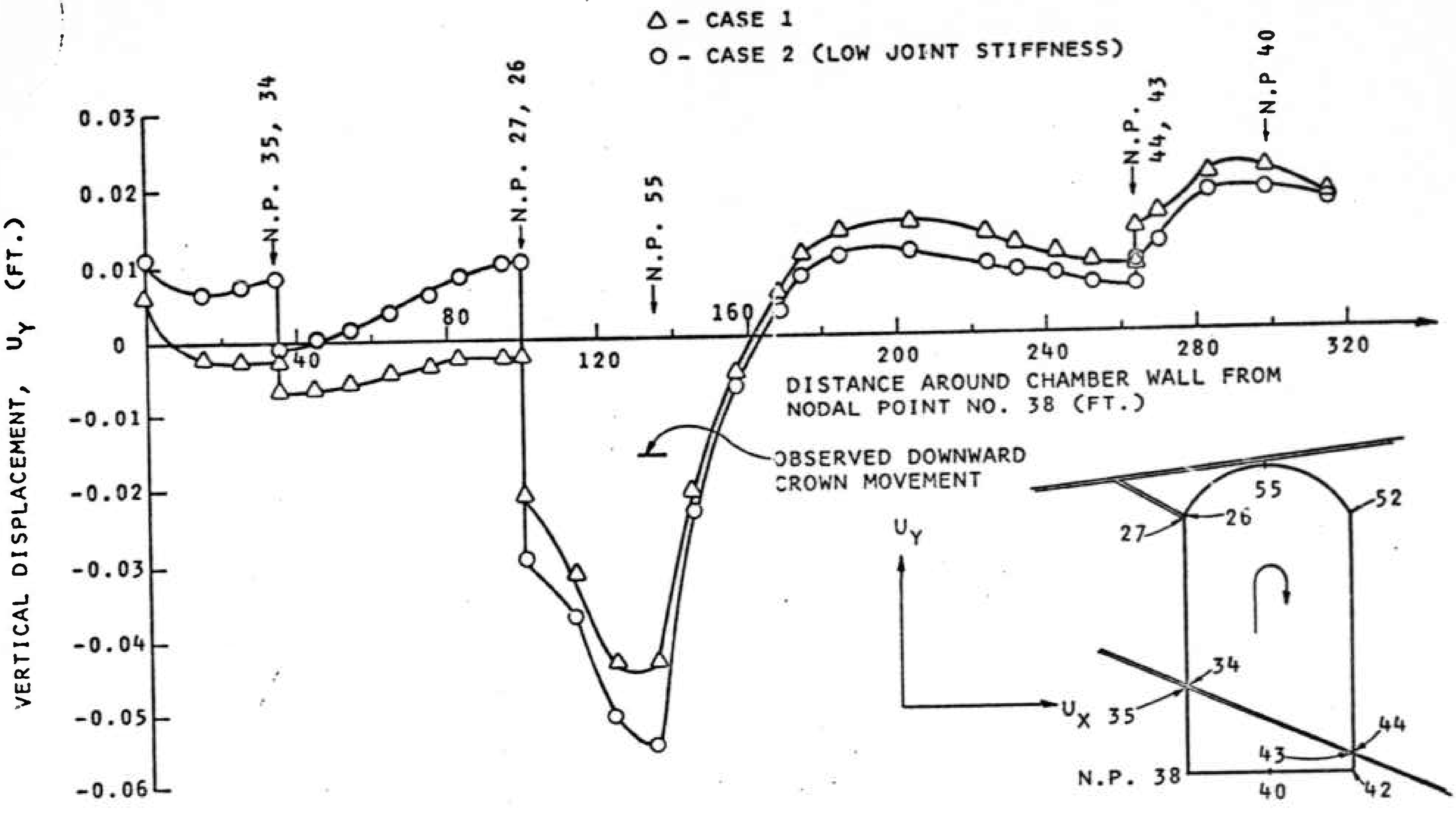


FIG. 17 - VERTICAL DISPLACEMENTS ALONG FACE OF POWERPLANT CHAMBER, MORROW POINT POWERPLANT

cases where the loading path has an influence on the results the effect of this assumption can be significant. High stress concentrations during the excavation process could cause displacements which would be larger than those computed on the assumption of an instantaneous creation of the opening.

CONCLUSIONS

Computational techniques which have the capability of including realistic idealizations of in situ rock conditions can provide a good qualitative prediction of the behavior of a rock mass in the vicinity of an excavation. Considering the availability and accuracy of input information the ability of the developed computational techniques to predict quantitative behavior is considered acceptable.

It was found that the effort necessary to idealize a practical problem for analysis is considerable. Since real problems, in general, include certain factors which cannot be modelled accurately, it may be necessary to utilize more than one idealization to obtain bounds on the actual behavior.

Theoretical concepts and computational techniques need to be developed for including (i) the development and propagation of fractures in a rock mass, and (ii) the influence of construction sequence and support systems.

It is recommended that a major effort should be directed towards the study of case histories for the purpose of establishing the reliability of various analytical methods to predict performance and to provide a basis for improvements. Such studies should emphasize the requirements for idealization and field input information. Without this information analytical techniques will not gain acceptance in the design profession.

ACKNOWLEDGMENTS

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by Bureau of Mines under Contract Number HO210046. The views and conclusions contained in this document are those of the author and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U.S. Government.

APPENDIX I

1. Elasto-plastic analysis of a thick-walled circular tube with the Von Mises yield criterion.
   A finite element idealization of the thick-walled circular tube with the material properties used in the analysis is shown in Figure I-1. Typical results of the analysis are presented in Figures I-2 to I-4.

2. Elasto-plastic analysis of a circular opening with the generalized Mohr-Coulomb yield criterion.
   A finite element idealization for the circular opening with the material properties used in the analysis is shown in Figure I-5. Figure I-6 shows the distribution of vertical and horizontal stresses along a horizontal section of the opening.

3. Combined no tension, joint perturnbation and elasto-plastic analysis of a rectangular underground opening.
   A finite element idealization of the rectangular opening with the material properties used in the analysis is shown in Figure I-7. Typical results of the analysis are presented in Figures I-8 and I-9.

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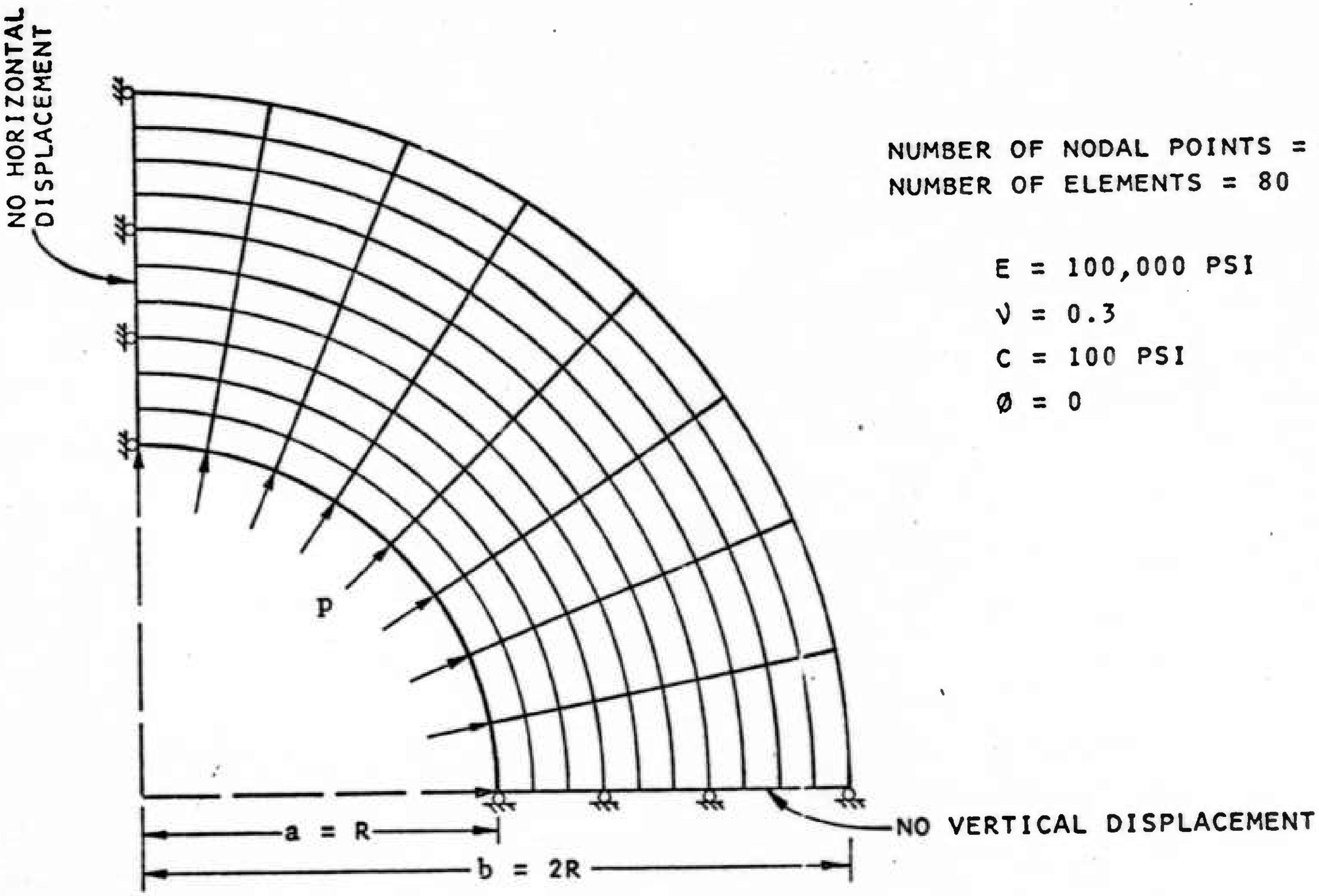
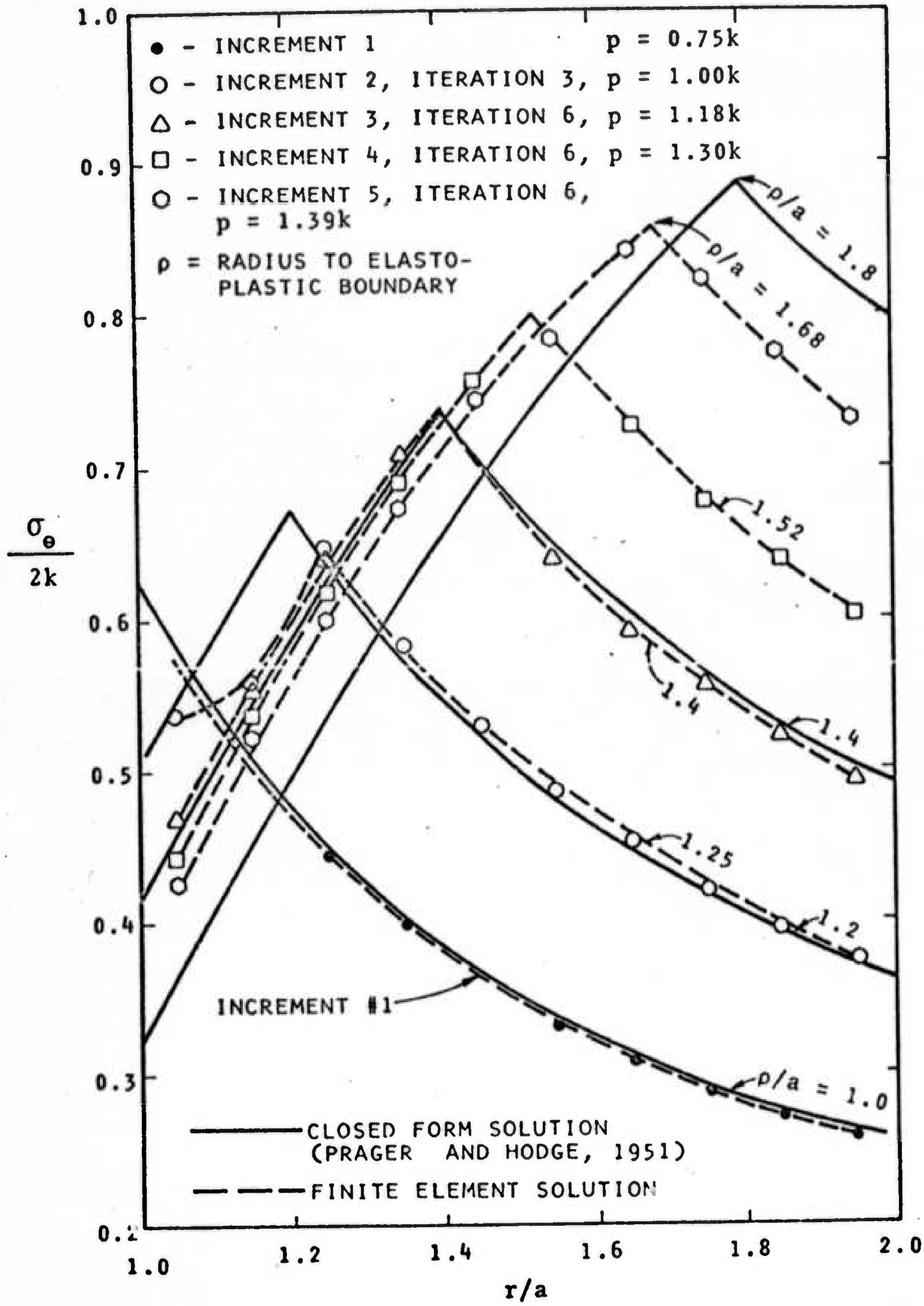


NUMBER OF NODAL POINTS = 99
NUMBER OF ELEMENTS = 80

E = 100,000 PSI
ν = 0.3
C = 100 PSI
ϕ = 0

FIG. I-1 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A THICK-WALLED CIRCULAR TUBE ($b = 2a$)



NOTE: THE APEX OF THE CURVES REPRESENTS THE BOUNDARY BETWEEN THE PLASTIC AND ELASTIC REGIONS. IN COMPARING CURVES IT IS NECESSARY TO EXAMINE CURVES WITH THE SAME ρ/a.

FIG. I-2 - DISTRIBUTION OF CIRCUMFERENTIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

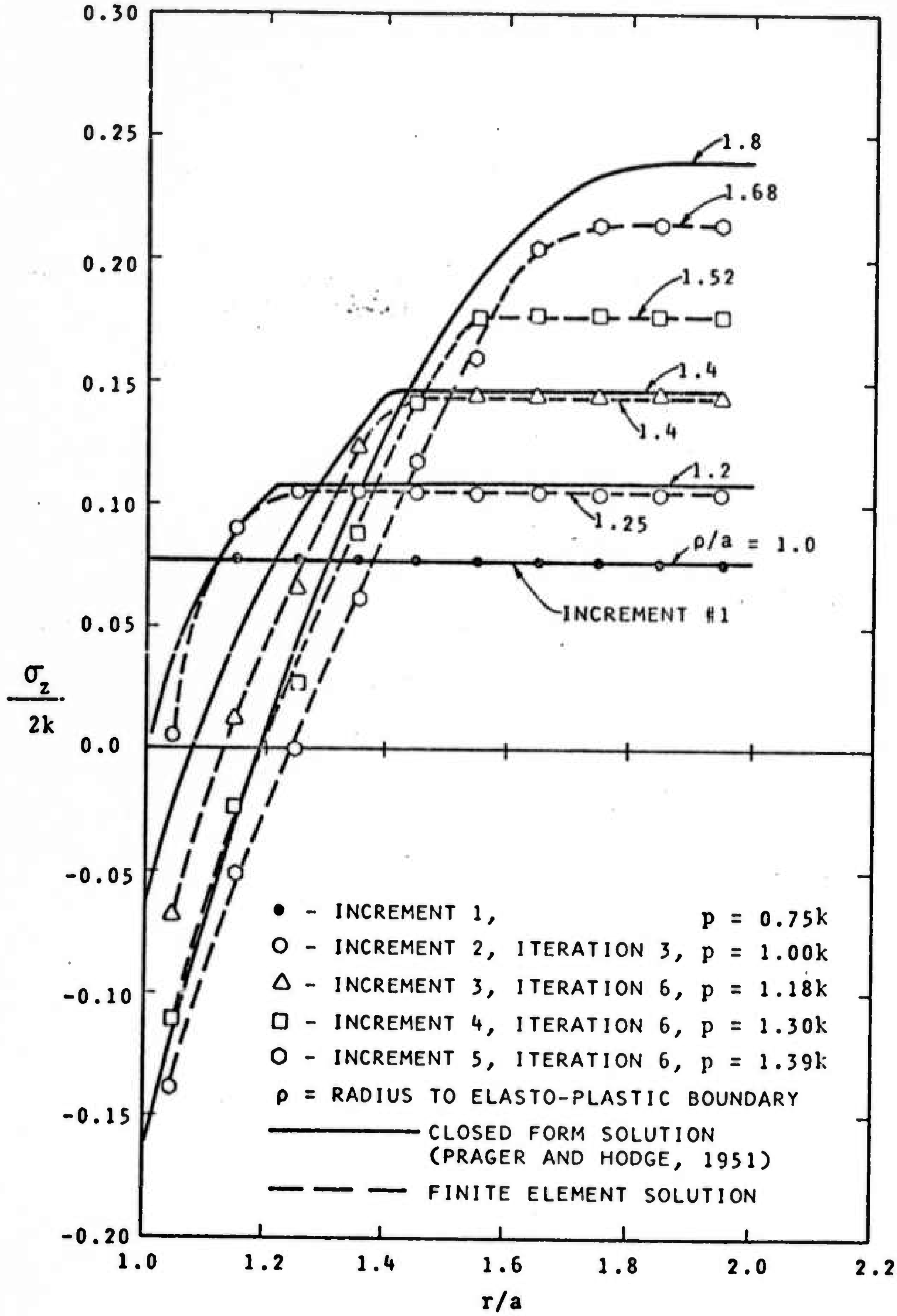


FIG. I-3 - DISTRIBUTION OF AXIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

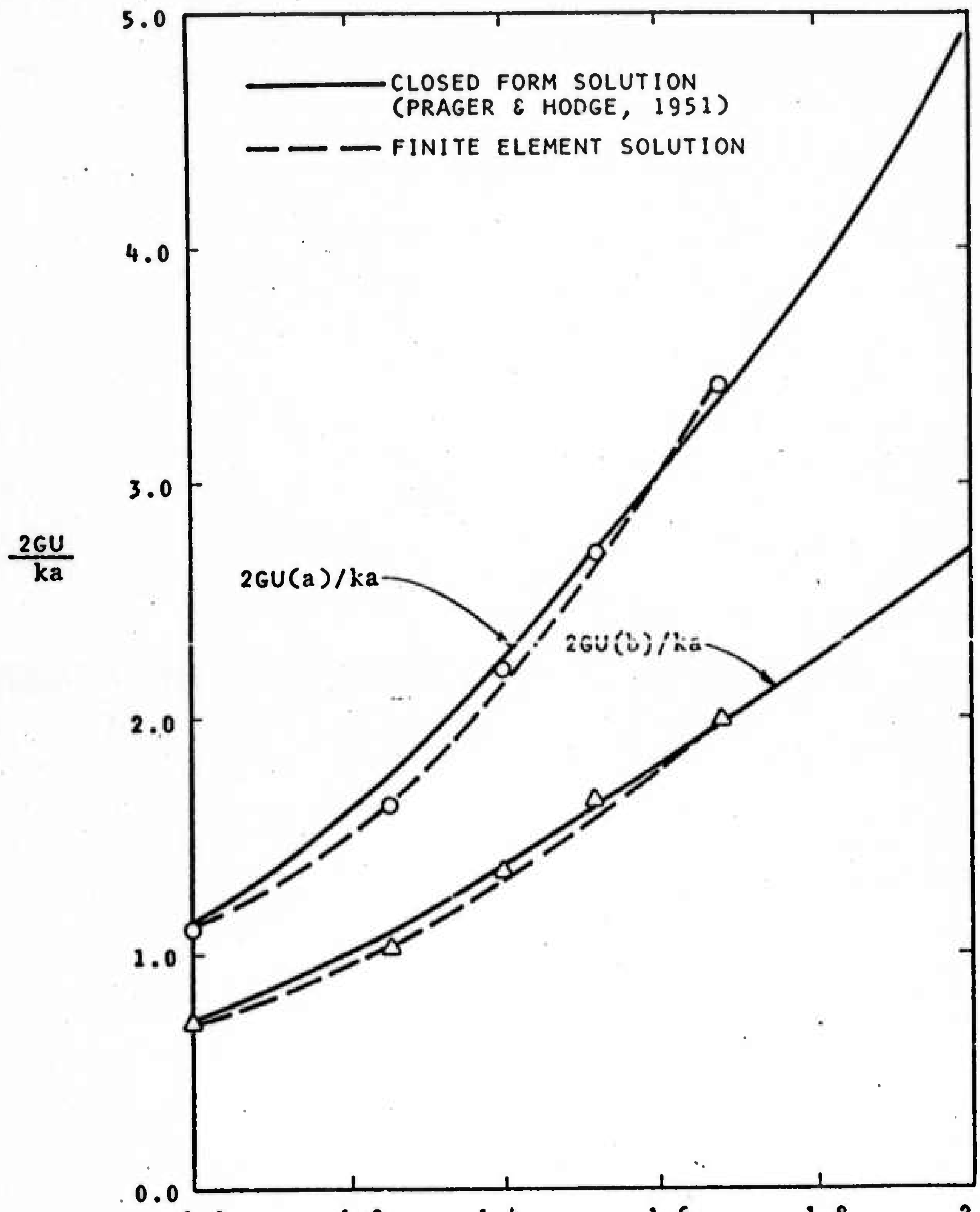
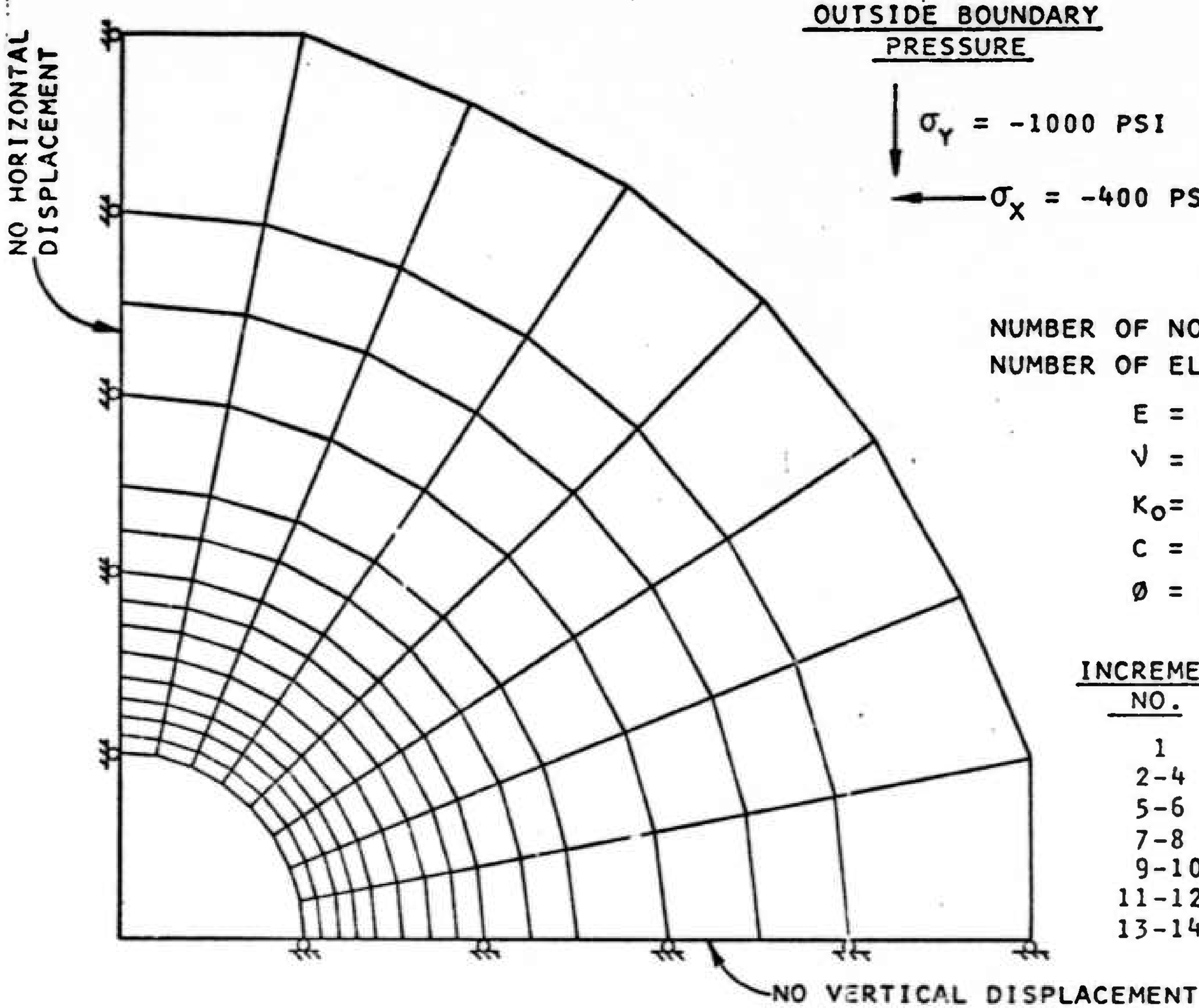


FIG. I-4 - RADIAL DISPLACEMENT U (a), U (b) VS. RADIUS ρ OF ELASTIC-PLASTIC BOUNDARY FOR A THICK-WALLED CIRCULAR TUBE



OUTSIDE BOUNDARY PRESSURE

σ_Y = -1000 PSI

σ_X = -400 PSI

0 10 FT.
SCALE

NUMBER OF NODAL POINTS = 135
NUMBER OF ELEMENTS = 112

E = 72,000 KSF

ν = 0.2

K_o = 0.4

C = 40.32 KSF

Ø = 30°

INCREMENT NO.	PERCENTAGE OF TOTAL BOUNDARY PRESSURE
1	0.370
2-4	0.256
5-6	0.105
7-8	0.089
9-10	0.073
11-12	0.058
13-14	0.049

FIG. I-5 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A CIRCULAR OPENING

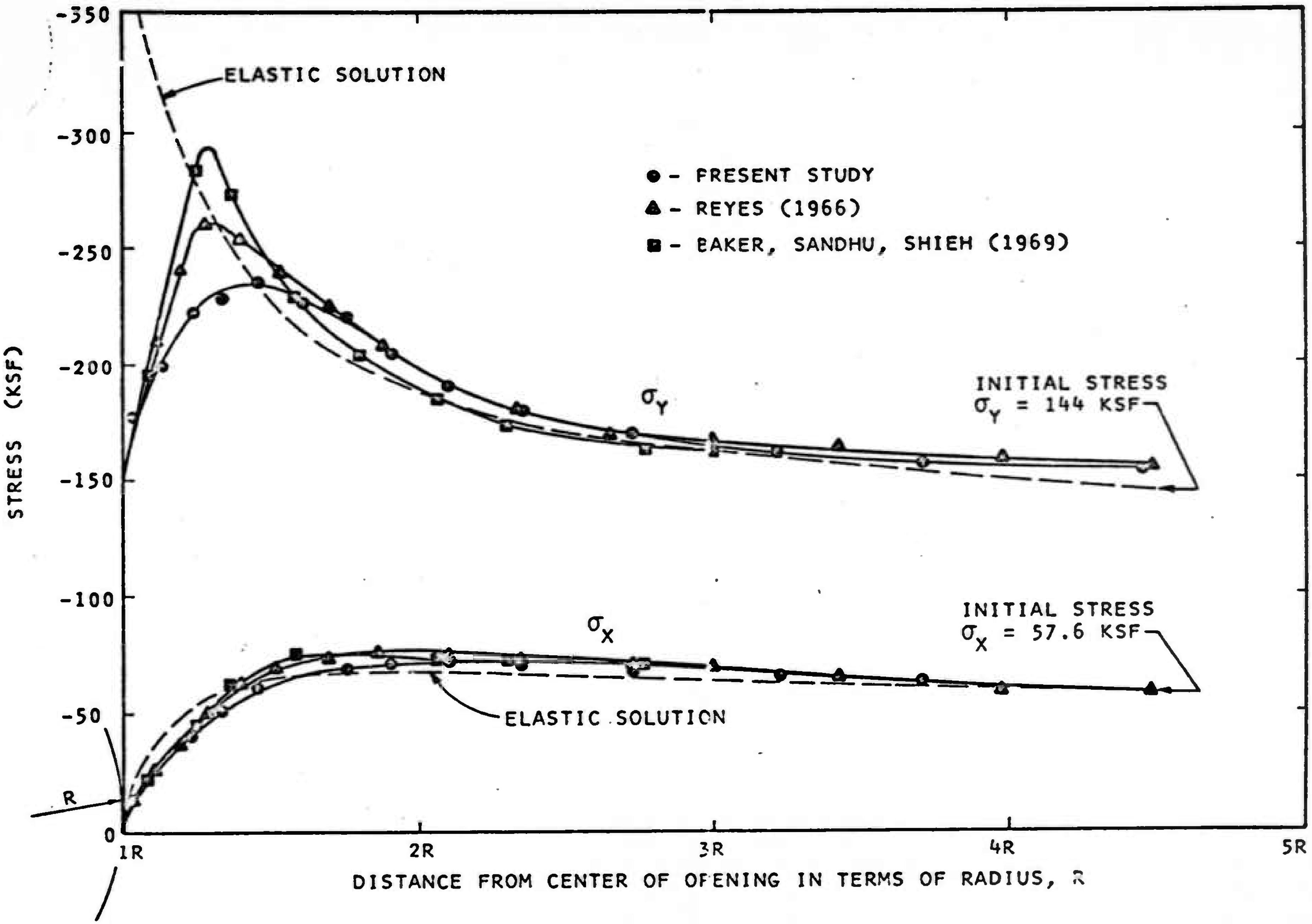


FIG. I-6 - VERTICAL AND HORIZONTAL STRESSES ALONG HORIZONTAL SECTION OF A CIRCULAR OPENING

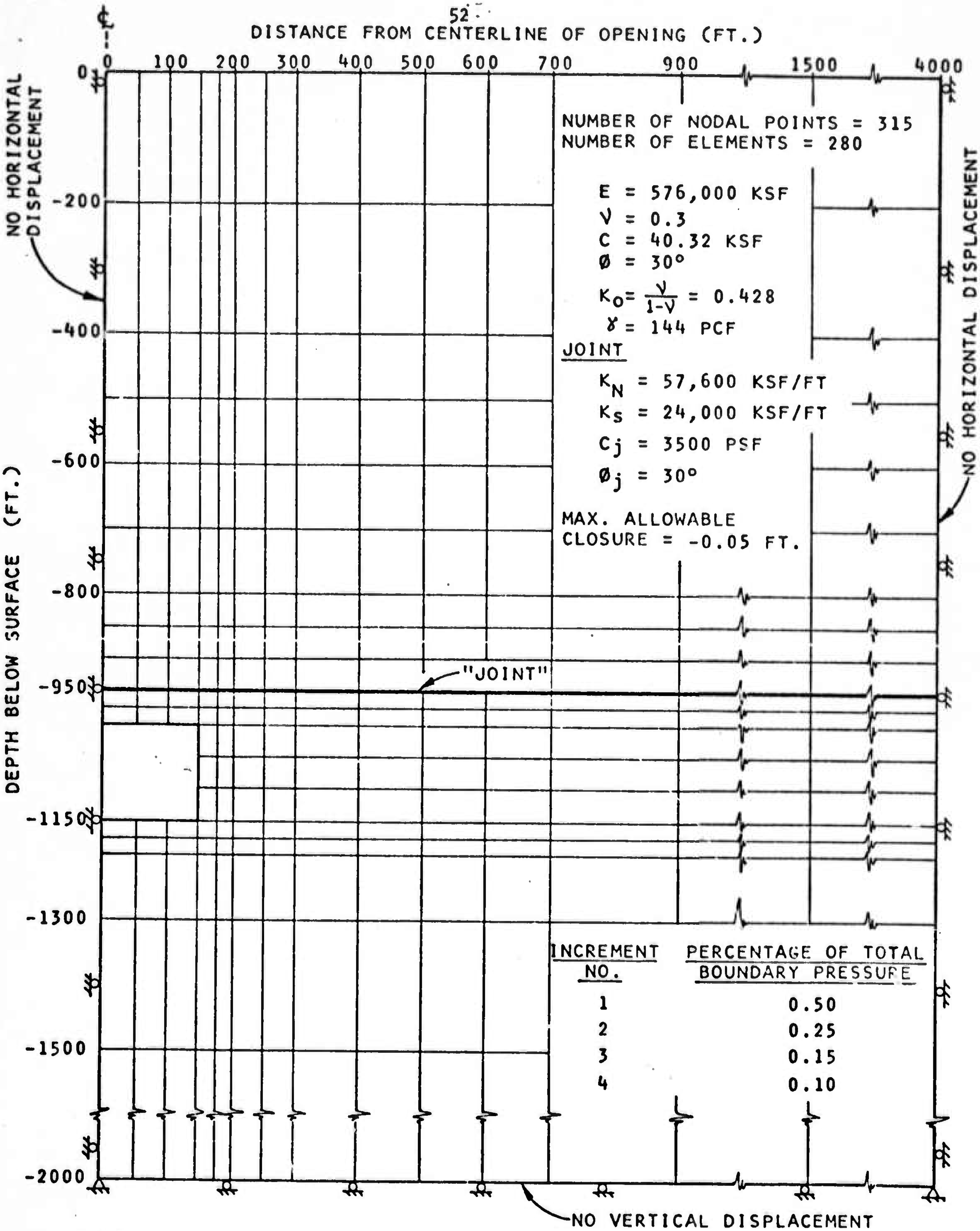


FIG. I-7 - FINITE ELEMENT MESH FOR ANALYSIS OF A RECTANGULAR OPENING

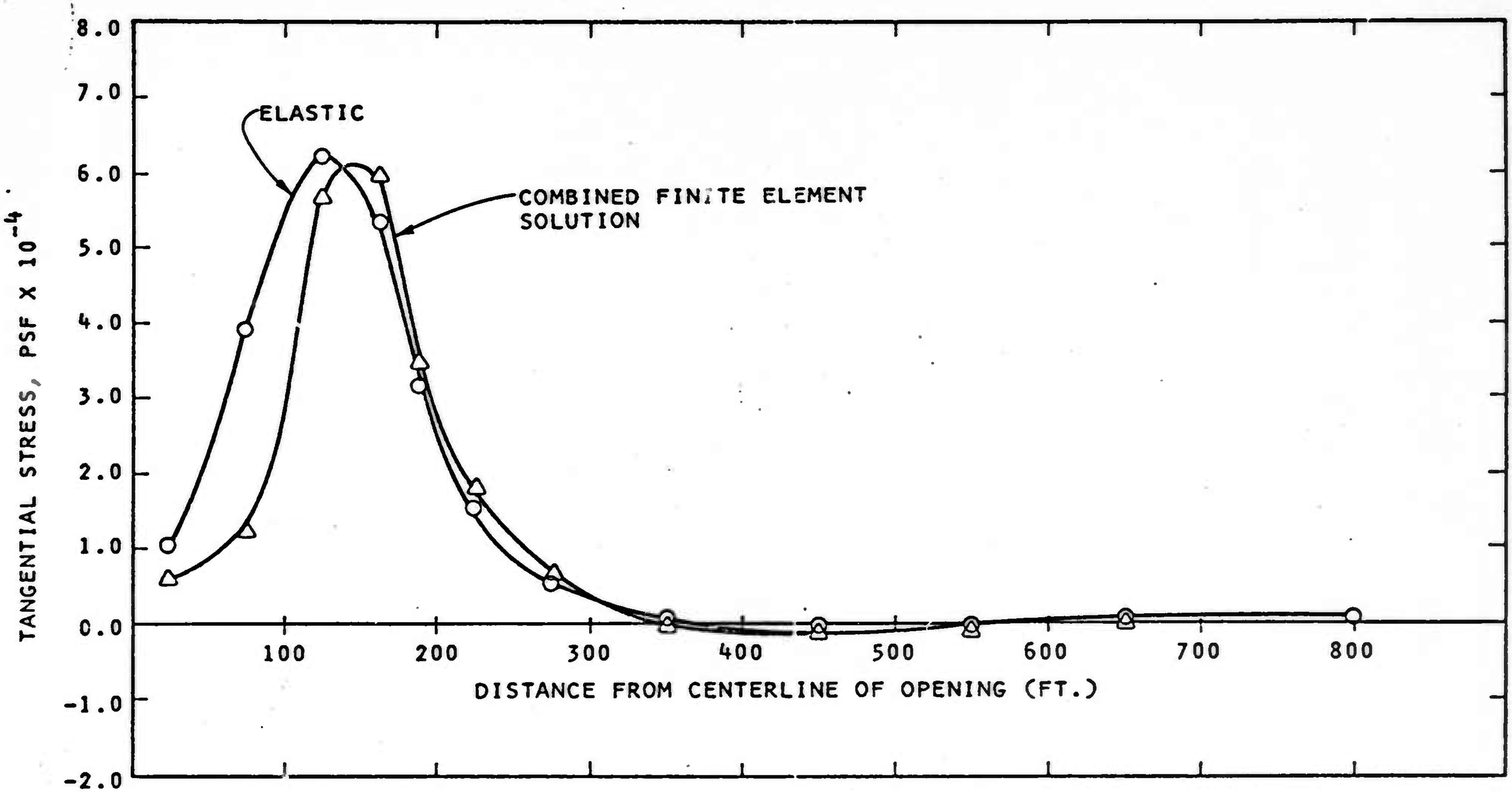


FIG. I-8 - DISTRIBUTION OF TANGENTIAL STRESS ALONG HORIZONTAL JOINT

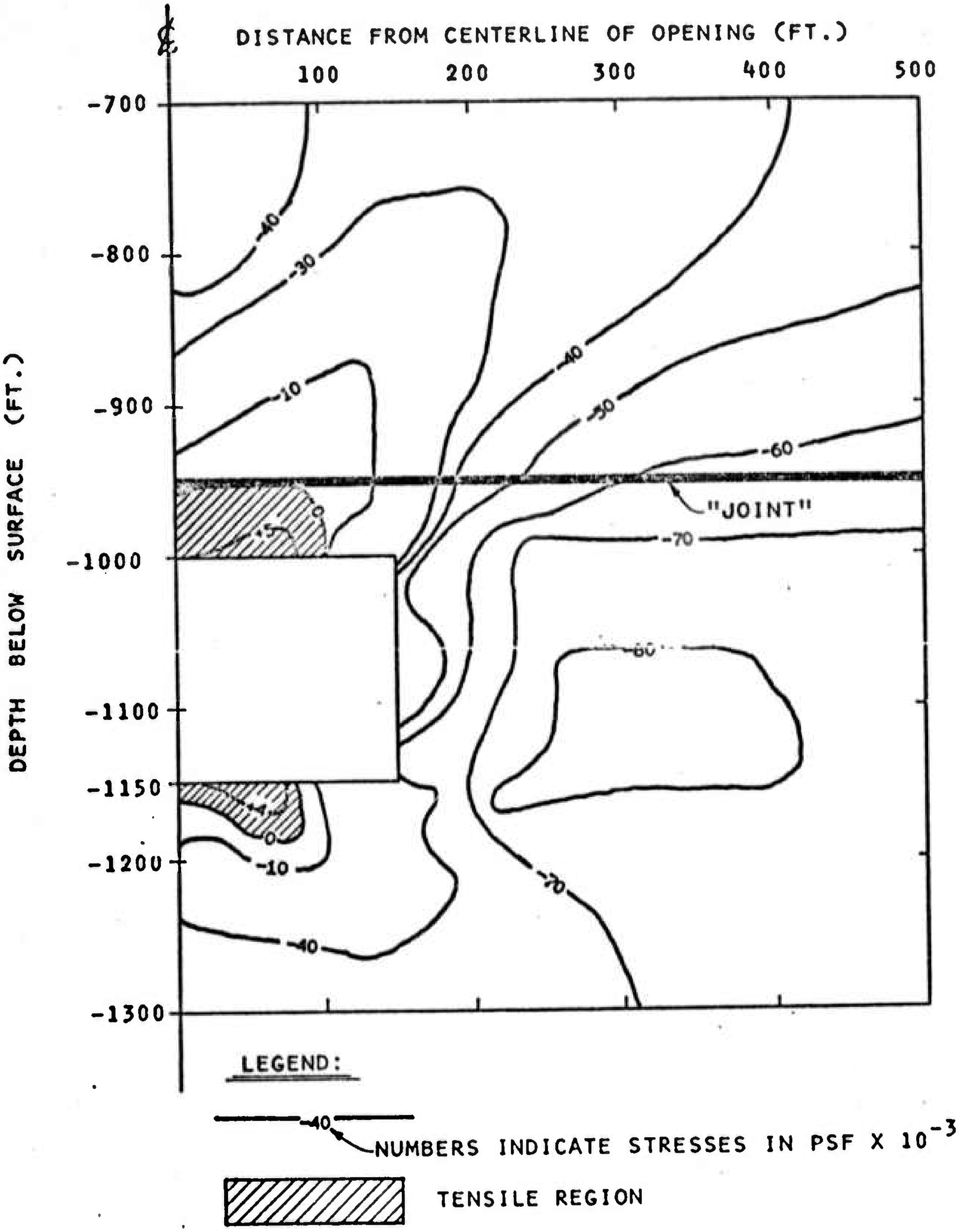


FIG. I-9 - DISTRIBUTION OF MAJOR PRINCIPAL STRESS AROUND A RECTANGULAR OPENING (COMBINED FINITE ELEMENT SOLUTION)

APPENDIX B

Abstract of oral presentation "Roof Movement Due to Increasing Roof Span in an Underground Opening" given at the Fourteenth Symposium on Rock Mechanics, held at the Pennsylvania State University, University Park, PA, June 11-14, 1972.

This presentation included results of research performed under contract H0210017 "Stresses, Deformations, and Progressive Failure in Non-Homogeneous Rock" with Ohio State University, Columbus, Ohio, Dr. R. S. Sandhu, Principal Investigator. Following is information on obtaining technical reports and a magnetic tape containing the computer programs developed under this contract from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, Va. 22151.

	AD No.	Price
Semiannual Technical Report	AD-731 688	$ 3.00[1]
Final Technical Report - Vol. 1	AD-768 648/8	5.50[1]
Final Technical Report - Vol. 2	AD-768 649/6	4.50[1]
Final Technical Report - Vol. 3	AD-768 650/4	2.75[1]
Magnetic tape containing computer programs developed under this contract	AD-768 651/2	250.00 Foreign $312.50

[1]Price shown is for paper copy. These reports are also available on microfiche at $1.45 per copy.

APPENDIX B

ROOF MOVEMENT DUE TO INCREASING ROOF SPAN IN AN UNDERGROUND OPENING

by

Dr. W. Karwoski[1], Dr. R. Sandhu[2], and Dr. R. Singh[3]

A finite element analysis is made of the roof displacements in an underground room as the roof span is incrementally increased.

The prototype opening was sited in a large tertiary gravel deposit. The material was described as cemented gravel with a bulk density of 130 lbs/ft^3. No other engineering properties were available.

Initially, two rooms were driven off an adit. The rooms were driven side-by-side on 40-foot centers to a length of 75 feet. One room was progresively widened from 10 feet to 15, 22, 30, and 40 feet in a sequence which increased the room span on each side of the initial room centerline. Each widening was done along the entire room length at once by blasting. This procedure left an irregular 5- to 10-foot pillar between rooms.

Roof to floor closure measurements were made at eight stations along the initial centerline of the room. In total, 24 closure measurements were made.

The closure measurements show the classical creep curve profile between widening events. From these measurements, the "instantaneous" sag in the roof due to each widening event was deduced. No attempt was made to calculate the creep deformation of the roof.

Finite elements were used to model a typical prototype cross-section as a 2-D plain strain condition and stressed by gravity induced loads. The material engineering properties used in the model were estimated. The room widening events were simulated in the model by decreasing the stiffness of

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elements which were to be removed by the excavation sequence. Results were output at locations in the model to compare with the prototype closure measurements.

The calculated closures compare favorably with the measured values. It is felt that to obtain closer comparison improvements must be made in the prototype measurement technique and in the material property estimates.

APPENDIX C

Manuscript of "A Parametric Study of Stresses in Steel Support for a Tunnel" presented at the Fifteenth Symposium on Rock Mechanics held at the South Dakota School of Mines, Rapid City, South Dakota, September 17-19, 1973. This presentation included results of research performed under contract H0210017 "Stresses, Deformations, and Progressive Failure in Non-Homogeneous Rock" with Ohio State University, Columbus, Ohio, Dr. R. S. Sandhu, Principal Investigator. Information on obtaining reports and computer programs developed under this contract are given on page 55.

APPENDIX C

A PARAMETRIC STUDY OF STRESSES IN STEEL SUPPORT FOR A TUNNEL

by

Dr. Ranbir S. Sandhu[1] and Dr. William J. Karwoski[2]

ACKNOWLEDGMENTS

This research was supported by the Advanced Research Projects Agency of the Department of the Defense under Contract number H0210017 with Ohio State University Research Foundation, monitored by the Bureau of Mines and Bureau of Mines internal research project, "Theoretical Investigation of Rock and Support Interaction," conducted at the Spokane Mining Research Center. Michael J. Beus of the Spokane Mining Research Center contributed the data on steel arch dimension and blocking configurations.

ABSTRACT

A two-dimensional finite element computer code is applied to the analysis of stress and bending moment in a steel arch and invert strut commonly used for ground support. Parametric investigations are made to delineate and rank rock physical properties that affect the loads imposed on the ground support structure. In the analysis it is assumed that elastic rock deformations have taken place prior to support installation. The support is loaded by assuming that zones of "failed rock" develop around the support and other mining loads are imposed as boundary loads on the computational mesh.

The effect of asymetrical blocking and blocking density on stresses and bending moments in the steel support is determined. The elastic modulus of the rock had the most significant effect on the state of stress in the support. Increased blocking density reduces bending moment stresses, and asymetrical blocking increases bending moment stresses.

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INTRODUCTION

The purpose of this investigation is to determine the influence of rock properties and the number and distribution of blocking points on stresses and bending moments in a steel arch used for mine support. Young's modulus, Poisson's ratio, cohesion, and the angle of internal friction are varied over a wide range of values to study the stress distribution in a continuously loaded steel arch. Using the combination of physical properties that produce the maximum bending moment and stresses, three cases with varying point load distributions on steel arches are investigated.

Figure 1 shows the configuration of the tunnel opening and the four different blocking systems studied. The initial stress field was specified as hydrostatic pressure corresponding to an overburden depth of 1,000 feet and material density of 165 pounds per cubic foot. The range of material property values is shown in table 1.

All data on the initial stress field, material properties, steel arches, blocking material, and locations were provided by the U.S. Bureau of Mines Spokane Mining Research Center. The finite element computer program used in these investigations was developed under ARPA contract number H0210017 with Ohio State University Research Foundation. The contract final report (reference 1) and a copy of the computer program may be obtained from the Defense Documentation Center, Alexandria, Virginia 22314. (See page 55)

Rational For Model Load Development

When a tunnel is excavated, the load carried by the material removed must be carried by the rock in the tunnel walls and by the unexcavated rock ahead of the face. Continued excavation at the face results in "loss of support" further increasing the stress in the walls. For linear homogeneous isotropic elastic rock, it has been shown (reference 2) that theoretically this effect is felt only in a region one diameter away from the face. If supports are installed immediately after excavation, they will share this transfer of load as the face is advanced. It is observed in field tests (reference 2) that rock movement continues for a long time before reaching stabilization. This results in continued growth in the load transferred to the supports. Also, exposure to atmosphere, loss of gouge material, and blasting damage may alter the mechanical properties of the rock mass, resulting in increasing deformation and increasing support stresses. The load development may be associated with one or more of the following mechanisms:

a. Upon continued excavation at the face, the removal of rock res 'ts in increased rock load being transferred to the walls of the portion already excavated and the supports in that portion.

b. Time-dependent deformation of rock is resisted by the supports resulting in their taking on increasing load.

c. Change in material properties after installation of supports will result in additional deformation which in turn will lead to increased stresses in the supports resisting such deformation.

d. By blocking, a prestress may be introduced to support rock. Wedging of the blocks will give equal and opposite forces acting on the tunnel surface and the support structure.

In the work reported herein, the influence of various rock properties (see Table 1) upon support stresses was studied, assuming load development primarily through mechanicism (a) described. The system in figure 1 (case a) was analyzed to study the effect of variation of parameters and to rank them in order of importance. Calculations were made varying each of the parameters between the limits given one at a time. The "constant" values of other parameters were taken as the midpoint of the specified range. The combination of parameter values corresponding to the worst stress in the supports was then used to analyze cases b, c, d shown in figure 1.

ASSUMPTIONS MADE IN THE ANALYSIS

Two Dimensional Finite Model

The assumption of two-dimensional symmetry can only be strictly true when the tunnel is continuously supported, for example, with concrete because every plane normal section through the tunnel is identical. When the tunnel is supported intermittently with steel supports, every plane normal section through the tunnel is not identical, and the analysis must be three-dimensional to be truly representative. Furthermore, rock reaction to line loads produced by steel sets is distinctly different to uniform loads produced by a continuous lining. Dixon (reference 3) discusses this in detail and reports a method whereby problems of intermittent supports in tunnels can be made amenable to two-dimensional analysis by scaling Young's modulus of elasticity for the rock mass.

In this paper the interest is in the trend of the support stresses as various rock mass properties are increased or decreased in value. The same trends are expected to exist when the three-dimensional aspects of the problem are accurately modelled. A two-dimensional model of the tunnel and its support is deemed suitable for these preliminary investigations.

Extent of the Finite Model

When an underground opening is excavated, changes in the stress field and associated deformations occur in the entire rock mass. The principle of local action implies that these changes diminish with increasing distance from the opening. In the finite element model, a finite region is generally considered. On the boundary of this finite model, force or displacement boundary conditions must be prescribed. These can be based on the assumption either of no change in the stress field or of no deformation. Neither of the two is true for finite distances from the opening, and the two assumptions in fact give bounds to the correct solution. Nair (reference 4) and Kulhawy (reference 5) among others, have studied the effect of lateral dimension of the finite model and of the choice of boundary conditions on stresses and deformations in the vicinity of underground openings. For the present study, allowing for the specified hydrostatic initial stress field, a preliminary analysis showed that it would be adequate to model a region extending approximately seven diameters above the roof of the opening, five diameters horizontally on each side of the center-line and five diameters below the invert of the opening. The boundary conditions for the finite element model are illustrated in figure 2. The overburden of the top 858 feet was replaced by an equivalent vertical load, and gravity was applied to the mesh interior. The stress field on the vertical faces AB, CD was assumed to be unaffected by the sequence of operations in the opening and the vertical displacement of the horizontal section BC was set equal to zero.

<u>Modelling of Support Structure and Sequence of Operations</u>

The steel supports were assumed to be in plane stress whereas the rock and shotcrete were in plane strain. The spacing of steel supports was specified as three feet. Thus, a three-foot length of the tunnel was supported by each support ring. Figure 3 shows a typical cross-section used in the finite element model. The cross-section of the steel rib was represented by five finite elements to obtain reasonably good distribution of stresses over the cross-section. The shotcrete and the steel rib were assumed to be bonded. If there is no bond between shotcrete and the rib, there is no load transferred through shear, and the load transfer is entirely radial. This situation would be similar to case d, in figure 1, except that the blocking would be continuous. For cases b, c, d, the wooden blocks were assumed to be axial members not transmitting any bending moments.

The shotcrete, the steel rib, and the timber blocks were assigned the properties given in table 1. It was assumed that C and ϕ, specified for the rock, were obtained in triaxial tests on cylindrical specimens. It was assumed that the rock properties do not change as a result of excavation of the opening.

Starting with the initial stress state, the sequence of opening excavation, installation of support, and support load development is simulated in the computation. As opening excavation is made, all deformation due to the interaction between the opening and the initial stress state takes place at that time. The steel support is placed in the deformed opening. At this point in the computation no loads would develop on the support unless the existing stress state is changed. This is done by increasing the density of the rock surrounding the support from 85.2 lbs/ft^3, used in the initial computation, to 165 lbs/ft^3, used in the computation with supports. The shaded areas in figure 4 are the regions in which this weight density change was made. This sequence of operations is rationalized as simulation of model load development mechanism (a) described earlier. The idea that the weight supported by the steel sets is limited to a finite region of rock around the tunnel is taken from Karl Terzaghi's "Rock Defects and Loads on Tunnel Supports" (Proctor and White, reference 6). The geometry of the shaded areas in figure 4 corresponds to some observed tunnel profiles where the rock has fallen out of the roof. Because good information on how rock loads develop is not available, it was arbitrarily assumed that the difference in excavation profiles in figure 1 case b and case a represented the rock load which could develop for cases a and d. For case b, the rock exerting the gravity load is shown shaded in figure 4b, the extent was arbitrarily taken as an average of 3.5 feet thickness beyond the excavation line. Similarly for case c, the extent was arbitrarily taken as an average of 3.5 feet thickness beyond the excavation line.

A number of important, and perhaps novel assumptions, was made in order to satisfy the problem input requirements. This is a particularly difficult task especially in areas such as rock load transfer or rock load development where knowledge is scarce. However, in view of the moderate goals of the research work, this is acceptable.

RESULTS

Preliminary

In studying the importance of material parameters, the quantities of interest were the maximum stresses in the steel member. It is customary to study the axial and shearing forces and bending moments in such structural components. Hence, these quantities were determined. It should be noted that the basic output from the finite element analysis is the components of stress evaluated at the center of each of the five elements into which the steel member is divided. The moments and forces were obtained by numerical integration of the stress values.

Figure 5 shows the cross-sections at which the stresses as well as the forces and moments in the steel rib were computed.

Influence of Material Properties

Four material parameters, viz., Young's modulus, Poisson's ratio, cohesion and angle of internal friction were to be considered to establish their order of importance in terms of their influence on the support forces.

For the specified range of the rock properties, the cohesion and the angle of internal friction did not influence the stresses in the structural supports. Figure 6 shows a plot of $J_1 = \sigma_1 + \sigma_2 + \sigma_3$, the first invariant of the stress tensor as the abcissa and $J_2^{\frac{1}{2}} = [\frac{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)}{6}]^{\frac{1}{2}}$, the second invariant of the stress deviator tensor as the ordinate. Generalized Mohr-Coulomb yield criteria corresponding to the prescribed range of values of cohesion and angle of internal friction are shown and also the stress paths traced by points of critical locations around the tunnel opening.

The generalized Mohr-Coulomb yield law is,

$$a\, J_1 + J_2^{\frac{1}{2}} = k$$

where: $a = \frac{1}{\sqrt{3}} \frac{2 \sin \phi}{(3 - \sin\phi)}$

$k = \frac{1}{\sqrt{3}} \frac{6C \cos \phi}{(3 - \sin\phi)}$

ϕ = angle of internal friction

C = cohesion

C and ϕ are determined from a triaxial test on a cylindrical specimen under axisymmetric radial stress. Derivation of the above equations is given by Singh (reference 5).

In Figure 6, paths A, B, C, D, E, are traced by elements located around the face of the underground opening. Points A, C refer to elements at the invert and the crown, respectively, and points B, D, E correspond to elements on the side of the opening. The locations are indicated in figure 5. The initial state in all cases is of hydrostatic stress. The terminal points represent the state after excavation. The development of rock load has little influence on the stress state in rock for the specified values of Young's modulus and Poisson's ratio. For all locations, the entire stress history was found to be well below the yield criteria corresponding to the range of parameter values specified in Table 1.

Figures 7(a) through 7(c) show the influence of variation of Young's modulus upon bending moments and upon axial and shearing forces. Only sections with the worst forces have been plotted. The maximum bending moments were at section 16 (crown), the maximum axial force at section 5 (side), and the maximum shearing force at section 10. In all cases, decrease in elastic modulus of rock was seen to result in increased moments and forces. This is indeed to be expected. The decrease in support forces, with increasing Young's modulus, is rapid at first and then is less pronounced. This is due to the fact that rock deformation is proportional to reciprocal of the modulus. Also, for very large moduli, the strains are extremely small, and difficulties arise with computational precision.

Figures 8(a) through 8(c) show the influence of the variation in the Poisson's ratio on the moments and axial and shear forces in the supports. Again, only results for the worst sections have been plotted. In the crown and the side (section 16 and 5), an increase in Poisson's ratio results in decreased bending moments whereas at section 10 the bending moment increases somewhat.

For tne specified range of values for the material parameters, the bending moments and the shearing forces were small. The major effect was the axial force in the member. An explanation for the bending effect being very small may be found in the assumption that the steel member is restrained by the shotcrete. Significant bending of the steel member must involve significant changes in curvature. This is not possible for a member restrained from radial movement by relatively unyielding rock. Also, the rock properties are assumed to be unaffected by the excavation process. This may not be true. There can be considerable change in deformability of a rock mass due to excavation of the underground opening.

In this study, it is evident that supports in lower elastic modulus rock will develop greater stresses. High value of Poisson's ratio has the effect of redistributing stresses decreasing the peaks and increasing the lowest values.

Using the lowest value, in the prescribed range, of Young's modulus and the minimum as well as the maximum values of Poisson's ratio, stresses for cases a, b, c, d of different excavation profiles and blocking arrangements were determined. The resulting stress distributions are shown in figure 9.

Additional Studies

Additional studies using values of Young's modulus less than the minimum of the range given in table 1 were carried out to consider situations where the elastic modulus of rock in the vicinity of an underground opening may be significantly reduced by damage during excavation or deterioration with exposure over a long time. The simulation assumed that after installation of supports and placement of shotcrete, Young's modulus may reduce from an initial value of 1×10^6 pounds per square inch. Different terminal values of Young's modulus used were 0.75×10^6, 0.67×10^6, 0.4×10^6, 0.25×10^6 pounds per square inch. Poisson's ratio was assumed to be 0.49 throughout. For case a, the results are plotted in figures 10(a) through 10(c). As might be expected, greater reduction in the elastic modulus was associated with greater bending moments and axial and shear forces. For reduction of Young's modulus to 0.25×10^6 psi, the maximum longitudinal stress would be over 20,000 psi. Considering that deterioration of rock is more likely to occur in cases b and d, analysis was performed corresponding to a reduction in Young's modulus from 1×10^6 to 0.4×10^6 pounds per square inch. Figure 11 shows the distribution of longitudinal stresses at critical sections for the cases a, b and d.

CONCLUSIONS

The tendency of the rock to deform is a key factor in load transfer to the structural support system. Higher modulus implies less deformation for the same rock load and consequently less load transferred to the steel member.

Higher values, tending towards .5, of Poisson's ratio are associated with redistribution of stress in the rock. This stress redistribution is reflected in a more uniform stress distribution in the steel support.

Sparce blocking increases bending moments in the support structure. The total load transferred to the support structure decreases as the amount of blocking between the steel set and rock increases. The wood blocking acts much like a "back packing" material.

The results of these analyses appear to be intuitively correct. The next step is to use the computer code to analyze a documented field case.

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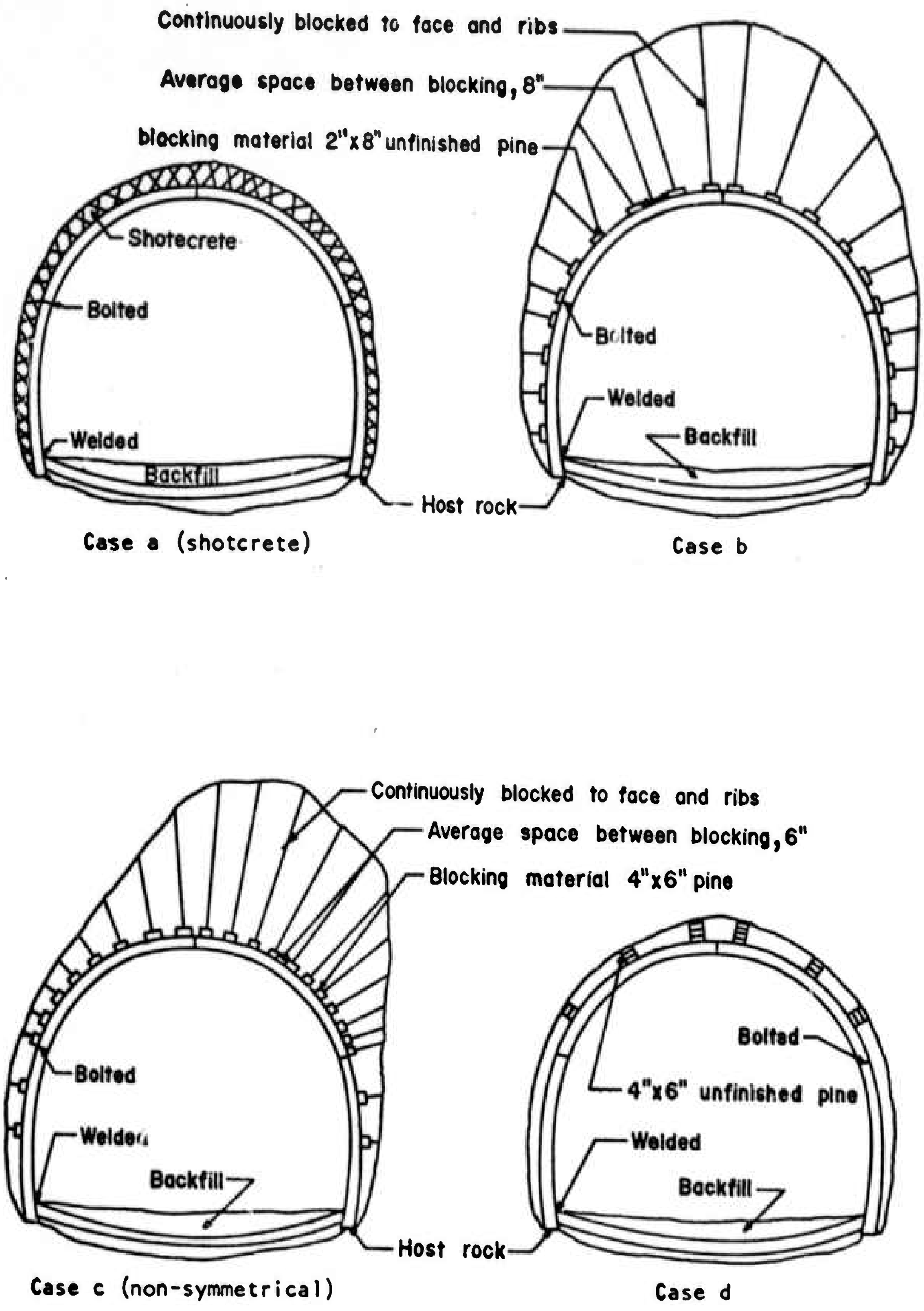


FIGURE 1. - Tunnel Excavation and Support Blocking Configuration.

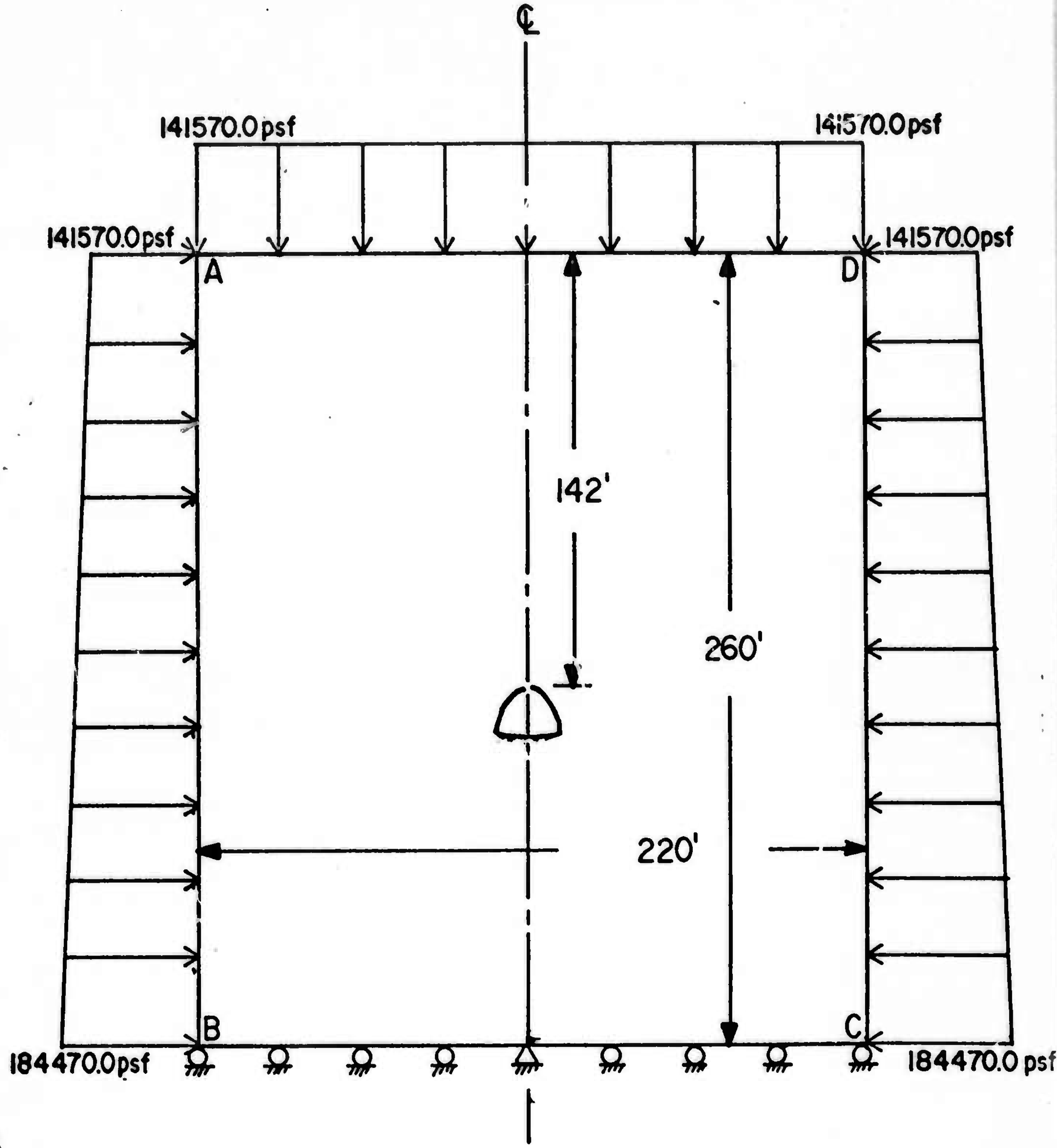
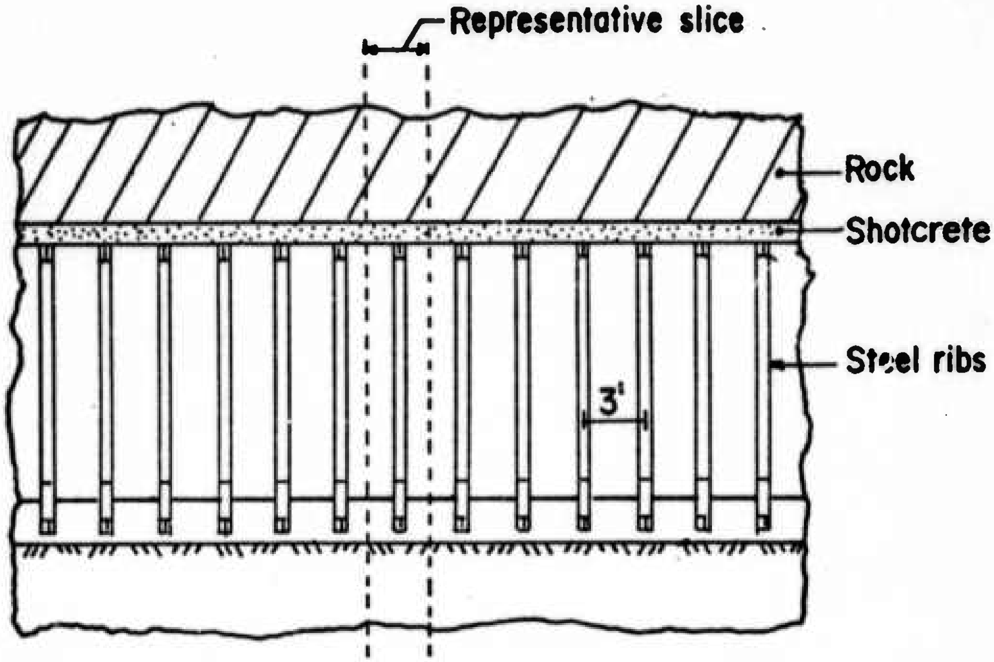
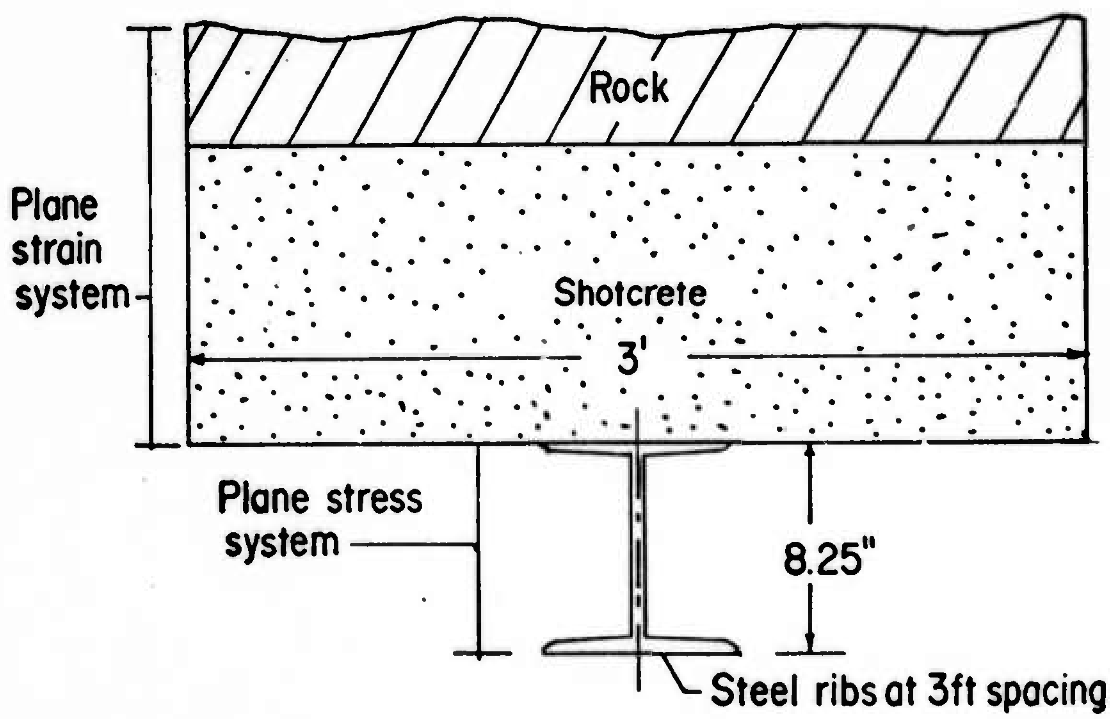


FIGURE 2. - Boundary Conditions on the Finite Element Model.

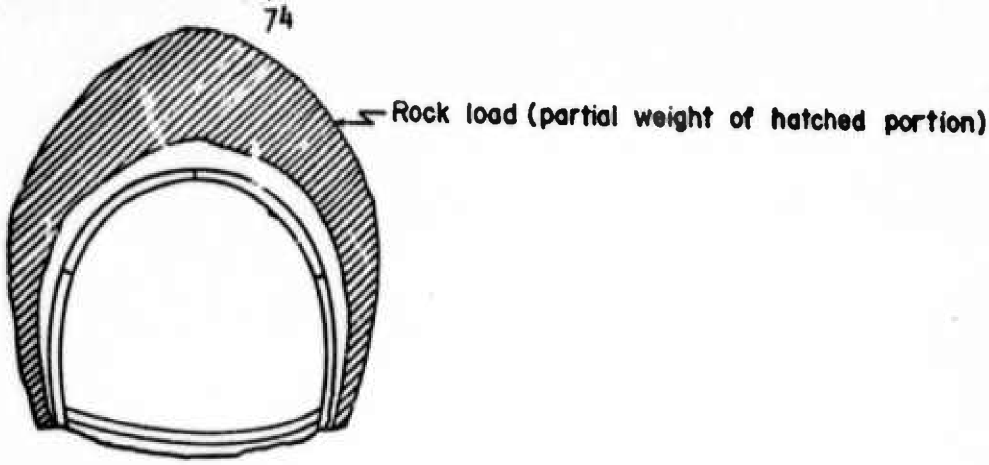


(a) Longitudinal Cross-Section of Tunnel.

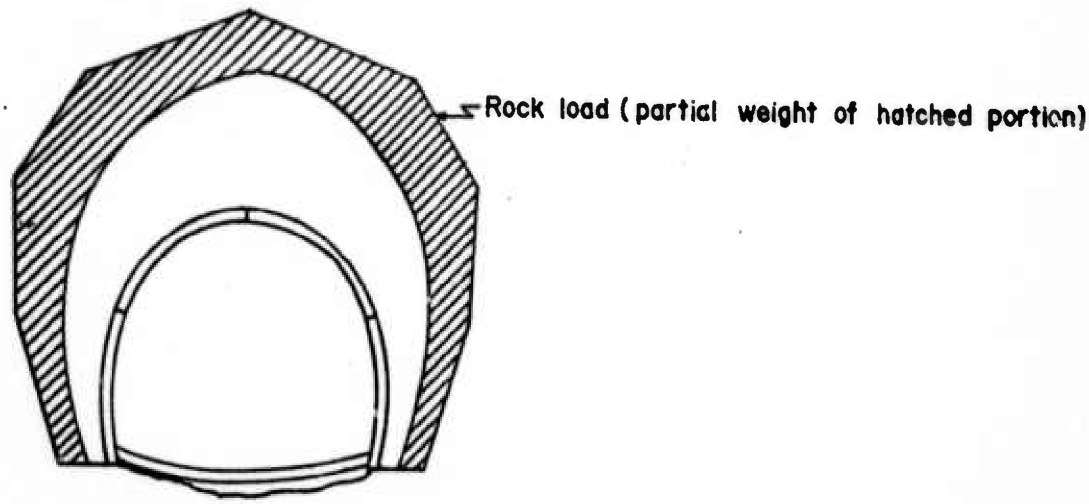


(b) Representative Slice.

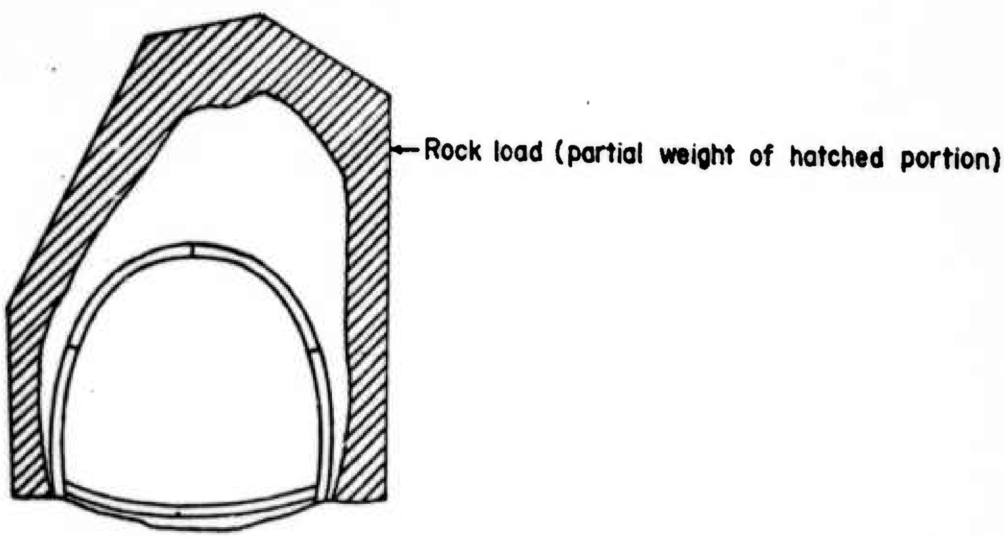
FIGURE 3. - Longitudinal Cross-Section of Tunnel and Representative Slice.



(a) Tunnel Cross-Section Showing Rock Load for Cases a and d.

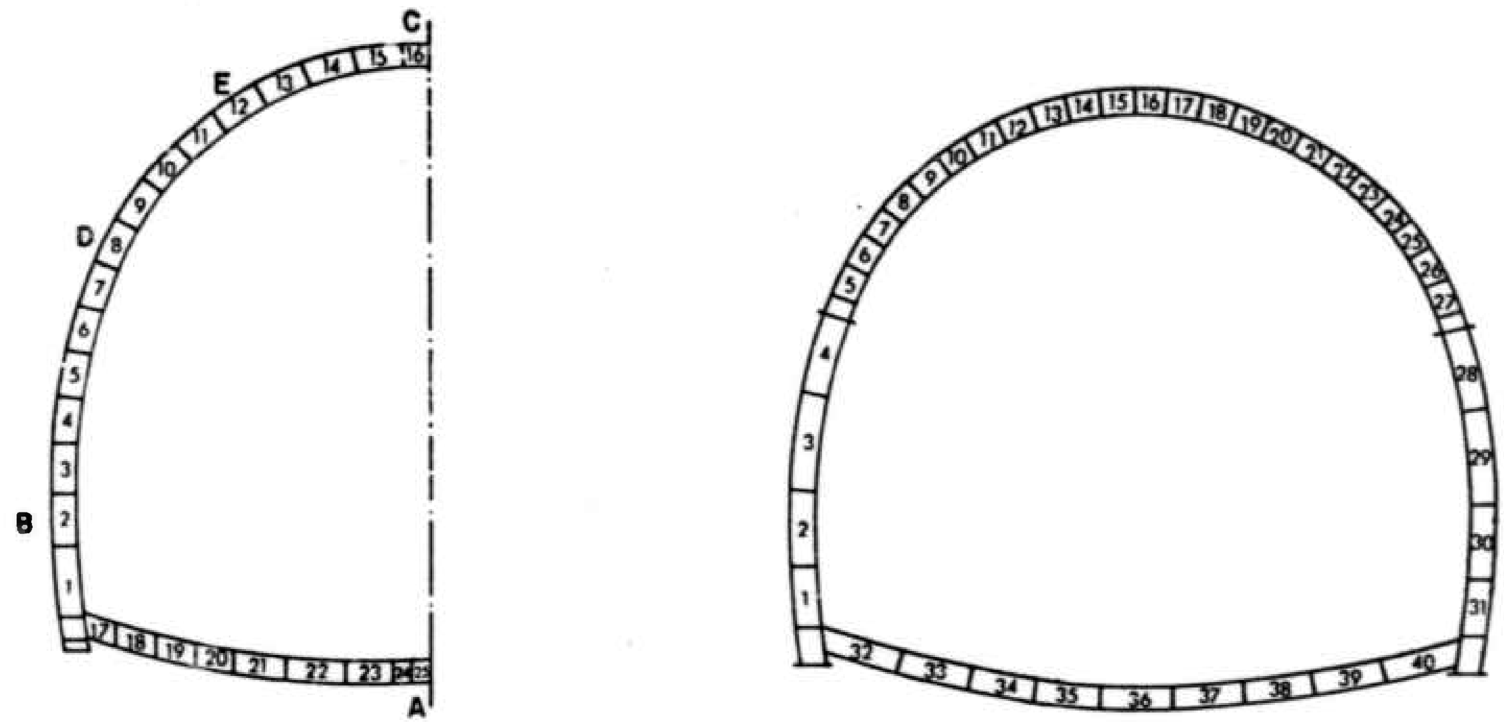


(b) Tunnel Cross-Section Showing Rock Load for Case b.



(c) Tunnel Cross-Section Showing Rock Load for Case c.

FIGURE 4. - Rock Load on Supports.



(a) Location of Sections Analyzed for Cases a, b and d.

(b) Location of Sections Analyzed for Case c.

FIGURE 5. - Location of Structure Sections.

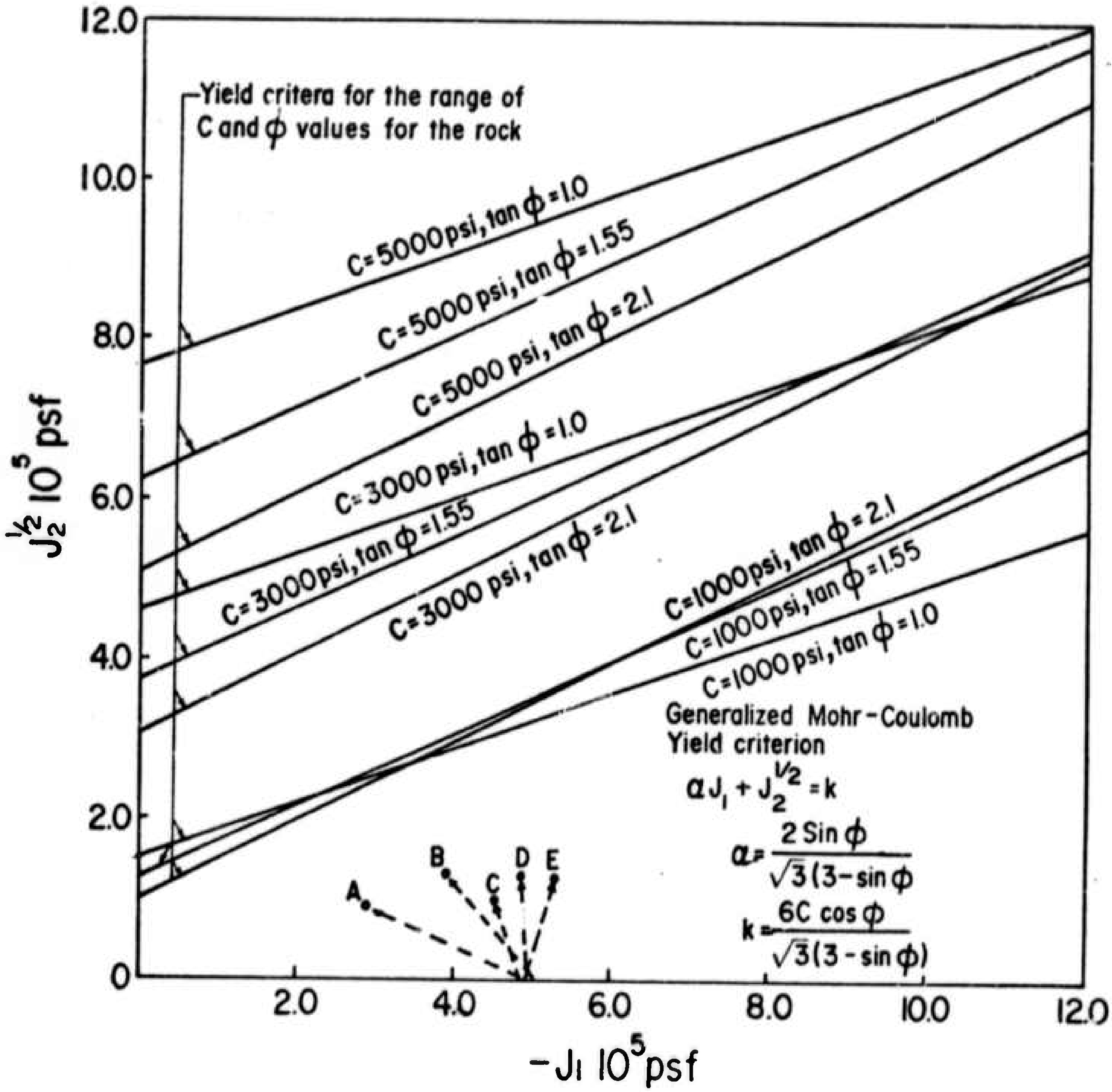
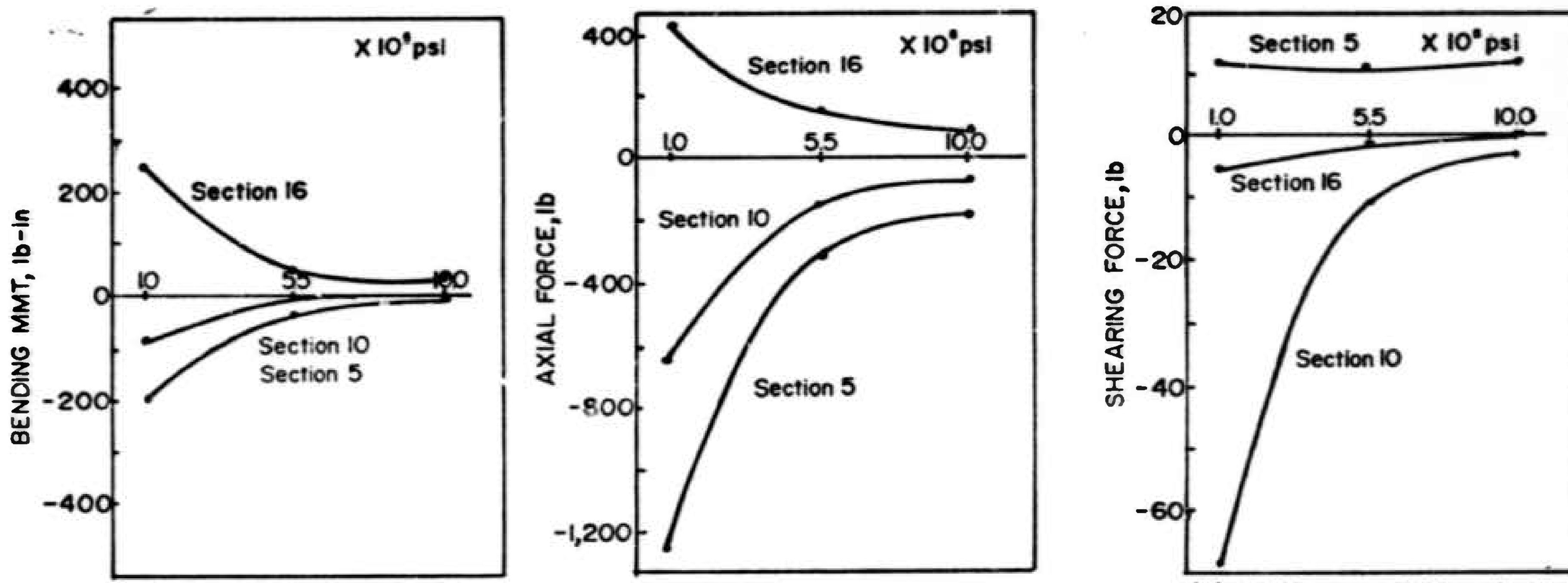


FIGURE 6. - Stress Paths for Points Around the Underground Opening (for location of points see FIG. 5).

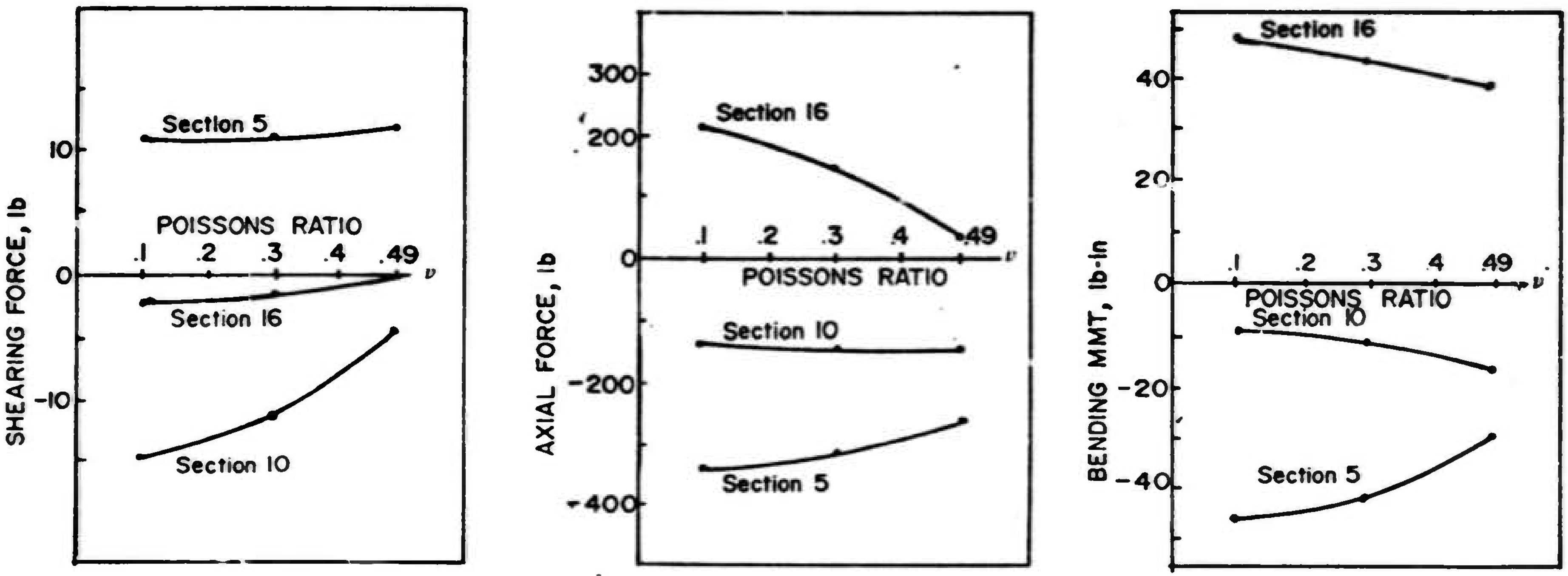


(a) Influence of Elastic Modulus on Bending Moments.

(b) Influence of Elastic Modulus on Axial Force.

(c) Influence of Elastic Modulus on Shearing Force.

FIGURE 7. - Influence of Elastic Modulus For Case(a).



(a) Influence of Poisson's Ratio on Shearing Force.

(b) Influence of Poisson's Ratio on Axial Force.

(c) Influence of Poisson's Ratio on Bending Moments

FIGURE 8. - Influence of Poisson's Ratio

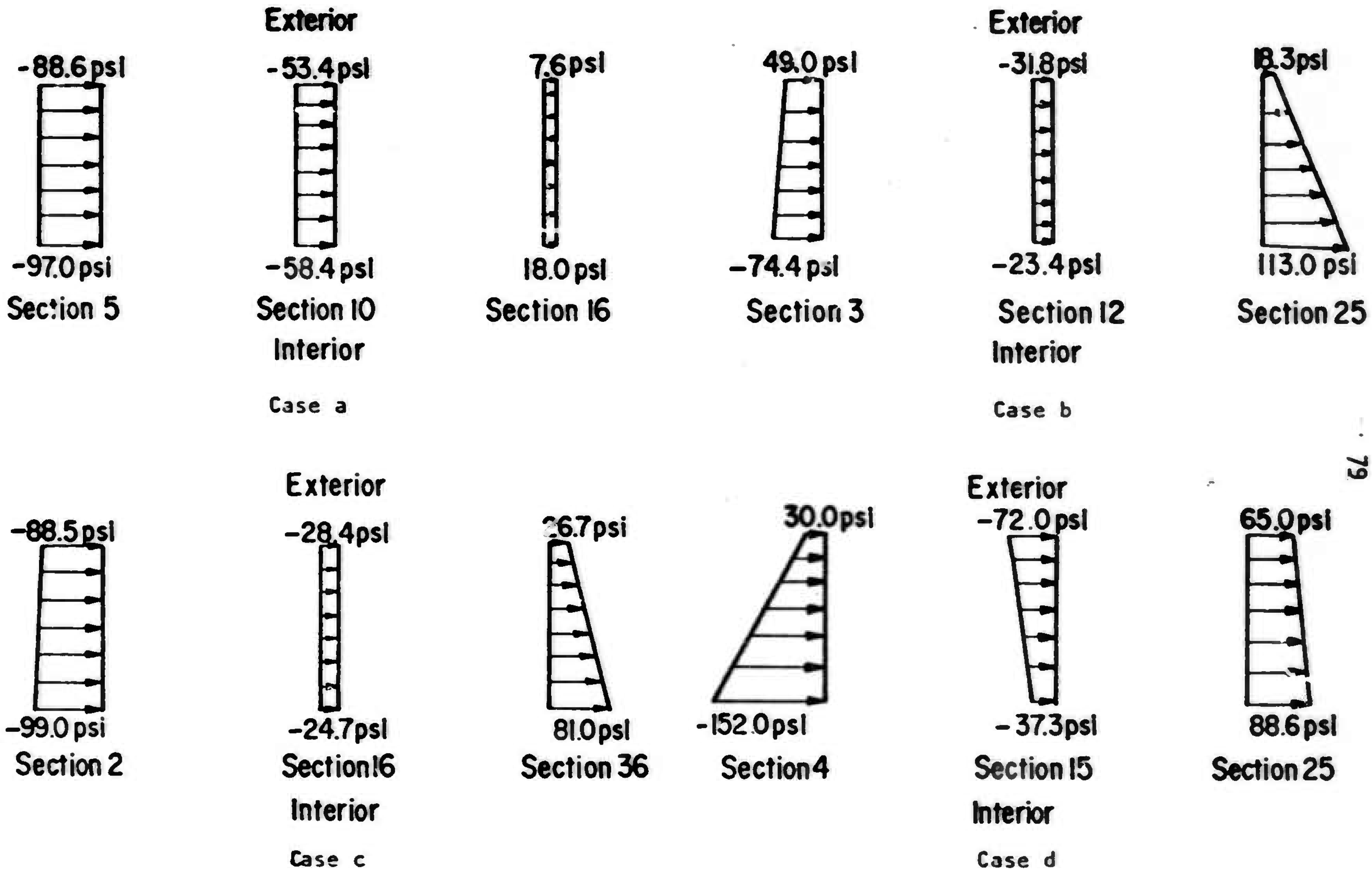
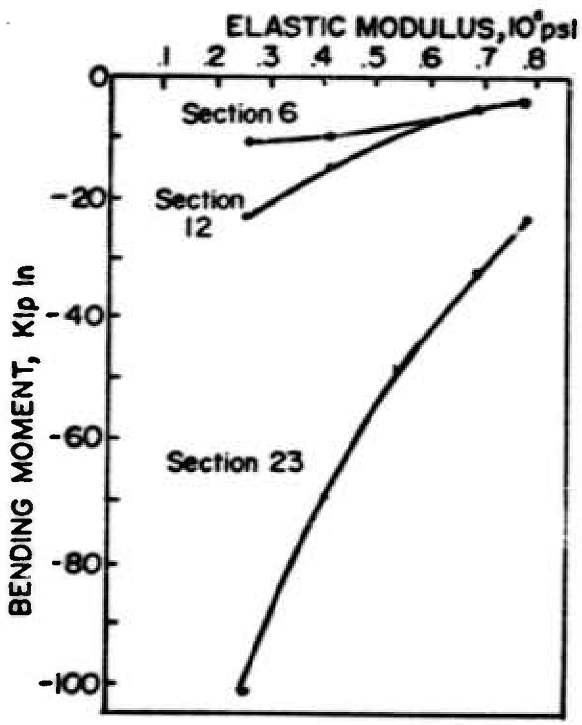
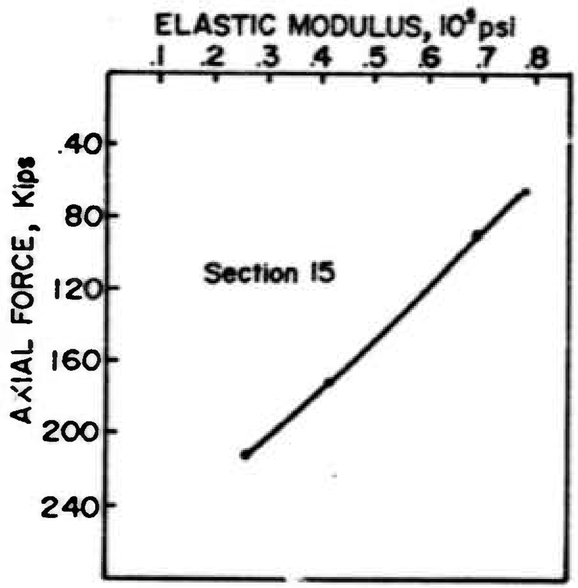


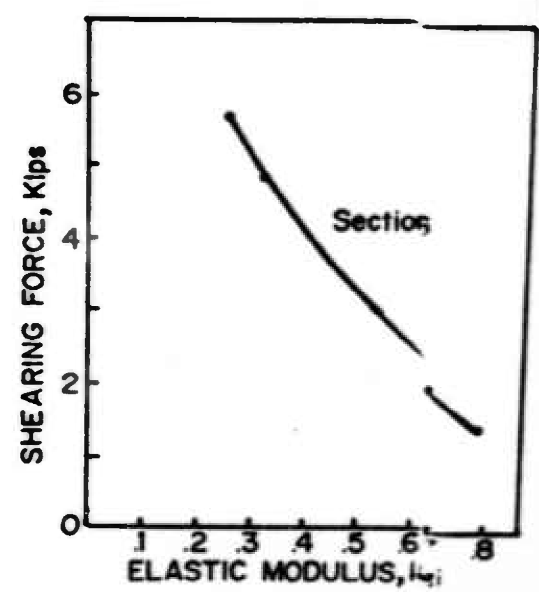
FIGURE 9. -Stress Distribution For Critical Combination of Rock Properties.



(a) Influence of Rock Deterioration on Bending Moment in Tunnel Supports.



(b) Influence of Rock Deterioration on Axial Force in Tunnel Supports.



(c) Influence of Rock -erioration on Shearingrce in Tunnel Suppor

FIGURE 10.- Influence of Rock Deterioration.

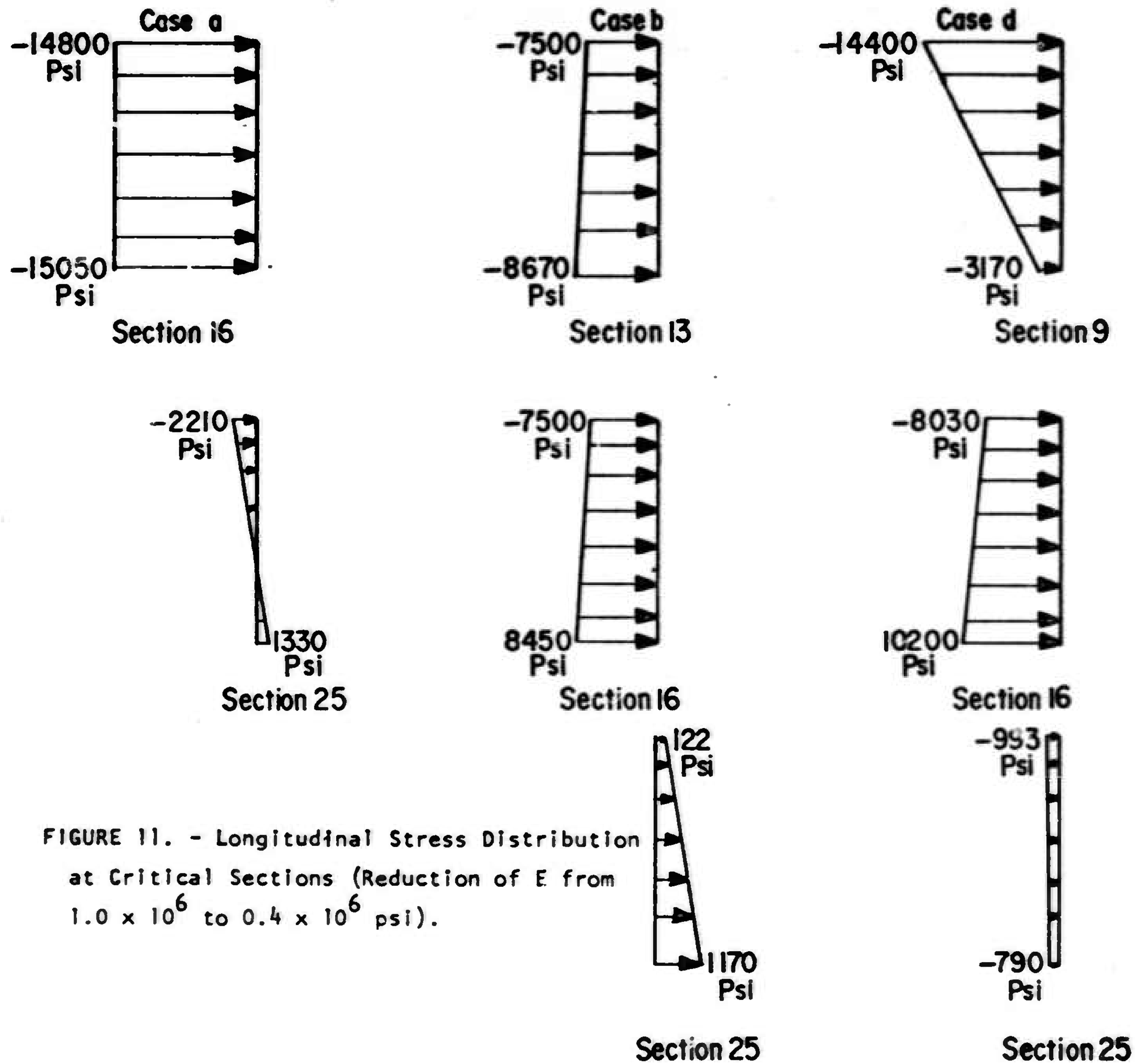


FIGURE 11. - Longitudinal Stress Distribution at Critical Sections (Reduction of E from 1.0×10^6 to 0.4×10^6 psi).

PARAMETERS	MATERIAL			
	(a) Rock	(b) Shotcrete	(c) Steel	(d) Timber
Young's Modulus, E (psi)	$1 \times 10^6 - 10 \times 10^6$	2.71×10^6	30×10^6	1.5×10^6
Poisson's Ratio, ν	0.1 - 0.5	0.1	0.231	0.03
Cohesion, c (psi)	1000 - 5000	3300	---	---
Angle of Internal Friction, ($\tan \phi$)	1.0 - 2.1	1.0	---	---
Material Density (pcf)	165.0	150.0	490.0	27.0

(a) Data furnished by sponsor.

(b) "Manual of Concrete Practice," Part 2, 1968, American Concrete Institute

$$E = w^{1.5} \; 33 \sqrt{f'_c}$$

where $w = 150$ lb./c.ft.

f'_c = compressive strength, assumed as 2000 psi

(c) "Manual of Steel Construction," AISC

(d) "Wood Handbook," Forest Products Laboratory, Forest Research. The properties are for western white pine.

Table 1. - Material Properties.

APPENDIX D

Summary of finite element computer code developed under ARPA contract H0220035 "Analytic Modeling of Rock-Structure Interaction" with Agbabian Associates, El Segundo, Calif., Dr. Jeremy Isenberg, Principal Investigator. The computer program and all technical reports generated under this contract may be obtained from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, Va. 22151. Information required for ordering these materials are given below.

	AD No.	Price
Semiannual Technical Report	AD-749 373	$ 3.00[1]
Annual Technical Report - Vol. 1 (describes range of possible application and discusses aspects of the program and its theoretical basis)	AD-761 648	6.75[1]
Annual Technical Report - Vol. 2 (explains the overall operation of the program)	AD-761 649	5.45[1]
Annual Technical Report - Vol. 3	AD-761 650	3.00[1]
Magnetic tape containing the computer program	AD-762 422	97.00 Foreign $122.50

[1]Price shown is for paper copy, microfiche copies are also available at $1.45 per report.

APPENDIX D

Summary of Finite Element Computer Code

Existing finite element technology was consolidated into a single, general purpose computer program. This computer program considers static two- and three-dimensional, nonlinear analysis. Gravity loading and sequence of excavation of construction are also considered. The following types of elements are available, (a) beam, (b) rod, (c) axisymetric quad, (d) plane and, (e) three-dimensional hexadedron, (f) thick shell, (g) plane slip or joint, (h) axisymetric slip or joint.

All elements except the beam and thick shell have nonlinear properties which may be assigned are: (a) variable moduli, (b) elastic or variable modulus with ideal plasticity. Yield function may be isotropic or anisotropic, (c) nonlinear elastic with ideally plastic fracture with anisotropic fracture criterion, (e) viscoelasticity (Kelvin, Maxwell, three-parameter fluid), (f) viscoplasticity.

The computer program was intended to solve problems in rock mechanics and mining engineering. Thus, the excavation and construction options are intended to aid in analysis of shafts or chambers in rock having nonlinear properties from the start of construction through excavation and installation of temporary supports to the installation of permanent supports.

The program uses the direct stiffness method of structural analysis. Degrees of freedom are defined at nodal points which are at corners of elements. Element stiffnesses are deposited in a global stiffness matrix. Nodal point forces, which include external loads and internal resisting forces are expressed in a global load vector. A form of Chloeski decomposition, involving triangularization of the global stiffness matrix, reduction of the load vector and back substitution of the reduced load vector into the triangularized stiffness matrix, is used to obtain incremental displacements. The program has several options for performing these operations from which it chooses depending on the available core. According to one option, the stiffness matrix is formed block by block (one bandwidth wide) and stored on peripheral units; it is then retrieved block by block and processed. Under a second option, the stiffness matrix and load vector are formed and reduced in a block-by-block manner and written on peripheral storage in a reduced form. Part of the reduction is performed in local double precision.

These operations require many accesses to peripheral storage. The number of accesses and hence the computer time required for large problems would become prohibitive on small or medium sized computers. In the present program, a multibuffering technique is used which allows data to be transferred to and from peripheral storage while computations are being performed in the main core area. This procedure virtually eliminates wait time for Input/Output operations.

The computer program was applied to the analysis of floor and roof movements during the construction of an underground hoist room. Good comparisons were made between computed and field measurements.