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ROLLING ELEMENT BEARING RETAINER ANALYSIS

Joseph A. Mauriello, et al

Avco Lycoming Division

Prepared for:

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13. ABSTRACT

An integrated analytical and experimental program was conducted to analyze the operation of rolling element bearing retainers and attendant lubrication effects. Analytical models to represent ball and roller bearing operation were derived with regard to the elastic deflections at the rolling element/race contacts, tractive forces generated at the bearing component interaction surfaces, and viscous drag effects.

Program objectives were conducted in two phases. The first phase was to analyze the forces and the lubrication conditions that exist between the rolling element and retainer pockets when the bearing is operating at high-load and high-speed conditions. Once completed, the derived analysis was incorporated into an iterative computer program to produce bearing solutions for arbitrary load and speed conditions

The second phase of the program was to conduct an experimental test program to validate the computer analysis. Bearing performance was investigated over a wide range of loads, speeds, and lubrication conditions. Also, data were acquired on ball bearings subjected to retainer skid at high-speed, low-load operating conditions. To accomplish the test program, it was necessary to modify an existing test rig and to design special instrumentation for acquisition of the desired operating data.

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The objectives of the program were to analyze the forces exerted on the retainer pockets of rolling element bearings during high-load, high-speed operating conditions that exist within the rolling-contact areas of the bearing.

Appropriate technical personnel of this Directorate have reviewed this report and concur with the conclusions contained herein.

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ROLLING ELEMENT BEARING RETAINER ANALYSIS

Final Report

Avco Lycoming Report No. 105.7.10

By

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FORT EUSTIS, VIRGINIA

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SUMMARY

An integrated analytical and experimental program was conducted to analyze the operation of rolling element bearing retainers and attendant lubrication effects. Analytical models to represent ball and roller bearing operation were derived with regard to the elastic deflections at the rolling element/race contacts, tractive forces generated at the bearing component interaction surfaces, and viscous drag effects.

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FOREWORD

This final report describes a program conducted by Avco Lycoming Division for the U. S. Army Air Mobility Research and Development Laboratory under Contract DAAJ02-69-C-0080, Task 1G162203D14414.

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LIST OF SYMBOLS

A	area of cage guiding surface, in.^{2}
A_1	axial distance between inner and outer race curvature centers, in.
A_2	radial distance between inner and outer race curvature centers, in.
A_3	drag area on rolling elements, in.^{2}
A_i	drag area on races, in.^{2}
$A_{Y,\ Z}$	pilot surface load parameters for Y and Z directions, lb
a_i	semimajor axis of contact ellipse, in.
b	semiwidth of roller contact, in.
b_i	semiminor axis of pressure ellipse, in.
$\overline{C}$	radial clearance of rolling element in retainer pocket, in.
C_D	drag coefficient
C_f	friction factor for flow between race and retainer
C_i	$C_1 = 1.0,\ C_2 = -1.0$
D_1	outside diameter of retainer or pilot surface, in.
D_2	inside diameter of retainer or pilot surface, in.
D'	retainer pilot diameter, in.
D_{C_1}	outside diameter of cage, in.
D_{C_2}	inside diameter of cage, in.
d	rolling element diameter, in.

E	pitch diameter, in.
$E(\epsilon_i)$	complete elliptical integral of the second kind
E_R, E	Young's modulus for bearing rings (R) and rolling elements (E), lb/in.^{2}
E'	instantaneous pitch diameter of ball, in.
E''	materials parameter in Dowson and Higginson equation, lb/in.^{2}
F	reaction of bearing, in.
$\bar{F}$	external radial load, lb
F_1, F_2	hydrodynamic forces in retainer pocket, lb
F_C	rolling element centrifugal force, lb
$\bar{F}_c$	centrifugal force on cage, lb
F_D	viscous drag force on rolling element, lb
F_F	retainer pilot surface Coulomb drag force, lb
F_P	normal force in roller/pocket contact, lb
F_{P_i}	total load of roller against retainer pocket, lb
F_{P_x}	friction force generated at retainer contact in pocket in X direction, lb
F_{P_z}	friction force generated at ball retainer contact in ball pocket in Z direction, lb
F_R	net viscous drag force on retainer, lb
F_T	tractive force in roller/race contact, lb
F_V	tangential force in roller/pocket contact, lb
F_X	bearing reaction force in X direction, lb

F_{X_i}	friction force directed along major axis (X) of pressure ellipse, lb
F_Y	bearing reaction force in Y direction, lb
F_{Y_i}	friction force directed along minor axis (Y) of pressure ellipse, lb
F_Z	bearing reaction force in Z direction, lb
F'_{p_x}	load per unit length of roller against retainer pocket, lb/in.
F'_Z	total tractive force between roller and retainer pocket, lb
F'_{Z_y}	tractive force per unit length between roller and retainer pocket, lb/in.
$\overline{F}_V$	roller bearing externally applied radial load in the Z direction, lb
$\tilde{F}_i$	dimensionless tractive forces
f	friction factor/coefficient - rolling element to race contacts
f_i	race curvature - groove radius divided by ball diameter
f_r	coefficient of sliding friction for pilot surface
G_1	shear rate parameter
G_2	thermal heating parameter
G_3	pressure-viscosity parameter
H	horizontal component of pilot surface load (P_c), lb
H_j	spacing between retainer and race, in.
h	elastohydrodynamic film thickness, in.
h_o	minimum elastohydrodynamic film thickness, in.

h_p	film thickness of midpoint of roller to cage contact, in.
h_{p_x}	film thickness at any point X along roller to cage contact, in.
$\tilde{h}$	dimensionless film thickness
I	mass moment of inertia of rolling element, in.-sec^2-lb
i	i = 1 for outer race contact; i = 2 for inner race contact
K	coefficient of friction for retainer-rolling element contacts
$K(\epsilon_1)$	complete elliptical integral of the first kind
K_c	Coulomb friction coefficient for roller-retainer contact
K_f	thermal conductivity of the lubricant, Btu/^{o}F-hr-ft
K'_l	constant = 0.5482
K'_w	constant = 2.447
ℓ_F	flat length of roller, in.
ℓ_T	total length of roller less two corner radii, in.
ℓ'	distance from roller dim point to contact pattern end, in.
M_c	total torque on retainer, lb-in.
m	mass of rolling element, lb-sec^2/in.
m_c	mass of cage, lb-sec^2/in.
$\nu_{R,\ E}$	Poisson's ratio for bearing rings (R) and rolling element (E)
M_V	viscous drag between rings and retainer, lb-in.
M_Y	inertial moment of ball about Y axis, lb-in.
M_Z	inertial moment of ball about Z axis, lb-in.
M'_Y	bearing reactive moment about Y axis, lb

n	number of rolling elements
P	race contact force, lb
P_c	retainer pilot surface to guiding race land load, lb
P_{c_r}	normal pilot contact force if $\bar{F}_c$ is absent, lb
P_D	diametral clearance of bearing, in.
P_{HZ}	Hertz pressure, lb/in.^{2}
P_i	race contact normal loads, lb
P_{i_x}	load per unit length of contact at X along roller contact, lb/in.
P_p	normal contact load between ball and retainer in pocket, lb
$\tilde{P}$	dimensionless load per unit length
p	pressure, lb/in.^{2}
p_x	roller loading, lb-in.
$\tilde{p}$	dimensionless pressure
Q_p	spin torque at ball to retainer pocket contact, lb-in.
Q_{s_i}	spin torque at race contact, lb-in.
Q_v	viscous drag torque on cage, lb-in.
$\bar{Q}$	torque unbalance on cage, lb-in.
$\underline{q}$	roller azimuth index
R	equivalent radius of contact against a plane surface, in.
R_c	roller crown radius, in.

R_E	radius of rolling element, in.
R_R	radius of race surface, in./sec
$\mathcal{R}$	Reynolds number
r	corner radius of roller, in.
r_r	roller radius, in.
S_m	maximum Hertz stress, psi
S_R, S_B	RMS surface roughness parameters
T_i	inlet lubricant temperature, ^oF
T_o	temperature of lubricant, ^oF
t_r	thickness of retainer, in.
U	velocity term in Dowson and Higginson equation, in./sec
U_i	sliding velocity of ring on cage, in./sec
U_R	linear velocity of roller, in./sec
U_y	linear velocity of race, in./sec
u_s	sliding speed at rolling element to race contact, in./sec
u_x, u_y	rolling element to retainer sliding velocities, in./sec
V	vertical component of pilot surface load (P_c), lb
V_{Bi}	linear velocity in the Y direction at a strip on a ball, in./sec
V_E	surface velocity of roller, in./sec
V_o	orbital velocity of rolling element, rad/sec
V_{Ri}	linear velocity in the Y direction at a strip on the race, in./sec

$V_{X_{R_i}}$	linear velocity on a strip on the race in the X direction, in. /sec
$V_{y_{B_i}}$	linear velocity of the ball in the Y direction at the center of the contact area, in. /sec
$V_{X_{B_i}}$	linear velocity on a strip on the ball in the X direction, in. /sec
V_{y_i}	surface velocity of ring relative to retainer, in. /sec
$V_{x_{B_i}}$	linear velocity on a strip on the ball in the X direction, in. /sec
$V_{y_{R_i}}$	linear velocity of a raceway in the Y direction at the center of the contact area, in. /sec
W	normal pocket load = P_p, lb
w	load per unit width of contact, lb/in.
$\tilde{W}$	dimensionless load
W'_i	total axial effective contact length between retainer and ring, in.
X_1	ball center (axial) coordinate relative to outer race center of curvature, in.
X_2	ball center (radial) coordinate relative to outer race center of curvature, in.
y^*	semistrip width, in.
Z	radial clearances at cage outside and inside diameter
α_o	pressure coefficient of viscosity, in.^{2}/lb
β_1^*	temperature viscosity coefficient used by Crook, ^oF
β_i	race contact angle, deg
γ_i	geometric parameter = $d/\cos\beta_i/E'$
Δ	total approach of inner and outer rings, in.

Δ_o	total compression at roller to race contact, in.
Δ_i	approach of ball to race at the ith contact, in.
Δ_{i_x}	approach of roller to race at X along roller contact, in.
δ	total approach of inner to outer race, in.
Δ_q	total compression of roller, in.
δ_o	maximum compressive stress in contact area, Hertz stress
δ_x	inner ring displacement in X direction, in.
δ_y	inner ring displacement in Y direction, in.
δ_z	inner ring displacement in Z direction, in.
ϵ_i	modular angle of a pressure ellipse, rad
ξ, η	dimensionless coordinates
σ	angular position of pilot surface load line, rad
μ	lubricant viscosity, lb-sec/in.^{2}
μ_o	lubricant viscosity in inlet region of elastohydrodynamic contacts, lb-sec/in.^{2}
ρ	mass density of lubricant, lb-sec^2/in.^{4}
τ	shear stress, lb/in.^{2}
$\tilde{\tau}$	dimensionless shear stresses
τ_i	Hertz auxiliary parameter
τ_y	shear stress in lubricant film in Y direction, lb/in.^{2}
ω	rolling element azimuth angle measured clockwise, deg

ω_i	angular velocity of the ith ring relative to the retainer, rad/sec
ω_{s_i}	spin velocity component at race contact, rad/sec
ω_x	rolling element angular velocity about X axis, rad/sec
ω_y	rolling element angular velocity about Y axis, rad/sec
ω_z	rolling element angular velocity about Z axis, rad/sec
$\dot{\omega}_x$	angular acceleration of rolling element about X axis, rad/sec^2
Ω_1	angular velocity of ball about X axis = ω_x rad/sec
Ω_2	angular velocity of ball about Z axis = ω_z rad/sec
Ω_E	retainer (cage) speed, rpm
Ω_i	absolute angular velocity of the bearing rings, rad/sec
ϑ	elastic constants, in.^{2}/lb

INTRODUCTION

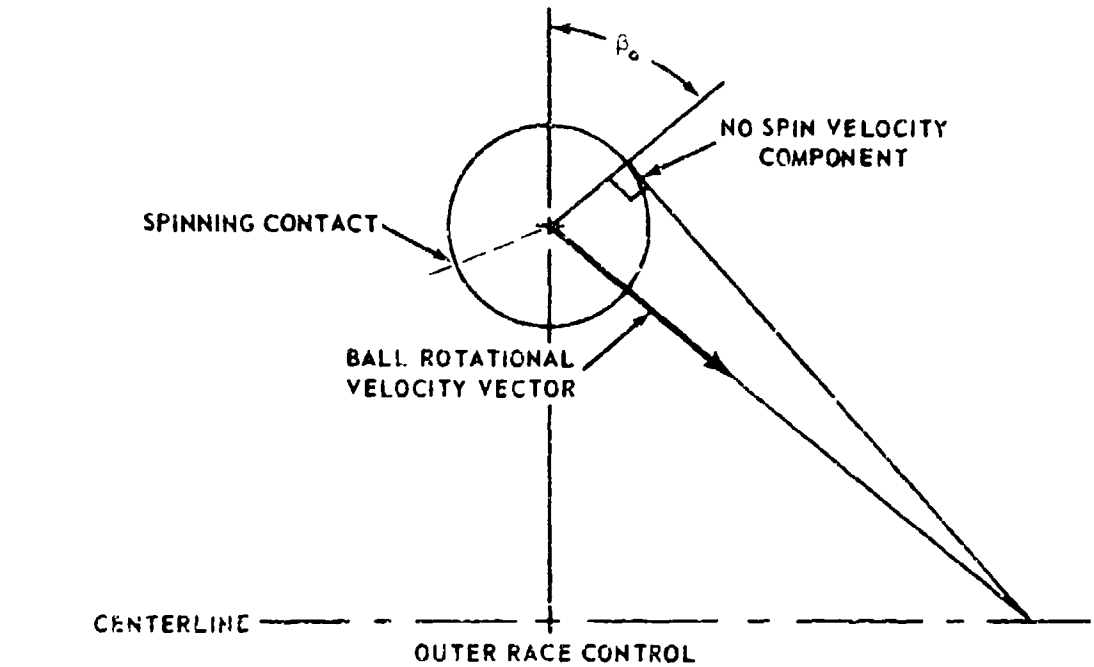
The analysis of rolling element bearing systems, which had earlier been prohibitive for manual calculation, has been facilitated by the advent of digital computers. Bearing systems present highly redundant, nonlinear mathematical models for which closed-form solutions are not available. Solutions must be obtained by numerical iterative techniques.

During the past ten years, many computer programs for bearing analysis have been created to meet the varying needs of industry. The best known is a five-degree-of-freedom program derived and written by A. B. Jones.[2] This program forms the basis of most of the computer work accomplished in the bearing analysis field, and is extensively used throughout the aerospace industry for the design and analysis of high-speed rolling element bearings.

The program uses Newton-Raphson iterative techniques to solve for equilibrium of the individual bearings and the entire bearing system simultaneously under the action of applied external loads while considering high-speed effects. Solutions are arrived at by systematically varying the shaft deflections until the integrated reaction forces of the rolling elements against the races balance the applied forces.

This analysis for ball bearings, however, does not consider the forces acting among the retainer, the rolling elements, and the piloting surfaces of the bearings, nor does it account for the tractive forces generated at the race contact areas. Bearing kinematics are established by constraining the balls' angular velocity vector to lie in a radial plane, i.e., no ball rotation about the axis dictated by the ball gyroscopic moment. Also, the analysis is subject to the race control hypothesis, which dictates that the ball will roll without spin at the race contact (inner or outer) that offers the greatest resistance to spinning. High-speed bearings with relatively large centrifugal ball loads against the outer race would, therefore, be predominantly outer race controlled. The significance of outer and inner race control is shown geometrically in Figure 1.

A race control assumption was imposed in the previous analysis because a ball in angular contact cannot exhibit pure rolling at both inner and outer race contacts simultaneously. Therefore, the ball must have a



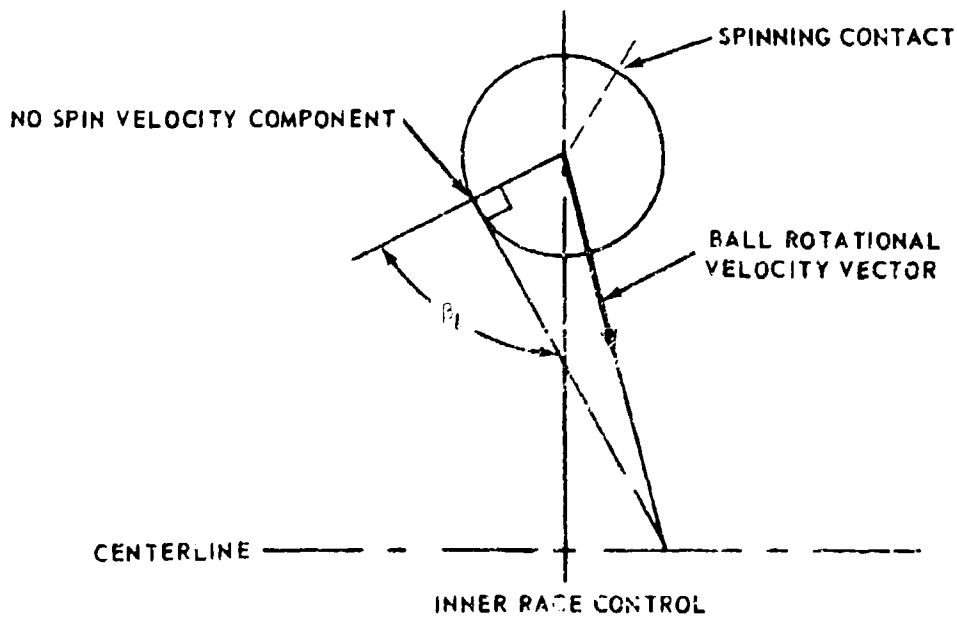


Figure 1. Race Control Concept.

spin velocity component at one or both race contacts which is equivalent to a twisting of the race body with respect to the ball about the normal to the center of the pressure area. Outer race control is considered to be advantageous for high-speed bearings because it results in minimum ball gyroscopic moments and maximum stability. It does have the disadvantage of placing the spin velocity component at the inner race with attendant heat generation, where oil cooling is most difficult.

Also implicit in the analysis is the concept that all friction resistance to the gyroscopic moment is generated at the controlling race and that tractive slip does not occur at any of the race contacts, which, of course, results in analytical solutions predicated upon zero retainer slip. The roller bearing analysis used is, in general, subject to the same limitations as those described for the ball bearing with the obvious exception of the race control and gyroscopic considerations, which are not applicable to rollers.

Recent work conducted by Harris,[5] Popiawski and Mauriello,[9] Boness,[3] and others has indicated that some of the assumptions discussed above and incorporated within the basic Jones analysis are only approximately valid. Specifically:

1. The ball angular velocity vector in an angular contact ball bearing does not lie exclusively in a radial plane. There is an angular velocity component in the circumferential direction.

2. The raceway control hypothesis is not applicable to all modes of bearing operation.

3. Tractive slip does occur at the rolling element-to-race contacts with attendant retainer slippage.

4. Resistance to the ball gyroscopic moment is not confined exclusively to the controlling race.

Some of these weaknesses have been resolved by various analysts in special computer models treating individual bearings. However, some fundamental problem areas have yet to be addressed, most notably the thermal effects due to contact area slip upon the tractions generated in an EHD (elastohydrodynamic) contact. This effect is of the utmost importance

in that the contact area traction is the key element in the establishment of the ball kinematics and thereby the entire solution.

The task undertaken as reported herein was to derive a new mathematical model for ball and roller bearings that is general and is not subject to the assumptions and restrictions described above. This undertaking required a reformulation of the existing Jones analytical model to include the following salient points:

1. EHD tractive forces and moments in the contact areas accounting for the thermal effects accompanying sliding

2. EHD film thickness accounting for spin as well as rolling velocities in the contact areas

3. Ball angular velocity in three degrees of freedom

4. Tractions and normal loads acting at the rolling element-to-retainer contacts

5. Viscous forces acting among the retainer inside diameter, outside diameter, and their respective bearing race surfaces

6. Rolling element viscous drag forces

7. Equilibrium of the retainer assembly to be established to predict retainer speed deviation from the theoretical epicyclic value.

Rig testing was conducted to verify and modify the theoretical analysis. The test program included the modification of a test rig and the design of special instrumentation to obtain the required verification data. Tests were run on a 100-millimeter ball bearing at speeds up to 20,000 rpm with axial and radial loads up to 1,000 pounds.

MATHEMATICAL ANALYSIS

BALL BEARING MATHEMATICAL MODEL

A complete presentation of the mathematical model developed for the ball bearing solution is presented in this section. The approach taken is an extension of the analysis related in Reference 2 to include retainer loads and lubrication considerations. The numerical procedure that has been employed is composed of essentially two major iterative loops, one within the other. Each loop has specific criteria which must be satisfied to obtain the desired solution. The innermost loop of the analysis deals with individual ball equilibrium which is achieved when Equations (26) through (30) are simultaneously satisfied. These equations represent force equilibrium in the X and Z directions, and moment equilibrium about the X, Y, and Z axes, respectively. The second loop of the analysis establishes equilibrium of the bearing rings with respect to externally applied loads. These criteria are achieved when Equations (34) through (37) are satisfied. These equations represent force equilibrium on the bearing races in the X, Y, and Z directions and moment equilibrium about the Y axis, respectively. A second function of this loop establishes retainer torque balance by iterating upon retainer speed accounting for ball-to-race tractive forces, ball-to-retainer loads, viscous drag between retainer and ring surfaces, and ball drag. In this manner, the retainer speed is established and its deviation from the theoretical epicyclic value can be determined. These considerations are expressed in Equations (46) through (49), which deal with force equilibrium in the X, Y, and Z directions and moment equilibrium about the X axis.

The balls in a high-speed ball bearing operating under other than a centric thrust load do not orbit at uniform speed if unrestrained by the cage. Consequently, they move fore and aft within the clearance of the pockets, and at times they may be in contact with the fore or aft wall of the retainer pocket.

When a ball is in contact with either the fore or aft wall of the pocket, its orbital speed is that of the cage and is generally different from its unconstrained orbital speed. Slippage between ball and race, always present to some degree, is greatly altered in form and intensity at this time. Also, frictional forces of the ball in the pocket are significantly increased.

At the instant a ball contacts a pocket wall, an impact force is produced. This force may be the maximum force exerted on the pocket when speeds are high and bearing loadings are light. As loading intensity is increased, the impact severity changes little, whereas the slippage-dependent forces increase greatly.

The mathematical model for this study neglects the impact forces and constrains the ball to orbit at the constant speed of the cage. Also, the fore and aft clearance in the pocket is zero to ensure the worst possible case of cage loading, neglecting impact forces; i.e., all balls are in contact with their pockets at all times. For purposes of assessing the properties of the film between ball and pocket, the true clearance is employed.

The ball referred to an XYZ coordinate system is shown in Figure 2. The X axis is parallel to the axis of the bearing and rotates about the latter with the angular cage (retainer) velocity Ω_E.

In the following, the subscript 1 refers to the outer ring or outer-ring contact. The subscript 2 refers to the inner ring or inner-ring contact.

$$\left.\begin{aligned} C_1 &= 1 \\ C_2 &= -1 \end{aligned}\right\} \qquad (1)$$

ω_i is the angular velocity of the ith ring relative to the cage and is related to the absolute angular velocity Ω_i of the ring and the cage velocity Ω_E.

$$\omega_i = \Omega_i - \Omega_E \qquad (2)$$

$i = 1$ for outer ring

$i = 2$ for inner ring

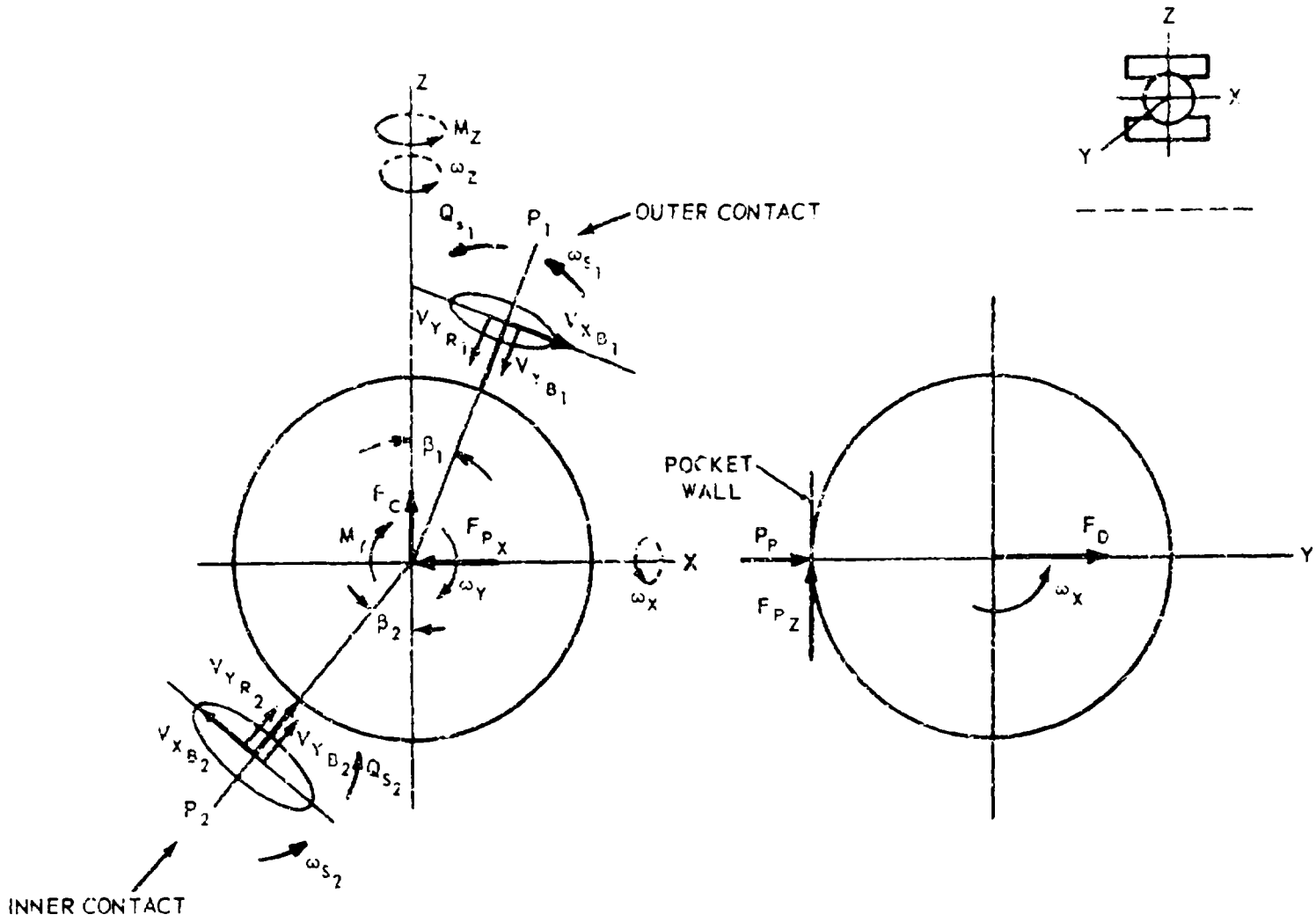


Figure 2. Ball Referred to XYZ Coordinate System.

ω_X, ω_Y, and ω_Z are the angular velocities of the ball about X, Y, and Z.

F_C is the centrifugal force, and M_Y and M_Z are the inertial moments about Y and Z.

$$M_Y = I\,\omega_Z\,\Omega_E \tag{3}$$

$$M_Z = -I\,\omega_Y\,\Omega_E \tag{4}$$

$$F_C = m\,\frac{E'}{2}\,\Omega_E^2 \tag{5}$$

I is the mass moment of inertia of the ball about its center, and m is the mass of the ball. E' is the instantaneous pitch diameter.

P_i is the contact force, and a_i and b_i are the semimajor and semiminor axes of the pressure ellipse.

$V_{Y_{R_i}}$ is the linear velocity of a raceway in the Y direction at the center of a pressure area.

$$V_{Y_{R_i}} = \frac{(\Omega_i - \Omega_E)\,(E' + c_i\,d\cos\beta_i)}{2} \tag{6}$$

$V_{Y_{B_i}}$ is the linear velocity of the ball at the ith contact in the Y direction.

$$V_{Y_{B_i}} = \frac{c_i\,d}{2}\,(\omega_X\cos\beta_i - \omega_Z\sin\beta_i) \tag{7}$$

β_i is the operating contact angle at the ith contact.

$V_{X_{B_i}}$ is the velocity of slip of race on ball in the X direction.

$$V_{X_{B_i}} = -\frac{d}{2}\,\omega_Y \tag{8}$$

ω_{S_i} is the angular velocity of spin of the ith race with respect to the ball and is

$$\omega_{S_i} = C_i \left[(\Omega_i - \Omega_E - \omega_X) \sin\beta_i - \omega_Z \cos\beta_i \right] \tag{9}$$

F_{X_i} and F_{Y_i} are friction forces directed along the major and minor axes of the pressure ellipse. Q_{S_i} is a friction moment about the normal at the center of the ellipse. These items are the result of sliding between the race and ball, which are separated by an EHD film. Their values and the values of their derivatives are calculated in a subroutine, BALFTJ.*

Input to subroutine BALFTJ[11] consists of:

**1. Pressure-viscosity coefficient of the lubricant - in.^{2}/lb
**2. Temperature-viscosity coefficient of the lubricant - /°F
3. Viscosity of the lubricant at inlet temperature - lb-sec/in.^{2}
**4. Thermal conductivity of the lubricant - Btu/°F-hr-ft
5. Inlet temperature - °F
6. Young's modulus for ball and for race - lb/in.^{2}
7. Poisson's ratio for ball and race
8. Dimensions a_i and b_i of the pressure area - in.
9. Load on the contact P_i - lb
10. Linear velocity of the race $V_{Y_{R_i}}$ - in./sec
11. Linear velocity of the ball $V_{Y_{B_i}}$ - in./sec
12. Velocity of slip of race on ball in the direction of the major axes $V_{X_{B_i}}$ - in./sec
13. Angular velocity of spin ω_{S_i} - rad/sec

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*Discussed under "Elastohydrodynamic Traction Coefficients"

**Values can be obtained from Appendix VII.

14. Ball diameter d - in.
15. Race groove radius $f_i d$ - in.
16. Pitch diameter E' - in.
17. Cosine of the contact angle $\cos \beta_i$
18. Value of C_i
19. Incrementing factor for $V_{Y_{Bi}}$
20. Incrementing factor for $V_{X_{Bi}}$
21. Incrementing factor for ω_{S_i}
22. Incrementing factor for P_i

BALFTJ returns F_{X_i}, F_{Y_i}, and Q_{S_i} and their derivatives with respect to V_{X_i}'s, V_{Y_i}'s, ω_{S_i}'s, and P_i's. It also returns minimum film thickness h_o.

P_P is the normal force of retainer on ball.

$$P_P = F_{Y_1} + F_{Y_2} - F_D \tag{10}$$

F_D is a viscous drag force.*

P_i is the contact force and is the result of the elastic approach Δ_i of ball and race body.

Figure 3 shows the initial and final positions of the inner and outer race curvature centers. β is the initial contact angle and f_i the race curvature factor.

From Figure 3,

$$\Delta_1 = \left[X_1^2 + X_2^2 \right]^{1/2} - (f_1 - .5)\, d \tag{11}$$

$$\Delta_2 = \left[(A_1 - X_1)^2 + (A_2 - X_2)^2 \right]^{1/2} - (f_2 - .5)\, d \tag{12}$$

A_1 and A_2 are defined analytically in Equations (31) and (32).

*Calculated as described under "Analysis of Shear Forces on a Lubricated, High-Speed Bearing Retainer,"

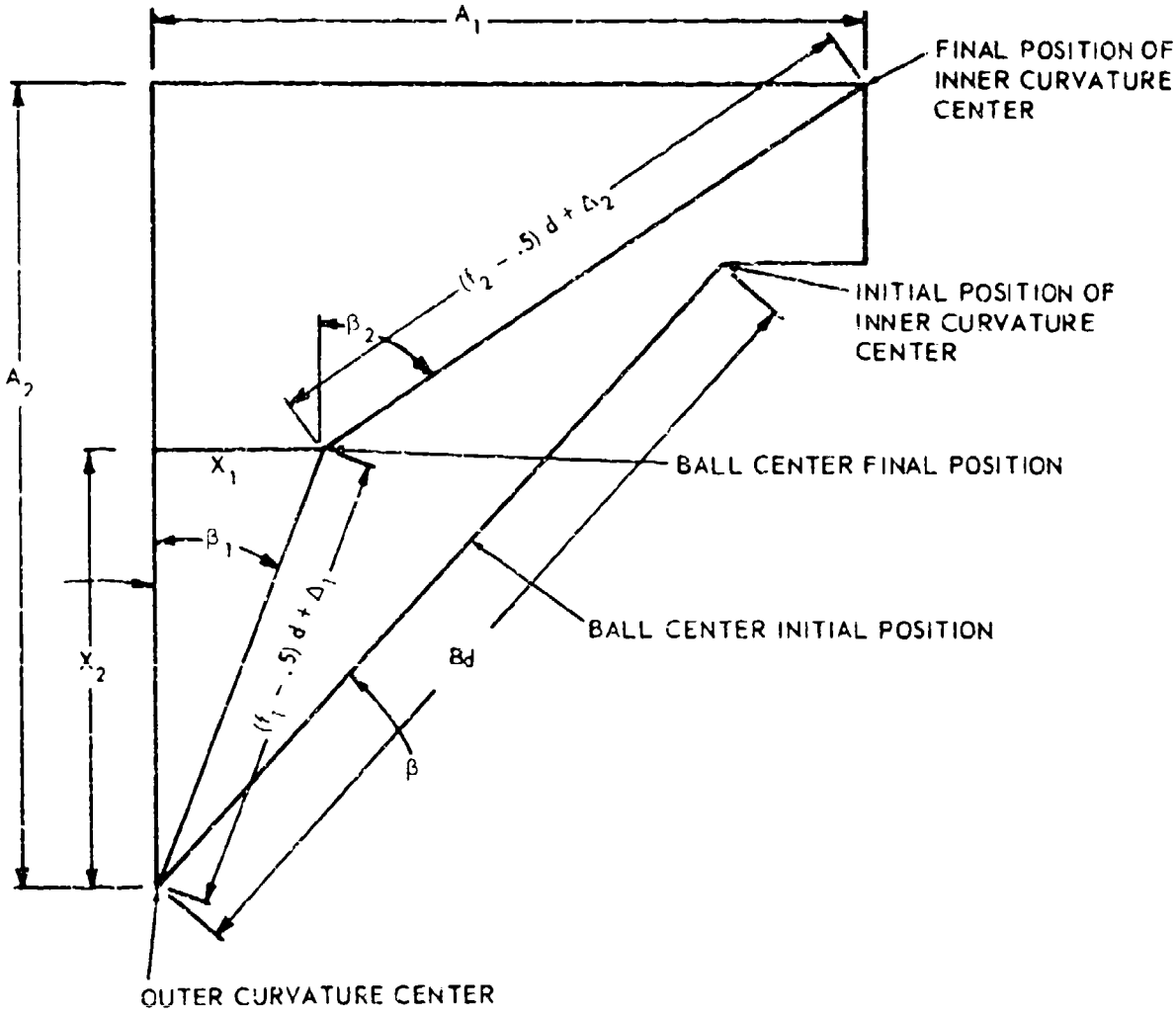

Figure 3. Initial and Final Positions of Inner and Outer Race Curvature Centers.

$$\tan \beta_1 = \frac{X_1}{X_2} \tag{13}$$

$$\tan \beta_2 = \frac{A_1 - X_1}{A_2 - X_2} \tag{14}$$

$$E' = E + 2 \left[X_2 - (f_1 - .5)\, d \cos \beta \right] \tag{15}$$

The dimensions of the pressure area are found as follows:

ϵ_i is the modular angle of the ellipse.

$$\epsilon_i = \sin^{-1} \left[1 - \left(\frac{b_i}{a_i} \right)^2 \right]^{1/2} \tag{16}$$

It is related to the geometries of the bodies in the neighborhood of the contact by an auxiliary parameter τ_i.

$$\cos \tau_i = \frac{\dfrac{1}{f_i} - \dfrac{2C_i \gamma_i}{1 + C_i \gamma_i}}{4 - \dfrac{1}{f_i} - \dfrac{2C_i \gamma_i}{1 + C_i \gamma_i}} \tag{17}$$

where

$$\gamma_i = \frac{d \cos \beta_i}{E'} \tag{18}$$

ϵ_i and τ_i are related by

$$\cot^2 \epsilon = \frac{(1 - \cos \tau_i)\, E(\epsilon_i)}{2 [K(\epsilon_i) - E(\epsilon_i)]} \tag{19}$$

$K(\epsilon_i)$ and $E(\epsilon_i)$ are the complete elliptic integrals of the first and second kind having the modular angle ϵ_i.

The semiminor axis of the pressure area is

$$b_i = \left[\frac{3(\vartheta_R + \vartheta_E) E(\epsilon_i) P_i d \cos \epsilon_i}{4\Pi \left(4 - \frac{1}{f_i} - \frac{2C_i \tau_i}{1 + C_i \tau_i} \right)} \right]^{1/3} \tag{20}$$

and the semimajor axis is

$$a_i = \frac{b_i}{\cos \epsilon_i} \tag{21}$$

ϑ_R and ϑ_E are elastic constants for race and ball, respectively.

$$\vartheta_R = \frac{4(1 - m_R^2)}{E_R} \tag{22}$$

$$\vartheta_E = \frac{4(1 - m_E^2)}{E_E} \tag{23}$$

The symbols m_R and m_E are Poisson's ratio for race and ball. E_R and E_E are Young's modulus for race and ball.

The relation between contact force and elastic deflection is nonlinear.

$$\Delta_i = \frac{3P_i(\vartheta_R + \vartheta_E) \quad K(\epsilon_i)}{8\Pi a} \tag{24}$$

The maximum normal pressure occurs at the center of the pressure area and is

$$\delta_{o_i} = \frac{3P_i}{2\Pi a_i b_i} \tag{25}$$

The equilibrium of the ball requires that

$$- P_1 \sin \beta_1 + P_2 \sin \beta_2 + F_{X_1} \cos \beta_1 - F_{X_2} \cos \beta_2 - F_{P_X} = 0 \tag{26}$$

$$- P_1 \cos \beta_1 + P_2 \cos \beta_2 - F_{X_1} \sin \beta_1 + F_{X_2} \sin \beta_2 + F_C + F_{P_Z} = 0 \tag{27}$$

$$(F_{Y_1} \cos \beta_1 - F_{Y_2} \cos \beta_2 - F_{P_Z}) \frac{d}{2} + Q_{S_1} \sin \beta_1 - Q_{S_2} \sin \beta_2 = 0 \tag{28}$$

$$(F_{X_1} + F_{X_2}) \frac{d}{2} + M_Y - Q_P = 0 \tag{29}$$

$$(- F_{Y_1} \sin \beta_1 + F_{Y_2} \sin \beta_2 - F_{P_X}) \frac{d}{2} + Q_{S_1} \cos \beta_1 - Q_{S_2} \cos \beta_2 + M_Z = 0 \tag{30}$$

F_{P_X} and F_{P_Z} are pocket frictional forces.*

The variables in the above set of equations are X_1 and X_2 and the angular velocities ω_X, ω_Y, and ω_Z. The equations are solved numerically.

Figure 4 shows the bearing referred to an XYZ coordinate system. The outer ring is fixed while the inner ring can move. The inner ring can have three linear displacements, δX, δY, and δZ, relative to the outer ring and is constrained from rotations about Y and Z.

The YZ plane in Figure 4 contains the locus of the inner race curvature centers.

For arbitrary values of the three displacements, there will be the reactions F_X, F_Z, and M_Y, which act in the directions shown in Figure 4.

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*Calculated as described under "Analysis of Shear Forces on a Lubricated, High-Speed Bearing Retainer."

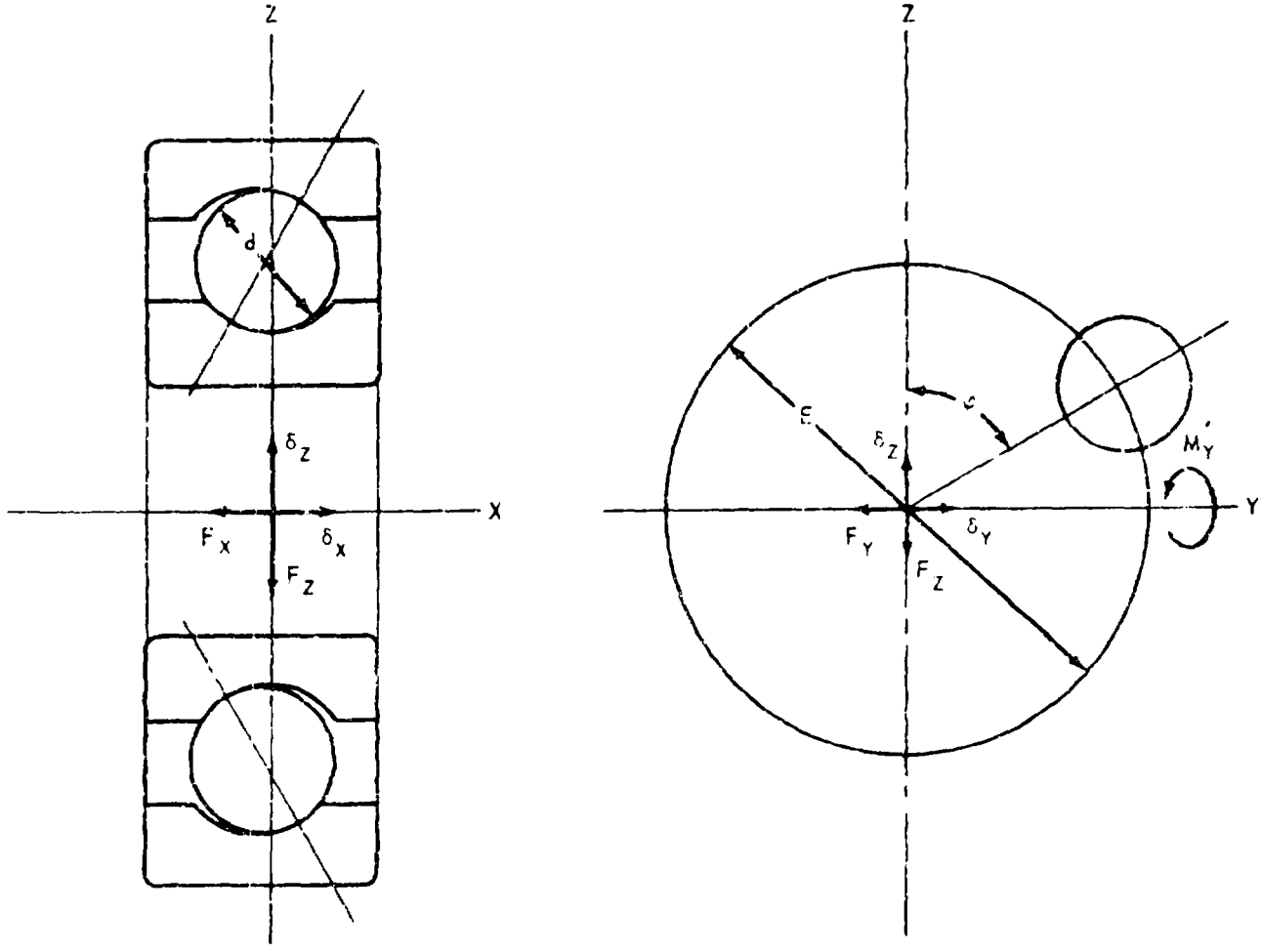

Figure 4. Bearing Referred to XYZ Coordinate System.

The values of A_1 and A_2 in Equation (14) depend on the displacements at the inner ring center.

$$A_1 = Bd \sin\beta + \delta_X \tag{31}$$

$$A_2 = Bd \cos\beta + \delta_Y \sin\varphi + \delta_Z \cos\varphi \tag{32}$$

$$B = f_1 + f_2 - 1 \tag{33}$$

φ is the ball azimuth measured clockwise from the YZ plane as viewed from +X.

The bearing reactions are:

$$F_X = \sum_{j=1}^{n} \left[P_{2_j} \sin\beta_{2_j} - F_{X_2} \cos\beta_2 \right] \tag{34}$$

$$F_Y = \sum_{j=1}^{n} \left[\left(P_{2_j} \cos\beta_{2_j} + F_{X_{2_j}} \sin\beta_{2_j} \right) \sin\varphi_j - F_{Y_{2_j}} \cos\varphi_j \right] + A_Y \tag{35}$$

$$F_Z = \sum_{j=1}^{n} \left[\left(P_{2_j} \cos\beta_{2_j} + F_{X_{2_j}} \sin\beta_{2_j} \right) \cos\varphi_j + F_{Y_{2_j}} \sin\varphi_j \right] + A_Z \tag{36}$$

$$M_Y' = \sum_{j=1}^{n} \left\{ \left[P_{2_j} R \sin\beta_{2_j} - F_{X_{2_j}} \left(R \cos\beta_{2_j} - f_{2_j} d \right) \right] \cos\varphi_j + F_{Y_{2_j}} f_2 d \sin\beta_{2_j} \sin\varphi_j \right\} \tag{37}$$

where n is the number of balls.

A_Y and A_Z are zero for an outer-piloted cage and are nonzero for inner-piloted cage; they are calculated as follows:

$$R = \frac{E}{2} + (f_2 - .5) d \cos\beta \tag{38}$$

M'_Y does not enter into the calculations but is an item of interest.

Figure 5 shows the forces and moments acting on an outer-piloted cage. Q_{V1} and Q_{V2} are viscous shear moments.* P_C is the reaction of the piloting ring against the cage. F_F is a Coulomb friction force between cage and guiding surface and acts at the angle θ.

$$P_c = \left[H^2 + V^2\right]^{1/2} \tag{39}$$

$$H = \sum_{j=1}^{n} \left[- P_{P_j} \cos \varphi_j - F_{P_{Z_j}} \sin \varphi_j \right] \tag{40}$$

$$V = \sum_{j=1}^{n} \left[-P_{P_j} \sin \varphi_j - F_{P_{Z_j}} \cos \varphi_j \right] \tag{41}$$

$$F_F = \mu P_c \tag{42}$$

$$\tan \theta = \frac{H}{V} \tag{43}$$

Figure 6 similarly illustrates an inner-piloted cage. Equations (37) through (41) apply here also.

With an inner-piloted cage, P_C and F_F are reactive on the inner ring, and A_Y and A_Z for use with Equations (35) and (36) are

$$A_Y = - P_c (\sin \theta - \mu \cos \theta) \tag{44}$$

$$A_Z = - P_c (\cos \theta + \mu \sin \theta) \tag{45}$$

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*Calculated as described under "Analysis of Shear Forces on a Lubricated, High-Speed Bearing Retainer."

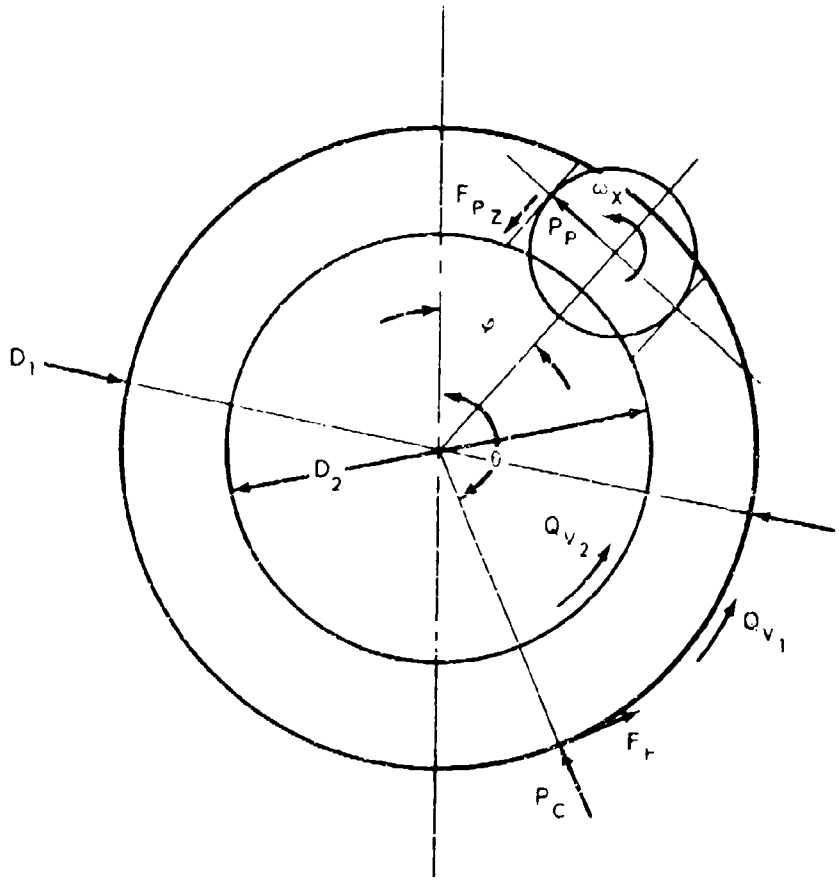


Figure 5. Forces and Moments Acting on Outer-Piloted Cage.

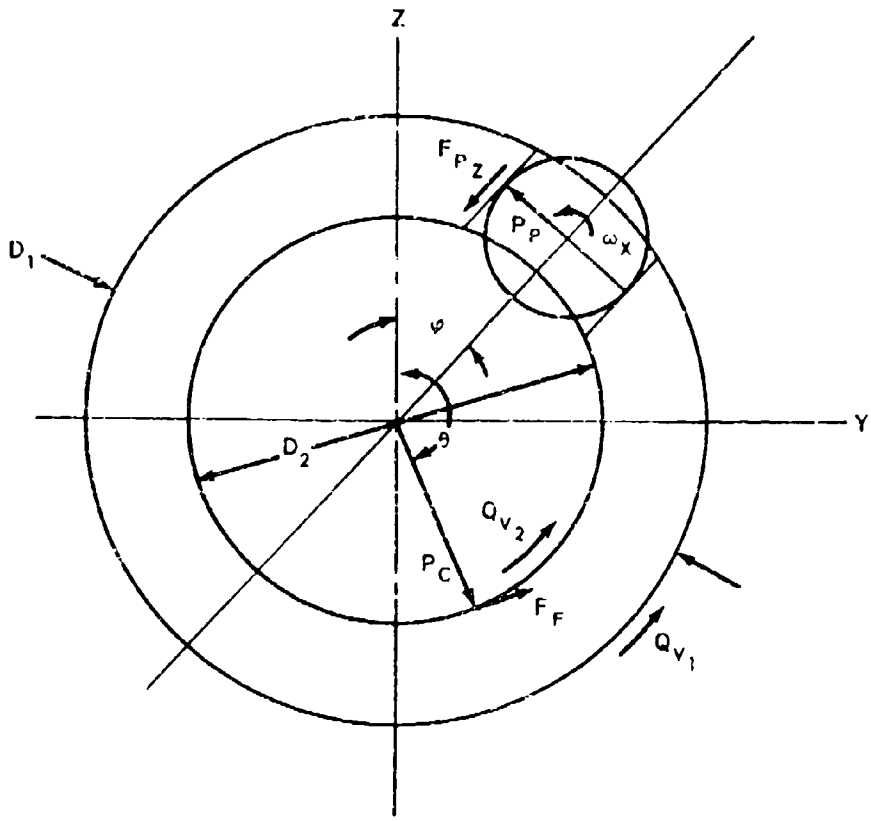


Figure 6. Forces and Moments Acting on Inner-Piloted Cage.

The requirements for equilibrium of the bearing and cage system may now be written:

$$F_X + \bar{F}_X = 0 \tag{46}$$

$$F_Y = 0 \tag{47}$$

$$F_Z + \bar{F}_Z = 0 \tag{48}$$

$$Q_{V_1} + Q_{V_2} + \frac{F_F D'}{2} + \frac{1}{2} \sum_{j=1}^{n} \left[P_{P_j} E' + F_{P_{Z_j}} d \right] = 0 \tag{49}$$

D' is D_1 for an outer-piloted cage and D_2 for an inner-piloted cage. $\bar{F}_X$ and $\bar{F}_Z$ are external thrust and radial loads.

Equations (46) through (49) are nonlinear in the variables δ_X, δ_Y, δ_Z, and Ω_E. They are solved numerically by iterative procedures.

ROLLER BEARING MATHEMATICAL MODEL

The determination of the forces on the cage of a high-speed roller bearing requires the evaluation of the internal load distribution, the friction forces in the EHD films at the race contacts, and the friction and normal forces in the roller to cage pocket contacts.

The calculation of individual roller loads is made without consideration of friction forces in the race contact film or in the pocket contact film. This procedure is valid since the tangential friction force of the roller against the pocket is small compared to the load at an outer-race contact.

For an assumed value of roller orbital speed (cage speed) the internal load distribution is obtained as follows. A first assumption of orbital speed is made close to the theoretical value for no slip.

$$\Omega_E = \frac{1}{2} \left[\Omega_1 \left(1 + \frac{d}{E}\right) + \Omega_2 \left(1 - \frac{d}{E}\right)\right] (1-S) \qquad (50)$$

where Ω_E = cage speed

Ω_1 = outer ring speed

Ω_2 = inner ring speed

d = roller diameter

E = pitch diameter

S = a slip factor

A suggested value of S for the first pass is 0.001.

Figure 7 illustrates a typical radial roller bearing and shows the detail of the roller's profile.

Let Δ_o be the approach of the roller to the race body measured at the midpoint of the roller's length. The approach is the same at all points of the cylindrical surface. The approach at a point on the crowned portion is less. At any location x along the roller's length, measured from the midpoint, the approach is

$$\Delta_x = \Delta_o \qquad -\ell_F \leq x \leq \ell_F \qquad (51)$$

$$\Delta_x = \Delta_o - \sqrt{R_c^2 - \left(\frac{\ell_F}{2}\right)^2} + \sqrt{R_c^2 - x^2} \qquad \frac{\ell_F}{2} < |x| < \frac{\ell_T}{2} \qquad (52)$$

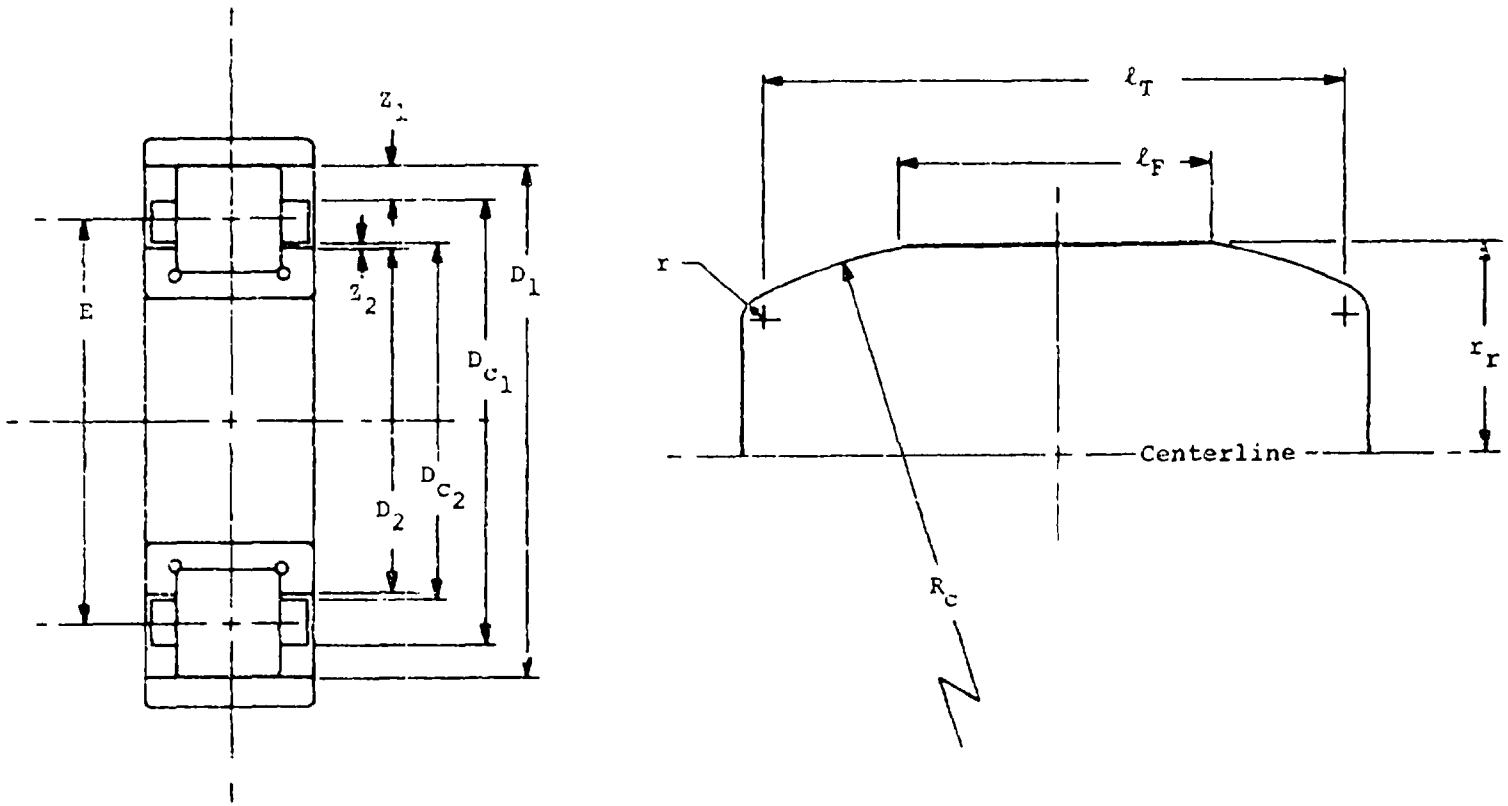


Figure 7. Bearing Details.

Lundberg, Reference 7, gives the relation between load and approach for a cylindrical roller of length ℓ as

$$\Delta = \frac{(\vartheta_R + \vartheta_E)\, P}{2 \pi \ell} \left[1.8864 + \ln \left(\frac{\ell}{2b}\right)\right] \tag{53}$$

where $\left.\begin{matrix}\vartheta_R \\ \vartheta_E\end{matrix}\right\}$ = elastic constants for race and roller, respectively, and are of the form

$$\vartheta = \frac{4\,(1-m^2)}{E} \tag{54}$$

m	=	Poisson's ratio
E	=	modulus of elasticity
P	=	total force on the contact
b	=	semiwidth of the contact area
ℓ	=	length of roller

Setting p_x as the lb/in. loading at point x, one may write

$$\Delta_x = \frac{(\vartheta_R + \vartheta_E)}{2 \pi} p_x \left[1.8864 + \ln \left(\frac{\ell'}{2b_x}\right)\right] \tag{55}$$

b_x is given by

$$b_x = \left[\frac{(\vartheta_R + \vartheta_E)\, p_{xd}\, \left(1 \pm \frac{d}{E}\right)}{2 \pi}\right]^{1/2} \tag{56}$$

The upper sign is used for an outer-race contact and the lower for an inner-race contact.

ℓ' is the distance from the roller midpoint to the end of the contact area.

$$\ell' = \sqrt{R_c^2 - \left[\sqrt{R_c^2 - \left(\frac{\ell_F}{2}\right)^2} - \Delta_o\right]^2} \tag{57}$$

If ℓ' from Equation (57) is greater than $\frac{\ell_T}{2}$, ℓ' is set equal to $\frac{\ell_T}{2}$.

The contact force resulting from an approach Δ_o is

$$P = 2 \int_0^{\ell'} p_x dx \tag{58}$$

Equation (58) is evaluated numerically by Simpson's rule. In the process of evaluating the integral it is necessary to assume discrete values of x and to calculate the value of p_x. From Equations (55) and (56) with Δ_x known,

$$\frac{d\Delta_x}{dp_x} = \frac{(\vartheta_R + \vartheta_E)}{2\Pi} \left[1.3864 + \ln\left(\frac{r'}{2b_x}\right)\right] \tag{59}$$

An assumption of p_x is made and the corresponding Δ_x calculated from Equation (55). An improved value of p_x is

$$p_x' = p_x - \frac{(\Delta_x' - \Delta_x)}{\left(\frac{d\Delta_x}{dp_x}\right)} \tag{60}$$

The process can be repeated until p_x' is to the desired accuracy.

Figure 8 shows a roller in contact with both races and acted upon by centrifugal force F_c and contact loads P_1 and P_2.

Assuming that the shear forces due to the pocket contact are small with respect to P_1,

$$- P_{1_q} + P_{2_q} + F_c = 0 \tag{61}$$

$$F_c = m_R \frac{E}{2} \Omega_E^2 \tag{62}$$

where m_R = mass of roller

Ω_E = orbital velocity of roller

$$\Delta_{o_1} + \Delta_{o_2} = \Delta_q \tag{63}$$

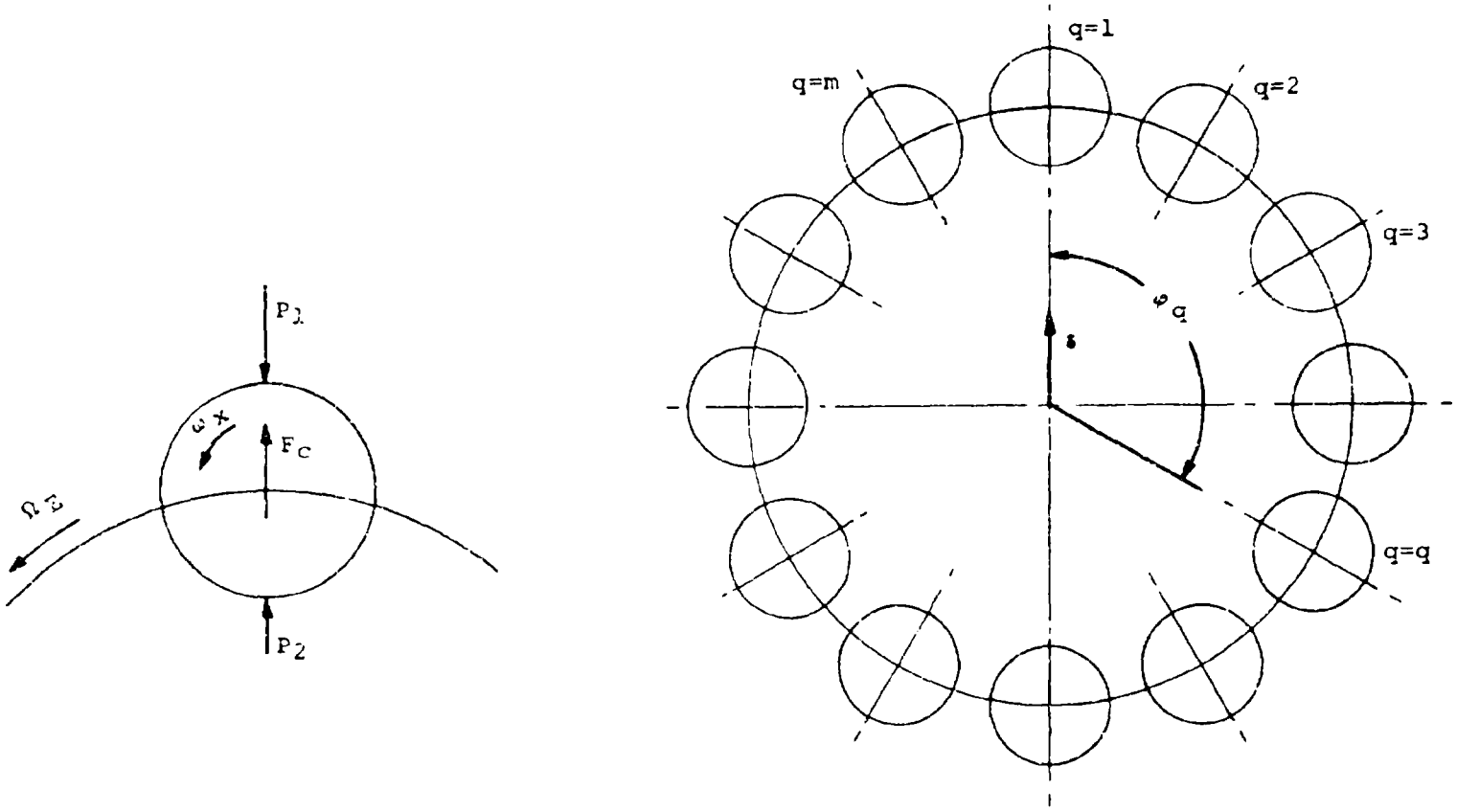


Figure 8. Roller Contact Forces and Roller Orientation Schematic.

Figure 8 also shows the numbering scheme for the rollers in relation to the displacement δ of the inner race with respect to the outer. The azimuth of the qth roller is

$$\varphi_q = \frac{2\Pi(q-1)}{n} \tag{64}$$

The total approach of inner race to outer at the qth roller is

$$\Delta_q = \delta \cos \varphi_q - \frac{P_D}{2} \tag{65}$$

where P_D = operating diametral clearance of bearing

Δ_{o_1} = elastic approach of roller and race at outer contact

Δ_{o_2} = elastic approach of roller and race at inner contact

For an assumed value of bearing displacement δ, Δ_q can be calculated. An estimate of Δ_{o_1} is made and Δ_{o_2} is given by Equation (63). P_{1_q} and P_{2_q} can be evaluated and substituted in Equation (61). In general, Equation (61) will not be satisfied and the residue ψ exists.

$$\Psi = -P_{1_q} + P_{2_q} + F_c \tag{66}$$

An improved value of Δ_{o_1} is

$$\Delta'_{o_1} = \Delta_{o_1} - \frac{\Psi}{\left(\frac{d\Psi}{d\Delta_{o_1}}\right)} \tag{67}$$

$$\frac{d\Psi}{d\Delta_{o_1}} = -\frac{dP_{1_q}}{d\Delta_{o_1}} - \frac{dP_{2_q}}{d\Delta_{o_2}} \tag{68}$$

The derivative of P with respect to the corresponding Δ_o is of the form

$$\frac{dP}{d\Delta_o} = 2 \int_o^{\ell} \frac{d_x}{\left(\frac{d\Delta_x}{dp_x}\right)} \tag{69}$$

Repetition of the process leading to Equation (67) enables Δ_{o_1} to be determined to any desired accuracy.

The reaction of the bearing to the displacement δ is

$$F = \sum_{q=1}^{n} P_{2_q} \cos \varphi_q \tag{70}$$

Equilibrium requires that

$$F - \bar{F} = 0 \tag{71}$$

where $\bar{F}$ = external radial load

For an assumption of δ, Equation (71) may not be satisfied. An improved value of δ is

$$\delta' = \delta - \frac{(F - \bar{F})}{\left(\frac{dF}{d\delta}\right)} \tag{72}$$

$$\frac{dF}{d\delta} = \sum_{q=1}^{n} \frac{dP_{2_q}}{d\Delta_{o_{2_q}}} \frac{d\Delta_{o_{2_q}}}{d\Delta_q} \cos^2 \varphi_q \tag{73}$$

Differentiating Equations (61) and (63) with respect to δ_q,

$$-\frac{dP_1}{d\Delta_{o_{1_q}}} \cdot \frac{d\Delta_{o_{1_q}}}{d\Delta_q} + \frac{dP_{2_q}}{d\Delta_{o_{2_q}}} \cdot \frac{d\Delta_{o_{2_q}}}{d\Delta_q} = 0 \tag{74}$$

$$\frac{d\Delta_{o_{1_q}}}{d\Delta_q} + \frac{d\Delta_{o_{2_q}}}{d\Delta_q} = 1 \tag{75}$$

From these is obtained

$$\frac{d\Delta_{o_{2_q}}}{d\Delta_q} = \frac{\dfrac{-dP_{1_q}}{d\Delta_{o_{2_q}}}}{-\dfrac{dP_{1_q}}{d\Delta_{o_{2_q}}} - \dfrac{dP_{2_q}}{d\Delta_{o_{2_q}}}} \tag{76}$$

Figure 9 shows the forces acting on the roller which arise from the lubricant effects. These are assumed small in comparison with P_1.

ω_1 and ω_2 are the velocities of the outer and inner races with respect to the cage:

$$\omega_i = \Omega_i - \Omega_E \qquad i = 1, 2 \tag{77}$$

where Ω_i = angular velocity of a ring relative to ground

Ω_E = angular velocity of the cage relative to ground

The linear velocity of a race relative to the cage is

$$U_{y_i} = \frac{(E + C_i d)}{2}\,\omega_i \qquad i = 1, 2 \tag{78}$$

where C_1 = + 1 for an outer contact

C_2 = − 1 for an inner contact

The evaluation of the integral in Equation (58) by Simpson's rule requires the establishment of an odd number of stations over the contact length. The stress conditions and the sliding velocities of race on roll at these stations will be utilized in calculating the friction forces in the EHD film.

The maximum Hertz pressure at any location x within the pressure area is

$$S_{m_x} = \frac{2\,p_x}{\Pi\,b} \tag{79}$$

For stations within the cylindrical portion of the roll, the radius of the roll is

$$r_r = \frac{d}{2} \tag{80}$$

For stations within the crowned portion of the roll,

$$r_{rx} = \frac{d}{2} - \sqrt{R^2 - \left(\frac{\ell F}{2}\right)^2} + \sqrt{R^2 - x^2} \tag{81}$$

The linear velocity of the roll at any station is

$$U_{R_{x_i}} = C_i\, r_x\, \omega_x \qquad i = 1, 2 \tag{82}$$

where ω_x is the angular velocity of roller about its center

The slip velocity of race on roll is

$$U_{S_{x_i}} = U_{y_i} - U_{R_{x_i}} \qquad i = 1, 2 \tag{83}$$

The minimum film thickness at any station is*

$$h_{x_i} = \frac{1.6 \quad R'_{x_i}{}^{.43} \quad \alpha^{.6} \quad \left(\mu_o \, |\bar{U}_{x_i}|\right)^{.7}}{P_{x_i}{}^{.13} \, (E'')^{.03}} \qquad i = 1, 2 \qquad (84)$$

where R_{x_i} = the radius of a roll having the same conformity with regard to a flat plate that the actual contact has

$$R'_{x_i} = \frac{1}{-\dfrac{2C_i}{(E + C_i d)} + \dfrac{1}{r_{x_i}}} \qquad i = 1, 2 \qquad (85)$$

α = the pressure-viscosity coefficent of the lubricant

μ_o = the lubricant viscosity at inlet temperature

$\bar{U}_{x_i}$ = the average rolling velocity at the station

$$\bar{U}_{x_i} = \frac{1}{2} \left(U_{y_i} + U_{R_{x_i}}\right) \qquad (86)$$

$$E'' = \frac{8}{\vartheta_R + \vartheta_E} \qquad (87)$$

The friction or tractive force generated in the EHD film depends on the coefficient of friction at the various stations along the contact length.

*Calculated as discussed under "Elastohydrodynamic Traction Coefficients"

A computer program provided by the Air Force Aero Propulsion Laboratory and Mechanical Technology, Inc. calculates the friction coefficient as a function of three parameters: G_1, G_2 and G_3.

$$G_{1_{x_i}} = \frac{G_A \left| U_{S_{x_i}} \right|}{S_{m_{x_i}} \; h_{x_i}} \tag{88}$$

$$G_{2_{x_i}} = G_B \, U_{S^2_{x_i}} \tag{89}$$

$$G_{3_{x_i}} = \alpha_3 \; S_{m_{x_i}} \tag{90}$$

where

$$G_A = \mu_3 \left(\frac{\alpha}{\alpha_3}\right)^{.6} \left(\frac{\mu_o}{\mu_3}\right)^{.7} \tag{91}$$

$$G_B = \frac{\mu_3 \beta}{1.728 \; K_F} \tag{92}$$

$$\alpha_3 = \frac{o + 930 \, \alpha \, (T_1 - 86)}{546 \, (T_1 + 460)} \tag{93}$$

$$\mu_3 = \mu_o \, e^{\beta \, (T_1 - 86)} \tag{94}$$

The tractive force is obtained by numerical integration of the following:

$$F_{T_i} = 2 \int_o^{\ell} f_{x_i} \; p_{x_i} \; dx \tag{95}$$

The derivative of F_{T_i} with respect to ω_x will be required. These are obtained by calculating F_{T_i} values for $\omega_x + \Delta\omega_x$ and $\omega_x - \Delta\omega_x$ and substituting them in:

$$\frac{dF_{T_i}}{d\omega_x} = \frac{(F_{T_i})_{\omega_x + \Delta\omega_x} - (F_{T_i})_{\omega_x - \Delta\omega_x}}{2\,(\Delta\omega_x)} \tag{96}$$

Turning now to the pocket contacts, the following expressions for F_p' and F_v' are obtained* (Figure 9 also shows a roller in a pocket and those forces acting on the roller from the cage):

$$F'_{p_x} = \frac{2.447\ \mu_o\ |\omega_x|\ \left(\frac{d_x}{2}\right)^2}{h_{p_x}} \tag{97}$$

$$F'_{v_x} = \frac{.5482\ \mu_o\ \omega_x\ \left(\frac{d_x}{2}\right)^{3/2}}{h_{p_x}^{1/2}} \tag{98}$$

Along the cylindrical portion, the film thickness is h_{p_o}. When $x > \frac{\ell_F}{2}$ the film thickness is

$$h_{p_x} = h_{p_o} + \sqrt{R_c^2 - \left(\frac{\ell_F}{2}\right)^2} - \sqrt{R_c^2 - x^2} \tag{99}$$

The pocket forces are obtained by numerical integration of

$$F_p = 2 \int_0^{\ell_T} F'_{p_x}\ dx \tag{100}$$

*Calculated as described under "Analysis of Shear Forces on a Lubricated, High-Speed Bearing Retainer."

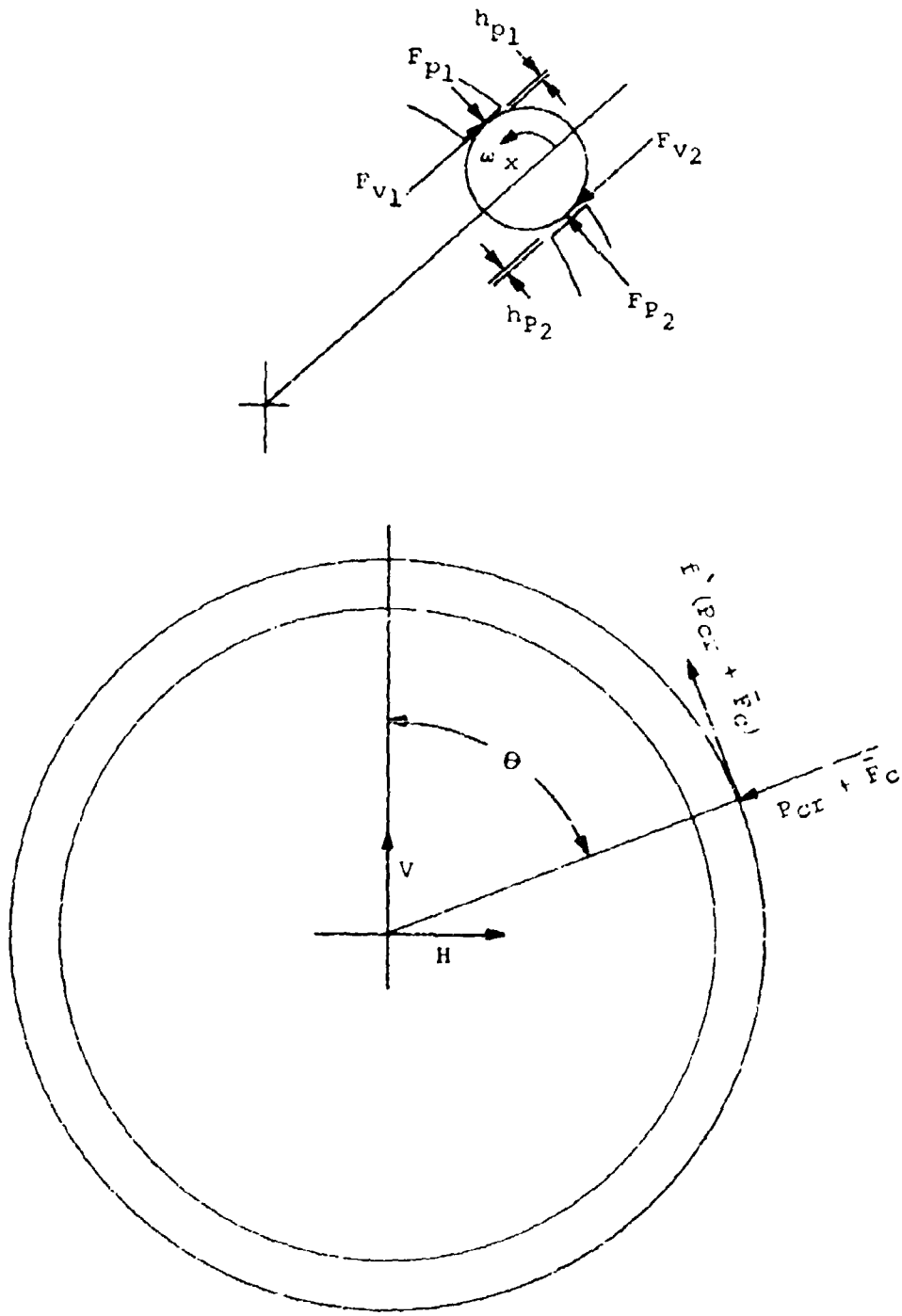


Figure 9. Roller Pocket Forces and Cage Whirl Forces.

and

$$F_v = 2 \int_0^{\ell_T} F'_{v_x} \, dx \tag{101}$$

The derivatives of F_p and F_v with respect to ω_x and h_o are required.

$$\frac{dF_p}{d\omega_x} = \frac{F_p}{|\omega_x|} \tag{102}$$

$$\frac{dF_v}{d\omega_x} = \frac{F_v}{\omega_x} \tag{103}$$

Differentiating Equations (97) and (98) gives

$$\frac{dF'_{p_x}}{dh_{p_x}} = \frac{-2.447 \, \mu \, |\omega_x| \left(\frac{d_x}{2}\right)^2}{h_{p_x}^2} \tag{104}$$

$$\frac{dF'_{v_x}}{dh_{p_x}} = \frac{-.2741 \, \mu \, \omega_x \left(\frac{d_x}{2}\right)^{3/2}}{h_{p_x}^{3/2}} \tag{105}$$

Then the required derivatives are

$$\frac{dF_p}{dh_{p_o}} = 2 \int_0^{\ell_T} \frac{dF'_{p_x}}{dh_{p_o}} \, dx \tag{106}$$

$$\frac{dF_v}{dh_{p_o}} = 2 \int_0^{\ell_T} \frac{dF'_{v_x}}{dh_{p_o}} \, dx \tag{107}$$

The viscous drag on a roller orbiting at high speed is*

$$F_D = \frac{\rho \ell_T}{64} \sum_{i=1}^{2} Z_i \left[|D_i - D_{c_i}| \; \Omega_E \right]^2 \tag{108}$$

where ρ = lubricant density

Z_1 = radial riding clearance at outer pilot surface

Z_2 = radial riding clearance at inner pilot surface

D_1 = bore of outer ring

D_{c_1} = outside diameter of cage

D_2 = outside diameter of inner ring

D_{c_2} = bore of retainer

If $Z_1 > Z_2$ the cage is inner-ring piloted. If $Z_1 < Z_2$ the cage is outer-ring piloted.

From Figure 9, assuming the forces F_{v_1} or F_{v_2} are small in comparison with P_1, let

$$\Phi = F_{T_1} + F_{T_2} - F_D \tag{109}$$

*Calculated as described under "Analysis of Shear Forces on a Lubricated, High-Speed Bearing Retainer."

If $\Phi > 0$, the roller contacts the leading wall of the pocket and drives the cage, and the value of an index, I, is 1.

If $\Phi < 0$, the roller contacts the trailing wall of the pocket and is driven by the cage. The value of the index is 2.

F_p and F_v need be calculated only for the end of the pocket corresponding to I. The forces at the other end of the pocket are negligible due to the pocket clearance, which is large compared to the lubricant film at the I end.

Two modes of operation are recognized. In the first mode the roller is in contact with both outer and inner races. In the second the roller contacts the outer race only. In neither mode is the angular acceleration of the roller in orbit or about its own axis considered. In other words the transition from motion with contact at both races to steady-state motion with outer-race contact only is assumed to be instantaneous.

When the roller is in contact with both races,

$$F_{p_I} - \left| F_{T_1} + F_{T_2} - F_D \right| = 0 \tag{110}$$

Equation (110) is calculated using an assumed value of ω_x. For the first pass, ω_x is obtained from

$$\omega_x = \frac{(\Omega_I - \Omega_E)\ (E + d)}{d} \tag{111}$$

Equation (110) is also evaluated using an assumed value of $h_{p_{o_I}}$, and

Equation (110) will probably have the residue Ψ.

An improved value of $h_{p_{o_I}}$ is

$$h'_{p_{o_I}} = h_{p_{o_I}} - \frac{\Psi}{\left(\frac{d\Psi}{dh_{p_{o_I}}}\right)} \tag{112}$$

where

$$\frac{d\Psi}{dh_{p_{o_I}}} = \frac{dF_{p_I}}{dh_{p_{o_I}}} \tag{113}$$

Repetition of the process yields $h_{p_{o_I}}$ to desired accuracy.

Taking moments about the roll center,

$$(F_{T_1} - F_{T_2} - F_{v_I})\frac{d}{2} = 0 \tag{114}$$

For the initial assumption of ω_x, Equation (114) will not be satisfied and there will be the residue Ψ. An improved value of ω_x is

$$\omega_x' = \omega_x - \frac{d\Psi}{\left(\frac{d\Psi}{d\omega_x}\right)} \tag{115}$$

$$\frac{d\Psi}{d\omega_x} = \frac{dF_{T_1}}{d\omega_x} - \frac{dF_{T_2}}{d\omega_x} - \frac{dF_{v_I}}{d\omega_x} - \frac{dF_{v_I}}{dh_{p_{o_I}}} \cdot \frac{dh_{p_{o_I}}}{d\omega_x} \tag{116}$$

where

$$\frac{dh_{p_{o_I}}}{d\omega_x} = -\frac{\left(\frac{dF_{p_I}}{d\omega_x}\right)}{\left(\frac{dF_{p_I}}{dh_{p_{o_I}}}\right)} \tag{117}$$

Repeated application of Equation (115) yields ω_x to desired precision. After each pass the process returns to Equation (82).

The solution for the case of outer contact only is solved in a similar manner except that F_{y_2} and its derivatives are nonexistent.

The viscous drag force between retainer and ring is, from Reference 3,

$$F_{R_i} = .008\, \rho^{3/4}\, \mu_o^{1/4} \sum_{i=1}^{2} \frac{|U_i|^{7/4}}{Z_i^{1/4}} \frac{A_i U_i}{|U_i|} \tag{118}$$

where ρ = lubricant density

μ = lubricant viscosity

U = sliding velocity of ring on cage

A = total guiding area

Z = 1, 2 cage ring diametral clearance

$$U_i = \frac{(\Omega_i - \Omega_E)\, D_i}{2} \qquad i = 1, 2 \tag{119}$$

The torque on the cage due to viscous drag is

$$Q_v = \frac{1}{2} \sum_{i=1}^{2} F_{R_i} D_i \tag{120}$$

The total horizontal and vertical loads on the cage due to pocket forces are

$$H = \sum_{q=1}^{n} \left[- C_{I_q} \left(F_{p_{I_q}} \cos \varphi_q + F_{v_{I_q}} \sin \varphi_q \right) \right] \tag{121}$$

$$V = \sum_{q=1}^{n} \left[C_{l_q} \left(F_{p_{r_q}} \sin \varphi_q - F_{v_{l_q}} \cos \varphi_q \right) \right] \tag{122}$$

Figure 9 shows the reactions of the piloting surface on the cage. The radial force consists of a component, P, and a centrifugal force due to cage whirl with a rotating load. From Figure 9,

$$V + f' (P + \bar{F}_c) \sin \Theta - (P + \bar{F}_c) \cos \Theta = 0 \tag{123}$$

$$H - f' (P + \bar{F}_c) \cos \Theta - (P + \bar{F}_c) \sin \Theta = 0 \tag{124}$$

$$f' = \frac{U_J}{|U_J|} f_r \tag{125}$$

where f_r = coefficient of sliding friction at the pilot interface

J = 1 for an outer-piloted retainer and 2 for an inner-piloted retainer

$$\bar{F}_c = \frac{m_c Z_J \omega_w^2}{2} \tag{126}$$

m_c = mass of cage

ω_w = speed of whirl

From Equations (120) and (121),

$$\tan \Theta = - \frac{(f' V - H)}{(f' H + V)} \tag{127}$$

$$P = \frac{H}{f \cos \Theta + \sin \Theta} - \bar{F}_c \tag{128}$$

The total torque on the cage may now be evaluated.

$$\bar{Q} = \frac{1}{2} \sum_{q=1}^{r} \left[C_{i_q} \left(F_{p_{I_q}} E + F_{v_{I_q}} D_p \right) \right] + \frac{f' \left(P + \bar{F}_c\right)}{2} D_{c_J} + Q_v \quad (129)$$

The cage speed is now decremented and the value of $\bar{Q}$ recalculated. This is equivalent to returning to Equation (77) with the new value of Ω_E.

The decrementing process is continued until $\bar{Q}$ changes sign. The method of false position is then applied to drive $|\bar{Q}|$ to a satisfactory minimum.

ELASTOHYDRODYNAMIC FILM THICKNESS CALCULATION

The minimum EHD film thickness is calculated within the computer program by a subroutine provided by the Air Force Aero Propulsion Laboratory and Mechanical Technology, Inc., in accordance with Reference 11. The fundamental relationship employed is the Dowson and Higginson[1] equation as indicated below for cylindrical contacts. Ball and roller race contacts are reduced to equivalent contacts in the analysis by dividing the contact zones into strips.

$$h_o = 1.6\,R \frac{\left[\alpha_o E''\right]^{.6} \left[\frac{u\,\mu_o}{E''R}\right]^{.7}}{\left[\frac{w}{E''R}\right]^{.13}} \tag{130}$$

where α_o pressure coefficient of viscosity $\left[\frac{in.}{lb}\right]^2$

$$E'' = \left[\frac{1-m_R^2}{2E_R} + \frac{1-m_E^2}{2E_E}\right] \left[\frac{lb}{in.}\right]^2$$

m Poisson's ratio (rolling element, race)

E Young's modulus (rolling element, race)

μ_o viscosity at ambient pressure and film inlet temperature $\left[\frac{lb\text{-}sec}{in.^2}\right]$

$u = \frac{1}{2}\left[V_R + V_E\right]$ velocity term $\left[\frac{in.}{sec}\right]$

$R = \left[\frac{1}{R_R} + \frac{1}{R_E}\right]^{-1}$ equivalent radius $\left[in.\right]$

w = load per unit width of contact $\left[\frac{lb}{in.}\right]$

h_o = minimum film thickness

The application of this equation to ball bearings requires the establishment of various kinematic and geometric parameters that will be developed here. The general case of ball motion occurring in a high-speed angular contact ball bearing is represented in Figure 10. The case depicted is with the ball fixed in the plane of the paper with relative outer and inner ring speeds of ω_1 and ω_2, respectively. As indicated, the ball is free to rotate about all three axes.

The operating contact angles at the inner and outer races are unequal owing to ball centrifugal forces and gyroscopic moments. As is evident, the relative motion between ball and race at each contact will be a combined rolling and spinning action. Proper calculation of the EHD film thickness in these contact areas must consider this combined motion. This was accomplished by dividing the contact area into a number of strips parallel to the y axis as shown in Figure 11. These strips are treated as individual cylindrical contacts rolling in the y direction.

Figure 11 represents the contact zone on the race, which has a velocity due to race rotation of

$$V_{Y_{R_i}} = \frac{\omega_i}{2} \left(E + C_i d \cos\beta_i \right) \tag{131}$$

where $i = 1$ designates outer contact

$i = 2$ designates inner contact

$$\left.\begin{aligned} C_1 &= +1 \\ C_2 &= -1 \end{aligned}\right\} \text{ sign terms}$$

and $$\omega_i = \Omega_i - \Omega_\ell$$

where Ω_1 = outer ring speed

Ω_2 = inner ring speed

Ω_ℓ = retainer speed

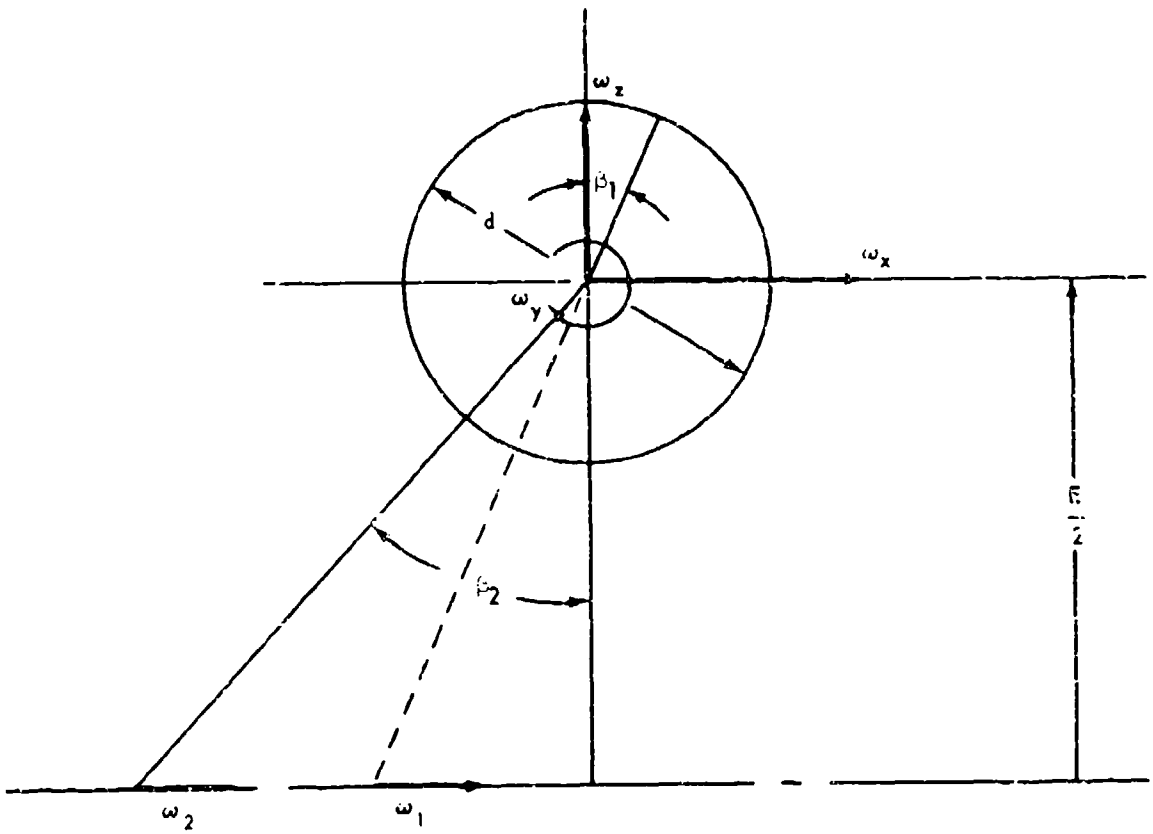


Figure 10. Ball Motion Occurring in High-Speed Angular-Contact Ball Bearing; General Case.

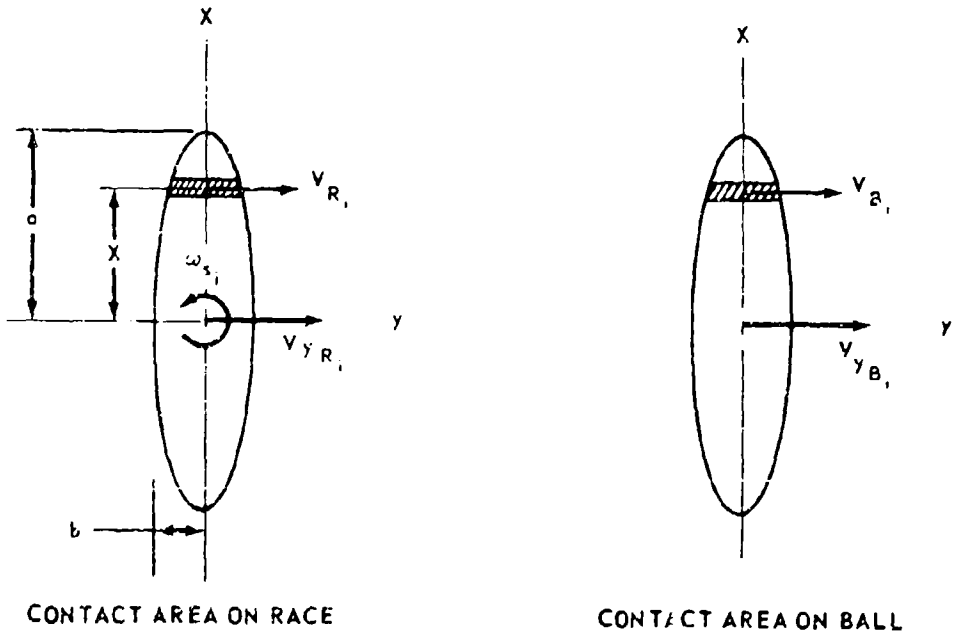


Figure 11. Contact Areas on Race and Ball.

and an angular spin velocity relative to the ball of

$$\omega_{s_i} = C_i \left[(\omega_i - \omega_x) \sin\beta_i - \omega_z \cos\beta_i \right] \tag{132}$$

Combining these two velocity components, we can now calculate the velocity in the y direction of any point on the race contact by

$$V_{R_i} = V_{Y_{R_i}} - X \omega_{s_i} \tag{133}$$

The velocities within the contact zone on the ball as shown in Figure 11 in the Y direction can be calculated from

$$V_{B_i} = V_{Y_{B_i}} = \frac{C_i d}{2} (\omega_x \cos\beta_i - \omega_z \sin\beta_i) \tag{134}$$

The race radii of curvature in the plane of rolling required to calculate the equivalent radius of contact against a flat surface can be stated as

$$R_{R_i} = -\frac{C_i}{2} \left(\frac{E}{\cos\beta_i} + C_i d \right) \tag{135}$$

and the ball radius of curvature

$$R_{B_i} = \frac{d}{2} \tag{136}$$

Finally, w, the load per unit width of contact, can be found from

$$w = \frac{3P}{4a} \left(1 - \frac{X^2}{a^2} \right) \tag{137}$$

The film thickness is calculated for each strip in the contact, $-a < x < a$, and the minimum value, all strips considered, is presented in the computer output as the minimum thickness for the contact.

The application of the Dowson and Higginson equation to cylindrical roller bearing contacts is treated in the same manner as in the above ball bearing method. The roller bearing case, however, is simpler because of the absence of spin velocities.

ELASTOHYDRODYNAMIC TRACTION COEFFICIENTS

Traction Model

The mechanics by which EHD oil films are generated in heavily loaded rolling element contacts such as those encountered in ball and roller bearings is well understood. The magnitude of the EHD film thicknesses can be accurately calculated using the Dowson and Higginson equation as indicated in the previous section. This is the case even though the analysis is predicated upon isothermal flow conditions, because the film thickness is determined primarily by the oil properties in the inlet region of the contact where the oil pressure is nearly ambient, and the oil temperature is approximately the same as that of the rolling surfaces.

Traction effects, however, present a more complicated picture to be analyzed in that the traction forces transmitted by the contacting surfaces are strongly influenced by the rheology of the fluid within the film. In the race contacts, high Hertz pressures are in effect accompanied by appreciable sliding with the attendant heating of the oil film. The rheological behavior of the lubricant film in the contact is strongly affected by these elevated pressures and temperatures. Currently, information regarding the shear properties of lubricant films under these high contact pressures and sliding velocities is incomplete, and sufficient understanding of the rheology of the lubricants in this regime is not available to produce analytical solutions.

The traction coefficients appearing within the computer programs were established with the use of computer program subroutines BALFTJ and TRAC provided by the Air Force Aero Propulsion Laboratory and Mechanical Technology, Inc. In these subroutines, sliding frictional coefficients were generated based upon the experimental data reported by Johnson and Cameron[6]. To apply this experimental data to a wide range of load, speed, and lubricant parameters, an analysis of friction similar to that developed by Crook[4] was conducted by Mechanical Technology, Inc., to identify the pertinent dimensionless parameters governing frictional behavior.

The results indicated that the friction coefficients are governed by three dimensionless parameters for a given inlet temperature and lubricant.

$$G_1 = \frac{\mu_o \; u_s}{P_{Hz} \; h_o}$$

$$G_2 = \frac{\beta_1^* \; \mu_o \; u_s}{8 \; K_f}$$

$$G_3 = \alpha_o \; P_{Hz}$$

where μ_o = inlet viscosity

u_s = sliding speed

P_{Hz} = maximum Hertz pressure

h_o = film thickness

β_1^* = temperature viscosity coefficient based on the function used by Crook

K_f = thermal conductivity of the lubricant

α_o = pressure viscosity coefficient based on the viscosity function used by Crook

Physically, G_1 is a direct measure of the shear rate effects, whereas G_2 and G_3 measure the thermal heating effects and the pressure-viscosity effects respectively. With the use of these three parameters, Johnson's experimental frictional coefficients were plotted against G_1 for a constant G_2 with G_3 as parameters. Typical graphs for a low and high value of G_2 are given in Figures 12 and 13. Portions of the curves are extrapolations of Johnson's data to cover broader ranges of speed, load, and lubricant properties.

It is seen that at small values of G_1, all curves have a constant slope. This corresponds to the fact that at small sliding speeds the frictional coefficient varies linearly with sliding speed. In each graph, all curves merge at large values of G_1 to a common value, which is the "ceiling" found by Johnson and Cameron and also by Plint.[8] The ceiling decreases with increasing G_2. The existence of a ceiling in friction suggests that there is a limiting shear stress in a lubricant film.

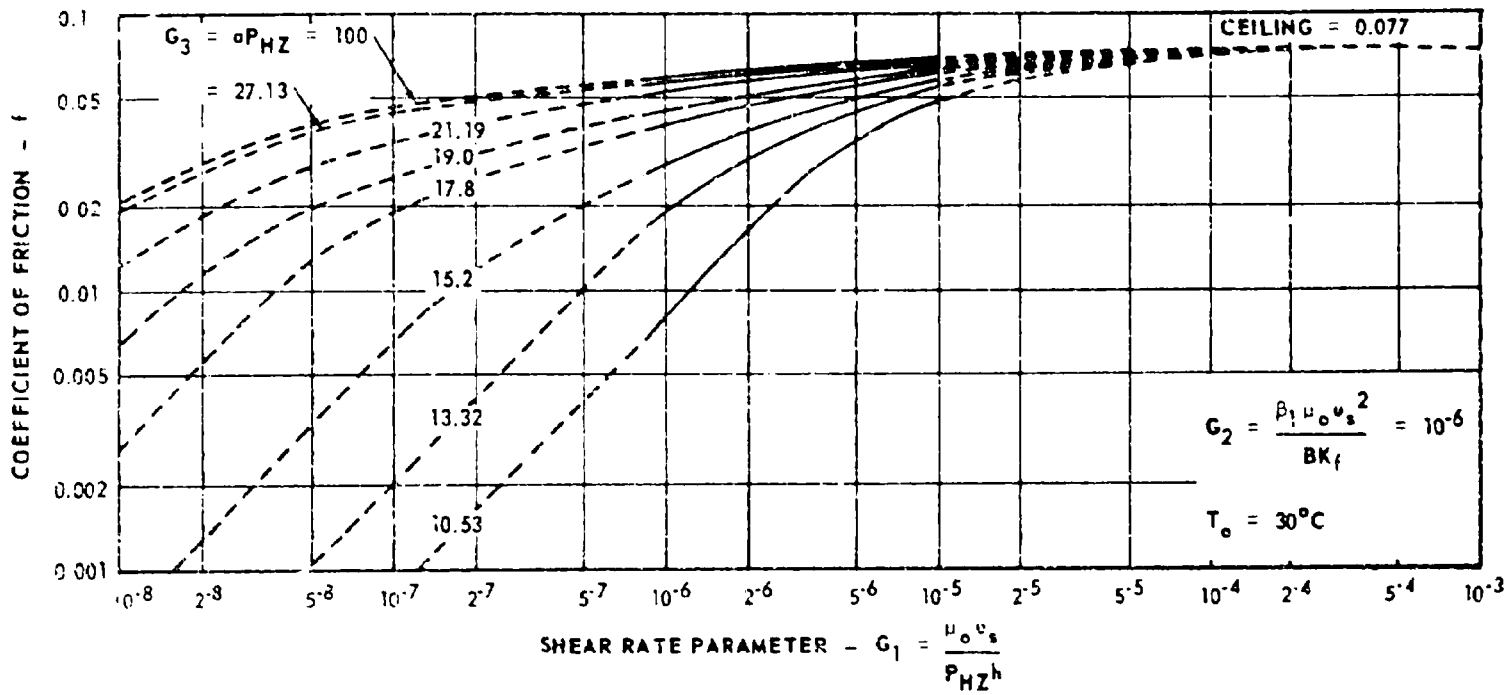


Figure 12. Coefficient of Friction Versus Shear Rate Parameter for Heating Parameter = 10^{-6}. Reference (11), Figure 93.

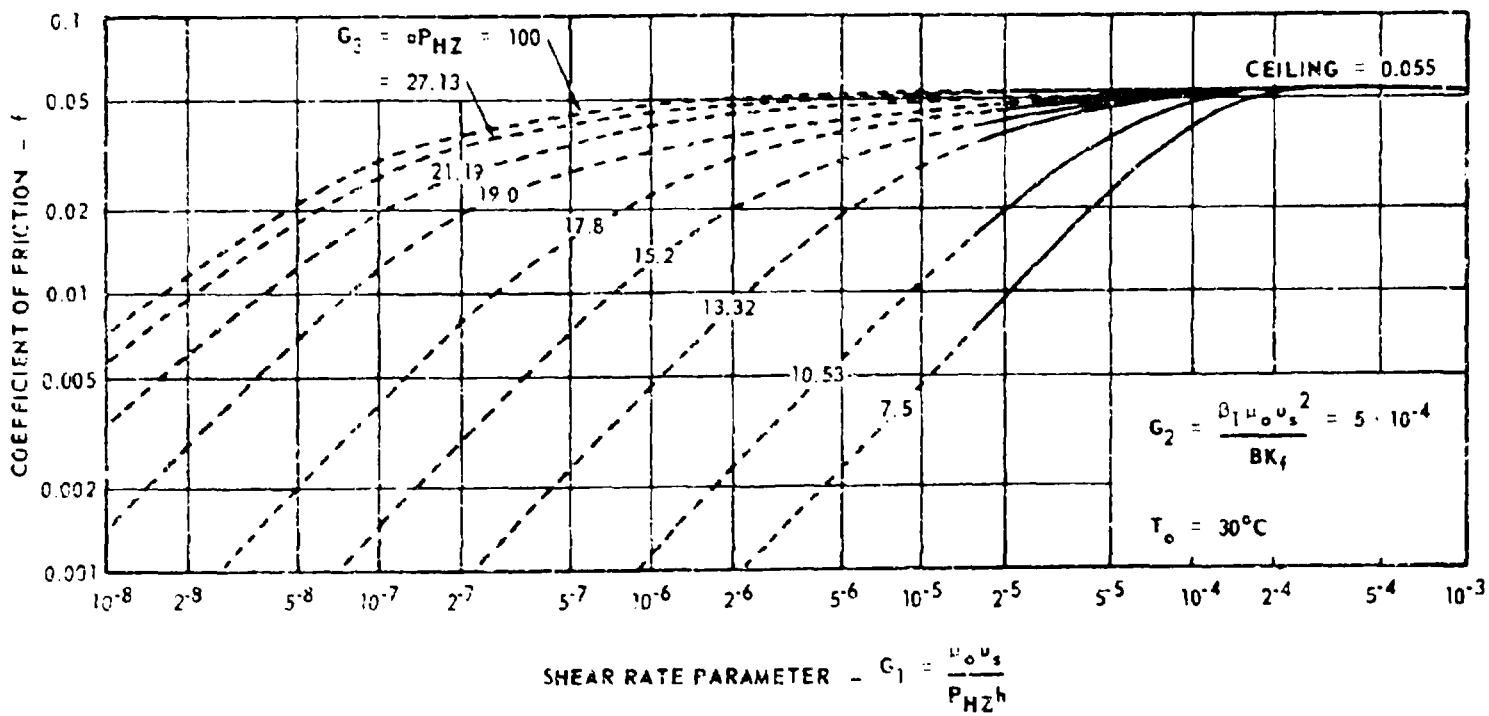


Figure 13. Coefficient of Friction Versus Shear Rate Parameter for Heating Parameter = 5×10^{-4}. Reference (11), Figure 97.

It should be noted that all the frictional coefficient graphs are for a given inlet lubricant temperature, 30° C. Change of inlet temperature affects the frictional coefficient due to the dependence of the limiting shear stress upon the inlet temperature. Johnson and Cameron and Plint investigated this inlet temperature effect. Pli 's experiment shows that the variation of frictional coefficient with inlet temperature is approximately linear, and the slope does not seem to vary much with either rolling or sliding speed. The slope was found to be -0.001 per degree Fahrenheit. This relationship is used to predict frictional coefficients for inlet temperatures other than 30° C.

Tractive Forces and Spin Torque

To facilitate the general kinematic solutions desired while accounting for gyroscopic slip and ball spin, it is necessary to calculate the tractive forces between the balls and races with the assumption of motion about all three ball axes. Figure 14 illustrates the velocity components active in the contact areas on the ball and race of a common contact. Each ellipse is divided into a number of strips parallel to the direction of rolling (Y axis). The dimensions a and b are the semimajor and the semiminor axes of the contacts, respectively. The sliding velocity in the Y direction at each strip can be found from Equations (133) and (134) and is

$$u_{s_y} = \left| V_{R_i} - V_{B_i} \right| \tag{138}$$

With this sliding velocity established and the Hertz stress, lubricant properties, and EHD film thickness known, the tractive coefficient f for the strip can be calculated with the procedure previously described. The tractive force acting on each strip in the y direction is

$$dF_y = 2\,\tau_y\,y^*\,dx \tag{139}$$

where y* is the semistrip width dimension as shown in Figure 14 and

$$\tau_y = \frac{fw}{2y^*}. \tag{140}$$

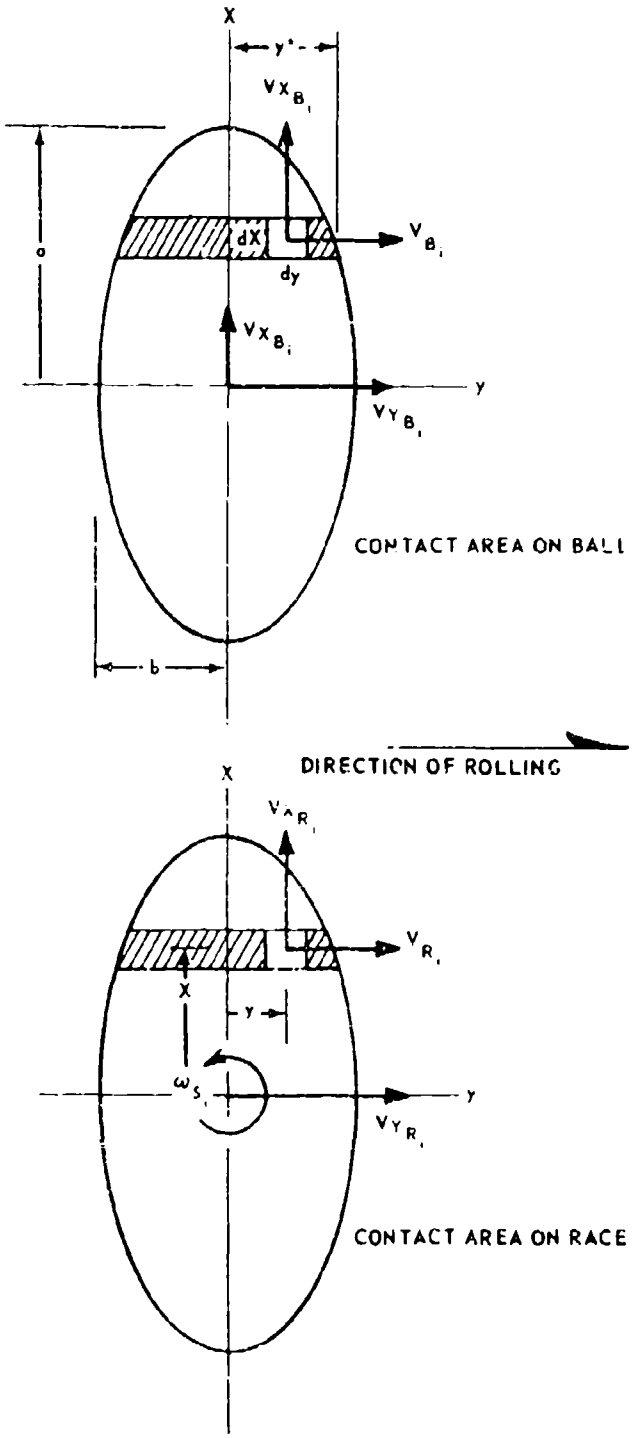


Figure 14. Velocity Components Active in Contact Areas.

The total tractive force for the contact at the y direction is the sum of the tractive forces within the individual strips, or

$$F_{y_i} = \int_{-a}^{a} 2\,\tau_y\, y^*\, dx = \int_{-a}^{a} f w\, dx \tag{141}$$

The shear stress in the x direction is calculated with the assumption that the ratio of the two mutually perpendicular shear stresses is proportional to their shear rates. The previously defined strip is then further divided into differential elements to evaluate the velocity variations along the Y axis as shown in Figure 14.

Therefore,

$$\frac{\tau_x}{\tau_y} = \frac{u_{s_x}}{u_{s_y}} \tag{142}$$

$$\tau_x = \left| \frac{V_{x_{R_i}} - V_{x_{B_i}}}{V_{R_i} - V_{B_i}} \right| \tau_y \tag{143}$$

where

$$V_{x_{R_i}} = y\, \omega_{s_i} \tag{144}$$

is the velocity at each element on the race relative to the ball owing to ball spin and

$$V_{x_{B_i}} = -\frac{d}{2}\, \omega_y \tag{145}$$

is the velocity of the ball relative to the race resulting from gyroscopic precession of the ball. The velocity component ω_y is the ball angular velocity about the axis dictated by the ball's gyroscopic movement and is one of the independent variables employed in the overall iterative bearing solution.

The total tractive forces in the x direction can be written

$$F_{x_i} = \int_{-a}^{a} \int_{-y^*}^{y^*} \tau_x \, dy \, dx \tag{146}$$

The spin torque normal to the contact ellipse can be expressed as

$$Q_{s_i} = \int_{-a}^{a} f_{wx} dx + \int_{-a}^{a} \int_{-y^*}^{y^*} \tau_x y \, dy \, dx \tag{147}$$

Each of these integrals, Equations (141), (146), and (147), was evaluated numerically in the subroutine BALFTJ to provide the tractive forces and moments along with their derivatives with respect to the velocity components and normal contact loads needed for the computer analysis.

Tractive forces and derivatives for the roller bearing contacts were calculated in the same manner in the subroutine ROLFTN subject to the simplification of zero spin velocity and torque.

Tractive Instability

The most significant difficulty encountered during the computer program development stems from the rheological behavior of the fluid and the resulting traction forces developed at high slip speeds. At high values of G_1, the slope of the traction curve in Figures 12 and 13 becomes progressively flatter, and finally reaches a zero slope at the ceiling, i.e., the limiting shear stress is reached. In this regime, changes in the sliding velocity do not strongly affect the tractive forces generated; consequently, all derivatives of forces and moments with respect to the velocity components are nearly zero. This behavior creates a breakdown within the iterative procedure because velocity changes are not then affecting bearing tractive forces, and the load adjustments with velocity changes required to achieve solutions are not forthcoming.

Another phenomenon that caused problems at high slip rates is illustrated in Figure 15, where tractive force is plotted against slip velocity directly. Note that at low slip rates the tractive forces increase with increasing slip rates, while at high slip rates the tractive forces decrease with increasing slip rates. This decrease in traction with increasing slip rate is primarily the result of heating of the oil film with an attendant temperature increase combined with a strong temperature dependence of the oil viscosity. In this regime, the heating effect overrides the increasing slip rate to produce a net decrease in traction.

This effect produces a surprising result when combined linear and spin slip velocities are superimposed on a race contact. In some cases, the computed reactive spin torque on the ball opposes the spinning velocity, as one would intuitively expect. However, in other cases, the reactive spin torque is in the same sense of the spinning velocity; i.e., the calculated reaction torque is in a direction so as to accelerate the spinning action of the ball. This anomaly can be explained by considering the two points A and B on the tractive curve, Figure 15. Point A is in the regime where shear rate predominates over tractive behavior, while point B is in the regime where thermal effects are predominant. Consider race contact as shown in Figure 16, with the ball sliding in the y direction with a sliding velocity V_y. The tractive force picture in the contact for this case would be as indicated in Figure 16. If angular velocity ω is now superimposed, different tractive profiles are obtained as indicated in Figure 17, depending upon whether we were initially operating at point A or B. In this manner, the torque reaction to ball spin can either oppose or reinforce the ball spin velocities, depending on whether the dominant slip velocities are on the increasing or decreasing section of the traction curve. This phenomenon is not fully understood at this point, and it must be remembered that the base friction data have been obtained in cylindrical contacts in the absence of spin.

This traction behavior certainly implies that the ball motion would be unstable in these regimes and solutions are unattainable. This phenomenon hampers convergence not only when the desired solution is in the unstable regime, but also if these instabilities are encountered during intermediate passes through the iterative loop for a case which ultimately is stable.

To study this traction-slip behavior in depth, an auxiliary computer program was created. The program treats a single ball-race contact for which the linear and spinning slip velocities are introduced. The program first calculates the tractive forces and moments resulting from

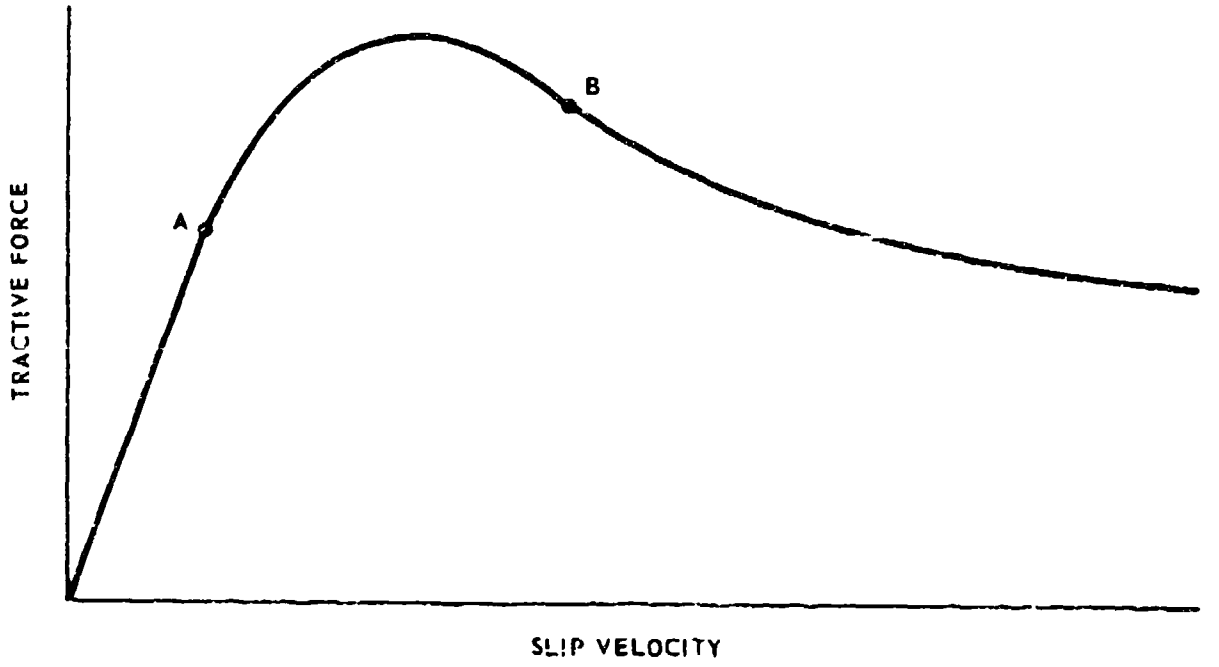


Figure 15. Tractive Force Versus Slip Velocity.

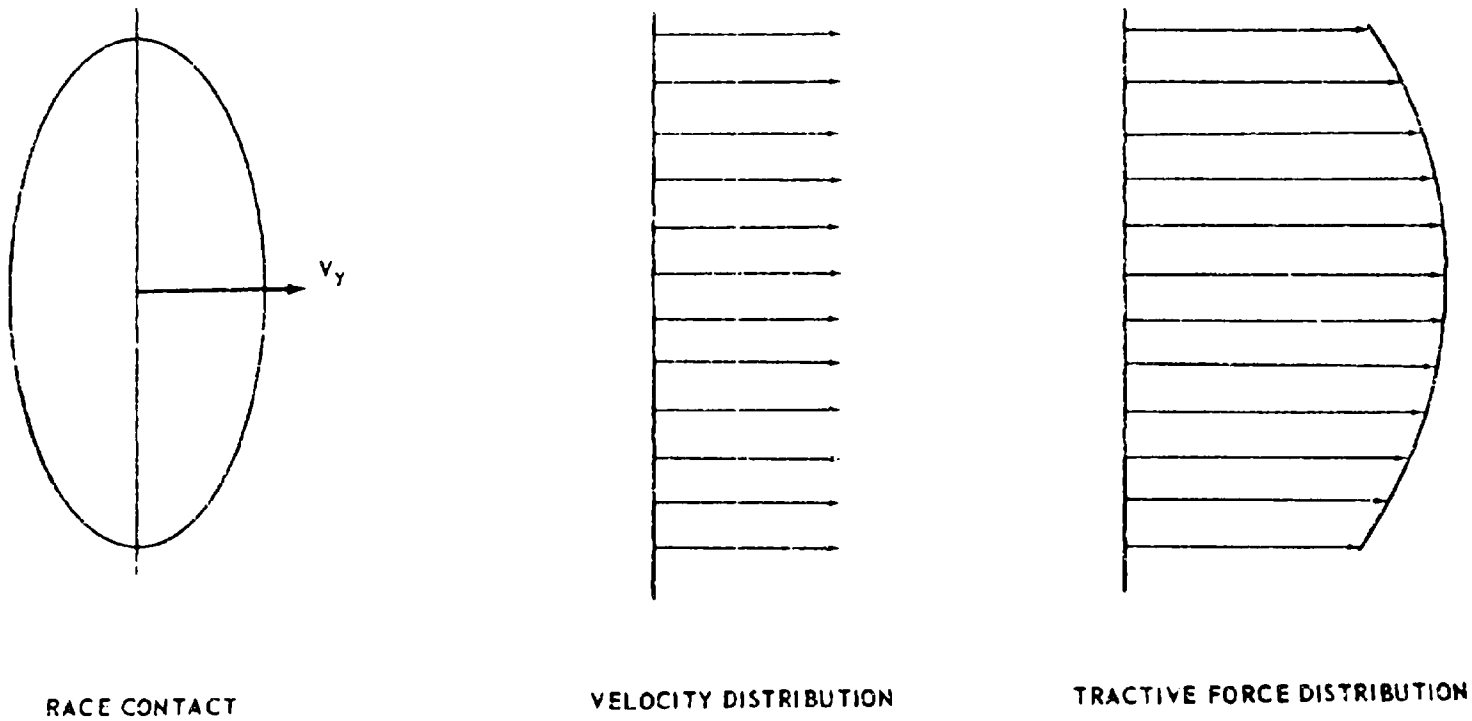


Figure 16. Race Contact, Velocity Distribution, and Tractive Force Distribution in a Ball-Race Contact.

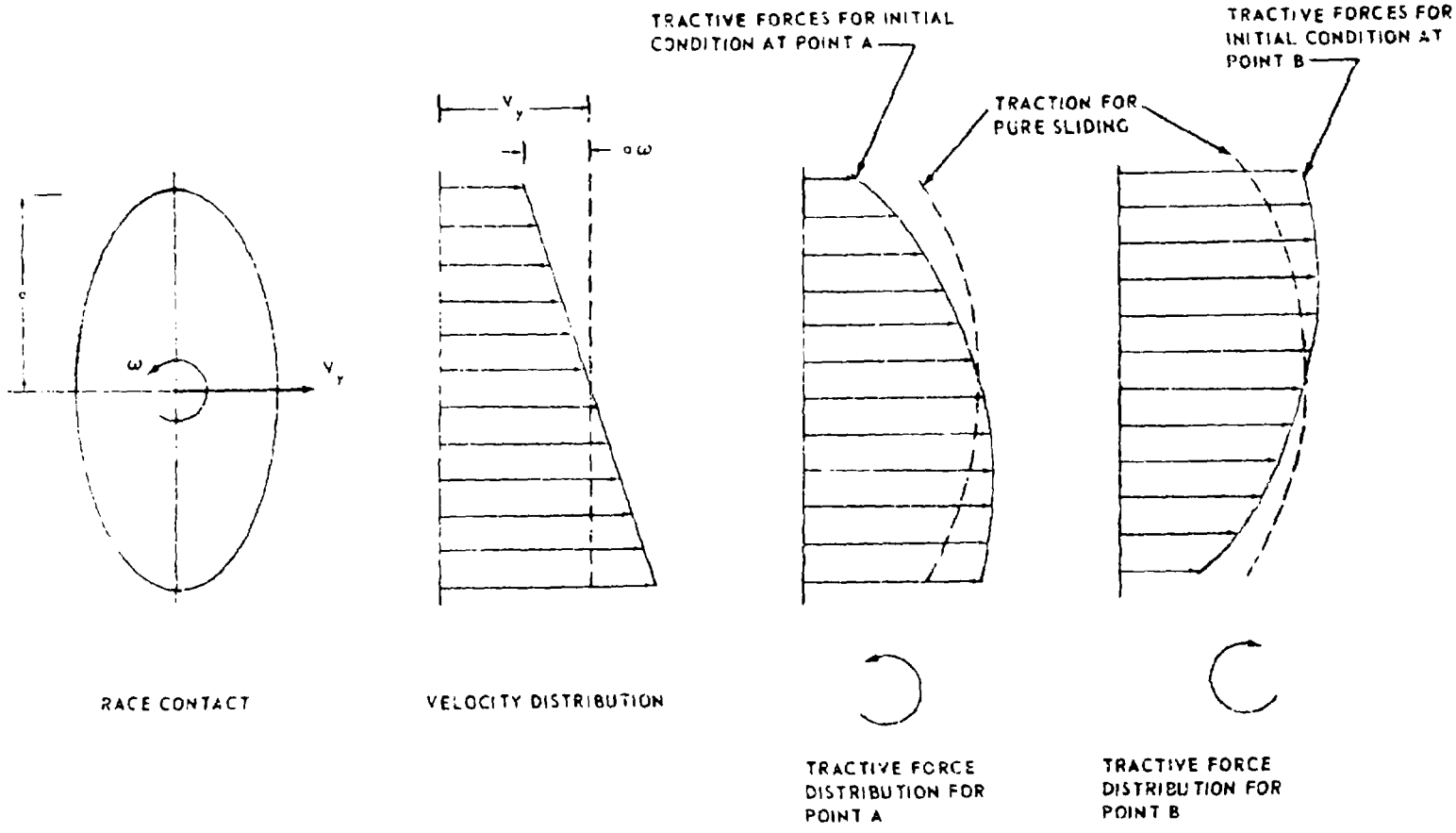


Figure 17. Sliding Velocity and Tractive Force Distribution in a Ball-Race Contact.

the input slip values, increments the input velocities a small amount, and then attempts to iterate back to the initial sliding velocities while knowing their corresponding tractive forces and moments. In this manner the areas of instability can be established as a function of linear and spin slip velocities.

The program incorporates three options to consider different traction versus slip relationships. The first relationship uses the aforementioned Johnson empirical data integrated into a table look-up/interpolation subroutine FRCTN written by Mechanical Technology, Inc. The second relationship is an analytical function that was derived to fit the FRCTN data by scaling and shifting of G_1, G_2, and G_3 terms. This smooth analytical function was formulated to eliminate any local numerical discontinuities that could creep into the table look-up interpolation procedure. The following analytical expression resulted:

$$f = \frac{.073\, G_1 \left(\frac{G_1}{G_2'}\right)^{.221}}{G_1 \left(\frac{G_1}{G_2'}\right)^{.171} + 2.65 \left(10^{-7.4 \log_{10} G_3 + 2.4}\right)} \tag{148}$$

where

$$G_2' = 1 \times 10^{-5}$$

which represents, relatively well, the trends of the charts prepared by Mechanical Technology, Inc. However, the deviation in the absolute value of the traction coefficients can be relatively large. The third relationship is a general power law function of the type

$$f = C_1 \; G_1^{\,c} \tag{149}$$

Cases of combined linear and spin slip up to 10,000 inches per second and 1,000,000 rpm, respectively, were investigated. Tables I through IV present the results in a matrix of linear versus spin

slip velocity indicating the number of iterations required to achieve the back solution. The letter "D" that appears in some positions designates a divergence in the iterative process and is indicative of the tractive instability.

The test case used for the study is typical of ball bearing contacts with a semimajor axis of 0.050, a semiminor axis of 0.005, and a Hertz pressure of 150,000 psi.

Table I presents the stability picture for the FRCTN subroutine. As is evident, instabilities for the case studied occur at moderate and high spin slip velocities and do not seem to be as strongly influenced by linear slip. The constant divergence at 1,000 inches per second sliding velocity at all spin slip velocities does not seem to follow the general trend depicted.

Table II is the result when the smooth analytical function is used instead of the interpolation procedure for the same empirical data. The result is generally the same with an apparent slight improvement at higher linear slip velocities. The strong instability previously seen at 1,000 inches per second is not present. The nature of the stability maps indicates that, in general, as the slope of the curve (Figures 12 and 13) diminishes, the instability tendency increases; therefore, some further studies were conducted with various slopes. The subject curves being log-log plots of traction versus G_1 implies that the slope of the curve is dictated by the exponent term e in Equation (149). Again referring to Figures 12 and 13, the slope of the straight-line portion of the curves at low values of G_1 has an e equal to 1.0. As the slope becomes progressively flatter, while G_1 is increasing, the value of e is decreasing and approaches 0. Tables III and IV give the results when the power law, Equation (149), is used with e values of 1/4 and 1/2 respectively. As is evident, the greater the value of e (greater slope), the greater the stability. In addition to investigating the effects of e, a number of numerical values for C_1 were also run. Results indicated that stability of the system is not a function of C_1, which establishes the magnitude of the traction coefficient, but, as intuitively expected, is a strong function of the traction coefficient's relationship to the slip velocity.

Roller Bearing Tractive Instability

Difficulties encountered with nonconvergence of the ball bearing program were also present in the roller bearing program. As a result, the power law function for friction, developed previously for the ball bearing program, was adopted. The roller bearing program was thus developed and solutions for all cases inputted were realized.

TABLE I . STABILITY MAP - FRCTN SUBROUTINE*

Linear Slip (in./sec)	Spin Slip (rpm)						
	0	10	10^2	10^3	10^4	10^5	10^6
0	-	1	3	D	D	D	D
1	1	1	2	D	D	D	D
10	4	4	6	D	D	D	D
10^2	3	4	4	4	13	D	D
10^3	D	D	D	D	D	D	D
10^4	6	8	7	7	7	D	D

* - Values indicate number of iterations required for solution
D - Divergence in iterative process

TABLE II. STABILITY MAP - ANALYTICAL FUNCTION FROM FRCTN SUBROUTINE*

Linear Slip (in./sec)	Spin Slip (rpm)						
	0	10	10^2	10^3	10^4	10^5	10^6
0	-	2	2	3	3	D	D
1	6	9	D	D	D	D	D
10	3	4	5	6	D	D	D
10^2	5	5	5	5	6	D	D
10^3	6	7	7	6	6	8	D
10^4	11	12	13	15	9	15	10

* - Values indicate number of iterations required for solution
D - Divergence in iterative process

TABLE III. STABILITY MAP - $f = C_1 G_1^{1/4}$ *							
Linear Slip (in./sec)	Spin Slip (rpm)						
	0	10	10^2	10^3	10^4	10^5	10^6
0	-	2	2	3	3	4	6
1	3	4	5	D	D	D	D
10	4	4	4	5	D	D	D
10^2	4	4	4	5	6	D	D
10^3	6	5	5	5	5	6	D
10^4	9	9	5	7	7	8	7

* - Values indicate number of iterations required for solution
D - Divergence in iterative process

TABLE IV. STABILITY MAP - $f = C_1 G_1^{1/2}$ *							
Linear Slip (in./sec)	Spin Slip (rpm)						
	0	10	10^2	10^3	10^4	10^5	10^6
0	-	2	2	3	3	3	6
1	3	3	4	8	9	9	9
10	4	4	4	4	9	11	13
10^2	4	4	4	4	5	9	13
10^3	9	4	4	4	5	5	13
10^4	14	5	6	15	5	5	8

* - Values indicate number of iterations required for solution

ANALYSIS OF SHEAR FORCES ON A LUBRICATED, HIGH-SPEED BEARING RETAINER

Until recently, very little attention has been paid to the effects of the retainer on the operating characteristics of a ball or roller bearing assembly. A knowledge of the forces on the retainer is important with respect to both their effect on the dynamics of the other components of the bearing and their influence on the life of the retainer itself.

In this section, simplified analyses are developed to estimate the shear forces imposed on the retainer by retainer-rolling element and retainer-race interactions in a high-speed oil-lubricated bearing assembly. These analyses require geometry, speeds, lubricant properties, and normal loading as inputs, and are intended to provide relationships to be incorporated as subroutines in a bearing dynamics analysis. Consideration is given to ball and roller bearings.

Traction and Load Analyses for Retainer-Ball Contact

Simplified relationships may be presented for predicting the shear forces on the retainer when in hydrodynamic contact with a ball. A schematic for a single ball in its pocket is shown in Figure 18. The ball imparts a normal load W to the retainer in the direction of orbital motion. For an angular contact bearing having angular velocity components Ω_1 and Ω_2 (Ω_3 is neglected) and a given lubricant and given geometry, it is desired to determine the relationship between the vertical and circumferential tractive forces F_1 and F_2 and the load W.

Development of Analysis

The analysis of the ball-retainer contact will start with a few basic assumptions:

1. Hydrodynamic forces at the contact can be described by Reynolds equation for steady, isoviscous, incompressible, laminar flow.

2. The radial clearance C is assumed large compared with the minimum film thickness h_0 and small compared with the ball radius R_B.

3. The ball-retainer contact will be considered rigid as opposed to EHD.

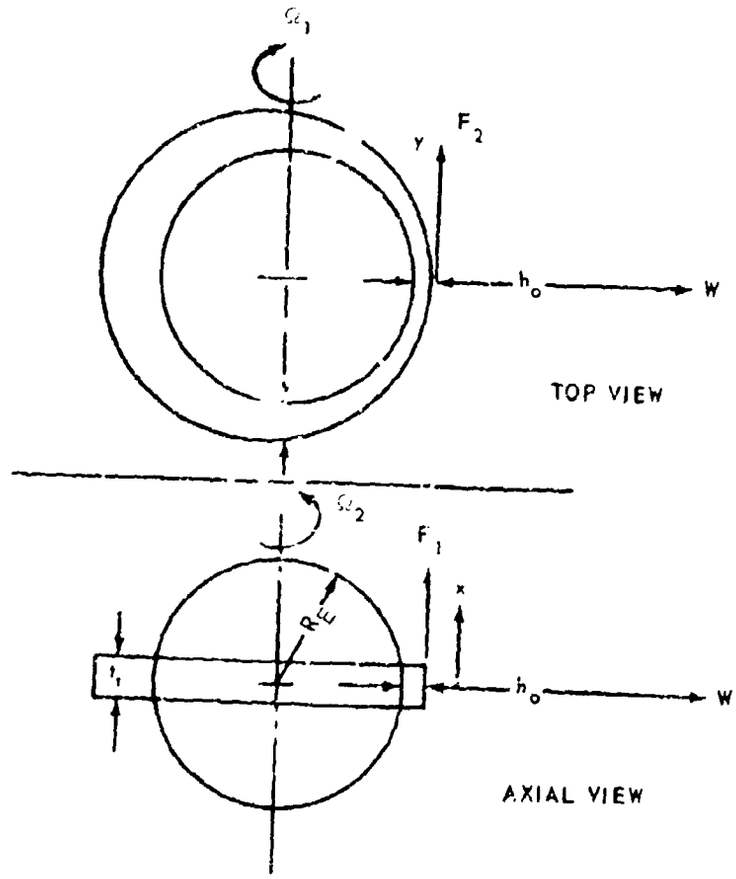


Figure 18. Analysis of Retainer-Ball Tractions; Schematic.

The Reynolds equation for steady, isoviscous, incompressible flow is given by

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu u_x \frac{\partial h}{dx} + 6\mu u_y \frac{\partial h}{dy} \tag{150}$$

If $h_o/C << 1$, the load will be supported in a very small region in the neighborhood of the point of contact ($x = y = 0$). Each of the surfaces can be represented by its quadratic approximation, thus giving a film thickness shape of the form

$$h = h_o + \frac{x^2}{2R_E} + \frac{y^2}{2}\left(\frac{1}{R_E} - \frac{1}{R_E + C}\right)$$

or approximately,

$$h = h_o + \frac{x^2}{2R_E} + \frac{y^2}{2R_E}\left(\frac{C}{R_E}\right) \tag{151}$$

The surface velocity u_y is given by $u_y = R\Omega_2$, and if h_o/C is small, the velocity u_y may be approximated by the constant value $u_y = R\Omega_1$.

Equations (150) and (151) may be put into convenient dimensionless form by substituting the dimensionless variables

$$\xi = \frac{x}{\sqrt{2R_E h_o}}$$

$$\eta = \frac{y}{\sqrt{2R_E h_o}}\sqrt{\frac{C}{R}}$$

$$p = \frac{6\mu R\Omega_1}{h_o}\sqrt{\frac{2R_E}{h_o}}\,\tilde{p}$$

$$\tilde{h} = h/h_o$$

for the respective dimensional quantities in Equations (150) and (151) to obtain the equation

$$\frac{\partial}{\partial \xi}\left(\tilde{h}^3 \frac{\partial \tilde{p}}{\partial \xi}\right) + \frac{C}{R_E}\frac{\partial}{\partial \eta}\left(\tilde{h}^3 \frac{\partial \tilde{p}}{\partial \eta}\right) = \frac{\partial \tilde{h}}{\partial \xi} + \frac{\Omega_2}{\Omega_1}\sqrt{\frac{C}{R_E}}\frac{\partial \tilde{h}}{\partial \eta}$$

where

$$\tilde{h} = 1 + \xi^2 + \eta^2 \tag{152}$$

In accordance with assumption (2), terms of order $\sqrt{C/R}$ are neglected to obtain the approximate form of Reynolds equation

$$\frac{d}{d\xi}\left(\tilde{h}^3 \frac{d\tilde{p}}{d\xi}\right) = \frac{d\tilde{h}}{d\xi} \tag{153}$$

The usual downstream separation condition is imposed on Equation (153), which requires the pressure and pressure gradient to vanish simultaneously at some point downstream of the center of contact $\xi = -\xi_1(\eta)$. Hence,

$$p = \frac{dp}{d\xi} = 0 \tag{154}$$

at $\xi = -\xi_1(\eta)$.

We may now integrate Equation (153) with respect to ξ subject to the boundary condition given by Equation (154) to obtain the equation

$$\frac{d\tilde{p}}{d\xi} = \frac{\xi_1^2 - \xi^2}{(1 + \xi^2 + \eta^2)^3} \tag{155}$$

It can be seen that the pressure gradient dies out at $1/\xi^4$ as ξ becomes large. The upstream edge of the retainer $x = -t_r/2$ corresponds to a value of $\xi = t_r/(2\sqrt{2}\,Rh_o)$, which will be much larger under the general range of conditions of interest than the value of ξ required for the pressure to approach its limiting value; hence, we may invoke the boundary condition

$$\lim_{\xi \to -\infty} \tilde{p} = 0 \tag{156}$$

The results of this analysis will thus be independent of the width of the retainer. The two constraints imposed by Equations (154) and (156) are sufficient for determining the constant of integration associated with Equation (155), the separation point $\xi_1(\eta)$, and hence the pressure profile $\tilde{p}(\eta)$.

The pressure profile satisfying Equations (154) and (155) is given by

$$\tilde{p}(\xi, \eta) = \int_{-\xi_1(\eta)}^{\xi} \frac{\xi_1^2(\eta) - \xi'^2}{(1 + \xi'^2 + \eta^2)^3} \, d\xi' \tag{157}$$

and $\xi_1(\eta)$ is determined from Equation (156). The result is given below:

$$\xi_1(\eta) = 0.4751 \sqrt{1 + \eta^2} \tag{158}$$

Film Thickness-Load Relationship for Retainer-Ball Contacts

The dimensionless pressure profile at $\eta = 0$ $\left[\tilde{p}(\xi, 0)\right]$ is shown graphically in Figure 19. It can be seen that the pressure dies out rapidly in the inlet region and at $\xi = 3$ is 10 percent of its peak value. Hence the error in pressure introduced by our upstream boundary approximation will be small if $t_r/(2/\sqrt{2}\,\overline{Rh_o}) > 3$.

The pressure at any value of η is given by

$$\tilde{p}(\xi, \eta) = \frac{1}{(1 + \eta^2)^{3/2}} \, \tilde{p}\left(\xi \sqrt{1 + \eta^2}, 0\right) \tag{159}$$

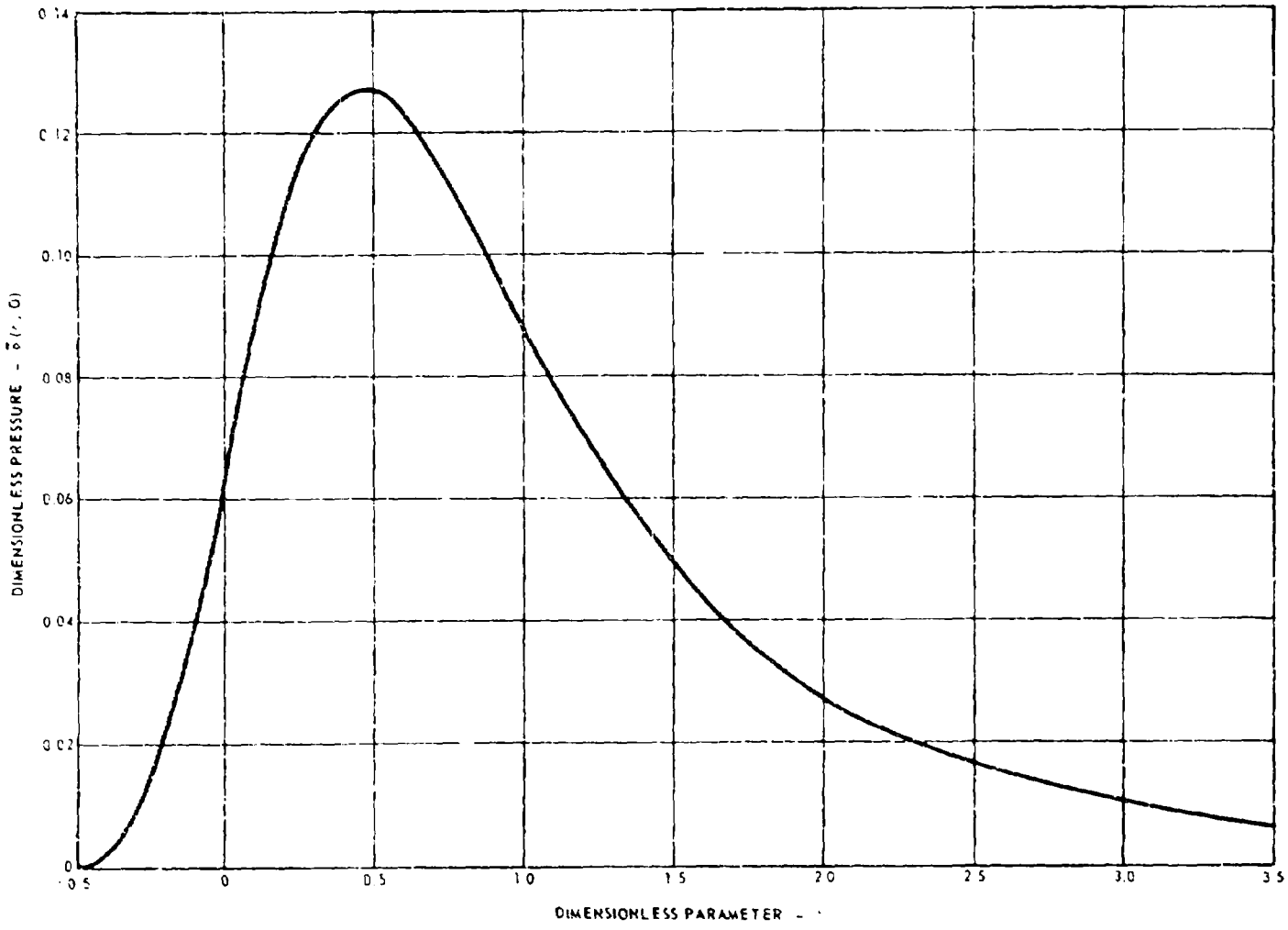


Figure 19. Dimensionless Pressure Profile for Ball-Retainer Contact ($\eta = 0$).

Equation (159) may be integrated with respect to ξ over the interval $-\xi_1 < \xi < \infty$ to obtain the dimensionless load per unit length $\tilde{P}(\eta)$, which is given by

$$\tilde{P}(\eta) = \int_{-\xi_1(\eta)}^{\infty} \tilde{p}(\xi, \eta)\, d\xi = 0.2040\,(1 + \eta^2)$$

and the total dimensionless load $\tilde{W}$ may be obtained by integrating $\tilde{P}(\eta)$ over all η $(-\infty < \eta < \infty)$. It can be shown that the error introduced here will be of the order of $\sqrt{h_o/C}$.

The dimensionless load $\tilde{W}$ is

$$\tilde{W} = \int_{-\infty}^{\infty} \tilde{p}(\eta)\, d\eta = 0.6407$$

and the load-film thickness relationship in terms of physical parameters is

$$\boxed{W = \frac{\mu \Omega_1 R^3}{\sqrt{h_o C}} K_w \qquad \text{where } K_w = 10.87} \tag{160}$$

Traction Relationships

The shear stress relationships on the retainer consistent with lubrication theory for isoviscous, incompressible flow are as follows:

$$\tau_{xz} = \frac{h}{2}\frac{\partial p}{\partial x} + \frac{\mu u_x}{h}$$

$$\tau_{yz} = \frac{h}{2}\frac{\partial p}{\partial y} + \frac{\mu u_y}{h}$$

The above equations may be put in their dimensionless form:

$$\tilde{\tau}_1 = \frac{h_o}{\mu R_E \Omega_1}\tau_{xz} = 3\tilde{h}\frac{\partial \tilde{p}}{\partial \xi} + \frac{1}{\tilde{h}} \tag{161}$$

$$\tilde{\tau}_2 = \frac{h_o}{\mu R_E \Omega_2}\tau_{yz} = 3\tilde{h}\frac{\sqrt{C}}{\sqrt{R_E}}\frac{\partial \tilde{p}}{\partial \eta}\frac{\Omega_1}{\Omega_2} + \frac{1}{\tilde{h}} \tag{162}$$

The first term on the right side of Equation (162) is of the order $\sqrt{C/R_E}$ and will be neglected. Equation (155) may be used to replace the pressure gradient appearing in Equation (161).

The resulting shear stress equations are

$$\tilde{\tau}_1 = 3\frac{(\xi^2 - \xi_1^2)}{(1 + \xi^2 + \eta^2)^2} + \frac{1}{(1 + \xi^2 + \eta^2)} \tag{163}$$

and

$$\tilde{\tau}_2 = \frac{1}{1 + \xi^2 + \eta^2} \tag{164}$$

The shear stresses die out as the reciprocal of the square of the coordinate variables as they get large. If integrated over an infinite area, the tractions will diverge logarithmically. Rather than introduce new parameters into the system to limit the area that stresses act upon, it is convenient to define an effective length in the y direction and to consider variations in shear stress in the x direction only. In other words, an effective length will be used to treat the traction problem as one of lubricated line (rather than point) contact.

For the case of a roller bearing, line contact would actually occur and the effective length would be the flat portion of the roller.

An effective length for a ball bearing will be selected by examining the variation of the dimensionless load per unit length $\tilde{P}$ with the dimensionless circumferential coordinate η, which is shown in Figure 20. An effective dimensionless length η_{eff} for calculating load can be defined such that $\tilde{P}_{max}\ \eta_{eff} = \tilde{W}$, which corresponds to a value of $\eta_{eff} = \pi$ or an effective length of $L_{eff} = \sqrt{2}\ \pi R \sqrt{h_o/C}$.

This effective circumferential length is used to calculate the dimensionless tractive forces:

$$\tilde{F}_1 = \eta_{eff} \int_{-\xi_o}^{\xi_o} \tilde{\tau}_1\, d\xi = 1.218$$

and

$$\tilde{F}_2 = \eta_{eff} \int_{-\xi_o}^{\xi_o} \tilde{\tau}_2\, d\xi = 6.328$$

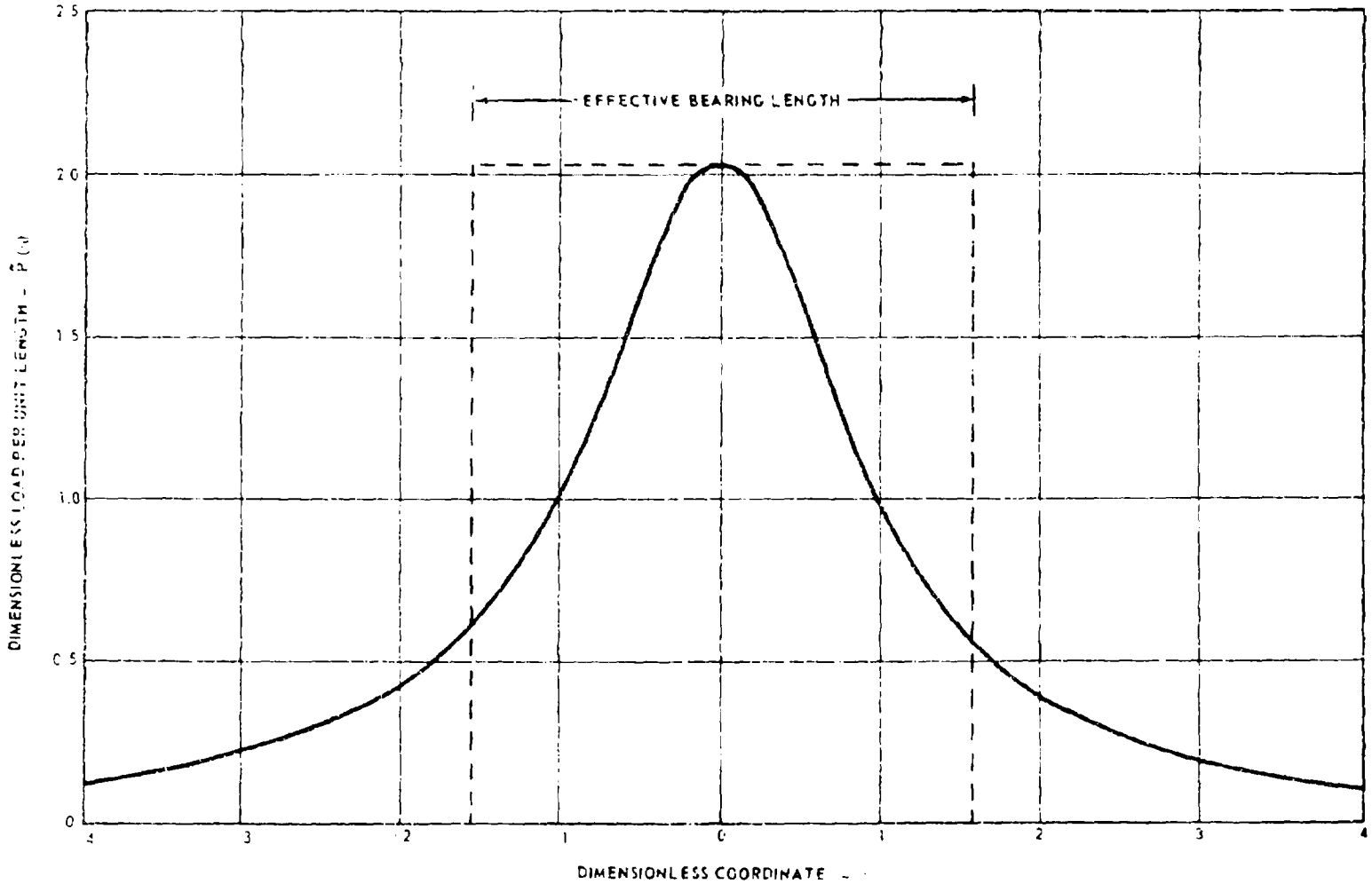


Figure 20. Dimensionless Load per Unit Length.

The hydrodynamic tractions may be expressed in terms of physical parameters as

$$F_i = \frac{\mu \Omega_i R_E^{5/2}}{C^{1/2}} K_i, \quad i = 1, 2 \tag{165}$$

where $K_1 = 2.436$ and $K_2 = 12.66$.

The traction formulas presented above are extremely simple to use for predicting hydrodynamic tractions. Some consideration should be given to the predicted film thickness and surface roughness to determine whether or not the contact is indeed hydrodynamic. Since the retainer ball contact is a concentrated contact, some consideration should also be given to the possibility of EHD effects. Further insight can be obtained here with the use of a numerical example.

Our numerical example will deal with a 0.5-inch-diameter ball spinning with an angular velocity of $\Omega_i = 50,000$ rpm in a retainer pocket having a 5-mil radial clearance, lubricated with a 2-centipoise-viscosity oil. When EHD effects are considered, a pressure coefficient of viscosity of 10^{-4} in.^{2}/lb will be used and elastic properties will be taken to correspond to those of steel. Film thickness and peak pressure versus load relationships are shown in Figure 21 for rigid-hydrodynamic and EHD contacts.

The EHD results were obtained using Hertz contact and Grubin's film thickness relationships. Both pressure and film thickness are shown to vary much more rapidly with load for rigid contact. At low loads, hydrodynamic effects spread the load out over a larger area than the Hertz contact zone, and EHD effects will be small. At loads where the rigid hydrodynamic pressures are predicted to be much higher than EHD pressures, EHD effects will be dominant and the analysis presented here will not be valid. It can be seen, however, that the film thickness at that point as predicted by either theory will be of the same order of magnitude as the surface roughness, and it is quite likely that the tractions will more nearly approach those resulting from dry friction.

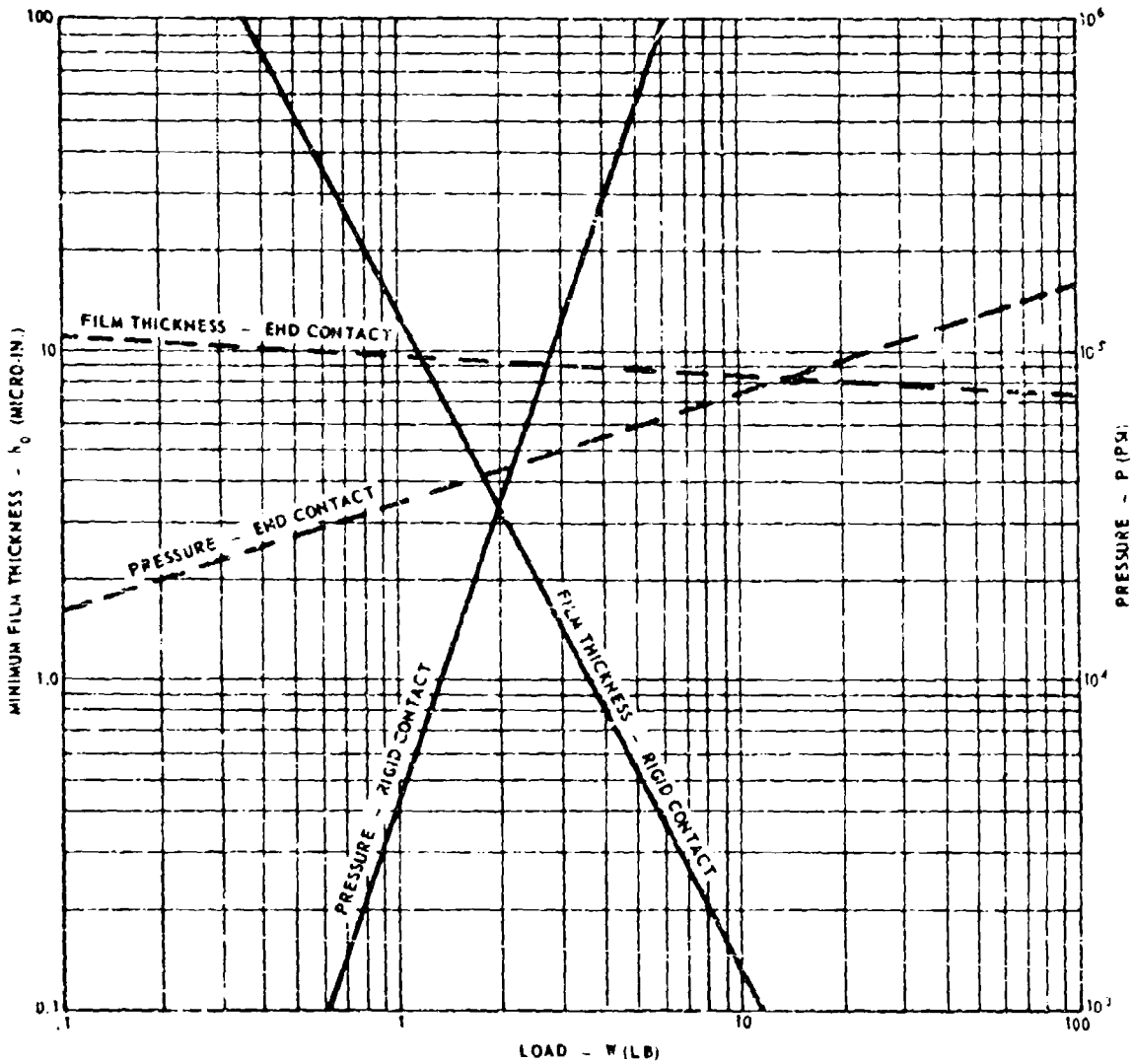


Figure 21. Film Thickness and Maximum Pressure for Retainer-Ball Contact.

The above arguments indicate that a reasonable result can be obtained by dividing the ball-retainer tractions into two regimes, rigid-hydrodynamic contact and dry friction, depending on the film thickness. A critical film thickness separating the two regimes may be determined from the surface roughness. We may define an effective surface roughness S_{eff} as

$$S_{eff} = \sqrt{S_R^2 + S_B^2}$$

where S_R and S_B refer to the RMS (root mean square) surface roughness of the retainer and ball respectively. When $h_o/S_{eff} < 4$, increased surface distress has been found to occur for bearings undergoing severe sliding. At a value of $h_o/S_{eff} = 1.5$, considerable surface distress is present as a result of asperity clashes. We will thus define our critical film thickness h_c for dry friction based on surface distress data as:

$$h_c = 1.5\ S_{eff}$$

The resulting ball-retainer traction formulas are

$$F_1 = \begin{cases} \dfrac{\mu \Omega_1 R_E^{5/2}}{C^{1/2}} K_1 , & h_o < h_c \\[2ex] K_1 W \dfrac{\Omega_1}{\sqrt{\Omega_1^2 + \Omega_2^2}} , & h_o > h_c \end{cases} \qquad (166)$$

$$K_1 = 2.436, \quad K_2 = 12.66, \quad i = 1,2$$

where K denotes coefficient of friction and h_o is calculated from Equation (160). A reasonable value for h_c is 10 microinches.

The variation of F_1 with W for our numerical example is shown in Figure 22. Inspection of this figure indicates that for practical purposes there is essentially no friction until a threshold load for Coulomb friction is reached. The retainer dynamics will thus probably be much more sensitive to the film thickness formula, Equation (160), than the hydrodynamic traction formulas. It should be noted that the film thickness formula does not contain the effective line contact assumption that was used in developing the traction formulas and hence should be less prone to error.

Tractions for Retainer-Roller Contact

The film thickness-load relationship and the traction relationship for the retainer-roller contact can be obtained from the formulas already presented for the retainer-ball contact. The pressures, shear stresses, and normal and tangential forces per unit length for the retainer-roller contact will be the same as those acting at y = 0 for the retainer-ball contact. The forces for the retainer-roller contact are simply the forces per unit length for the retainer-ball contact at y = 0 multiplied by the length ℓ of the roller. The equations obtained in this manner are:

$$W = \frac{\mu \Omega_1 R_E^2 L}{h_o} K_w' , \tag{167}$$

$$F_1 = \mu \Omega_1 R_E L \sqrt{\frac{R}{h_o}} K_1' , \tag{168}$$

where $K_w' = 2.447$, $K_1' = 0.5482$.

Film thickness and peak pressure versus load relationships for a roller-retainer contact are shown in Figure 23. The lines are drawn in a manner analogous to those in Figure 21, and the input parameters are the same except for the additional input of the length of the roller $\ell = 3/8$ inch. Comparison of Figures 21 and 23 indicates that the pressures

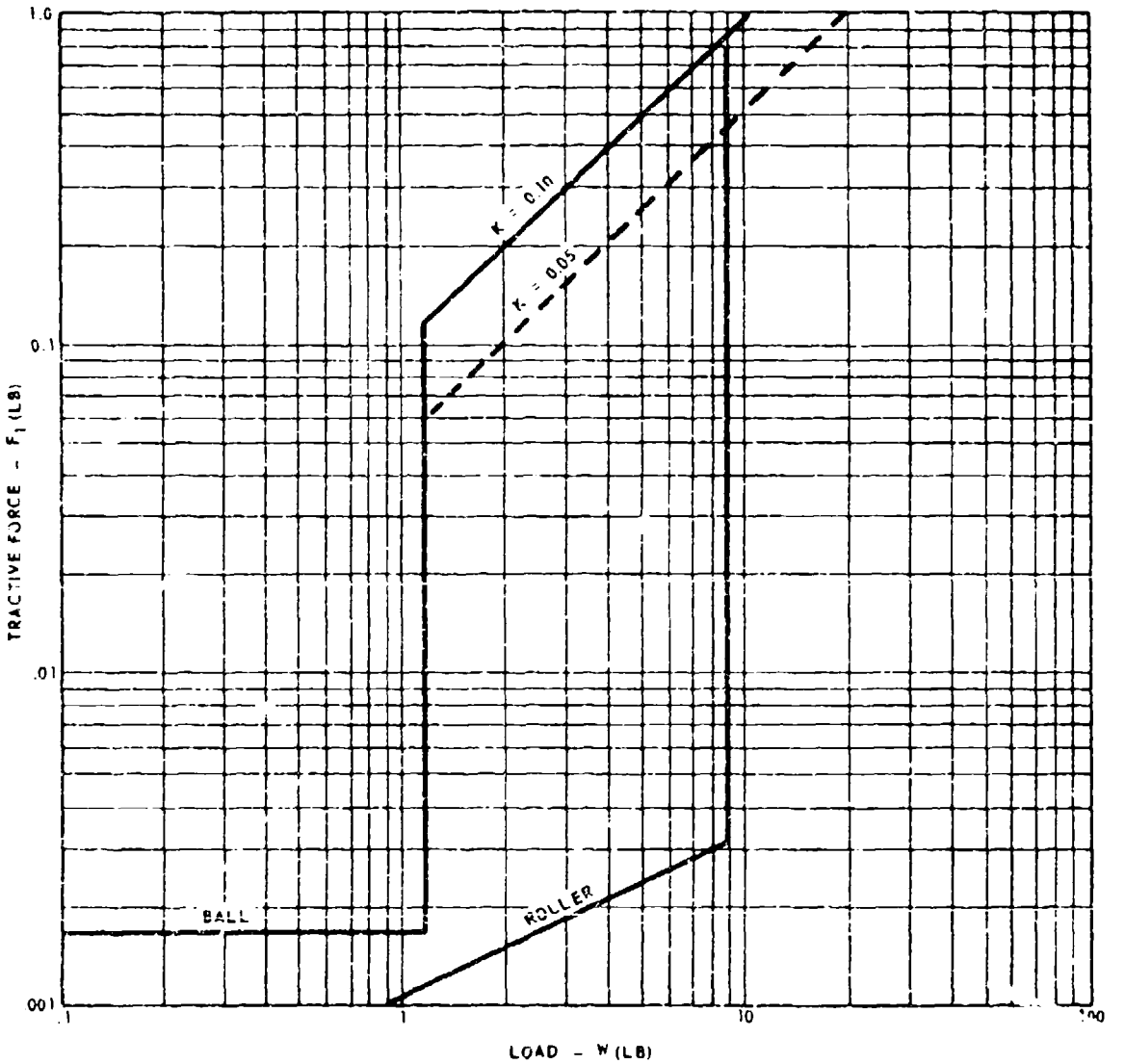


Figure 22. Traction Versus Load for Ball and Roller Contacts.

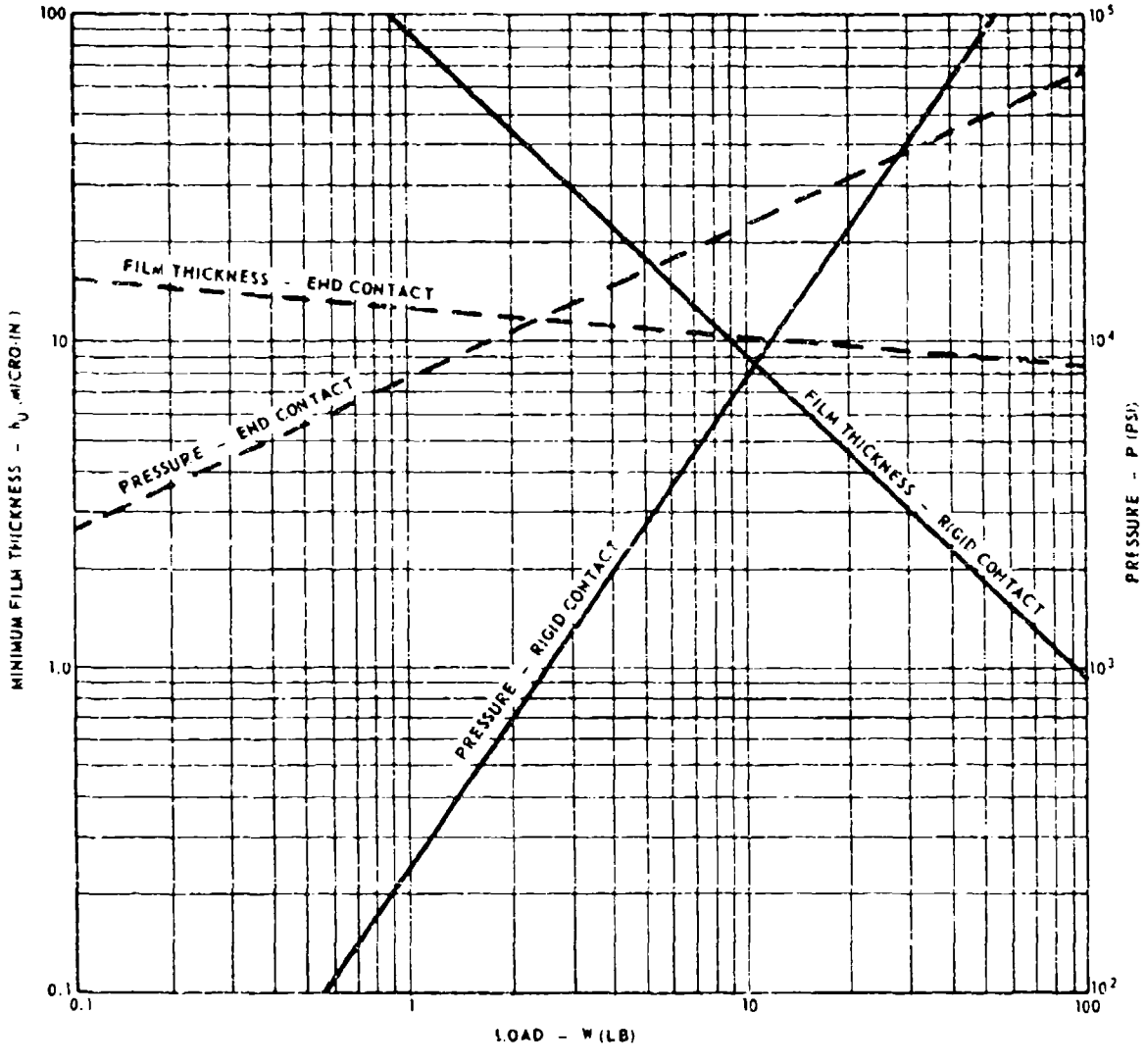


Figure 23. Film Thickness and Maximum Pressure for Retainer-Roller Contact.

will be lower and the retainer-roller contact will tend to be hydrodynamic at somewhat higher loads than in the retainer-ball contact. In other respects the two figures are qualitatively similar, and it should suffice to use the same criteria used in arriving at Equation (166) to obtain the relationship

$$F_1 = \begin{cases} \mu\Omega_1 R_E L \sqrt{\dfrac{R_E}{h_o}}\, K'_1 , & h_o < h_c \\ KW , & h_o > h_c \end{cases} \tag{169}$$

Tractions calculated from Equation (166) are shown in Figure 22.

Lubricant Drag Between Retainer and Race

A cross-sectional schematic of the retainer and races is shown in Figure 24. At the high speeds of interest here, the flow between the retainer and the races should be turbulent and the shear stress on either surface can be approximated by

$$|\tau_j| = \frac{1}{8} C_f \rho V_j^2 \qquad j = 1,2 \tag{170}$$

where ρ is the fluid density, V_j is the absolute value of the relative velocity between the retainer and race, j, and C_f is the friction factor for flow between the retainer and race, j. The friction factor will in general be a function of the Reynolds number $\mathcal{R}_j$.

$$\mathcal{R}_j = \frac{V_j H_j \rho}{\mu}$$

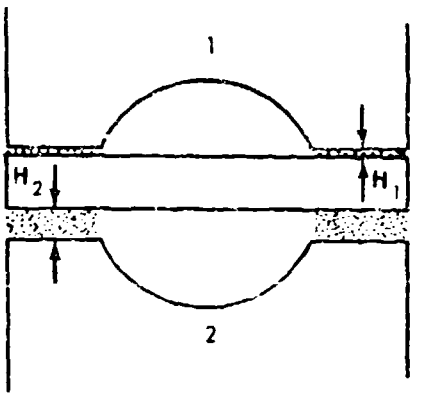


Figure 24. Geometry Used in Calculating Retainer-Race Tractions; Schematic.

It will be assumed that the spacing between the retainer and the guiding surfaces of each race is flooded with oil. We will use the friction factor relationship for Couette flow given by

$$C_f = \frac{0.064}{(\mathcal{R}_j)^{1/4}} \tag{171}$$

The net drag force on the retainer in the grooved region should be small in comparison with the drag forces between the guiding surfaces, since the flow cross section is larger and the region will probably not be flooded. The drag force on the retainer will thus be calculated based on the area of the guiding surfaces (A_j).

The net force on the retainer, F_R, is given by

$$|F_R| = 0.008 \left| \frac{V_1^2 \; A_1}{\mathcal{R}_2^{1/4}} - \frac{V_2^2 \; A_2}{\mathcal{R}_2^{1/4}} \right| \tag{172}$$

and will in general act in the direction of the relative velocity of the guiding race with respect to the retainer.

It should be noted that when the retainer is under load it will be operating with a high eccentricity with respect to the guiding race. It is doubtful that the retainer-race contact can support a significant amount of load hydrodynamically (as appears to be the case for the retainer-ball contact) because of the large clearances and resulting small area of effective contact. It is thus recommended that a tractive force due to Coulomb friction be added to F_R to obtain the total tractive force on the retainer.

Drag on Ball/Roller

The accurate computation of viscous drag on the ball requires a somewhat more detailed knowledge of the flow field than is currently available. Drag formulas are available for unconfined motions of bodies of various shapes. The flow in a ball bearing assembly is highly confined; however, it is quite likely that both raceways are not flooded and the flow pattern will be quite complex.

Figure 25 shows the cross section to be considered in estimating drag for the case of a ball bearing. In general, conformity is such that the major open channel for flow will be that existing between the retainer and race 2 shown in the illustration. If we assume that all the flow past the ball occurs around the shaded region and treat this region as a cylinder, the drag force on the ball F_D will be given by

$$F_D = C_D A_3 \frac{\rho V_3^2}{2} \tag{173}$$

where A_3 is the projected shaded region and V_3 is the linear orbital velocity of area A_3. As a result of the highly approximate nature of these calculations, it will suffice to take A_3 to be $A_3 \simeq 2R_E H_2$, V_3 to be the orbital velocity of the ball V_o and a characteristic drag coefficient of $C_D = 1$ for flow past a cylinder to obtain the formula

$$\boxed{F_D = \rho R_E H_2 V_o^2} \tag{174}$$

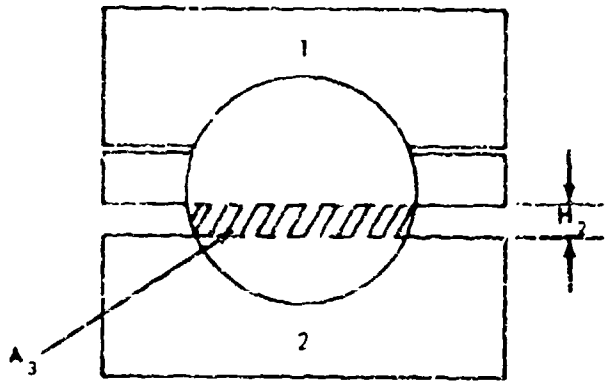


Figure 25. Section Used in Calculating Drag on Ball; Schematic.

VERIFICATION TEST PROGRAM

TEST RIG DESCRIPTION AND OPERATION

The test rig used in this program is shown in Figure 26. The rig simulates a 100-millimeter power turbine bearing and seal package, and it is capable of operating at speeds to 30,000 rpm with 1500° F air temperature and 250 psig air pressure outside the package. The test rig is driven by an 8-inch steam turbine rated at 100 horsepower at 30,000 rpm. A 2:1 speed increaser was installed between the turbine and the test rig. A picture of the test facility is shown in Figure 27. Photographs of the test rig are shown in Figures 28 and 29.

The test rig, Figure 26, incorporates the 100-millimeter bearing, a 100-millimeter radial load bearing, and a 55-millimeter support bearing package. When a roller bearing was installed in the test position, a ball bearing was used in the support position for axial positioning of the shaft.

The axial load for the ball bearing was applied during the test by creating an air pressure differential across the thrust load wheel. The magnitude of the axial load was accurately controlled by modulating the differential pressure across the wheel area. Axial thrust load as a function of air pressure differential is plotted in Figure 30. The radial load on the test bearing was obtained by applying a force to the load bearing. The housing of the load bearing is attached through a rod to a hydraulic piston. The housing is not restrained radially but is prevented from rotating by the loading rod, which is strain gaged to measure the applied radial load. The support bearing package of the test rig was adapted from existing engine hardware (T53-L-13, No. 2 bearing package).

In the main bearing test package, a carbon labyrinth was used to restrict the thrust wheel pressurization air from flowing into the bearing package. The load bearing was lubricated by one jet of 0.050 inch diameter, located on the forward side of the load bearing. The test bearing was lubricated by two jets of 0.035 inch diameter, one located on each side of the test bearing. The oil supply to the load and to the test bearings was controlled independently.

The oil used for the testing was MIL-L-23699, supplied by the Air Force Aero Propulsion Laboratory, oil code 0-64-2. Physical properties for this batch are listed in Table V.

The following parameters were measured and controlled during test rig operation:

1. Oil flow to the bearings, pph
2. Oil temperature in and out, ^{o}F } (test bearing)
3. Oil pressure in and out, psig }
4. Bearing cavity pressure, psig
5. Bearing cavity temperature, ^{o}F
6. Air temperature, ^{o}F } (outside test bearing package)
7. Air pressure, psig }
8. Inner and outer race temperatures, ^{o}F
9. Axial load, lb
10. Radial load, lb
11. Shaft speed, rpm
12. Vibration level (at test bearing housing)
13. Chip detection
14. Cage speed, rpm

The special instrumentation package for measuring retainer data was cantilevered off the rig housing (Figure 26). The retainer drives the instrumentation package, which consists of a multiplexer and mercury slip rings. The multiplexer bearings are oil mist lubricated and air cooled. The mercury slip ring bearings are self-contained and grease packed.

A schematic of the test rig facilities is shown in Figure 31. Typical data sheets are shown in Figure 32.

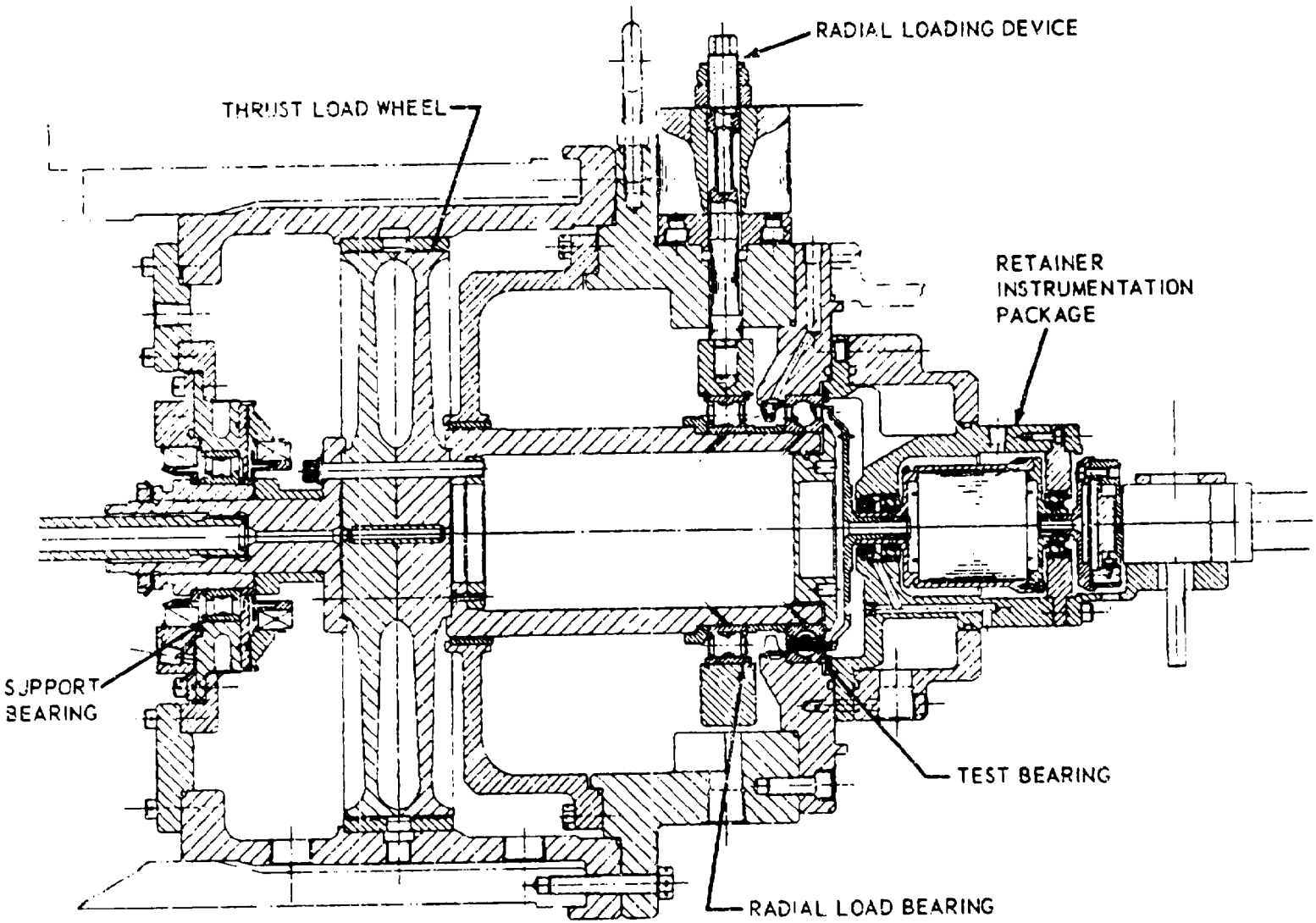


Figure 26. Bearing Test Rig; Cross-Sectional View.

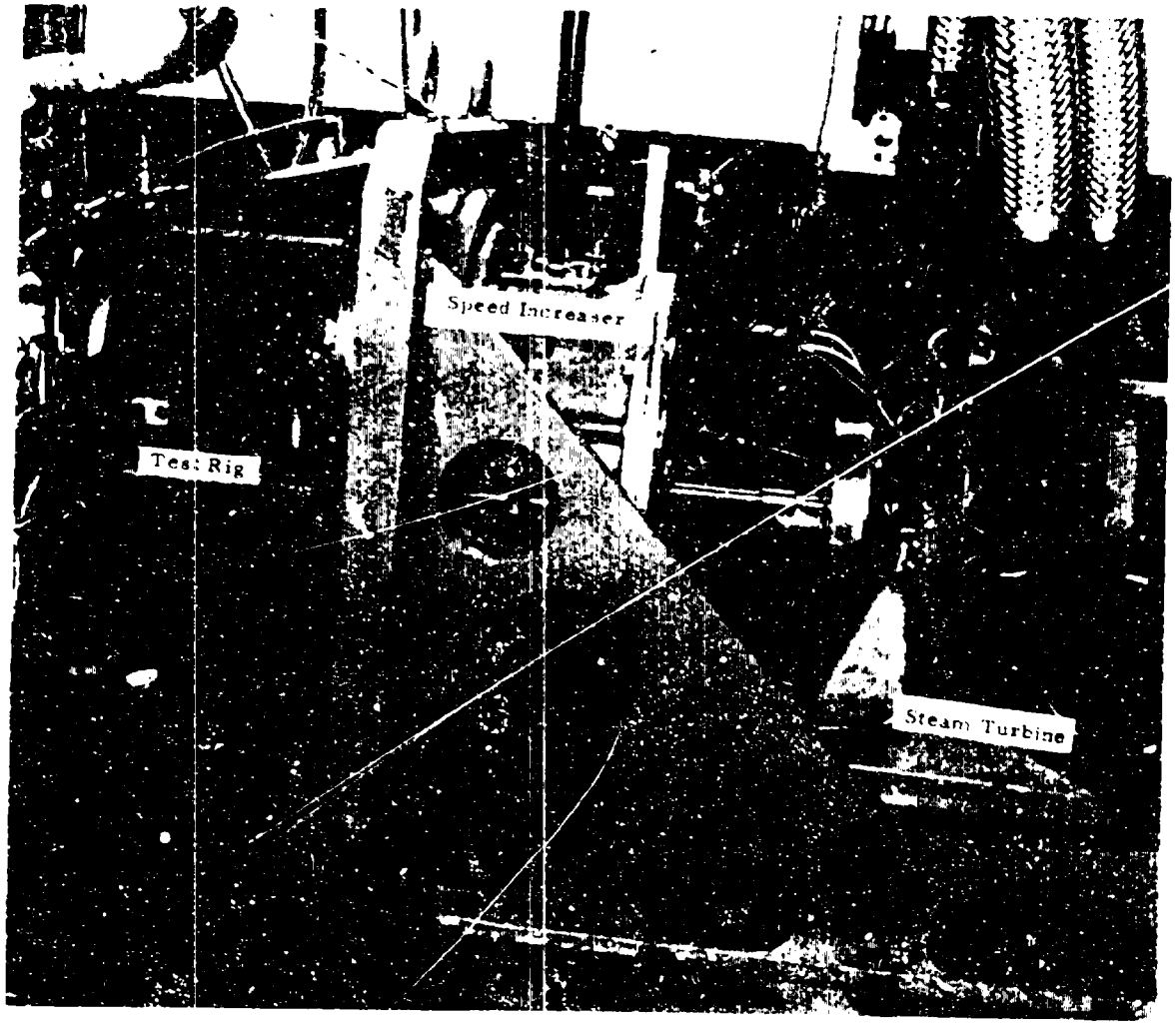


Figure 27. Bearing Test Facility.

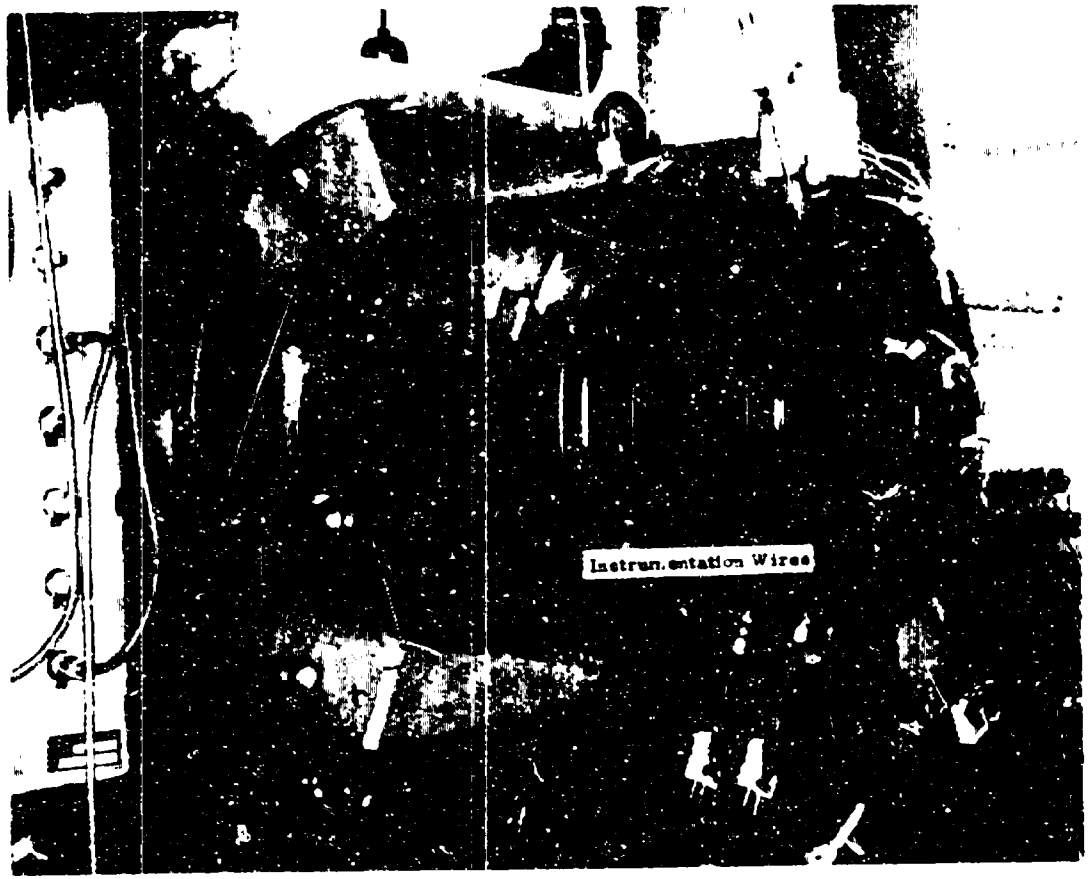


Figure 28. Bearing Test Rig: Right Front View.

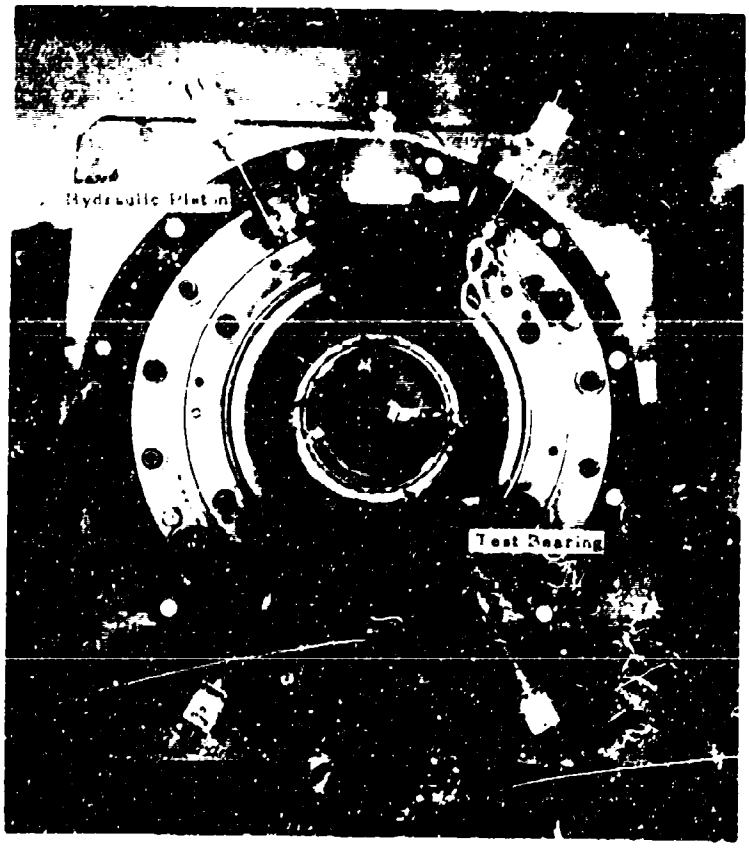


Figure 29. Bearing Test Rig; Front View.

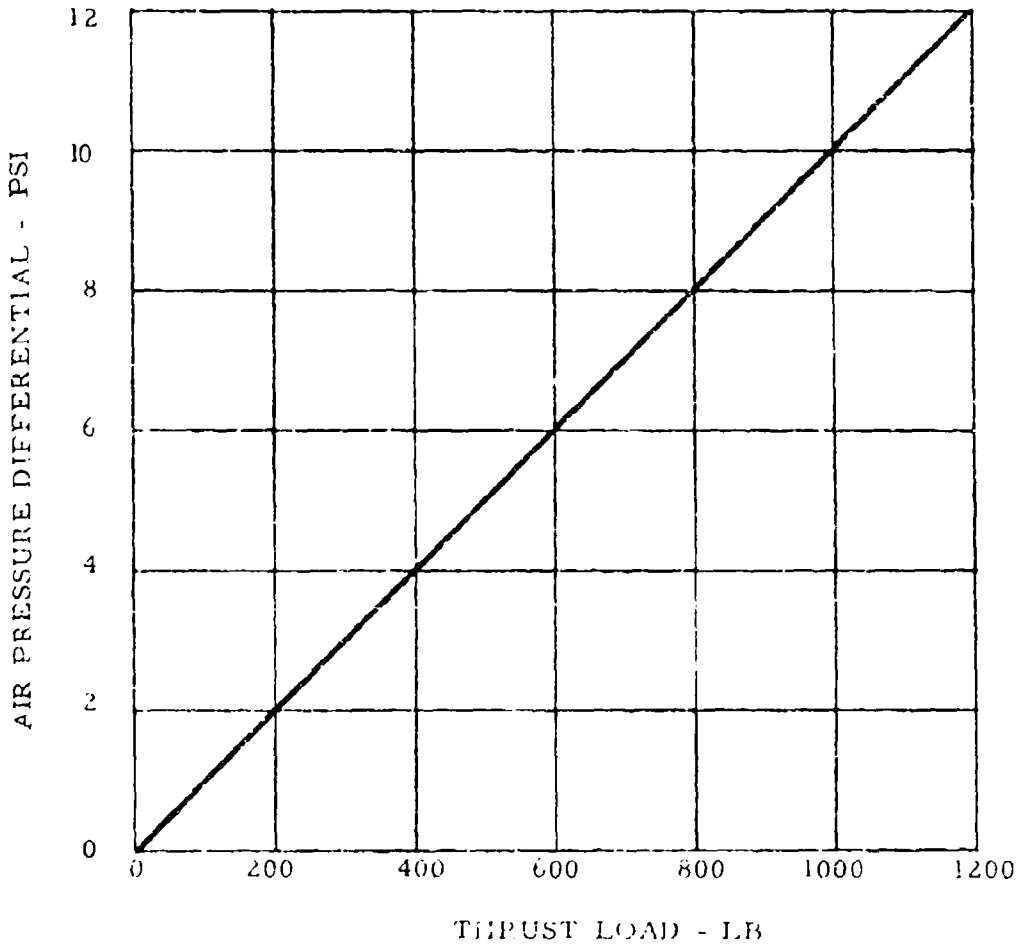


Figure 30. Air Pressure Differential Versus Bearing Thrust Load.

TABLE V. PHYSICAL PROPERTIES OF MIL-L-23699 OIL, CODE 0-64-2

MATERIAL EXAMINATION REPORT

MATERIALS LABORATORY

AVCO LYCOMING DIVISION

0-64-2

Lab No. _____

Lot No. _____

P. O. _____ R. R. _____ Quantity 1 qt Date 4/28/70

Batch No. _____ Mfr's Dsgn. _____ Date of Synthesis _____

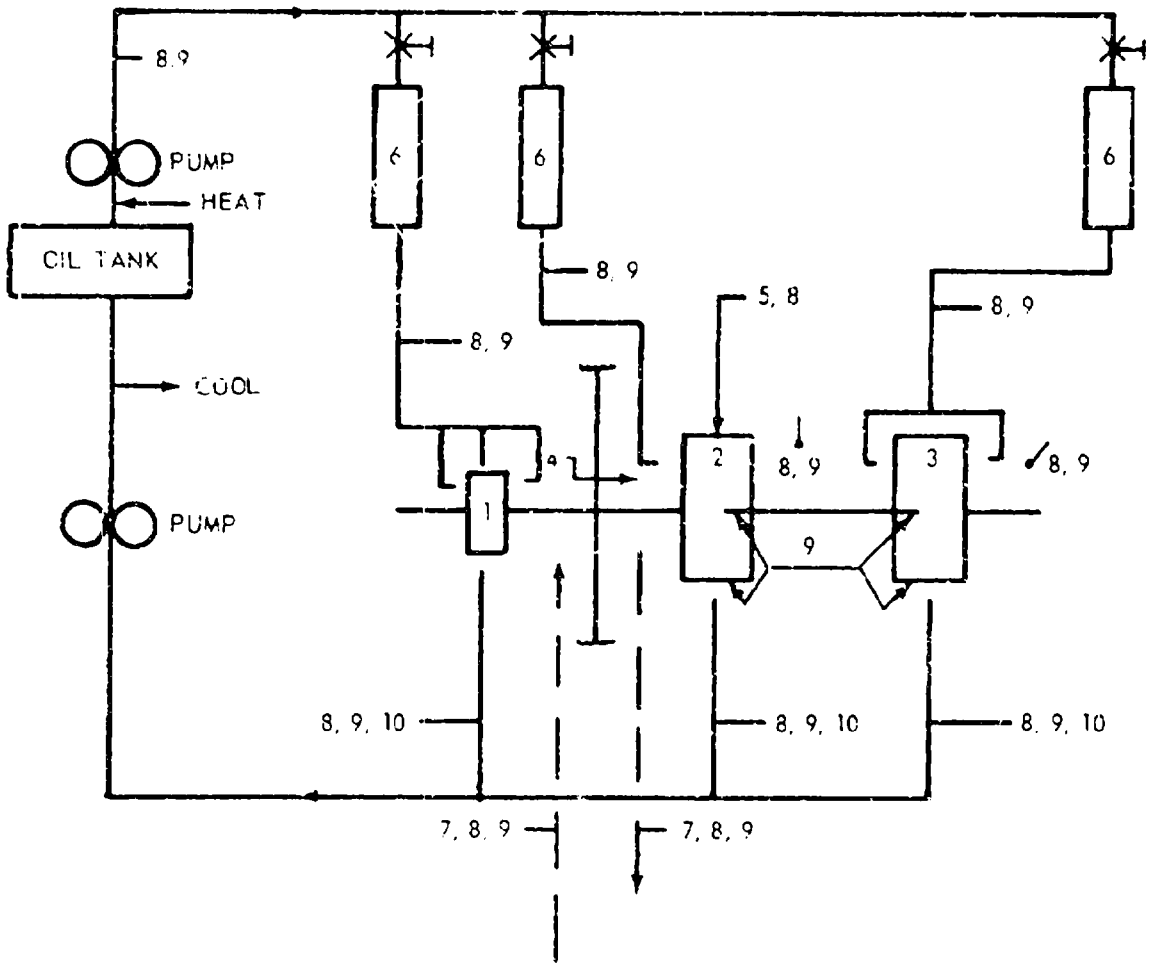
Specification MIL-L-23699 Vendor _____

From _____ Condition _____

TEST	REQUIREMENTS	VENDOR	LYCOMING	DISP.
Viscosity at 100° F	25 cs min		27.71	Accept
Viscosity at 210° F	5.0 to 5.5 cs		5.91	Reject
Flash Point, min	450° F		490	See Note
Pour Point, max	-65° F		L-65	
Evap. Loss, 400° F/6 hr	10% max by wt		8.3	
Color, ASTM	Report		3	
	Volume/collapse			
Foam, max, 75° F	25 ml/1 minute		15/45 sec	
200° F	25 ml/1 minute		5/4 sec	
75° F after 200° F	25 ml/1 minute		10/20 sec	
Visual	Transparent, Uniform No sediment, etc.		Clear	
Neutralization No.	0.50 max		.12	
"F" Rubber Swell, % 400° F/72 hr	+10 to +25		23.5	
Cracking Under Tension	None		None	
Trace Sediment	.005 ml		< .005	

By _____ Date 5/25/70

NOTE: Infrared indicates material to be similar to Mobil Jet II



1. SUPPORT BEARING
2. LOAD BEARING
3. TEST BEARING
4. AXIAL LOAD WHEEL
5. RADIAL LOAD ROD
6. OIL FLOWMETER
7. AIR
8. PRESSURE PROBE
9. TEMPERATURE PROBE
10. CHIP DETECTOR

Figure 31. Test Rig Facilities: Schematic Diagram.

LYCOMING GAS TURBINE TEST DATA

Sheet No. 1

ENGINE N/A CONFIGURATION Ball Brg CONTRACT _____ DATE _____ TEST CELL 4

TEST TITLE Bearing Retainer Analysis Test Rig OPERATOR _____ OBSERVER _____

RUN NO.	TIME	Temperature																		
		Brg Output G. B. 2	G. B. Oil Out 10	11	Slip Ring 12	13	14	Oil In Test Brg 15	Oil Out Test Brg 16	Oil In Sup Brg 17	Oil Out Sup Brg 18	Roller Brg 19	20	Test Ball Brg 21	22	23	24			
1																				
2																				
3																				

LYCOMING GAS TURBINE TEST DATA

Sheet No. 2

ENGINE N/A CONFIGURATION Ball Brg CONTRACT _____ DATE _____ TEST CELL 4

TEST TITLE Bearing Retainer Analysis Test Rig OPERATOR _____ OBSERVER _____

RUN NO.	TIME	Pressure																		
		Test Brg Cav Press 17	Oil Mist Multi Plexer 18	Oil In Sup Brg 19	Supply Air 20	Air Oil Mist 21	Gear Box Oil 22	Oil In Test Brg 23	Oil Out Test Brg 24	Fwd Wheel Thrust Load 25	Aft Wheel Press 26		Oil Flow Test Brg + Roller	Oil Flow Sup Brg				Vib. Turb Dis/Vel	Vib. G. B. Dis/Vel	Vib Rig Dis/Vel
1																				
2																				
3																				

Figure 32. Typical Data Sheets.

TEST BEARING DESCRIPTION

Ball Bearings

The ball bearings used for the test program are typical of main shaft thrust bearing designs of aircraft gas turbine engines. The bearing design is of an angular contact, split inner ring configuration with a one-piece, machined, outer-land-piloted retainer. Complete design details are given in Figure 33 (PSK 18123). Some modifications to the test bearing were made to obtain desired inner and outer race surface texture.

One test bearing, S/N 003, was reworked to produce a race surface roughness of 5AA maximum; another, S/N 006, was reworked to produce a race surface roughness of 24AA maximum. This rework was done in order to evaluate the effect of surface texture/EHD film interaction upon bearing performance. Calculations indicate that the film thickness-to-composite surface roughness factor for the 5AA finish bearing at 100° F oil-in temperature and 20,000 rpm is greater than 4.0, and consequently the bearing would operate with full film separation of the contacting surfaces. In contrast, the bearing with the 24AA surface finish would have a film thickness-to-composite surface finish factor of less than 1.0 at 200°F oil-in and 20,000 rpm, and it should operate without full film lubrication, i.e., with boundary lubrication.

Modifications to the retainer were also required to facilitate the placement of strain gages for pocket load measurements Undercuts were machined into three adjacent pockets as indicated in Figure 34 (PSK 17438). The undercuts provide a recess in the pocket to prevent contact of the strain gages with the ball, and also increase the flexibility of the cross beam section to produce measurable strain levels for load measurements under the anticipated force levels.

A complete pretest dimensional inspection of the test bearings was performed on critical dimensions and characteristics. Clearances, surface textures, groove radii, race runouts, etc., were measured and recorded, and are presented in Table VI and Figures 35 through 42.

Roller Bearings

The roller bearings employed for the test program are also typical of main shaft gas turbine designs. The bearings incorporate cylindrical rollers in a one-piece, machined retainer that is piloted on inner ring shoulders. Bearing details are presented in Table VII.

Test retainer modifications similar to the ball bearing retainer modifications were required to provide for strain gage placement and suitable beam flexibility to facilitate measurement of the pocket loads, as depicted in Figure 43. Here, also, a complete pretest dimensional inspection was performed on the critical characteristics of the bearing. Results are presented in Figures 44 through 47.

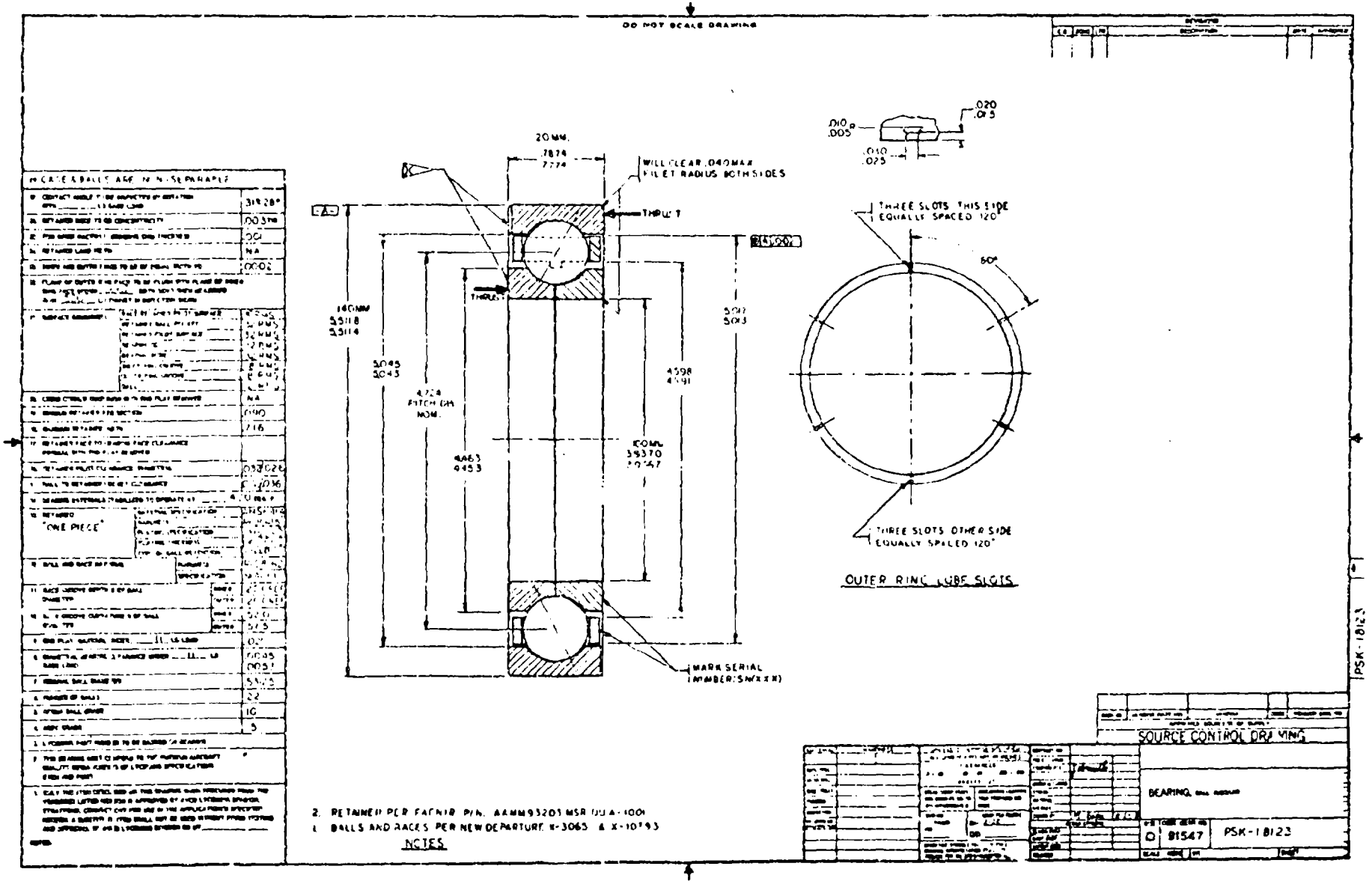


Figure 33. Ball Bearing PSK 18123.

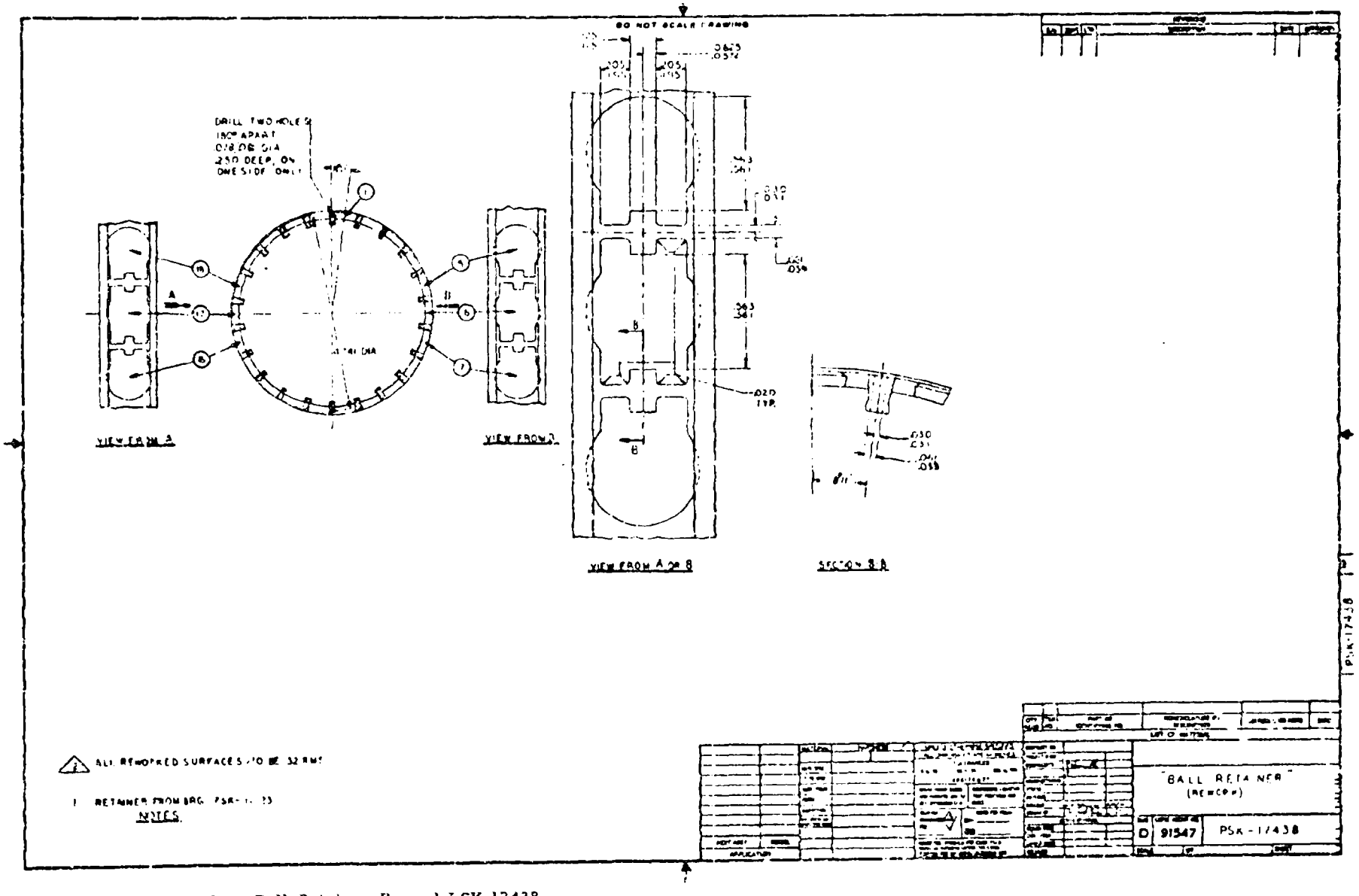


Figure 34. Ball Retainer Rework PSK 17438.

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TABLE VI. TEST BALL BEARING INSPECTION DATA, DETAIL INTERNAL GEOMETRY

	Bearing S/N	
	003	006
Outside Diameter (in.)	5.5110/5.5109	5.51163/5.51162
Inside Diameter (in.)	3.9360/3.93596	3.93666/3.93665
Ball Diameter (in.)	.53105	.53103
Inner Race Finish (AA)	4-5	30-35
Outer Race Finish (AA)	4-5	24
Ball Finish (AA)	3	2.5
Internal Clearance (in.)	.0050	.0045
Inner Race Groove Radius (thrust)	.2728	.27676
Outer Race Groove Radius	.2728	.2753
Inner Race Groove Radius (nonthrust)	.2741	.27676

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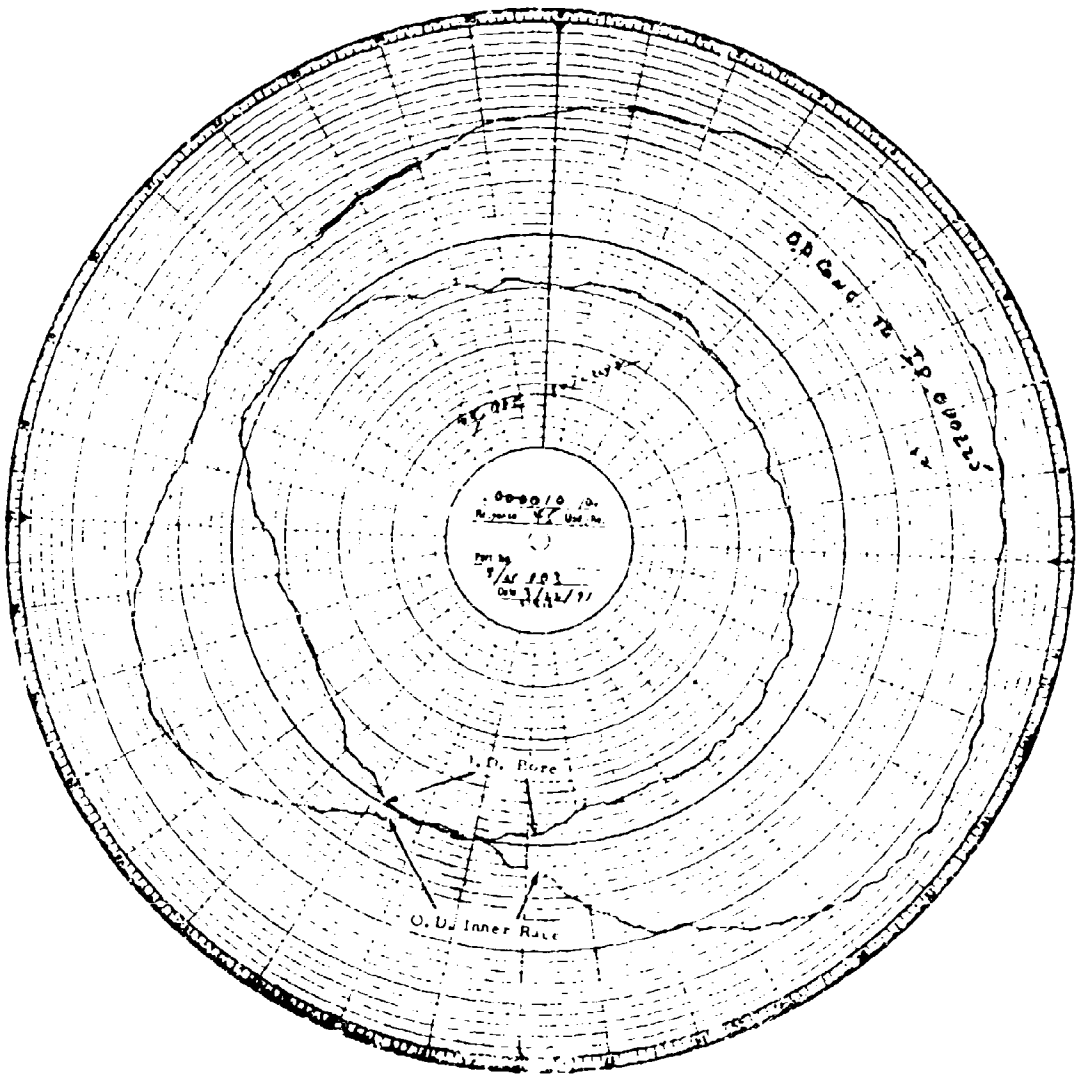

Figure 35. Ball Bearing S/N 003 Inner Race Roundness and Concentricity.

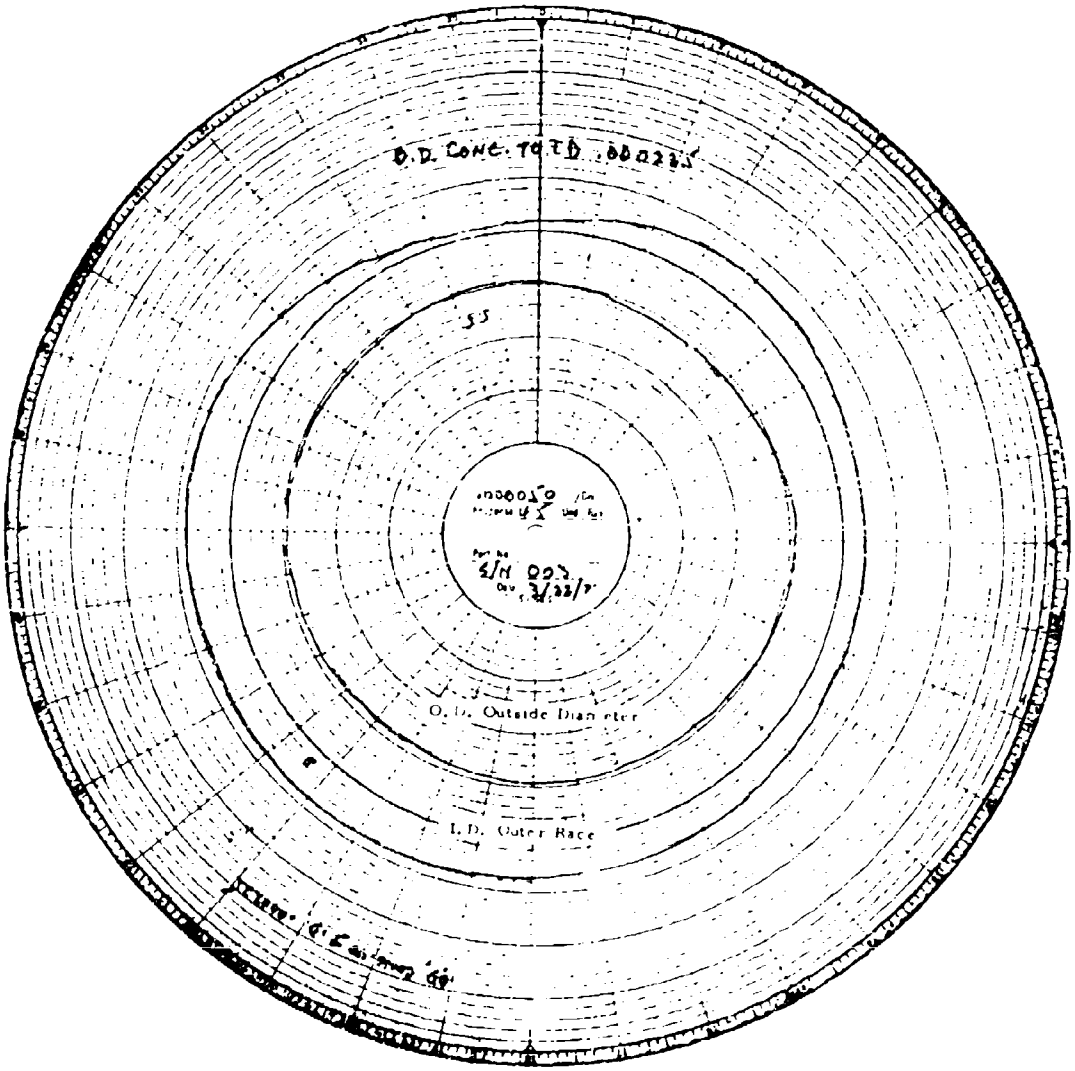

Figure 36. Ball Bearing S/N 003 Outer Race Roundness and Concentricity.

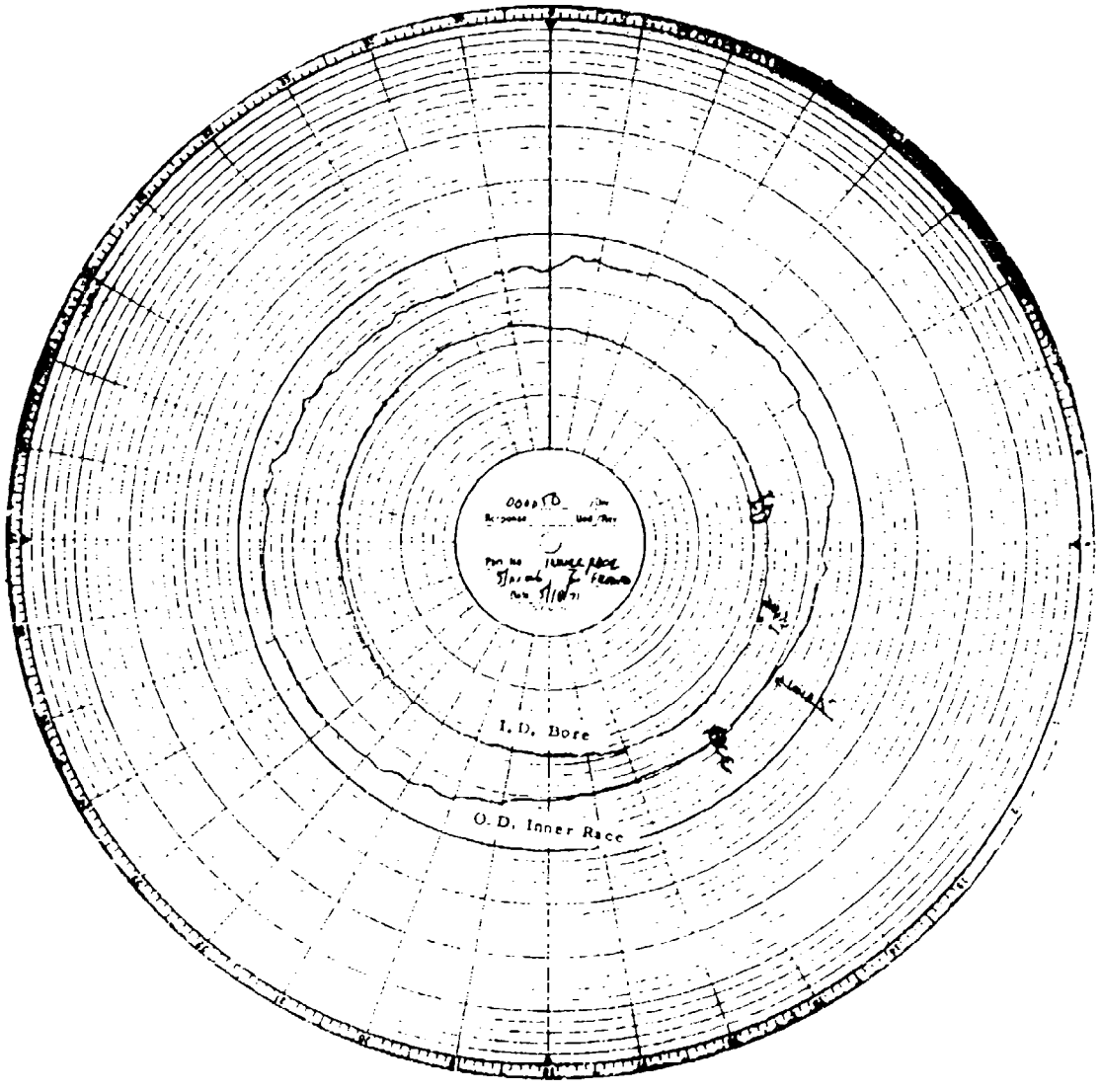

Figure 37. Ball Bearing S/N 006 Inner Race Roundness and Concentricity.

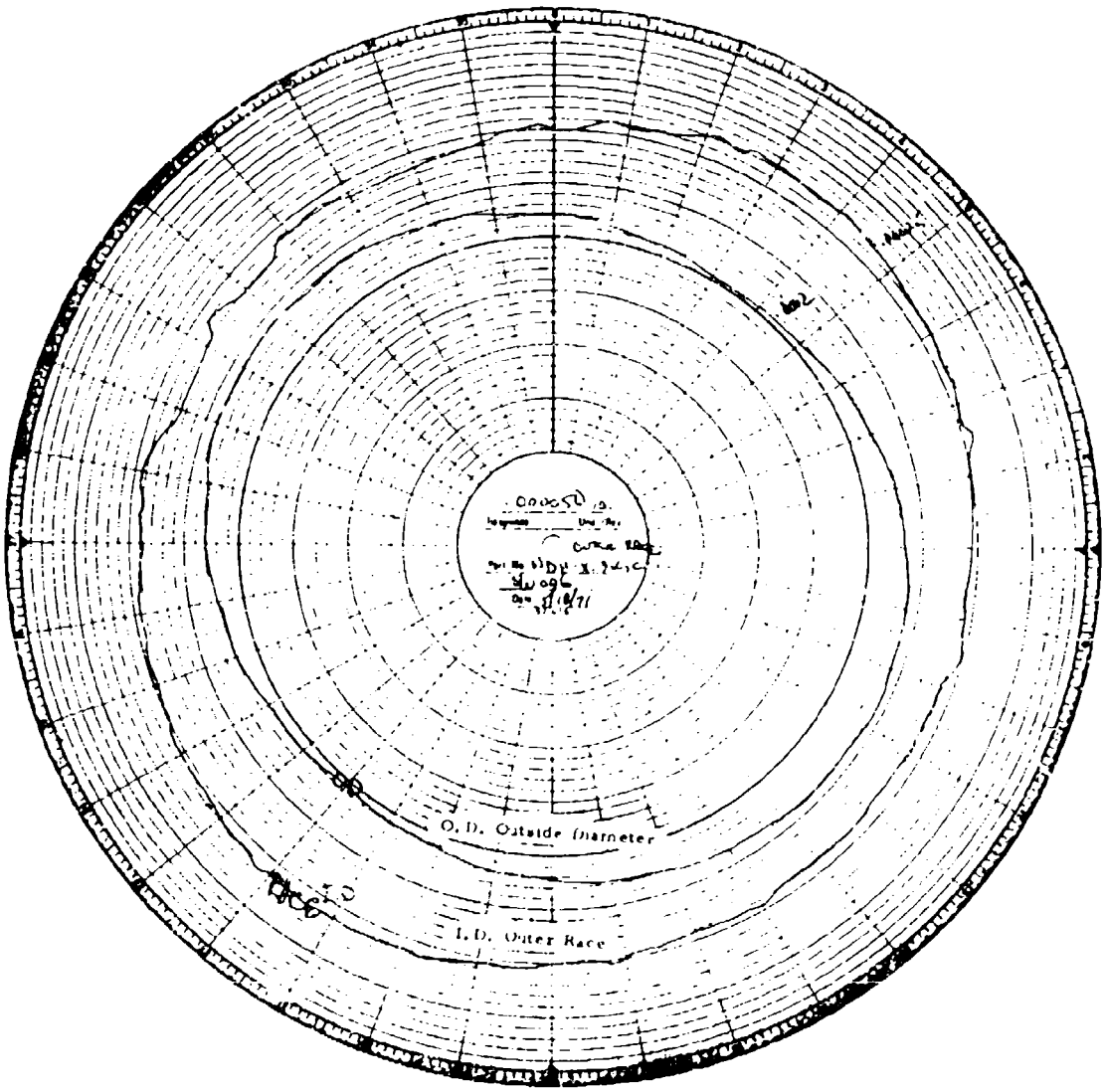

Figure 38. Ball Bearing S/N 006 Outer Race Roundness and Concentricity.

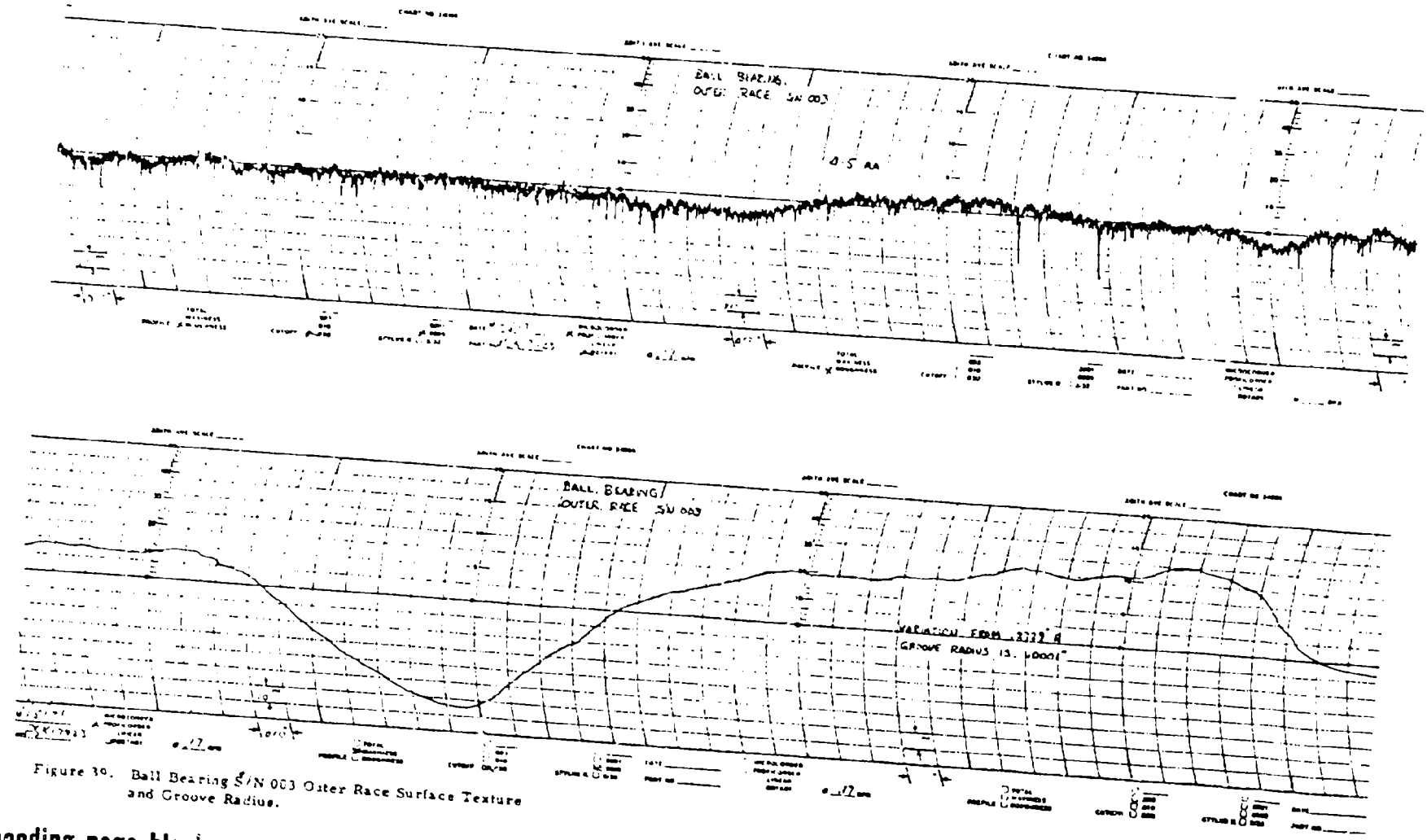

Figure 30. Ball Bearing S/N 003 Outer Race Surface Texture and Groove Radius.

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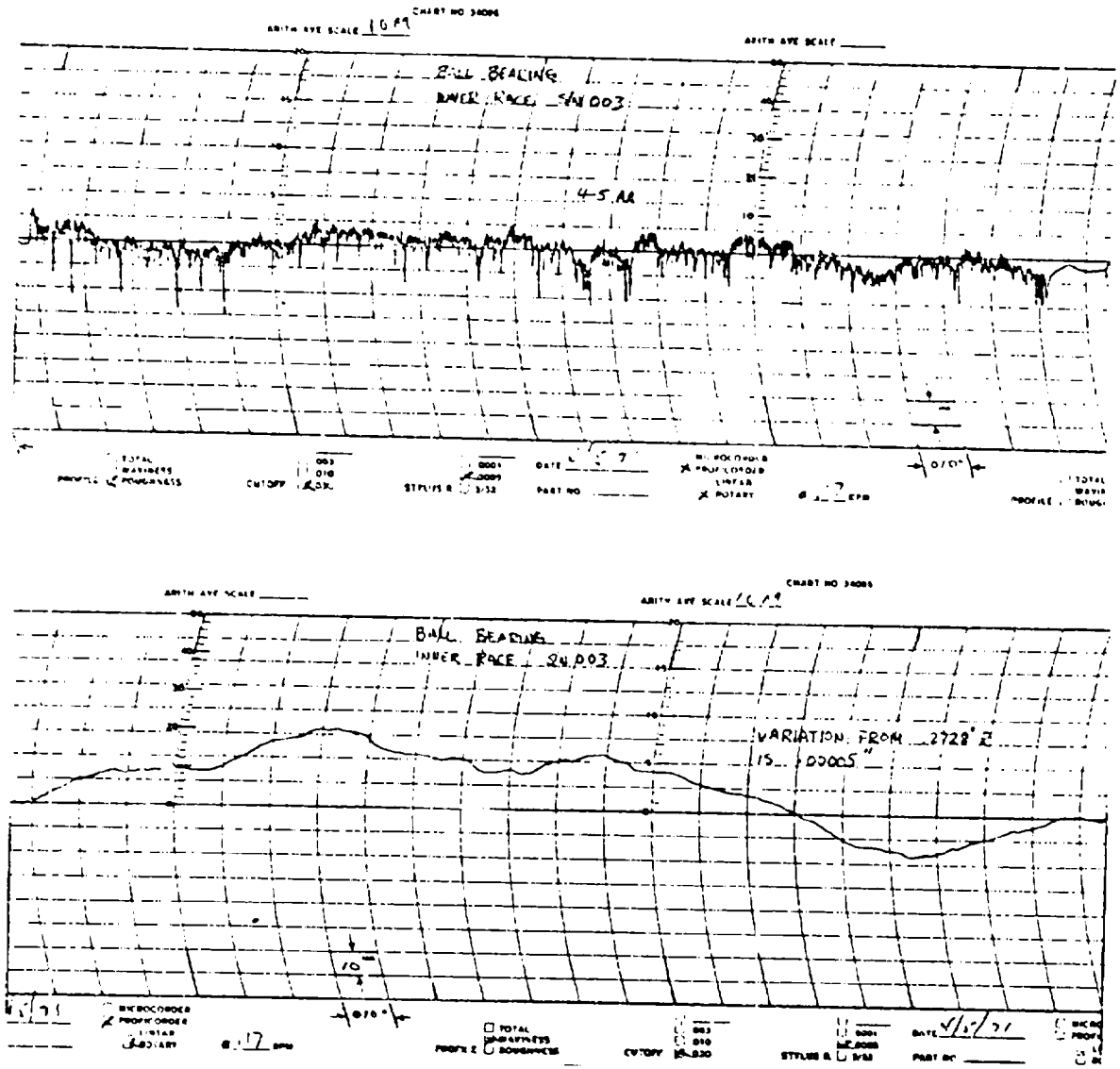

Figure 40. Ball Bearing S/N 003 Thrust Inner Race Surface Texture and Groove Radius.

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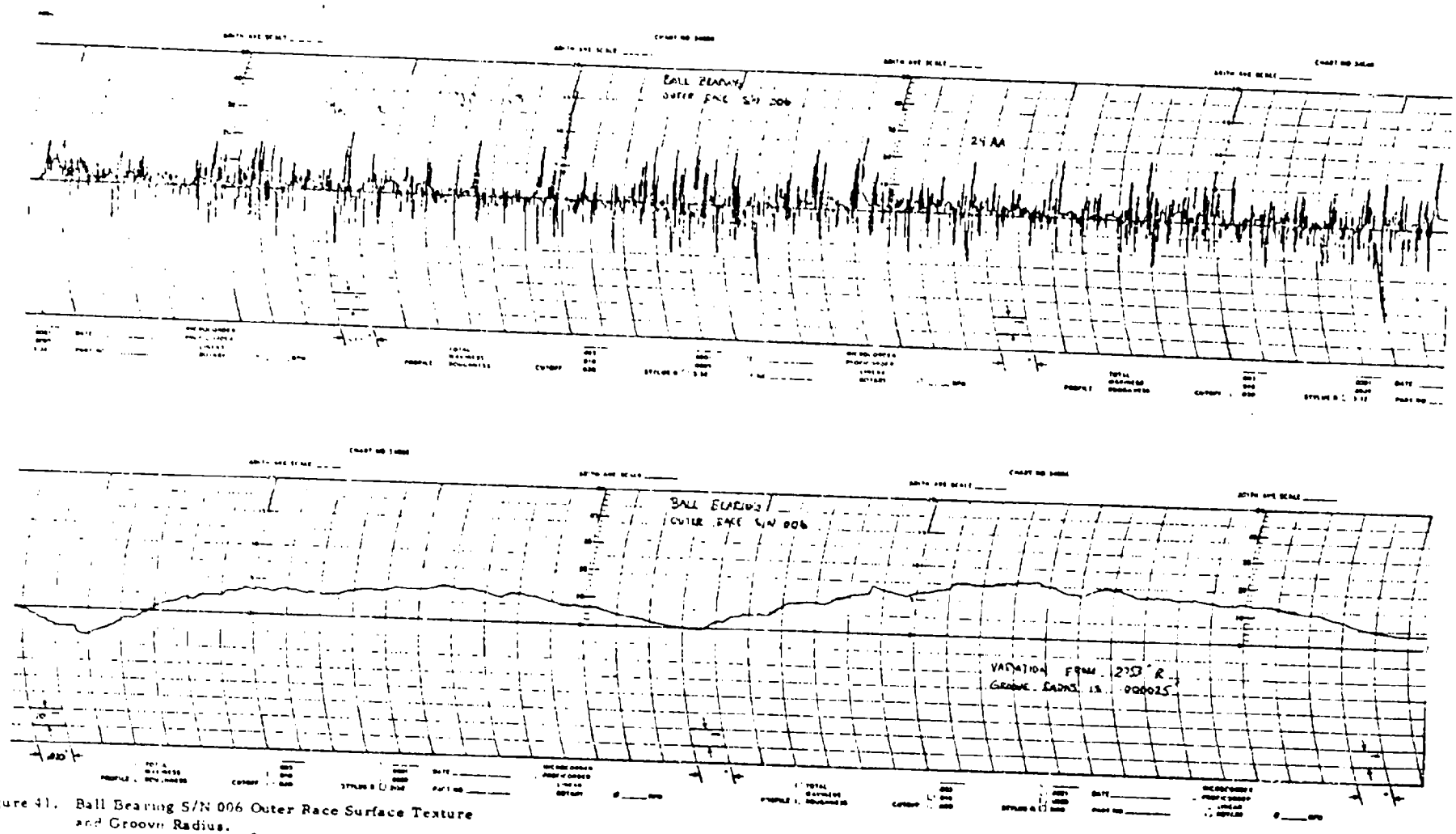

Figure 41. Ball Bearing S/N 006 Outer Race Surface Texture and Groove Radius.

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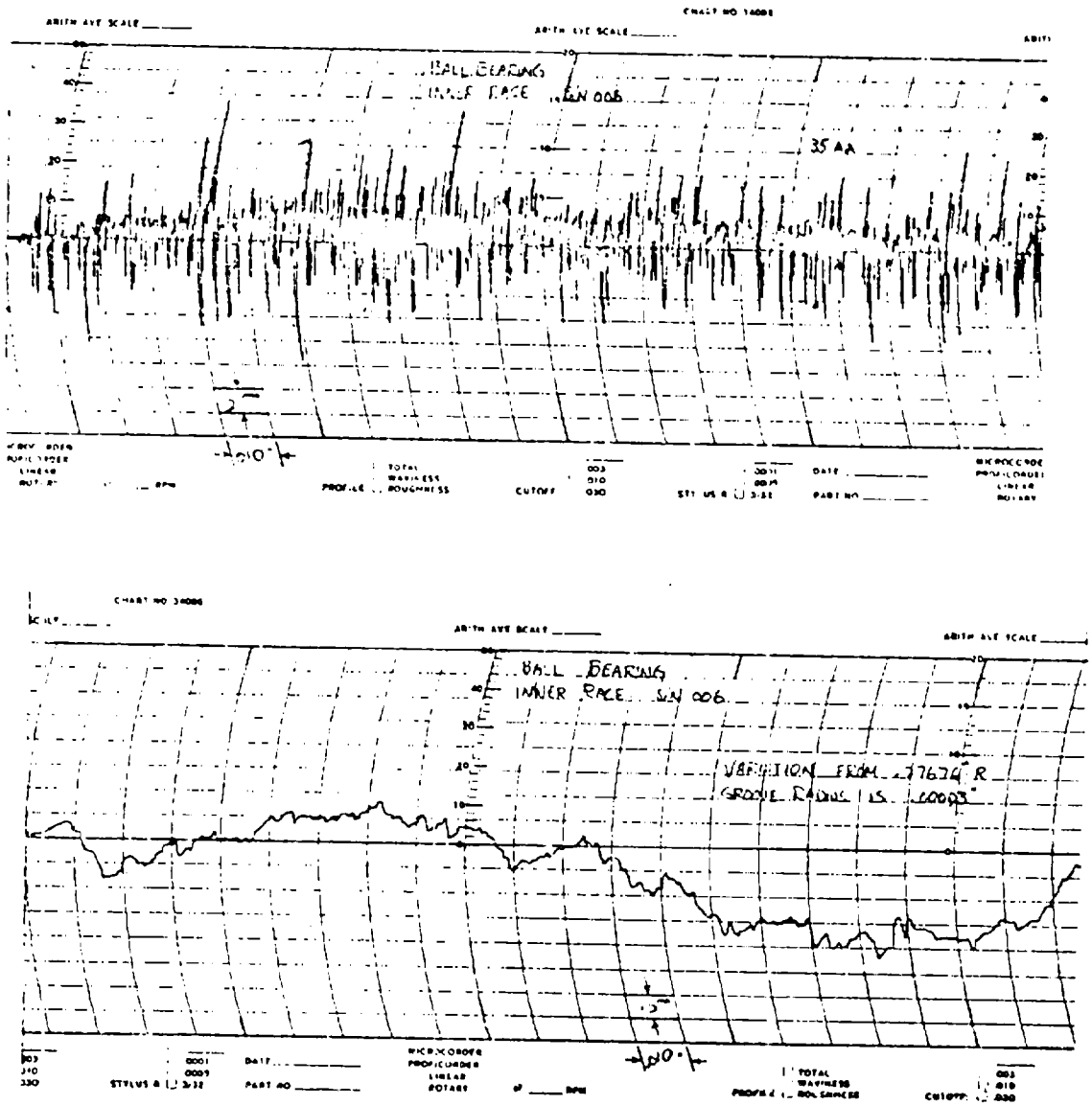

Figure 42. Ball Bearing S/N 006 Thrust Inner Race Surface Texture and Groove Radius.

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TABLE VI. ROLLER BEARING ENGINEERING DATA	
Lycoming Identification	MRC R-1920-C
Bearing Application	Test Rig
Bearing Type	Straight Thru Outer
RBEC Grade	5
Bearing to Conform to Lycoming Specifications	P6903, E-1004
Bore	100 MM
Outside Diameter	140 MM
Width	20 MM
Number of Rollers	28
Roller Diameter	11 MM = .4331
Roller Length	11 MM = .4331
Roller Pitch Diameter	4.7244 Ref.
Roller Diameter Variation	± .000025
Roller Length Variation	± .0001
Roller End Radius	.030/.040
Roller Crown Radius	35.0
Roller Flat Length	.144/.271
Roller Ends Square to Outside Diameter	.0001
Parallelism of Roller Active Surface	.000050
Diametral Clearance under 11-lb Gage Load	.0055
Inner Ring Shoulder Height, % of Roller Diameter	22%
Outer Ring Shoulder Height, % of Roller Diameter	NA
Inner Ring Shoulder Diameter	4.479/4.469
Outer Ring Shoulder Diameter	NA
Cross Corner Dimension	NA
Roller to Shoulders End Play	.0005
Raceway Surface Finish	6RMS
Roller Diameter Surface Finish	3RMS
Roller Ends Surface Finish	3RMS
Ring Pilot Surface Finish	16RMS
Retainer Pilot Surface Finish	20RMS

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TABLE VII - Continued	
Roller and Ring Material	SAE 52100 Steel
Roller and Ring Hardness	R_c 58-62
Retainer Material	Bronze
Retainer Hardness	
Retainer Plating	AMS 2412
Retainer Plating Thickness	.0005/.0015
Material Stabilized to Operate at	-65° to +350°F
Pilot Surface	Inner
Pilot Clearance (Total Diametral)	.025 - .035
Roller Axial Pocket Clearance	.010/.015
Roller Circumferential Pocket Clearance	.007/.012
Minimum Retainer Pocket Corner Radii	
Retainer Face to Bearing Face Clearance, Min. End Play Removed	
Roller Pocket Location in Retainer - Axially	.001 min
Roller Pocket Location in Retainer - Circumferentially	
Minimum Retainer Web Section	
Minimum Retainer Land Width	
Retainer Pilot Surface Runout	
Retainer O.D. to Bore Concentricity	
Type of Retention	Snap-In Type
Max/Min Diameter over/Under Roller	
Minimum Roller Drop	.035 max
Runout of Outer Ring Shoulder Diameter	
Runout of Inner Ring Shoulder Diameter	
Note: Cage, Inner Race and Rollers are Nonseparable.	

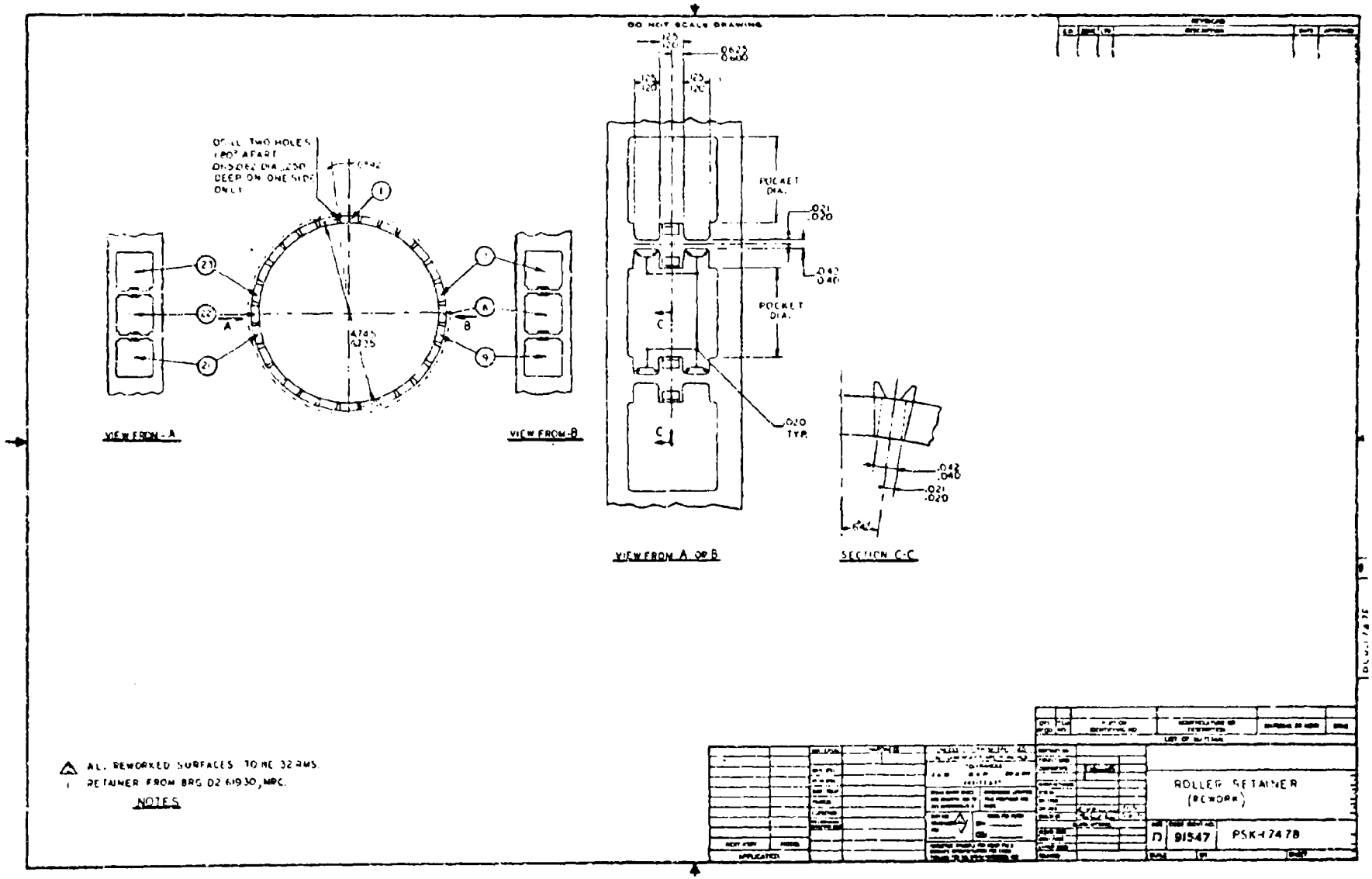

Figure 43. Roller Retainer Rework PSK 17478.

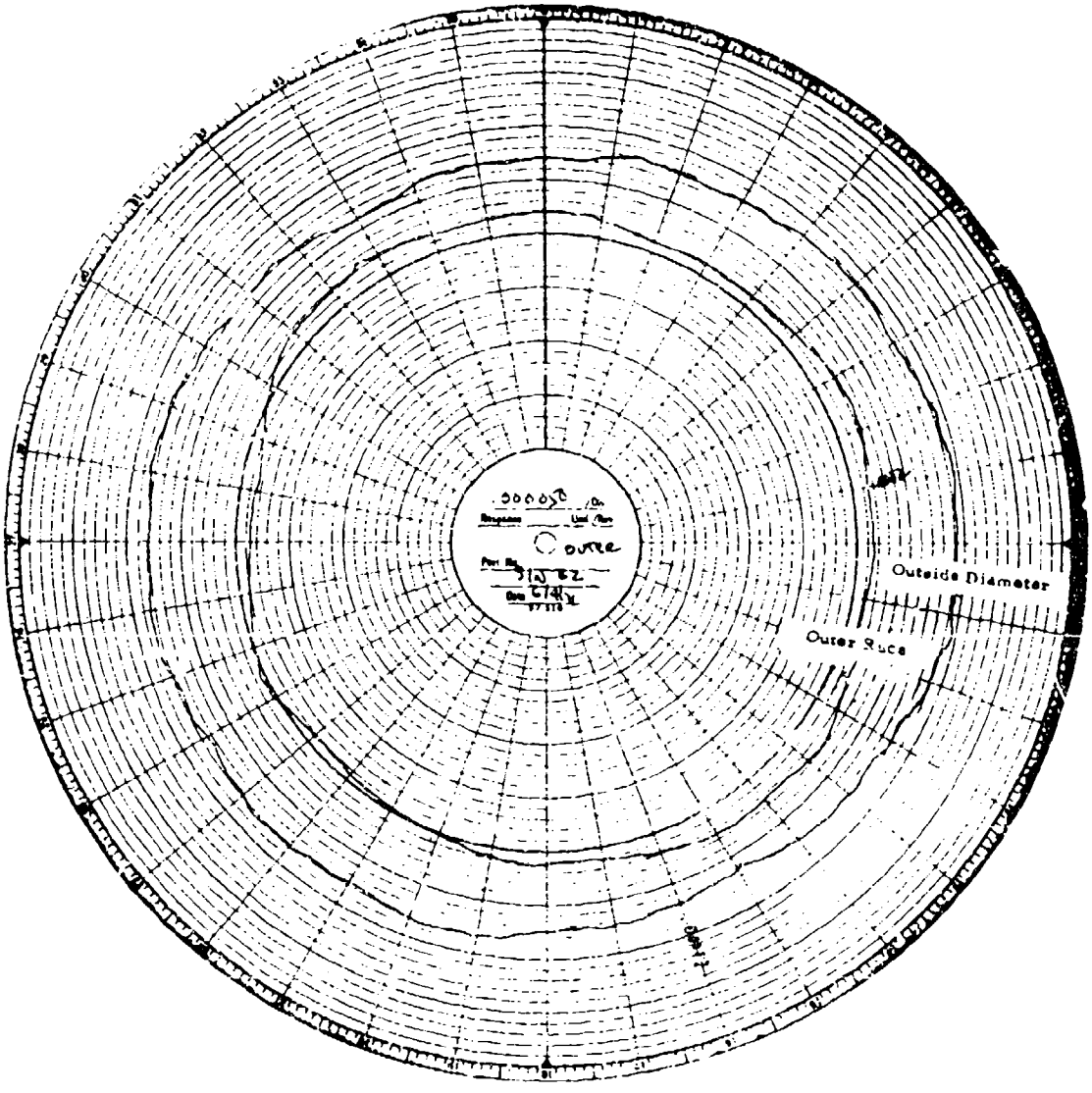

Figure 44. Roller Bearing Outer Ring Roundness and Concentricity.

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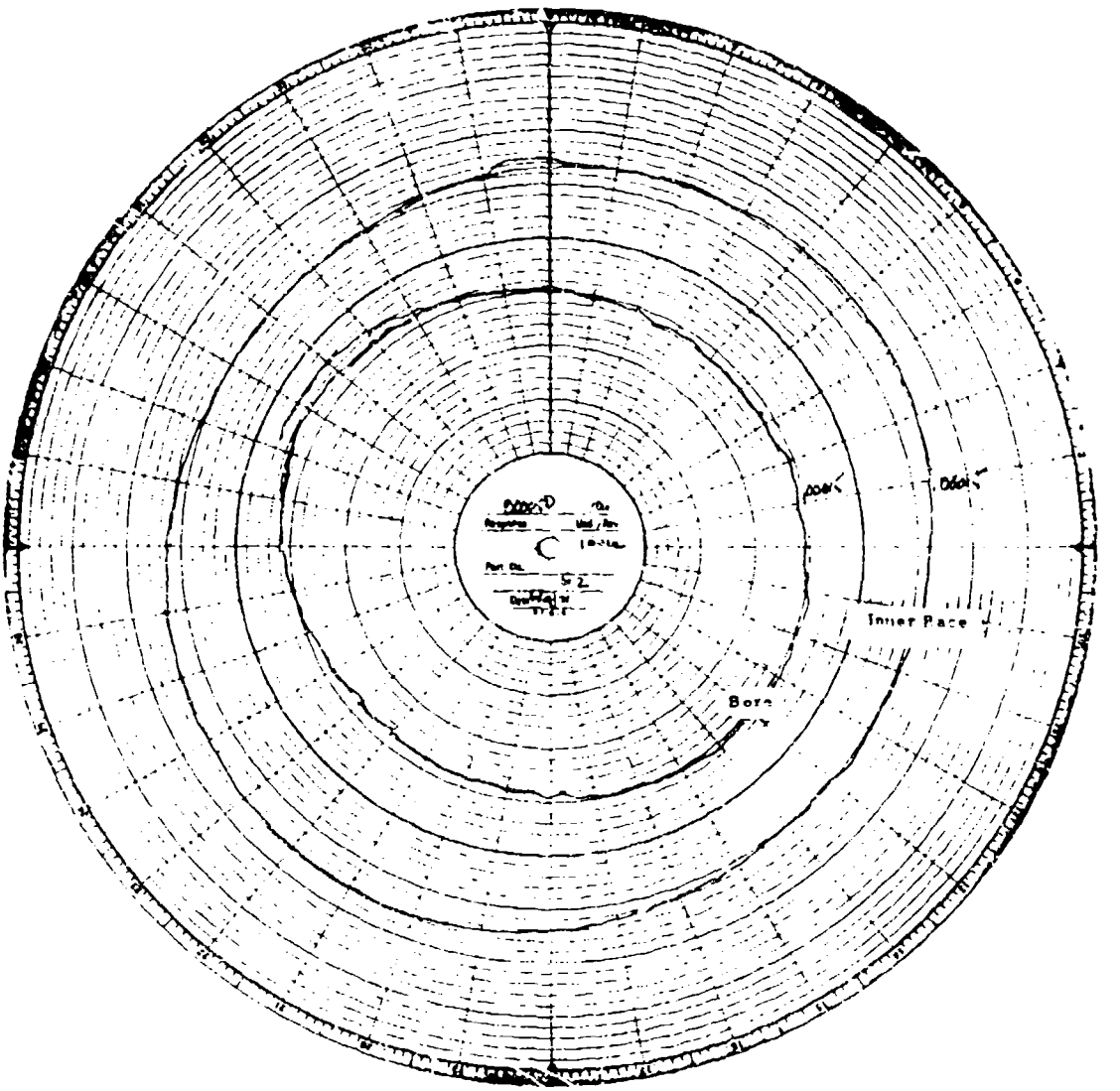


Figure 45. Roller Bearing Inner Ring Roundness and Concentricity.

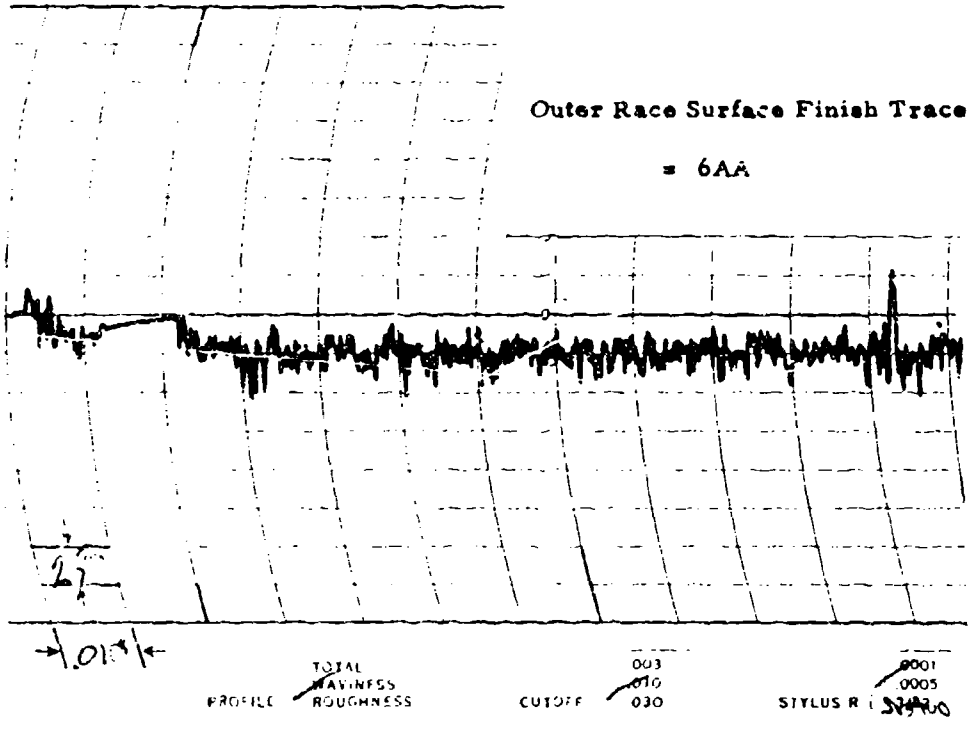


Figure 46. Roller Bearing Outer Race Surface Finish.

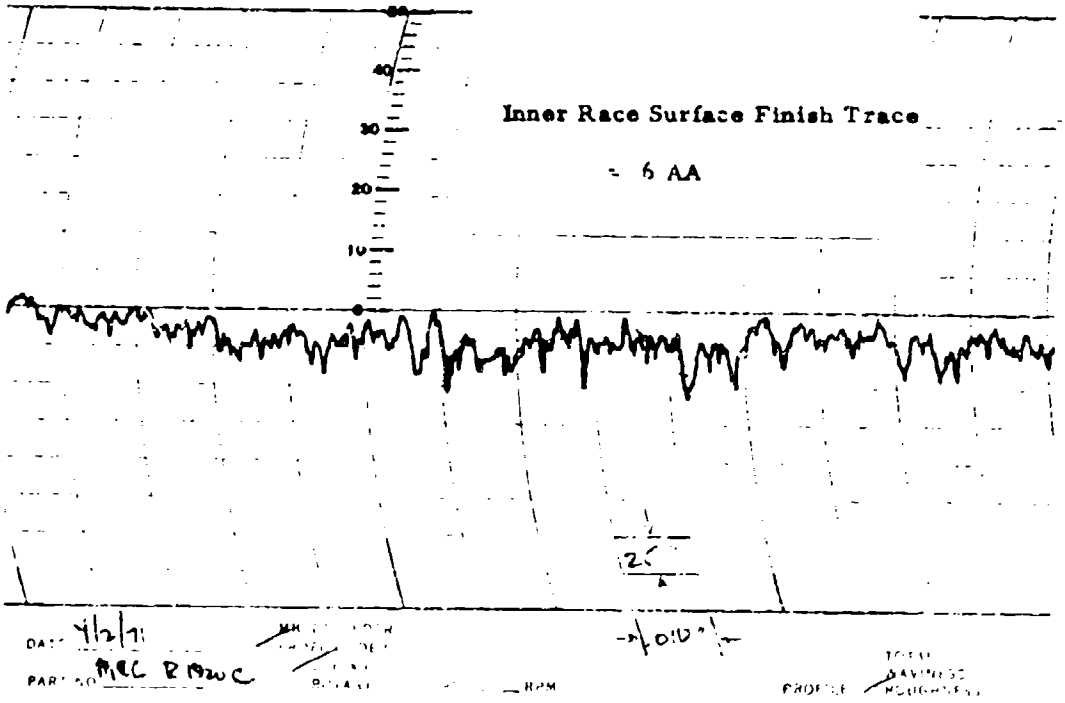


Figure 47. Roller Bearing Inner Race Surface Finish.

INSTRUMENTATION

Instrumentation was incorporated in the bearing retainer test vehicle (F/N TE 21869), Figure 48, to measure the following key parameters:

1. Inner race temperature - °F
2. Outer race temperature - °F
3. Bearing retainer speed - rpm
4. Inner race speed - rpm
5. Radial load on bearing - lb
6. Force exerted by ball/roller on the bearing retainer pocket - lb
7. Ball attitude and three-axis rotational velocity (ball bearing test only)
8. Torque exerted on the retainer by slip ring drag and inertia - in.-lb
9. Bearing retainer and torque element temperature - °F

Items 1 through 7 were required to compare test results with the computer program output. Item 5 was a measurement duplication. Radial force was applied through a hydraulic ram with a known cross-sectional area with a known pressure. Item 7 was to provide an instantaneous ball attitude readout. Item 8 was to measure and divorce the effects of slip ring and associated hardware from the retainer data required by Item 6. Item 9 was to evaluate the effects of the retainer environment on Items 6 through 8.

A description of each item of instrumentation follows:

1. Inner Race Temperature - Inner race temperature measurement on both the bearing under study and the loading bearing was accomplished by embedding thermocouples in the shaft, which provided intimate contact between the thermocouples and the bearing inner race bore.

2. Outer Race Temperature - Outer race temperatures were measured using standard bayonet-type thermocouples in intimate contact with the outer race outer diameter.

3. Ball Retainer Speed - Speed was measured using a magnetic pickup sensing rotation of the slip ring shaft, which was directly coupled to the slip ring driving plate. The slip ring shaft was manufactured with five small lobes and one large lobe. This large lobe produced a magnetic pickup output greater than the other five. The angular relationship between the large lobe and the location of the instrumented

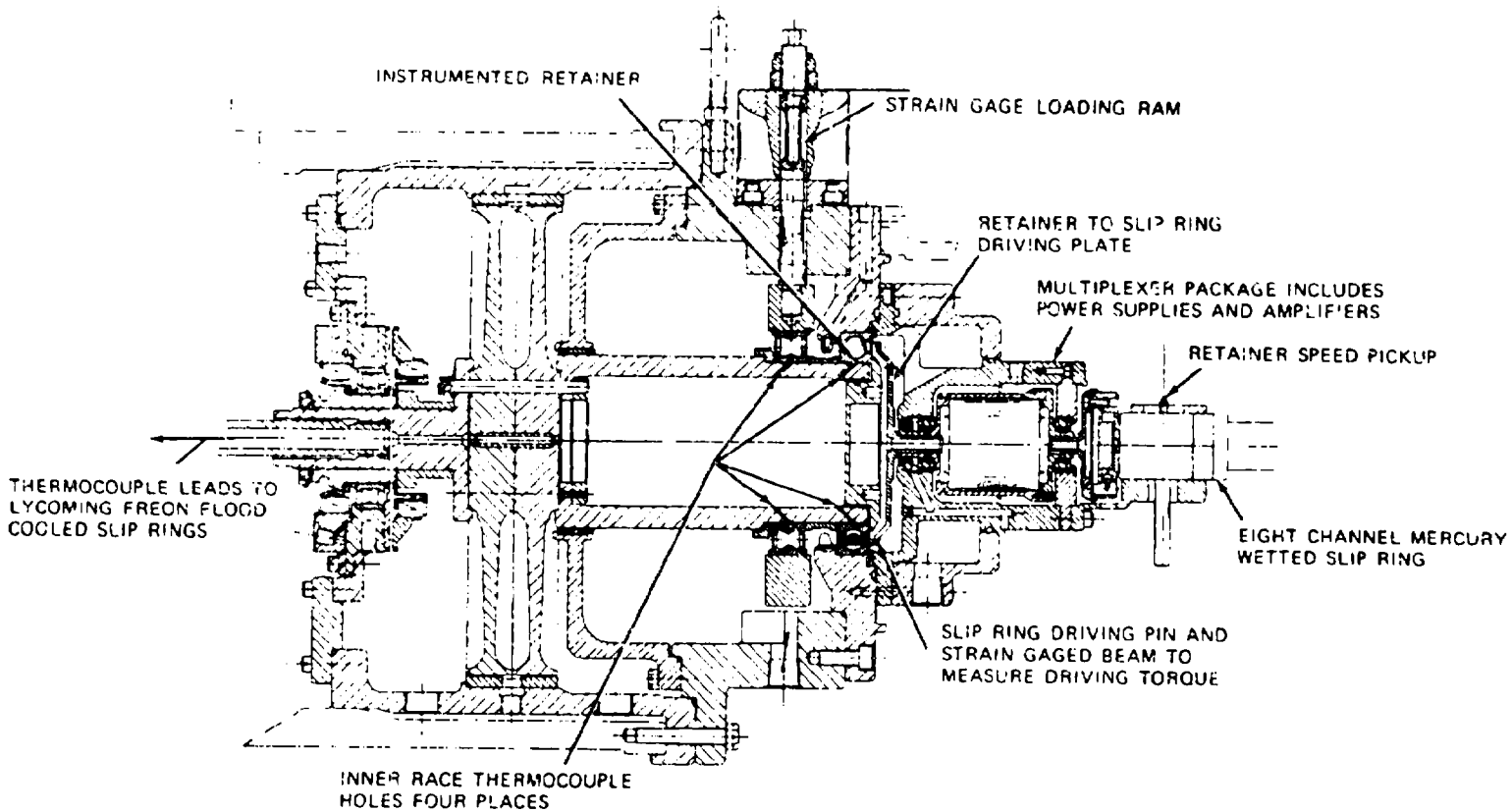


Figure 48. Instrumentation Location.

ball bearing pockets was known, as was the relationship between the stationary magnetic pickup and the point of application of the radial load. A means was thereby provided to relate the speed signal to the angular position of the retainer pockets under study and consequently the ball loading cycle. In the first tests, a strain gage was attached to the outer race to detect ball passing frequency, which is also a function of retainer speed.

4. Inner Race Speed - A magnet pickup was used to detect the speed of a gear directly coupled to the shaft supporting the rotating inner race.

5. Radial Load on Bearing - A hydraulic piston was used to apply radial loads to the bearing. The shaft between the piston loading bearing was instrumented with a four-active-arm strain-gage bridge and was deadweight calibrated. A pressure gage was installed in the line to the piston for cross checking the force on the loading bearing.

6a. Ball Force on Bearing Retainer - To measure the force exerted by the ball on the retainer web, two webs were reworked to accommodate the strain gages and increase the strain sensitivity. The two reworked webs were strain gaged with two fully active four-arm bending bridges, Figure 49. The inside of the reworked web was chosen for strain gaging over the outside, as shown. A photoelastic model was used to determine the optimum location. The model showed the outside to be more cross sensitive to other cage loadings (i.e., a ball pushing axially on the cage would cause larger outputs at the outside of the web than at the inside). The strain gage bridges were calibrated. The first retainers were designed for a maximum strain sensitivity for a 50-pound ball load. At the time the retainer instrumentation was designed, accurate estimates of ball forces were not available, and 50 pounds was selected arbitrarily with the knowledge that the retainer could, if found too insensitive, be reworked and made more sensitive. Later retainers were modified for a maximum strain sensitivity at 25 pounds, based on preliminary results of retainer ball load measurements.

6b. Roller Force - The same type of strain gage instrumentation employed in the ball force measurements was used in the roller force measurement except that the rework to the retainer was only to accommodate the strain gages, not to increase its strain sensitivity, as the webs were already thin enough for good strain sensitivity.

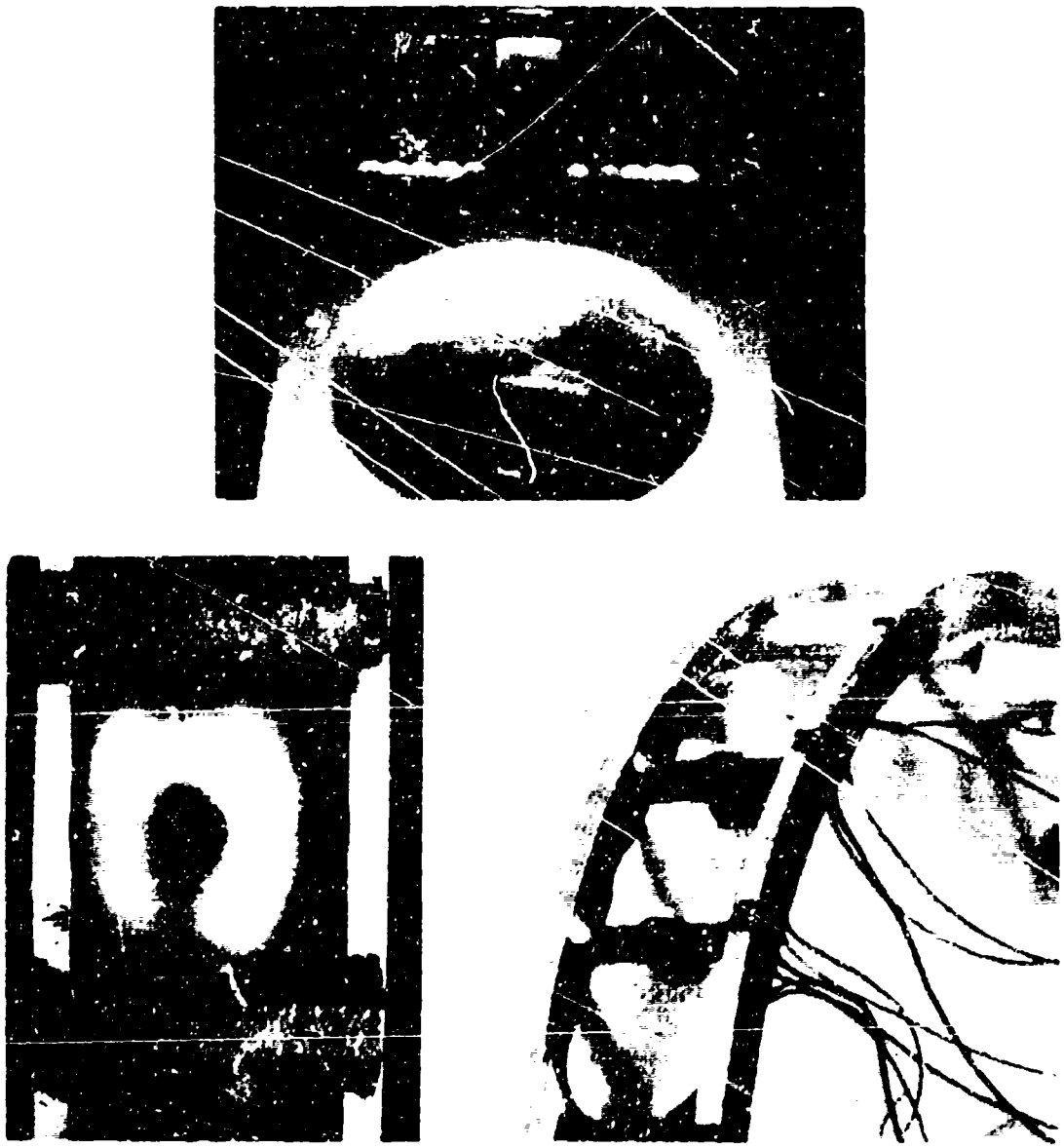


Figure 49. Ball Bearing Retainer Strain-Gage Location and Rework. Top: Two additional strain gages are opposite the two shown, and the four gages are wired on a four-arm bending bridge. Bottom Left: View looking down through retainer ball pocket shows two instrumented retainer webs, each having a four-active-arm bending bridge. Bottom Right: Overall view of strain-gaged ball retainer pocket webs.

7. Ball Bearing Attitude - Each test ball bearing retainer was instrumented with two sets, three per set, of Hall generators (F. W. Bell Inc. Model 105-225). These generators were not standard. They were built for Lycoming in the smallest size practical (0.050 by 0.060 inch). The temperature resistance was also increased from 200° F for standard items to 248° F for this application. The vendor supplied the characteristics for each generator.

A Hall generator is a device whose voltage output is proportional to a current input and the magnetic field to which the Hall generator is subjected (see sketch below):

$$V_H \propto I_c \; B \; (\cos \phi)$$

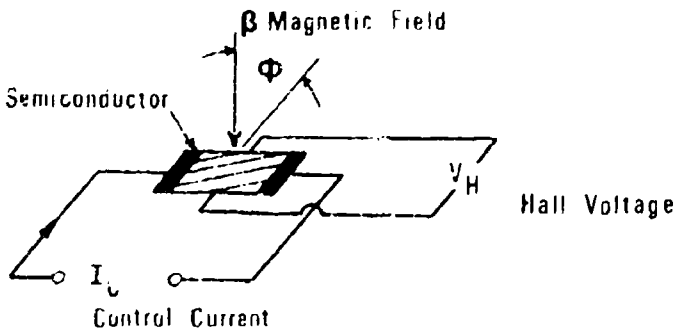
where V_H = Hall voltage

I_c = Hall control current

B = magnetic flux

ϕ = angle between I_c and B

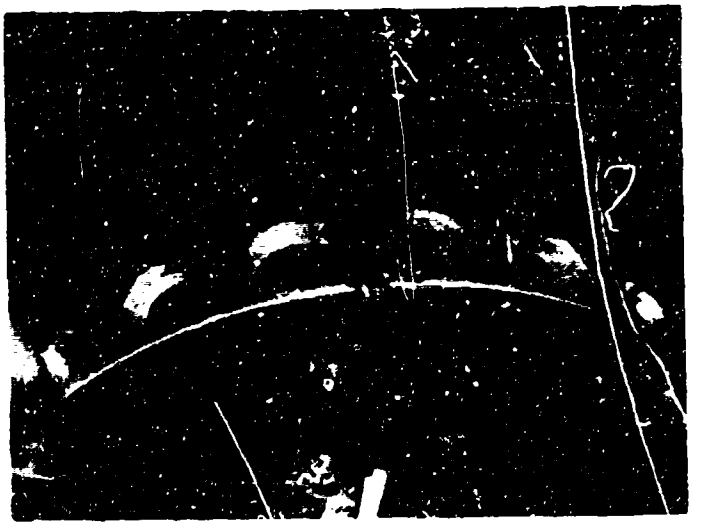
If the control current (I_c) is held constant, the voltage (V_H) is proportional to the magnetic field to which the Hall generator is subjected. Unlike generation of a voltage in a conductor through changing flux fields, the Hall requires no change in flux.



As originally proposed, three mutually perpendicular Hall generators would sense the magnetic field strength of a magnetized ball, making it possible to determine angular location at any instant in time. This was accomplished by producing magnetic poles at 90 degrees to one another. While more than one set of 90-degree poles is possible, it was believed that the resulting confusion and effort required to keep track of similar poles would not be justified. With this arrangement (one set of 90-degree poles), only about 25 percent of the surface of the ball has a field strength sufficient to be detectable by the Hall generators, using the planned generators and ball material. To increase the time during which the ball magnetic field could be sensed, the Hall generators were installed in line at the ball contact point on the retainer, in slightly recessed slots. With the Hall generator and ball bearing material used (M50), sensitivities were so low that the ball clearance had to be reduced to 0.005 inch. Figure 50 shows the Hall generators in place.

Prior to installing the instrumented retainer and magnetized balls in the test rig, each ball was mapped to describe the magnetic field in terms of a Hall generator output versus ball position (Figure 51). This mapping was to be transferred to a large three-dimensional model of the ball. Scale-model Hall generators could then be used to determine the angular position of the ball by rotating the model ball until three voltages obtained from testing matched the three model ball voltages. This position described by three voltages would be unique. It would describe the angular position of the ball with respect to the retainer. This process repeated at defined small time intervals over small azimuth angles would provide three-axis rotational velocities at these angles.

Preliminary static and low-speed laboratory tests indicated the idea to be feasible. In one test a magnetic ball was calibrated as outlined in Figure 51. The ball was spun in a lathe and the output was observed. The output was as expected. In another test the magnetic ball was placed in an instrumented retainer pocket and rotated slowly by hand using a Plexiglas rod bonded to the ball. The three resulting Hall voltages were observed on three oscilloscope channels.



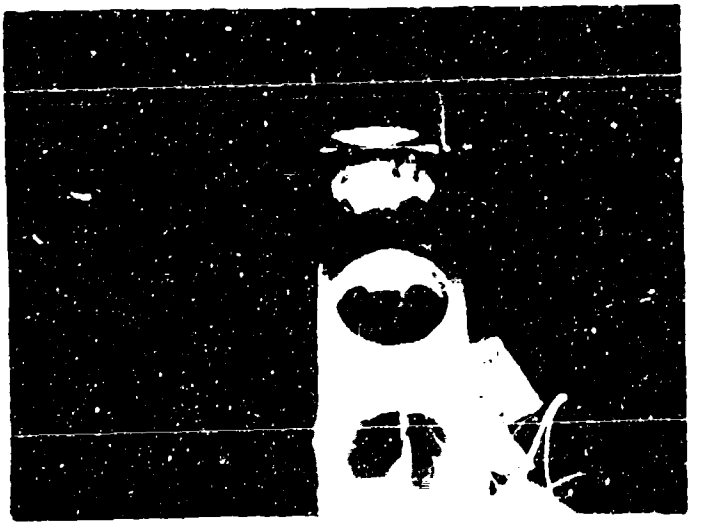


Figure 50. Hall Generators Installed in Ball Bearing Retainer Pockets. Top: Reworks and Hall lead wire routing. Bottom: Location of Hall generators.

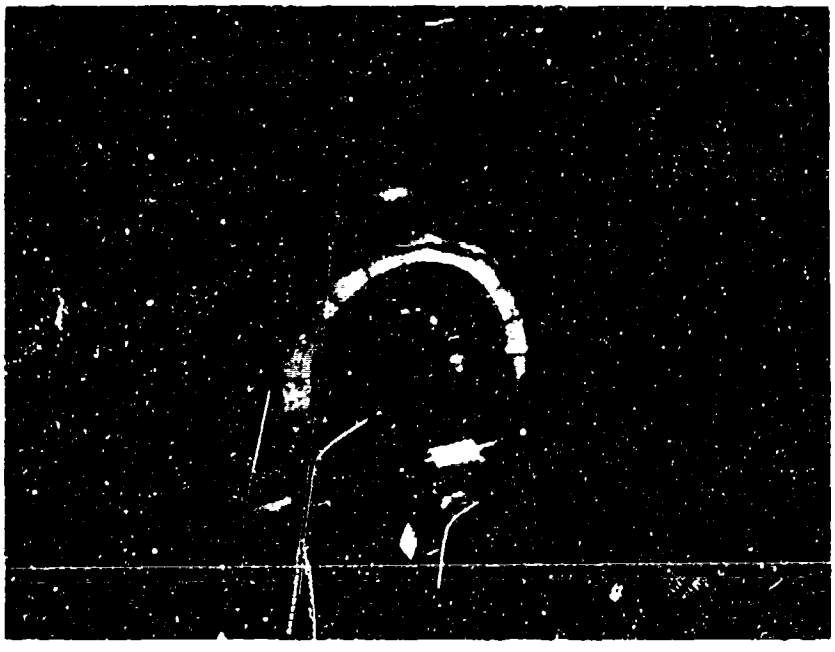


Figure 51. Ball Bearing Magnetic Field Mapping. The magnetic ball is bonded to a Plexiglas rod mounted in a lathe chuck. A Hall generator is mounted on a Plexiglas rod, held in the tool post. In the position shown, the ball is rotated 360 degrees in 20 degree increments. At each increment a reading from the Hall generator is taken. The tool post is then rotated 20 degrees and again the ball is rotated 360 degrees in 20-degree increments. The process is repeated until the mapping is complete.

8. Slip Ring Driving Torque - The bearing retainer would be driving a slip ring and multiplexer system, which it would not normally do. The torque required to drive the slip ring was measured by instrumenting the slip ring driving plate with two strain-gaged cantilever beams. The gaged beams were calibrated for torque and for temperature. The slip ring assembly and driving plate are shown in Figure 52.

   The torque system was designed to read torque in one direction, but these tests showed that significant vibratory torques occur.

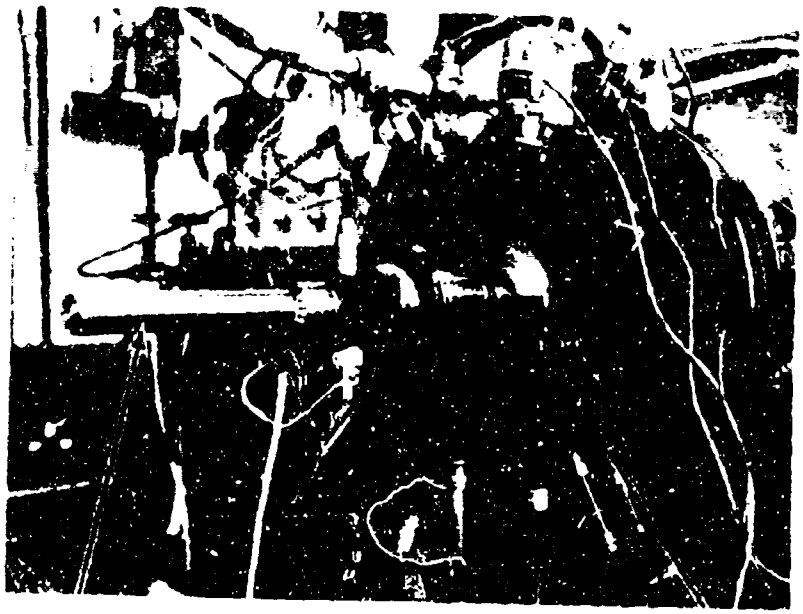
9. Retainer and Torque Element Temperature - To be able to compensate for temperature effects on the retainer strain-gaged bridges, torque element strain-gaged bridges, and Hall generators, thermocouples were installed at these locations.

DATA ACQUISITION

Retainer Data Transmission

The large number of signal leads from the retainer, 12 leads for the Hall generators, 8 leads from the ball force bridges, 4 leads from the torque bridge, and 6 leads from the thermocouples, would have required 30 slip ring channels. If common grounding were used, 26 channels would be required. Slip rings of this size would inherently have large drag characterisitics. A mercury-wetted slip ring with 16 channels was proposed, but this is not a developed item and nonrecurring engineering cost would have been excessive. An eight-channel mercury-wetted slip ring was available, but if this were employed, data could not be taken simultaneously from Hall generators, ball force gages, and the torque driving plate. Simultaneous data acquisition was a goal. The problem of a large number of signals and the requirement for simultaneity suggested the use of a multiplexer.

A multiplexer system, which was designed and built at Lycoming, incorporated constant-current power supplies for the Hall generators, constant-voltage power supplies for the strain-gaged bridges, compensating networks and cold-junction reference for the thermocouples, and buffer amplifiers with a gain of 100 for the output from all the previously mentioned channels. Coupled with an eight-channel mercury-wetted slip ring (Vibro-Meter SA Model MT8/A), this system provided close to the

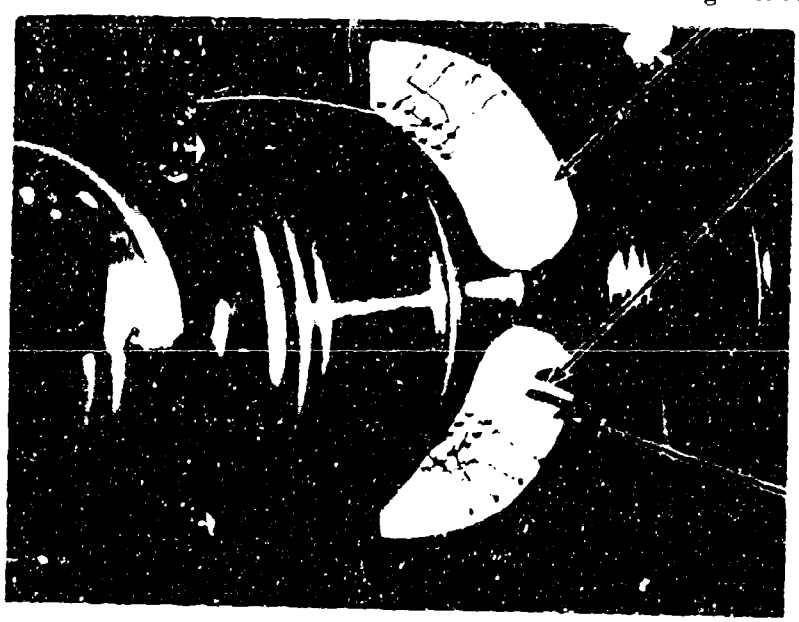


Slip Ring

Driving Plate

Lead Wires From
Bearing Retainer

Driving Plate



Instrumented Cantilever To Measure Slip Ring Driving Torque

Driving Pin From Retainer - A Second Pin Is 180 Degrees From One Shown

Figure 52. Slip Ring and Driving Plate. Top: Overall view. Bottom: Closeup of driving plate. Rotation is counterclockwise.

desired goal for obtaining simultaneous data from the rotating retainer instrumentation. The 30 leads were reduced to four through multiplexing, and three slip ring channels were required to power and sequence the multiplexer. The multiplexer was time divided, with the output from the Hall generators being multiplexed simultaneously, the output from the strain-gaged bridges being displayed simultaneously, and the output from thermocouples being displayed simultaneously. The switching rate of the multiplexer could be varied from 0.1 time per second to in excess of 10,000 times per second. Figure 53 shows typical outputs from the multiplexer.

Inner Race Temperature Data Transmission

The rotating inner race temperatures were transmitted through Lycoming-developed Freon flood-cooled slip rings coupled to the rear of the shaft supporting the test bearing inner race.

Recording

All signals except the inner race temperatures were recorded on magnetic tape. Lockheed tape recorder Model 417D was used. The frequency response of this sytem is 10,000 Hertz. Signals from the inner race were displayed on instrumentation in the test cell and were manually recorded.

Monitoring

During all testing, inner race speed, retainer speed, inner race speed ratio, ball force, radial load, and driving torque were monitored before and after magnetic tape recording. This procedure assured that the quality of the signals was maintained throughout the recording process.

Problem Areas

During this test program three bearing failures occurred which resulted in the loss of data and the elimination of certain instrumentation.

The first failure (a smooth-race ball bearing) was due to shaft unbalance. This failure also resulted in the loss of a Hall generator-instrumented retainer and multiplexer (Figure 54).

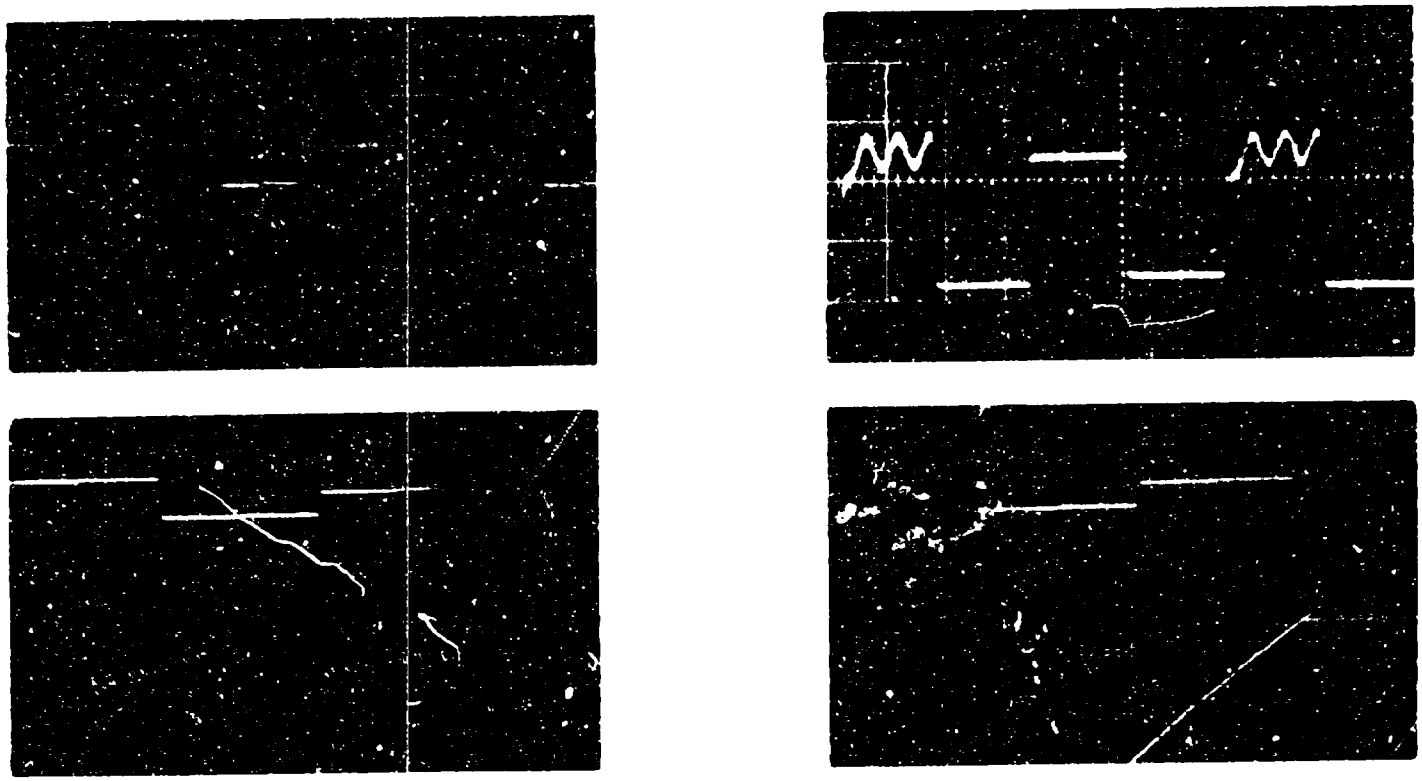


Figure 53. Typical Multiplexer Outputs. Top Left: Output from multiplexer when four different DC voltages are input to the four multiplex channels. Top Right: Output from multiplex when (from left to right) a sine wave is input to the first channel, and a DC voltage is input to the second channel. Zero input to channel (to be used as a reference channel) and same voltage input to channel two is input to channel four. Bottom Left and Right: Outputs from multiplexer just prior to running. Note shift in output of channel four during calibration of one of the web force strain-gage bridges. The shift is approximately 0.2 volt and is equal to 9 pounds of force acting on the retainer pocket web.

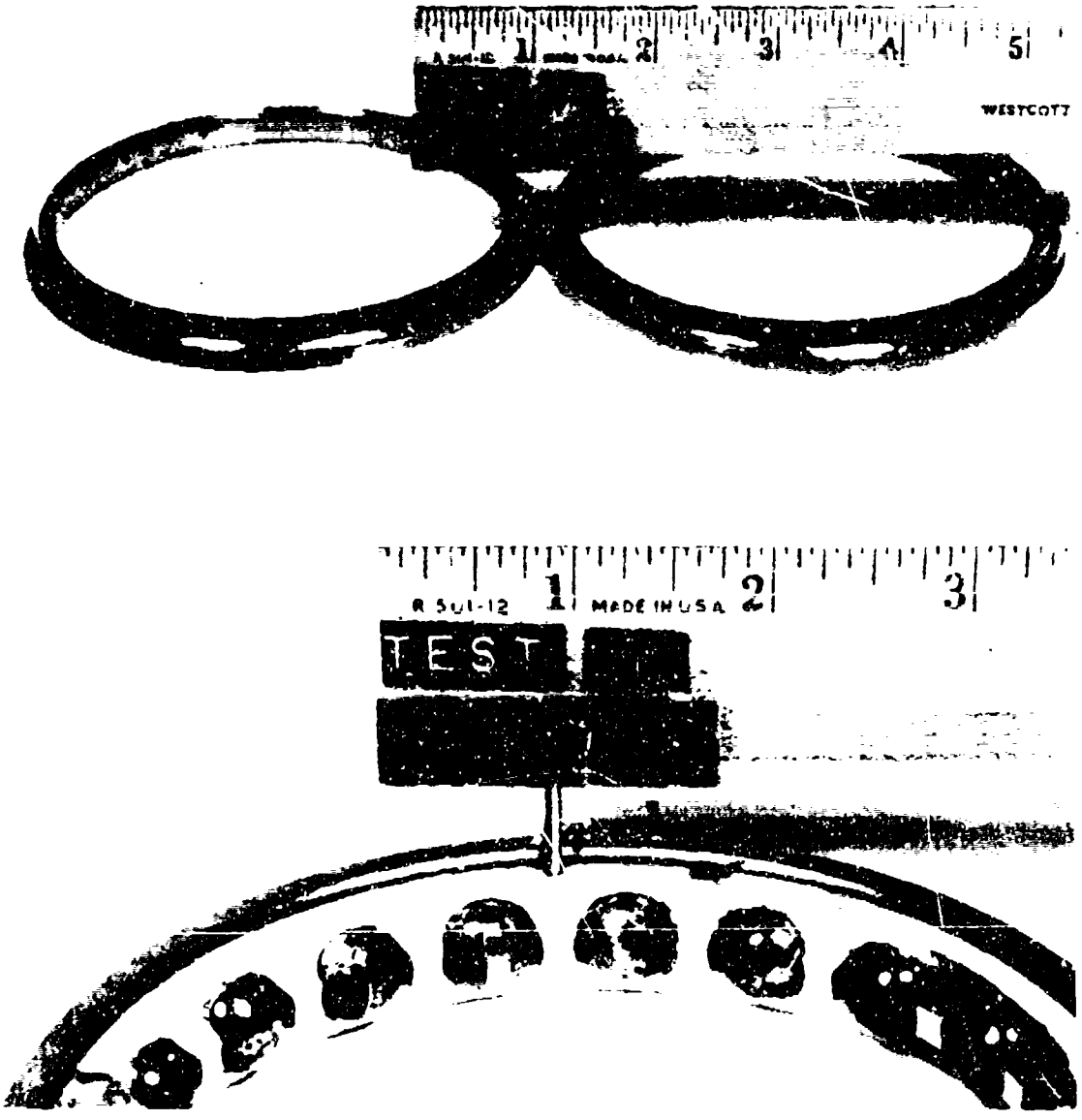


Figure 54. Build No. 2 Bearing Failure; Shaft Unbalance. Top: Inner races. Bottom: Cage, balls, and outer race.

The second failure (a smooth-race ball bearing) was the result of reducing the pocket clearance from 0.022 inch for the normal bearing to 0.005 inch to achieve the necessary Hall generator sensitivity. This failure is shown in Figure 55. When the Hall generators were eliminated to provide the necessary clearance, the multiplexer system was no longer required.

The third failure (a smooth-race roller bearing) was the result of excessive roller skewing during attempts to produce a skid map.

During early test runs, the wire routing from the retainer to the slip ring driving plate was adequate; but during later runs, there were lead wire failures resulting in the loss of data, and improved techniques were devised. The original and the improved lead paths are shown in Figure 56.

TEST PROCEDURE

The test program was accomplished in four builds as follows:

Build	Test Bearing	Lubrication Condition
1	Ball	EHD
2	Ball	EHD
3	Ball	Boundary
4	Roller	EHD

A full EHD oil film was achieved by operating at high speed and moderate loads.

Boundary lubrication was achieved by utilizing races with poor surface finish (24 AA). Experience has shown that this degree of roughness allows metal-to-metal contact.

Load and speed test conditions for the ball and roller bearing tests are defined in Tables VIII and IX respectively.

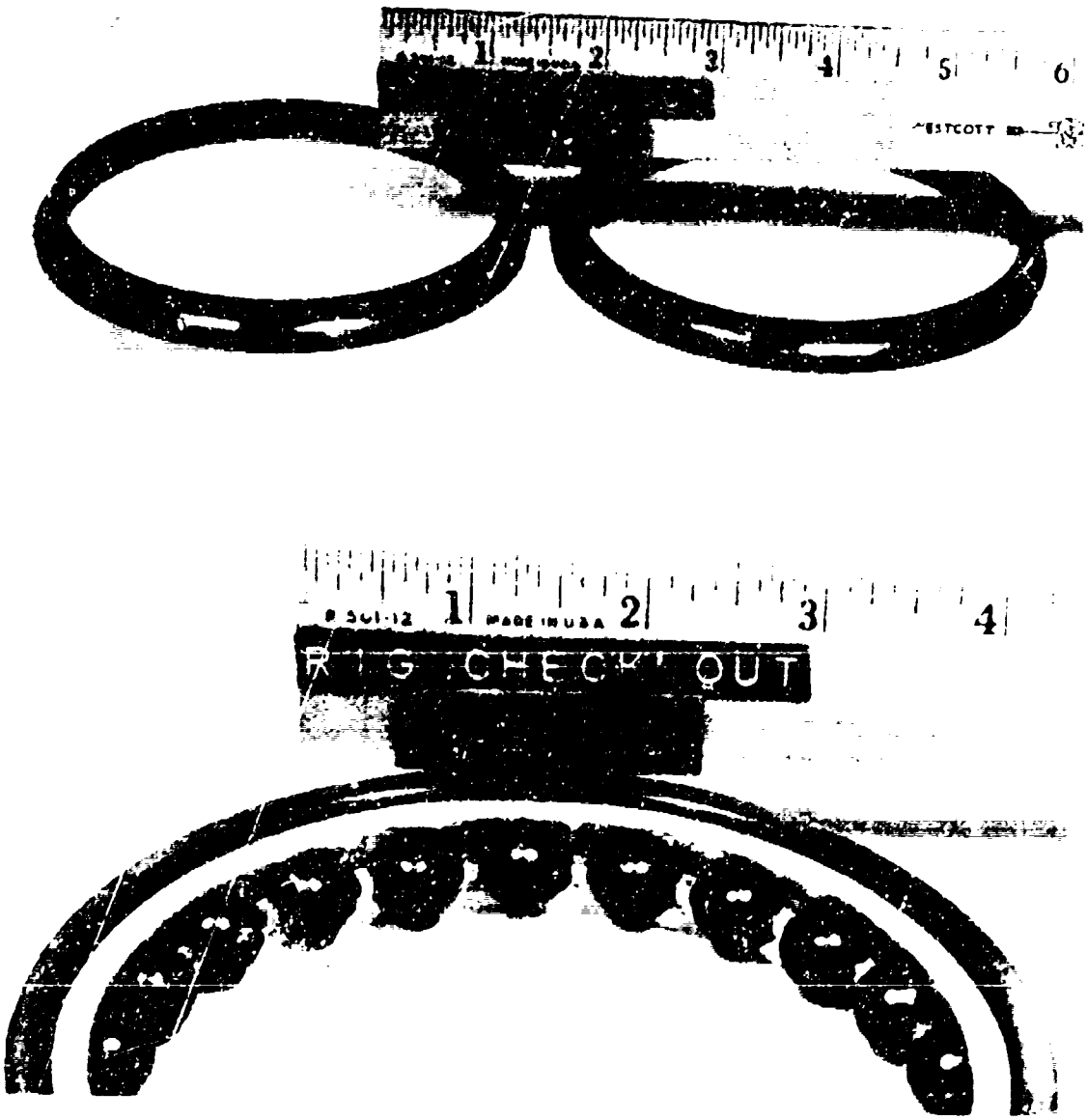


Figure 55. Build No. 2 Bearing Failure; Insufficient Pocket Clearance. Top: Inner races. Bottom: Cage, balls, and outer race.

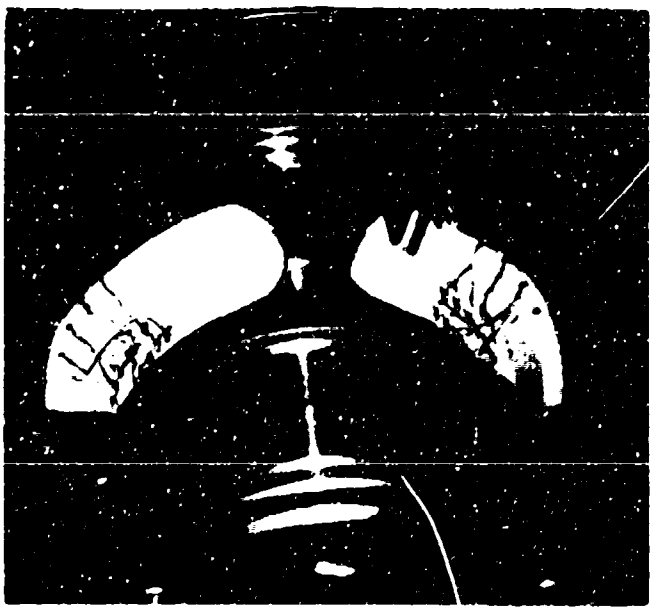


Figure 56. Improved and Original Retainer to Driving Ring Strain Gage Lead Paths. Top: Improved lead path showing gage leads fixed to pins attached on retainer; leads are spot welded to plate riveted to driving plate, and the leads are larger than originally. Bottom: Original lead path, unsupported from retainer, epoxy bonded to driving plate.

TABLE VIII. BALL BEARING TEST CONDITIONS			
Data Point	Inner Ring Speed (rpm)	Axial Load (lb)	Radial Load (lb)
1	16,000	1000	0
2	16,000	1000	500
3	16,000	500	500
4	16,000	100	0
5	20,000	1000	0
6	20,000	1000	500
7	20,000	500	500
8	20,000	100	0

TABLE IX. ROLLER BEARING TEST CONDITIONS			
Data Point	Inner Ring Speed (rpm)	Axial Load (lb)	Radial Load (lb)
1	16,000	0	1000
2	16,000	0	600
3	16,000	0	200
4	16,000	0	0
5	20,000	0	1000
6	20,000	0	600
7	20,000	0	200
8	20,000	0	0

Build No. 1, Rig Checkout

The objective of this build was to check out the test rig for the test program. Standard rig instrumentation was used. Retainer instrumentation and inner race temperature probes were not installed for this build. A standard ball bearing was installed in the test position.

The test rig was operated to 23,000 rpm. Oil flows to the test bearing and two support bearings were adjusted at various speed increments to note the effect on scavenge temperature. Radial and axial thrust loads were also varied at different shaft speed settings to determine load effects on vibration. The rig checkout was concluded with a 1-hour run at 23,000 rpm.

Build No. 2, Smooth-Race Ball Bearing

The test rig was rebuilt with bearing instrumentation to take data on the ball bearing in the EHD region.

The oil-in temperature to the test bearing was maintained at 100 ± 5° F during the test.

The rig was operated through all of the data points defined in Table VIII.

To generate a bearing skid map, the test rig was brought to 20,000 rpm, the radial load was reduced to zero, and the thrust (axial) load was reduced in increments until skidding occurred.

Build No. 3, Rough-Race Ball Bearing

This build was the same as Build No. 2 except that the test bearing had a rough surface finish on the inner and outer races to promote boundary lubrication at the rolling contacts. The same data points defined in Table VIII were evaluated. The oil-in temperature during the test was 200 ± 5°F.

An attempt was made to produce a skid map, but skidding would not occur.

Build No. 4, Smooth-Race Roller Bearing

The test rig was rebuilt with bearing instrumentation to take data on the roller bearing in the EHD region.

The oil-in temperature to the test bearing was maintained at 100 ± 5°F during the test.

The eight data points to be evaluated during this build are defined in Table IX. The test rig was operated through all of these data points.

TEST RESULTS

Build No. 1, Rig Checkout

This build resulted in successful operation of the test rig, without retainer or race instrumentation, to speeds of 23,000 rpm under test load conditions.

Disassembly and inspection of the test rig after checkout operation showed that both the test and facility bearings were in excellent condition.

Build No. 2, Smooth-Race Ball Bearing

The first ball bearing configuration tested was with a smooth-race surface finish (5AA). Test results are summarized in Table X, which includes test race temperatures, measured radial load, slip ring driving torque, radial load frequency, and retainer-to-inner race speed ratio.

These results show that radial load was composed of a steady and an oscillatory component. Retainer driving torque was found to be oscillatory also. When drag was introduced into the driving system by "holding back" the slip ring, torque was still oscillatory, which suggested that the cage motion was not steady but accelerated and decelerated once per revolution. Maximum torque required to drive the slip ring was measured at 7 inch-pounds steady load, ± 7 inch-pounds vibratory.

Samples of the outputs from both ball force strain gage bridges and the cage speed signal as a function of time are shown in Figures 57 through 64. These figures show large pulse-type forces being applied to the cage at a nonuniform amplitude and nonperiodic frequency. In addition, they show a small vibratory (approximately ± 3 to ± 4 pounds) force. This smaller vibratory signal is expanded five times and presented in Figures 65 through 72.

TABLE X. SUMMARY OF TEST RESULTS, BUILD NO. 2

Data Points[1]	Inner and Outer Race Temperatures: Inner Race (°F)	Outer Race (°F)	ΔT (°F)	Radial Load[2] (lb)	Radial Load Frequency[3] (Hz)	Retainer to Ring Speed Ratio	Driving Torque[4] (in.-lb)
1				0 ± 100	267	.46	
	320	209	-126	0 ± 20	(5)	.46	6 ± 3
2				470 ± 80	281	.46	
	-	205	-	470 ± 120	254	.46	6 ± 3
3				470 ± 140	279	.46	
	340	205	-135	510 ± 130	(5)	.45	5 ± 3
4				0 ± 20	(5)	.31[6]	
	260	195	-65	0 ± 20	(5)	.45	5 ± 3
5				0 ± 20	(5)	.46	
	320	245	-75	0 ± 20	(5)	.45	6 ± 6
6				500 ± 120	356	.45	
	250	230	-20	480 ± 95	334	.45	7 ± 7
7				500 ± 120	348	.45	
	360	225	-135	480 ± 100	331	.45	7 ± 6
8				0 ± 20	(5)	.45	
	260	220	-40	0 ± 20	(5)	.46	3 ± 3

1. Each test was run twice (once with two web force strain gage bridges, once with one web force and one torque measurement); inner and outer race temperatures were determined during the second run only. Upper values are for test No. 1; lower values are for test No. 2.
2. From strain-gaged radial loading rail; static and dynamic components.
3. Frequency of oscillation of radial loading rail; caused by unbalance and/or eccentricity of shaft supporting bearing.
4. Vibratory component not sinusoidal during condition where torque goes to zero.
5. Ability to determine primary frequency limited by low-level dynamic component.
6. Indicate slip during first run but no slippage during second run; also shown in harmonic analysis.

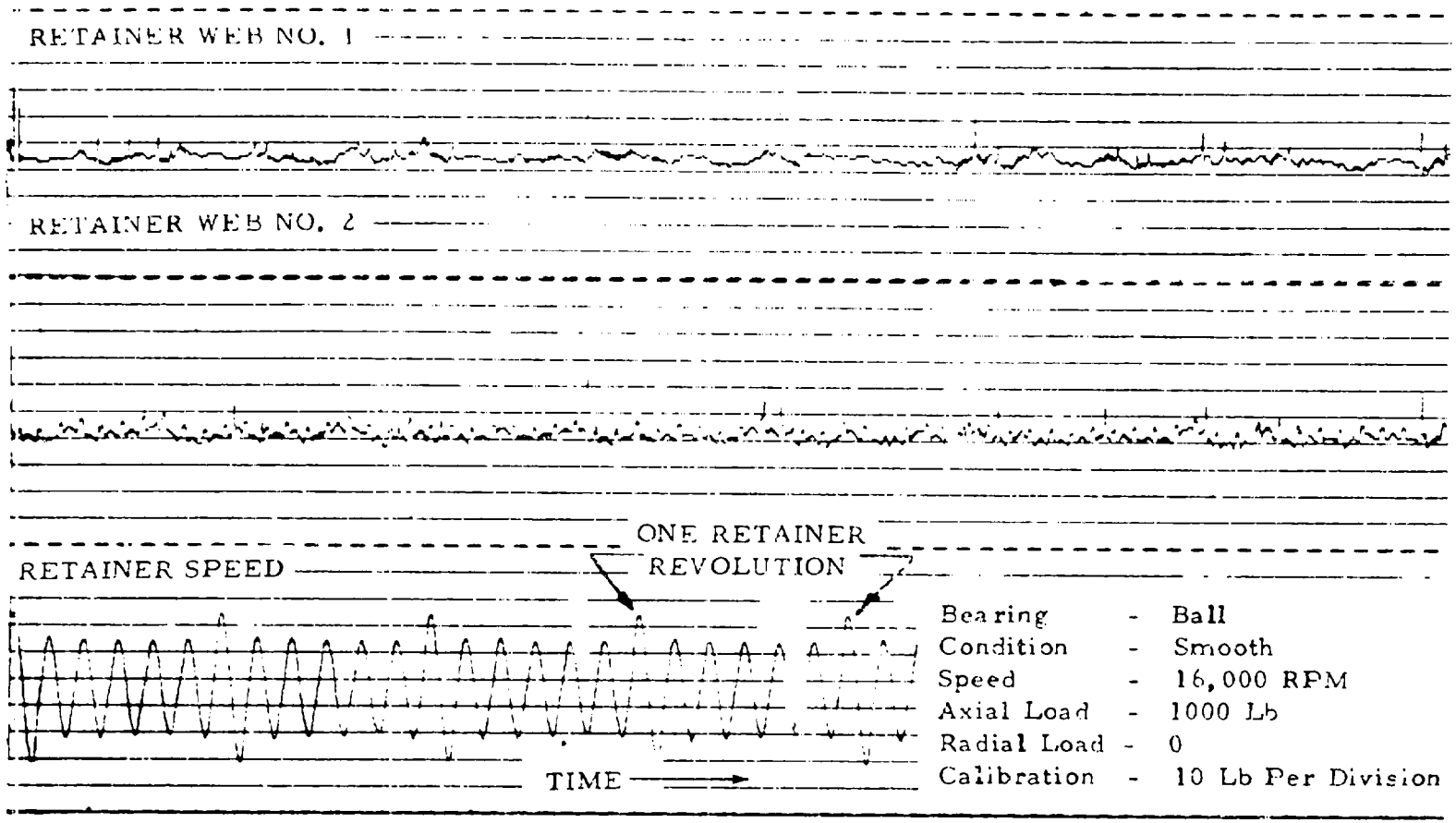


Figure 57. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

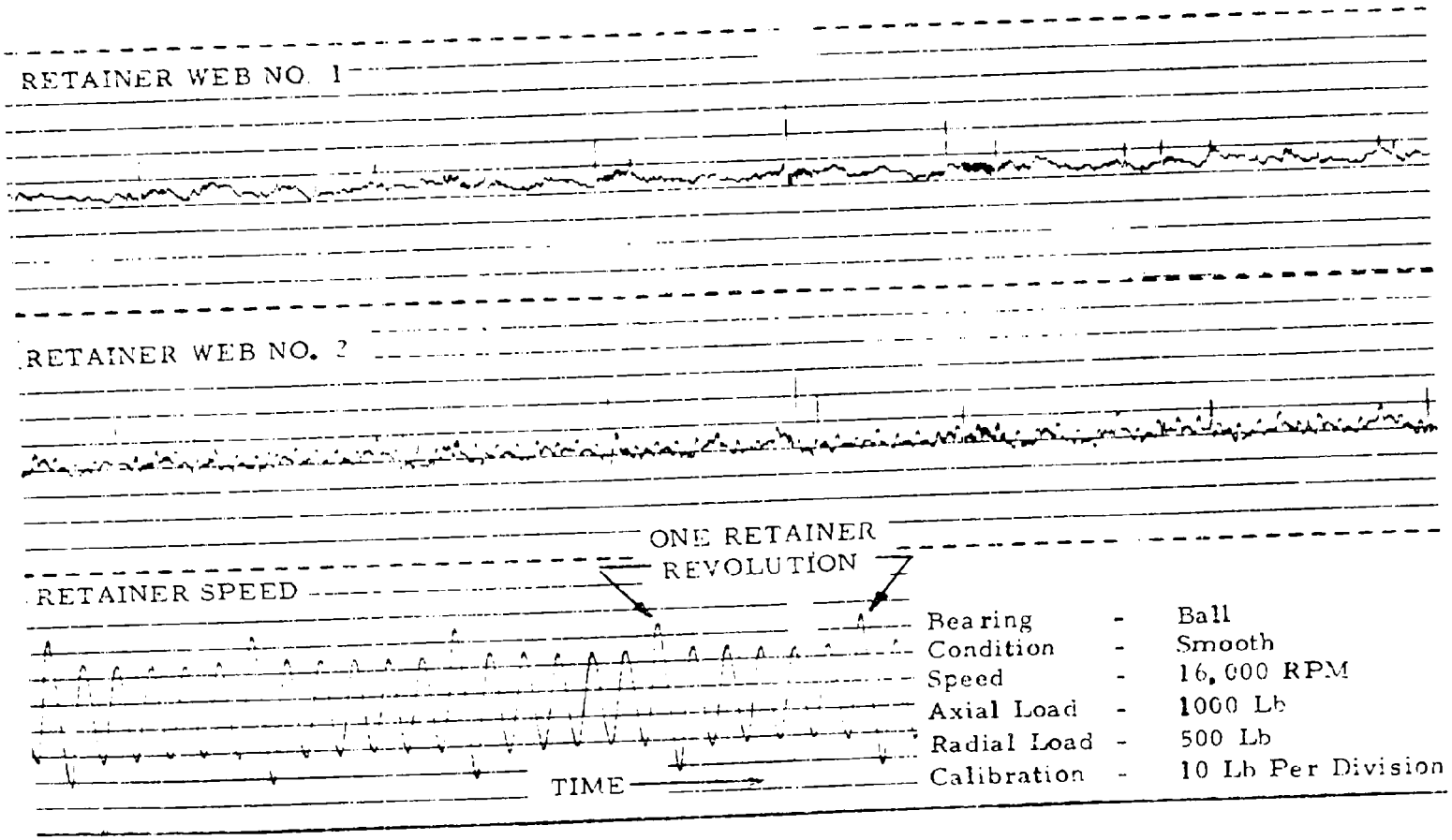


Figure 58. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

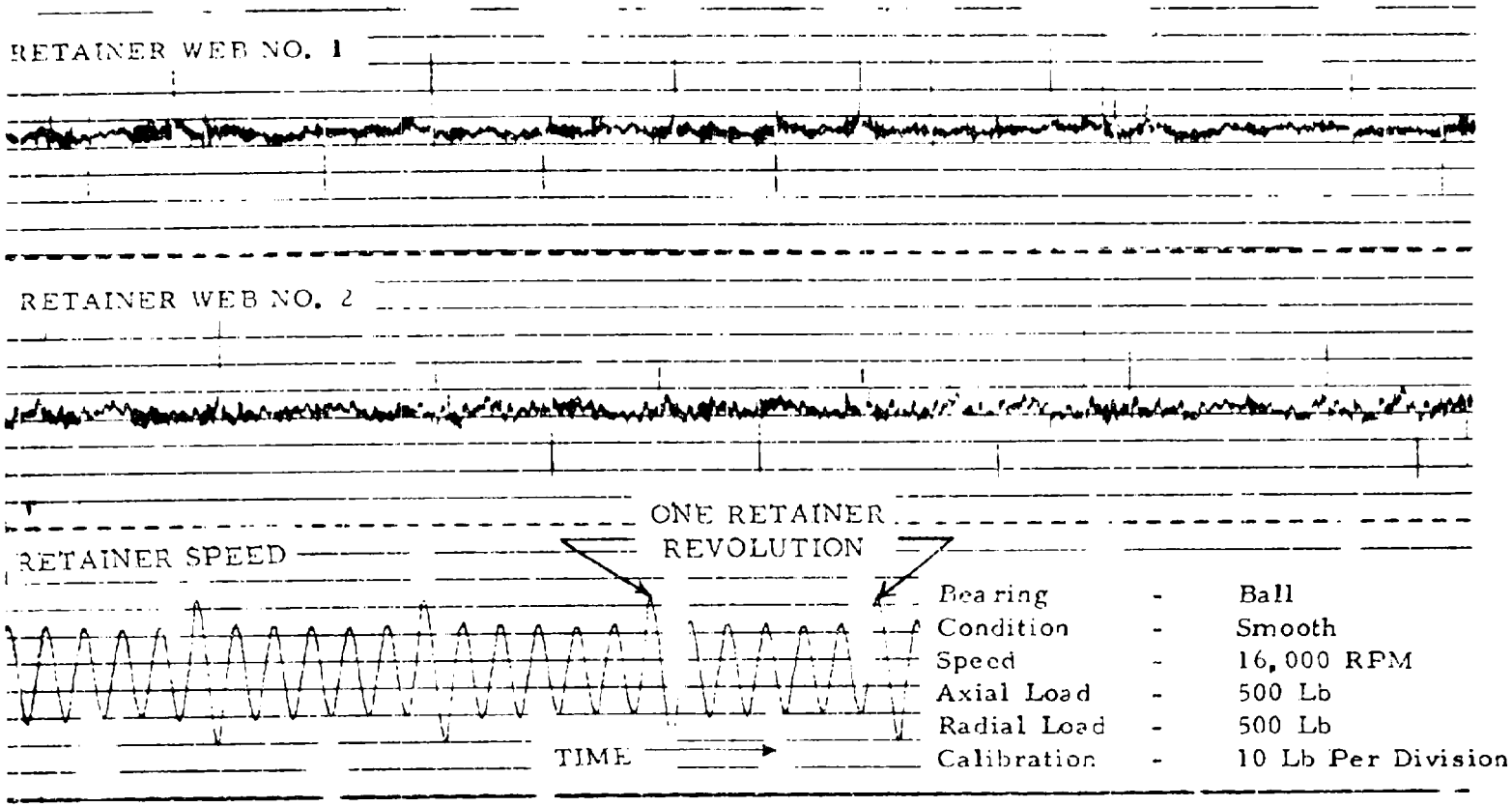


Figure 59. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

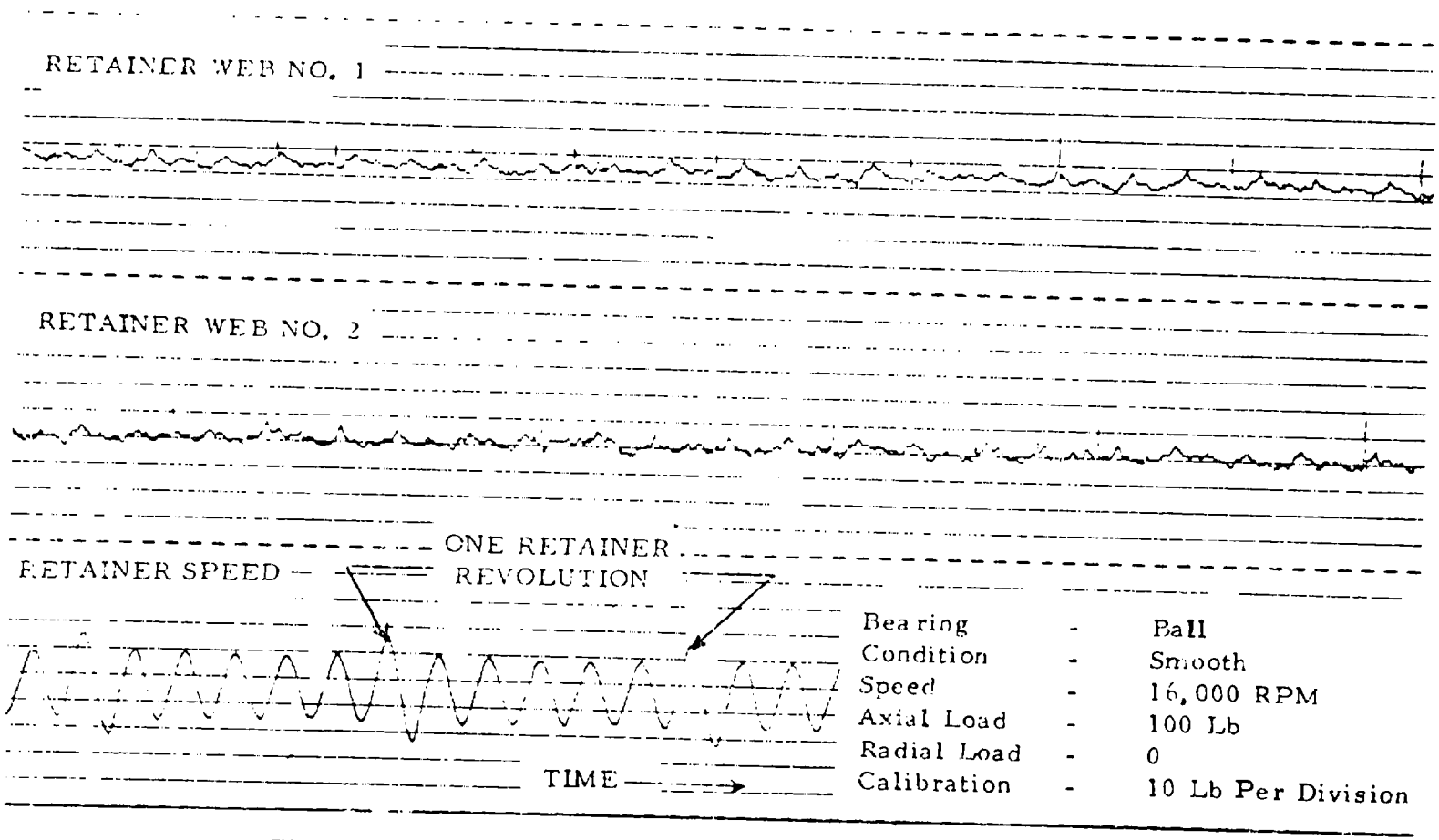


Figure 60. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

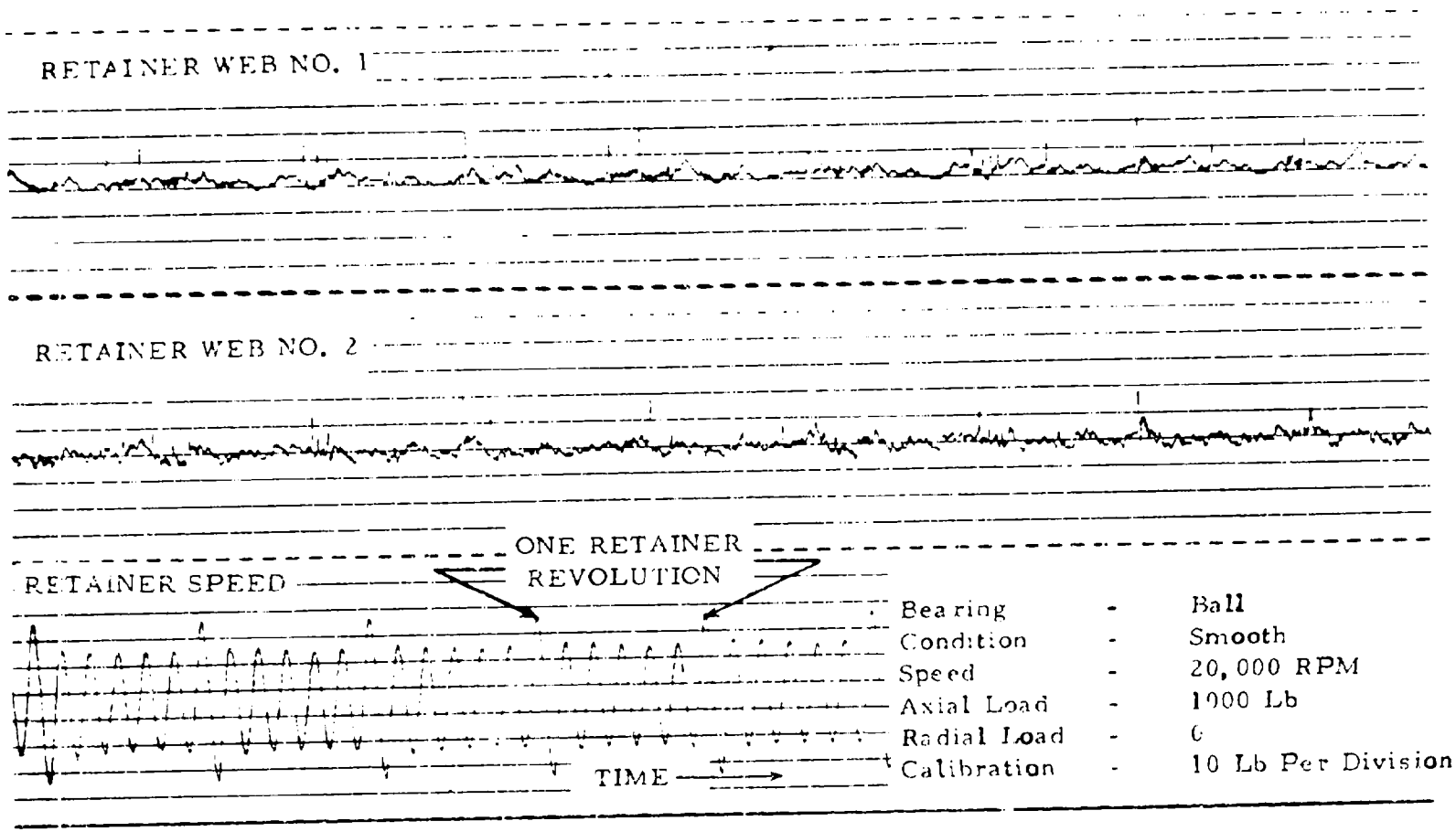


Figure 61. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

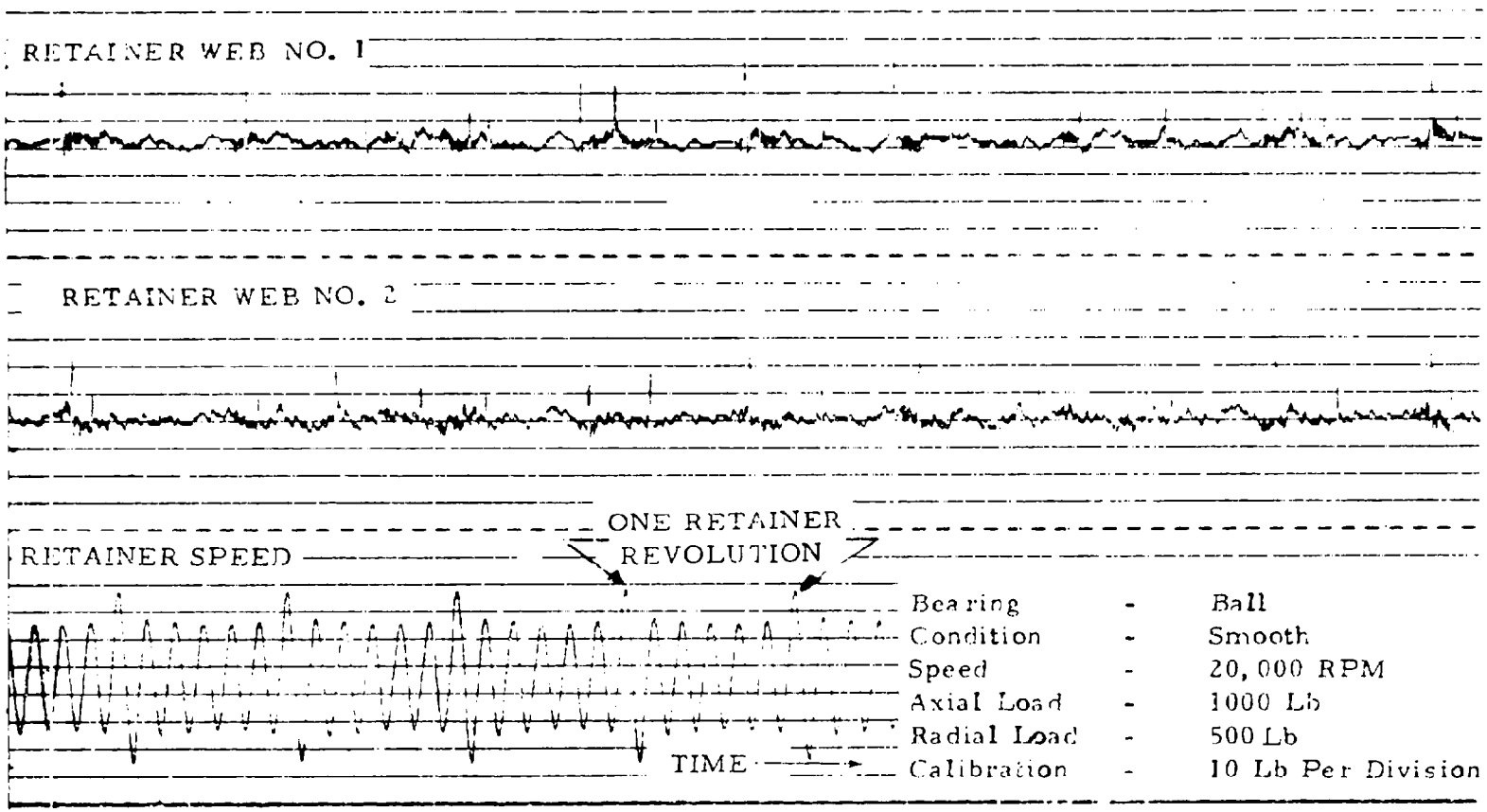


Figure 62. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

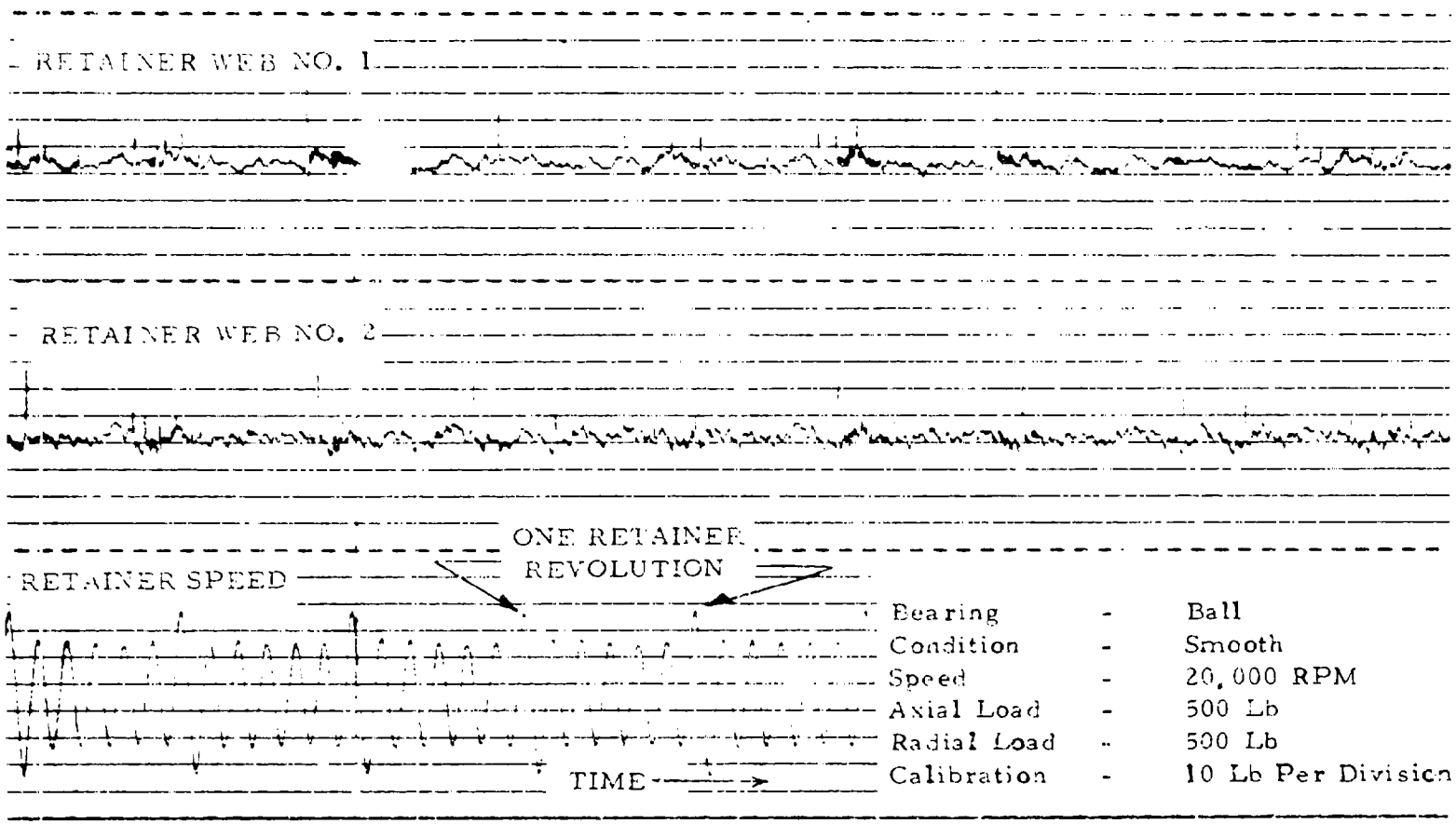


Figure 63. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

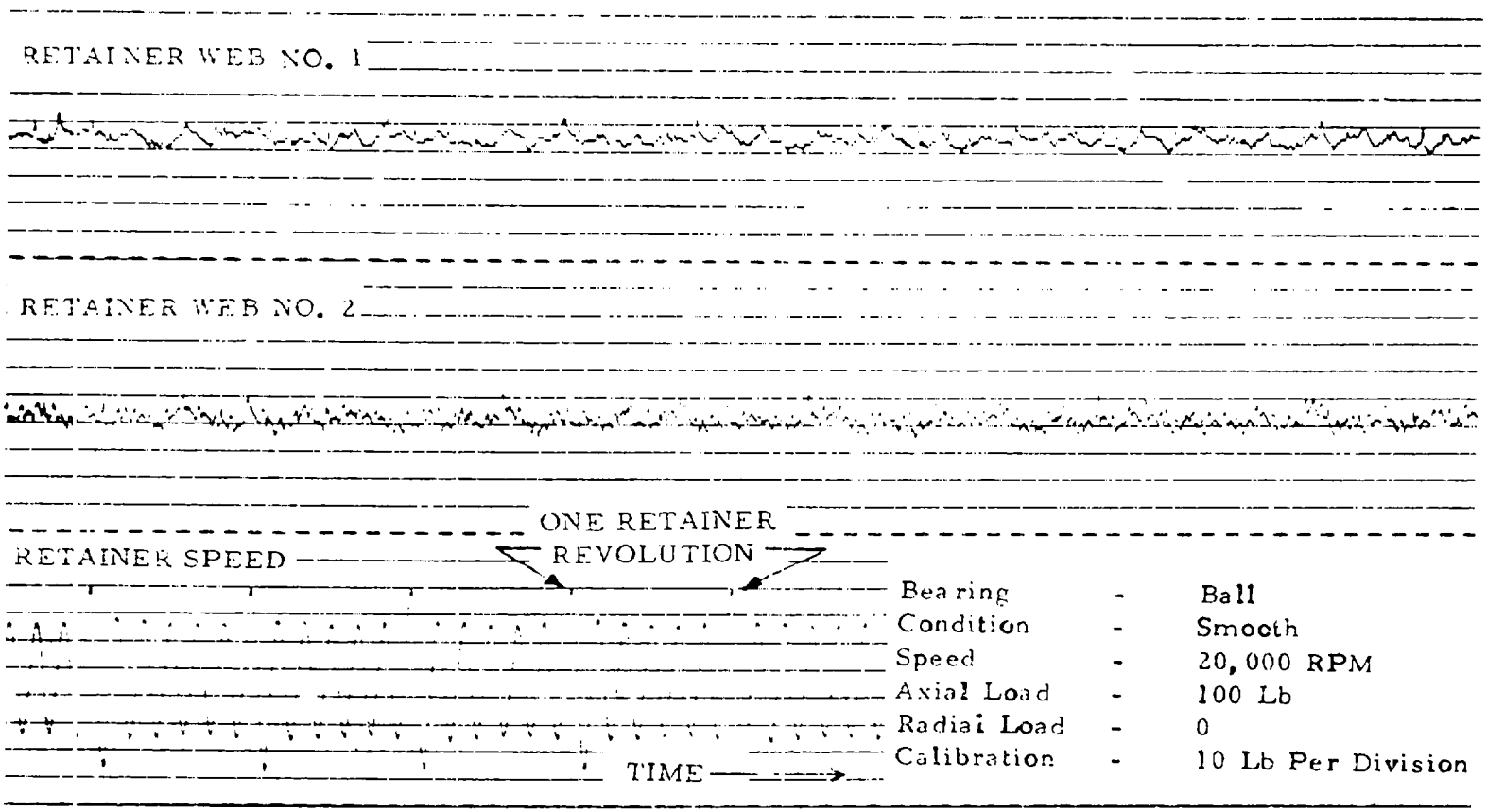


Figure 64. Retainer Web Force and Retainer Speed Versus Time, Showing Nature of Impact Loads.

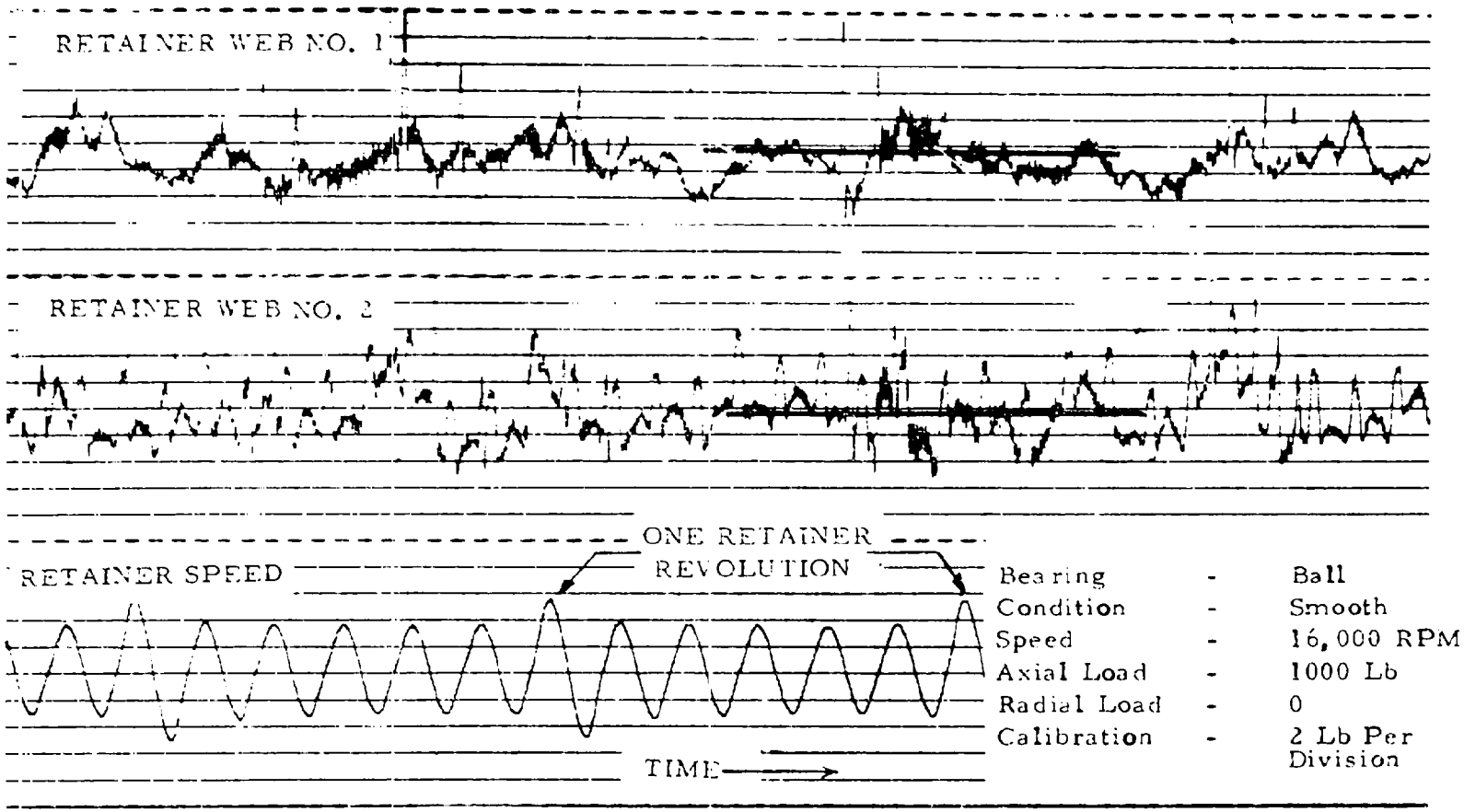


Figure 65. Retainer Web Force Expanded With Computed Web Force Superimposed.

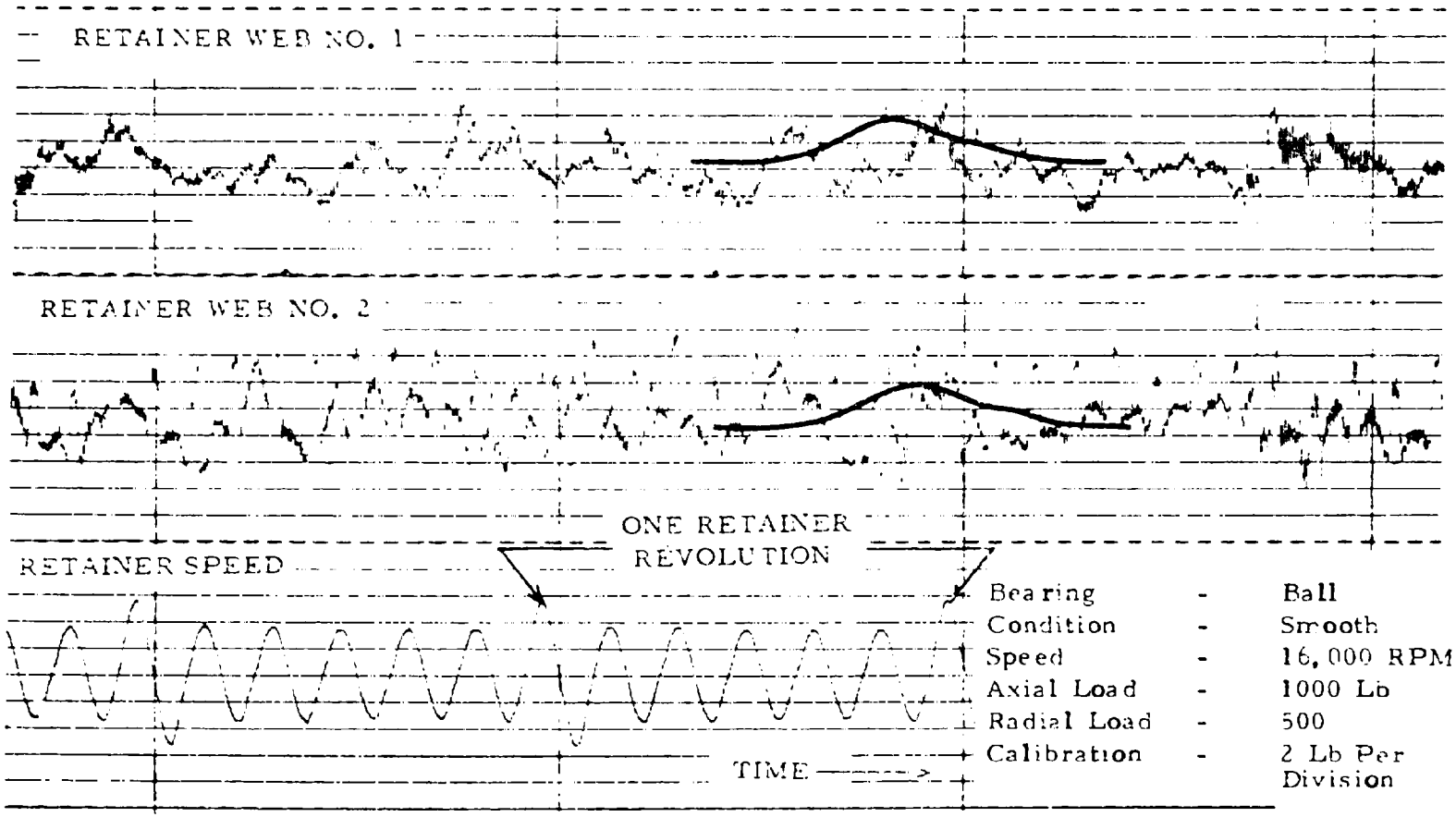


Figure 66. Retainer Web Force Expanded With Computed Web Force Superimposed.

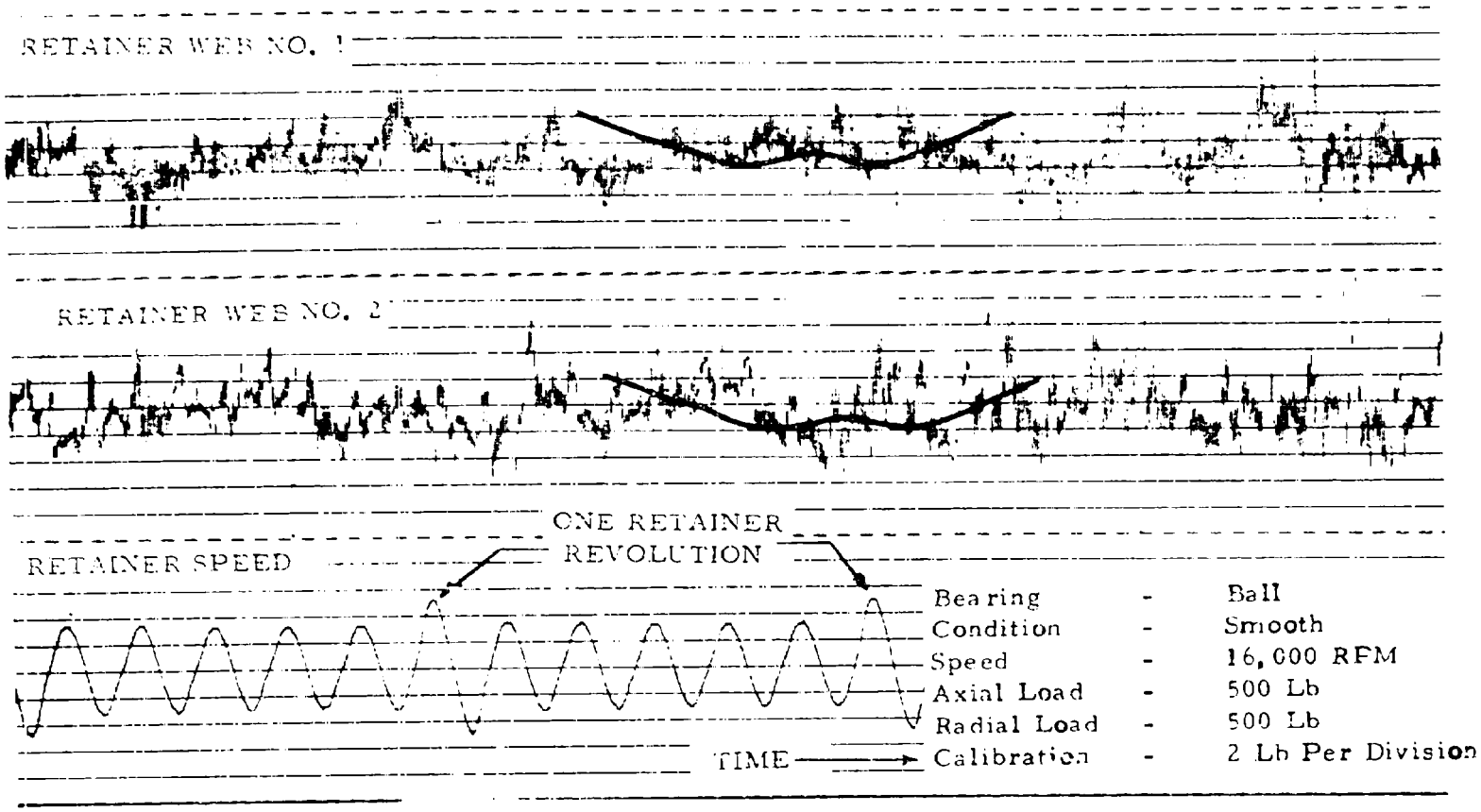


Figure 67. Retainer Web Force Expanded With Computed Web Force Superimposed.

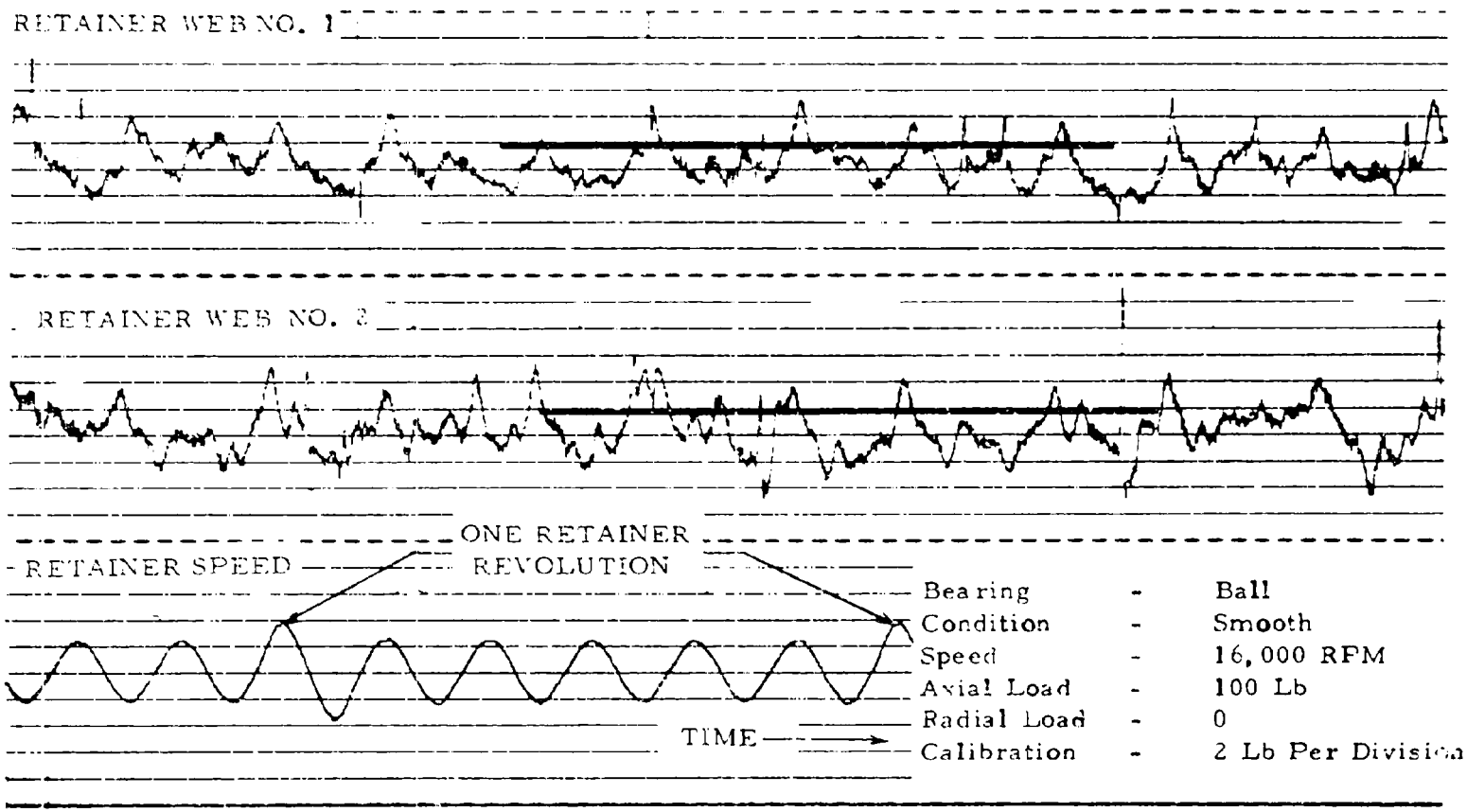


Figure 68. Retainer Web Force Expanded With Computed Web Force Superimposed.

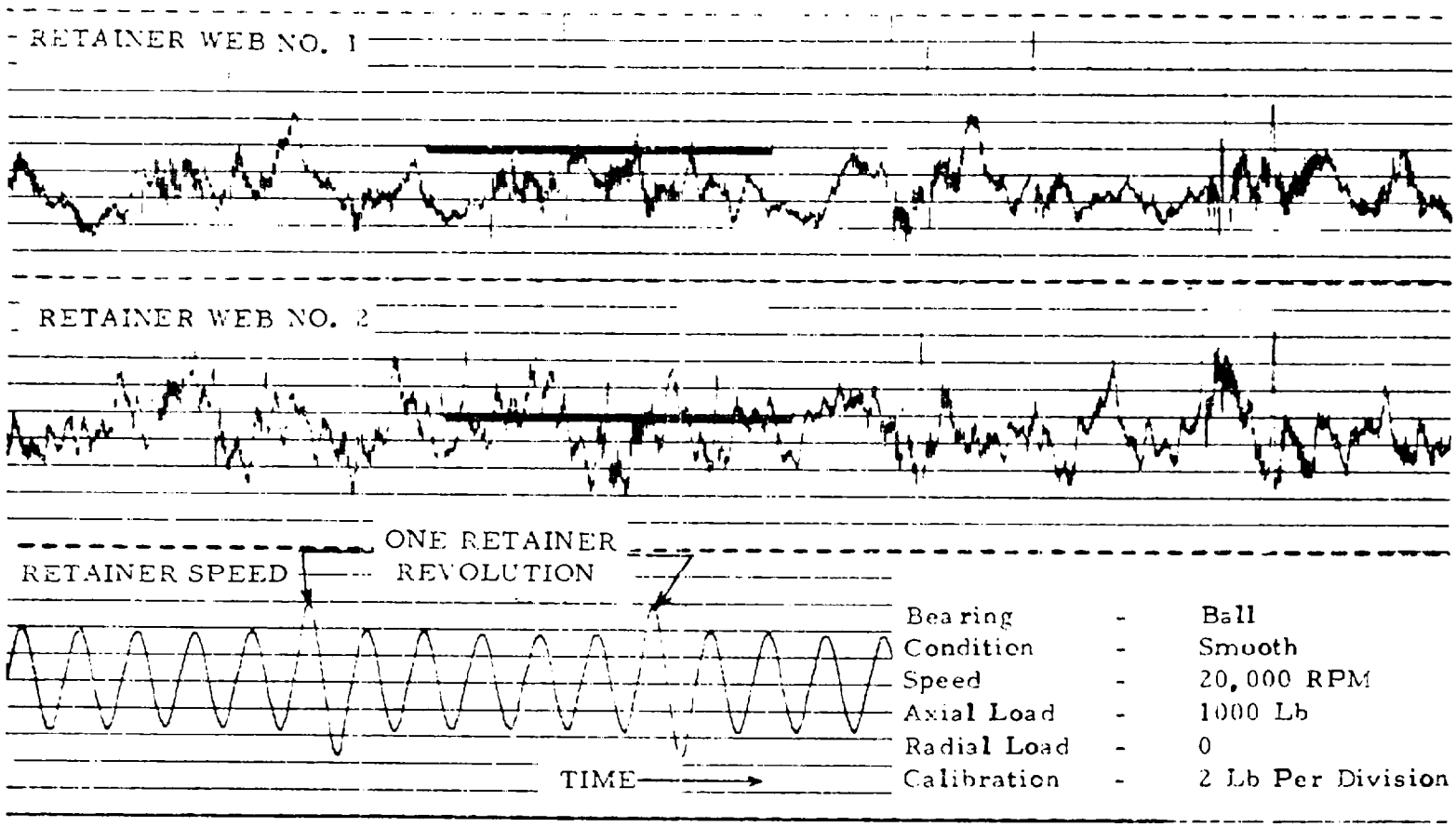


Figure 69. Retainer Web Force Expanded With Computed Web Force Superimposed.

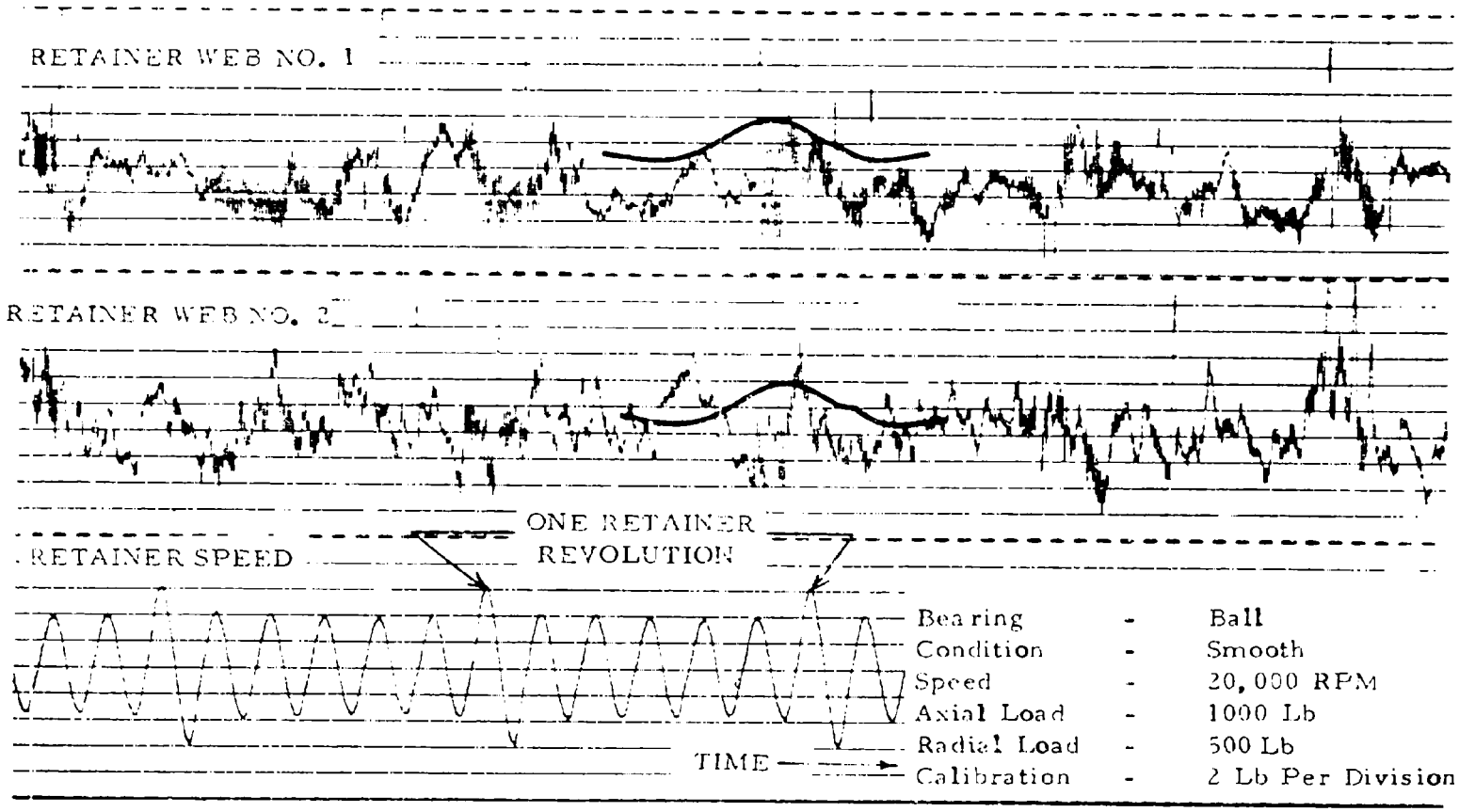


Figure 70. Retainer Web Force Expanded With Computed Web Force Superimposed.

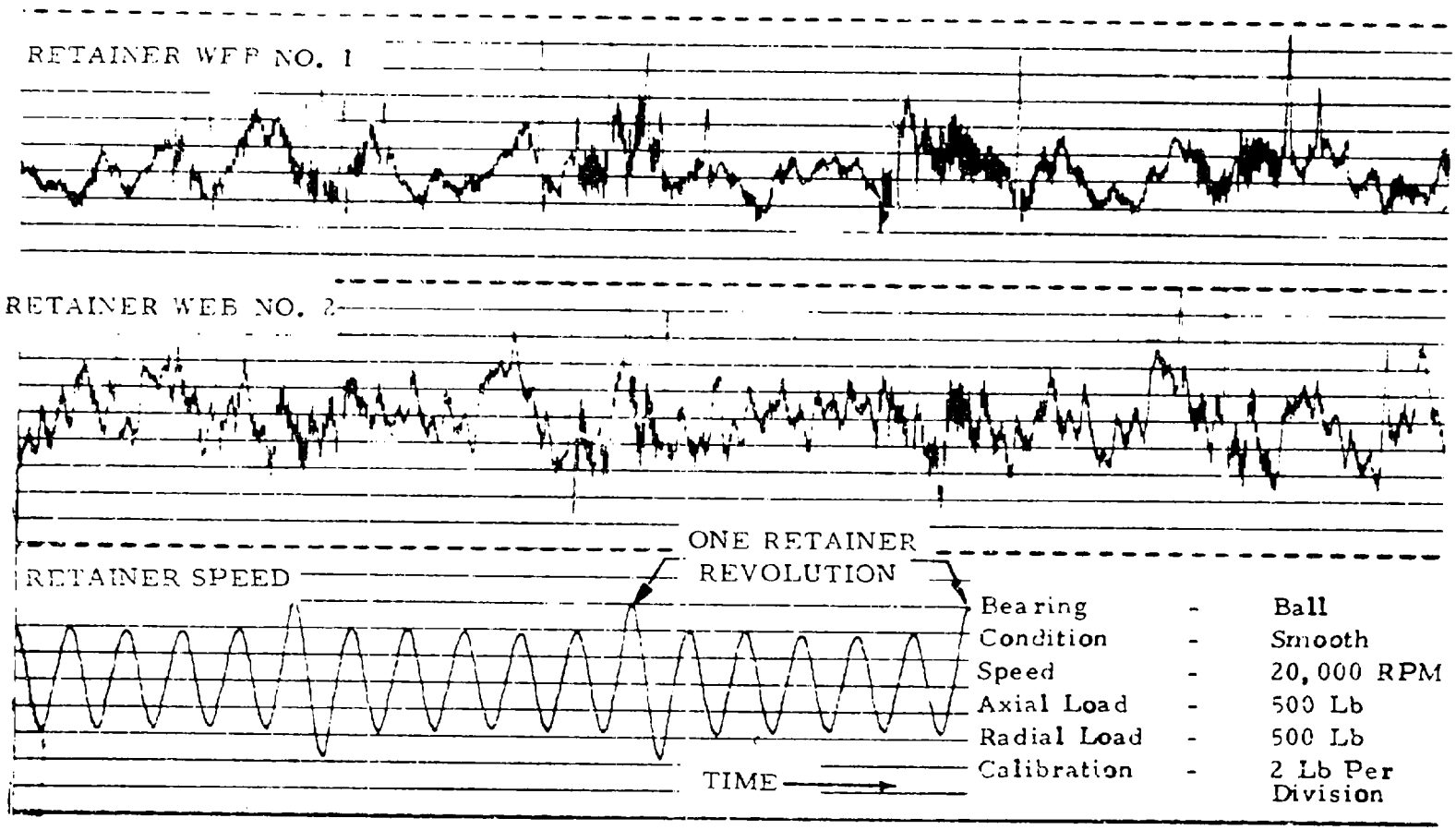


Figure 71. Retainer Web Force Expanded With Computed Web Force Superimposed.

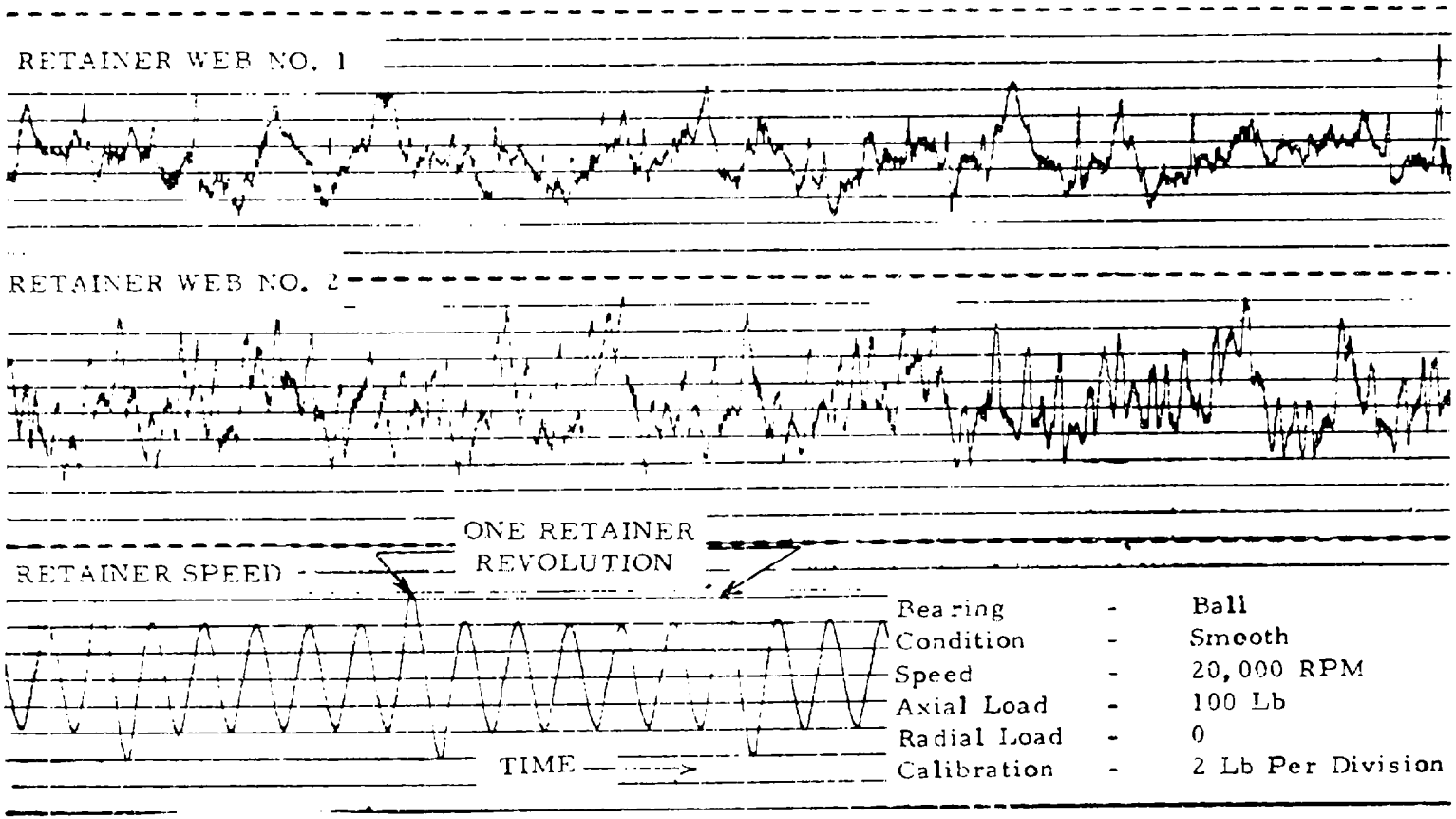


Figure 72. Retainer Web Force Expanded With Computer Web Force Superimposed.

The large, pulse-like force was later simulated in the laboratory by impact loading a ball against a retainer web. Results suggested the presence of dynamic ball/pocket forces resulting from accelerations of the ball within the pocket clearance or of the retainer itself. These dynamic effects were not predicted by the computer program.

With respect to the smaller vibratory signal, two approaches were taken in the analysis of the test data. First, ball azimuth versus retainer web force plots as determined by the computer program were superimposed on similar plots from the test program data (Figures 65 through 72). Second, a harmonic analysis of the computer-determined output was compared with similar analysis from the test data (Figures 73 through 75). Figure 75 shows the computer-determined harmonic analysis of the computed web force wave, and is normalized here to an arbitrary unit to compare ratios of harmonics rather than absolute values. Figures 73 and 74 show the average of 16 spectra obtained from experimental test tape recordings. Spectrum averaging was used to eliminate random signals that might be present in the experimental wave forces, since the harmonic analysis of any selected cycle could be distorted by random occurrences.

In the cases of pure thrust bearing loading (Figures 65, 68, and 69), where computed pocket force is a constant value of approximately 0.25 pound, the dynamic component determined experimentally (neglecting impact peaks) was between ± 3 and ± 4 pounds. It is suggested that the difference between the computed and measured force levels at these conditions, where the predicted force is nearly zero, is background or additional forces generated by conditions not considered in the computer analysis. Practically speaking, a 3- to 4-pound background force is of little consequence.

In the cases of combined thrust and radial loading (Figures 66, 67, and 70), the predicted and measured force levels are of the same magnitude, neglecting impact peaks. The computer program predicted approximately ± 1.5 to ± 2.0 pounds; the measured values were from ± 4 to ± 4.5 pounds. Note that the difference is not large in view of the background levels determined during pure thrust load tests.

Two of the test conditions, filtered and unfiltered, are shown in Figures 76 through 79. Frequencies higher than seven times retainer speed (seventh order) have been eliminated in the filtered wave.

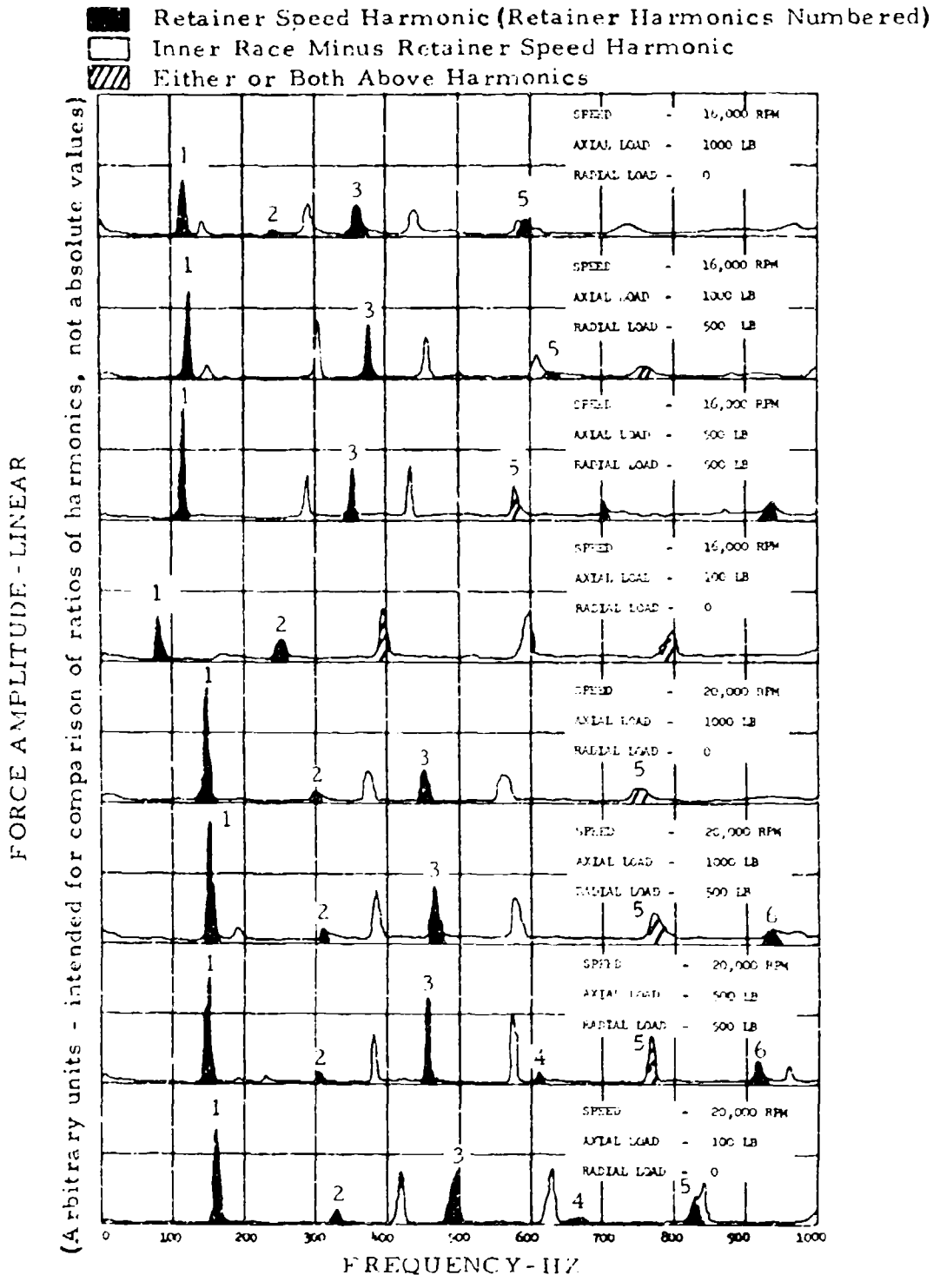


Figure 73. Harmonic Analysis of Experimentally Determined Retainer Web Force (Average of 16 Spectra) for Ball Bearing Smooth Race. Bearing Retainer Web No. 1.

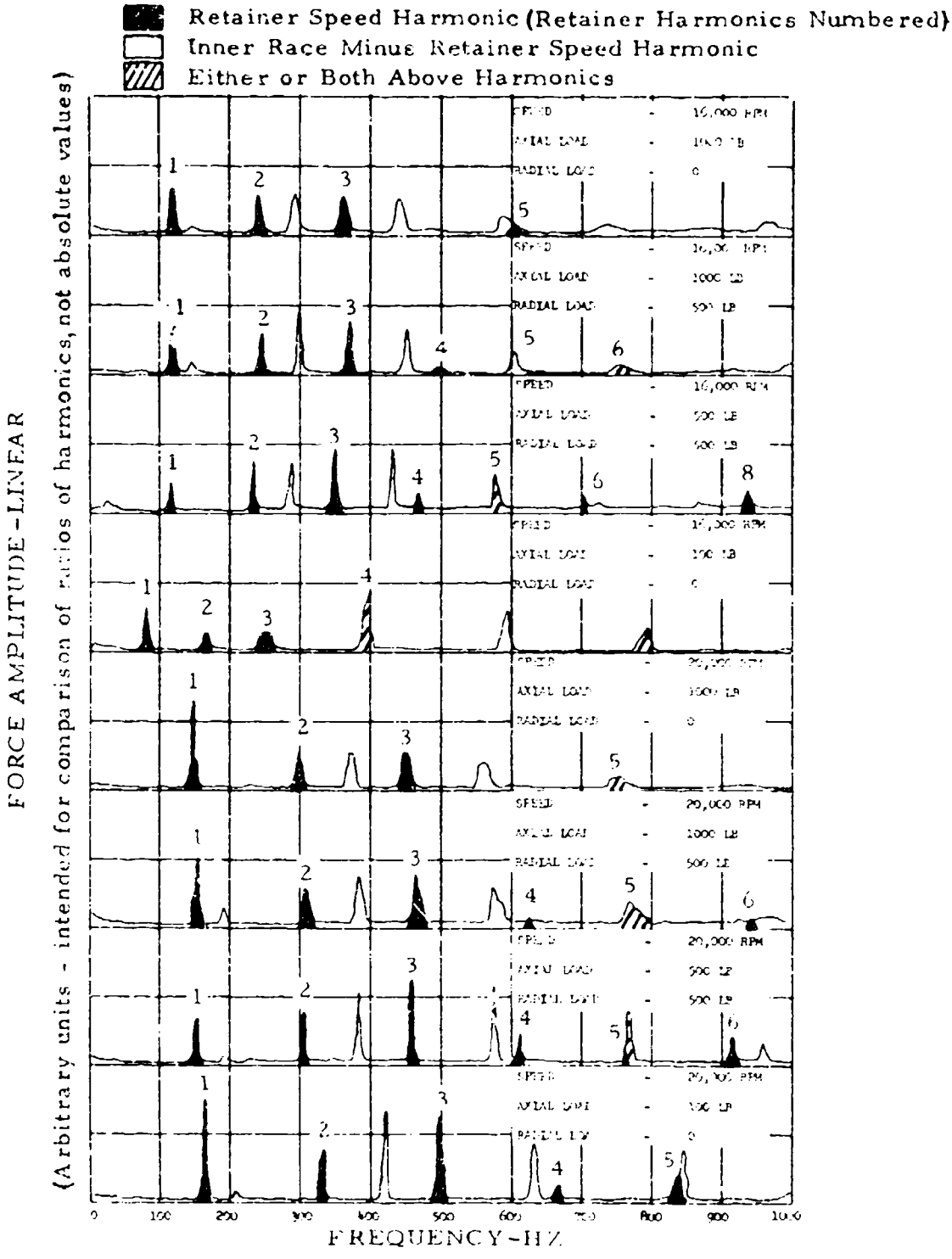


Figure 74. Harmonic Analysis of Experimentally Determined Retainer Web Force (Average of 16 Spectra) for Ball Bearing Smooth Race, Bearing Retainer Web No. 2.

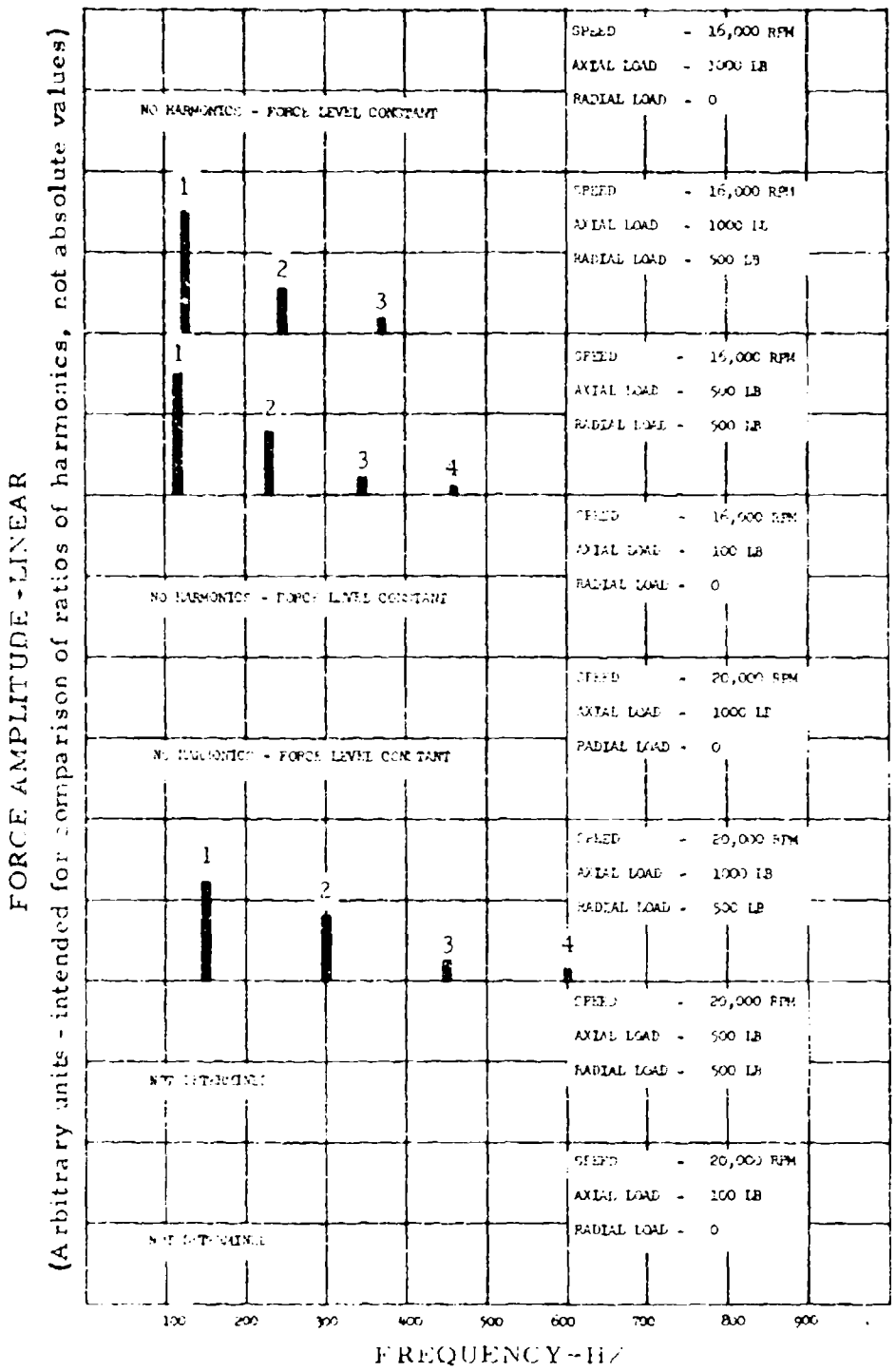


Figure 75. Harmonic Analysis of Computed Retainer Web Force for Ball Bearing Smooth Race.

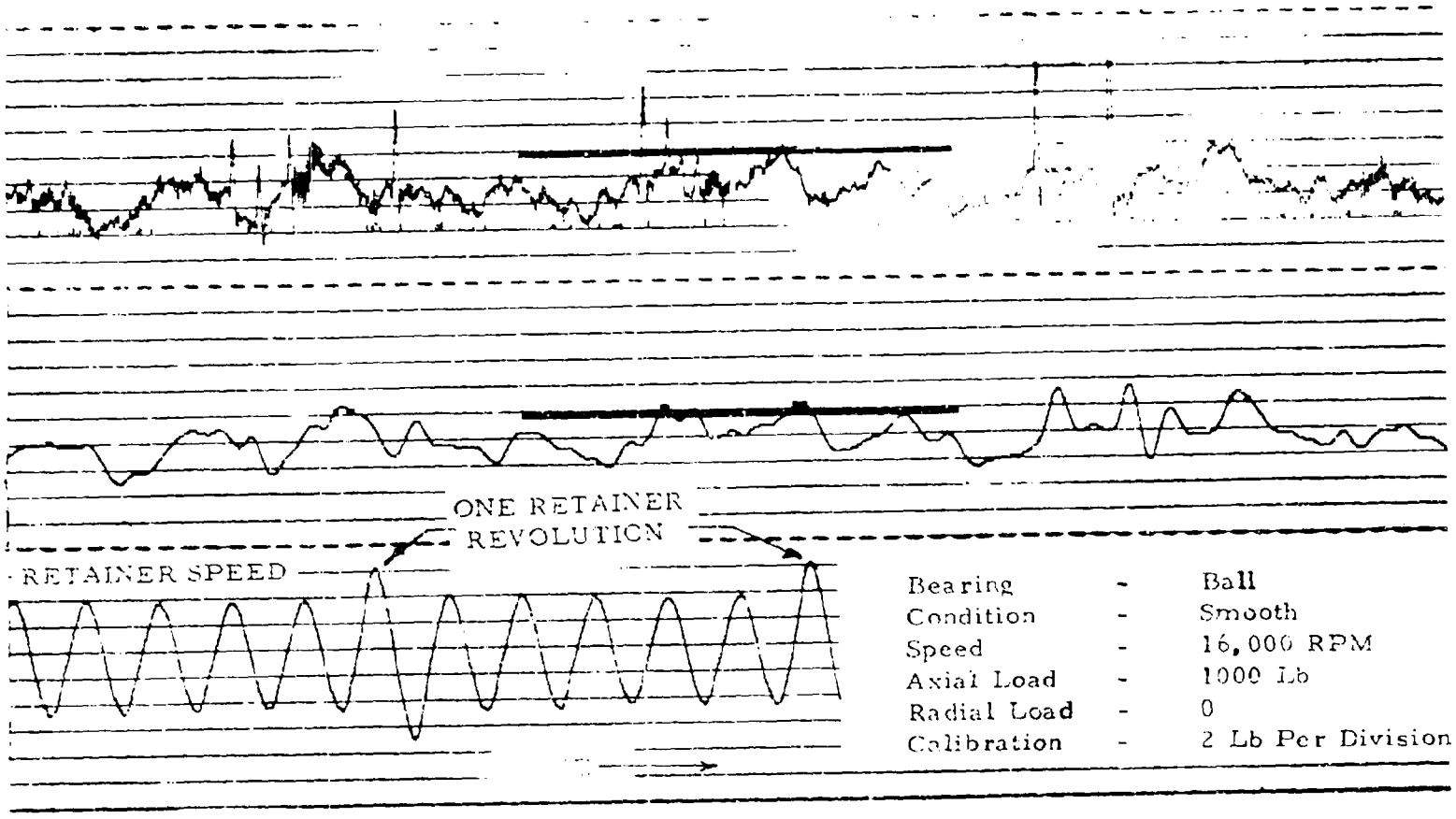


Figure 76. Retainer Web No. 1, Filtered and Unfiltered Retainer Force Wave Form With Computed Forces Superimposed. Frequencies Higher Than Seventh Order Retainer Speed Removed in Filtered Wave.

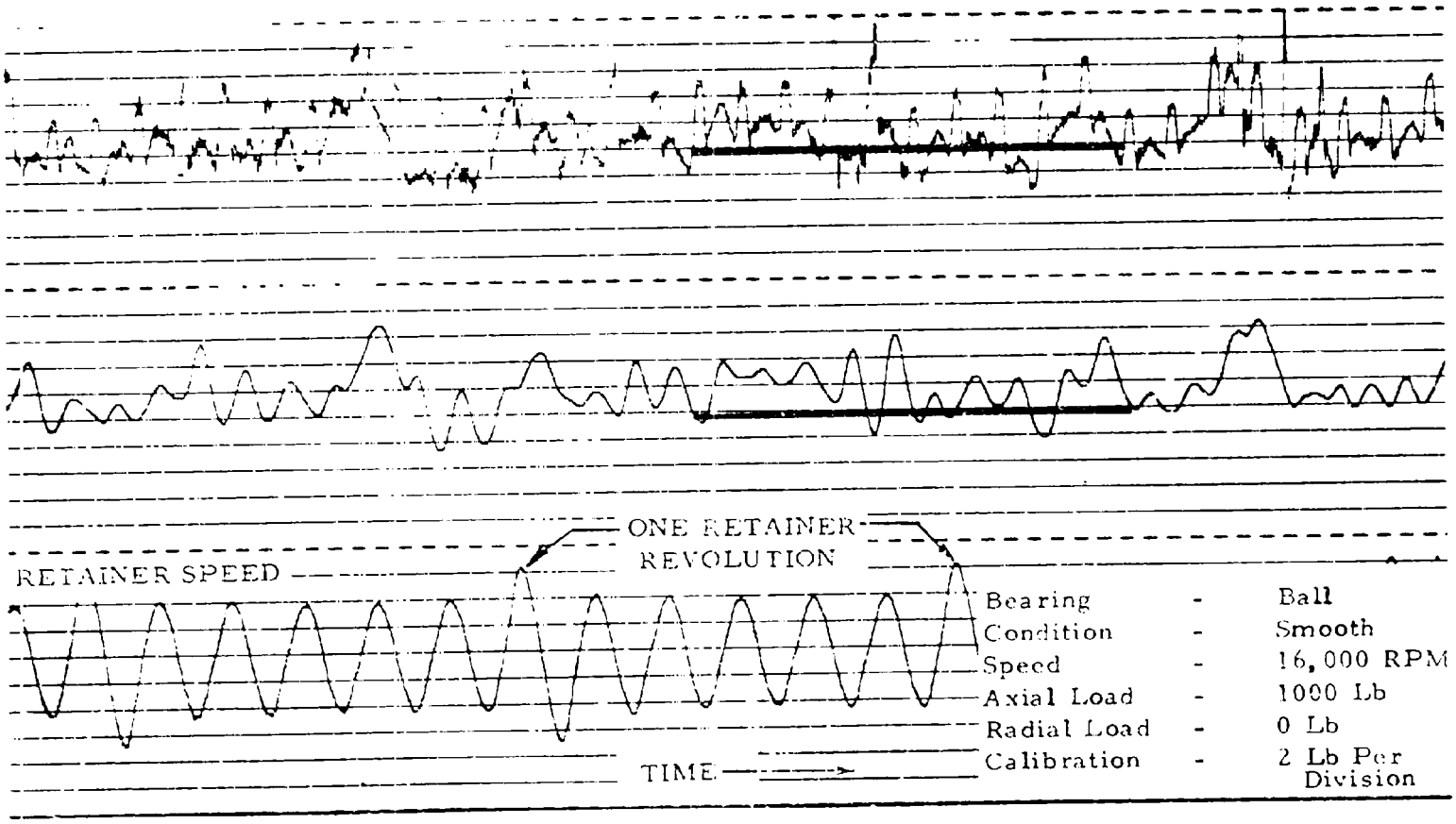


Figure 77. Retainer Web No. 2, Filtered and Unfiltered Retainer Force Wave Form With Computed Forces Superimposed. Frequencies Higher Than Seventh Order Retainer Speed Removed in Filtered Wave.

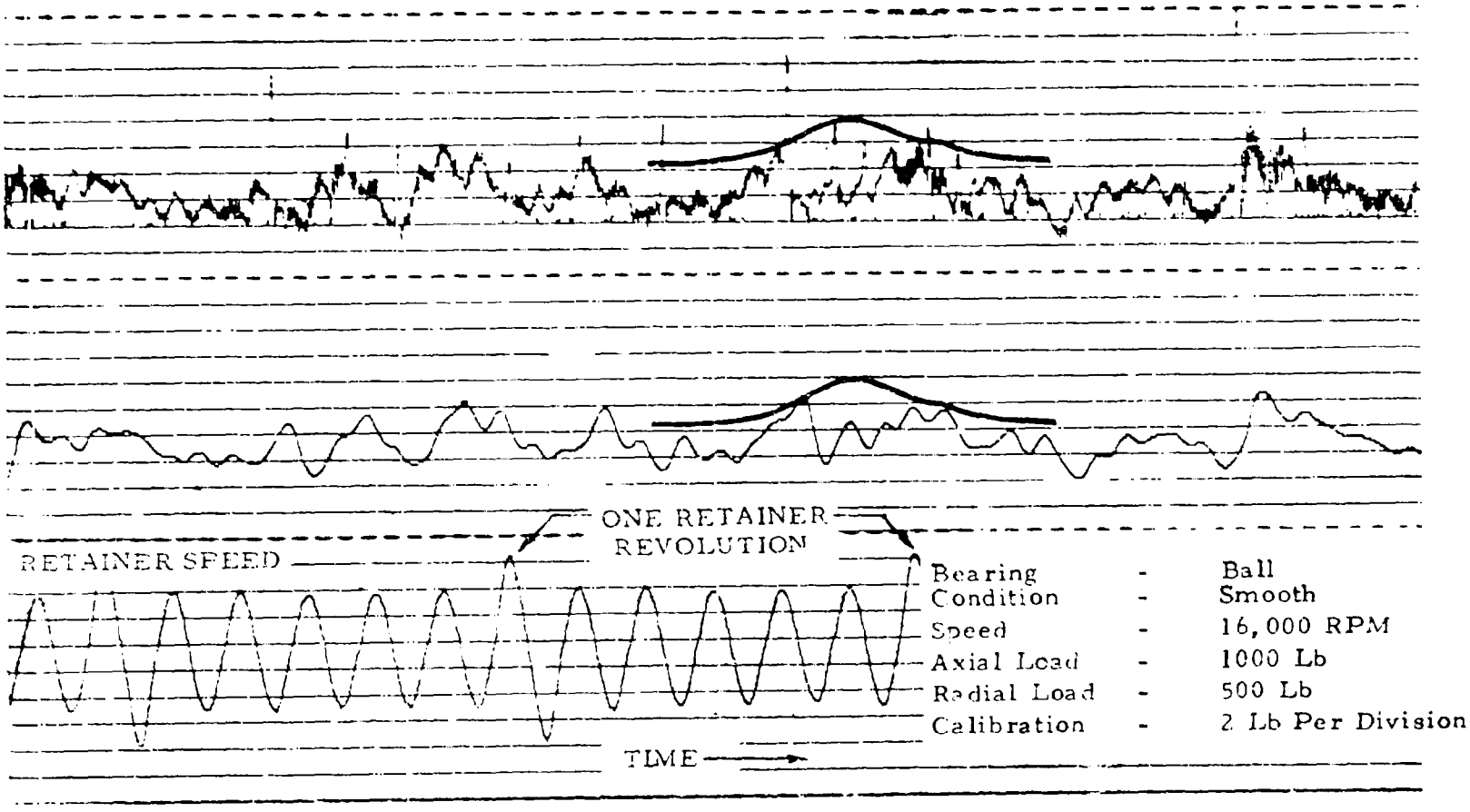


Figure 78. Retainer Web No. 1, Filtered and Unfiltered Retainer Force Wave Form With Computed Forces Superimposed. Frequencies Higher Than Seventh Order Retainer Speed Removed in Filtered Wave.

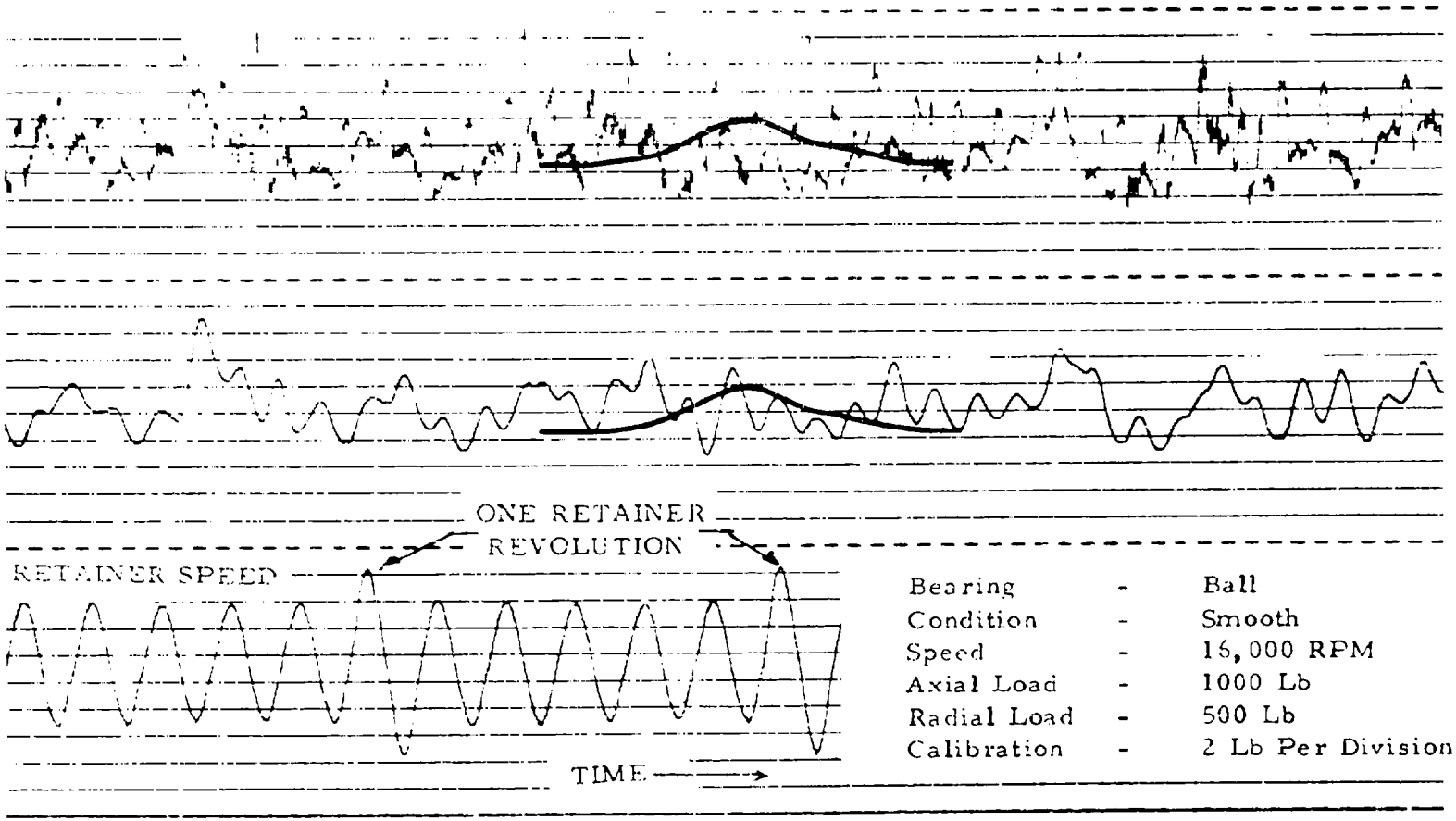


Figure 79. Retainer Web No. 2, Filtered and Unfiltered Retainer Force Wave Form With Computed Forces Superimposed. Frequencies Higher Than Seventh Order Retainer Speed Removed in Filtered Wave.

Harmonic analysis (Figures 73 through 75) shows similar frequency components for computer and test results, but proportions of the harmonics are not comparable; in fact, two different retainer webs show different harmonic amplitudes. There are also more frequency components in the experimental data. Further experimentation and analysis are required to understand these differences between predicted and measured values and to explain the origin of the background force level described above.

A skid map is shown in Figure 80.

Build No. 3, Rough-Race Ball Bearing

Inner and outer race temperatures are summarized in Table XI. Ball force data from the rough bearing generally looked similar to that of the smooth bearing with pulse magnitudes reduced. Samples of data from the first two test points are shown in Figures 81 and 82. The distorted retainer speed signal is the result of magnetic tape saturation in one direction, but since the large peak is still discernible, the signal is still useful in establishing the time of a single ball cycle and the relationship between the location of the retainer and the leading zone. Figure 82 also shows the beginning of the deterioration of the No. 2 retainer-web strain-gage bridge.

Build No. 4, Smooth-Race Roller Bearing

Inner and outer race temperatures are summarized in Table XII.

Traces from the only functioning retainer-web strain-gage bridge for four test conditions are shown in Figures 83 through 86.

The roller force traces do not show the large pulse-like forces found in the ball force traces at the test conditions required for computer verification; however, these peaks were observed while sweeping to the test speed.

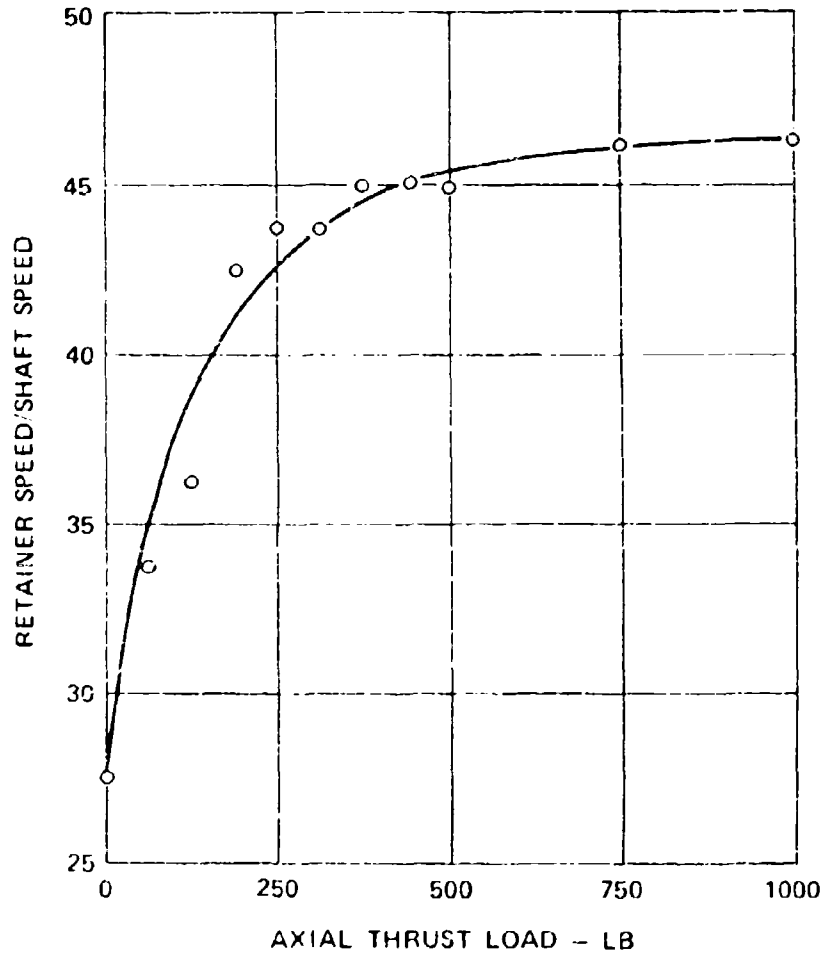


Figure 80. Retainer Speed Ratio Versus Axial Thrust Load.

TABLE XI. INNER AND OUTER RACE TEMPERATURES, BUILD NO. 3

Data Point	Inner Race (°F)	Outer Race (° F)	ΔT (° F)
1	275	250	-25
2	340	265	-75
3	260	260	0
4	290	265	-25
5	265	275	10
6	320	285	-35
7	300	285	15
8	-	275	-

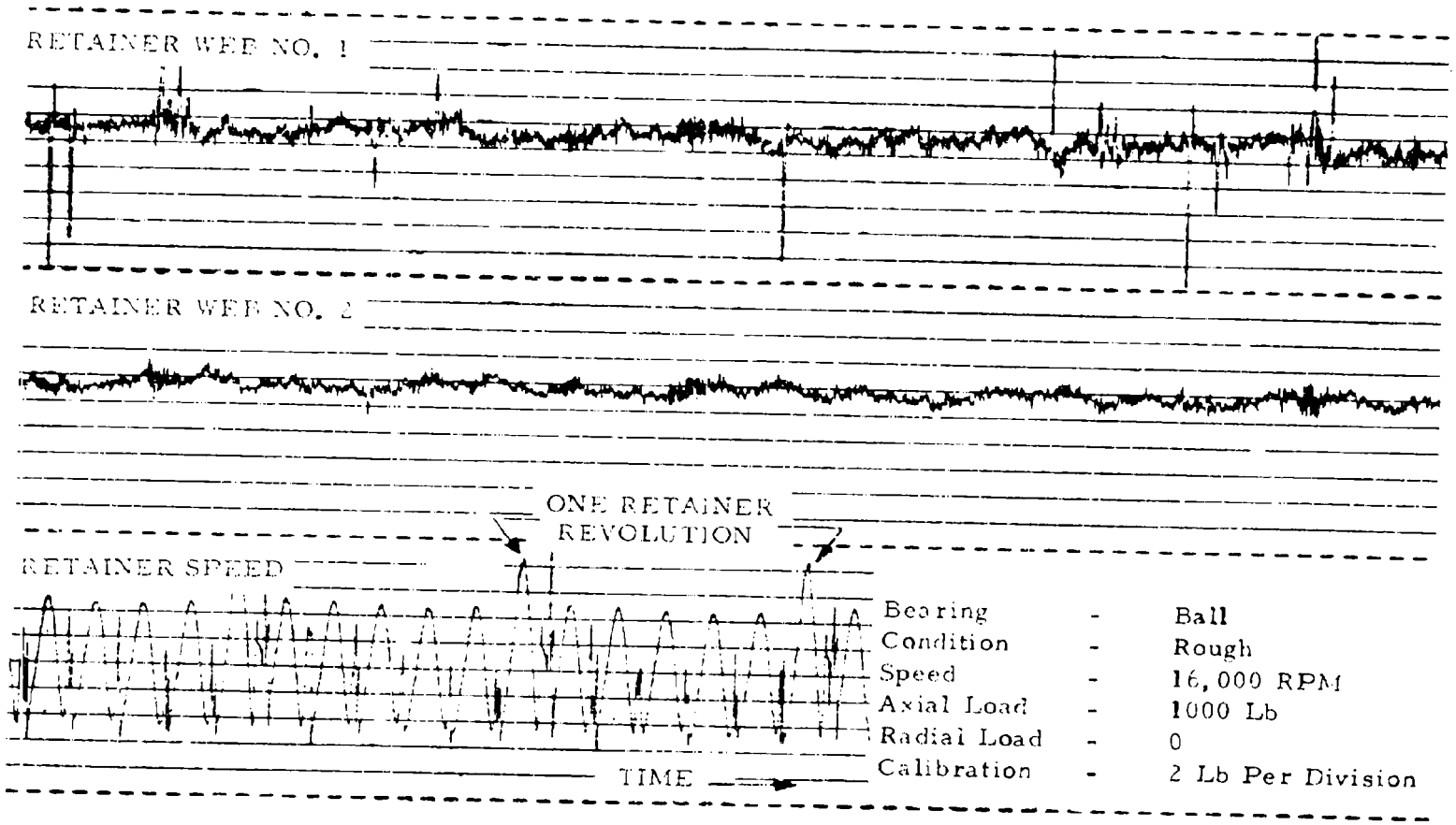


Figure 81. Expanded Trace of Rough-Race Bearing Retainer Web Forces. Speed signal distortion is result of magnetic tape saturation.

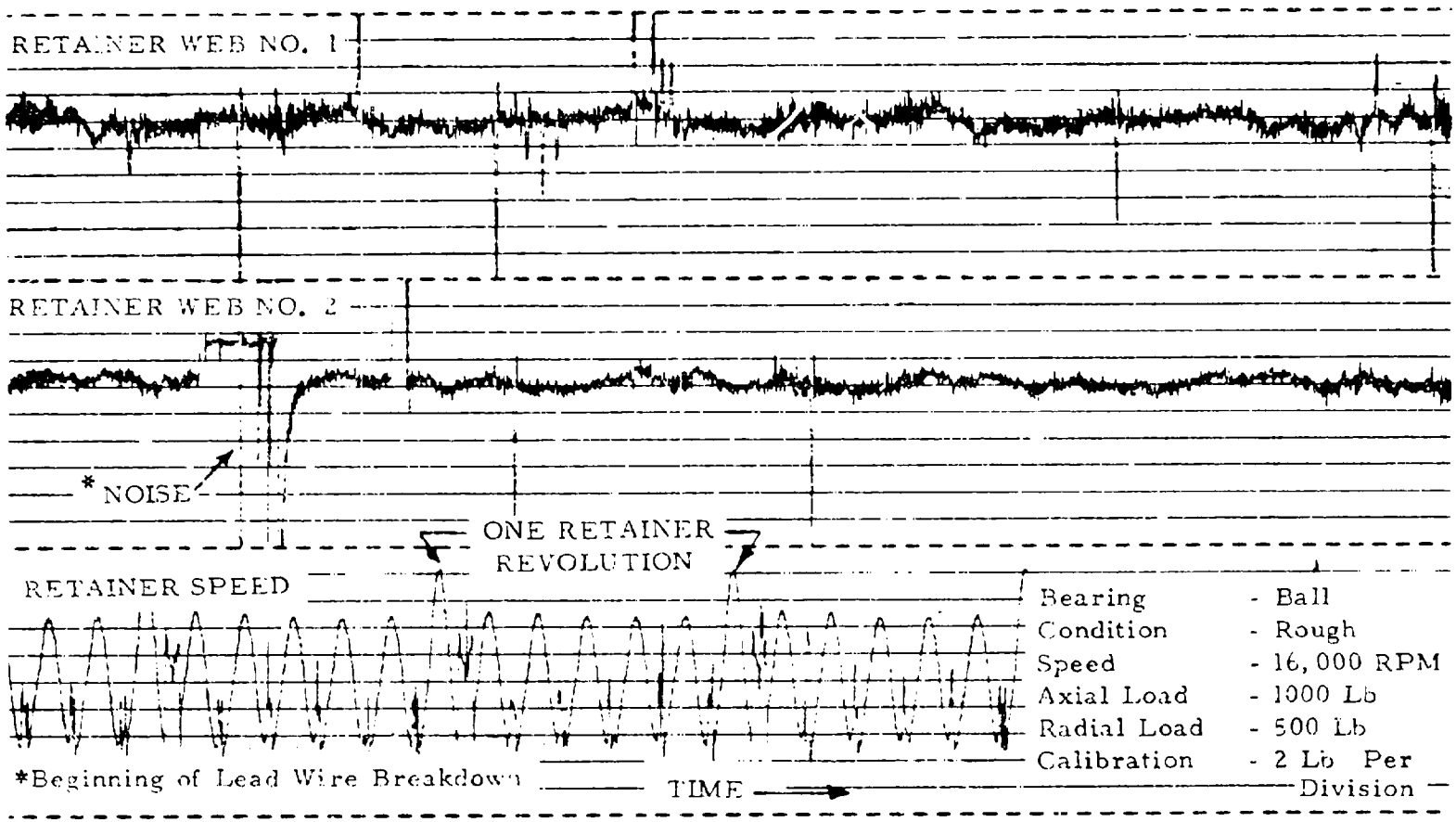


Figure 82. Expanded Trace of Rough-Race Bearing Retainer Web Forces. Speed signal distortion is result of magnetic tape saturation.

TABLE XII. INNER AND OUTER RACE TEMPERATURES, BUILD NO. 4

Data Point	Inner Race (° F)	Outer Race (° F)	ΔT (° F)
1	200	225	25
2	180	215	25
3	168	195	27
4	130	165	35
5	230	225	-5
6	245	245	0
7	180	255	75
8	140	265	125

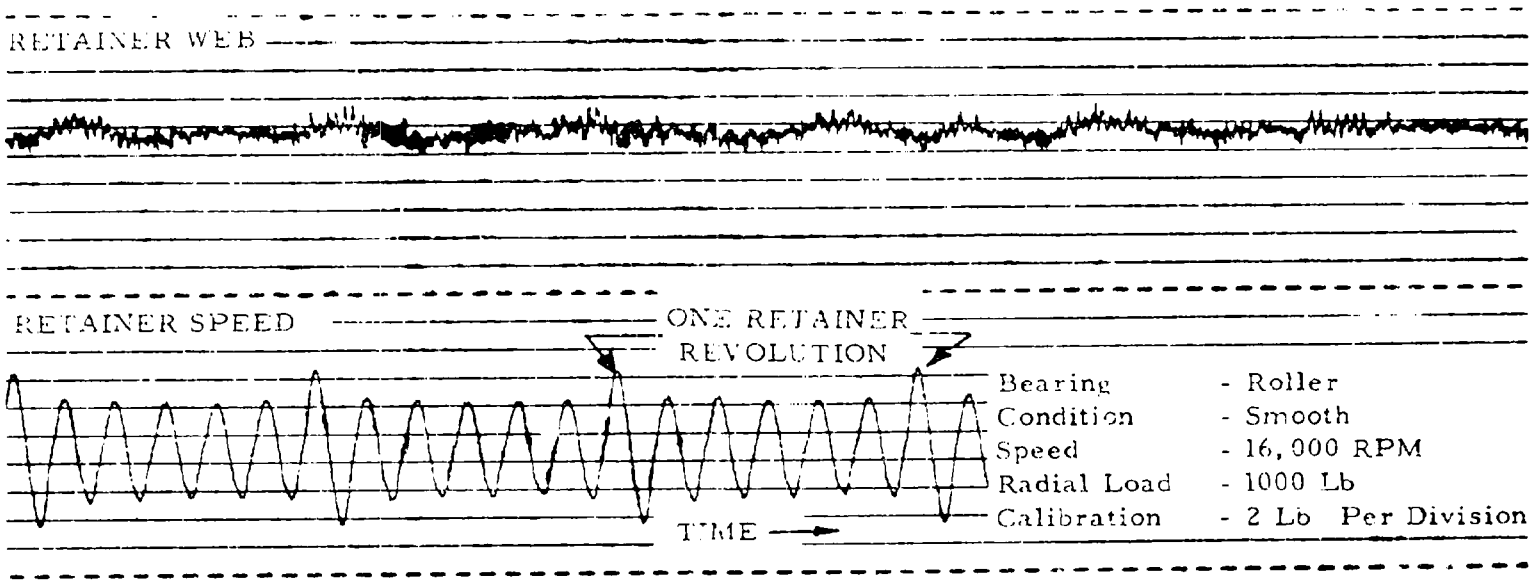


Figure 83. Bearing Retainer Web Force and Speed Versus Time.

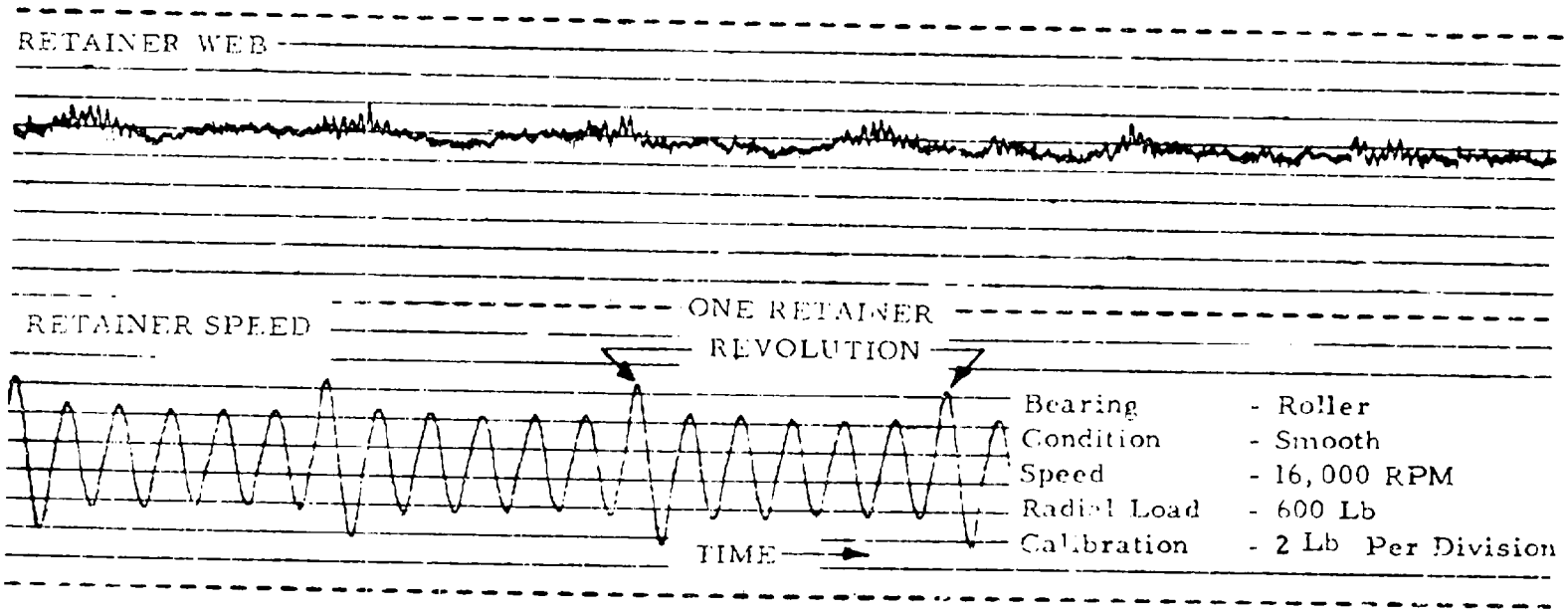


Figure 84. Bearing Retainer Web Force and Speed Versus Time.

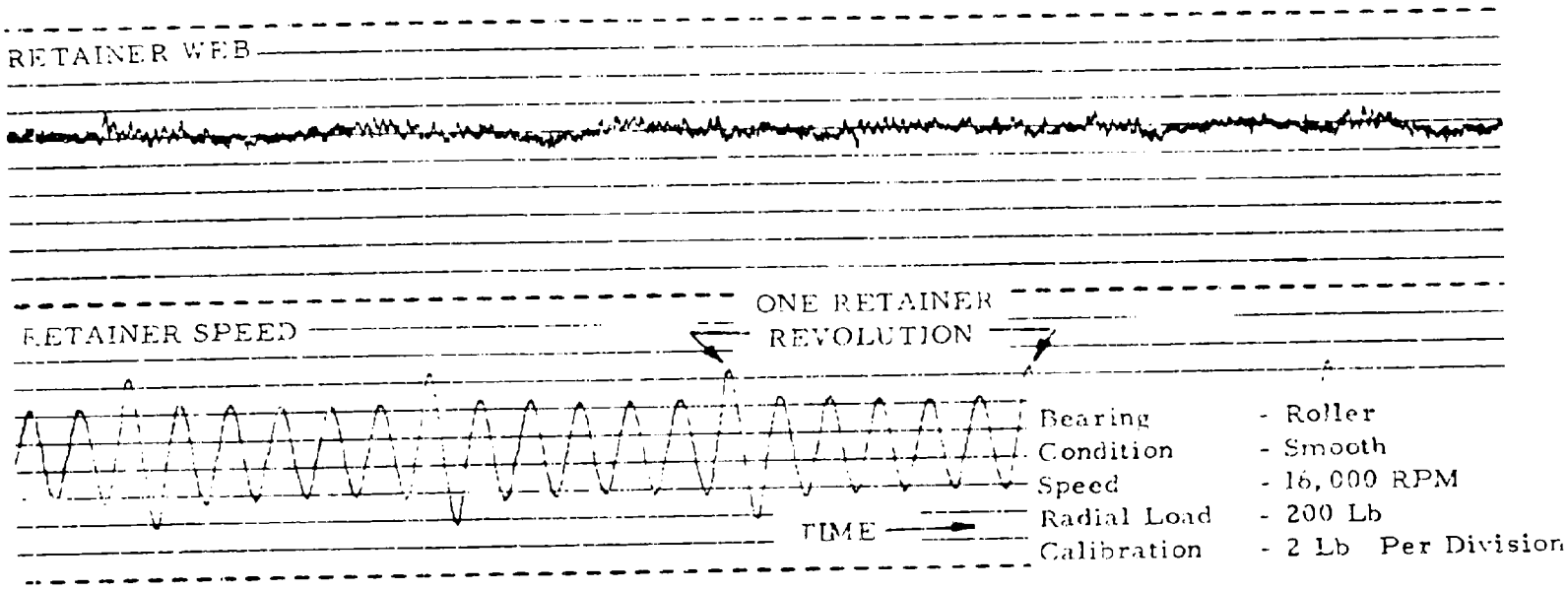


Figure 85. Bearing Retainer Web Force and Speed Versus Time.

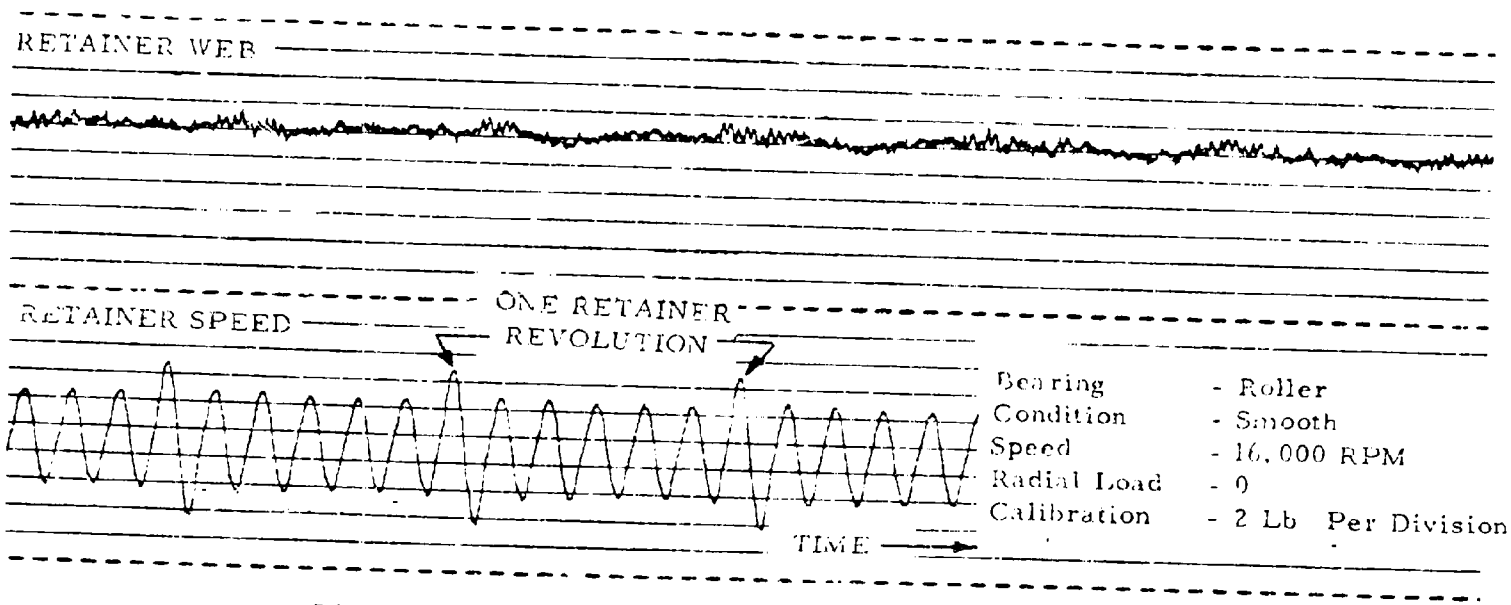


Figure 86. Bearing Retainer Web Force and Speed Versus Time.

COMPUTER PROGRAM RESULTS

BALL BEARING COMPUTER DECK

A listing of the ball bearing computer program FORTRAN source deck is presented in Appendix I. The program consists of a main section and eleven subroutines, each of which performs a specific function. This modular organization of the program facilitates the incorporation of alternate or improved mathematical models at a later date if revisions or modifications are indicated by future advances in the technology. For example, if it is desired to employ traction-slip data other than that reflected, only the subroutine FRCTN need be replaced.

The format for the input information required in the program is given in Appendix II. Cards signifying appropriate units for the input parameters and recommended values of convergence limits are included. Available options concerning the sequence of running numerous load cases with varying bearing definitions, lubricant descriptors, etc., are explained.

A typical problem with the input restated and the program output conveniently labeled for easy interpretation is presented in Appendix III.

BALL BEARING TYPICAL RESULTS

Retainer Pocket Loading and Lubrication

To illustrate the results desired of the computer program, the bearing problem shown in Table XIII, based on the test bearing in the experimental program, was adopted.

The bearing is analyzed at a constant inner ring speed of 20,000 rpm (2.0×10^6 DN) and an axial thrust load of 1,000 pounds. Externally applied radial loads were varied to determine their effect on pocket loading and lubrication. The effects of applied radial loads on the ball pocket load distribution are shown in Figure 87.

A positive pocket force indicates that the ball is driving the retainer assembly; conversely, a negative pocket force indicates that the ball is retarding the rotation of the retainer assembly. The results clearly indicate, as intuition dictates, that ball pocket loading is strongly dependent upon the relationship between the thrust and the radial load when operating under combined axial and radial externally applied loads. This dependency occurs because the unconstrained orbital velocity of each ball

TABLE XIII. BEARING PROBLEM PARAMETERS	
Number of Balls	22
Ball Diameter	.531 in.
Pitch Diameter	4.724 in.
Nominal Contact Angle	30 deg
Outer Race Curvature	.515
Inner Race Curvature	.520
Clearance Change	-.0015 in.
Pilot Clearance (outer)	.029 in.
Pocket Clearance	.031 in.
Pressure Viscosity Coefficient	1.18×10^{-4} in.^{2}/lb
Temperature Viscosity Coefficient	1.53×10^{-2} $^{o}F^{-1}$
Viscosity at Inlet	3.86×10^{-6} lb-sec/in.^{2}
Thermal Conductivity	7.74×10^{-2} Btu/^{o}F-hr/ft
Lubricant Density	8.97×10^{-5} lb-sec^2/in.^{4}

position would be different, owing to contact angle variations from position to position. Contact angle variations are strongly dependent on the magnitude of radial load relative to the axial load acting on the bearing. The calculated radial force acting between the pilot surface and the outer race land is shown as a parenthetic quantity in Figure 87. It is interesting to note the variation in this load component for the equal increments of externally applied radial loads. Note that for equal externally applied radial load increments from 500 to 750 pounds and from 750 to 1,000 pounds, the pilot surface contact force increase is 2.2 and 17.3 pounds, respectively. Additional cases were investigated with a thrust load of 500 pounds and a radial load of 1,000 pounds, and a pure radial load of 1,000 pounds with resulting pilot surface contact forces of 7.76 and 1.91 pounds, respectively. An axial-radial load ratio appears to exist (approximately 1:1) at which the pilot load is at a maximum value. This load component, in the case of an outer-piloted retainer, results in a retainer drag force; in the case of an inner-piloted retainer, it creates a retainer driving force. This effect might be theorized to be responsible for the skid recovery phenomenon often experienced on ball bearings at high speeds and extremely low thrust loads. Figure 88 illustrates the hydrodynamic ball-pocket film thickness as a function of azimuth angle for a thrust load of 1000 pounds and a radial load of 500 pounds. Also plotted on the same azimuth scale is the pocket loading to illustrate the strong dependence of film thickness upon load. For light loads, pocket film thicknesses are limited to pocket clearances in the analysis because often, with extremely light pocket loads, unrealistically large estimates will result.

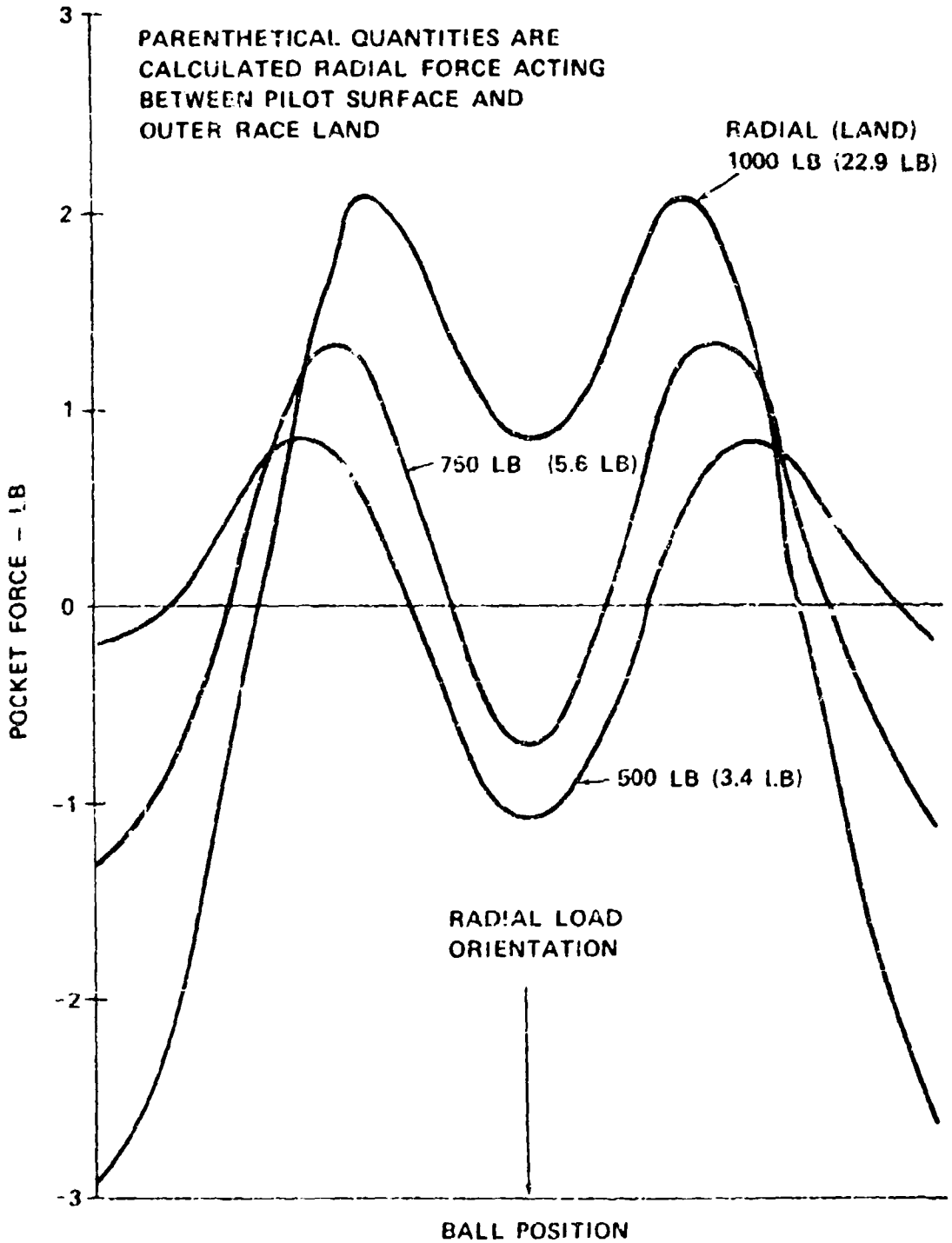


Figure 87. Pocket Loading Versus Radial Load (Axial Load = 1000 Pounds).

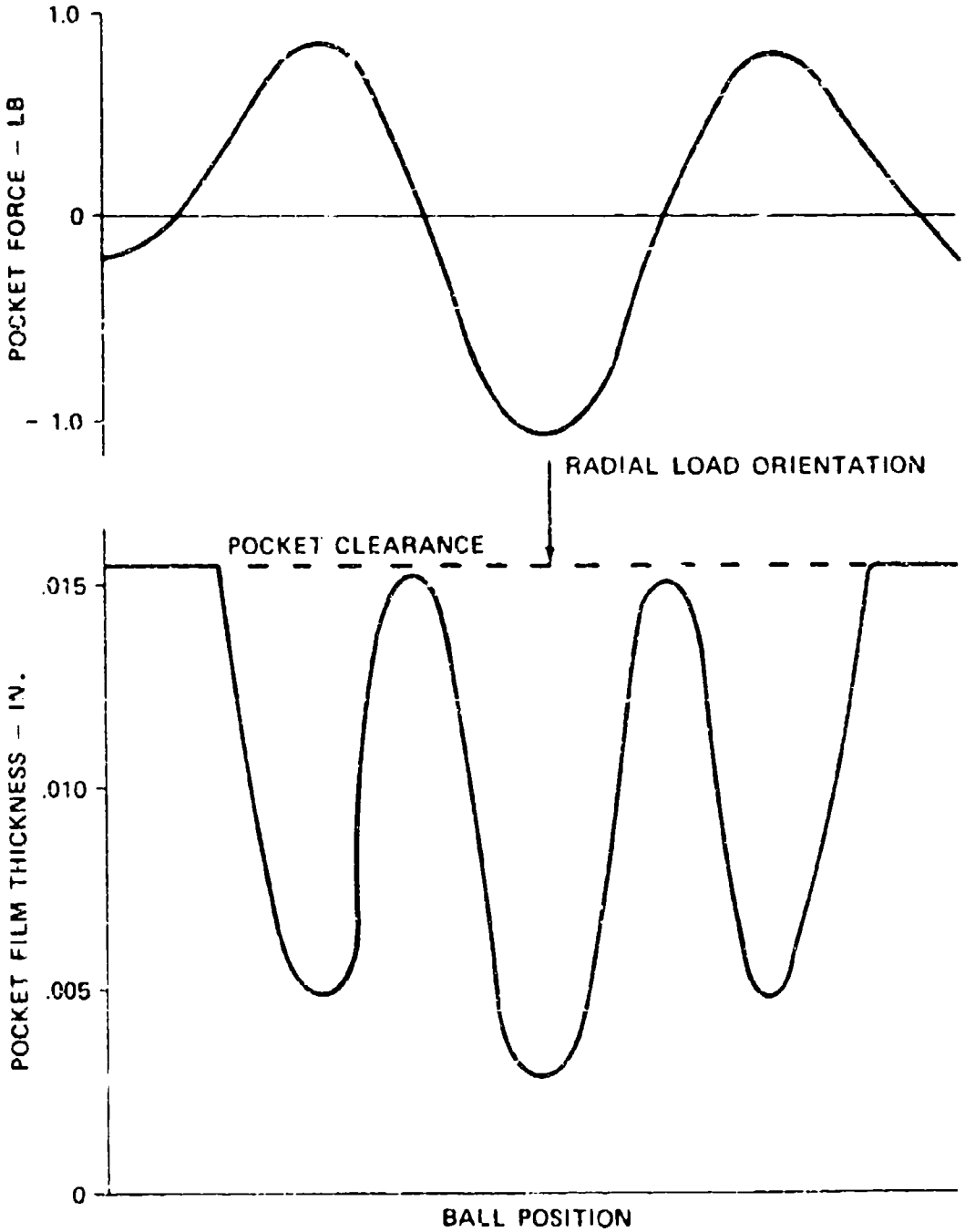


Figure 88. Pocket Film Thickness Versus Pocket Load (Axial Load = 1000 Pounds, Radial Load = 500 Pounds).

Retainer Speed Variations (Skidding)

An important spinoff benefit derived from the retainer analysis reported herein is the ability to predict retainer skidding as a function of externally applied loads for high-speed angular-contact ball bearings. Figure 89 compares retainer speed results derived from the computer program with data obtained during the verification program. Also shown are the retainer speeds calculated using the elastic analysis (no slip), which is extensively used for bearing design today throughout industry. The zone depicted between the dashed lines in the gross skid regime is indicative of retainer speed variations encountered. Unlike previous results,[9] retainer rotational velocities in this skid regime were not constant, and significant excursions were observed at constant thrust loads. This phenomenon is not considered to be of great importance from a design standpoint; what is significant is the break in the curve below which gross skidding occurs.

With respect to the difference between the elastic solution prediction and the EHD solution results, the difference in retainer speeds calculated at high thrust loads is predominantly the result of ball slip at the race contacts required to generate traction. Inner and outer race contact angles for this load case were essentially the same for the two different analytical solutions.

Ball Kinematics

The kinematics of a ball in a high-speed angular-contact bearing has provided bearing engineers and analysts with a wide area for disagreement over the last decade. The reason is that in an angular-contact ball bearing, pure rolling cannot occur simultaneously at both inner and outer race contacts.[2] Consequently, assumptions were required concerned with the spin velocities prevalent at the race contacts and the distribution of tractive force components required to offset ball gyroscopic moments. The major controversy centers about the race control concept previously discussed under "Introduction."

This concept is based upon the supposition that all the spin occurs at the contact where the resistance to spin (spin torque) is minimum, and that all traction in the gyroscopic plane occurs at the race contact which exhibits pure rolling (no slip). For example, a bearing with outer race control would have zero spin at the outer contact with the inner contact operating at the geometrically required spin level, while all the required traction force to resist gyroscopic precession would be developed at the outer contact.

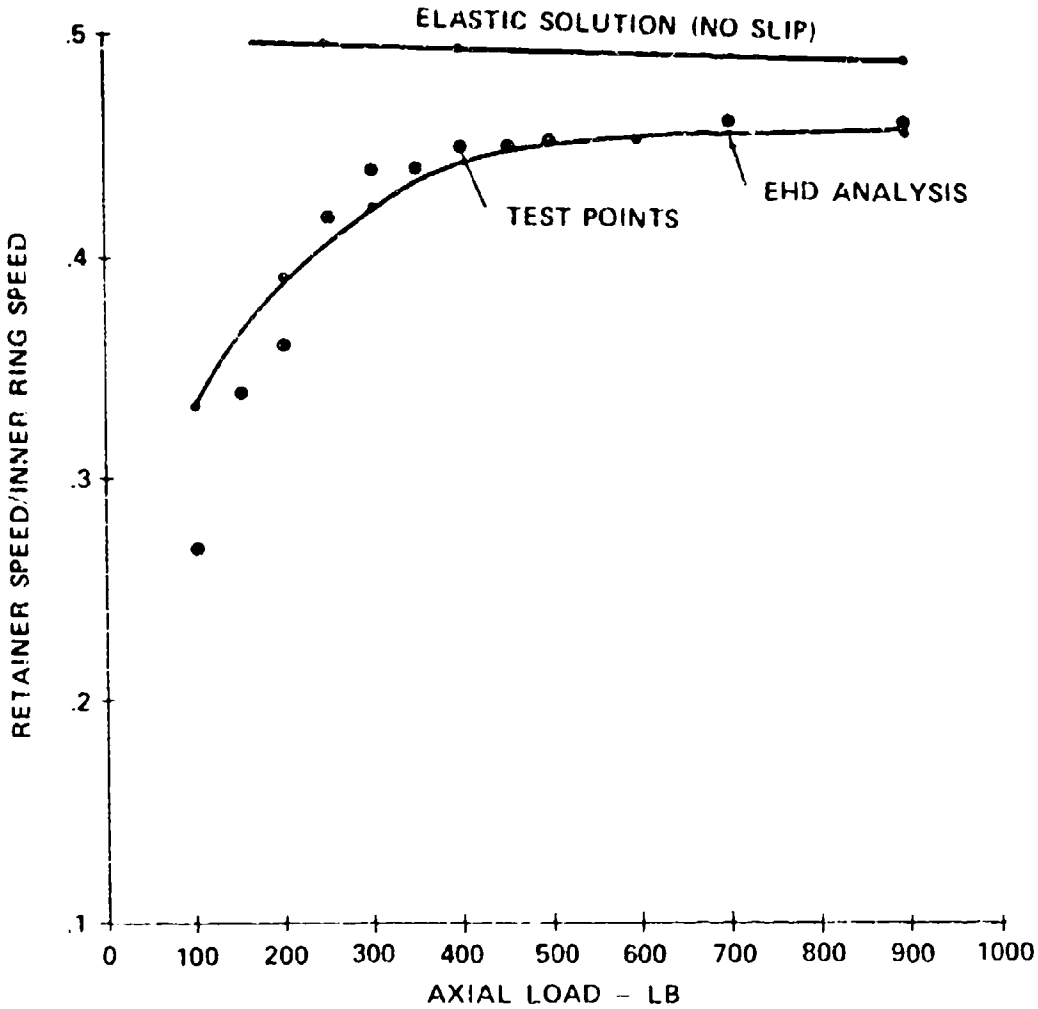


Figure 89. Retainer Speed Versus Axial Load.

The analysis developed herein is general and does not require these assumptions, and unbiased solutions can be obtained. Table XIV presents computer results pertaining to ball kinematics for varying thrust loads to explore this phenomenon in the skid regime and at high thrust loads. The cases studied are the same as those used for the skid curve, Figure 89. Also shown at the bottom of the table is the result of an elastic solution in which outer race control is assumed.

In Table XIV, ω_x, ω_y, and ω_z are the angular velocity components about the indicated axes. The angular velocity component ω_y has a circumferential vectorial direction, i.e., gyroscopic precession. F_{X1} and F_{X2} are the tractive forces in the outer and inner contact areas, which are opposing the rotation ω_y. The ratio F_{X2} divided by F_{X1} compares these two force components. Outer race control would have $F_{X2} = F_{X2}/F_{X1} = 0$. The computer results do not rigorously substantiate this assumption, but F_{X2} is significantly less than F_{X1}, which indicates that the race control concept is, if not absolutely correct, practically useful from this standpoint. It is interesting to note that from a traction standpoint, bearings in the skid regime tend more toward race control than those that are operating at high thrust loads. Also, by comparing the 800-pound thrust load cases, it is seen that a greater value of ω_z is predicted with the EHD analysis with a corresponding increase in gyroscopic moment and tractive force.

Another relationship that can be used to test the accuracy of the race control concept is the spin velocities at the race contact. Again, referring to Table XIV, it is evident that spin occurs predominantly at the inner contact as the control theory predicts. With regard to the behavior in the skid regime, values of ω_{s1} divided by ω_{s2} increase as gross skidding increases -- the opposite effect relative to race control behavior as observed with the tractive forces.

In general, kinematic behavior that is in relative agreement with the race control concept is not surprising, in that many successful bearings have been designed with the use of analytical techniques predicated upon its precepts.

TABLE XIV. BALL KINEMATICS; INNER RING SPEED = 20,000 RPM, RADIAL LOAD = 0

Thrust (lb)	ω_X (rpm)	ω_Y (rpm)	ω_Z (rpm)	F_{X_1} (lb)	F_{X_2} (lb)	F_{X_2}/F_{X_1}	ω_{s1} (rpm)	ω_{s2} (rpm)	ω_{s1}/ω_{s2}
100	-67,640	1,696	8,470	-3.78	-0.05	.013	-7,854	-39,753	.198
200	-78,834	1,960	10,030	-5.30	-0.15	.028	-7,764	-44,826	.173
300	-84,104	2,052	11,641	-6.32	-0.27	.043	-7,210	-46,953	.154
500	-88,447	2,075	13,930	-7.75	-0.59	.076	-5,945	-47,820	.124
800	-89,566	1,990	17,210	-9.29	-1.25	.135	-4,540	-45,800	.099
800 (Elastic Solution)	-94,800	0	12,600	-8.2	0	0	0	-52,669	0

ROLLER BEARING COMPUTER DECK

A listing of the roller bearing computer program FORTRAN source deck is presented in Appendix IV. The program is structured in a manner similar to the ball bearing deck and consists of a main section and nine subroutines. Four subroutines are inactive in this listing as a result of the convergence problems encountered with the COEFCT routine. They are included in this listing since they were part of the initial effort.

The format for the program input is given in Appendix V. Directions for exercising various program input options are also given. A particular input requirement for the roller bearing program is found on card 9, columns 41 through 50. This input item causes the program to execute the specific case for the cage slip inputted. It is therefore required that the user input an appropriate series of cases to achieve a desired solution. The solution is obtained when the "Torque on Cage" output value approaches zero.

A typical problem and corresponding output is presented in Appendix VI.

ROLLER BEARING TYPICAL RESULTS

The bearing data shown in Table XV was used to illustrate the desired program output. This data represents the test roller bearing used during the experimental program.

The bearing was analyzed at an inner ring speed of 20,000 rpm, which results in a DN speed value of 2×10^6. Radial loads were varied to determine slip speeds of the cage and other bearing outputs. To arrive at the desired solution, various cage slip speed assumptions were inputted. Each output case yielded residual cage torques which were plotted as shown in Figure 90. The final solutions for slip are obvious and can be inputted to obtain final results for each load case. Final results for cage slip speeds as a function of radial loading are shown in Figure 91. The iterative procedure was pursued for the 200-pound load case, the solution for which is shown in Appendix VIII.

TABLE XV. ROLLER BEARING PROBLEM PARAMETERS	
Number of Rollers	28
Roller Diameter - in.	.4331
Roller Length - in.	.4331
Pitch Diameter - in.	4.7244
Diametral Clearance - in.	.0025
Retainer Pilot Clearance (Inner) - in.	.030
Pocket Clearance - in.	.0125
Pressure Viscosity Coefficient - $in.^2/lb$	9.6×10^{-5}
Temperature Viscosity Coefficient - $^oF^{-1}$	1.47×10^{-2}
Viscosity at Inlet - $lb\text{-}sec/in.^2$	7.35×10^{-7}
Thermal Conductivity - $Btu/^oF\text{-}hr/ft$	7.40×10^{-2}
Lubricant Density - $lb\text{-}sec^2/in.^4$	8.25×10^{-5}

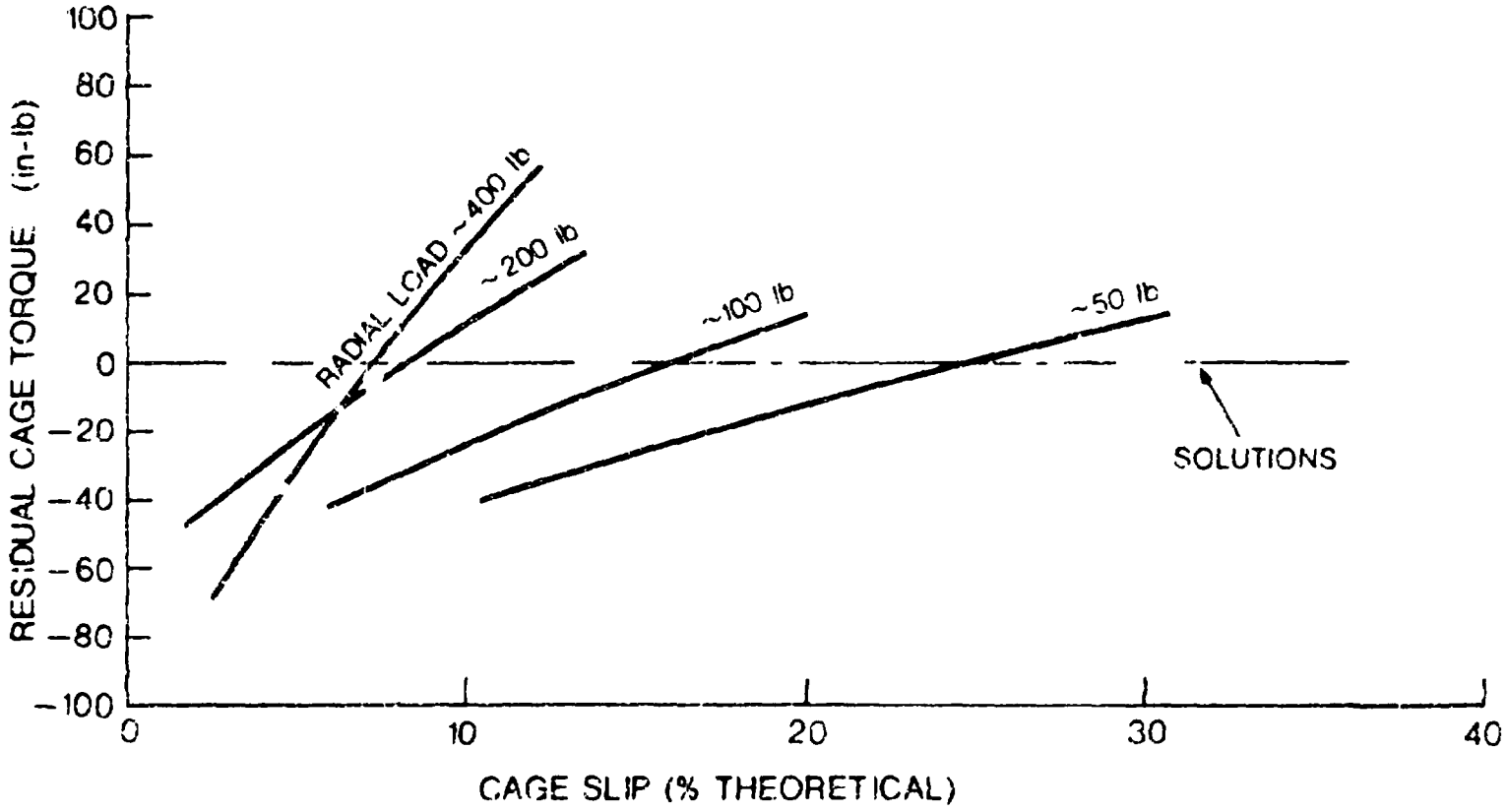


Figure 90. Roller Bearing Program Data: Residual Cage Torque Versus Cage Slip (Inner Race Speed = 20,000 RPM).

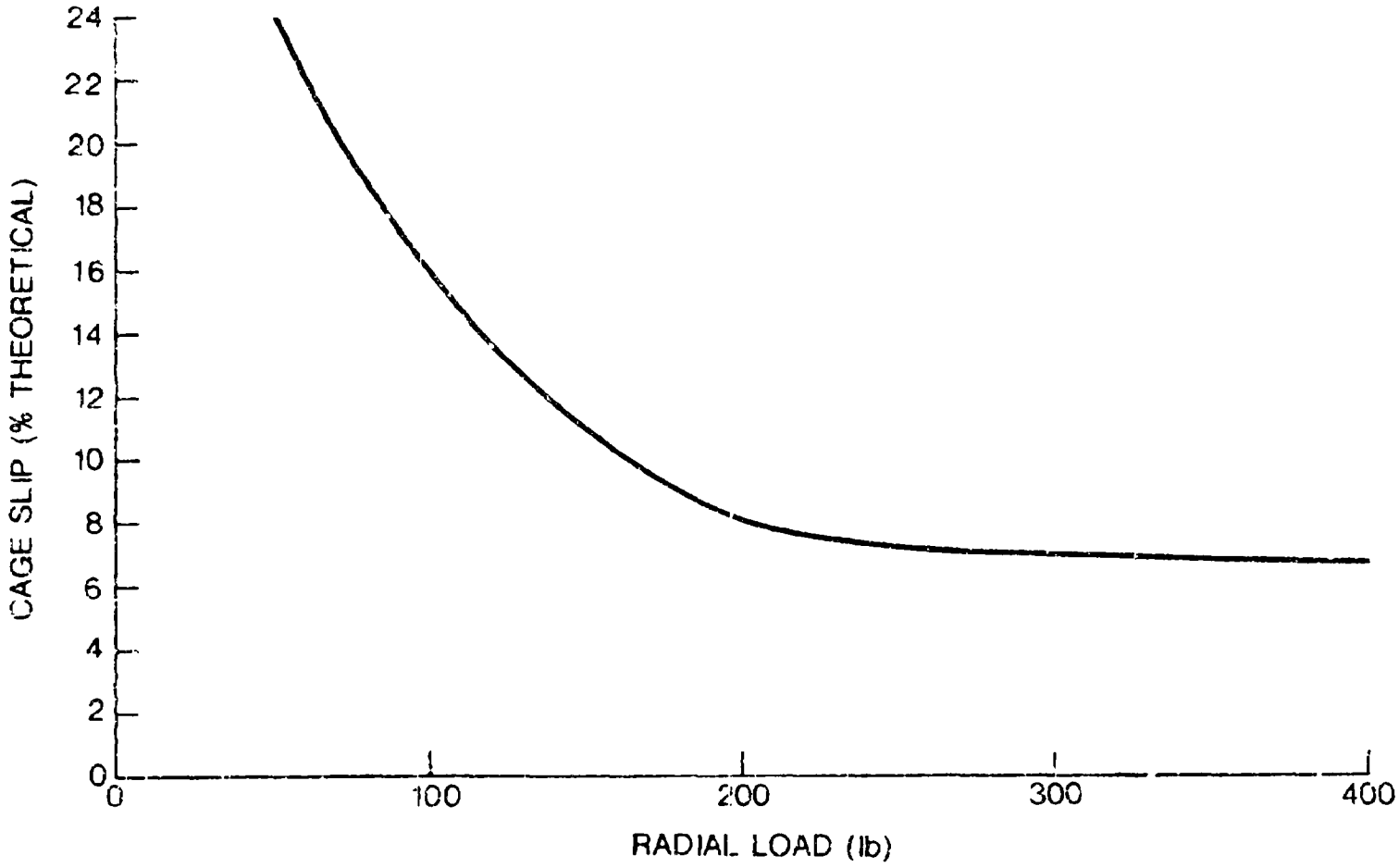


Figure 91. Roller Bearing Program Data; Cage Slip Versus Radial Load (Inner Race Speed = 20,000 RPM).

CONCLUSIONS

1. Retainer pocket forces predicted by the computer analysis for the cases studied are small, approximately 5 pounds, and are in quantitative agreement with the basic pocket force cycles measured during the experimental test program. Existing differences between measured and predicted values can be attributed to dynamic shaft unbalance, imperfect bearing geometry, etc. However, superimposed upon these force levels are pulse-type forces that were measured at the retainer pockets. These forces were significantly larger than the former, on the order of 50 pounds, and are believed to be the result of impacts between the rolling elements and retainer pockets resulting from acceleration/deceleration of the rolling elements and/or retainer. Impact force levels dominate and constitute the major criteria in the establishment of the structural integrity of the high-speed retainer.

2. Lubricant film thicknesses developed at the rolling element-to-retainer pocket contacts are hydrodynamic in nature, and they are strongly dependent upon the contact force magnitudes.

3. Bearing kinematics resulting from the computer solutions that consider traction at the rolling element-to-race contacts are not widely different from analyses conducted hitherto that employ the race control assumptions. However, the new model does predict ball spin at both inner and outer contacts, gyroscopic precession, and resistance to gyroscopic precession at each of the inner and outer race contacts.

4. The computer analysis predicts regions of tractive ball instability at some load and speed conditions resulting from thermal effects in the EHD contact between balls and races.

5. Retainer slip at low-load conditions can be predicted by the computer program. Confirmation of the analysis for the test ball bearing employed during the program was obtained. Retainer pilot surface-to-race land contact force, which affects skidding characteristics, was found to be strongly dependent upon the ratio of the axial to radial loads applied to the bearing.

RECOMMENDATIONS

1. It is recommended that further work conducted to achieve a better understanding of overall bearing operation address the rheological behavior of EHD lubrication with particular emphasis on traction.

   The effects of lubrication upon bearing operating characteristics are central to the analysis of rolling element bearings. The dynamics and kinematics of bearing operation are well understood in comparison with the equally important effects of the rheological behavior of the lubricant. Much of the lubricant information drawn upon in the analysis is an extrapolation of empirical data obtained in different operating regimes. The development of more extensive and sophisticated bearing analyses based upon the relatively underdeveloped technology of EHD lubrication will not substantially increase our overall understanding of high-speed bearing performance, especially in the light of the convergence problems encountered with available data as described in "Tractive Instability," page 53.

2. Attention should be directed toward further investigation of the cage pocket impact forces observed. Tests should be conducted on a greater variety of bearing configurations and operating conditions to achieve a more general verification of the observed results.

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APPENDIX I

BALL BEARING COMPUTER PROGRAM FORTRAN SOURCE DECK

```
C     MAIN PROGRAM
      REAL LUBDEN
      COMMON HSO(40),HSI(40)
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),        BET,BETA
     1,B(2),B3(2),BPOCK(40),  CAGEW,     CRV(2),    CRAD, COSB,CPH,CTHET,
     2CORR(5),CB(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,  DP(2,5),
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DFL(4),DTV(5,5),
     4DETER,DEL(2),DDELX(2,2),DXKDG(2),DFXVX(2),DFXVY(2),DFXWS(2),
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWS
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(
     75),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYA
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2
     9),DVARA1(5),DVARA2(5),DVOME(5), DGA(2),                    E,EL
      COMMON ERR(5),E2,ESAVE(2),                          FLAT,F(2),FK,FRAD
     1,FTAN,FFX(2,40),FFY(2,40),                          GAGE,G2(2),GG(2),
     2                    HORIZ,HOR1,HOR2,HOR3,HOR4,                IQUIT,
     3JPASS,              LPASS,LUBDEN,          N,            IMEGAE,OMY,OMX,
     4OMZ,OMS(2),OMEX(40),OMEY(40),OMEZ(40),                  PPOC(40),PL,
     5PR,PE,PC,PD,PCCKET,PHI,PILOT,PX(2),PSI(5),P(2,40),              OMV,
     6QS(2),QQS(2,40),                R,RPM(2),              SINB,SPH,STHET,
     7SB(2),SQ(2),                      TOTL,TI,THET,TOL(11),TCRV,TSAVE(2),
     8THICKR(2,40) ,THICKP(40),     VIS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2),
     9VYB(2),VXB(2),VXP(40),VZP(40),VEC(5)
      COMMON X(5),XN,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMS
     1(2,40),XML,XMU1,     YE,YR,YC,YNUM
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS
      COMMON FX(2),FY(2)
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,IQ(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),
     2CBAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2),
     3XA(2),  KONV,DAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4,
     4DET,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)
     5,CAGEDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),
     6FPZDA1,FPZDA2,FPZOME,DBDRAG(4)
      COMMON/TSTCOM/TEST1,TEST2
      COMMON/BALTST/PLTEST
      COMMON/XTRA/VDRAG(2),CDRAG(2) ,CFORCE
      CALL ZERO(ALPHA,DBDRAG(4))
      GO TO 999
  998 CONTINUE
      DAD(1,1)=1.
      DAD(2,1)=0.
      DAD(1,2)=0.
      BA(1,1)=0.
      BA(1,2)=0.
      DEX(1)=0.
      DEX(2)=2.
      DELA(1,1)=0.
      DELA(1,2)=0.
      H(2)=0.
      HH(1)=0.
      H1(1)=1.
      H1(2)=0.
      H2(1)=0.
      H2(2)=1.
      GA(1,1)=0.
      GA(1,2)=0.
      DCF(1)=0.
      DCF(3)=0.
```

```
      DCF(4)=0.
      DCF(5)=0.
      C(1)=1.
      C(2)=-1.
    2 READ(5,80)
      WRITE(6,80)
      CALL INPT
      XMASS=1.3550693E-3*DENS*D**3
      XIN=.1*XMASS* D**2
10    READ(5,1000)ALPHA,BETA,VIS,TI,FK,LUBDEN,D2VYR,D2VX
      LCASE=0
      IF(ALPHA)1,9,1
    9 WRITE(6,81)
      GO TO 2
1     WRITE(6,3)
    3 FORMAT(1H /1H /1H /1H /1H /1H /23H0  LUBRICANT PROPERTIES)
      READ(5,1000)D2WS,D2P2,(TOL(K),K=1,9),CONST1,CONST2
      WRITE(6,4)ALPHA
    4 FORMAT(40H0  PRESSURE-VISCOSITY COEFFICIENT        ,1P1E11.4)
      WRITE(6,5)BETA
      WRITE(6,6)VIS
    5 FORMAT(40H   TEMPERATURE-VISCOSITY COEFFICIENT     ,1P1E11.4)
    6 FORMAT(40H   VISCOSITY AT INLET TEMPERATURE        ,1P1E11.4)
      WRITE(6,7)TI
    7 FORMAT(40H   INLET TEMPERATURE                     ,1P1E11.4)
      WRITE(6,8)FK
    8 FORMAT(40H   THERMAL CONDUCTIVITY                  ,1P1E11.4)
      WRITE(6,1001)LUBDEN
1001  FORMAT(40H   LUBRICANT DENSITY                     ,1P1E11.4)
      WRITE(6,5000)XMU
5000  FORMAT(40H   BALL POCKET FRICTION COEFFICIENT      ,1P1E11.4)
      WRITE(6,5001)XMU1
5001  FORMAT(40H   CAGE PILOT FRICTION COEFFICIENT       ,1P1E11.4)
30    READ(5,1000)RPM(1),RPM(2),XF(1),XF(3),DDL(4),DDL(1),DDL(3),VLCTL
      LCASE=LCASE+1
      LCTL=VLCTL
      IF(D2VYR.EQ.0.)D2VYR=.05
      IF(D2VX.EQ.0.)D2VX=.05
      IF(D2WS.EQ.0.)D2WS=.1
      IF(D2P2.EQ.0.)D2P2=.05
1000  FORMAT(7F10.0,F9.0,I1)
      IF(LCTL.LE.0)LCTL=1
      GGG=D*COS9/E
      OMEGAE=.5*(RPM(1)*(1.+GGG)+RPM(2)*(1.-GGG))
4000  ZAPS=OMEGAE
1000  FORMAT(8F10.0)
      IF((RPM(1)-RPM(2)).EQ.0.) GO TO 10
      WRITE(6,81)
      WRITE(6,80)
      WRITE(6,71)LCASE
      WRITE(6,72)RPM(1)
      WRITE(6,73)RPM(2)
      WRITE(6,82)OMEGAE
      WRITE(6,74)XF(1)
      WRITE(6,75)XF(3)
      WRITE(6,76)DDL(4)
      WRITE(6,77)DDL(1)
      WRITE(6,78)DDL(3)
      WRITE(6,79)VLCTL
```

```
 71 FORMAT(40H0  OPERATION DATA FOR LOAD CASE NUMBER   ,I2)
 72 FORMAT(40H0  OUTER RACE SPEED                      ,1P1E11.4)
 73 FORMAT(40H   INNER RACE SPEED                      ,1P1E11.4)
 74 FORMAT(40H   THRUST LOAD                           ,1P1E11.4)
 75 FORMAT(40H   RADIAL LOAD                           ,1P1E11.4)
 76 FORMAT(40H   MISALIGNMENT                          ,1P1E11.4)
 77 FORMAT(40H   INITIAL AXIAL DEFLECTION              ,1P1E11.4)
 78 FORMAT(40H   INITIAL RADIAL DEFLECTION             ,1P1E11.4)
 79 FORMAT(40H   RACE CONTROL INDEX                    ,1P1E11.4)
 80 FORMAT(71H
   1                                                   )
 81 FORMAT(1H1)
 82 FORMAT(40H   THEORETICAL CAGE SPEED                ,1P1E11.4)
 83 FORMAT(1P3E11.4)
    OMEGAE=.10471976*OMEGAE
    IF(ABS(DDL(1))+ABS(DDL(3))+ABS(DDL(4)))13,13,14
13  DDL(1)=DFL(1)
    DDL(3)=DFL(3)
    DDL(4)=DFL(4)
14  N=XN
    IF(TOL(1).EQ.0.)TOL(1)=1.E-6
    IF(TOL(2).EQ.0.)TOL(2)=1.E-6
    IF(TOL(3).EQ.0.)TOL(3)=10.
    TOL(3)=TOL(3)*.1047198
    IF(TOL(4).EQ.0.)TOL(4)=1.
    TOL(4)=TOL(4)*.1047198
    IF(TOL(5).EQ.0.)TOL(5)=5.
    TOL(5)=TOL(5)*.1047198
    IF(TOL(6).EQ.0.)TOL(6)=1.E-6
    IF(TOL(7).EQ.0.)TOL(7)=1.E-6
    IF(TOL(8).EQ.0.)TOL(8)=1.E-6
    IF(TOL(9).EQ.0.)TOL(9)= .5
    TOL(9)=TOL(9)*.1047198
    CONST1=.2
    CONST2=1.
    DO 34 K=1,4
34  DFL(K)=DDL(K)
    KINV=0
    RPM(1)=RPM(1)*.10471976
    RPM(2)=RPM(2)*.10471976
    PL=4.*((1.-PE**2)/YE+(1.-PC**2)/YC)
    DO 43 J=1,N
43  CTL(J)=LCTL
48  CONTINUE
    DO 15 ITER=1,15
    RTEST=0.0
    DO 481 K=1,4
481 BBDRAG(K)=0.
    ITPASS=ITER
    TEMP1=.008*LUBDEN**.75*SQRT(SQRT(VIS))
    DO 400 K=1,2
    VEL(K)=(RPM(K)-OMEGAE)*CAGEDI(K)*.5
    SDRAG(K)=TEMP1*AREA(K)*ABS(VEL(K))**1.75/CLEARS(K)**.25*.5*CAGEDI(
   1K)
    SDRAG(K)=SIGN(SDRAG(K),VEL(K))
400 VDRAG(K)=SDRAG(K)
    BDRAG=LUBDEN*D*(CLEARS(1)+CLEARS(2))*(E*OMEGAE)**2*.062
    BDRAG=SIGN(BDRAG,OMEGAE)
    DO 16 K=1,4
```

```
16    XF1(K)=0.
      DO 17 K=1,4
      EFR(K)=0.
      DO 17 L=1,4
17    DTV(L,K)=0.
      HORIZ=0.
      VERT=0.
      HOR1=0.
      HOR2=0.
      HOR3=0.
      HOR4=0.
      VRT1=0.
      VRT2=0.
      VRT3=0.
      VRT4=0.
      DO 19 J=1,N
      NUM=0
      ICNT=0
      LOOP=0
      JPASS=J
      DMF(J)=0.
      XJ=J
      IF(KONV)52,52,53
53    IF(ITER-1)52,55,52
55    X(3)=RFVX(J)
      X(4)=0.
      X(5)=RFVZ(J)
52    PHI=6.283185*(XJ-1.5)/XN
      SPH=SIN(PHI)
      CPH=COS(PHI)
      DAC(2,2)=CPH
      CALL BALL(CONST1,CONST2,FPZ,DVX)
      IF(TEST1.EQ.0.0) GO TO 2054
      WRITE(6,2052)TEST1
2052  FORMAT(1X,9H TEST1 = ,F5.1)
      GO TO 30
2054  CONTINUE
      IF(P(2,J))18,18,54
54    FPR(4)=FPR(4)+PPOC(J)*.5*E2+FPZ*.5*D
      IF(PLTEST.EQ.1.0)PTEST=1.0
18    CONTINUE
      IF(KONV)44,44,45
44    DET=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
      FPR(1)=XF1(1)+XF(1)
      FPR(3)=XF1(3)+XF(3)
      CORR(1)=(FPR(1)*DTV(2,2)-FPR(3)*DTV(1,2))/DET
      CORR(3)=(DTV(1,1)*FPR(3)-DTV(2,1)*FPR(1))/DET
      IF(ABS(CORR(1))-TOL(1))46,1515,1515
46    IF(ABS(CORR(3))-TOL(3))47,1515,1515
1515  CONTINUE
      DEL(1)=DEL(1)-CORR(1)
      DEL(3)=DEL(3)-CORR(3)
      GO TO 15
45    FPR(1)=XF1(1)+XF(1)
      FPR(2)=XF1(2)
      FPR(3)=XF1(3)+XF(3)
            IF(HORIZ.EQ.0.0)HORIZ=.01
            IF(VERT.EQ.0.0)VERT=.01
      TEMP2=SQRT(HORIZ**2+VERT**2)
```

```
      CFORCE=TEMP2
      KPLT=PILOT+.1
      DO 402 K=1,2
      DO 403 L=1,4
403   DSDRG(K,L)=0.
      IF(K.NE.KPLT)GO TO 1402
      TEMP=.5*XMU1*CAGEDI(K)*TEMP2
      CDRAG(K)=SIGN(TEMP,VEL(K))
      SDRAG(K)=SERAG(K)*CDRAG(K)
      TEMP=XMU1*CAGEDI(K)/TEMP2
      TEMP=SIGN(TEMP,VEL(K))
      DSDRG(K,1)=(HORIZ*HOR1+VERT*VRT1)*TEMP
      DSDRG(K,2)=(HORIZ*HOR2+VERT*VRT2)*TEMP
      DSDRG(K,3)=(HORIZ*HOR3+VERT*VRT3)*TEMP
      DSDRG(K,4)=(HORIZ*HOR4+VERT*VRT4)*TEMP
      TEMP=CAGEDI(K)**2*TEMP1*AREA(K)*ABS(VEL(K))**.75*.4375/CLEARS(K)**
     1.75
      TEMP=SIGN(TEMP,VEL(K))
      DSDRG(K,4)=DSDRG(K,4)+TEMP
1402  CONTINUE
      ERR(4)=ERR(4)+SDRAG(K)
402   CONTINUE
      TEMP=ATAN2(HORIZ,VERT)
      SNTH=SIN(TEMP)
      CSTH=COS(TEMP)
      TMP1=SNTH-XMU1*CSTH
      TMP2=CSTH+XMU1*SNTH
      TMP3=CSTH+XMU1*SNTH
      TMP4=-SNTH+XMU1*CSTH
      AY=-TEMP2*TMP1
      AZ=-TEMP2*TMP3
      DTMP21= (HORIZ*HOR1+VERT*VRT1)/TEMP2
      DTMP22=(HORIZ*HOR2+VERT*VRT2)/TEMP2
      DTMP23=(HORIZ*HOR3+VERT*VRT3)/TEMP2
      DTMP24=(HORIZ*HOR4+VERT*VRT4)/TEMP2
      DAY1=-TEMP2*TMP2*(VERT*HOR1-HORIZ*VRT1)/(VERT*CSTH)**2-TMP1*DTMP21
      DAY2=-TEMP2*TMP2*(VERT*HOR2-HORIZ*VRT2)/(VERT*CSTH)**2-TMP1*DTMP22
      DAY3=-TEMP2*TMP2*(VERT*HOR3-HORIZ*VRT3)/(CSTH*VERT)**2-TMP1*DTMP23
      DAY4=-TEMP2*TMP2*(VERT*HOR4-HORIZ*VRT4)/(CSTH*VERT)**2-TMP1*DTMP24
      DAZ1=-TEMP2*TMP4*(VERT*HOR1-HORIZ*VRT1)/(CSTH*VERT)**2-TMP3*DTMP21
      DAZ2=-TEMP2*TMP4*(VERT*HOR2-HORIZ*VRT2)/(CSTH*VERT)**2-TMP3*DTMP22
      DAZ3=-TEMP2*TMP4*(VERT*HOR3-HORIZ*VRT3)/(CSTH*VERT)**2-TMP3*DTMP23
      DAZ4=-TEMP2*TMP4*(VERT*HOR4-HORIZ*VRT4)/(CSTH*VERT)**2-TMP3*DTMP24
      DO 404 K=1,4
404   DTV(4,K)=DTV(4,K)+DSDRG(1,K)+DSDRG(2,K)
      IF(KPLT-2)33,42,33
42    XF1(2)=XF1(2)+AY
      XF1(3)=XF1(3)+AZ
      DTV(2,1)=DTV(2,1)+DAY1
      DTV(2,2)=DTV(2,2)+DAY2
      DTV(2,3)=DTV(2,3)+DAY3
      DTV(2,4)=DTV(2,4)+DAY4
      DTV(3,1)=DTV(3,1)+DAZ1
      DTV(3,2)=DTV(3,2)+DAZ2
      DTV(3,3)=DTV(3,3)+DAZ3
      DTV(3,4)=DTV(3,4)+DAZ4
33    NN=4
      CALL SIMULT(DTV,NN,ERR,COFR,IQUIT)
      IF(IQUIT)24,24,25
```

```
 25   WRITE(6,26)
 26   FORMAT(14HOSTAT 25, MAIN)
      GO TO 30
 24   DO 27 K=1,3
 27   DFL(K)=DFL(K)-CORR(K)
      OMEGAF=OMEGAF-CORR(4)
      DO 28 K=1,4
      IF(ABS(CORR(K))-TOL(K+5))28,15,15
 28   CONTINUE
      GO TO 31
 15   CONTINUE
      WRITE(6,29)
 29   FORMAT(1X,21HRETAINER LOOP FAILURE)
C     STOP
 31   CALL OUTPT
      IF(RTEST.EQ.1.0) WRITE(6,131)
 131  FORMAT(1X,17H        B-L        )
      GO TO 30
 47   KONV=1
      IF(OMEGAF)48,49,48
 49   VK=0.
      XMEGA=0.
      DO 50 J=1,N
      IF(P(2,J))50,50,51
 51   VK=VK+1.
      XMEGA=XMEGA+CMF(J)
 50   CONTINUE
      OMEGAF=XMEGA/VK
      MCF(2)=XMASS*OMEGAF**2
      GO TO 48
 999  CALL ZERO(K,XMEGA)
      GO TO 999
 2002 FORMAT(1P11E12.4)
      END
```

```
      SUBROUTINE BALL(CONST1,CONST2,FPZ,QVX)
      REAL LUBDEN
      COMMON HSC(40),HSI(40)
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),        BET,BETA
     1,B(2),BB(2),BPOCK(40),  CAGEW,     CRV(2),    CRAD, COSB,CPH,CTHET,
     2CORR(5),CB(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,   DP(2,5),
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DFL(4),DTV(5,5),
     4DETER,DFL(2),DDELX(2,2),DXKDC(2),DFXVX(2),DFXVY(2),DFXWS(2),
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWS
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(
     75),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYA
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2
     9),DVARA1(5),DVARA2(5),DVOME(5), DGA(2),                    E,EL
      COMMON ERR(5),F2,FSAVE(2),                          FLAT,F(2),FK,FRAD
     1,FTAN,FFX(2,40),FFY(2,40),                          GAGE,G2(2),GG(2),
     2                    HORIZ,HOR1,HOR2,HOR3,HOR4,                IQUIT,
     3JPASS,              LPASS,LUBDEN,          N,          OMEGAF,OMY,OMX,
     4OMZ,OMS(2),OMFX(40),OMFY(40),OMFZ(40),                  PPOC(40),PL,
     5PR,PE,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),PI2,40),              QMV,
     6QS(2),QQS(2,40),                P,RPM(2),              SINP,SPH,STHET,
     7SB(2),SQ(2)                     TOTL,TI,THET,TOL(11),TCRV,TSAVE(2),
     8THICKR(2,40) ,THICKP(40),       ..S,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2),
     9VYB(2),VXB(2),VXP(40),VZP(40),VEC(5)
      COMMON X(5),XA,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMS
     1(2,40),XMU,XMU1,     YE,YR,YC   NUM
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS
      COMMON FX(2),FY(2)
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,TQ(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),
     2SBAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2),
     3XA(2),   KONV,DAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4,
     4DET,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)
     5,CAGEDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),
     6FPZDA1,FPZDA2,FPZOME,DBDRAG(4)
      DIMENSION BQB(2)
      COMMON/TSTCOM/TEST1,TEST2
      COMMON/BALTST/BLTEST
      TEST1=0.0
      J=JPASS
      PPOC(J)=0.
      BPOCK(J)=0.
      APOCK(J)=0.
      DQVXWX=0.
      DQVYWY=0.
      DQVZWZ=0.
      FPX=0.
      FPZ=0.
      DSP=0.
      VXP(J)=0.
      VZP(J)=0.
      DFPXWZ=0.
      DFPXWY=0.
      DFPZWX=0.
      DFPZWY=0.
      DQSPP=0.
      DQSPWX=0.
      DQSPWZ=0.
      DQSPWY=0.
      PX(1)=0.
```

```
      DX(2)=0.
      DO 52 K=3,5
      DO 52 L=1,2
      DB(L,K)=0.
   52 DB(L,K)=0.
      DEX(1)=0.
      DEX(2)=2.
  410 A1=TCRV*SINB +DFL(1)+B* DFL(4)*CPH
      A2=TCRV*COSB+DFL(3)*CPH+DFL(2)*SPH-.5*PD
C     IF%KONV<409,409,80
C80   X%1<#XA1%JK
C     X%2<#XA2%JK
C     GO TO 8
  409 TEMP=A1
      A1A2=A1**2+A2**2
      ART=SQRT(A1A2)
      IF(ABS(A1)-1.E-6)1,1,2
    2 IF(A1)3,3,4
    3 TAG=-1.
      GO TO 5
    1 IF(A2-TCRV-1.E-8)9,9,7
    7 X(1)=0.
      X(2)=.55*(A2-TCRV)+CRV(1)
      GO TO 17
    4 TAG=1.
    5 IF((A2-CRV(1))**2+A1**2-CRV(2)**2)9,9,10
    9 NOLOAD=NOLOAD+1
      P(2,J)=0.
      RETURN
   10 A1=ABS(A1)
      TB=A1/A2
      IF(ART-TCRV)11,12,13
   12 AX=A1*CRV(1)/ART
      AY=A2*CRV(1)/ART
   16 XX=(AX*TB+AY-A2)/TB
      IF(XX)14,15,15
   15 X(1)=(XX+.6*(AX-XX))*TAG
      X(2)=A2-.6*(A2-AY)
   17 A1=TEMP
      GO TO 8
   11 CLAM=(CRV(1)**2+A1A2-CRV(2)**2)/(2.*CRV(1)*ART)
      ALAM=ATAN(SQRT(1.-CLAM**2)/CLAM)
      B1=ATAN(TB)-ALAM
      AX=CRV(1)*SIN(B1)
      AY=SQRT(CRV(1)**2-AX**2)
      GO TO 16
   14 X(1)=.6*AX*TAG
      X(2)=AY+.4*AX*TB
      GO TO 17
   13 DO 18 L=1,2
      G=D*SQRT(1./(1.+TB**2))*C(L)/E
      T1=1./F(L)-2.*G/(1.+G)
      T2=4.-2./F(L)+T1
      CALL ELIPIN(T1,T2,FE,FK,CSE2,IQUIT)
      IF(IQUIT)18,18,19
   19 CONTINUE
      TEST1=19.0
      RETURN
   18 XK(L)=11.847711/EL*SQRT(FE*D/(EK**3*CSE2*T2))
```

```
      APIN=(ART-TCRV)/(1.+(XK(2)/XK(1))**.66666667)                     BALL1190
      APO=ART-TCRV-APIN                                                 BALL1200
      AX=(CRV(1)+APO)*TA/SQRT(1.+TA**2)                                 BALL1210
      AY=SQRT((CRV(1)+APO)**2-AX**2)                                    BALL1220
      GO TO 16                                                          BALL1230
9     CVRG2=0.                                                          BALL1240
2000  FORMAT(1X,1P10E12.4)                                              BALL1250
      DO 21 IT=1,15                                                     BALL1260
      XMY=XIN*CMECAE*X(5)                                               BALL1270
      XMZ=-XIN*CMECAE*X(4)                                              BALL1280
      CVRG2=CVRG2+CONST1                                                BALL1290
      IF(CVRG2-CONST2)22,22,23                                          BALL1300
23    CVRG2=CONST2                                                      BALL1310
22    B(1)=ATAN(X(1)/X(2))                                              BALL1320
      B(2)=ATAN((A1-X(1))/(A2-X(2)))                                    BALL1330
      DO 24 K=1,2                                                       BALL1340
      SB(K)=SIN(B(K))                                                   BALL1350
24    CB(K)=COS(B(K))                                                   BALL1360
      DB(1,1)=CB(1)**2/X(2)                                             BALL1370
      DB(1,2)=-X(1)*DB(1,1)/X(2)                                        BALL1380
      DB(2,1)=-CB(2)**2/(A2-X(2))                                       BALL1390
      DB(2,2)=-DB(2,1)*(A1-X(1))/(A2-X(2))                              BALL1400
      E2=E+2.*(X(2)-CRV(1)*COS9)                                        BALL1410
      DEL(1)=SQRT(X(1)**2+X(2)**2)-CRV(1)                               BALL1420
      DEL(2)=SQRT((A1-X(1))**2+(A2-X(2))**2)-CRV(2)                     BALL1430
      DO 25 K=1,2                                                       BALL1440
      IF(DEL(K)-1.E-7)9,25,25                                           BALL1450
25    CONTINUE                                                          BALL1460
      DO 26 K=1,2                                                       BALL1470
      G2(K)=D*CB(K)/E2                                                  BALL1480
      GG(K)=1.+C(K)*G2(K)                                               BALL1490
      T1=1./F(K)-2.*C(K)*G2(K)/GG(K)                                    BALL1500
      T2=4.-2./F(K)+T1                                                  BALL1510
      CALL ELIPIN(T1,T2,EE,EK,CSE2,IQUIT)                               BALL1520
      IF(IQUIT)28,28,29                                                 BALL1530
29    CONTINUE                                                          BALL1540
      TEST1=29.0                                                        BALL1550
      RETURN                                                            BALL1560
28    CSEP=SQRT(CSE2)                                                   BALL1570
      CSP(K)=CSEP                                                       BALL1580
      ESAVE(K)=EE                                                       BALL1590
      SNEP=SQRT(1.-CSE2)                                                BALL1600
      TSAVE(K)=T2                                                       BALL1610
      XK(K)=11.847711/FL*SQRT(D*EE/(FX**3*CSE2*T2))                     BALL1620
      SD(K)=XK(K)*SQRT(DEL(K))                                          BALL1630
      DDELX(K,1)=C(K)*SB(K)                                             BALL1640
      DDELX(K,2)=C(K)*CB(K)                                             BALL1650
      CTAU=T1/T2                                                        BALL1660
      STAU=SQRT(1.-CTAU**2)                                             BALL1670
      DTDG=2.*(T2-T1)*C(K)/((T2*GG(K))**2*STAU)                         BALL1680
      DKDEP=(EE-EK*CSE2)/(SNEP*CSEP)                                    BALL1690
      DEDEP=(EE-EK)/SNEP*CSEP                                           BALL1700
      DEDTAU=(EE*SNEP)**2*STAU*.5/(CSEP*(-2.*EE*(EK-EE)/SNEP*(EE*DKDEP- BALL1710
     1EK*DEDEP)*CSEP))                                                  BALL1720
      DXKDEP=5.9238555*SQRT(D/(T2*EE*EK**5))/(FL*CSE2)*((EK*DEDEP-3.*EE*BALL1730
     1KDEP)*CSEP+2.*EK*EE*SNEP)                                         BALL1740
      DXKDT2=-XK(K)*.5/T2                                               BALL1750
26    DXKDG(K)=DXKDEP*DEDTAU*DTDG-DXKDT2*2.*C(K)/GG(K)**2               BALL1760
      DO 31 L=1,2                                                       BALL1770
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      LPASS=1                                                           BALL1780
      PX(L)=DEL(L)*SC(L)                                                BALL1790
      P(L,J)=PX(L)                                                      BALL1800
      BB(L)=(.23973241*ESAVE(L)*CSP(L)*EL*PX(L)*D/TSAVE(L))**.33333333  BALL1810
      AA(L)=BB(L)/CSP(L)                                                BALL1820
      DO 32  K=1,2                                                      BALL1830
      DG(L,K)=-D/E2**2*(E2*SB(L)*DB(L,K)+CB(L)*DEX(K))                  BALL1840
32    DP(L,K)=1.5*SC(L)*DDELX(L,K)*DEL(L)*SQRT(DEL(L))*DXKDG(L)*DG(L,K) BALL1850
      IF(KONV)31,31,403                                                 BALL1860
403   VYR(L)=(RPM(L)-OMEGAE)*.5*(E2+C(L)*D*CB(L))                       BALL1870
      VYB(L)=.5*C(L)*D*(X(3)*CB(L)-X(5)*SB(L))                          BALL1880
      VXB(L)=-.5*D*X(4)                                                 BALL1890
      OMS(L)=C(L)*((RPM(L)-OMEGAE-X(3))*SB(L)-X(5)*CB(L))               BALL1900
      RG=-F(L)*D                                                        BALL1910
      CF=XMASS*OMEGAE**2*.5*E2                                          BALL1920
      CALL BALETJ(RG)                                                   BALL1930
31    CONTINUE                                                          BALL1940
      IF(KONV)404,404,405                                               BALL1950
404   CALL BALL1                                                        BALL1960
      DO 406 K=1,2                                                      BALL1970
      IF(ABS(COR(K))-5.E-6)406,406,21                                   BALL1980
406   CONTINUE                                                          BALL1990
      XA1(J)=X(1)                                                       BALL2000
      XA2(J)=X(2)                                                       BALL2010
407   CALL BALL2                                                        BALL2020
      IF(NUM)408,408,409                                                BALL2030
408   OME(J)=OMEGA                                                      BALL2040
      RETURN                                                            BALL2050
409   PPOC(J)=FY(1)+FY(2)-PDRAG                                         BALL2060
      BGB(1)=B(1)*57.2958                                               BALL2070
      BGB(2)=B(2)*57.2958                                               BALL2080
      IF(PPOC(J))36,38,36                                               BALL2090
36    T1=2.*D/POCKET                                                    BALL2100
      T2=4.-T1                                                          BALL2110
      CLEARP=.5*(POCKET-D)                                              BALL2120
      VZP(J)=X(3)*.5*D                                                  BALL2130
      VXP(J)=-X(5)*.5*D                                                 BALL2140
      CALL ELIPIN(T1,T2,EE,EK,CSE2,IQUIT)                               BALL2150
      IF(IQUIT)33,33,34                                                 BALL2160
34    CONTINUE                                                          BALL2170
      TEST1=35.0                                                        BALL2180
      RETURN                                                            BALL2190
33    PPOCK(J)=(.23973241*EE*SQRT(CSE2)*PL*ABS(PPOC(J))*D/T2)**.33333333BALL2200
      APOCK(J)=PPOCK(J)/SQRT(CSE2)                                      BALL2210
      THICKP(J)=(VIS*X(3)*.125*D**3*10.87/PPOC(J))**2/CLEARP            BALL2220
      IF(THICKP(J)-10.E-6)401,402,402                                   BALL2230
402   FPX=VIS*X(5)*(.5*D)**2.5*12.56/SQRT(CLEARP)                       BALL2240
      FPZ=VIS*X(3)*(.5*D)**2.5*2.436/SQRT(CLEARP)                       BALL2250
      DFPXP=0.                                                          BALL2260
      DFPZP=0.                                                          BALL2270
      DFPXWX=0.                                                         BALL2280
      DFPZWZ=0.                                                         BALL2290
      DFPXWZ=FPX/X(5)                                                   BALL2300
      DFPZWX=FPZ/X(3)                                                   BALL2310
      GO TO 38                                                          BALL2320
401   TEMP=XMU/SQRT(X(3)**2+X(5)**2)                                    BALL2330
      FPX=TEMP*X(5)*PPOC(J)                                             BALL2340
      FPZ=TEMP*X(3)*PPOC(J)                                             BALL2350
      TEMP1=TEMP*PPOC(J)/(X(3)**2+X(5)**2)                              BALL2360
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      DFPXWX=-TEMP1*X(3)*X(5)                                           BALL2370
      DFPXWZ=TEMP1*X(3)**2                                              BALL2380
      DFPXP=TEMP*X(5)                                                   BALL2390
      DFPZWX=TEMP1*X(5)**2                                              BALL2400
      DFPZWZ=-TEMP1*X(3)*X(5)                                           BALL2410
      DFPZP=TEMP*X(3)                                                   BALL2420
38    PSI(1)=-PX(1)*SB(1)+PX(2)*SB(2)+FX(1)*CB(1)-FX(2)*CB(2)-FPX       BALL2430
      PSI(2)=-PX(1)*CB(1)+PX(2)*CB(2)-FX(1)*SB(1)+FX(2)*SB(2)+CF+FPZ    BALL2440
      PSI(3)=(FY(1)*CB(1)-FY(2)*CB(2)-FPZ)*D*.5+QS(1)*SB(1)-QS(2)*SB(2) BALL2450
      PSI(4)=(FX(1)+FX(2))*D*.5+XMY-QSP                                 BALL2460
      PSI(5)=(-FY(1)*SB(1)+FY(2)*SB(2)-FPX)*D*.5+QS(1)*CB(1)-QS(2)*CB(2)BALL2470
     1+XMZ                                                              BALL2480
      DO 42 K=1,2                                                       BALL2490
      DO 43 L=1,2                                                       BALL2500
      DFX(K,L)=DFXP(K)*DP(K,L)+.5*(RPM(K)-OMEGAE)*(DFX(L)-C(K)*D*SB(K)*DBALL2510
     1B(K,L))*DFXVY(K)+.5*C(K)*D*(X(3)*SB(K)+X(5)*CB(K))*DFXVY(K)*DB(K,LBALL2520
     2)+C(K)*((RPM(K)-OMEGAE-X(3))*CB(K)+X(5)*SB(K))*DFXWS(K)*DB(K,L)   BALL2530
      DFY(K,L)=DFYP(K)*DP(K,L)+.5*(RPM(K)-OMEGAE)*(DFX(L)-C(K)*D*SB(K)*DBALL2540
     1B(K,L))*DFYVY(K)+.5*C(K)*D*(X(3)*SB(K)+X(5)*CB(K))*DFYVY(K)*DB(K,LBALL2550
     2)+C(K)*((RPM(K)-OMEGAE-X(3))*CB(K)+X(5)*SB(K))*DFYWS(K)*DB(K,L)   BALL2560
43    DQS(K,L)=DQSP(K)*DP(K,L)+.5*(RPM(K)-OMEGAE)*(DFX(L)-C(K)*D*SB(K)*DBALL2570
     1B(K,L))*DQSVY(K)+.5*C(K)*D*(X(3)*SB(K)+X(5)*CB(K))*DQSVY(K)*DB(K,LBALL2580
     2)+C(K)*((RPM(K)-OMEGAE-X(3))*CB(K)+X(5)*SB(K))*DQSWS(K)*DB(K,L)   BALL2590
      DFX(K,3)=-C(K)*(.5*D*CB(K)*DFXVY(K)+SB(K)*DFXWS(K))               BALL2600
      DFY(K,3)=-C(K)*(.5*D*CB(K)*DFYVY(K)+SB(K)*DFYWS(K))               BALL2610
      DQS(K,3)=-C(K)*(.5*D*CB(K)*DQSVY(K)+SB(K)*DQSWS(K))               BALL2620
      DFY(K,4)=-.5*D*DFYVX(K)                                           BALL2630
      DFX(K,4)=-.5*D*DFXVX(K)                                           BALL2640
      DQS(K,4)=-.5*D*DQSVX(K)                                           BALL2650
      DFX(K,5)=C(K)*(.5*D*SB(K)*DFXVY(K)-CB(K)*DFXWS(K))                BALL2660
      DFY(K,5)=C(K)*(.5*D*SB(K)*DFYVY(K)-CB(K)*DFYWS(K))                BALL2670
42    DQS(K,5)=C(K)*(.5*D*SB(K)*DQSVY(K)-CB(K)*DQSWS(K))                BALL2680
      DFPX(1)=DFPXP*(DFY(1,1)+DFY(2,1))                                 BALL2690
      DFPX(2)=DFPXP*(DFY(1,2)+DFY(2,2))                                 BALL2700
      DFPX(3)=DFPXP*(DFY(1,3)+DFY(2,3))+DFPXWX                          BALL2710
      DFPX(4)=DFPXP*(DFY(1,4)+DFY(2,4))+DFPXWY                          BALL2720
      DFPX(5)=DFPXP*(DFY(1,5)+DFY(2,5))+DFPXWZ                          BALL2730
      DFPZ(1)=DFPZP*(DFY(1,1)+DFY(2,1))                                 BALL2740
      DFPZ(2)=DFPZP*(DFY(1,2)+DFY(2,2))                                 BALL2750
      DFPZ(3)=DFPZP*(DFY(1,3)+DFY(2,3))+DFPZWX                          BALL2760
      DFPZ(4)=DFPZP*(DFY(1,4)+DFY(2,4))+DFPZWY                          BALL2770
      DFPZ(5)=DFPZP*(DFY(1,5)+DFY(2,5))+DFPZWZ                          BALL2780
      DQSP(1)=DQSPP*(DFY(1,1)+DFY(2,1))                                 BALL2790
      DQSP(2)=DQSPP*(DFY(1,2)+DFY(2,2))                                 BALL2800
      DQSP(3)=DQSPP*(DFY(1,3)+DFY(2,3))+DQSPWX                          BALL2810
      DQSP(4)=DQSPP*(DFY(1,4)+DFY(2,4))+DQSPWY                          BALL2820
      DQSP(5)=DQSPP*(DFY(1,5)+DFY(2,5))+DQSPWZ                          BALL2830
      DMY(1)=0.                                                         BALL2840
      DMY(2)=0.                                                         BALL2850
      DMY(3)=0.                                                         BALL2860
      DMY(4)=0.                                                         BALL2870
      DMY(5)=XIN*OMEGAE                                                 BALL2880
      DMZ(1)=0.                                                         BALL2890
      DMZ(2)=0.                                                         BALL2900
      DMZ(3)=0.                                                         BALL2910
      DMZ(4)=-XIN*OMEGAE                                                BALL2920
      DMZ(5)=0.                                                         BALL2930
      DCF(1)=0.                                                         BALL2940
      DCF(2)=XMASS*OMEGAE**2                                            BALL2950
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      DCF(3)=0.                                                         BALL2960
      DCF(4)=0.                                                         BALL2970
      DCF(5)=0.                                                         BALL2980
      DO 40 K=1,5                                                       BALL2990
      DPSI(1,K)=-PX(1)*CB(1)*DB(1,K)-SB(1)*DP(1,K)+PX(2)*CB(2)*DB(2,K)+ BALL3000
     1SB(2)*DP(2,K)-FX(1)*SB(1)*DB(1,K)+CB(1)*DFX(1,K)+FX(2)*SB(2)*DB(2,BALL3010
     2K)-CB(2)*DFX(2,K)-DFPX(K)                                         BALL3020
      DPSI(2,K)=PX(1)*SB(1)*DB(1,K)-CB(1)*DP(1,K)-PX(2)*SB(2)*DB(2,K)+CBBALL3030
     1(2)*DP(2,K)-FX(1)*CB(1)*DB(1,K)-SB(1)*DFX(1,K)+FX(2)*CB(2)*DB(2,K)BALL3040
     2+SB(2)*DFX(2,K)+DFPZ(K)+DCF(K)                                    BALL3050
      DPSI(3,K)=(-FY(1)*SB(1)*DB(1,K)+CB(1)*DFY(1,K)+FY(2)*SB(2)*DB(2,K)BALL3060
     1-CB(2)*DFY(2,K)-DFPZ(K))*.5*D+DS(1)*CB(1)*DB(1,K)+SB(1)*DDS(1,K)- BALL3070
     2DS(2)*CB(2)*DB(2,K)-SB(2)*DDS(2,K)                                BALL3080
      DPSI(4,K)=(DFX(1,K)+DFX(2,K))*.5*D+DMY(K)-DDP(K)                  BALL3090
   40 DPSI(5,K)=(-FY(1)*CB(1)*DB(1,K)-SB(1)*DFY(1,K)+FY(2)*CB(2)*DB(2,K)BALL3100
     1+SB(2)*DFY(2,K)-DFPX(K))*.5*D-DS(1)*SB(1)*DB(1,K)+CB(1)*DDS(1,K)+ BALL3110
     2DS(2)*SB(2)*DB(2,K)-CB(2)*DDS(2,K)+DMZ(K)                         BALL3120
C     DPSI(2,2)=DPSI(2,2)*XMASS*OMEGAE**2                               BALL3130
      NN=5                                                              BALL3140
      DO 53 K=1,5                                                       BALL3150
      DO 53 L=1,5                                                       BALL3160
   53 DPSI1(L,K)=DPSI(L,K)                                              BALL3170
      CALL SIMULT(DPSI,NN,PSI,COR,IQUIT)                                BALL3180
      IF(IQUIT)44,44,45                                                 BALL3190
   45 CONTINUE                                                          BALL3200
      TEST1=45.0                                                        BALL3210
      RETURN                                                            BALL3220
   44 DO 47 K=1,5                                                       BALL3230
   47 X(K)=X(K)-COR(K)*CVPG2                                            BALL3240
      DO 48 K=1,5                                                       BALL3250
      IF(ABS(COR(K))-TOL(K))48,21,21                                    BALL3260
   48 CONTINUE                                                          BALL3270
      HORIZ=HORIZ-PPOC(J)*CPH-FPZ*SPH                                   BALL3280
      VERT=VERT+PPOC(J)*SPH-FPZ*CPH                                     BALL3290
      BLTEST=0.0                                                        BALL3300
      GO TO 54                                                          BALL3310
   21 CONTINUE                                                          BALL3320
      BLTEST=1.0                                                        BALL3330
   54 DBA(1)=CB(2)**2/(A2-X(2))                                         BALL3340
      DBA(2)=-(A1-X(1))*(CB(2)/(A2-X(2)))**2                            BALL3350
      DSA(1)=-D*DBA(1)*SB(2)/E2                                         BALL3360
      DSA(2)=-D*DBA(2)*SB(2)/E2                                         BALL3370
      DDELA(1)=SB(2)                                                    BALL3380
      DDELA(2)=CB(2)                                                    BALL3390
      V1=-PX(2)*CB(2)-FX(2)*SB(2)                                       BALL3400
      V2=PX(2)*SB(2)-FX(2)*CB(2)                                        BALL3410
      V3=-.5*D*FY(2)*SB(2)+DS(2)*CB(2)                                  BALL3420
      V4=-.5*D*FY(2)*CB(2)-DS(2)*SB(2)                                  BALL3430
      KOUNT=0                                                           BALL3440
      XPHI(J)=PHI*57.295780                                             BALL3450
      OMEX(J)=X(3)/.10471976                                            BALL3460
      OMEY(J)=X(4)/.10471976                                            BALL3470
      OMEZ(J)=X(5)/.10471976                                            BALL3480
      DO 60 K=1,2                                                       BALL3490
      P(K,J)=PX(K)                                                      BALL3500
      XBET(K,J)=B(K)*57.295780                                          BALL3510
      AMAJ(K,J)=AA(K)*2.0                                               BALL3520
      AMIN(K,J)=BB(K)*2.0                                               BALL3530
      FFX(K,J)=FX(K)                                                    BALL3540
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      FEY(K,J)=FY(K)                                                    BALL3550
      DDS(K,J)=DS(K)                                                    BALL3560
      XDMS(K,J)=DMS(K)/.10471976                                        BALL3570
      DPA(K)=1.5*SQ(2)*DDFLA(K)+PX(2)*DXKDG(2)*DGA(K)/XK(2)             BALL3590
      DFXA(K)=DFXP(2)*DPA(K)-(.5*D*(-(RPM(2)-OMEGAE-X(3))*SB(2)+X(5)*CB(BALL3590
     12))*DFXVY(2)+((RPM(2)-OMEGAE-X(3))*CB(2)+X(5)*SB(2))*DFXWS(2))*DBABALL3600
     2(K)                                                               BALL3610
      DFYA(K)=DFYP(2)*DPA(K)-(.5*D*(-(RPM(2)-OMEGAE-X(3))*SB(2)+X(5)*CB(BALL3620
     12))*DFYVY(2)+((RPM(2)-OMEGAE-X(3))*CB(2)+X(5)*SB(2))*DFYWS(2))*DBABALL3630
     2(K)                                                               BALL3640
      DDSA(K)=DDSP(2)*DPA(K)-(.5*D*(-(RPM(2)-OMEGAE-X(3))*SB(2)+X(5)*CB(BALL3650
     12))*DDSVY(2)+((RPM(2)-OMEGAE-X(3))*CB(2)+X(5)*SB(2))*DDSWS(2))*DBABALL3660
     2(K)                                                               BALL3670
      DFXDME(K)=-(E2+C(K)*D*CB(K))*.5*DFXVY(K)-C(K)*SB(K)*DFXWS(K)      BALL3680
      DFYDME(K)=-(E2+C(K)*D*CB(K))*.5*DFYVY(K)-C(K)*SB(K)*DFYWS(K)      BALL3690
      DDSDME(K)=-(E2+C(K)*D*CB(K))*.5*DDSVY(K)-C(K)*SB(K)*DDSWS(K)      BALL3700
      DFPXA(K)=DFPXP*DFYA(K)                                            BALL3710
      DFPZA(K)=DFPZP*DFYA(K)                                            BALL3720
      DDPA(K)=DDSPP*DFYA(K)                                             BALL3730
      DFPDME=DFYDME(1)+DFYDME(2)                                        BALL3740
      XMYDME=XIN*X(5)                                                   BALL3750
      XMZDME=-XIN*X(4)                                                  BALL3760
      VEC(1)=V1*DBA(1)-SB(2)*DPA(1)+CB(2)*DFXA(1)+DFPXA(1)              BALL3770
      VEC(2)=V2*DBA(1)-CB(2)*DPA(1)-SB(2)*DFXA(1)-DFPZA(1)              BALL3780
      VEC(3)=V3*DBA(1)+.5*D*CB(2)*DFYA(1)+SB(2)*DDSA(1)+.5*D*DFPZA(1)   BALL3790
      VEC(4)=-.5*D*DFXA(1)+DDPA(1)                                      BALL3800
      VEC(5)=V4*DBA(1)-.5*D*SB(2)*DFYA(1)+CB(2)*DDSA(1)+.5*D*DFPXA(1)   BALL3810
      DO 66 K=1,5                                                       BALL3820
      DVARA1(K)=0.                                                      BALL3830
      DO 66 L=1,5                                                       BALL3840
66    DPSI(L,K)=DPSI1(L,K)                                              BALL3850
      CALL SIMULT(DPSI,NN,VEC,DVARA1, IQUIT)                            BALL3860
      KOUNT=KOUNT+1                                                     BALL3870
      IF(IQUIT)61,61,62                                                 BALL3880
62    CONTINUE                                                          BALL3890
      TEST1=62.0                                                        BALL3900
      RETURN                                                            BALL3910
61    VEC(1)=V1*DBA(2)-SB(2)*DPA(2)+CB(2)*DFXA(2)+DFPXA(2)              BALL3920
      VEC(2)=V2*DBA(2)-CB(2)*DPA(2)-SB(2)*DFXA(2)-DFPZA(2)              BALL3930
      VEC(3)=V3*DBA(2)+.5*D*CB(2)*DFYA(2)+SB(2)*DDSA(2)+.5*D*DFPZA(2)   BALL3940
      VEC(4)=-.5*D*DFXA(2)+DDPA(2)                                      BALL3950
      VEC(5)=V4*DBA(2)-.5*D*SB(2)*DFYA(2)+CB(2)*DDSA(2)+.5*D*DFPXA(2)   BALL3960
      DO 67 K=1,5                                                       BALL3970
      DVARA2(K)=0.                                                      BALL3980
      DO 67 L=1,5                                                       BALL3990
67    DPSI(L,K)=DPSI1(L,K)                                              BALL4000
      CALL SIMULT(DPSI,NN,VEC,DVARA2,IQUIT)                             BALL4010
      KOUNT=KOUNT+1                                                     BALL4020
      IF(IQUIT)64,64,62                                                 BALL4030
64    VEC(1)=-CB(1)*DFXDME(1)+CB(2)*DFXDME(2)+DFPXP*(DFYDME(1)+DFYDME(2)BALL4040
     1)                                                                 BALL4050
      VEC(2)=SB(1)*DFXDME(1)-SB(2)*DFXDME(2)-XMASS*E2*OMEGAE-DFPZP*(DFYDBALL4060
     1ME(1)+DFYDME(2))                                                  BALL4070
      VEC(3)=-.5*D*(CB(1)*DFYDME(1)-CB(2)*DFYDME(2))-SB(1)*DDSDME(1)+SB(BALL4080
     12)*DDSDME(2)+.5*D*DFPZP*(DFYDME(1)+DFYDME(2))                     BALL4090
      VEC(4)=-.5*D*(DFXDME(1)+DFXDME(2))-XMYDME+DDSPP*(DFYDME(1)+DFYDME(BALL4100
     12))                                                               BALL4110
      VEC(5)=.5*D*(DFYDME(1)*SB(1)-DFYDME(2)*SB(2))-CB(1)*DDSDME(1)+CB(2BALL4120
     1)*DDSDME(2)-XMZDME+.5*D*DFPXP*(DFYDME(1)+DFYDME(2))               BALL4130
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      DO 68 K=1,5                                                       BALL4140
      DVOME(K)=0.                                                       BALL4150
      DO 68 L=1,5                                                       BALL4160
68    DPSI(L,K)=DPSI1(L,K)                                              BALL4170
      CALL SIMULT(DPSI,NN,VEC,DVOME,IQUIT)                              BALL4180
      KOUNT=KOUNT+1                                                     BALL4190
      IF(IQUIT)65,65,62                                                 BALL4200
65    DB2DA1=DBA(1)+DVARA1(1)*DB(2,1)+DVARA1(2)*DB(2,2)                 BALL4210
      DB2DA2=DBA(2)+DVARA2(1)*DB(2,1)+DVARA2(2)*DB(2,2)                 BALL4220
      DB2DME=DVOME(1)*DB(2,1)+DVOME(2)*DB(2,2)                          BALL4230
      DP2DA1=DPA(1)+DVARA1(1)*DP(2,1)+DVARA1(2)*DP(2,2)                 BALL4240
      DP2DA2=DPA(2)+DVARA2(1)*DP(2,1)+DVARA2(2)*DP(2,2)                 BALL4250
      DP2DME=DVOME(1)*DP(2,1)+DVOME(2)*DP(2,2)                          BALL4260
      DEPA1=DEYA(1)+(DEY(1,1)+DEY(2,1))*DVARA1(1)+(DEY(1,2)+DEY(2,2))*  BALL4270
     1DVARA1(2)+(DEY(1,3)+DEY(2,3))*DVARA1(3)+(DEY(1,4)+DEY(2,4))*DVARA1BALL4280
     2(4)+(DEY(1,5)+DEY(2,5))*DVARA1(5)                                 BALL4290
      DEPA2=DEYA(2)+(DEY(1,1)+DEY(2,1))*DVARA2(1)+(DEY(1,2)+DEY(2,2))*  BALL4300
     1DVARA2(2)+(DEY(1,3)+DEY(2,3))*DVARA2(3)+(DEY(1,4)+DEY(2,4))*DVARA2BALL4310
     2(4)+(DEY(1,5)+DEY(2,5))*DVARA2(5)                                 BALL4320
      DEPOME=DEPOME+(DEY(1,1)+DEY(2,1))*DVOME(1)+(DEY(1,2)+DEY(2,2))*   BALL4330
     1DVOME(2)+(DEY(1,3)+DEY(2,3))*DVOME(3)+(DEY(1,4)+DEY(2,4))*DVOME(4)BALL4340
     2+(DEY(1,5)+DEY(2,5))*DVOME(5)-2.*DDRAG*(2.*OMEGAF*DVOME(2)+E2)/(E2BALL4350
     3*OMEGAF)                                                          BALL4360
      EPZDA1=DEPZP*DEPA1+DEPZ(3)*DVARA1(3)                              BALL4370
      EPZDA2=DEPZP*DEPA2+DEPZ(3)*DVARA2(3)                              BALL4380
      EPZOME=DEPZP*DEPOME+DEPZ(3)*DVOME(3)                              BALL4390
      HORA1=-DEPA1*CPH-EPZDA1*SPH                                       BALL4400
      HORA2=-DEPA2*CPH-EPZDA2*SPH                                       BALL4410
      HOROME=-DEPOME*CPH-EPZOME*SPH                                     BALL4420
      VRTA1=DEPA1*SPH-EPZDA1*CPH                                        BALL4430
      VRTA2=DEPA2*SPH-EPZDA2*CPH                                        BALL4440
      VRTOME=DEPOME*SPH- EPZOME*CPH                                     BALL4450
      HOR1=HOR1+HORA1                                                   BALL4460
      HOR2=HOR2+HORA2*SPH                                               BALL4470
      HOR3=HOR3+HORA2*CPH                                               BALL4480
      HOR4=HOR4+HOROME                                                  BALL4490
      VRT1=VRT1+VRTA1                                                   BALL4500
      VRT2=VRT2+VRTA2*SPH                                               BALL4510
      VRT3=VRT3+VRTA2*CPH                                               BALL4520
      VRT4=VRT4+VRTOME                                                  BALL4530
      XF1(1)=XF1(1)+PX(2)*SB(2)-FX(2)*CB(2)                             BALL4540
      XF1(2)=XF1(2)+(PX(2)*CB(2)+FX(2)*SB(2))*SPH-FY(2)*CPH             BALL4550
      XF1(3)=XF1(3)+(PX(2)*CB(2)+FX(2)*SB(2))*CPH+FY(2)*SPH             BALL4560
      XF1(4)=XF1(4)+(R*PX(2)*SB(2)-FX(2)*(R*CB(2)-E(2)*D))*CPH+FY(2)*E(2BALL4570
     1)*D*SB(2)*SPH                                                     BALL4580
      DFXA(1)=DFXA(1)+DVARA1(1)*DFX(2,1)+DVARA1(2)*DFX(2,2)+DVARA1(3)*  BALL4590
     1DFX(2,3)+DVARA1(4)*DFX(2,4)+DVARA1(5)*DFX(2,5)                    BALL4600
      DFXA(2)=DFXA(2)+DVARA2(1)*DFX(2,1)+DVARA2(2)*DFX(2,2)+DVARA2(3)*  BALL4610
     1DFX(2,3)+DVARA2(4)*DFX(2,4)+DVARA2(5)*DFX(2,5)                    BALL4620
      DFXOME(2)=DFXOME(2)+DVOME(1)*DFX(2,1)+DVOME(2)*DFX(2,2)+DVOME(3)* BALL4630
     1DFX(2,3)+DVOME(4)*DFX(2,4)+DVOME(5)*DFX(2,5)                      BALL4640
      DFYA(1)=DFYA(1)+DVARA1(1)*DFY(2,1)+DVARA1(2)*DFY(2,2)+DVARA1(3)*  BALL4650
     1DFY(2,3)+DVARA1(4)*DFY(2,4)+DVARA1(5)*DFY(2,5)                    BALL4660
      DFYA(2)=DFYA(2)+DVARA2(1)*DFY(2,1)+DVARA2(2)*DFY(2,2)+DVARA2(3)*  BALL4670
     1 DFY(2,3)+DVARA2(4)*DFY(2,4)+DVARA2(5)*DFY(2,5)                   BALL4680
      DFYOME(2)=DFYOME(2)+DVOME(1)*DFY(2,1)+DVOME(2)*DFY(2,2)+DVOME(3) *BALL4690
     1DFY(2,3)+DVOME(4)*DFY(2,4)+DVOME(5)*DFY(2,5)                      BALL4700
      A1=-V1*DB2DA1+SB(2)*DP2DA1-CB(2)*DFXA(1)                          BALL4710
      A2=-V1*DB2DA2+SB(2)*DP2DA2-CB(2)*DFXA(2)                          BALL4720
```

```
      A3=-V1*DB2OME+SB(2)*DP2OME-CB(2)*DFXOME(2)
      DTV(1,1)=DTV(1,1)+A1
      DTV(1,2)=DTV(1,2)+A2*SPH
      DTV(1,3)=DTV(1,3)+A2*CPH
      DTV(1,4)=DTV(1,4)+A3
      A1=-V2*DB2DA1+CB(2)*DP2DA1+SB(2)*DFXA(1)
      A2=-V2*DB2DA2+CB(2)*DP2DA2+SB(2)*DFXA(2)
      A3=-V2*DB2OME+CB(2)*DP2OME+SB(2)*DFXOME(2)
      DTV(2,1)=DTV(2,1)+A1*SPH-DFYA(1)*CPH
      DTV(2,2)=DTV(2,2)+A2*SPH**2-DFYA(2)*SPH*CPH
      DTV(2,3)=DTV(2,3)+A2*SPH*CPH-DFYA(2)*CPH**2
      DTV(2,4)=DTV(2,4)+A3*SPH-DFYOME(2)*CPH
      DTV(3,1)=DTV(3,1)+A1*CPH+DFYA(1)*SPH
      DTV(3,2)=DTV(3,2)+A2*SPH*CPH+DFYA(2)*SPH**2
      DTV(3,3)=DTV(3,3)+A2*CPH**2+DFYA(2)*SPH*CPH
      DTV(3,4)=DTV(3,4)+A3*CPH+DFYOME(2)*SPH
      DTV(4,1)=DTV(4,1)+.5*(DFPA1*E2+FPZDA1*D)+DQVXWX*DVARA1(3)
      DTV(4,2)=DTV(4,2)+(.5*(DFPA2*E2+FPZDA2*D)+DQVXWX*DVARA2(3))*SPH
      DTV(4,3)=DTV(4,3)+(.5*(DFPA2*E2+FPZDA2*D)+DQVXWX*DVARA2(3))*CPH
      DTV(4,4)=DTV(4,4)+.5*(DFPOME*E2+FPZOME*D)+DQVXWX*DVOME(3)
      RETURN
      END
```

```
      SUBROUTINE ELIPIN(TERM1,TERM2,EE,EK,CSE2,IER)
      CTAU=TERM1/TERM2
      SNE=.99999
      IER=0
      DO 1 I=1,30
      SNES=SNE*SNE
      CSE2=1.-SNES
      IF(CSE2)3,3,4
3     WRITE(6,5)
5     FORMAT(9H0ELIPIN-1)
      IER=1
      RETURN
4     CTE2=CSE2/SNES
      XLG=ALOG(1./CSE2)
      EE=1.+CSE2*(.444792040+CSE2*(.085099193+CSE2*.040905094))+CSE2*(.2
     1496597949+CSE2*(.081502240+CSE2*.013929990))*XLG
      EK=1.38629436 +CSE2*(.097932891+CSE2*(.054544609+.032024666*CSE2))
     1+(.5+CSE2*(.124750742+CSE2*(.060118519+.010944912*CSE2)))*XLG
      EUK=EK/EE
      COR=(1.-CTAU-2.*(EUK-1.)*CTE2)*SNES*SNE/((8.-4.*SNES)*EUK-2.*CSE2*
     1EUK**2-6.)
      SNE=SNE-COR
      IF(ABS(COR)-5.E-7)2,1,1
1     CONTINUE
      WRITE(6,6)
6     FORMAT(9H0ELIPIN-2)
      IER=1
2     RETURN
      END
```

```
      SUBROUTINE INPT                                                   INPT  10
      REAL LUBDEN                                                       INPT  20
      REAL*BW1,W2,WORD                                                  INPT  30
      COMMON HSO(40),HSI(40)                                            INPT  40
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),        BET,BETAINPT  50
     1,B(2),BB(2),BPOCK(40),  CAGEW,     CRV(2),    CRAD, COSB,CPH,CTHET, INPT  60
     2CORR(5),CB(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,  DP(2,5),  INPT  70
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DEL(4),DTV(5,5), INPT  80
     4DETER,DEL(2),DDELX(2,2),DXKDG(2),DFXVX(2),DFXVY(2),DFXWS(2),      INPT  90
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWSINPT 100
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(INPT 110
     75),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYAINPT 120
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2INPT 130
     9),DVARA1(5),DVARA2(5),DVOME(5), DGA(2),                   E,EL    INPT 140
      COMMON ERR(5),E2,FSAVE(2),                          FLAT,F(2),FK,FRADINPT 150
     1,FTAN,FFX(2,40),FFY(2,40),                          GAGE,G2(2),GG(2),INPT 160
     2                  HORIZ,HOR1,HOR2,HOR3,HOR4,                IQUIT,  INPT 170
     3JPASS,            LPASS,LUBDEN,          N,          OMEGAF,OMY,OMX, INPT 180
     4OMZ,OMS(2),OMEX(40),OMEY(40),OMEZ(40),                 PPOC(40),PL,INPT 190
     5PR,PE,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),P(2,40),             OMV,INPT 200
     6QS(2),QQS(2,40),              R,RPM(2),              SINB,SPH,STHET, INPT 210
     7SB(2),SQ(2),                    TOTL,TI,THET,TOL(11),TCRV,TSAVE(2), INPT 220
     8THICKR(2,40) ,THICKP(40),     VIS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2), INPT 230
     9VYR(2),VXR(2),VXP(40),VZP(40),VEC(5)                              INPT 240
      COMMON X(5),XN,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMSINPT 250
     1(2,40),XMU,XMU1,     YE,YR,YE,YNUM                                INPT 260
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS                             INPT 270
      COMMON FX(2),FY(2)                                                INPT 280
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,TO(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9INPT 290
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),    INPT 300
     2SBAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2), INPT 310
     3XA(2),  KONV,DAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4, INPT 320
     4DET,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)                  INPT 330
     5,CAGEDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),   INPT 340
     6FPZDA1,FPZDA2,FPZOME                                              INPT 350
      DATA W1,W2/5HOUTER,5HINNER/                                       INPT 360
1     FORMAT(8F10.0)                                                    INPT 370
      READ(5,1)XN,D,E,BET,F(1),F(2),PD,POCKET                           INPT 380
      BAT=BET                                                           INPT 390
      IF(XN)2,2,5                                                       INPT 400
2     STOP                                                              INPT 410
5     READ(5,1)YC,PC,YE,PE,YR,PR,DENS,XMU,XMU1,CAGEW,CAGEDI(1),CAGEDI(2)INPT 420
     1,RINGDI(1),RINGDI(2)                                              INPT 430
      BET=BET/57.295780                                                 INPT 440
      SINB=SIN(BET)                                                     INPT 450
      COSB=COS(BET)                                                     INPT 460
      TCRV=(F(1)+F(2)-1.)*D                                             INPT 470
      WORD=W2                                                           INPT 480
      IF(YE)6,6,7                                                       INPT 490
6     YE=29.E6                                                          INPT 500
7     IF(YR)8,8,9                                                       INPT 510
8     YR=29.E6                                                          INPT 520
9     IF(PE)10,10,11                                                    INPT 530
10    PE=.25                                                            INPT 540
11    IF(PR)12,12,13                                                    INPT 550
12    PR=.25                                                            INPT 560
13    EL=4.*((1.-PR**2)/YR+(1.-PE**2)/YE)                               INPT 570
      DO 40 K=1,2                                                       INPT 580
40    CRV(K)=(F(K)-.5)*D                                                INPT 590
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      R=.5*E+CRV(2)*COSB                                                INPT 600
      DO 41 K=1,2                                                       INPT 610
      DIA=E+2.*C(K)*(F(K)*D-CRV(K)*COSB)                                INPT 620
      DEPTH=ABS(DIA-RINGDI(K))*.5                                       INPT 630
      WIDTH=SQRT((F(K)*D)**2-(F(K)*D-DEPTH)**2)*2.                      INPT 640
      AREA(K)=3.14159*CAGEDI(K)*(CAGEW-WIDTH)                           INPT 650
41    CLEARS(K)=ABS(RINGDI(K)-CAGEDI(K))*0.5                            INPT 660
      PILOT=1.                                                          INPT 670
      IF(CLEARS(2).LT.CLEARS(1))PILOT=2.                                INPT 680
21    WRITE(6,18)                                                       INPT 690
18    FORMAT(22H   BEARING DESIGN DATA)                                 INPT 700
      WRITE(6,19)                                                       INPT 710
19    FORMAT(129H0   NO. OF   ELEMENT    PITCH    CONTACT    RAINPT 720
     1OF CURVATURES      CLEARANCE    TYPE OF      PILOT      POCKET   INPT 730
     2    CAGE/129H   ELEMENTS    DIAMETER    DIAMETER    ANGLE      CINPT 740
     3UTER       INNER      INCREMENT      PILOT      CLEARANCE   CLEARANCEINPT 750
     6      WIDTH)                                                     INPT 760
      WORD=W1                                                           INPT 770
      IF(PILOT.EQ.2.)WORD=W2                                            INPT 780
      TEMP=2.*CLEARS(1)                                                 INPT 790
      IF(PILOT.EQ.2.0)TEMP=CLEARS(2)*2.0                                INPT 800
      TEMP1=POCKET-D                                                    INPT 810
      IF(DENS.EQ.0.)DENS=.283                                           INPT 820
      WRITE(6,50)XN,D,E,BAT,F(1),F(2),PD,WORD,TEMP,TEMP1,CAGEW          INPT 830
50    FORMAT(1P7E12.4,4X,A5,3X,1P3E12.4)                                INPT 840
      WRITE(6,51)                                                       INPT 850
51    FORMAT(130H0  I. D. OF    O. D. OF    I. D. OF    O. D. OF       INPT 860
     1 MODULUS OF ELASTICITY                POISSON@S RATIO             INPT 870
     2 ELEMENT/130H    OUTER       INNER       CAGE        CAGE        INPT 880
     3RINGS       CAGE       ELEMENTS      RINGS       CAGE       ELEMENTINPT 890
     4S    DENSITY)                                                    INPT 900
      WRITE(6,52)RINGDI(1),RINGDI(2),CAGEDI(2),CAGEDI(1),YR,YC,YE,PR,PC,INPT 910
     1PE,DENS                                                           INPT 920
52    FORMAT(1P11E12.4)                                                 INPT 930
      RETURN                                                            INPT 940
      END                                                               INPT 950

      SUBROUTINE OUTPT                                                  OUTP  10
      REAL LUBDEN                                                       OUTP  20
      COMMON HSC(40),HSI(40)                                            OUTP  30
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),       RET,BETAOUTP  40
     1,B(2),BO(2),PPOCK(40),  CAGEW,    CRV(2),   CRAD, COSB,CPH,CTHET, OUTP  50
     2CORR(5),CP(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,  DP(2,5),  OUTP  60
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DFL(4),DTV(5,5), OUTP  70
     4DETER,DFL(2),DDFLX(2,2),DXKDC(2),DEXVX(2),DEXVY(2),DEXWS(2),      OUTP  80
     5DEXP(2),DEYVX(2),DEYVY(2),DEYWS(2),DEYP(2),DDSVX(2),DDSVY(2),DDSWSOUTP  90
     6(2),DDSP(2),DEX(2,5),DEY(2,5),DDS(2,5),DERX(5),DERZ(5),DDR(5),DMY(OUTP 100
     75),DMZ(5),DDSI(5,5),DDSII(5,5),DBA(2),DDELA(2),DPA(2),DEXA(2),DEYAOUTP 110
     8(2),DDSA(2),DEXDME(2),DEYDME(2),DDSDME(2),DERXA(2),DERZA(2),DDRA(2OUTP 120
     9),DVARA1(5),DVARA2(5),DVDME(5), DGA(2),                   E,EL    OUTP 130
      COMMON ERR(5),F2,FSAVE(2),                          FLAT,F(2),FK,FRADOUTP 140
     1,STAN,EFX(2,40),EFY(2,40),                          GAGE,G2(2),GG(2),OUTP 150
     2                  HDR17,HDR1,HDR2,HDR3,HDR4,                IQUIT,  OUTP 160
     3IPASS,            LPASS,LUBDEN,        N,          OMEGAE,OMY,OMX, OUTP 170
     4DAZ,DMS(2),DMEX(40),DMEY(40),DMEZ(40),                 PPOC(40),PL,OUTP 180
     5PR,PE,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),P(2,40),            OMV,OUTP 190
     6DS(2),DDS(2,40),                R,RPM(2),                SINP,SPH,STHET, OUTP 200
     7SR(2),SO(2),                      TOTL,TI,THCK,TOL(11),YCRV,TSAVE(2), OUTP 210
     8THICKP(2,40) ,THICKP(40),     VIS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2), OUTP 220
     9VZR(2),VXR(2),VXP(40),VZP(40),VEC(5)                              OUTP 230
      COMMON X(4),XN,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XRET(2,40),XDISOUTP 240
     1(2,40),XMU,XMU1,     YE,YR,YC,YNUM                                OUTP 250
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS                             OUTP 260
      COMMON FX(2),FY(2)                                                OUTP 270
      COMMON  TC  TL(40),LCTL,ICNT,LOOP,TQ(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9OUTP 280
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     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),     OUTP 290
     2CBAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2), OUTP 300
     3XA(2),  KONV,CAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4, OUTP 310
     4DFT,NUM,XA1(40),XA2(40),TYPASS,REVX(40),REVZ(40)                   OUTP 320
     5,CACCDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),    OUTP 330
     6EPZDA1,EPZDA2,EPZOME                                               OUTP 340
      COMMON/XTRA/VDRAG(2),CDRAG(2) ,CFORCE                              OUTP 350
      PIE=3.14159                                                        OUTP 360
      RPM(1)=RPM(1)/.10471976                                            OUTP 370
      RPM(2)=RPM(2)/.10471976                                            OUTP 380
      OMEGAE=OMEGAE/.10471976                                            OUTP 390
      WRITE(6,1111)                                                      OUTP 400
      WRITE(6,1112)                                                      OUTP 410
 1111 FORMAT(130H1  RPM. OF      RPM. OF         EXTERNAL LOADS        INITOUTP 420
     1EAL      STARTING DEFLECTIONS            FINAL DEFLECTIONS        OUTP 430
     2  RPM. OF)                                                         OUTP 440
 1112 FORMAT(129H     OUTER        INNER        AXIAL        RADIAL  MISALIGOUTP 450
     1NMENT    ALONG X      ALONG Z      ALONG X      ALONG Y      ALONG Z  OUTP 460
     2    CAGE)                                                          OUTP 470
      WRITE(6,1)RPM(1),RPM(2),XF(1),XF(3),DDL(4),DDL(1),DDL(3),DFL(1),   OUTP 480
     1DFL(2),DFL(3),OMEGAE                                               OUTP 490
 1    FORMAT(1P11E12.4)                                                  OUTP 500
      WRITE(6,50)                                                        OUTP 510
 50   FORMAT(39H0          REACTIONS OF BEARING ON SHAFT/46H0  ALONG X   OUTP 520
     1  ALONG Y      ALONG Z      ABOUT Y)                               OUTP 530
      WRITE(6,1)XF1(1),XF1(2),XF1(3),XF1(4)                              OUTP 540
      WRITE(6,11)                                                        OUTP 550
 11   FORMAT(25H0  BALL/RACE CONTACT DATA)                               OUTP 560
      WRITE(6,2)                                                         OUTP 570
 2    FORMAT(119H0  ELEMENT      ELEMENT          CONTACT LOAD           COUTP 580
     1ONTACT ANGLE          PRESSURE AREA LENGTH    PRESSURE AREA WIDTH) OUTP 590

      WRITE(6,3)                                                         OUTP 600
 3    FORMAT(117H      NO.        AZIMUTH       OUTER        INNER       OUTOUTP 610
     1ER        INNER        OUTER        INNER        OUTER        INNER)  OUTP 620
      DO 4 J=1,N                                                         OUTP 630
      IF(P(2,J))4,4,5                                                    OUTP 640
 5    WRITE(6,6)J,XPHI(J),P(1,J),P(2,J),XBET(1,J),XBET(2,J),AMAJ(1,J),   OUTP 650
     1AMAJ(2,J),AMIN(1,J),AMIN(2,J)                                      OUTP 660
 4    CONTINUE                                                           OUTP 670
 6    FORMAT(I7,5X,1P10E12.4)                                            OUTP 680
      WRITE(6,7)                                                         OUTP 690
 7    FORMAT(116H0  ELEMENT          BALL VELOCITY COMPONENTS          TROUTP 700
     1CTION ALONG X          TRACTION ALONG Y          SPINNING MOMENT)   OUTP 710
      WRITE(6,8)                                                         OUTP 720
 8    FORMAT(117H      NO.        ABOUT X      ABOUT Y      ABOUT Z      OUTOUTP 730
     1ER        INNER        OUTER        INNER        OUTER        INNER)  OUTP 740
      DO 9 J=1,N                                                         OUTP 750
      IF(P(2,J))9,9,10                                                   OUTP 760
 10   WRITE(6,6)J,OMEX(J),OMEY(J),OMEZ(J),EEX(1,J),EEX(2,J),EEY(1,J),    OUTP 770
     1EEY(2,J),QQS(1,J),QQS(2,J)                                         OUTP 780
 9    CONTINUE                                                           OUTP 790
      WRITE(6,12)                                                        OUTP 800
 12   FORMAT(94H0  ELEMENT       SPINNING VELOCITY      MINIMUM FILM THICKOUTP 810
     1NESS    MEAN HERTZ STRESS    )                                     OUTP 820
      WRITE(6,13)                                                        OUTP 830
 13   FORMAT(84H      NO.        OUTER        INNER        OUTER        INNEOUTP 840
     1R        OUTER        INNER     )                                 OUTP 850
      DO 80 J=1,N                                                        OUTP 860
      IF(P(2,J))80,80,81                                                 OUTP 870
 81   HSO(J)=4.0*P(1,J)/(PIE*AMAJ(1,J)*AMIN(1,J))                        OUTP 880
      HSI(J)=4.0*P(2,J)/(PIE*AMAJ(2,J)*AMIN(2,J))                        OUTP 890
 80   CONTINUE                                                           OUTP 900
      DO 14 J=1,N                                                        OUTP 910
      IF(P(2,J))14,14,15                                                 OUTP 920
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 15   WRITE(6,6)J,XOMS(1,J),XOMS(2,J),THICKR(1,J),THICKR(2,J),HSO(J),HSIOUTP 930
     1(J)                                                                OUTP 940
 14   CONTINUE                                                           OUTP 950
      WRITE(6,16)                                                        OUTP 960
 16   FORMAT(27H0  BALL/POCKET CONTACT DATA)                             OUTP 970
      WRITE(6,17)                                                        OUTP 980
 17   FORMAT(83H0  ELEMENT       POCKET        PRESSURE ELLIPSE        SLIOUTP 990
     1DING VELOCITY      MIN. FILM)                                      OUTP1000
      WRITE(6,18)                                                        OUTP1010
 18   FORMAT(83H     NO.          FORCE        LENGTH       WIDTH       RADIOUTP1020
     1AL        AXIAL       THICKNESS)                                   OUTP1030
      DO 19 J=1,N                                                        OUTP1040
      IF(P(2,J))19,19,20                                                 OUTP1050
 20   APOCK(J)=2.*APOCK(J)                                               OUTP1060
      BPOCK(J)=2.*BPOCK(J)                                               OUTP1070
      IF(THICKP(J).GT.CLEARP)THICKP(J)=CLEARP                            OUTP1080
      WRITE(6,6)J,PPOC(J),APOCK(J),BPOCK(J),VZP(J),VXP(J),THICKP(J)      OUTP1090
 19   CONTINUE                                                           OUTP1100
      WRITE(6,70) BDRAG                                                  OUTP1110
      WRITE(6,60)                                                        OUTP1120
   60 FORMAT(29H0  CAGE/PILOTING SURFACE DATA/34H0  VISCOUS      COULOMB OUTP1130
     1      NORMAL/33H     TORQUE       TORQUE        LOAD)               OUTP1140
   70 FORMAT(25H0  VISCOUS DRAG ON BALL #,1P1E11.4)                      OUTP1150
      TEMP=VDRAG(1)+VDRAG(2)                                             OUTP1160
      KPLT=PILOT+.1                                                      OUTP1170
      TEMP1=CDRAG(KPLT)                                                  OUTP1180
      WRITE(6,1)TEMP,TEMP1   ,CFORCE                                     OUTP1190
      RETURN                                                             OUTP1200
      END                                                                OUTP1210


      SUBROUTINE SIMULT(A,N,B,X,KX)                                      SIMU  10
      DOUBLE PRECISION AMPY,EL,V,ROW,AMPYI                               SIMU  20
    1 FORMAT( ' ELEMENTS ALL ZERO. ROW NO.9 ,I6 /)                       SIMU  30
      IW=6                                                               SIMU  40
      DIMENSION A(5,5),B(5),X(5),ROW(5),KOL(5)                           SIMU  50
      DO 5050 J=1,N                                                      SIMU  60
      TEMP=0.                                                            SIMU  70
      DO 5051 K=1,N                                                      SIMU  80
      IF(ABS(A(J,K))-TEMP)5051,5051,5052                                 SIMU  90
 5052 TEMP=ABS(A(J,K))                                                   SIMU 100
 5051 CONTINUE                                                           SIMU 110
      IF(TEMP.NE.0.) GO TO 15051                                         SIMU 120
      WRITE(IW,1) J                                                      SIMU 130
      GO TO 5050                                                         SIMU 140
C     DO 5053 K=1,N                                                      SIMU 150
15051 DO 5053 K=1,N                                                      SIMU 160
 5053 A(J,K)=A(J,K)/TEMP                                                 SIMU 170
      B(J)=B(J)/TEMP                                                     SIMU 180
 5050 CONTINUE                                                           SIMU 190
 5000 KOL(1)=1                                                           SIMU 200
 5001 DO 5002  IROW=2,N                                                  SIMU 210
 5002 KOL(IROW)=KOL(IROW-1)+1                                            SIMU 220
 5004 DO 5025  KOUNT=1,N                                                 SIMU 230
      LARGST=N-KOUNT+1                                                   SIMU 240
      I  ASE=KOL(1)                                                      SIMU 250
      JOOL=1                                                             SIMU 260
 5005 IF (N-KOUNT)  5035,5014,5006                                       SIMU 270
 5006    AMPY = ABS ( A(1,1) )                                           SIMU 280
 5007 DO 5010  IROW=2,LARGST                                             SIMU 290
      AMPYI=ABS(A(IROW,1))                                               SIMU 300
 5008    IF ( AMPY - AMPYI            ) 5009, 5010, 5010                 SIMU 310
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 5009 JCOL=IROW                                                         SIMU 320
          AMPY = ABS ( A(IROW,1) )                                      SIMU 330
      IERASE=KOL(IROW)                                                  SIMU 340
 5010 CONTINUE                                                          SIMU 350
 5011 IF (KOL(1)-IERASE)  5012,5014,5012                                SIMU 360
 5012 KOL(JCOL)=KOL(1)                                                  SIMU 370
      KOL(1)=IERASE                                                     SIMU 380
 5014 IF(A(JCOL,1))5015,5035,5015                                       SIMU 390
 5015 AMPY=A(JCOL,1)                                                    SIMU 400
 5017 DO 5019  IROW=2,N                                                 SIMU 410
      EL=A(JCOL,IROW)                                                   SIMU 420
C     ROW(IROW-1)=A(JCOL,IROW)/AMPY                                     SIMU 430
      ROW(IROW-1)=EL/AMPY                                               SIMU 440
 5018 A(JCOL,IROW-1)=A(1,IROW-1)                                        SIMU 450
      ROW(N)=1./AMPY                                                    SIMU 460
      A(JCOL,N)=A(1,N)                                                  SIMU 470
 5019 DO 5022  IROW=2,N                                                 SIMU 480
      AMPY=A(IROW,1)                                                    SIMU 490
 5020 DO 5021  JCOL=2,N                                                 SIMU 500
      EL=A(IROW,JCOL)                                                   SIMU 510
C5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(JCOL-1)                    SIMU 520
 5021 A(IROW-1,JCOL-1)=EL-AMPY*ROW(JCOL-1)                              SIMU 530
 5022 A(IROW-1,N)=-AMPY*ROW(N)                                          SIMU 540
 5023 DO 5024  JCOL=1,N                                                 SIMU 550
      KOL(JCOL)=KOL(JCOL+1)                                             SIMU 560
 5024 A(N,JCOL)=ROW(JCOL)                                               SIMU 570
 5025 KOL(N)=IERASE                                                     SIMU 580
 5026 DO 5034  KOUNT=1,N                                                SIMU 590
 5027 IF (KOL(KOUNT)-KOUNT)  5035,5034,5028                             SIMU 600
 5028 DO 5032  IROW=KOUNT,N                                             SIMU 610
 5029 IF (KOL(IROW)-KOUNT)  5035,5030,5032                              SIMU 620
 5030 DO 5031  JCOL=1,N                                                 SIMU 630
      ROW(1)=A(JCOL,IROW)                                               SIMU 640
      A(JCOL,IROW)=A(JCOL,KOUNT)                                        SIMU 650
 5031 A(JCOL,KOUNT)=ROW(1)                                              SIMU 660
      IERASE=KOL(KOUNT)                                                 SIMU 670
      KOL(KOUNT)=KOL(IROW)                                              SIMU 680
      KOL(IROW)=IERASE                                                  SIMU 690
      GO TO 5034                                                        SIMU 700
 5032 CONTINUE                                                          SIMU 710
      GO TO 5035                                                        SIMU 720
 5034 CONTINUE                                                          SIMU 730
 997  IF(KX-31599,9000,998                                              SIMU 740
 9000 KX=0                                                              SIMU 750
      RETURN                                                            SIMU 760
 999   DO 5042 IROW=1,N                                                 SIMU 770
C5040     X(IROW) #0.0                                                  SIMU 780
 5040 AMPY=0.                                                           SIMU 790
C5041     DO 5042  JKOL=1,N                                             SIMU 800
 5041 DO 15042 JKOL=1,N                                                 SIMU 810
      EL=A(IROW,JKOL)                                                   SIMU 820
      V=B(JKOL)                                                         SIMU 830
C5042     X(IROW) # X(IROW)&A(IROW,JKOL)  *  B(JKOL)                    SIMU 840
15042 AMPY=AMPY+EL*V                                                    SIMU 850
 5042 X(IROW)=AMPY                                                      SIMU 860
      KX=0                                                              SIMU 870
                RETURN                                                  SIMU 880
 5035 KX=1                                                              SIMU 890
      RETURN                                                            SIMU 900
      END                                                               SIMU 910
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      SUBROUTINE BALFTJ(RG)                                             BALF  10
      REAL LUBDEN                                                       BALF  20
      COMMON HSO(40),HSI(40)                                            BALF  30
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),        BET,BETABALF  40
     1,B(2),BB(2),BPOCK(40),  CAGEW,     CRV(2),    CRAD, COSB,CPH,CTHE), BALF  50
     2CORR(5),CB(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,  DP(2,5),  BALF  60
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DFL(4),DTV(5,5), BALF  70
     4DETER,DFL(2),DDELX(2,2),DXKOG(2),DFXVX(2),DFXVY(2),DFXWS(2),      BALF  80
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWSBALF  90
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(BALF 100
     75),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYABALF 110
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2BALF 120
     9),DVARA1(5),DVARA2(5),DVOME(5), DGA(2),                   E,EL    BALF 130
      COMMON ERR,5),F2,FSAVE(2),                          FLAT,F(2),FK,FRADBALF 140
     1,FTAN,FFX(2,40),FFY(2,40),                          GAGE,G2(2),GG(2),BALF 150
     2                  HORIZ,HOR1,HOR2,HOR3,HOR4,                 QUIT,  BALF 160
     3J ASS,            LPASS,LUBDEN,        N,          OMEGAF,OMY,OMX, BALF 170
     4OMZ,OMS(2),OMFX(40),OMEY(40),OMEZ(40),                 PPOC(40),PL,BALF 180
     5PR,PE,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),P(2,40),            OMV,BALF 190
     6QS(2),QQS(2,40),              R,RPM(2),              SINB,SPH,STHET, BALF 200
     7SB(2),SQ(2),                    TOTL,TI,THET,TOL(11),TCRV,TSAVE(2), BALF 210
     8THICKR(2,40) ,THICKP(40),     VIS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2), BALF 220
     9VYB(2),VXB(2),VXP(40),VZP(40),VEC(5)                              BALF 230
      COMMON X(5),XA,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMSBALF 240
     1(2,40),XMU,XMU1,     YE,YR,YC,YNUM                                BALF 250
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS                             BALF 260
      COMMON FX(2),FY(2)                                                BALF 270
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,TQ(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9BALF 280
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),    BALF 290
     2CBAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2), BALF 300
     3XA(2),  KONV,DAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4, BALF 310
     4DET,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)                  BALF 320
     5,CAGEDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),   BALF 330
     6FPZDA1,FPZCA2,FPZOME                                              BALF 340
      DIMENSION U(3),V(3),WSS(3),WIT(3),SW(3),S1(3),S2(3),PHZ(3),Y(60), BALF 350
     1RATIO(60),FFXX(3,3,6),FFYY(3,3,6),TMM(3,3,6)                      BALF 360
      J=JPASS                                                           BALF 370
      L=LPASS                                                           BALF 380
      DVYR=D2VYR*ABS(VYR(L)-VYB(L))                                     BALF 390
      DVX=D2VX*ABS(VXB(L))                                              BALF 400
      DWS=D2WS*ABS(OMS(L)/.10472)                                       BALF 410
      D7=D2P2*PX(L)                                                     BALF 420
      IF(DVYR.LT.0.1)DVYR=0.1                                           BALF 430
      IF(DVX.LT.0.1)DVX=0.1                                             BALF 440
      IF(DWS.LT.0.1)DWS=0.1                                             BALF 450
      IF(D7.LT.0.1)D7=0.1                                               BALF 460
      IF(D7.GE.PX(L)) D7=0.9*PX(L)                                      BALF 470
      THICKR(L,J)=1.E6                                                  BALF 480
      S1(1)=AA(L)                                                       BALF 490
      S2(1)=BB(L)                                                       BALF 500
      IDY=6                                                             BALF 510
      IDY=20                                                            BALF 520
      XIDY=IDY                                                          BALF 530
      RJ=.5*D                                                           BALF 540
      RR=-C(L)*.5*(E2/CB(L)+C(L)*D)                                     BALF 550
      WT=PX(L)                                                          BALF 560
      WWS=-(OMS(L)/.10471976)                                           BALF 570
      WS=WWS                                                            BALF 580
      U1=VYB(L)                                                         BALF 590
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      U2=VYR(L)                                                         BALF 600
      V1=-VXR(L)                                                        BALF 610
      DU=-DVYR                                                          BALF 620
      DV=-DVX                                                           BALF 630
      DDWS=-DWS                                                         BALF 640
      DWS=DDWS                                                          BALF 650
      DWT=DT                                                            BALF 660
      U(2)=U1+DU                                                        BALF 670
      U(3)=U1-DU                                                        BALF 680
      V(2)=V1+DV                                                        BALF 690
      V(3)=V1-DV                                                        BALF 700
      WSS(2)=WS+DWS                                                     BALF 710
      WSS(3)=WS-DWS                                                     BALF 720
      WTT(2)=WT+DWT                                                     BALF 730
      WTT(3)=WT-DWT                                                     BALF 740
      THETA=(TI-32.)/1.8                                                BALF 750
      RXP=1./RJ+1./RP                                                   BALF 760
      RYP=1./RJ+1./RG                                                   BALF 770
      AAA=.5*RXP                                                        BALF 780
      DDD=.5*RYP                                                        BALF 790
      RXP=1./RXP                                                        BALF 800
      RYP=1./RYP                                                        BALF 810
      AMC=(AAA-DDD)/(AAA+DDD)                                           BALF 820
      ED=3.14159265/((1.-PR**2)/YE+(1.-PR**2)/YG)                       BALF 830
      U(1)=U1                                                           BALF 840
      V(1)=V1                                                           BALF 850
      WSS(1)=WS                                                         BALF 860
      WTT(1)=WT                                                         BALF 870
      DY=2./XIDY                                                        BALF 880
      DO 200 K=2,3                                                      BALF 890
      TEMP=(WTT(K)/WTT(1))**.33333333                                   BALF 900
      S1(K)=S1(1)*TEMP                                                  BALF 910
200   S2(K)=S2(1)*TEMP                                                  BALF 920
      DO 45 I=1,3                                                       BALF 930
      RH2(I)=1.5*WTT(I)/(3.14159265 *S1(I)*S2(I))                       BALF 940
      SW(I)=.75*WTT(I)/S1(I)                                            BALF 950
45    CONTINUE                                                          BALF 960
35    VIS3=VIS*EXP(BETA*(TI-96.))                                       BALF 970
      AL3=ALPHA+930.*ALPHA*(TI-86.)/(546.*(TI+460.))                    BALF 980
      HHH=1.58*ALPHA**.6*VIS**.7*ED**.03*RXP**.43                       BALF 990
      G1T=VIS3*(ALPHA/AL3)**.6*(VIS/VIS3)**.7                           BALF1000
      G2T=VIS3*BETA/(8.*EK*.216)                                        BALF1010
      II=1                                                              BALF1020
      JJ=1                                                              BALF1030
      KK=1                                                              BALF1040
      LL=1                                                              BALF1050
      IN=1                                                              BALF1060
49    EE1Y=0.                                                           BALF1070
      EE1X=0.                                                           BALF1080
      TM2=0.                                                            BALF1090
      TM1=0.                                                            BALF1100
      SS=WSS(KK)*.10471976*S1(LL)                                       BALF1110
      IF((KK.EQ.1).AND.(LL.EQ.1).AND.((II.NE.1).OR.(JJ.NE.1)))GO TO 49  BALF1120
      IF(SS.EQ.0.)GO TO 43                                              BALF1130
      YS=(U(II)-U2)/SS                                                  BALF1140
      IF((YS.GE.1.).OR.(YS.LE.(-1.)))GO TO 43                           BALF1150
      GO TO 44                                                          BALF1160
43    Y(II)=1.+DY*.5                                                    BALF1170
      M4=1                                                              BALF1180
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      GO TO 42                                                          BALF1190
 44   NNN=1                                                             BALF1200
      Y(1)=YS                                                           BALF1210
 41   Y(NNN+1)=Y(NNN)+DY                                                BALF1220
      RATIO(NNN+1)=1.-Y(NNN+1)**2                                       BALF1230
      NNN=NNN+1                                                         BALF1240
      IF(Y(NNN).LT.(1.-.5*DY))GO TO 41                                  BALF1250
      IF(Y(NNN).GE.1.)NNN=NNN-1                                         BALF1260
      MM=NNN+1                                                          BALF1270
      Y(MM)=YS-DY                                                       BALF1280
      RATIO(MM)=1.-Y(MM)**2                                             BALF1290
      IF(Y(MM).LE.(-1.))MM=MM-1                                         BALF1300
      IF(Y(MM+1).LE.(-1.0))GO TO 42                                     BALF1310
 42   Y(MM+1)=Y(MM)-DY                                                  BALF1320
      RATIO(MM+1)=1.-Y(MM+1)**2                                         BALF1330
      MM=MM+1                                                           BALF1340
      IF(Y(MM).GT.(-1.+.5*DY))GO TO 42                                  BALF1350
      IF(Y(MM).LE.(-1.))MM=MM-1                                         BALF1360
 49   DO 50 M=2,MM                                                      BALF1370
      UBAR=ABS((U(II)+U2-SS*Y(M))*.5)                                   BALF1380
      W=SW(LL)*RATIO(M)                                                 BALF1390
      HQ=HHH*UBAR**.7/W**.13                                            BALF1400
      PPHZ=PHZ(LL)*SQRT(RATIO(M))                                       BALF1410
      US=ABS(U2-U(II)+SS*Y(M))                                          BALF1420
      G11=G1T*US/(HQ*PPHZ)                                              BALF1430
      G22=G2T*US**2                                                     BALF1440
      G33=AL3*PPHZ                                                      BALF1450
C     CALL FRCTN (G11,G22,G33,FCOF,THETA)                               BALF1460
C     FCOF= 7.5*SQRT(G11)                                               BALF1470
      FCOF= 7.5*SQRT(G11)                                               BALF1480
      F1X=-FCOF*W*DY*S1(LL)                                             BALF1490
      IF((U(II)-SS*Y(M)).LT.U2)F1X=-F1X                                 BALF1500
      F1Y=-FCOF*W*V(JJ)/US*DY*S1(LL)                                    BALF1510
      FF1X=FF1X+F1X                                                     BALF1520
      FF1Y=FF1Y+F1Y                                                     BALF1530
      TFX=-F1X*Y(M)*S1(LL)                                              BALF1540
      TM1=TM1+TFX                                                       BALF1550
      TFY=-FCOF*W*SS*S2(LL)**2*RATIO(M)*DY/(3.*US)                      BALF1560
      TM2=TM2+TFY                                                       BALF1570
      IF(HQ-THICKR(L,J))201,201,50                                      BALF1580
 201  THICKR(L,J)=HQ                                                    BALF1590
 50   CONTINUE                                                          BALF1600
      TM=TM1+TM2                                                        BALF1610
      FFXX(II,JJ,NN)=FF1X                                               BALF1620
      FFYY(II,JJ,NN)=FF1Y                                               BALF1630
      TMM(II,JJ,NN)=TM                                                  BALF1640
      IF((II.GE.3).OR.(JJ.GT.1).OR.(KK.GT.1).OR.(LL.GT.1))GO TO 55      BALF1650
      II=II+1                                                           BALF1660
      GO TO 48                                                          BALF1670
 55   II=1                                                              BALF1680
      IF((JJ.GE.3).OR.(KK.GT.1).OR.(LL.GT.1))GO TO 56                   BALF1690
      JJ=JJ+1                                                           BALF1700
      GO TO 48                                                          BALF1710
 56   JJ=1                                                              BALF1720
      IF((KK.GE.3).OR.(LL.GT.1))GO TO 57                                BALF1730
      KK=KK+1                                                           BALF1740
      NN=NN+1                                                           BALF1750
      GO TO 48                                                          BALF1760
 57   KK=1                                                              BALF1770
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      IF(LL.GE.3)GO TO 58                                               BALF1780
      LL=LL+1                                                           BALF1790
      NN=NN+1                                                           BALF1800
      GO TO 49                                                          BALF1810
58    CONTINUE                                                          BALF1820
      FXDU=(FFXX(2,1,1)-FFXX(3,1,1))/(2.*DU)                            BALF1830
      FXDV=(FFXX(1,2,1)-FFXX(1,3,1))/(2.*DV)                            BALF1840
      FXDWS=(FFXX(1,1,2)-FFXX(1,1,3))/(2.*DWS)                          BALF1850
      FXDW=(FFXX(1,1,4)-FFXX(1,1,5))/(2.*DWT)                           BALF1860
      FYDU=(FFYY(2,1,1)-FFYY(3,1,1))/(2.*DU)                            BALF1870
      FYDV=(FFYY(1,2,1)-FFYY(1,3,1))/(2.*DV)                            BALF1880
      FYDWS=(FFYY(1,1,2)-FFYY(1,1,3))/(2.*DWS)                          BALF1890
      FYDW=(FFYY(1,1,4)-FFYY(1,1,5))/(2.*DWT)                           BALF1900
      TMDU=(TMM(2,1,1)-TMM(3,1,1))/(2.*DU)                              BALF1910
      TMDV=(TMM(1,2,1)-TMM(1,3,1))/(2.*DV)                              BALF1920
      TMDWS=(TMM(1,1,2)-TMM(1,1,3))/(2.*DWS)                            BALF1930
      TMDW=(TMM(1,1,4)-TMM(1,1,5))/(2.*DWT)                             BALF1940
      FX(L)=FFYY(1,1,1)                                                 BALF1950
      FY(L)=FFXX(1,1,1)                                                 BALF1960
      DS(L)=TMM(1,1,1)                                                  BALF1970
      DFXVX(L)=-FYDV                                                    BALF1980
      DFXVY(L)=-FYDU                                                    BALF1990
      DFXWS(L)=-FYDWS/.10471976                                         BALF2000
      DFXP(L)=FYDW                                                      BALF2010
      DFYVX(L)=-FXDV                                                    BALF2020
      DFYVY(L)=-FXDU                                                    BALF2030
      DFYWS(L)=-FXDWS/.10471976                                         BALF2040
      DFYP(L)=FXDW                                                      BALF2050
      DQSVX(L)=-TMDV                                                    BALF2060
      DQSVY(L)=-TMDU                                                    BALF2070
      DQSWS(L)=-TMDWS/.10471976                                         BALF2080
      DQSP(L)=TMDW                                                      BALF2090
      RETURN                                                            BALF2100
      END                                                               BALF2110

      SUBROUTINE BALL1                                                  BALL  10
      REAL LUBDEN                                                       BALL  20
      COMMON HSO(40),HSI(40)                                            BALL  30
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),       BET,BETABALL  40
     1,B(2),BP(2),BPOCK(40),  CAGEW,    CRV(2),   CRAD, CDSB,CPH,CTHET, BALL  50
     2CORR(5),CP(2),CF,C(2),CSP(2),CDR(5), CLEARS(2),CLEARP,  DP(2,5),  BALL  60
     3DC(2,2),DCF(5),DP(2,5),DEX(2),DRDP,D,DENS,DDL(4),DFL(4),DTV(5,5), BALL  70
     4DETEP,DFL(2),DDELX(2,2),DXKDG(2),DFXVX(2),DFXVY(2),DFXWS(2),      BALL  80
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWSBALL  90
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(BALL 100
     7S),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYABALL 110
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2BALL 120
     9),DVAPA1(5),DVAPA2(5),DVOME(5), DGA(2),                  F,EL     BALL 130
      COMMON ERR(5),E2,ESAVE(2),                          FLAT,F(2),FK,FRADBALL 140
     1,FTAN,FFX(2,40),FFY(2,40),                          GAGF,G2(2),GG(2),BALL 150
     2                    HOR1(2,HOR1,HOR2,HOR3,HOR4,                IOUTT,  BALL 160
     3JPASS,              LPASS,LUBDEN,        N,          OMEGAE,OMY,OMX, BALL 170
     4OMZ,OMS(2),OMFX(40),OMFY(40),OMEZ(40),                  PPOC(40),PL,BALL 190
     5PR,PF,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),PI2,40),              OMV,BALL 190
     6QS(2),QQS(2,40),                R,RPM(2),              SINB,SPH,STHET, BALL 200
     7SP(2),SQ(2),                    IOTL,TI,THET,TOLI(1),TORV,TSAVE(2), BALL 210
     8THICKR(2,40) ,THICKP(40),     VTS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2), BALL 220
     9VYB(2),VXB(2),VXP(40),VZP(40),VEC(5)                              BALL 230
      COMMON X(5),XA,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMSBALL 240
     1(2,40),XMU,XMU1,     YF,YR,YC,YNUM                                BALL 250
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS                             BALL 260
      COMMON FX(2),FY(2)                                                BALL 270
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,TO(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9BALL 280
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),    BALL 290
     2CPAL(2),OMEGA,OME(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2), BALL 300
     3XA(2), KONV,EAD(2,2),RFVR,DGYRO(2),CAL,SAL,A1,A2,AN,I1,I4,I3,I4, BALL 310
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```
     4DEL,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)
     5,CACFDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),
     6FPZCA1,FPZCA2,FPZOME
      J=JPASS
      IC=CTL(J)
      TAL=SF(IC)/(CF(IC)+C(IC)*D/E2)
      ALX=ATAN(TAL)
      SAL=SIN(ALX)
      CAL=COS(ALX)
      DO 1K=1,2
      TEMP=B(K)-ALX
      SBAL(K)=SIN(TEMP)
1     CBAL(K)=COS(TEMP)
      T1=GG(1)*CBAL(2)
      T2=GG(2)*CBAL(1)
      DN=T1+T2
      REVB=E2*(RPM(1)-RPM(2))*GG(1)*GG(2)/(D*DN)
      REVX(J)=REVB*CAL
      REVZ(J)=-REVB*SAL
      OMEGA=(T1*RPM(1)+T2*RPM(2))/DN
      CF=XMASS*.5*E2*OMEGA**2
      XMY=XIN*OMEGA*REVB*SAL
      H(1)=2.*XMY/D
      HH(2)=H(1)
      Y1=CB(IC)**2+C(IC)*G2(IC)
      Y2=CB(IC)**2-SB(IC)**2
      Y3=SB(IC)*CB(IC)
      Y4=(CAL/Y1)**2

      Y5=E2*(RPM(1)-RPM(2))/(D*DN**2)
      Y6=OMEGA*DN
      Y7=2./D*(H1(IC)*CB(1)-H2(IC)*CB(2))
      Y8=2./D*(H1(IC)*SB(1)-H2(IC)*SB(2))
      Y9=.5*D*Y7
      Y10=.5*D*Y8
      DO 2 K=1,2
      DALX=(Y1*Y2*DB(IC,K)-Y3*(-2.*Y3*DB(IC,K)+C(IC)*DG(IC,K)))*Y4
      Z1=GG(1)*SBAL(2)*(DB(2,K)-DALX)
      Z2=GG(2)*SBAL(1)*(DB(1,K)-DALX)
      DOMBX=Y5*(DN*(-GG(1)*DG(2,K)+GG(2)*DG(1,K))-GG(1)*GG(2)*(-Z1+CBAL(
     12)*DG(1,K)-Z2-CBAL(1)*DG(2,K)))+REVB*DEX(K)/E2
      DOMEX=(DN*(RPM(1)*(-Z1+CBAL(2)*DG(1,K))-RPM(2)*(Z2+CBAL(1)*DG(2,K)
     1))-Y6*(-Z1+CBAL(2)*DG(1,K)-Z2-CBAL(1)*DG(2,K)))/DN**2
      DCX(K)=XMASS*(E2*OMEGA*DOMEX+.5*OMEGA**2*DEX(K))
2     DGYRD(K)=XIN*((REVB*DOMEX+OMEGA*DOMBX)*SAL+REVB*OMEGA*CAL*DALX)
      T1=PX(1)*CB(1)+H(IC)*SB(1)
      T2=PX(2)*CB(2)+HH(IC)*SB(2)
      T3=PX(1)*SB(1)-H(IC)*CB(1)
      T4=PX(2)*SB(2)-HH(IC)*CB(2)
      DO 3 K=1,2
      DPSIX(1,K)=T1*DB(1,K)-T2*DB(2,K)+DP(1,K)*SB(1)-DP(2,K)*SB(2)-Y7*
     1DGYRD(K)
3     DPSIX(2,K)=-T3*DP(1,K)+T4*DB(2,K)+DP(1,K)*CB(1)-DP(2,K)*CB(2)+Y8
     1*DGYRD(K)-DCX(K)
      PSI(1)=PX(1)*SB(1)-PX(2)*SB(2)-H(1)*Y9
      PSI(2)=PX(1)*CB(1)-PX(2)*CB(2)+H(1)*Y10-CF
      DET=DPSIX(1,1)*DPSIX(2,2)-DPSIX(2,1)*DPSIX(1,2)
      COR(1)=(PSI(1)*DPSIX(2,2)-PSI(2)*DPSIX(1,2))/DET
      COR(2)=(DPSIX(1,1)*PSI(2)-DPSIX(2,1)*PSI(1))/DET
      DO 4 K=1,2
4     X(K)=X(K)-COR(K)
      RETURN
      END
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      SUBROUTINE BALL2                                                  BALL  10
      REAL LUBDEN                                                       BALL  20
      COMMON HSO(40),HSI(40)                                            BALL  30
      COMMON ALPHA,AA(2),AMAJ(2,40),AMIN(2,40),APOCK(40),        BET,BETABALL  40
     1,B(2),BB(2),BPOCK(40),  CAGEW,    CRV(2),   CRAD, COSB,CPH,CTHET, BALL  50
     2CORR(5),CP(2),CF,C(2),CSP(2),COR(5), CLEARS(2),CLEARP,  DP(2,5),  BALL  70
     3DG(2,2),DCF(5),DB(2,5),DEX(2),DROP,D,DENS,DDL(4),DFL(4),DTV(5,3), BALL  80
     4DETER,DFL(2),DDELX(2,2),DXKDG(2),DFXVX(2),DFXVY(2),DFXWS(2),      BALL  90
     5DFXP(2),DFYVX(2),DFYVY(2),DFYWS(2),DFYP(2),DQSVX(2),DQSVY(2),DQSWSBALL 100
     6(2),DQSP(2),DFX(2,5),DFY(2,5),DQS(2,5),DFPX(5),DFPZ(5),DQP(5),DMY(BALL 110
     75),DMZ(5),DPSI(5,5),DPSI1(5,5),DBA(2),DDELA(2),DPA(2),DFXA(2),DFYABALL 120
     8(2),DQSA(2),DFXOME(2),DFYOME(2),DQSOME(2),DFPXA(2),DFPZA(2),DQPA(2BALL 130
     9),DVARA1(5),DVARA2(5),DVOME(5), DGA(2),                  E,EL     BALL 140
      COMMON ERR(5),E2,ESAVE(2),                        FLAT,F(2),FK,FRADBALL 150
     1,FTAN,FFX(2,40),FFY(2,40),                        GAGE,G2(2),GG(2),BALL 160
     2                  HORIZ,HOR1,HOR2,HOR3,HOR4,                IQUIT,  BALL 170
     3JPASS,            IPASS,LUBDEN,        N,          OMEGAE,OMY,OMX, BALL 180
     4OMZ,OMS(2),OMEX(40),OMEY(40),OMEZ(40),                PPOC(40),PL,BALL 190
     5PR,PE,PC,PD,POCKET,PHI,PILOT,PX(2),PSI(5),P(2,40),            OMV,BALL 200
     6QS(2),QQS(2,40),              R,RPM(2),              SINB,SPH,STHET, BALL 210
     7SB(2),SO(2),                  TOTL,TI,THET,TOL(11),TCPV,TSAVE(2), BALL 220
     8THICKR(2,40) ,THICKP(40),     VIS,VERT,VRT1,VRT2,VRT3,VRT4,VYR(2), BALL 230
     9VYB(2),VXB(2),VXP(40),VZP(40),VEC(5)                              BALL 240
      COMMON X(5),XN,XF(4),XF1(4),XK(2),XMZ,XMY,XPHI(40),XBET(2,40),XOMSBALL 250
     1(2,40),XMU,XMU1,     YE,YR,YC,YNUM                                BALL 260
      COMMON D2VYR,D2VX,D2WS,D2P2,XIN,XMASS                             BALL 270
      COMMON FX(2),FY(2)                                                BALL 280
      COMMON  IC,CTL(40),LCTL,ICNT,LOOP,TQ(2),Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9BALL 290
     1,Y10,GA(2,2),BA(2,2),DELA(2,2),H(2),HH(2),H1(2),H2(2),SBAL(2),    BALL 300
     2CBAL(2),OMEGA,OMS(40),DCX(2),DPSIX(2,2),DGYA(2),PA(2,2),DPSIA(2), BALL 310
     3XA(2),  KONV,DAD(2,2),REVB,DGYRO(2),CAL,SAL,A1,A2,DN,T1,T2,T3,T4, BALL 320
     4DET,NUM,XA1(40),XA2(40),ITPASS,REVX(40),REVZ(40)                  BALL 330
     5,CAGEDI(2),RINGDI(2),AREA(2), VEL(2),SDRAG(2),BDRAG,DSDRG(2,4),   BALL 340
     6FPZDA1,FPZDA2,FPZOME                                              BALL 350
      J=JPASS                                                           BALL 360
      ICNT=ICNT+1                                                       BALL 370
      DO 1 K=1,J                                                        BALL 380
1     TQ(K)=AA(K)*PX(K)*ESAVE(K)/CSP(K)                                 BALL 390
      B2B1=COS(B(2)-B(1))                                               BALL 400
      IF(ICNT-3)2,3,3                                                   BALL 410
3     IF(LOOP)4,4,5                                                     BALL 420
4     LOOP=1                                                            BALL 430
      IC=1                                                              BALL 440
      NUM=1                                                             BALL 450
      RETURN                                                            BALL 460
2     IF(IC-1)6,6,7                                                     BALL 470
6     IF(TQ(2)*B2B1-TQ(1))8,8,9                                         BALL 480
9     IC1=2                                                             BALL 490
      GO TO 10                                                          BALL 500
8     IC1=1                                                             BALL 510
      GO TO 10                                                          BALL 520
7     IF(TQ(1)*B2B1-TQ(2))11,11,12                                      BALL 530
12    IC1=1                                                             BALL 540
      GO TO 10                                                          BALL 550
11    IC1=2                                                             BALL 560
10    IF(IC1-IC)13,5,13                                                 BALL 570
13    IC=IC1                                                            BALL 580
      NUM=1                                                             BALL 590
      RETURN                                                            BALL 600
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5     CTL(J)=IC                                                         BALL 610
      NUM=0                                                             BALL 620
      DELA(2,1)=SB(2)                                                   BALL 630
      DELA(2,2)=CB(2)                                                   BALL 640
      BA(2,1)=CB(2)**2/(A2-X(2))                                        BALL 650
      BA(2,2)=-(A1-X(1))*(CB(2)/(A2-X(2)))**2                           BALL 660
      GA(2,1)=-D*BA(2,1)*SB(2)/E2                                       BALL 670
      GA(2,2)=-D*BA(2,2)*SB(2)/E2                                       BALL 680
      DO 14 K=1,2                                                       BALL 690
      DALA=(Y1*Y2*BA(IC,K)-Y3*(-2.*Y3*BA(IC,K)+C(IC)*GA(IC,K)))*Y4      BALL 700
      DALA=(Y1*Y2*BA(IC,K)-Y3*(-2.*Y3*BA(IC,K)+C(IC)*GA(IC,K)))*Y4      BALL 710
      Z1=GG(1)*SBAL(2)*(BA(2,K)-DALA)                                   BALL 720
      Z2*GG(2)*SBAL(1)*(BA(1,K)-DALA)                                   BALL 730
      DOMEA=Y5*(DN*(-GG(1)*GA(2,K)+GG(2)*GA(1,K))-GG(1)*GG(2)*(-Z1+CBAL(BALL 740
     1 2)*GA(1,K)-Z2-CBAL(1)*GA(2,K)))                                  BALL 750
      DOMEA=(DN*(BPM(1)*(-Z1+CBAL(2)*GA(1,K))-BPM(2)*(Z2+CBAL(1)*GA(2,K)BALL 760
     1 1))-Y6*(-Z1+CBAL(2)*GA(1,K)-Z2-CBAL(1)*GA(2,K)))/DN**2           BALL 770
      DCA=XMASS*E2*OMEGA*DOMEA                                          BALL 780
      DCYA(K)=XIN*((REVP*DOMEA+OMEGA*DOMBA)*SAL+REVB*OMEGA*CAL*DALA)    BALL 790
      DO 15 L=1,2                                                       BALL 800
15    BA(L,K)=1.5*SQ(L)*DELA(L,K)+DEL(L)*SQRT(DEL(L))*DXKDG(L)*GA(L,K)  BALL 810
      DPSIA(1)=T1*BA(1,K)-T2*BA(2,K)+BA(1,K)*SB(1)-BA(2,K)*SB(2)-Y7*DGYABALL 820
     1(K)                                                               BALL 830
      DPSIA(2,=-T3*BA(1,K)+T4*BA(2,K)+BA(1,K)*CB(1)-BA(2,K)*CB(2)+Y8*   BALL 840
     1DGYA(K)-DCA                                                       BALL 850
      XA(1)=(-DPSIA(1)*DPSIX(2,2)+DPSIA(2)*DPSIX(1,2))/DET              BALL 860
      XA(2)=(-DPSIA(2)*DPSIX(1,1)+DPSIA(1)*DPSIX(2,1))/DET              BALL 870
      BA(2,K)=BA(2,K)+DB(2,1)*XA(1)+DB(2,2)*XA(2)                       BALL 880
      DELA(2,K)=DELA(2,K)+DDELX(2,1)*XA(1)+DDELX(2,2)*XA(2)             BALL 890
      BA(2,K)=1.5*SQ(2)*DELA(2,K)+DEL(2)*SQRT(DEL(2))*DXKDG(2)*(GA(2,K)+BALL 900
     1DG(2,1)*XA(1)+DG(2,2)*XA(2))                                      BALL 910
14    DGYA(K)=DGYA(K)+DGYB(1)*XA(1)+DGYB(2)*XA(2)                       BALL 920
      XF1(1)=XF1(1)+T4                                                  BALL 930
      XF1(3)=XF1(3)+T2*CPH                                              BALL 940
      DO 16 K=1,2                                                       BALL 950
      Y1=BA(2,1)*DAD(1,K)+BA(2,2)*DAD(2,K)                              BALL 960
      Y2=BA(2,1)*DAD(1,K)+BA(2,2)*DAD(2,K)                              BALL 970
      Y3=DGYA(1)*DAD(1,K)+DGYA(2)*DAD(2,K)                              BALL 980
      Y4=T2*Y1+Y2*SB(2)-H2(IC)*Y3*CB(2)/D**2.                           BALL 990
      Y5=-T4*Y1+Y2*CB(2)+H2(IC)*Y3*SB(2)/D**2.                          BALL1000
      DTV(1,K)=DTV(1,K)+Y4                                              BALL1010
16    DTV(2,K)=DTV(2,K)+Y5*CPH                                          BALL1020
      RETURN                                                            BALL1030
      END                                                               BALL1040
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 SUBROUTINE FRCTN (G1,G2,G3,FCOF,THETA)                                 FRCT  10
 DIMENSION AG1(13),AG2(7),AG3(7),FTN(13,7,7),U7(7,7),V7(7),VTN(637)FRCT  20
 DIMENSION A1(13),A2(13),A3(13),A4(13),A5(13),A6(13),A7(13),A8(13),FRCT  30
1A9(13),A10(13),A11(13),A12(13),A13(13),A14(13),A15(13),A16(13),   FRCT  40
2A17(13),A18(13),A19(13),A20(13),A21(13),A22(13),A23(13),A24(13),  FRCT  50
3A25(13),A26(13),A27(13),A28(13),A29(13),A30(13),A31(13),A32(13),  FRCT  60
4A33(13),A34(13),A35(13),A36(13),A37(13),A38(13),A39(13),A40(13),  FRCT  70
5A41(13),A42(13),A43(13),A44(13),A45(13),A46(13),A47(13),A48(13),  FRCT  80
6A49(13)                                                            FRCT  90
 EQUIVALENCE(VTN(1),FTN(1,1,1))                                     FRCT 100
 EQUIVALENCE(VTN(1),A1(1)),(VTN(14),A2(1)),(VTN(27),A3(1)),         FRCT 110
1(VTN(40),A4(1)),(VTN(53),A5(1)),(VTN(66),A6(1)),(VTN(79),A7(1)),   FRCT 120
2(VTN(92),A8(1)),(VTN(105),A9(1)),(VTN(118),A10(1)),(VTN(131),A11(1FRCT 130
3)),(VTN(144),A12(1)),(VTN(157),A13(1)),(VTN(170),A14(1)),(VTN(183)FRCT 140
4,A15(1)),(VTN(196),A16(1)),(VTN(209),A17(1)),(VTN(222),A18(1)),   FRCT 150
5(VTN(235),A19(1)),(VTN(248),A20(1)),(VTN(261),A21(1)),(VTN(274),  FRCT 160
6A22(1)),(VTN(287),A23(1)),(VTN(300),A24(1)),(VTN(313),A25(1)),    FRCT 170
7(VTN(326),A26(1)),(VTN(339),A27(1)),(VTN(352),A28(1)),(VTN(365),  FRCT 180
8A29(1)),(VTN(378),A30(1)),(VTN(391),A31(1)),(VTN(404),A32(1)),    FRCT 190
9(VTN(417),A33(1)),(VTN(430),A34(1)),(VTN(443),A35(1)),(VTN(456),  FRCT 200
1A35(1)),(VTN(469),A37(1)),(VTN(482),A38(1)),(VTN(495),A39(1)),    FRCT 210
2(VTN(508),A40(1)),(VTN(521),A41(1)),(VTN(534),A42(1)),(VTN(547),  FRCT 220
3A43(1)),(VTN(560),A44(1)),(VTN(573),A45(1)),(VTN(586),A46(1)),(VTNFRCT 230
4(599),A47(1)),(VTN(612),A48(1)),(VTN(625),A49(1))                  FRCT 240
 DATA A1            /0.00025,0.0007,   0.0014, 0.0023, 0.0067,      FRCT 250
1 0.0135,  0.021, 0.037, 0.046, 0.052, 0.0615, 0.0685, 0.0775/      FRCT 260
 DATA A2                 /0.0007, 0.002, 0.0039, 0.0065, 0.0150,    FRCT 270
1 0.022, 0.029, 0.042, 0.0495, 0.054, 0.0625, 0.069, 0.0775/        FRCT 280
 DATA A3            /        0.0026, 0.0079, 0.014, 0.018, 0.028,   FRCT 290
1 0.034, 0.038, 0.0485, 0.054, 0.059, 0.064, 0.0695, 0.0775/        FRCT 300
 DATA A4                   / 0.0077, 0.017,0.023, 0.027, 0.035,     FRCT 310
1 0.04, 0.044, 0.051, 0.0555, 0.0605, 0.066, 0.071, 0.0775/         FRCT 320
 DATA A5            /0.019, 0.0285, 0.034, 0.037, 0.044, 0.048,     FRCT 330
1 0.05 , 0.056, 0.06, 0.0635, 0.069, 0.0725, 0.0775/                FRCT 340
 DATA A6                   / 0.0215, 0.032, 0.037, 0.0415, 0.0475,FRCT 350
1 0.051, 0.054, 0.06, 0.0635, 0.066, 0.07, 0.073, 0.0775/           FRCT 360
 DATA A7                   / 0.0245,0.034, 0.04, 0.0435, 0.05,      FRCT 370
1 0.054, 0.0565, 0.062, 0.0655, 0.0675, 0.071, 0.074, 0.0775/       FRCT 380
 DATA A8            /0.0002,0.0006, 0.0    , 0.002, 0.006, 0.012 ,FRCT 390
1 0.019, 0.036, 0.047, 0.054, 0.0645, 0.0    0.077/                 FRCT 400
 DATA A9            /0.0006,0.002, 0.0    0.0065, 0.015, 0.024, FRCT 410
1 0.028, 0.043, 0.052, 0.0575, 0.0665, 0.072, 0.077/                FRCT 420
 DATA A10           /  0.0027, 0.0081, 0.014, 0.019, 0.0285,        FRCT 430
10.035, 0.0395, 0.051, 0.057, 0.061, 0.068, 0.0725, 0.077/          FRCT 440
 DATA A11           /0.0066, 0.015, 0.021, 0.025, 0.034, 0.04 ,     FRCT 450
1 0.0445, 0.054, 0.06, 0.064, 0.069, 0.073, 0.077/                  FRCT 460
 DATA A12           /   0.017, 0.0225, 0.029, 0.0345, 0.043,        FRCT 470
1 0.049, 0.052, 0.059, 0.063, 0.066, 0.070, 0.073, 0.077/           FRCT 480
 DATA A13           /   0.019, 0.032, 0.039, 0.0415, 0.05,0.054,FRCT 490
1 0.056, 0.061, 0.064, 0.067, 0.0725, 0.075, 0.077/                 FRCT 500
 DATA A14           /   0.021, 0.0335, 0.041, 0.0455, 0.053,        FRCT 510
1 0.056, 0.059, 0.064, 0.068, 0.07, 0.073, 0.076, 0.077/            FRCT 520
 DATA A15           /.00016,.00055,0.0011,0.0019, 0.0059,0.011,     FRCT 530
1 0.016, 0.033, 0.042, 0.048, 0.06, 0.068, 0.075/                   FRCT 540
 DATA A16           /.0007,.0021, 0.0043, 0.007, 0.015, 0.023,      FRCT 550
1 0.027, 0.039, 0.046, 0.052, 0.062, 0.07, 0.075/                   FRCT 560
 DATA A17           /    0.0018, 0.0053, 0.01, 0.015, 0.0255,       FRCT 570
1 0.034, 0.037, 0.049, 0.054, 0.058, 0.066, 0.073, 0.075/           FRCT 580
 DATA A18           /   0.0046, 0.011, 0.018, 0.024, 0.037,         FRCT 590
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0 0.039, 0.044, 0.0525, 0.057, 0.061, 0.068, 0.074, 0.075/          FRCT 600
 DATA A19                 /   0.0082, 0.017, 0.025, 0.03, 0.042,    FRCT 610
10.048, 0.052, 0.06, 0.064, 0.066, 0.070, 0.074, 0.075/             FRCT 620
 DATA A20                 / 0.015, 0.026, 0.034, 0.04, 0.049,       FRCT 630
1 0.054,0.058, 0.063, 0.067, 0.069, 0.073, 0.075, 0.075/            FRCT 640
 DATA A21                 /   0.018, 0.028, 0.036, 0.043, 0.052,    FRCT 650
1 0.057, 0.06, 0.065, 0.068, 0.07, 0.074, 0.075, 0.075/             FRCT 660
 DATA A22                 /.00015,.00046,.001, 0.0017, 0.005, 0.01, FRCT 670
1 0.016, 0.034, 0.04, 0.046, 0.059, 0.067, 0.073/                   FRCT 680
 DATA A23                 /0.00064,0.002, 0.0039, 0.0064, 0.015,    FRCT 690
1 0.021, 0.026, 0.039, 0.046, 0.052, 0.062, 0.068, 0.073/           FRCT 700
 DATA A24                 /   0.0017, 0.0048, 0.009, 0.014, 0.025,  FRCT 710
1 0.031, 0.036, 0.047, 0.053, 0.057, 0.065, 0.0695, 0.073/          FRCT 720
 DATA A25                 /    0.043, 0.01, 0.017, 0.023, 0.033,    FRCT 730
1  .039, 0.043, 0.052, 0.059, 0.061, 0.066, 0.07, 0.073/            FRCT 740
 DATA A26                 / 0.0078, 0.015, 0.024, 0.029, 0.04,      FRCT 750
1 0.046, 0.051, 0.057, 0.062, 0.064, 0.068, 0.071, 0.073/           FRCT 760
 DATA A27                 /   0.0145, 0.024, 0.033, 0.039, 0.049,   FRCT 770
1 0.053, 0.056, 0.062, 0.066, 0.067, 0.07, 0.072, 0.073/            FRCT 780
 DATA A28                 / 0.016, 0.027, 0.0355, 0.041, 0.052,     FRCT 790
1 0.056, 0.059, 0.064, 0.067, 0.068, 0.071, 0.073, 0.073/           FRCT 800
 DATA A29                 /.0001,.00027,.00055,0.0009,0.0029,       FRCT 810
1 0.0059, 0.01, 0.025, 0.036, 0.043, 0.054, 0.062, 0.068/           FRCT 820
 DATA A30                 /.00044,.0013,0.0026, 0.0044, 0.012,      FRCT 830
1 0.018, 0.024, 0.036, 0.044, 0.048, 0.057, 0.063, 0.068/           FRCT 840
 DATA A31                 /0.001, 0.0029, 0.0056, 0.009, 0.0185,    FRCT 850
1 0.0255, 0.03, 0.042, 0.0485, 0.053, 0.06, 0.064, 0.068/           FRCT 860
 DATA A32                 /0.0015, 0.0045, 0.0095, 0.013, 0.024,    FRCT 870
1 0.032, 0.036, 0.047, 0.053, 0.057, 0.063, 0.065, 0.068/           FRCT 880
 DATA A33                 /0.0039, 0.0093, 0.0155, 0.021, 0.032,    FRCT 890
1 0.038, 0.042, 0.052, 0.056, 0.06, 0.064, 0.066, 0.068/            FRCT 900
 DATA A34                 /0.0039, 0.017, 0.0245, 0.03, 0.04,       FRCT 910
1 0.0465, 0.051, 0.058, 0.061,0.063, 0.066, 0.067, 0.068/           FRCT 920
 DATA A35                 /0.012, 0.022, 0.0295, 0.035, 0.0455,     FRCT 930
1 0.051, 0.055, 0.061, 0.063, 0.065, 0.067, 0.068, 0.068/           FRCT 940
 DATA A36                 /.00005,.00014,.00027,.00065,.0014,       FRCT 950
1 0.0027, 0.0046, 0.013, 0.023, 0.029, 0.042, 0.052, 0.055/         FRCT 960
 DATA A37                 /.00012,.00042,.00085,.0014,.0044,        FRCT 970
1 0.0084, 0.013, 0.0245, 0.032, 0.037, 0.0455, 0.053, 0.055/        FRCT 980
 DATA A38                 /.0004,.0012, 0.0024, 0.004, 0.011,       FRCT 990
1 0.017, 0.023, 0.0345, 0.039, 0.0425, 0.049, 0.0535, 0.055/        FRCT1000
 DATA A39                 /0.0015, 0.0043, 0.008, 0.012, 0.023,     FRCT1010
1 0.029, 0.033, 0.0395, 0.043, 0.045, 0.050, 0.0535, 0.055/         FRCT1020
 DATA A40                 /0.0034, 0.0084, 0.014,  0.0195, 0.03,    FRCT1030
1 0.036, 0.04, 0.046, 0.049, 0.049, 0.051, 0.054, 0.055/            FRCT1040
 DATA A41                 /0.0056, 0.013, 0.02, 0.026, 0.0375,      FRCT1050
1 0.042, 0.045, 0.049, 0.05, 0.052, 0.0535, 0.054, 0.055/           FRCT1060
 DATA A42                 /0.0071, 0.015, 0.023, 0.03, 0.04,        FRCT1070
1 0.045, 0.049, 0.051, 0.052, 0.053, 0.054, 0.055, 0.055/           FRCT1080
 DATA A43                 /.00003,.00009,.00019,.0003,.0009,        FRCT1090
1 0.0019, 0.003, 0.009, 0.016, 0.023, 0.037, 0.045, 0.051/          FRCT1100
 DATA A44                 /.0001,.00026,.00059,.0009,.0029,         FRCT1110
1 0.0057, 0.0092, 0.021, 0.029, 0.034, 0.041, 0.047, 0.051/         FRCT1120
 DATA A45                 /.00025,.00074,0.0015, 0.0024, 0.0072,    FRCT1130
1 0.013, 0.019, 0.028, 0.035, 0.039, 0.045, 0.048, 0.051/           FRCT1140
 DATA A46                 /0.00054, 0.0016, 0.0033, 0.0052, 0.014,  FRCT1150
1 0.022, 0.0265, 0.035, 0.04, 0.042, 0.0465, 0.049, 0.051/          FRCT1160
 DATA A47                 /0.0013, 0.0036, 0.0069, 0.011, 0.023,    FRCT1170
1 0.029, 0.0325, 0.0395, 0.044, 0.046, 0.0485, 0.050, 0.051/        FRCT1180
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      DATA A49                /0.0015, 0.041, 0.009, 0.012, 0.0235,       FRCT1190
     1 0.0295, 0.033, 0.04, 0.044, 0.0465, 0.049, 0.0505, 0.051/          FRCT1200
      DATA A49                /0.0018, 0.0052, 0.0094, 0.014, 0.0255,     FRCT1210
     1 0.031, 0.035, 0.042, 0.045, 0.047, 0.05, 0.051, 0.051/             FRCT1220
      DATA AG1             /1.E-8, 3.E-8, 6.E-8, 1.E-7, 3.E-7, 6.E-7,     FRCT1230
     1 1.E-6, 3.E-6, 6.E-6, 1.E-5, 3.E-5,          1.E-4, 1.E-3/          FRCT1240
      DATA AG2              /5.E-7, 1.E-6, 5.E-6, 1.E-5, 5.E-5, 5.E-4, 1.E-3FRCT1250
     1 /                                                                  FRCT1260
      DATA AG3             /13.32, 15.2, 17.8, 19., 21.19, 27.13, 100./   FRCT1270
      IF(G1.EQ.0.) GO TO 240                                              FRCT1280
      IF(G2.EQ.0.) GO TO 240                                              FRCT1290
      DO 100 I=1,7                                                        FRCT1300
      DO 100 J=1,7                                                        FRCT1310
      IF(G1-AG1(13)) 40,60,60                                             FRCT1320
   40 IF(G1.LT.AG1(1)) GO TO 70                                           FRCT1330
      CALL TLUG(G1,UZ(I,J),AG1,FTN(1,I,J),13)                             FRCT1340
      GO TO 100                                                           FRCT1350
   60 UZ(I,J)=FTN(13,I,J)                                                 FRCT1360
      GO TO 100                                                           FRCT1370
   70 UZ(I,J)=EXP(ALOG(FTN(1,I,J))+(ALOG(FTN(2,I,J))-ALOG(FTN(1,I,J)))/(FRCT1380
     1ALOG(AG1(2))-ALOG(AG1(1)))*(ALOG(G1)-ALOG(AG1(1))))                 FRCT1390
  100 CONTINUE                                                            FRCT1400
      DO 200 K=1,7                                                        FRCT1410
      IF(G3-AG3(7)) 140,160,160                                           FRCT1420
  140 IF(G3.LT.AG3(1)) GO TO 170                                          FRCT1430
      CALL TLU(G3,VZ(K),AG3,UZ(1,K),7)                                    FRCT1440
      GO TO 200                                                           FRCT1450
  160 VZ(K)=UZ(7,K)                                                       FRCT1460
      GO TO 200                                                           FRCT1470
  170 VZ(K)=UZ(1,K)-(UZ(2,K)-UZ(1,K))/(AG3(2)-AG3(1))*(AG3(1)-G3)         FRCT1480
      IF (VZ(K).LE.0.0) VZ(K)=1.E-5                                       FRCT1490
  200 CONTINUE                                                            FRCT1500
      IF(G2-AG2(7)) 204,206,206                                           FRCT1510
  204 IF(G2.LT.AG2(1)) GO TO 207                                          FRCT1520
      CALL TLUG(G2,FR3,AG2,VZ,7)                                          FRCT1530
      GO TO 210                                                           FRCT1540
  206 FR3=EXP(ALOG(VZ(6))+(ALOG(VZ(7))-ALOG(VZ(6)))/(ALOG(AG2(7))-ALOG(AFRCT1550
     1G2(6)))*(ALOG(G2)-ALOG(AG2(6))))                                    FRCT1560
      GO TO 210                                                           FRCT1570
  207 FR3=EXP(ALOG(VZ(2))+(ALOG(VZ(1))-ALOG(VZ(2)))/(ALOG(AG2(1))-ALOG(AFRCT1580
     1G2(2)))*(ALOG(G2)-ALOG(AG2(2))))                                    FRCT1590
  210 FCOE=ALOG(FR3)-0.149*(ALOG(THE...)-ALOG(30.))                       FRCT1600
      FCOE=EXP(FCOE)                                                      FRCT1610
      GO TO 250                                                           FRCT1620
  240 FCOE=0.                                                             FRCT1630
  250 CONTINUE                                                            FRCT1640
      RETURN                                                              FRCT1650
      END                                                                 FRCT1660
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      SUBROUTINE TLU(A,B,C,D,N)                                         TLU%  10
      DIMENSION C(1),D(1)                                               TLU%  20
      IF(N-1)1,2,3                                                      TLU%  30
    1 B=0.                                                              TLU%  40
      GO TO 100                                                         TLU%  50
    2 B=D(1)                                                            TLU%  60
      GO TO 100                                                         TLU%  70
    3 ML=1                                                              TLU%  80
      MU=N                                                              TLU%  90
    8 IF(MU-ML-1) 15,15,9                                               TLU% 100
    9 M=(MU+ML)/2                                                       TLU% 110
      IF(C(1)-C(2))11,2,10                                              TLU% 120
   10 IF(C(M)-A)13,12,14                                                TLU% 130
   11 IF(A-C(M))13,12,14                                                TLU% 140
   12 B=D(M)                                                            TLU% 150
      GO TO 100                                                         TLU% 160
   13 MU=M                                                              TLU% 170
      GO TO 8                                                           TLU% 180
   14 ML=M                                                              TLU% 190
      GO TO 8                                                           TLU% 200
   15 B=D(ML)+(D(MU)-D(ML))*((A-C(ML))/(C(MU)-C(ML)))                   TLU% 210
  100 RETURN                                                            TLU% 220
      END                                                               TLU% 230
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      SUBROUTINE TLUG(A,B,C,D,N)                                        TLUG  10
      DIMENSION C(1),D(1)                                               TLUG  20
      IF(N-1)1,2,3                                                      TLUG  30
    1 B=0.                                                              TLUG  40
      GO TO 100                                                         TLUG  50
    2 B=D(1)                                                            TLUG  60
      GO TO 100                                                         TLUG  70
    3 ML=1                                                              TLUG  80
      MU=N                                                              TLUG  90
    8 IF(MU-ML-1) 15,15,9                                               TLUG 100
    9 M=(MU+ML)/2                                                       TLUG 110
      IF(C(1)-C(2))11,2,10                                              TLUG 120
   10 IF(C(M)-A)13,12,14                                                TLUG 130
   11 IF(A-C(M))13,12,14                                                TLUG 140
   12 B=D(M)                                                            TLUG 150
      GO TO 100                                                         TLUG 160
   13 MU=M                                                              TLUG 170
      GO TO 8                                                           TLUG 180
   14 ML=M                                                              TLUG 190
      GO TO 8                                                           TLUG 200
   15 B=EXP(ALOG(D(ML))+(ALOG(D(MU))-ALOG(D(ML)))*((ALOG(A)-ALOG(C(ML)))TLUG 210
     1 /(ALOG(C(MU))-ALOG(C(ML)))))                                     TLUG 220
  100 RETURN                                                            TLUG 230
      END                                                               TLUG 240
```

APPENDIX II

FORMAT FOR BALL BEARING COMPUTER INPUT INFORMATION

CARD NO.	COL. NO.	ITEM
1	1-10	NUMBER OF BALLS
	11-20	BALL DIAMETER - IN.
	21-30	PITCH DIAMETER - IN.
	31-40	CONTACT ANGLE - DEG.
	41-50	OUTER RACE CURVATURE FACTOR.
	51-60	INNER RACE CURVATURE FACTOR.
	61-70	CHANGE IN INTERNAL CLEARANCE (+ IF INCREASED)
	71-80	DIAMETER OF CYLINDRICAL CAGE POCKET - IN.
2	1-10	YOUNG'S MODULUS FOR CAGE - PSI.
	11-20	POISSON'S RATIO FOR CAGE.
	21-30	YOUNG'S MODULUS FOR BALLS - PSI. IF BLANK PROGRAM ASSUMES 29000000.
	31-40	POISSON'S RATIO FOR BALLS. IF BLANK PROGRAM ASSUMES .25 .
	41-50	YOUNG'S MODULUS FOR RINGS. - PSI. IF BLANK PROGRAM ASSUMES 29000000.
	51-60	POISSON'S RATIO FOR RINGS. IF BLANK PROGRAM ASSUMES .25 .
	61-70	BALL MATERIAL DENSITY - LB/IN**2 .
	71-80	COEFFICIENT OF SLIDING FRICTION OF BALL IN POCKET.
3	1-10	COEFFICIENT OF SLIDING FRICTION OF CAGE ON PILOTING SURFACE.
	11-20	WIDTH OF CAGE - IN.
	21-30	O. D. OF CAGE - IN.
	31-40	I. D. OF CAGE - IN.
	41-50	I. D. OF OUTER RING - IN.
	51-60	O. D. OF INNER RING - IN.
	61-80	LEAVE BLANK
4	1-10	PRESSURE-VISCOSITY COEFFICIENT OF THE LUBRICANT - IN**2/LB.
	11-20	TEMPERATURE-VISCOSITY COEFFICIENT OF LUBRICANT - /F DEG.
	21-30	VISCOSITY OF LUBRICANT - LB-SEC/IN**2.
	31-40	INLET TEMPERATURE - F DEG.
	41-50	THERMAL CONDUCTIVITY OF LUBRICANT - BTU/F DEG - HR - FT.
	51-60	DENSITY OF LUBRICANT - LB*SEC**2/IN**4
	61-70	PROPORTIONATE INCREMENT FOR THE VELOCITY OF THE RACE IN THE Y DIRECTION. SET TO .05 IF LEFT BLANK
	71-80	PROPORTIONATE INCREMENT FOR THE VELOCITY OF THE BALL IN THE X DIRECTION. SET TO .05 IF LEFT BLANK
5	1-10	PROPORTIONATE INCREMENT FOR ANGULAR VELOCITY OF SPIN. SET TO .1 IF LEFT BLANK
	11-20	PROPORTIONATE INCREMENT FOR CONTACT LOAD. SET .05 IF BLANK
	21-30	TOLERANCE ON X(1) BALL CENTER COORDINATE. IF BLANK PROGRAM ASSUMES 1.E-6
	31-40	TOLERANCE ON X(2) BALL CENTER COORDINATE. IF BLANK PROGRAM ASSUMES 1.E-6 .
	41-50	TOLERANCE ON BALL'S ANGULAR VELOCITY ABOUT X. IF BLANK PROGRAM ASSUMES 10 RPM.
	51-60	TOLERANCE ON BALL'S ANGULAR VELOCITY ABOUT Y. IF BLANK PROGRAM ASSUMES 1 RPM.
	61-70	TOLERANCE ON BALL'S ANGULAR VELOCITY ABOUT Z. IF BLANK

		PROGRAM ASSUMES 5 RPM.
	71-80	TOLERANCE ON DEFLECTION ALONG X - IN. IF BLANK PROGRAM ASSUMES 1.E-6
6	1-10	TOLERANCE ON DEFLECTION ALONG Y - IN. IF BLANK PROGRAM ASSUMES 1.E-6
	11-20	TOLERANCE ON DEFLECTION ALONG Z - IN. IF BLANK PROGRAM ASSUMES 1.E-6
	21-30	TOLERANCE ON ANGULAR VELOCITY OF CAGE - RPM. IF BLANK PROGRAM ASSUMES .5 RPM
	31-80	LEAVE BLANK
7	1-10	RPM OF OUTER RING.
	11-20	RPM OF INNER RING.
	21-30	THRUST LOAD - LB.
	31-40	RADIAL LOAD - LB.
	41-50	MISALIGNMENT ABOUT Y - IN/IN.
	51-60	ESTIMATE OF DEFLECTION ALONG X - IN.
	61-70	ESTIMATE OF DEFLECTION ALONG Z - IN.
	71-80	PUNCH 2. IF BEARING WHEN RUNNING WITH BOUNDARY LUBRICATION COULD BE EXPECTED TO HAVE 'INNER-RACE CONTROL'. IF BLANK PROGRAM ASSUMES THAT 'OUTER-RACE CONTROL' WOULD EXIST.

NOTES

TO RUN ADDITIONAL LOAD CASES WITH THE SAME SYSTEM REPEAT CARD NO. 7 FOR EACH NEW LOAD CASE

TO RUN NEW LUBRICANT PROPERTIES WITH SAME BEARING PLACE ONE BLANK AFTER LAST CARD NO. 7 AND READ IN NEW CARD NO. 4 ET SEQ.

TO READ NEW BEARING SYSTEM PLACE TWO BLANKS AFTER LAST CARD NO. 7 AND READ IN CARD NO. 1 ET SEQ.

APPENDIX III

TYPICAL BALL BEARING PROBLEM FOR COMPUTER PROGRAM

BEARING DESIGN DATA

NO. OF ELEMENTS	ELEMENT DIAMETER	PITCH DIAMETER	CONTACT ANGLE	RACE CURVATURES OUTER	RACE CURVATURES INNER	CLEARANCE INCREMENT	TYPE OF PILOT	PILOT CLEARANCE	POCKET CLEARANCE	CAGE WIDTH
2.2200E 01	5.1100E-01	4.7240E 00	3.0000E 01	5.1500E-01	5.2000E-01	-1.5000E-03	OUTER	2.9000E-02	3.1000E-02	7.1600E-01

I. D. OF OUTER	O. D. OF INNER	I. D. OF CAGE	O. D. OF CAGE	MODULUS OF ELASTICITY RINGS	MODULUS OF ELASTICITY CAGE	MODULUS OF ELASTICITY ELEMENTS	POISSONS RATIO RINGS	POISSONS RATIO CAGE	POISSONS RATIO ELEMENTS	ELEMENT DENSITY
5.2840E 02	4.5330E 02	4.5950E 00	5.0150E 00	2.9000E 07	2.9000E 07	2.9000E 07	2.5300E-01	3.0000E-01	2.5000E-01	2.0300E-01

LUBRICANT PROPERTIES

PRESSURE-VISCOSITY COEFFICIENT = 1.1800E-04
TEMPERATURE-VISCOSITY COEFFICIENT = 1.5300E-02
VISCOSITY AT INLET TEMPERATURE = 3.8600E-06
INLET TEMPERATURE = 1.0000E 02
THERMAL CONDUCTIVITY = 7.7400E-02
DENSITY = 3.9700E-05

BALL POCKET FRICTION COEFFICIENT = 1.0000E-02
CAGE PILOT SURFACE FRICTION COEFFICIENT = 1.1000E-02

RPM. OF OUTER	RPM. OF INNER	EXTERNAL LOADS AXIAL	RADIAL	INITIAL MISALIGNMENT	STARTING DEFLECTIONS ALONG X	ALONG Z	FINAL DEFLECTIONS ALONG X	ALONG Y	ALONG Z	RPM. OF CAGE
0.0	1.5000E 04	-1.0000E 03	5.0000E 02	0.0	-1.0000E-03	-1.0000E-03	-1.1463E-03	7.0700E-06	-4.4807E-04	7.2689E 03

REACTIONS OF BEARING ON SHAFT

ALONG X	ALONG Y	ALONG Z	ABOUT Y
9.9998E 02	-5.6314E-03	-5.0001E 02	-6.3303E 02

BALL/RACE CONTACT DATA

ELEMENT NO.	ELEMENT AZIMUTH	CONTACT LOAD OUTER	CONTACT LOAD INNER	CONTACT ANGLE OUTER	CONTACT ANGLE INNER	PRESSURE AREA LENGTH OUTER	PRESSURE AREA LENGTH INNER	PRESSURE AREA WIDTH OUTER	PRESSURE AREA WIDTH INNER
1	0.0	1.1059E 02	3.8615E 01	9.4963E 00	3.9241E 01	1.0911E-01	6.9530E-02	1.2072E-02	8.0931E-03
2	1.5364E 01	1.1145E 02	3.9546E 01	9.6754E 00	3.9058E 01	1.0939E-01	7.0155E-02	1.2103E-02	8.1565E-03
3	3.2727E 01	1.1394E 02	4.2159E 01	1.0173E 01	3.8551E 01	1.1025E-01	7.1572E-02	1.2191E-02	8.3293E-03
4	4.9091E 01	1.1835E 02	4.6770E 01	1.0964E 01	3.7751E 01	1.1161E-01	7.4201E-02	1.2345E-02	8.6176E-03
5	6.5455E 01	1.2503E 02	5.3653E 01	1.2012E 01	3.6703E 01	1.1367E-01	7.7844E-02	1.2571E-02	9.0147E-03
6	8.1818E 01	1.3460E 02	6.3425E 01	1.3241E 01	3.5485E 01	1.1553E-01	8.2149E-02	1.2880E-02	9.5239E-03
7	9.8182E 01	1.4767E 02	7.6654E 01	1.4537E 01	3.4208E 01	1.2016E-01	8.7514E-02	1.3280E-02	1.0136E-02
8	1.1455E 02	1.6410E 02	9.3191E 01	1.5762E 01	3.2958E 01	1.2447E-01	9.3413E-02	1.3752E-02	1.0810E-02
9	1.3091E 02	1.8209E 02	1.1132E 02	1.6785E 01	3.1981E 01	1.2836E-01	9.9185E-02	1.4241E-02	1.1469E-02
10	1.4727E 02	1.9939E 02	1.2853E 02	1.7524E 01	3.1234E 01	1.3293E-01	1.0400E-01	1.4667E-02	1.2020E-02
11	1.6364E 02	2.1135E 02	1.4049E 02	1.7957E 01	3.0793E 01	1.3544E-01	1.0713E-01	1.4953E-02	1.2378E-02
12	1.8000E 02	2.1536E 02	1.4450E 02	1.8088E 01	3.0655E 01	1.3529E-01	1.0814E-01	1.5046E-02	1.2494E-02
13	1.9636E 02	2.1039E 02	1.3952E 02	1.7924E 01	3.0824E 01	1.3523E-01	1.0688E-01	1.4930E-02	1.2350E-02
14	2.1273E 02	1.9769E 02	1.2683E 02	1.7458E 01	3.1302E 01	1.3245E-01	1.0353E-01	1.4626E-02	1.1967E-02
15	2.2909E 02	1.8036E 02	1.0949E 02	1.6685E 01	3.2081E 01	1.2846E-01	9.8576E-02	1.4188E-02	1.1400E-02
16	2.4545E 02	1.6215E 02	9.1229E 01	1.5635E 01	3.3124E 01	1.2398E-01	9.2751E-02	1.3697E-02	1.0734E-02
17	2.6182E 02	1.4606E 02	7.5027E 01	1.4394E 01	3.4348E 01	1.1973E-01	8.6389E-02	1.3237E-02	1.0065E-02
18	2.7818E 02	1.3337E 02	6.2180E 01	1.3100E 01	3.5625E 01	1.1615E-01	8.1507E-02	1.2861E-02	9.4621E-03
19	2.9455E 02	1.2417E 02	5.2771E 01	1.1885E 01	3.6829E 01	1.1341E-01	7.7255E-02	1.2542E-02	8.9658E-03
20	3.1091E 02	1.1776E 02	4.6149E 01	1.0864E 01	3.7852E 01	1.1142E-01	7.3970E-02	1.2325E-02	8.5800E-03
21	3.2727E 02	1.1338E 02	4.1782E 01	1.0103E 01	3.8622E 01	1.1008E-01	7.1654E-02	1.2179E-02	8.3046E-03
22	3.4364E 02	1.1127E 02	3.9354E 01	9.6402E 00	3.9094E 01	1.0933E-01	7.0341E-02	1.2097E-02	8.1435E-03

ELEMENT NO.	BALL VELOCITY COMPONENTS ABOUT X	ABOUT Y	ABOUT Z	TRACTION ALONG X OUTER	INNER	TRACTION ALONG Y OUTER	INNER	SPINNING MOMENT OUTER	INNER
1	-7.0861E 04	2.1481E 03	1.6292E 04	-6.9832E 00	-9.3709E-01	1.7615E 00	2.2722E 03	-2.7635E-02	-8.4123E-03
2	-7.0318E 04	2.1345E 03	1.6419E 04	-7.0087E 00	-9.7138E-01	1.7719E 00	2.2311E 03	-2.7781E-02	-8.7991E-03
3	-7.0697E 04	2.0893E 03	1.6771E 04	-7.0757E 00	-1.0764E 00	1.8010E 00	2.2993E 03	-2.7918E-02	-9.9667E-03
4	-7.0499E 04	2.0039E 03	1.7288E 04	-7.1877E 00	-1.2153E 00	1.8321E 00	2.3113E 03	-2.8188E-02	-1.2453E-02
5	-7.0317E 04	1.9505E 03	1.7928E 04	-7.3067E 00	-1.4069E 00	1.8765E 00	2.2746E 03	-2.8911E-02	-1.7640E-02
6	-6.9872E 04	1.6393E 03	1.9628E 04	-7.4550E 00	-1.5961E 00	1.7634E 00	2.1675E 00	-3.0098E-02	-2.6039E-02
7	-6.9491E 04	1.3834E 03	1.9345E 04	-7.6752E 00	-1.7281E 00	1.6155E 00	1.9739E 03	-3.3637E-02	-3.8383E-02
8	-6.9125E 04	1.1435E 03	2.0014E 04	-7.9409E 00	-1.7867E 00	1.3930E 00	1.7072E 00	-3.7410E-02	-5.3410E-02
9	-6.8920E 04	9.3764E 02	2.0584E 04	-8.2233E 00	-1.7795E 00	1.1143E 00	1.3919E 00	-4.1726E-02	-6.9554E-02
10	-6.8592E 04	7.9659E 02	2.1019E 04	-8.4658E 00	-1.7484E 00	8.3181E-01	1.0921E 00	-4.5435E-02	-8.4286E-02
11	-6.8466E 04	7.1266E 02	2.1288E 04	-8.6236E 00	-1.7241E 00	6.2575E-01	8.6662E-01	-4.7634E-02	-9.3737E-02
12	-6.8418E 04	6.9442E 02	2.1361E 04	-8.6729E 00	-1.7091E 00	5.6357E-01	7.9128E-01	-4.8061E-02	-9.6247E-02
13	-6.8457E 04	7.2501E 02	2.1267E 04	-8.6110E 00	-1.7255E 00	6.4338E-01	8.8472E-01	-4.7641E-02	-9.3079E-02
14	-6.8613E 04	8.0937E 02	2.0978E 04	-8.4431E 00	-1.7569E 00	8.5740E-01	1.1106E 00	-4.4912E-02	-8.2713E-02
15	-6.8849E 04	9.5839E 02	2.0530E 04	-8.1899E 00	-1.7851E 00	1.1434E 00	1.4252E 00	-4.1066E-02	-6.7895E-02
16	-6.9135E 04	1.1569E 03	1.9946E 04	-7.9073E 00	-1.7860E 00	1.4185E 00	1.7380E 00	-3.6780E-02	-5.1728E-02
17	-6.9335E 04	1.4100E 03	1.9258E 04	-7.6468E 00	-1.7132E 00	1.6345E 00	1.9388E 00	-3.3149E-02	-3.6603E-02
18	-6.9310E 04	1.6657E 03	1.8557E 04	-7.4368E 00	-1.5809E 00	1.7704E 00	2.1330E 00	-3.0026E-02	-2.5182E-02
19	-7.0253E 04	1.8685E 03	1.7842E 04	-7.2906E 00	-1.3784E 00	1.8310E 00	2.2812E 00	-2.8713E-02	-1.6642E-02
20	-7.0524E 04	2.0169E 03	1.7230E 04	-7.1742E 00	-1.2315E 00	1.8331E 00	2.4122E 00	-2.8143E-02	-1.2158E-02
21	-7.0714E 04	2.0964E 03	1.6722E 04	-7.0690E 00	-1.0603E 00	1.7942E 00	2.2979E 00	-2.7379E-02	-9.7891E-03
22	-7.0826E 04	2.1470E 03	1.6395E 04	-7.0061E 00	-9.6433E-01	1.7710E 00	2.2795E 00	-2.7734E-02	-8.7175E-03

ELEMENT NO.	SPINNING VELOCITY OUTER	SPINNING VELOCITY INNER	MINIMUM FILM THICKNESS OUTER	MINIMUM FILM THICKNESS INNER
1	-5.5744E 03	-3.7725E 04	8.9414E-05	8.5188E-05
2	-5.5390E 03	-3.7371E 04	8.9345E-05	8.5063E-05
3	-5.3250E 03	-3.6384E 04	8.9153E-05	8.4723E-05
4	-4.9501E 03	-3.4834E 04	8.8823E-05	8.4124E-05
5	-4.4348E 03	-3.2811E 04	8.8339E-05	8.3357E-05
6	-3.7059E 03	-3.0459E 04	8.7692E-05	8.2329E-05
7	-3.1068E 03	-2.7979E 04	8.6895E-05	8.1179E-05
8	-2.5554E 03	-2.5619E 04	8.6012E-05	7.9923E-05
9	-1.9299E 03	-2.3617E 04	8.5134E-05	7.8777E-05
10	-1.5746E 03	-2.2126E 04	8.4410E-05	7.7887E-05
11	-1.3239E 03	-2.1277E 04	8.3937E-05	7.7303E-05
12	-1.3196E 03	-2.0960E 04	8.3754E-05	7.7126E-05
13	-1.3383E 03	-2.1296E 04	8.3974E-05	7.7347E-05
14	-1.5089E 03	-2.2258E 04	8.4473E-05	7.7979E-05
15	-1.9330E 03	-2.3911E 04	8.5218E-05	7.8899E-05
16	-2.5735E 03	-2.5863E 04	8.6104E-05	8.0065E-05
17	-3.1265E 03	-2.8256E 04	8.6995E-05	8.1301E-05
18	-3.8795E 03	-3.0720E 04	8.7771E-05	8.2453E-05
19	-4.3855E 03	-3.3067E 04	8.8400E-05	8.3436E-05
20	-4.9983E 03	-3.5029E 04	8.8867E-05	8.4203E-05
21	-5.3304E 03	-3.6526E 04	8.9181E-05	8.4771E-05
22	-5.5171E 03	-3.7447E 04	8.9359E-05	8.5088E-05

BALL/POCKET CONTACT DATA

ELEMENT NO.	POCKET FORCE	PRESSURE ELLIPSE LENGTH	PRESSURE ELLIPSE WIDTH	SLIDING VELOCITY RADIAL	SLIDING VELOCITY AXIAL	MIN. FILM THICKNESS
1	8.3836E-01	1.6521E-02	2.5333E-03	-1.9702E 03	-4.5293E 02	3.8167E-03
2	8.5100E-01	1.6649E-02	2.5529E-03	-1.9689E 03	-4.5661E 02	2.9720E-03
3	9.0533E-01	1.6950E-02	2.5990E-03	-1.9656E 03	-4.6628E 02	2.6603E-03
4	9.4844E-01	1.7215E-02	2.5396E-03	-1.9601E 03	-4.8077E 02	2.6103E-03
5	9.5520E-01	1.6955E-02	2.5998E-03	-1.9523E 03	-4.9845E 02	2.6193E-03
6	7.3291E-01	1.5797E-02	2.4222E-03	-1.9426E 03	-5.1795E 02	3.9653E-03
7	3.9944E-01	1.2850E-02	1.5703E-03	-1.9321E 03	-5.3782E 02	1.3541E-02
8	-9.4733E-02	7.9884E-03	1.2249E-03	-1.9220E 03	-5.5642E 02	1.5500E-02
9	-6.8858E-01	1.5575E-02	2.3709E-03	-1.9134E 03	-5.7222E 02	4.3527E-03
10	-1.2811E 00	1.9029E-02	2.9179E-03	-1.9071E 03	-5.8629E 02	1.2507E-03
11	-1.7026E 00	2.0922E-02	3.2080E-03	-1.9033E 03	-5.9180E 02	7.0526E-04
12	-1.8402E 00	2.1471E-02	3.2922E-03	-1.9023E 03	-5.9331E 02	6.3337E-04
13	-1.5569E 00	2.0775E-02	3.1954E-03	-1.9036E 03	-5.9123E 02	7.2605E-04
14	-1.2270E 00	1.8759E-02	2.8762E-03	-1.9076E 03	-5.8327E 02	1.3641E-03
15	-6.2638E-01	1.4991E-02	2.2987E-03	-1.9142E 03	-5.7074E 02	5.2707E-03
16	-3.8503E-02	5.9164E-03	9.0719E-04	-1.9230E 03	-5.5454E 02	1.5500E-02
17	4.3841E-01	1.3310E-02	2.0409E-03	-1.9333E 03	-5.3549E 02	1.0975E-02
18	7.5940E-01	1.5976E-02	2.4500E-03	-1.9437E 03	-5.1601E 02	3.7070E-03
19	9.1719E-01	1.7024E-02	2.6103E-03	-1.9533E 03	-4.9500E 02	2.5596E-03
20	9.4733E-01	1.7208E-02	2.6386E-03	-1.9608E 03	-4.7902E 02	2.4177E-03
21	9.0369E-01	1.6921E-02	2.5945E-03	-1.9661E 03	-4.6486E 02	2.6872E-03
22	8.5550E-01	1.6533E-02	2.5504E-03	-1.9692E 03	-4.5575E 02	2.9902E-03

VISCOUS DRAG ON BALL = 3.1950E 00

CAGE/PILOTING SURFACE DATA

VISCOUS TORQUE	COULOMB TORQUE	NORMAL LOAD
-2.4481E 00	-3.9156E-01	1.4196E 01

[HC900] EXECUTION TERMINATING DUE TO ERROR COUNT FOR ERROR NUMBER 217

APPENDIX IV

ROLLER BEARING COMPUTER PROGRAM FORTRAN SOURCE DECK

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      COMMON ALPHA,AL3,     BETA,     C(2),CAGEDI(2),CAGEW,CAGEWT,CLCAGE(2000008700
     X),CORNER,CP,CZ,CP1,CZ1,          D,DROP,DENS,DENLUB,DEL2,DEL(2),     00008800
     XDEDEL(2),DEYWX(2),DEPDHI(2),DEZDHI(2),DEPWXI(2),DEZWX(2),             00008900
     XE,EL,EP,      FLAT,FLAT02,FK,FBARV,FD,FC,FY(2),FP(2),FZ(2),FY11,      00009000
     XFY12,FP11,FP12,FZ11,FZ12,FILM,FFY(3),          GAM,GAGE,G1T,G2T,      00009100
     XHF,HHH,HP(2),HMN1,HMN2,HMN(2),HMRAC(2),      ISYS,ILOAD,ILUB,ISIGN,   00009200
     XIQUIT,        JPASS,      KPLT,KPASS,        NUM,NUMY,       OMEDCR,  00009300
     XOMEGAE,OMEI,OMEGAX,OMX1,OMEX,              PD,POCKET,PILOT,POCCLR,    00009400
     XPX(2),PPI(9),                    RADIUS,RINGDI(2),RINGYM,ROLLYM,      00009500
     XRINGPR,ROLLPR,RMH,RPM(2),ROT,REV,R(9),            STR,STS(9)          00009600
      COMMON TOTL,TOL(4),TOTL02,TI,THETA,TORQ,TORK,      VHMIN,VIS,VEINC,00009700
     XVIS3,VY(2),VIXC(3),VSS,V(2),        XMU1,XMASS,XTM,XTREM,XN,          00009800
     XYE,YNUM,YINC(2,50)                                                    00009900
      COMMON CPH,CAGECF,          DXFZ,DELTA,DRAG,        FY1(2,50),FP1(2,5000010000
     X),FZ1(2,50),FLIM(2,50),     HOR,H1(2,50),     ITER,      NOLOAD,      00010100
     XOMEGX(50),   P(2,50),PPP(9,2,50),PN,PXOUT,      RR(9,2,50),           00010200
     XSSTS(9,2,50),SPH,        VERT,VY1(2,50),        XFZ,XTRM(2,50)         00010300
      COMMON /CHECK/IV                                                      00010400
      DIMENSION WORD(2)                                                     00010500
      DATA WORD(1),WORD(2)/4H OUT,4H IN /                                   00010600
      ISYS=0                                                                00010700
      C(1)=1.                                                               00010800
      C(2)=-1.                                                              00010900
      GO TO 5
    6 WRITE(6,650)
5     READ(5,10)XN,D,E,TOTL,FLAT,CORNER,RADIUS,DROP                         00011000
      IF(XN.EQ.0.)STOP                                                      00011100
10    FORMAT(8E10.0)                                                        00011200
      NUM=XN                                                                00011300
      ILOAD=0                                                               00011400
      ILUB=0                                                                00011500
      ISYS=ISYS+1                                                           00011600
      GAM=D/E                                                               00011700
      READ(5,10)GAGE,PD,RINGDI(1),RINGDI(2),CAGEDI(1),CAGEDI(2),POCKET,     00011800
     1CAGEW,RINGYM,ROLLYM,RINGPR,ROLLPR,      XMU1,CAGEWT,DENS,YNUM,(TOL(K00011900
     2),K=1,4)                                                              00012000
      IF(DENS.EQ.0.)DENS=.283                                               00012100
      IF(TOL(1).EQ.0.)TOL(1)=.000001                                        00012200
      IF(TOL(2).EQ.0.)TOL(2)=.0000005                                       00012300
      IF(TOL(3).EQ.0.)TOL(3)=1.                                             00012400
      IF(TOL(4).EQ.0.)TOL(4)=.1                                             00012500
C     TOL(3)=TOL(3)*.1047198                                                00012600
      TOL(4)=TOL(4)*.1047198                                                00012700
      IF(XMU1.EQ.0.)XMU1=.1                                                 00012800
      IF(YNUM.EQ.0.)YNUM=5.                                                 00012900
      NUMY=YNUM                                                             00013000
      CALL        RCALC(TOTL,FLAT,GAGE,RADIUS,DROP)                         00013100
C     XN = NO. OF ROLLS                                                     00013200
C     D = ROLL DIA. - IN                                                    00013300
C     E = PITCH DIA. - IN                                                   00013400
C     TOTL = TOTAL LENGTH OF ROLL - IN                                      00013500
C     FLAT = LENGTH OF FLAT PORTION OF ROLL - IN                            00013600
C     CORNER = CORNER RADIUS OF ROLL - IN                                   00013700
C     RADIUS = CROWN RADIUS - IN                                            00013800
C     DROP = DROP OF ROLL CROWN - IN                                        00013900
C     GAGE = DISTANCE FROM END OF ROLL TO CROWN DROP REFERENCE POINT        00014000
C            - IN                                                           00014100
C     PD = DIAMETRAL CLEARANCE - IN (+ IF BEARING IS LOOSE)                 00014200
C     RINGDI(1) = I.D. OF OUTER RING - IN                                   00014300
C     RINGDI(2) = O.D. OF INNER RING - IN                                   00014400
C     CAGEDI(1) = O.D. OF CAGE - IN                                         00014500
C     CAGEDI(2) = I.D. OF CAGE - IN                                         00014600
C     POCKET = POCKET LENGTH IN ROLLING DIRECTION - IN                      00014700
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C     CAGEW = WIDTH OF CAGE - IN
C     RINGYM = YOUNG'S MODULUS FOR RINGS - PSI
C     ROLLYM = YOUNG'S MODULUS FOR ROLLS - PSI
C     RINGPR = POISSON'S RATIO FOR RINGS
C     ROLLPR = POISSON'S RATIO FOR ROLLS
C     CAGEWT = WEIGHT OF CAGE - LB
C     XMU1 = COEFFICIENT OF COULOMB FRICTION FOR PILOT SURFACES
      FLATO2=.5*FLAT
      TOTLO2=.5*TOTL-CORNER
      RNH=SQRT(RADIUS**2-FLATO2**2)
      READ (5,20)
 20   FORMAT(1H1,79H
     1                                  /1X,79H
     2                                                  )
      WRITE(6,20)
      WRITE(6,30)ISYS
 30   FORMAT(30H0  DESIGN DATA FOR BEARING NO.,I3)
      WRITE(6,40)
 40   FORMAT(130H0   NO. OF        ROLL        PITCH        TOTAL        FL
     1AT        CROWN        CROWN        GAGE        DIAMETRAL     I.D. OF
     2  O.D. OF/132H   ROLLERS      DIAMETER     DIAMETER      LENGTH
     3LENGTH        DROP        RADIUS        POINT       CLEARANCE   OUTER RI
     4NG   INNER RING)
      WRITE(6,50)XN,D,F,TOTL,FLAT,DROP,RADIUS,GAGE,PO,RINGDI(1),RINGDI(2
     1)

 50   FORMAT(1P11E12.4)
      WRITE(6,60)
 60   FORMAT(128H0  O.D. OF      I.D. OF      TYPE OF      CAGE/RING CLEARA
     1NCE        POCKET        CAGE        YOUNG'S MODULUS        POISSON'
     2S RATIO/129H      CAGE        CAGE        PILOT        OUTER        IN
     3NER      CLEARANCE      WIDTH        RINGS        ROLLS        RINGS
     4     ROLLS)
      PILOT=1.
      IF(ABS(RINGDI(2)-CAGEDI(2)).LT. ABS(RINGDI(1)-CAGEDI(1)))PILOT=2.
      KPLT=PILOT
      CLCAGE(1)=ABS(RINGDI(1)-CAGEDI(1))
      CLCAGE(2)=ABS(RINGDI(2)-CAGEDI(2))
      WRD=WORD(KPLT)
      POCCLR=POCKET-D
      IF(RINGYM.EQ.0.)RINGYM=29.E6
      IF(ROLLYM.EQ.0.)ROLLYM=29.E6
      IF(RINGPR.EQ.0.)RINGPR=.25
      IF(ROLLPR.EQ.0.)ROLLPR=.25
      FL=.6366198*((1.-RINGPR**2)/RINGYM+(1.-ROLLPR**2)/ROLLYM)
      WRITE(6,70)CAGEDI(1),CAGEDI(2),WRD,CLCAGE(1),CLCAGE(2),POCCLR,CAGE
     1W,RINGYM,ROLLYM,RINGPR,ROLLPR
 70   FORMAT(1P2E12.4,3X,A6,3X,1P8E12.4)
      WRITE(6,75)
 75   FORMAT(45H0  FRICTION     MATERIAL      CORNER        CAGE/46H     COE
     1F.        DENSITY        RADIUS        WEIGHT)
      WRITE(6,50)     XMU1,DENS,CORNER,CAGEWT
      XMASS=D**2*TOTL*DENS/491.9797
 80   READ(5,10)ALPHA,BETA,VIS,TI,FK,DENLUB
      IF(ALPHA.LE.0.)GO TO 6
      ILUB=ILUB+1
C     ALPHA = PRESSURE-VISCOSITY COEFFICIENT - IN**2/LB
C     BETA = TEMPERATURE-VISCOSITY COEFFICIENT - 1/(DEG F)
C     VIS = VISCOSITY AT INLET TEMPERATURE - LB*SEC/IN**2
C     TI = INLET TEMPERATURE - DEG F
C     FK = THERMAL CONDUCTIVITY - BTU/(DEG F)/HR/FT
C     DENLUB = LUBRICANT DENSITY - LB SEC**2/IN**4
      READ(5,90)
 90   FORMAT(1H0,79H
     1                                  )
```

```
      WRITE(6,81)
   81 FORMAT(1H /1H /1H /1H /1H /1H /23H0  LUBRICANT PROPERTIES)
      WRITE(6,90)
      WRITE(6,82)ALPHA
   82 FORMAT(40H0  PRESSURE-VISCOSITY COEFFICIENT          ,1P1E11.4)
      WRITE(6,83)BETA
   83 FORMAT(40H   TEMPERATURE-VISCOSITY COEFFICIENT       ,1P1E11.4)
      WRITE(6,84)VIS
   84 FORMAT(40H   VISCOSITY AT INLET TEMPERATURE          ,1P1E11.4)
      WRITE(6,85)TI
   85 FORMAT(40H   INLET TEMPERATURE                       ,1P1E11.4)
      WRITE(6,86)FK
   86 FORMAT(40H   THERMAL CONDUCTIVITY                    ,1P1E11.4)
      WRITE(6,87)DENLUB
   87 FORMAT(40H   LUBRICANT DENSITY                       ,1P1E11.4)
  115 READ(5,10)RPM(1),RPM(2),FBARV,ROT,OMEDCR,VEINC,HF
      IF((RPM(1).EQ.0.).AND.(RPM(2).EQ.0.))GO TO 80
      ILOAD=ILOAD+1
      WRITE(6,120)ILOAD,ISYS
  120 FORMAT(26H1  INPUT DATA FOR LOAD NO.,I3,13H, BEARING NO.,I3/34H0
     1RPM. OF      RPM. OF       RADIAL/33H    OUTER        INNER        LO
     2AD)
      IF(VEINC.EQ.0.)VEINC=.0025
C
      IF(HF.EQ.0.)HF=.00001
C
      WRITE(6,50)RPM(1),RPM(2),FBARV
      WRITE(6,7551)
 7551 FORMAT(1X)
      WRITE(6,7550)OMEDCR
 7550 FORMAT(1X,17HPERCENT CAGE SLIP,1P1E15.4)
      RPM(1)=RPM(1)*.1047198
      RPM(2)=RPM(2)*.1047198
      OMEGAE=.5*(RPM(1)*(1.+GAM)+RPM(2)*(1.-GAM))*(1.-OMEDCR)
      IF(OMEDCR.EQ.0.)OMEDCR=.1*OMEGAE
      OMEGE=OMEGAE
      OMEGAE=OMEGAE-OMEDCR
      ISIGN=1
      REV=0.
      IF(ROT)121,123,121
  121 REV=ROT*.1047198
      WRITE(6,122)ROT
  122 FORMAT(18H0  LOAD ROTATES AT,1P1E11.4,4H RPM)
      GO TO 125
  123 WRITE(6,124)
  124 FORMAT(21H0  LOAD IS STATIONARY)
  125 EP=2./((1.-RINGPR**2)/RINGYM+(1.-ROLLPR**2)/ROLLYM)
      THETA=(TI-32.)/1.8
      VIS3=VIS*EXP(BETA*(TI-86.))
      AL3=ALPHA+930.*ALPHA*(TI-86.)/(546.*(TI+460.))
      G1T=VIS3*(ALPHA/AL3)**.6*(VIS/VIS3)**.7
      G2T=VIS3*BETA/(1.728*FK)
      HNH=1.58*ALPHA**.6*VIS**.7*EP**.03
      IF(PD.GT.0.)DELZ=DELZ+.5*PD
      III=0
      TORQ=0.
      TORK=-0.0
      OME1=OMEGAE
  126 FD=0.
      IQUIT=0
      TORQ=TORK
      TORK=0.
      DO 130 K=1,2
      VY(K)=.5*(E+C(K)*D)*(RPM(K)-OMEGAE)
```

```
130   FD=FD+.25*CLCAGE(K)*TOTL*DENLUB*(.25*(RINGDI(K)+CAGEDI(K))*OMEGAE 00027300
     1)**2*OMEGAE/ABS(OMEGAE)                                           00027400
      FC=XMASS*.5*F*OMEGAE**2                                           00027500
      IV=0                                                              00027600
      CALL NOCON                                                        00027700
      IF(IV)132,132,131                                                 00027800
 131  WRITE(6,8001)                                                     00027900
8001  FORMAT(1X,30H**HP EXCEEDED 1.0E2 IN NOCON**)                      00028000
      GO TO 115                                                         00028100
 132  DELZ=.005*D                                                       00028200
C     IF(IOUT)140,140,115                                               00028300
 140  DO 240 ITER=1,15                                                  00028400
      XFZ=0.                                                            00028500

      DXFZ=0.                                                           00028600
      NOLOAD=0                                                          00028700
      DO 230 J=1,NUM                                                    00028800
      JPASS=J                                                           00028900
      XJ=J                                                              00029000
      PHI=6.283185*(XJ-1.)/XN                                           00029100
      CPH=COS(PHI)                                                      00029200
      DELTA=DELZ*CPH-.5*PD                                              00029300
      DEL(1)=.55*DELTA                                                  00029400
      DEL(2)=DELTA-DEL(1)                                               00029500
      DO 200 IT=1,15                                                    00029600
      IF(DEL(2)-1.E-8)141,141,142                                       00029700
 141  NOLOAD=NOLOAD+1                                                   00029800
      P(2,J)=0.                                                         00029900
      GO TO 230                                                         00030000
 142  DO 190 K=1,2                                                      00030100
      KPASS=K                                                           00030200
 145  CALL RCLL                                                         00030300
      IF(IOUT)170,170,150                                               00030400
 150  WRITE(6,160)J,IT,ITER,K                                           00030500
 160  FORMAT(9H0MAIN 145,4I4)                                           00030600
C     GO TO 115                                                         00030700
 170  IF(PX(K))171,171,172                                              00030800
 171  P(2,J)=0.                                                         00030900
      NOLOAD=NOLOAD+1                                                   00031000
      GO TO 230                                                         00031100
 172  DO 180 L=1,NLMY                                                   00031200
      RR(L,K,J)=R(L)                                                    00031300
      PPP(L,K,J)=PPI(L)                                                 00031400
 180  SSTS(L,K,J)=STS(L)                                                00031500
      XTRM(K,J)=XTREM                                                   00031600
      P(K,J)=PX(K)                                                      00031700
 190  CONTINUE                                                          00031800
      PSI1=-P(1,J)+P(2,J)+FC                                            00031900
      X1=-DFDEL(1)-DFDEL(2)                                             00032000
      X1=PSI1/X1                                                        00032100
      DEL(1)=DEL(1)-X1                                                  00032200
      DEL(2)=DELTA-DEL(1)                                               00032300
      IF(ABS(X1)-TOL(1))220,200,200                                     00032400
 200  CONTINUE                                                          00032500
      WRITE(6,210)J                                                     00032600
 210  FORMAT(9H0MAIN 200,I4)                                            00032700
      GO TO 115                                                         00032800
 220  DET=-DFDEL(1)-DFDEL(2)                                            00032900
      DP2DEL=-DFDEL(1)/DET                                              00033000
      XFZ=XFZ+P(2,J)*CPH                                                00033100
      DXFZ=DXFZ+DP2DEL*DFDEL(2)*CPH**2                                  00033200
 230  CONTINUE                                                          00033300
      X1=(XFZ-FBARW)/DXFZ                                               00033400
      DELZ=DELZ-X1                                                      00033500
      IF(ABS(X1)-5.*TOL(1))255,240,240                                  00033600
```

```
  240   CONTINUE                                                        00033700
        WRITE(6,250)X1                                                  00033800
  250   FORMAT(9HOMAIN 240,1P1E12.4)                                    00033900
C       GO TO 115                                                       00034000
C 255   OMEGE=OMEGAF
  255   TORK=0.
        TORK=0.                                                         00034200
        MOR=0.                                                          00034300
        VERT=0.                                                         00034400
        DO 310 J=1,NUM                                                  00034500
        IF(P(2,J))310,310,2551                                          00034600
 2551   XJ=J                                                            00034700
        PHI=6.283185*(XJ-1.)/XN                                         00034800
        SPH=SIN(PHI)                                                    00034900
        CPH=COS(PHI)                                                    00035000
        OMEGAX=(RPM(J)-OMEGAF)*(E+D)/D                                  00035100
        JPASS=J                                                         00035200
        DO 261 ITA=1,30                                                 00035300
        CPI=CP*ABS(OMEGAX)                                              00035400
        C71=C7*OMEGAX                                                   00035500
        DO 261 K=1,2                                                    00035600
        HMRAC(K)=0.0                                                    00035700
        KPASS=K                                                         00035800
        DO 260 L=1,NUMY                                                 00035900
        RI(L)=RR(L,K,J)                                                 00036000
        PPI(L)=PPP(L,K,J)                                               00036100
  260   STS(L)=SSTS(L,K,J)                                              00036200
        CALL TRAC                                                       00036300
  261   CONTINUE                                                        00036400
        TEMP=FY(1)+FY(2)-FD                                             00036500
        IH=1                                                            00036600
        IF(TEMP.LT.0.0)IH=2
        HP(1)=HF                                                        00036700
        HP(2)=HF
        DO 262 ITB=1,30                                                 00036800
        CALL POCFLM(IH)                                                 00036900
        PSI1=FP(IH)-ABS(TEMP)
        DPSI1H=DFPDH(IH)                                                00037100
        TEMP1=PSI1/DPSI1H                                               00037200
        HP(IH)=HP(IH)-TEMP1                                             00037300
        IF(HP(1).GT.1.0E2)GO TO 9000                                    00037400
        IF(HP(2).GT.1.0E2)GO TO 9000                                    00037500
        GO TO 2650                                                      00037600
 9000   WRITE(6,7500)                                                   00037700
 7500     FORMAT(1X,21H**HP EXCEEDED 1.0E2**)                           00037800
        GO TO 115                                                       00037900
 2650     ABS(T.HP1).LE.TOL(2)GO TO 264                                 00038000
  262   CONTINUE                                                        00038100
        WRITE(6,263)J                                                   00038200
  263   FORMAT(1X,9HOMAIN 262,I3)                                       00038300
C       GO TO 115                                                       00038400
  264   PSI2=(FY(1)-FY(2)-FZ(IH))*.5*D                                  00038500
        DHDWX=-DFPWX(IH)/DFPDH(IH)                                      00038600
        DPSI2W=(DFYWX(1)-DFYWX(2)-DFZWX(IH)-DFZDH(IH)*DHDWX)*.5*D       00038700
        TEMP=PSI2/DPSI2W                                                00038800
        OMEGAX=OMEGAX-TEMP                                              00038900
        IF(ABS(TEMP).LE.TOL(3))GO TO 300                                00039000
  265   CONTINUE                                                        00039100
        WRITE(6,266)J                                                   00039200
  266   FORMAT(1X,9HOMAIN 265,I3                                        00039300
C       GO TO 115                                                       00039400
  300   IF(IH.EQ.1)HP(2)=POCCLR-HP(1)                                   00039500
        IF(IH.EQ.2)HP(1)=POCCLR-HP(2)                                   00039600
        DO 305 K=1,2                                                    00039700
        FY1(K,J)=FY(K)                                                  00039800
```

```
      FP1(K,J)=FP(K)
      FZ1(K,J)=FZ(K)
      VY1(K,J)=V(K)
      FLIM(K,J)=HMRAC(K)
      H1(K,J)=HP(K)
  305 CONTINUE
      OMEGX(J)=OMEGAX
      X1=-C(IH)*(FP(IH)*CPH+FZ(IH)*SPH)
      X2=C(IH)*(FP(IH)*SPH-FZ(IH)*CPH)
      HOR=HOR+X1
      VERT=VERT+X2
      TORK=TORK+C(IH)*(FP(IH)*E+FZ(IH)*POCKET)*.5
  310 CONTINUE
      VEL=(RPM(KPLT)-OMEGAE)*RINGDI(KPLT)*.5
      XMUP=XMU1*VEL/ABS(VEL)
      X1=-XMUP*VERT+HOR
      X2=XMUP*HOR+VERT
      ANG=ATAN2(X1,X2)
      CAGECF=CAGEWT*REV**2*CLCAGE(KPLT)/772.8
      PN=HOR/(XMUP*COS(ANG)+SIN(ANG))-CAGECF
      DRAG=0.
      DO 320 K=1,2
      AREA=3.141593*RINGDI(K)*(CAGEW-TOTL)
      VEL=(RPM(K)-OMEGAE)*CAGEDI(K)*.5
  320 DRAG=DRAG+.008*SQRT(SQRT(VIS*ABS(VEL)**7*DENLUB**3/(.5*CLCAGE(K)))
     1)*VEL/ABS(VEL)*AREA
      TORK=TORK+(XMUP*(PN+CAGECF)+DRAG)*CAGEDI(KPLT)*.5
 7900 FORMAT(1X, 9HMAIN ROOT,I4,1P5E13.5)
C     TOL(5)=.001
C     IF(ABS(TORK)-TOL(5))390,390,330
C 330 IF(TORK)331,331,333
C 331 OMEGAE=OMEGAE-OMEDCR
C     GO TO 126
C 335 OMEDCR=OMEDCR/5.0
C     TORK=TORQ
C     OMEGAE=OMEGE-OMEDCR
C     WRITE(6,7900)III,OMEGAE,OME1,OMEDCR,TORK,TORQ
C     GO TO 126
  390  WRITE(6,2000)
 2000  FORMAT(1H1)
       WRITE(6,400)ILOAD,ISYS
  400 FORMAT(27H   OUTPUT DATA FOR LOAD NO.,I3,12H BEARING NO.,I3)
      WRITE(6,410)
  410 FORMAT(60H0  BEARING       RPM OF      PERCENT       TORQUE     CAGE P
     1ILOT)
      WRITE(6,420)
  420 FORMAT(60H  REACTION       CAGE      CAGE SLIP    ON CAGE    NORMAL
     1LOAD)
      OMEGAE=OMEGAE/.1047198
      TEMP=(RPM(1)*(1.+GAM)+RPM(2)*(1.-GAM))/.2094396
      TEMP=(TEMP-OMEGAE)/TEMP*100.
      WRITE(6,430)XFZ,OMEGAE,TEMP,TORK,PN
  430 FORMAT(1P2E12.4,0P1F9.2,3X,1P2E12.4)
      WRITE(6,440)
  440 FORMAT(32H0  OUTPUT DATA FOR RACE CONTACTS)
      WRITE(6,450)
  450 FORMAT(115H0  ELEMENT      ELEMENT          CONTACT LOAD       MEAN C
     1OMPRESSIVE STRESS     PATTERN EXTREMITY          SLIP VELOCITY)
```

```
      WRITE(6,460)
460   FORMAT(117H    NUMBER      AZIMUTH       OUTER        INNER        OUT
     1ER        INNER        OUTER        INNER        OUTER        INNER)
      DO 490 J=1,NUM
      IF(P(2,J))490,490,470
470   XJ=J
      PHI=360.*(XJ-1.)/XN
      SSTS(9,2,J)=.7853982*SSTS(NUMY,2,J)
      SSTS(9,1,J)=.7853982*SSTS(NUMY,1,J)
      WRITE(6,480)J,PHI,P(1,J),P(2,J),SSTS(9,1,J),SSTS(9,2,J),XTRM(1,J),
     1XTRM(2,J),VY1(1,J),VY1(2,J)
480   FORMAT(I7,5X,1P9E12.4)
490   CONTINUE
      IF(NOLOAD)515,515,500
500   STR=.7853982*STR
      WRITE(6,510)PXOUT,STR,XTM,VSC
510   FORMAT(12H   THE REST ,12X,1P1E12.4,12X,1P1E12.4,12X,1P1E12.4,12X,
     11P1E12.4)
515   WRITE(6,520)
520   FORMAT(56H0  ELEMENT      MIN. FILM THICKNESS        TRACTIVE FORCE
     1)
      WRITE(6,530)
530   FORMAT(57H    NUMBER       OUTER        INNER        OUTER        INNE
     1R)
      DO 550 J=1,NUM
      IF(P(2,J))550,550,540
540   WRITE(6,480)J,FLIM(1,J),FLIM(2,J),FY1(1,J),FY1(2,J)
550   CONTINUE
      IF(NOLOAD)565,565,560
560   WRITE(6,640)FILM,YF
565   WRITE(6,570)
570   FORMAT(34H0  OUTPUT DATA FOR POCKET CONTACTS)
      WRITE(6,580)
580   FORMAT(94H0  ELEMENT       RPM OF       NORMAL POCKET LOAD    TANGEN
     1TIAL POCKET LOAD   MIN. FILM THICKNESS)
      WRITE(6,590)
590   FORMAT(92H    NUMBER       ROLLER        FORE         AFT          FOR
     1E         AFT          FORE         AFT)
      DO 610 J=1,NUM
      IF(P(2,J))610,610,600
 600  OMEGX(J)=OMEGX(J)/.2047198
      WRITE(6,480)J,OMEGX(J),FP1(1,J),FP1(2,J),FZ1(1,J),FZ1(2,J),H1(1,J)
     1,H1(2,J)
610   CONTINUE
      IF(NOLOAD)115,115,620
620   OMX1=OMX1/.1047198
      WRITE(6,630)OMX1,FP11,FP12,FZ11,FZ12,HMN1,HMN2
630   FORMAT(12H   THE REST ,1P8E12.4)
      GO TO 115
640   FORMAT(12H   THE REST ,1P1E12.4,12X,1P1E12.4)
 650  FORMAT(1H1)
      END
```

```
      SUBROUTINE RCALC(TOTL,FLAT,GAGE,RADIUS,DROP)                      00056300
      DOUBLE PRECISION DRP,RDIUS,FLT,TTL,GAG,A1,A2,DSQRT                00056400
      RDIUS=RADIUS                                                      00056500
      DRP=DROP
      FLT=FLAT                                                          00056800
      TTL=TOTL                                                          00056900
      GAG=GAGE                                                          00057000
      IF(RADIUS)10,10,20                                                00056600
   10 A1=(.5D0*TTL-GAG)**2
      A2=(.5D0*FLT)**2                                                  00057200
      RDIUS=DSQRT(((A1-A2-DRP**2)/(2.D0*DRP))**2+A1)                    00057300
      RADIUS=RDIUS                                                      00057400
      RETURN                                                            00057500
   20 A1=DSQRT(RDIUS**2-(.5D0*FLT)**2)                                  00057600
      A2=DSQRT(RDIUS**2-(.5D0*TTL-GAG)**2)                              00057700
      DRP=A1-A2                                                         00057800
      DROP=DRP                                                          00057900
      RETURN                                                            00058000
      END                                                               00058100
```

```
      SUBROUTINE RCLL                                                   00058200
      COMMON ALPHA,AL2,     BETA,     C(2),CAGEDI(2),CAGEW,CAGEWT,CLCAGE(200058300
     X),CORNER,CD,CZ,CP1,CZ1,        D,DROP,DENS,DENLUB,DELZ,DEL(2),    00058400
     XDEDEL(2),DEYWX(2),DERDH(2),DEZDH(2),DERWX(2),DEZWX(2),            00058500
     XC,EL,ED,      FLAT,FLATD2,FK,FBARV,FD,FC,FY(2),FP(2),FZ(2),FY11,  00058600
     XFY12,FP11,FP12,FZ11,FZ12,FILM,FFY(3),          GAM,GAGE,GLT,GZT,  00058700
     XHE,HHH,HP(2),HMN1,HMN2,HMN(2),HMRAC(2),     ISYS,ILOAD,ILUB,ISIGN, 00058800
     XIOUTT,       JPASS,      KPLT,KPASS,       NUM,NUMY,       OMEDCR, 00058900
     XOMEGAC,OMEI,OMEGAX,OMXI,OMEX,                PD,POCKET,PILOT,POCCLR, 00059000
     XPX(2),PPI(9),                     RADIUS,RINGDI(2),RINGYM,ROLLYM, 00059100
     XRINGPR,RCLLPR,RMH,REM(2),ROT,REV,R(9),            STR,STS(9)      00059200
      COMMON TOTL,TCL(4),TOTLD2,TI,THETA,TORC,TORK,      VHMIN,VIS,VEINC,00059300
     XVIS3,VY(2),VINC(2),VSS,V(2),        XMU1,XMASS,XTM,XTREM,XN,      00059400
     XYF,YNUM,YINC(2,50)                                                00059500
      COMMON CPH,CAGECF,        DXEZ,DELTA,DRAG,        FY1(2,50),FP1(2,50000059600
     X),FZ1(2,50),FLIM(2,50),     HDR,H1(2,50),     ITER,      NOLOAD,  00059700
     XOMEGX(50),  P(2,50),PPP(9,2,50),PN,PXOUT,     RR(9,2,50),         00059800
     XSSTS(9,2,50),SPH,        VERT,VY1(2,50),        XFZ,XTRM(2,50)    00059900
      J=JPASS                                                           00060000
      K=KPASS                                                           00060100
      DX(K)=0.                                                          00060200
      DEDEL(K)=0.                                                       00060300
      XTREM=SQRT(2.*RADIUS*DEL(K)+FLATD2**2)                            00060400
      IF(XTREM-TOTLD2)15,15,10                                          00060500
 10   XTREM=TOTLD2                                                      00060600
 15   EEL=     XTREM-FLATD2                                             00060700
      YINC=EEL/(YNUM-1.)                                                00060800
      EEL=XTREM                                                         
      YINC(K,J)=YINC                                                    00060900
      YH=XTREM+YINC                                                     00061000
      SMINC=1.                                                          00061100
      DO 80 L=1,NUMY                                                    00061200
      PPI(L)=0.                                                         00061300
      STS(L)=0.                                                         00061400
      YH=YH-YINC                                                        00061500
      SM=3.-SMINC                                                       00061600
      SMINC=-SMINC                                                      00061700
      IF(L-1)30,20,30                                                   00061800
 20   SM=1.                                                             00061900
      GO TO 40                                                          00062000
 30   IF(L-NUMY)40,20,40                                                00062100
 40   DELX=DEL(K)+.5*(FLATD2**2-YH**2)/RADIUS                           00062200
       IF((DELX-1.E-8).LE.0.0)GO TO 80                                  
   43 P(L)=.5*D-DEL(K)+DELX                                             00062300
      GMA=2.*P(L)/E                                                     00062400
      TEMP=5.E7*DELX**1.111111/EEL**.111111                             00062500
      DO 50 IIT=1,15                                                    00062600
      IF(TEMP)80,80,45                                                  00062700
 45   A1=SQRT(EL*TEMP*D*(1.+C(K)*GMA))                                  00062800
      A2=1.8854+ALOG(EEL/(2.*A1))                                       00062900
      A3=A2*EL*TEMP                                                     00063000
      A4=(A3-DELX)/(EL*(A2-.5))                                         00063100
      TEMP=TEMP-A4                                                      00063200
      IF(ABS(A3-DELX)-TOL(1))70,70,50                                   00063300
 50   CONTINUE                                                          00063400
      WRITE(6,60)                                                       00063500
 60   FORMAT(9HCRCLL 50)                                                00063600
C     IOUTT=1                                                           00063700
C     RETURN                                                            00063800
```

```
70    PX(K)=PX(K)+TEMP*SM                                              00063900
      DEDEL(K)=DEDEL(K)+SM/(EL*(A2-.5))                                00064000
      DPI(L)=TEMP                                                      00064100
      STS(L)=.6366198*TEMP/A1                                          00064200
80    CONTINUE                                                         00064300
      A3=XINC/3.                                                       00064400
      PX(K)=PX(K)*A3*2.+FLAT*TEMP                                      00064500
      DEDEL(K)=DEDEL(K)*A3*2.+FLAT/(EL*(A2-.5))                        00064600
      PXOUT=PX(1)                                                      00064700
      RETURN                                                           00064800
      END                                                              00064900


      SUBROUTINE PCCCN                                                 00065000
      COMMON ALPHA,A13,     BETA,     C(2),CAGED1(2),CAGEW,CAGEWT,CLCAGE(200065100
     X),CCEASW,CP,C2,CP1,C2,        D,DDOP,DENS,DENLUB,DEL/,DEL(2),   00065200
     XDEDEL(2),DEYWX(2),DEZOW(2),DEZOW(2),DEPWX(2),DEZWX(2),           00065300
     X2,EL,EP,     ELAT,ELAT02,EK,EBARV,ED,EC,EY(2),EP(2),EZ(2),EY11,  00065400
     XEY12,EP11,EP12,EZ11,EZ12,FILM,FEY(2),        CAM,CAGE,G1T,G2T,   00065500
     XHF,HHH,HP(2),HYN1,HHN2,HYN(2),HMP3C(2),     ISYS,ILOAD,ILUB,ISIGN, 00065600
     XIOUIT,       JPASS,      KPIT,KPASS,       NUM,NUMY,      OMEDCP, 00065700
     XOMEGAF,OMEI,OMEGAX,OMY,OMEX,              PD,POCKET,PILOT,POCCLR, 00065800
     XPX(2),PPI(9),                   PRONIS,RINGOT(2),RINGYM,POLLYM,  00065900
     XRINGOF,POLLPR,RWW,RPM(2),RDT,REV,R(9),          STR,STS(9)       00066000
      COMMON TCTL,T1(14),TOTL02,T1,TREF1,T030,TORK,      VHMIN,VIS,VEINC,00066100
     XVUE2,VY(2),VLJC(2),VSS,V(2),        XMU1,XMASS,XTM,XTREM,XN,      00066200
     XXF,XMIN,XINC(2,50)                                                00066300
      COMMON CCR,CACECC,          DXEZ,DELTZ,DRAG,        EY1(2,50),EP1(2,500066400
     X1,EZ1(2,50),EL1M(2,50),     HDP,H1(2,50),      ITER,      NOLOAD,  00066500
     XCHECKY(50),   P(2,50),CP2(9,2,50),PM,PXOUT,      PP(9,2,50),     00066600
     XSSTS(2,2,50),SPH,          VERT,VY1(2,50),        XEZ,XTRM(2,50)  00066700
      COMMON /CHECC/IV
      JPASS=1                                                          00066800
      KPASS=1                                                          00066900
      DEL(1)=.0330                                                     00067000
      HP(2)=HC                                                         00067200
      DEZ,DY=(EDX(1)-EXP(DE)*(E+D)*.5007/C
      CP=2.76/EVIS                                                     00067400
      C2=.5422*VIS                                                     00067500
      DO 10 IT=1,20                                                    00067600
      CALL PCCL                                                        00067700
      IF(IOUIT)12,2,1                                                  00067800
1     WRITE(6,3)                                                       00067900
3     FORMAT(11HCONVERGENCE)                                           00068000
2     TEMP=(PX(1)-E)/DPDEL(1)                                          00068100
      DEL(1)=DEL(1)-TEMP                                               00068200
      IF(ABS(TEMP)-10E(1))11,10,10                                     00068300
10    CONTINUE                                                         00068400
      WRITE(6,20)TEMP,DEL(1)                                           00068500
20    FORMAT(20HNOCONVERGENCE 10,2PE11.4)                              00068600
      IOUIT=1                                                          00068700
12    VINC(1)=0.                                                       00068800
      VINC(2)=VEINC                                                    00068900
      VINC(3)=-VEINC                                                   00069000
      X1=0.0
      X2=0.0
      HMN(1)=0.0
      HMN(2)=0.0
      DO 60 IT=1,40
      CP1=CDABS(EXPCAY)
      CP1=CP1-XEZ/X
      CALL TRAC
```

```
      TEMP=FY(1)-FC
      I4=2
      HP(2)=HF
      DO 43 ITA=1,40
      CALL DOCFLM(IH)
      PSI1=FP(IH)-ABS(TEMP)
      DPSI1H=DFPDH(IH)
      TEMP1=PSI1/DPSI1H
      HP(IH)=HP(IH)-TEMP1
      IF(ABS(TEMP1).LE.TOL(2))GO TO 45

   43 CONTINUE
   45 PSI2=(FY(1)-FZ(IH))*.5*D
      DHDWX=-DFDWX(IH)/DFDDH(IH)
      DPSI2W=(DFYWX(1)-DFZWX(IH)-DFZDH(IH)*DHDWX)*.5*D
C     TEMP=PSI2/DPSI2W
      TEMP=.5*PSI2/DPSI2W
      OMEGAX=OMEGAX-TEMP
      IF(ABS(TEMP).LE.TOL(3))GO TO 55
   50 CONTINUE
   55 HP(1)=DOCCLR-HP(2)
      FY12=0.                                                          00072900
      FP11=FP(1)                                                       00073000
      FP12=FP(2)                                                       00073100
      FZ11=FZ(1)                                                       00073200
      FZ12=FZ(2)                                                       00073300
      XTM=XTREM                                                        00073400
      STR=STS(NUMY)                                                    00073500
      VSS=V(1)                                                         00073600
      FILM=VHMIN                                                       00073700
      YF=FY(1)                                                         00073800
      HMN1=HMN(1)                                                      00073900
      HMN2=HMN(2)                                                      00074000
      HMN1=DOCCLR-HMN2
      OMX1=OMEGAX                                                      00074300
      RETURN
      END                                                              00074500

      SUBROUTINE DOCFLM(IH)                                            00074600
      COMMON ALPHA,ALF,     BETA,     C(2),CAGED(2),CAGEW,CAGEWT,CLCAGE(200074700
     X1,CDRLFD,CD,CZ,CD1,CZ1,          D,DFDP,DENS,DENLUB,DELZ,DEL(2),  00074800
     XDFDE(2),DFYWX(2),DFDDH(2),DFZDH(2),DFDWX(2),DFZWX(2),            00074900
     XE,EL,EO,      FLAT,FLAT02,FK,FRARV,FO,FC,FY(2),FP(2),FZ(2),FY11,  00075000
     XFY12,FP11,FP12,FZ11,FZ12,FILM,FFY(3),          GAM,GAGE,G1T,G2T,  00075100
     XHF,HPH,HP(2),HMN1,HMN2,HMN(2),HMPAC(2),     ISYS,ILOAD,ILUB,ISIGN, 00075200
     XIOUT,        IPASS,      KPLT,KPASS,       NUM,NUMY,       OMEDCR, 00075300
     XOMEGAF,OMEI,OMEGAX,OMY1,OMEX,            PD,POCKET,PILOT,POCCLR.  00075400
     XPY(2),PP1(9),                  RADIUS,RINGDI(2),RINGYM,ROLLYM,   00075500
     XRINGPS,RELLPD,RMH,RPM(2),RPT,REV,R(9),          STR,STS(9)        00075600
      COMMON TOTL,TOL(4),TOTLO2,TI,THETA,TORQ,TORK,      VHMIN,VIS,VFINC,00075700
     XV(53,VY(2),VINC(2),VSS,V(2),        XMUT,XMASS,XTM,XTREM,XN,      00075800
     XYF,YNUM,YINC(2,50)                                               00075900
      COMMON CPH,CAGECF,        DXFZ,DELTA,DRAG,        FY1(2,50),FP1(2,5000076000
     X1,FZ1(2,50),FLIM(2,50),     HCR,H1(2,50),     ITER,     NOLOAD,   00076100
     XOMEGX(50),   F(2,50),FPP(5,2,50),PN,PXOUT,      PP(9,2,50),       00076200
     XSSTS(9,2,50),SPH,        VERT,VY1(2,50),        XFZ,XTRM(2,50)    00076300
      FFL=TOTLO2-FLATO2                                                00076400
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```
      ZINC=FFI/(YNUM-1.)                                                00076500
      TEMP1=ZINC/3.                                                     00076600
      DO 5 K=1,2                                                        00076700
      FP(K)=0.                                                          00076800
      FZ(K)=0.                                                          00076900
      DFPDH(K)=0.                                                       00077000
       DFPWX(K)=0.0
       DFZWX(K)=0.0
5     DFZDH(K)=0.                                                       00077100
      K=IH                                                              00077200
      SMINC=1.                                                          00077300
      XH=TOTL02+ZINC                                                    00077400
      DO 60 I=1,NUMY                                                    00077500
      SM=3.-SMINC                                                       00077600
      IF(I-1)20,10,20                                                   00077700
10    SM=1.                                                             00077800
      GO TO 30                                                          00077900
20    IF(I-NUMY)30,10,30                                                00078000
30    SMINC=-SMINC                                                      00078100
      XH=XH-ZINC                                                        00078200
      A1=RMH-SQRT(RADIUS**2-XH**2)                                      00078300
       A1=.5*(XH**2-FLAT02**2)/RADIUS
      HPX=HP(K)+A1                                                      00078400
       IF(HPX - 1.E-6)40,40,31
31    A2=.5*A1                                                          00078500
      CCP1=CCP1*A2**2                                                   00078600
      CCZ1=CCZ1*A2*SQRT(A2)                                             00078700
      FP(K)=FP(K)+SM*CCP1/HPX                                           00078800
      TEMP=SQRT(HPX)                                                    00078900
      FZ(K)=FZ(K)+SM*CCZ1/TEMP                                          00079000
      DFPDH(K)=DFPDH(K)-SM*CCP1/HPX**2                                  00079100
      DFZDH(K)=DFZDH(K)-.5*SM*CCZ1/(HPX*TEMP)                           00079200
40    CONTINUE
      HMN(K)=HPX                                                        00079300
      FP(K)=2.*FP(K)*TEMP1+CCP1*FLAT/HPX                                00079400
      FZ(K)=2.*FZ(K)*TEMP1+CCZ1*FLAT/TEMP                               00079500
      DFPDH(K)=2.*DFPDH(K)*TEMP1-CCP1*FLAT/HPX**2                       00079600
      DFZDH(K)=2.*DFZDH(K)*TEMP1-CCZ1*FLAT/(HPX*TEMP)*.5                00079700
      DFPWX(K)=FP(K)/ABS(OMEGAX)                                        00079800
50    DFZWX(K)=FZ(K)/OMEGAX                                             00079900

      RETURN                                                            00080000
      END                                                               00080100

      SUBROUTINE TRPC                                                   00080200
      COMMON ALPHA,FL2,     BETA,     C(2),CAGEDI(2),CAGEW,CAGEWT,CLCAGE(200080300
     X1,CORNER,CP,CZ,CP1,CZ1,        D,DDOP,DENS,DENLOB,DELZ,DEL(2),    00080400
     XDFPDH(2),DFPWX(2),DFPDH(2),DFZDH(2),DFZWX(2),DFZWX(2),            00080500
     XF,F1,F2,     FLAT,FLAT02,FK,FHARV,FD,FF,FY(2),FP(2),FZ(2),FY11,   00080600
     XFY12,FP11,FP12,FZ11,FZ12,FIIH,FFY(3),        GAM,GAGE,GIT,G2T,    00080700
     XHF,HPR,HO(2),HMA1,HMN2,HMN(2),HMPAC(2),     ISYS,ILOAD,ILOB,ISIGN, 00080800
     XIOUT,     JPASS,     KPIT,KPASS,     NUM,NUMY,     OMEDER,         00080900
     XOMEGAF,OMEI,OMEGAX,OMX,OMEX,          PD,POCKET,PILOT,POCCLR,     00081000
     XPX(2),PPI(9),                  RADIUS,RINGOI(2),RINGYM,ROLLYM,    00081100
     XRINGPR,RPILOP,RMH,REM(2),RCT,REV,R(9),          STR,STS(9)        00081200
      COMMON TOTL,TOTL(4),TOTLO2,T1,THETA,TORD,TORK,     VHMIN,VIS,VFINC,00081300
     XVISS,VY(2),VINC(2),VCS,V(2),       XMU1,XMASS,XTM,XTREM,XN,       00081400
     XYF,YNUM,YINC(2,50)                                                00081500
      COMMON CPM,CAGEDF,        DXFZ,DELTA,DRAG,     FY1(2,50),FP1(2,50)00081600
     X1,FZ1(2,50),FLIM(2,50),     HOR,H1(2,50),     TIER,     NOLOAD,   00081700
     XOMEGX(50),   P(2,50),PPP(9,2,50),PN,PXOUT,     PR(9,2,50),        00081800
     XSSTS(9,2,50),SPH,        VERY,VY1(2,50),        XFZ,XTRM(2,50)    00081900
      I=JPASS                                                           00082000
      K=KPASS                                                           00082100
      PP=-P(K)*9.E-6(C*C(K)*PD)                                         00082200
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      DEN=2.*VFINC*ABS(OMEGAX)                                          00082300
      DO 80 I=1,3                                                       00082400
      OMEX=OMEGAX+VINC(I)*ABS(OMEGAX)                                   00082500
      SMINC=1.                                                          00082600
      FFY(I)=0.                                                         00082700
      DO 70 L=1,NUMY                                                    00082800
      SM=3.-SMINC                                                       00082900
      RXP=1./(1./R2+1./R(L))                                            00083000
      IF(L-1)20,10,20                                                   00083100
   10 SM=1.                                                             00083200
      GO TO 30                                                          00083300
   20 IF(L-NUMY)30,10,30                                                00083400
   30 SMINC=-SMINC                                                      00083500
      IF(RPI(L)-.01)70,70,40
   40 VROL=OMEX*R(L)*C(K)                                               00083600
      VS=VY(K)-VROL                                                     00083700
      IF(VS)51,50,51
   50 WRITE(6,53)VY(K),VROL,VS,OMEX,R(L),C(K)
   53 FORMAT(1X,4HTRAC,1PE15.4)
   51 VRAR=(VY(K)+VROL)*.5                                              00083800
      URAR=ABS(VRAR)                                                    00083900
      US=ABS(VS)                                                        00084000
      VHMIN=HHH*(URAR**.7/RPI(L)**.13*RXP**.43                          00084100
      G1=G1T*US/(VHMIN*STS(L))                                          00084200
      G2=G2T*VS**2                                                      00084300
      G3=AL3*STS(L)                                                     00084400
C     CALL COEFCT(G1,G2,G3,COE,THETA)                                   00084500
C     CALL COEFCT(G1,G2,G3,COE,THETA)                                   00084500
C     COE=5.)*SQRT(G1)
      COE=7.5*SQRT(G1)                                                  00084600
      FFY(I)=FFY(I)+ SM*RPI(L)*COE                                      00084700
   70 CONTINUE                                                          00084800
      FFY(I)=2.*(FFY(I)+YINC(K,I)/3.+RPI(NUMY)*FLAT02*COE)*US/VS        00084900
   80 CONTINUE                                                          00085000
      FY(K)=FFY(1)                                                      00085100
      DFYDY(K) =(FFY(2)-FFY(3))/DEN                                     00085200
      HMRAC(K)=VHMIN                                                    00085300
      V(K)=VS                                                           00085400

      RETURN                                                            00085500
      END                                                               00085600

      SUBROUTINE TLUA(A,B,C,D,N)
      DIMENSION C(1),D(1)
      IF(N-1)11,2,3
    1 B=0.
      GO TO 100
    2 B=D(1)
      GO TO 100
    3 ML=1
      MU=N
    8 IF(MU-ML-1) 15,15,9
    9 M=(MU+ML)/2
      IF(C(1)-C(2))1,2,10
   10 IF(C(M)-A)13,12,14
   11 IF(A-C(M))13,12,14
   12 B=D(M)
      GO TO 100
   13 MU=M
      GO TO 8
   14 ML=M
      GO TO 8
   15 B=D(ML)+(D(MU)-D(ML))*(A-C(ML))/(C(MU)-C(ML))
  100 RETURN
      END
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      SUBROUTINE COEFCT(G1,G2,G3,ECOE,THETA)
      DIMENSION AG1(13),AG2(7),AG3(7),FTN(13,7,7),UZ(7,7),VZ(7),VTN(637)FRCT   2
      DIMENSION A1(13),A2(13),A3(13),A4(13),A5(13),A6(13),A7(13),A8(13),FRCT   3
     1A9(13),A10(13),A11(13),A12(13),A13(13),A14(13),A15(13),A16(13),   FRCT   4
     2A17(13),A18(13),A19(13),A20(13),A21(13),A22(13),A23(13),A24(13),  FRCT   5
     3A25(13),A26(13),A27(13),A28(13),A29(13),A30(13),A31(13),A32(13),  FRCT   6
     4A33(13),A34(13),A35(13),A36(13),A37(13),A38(13),A39(13),A40(13),  FRCT   7
     5A41(13),A42(13),A43(13),A44(13),A45(13),A46(13),A47(13),A48(13),  FRCT   8
     6A49(13)                                                           FRCT   9
      EQUIVALENCE(VTN(1),FTN(1,1,1))
      EQUIVALENCE(VTN(1),A1(1)),(VTN(14),A2(1)),(VTN(27),A3(1)),
     1(VTN(40),A4(1)),(VTN(53),A5(1)),(VTN(66),A6(1)),(VTN(79),A7(1)),  FRCT  11
     2(VTN(92),A8(1)),(VTN(105),A9(1)),(VTN(118),A10(1)),(VTN(131),A11(1FRCT
     3)),(VTN(144),A12(1)),(VTN(157),A13(1)),(VTN(170),A14(1)),(VTN(183)FRCT
     4,A15(1)),(VTN(196),A16(1)),(VTN(209),A17(1)),(VTN(222),A18(1)),   FRCT
     5(VTN(235),A19(1)),(VTN(248),A20(1)),(VTN(261),A21(1)),(VTN(274),  FRCT
     6A22(1)),(VTN(287),A23(1)),(VTN(300),A24(1)),(VTN(313),A25(1)),    FRCT
     7(VTN(326),A26(1)),(VTN(339),A27(1)),(VTN(352),A28(1)),(VTN(365),  FRCT
     8A29(1)),(VTN(378),A30(1)),(VTN(391),A31(1)),(VTN(404),A32(1)),    FRCT
     9(VTN(417),A33(1)),(VTN(430),A34(1)),(VTN(443),A35(1)),(VTN(456),  FRCT
     1A36(1)),(VTN(469),A37(1)),(VTN(482),A38(1)),(VTN(495),A39(1)),    FRCT
     2(VTN(508),A40(1)),(VTN(521),A41(1)),(VTN(534),A42(1)),(VTN(547),  FRCT
     3A43(1)),(VTN(560),A44(1)),(VTN(573),A45(1)),(VTN(586),A46(1)),(VTNFRCT
     4(599),A47(1)),(VTN(612),A48(1)),(VTN(625),A49(1))                 FRCT
      DATA A1          /0.00075,0.0007,  0.0014, 0.0023, 0.0067,        FRCT  25
     1 0.0135,  0.021, 0.037, 0.046, 0.057, 0.0615, 0.0685, 0.0775/     FRCT  26
      DATA A2                 /0.0007, 0.002, 0.0039, 0.0065, 0.0150,   FRCT  27
     1 0.022, 0.029, 0.042, 0.0495, 0.054, 0.0625, 0.069, 0.0775/       FRCT  28
      DATA A3                 /       0.0026, 0.0078, 0.014, 0.018, 0.028,FRCT  29
     1 0.034, 0.0395, 0.0485, 0.054, 0.058, 0.064, 0.0695, 0.0775/      FRCT  30
      DATA A4                 / 0.0077, 0.017, 0.023, 0.027, 0.035,     FRCT  31
     1 0.04, 0.044, 0.051, 0.0565, 0.0605, 0.066, 0.071, 0.0775/        FRCT  32
      DATA A5                 /0.019, 0.0285, 0.034, 0.037, 0.044, 0.048,FRCT  33
     1 0.05 , 0.056, 0.06, 0.0635, 0.069, 0.0725, 0.0775/               FRCT  34
      DATA A6                 / 0.0215, 0.032, 0.037, 0.0415, 0.0475,FRCT  35
     1 0.051, 0.054, 0.06, 0.0635, 0.066, 0.07, 0.073, 0.0775/          FRCT  36
      DATA A7                 / 0.0245,0.034, 0.04, 0.0435, 0.05,       FRCT  37
     1 0.054, 0.0565, 0.062, 0.0655, 0.0675, 0.071, 0.074, 0.0775/      FRCT  38
      DATA A8                 /0.0002,0.0005, 0.0012, 0.002, 0.006, 0.012 ,FRCT  39
     1 0.019, 0.036, 0.047, 0.054, 0.0645, 0.072, 0.077/                FRCT  40
      DATA A9                 /0.0006,0.002, 0.004, 0.0065, 0.015, 0.024,FRCT  41
     1 0.028, 0.043, 0.052, 0.0575, 0.0665, 0.072, 0.077/               FRCT  42
      DATA A10                /  0.0027, 0.0081, 0.014, 0.019, 0.0295,  FRCT  43
     1 0.035, 0.0395, 0.051, 0.057, 0.061, 0.069, 0.0725, 0.077/        FRCT  44
      DATA A11                /0.0066, 0.015, 0.021, 0.025, 0.034, 0.04 ,FRCT  45
     1 0.0445, 0.054, 0.06, 0.064, 0.069, 0.073, 0.077/                 FRCT  46
      DATA A12                /   0.012, 0.0225, 0.029, 0.0345, 0.043,  FRCT  47
     1 0.049, 0.052, 0.059, 0.063, 0.066, 0.070, 0.073, 0.077/          FRCT  48
      DATA A13                /   0.010, 0.022, 0.039, 0.0415, 0.05,0.054,FRCT  49
     1 0.056, 0.061, 0.064, 0.067, 0.0715, 0.075, 0.077/                FRCT  50
      DATA A14                /   0.021, 0.0335, 0.041, 0.0455, 0.053,  FRCT  51
     1 0.056, 0.059, 0.064, 0.068, 0.07, 0.073, 0.076, 0.077/           FRCT  52
      DATA A15                /.00016,.00055,0.0011,0.0019, 0.0059,0.011,FRCT  53
     1 0.018, 0.023, 0.042, 0.048, 0.06, 0.068, 0.075/                  FRCT  54
      DATA A16                /.0007,.0021, 0.0043, 0.007 , 0.015, 0.023,FRCT  55
     1 0.027, 0.039, 0.046, 0.052, 0.062, 0.07, 0.075/                  FRCT  56
      DATA A17                /   0.0018, 0.0053, 0.01, 0.015, 0.0255,  FRCT  57
     1 0.034, 0.037, 0.048, 0.054, 0.058, 0.066, 0.073, 0.075/          FRCT  58
      DATA A18                /   0.0046, 0.011, 0.018, 0.024, 0.037,   FRCT  59
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U 0.039, 0.044, 0.0525, 0.057, 0.061, 0.068, 0.074, 0.075/          FRCT  60
DATA A19                  /   0.0082, 0.017, 0.025, 0.03, 0.042,    FRCT  61
1 0.048, 0.052, 0.06, 0.064, 0.066, 0.071, 0.074, 0.075/            FRCT  62
DATA A20                  / 0.015, 0.024, 0.034, 0.04, 0.049,       FRCT  63
1 0.054,0.058, 0.063, 0.067, 0.069, 0.073, 0.075, 0.075/            FRCT  64
DATA A21                  /   0.015, 0.028, 0.036, 0.043, 0.052,    FRCT  65
1 0.057, 0.06, 0.065, 0.068, 0.07, 0.074, 0.075, 0.075/             FRCT  66
DATA A22                  /.00015,.0006,.001, 0.0017, 0.005, 0.01,  FRCT  67
1 0.016, 0.034, 0.04, 0.046, 0.059, 0.067, 0.073/                   FRCT  68
DATA A23                  /0.00064,0.002, 0.0139, 0.0064, 0.015,    FRCT  69
1 0.021, 0.026, 0.039, 0.046, 0.052, 0.062, 0.068, 0.073/           FRCT  70
DATA A24                  /   0.0017, 0.0048, 0.009, 0.014, 0.025,  FRCT  71
1 0.031, 0.036, 0.047, 0.053, 0.057, 0.065, 0.0685, 0.073/          FRCT  72
DATA A25                  /   0.043, 0.01, 0.017, 0.023, 0.033,     FRCT  73
1 .039, 0.043, 0.052, 0.058, 0.061, 0.066, 0.07, 0.073/             FRCT  74
DATA A26                  / 0.0078, 0.016, 0.024, 0.029, 0.04,      FRCT  75
1 0.044, 0.051, 0.057, 0.062, 0.064, 0.068, 0.071, 0.073/           FRCT  76
DATA A27                  /   0.0145, 0.024, 0.033, 0.039, 0.049,   FRCT  77
1 0.053, 0.056, 0.062, 0.066, 0.067, 0.07, 0.072, 0.073/            FRCT  78
DATA A28                  / 0.016, 0.027, 0.0355, 0.041, 0.052,     FRCT  79
1 0.056, 0.059, 0.064, 0.067, 0.068, 0.071, 0.073, 0.073/           FRCT  80
DATA A29                  /.0001,.0027,.0055,0.0009,0.0029,         FRCT  81
1 0.0059, 0.01, 0.025, 0.036, 0.043, 0.054, 0.062, 0.068/           FRCT  82
DATA A30                  /.00044,.0013,0.0026, 0.0044, 0.012,      FRCT  83
1 0.019, 0.024, 0.036, 0.044, 0.049, 0.057, 0.063, 0.068/           FRCT  84
DATA A31                  /0.001, 0.0029, 0.0255, 0.019, 0.0185,    FRCT  85
1 0.0255, 0.03, 0.042, 0.0495, 0.053, 0.06, 0.064, 0.068/           FRCT  86
DATA A32                  /0.0015, 0.0045, 0.0085, 0.013, 0.024,    FRCT  87
1 0.032, 0.036, 0.047, 0.053, 0.057, 0.063, 0.065, 0.068/           FRCT  88
DATA A33                  /0.0036, 0.0093, 0.0155, 0.021, 0.032,    FRCT  89
1 0.038, 0.042, 0.052, 0.056, 0.06, 0.064, 0.066, 0.068/            FRCT  90
DATA A34                  /0.0098, 0.017, 0.0245, 0.03, 0.04,       FRCT  91
1 0.0445, 0.051, 0.058, 0.061,0.063, 0.066, 0.067, 0.068/           FRCT  92
DATA A35                  /0.012, 0.022, 0.0295, 0.035, 0.0455,     FRCT  93
1 0.051, 0.055, 0.061, 0.063, 0.065, 0.067, 0.068, 0.068/           FRCT  94
DATA A36                  /.00005,.00014,.00027,.00065,.0014,       FRCT  95
1 0.0027, 0.0044, 0.013, 0.023, 0.029, 0.042, 0.052, 0.055/         FRCT  96
DATA A37                  /.0012,.0042,.0085,.0014,.0044,           FRCT  97
1 0.0084, 0.013, 0.0245, 0.032, 0.037, 0.0455, 0.053, 0.055/        FRCT  98
DATA A38                  /.0014,.0012, 0.0074, 0.004, 0.011,       FRCT  99
1 0.017, 0.023, 0.0345, 0.039, 0.0425, 0.049, 0.0535, 0.055/        FRCT 100
DATA A39                  /0.0015, 0.0042, 0.008, 0.012, 0.023,     FRCT 101
1 0.025, 0.032, 0.0395, 0.043, 0.045, 0.050, 0.0535, 0.055/         FRCT 102
DATA A40                  /0.0034, 0.0084, 0.014, 0.0195, 0.03,     FRCT 103
1 0.024, 0.04, 0.046, 0.048, 0.049, 0.051, 0.054, 0.055/            FRCT 104
DATA A41                  /0.0056, 0.013, 0.02, 0.026, 0.0375,      FRCT 105
1 0.042, 0.045, 0.049, 0.05, 0.052, 0.0525, 0.054, 0.055/           FRCT 106
DATA A42                  /0.0071, 0.015, 0.023, 0.03, 0.04,        FRCT 107
1 0.045, 0.048, 0.051, 0.052, 0.053, 0.054, 0.055, 0.055/           FRCT 108
DATA A43                  /.00003,.00009,.00013,.0003,.0009,        FRCT 109
1 0.0019, 0.003, 0.009, 0.016, 0.023, 0.037, 0.045, 0.051/          FRCT 110
DATA A44                  /.0001,.0026,.0058,.0009,.0029,           FRCT 111
1 0.0057, 0.0092, 0.021, 0.029, 0.034, 0.041, 0.047, 0.051/         FRCT 112
DATA A45                  /.00025,.0074,0.0015, 0.0024, 0.0072,     FRCT 113
1 0.012, 0.018, 0.028, 0.035, 0.039, 0.045, 0.048, 0.051/           FRCT 114
DATA A46                  /0.0054, 0.0116, 0.0033, 0.0052, 0.014,   FRCT 115
1 0.022, 0.0265, 0.035, 0.04, 0.042, 0.0465, 0.049, 0.051/          FRCT 116
DATA A47                  /0.0013, 0.0036, 0.0069, 0.011, 0.023,    FRCT 117
1 0.029, 0.0325, 0.0335, 0.044, 0.046, 0.0495, 0.050, 0.051/        FRCT 118
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      DATA A48              /0.0015, 0.041, 0.008, 0.012, 0.0235,        FRCT 119
     1 0.0295, 0.033, 0.04, 0.044, 0.0465, 0.049, 0.0505, 0.051/          FRCT 120
      DATA A49              /0.0018, 0.0072, 0.0094, 0.014, 0.0255,      FRCT 121
     1 0.031, 0.035, 0.042, 0.045, 0.047, 0.05, 0.051, 0.051/             FRCT 122
      DATA AG1              /1.E-8, 3.E-8, 6.E-8, 1.E-7, 3.E-7, 6.E-7,   FRCT 123
     1 1.E-6, 3.E-6, 6.E-6, 1.E-5, 3.E-5,          1.E-4, 1.E-3/          FRCT 124
      DATA AG2              /5.E-7, 1.E-6, 5.E-6, 1.E-5, 5.E-5, 5.E-4, 1.E-3FRCT 125
     1 /                                                                  FRCT 126
      DATA AG3              /13.32, 15.2, 17.8, 19., 21.19, 27.13, 100./ FRCT 127
      IF(G1.EQ.0.) GO TO 240
      IF(G2.EQ.0.) GO TO 240
      DO 100 I=1,7
      DO 100 J=1,7
      IF(G1-AG1(13)) 40,60,60
   40 IF(G1.LT.AG1(1)) GO TO 70
      CALL TLU(G1,UZ(I,J),AG1,ETN(1,I,J),13)
      GO TO 100
   60 UZ(I,J)=ETN(13,I,J)
      GO TO 100
   70 UZ(I,J)=EXP(ALOG(ETN(1,I,J))+(ALOG(ETN(2,I,J))-ALOG(ETN(1,I,J)))/(
     1ALOG(AG1(2))-ALOG(AG1(1)))*(ALOG(G1)-ALOG(AG1(1))))
  100 CONTINUE
      DO 200 K=1,7
      IF(G3-AG3(7)) 140,160,160
  140 IF(G3.LT.AG3(1)) GO TO 170
      CALL TLU(G3,VZ(K),AG3,UZ(1,K),7)
      GO TO 200
  160 VZ(K)=UZ(7,K)
      GO TO 200
  170 VZ(K)=UZ(1,K)-(UZ(2,K)-UZ(1,K))/(AG3(2)-AG3(1))*(AG3(1)-G3)
      IF (VZ(K).LE.0.0) VZ(K)=1.E-5
  200 CONTINUE
      IF(G2-AG2(7)) 204,206,206
  204 IF(G2.LT.AG2(1)) GO TO 207
      CALL TLU(G2,FR3,AG2,VZ,7)
      GO TO 210
  206 FR3=EXP(ALOG(VZ(6))+(ALOG(VZ(7))-ALOG(VZ(6)))/(ALOG(AG2(7))-ALOG(A
     1G2(6)))*(ALOG(G2)-ALOG(AG2(6))))
      GO TO 210
  207 FR3=EXP(ALOG(VZ(2))+(ALOG(VZ(1))-ALOG(VZ(2)))/(ALOG(AG2(1))-ALOG(A
     1G2(2)))*(ALOG(G2)-ALOG(AG2(2))))
  210 FCOE=ALOG(FR3)-0.149*(ALOG(THETA)-ALOG(30.))
      FCOE=EXP(FCOE)
      GO TO 250
  240 FCOE=0.
  250 CONTINUE
      RETURN
      END

      SUBROUTINE TLU(A,B,C,D,N)                                         TLU00010
      DIMENSION C(1),D(1)                                               TLU00020
      IF(A-1)1,2,3                                                      TLU00030
    1 B=0.                                                              TLU00040
      GO TO 100                                                         TLU00050
    2 B=D(1)                                                            TLU00060
      GO TO 100                                                         TLU00070
    3 ML=1                                                              TLU00080
      MU=N                                                              TLU00090
    8 IF(MU-ML-1) 15,15,9                                               TLU00100
    9 M=(MU+ML)/2                                                       TLU00110
      IF(C(1)-C(2))1,2,10                                               TLU00120
   10 IF(C(M)-A)12,12,14                                                TLU00130
```

```
 11 IF(A-C(M))13,12,14                                                  TLU00140
 12 B=D(M)                                                              TLU00150
    GO TO 100                                                           TLU00160
 13 MU=M                                                                TLU00170
    GO TO 8                                                             TLU00180
 14 ML=M                                                                TLU00190
    GO TO 8                                                             TLU00200
 15 B=D(ML)+(D(MU)-D(ML))*((A-C(ML))/(C(MU)-C(ML)))                     TLU00210
100 RETURN                                                              TLU00220
    END                                                                 TLU00230

    SUBROUTINE TLUG(A,B,C,D,N)                                          TLUG0010
    DIMENSION C(1),D(1)                                                 TLUG0020
    IF(N-1)1,2,3                                                        TLUG0030
  1 B=0.                                                                TLUG0040
    GO TO 100                                                           TLUG0050
  2 B=D(1)                                                              TLUG0060
    GO TO 100                                                           TLUG0070
  3 ML=1                                                                TLUG0080
    MU=N                                                                TLUG0090
  8 IF(MU-ML-1) 15,15,9                                                 TLUG0100
  9 M=(MU+ML)/2                                                         TLUG0110
    IF(C(1)-C(2))11,2,10                                                TLUG0120
 10 IF(C(M)-A)13,12,14                                                  TLUG0130
 11 IF(A-C(M))13,12,14                                                  TLUG0140
 12 B=D(M)                                                              TLUG0150
    GO TO 100                                                           TLUG0160
 13 MU=M                                                                TLUG0170
    GO TO 8                                                             TLUG0180
 14 ML=M                                                                TLUG0190
    GO TO 8                                                             TLUG0200
 15 B=EXP(ALOG(D(ML))+(ALOG(D(MU))-ALOG(D(ML)))*((ALOG(A)-ALOG(C(ML)))TLUG0210
   1 /(ALOG(C(MU))-ALOG(C(ML))))                                        TLUG0220
100 RETURN                                                              TLUG0230
    END                                                                 TLUG0240
```

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APPENDIX V
FORMAT FOR ROLLER BEARING
COMPUTER PROGRAM INPUT INFORMATION

```
MAIN PROGRAM                                                          00000100
CARD    COL.          ITEM                                            00000200
 1      1-10    NO. OF ROLLS (50 MAXIMUM)                             00000300
        11-20   ROLL DIA. - IN.                                       00000400
        21-30   PITCH DIA. - IN.                                      00000500
        31-40   TOTAL LENGTH OF ROLL - IN.                            00000600
        41-50   LENGTH OF FLAT PORTION OF ROLL - IN.                  00000700
        51-60   ROLL CORNER RADIUS - IN.                              00000800
        61-70   ROLL CROWN RADIUS - IN. DO NOT READ IF CROWN DROP IS00000900
                GIVEN                                                 00001000
        71-80   ROLL CROWN DROP - IN. DO NOT READ IF CROWN RADIUS IS00001100
                                                                      00001200
 2      1-10    DISTANCE FROM ROLL END TO POINT WHERE CROWN DROP IS 00001300
                MEASURED - IN.                                        00001400
        11-20   DIAMETRAL CLEARANCE - IN. + IF LOOSE.                 00001500
        21-30   I.D. OF OUTER RING - IN.                              00001600
        31-40   O.D. OF INNER RING - IN.                              00001700
        41-50   O.D. OF CAGE - IN.                                    00001800
        51-60   I.D. OF CAGE - IN.                                    00001900
        61-70   LENGTH OF CAGE POCKET IN THE ROLLING DIRECTION - IN.00002000
        71-80   WIDTH OF CAGE - IN.                                   00002100
 3      1-10    MODULUS OF ELASTICITY FOR RINGS - PSI. IF BLANK       00002200
                PROGRAM ASSUMES 29,000,000.                           00002300
        11-20   SAME FOR ROLLS.                                       00002400
        21-30   POISSON'S RATIO FOR RINGS. IF BLANK PROGRAM ASSUMES 00002500
                .25                                                   00002600
        31-40   SAME FOR ROLLS.                                       00002700
        41-50   COEFFICIENT OF SLIDING FRICTION FOR CAGE/PILOT        00002800
                CONTACT.                                              00002900
        51-60   WEIGHT OF CAGE - LB.                                  00003000
        61-70   ROLL DENSITY - LB/IN**3. IF BLANK PROGRAM ASSUMES     00003100
                .283                                                  00003200
        71-80   TOLERANCE ON ELASTIC APPROACH OF ROLL TO RACE - IN. 00003300
                IF BLANK PROGRAM ASSUMES .000,000,1                   00003400
 4      1-10    TOLERANCE ON FILM THICKNESS AT ROLL/POCKET CONTACT    00003500
                - IN. IF BLANK PROGRAM ASSUMES.000,000,5 IN.          00003600
        11-20   TOLERANCE ON ANGULAR VELOCITY OF ROLL ABOUT OWN       00003700
                CENTER - RPM. IF BLANK PROGRAM ASSUMES 1.             00003800
        21-30   TOLERANCE ON ANGULAR VELOCITY OF CAGE - RPM.          00003900
                IF(BLANK PROGRAM ASSUMES .1                           00004000
        31-80   LEAVE BLANK.                                          00004100
        1       PUNCH 1 (NO DECIMAL POINT)                            00004200
        2-80    TITLE CARD. WRITE ANYTHING.                           00004300
 6      1       LEAVE BLANK.                                          00004400
        2-80    WRITE ANYTHING.                                       00004500
 7      1-10    PRESSURE-VISCOSITY COEFFICIENT OF LUBRICANT.          00004600
                IN**2/LB.                                             00004700
        11-20   TEMPERATURE-VISCOSITY COEFFICIENT OF LUBRICANT -      00004800
                1/(DEG F)                                             00004900
        21-30   VISCOSITY OF LUBRICANT AT INLET TEMPERATURE -         00005000
                LB*SEC/IN**2.                                         00005100
        31-40   INLET TEMPERATURE OF LUBRICANT - DEG F.               00005200
        41-50   THERMAL CONDUCTIVITY OF LUBRICANT -                   00005300
                BTU/(DEG F)/HR/FT.                                    00005400
        51-60   LUBRICANT DENSITY - LB*SEC**2/IN**4.                  00005500
        61-80   LEAVE BLANK.                                          00005600
 8      1       PUNCH ZERO                                            00005700
        2-80    IDENTIFICATION DATA FOR LUBRICANT. PUNCH ANYTHING.    00005800
```

```
9     1-10   RPM OF OUTER RING.                                        00005900
      11-20  RPM OF INNER RING.                                        00006000
      21-30  RADIAL LOAD - LB. (POSITIVE SIGN)                         00006100
      31-40  RPM OF ROTATING LOAD. IF LOAD IS STATIONARY LEAVE         00006200
             BLANK.                                                    00006300
      41-50  FACTOR WHICH MULTIPLIES CAGE SPEED TO GET CAGE SPEED00006400
             DECREMENT. IF BLANK PROGRAM ASSUMES .1                    00006500
      51-60  PROPORTIONALITY FACTOR FOR FINITE DIFFERENCE IN THE 00006600
             ANGULAR VELOCITY OF A ROLL ABOUT ITS OWN CENTER.          00006700
             IF BLANK PROGRAM ASSUMES .05                              00006800
      61-70  STARTING VALUE FOR MINIMUM FILM THICKNESS AT              00006900
             ROLL/POCKET CONTACT - IN. IF BLANK PROGRAM ASSUMES        00007000
             .0005 IN.                                                 00007100
      71-80  LEAVE BLANK.                                              00007200
                                                                       00007300
             TO RUN ADDITIONAL LOAD CASES WITH SAME LUBRICANT          00007400
             REPEAT CARD 9 AS REQUIRED
                                                                       00007600
             TO RUN ADDITIONAL LUBRICANTS PLACE ONE BLANK AFTER        00007700
             LAST CARD 7 AND READ CARD 5 ET. SEQ.                      00007800
                                                                       00007900
             TO RUN NEW BEARING CONFIGURATION PLACE TWO BLANKS         00008000
             AFTER LAST CARD 7 AND READ CARD 1 ET. SEQ.                00008100
                                                                       00008200
             TO STOP PLACE THREE BLANK CARDS AFTER LAST CARD 7.        00008300
                                                                       00008400
                                                                       00008500
```

APPENDIX VI
TYPICAL ROLLER BEARING PROBLEM FOR COMPUTER PROGRAM

TEST CASE

DESIGN DATA FOR BEARING NO. 1

NO. OF ROLLERS	ROLL DIAMETER	PITCH DIAMETER	TOTAL LENGTH	FLAT LENGTH	CROWN DROP	CROWN RADIUS	GAGE POINT	DIAMETRAL CLEARANCE	I.D. OF OUTER RING	O.D. OF INNER RING
6.0000E 00	4.3310E-01	4.7224E 00	4.3310E-01	2.5000E-01	2.0000E-04	4.7940E 01	3.0000E-02	0.0	5.1570E 00	4.4900E 00

O.D. OF CAGE	I.D. OF CAGE	TYPE OF PILOT	CAGE/RING CLEARANCE OUTER	CAGE/RING CLEARANCE INNER	POCKET CLEARANCE	CAGE WIDTH	YOUNG'S MODULUS RINGS	YOUNG'S MODULUS ROLLS	POISSON'S RATIO RINGS	POISSON'S RATIO ROLLS
4.8000E 00	4.5000E 00	IN	3.5700E-01	1.0000E-02	9.9000E-03	7.5000E-01	2.9000E 07	2.9000E 07	2.5000E-01	2.5000E-01

FRICTION COEF.	MATERIAL DENSITY	CORNER RADIUS	CAGE WEIGHT
1.0000E-01	2.8100E-01	2.0000E-02	4.0000E-01

LUBRICANT PROPERTIES

PRESSURE-VISCOSITY COEFFICIENT	1.1900E-04
TEMPERATURE-VISCOSITY COEFFICIENT	1.5300E-02
VISCOSITY AT INLET TEMPERATURE	3.9600E-06
INLET TEMPERATURE	1.0000E 02
THERMAL CONDUCTIVITY	7.7400E-02
LUBRICANT DENSITY	8.9700E-05

INPUT DATA FOR LOAD NO. 2, BEARING NO. 1

RPM. OF OUTER	RPM. OF INNER	RADIAL LOAD
0.0	1.0000E 04	2.0000E 02

PERCENT CAGE SLIP 1.0000E-01

LOAD IS STATIONARY

OUTPUT DATA FOR LOAD NO. 1 BEARING NO. 2

BEARING REACTION	RPM OF CAGE	PERCENT CAGE SLIP	TORQUE ON CAGE	CAGE PILOT NORMAL LOAD
2.031[illegible]E 02	4.0963E 03	10.02	6.6891E 01	1.9838E 01

OUTPUT DATA FOR RACE CONTACTS

ELEMENT NUMBER	ELEMENT AZIMUTH	CONTACT LOAD OUTER	CONTACT LOAD INNER	MEAN COMPRESSIVE STRESS OUTER	MEAN COMPRESSIVE STRESS INNER	PATTERN EXTREMITY OUTER	PATTERN EXTREMITY INNER	SLIP VELOCITY OUTER	SLIP VELOCITY INNER
1	0.0	1.6265E 02	1.4205E 02	8.5242E 04	8.7831E 04	1.6415E-01	1.6092E-01	9.7878E 01	1.2718E 02
2	6.0000E 01	7.8287E 01	5.8128E 01	6.0951E 04	5.8051E 04	1.4723E-01	1.4271E-01	8.3379E 01	1.4168E 02
6	3.0000E 02	7.8286E 01	5.[illegible]28E 01	6.0950E 04	5.8051E 04	1.4723E-01	1.4271E-01	8.3380E 01	1.4168E 02
THE REST		7.8286E 01		3.1867E 04		1.3213E-01		2.7620E 00	

ELEMENT NUMBER	MIN. FILM THICKNESS OUTER	MIN. FILM THICKNESS INNER	TRACTIVE FORCE OUTER	TRACTIVE FORCE INNER
1	6.2125E-05	6.2265E-05	9.2945E 00	9.3489E 00
2	6.7478E-05	6.9053E-05	4.6495E 00	4.6826E 00
6	6.7478E-05	6.9053E-05	4.6495E 00	4.6826E 00
THE REST	7.7876E-05		1.1860E-02	

OUTPUT DATA FOR POCKET CONTACTS

ELEMENT NUMBER	RPM OF ROLLER	NORMAL POCKET LOAD FORE	NORMAL POCKET LOAD AFT	TANGENTIAL POCKET LOAD FORE	TANGENTIAL POCKET LOAD AFT	MIN. FILM THICKNESS FORE	MIN. FILM THICKNESS AFT
1	-2.7022E 04	1.4628E 01	0.0	-5.4400E-02	0.0	5.0747E-05	9.8492E-03
2	-2.6696E 04	5.3084E 00	0.0	-3.3110E-02	0.0	1.5073E-04	9.7492E-03
6	-2.6696E 04	5.3084E 00	0.0	-3.3110E-02	0.0	1.5074E-04	9.7492E-03
THE REST	-3.8634E 04	0.0	4.0136E 00	0.0	-2.7844E-02	9.7111E-03	1.9804E-04

APPENDIX VII
MIL-L-23699-TYPE OIL PROPERTIES
AVAILABLE FOR USE IN PROGRAM INPUTS

The following pressure-viscosity data* for Aeroshell Turbine Oil 555 is typical of the Mil-L-23699 class of lubricants. Pertinent properties for the formulated fluid are as follows:

	Thermal Conductivity (Btu/ft^2-hr-^oF-ft)	Kinematic Viscosity (cs)	Density (g/ml)	Reciprocal Asymptotic Isoviscous Pressure, $((psi)^{-1})$
100^oF	.098	28.2	.980	.942
210^oF	-	5.3	.935	.691
300^oF	.078	2.4	.897	.602
500^oF	.075	-	-	-

*Data provided by William R. Jones, Jr., Research Engineer, NASA Lewis Research Center, Cleveland, Ohio.

APPENDIX VIII

SOLUTION FOR ROLLER BEARING CASE OPERATING AT 20,000 RPM AND RADIAL LOAD OF 200 POUNDS

```
TEST PROGRAM ROLLER BEARING INNER LOAD PILOTING

DESIGN DATA FOR BEARING NO. 1

 NO. OF      ROLL      PITCH      TOTAL      FLAT      CROWN      CROWN      GAGE      DIAMETRAL    I.D. OF      O.D. OF
 ROLLERS     DIAMETER  DIAMETER   LENGTH     LENGTH    DROP       RADIUS     POINT     CLEARANCE    OUTER RING   INNER RING
 2.9000E 01  4.3[illegible]E-01  4.[illegible]E-01  [illegible]E-01  [illegible]E-01  [illegible]E-01  [illegible]  2.5000E-01  5.1575E 00  4.4445E 00

 O.D. OF     I.D. OF     TYPE OF     CAGE/RING CLEARANCE     POCKET     CAGE      YOUNG'S MODULUS          POISSON'S RATIO
 CAGE        CAGE        PILOT       OUTER      INNER        CLEARANCE  WIDTH     RINGS      ROLLS         RINGS      ROLLS
 4.[illegible]E 00  4.4945E 00  [illegible]  [illegible]E-01  3.0000E-02  [illegible]E-02  6.5000E-01  2.9000E 07  2.9000E 07  2.5000E-01  2.5000E-01

 FRICTION    MATERIAL    CORNER     CAGE
 COEF.       DENSITY     RADIUS     WEIGHT
 1.[illegible]E-01  [illegible]E-01  3.5000E-02  [illegible]E-01

LUBRICANT PROPERTIES

 PRESSURE-VISCOSITY COEFFICIENT          [illegible]E-05
 TEMPERATURE-VISCOSITY COEFFICIENT       [illegible]E-02
 VISCOSITY AT INLET TEMPERATURE          [illegible]E-02
 INLET TEMPERATURE                       [illegible]E-02
 THERMAL CONDUCTIVITY                    [illegible]E-02
 LUBRICANT DENSITY                       [illegible]E-05
```

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INPUT DATA FOR LOAD NO. 9, BEARING NO. 3

RPM. OF OUTER	RPM. OF INNER	RADIAL LOAD
0.0	2.0033E 04	2.3333E 02

PERCENT CAGE SLIP 9.5333E-02

LOAD IS STATIONARY

3

OUTPUT DATA FOR LOAD NO. 9 BEARING NO. 3

BEARING REACTION	RPM OF CAGE	PERCENT CAGE SLIP	TORQUE ON CAGE	CAGE PILOT NORMAL LOAD
2.3648E 02	8.3134E 03	9.51	1.5432E-01	9.4975E-01

OUTPUT DATA FOR RACE CONTACTS

ELEMENT NUMBER	ELEMENT AZIMUTH	CONTACT LOAD OUTER	CONTACT LOAD INNER	MEAN COMPRESSIVE STRESS OUTER	MEAN COMPRESSIVE STRESS INNER	PATTERN EXTREMITY OUTER	PATTERN EXTREMITY INNER	SLIP VELOCITY OUTER	SLIP VELOCITY INNER
1	0.0	1.6859E 02	8.6223E 01	9.4384E 04	7.6152E 04	1.4248E-01	1.2747E-01	9.4479E 01	2.8790E 02
2	1.2857E 01	1.4397E 02	6.1674E 01	8.7719E 04	5.5189E 04	1.3826E-01	1.2190E-01	7.6020E 01	3.0636E 02
28	3.4714E 02	1.4397E 02	6.1674E 01	8.7719E 04	5.5189E 04	1.3826E-01	1.2190E-01	7.6021E 01	3.0636E 02
THE BEST		1.4397E 02		6.8554E 04		1.2655E-01		5.2067E 02	

ELEMENT NUMBER	MIN. FILM THICKNESS OUTER	MIN. FILM THICKNESS INNER	TRACTIVE FORCE OUTER	TRACTIVE FORCE INNER
1	2.7117E-05	2.8672E-05	6.3434E 00	6.8547E 00
2	2.7536E-05	2.9775E-05	5.3390E 00	5.3492E 00
28	2.7536E-05	2.9775E-05	5.3390E 00	5.3492E 00
THE BEST	2.9336E-05		-2.6711E-01	

OUTPUT DATA FOR POCKET CONTACTS

ELEMENT NUMBER	RPM OF ROLLER	NORMAL POCKET LOAD FORE	NORMAL POCKET LOAD AFT	TANGENTIAL POCKET LOAD FORE	TANGENTIAL POCKET LOAD AFT	MIN. FILM THICKNESS FORE	MIN. FILM THICKNESS AFT
1	-5.2622E 04	1.6673E 03	0.0	-1.1143E-02	0.0	1.6235E-04	1.2338E-02
2	-5.2206E 04	0.0	1.3439E 00	0.0	-9.9983E-03	1.2295E-02	2.0490E-04
28	-5.2207E 04	0.0	1.3439E 03	0.0	-9.9982E-03	1.2295E-02	2.0490E-04
THE BEST	-9.8976E 04	0.0	1.7625E 01	0.0	-2.7964E-02	1.2484E-02	1.6458E-05