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DYNAMIC ANALYSIS AND OPTIMAL DESIGN FORMULATION OF A WEAPON-VEHICLE SYSTEM

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A dynamic model of an automatic weapon-vehicle system is analyzed and formulated as an optimal design problem. The design concept adopted is to find the design parameters such that the gun barrel has a desirable property of least possible disturbance from a perturbation due to continuous bursts under uncertainty of environments such as road conditions and the azimuth angle. Thus, the optimal design problem becomes a "parametric optimal design" problem with a system of highly nonlinear differential equations that describe

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20. (cont.)

the state of the system. A numerically implementable transcription is given such that the formulation fits into the finite dimensional parametric optimal design problem.

TABLE OF CONTENTS

1. INTRODUCTION

Like most engineering design problems, the design of a weapon-vehicle system for given objectives is first subjected to an analysis of a model. The model should be such that it describes the actual system as closely as possible and be still within the analysis capabilities. The model of a weapon-vehicle system considered in this report is, in some respects, similar to the models presented in previous Themis Reports [1,2]. However, it differs from the previous models in that it includes the simulation of some environmental conditions. The primary interest is to study the role of these environmental conditions in the design problems.

In Section 2 of this report, the weapon-vehicle model is described. A detailed analysis and equations of motion of the system are presented. Then the problem is formulated as a parametric optimal design problem in Section 3. A complete analysis of environmental parameters and a target analysis are presented there. No numerical results are presented in this report. This is due to the fact that the problem of parametric optimization has not been explored in any detail in the past. Currently, this problem is receiving considerable attention at the University of Iowa. So, any developments from this effort will be reported at a later time.

2. DYNAMIC ANALYSIS OF THE WEAPON-VEHICLE MODEL

2.1 Description of the Model

Figure 1 shows the model of weapon-vehicle system considered in this report. The gun and its flexible mount are attached to a vehicle supported by a suspension system that is simulated by a model similar to the one presented in [3,4]. Here, the suspension model is a two-dimensional analog of the three-dimensional model presented in [3]. The mount is considered as an elastic structure with a finite number of elastic elements. Furthermore, the gun is assumed to have a barrel moving back and forth due to a recoil mechanism simulated by a reaction force $\bar{R}(t)$, a spring and a damper.

The masses m_1 and m_2 represent wheels, axles and associated mechanism of the rear and the front of the vehicle, respectively. The equivalent spring and damper coefficients for the rear suspension are denoted by k_1 and c_1, respectively, and the spring k_{10} is included to denote a spring-like property of the tire. As will be discussed later, inclusion of the variable force f_1 acting on m_1 will explain the base displacement due to various road conditions. Similar variables with subscript 2 represent properties of the front suspension of the vehicle.

The main body of the vehicle is idealized as a rigid body A on which the weapon C + D is connected through a flexible mount B. The weapon is composed of a rigid housing assembly C, a rigid gun barrel D, and a recoil mechanism whose presence is denoted by the reaction force $\bar{R}(t)$ acting at the points Q_1 and Q_2, and a spring and damper system k_r and c_r, respectively. For the purpose of representing other types of recoil mechanisms,

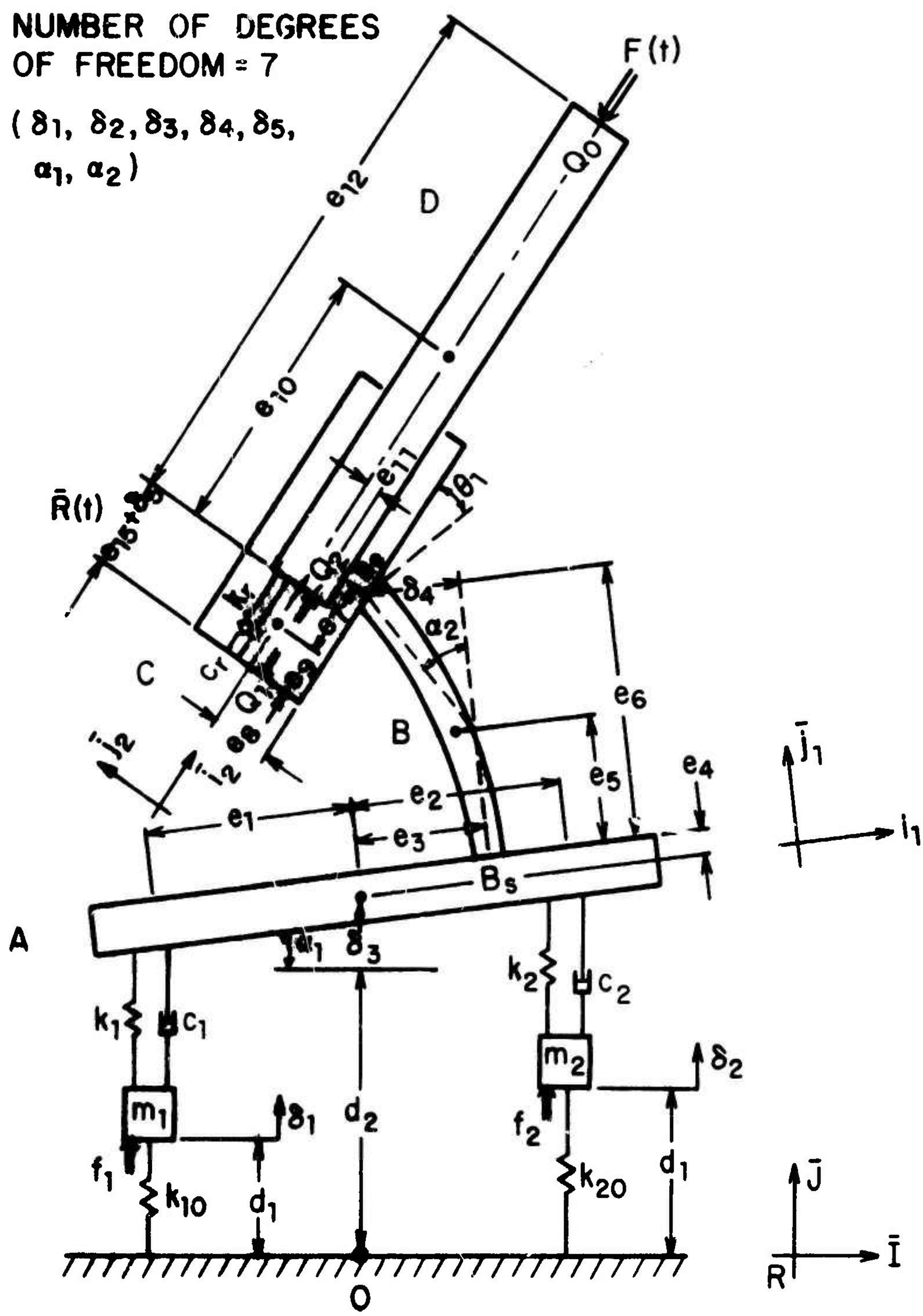

FIGURE 1. A MODEL OF WEAPON-VEHICLE SYSTEM.

the reaction force $\bar{R}(t)$ is assumed to be a known function of the relative displacement of the gun barrel and its housing. The flexible part B represents a set of structural elements such as truss, beam, plate, and shell. In the present formulation, B is assumed to be a beam element with both ends clamped at the points B_s and B_e of bodies A and C.

Following assumptions are made in the analysis:

1) The weapon is automatic with relatively high number of bursts.

2) The aiming by the operator is perfect.

3) Every motion is confined in a plane.

4) The whole system is located in an inertial frame R translating with a uniform velocity in the plane considered. The notation R should not be confused with the reaction force $\bar{R}(t)$ or its magnitude $R(t)$.

5) The point masses m_1, m_2 and the mass center of A can move only perpendicular to a flat surface fixed in R.

6) The flexible mount B is composed of linearly elastic structural elements.

7) No dry friction and no mass variation exist.

8) The axis of the gun barrel D passes through the mass center of the housing assembly C.

9) The flexible mount B can be subjected to a finite element analysis. This is discussed in a later section for a general approach.

Other assumptions will be made whenever necessary during the analysis. With these assumptions, the model is a holonomic mechanical system having seven degrees of freedom, since infinite degrees of freedom of the flexible mount B can be assumed to be represented by a finite number of boundary displacements. The generalized coordinates chosen are; 1) the displacements δ_1 and δ_2 measured from the unstretched lengths d_1 and d_2 of the springs k_{10} and k_{20}, respectively, 2) the rotation α_1 in R and linear displacement δ_3 of the mass center of body A measured from a configuration represented by the unstretched springs k_{10}, k_{20}, k_1 and k_2, 3) the end deflection δ_4 and the slope α_2 of the tip of the beam element with the displacement along the central axis neglected, and 4) a relative displacement δ_5 of the gun barrel D with respect to the housing assembly C. The other necessary geometrical dimensions are denoted by d_i, θ_i and e_i $i = 1, 2, \ldots$ The inertia properties of the rigid bodies are assumed to be known and will be denoted by M and I with appropriate subscripts.

2.2 Analysis of Flexible Mount

An exact dynamic analysis of the flexible mount should be based on a continuous model, but for the purpose of this report, a finite element model is used. For static problems, the displacements in a continuous structure can be related to a finite number of displacements selected at some arbitrary points on the structure, usually the boundary points intersecting with the surrounding systems. For dynamic problems, the displacements and velocities of any point are implicitly related through the differential equations of motion to the history of the displacements and velocities of boundary points interacting with surroundings. No explicit expression can be found for general dynamic loadings. For a simple analsis, therefore, an approximate treatment is necessary to reduce the system to a finite dimensional one.

Recently, some approximate techniques have been used in the analysis of a flexible satellite. In Ref. [5],the authors used synthetic modes in the process of truncating the modal coordinates obtained from a linearized system, assuming small motions. Robe and Kane [6] considered two symmetric rigid bodies connected by a flexible beam. They neglected masses of the connecting structures, and linearized the equations with a small angle restriction.

Another approach is to assume that the accelerations are small and use static displacement distributions for the dynamic case. This approach is conventional [7], and is suitable for matrix dynamic structural analysis. In this method, the element stiffness and mass matrices for each element are generated in a local coordinate system. These are then transformed to a datum coordinate system, and combined to form structure stiffness and mass matrices. To remove the internal degrees of freedom, the virtual work principle is applied. For details, the reader is referred to [7,8]. Let the stiffness matrix K and the displacement vector u be partitioned as

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \tag{2.1}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2.2}$$

where the vector u_1 refers to the boundary displacements to be retained as the degrees of freedom for the dynamic analysis, and u_2 denotes all the remaining displacements, i.e., the internal degrees of freedom. Then the resulting condensed stiffness and mass matrices corresponding to u_1 are given by

$$K^c = K_{11} - K_{12} K_{22}^{-1} K_{21} \tag{2.3}$$

$$M^c = L^T M L \tag{2.4}$$

where

$$L = \begin{bmatrix} I \\ -(K_{22})^{-1} \quad K_{21} \end{bmatrix} \tag{2.5}$$

and I is an identity matrix. Note that these matrices are time dependent, since the transformation matrix from the local system to datum system contains time variable. The internal displacements u_2 are related to the boundary displacements u_1 by

$$u_2 = - K_{22}^{-1} K_{21} u_1 \quad . \tag{2.6}$$

In the present problem, some further approximations will be made concerning the flexible beam element. In obtaining a strain energy expression, the static configuration will be used and for kinetic energy, and potential energy due to gravity, a lumped mass system will be used.

2.3 Equations of Motion of the System

The equations of motion of the system will be derived using Lagrangian formulation. The notation used is the same as in Ref. [9]. For convenience of easy modifications of the resulting equations in case of some adjustments in the model at a later stage of numerical solutions, the expressions of kinematic analysis are written in detail followed by dynamic analysis leading to the equations of motion.

2.3.1 Kinematic Analysis

For the kinematic analysis of the mechanical system, let ($\bar{I}$, $\bar{J}$, $\bar{K}$) be the right-handed orthonormal vectors fixed in the inertial reference frame R, and let the right-handed orthonormal vectors ($\bar{i}_1$, $\bar{j}_1$, $\bar{k}_1$) and ($\bar{i}_2$, $\bar{j}_2$, $\bar{k}_2$) be fixed in the bodies A and C, respectively, as shown in Fig. 1; here $\bar{k}_1 = \bar{k}_2 = \bar{K}$. In order to abbreviate kinematical equations, let cosine and sine functions be denoted by c and s, respectively. Then, for the base vectors,

$$\begin{aligned} \bar{I} &= c\,\alpha_1\,\bar{i}_1 - s\,\alpha_1\,\bar{j}_1 \\ \bar{J} &= s\,\alpha_1\,\bar{i}_1 + c\,\alpha_1\,\bar{j}_1 \end{aligned} \tag{2.7}$$

$$\begin{aligned} \bar{i}_2 &= c(\alpha_2 + \theta_1)\bar{i}_1 + s(\alpha_2 + \theta_1)\bar{j}_1 \\ \bar{j}_2 &= -\,s(\alpha_2 + \theta_1)\bar{i}_1 + c(\alpha_2 + \theta_1)\bar{j}_1\ . \end{aligned} \tag{2.8}$$

The angular velocities of the rigid bodies are,

$$^{R}\bar{\omega}^{A} = \dot{\alpha}_1\,\bar{K}\quad , \tag{2.9}$$

$$^{R}\bar{\omega}^{C} = (\dot{\alpha}_1 + \dot{\alpha}_2)\bar{K}\quad , \tag{2.10}$$

$$ {}^{R}\bar{\omega}^{D} = {}^{R}\bar{\omega}^{C} \quad . \tag{2.11} $$

Now, the position and velocity vectors of various points of interest are obtained. A superscript * on an entity refers to the position of mass center of a body. For Body A,

$$ \bar{r}^{A*/0} = (d_2 + \delta_3)\bar{J} \quad , \tag{2.12} $$

$$ {}^{R}\bar{v}^{A*} = \dot{\delta}_3(s\ \alpha_1\ \bar{i}_1 + c\ \alpha_1\ \bar{j}_1) \quad . \tag{2.13} $$

For a simplified analysis assume that the relative displacements of Body B with respect to Body A are small. From the given end deflection δ_4 and its end rotation α_2, one can write down the position vector of B* as follows [10]:

$$ \bar{r}^{B*/B_s} = e_5\ \bar{j}_1 - (h_1\ \delta_4 + h_2\ \alpha_2)\bar{i}_1 \quad , \tag{2.14} $$

where

$$ h_1 = \frac{e_5^2}{e_6^2}\left(3 - 2\frac{e_5}{e_6}\right) \quad , \tag{2.15} $$

$$ h_2 = \frac{e_5^2}{e_6}\left(\frac{e_5}{e_6} - 1\right) \quad . \tag{2.16} $$

The angle of rotation at any point from the original axis is obtained by differentiation of $(h_1\ \delta_4 + h_2\ \alpha_2)$ with respect to e_5. The velocities of the location e_5 is given by

$$R_{\bar{V}}^{B*} = \{\delta_3\ s\ \alpha_1 - h_1\ \dot{\delta}_4 - h_2\ \dot{\alpha}_2 - \dot{\alpha}_1(e_4 + e_5)\}\bar{i}_1$$

$$+ \{\dot{\delta}_3\ c\ \alpha_1 + \dot{\alpha}_1(e_3 - h_1\ \delta_4 - h_2\ \alpha_2)\}\bar{j}_1 \quad . \qquad (2.17)$$

For the point B_e,

$$\bar{r}^{B_e/0} = (d_2 + \delta_3)\bar{J} + (e_3 - \delta_4)\bar{i}_1 + (e_4 + e_6)\bar{j}_1 \quad , \qquad (2.18)$$

$$R_{\bar{V}}^{B_e} = \{\dot{\delta}_3\ s\ \alpha_1 - \dot{\delta}_4 - \dot{\alpha}_1(e_4 + e_6)\}\bar{i}_1$$

$$+ \{\dot{\delta}_3\ c\ \alpha_1 + \dot{\alpha}_1(e_3 - \delta_4)\}\bar{j}_1 \quad . \qquad (2.19)$$

Similarly, for the Bodies C and D, one can obtain the following position and velocity vectors:

$$\bar{r}^{C*/0} = \bar{r}^{B_e/0} - \{e_7\ c(\alpha_2 + \theta_1) + e_8\ s(\alpha_2 + \theta_1)\}\bar{i}_1$$

$$- \{e_7\ s(\alpha_2 + \theta_1) - e_8\ c(\alpha_2 + \theta_1)\}\bar{j}_1 \quad , \qquad (2.20)$$

$$^{R}\bar{V}^{C^*} = [\dot{\delta}_3 \; s \; \alpha_1 - \dot{\delta}_4 - \dot{\alpha}_1(e_4 + e_6) + (\dot{\alpha}_1 + \dot{\alpha}_2)\{e_7 \; s(\alpha_2 + \theta_1) - e_8 \; c(\alpha_2 + \theta_1)\}]\bar{i}_1 + [\dot{\delta}_3 \; c \; \alpha_1 + \dot{\alpha}_1(e_3 - \delta_4) - (\dot{\alpha}_1 + \dot{\alpha}_2) \{e_7 \; c(\alpha_2 + \theta_1) + e_8 \; s(\alpha_2 + \theta_1)\}]\bar{j}_1 \; , \tag{2.21}$$

$$\bar{r}^{D^*/O} = \bar{r}^{C^*/O} + (\delta_5 + e_{15} - e_9 + e_{10})\{c(\alpha_2 + \theta_1)\bar{i}_1 + s(\alpha_2 + \theta_1)\bar{j}_1\} \; , \tag{2.22}$$

$$^{R}\bar{V}^{D^*} = {}^{R}\bar{V}^{C^*} + \{\dot{\delta}_5 \; c(\alpha_2 + \theta_1) - (\dot{\alpha}_1 + \dot{\alpha}_2)(\delta_5 + e_{15} - e_9 + e_{10})s(\alpha_2 + \theta_1)\}\bar{i}_1 + \{\dot{\delta}_5 \; s(\alpha_2 + \theta_1) + (\dot{\alpha}_1 + \dot{\alpha}_2)(\delta_5 + e_{15} - e_9 + e_{10}) c(\alpha_2 + \theta_1)\}\bar{j}_1 \; . \tag{2.23}$$

2.3.2 Potential and Dissipation Energies

The potential energy of the system in R, disregarding irrelevant constants, consists of potential functions of the gravitational and spring forces, and the strain energy stored in the flexible mount B.

Using the position vectors derived in kinematic analysis, the total potential energy P is obtained as:

$$P = m_1 \; g \; \delta_1 + m_2 \; g \; \delta_2 + M_A \; g \; \delta_3 + M_B \; g\{\delta_3 + (e_3 - h_1 \; \delta_4 - h_2 \; \alpha_2)s \; \alpha_1 + (e_4 + e_5)c \; \alpha_1\} + (M_C + M_D)g\{\delta_3 + (e_3 - \delta_4)s \; \alpha_1 + (e_4 + e_6)c \; \alpha_1 - e_7 \; s(\alpha_1 + \alpha_2 + \theta_1) + e_8 \; c(\alpha_1 + \alpha_2 + \theta_1)\}$$

$$+ M_D\, g(\delta_5 + e_{15} - e_9 + e_{10})s(\alpha_1 + \alpha_2 + \theta_1)$$

$$+ 1/2\ k_{10}\ \delta_1^2 + 1/2\ k_{20}\ \delta_2^2 + 1/2\ k_1(\delta_3 - e_1\ s\ \alpha_1 - \delta_1)^2$$

$$+ 1/2\ k_2(\delta_3 + e_2\ s\ \alpha_1 - \delta_2)^2 + 1/2\ k_r\ \delta_5^2$$

$$+ \frac{2EI}{e_6}\left(\alpha_2^2 - 3\,\frac{\alpha_2\delta_4}{e_6} + 3\,\frac{\delta_4^2}{e_6^2}\right)\ , \tag{2.24}$$

where g is the gravitational constant, and $\frac{2EI}{e_6}$ is a stiffness factor for the beam. The strain energy of the beam was obtained by integration of the square of second derivative of the deflection curve, from Eq. (2.14).

Similarly, the dissipation energy is,

$$\mathcal{F} = 1/2\ c_1(\dot{\delta}_3 - \dot{\alpha}_1\ e_1\ c\ \alpha_1 - \dot{\delta}_1)^2 + 1/2\ c_2(\dot{\delta}_3 + \dot{\alpha}_1\ e_2\ c\ \alpha_1 - \dot{\delta}_2)^2$$

$$+ 1/2\ c_r\ \dot{\delta}_5^2\ . \tag{2.25}$$

2.3.3 Kinetic Energy

The translational kinetic energy is obtained from velocity of the mass center. For the beam element an integration of the square of velocity along the axis of beam will give the total kinetic energy. As outlined before, this energy is obtained by assuming that the entire mass is concentrated at the mass center of the beam. The angular velocity of the mass is approximated as

$$^{R}\bar{\omega}^{B} \approx (\dot{\alpha}_1 + \dot{\alpha}_2/2)\bar{K}\ . \tag{2.26}$$

The total kinetic energy of the system is then obtained as

$$
\begin{aligned}
T = {} & 1/2\, m_1\, \dot\delta_1^2 + 1/2\, m_2\, \dot\delta_2^2 + 1/2\, M_A\, \dot\delta_3^2 + 1/2\, I_A\, \dot\alpha_1^2 \\
& + 1/2\, M_B[\{\dot\delta_3\, s\, \alpha_1 - h_1\, \dot\delta_4 - h_2\, \dot\alpha_2 - \dot\alpha_1(e_4 + e_5)\}^2 \\
& \qquad + \{\dot\delta_3\, c\, \alpha_1 + \dot\alpha_1(e_3 - h_1\, \delta_4 - h_2\, \alpha_2)\}^2] \\
& + 1/2\, I_B(\dot\alpha_1 + \dot\alpha_2/2)^2 \\
& + 1/2\, M_C[\{\dot\delta_3\, s\, \alpha_1 - \dot\delta_4 - \dot\alpha_1(e_4 + e_6) + (\dot\alpha_1 + \dot\alpha_2)(e_7\, s(\alpha_2 + \theta_1) \\
& \qquad - e_8\, c(\alpha_2 + \theta_1))\}^2 \\
& \qquad + \{\dot\delta_3\, c\, \alpha_1 + \dot\alpha_1(e_3 - \delta_4) - (\dot\alpha_1 + \dot\alpha_2)(e_7\, c(\alpha_2 + \theta_1) \\
& \qquad + e_8\, s(\alpha_2 + \theta_1))\}^2] \\
& + 1/2\, I_C(\dot\alpha_1 + \dot\alpha_2)^2 \\
& + 1/2\, M_D[\{\dot\delta_3\, s\, \alpha_1 - \dot\delta_4 - \dot\alpha_1(e_4 + e_6) + (\dot\alpha_1 + \dot\alpha_2)(e_7\, s(\alpha_2 + \theta_1) \\
& \qquad - e_8\, c(\alpha_2 + \theta_1)) \\
& \qquad + \dot\delta_5\, c(\alpha_2 + \theta_1) - (\dot\alpha_1 + \dot\alpha_2)(\delta_5 + e_{15} - e_9 + e_{10})s(\alpha_2 + \theta_1)\}^2 \\
& \qquad + \{\dot\delta_3\, c\, \alpha_1 + \dot\alpha_1(e_3 - \delta_4) - (\dot\alpha_1 + \dot\alpha_2)(e_7\, c(\alpha_2 + \theta_1) \\
& \qquad + e_8\, s(\alpha_2 + \theta_1)) \\
& \qquad\quad + \dot\delta_5\, s(\alpha_2 + \theta_1) + (\dot\alpha_1 + \dot\alpha_2)(\delta_5 + e_{15} - e_9 + e_{10}) \\
& \qquad\quad c(\alpha_2 + \theta_1)\}^2] \\
& + 1/2\, I_D(\dot\alpha_1 + \dot\alpha_2)^2 \; . \qquad (2.27)
\end{aligned}
$$

where

I_A = moment of inertia of Body A along $\bar{K}$ direction for the mass center of A, etc.

2.3.4 Generalized Active Force

Now, adopting the terminology and definition of Kane [9], one needs to find generalized active forces of the system. Since no external couple is assumed, forces that contribute to the generalized active force are the breech force acting on the gun barrel through its axis, the reaction force from the recoil mechanism acting on the housing assembly, and the forces f_1 and f_2 acting on masses m_1 and m_2, respectively. The breech force is given by

$$\bar{F}(t) = -F(t)\ \{c(\alpha_2 + \theta_1)\bar{i}_1 + s(\alpha_2 + \theta_1)\bar{j}_1\} \quad , \tag{2.28}$$

and the reaction force is given by

$$\bar{R}(t) = R(t)\ \{c(\alpha_2 + \theta_1)\bar{i}_1 + s(\alpha_2 + \theta_1)\bar{j}_1\} \quad . \tag{2.29}$$

Since $\bar{F}(t)$ can be considered acting at the mass center of D, one has the partial rates of change of point D*, as

$$^{R}\bar{v}^{D^*}_{\dot{\delta}_1} = \bar{0} \quad ,$$

$$^{R}\bar{v}^{D^*}_{\dot{\delta}_2} = \bar{0} \quad ,$$

$${}^{R}\bar{v}^{D*}_{\dot{\delta}_3} = s\,\alpha_1\,\bar{i}_1 + c\,\alpha_1\,\bar{j}_1 \quad ,$$

$$\begin{aligned}{}^{R}\bar{v}^{D*}_{\dot{\alpha}_1} = &- (e_4 + e_6)\bar{i}_1 + (e_3 - \delta_4)\bar{j}_1 \\ &- e_8\,\bar{i}_2 + (\delta_5 + e_{15} - e_9 - e_7 + e_{10})\bar{j}_2 \quad ,\end{aligned}$$

$${}^{R}\bar{v}^{D*}_{\dot{\delta}_4} = - \bar{i}_1 \quad ,$$

$${}^{R}\bar{v}^{D*}_{\dot{\alpha}_2} = - e_8\,\bar{i}_2 + (\delta_5 + e_{15} - e_7 - \epsilon_9 + e_{10})\bar{j}_2 \quad ,$$

$${}^{R}\bar{v}^{D*}_{\dot{\delta}_5} = \bar{i}_2 \quad .$$

Hence, contribution of the breech force $\bar{F}(t)$ to the generalized active force vector is:

$$\begin{aligned}&[0,\ 0,\ - F(t)s(\alpha_1 + \alpha_2 + \theta_1) \quad , \\ &F(t)\{e_8 + (e_4 + e_6)c(\alpha_2 + \theta_1) - (e_3 - \delta_4)s(\alpha_2 + \theta_1)\} \quad , \\ &F(t)\ c(\alpha_2 + \theta_1) \quad , \quad F(t)\ e_8 \quad , \quad - F(t)]^T \quad .\end{aligned} \tag{2.30}$$

For the forces f_1 and f_2, the only nonzero components are

$$F^{f_1}_{\delta_1} = f_1 \quad , \qquad F^{f_2}_{\delta_2} = f_2 \quad , \tag{2.31}$$

respectively. For $\bar{R}(t)$, the partial rates of change of positions of Q_1 and Q_2 are the same, except those for the generalized coordinate δ_5, and a force of the same magnitude acts in opposite directions at Q_1 and Q_2, respectively. Hence, a net contribution to the generalized active force is

$$F_{\delta_5}^{R(t)} = R(t) \quad , \tag{2.32}$$

since $\quad {}^{R}\bar{v}_{\delta_5}^{Q_2} = \bar{i}_2 \quad$ and $\quad {}^{R}\bar{v}_{\delta_5}^{Q_1} = \bar{0}$.

Therefore, the generalized active force vector of the system in R is given by

$$
\begin{aligned}
F_{\delta_1} &= f_1 \quad , \\
F_{\delta_2} &= f_2 \quad , \\
F_{\delta_3} &= - F(t)\ s(\alpha_1 + \alpha_2 + \theta_1) \quad , \\
F_{\alpha_1} &= F(t)\{e_8 + (e_4 + e_6)c(\alpha_2 + \theta_1) - (e_3 - \delta_4)s(\alpha_2 + \theta_1)\} \quad , \\
F_{\delta_4} &= F(t)\ c(\alpha_2 + \theta_1) \quad , \\
F_{\alpha_2} &= F(t)\ e_8 \quad , \\
F_{\delta_5} &= R(t) - F(t) \quad .
\end{aligned} \tag{2.33}
$$

2.3.5 The Equations of Motion

The equations of motion of the system are obtained by the Lagrange's equation:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial (T - P)}{\partial q_i} + \frac{\partial \mathcal{F}}{\partial \dot{q}_i} = F_{q_i} \tag{2.34}$$

where

$$q_{1,\, 2,\, \ldots,\, 7} \equiv [\delta_1, \delta_2, \delta_3, \alpha_1, \delta_4, \alpha_2, \delta_5] \ . \tag{2.35}$$

As seen from the expressions derived before, the kinetic energy of the system is a homogeneous quadratic function of the variables $\dot{q}_i$, that is,

$$T = 1/2\ \dot{q}_i\ H_{ij}\ \dot{q}_j{}^{\dagger} \tag{2.36}$$

where H_{ij} is an inertia coefficient of the system in R, and is a symmetric matrix of functions of the generalized coordinates q_i only. Therefore,

$$\frac{\partial T}{\partial \dot{q}_i} = H_{ij}(q_\ell)\dot{q}_j \ . \tag{2.37}$$

Defining

$$G_i(\dot{q}_\ell, q_m) \equiv F_i - \frac{\partial \mathcal{F}}{\partial \dot{q}_i} + \frac{\partial T}{\partial q_i} - \frac{\partial P}{\partial q_i} \ , \tag{2.38}$$

†A repeated subscript indicates summation having a range of 1 to 7.

the equations of motion now become

$$H_{ij}\, \ddot{q}_j + \dot{H}_{ij}\, \dot{q}_j = G_i \quad , \qquad i = 1, 2, \ldots, 7 \ . \tag{2.39}$$

Equation (2.39) can be put in a first order form as follows:

$$\dot{q}_i = u_i \quad , \tag{2.40}$$

$$H_{ij}(q_\ell)\dot{u}_i = G_i(u_\ell, q_m) - \dot{H}_{ij}(u_\ell, q_m)u_j \quad , \tag{2.41}$$

where $i, j = 1, 2, \ldots, 7$.

After a straightforward differentiation, the elements of the matrix H and vector G can be obtained. Tables 1 and 2 give expressions for the elements of G and H in the original notation of the generalized coordinates. For abbreviation, the following notation is introduced:

$$h = e_3 - h_1\, \delta_4 - h_2\, \alpha_2 \quad ,$$

$$\delta_t = \delta_5 + e_{15} - e_9 + e_{10} \quad ,$$

$$e_s = e_7\, s(\alpha_2 + \theta_1) - e_8\, c(\alpha_2 + \theta_1) \quad ,$$

$$e_c = e_7\, c(\alpha_2 + \theta_1) + e_8\, s(\alpha_2 + \theta_1) \quad ,$$

$$e_t = e_7\, c(\alpha_1 + \alpha_2 + \theta_1) + e_8\, s(\alpha_1 + \alpha_2 + \theta_1) \quad ,$$

$$e_{45} = e_4 + e_5 \quad ,$$

$$e_{46} = e_4 + e_6 \quad ,$$

$$v_1 = \dot{\delta}_3 \, s \, \alpha_1 - h_1 \, \dot{\delta}_4 - h_2 \, \dot{\alpha}_2 - \dot{\alpha}_1 \, e_{45} \quad ,$$

$$v_2 = \dot{\delta}_3 \, c \, \alpha_1 + \dot{\alpha}_1 \, h \quad ,$$

$$v_3 = \dot{\delta}_3 \, s \, \alpha_1 - \dot{\delta}_4 - \dot{\alpha}_1 \, e_{46} + (\dot{\alpha}_1 + \dot{\alpha}_2) e_s \quad ,$$

$$v_4 = \dot{\delta}_3 \, c \, \alpha_1 + \dot{\alpha}_1 (e_3 - \delta_4) - (\dot{\alpha}_1 + \dot{\alpha}_2) e_c \quad ,$$

$$v_5 = v_3 + \dot{\delta}_5 \, c(\alpha_2 + \theta_1) - (\dot{\alpha}_1 + \dot{\alpha}_2) \delta_t \, s(\alpha_2 + \theta_1) \quad ,$$

$$v_6 = v_4 + \dot{\delta}_5 \, s(\alpha_2 + \theta_1) + (\dot{\alpha}_1 + \dot{\alpha}_2) \delta_t \, c(\alpha_2 + \theta_1) \quad .$$

To complete the system of the differential equations, initial conditions should be given. As noted earlier, the weapon is assumed to be fired automatically at the equilibrium configuration under gravity. Hence the initial conditions for the differential equations are prescribed such that $q_i(0)$ satisfy the equilibrium configuration under gravity, and the velocities are zero. To get the initial configurations one has to solve the static problem

$$\frac{\partial P}{\partial q_i} = 0 \quad , \qquad i = 1, 2, \ldots, 7 \quad . \tag{2.42}$$

In some instances, one may impose certain initial values to specific generalized coordinates; for example, $q_7 = \delta_5$ may usually be specified initially from the actual measurement. In such a case, the number of equations in Eq. (2.42) is reduced, accordingly. The equilibrium configuration will be denoted by a superscript e.

3. FORMULATION AS OPTIMAL DESIGN PROBLEM

3.1 Introduction

The ultimate goal of engineering design may be stated as that of synthesizing a system for the stated needs and objectives. A conceptual model for the system is usually used for analysis and a quantitative identification of the objectives. The model should be designed in such a way that it represents the system as closely as possible and is still within the analysis capabilities. In the previous section, such a model was described and the governing equations of motion were derived. In this section the problem is posed as a "parametric optimal design" problem. In this problem, the maximum of cost function over a conceivable range of environmental parameter is to be minimized by choosing the design variables.

3.2 Classification of Parameters

The first step in an optimal design algorithm is to identify various parameters that describe the system. These system parameters may be divided into three groups, and this classification is described for the model at hand. The "state parameters" are the generalized coordinates of the mechanical system and these are δ_1, δ_2, δ_3, α_1, δ_4, α_2, and δ_5.

"System design parameters" are those describing the system geometries and properties of the specific elements, such as, k_{10}, k_{20}, k_1, k_2, c_1, c_2, c_r, EI, m_1, m_2, M_A, M_B, M_C, M_D, I_A, I_B, I_C, I_D, d_1, d_2, e_1, e_2, and so on. The system design parameters, to be determined by an optimization technique, are chosen by the designer. The last group of parameters, called the "environmental parameters," is that group which interacts with the environment or external agencies. Hence, the environmental parameters can be characterized as having some degree of uncertainty during performance of the system. For the present model, f_1, f_2, θ_1, F(t) and R(t) can be considered as environmental parameters. It may be noted that this classification is not necessarily unique, but depends on the designer's choice, especially for the latter two classes of parameters. For example, if the recoil mechanism is not specified explicitly and the reaction force $\bar{R}(t)$ is known to vary within some given range, one could choose $\bar{R}(t)$ as an environmental parameter (in this case one may call it an environmental variable). On the other hand, if a recoil mechanism is to be designed such that the system performance is optimized by choosing $\bar{R}(t)$, then R(t) is classified as a system design variable.

In the following formulation, k_1 $(= k_2)$, k_r, c_1 $(= c_2)$, and EI are chosen as design variables, while the other system design parameters are assumed to be specified. f_1 and f_2, related to the road conditions, and θ_1 related to operator's choice, are chosen as the environmental parameters.

3.3 Environmental Parameters from Road Conditions

It is assumed that the road shape is sinusoidal with amplitude and wavelength of a and X, respectively. The vertical displacement is then given by

$$v = a \sin \frac{2\pi x}{X} \quad , \tag{3.1}$$

where x is a coordinate in the direction of road. Let speed s of the vehicle be uniform, and the distance between front and rear wheels be $b(= e_1 + e_2)$. Then,

$$x = st \quad , \tag{3.2}$$

$$v_r = a \sin \gamma t \quad , \tag{3.3}$$

where $\gamma = \frac{2\pi s}{X}$, and v_r is the vertical displacement of the rear wheel bottom. Since the time difference between the rear and front wheel passing a point on the road is b/s, the vertical displacement of front wheel bottom is

$$v_f = a \sin \gamma(t - b/s) \quad . \tag{3.4}$$

Denoting the time difference between initial firing and the instant of zero vertical displacement of rear wheel bottom by f, one obtains expressions for the vertical displacements (Fig. 2) as

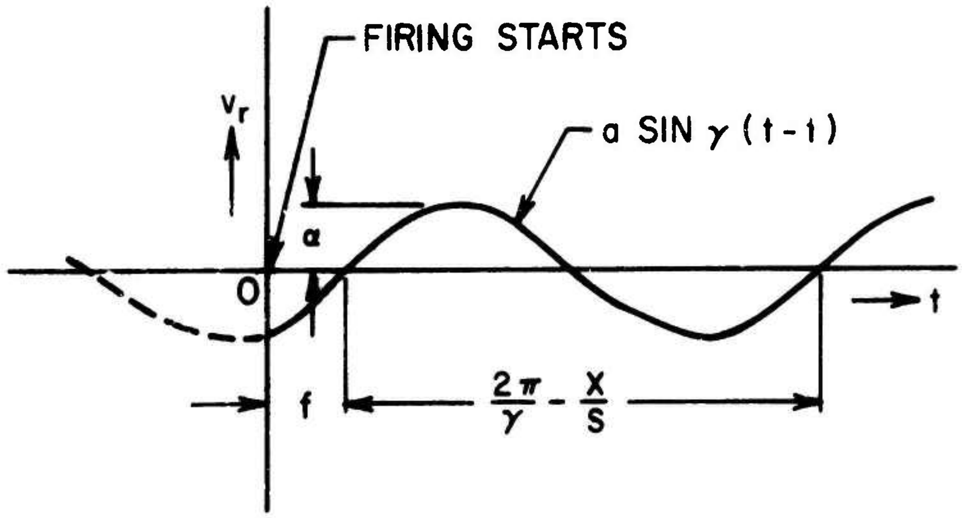

FIGURE 2. DISPLACEMENT OF REAR WHEEL BOTTOM.

$$v_r = a \sin \gamma(t - f) \quad , \tag{3.5}$$

$$v_f = a \sin \gamma(t - b/s - f) \quad . \tag{3.6}$$

It may be noted that a, γ, f, and s are the environmental parameters related to the road conditions. Assuming the amplitude is also fixed, one has γ, f, and s (or correspondingly X, f, and s) as the environmental parameters.

By defining f_1 and f_2 such that

$$f_1 = k_{10} v_r = k_{10}\, a \sin \gamma(t - f) \tag{3.7}$$

$$f_2 = k_{20} v_f = k_{20}\, a \sin \gamma(t - b/s - f) \quad , \tag{3.8}$$

the masses m_1 and m_2 of the model on an even surface will have same response as that of the vehicle running on a sinusoidal road [11].

3.4 Objective Function

As described earlier, an objective for the system must be formulated quantitatively, possibly as a function of the parameters. Sometimes, the choice of a function form for the objective is obvious, but in general, it depends on the designer.

For the weapon-vehicle system, it is assumed that the motion of the system is induced only by a cyclic breech force due to continuous automatic bursts, starting at time, $t = 0$. Then the system will get into a steady state motion after a transient period. Neglecting the factors such as, wind velocity and gravitational force, trajectory of the bullet can be assumed to be a straight line. A plane that is normal to the axis of gun barrel at equilibrium position is defined as a target plane. A set of bullets on target plane at a distance ℓ from the mass center of gun barrel is shown in Fig. 3, for a two-dimensional case and for a one-dimensional case. For general environmental conditions with irregular road shape, the shot pattern would be more irregular. In any case, after firing for a sufficiently long period of time, the shot pattern will be bounded by a finite radius. The design objective is to have an optimal system property that minimizes the disturbance i.e., the radius of the shot pattern of continuous bursts. The idea here is that the initial few shots are more important than the rest of the shots. Also, the deviation of the first shot from the aimed point is not important, for the deviation is a constant property of the gun and can be assumed fixed for a given gun. Hence, without loss of generality one can assume that the aimed point is the same as the point where first bullet lands.

Now, some objective functions will be described. The choice is dependent on the designer's view. The simplest one may be,

$$J_a = \frac{\sum^N \omega_i \, r_i}{N} \quad , \qquad \sum^N \omega_i = 1 \quad , \tag{3.9}$$

where r_i is the distance from the aimed point to the ith point of shot pattern, and ω_i is a weighting factor. This objective function gives the weighted average distance of the first N shots from the aimed point. The second objective function may be

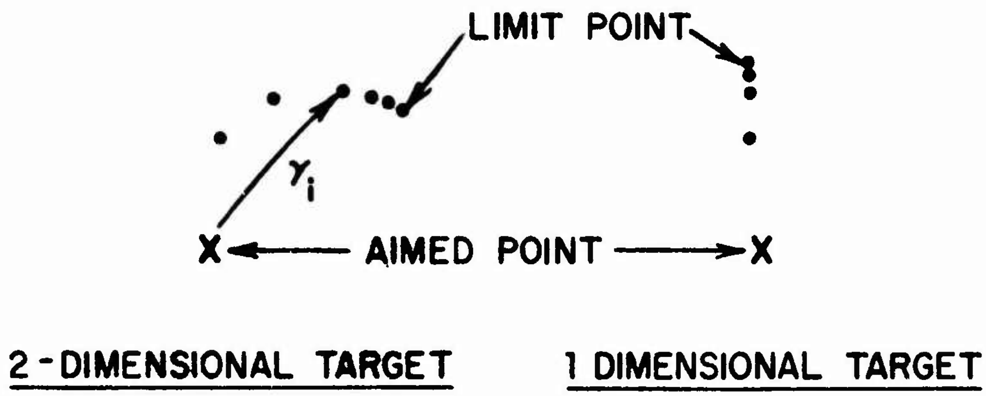

FIGURE 3. SHOT PATTERNS

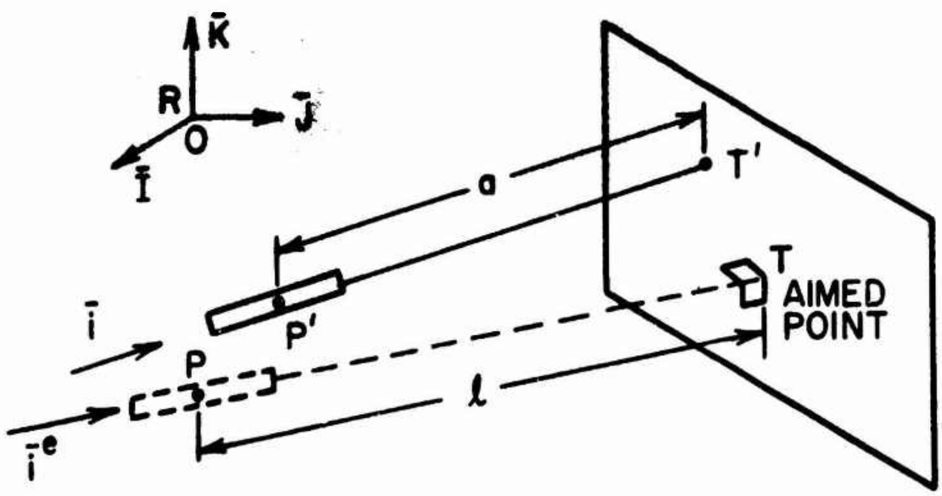

FIGURE 4. GEOMETRY FOR THE TARGET ANALYSIS

$$J_d = \sum_{i=1}^{N-1} \omega_i(\bar{r}_{i+1} - \bar{r}_i)^2 \quad , \tag{3.10}$$

denoting the weighted sum of the squares of the consecutive deviations of the first N shots. This form of the objective function may be good if a fast steady state is needed. A third objective function may be

$$J_v = \sum_{i=1}^{N} \omega_i(r_i - r_a)^2 \quad , \tag{3.11}$$

where

$$r_a = \sum^{N} r_i/N \quad . \tag{3.12}$$

Still another type of objective may be defined as a functional,

$$J_c = \int_0^T \omega(t)\ r(t)dt \quad , \tag{3.13}$$

where T is the duration of operation, $\omega(t)$ is a weighting function, and $r(t)$ is the distance from the aimed point to the point of intersection of the gun barrel axis with the target plane. Other objective functions are also possible. In the next subsection an explicit expression for r_i will be derived from geometrical considerations.

3.5 Target Analysis

The target plane was defined such that its normal was along the barrel axis at equilibrium. The aimed point is the intersection of the barrel axis with the target plane. In Fig. 4, the following vectors are known, after the equations of motion have been solved:

$$\bar{r}^{P/0} = a_1^e \bar{I} + a_2^e \bar{J} + a_3^e \bar{K} \qquad (3.14)$$

$$\bar{r}^{P'/0} = a_1 \bar{I} + a_2 \bar{J} + a_3 \bar{K} \qquad (3.15)$$

$$\bar{i}^e = \alpha_1^e \bar{I} + \alpha_2^e \bar{J} + \alpha_3^e \bar{K} \qquad (3.16)$$

$$\bar{i} = \alpha_1 \bar{I} + \alpha_2 \bar{J} + \alpha_3 \bar{K} \qquad (3.17)$$

where $\bar{i}^e$ and $\bar{i}$ are the orientation of the barrel at equilibrium and at arbitrary time t, respectively, and $(\bar{I}, \bar{J}, \bar{k})$ are same as defined before. The distance ℓ of the target plane to the mass center of the gun barrel is known. Then from the geometry, one has

$$\bar{r}^{P'/P} - \bar{r}^{T/P} + \bar{r}^{T'/P} - \bar{r}^{T'/T} = 0 \quad , \qquad (3.18)$$

$$\bar{r}^{T'/P'} = a\bar{i} \quad , \qquad \bar{r}^{T'/T} \cdot \bar{i}^e = 0 \quad . \qquad (3.19)$$

Let the unknown vector $\bar{r}^{T'/T}$ be denoted as

$$\bar{r}^{T'/T} = x_1 \bar{I} + x_2 \bar{J} + x_3 \bar{K} \quad . \qquad (3.20)$$

By substitution, one obtains the following set of equations:

$$a_i - a_i^e - \ell\, \alpha_i^e + a\, \alpha_i - x_i = 0 \quad , \qquad i = 1, 2, 3 \qquad (3.21)$$

$$\sum^{3} x_i \, \alpha_i^e = 0 \quad . \tag{3.22}$$

Solving for a, x_1, x_2, x_3, one obtains

$$a = \frac{\sum^{3}\left(\alpha_i^e \, a_i^e + \ell \, \alpha_i^{e^2} - \alpha_i^e \, a_i\right)}{\sum^{3} \alpha_i^e \, \alpha_i} \quad , \tag{3.23}$$

or,

$$a = \frac{\bar{i}^e \cdot \bar{r}^{P/0} + \ell \, \bar{i}^e \cdot \bar{i}^e - \bar{i}^e \cdot \bar{r}^{P'/0}}{\bar{i} \cdot \bar{i}^e} \quad . \tag{3.24}$$

Hence

$$x_i = a_i - a_i^e - \ell \, \alpha_i^e + a \, \alpha_i \quad , \qquad i = 1, 2, 3 \quad . \tag{3.25}$$

One may also write,

$$\bar{r}^{T'/T} = \bar{r}^{P'/P} - \bar{r}^{T/P} + \bar{r}^{T'/P'} = \bar{r}^{P'/0} - \bar{r}^{T/0} + a\bar{i} \quad . \tag{3.26}$$

Since $\bar{r}^{T/0}$ is a fixed vector, $\bar{r}^{T'/T}$ is a function of only the position of mass center and the orientation of the barrel.

3.6 Formulation as an Optimal Design Problem

After identification of the parameters and the objective function, one is ready to formulate the optimal design problem. Redefine the parameters as follows:

Design Variable, $b \equiv [k_1(= k_2),\ c_1(= c_2),\ EI]$, (3.27)

Environmental Parameter, $\alpha \equiv [\theta_1,\ s,\ X,\ f]$, (3.28)

State Variable, $z \equiv [q_m,\ u_m]$, $m = 1, 2, 3, \ldots, 7$ (3.29)

Then the optimal design problem will be in the following form:

$$\min \ \max_{\alpha \epsilon A} J \tag{3.30}$$

subject to

$$h(z,\ b,\ \alpha) = 0 \ , \qquad z(0) \text{ given} \ , \tag{3.31}$$

and

$$g(z,\ b,\ \alpha) \leq \text{ for all } \alpha \epsilon A \ , \tag{3.32}$$

where

$$A \equiv \{\alpha | q(\alpha) \leq 0\} \ . \tag{3.33}$$

J represents one of the objective functions from Section 3.4, and $h(z,\ b,\ \alpha) = 0$ represents the equations of motion from Section 2.3.5.

This formulation is more general than the conventional nonlinear programming problem or the minimax problem, and is termed as "parametric optimal design" problem. The g-constraint which depends on the environmental parameter α is called "parametric constraint." The α-constraints define a constrained set in the space of environmental parameters. Here one may choose,

$$- \theta_{min} \leq \theta_1 \leq \theta_{max} \quad , \tag{3.34}$$

$$s \leq s_{max} \quad , \tag{3.35}$$

$$X_{min} \leq X \quad , \tag{3.36}$$

$$f^2 \leq (X/2s)^2 \quad , \tag{3.37}$$

where the subscripts min and max denote the given lower and upper bounds. The last inequality indicates that the time difference f is restricted by the period.

A transcription of the problem is made into a more manageable form. The state equations, being a set of differential equations, are discretized for numerical solution. Also, instead of the min-max type of objective, an artificial design variable b_{n+1} is introduced. A transcribed optimal design problem is

$$\min b_{n+1} \tag{3.38}$$

subject to

$$\max_{\alpha \epsilon A} J(\bar{z}^i, b, \alpha) \leq b_{n+1} \tag{3.39}$$

$$z^{i+1} - z^i = S(z^i, b, \alpha) \quad , \quad z^0 \text{ given} , \tag{3.40}$$

$$g(b) \leq 0 \tag{3.41}$$

where

$$z^i \equiv [q_m(t_i), u_m(t_i)]^T \quad , \qquad m = 1, 2, \ldots, 7 \quad , \tag{3.42}$$

and t_i is a discretized time point which is chosen such that the grid contains the firing instant where the values of z are denoted by an upper bar. S is symbolically used as an operator giving the numerical solution to the initial problem. In Euler's method, this would be

$$S \equiv \Delta t \begin{bmatrix} u \\ H^{-1} (G - \dot{H}u) \end{bmatrix} . \tag{3.43}$$

Other methods of solving the equations of motion are possible and can be easily implemented in the solution algorithm for "parametric optimal design" problem.

4. SUMMARY

In this report, a model for conceptual design of an automatic weapon-vehicle system is suggested and analyzed such that a study of the interaction of the environmental conditions is possible. Although the motion is constrained in a plane, the model has several important features, such as, simulation of the road conditions by externally varying forces f_1 and f_2, inclusion of the control of the azimuth angle which may be randomly varied by the operation, simulation of the recoil mechanism having a comparatively large mass moving back and forth, and the flexible mount. Due to the flexible part, the model has infinite degrees of freedom. The internal degrees of freedom of the flexible part are removed by the assumption that the dynamic configuration is the same as the static configuration which can be related to the finite number of generalized coordinates at the interface. Thus, the infinite degrees of freedom system reduces to a finite degrees of freedom system. An explicit form of the equations of motion are then derived for the proposed model. The system has seven generalized coordinates in all, and the governing equations of motion are highly nonlinear because of a relatively large oscillating mass. The system may be linearized by the usual method of linearization in the neighborhood of equilibrium configuration except for the portion involving large oscillatory mass.

The design concept adopted is to find the design variables such that the system has a desirable property of least possible disturbance from the perturbation due to a continuous burst under uncertainty of the environment such as, road conditions and the azimuth angle. Some expressions for this objective are given after an identification of the parameters. In the formulation, it is assumed that only the first N shots are important, or a finite duration of the operation from the start of bursts is of importance. The objective functions considered are functions of only the position of the mass center of the gun barrel and its orientation.

The conceptual optimal design problem is, then, formulated mathematically as a "parametric optimal design" problem. A numerically implementable transcription is given such that the formulation fits into the finite dimensional parametric optimal design problem.

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TABLE 1. ELEMENTS OF H

$$H_{11} = m_1 \quad ,$$

$$H_{12} = \ldots = H_{17} = 0 \quad ,$$

$$H_{22} = m_2 \quad ,$$

$$H_{23} = \ldots = H_{27} = 0 \quad ,$$

$$H_{33} = M_A + M_B + M_C + M_D \quad ,$$

$$H_{34} = M_B(h \; c \; \alpha_1 - e_{45} \; s \; \alpha_1)$$

$$+ (M_C + M_D)\{(e_3 - \delta_4)c \; \alpha_1 - \epsilon_{46} \; s \; \alpha_1 - e_t\}$$

$$+ M_D \; \delta_t \; c(\alpha_1 + \alpha_2 + \theta_1) \quad ,$$

$$H_{35} = - (h_1 \; M_B - M_C - M_D)s \; \alpha_1 \quad ,$$

$$H_{36} = - h_2 \; M_B \; s \; \alpha_1 - (M_C + M_D)e_t + M_D \; \delta_t \; c(\alpha_1 + \alpha_2 + \theta_1) \quad ,$$

$$H_{37} = M_D \; s(\alpha_1 + \alpha_2 + \theta_1) \quad ,$$

$$H_{44} = I_A + I_B + I_C + I_D + M_B(e_{45}^2 + h^2)$$

$$+ (M_C + M_D)\{e_{46}^2 + (e_3 - \delta_4)^2 + e_7^2 + e_8^2 - 2e_{46} \; e_s - 2(e_3 - \delta_4)e_c\}$$

$$+ M_D[\delta_t^2 - 2\delta_t\{e_7 - \text{[illegible]} \; s(\alpha_2 + \theta_1) - (e_3 - \delta_4)c(\alpha_2 + \theta_1)\}] \quad ,$$

$$H_{45} = h_1 \; M_B \; e_{45} + (M_C + M_D)(e_{46} - e_s) + M_D \; \delta_t \; s(\alpha_2 + \theta_1) \quad ,$$

TABLE 2. ELEMENTS OF G

$$G_1 = f_1 + c_1(\dot{\delta}_3 - \dot{\alpha}_1\, e_1\, c\, \alpha_1 - \dot{\delta}_1)$$

$$- m_1\, g - k_{10}\, \delta_1 + k_1(\delta_3 - e_1\, s\, \alpha_1 - \delta_1)$$

$$G_2 = f_2 + c_2(\dot{\delta}_3 + \dot{\alpha}_1\, e_2\, c\, \alpha_1 - \dot{\delta}_2)$$

$$- m_2\, g - k_{20}\, \delta_2 + k_2(\delta_3 + e_2\, s\, \alpha_1 - \delta_2)$$

$$G_3 = - F(t)\, s(\alpha_1 + \alpha_2 + \theta_1) - c_1(\dot{\delta}_3 - \dot{\alpha}_1\, e_1\, c\, \alpha_1 - \dot{\delta}_1)$$

$$- c_2(\dot{\delta}_3 + \dot{\alpha}_1\, e_2\, c\, \alpha_1 - \dot{\delta}_2)$$

$$- (M_A + M_B + M_C + M_D)g - k_1(\delta_3 - e_1\, s\, \alpha_1 - \delta_1)$$

$$- k_2(\delta_3 + e_2\, s\, \alpha_1 - \delta_2)$$

$$G_4 = F(t)\{e_8 + e_{46}\, c(\alpha_2 - \theta_1) - (e_3 - \delta_4)s(\alpha_2 + \theta_1)\}$$

$$+ c_1\, e_1(\dot{\delta}_3 - \dot{\alpha}_1\, e_1\, c\, \alpha_1 - \dot{\delta}_1)c\, \alpha_1$$

$$- c_2\, e_2(\dot{\delta}_3 + \dot{\alpha}_1\, e_2\, c\, \alpha_1 - \dot{\delta}_2)c\, \alpha_1$$

$$+ M_B\, \dot{\delta}_3(v_1\, c\, \alpha_1 - v_2\, s\, \alpha_1)$$

$$+ M_C\, \dot{\delta}_3(v_3\, c\, \alpha_1 - v_4\, s\, \alpha_1)$$

$$+ M_D\, \dot{\delta}_3(v_5\, c\, \alpha_1 - v_6\, s\, \alpha_1)$$

$$- M_B\, g(h\, c\, \alpha_1 - e_{45}\, s\, \alpha_1)$$

$$- (M_C + M_D)g\{(e_3 - \delta_4)c\, \alpha_1 - e_{46}\, s\, \alpha_1 - e_t\}$$

$$- M_D\, g\, \delta_t\, c(\alpha_1 + \alpha_2 + \theta_1)$$

$$+ k_1(\delta_3 - e_1\, s\, \alpha_1 - \delta_1)e_1\, c\, \alpha_1$$

$$- k_2(\delta_3 + e_2\, c\, \alpha_1 - \delta_2)e_2\, c\, \alpha_1$$

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TABLE 2 (cont'd)

$$G_5 = F(t)c(\alpha_2 + \theta_1) - (M_B\, h_1\, v_2 + M_C\, v_4 + M_D\, v_6)\dot{\alpha}_1$$
$$+ h_1\, M_B\, g\, s\, \alpha_1 + (M_C + M_D) g\, s\, \alpha_1$$
$$+ \frac{6EI}{e_6^2}\alpha_2 - \frac{12EI}{e_6^3}\delta_4$$

$$G_6 = F(t)e_8 - M_B\, v_2\, h_2\, \dot{\alpha}_1 + M_C(\dot{\alpha}_1 + \dot{\alpha}_2)(v_3\, e_c + v_4\, e_s)$$
$$+ M_D(\dot{\alpha}_1 + \dot{\alpha}_2)\{v_5\, e_c - \delta_t\, v_5\, c(\alpha_2 + \theta_1) + v_6\, e_s - \delta_t\, v_6\, s(\alpha_2 + \theta_1)\}$$
$$- M_D\, \dot{\delta}_5\{v_5\, s(\alpha_2 + \theta_1) - v_6\, c(\alpha_2 + \theta_1)\}$$
$$+ M_B\, g\, h_2\, s\, \alpha_1 + (M_C + M_D)e_t$$
$$- M_D\, g\, c(\alpha_1 + \alpha_2 + \theta_1)\delta_t$$
$$- \frac{4EI}{e_6}\alpha_2 + \frac{6EI}{e_6^2}\delta_4$$

$$G_7 = R(t) - F(t) - c_r\, \dot{\delta}_5$$
$$+ M_D\, v_6(\dot{\alpha}_1 + \dot{\alpha}_2)\{c(\alpha_2 + \theta_1) - s(\alpha_2 + \theta_1)\}$$
$$- M_D\, g\, s(\alpha_1 + \alpha_2 + \theta_1) - k_r\, \delta_5 \quad .$$