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EFFECT OF BACKGROUND ESTIMATION ON THE SENSITIVITY OF SONAR OR RADAR RECEIVERS

C. Nicholas Pryor, et al

Naval Ordnance Laboratory
White Oak, Maryland

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Prepared by:
C. Nicholas Pryor
Gary Hess

ABSTRACT: Radar and sonar receivers usually operate in a noise background of unknown level. In order to maintain a constant false alarm rate, they often include circuits to estimate this background level. A technique is derived for determining the performance loss caused by background estimation. The loss is evaluated for several systems with Gaussian statistics and for narrow-band systems with square law, envelope, and logarithmic detectors. The loss depends on the type of system, the false alarm probability desired, and the size of the sample used in the background estimation.

NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

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EFFECT OF BACKGROUND ESTIMATION ON THE SENSITIVITY OF SONAR OR RADAR RECEIVERS

This report describes a technique for determining the performance loss caused by different background estimation systems. The material should be of interest to those engaged in random signal analysis or signal processing system design and evaluation. The work leading to this report was performed in the Signal Processing Division of the Physics Research Department and was performed under Task A370-370A/WF11-121-703.

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INTRODUCTION

Most sonar or radar receiving systems operate by measuring the energy received as a function of such parameters as time, frequency, range, or direction of arrival and comparing this measurement against a decision threshold for each combination of parameters. A detection is considered to occur whenever the measured energy exceeds this threshold. The threshold is chosen so that only a limited number of false detections (false alarms) occur due to random fluctuations of the energy measurement when only background noise and no target is present at the input. Unfortunately the required value of the decision threshold for a given false alarm rate depends on the level of the background noise, and this level is seldom known a priori. Thus most actual receiving systems also contain some means for measuring the background level and adjusting the decision threshold according to this measurement to provide a "constant false alarm rate" (CFAR) system independent of background level. This measurement is usually made by averaging the energy values in a number of neighboring cells (in the sense of having nearly the same range, frequency, etc. parameters). If the background noise is not expected to be perfectly uniform over the entire range of the measurement parameters, it is desirable to make the region over which the background average is taken as small as possible. However, once the number of points used in the average becomes small, the statistical fluctuations in the background measurement become significant and degrade the performance of the receiver compared to one with a fixed decision threshold. The purpose of this report is to evaluate this performance degradation for several different types of detector and background compensation. A general model is first developed which is applicable to a wide class of systems, and then specific results are obtained for several commonly used techniques.

GENERAL MODEL FOR BACKGROUND COMPENSATION

The specific form used for a background compensation system depends somewhat on the statistical behavior of the detector output as a function of the background level. If only the mean value of the detector output changes with changing background and the variance and other moments remain constant at a known value (as is true in certain systems using clipped data and in "logarithmic" receivers), the background compensation usually takes the form shown in Figure 1a. Here an averager of some sort estimates the mean, based on samples taken from the detector output, and this estimate is subtracted from each output of the detector. The resulting zero-mean function has a known distribution and can be tested against a fixed threshold to make the detection decision with a constant false alarm rate. A second common case is the one in which the shape of the detector output distribution remains constant, but its scaling varies in direct proportion to the mean output. This is true for systems such as the square-law detector and various linear envelope detectors. In this case normalization can be done by dividing the detector output by the estimated mean rather than by subtracting the mean as was done when the higher moments remained constant. An entirely equivalent, but easier to analyze, approach is shown in Figure 1b. If the desired decision threshold is G times the mean (corresponding to dividing by the mean and comparing the result to G), then the same test can be performed by multiplying the mean estimate by G, subtracting this from the detector output, and comparing the result to zero. Finally in some systems there is no definite relationship between the mean and the higher moments of the detector output distribution, but measurement of the variance is sufficient to characterize the rest of the distribution. This is true in certain systems which average a number of samples before a detection decision is to be made and where the mean may be fluctuating. In this case both the mean and the mean square may be estimated as shown in Figure 1c, and the sum of the mean estimate and G times the estimated standard deviation is subtracted from the detector output before comparing with zero.

All of the above forms may be characterized by the model shown in Figure 2a, where the background compensation system forms a function $A\hat{\mu} + B\hat{\sigma} + C$, subtracts this from the detector outputs, and compares the result to zero. If the estimates $\hat{\mu}$ and $\hat{\sigma}$ of the mean and standard deviation were perfect, this would perform the desired normalization without degradation. However, with a finite number of data samples used to form these estimates, fluctuations will appear at the output of the background estimator and these appear as additional noise which is passed to the decision element. Several assumptions will be made about the nature of this estimation noise throughout this report. These are

(a) The estimators of μ and σ are unbiased so that the errors in estimation have zero mean.

(b) The noise in the background estimation has a Gaussian distribution by virtue of the central limit theorem and the fact that a reasonable number of samples are used in estimating the background. The variance of this estimate is σ_ϵ^2. This assumption clearly becomes suspect for small numbers of samples averaged if the input noise is non-Gaussian.

(c) The background estimation errors are uncorrelated with the output of the detector for a given cell in the measurement space (range, frequency, etc.) of the system. This should be true for any properly designed background estimator and can be accomplished by pre-whitening the inputs to the averager.

(d) The amount of smoothing used in the background estimator is equivalent to that obtained by using N <u>independent</u> samples of the detector output in forming the estimate. When the samples used are correlated, this serves as a definition of N.

Under these assumptions, an equivalent system may be drawn as shown in Figure 2b. Here it is seen that the estimation noise functions as a zero-mean Gaussian fluctuation added to the output of the detector before the detector output is compared to the threshold $T = A\mu + B\sigma + C$, where μ and σ are the true parameters of the detector distribution. Under the assumption of independence it may be shown that the probability density function $p_z(z)$ of the detector output as corrupted by the estimation noise is the convolution of the detector output density function $p_x(x)$ and the density function $p_y(y)$ of the estimation noise. Thus

$$p_z(z) = \int_{-\infty}^{\infty} p_x(x)\, p_y(z-x)\, dx$$

$$= \int_{-\infty}^{\infty} p_x(z-y)\, p_y(y)\, dy \qquad (1)$$

The probability of a false alarm PFA is equal to the probability that z exceeds the threshold or

$$PFA = \int_{T}^{\infty} p_z(z)\, dz \qquad (2)$$

Combining these integral expressions and reversing the order of integration gives

$$\mathrm{PFA} = \int_{T}^{\infty} \left[\int_{-\infty}^{\infty} p_x(x)\, p_y(z-x)\, dx \right] dz$$

$$= \int_{-\infty}^{\infty} p_x(x) \left[\int_{T}^{\infty} p_y(z-x)\, dz \right] dx$$

$$= \int_{-\infty}^{\infty} p_x(x) \left[\int_{T-x}^{\infty} p_y(z')\, dz' \right] dx \quad . \qquad (3)$$

Since the estimation noise was assumed to be Gaussian with zero mean and a variance of σ_e^2, the density function $p_y(z')$ is

$$p_y(z') = (1/\sqrt{2\pi}\,\sigma_e)\ \exp(-z'^2/2\,\sigma_e^2) \qquad (4)$$

and the inner integral becomes

$$\int_{T-x}^{\infty} p_y(z')\, dz' = (1/\sqrt{2\pi}\,\sigma_e) \int_{T-x}^{\infty} \exp(-z'^2/2\,\sigma_e^2)\, dz'$$

$$= (1/\sqrt{\pi}) \int_{\frac{(T-x)}{\sqrt{2}\,\sigma_e}}^{\infty} \exp(-t^2)\, dt$$

$$= (1/2)\ \mathrm{erfc}((T-x)/\sqrt{2}\,\sigma_e) \qquad (5)$$

where erfc represents the complementary error function defined as

$$\mathrm{erfc}(x) = (2/\sqrt{\pi}) \int_{x}^{\infty} \exp(-t^2)\, dt \quad . \qquad (6)$$

This function is defined by equation (6) for all x and ranges from 2.0 at $x = -\infty$ through 1.0 at $x = 0$ to 0 at $x = \infty$. The probability of false alarm can thus be written as a single integral

$$PFA = (1/2) \int_{-\infty}^{\infty} p_x(x) \, \mathrm{erfc}((T-x/\sqrt{2}\,\sigma_e) \, dx \qquad (7)$$

and for any $p_x(x)$ is a function of the threshold T and the estimation noise variance σ_e^2.

The above equation can be used to evaluate the effect of estimation noise on the false alarm probability (or the required change in T to maintain a constant PFA) for any given distribution $p_x(x)$ from the detector. This function varies with the type of detector used in the system, and the following sections use this general result to evaluate the effects of background estimation for each of several common detectors. Numerical integration is required in most cases to evaluate equation (7). The Fortran programs in Appendix A were developed for this purpose.

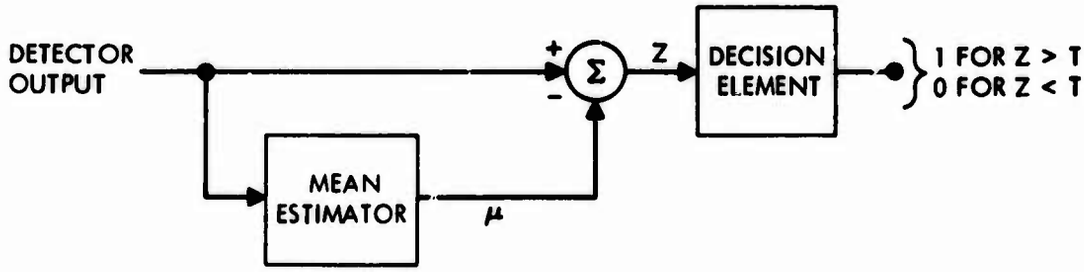


FIG. 1 (a) BACKGROUND MEAN ESTIMATION

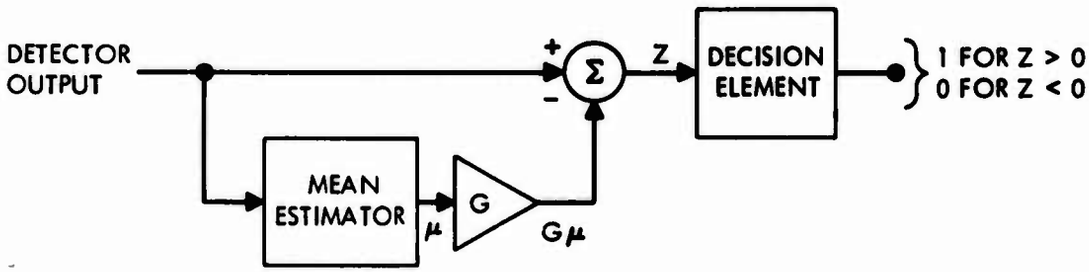


FIG. 1 (b) BACKGROUND MEAN ($T = G\mu$) ESTIMATION

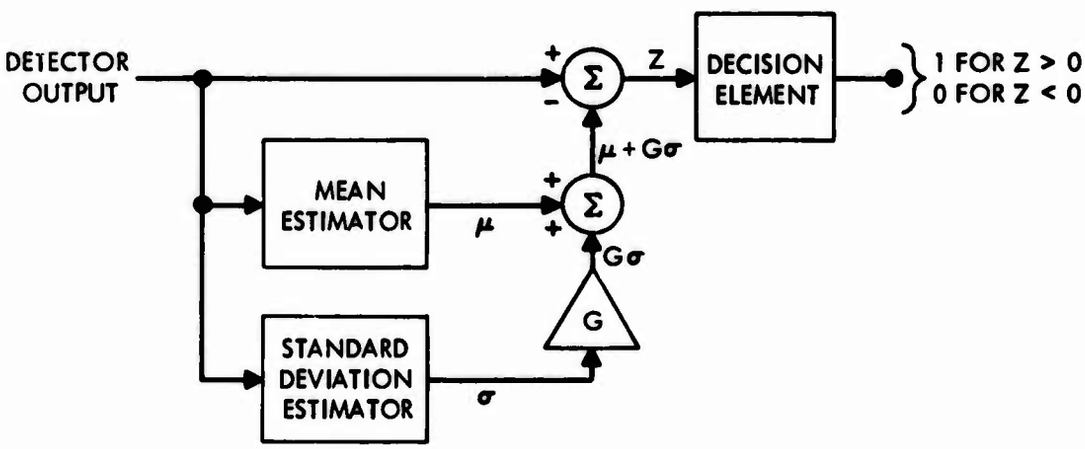


FIG. 1 (c) BACKGROUND MEAN PLUS STANDARD DEVIATION ($T = \mu + G\sigma$) ESTIMATION

FIG. 1 TYPICAL SYSTEMS FOR BACKGROUND ESTIMATION

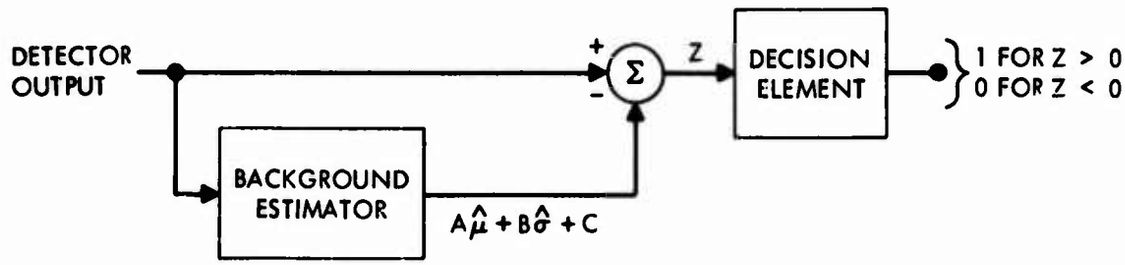


FIG. 2(a) GENERAL FORM FOR BACKGROUND COMPENSATION

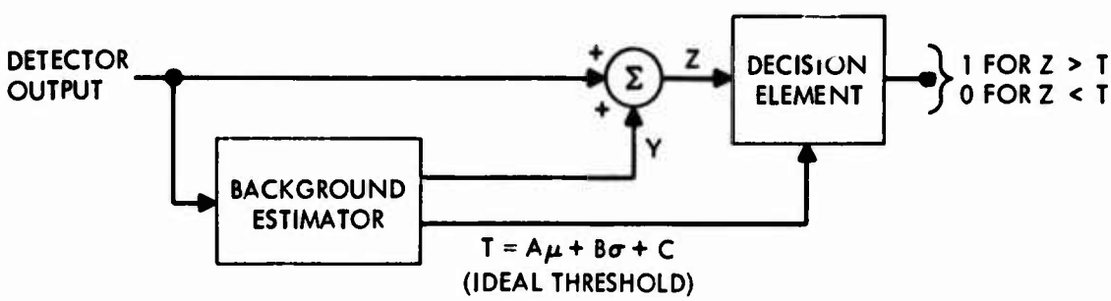


FIG. 2 (b) EQUIVALENT MODEL FOR BACKGROUND COMPENSATION

FIG. 2 ANALYTICAL MODELS FOR THE BACKGROUND ESTIMATION SYSTEM

GAUSSIAN CASES

In many systems the assumption that the input to the decision element has a Gaussian probability density function may be justified. This is true particularly when some averaging has been performed on the output of the detector, as is the case in a spectrum analyzer or a multibeam sonar with a low-pass filter following the energy detector in each channel output. Coherent processors or matched filters for detecting signals of known waveform and known phase also tend to have Gaussian output statistics. A major advantage of the Gaussian assumption in this instance is that analytical expressions can be obtained for the loss due to background estimation, thus avoiding the need for numerical integration. Three cases are studied in the following sections, depending on the relation assumed between the mean and the standard deviation of the input to the decision element. In the first case the standard deviation is assumed to be constant and known, in the second case it is assumed to be in known proportion to the mean, and in the third case both the mean and standard deviation are assumed to be unknown and separately estimated. Each of these three cases has application in signal processing systems, and the effects of background estimation are somewhat different in each.

GAUSSIAN DETECTOR WITH CONSTANT STANDARD DEVIATION

The detection of a shift in the mean of a Gaussian random variable is one of the classical problems in detection theory. It is also a reasonable approximation to physical systems in which strong AGC (such as hard clipping) has been applied at a point which keeps the output variance constant and in which a substantial amount of averaging has been performed (as in wideband correlation systems) before a detection decision is made. Since background compensation for such systems consists simply of subtracting an estimate of the mean and comparing to a constant threshold (as in Figure 1a), the appropriate functional form for T in Figure 2 is $T = 1 \cdot \mu + 0 \cdot \sigma + C$. If the input variance from the detector is σ^2, the variance in $\hat{\mu}$ resulting from averaging N independent samples is $\sigma_e^2 = \sigma^2/N$. Inserting the Gaussian distribution with mean μ and variance σ^2 for $p_x(x)$ in equation (7) then gives

$$PFA = (1/2\sqrt{2\pi}\,\sigma) \int_{-\infty}^{\infty} \exp(-(x-\mu)^2/2\sigma^2)\ \mathrm{erfc}((\mu+C-x)/(\sqrt{2}\,\sigma/\sqrt{N}))dx \tag{8}$$

which may be simplified to

$$PFA = (1/2\sqrt{2\pi}) \int_{-\infty}^{\infty} \exp(-x'^2/2)\ \mathrm{erfc}(\sqrt{N/2}\,(c-x'))dx' \tag{9}$$

where $c = C/\sigma$ is a normalized threshold value and the resulting expression is independent of μ and σ as expected.

A second, more direct, way exists for obtaining the PFA in this special case of Gaussian input distribution. Since the sum of two independent Gaussian variates is also Gaussian and has a variance equal to the sum of the variances of variates being added, the detector output after background compensation is Gaussian with zero mean and a variance of $(1 + 1/N)\sigma^2$. This is compared with the threshold $C = c\sigma$, and the probability of exceeding this threshold is simply

$$PFA = (1/2)\ \mathrm{erfc}(c/\sqrt{2(N+1)/N}) \tag{10}$$

This direct result in this case permits analytical evaluation of the degradation due to background estimation and also provides a means of confirming the accuracy of the numerical approach.

Figure 3 shows the resulting PFA as a function of the threshold parameter c, plotted for a number of values of N. The curve for $N = \infty$ represents the case where the mean is perfectly known and the estimation noise is zero. As one would expect, any reduction in the number of samples used in estimating the mean results in a higher false alarm rate for a given threshold or a higher threshold for a given false alarm rate.

If the threshold is adjusted to maintain the desired false alarm rate, in spite of the background estimation noise, a larger signal (change in the mean) is required in order to reach a desired probability of detection. In particular if a 50% probability of detection is desired, the mean of the detector output must change by an amount just equal to the threshold $C = c\sigma$. It is traditional in discussions of this problem (detecting a change in the mean of a Gaussian variate) to define a detector output signal to noise ratio (SNRO) in which the "signal" is equal to the square of the required change in the mean and the "noise" is equal to the variance at the detector output. Suppose c_o is the normalized threshold required for a given false alarm probability with $N = \infty$. Then the threshold c required for the same false alarm probability with a finite N is given by

$$(1/2)\ \mathrm{erfc}(c_o/\sqrt{2}) = \mathrm{PFA} = (1/2)\ \mathrm{erfc}(c/\sqrt{2(N+1)/N}) \qquad (11)$$

$$\text{or} \qquad c_o = c/\sqrt{(N+1)/N} \qquad (12)$$

$$\text{so} \qquad (c/c_o)^2 = (N+1)/N \qquad (13)$$

This ratio represents the increase in detector output signal-to-noise ratio required because of the background estimation noise, and it may be expressed in decibels as

$$\Delta(\mathrm{SNRO})_{est} = 10 \log[(N + 1)/N] \qquad (14)$$

This degradation (increase in required SNRO) is plotted as a function of the number N of independent samples in Figure 4. It varies from an extreme of 3 db if only one sample was used in estimating the background, through just less than 1 db with $N = 4$, to less than 1/4 db if more than 16 samples are used in the background average. It is interesting that in this special case the degradation depends only on N and is not a function of the desired false alarm rate.

It is important to note that $\Delta(SNRO)_{est}$ as defined above refers to a signal to noise ratio at the output of some sort of detector, and not at the processing system input. Relating this correction back to the system input to determine the amount of additional signal required for a given performance requires knowing the input/output relationship of the detector. In general this is a rather complex step, requiring one to determine the amount of signal input required to produce the deflection c_o and again for the deflection c, then to determine the SNR correction from the ratio of these two required signal powers. Generally the relationship between the $\Delta(SNRO)_{est}$ figured at the detector output and the equivalent degradation $\Delta(SNRI)_{est}$ expressed in terms of the input signal required depends on several factors such as the amount of post-detection averaging done and the false alarm and detection probabilities. However for the square law detector, and for most detector functions when the SNR at the detector is small, the deflection at the detector output is directly proportional to the input signal power. Thus the ratio of input signal powers is simply c/c_o rather than the square of this quantity, and the correction $\Delta(SNRI)_{est}$ expressed in decibels at the system input is just half of the $\Delta(SNRO)_{est}$ at the detector output. The vertical axis of Figure 4 is marked in terms of both $\Delta(SNRI)_{est}$ and $\Delta(SNRO)_{est}$, but it must be remembered that the $\Delta(SNRI)_{est}$ scale is limited in application to square law or equivalent detectors while the $\Delta(SNRO)_{est}$ scale is applicable in the general case.

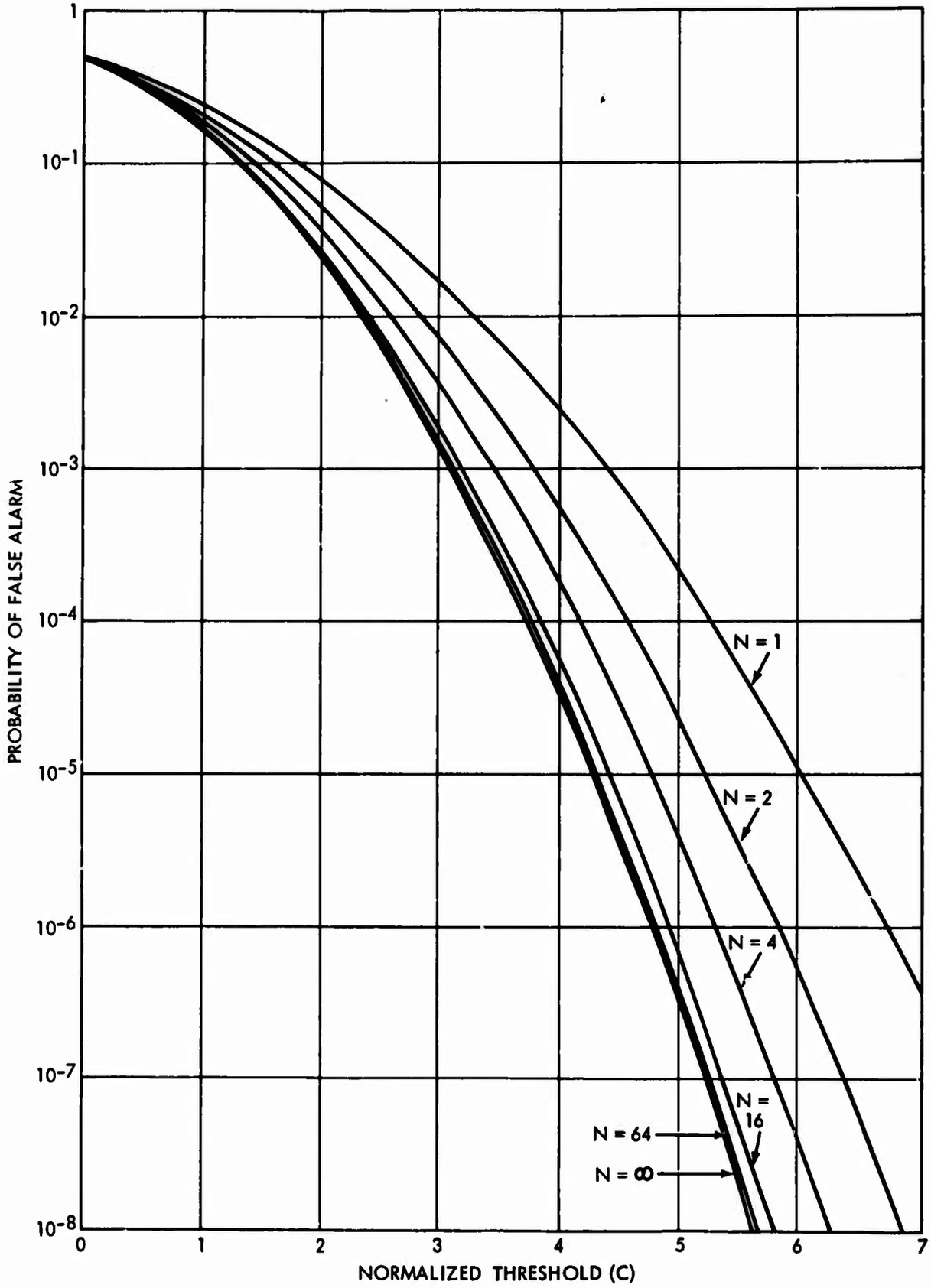


FIG. 3 PFA VERSUS THRESHOLD FOR CONSTANT STANDARD DEVIATION

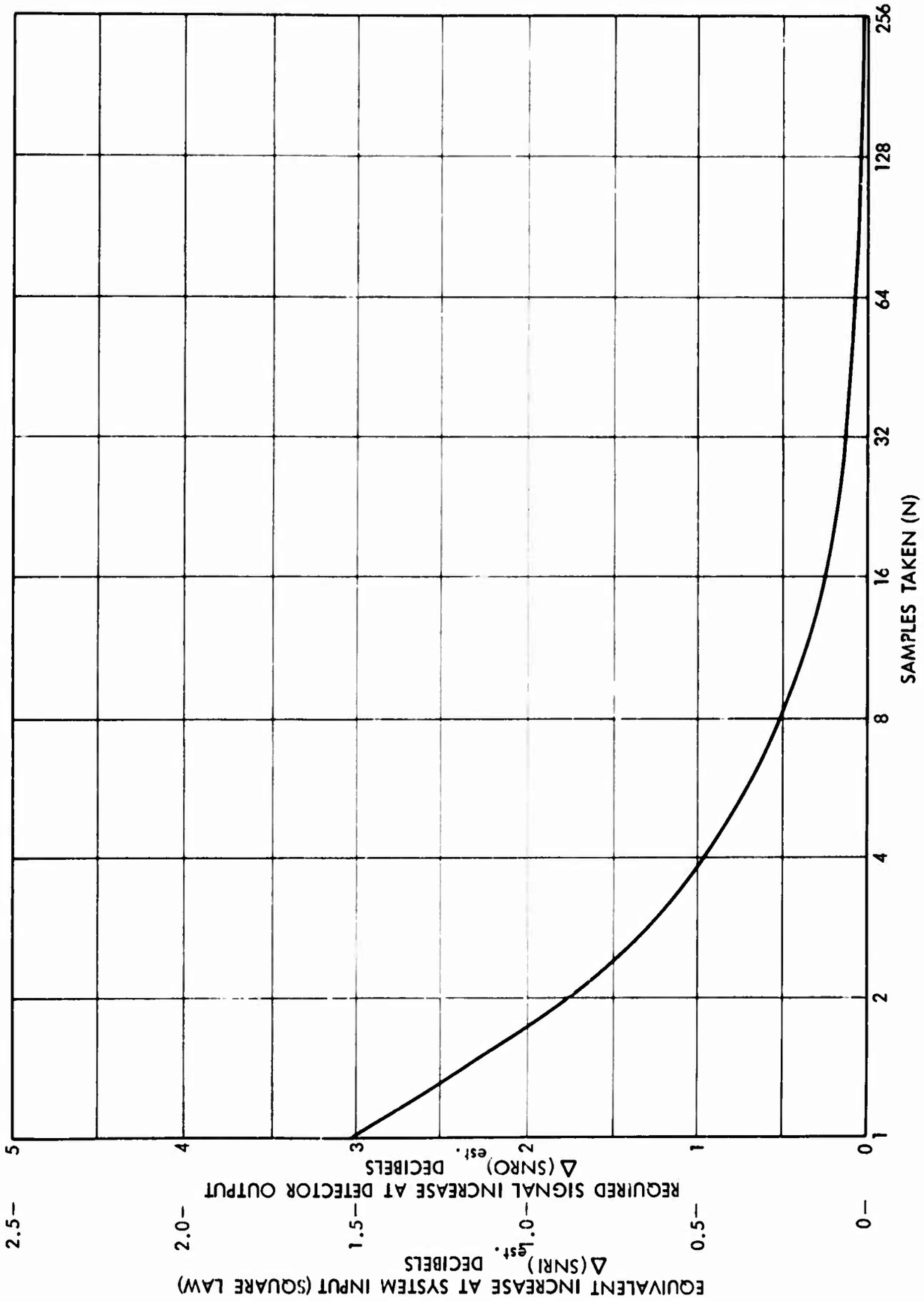


FIG. 4 LOSS DUE TO ESTIMATION FOR CONSTANT STANDARD DEVIATION

GAUSSIAN DETECTOR WITH PROPORTIONAL STANDARD DEVIATION

Another common situation is a detection system whose output distribution is Gaussian but with a standard deviation proportional to its mean output. This occurs (or is approximated) whenever a substantial amount of post-detection averaging (or non-coherent processing) is performed on the output of a square law or a linear detector before a detection decision is made. Suppose the standard deviation σ is known to be equal to K times the mean μ of the detection system output, on the basis of the amount of non-coherent averaging being performed. With this ratio known, a measurement of μ is all that is required to completely characterize the distribution and set the decision threshold. If a threshold of c standard deviations above the mean is required to give the desired false alarm probability, then the general form of Figure 1b can be used with 1+cK times the mean subtracted from the detector output and the result compared to zero. Thus the function to be formed by the background estimator is $(1+cK)\cdot\hat{\mu} + 0\cdot\hat{\sigma} + 0$, as shown in Figure 2a. When this function is broken into an ideal threshold T and a noise component as shown in Figure 2b, the noise due to estimation of the mean has a variance of $\sigma_e^2 = (1+cK)^2\sigma^2/N$. Note that this is larger by the factor $(1+cK)^2$ than the estimation variance when the variance of the input was constant. If the substitutions $\sigma = K\mu$, $T = (1+cK)\mu$, and $\sigma_e^2 = (1+cK)^2K^2\mu^2/N$ are made into equation (7), the result for the probability of false alarm is

$$PFA = (1/2\sqrt{2\pi}\,K\mu) \int_{-\infty}^{\infty} \exp(-(x-\mu)^2/2K^2\mu^2)\cdot$$

$$\operatorname{erfc}(((1+cK)\mu-x)/\sqrt{2}\,(1+cK)(K\mu/\sqrt{N}))\,dx \qquad (15)$$

or by simplifying

$$PFA = (1/2\sqrt{2\pi}) \int_{-\infty}^{\infty} \exp(-x'^2/2)\ \operatorname{erfc}(\sqrt{N/2}\,(c-x')/(1+cK))\,dx' \qquad (16)$$

This result again is independent of μ and may be evaluated numerically to find the PFA as a function of the threshold parameter c for any desired K and N.

Again because of the additive property of Gaussian distributions, a more direct means is available for finding the false alarm

probability without requiring a numerical integration. The input to the decision element of Figure 2b is Gaussian with a mean of μ and a variance of $\sigma^2 + \sigma_e^2 = (K\mu)^2(1 + (1+cK)^2/N)$, and a false alarm occurs whenever this exceeds a threshold of $T = (1+cK)\mu$. This probability is given directly from the error function as

$$PFA = (1/2)\ \mathrm{erfc}(c/\sqrt{2(1 + (1+cK)^2/N)}) \qquad (17)$$

This PFA as a function of c depends on both the number of samples N used in the background average and on the ratio K between the standard deviation and the mean of the input data. Figure 5 gives a plot of this function with N as a parameter for K equal to .03 and .3. The dependence on K is seen to be strong only for fairly large c (small PFA). In the limit as K approaches zero the function approaches that given in Figure 3 for the constant variance case.

Again we can define a signal to noise ratio as the square of the change in mean required to give 50% detection probability divided by the input variance, or $(cK\mu)^2/(K\mu)^2 = c^2$. If c_o is the threshold parameter required to give the desired PFA for $N = \infty$, then

$$(1/2)\ \mathrm{erfc}(c_o/\sqrt{2}) = PFA = (1/2)\ \mathrm{erfc}(c/\sqrt{2(1 + (1+cK)^2/N)}) \qquad (18)$$

or

$$c_o = c/\sqrt{(1 + (1+cK)^2/N)} \qquad (19)$$

This equation may be solved for the ratio c/c_o to give the expression

$$c/c_o = \frac{c_oK + N\sqrt{1 + (1-c_o^2K^2)/N}}{N - c_o^2K^2} \qquad (20)$$

This ratio of thresholds required for the same false alarm probability can be interpreted as an increase in required signal-to-noise ratio at the detector output, where the signal is again

defined as the square of the change in mean required to reach 50% probability of detection and the noise is the variance of the detector output in the no-signal case. Expressed in decibels, this increase is

$$\Delta(\mathrm{SNRO})_{est} = 10 \log(c/c_o)^2 = 20 \log\left[\frac{c_oK + N\sqrt{1 + (1-c_o{}^2K^2)/N}}{N - c_o{}^2K^2}\right] \tag{21}$$

This expression has several interesting properties. First is that the required SNR increase is a function only of the product c_oK of the threshold parameter c_o and the ratio K between the mean and the standard deviation, rather than depending on these two parameters individually. Figure 6 shows how $\Delta(\mathrm{SNRO})_{est}$ varies with N for various values of c_oK and also contains a table giving values of c_oK for a number of false alarm rates (which defines c_o) and K values. The curve for $c_oK = 0$ is identical to the curve in Figure 4 for the case with constant standard deviation, indicating that the SNR change is always larger when the standard deviation changes in proportion to the mean.

Another interesting property of the above result is that the required increase in SNRO goes to infinity when N equals $c_o{}^2K^2$. This represents the minimum value of N for which the desired false alarm probability can be achieved while estimating the background in this manner. Fortunately this minimum value of N is near unity, since c_o in any practical case is less than about 6 and K generally is appreciably smaller than unity. Whenever N is appreciably greater than $c_o{}^2K^2$ an approximate form can be derived for the SNRO change of the form

$$\Delta(\mathrm{SNRO})_{est} \approx 20 \log[1 + (1+c_oK)^2/2N] \qquad N \gg c_o{}^2K^2 \tag{22}$$

This form lends some insight into the behavior of the SNRO change in the region which is usually used in practice.

A second scale in terms of the equivalent degradation $\Delta(\mathrm{SNRI})_{est}$ at the processing system input is again provided in Figure 6, for use only in the case of square-law detectors or low signal to noise ratio at the detector itself. Limitations on the use of this scale are discussed in the previous section.

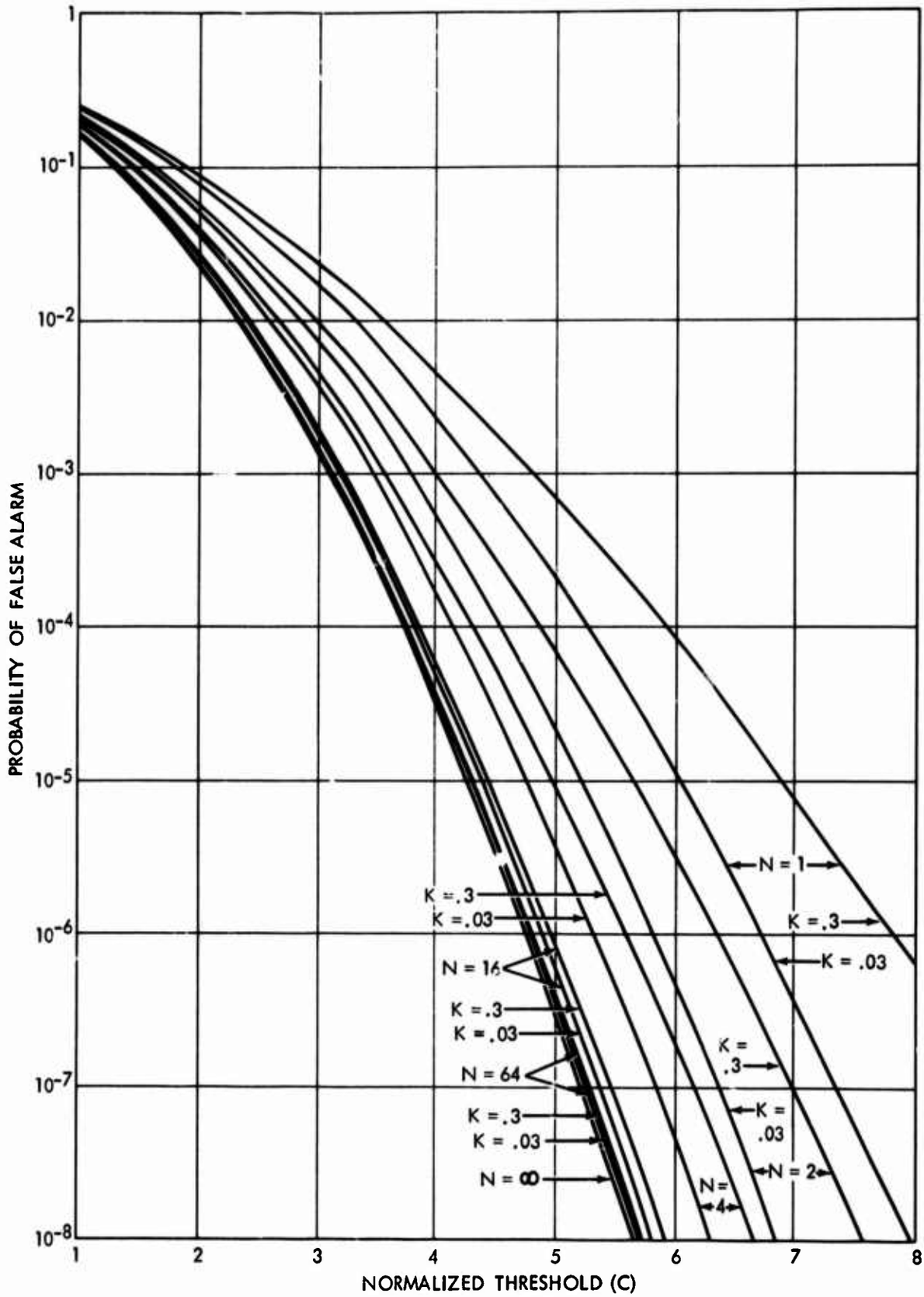
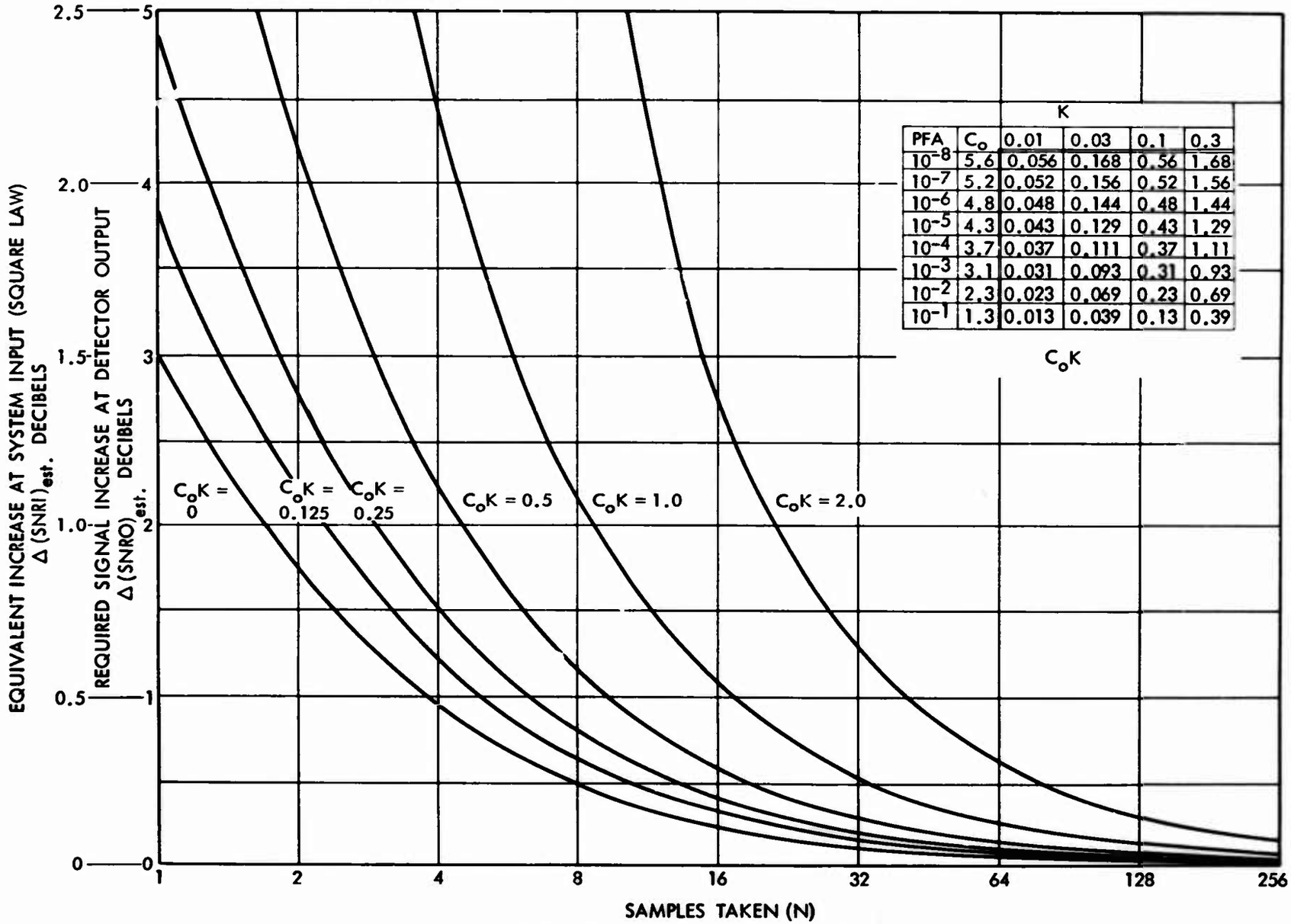


FIG. 5 PFA VERSUS THRESHOLD FOR PROPORTIONAL STANDARD DEVIATION



PFA	C_o	K = 0.01	0.03	0.1	0.3
10^{-8}	5.6	0.056	0.168	0.56	1.68
10^{-7}	5.2	0.052	0.156	0.52	1.56
10^{-6}	4.8	0.048	0.144	0.48	1.44
10^{-5}	4.3	0.043	0.129	0.43	1.29
10^{-4}	3.7	0.037	0.111	0.37	1.11
10^{-3}	3.1	0.031	0.093	0.31	0.93
10^{-2}	2.3	0.023	0.069	0.23	0.69
10^{-1}	1.3	0.013	0.039	0.13	0.39

C_oK

FIG. 6 LOSS DUE TO ESTIMATION FOR PROPORTIONAL STANDARD DEVIATION

GAUSSIAN DETECTOR WITH UNKNOWN STANDARD DEVIATION

A third common situation when dealing with Gaussian inputs is that where the relationship between the mean μ and the variance σ^2 of the detector output is not known. In this case the background estimator must estimate both the mean and the variance. If a threshold c standard deviations above the mean is necessary for the desired false alarm probability, then the function generated by the estimator in Figure 2a is $(1)\hat{\mu} + (c)\hat{\sigma} + 0$. Expressing this operation as in Figure 2b, the ideally estimated threshold would be $T = \mu + c\sigma$, and the variance σ_e^2 of the estimation noise is the variance of the estimate of μ plus c^2 times the variance of the estimate of σ(if $\hat{\mu}$ and $\hat{\sigma}^2$ have uncorrelated estimation errors).

Unbiased estimators for μ and σ^2, based on N independent samples, are

$$\hat{\mu} = \sum_{i=1}^{N} x_i/N \tag{23}$$

and

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N} x_i^2 - N\hat{\mu}^2}{N - 1} \tag{24}$$

The variances of these estimates are σ^2/N and $2\sigma^4/(N-1)$ respectively, and the errors can be shown to be uncorrelated. The estimate $\hat{\sigma}$ of the standard deviation, obtained by taking the square root of $\hat{\sigma}^2$, is not necessarily unbiased nor is it easy to show that its errors are uncorrelated with those in $\hat{\mu}$. However we will ignore these difficulties here. Assuming $N >> 1$ so that the standard deviation of $\hat{\sigma}^2$ is small compared to σ^2, the variance of $\hat{\sigma}$ may be shown to be $\sigma^2/2(N-1)$. This estimate of σ is also not quite Gaussian because of the nonlinearity of the square root operation, but we will ignore this problem too.

Based on the above, we find that the variance on the threshold estimate $\hat{\mu} + c\hat{\sigma}$ is $(\sigma^2/N) + c^2(\sigma^2/2(N-1))$ or

$$\sigma_e^2 = (\sigma^2/N)(1 + Nc^2/2(N-1)) \tag{25}$$

Adding this estimation variance to the variance of the input from the detector gives a total variance of $\sigma^2(1 + (1/N) + c^2/2(N-1))$,

and the detection process then becomes equivalent to testing a zero-mean process of this variance against a threshold $c\sigma$. As shown in previous sections, this leads to a false alarm probability

$$PFA = (1/2)\ \mathrm{erfc}(c/\sqrt{2(1 + 1/N + c^2/2(N-1)}\ ) \tag{26}$$

The threshold c_o required for perfectly known μ and σ (that is, $N = \infty$) is obtained from $PFA = (1/2)\ \mathrm{erfc}(c_o/\sqrt{2})$, so for the same false alarm rate

$$c_o = c/\sqrt{1 + 1/N + c^2/2(N-1)} \tag{27}$$

or, solving for $(c/c_o)^2$

$$(c/c_o)^2 = \frac{1 + 1/N}{1 - c_o^2/2(N-1)} \tag{28}$$

Again we can define a detector output signal-to-noise ratio required for detection as the square of the shift in the detector mean required to reach 50% detection probability divided by the detector output variance. Since the required shift in the mean is just equal to c, the increase in signal-to-noise ratio required because of imperfect estimation of μ and σ is given in decibels as

$$\Delta(SNRO)_{est} = 10 \log_{10} \left[\frac{1 + 1/N}{1 - c_o^2/2(N-1)} \right] \tag{29}$$

This increase in required SNRO is plotted in Figure 7 as a function of N for several values of c_o, where the values of c_o are identified by their corresponding values of the false alarm probability. The dashed curve in Figure 7 is duplicated from Figure 4 for reference and is the result for constant standard deviation. This is the curve which would be obtained above by setting c_o equal to zero, and all cases where the variance must be estimated involve a larger SNRO correction than the constant variance case. It is also interesting that for any value of c_o there is a minimum permitted N,

$$N_{min} = 1 + c_o^2/2 \tag{30}$$

Decreasing N below this limit causes the variance in the estimate to rise faster than can be compensated by increasing c.

As in the previous two cases a second scale showing the equivalent degradation $\Delta(SNRI)_{est}$ at the system input is provided in Figure 7, but its use is limited to square law detectors or systems in which the signal to noise ratio at the detector itself is small.

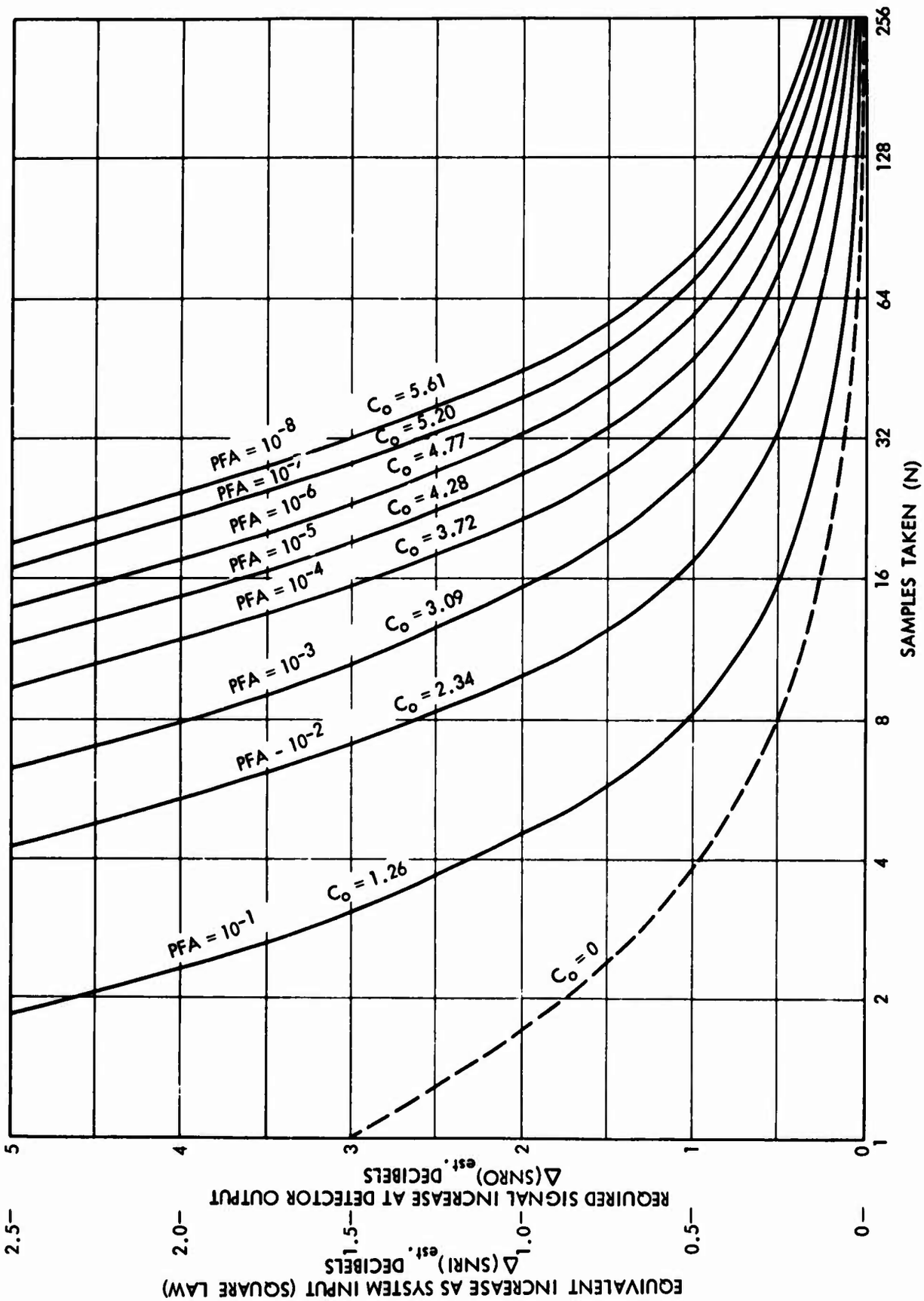


FIG. 7 LOSS DUE TO ESTIMATION FOR UNKNOWN STANDARD DEVIATION

NARROW BAND SYSTEMS WITHOUT POST DETECTION AVERAGING

In a wide class of radar and active sonar applications the receiver must be designed to detect a signal of known waveform modulating a carrier of unknown phase. These are generally termed narrow-band systems, and the statistics of the detector outputs may be derived from a narrow-band Gaussian assumption for the noise on each of the two orthogonal carrier phases. Often no further averaging is done on the detector outputs before the input to the decision element, and the statistics of these detector outputs are distinctly non-Gaussian. Three common detector types are studied in the following sections, and the probability density functions of their noise-only outputs are shown for reference in Figure 8. The first is a square-law detector whose output is equal to the instantaneous input power. The second is an envelope or linear detector whose output is equal to the envelope amplitude of the narrow-band input. The third system is a logarithmic detector whose output is equal to the measure in decibels of the narrow-band input. While these three detector types have similar performance in the absence of further averaging or background estimation, they are affected somewhat differently when their outputs must be used to estimate the background level.

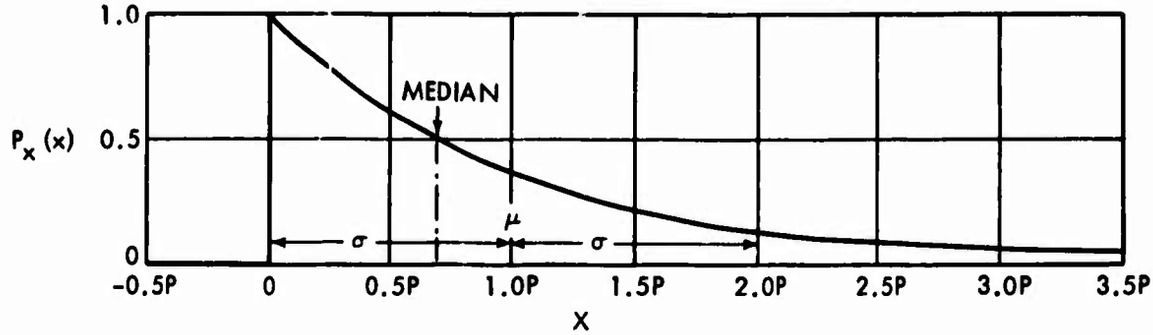


FIG. 8 (a) SQUARE LAW DETECTOR

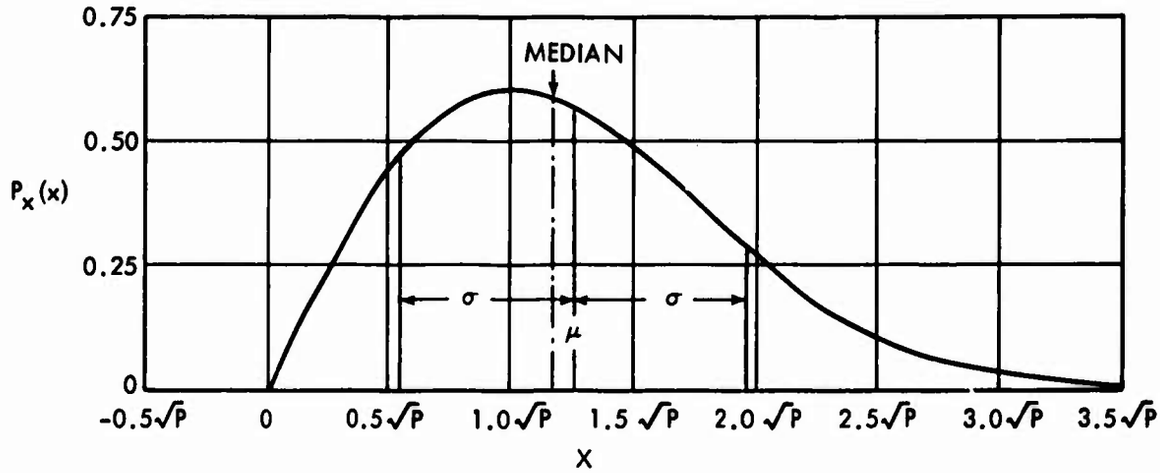


FIG. 8 (b) ENVELOPE DETECTOR

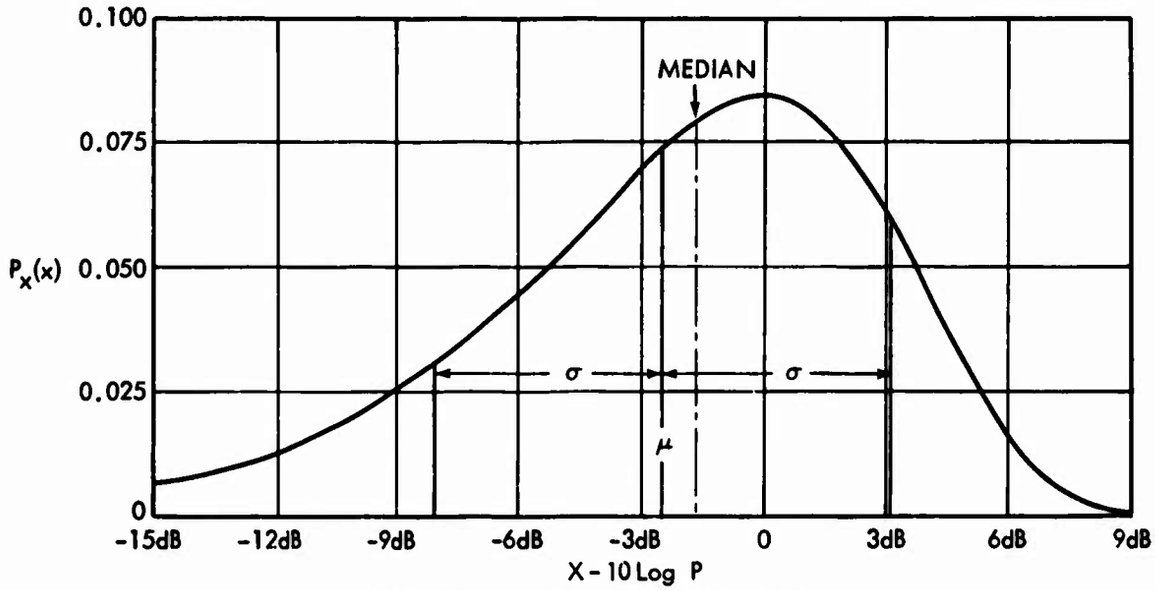


FIG. 8(c) LOGARITHMIC DETECTOR

FIG. 8 PROBABILITY DENSITY FUNCTIONS FOR NARROW-BAND DETECTORS

NARROW-BAND SYSTEM WITH SQUARE-LAW DETECTOR

Narrow-band systems may be defined as those in which the bandwidth of the signal to be detected is small compared to its center frequency. Most radar and sonar systems fall into this category. The usual noise model for a random narrow-band process consists of a low-pass Gaussian random function modulating each of the two orthogonal phases (sine and cosine) of the carrier. If a square-law detector is used to measure the power in this signal (or if coherent detection is performed on each carrier phase, as in detecting a known pulse waveform of unknown phase, and the squares of the two results are added) the detector output statistics may be shown to be chi-squared with two degrees of freedom. This density function is of the form

$$p_x(x) = \begin{cases} (1/P)\ \exp(-x/P) & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{31}$$

where P is the average power in the noise process. The mean of this distribution is P and the variance is P^2. The shape of the distribution is shown in Figure 8a.

Since the mean and the standard deviation are proportional in this type of detector, the estimate of σ is usually derived from the estimate of μ in constant false alarm rate detectors so that the decision threshold $T = (1+c)\hat{\mu}$. Written in this form, the decision threshold may be thought of as being c standard deviations above the estimated mean, since $\hat{\sigma} = \hat{\mu}$. Putting this system into the context of Figure 2b, the ideal threshold is $T = (1+c)\ P$ and an equivalent noise of variance $\sigma_e^2 = (1+c)^2P^2/N$ is added to the detector output if N independent samples are averaged in forming the background estimate.

With the above information of $P_x(x)$, T, and σ_e^2 we can now use equation (7) to evaluate the probability of false alarm as

$$\begin{aligned} PFA &= \frac{1}{2} \int_{-\infty}^{\infty} p_x(x)\ \mathrm{erfc}((T-x)/\sqrt{2}\ \sigma_e)dx \\ &= \frac{1}{2P} \int_{0}^{\infty} \exp(-x/P)\ \mathrm{erfc}(((1+c)P-x)/\sqrt{2/N}(1+c)P)dx \\ &= (1/2) \int_{0}^{\infty} \exp(-x')\ \mathrm{erfc}((1+c-x')/\sqrt{2/N}(1+c))dx' \end{aligned} \tag{32}$$

where the substitution of variables $x' = x/P$ shows that the resultant false alarm rate is indeed independent of P as expected. The above expression for PFA must be evaluated numerically as a function of c and N, and the result is shown in Figure 9. The curve for $N=\infty$ represents the case where the threshold is perfectly determined and the false alarm probability reduces to

$$PFA = \int_{1+c}^{\infty} \exp(-x)dx = \exp(-(1+c)) \quad (N=\infty) \tag{33}$$

For any finite N, a larger value of c is required for the same false alarm probability. This is further shown by Figure 10, in which the same data used in Figure 9 are replotted to show the dependence of c on N for several false alarm rates.

This increase in the required decision threshold due to background estimation may be related to an increase in signal to noise ratio required at the system input in the following manner. If a signal with bandwidth comparable to the narrow-band noise is added to the noise input, the resulting output of the detector is still chi-squared but with a total power parameter P(1+SNR) where SNR is the signal to noise power ratio.* The median output M of the detector can be obtained by integrating the distribution and setting this integral equal to 1/2 as follows

$$\int_{M}^{\infty} (1/P(1+SNR)) \exp(-x/P(1+SNR))dx$$

$$= \exp(-M/P(1+SNR)) = 1/2 \quad \text{or}$$

$$M_{square} = P(1+SNR)(\log_e 2) \tag{34}$$

If the decision threshold is set at this median value M, then a 50% probability of detection occurs. The probability of detection may be modified slightly from 50% by the fluctuation in the threshold due to background estimation, but this is only a second order effect.

*Note that this result is slightly different from that obtained if the signal is assumed to be a sinusoid. In that case the resulting distribution is a non-central chi-square and the mathematics is considerably more involved. In actual systems the processor bandwidth should approximate the signal bandwidth, and it is rather a tossup as to which model is more realistic.

Once c is determined for the required false alarm probability we can equate the decision threshold $T = (1+c)P$ with the median M and solve for the SNR which results in 50% detection probability. This yields

$$(1+c)P = P(1+SNR)(\log_e 2) \tag{35}$$

or

$$SNR = (1 - \log_e 2 + c)/\log_e 2 \ . \tag{36}$$

Now in order to determine the increase in SNR required due to estimation of the background, the SNR can be computed as above both with c_o for perfect estimation and with the c required due to finite N. Taking the ratio of the resulting values of SNR and converting the result to decibels yields

$$\Delta(SNRI)_{est} = 10 \log_{10} \frac{(1 - \log_e 2 + c)}{(1 - \log_e 2 + c_o)} \quad \text{decibels} \ . \tag{37}$$

Figure 11 shows this increase in SNR as a function of N for several values of false alarm probability. Note that the behavior is similar to that for the Gaussian case with proportional standard deviation in that the correction for a given N is larger for smaller allowed false alarm probabilities and that for any given PFA the loss increases rapidly as some critical value of N is approached.

In using the results for small N (below about N=16) it must be remembered that the assumption of Gaussian statistics for the estimation noise begins to fail because of the non-Gaussian inputs to the detector. The error is in the direction of over-estimating the loss, so that actual system degradation is not as bad as that shown in Figure 11.

It is important to note that the definition of signal to noise ratio is different in this and the following sections than that used in the Gaussian cases. In each of the Gaussian cases a signal to noise ratio loss was defined at the detector output, and it was shown that this could be related to an equivalent loss at the processing system input only for a limited set of cases. However in this example the characteristics of a square law detector operating on narrow band information are known and it is possible to relate the effect of background estimation directly to a change in required signal to noise ratio at the system input.

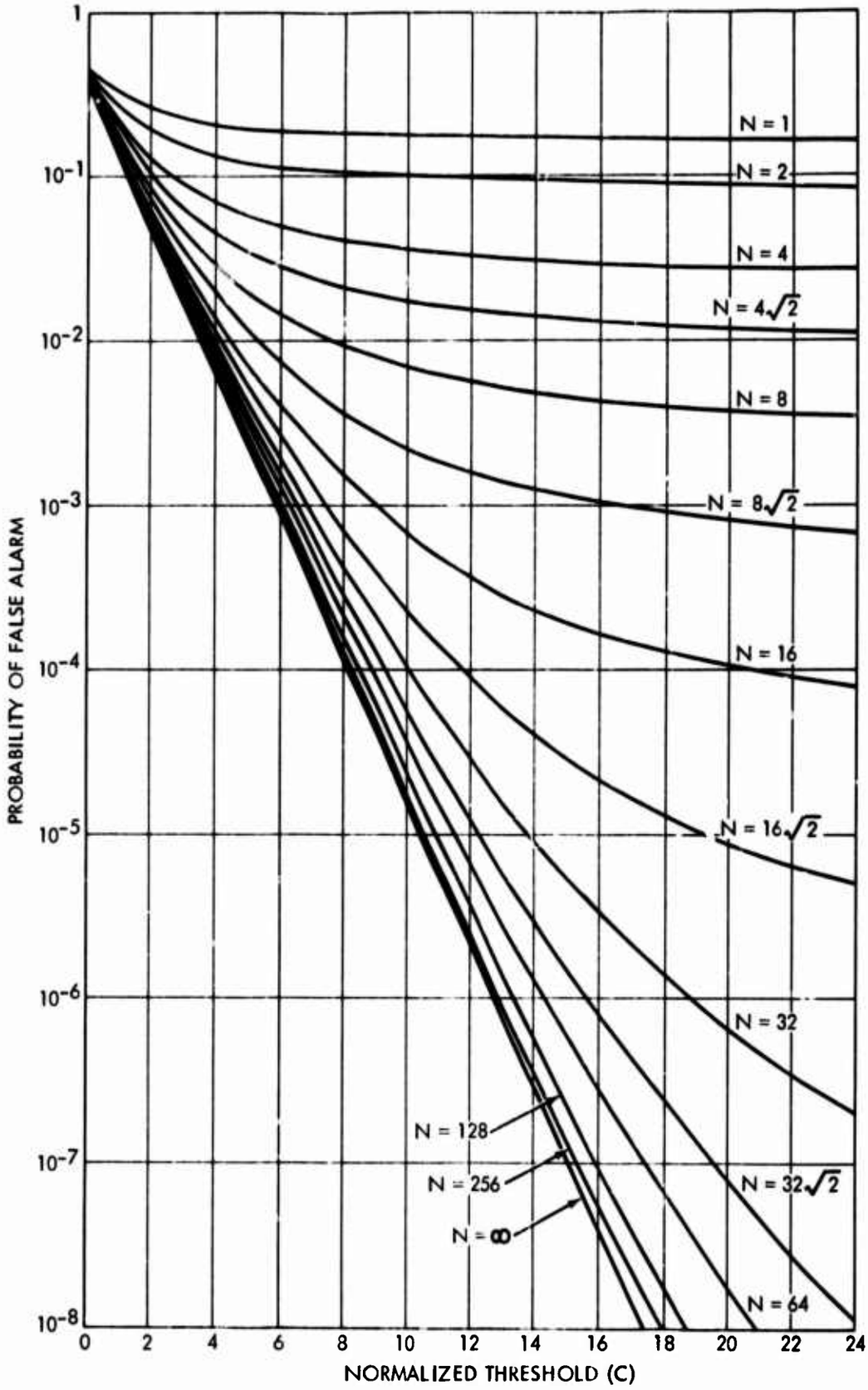


FIG. 9 PFA VERSUS THRESHOLD FOR SQUARE LAW DETECTOR

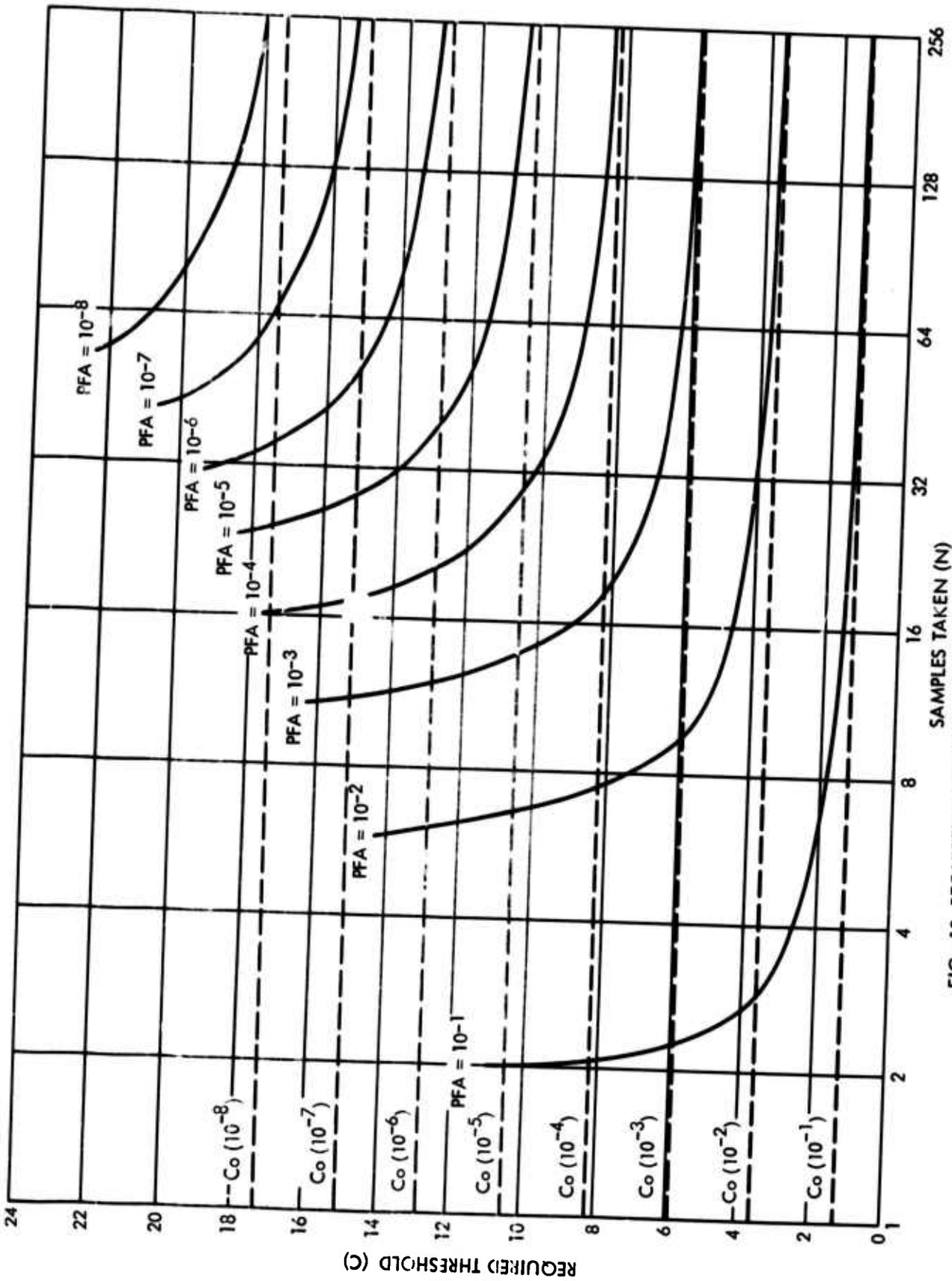


FIG. 10 REQUIRED THRESHOLD VERSUS N FOR SQUARE LAW DETECTOR

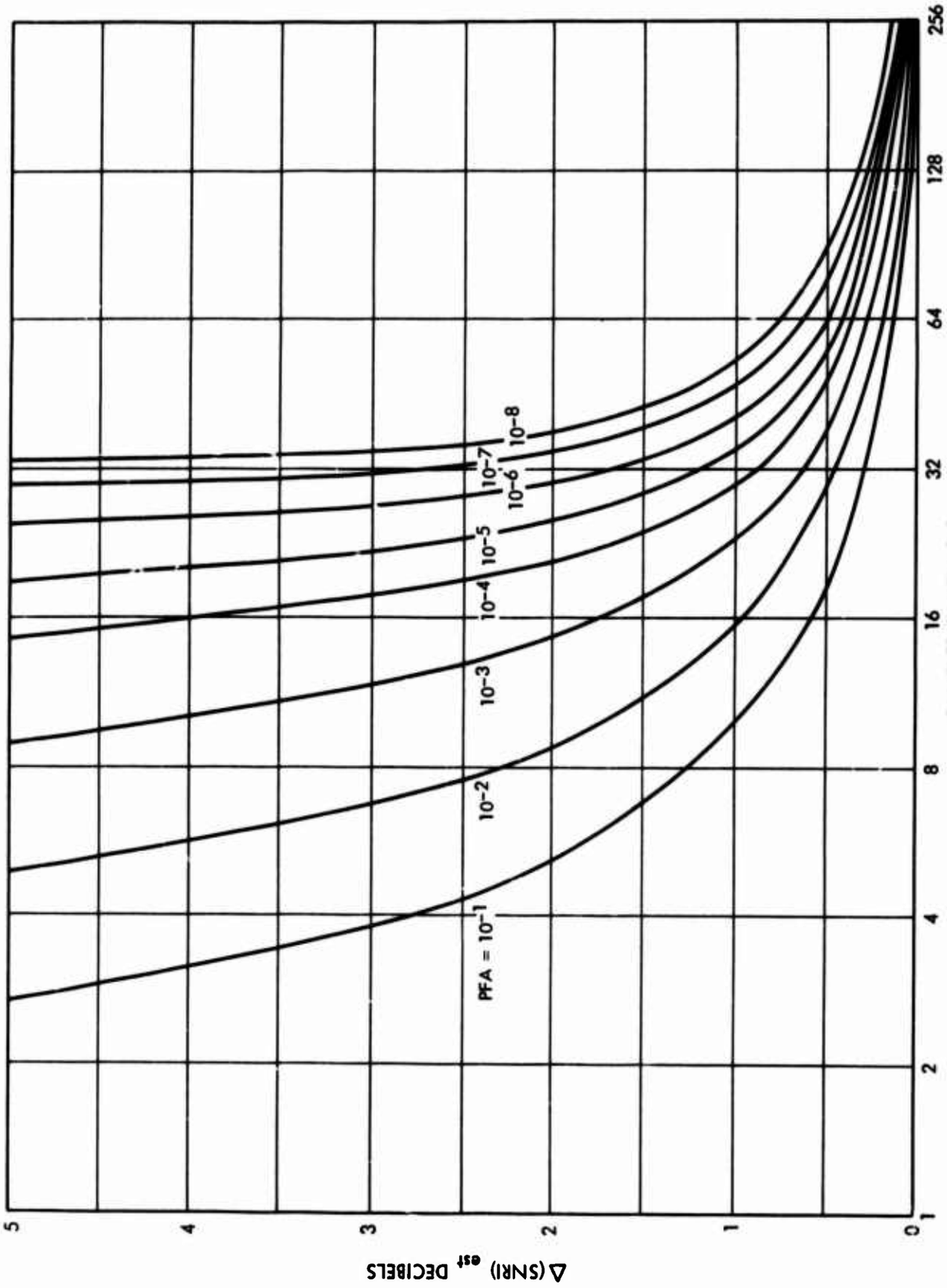


FIG. 11 LOSS DUE TO ESTIMATION FOR SQUARE LAW DETECTOR

NARROW-BAND SYSTEM WITH ENVELOPE DETECTOR

Many narrow-band systems use linear detectors, such as full or half wave rectifiers or envelope detectors, in place of the square-law detector assumed in the previous section. This is done either for convenience in detector design or to reduce the dynamic range which must be handled beyond the detector. Since the instantaneous detector output is related in a known way to the output of the square law detector, a system which makes detection decisions directly on the detector output (that is, with no further smoothing) using a perfectly selected threshold can have identical performance to a similar system using a square-law detector. However when the background must be estimated by averaging samples of the detector output, some differences in performance occur.

The instantaneous output of an envelope detector operating on a narrow-band signal may be related to that of a square-law detector by $x' = \sqrt{2x}$, where x' is the envelope detector output and x is the square-law detector output. By using this transformation of variables and the previously found distribution for the square law detector, the density function at the envelope detector output may be shown to be (dropping the primes on the variable)

$$p_x(x) = \begin{cases} (x/P)\exp(-x^2/2P) & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad (38)$$

This density function is shown in Figure 8b and has a mean $\sqrt{\pi P/2} = 1.253\sqrt{P}$ and variance $(2-\frac{\pi}{2})P = .4292P$.

Again the mean and standard deviation are proportional, so the mean alone can be estimated and the threshold set to $T = (1+c)\hat{\mu}$. Note that c no longer represents the number of standard deviations the threshold lies above the mean. According to the model used in Figure 2b, this corresponds to an ideal threshold of $T = (1+c)\sqrt{\pi P/2}$ and an estimation noise whose variance is equal to $(1+c)^2$ times the detector output variance divided by the number of independent samples N used in estimation, or $\sigma_e^2 = (1+c)^2(2-\frac{\pi}{2})P/N$. Now using equation (7) to determine the false alarm probability, including the effects of background estimation, gives

$$PFA = \frac{1}{2}\int_{-\infty}^{\infty} p_x(x)\ \mathrm{erfc}((T-x)/\sqrt{2}\,\sigma_e)dx$$

$$= (1/2P)\int_{0}^{\infty} x\ \exp(-x^2/2P)\ \mathrm{erfc}(((1+c)\sqrt{\pi P/2}-x)/$$

$$(1+c)\sqrt{(4-\pi)P/N})dx$$

$$= (1/2)\int_{0}^{\infty} x'\ \exp(-(x')^2/2)\ \mathrm{erfc}((1+c)\sqrt{\pi/2}-x')/$$

$$(1+c)\sqrt{(4-\pi/N)})dx' \qquad (39)$$

Again the substitution of variables $x' = x/\sqrt{P}$ shows that the false alarm probability is indeed independent of the background noise power P and depends only on the selected threshold ratio c and the number N of samples usedin estimation. For the case $N=\infty$ which corresponds to perfect background estimation this expression simplifies to

$$PFA = \int_{(1+c)\sqrt{\frac{\pi}{2}}}^{\infty} x\ \exp(-x^2/2)\ dx$$

$$= \exp(-(1+c)^2(\pi/4)) \qquad (N=\infty) \qquad (40)$$

For finite N the expression must be integrated numerically, and the results are shown in Figure 12. Again to maintain any given false alarm probability, an increase in the threshold ratio c is required to compensate for any decrease in the number of samples N used to estimate the background. This is replotted in Figure 13 to show the dependence of c on N, with the PFA as a parameter.

Following the same approach as in the previous section, we can relate the required increase in c to an increase in SNR required at the processor input. Since the instantaneous output x' of the envelope detector is related to the output x

of the square law detector by $x' = \sqrt{2x}$, the median outputs are related in the same way. Thus from equation (34) we can obtain the median output of the envelope detector as a function of SNR as

$$M_{env} = \sqrt{2P\ (1+SNR)(\log_e 2)} \tag{41}$$

Equating this to the threshold $T = (1+c)\sqrt{\pi P/2}$ gives

$$(1+c)^2(\pi P/2) = 2P\ (1+SNR)(\log_e 2)$$

or

$$SNR = ((1+c)^2\pi - 4(\log_e 2))/(4(\log_e 2)) \tag{42}$$

Again the increase in SNR due to background estimation can be expressed in decibels as

$$\Delta(SNRI)_{est} = 10 \log_{10} \frac{((1+c)^2\pi - 4(\log_e 2))}{((1+c_o)^2\pi - 4(\log_e 2))} \text{decibels} \tag{43}$$

where c and c_o are obtained from Figure 13 for the desired PFA and the selected N. This SNR correction is plotted in Figure 14 as a function of N for several values of false alarm probability. It may be seen to have the same general properties as that for the square law detector in Figure 11. The loss does not appear to rise as rapidly for small values of N as in the square-law detector case. However this result is again influenced by the breakdown of the Gaussian assumption for small N (again over-estimating the loss), so no conclusions should be drawn about the relative loss for square-law and linear detectors without more careful analysis.

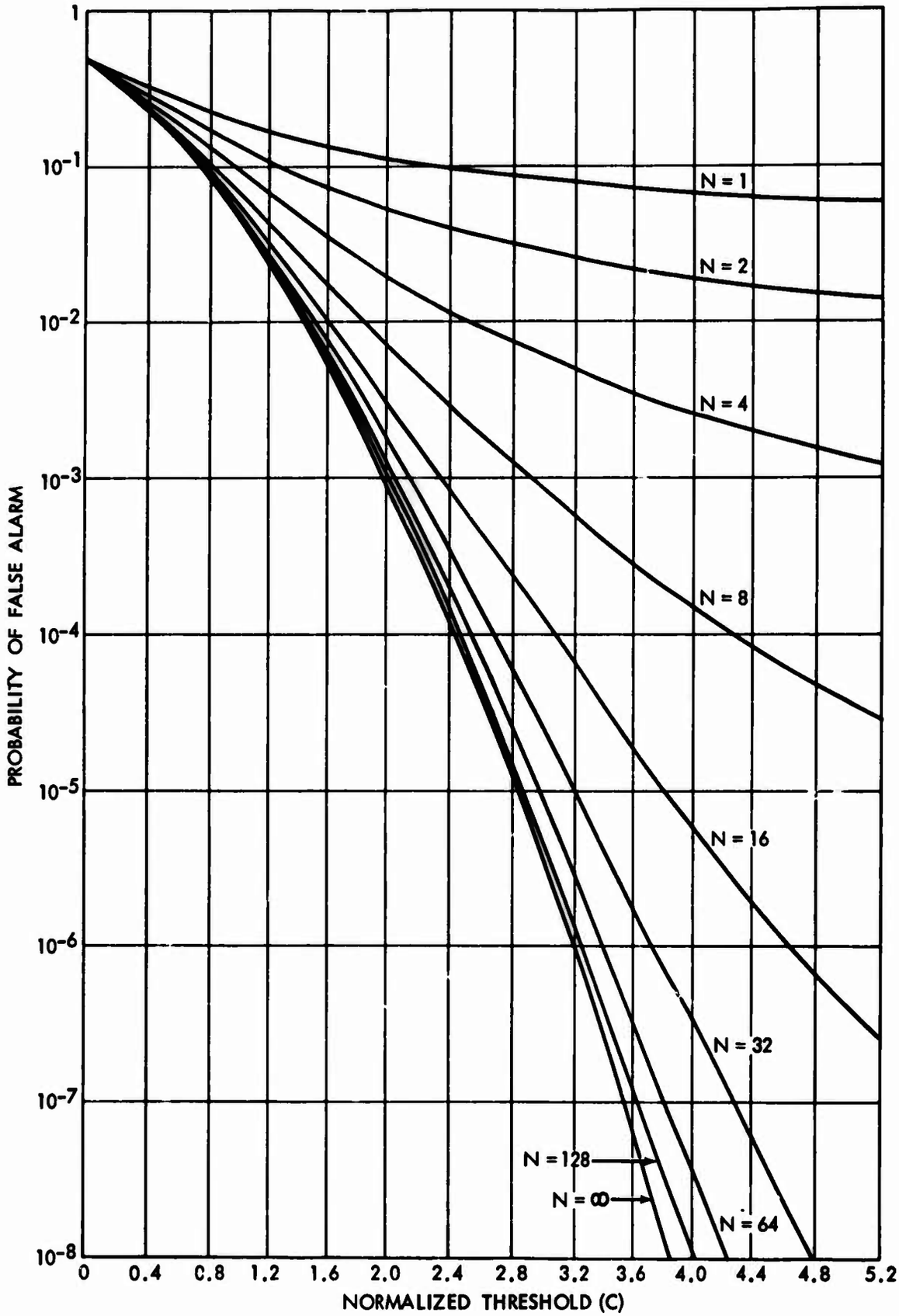


FIG. 12 PFA VERSUS THRESHOLD FOR ENVELOPE DETECTOR

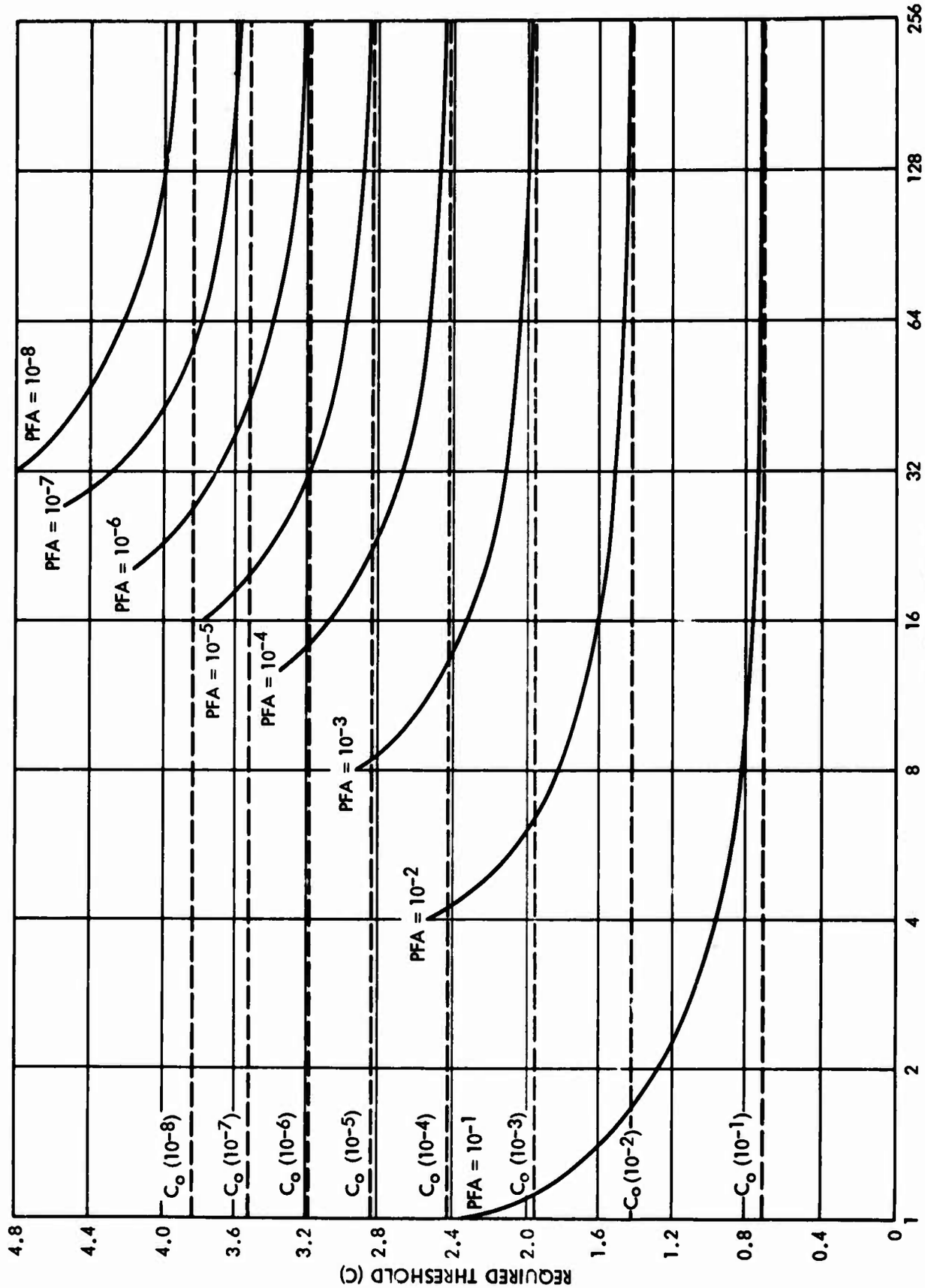


FIG. 13 REQUIRED THRESHOLD VERSUS N FOR ENVELOPE DETECTOR

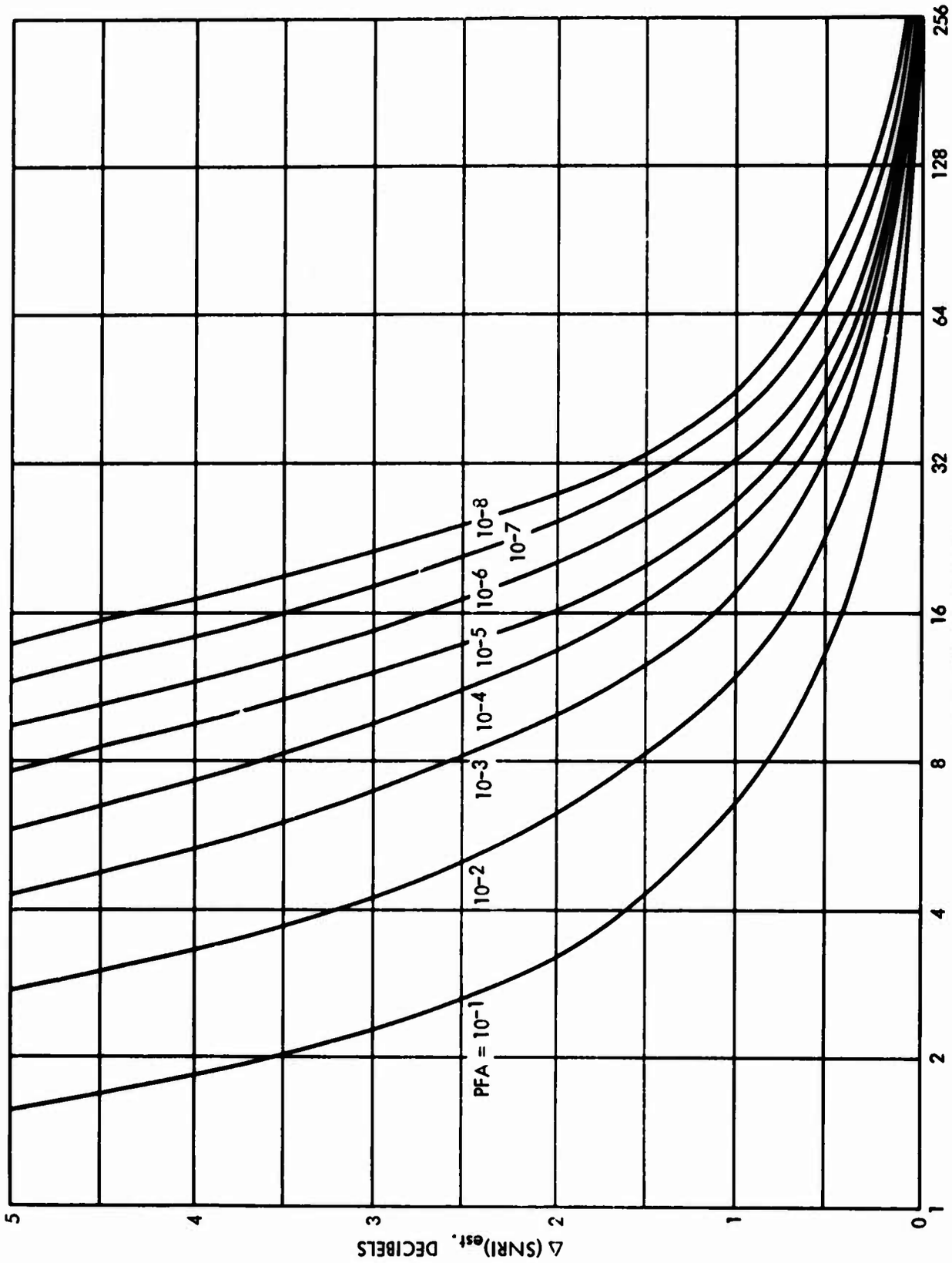


FIG. 14 LOSS DUE TO ESTIMATION FOR ENVELOPE DETECTOR

NARROW-BAND SYSTEM WITH LOGARITHMIC DETECTOR

Another type of detector sometimes used in narrow-band systems produces an output proportional to the logarithm of the measured power (in other words, a measurement in decibels). This further reduces the dynamic range to be handled at the detector output and it has other interesting statistical properties. The base of the logarithm is not important (since it represents only a gain factor at the output), so we will assume here a detector characteristic $x' = 10 \log_{10} x = 4.3429 \log_e x$, where x is the instantaneous power as measured by a square law detector. This choice is made so the detector output may be interpreted directly in decibels. Applying this transformation of variables to the density function given previously for the square-law detector gives a new density function for the logarithmic detector of (dropping the primes)

$$p_x(x) = (1/aP) \exp(x/a) \exp(-\exp(x/a)/P)$$

$$a = 10 \log_{10} e = 4.3429 \tag{44}$$

This may be written in the form

$$p_x(x') = (1/a) \exp(x'/a) \exp(-\exp(x'/a))$$

$$a = 10 \log_{10} e = 4.3429 \tag{45}$$

where $x' = x - 10 \log_{10} P$. Thus the probability density function at the output of the logarithmic detector is independent of P except for a pure translation equal to the measure of P in decibels. This distribution is plotted in Figure 8c. The mean is $a(\log_e P - \gamma) = 10 \log_{10} P - 2.5068$ and the variance is constant at $\frac{a^2\pi^2}{6} = 31.0247$.

Since the variance does not depend on P, only the mean must be compensated to provide a constant false alarm rate decision function. Thus an appropriate threshold function is $T = \hat{\mu} + c$, where c is selected to give the desired false alarm probability. The ideal threshold in the model of Figure 2b is thus $T = 10 \log_{10} P - 2.5068 + c$ while the estimation noise variance for an average of N samples is $\sigma_e^2 = 31.0247/N$. Writing the false alarm probability as in equation (7) gives

$$PFA = \frac{1}{2}\int_{-\infty}^{\infty} p_x(x)\ \mathrm{erfc}((T-x)/\sqrt{2}\,\sigma_e)dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} (1/a)\ \exp(x'/a)\ \exp(-\exp(x'/a))\cdot$$

$$\mathrm{erfc}((c-2.5068-x')/\sqrt{62.0294\ N})dx \qquad (46)$$

where the expression is written in terms of the translated function x' with the dependence on P removed.

Again for the case N = ∞ this expression can be evaluated in the form

$$PFA = \exp(-\exp(-\gamma)\ \exp(c/a))$$

$$= \exp(-0.5615(10^{c/10})) \qquad N=\infty \qquad (47)$$

For finite values of N the false alarm probability must be computed by numerical evaluation of equation (46), and these results are shown in Figure 15 for various values of threshold c and number N of samples averaged in forming the background estimate. These same results are replotted in Figure 16 showing the required threshold as a function of N, with the false alarm probability as a parameter.

Using the same technique as in the previous sections, this increase in c to maintain a given false alarm probability may be equated to an increase in required SNR at the processor input. The median output of the logarithmic detector may be found by taking 10 $\log_{10}$ of the median output of the square law detector, to yield

$$M_{log} = 10\ \log_{10}P + 10\ \log_{10}(1+SNR) + 10\ \log_{10}(\log_e 2) \qquad (48)$$

Equating this median output with the threshold T = μ+c gives

$$10\ \log_{10}P - 2.5068 + c = 10\ \log_{10}P + 10\ \log_{10}(1+SNR) - 1.5917 \qquad (49)$$

or

$$10 \log_{10}(1+SNR) = c - .9151 \tag{50}$$

so that

$$SNR = 10^{(c/10-.09151)} - 1 = .8100(10^{c/10}) - 1 \tag{51}$$

is the signal to noise ratio required to reach 50% detection probability for a given value of threshold c.

The increase in SNR required for 50% detection probability as a result of background estimation can be determined from the ratio of the SNR's corresponding to c (for a given N) and c_o (for $N=\infty$) and may be expressed in decibels as

$$\Delta(SNRI)_{est} = 10 \log_{10} \frac{.81(10^{c/10})-1}{.81(10^{c_o/10})-1} \tag{52}$$

where c and c_o may be obtained from Figure 16 for the chosen N and false alarm probability.

The SNR correction for the logarithmic detector is plotted in Figure 17 as a function of N for several false alarm probabilities. Again it is generally similar to that for the square law and envelope detectors except for seeming to indicate a lower loss in most cases. However this result is also influenced by the non-Gaussian nature of the input statistics for small N, and the error is this time in the direction of under-estimating the loss (because, as may be seen from Figure 8, the extended tail of the logarithmic detector density function is on the opposite side from that for the square-law or envelope detectors). Thus the results from the second moment analysis used in this report do not necessarily reflect the relative performance of the log, square-law, end envelope detectors in this respect. In fact Hansen[1] has shown through Monte Carlo analysis that the estimation loss for the logarithmic detector is actually somewhat worse than that for the square-law or envelope detectors, and the results shown here in Figure 17 fall fairly close to his results for the square-law or envelope detectors.

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[1]Hansen, G. S. and Ward, H. R., "Detection Performance of the Cell Averaging LOG/CFAR Receiver," IEEE Trans. Aerospace and Electronic Systems, Vol. AES-8, No. 5, September 1972.

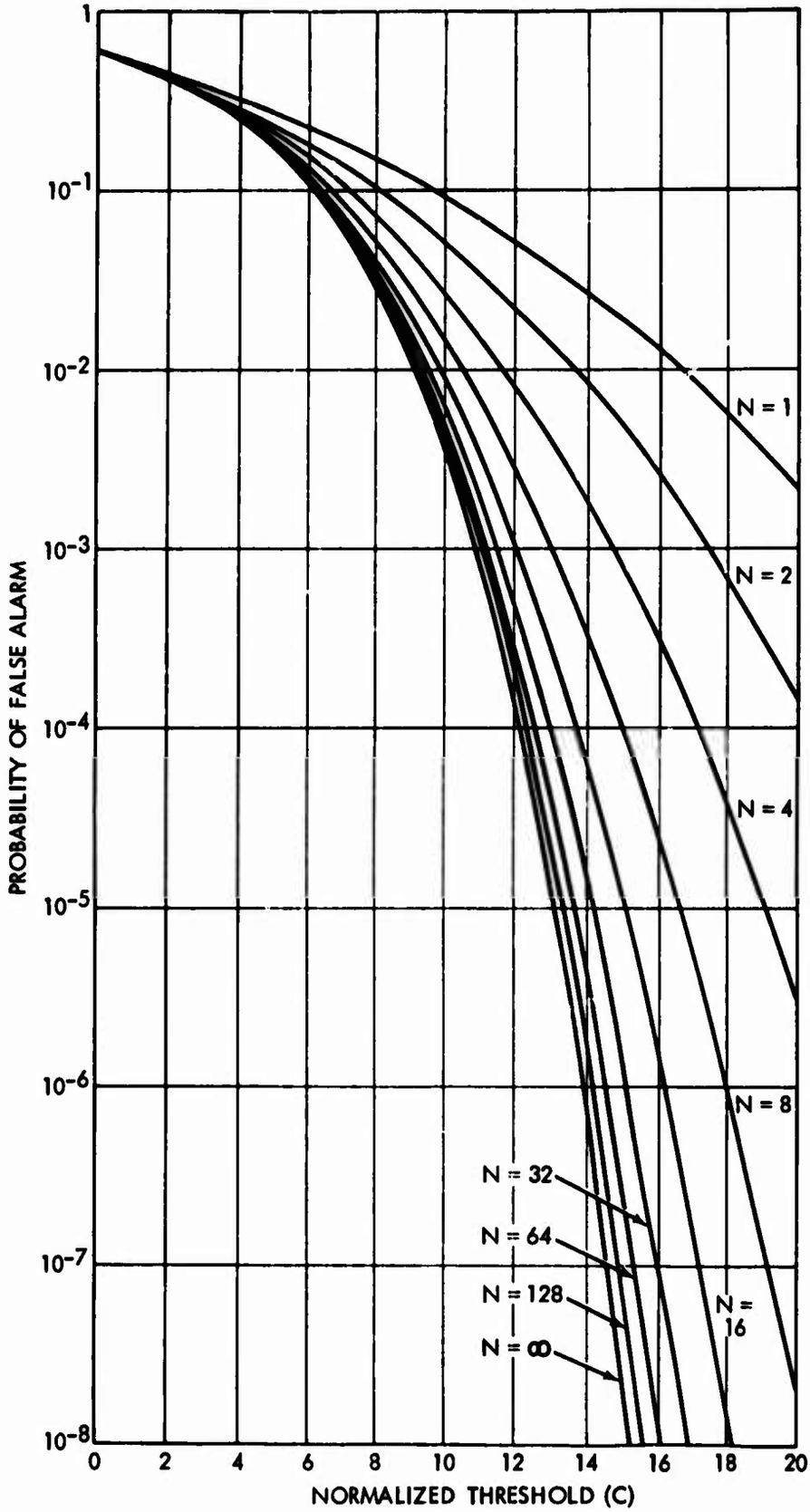


FIG. 15 PFA VERSUS THRESHOLD FOR LOGARITHMIC DETECTOR

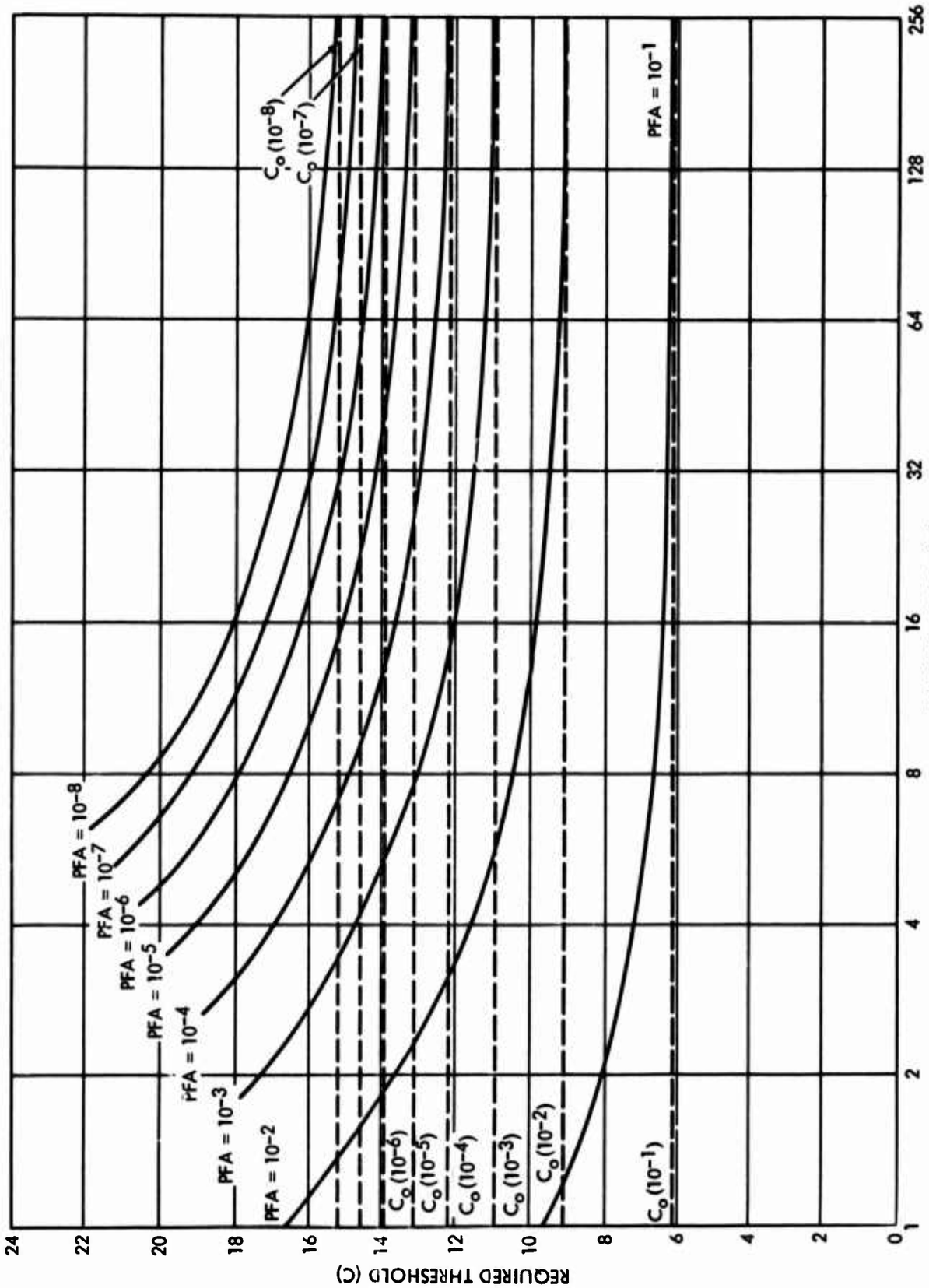


FIG. 16 REQUIRED THRESHOLD VERSUS N FOR LOGARITHMIC DETECTOR

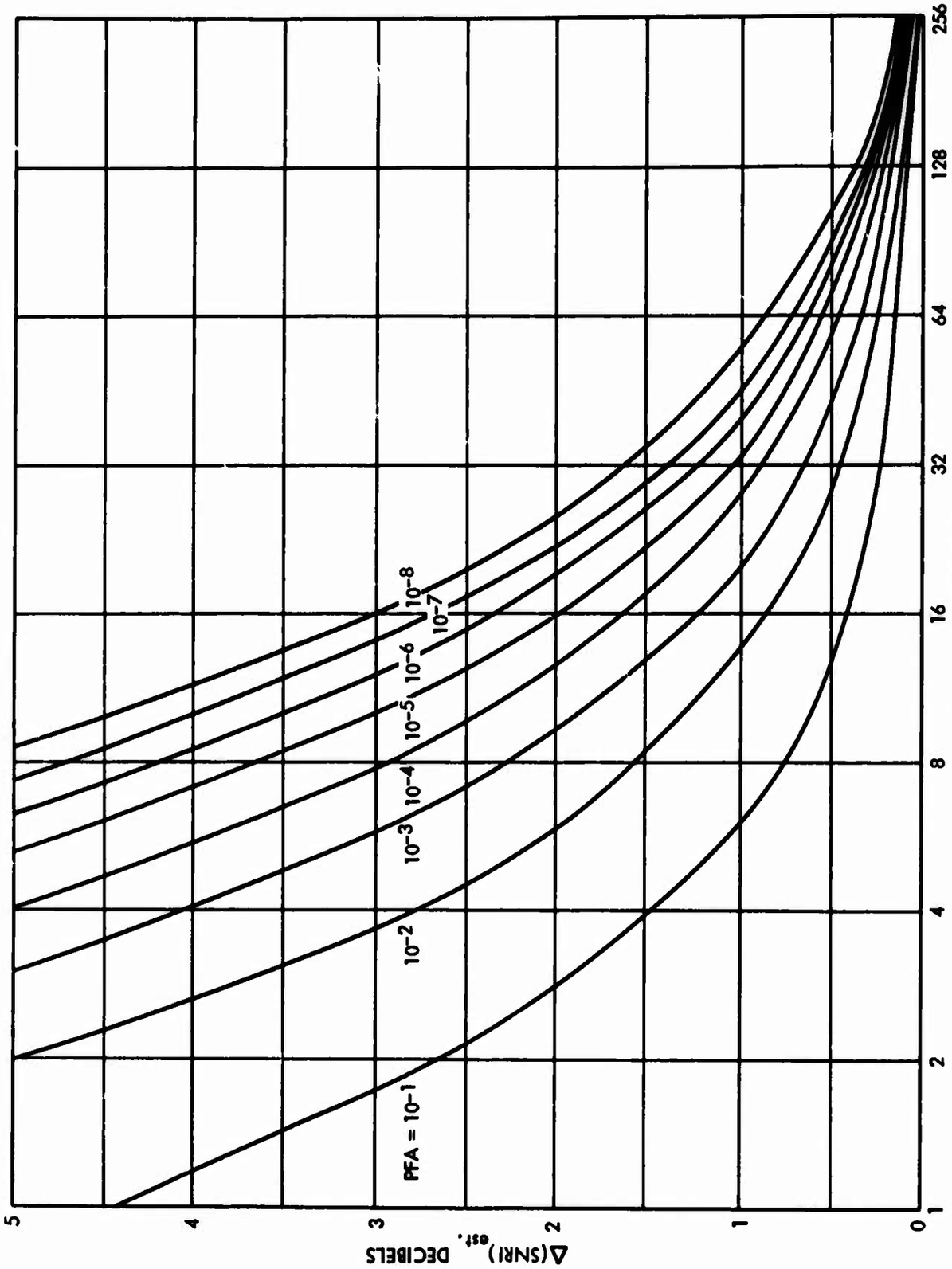


FIG. 17 LOSS DUE TO ESTIMATION FOR LOGARITHMIC DETECTOR

SUMMARY

The estimation of the background noise level which is performed in constant-false-alarm-rate detection systems has been shown to degrade the performance of the system. This degradation can be determined analytically when Gaussian statistics are applicable, while a numerical integration is required for other detector output statistics. The amount of the degradation depends on the form of the detector and the parameters being estimated, and in general depends inversely on the false alarm rate and on the number of samples used in the background estimate.

In systems with Gaussian statistics at the detector output the degradation depends significantly on the relationship between the mean and the standard deviation of the detector output. For example, in the case where sixteen samples are averaged to form the background estimate and a false alarm probability of 10^{-4} is desired, the loss (referenced to the system input) rises from 0.125 decibels if the standard deviation is known to 1.5 decibels if the standard deviation as well as the mean must be estimated.

The non-Gaussian cases studied (narrow band systems with various energy detectors and no post-detection averaging) showed results generally similar to those for the Gaussian cases, in that the system performance loss increases with a lowering of either the number of samples averaged or the false alarm probability allowed. Generally once a degradation of about one decibel is reached, any further reduction in the number of samples averaged causes the loss to increase rapidly. Unfortunately the approach used here, using only the second moment of the input statistics to characterize the estimation error, is not sufficiently accurate to indicate the differences in performance among the various detector types correctly. Thus the losses derived for the non-Gaussian cases must be considered approximate and a more elegant method, taking into account the higher moments of the estimation error, used to determine exact losses.

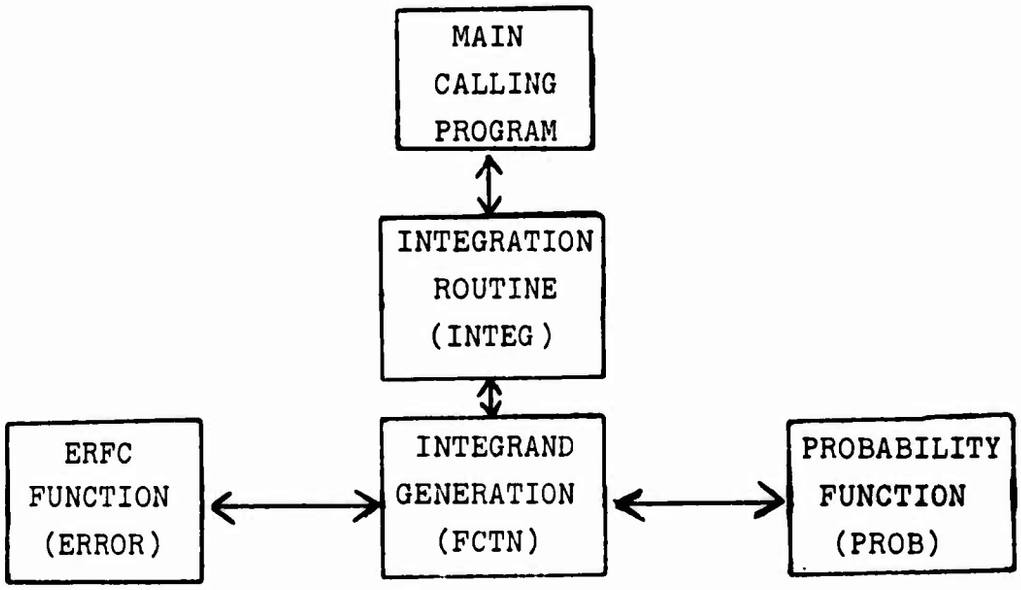
LIST OF SYMBOLS USED

c	Normalized threshold parameter of decision element
c_o	Normalized threshold parameter without background estimation
$\Delta(SNRI)_{est}$	System performance loss in decibels due to estimation, referenced to system input
$\Delta(SNRO)_{est}$	System performance loss in decibels due to estimation, referenced to detector output
γ	Euler's constant = 0.57721566
e	Base of natural logarithms = 2.71828
erf( )	Error function
erfc( )	Complementary error function
exp( )	Exponential function
K	Ratio of standard deviation to mean
M	Median output of detector
μ	Mean of detector output
$\hat{\mu}$	Estimate of mean of detector output
N	Number of independent samples used in background estimation
N_{min}	Minimum number of samples in estimation required for finite SNR
P	Noise power input to narrow-band detector
$p_x(x)$	Probability density function for detector output
$p_y(y)$	Probability density function of background estimation noise
$p_z(z)$	Probability density function for sum of detector and estimation noise
SNR	Input signal to noise ratio, expressed as a power ratio
SNRI	Signal to noise ratio at system input
SNRO	Signal to noise ratio at detector output

σ	Standard deviation of detector output
$\hat{\sigma}$	Estimate of detector output standard deviation
$\hat{\sigma}^2$	Estimate of detector output variance
σ_e	Standard deviation of background estimator output
T	Decision element threshold

APPENDIX A

The following computer programs were developed for the numerical integration necessary to evaluate the previously discussed narrow band systems. These programs model equation (7) of the text as shown below:



A main routine initializes and varies parameters, contains formatting statements, and calls the integration subroutine, INTEG. This subroutine calls the integrand generating subroutine, FCTN, which formulates the integrand required by INTEG. For the prescribed probability function FCTN calls the subroutine PROB and for the erfc function FCTN calls the local system library subroutine, ERROR.

To use the program for the three cases cited (square law, envelope and logarithmic detectors) it is necessary to alter only the main routine and the PROB subroutine. The former requires that the limits of integration and appropriate constants peculiar to the case considered be varied and the latter requires entry of the probability function of the particular case.

Routines 1 through 4 represent the program for the numerical integration involved in the evaluation of the square law detection system. Routines 5 and 6 contain the necessary alterations for the envelope detection case. The final two routines, 7 and 8, represent the changes in the initial program to provide for the numerical integration in the logarithmic detection system.

```
      PROGRAM MAIN(OUTPUT,TAPE6=OUTPUT)
      COMMON/A/ AREA,STEP,FOLD,T,SIGE
      AN=1.
      STEP=.01
      SRT = SQRT(2.)
      DO 90 NN=1,18
      C=0.
      DO 89 NT=1,25
      SIGE=(1.+C)/SQRT(AN)
      T=1.+C
      FOLD=10.
      CALL INTEG
      WRITE(6,110) AN,C,AREA
  110 FORMAT(1X,*PFA(*,I4,F6.2,*)=*,F20.10)
      C=C+1.
   89 CONTINUE
      AN = SRT*AN
   90 CONTINUE
      STOP
      END
```

Routine 1

Main Routine for the Square Law Detector

```
      SUBROUTINE INTEG
      COMMON/A/ AREA,STEP,FOLD,T,SIGE
      S=STEP
      F=ABS(FOLD)
      X=S/2.
      D=X**2
      SN = 1.
      AREA = FCTN(X)
      IF(FOLD.LT.0.) AREA = FCTN(-X)
   10 X = X+S
      IF (X.GE.F) GO TO 20
      K=F**2/(X**2-D)
      IF(FOLD.LT.0.) SN = -1.
      AREA = AREA+FCTN(SN*X)+K*FCTN(SN*F**2/X)
      GO TO 10
   20 AREA = S*AREA
      RETURN
      END
```

Routine 2

Integration Routine for the Square Law Detector

```
      FUNCTION FCTN(X)
      COMMON/A/ AREA,STEP,FOLD,T,SIGE
      Z=(T-X)/(1.414213562*SIGE)
      R=ABS(Z)
      CALL ERROR(R,ERF,ERFC,EXERFC)
      IF(Z.LT.0.) ERFC = ERF + 1.
      FCTN = 0.
      IF(ERFC.NE.0.) FCTN = .5*ERFC*PROB(X)
      RETURN
      END
```

Routine 3

Integrand Generation Routine for the Square Law Detector

```
      FUNCTION PROB(X)
      PROB=EXP(-X)
      RETURN
      END
```

Routine 4

Probability Function Routine for the Square Law Detector

```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON/A/ AREA,STEP,FOLD,T,SIGE
      N=1
      STEP = .004
      DO 90 NN=1,8
      AN = N
      C=4.
      DO 89 NT=1,6
      SIGE= (1.+C)*.6551363774/SQRT(AN)
      T=(1.+C)*1.253314137
      FOLD = 5.
      CALL INTEG
      WRITE(6,110) N,C,AREA
  110  FORMAT(1X,*PFA(*,I4,F9.2,*)=*,F20.10)
      C = C+.2
   89 CONTINUE
      N=2*N
   90 CONTINUE
  140  STOP
      END
```

Routine 5

Main Routine for the Envelope Detector

```
      FUNCTION PROB(X)
      PROB = X*EXP(-X**2/2.)
      RETURN
      END
```

Routine 6

Probability Function Routine for the Envelope Detector

```
PROGRAM      MAIN                                    CDC 6400 FTN V3.0-P316 OPT=2   03/0

                 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
                 COMMON/A/ AREA,STEP,FOLD,T,SIGE
                 AN=1.
                 STEP =.01
                 DO 90 NN=1,9
                 C=4.
                 DO 89 NT=1,17
                 T=C-2.5068
                 SIGE = SQRT(31.0247/AN)
                 FOLD=10.
                 CALL INTEG
                 AR = AREA
                 FOLD = -10.
                 CALL INTEG
                 AREA = AR + AREA
                 WRITE(6,110) AN,C,AREA
             110 FORMAT(1X,*PFA(*,F4.2,F6.2,*)=*,F20.10)
                 C=C+1.
              89 CONTINUE
                 AN = 2.*AN
              90 CONTINUE
             140 STOP
                 END

OF CHANGES MADE BY THE OPTIMIZER
DS OF INVARIANT RLIST REMOVED FROM THE LOOP STARTING AT LINE      7
```

Routine 7

Main Routine for the Logarithmic Detector

```
      FUNCTION PROB(X)
      A = 4.3429
      XA =X/A
      PROB = 0.
      IF(XA.GT.4.) GO TO 10
      PROB = (1./A)*EXP(XA)*EXP(-EXP(XA) )
   10 CONTINUE
      RETURN
      END
```

Routine 8

Probability Function Routine for the Logarithmic Detector