QUANTO - A CODE TO OPTIMIZE WEAPON ALLOCATIONS

Karl T. Benson, et al

Air Force Weapons Laboratory
Kirtland Air Force Base, New Mexico

January 1974
**UNCLASSIFIED**

**DOCUMENT CONTROL DATA - R & D**

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<tr>
<th>1. ORIGINATING ACTIVITY* (Corporate author)</th>
<th>2a. REPORT SECURITY CLASSIFICATION</th>
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<tr>
<td>Air Force Weapons Laboratory (SAB)</td>
<td>UNCLASSIFIED</td>
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<td>Kirtland Air Force Base, New Mexico 87117</td>
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<th>3. REPORT TITLE</th>
<th>4. DESCRIPTIVE NOTES (Type of report and inclusive dates)</th>
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<td>QUANTO--A CODE TO OPTIMIZE WEAPON ALLOCATIONS</td>
<td>1 September 1971 through 1 October 1973</td>
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<th>5. AUTHOR(S) (First name, middle initial, last name)</th>
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<td>Karl T. Benson, Capt, USAF, Arthur R. Geidbach, Maj, USAF, and Craig E. Miller, Capt, USAF</td>
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<th>6. REPORT DATE</th>
<th>7a. TOTAL NO. OF PAGES</th>
<th>7b. NO. OF REFS</th>
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<tr>
<th>8a. CONTRACT OR GRANT NO</th>
<th>8b. PROJECT NO</th>
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<th>8d. DISTRIBUTION STATEMENT</th>
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<td>Approved for public release; distribution unlimited.</td>
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### ABSTRACT

(Distribution Limitation Statement A)

An advanced computer model has been developed within the Air Force Weapons Laboratory (AFWL) to study the effects of a sea-launched ballistic missile (SLBM) attack on targets consisting of a flushing aircraft force. Using the technique of Lagrangian multiplier optimization, a near-optimal allocation of SLBMs to targets is produced. Although a number of other computer codes exist which model the same situation, the AFWL code is believed to be the only one possessing all of the following features: (1) the determination of the SLBM laydown against a mixed aircraft force (for instance, bombers and tankers); (2) the automatic consideration of differing aircraft hardinesses, flyout profiles, level-off altitudes, and kill values; (3) the treatment of an SLBM attack by multiple types of SLBMs, accounting for differing missile trajectories, yields, launch intervals, reliabilities, and numbers of missiles per submarine; (4) automated relocation of the submarines and/or the aircraft, if desired, to enhance the goals of each; and (5) thermal as well as overpressure kills, accounting for such additional nuclear effects parameters as detonation altitude, haze level, water vapor, visibility, and albedo, through the use of the most current advanced nuclear effects codes developed by AFWL.
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QUANTO--A CODE TO OPTIMIZE WEAPON ALLOCATIONS

Karl T. Benson, Capt, USAF
Arthur R. Geldbach, Maj, USAF
Craig E. Miller, Capt, USAF

Final Report for Period 1 September 1971 through 1 October 1973

Approved for public release; distribution unlimited.
FOREWORD

The research was performed under Program Element 62601F, Project 8809, Task 04.

Inclusive dates of research were 1 September 1971 through 1 October 1973. The report was submitted 25 October 1973 by the Air Force Weapons Laboratory Project Officer, Major Arthur R. Geldbach (SA3).

The advanced computer model QUANTO has been developed within the Air Force Weapons Laboratory to study various scenarios involving sea-launched ballistic missile attacks on bomber air bases. The QUANTO model has been reviewed by interested Air Staff agencies, the Air Force Systems Command, and the Strategic Air Command, and is considered appropriate for use in activities relating to bomber force prelaunch survival. However, prudence should be exercised in its use, because of its sensitivity and the dynamic nature of the problem.

The basic model was developed by Major Richard Conway. A large portion of the debugging and exercising of the code was done by Mr. Eugene Omoda and Mr. William Peay. The assistance of Mr. Harry Murphy in utilizing the operating system and remote terminal, and that of Mr. Al Sharp in incorporating the thermal and overpressure routines into QUANTO were also invaluable in the development process.

This technical report has been reviewed and is approved.

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ABBREVIATIONS AND SYMBOLS

- $P_k$: Probability of destroying the target or any single aircraft at the target, given that the weapon successfully detonates.
- $p$: Number of candidate submarine locations.
- $A_a$: Candidate location $a$ ($a = 1, 2, ..., p$) of attacking submarine(s).
- $S_a$: Number of missiles on all submarines at point $A_a$.
- $t_a$: Number of submarines at point $A_a$.
- $T_i$: Target $i$ consisting of aircraft with value $V_i$.
- $V_i$: Value of the aircraft on target $i$.
- $L$: Number of weapon groups.
- $n_{ij}$: Number of weapons on target $i$ from a weapon in group $j$, $j = 1, 2, ..., L$.
- $f(n_{ij})$: The objective function to be maximized.
- $M$: Number of targets with values $V_i$.
- $S_{ij}$: Probability of survival of aircraft on target $i$ from $n_{ij}$ weapons in weapon group $j$.
- $N_j$: Number of weapons in group $j$.
- $\lambda_j$: Constants (the Lagrange Multipliers).
- $h(n_{ij}, \lambda_j)$: The Lagrangian function.
- $\lambda_{ij}$: Variables dependent on $n_{kj}$ ($k = 1, 2, ..., M$) which assist in determining the Lagrange Multipliers $\lambda_j$ and the optimal laydown $n_{ij}$.
- $\Delta n$: The number of weapons shifted with each iteration in the convergence to the optimal laydown.
- $r(\Delta n)$: A function representing the kill contribution to the objective function $f(n_{ij})$ from targets $k$ and $m$ after $\Delta n$ weapons are moved from target $k$ to target $m$.
- $\lambda_{ik}$: $\min \left\{ \lambda_{ik} \text{ such that } n_{ik} \geq 0.0001 \right\}$, $i = 1, 2, ..., M$. 

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ABBREVIATIONS AND SYMBOLS (cont'd)

\[ \lambda_{m\xi} = \max_i \{\lambda_{i\xi}\}, i = 1, 2, \ldots, M \]

\( \epsilon \) The tolerance used to test for convergence of the \( \lambda_{ij} \)'s in obtaining the optimal laydown \( n_{ij} \)

\( s \) The number of salvos of SLBMs on a submarine which is a candidate for relocation

\( R_1 \) The radial distance of the most distant aircraft from the centroid at the time of a given weapon arrival

\( R_N \) The radial distance of the least distant aircraft from the centroid at the time of a given weapon arrival

\( A_L[\text{for } x] \) The circular lethal area when the detonation point is at distance \( x \) from the centroid

\( A_{LAN}[\text{for } x] \) The lethal area occupied by aircraft in the annulus with radii \( R_1 \) and \( R_N \) when the detonation point is at distance \( x \) from the centroid

\( R_{LR}[\text{for } x] \) The distance from detonation point to lethal region boundary, in a direction away from the centroid

\( V_{iB} \) Total bomber value on base \( i \)

\( V_{iT} \) Total tanker value on base \( i \)

\( S_{ijB} \) Survival probability of bombers on target \( i \) from one weapon in weapon group \( j \)

\( S_{ijT} \) Survival probability of tankers on target \( i \) from one weapon in weapon group \( j \)

\( P_{kB} \) Probability of destroying bombers at a target, given that the weapon successfully detonates at the target

\( P_{kT} \) Probability of destroying tankers at a target, given that the weapon successfully detonates at the target

\( R_{LR/\text{MAX}} \) The distance from detonation point to the farthest lethal region boundary (for all aircraft types), in a direction away from the centroid

\( \ln \) Napierian base logarithm
SECTION I
INTRODUCTION

The theory of the allocation of the sea-launched ballistic missiles (SLBM) against a force of aircraft flushing from their respective airbases and the defensive reactions to given threat levels is discussed. The analysis, which led to the models discussed later, has culminated in a computer program called QUANTO. The model used in QUANTO has as its inputs latitude and longitude coordinates of target and submarine locations, aircraft beddowns, aircraft and missile flight parameters, and aircraft vulnerability levels. Consequently, the code is useful for studying the effects of variations in a number of parameters.

QUANTO analyzes three types of problems important to strategic planners:

Case I: Given specific locations (Aa) for a fixed number of attacking submarines and a specific beddown for aircraft at locations Ti, QUANTO can compute where the assigned missiles from Aa should go.

Case II: Given specific beddown for aircraft at locations Ti, QUANTO can optimize the locations for the submarines among a set of candidate locations Aa.

Case III: Given specific submarine locations Aa, QUANTO can optimize the beddown of aircraft at Ti.

Lagrange multipliers are used in the optimization procedures of QUANTO. A brief review of this technique is presented in appendix I and is intended to acquaint the reader with the basic mathematics involved.

The QUANTO code has been developed within the Air Force Weapons Laboratory (AFWL). It was intended originally as a vehicle for increasing the understanding of the operation of a computer program called COG, which dealt only with Case I (as of May 1971), that was written by the Lambda Corporation (ref. 1). Compared to other codes, QUANTO permits a more detailed and accurate analysis, because weapons and their detonations are handled individually, rather than as members of fixed weapon patterns. Studies show that QUANTO produces a considerably better
allocation than does COG. Further investigation, substantiated by simulation of the attack through the use of another AFWL code, supports the assumptions and models used in QUANTO. Hence, QUANTO provides a means for comparing and evaluating the effectiveness of other weapon allocation codes. More importantly, QUANTO provides a framework for modification and extension in further studies of total bomber/tanker force survivability.
SECTION II
BASIC WEAPON ALLOCATION PROBLEM

In this study, the attacking force of submarines (figure 1) is distributed among points $A_a, a = 1, 2, ..., p$, where submarines at $A_a$ each carry $s_a$ SLBM weapons. At the same time, suppose that the targets, $T_i, i = 1, 2, ..., M$, have values $V_i$. If one were to visualize this engagement as in figure 1, it becomes apparent that many strategies are open to the attacker and defender. For example, the attacking force could put all missiles on target $T_1$. On the other hand, the missiles could be distributed among all targets. As for the defender, he could place his bombers and tankers throughout the target area evenly or perhaps all on the same base. The multiplicity of possibilities increases with each new missile or aircraft, making hand calculations impractical. The approach taken to solve this problem is to use the method of Lagrange Multipliers to produce a near-optimal allocation of SLBMs to targets consisting of escaping aircraft. To construct the objective function which describes the expected value killed, one must first develop the survival probability $S_j$ for each weapon. This figure is the probability that a single aircraft, given an escaping time-dependent pattern of aircraft taking off from the airfield, survives one incoming SLBM. The probability of kill is then

$$P_k = 1 - S_j$$  \hspace{1cm} (1)

Suppose now that $n$ weapons are delivered to a target, and the survival probabilities $S_j$ of the target from each weapon, $j = 1, 2, ..., n$, are independent; then the probability of destroying the target is

$$P_k = 1 - \prod_{j=1}^{n} S_j$$  \hspace{1cm} (2)

For a system of $M$ targets, each having a value $V_i$, the expected return from delivery of all weapons is
\( A_0 \): The attacking force of submarines (could be zero or more than one at each location).

\( T_i \): Targets

\( V_i \): Value of targets \( T_i \)

Figure 1. Attacker versus Attacked Forces
where $S_{ij}$ is the probability of survival of target $i$ from weapon $j$. The product in expression (3) for target $i$ includes only those survivabilities corresponding to the weapons which are aimed at target $i$. In practice, there are weapon "groups," where the weapons in each group are so nearly identical in characteristics and location that no distinction between them is required for purposes of allocation. Hence, in practice, $S_{ij}$ is raised to the $n_{ij}$ power, where $n_{ij}$ is the number of weapons from weapon group $j$ that are targeted against target $i$. It is easy to see that the product 

$$
\prod_{j} S_{ij}^{n_{ij}}
$$

is not changed if the weapon groups $j$ are included for which $n_{ij} = 0$. Consequently, if $L$ is the total number of weapon groups, the expected aircraft kill may be written

$$
\sum_{i=1}^{M} V_i \left( 1 - \prod_{j=1}^{L} S_{ij}^{n_{ij}} \right)
$$

Table I clarifies the submarine input parameters used in QUANTO. The table has six columns, the first of which is submarine locations, given to QUANTO in terms of latitude and longitude coordinates, surrounding a given target country. The second and fifth columns contain the same information and are presented separately to emphasize the fact that the number of submarines and number of missiles per group are the same since all the submarines at a given location are assumed to fire a missile apiece at the same time. Note here that zero submarines are allowed at a given location. In column four the numbering 1 to 4 is applied to two types of weapons, each of which is restricted to either the Atlantic or Pacific Ocean. Numbers 1 and 2 may identify weapon types 1 and 2 in the Pacific, whereas 3 and 4 may represent weapon types 1 and 2 in the Atlantic. Since submarines may be shifted only among locations which have like missile type identifiers in QUANTO, such a numbering system prevents submarines from relocating to a different ocean. The last column is the numbering given to the
Table I

SUBMARINE INPUT DATA TO QUANTO (EXAMPLE)

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<tr>
<th>Location</th>
<th>Number Submarines</th>
<th>Number Missiles/Sub (L = 42)</th>
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<td>3</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>0</td>
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<tr>
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<td>4</td>
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<td>3</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>37 - 42</td>
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weapon groups. Note here that the numbering in row 2 goes from 9 through 14. Each weapon group here consists of two missiles in the same salvo since there are two submarines at this location. Missiles may be placed in the same group if they have identical trajectories and are launched at the same time from the same point. Also, row 3 allows for a set of weapon groups even though no submarines are initially placed at submarine location 3 (although there may be subsequently, if the submarine-placement optimizer of Case II is exercised).

The basic allocation problem is to maximize the expected kill given by expression (4) by sending the missiles to the proper targets. Since the allocation of missiles to targets is expressed by the values n_ij, the problem is to find the integer values n_ij which result in the greatest kill while satisfying constraints on the number of weapons available in each group.
SECTION III
USE OF LAGRANGE MULTIPLIERS IN THE QUANTO COMPUTER CODE

1. FORMULATION FOR SOLUTION

The weapon allocation problem is one of determining the optimal allocations \( n_{ij} \) of weapons to targets to maximize the expected kill value

\[
f(n_{ij}) = \sum_{i=1}^{M} V_i \left[ 1 - \prod_{j=1}^{L} S_{ij}^{n_{ij}} \right] \tag{5}\]

subject to the stockpile constraints

\[
\sum_{i=1}^{M} n_{ij} = N_j, \quad j = 1, 2, \ldots, L \tag{6}\]

where \( N_j \) is the number of weapons in weapon group \( j \). Fractional allocations in \( n_{ij} \) are permitted in the solution of this problem, but each \( n_{ij} \) must satisfy

\[
0 \leq n_{ij} \leq N_j \tag{7}\]

As described in appendix I, this problem gives rise to the Lagrangian function

\[
h(n_{ij}, \lambda_j) = f(n_{ij}) + \sum_{j=1}^{L} \lambda_j \left[ \sum_{i=1}^{M} (n_{ij}) - N_j \right] \tag{8}\]

In seeking the extremum of the Lagrangian function \( h(n_{ij}, \lambda_j) \), the values of \( (n_{ij}, \lambda_j) \) are sought which satisfy the following necessary conditions for a solution using this Lagrangian Multiplier technique

\[
\frac{\partial h}{\partial n_{kj}} = -V_k (\sum n_{kj} \prod_{j=1}^{L} S_{kj}^{n_{kj}} + \lambda_k = 0 \tag{9}\]

\[
k = 1, 2, \ldots, M; \quad \varepsilon = 1, 2, \ldots, L\]
If variable \( \lambda_{k\ell} \) (dependent on \( n_{kj} \), \( j = 1, 2, \ldots, L \)) are defined as
\[
\lambda_{k\ell} = -V_k \left( \prod_{j=1}^L S_{kj} \right)^{\frac{n_{kj}}{S_{k\ell}}}
\]
(10)
the system (equation (9)) of \((M \times L)\) equations becomes
\[
\lambda_{k\ell} = -\lambda_{\ell}, \quad k = 1, 2, \ldots, M; \ell = 1, 2, \ldots, L
\]
Now fix \( \ell \) and consider the subsystem of \( M \) equations
\[
\lambda_{1\ell} = -\lambda_{\ell}
\]
\[
\lambda_{2\ell} = -\lambda_{\ell}
\]
\[
\vdots
\]
\[
\lambda_{M\ell} = -\lambda_{\ell}
\]
(11)
A word is in order concerning notation. In equations (11), \( \lambda_{\ell} \) is one of the unknown Lagrange multipliers. The variables \( \lambda_{k\ell} \) \((k = 1, 2, \ldots, M)\) are computable if one has the values of \( n_{kj} \) \((j = 1, 2, \ldots, L)\). The technique used for finding the values of \( \lambda_{\ell} \) and \( n_{kj} \) \((k = 1, 2, \ldots, M; j = 1, 2, \ldots, L)\) which satisfy the system (equations (11)) of \( M \) equations takes advantage of the fact that all the \( \lambda_{k\ell} \) should equal the same quantity, namely \(-\lambda_{\ell}\). The method chooses values of \( n_{kj} \) iteratively, subject to the constraints, so that the values of \( \lambda_{k\ell} \) \((k = 1, 2, \ldots, M)\) approach a single value, namely \(-\lambda_{\ell}\).

2. ITERATIVE PROCEDURE

An initial allocation of weapons to targets \( n_{ij} \) is input to QUANTO, and the variables \( \lambda_{ij} \) are computed. Suppose for a given weapon group \( \ell \) that \( \lambda_{k\ell} < \lambda_{m\ell} \) and \( n_{kj} \geq 0.0001 \). Then by moving an appropriate number \( \Delta n \) of weapons in group \( \ell \) from target \( k \) to target \( m \), \( \lambda_{k\ell} \) and \( \lambda_{m\ell} \) may be made more nearly equal. Note
that \( n_{k\ell} \) must be initially positive or there would be no weapons to shift. In fact, were it not for the restriction that \( n_{k\ell} \) may not be reduced to a negative amount (i.e., \( \Delta n \leq n_{k\ell} \)), \( \lambda_{k\ell} \) and \( \lambda_{m\ell} \) could be made equal in all cases. The value of \( \Delta n \) which would make the new values of \( \lambda_{k\ell} \) and \( \lambda_{m\ell} \), say \( \hat{\lambda}_{k\ell} \) and \( \hat{\lambda}_{m\ell} \), equal is the value of \( \Delta n \) which satisfies

\[
\hat{\lambda}_{k\ell} = V_k \left( \frac{\ln S_{k\ell}}{S_{k\ell}} \right) S_{k1}^{n_{k1}} S_{k2}^{n_{k2}} \cdots S_{k\ell}^{n_{k\ell} - \Delta n} \cdots S_{kL}^{n_{kL}}
\]

\[
= V_m \left( \frac{\ln S_{m\ell}}{S_{m\ell}} \right) S_{m1}^{n_{m1}} S_{m2}^{n_{m2}} \cdots S_{m\ell}^{n_{m\ell} + \Delta n} \cdots S_{mL}^{n_{mL}} = \hat{\lambda}_{m\ell}
\]

This may be written as

\[
\hat{\lambda}_{k\ell} = \lambda_{k\ell} S_{k\ell}^{-\Delta n} = \lambda_{m\ell} S_{m\ell}^{\Delta n} = \hat{\lambda}_{m\ell}
\]

Therefore,

\[
\Delta n = \frac{\ln \lambda_{k\ell}}{\ln S_{k\ell} S_{m\ell}}
\]

(12)

Since \( \Delta n \) is not permitted to be so large that \( (n_{k\ell} - \Delta n) \) becomes negative, the actual number of weapons shifted is

\[
\Delta n = \min \left\{ n_{k\ell} \left( \frac{\ln \left( \frac{\lambda_{k\ell}}{\lambda_{m\ell}} \right)}{\ln S_{k\ell} S_{m\ell}} \right) \right\}
\]

This shift of weapons gives rise to a new \( \hat{j}_{ij} \) and new \( \hat{\lambda}_{ij} \). Repeated shifts ultimately force each pair, \( (\lambda_{k\ell}, \lambda_{m\ell}) \), for each weapon group \( \ell \), to be equal (for those targets \( k \) and \( m \) for which weapons from group \( \ell \) end up being allocated).

Although the restriction \( \Delta n \leq n_{k\ell} \) makes it impossible to force the equality of every pair \( (\lambda_{k\ell}, \lambda_{m\ell}) \), the preceding choice of \( \Delta n \) does result in the greatest increase in the objective function which can result from such a shift of weapons in group \( \ell \) from target \( k \) to target \( m \). To see this consider the function
which represents the kill contribution to the objective function $f(n_{ij})$ (equation (5)) from targets $k$ and $m$ after $\Delta n$ weapons are moved from target $k$ to target $m$. The best choice of $\Delta n$ is where $r(\Delta n)$ achieves its maximum within the interval $0 \leq \Delta n \leq n_{k2}$. The unrestricted maximum of $r(\Delta n)$ occurs where

$$
\frac{dr(\Delta n)}{d(\Delta n)} = 0
$$

i.e.,

$$
r'(\Delta n) = \lambda_{m2} S_{m2}^{+\Delta n} - \lambda_{k2} \omega_{k2} \Delta n = 0
$$

or

$$
\Delta n^* = \frac{\ln\left(\frac{\lambda_{k2}/\lambda_{m2}}{\lambda_{m2}/S_{m2}}\right)}{\ln\left(S_{k2}/S_{m2}\right)}
$$

If this value is greater than $n_{k2}$, the constrained maximum of $r(\Delta n)$ occurs at $\Delta n = n_{k2}$. This follows from

$$
r'(0) = \lambda_{m2} - \lambda_{k2} > 0
$$

and

$$
r''(\Delta n) = \lambda_{m2} \left(\ln S_{k2}\right) S_{m2}^{+\Delta n} + \lambda_{m2} \left(\ln S_{m2}\right) S_{k2}^{\Delta n} \leq 0
$$

for all $\Delta n$ in the range $0 \leq \Delta n \leq n_{k2}$ (since $0 \leq S_{ij} \leq 1$, $\ln S_{ij} \leq 0$ and $\lambda_{ij} \geq 0$ for all $i, j$). Thus, $r(\Delta n)$ appears as in figure 2 or figure 3. Note that the curvature is always downward and that the maximum occurs at the point $\Delta n^*$. If the situation of figure 3 occurs, it is impossible to choose $\Delta n = \Delta n^*$ to force $\lambda_{k2}$ and $\lambda_{m2}$ to be equal. Consequently, equations (11) will not be satisfied. However, the optimal value of $f(n_{ij})$, where the $n_{ij}$ are constrained by equation
(6) and equation (7), is still found due to the preceding comments concerning \( r(\Delta n) \). Thus, the \( \lambda_{ij} \)'s merely serve as a means of adjusting the \( n_{ij} \)'s to approach optimality. The optimal \( n_{ij} \) are, of course, nonintegral and, therefore, not physically possible. Consequently, the optimal nonintegral allocation is integerized to give an integral allocation which satisfies the constraints.

This integerization is performed for each weapon group \( j \) by rounding those \( n_{ij} \)'s with the largest fractional parts up and rounding the remaining \( n_{ij} \)'s down. Of course, the constraints

\[
\sum_i n_{ij} = N_j
\]

(in which the \( N_j \)'s are integers) are satisfied by the real \( n_{ij} \)'s before integerization and must be satisfied by the integer \( n_{ij} \)'s also. Therefore, the number of \( n_{ij} \)'s rounded up is determined so that, for each weapon group \( j \), the sum of those \( n_{ij} \)'s rounded up and those rounded down equals \( N_j \). In practice, the expected kill resulting from this integerized allocation is not significantly different from the expected kill computed from the nonintegral allocation, since the difference in kills is usually only a fraction of an aircraft. Integerization of the optimal nonintegral allocation need not produce the optimal integral allocation, but it does produce at least a near-optimal integral allocation, with the difference in kills being the upper bound of how far from optimal the kill of the integerized allocation could be.

It has been indicated how \( n_{kj} \) and \( n_{mj} \) may be adjusted to increase the expected kill value when \( \lambda_{kj} < \lambda_{mj} \) and \( n_{kj} < 0.0001 \) for some weapon group \( k \).
In practice, a tolerance level, \( \epsilon \), is set in QUANTO, so that convergence is said to occur when
\[
\lambda_{mz} - \epsilon \leq \lambda_{kz} \leq \lambda_{mz}
\]
for all \( n_{kz} \geq 0.0001 \) for all values of \( z \), where \( \lambda_{mz} = \max \{ \lambda_{iz} \} \), \( i = 1, 2, \ldots, M \).

Specifically, the weapon group \( z \), upon which each allocation adjustment is based, is selected in a cyclical manner. The first allocation adjustment is made within weapon group one \((z = 1)\) if
\[
\lambda_{kz} < \lambda_{mz} - \epsilon
\]
where
\[
\lambda_{kz} = \min_i \{ \lambda_{iz} \text{ such that } n_{iz} \geq 0.0001 \}, \quad i = 1, 2, \ldots, M
\]

(16)

and
\[
\lambda_{mz} = \max_i \{ \lambda_{iz} \}, \quad i = 1, 2, \ldots, M
\]

(17)

If this situation does not exist for \( z = 1 \), successive weapon groups are inspected in sequential order until one is found in which the highest \( \lambda_{iz} \) exceeds the lowest \( \lambda_{iz} \) with a corresponding positive allocation \((n_{iz} \geq 0.0001)\) by more than the tolerance \( \epsilon \). Successive allocation adjustments are accomplished in a repeating cycle through the values of \( z \) (i.e., 1, 2, \ldots, \( L \)), 1, 2, \ldots, \( L \), 1, 2, \ldots). Convergence occurs when all weapon groups are inspected without finding one which initiates an allocation adjustment. In practice, the \( \lambda_{ij} \) matrix if first converged to a tolerance of \( \epsilon = 0.1 \), then \( \epsilon = 0.01 \), then \( \epsilon = 0.001 \) and so forth, with the final tolerance under the control of the user. This process results in a faster overall convergence to the final tolerance level. An additional cutoff of the convergence occurs if a given number (specified by the user, say 100) of allocation adjustments are performed without increasing the kill value above some user-selected amount (say 0.01). The user may also simply specify a maximum number of allocation adjustments to be made.

The iterative procedure is illustrated in the flow chart of figure 4, with several additional details appearing in the figure. When \( \lambda_{kz} < \lambda_{mz} - \epsilon \) is found for some weapon group \( z \), \( \Delta n \) must be computed. When only one type of aircraft is considered in the model, \( \Delta n \) is computed according to equation (12). However, when more than one type of aircraft is considered, \( \Delta n \) must be computed by a Newton iterative procedure, described in the mixed force allocation problem discussed in section VI.
SET CONVERGENCE TOLERANCE $\epsilon = 0.1$

- Compute $f(n_{ij}), \lambda_{ij}, \pi S_{ij}$ for initial $n_{ij}$

INCREMENT ITERATION COUNTER

INCREMENT WEAPON GROUP NUMBER $i$

FIND $\lambda_{k,i} = \min \left\{ \lambda_{i,j} \text{ such that } n_{i,j} \leq 0.0001 \right\}$ and $\lambda_{m,i} = \max \left\{ \lambda_{i,j} \right\}$

- $\lambda_{k,i} < \lambda_{m,i} - \epsilon$?
  - NO
  - ALL WPNS GPS INSPECTED?
    - NO
    - TEST NO. OF ACFT TYPES
      - YES
      - $\epsilon \leq \text{CUTOFF } \epsilon$?
        - NO
        - SET $\epsilon = \epsilon / 10$
          - MSG
        - YES
          - COMPUTE $\Delta n$ BY NEWTON ITERATION
            - NO
            - LIMIT OF ITERATION SUBGROUP?
              - YES
              - SAVE KILL AND OUTPUT
                - MSG
              - NO
                - RECOMPUTE $n_{ij}, \pi S_{ij, \lambda_{ij}, f(n_{ij})}$.
                  - YES
                  - LIMIT OF SUBGROUP?
                    - YES
                    - SAVE KILL AND OUTPUT
                      - MSG
                    - NO
                      - SIGNIFICANT CHANGE IN KILL?
                        - NO
                        - EXIT
                        - MSG
                        - YES
                          - ITERATION CUTOFF?
                            - YES
                            - MSG
                            - NO
                              - PRINT $n_{ij}$, KILL SUMMARY
                                - EXIT

Figure 4. QUANTO'S ITERATIVE PROCEDURE

13
The amount of output may be controlled to some extent by a control variable set by the user. Output on each iteration may be suppressed and only a limited output obtained after a "subgroup" of iterations. If the expected kill has not increased significantly for the subgroup of iterations, the procedure is terminated. Two other conditions may terminate the procedure, as shown in figure 4 and previously described.

3. OPTIMIZATION OF SUBMARINE LOCATIONS IN QUANTO

In the weapon allocation problem, the Lagrange multiplier $\lambda_j$ represents the shadow value associated with weapon group $j$. In notation

$$\frac{\partial h}{\partial N_j} = -\lambda_j \geq 0$$

where $h(n_{ij}, \lambda_j)$ is the Lagrangian function. Therefore, increasing $N_j$ has the instantaneous effect of permitting an increase in $h$ (and, therefore, $f$) at the rate of $-\lambda_j$ units of $f$ per unit of $N_j$. Thus, one can get some feel for the value of an additional weapon in group $j$ by observing the magnitude of $\lambda_j$.

A heuristic rule has been used in QUANTO to relocate submarines among the input candidate submarine locations so as to improve the expected kill value. The value of a submarine at a given location bears some relation to the magnitudes of $\lambda_{kj}$ for those weapon groups $j$ corresponding to salvos from a submarine at that location. A submarine location input to QUANTO is characterized not only by its geographical coordinates but also by the type of submarine (and its number of salvos, $s$) which can be located there.

For each submarine, the quantity

$$\sum_j \lambda_{kj}$$

is calculated where the sum is over the $s$ values of $j$ corresponding to that submarine's salvos, and each

$$\lambda_{kj} = \min \left\{ \lambda_{ij} \text{ such that } n_{ij} \geq 0.0001 \right\}, \; i = 1, 2, \ldots, M$$

for each weapon group $j$ at that submarine's current location. Similarly, for every other location at which that submarine can operate, the quantity
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\[ \sum_{j} \lambda_{mj} \]

is calculated, where

\[ \lambda_{mj} = \max_{i} \{ \lambda_{ij} \}, \quad i = 1, 2, \ldots, M \]

with a view to possibly moving the submarine to a location where it can be expected to kill more value. When there are several types of submarines, the type of submarine moved is the one having the largest average difference

\[ \frac{1}{s} \left\{ \sum_{j} \lambda_{mj} - \sum_{j} \lambda_{kj} \right\}, \quad \lambda_{kj} \text{ such that } n_{kj} \geq 0.0001 \]

Within this submarine type, the submarine relocated is the one corresponding to the lowest quantity

\[ \sum_{j} \lambda_{kj} \]

and it is placed in the location having the highest value of

\[ \sum_{j} \lambda_{mj} \]

Relocation of a submarine in QUANTO is accomplished by moving the s missiles on that submarine to another location. Consequently, \( n_{ij} \) is increased by one missile in the s weapon groups j corresponding to the submarine location to which that submarine is moved and those additional SLBMs are assigned to targets i having the largest value \( \lambda_{ij} \) (for each j). Similarly, for the s weapon groups j from which a missile is removed, \( n_{ij} \) is reduced for the targets i corresponding to the lowest \( \lambda_{ij} \)'s until a total of one missile is removed from each weapon group. In this way, a rational guess is made at where the missiles from the relocated submarine should go in order to obtain an initial allocation prior to re-entering the laydown optimization procedure.

This submarine relocation process in no way guarantees an increase in value killed. This is because the relocation is accomplished by moving an integral number of missiles, not a \( \Delta n \) computed to maximize kill. Also, s missiles (not just one) are moved before the \( \lambda_{ij} \) are recomputed from the new \( n_{ij} \)'s.
Although the heuristic rule does not always increase the kill, experience with the procedure reveals that the kill usually increases with every more until a decrease occurs, after which the kill varies with additional moves without significant gains or losses. Consequently, submarine moving is terminated in QUANTO after the first move which results in a decreased kill.

4. BEDDOWN OPTIMIZATION

A heuristic routine for shifting aircraft from base to base has been supplied in the QUANTO weapon allocation code with the intent of determining better aircraft beddowns for a given positioning of submarines. The procedure shifts aircraft from the base having aircraft value greater than 0.0001 with the lowest survivability product

$$\prod_{j=1}^{L} s_{ij}^{n_{ij}}$$

to the base with the highest survivability product, provided the losing base starts with an aircraft value greater than 0.0001. Otherwise, the survivability products are inspected in ascending order until a corresponding aircraft value greater than 0.0001 is found, and the corresponding base is selected as the losing base. The amount of value shifted is the nonintegral product

$$\Delta V = V_k \cdot \left( \prod_{j=1}^{L} s_{mj}^{n_{mj}} - \prod_{j=1}^{L} s_{kj}^{n_{kj}} \right)$$

where bases m and k are those having the highest and lowest survivability products, respectively, where $V_k > 0.0001$. If $V_k < 0.1$, then all the value $V_k$ is moved from base k to base m regardless of the $\Delta V$ computed.

A with the heuristic submarine relocation routine, each shift of beddown value in accordance with the above formula does not guarantee a decrease in the overall expected kill value, although the general trend is toward a lowering of the kill. Occasionally, an overall kill increase may occur as the result of individual shifts considerably before the process has exhausted the gains to be made in aircraft surviving. The shifting of value terminates if the value $\Delta V$ to be moved is less than 0.05 (specified by a program statement), at which point the survivability products have essentially converged and the beddown is not changing significantly.
Shifts of aircraft do not cause a recomputation of the survivabilities $S_{ij}$ in QUANTO, since only rarely does the computed survivability depend upon the number of aircraft present at the target. The methods of computing the survivabilities $S_{ij}$ are described in the next section of this report.

If both beddown optimization and optimization of submarine locations are requested by the user of QUANTO, the beddown optimization is performed last. Of course, if the user wants the submarines to have the last move, he may request beddown optimization only, and in a subsequent run, input the optimal beddown and request submarine optimization.

The beddown optimization procedure shifts nonintegral numbers of aircraft, and thus results in a beddown which has fractional numbers of aircraft at the various bases. After the termination of aircraft moves, the beddown is integerized along with the missile laydown and the results are output. Integerization of the beddown has the effect of increasing the kill by a negligible amount over the expected kill computed on the basis of nonintegral beddown.

The following discussion, in the form of a critiqued proof, is presented as a partial justification for the heuristic rule for improving the beddown.

Suppose $(V_1, V_2, \ldots, V_k, \ldots, V_m, \ldots, V_M)$ represents the values of the aircraft bedded down on the $M$ bases for which the optimal missile laydown is $[n_{ij}]$ and

$$\prod_{i=1}^{L} S_{kj} < \prod_{j=1}^{L} S_{mj}$$  \hspace{1cm} (20)

Next suppose the beddown is changed by subtracting some small value $\epsilon > 0$, from $V_k$ and adding $\epsilon$ to $V_m$. Thus, $(V_1, V_2, \ldots, V_k - \epsilon, \ldots, V_m + \epsilon, \ldots, V_M)$ represents the new beddown, and the new optimal laydown $[\hat{n}_{ij}]$ could be determined. The new value surviving in the new beddown is then

$$\text{New Surviving Value} = \sum_{i=1}^{M} V_i \prod_{j=1}^{L} S_{ij} \hat{n}_{ij} + \epsilon \prod_{j=1}^{L} S_{mj} \hat{n}_{mj} - \prod_{j=1}^{L} S_{kj} \hat{n}_{kj}$$  \hspace{1cm} (21)

Now if $\epsilon$ is sufficiently small, it is reasonable to expect that $[\hat{n}_{ij}]$ is close to $[n_{ij}]$, so that
Therefore, for some choice of $\varepsilon$

$$\text{New Surviving Value} > \sum_{i=1}^{M} \prod_{j=1}^{L} V_i \prod_{j=1}^{L} S_{ij} \hat{n}_{ij}$$

(22)

Furthermore,

$$\sum_{i=1}^{M} \prod_{j=1}^{L} S_{ij} \hat{n}_{ij} > \sum_{i=1}^{M} \prod_{j=1}^{L} S_{ij} n_{ij} = \text{Old Surviving Value}$$

(23)

since $[n_{ij}]$ is the optimal laydown for the old beddown ($V_i$) and therefore minimizes the survivors. Consequently, a shift of value (sufficiently small) from base $k$ to base $m$, when

$$\prod_{j=1}^{L} S_{mj} > \prod_{j=1}^{L} S_{kj}$$

result in a reduced expected kill. Note that this proof does not indicate the best amount of value $\varepsilon$ to shift, but merely that value should be shifted to bases having high survivability products from those with low products.

5. LETHAL AREA DETERMINATION

The determination of lethal area (i.e., the region within which aircraft are destroyed) resulting from a nuclear weapon detonation is an integral part of the QUANTO code. The lethal areas are required in the computations of the survivabilities, $S_{ij}$, of aircraft flushing each target area. This subsection discusses the assumptions, assertions, models, and methods used in the lethal area determination.

The nuclear environment created by the detonation of nuclear weapons is discussed in AFSCM 500-1. This section is concerned with the fireball effect (thermal) and the blast (overpressure) effect which are considered to be the only two structural kill mechanisms which can destroy an aircraft for this model.
A superheated region, the fireball, cools while expanding and radiates thermal energy (heat). At the expanding edge of the fireball, tremendous pressures are created and form a shock front. The shock front propagates approximately spherically at supersonic speeds and produces a crushing overpressure force with accompanying gusts of dynamic forces. The thermal energy effect is measured in calories per square centimeter (cal/cm²), and blast effects in pounds per square inch (psi).

Mathematical models are available to study the thermal and blast effects. Computer codes for these models require many hours of computer time; consequently, the precise codes are not suited for systems analysis or war games. Reliable models based on the precise hydrodynamic and radiation hydrodynamic models have been developed, tested, and improved by the Air Force Weapons Laboratory. Codes for these models require only milliseconds of computer time and, therefore, are suitable for systems analysis. The computerized versions of these codes bear the names SABER and SNAFT, and are used to approximate the blast and thermal environments, respectively.

The Systems Analysis Blast Environment Routine (SABER) has been modified to determine only the ranges of given levels of overpressure and the times of shock arrival at those ranges. This is a restricted use of the multipurpose program. The modified version is called SABERCM. Inputs, in addition to specified peak overpressure, are nuclear weapon yield, height of burst, terrain height, and aircraft altitude. Outputs are overpressure range and time of shock arrival.

SNAFT is a computerized model which can be used to calculate the free-field thermal energy resulting from the detonation of a nuclear weapon or to calculate the range at which a given level of free-field thermal energy occurs. SNAFT has been modified to perform only the latter calculation as a subroutine named SNAFTCM. Necessary input data, other than the free-field energy level, consist of nuclear weapon yield, height of burst, terrain height, aircraft altitude, and pertinent atmospheric conditions. The atmospheric parameters include haze layer height, water vapor pressure, ground reflectance (albedo), and visibility. The horizontal range at which the desired free-field thermal energy level occurs is output from SNAFTCM.
The cookie-cutter assumption has been made to distinguish between regions of lethality and nonlethality to aircraft. For these purposes, aircraft vulnerability levels for the thermal and overpressure kill mechanisms are specified in cal/cm² and psi, respectively. Under the cookie-cutter assumption, aircraft are assumed to be killed if either specified vulnerability level is exceeded, and safe otherwise.

Vulnerability levels are input to the nuclear routines along with atmospheric conditions, terrain height, and weapon characteristics in order to calculate the lethal nuclear environmental ranges for a fixed height of receiver. A point of detonation is first specified for each missile at each potential target, based upon where the aircraft from that airfield are located when the missile arrives. The routines SABERCM and SNAPTCM require a height of receiver to compute the horizontal ranges of the lethal nuclear environment. The height of receiver is taken as the altitude of the aircraft, according to its flight profile, at the time of weapon detonation (relative to the brake release time). This is equivalent to slicing the spherically propagating shock front (or overpressure contour) and the thermal contour with a plane parallel to the ground at a distance above the ground equal to the altitude of the aircraft at the above. This horizontal plane, called the lethal plane, is the geometrical structure in which the determination of lethal area is accomplished.

The general appearance in the lethal area plane of the overpressure and thermal contours, at the lethal levels specified by the vulnerability levels and relative to the other input data, is that of two concentric circles centered at the perpendicular point projection of the burst center onto the plane. Figure 5 depicts the intersection of the lethal area plane with the lethal overpressure contour. The horizontal range associated with the lethal overpressure contour is the lethal overpressure radius, and is measured from the perpendicular point projection of the burst center onto the plane to the lethal overpressure contour. Similarly, the lethal thermal radius is that horizontal range associated with the lethal thermal contour. Thus, the nuclear routines are used to compute the lethal contours needed to compute lethal area.

The orientation of the aircraft is not considered in computing the horizontal ranges associated with the overpressure or thermal kill mechanisms. The lethal thermal radius is computed under the assumption that the aircraft is oriented
so as to receive the maximum amount of thermal energy. Similarly, the fact that the aircraft is better equipped to withstand the overpressure shock front in one position, as opposed to another, is also not taken into account in the computation of the lethal overpressure radius.

The lethal radii which are outputs from the nuclear routines are computed for a stationary receiver (aircraft). Thus, the lethal contours defined by the lethal radii must be adjusted to account for a moving aircraft. This is accomplished in the lethal area plane.

The lethal area (within the lethal area plane) resulting from the detonation of a nuclear weapon is that area within which the aircraft cannot survive if located there at the onset of the detonation. Tabulated data for the aircraft flight profile is used in conjunction with the distance to the centroid (defined in section V, fifth assumption) to transform the lethal overpressure contour in the lethal area plane into a locus of points describing the boundary of the area reflecting aircraft kill from overpressure. The lethal thermal...
contour remains unchanged since the thermal energy propagation time is negligible. The entire process of lethal area determination takes place in the lethal area plane.

A top view of a typical lethal area plane containing a lethal overpressure contour, a lethal thermal contour, and a centroid (designated by an asterisk) is offered in figure 6. The lethal overpressure contour is represented by the dashed-line circle, the lethal thermal contour by the solid-line circle, and the perpendicular point projection of the burst center onto the lethal area plane by the symbol X. The remaining task is to adjust the overpressure contour to account for the movement of the aircraft.

![Figure 6. Top View of Lethal Area Plane](image)

Points lying on the lethal overpressure contour are used to determine the boundary for the overpressure lethal locus, that is, the lethal area at the onset of the burst associated with the overpressure kill mechanism. The aircraft are assumed to be emanating radially from a point called the centroid. The points on the lethal overpressure contour are backed up radially toward the centroid by the distance flown between detonation and the arrival of the shock wave at the overpressure contour. This distance is obtained by interpolation from the distance/time coordinates representing the aircraft flight profile. This radial translation of the overpressure contour toward the
centroid usually results in a petal or egg-shaped overpressure lethal locus as shown in figure 7. More complex shapes may result when the centroid is within the lethal overpressure contour.

The overpressure lethal locus encompasses the overpressure kill region at the time of detonation under the cookie-cutter assumption for a moving aircraft. An underlying assumption is that the aircraft maintains radial flight from the centroid. Since the aircraft is constrained by the aircraft flight profile, if within the overpressure lethal locus at the onset of the burst, it will be intercepted by the supersonically propagating shock front at a higher level of overpressure than it can withstand. A possibility exists that the aircraft could be located within the lethal thermal contour, as well as within the overpressure lethal locus, at the onset of the detonation.

The overpressure lethal locus is combined with the lethal thermal contour to produce the boundary of the lethal area. Figure 8 gives an example of a lethal circle/petal area with respect to a moving aircraft. Numerical integration is used to compute the area within this lethal region.
6. DETERMINATION OF SURVIVABILITIES

The values $S_{ij}$, which appear in the objective function (maximizing the expected value killed), must be computed for each weapon in group $j$ against each target $i$. Assuming that the aircraft at each target are uniformly distributed over some area at each weapon arrival time, the probability of kill, $P_k$, of each aircraft at that target is the quotient of the lethal area divided by the area in which the aircraft could be located when the weapon arrives. In this report, the $P_k$ will be defined as the probability of kill given that the weapon successfully detonated at the target. In QUANTO, reliability factors are given for each missile type for the probabilities that (1) the missile is successfully launched, (2) the missile successfully reaches the target, and (3) the warhead successfully detonates. The overall reliability of a missile is the product of these three reliabilities. The survivability of a target from a single weapon is then

\[
\text{Survivability} = 1 - P_k \times \text{(reliability)}
\]

The $P_k$ of each weapon versus each target is computed from input data in QUANTO. Initially, the arrival time of the missile on the target is computed. Time zero is the time at which all of the first missiles from each submarine are simultaneously launched. Subsequent salvos from the submarines are launched after time zero, as determined by the salvo number and the missile launch.
interval. The flight time of a missile to a given target is interpolated from input distance/time missile trajectory information after the distance from launch point to target (i.e., the coordinates of the base) is computed. The arrival time of the weapon on the target is the sum of the launch time and the flight time.

The location of the aircraft at the time of weapon arrival may be computed from the input aircraft flight profile and the brake release times. The aircraft are assumed to disperse radially from a single point, called the centroid. Unless aircraft are assumed to be departing in both directions from a base (from dual runways, for instance), the centroid will not be on the runway, for the centroid's location is a function of the time it takes an aircraft to raise its gear and flaps, reach a turn altitude, etc., and then make a turn to its fly-out direction. The distance an aircraft will be from the centroid at weapon arrival time is computed by (1) subtracting the brake release time from the weapon arrival time to obtain the time the aircraft has had to escape before the weapon arrives, (2) interpolating in the aircraft flight profile to obtain the distance the aircraft has traveled from brake release, and (3) subtracting the distance from brake release to centroid from the total distance traveled. The distance to the centroid from brake release point is input for each target. In this manner, QUANTO computes:

\[ R_1 = \text{the radial distance from the centroid of the first aircraft at weapon arrival time} \]

and

\[ R_N = \text{the radial distance from the centroid of the last aircraft at weapon arrival time} \]

If an aircraft has either not begun its takeoff or not reached the centroid, its radial distance from the centroid is set to zero. The intent here is to treat all aircraft which have not reached the centroid as essentially undispersed aircraft which can be targeted with a single SLBM.

The area of kill generated by a warhead detonation for aircraft of a given type is dependent on many parameters, as described in the preceding section of this report. Many of these parameters are needed to describe the nuclear environment and are directly supplied by user inputs. First, the horizontal
ranges are determined at which a stationary receiver would experience a lethal overpressure or thermal effect. Then the assumed detonation point of the SLBM, the aircraft climb profile, and the distance from brake release point to the centroid are used as described in the previous section to determine the shape of the circle/petal thermal/overpressure lethal area at detonation time, thus taking into account the moving receiver (aircraft). The lethal area varies somewhat with the distance of the detonation from the centroid, because the aircraft are at different altitudes and velocities at different points in the climb profile. Hence, an approximation must be made of the lethal area used in the calculation of $P_k$, and QUANTO must make some assumption about where the weapon might land without having determined yet how many total SLBMs will be allocated to the target.

QUANTO assumes that the attacker can compute $R_1$, assuming a certain brake release time, and accepts this as the farthest distance that the first aircraft on a base can achieve by weapon arrival time. However, although the attacker might also be able to compute $R_N$ based on the stipulated aircraft takeoff intervals and brake release time, he would realize that unanticipated delays could occur (in detection and warning of attack, etc.) and might consider it equally likely to find aircraft at any point within the circle of radius $R_1$. Consequently, QUANTO attacks that area, in general, with a uniformly dense distribution of weapons. Thus, an average weapon might land at a distance $(R_1/\sqrt{2})$ from the centroid because a circle of radius $(R_1/\sqrt{2})$ contains half the area within the circle of radius $R_1$. One benefit of this uniform attack is that a delay in the brake release time will not significantly decrease the expected kill and may result in a large increase in kill. When the SLBM detonates at distance $(R_1/\sqrt{2})$ from the centroid and the entire lethal circle/petal area falls within the circle of radius $R_1$, the $P_k$ is simply

$$P_k = \frac{A_L}{\pi R_1^2} \text{ for } R_1/\sqrt{2}$$

where

$$A_L \text{ for } R_1/\sqrt{2}$$

indicates the lethal area when the detonation point is at range $(R_1/\sqrt{2})$ from the centroid. This situation is illustrated in figure 9.
The use of equation (24) results in an overestimate of the expected kill of aircraft in the case of very few aircraft on a base and a lethal area which is a large portion of the circular area $\pi R_1^2$. With few (or one) aircraft, $R_N$ may be only slightly less than (or equal to) $R_1$. This situation is shown in Figure 10 for the lethal area labeled A. In this case, equation (24) predicts a large percentage of the aircraft killed, although when the brake release time is certain, no kills result. To guard against this possibility, the attacker would wish to reduce his estimate of $P_k$ in allocating his attacking weapons. One way in which he could do this would be to replace the procedure of the previous paragraph with one in which the weapon was placed at distance $(R_1 + R_N)/2$ from the centroid, assuming the aircraft were uniformly distributed throughout the annulus of radii $R_1$ and $R_N$. This $P_k$ is the shaded area within the circle/petal labeled B in Figure 10, divided by the annulus area $\pi (R_1^2 - R_N^2)$. The shaded area within B may be approximated by considering the circle/petal B as a circle of equivalent area centered at $(R_1 + R_N)/2$ radial distance from the centroid, and computing the area within both the equivalent circle and the annulus thickness. This latter common area, which will be labeled $A_{LAN}$ [for $(R_1 + R_N)/2$], may be computed from closed-form geometric expressions. The single aircraft situation is handled in this manner by artificially setting $R_N = R_1 - 0.01$ so that the annulus has a small positive area. The $P_k$ formula thus becomes

$$P_k = \min \left( \frac{A_L \ [\text{for } R_1/\sqrt{2}]}{\pi R_1^2}, \frac{A_{LAN} \ [\text{for } (R_1 + R_N)/2]}{\pi R_1^2 - R_N^2} \right)$$

(24)
The use of this formula, when the second quotient is the minimum of the two, encourages QUANTO to allocate a second weapon to the target since the first weapon's $P_k$ is lowered.*

As shown in the previous figures, the thermal lethal circle usually extends farther from the centroid than does the overpressure lethal petal. Likewise, when the SLBM detonates at the centroid, the thermal circle usually extends farther from the centroid than does the region of overpressure kill, which in this special case is also a circle. "Usually" in this context means for those combinations of flight profile and overpressure/thermal vulnerability levels normally of interest. However, for some hardness levels and aircraft flight profiles, the region of overpressure kill may totally encompass the thermal lethal circle.

The farthest reach of the lethal region in a direction away from the centroid becomes a concern when $R_i$ is small enough that the lethal area may protrude beyond $R_i$ for a given weapon placement. When the lethal area so protrudes, the

*The validity of the method implicit in the second term, and indeed of the whole procedure in equation (24), has been confirmed by comparing answers obtained from QUANTO with those obtained using a Monte Carlo simulation model.
first expression competing for the minimum in equation (24) is in error because the aircraft cannot be located in the protruding portion of the lethal area. To handle these cases, the weapon is assumed to detonate at a position for which no protrusion occurs, if possible.

Therefore, QUANTO computes

\[ R_{LR} \text{[for } R_1/\sqrt{2} \text{]} = \text{the distance from detonation point to lethal region boundary, in a direction away from the centroid, for a detonation at distance } (R_1/\sqrt{2}) \text{ from the centroid.} \]

It will usually be true that

\[ R_{LR} \text{[for } R_1 - R_{LR} \text{[for } R_1/\sqrt{2} \text{]} \approx R_{LR} \text{[for } R_1/\sqrt{2} \text{]} \]

i.e., \( R_{LR} \) varies little as the detonation point is adjusted to avoid protrusion.

Now, if

\[ R_1 \leq R_{LR} \text{[for } 0 \text{]}, \]

then QUANTO sets \( P_k = 1 \)

But if

\[ R_1 > R_{LR} \text{[for } 0 \text{]} \]

and

\[ (R_1/\sqrt{2}) + R_{LR} \text{[for } R_1/\sqrt{2} \text{]} > R_1 \text{ (i.e., the lethal area protrudes beyond } R_1) \]

then QUANTO computes the \( P_k \) as

\[
P_k = \min \left\{ \frac{A_L \text{[for } R_1 - R_{LP} \text{[for } R_1/\sqrt{2} \text{]}]}{\pi R_1^2}, \frac{A_{LAN} \text{[for } (R_1 + R_N)/2 \text{]}]}{\pi (R_1^2 - R_N^2)} \right\}
\]

where the first quotient assumes placement of the weapon at

\[ (R_1 - R_{LR} \text{[for } R_1/\sqrt{2} \text{]}) \]

in order to avoid protrusion.
Thus, the complete formula for $P_k$ is

$$P_k = \begin{cases} 
1 & \text{if } R_1 \leq R_{LR} \text{ [for 0]} \\
\min \left\{ \frac{A_L \text{ [for } R_1 - R_{LR} \text{ [for } R_1/\sqrt{2} \text{]}}{\pi R_1^2}, \frac{A_{LAN} \text{ [for } (R_1 + R_N)/2 \text{]}}{\pi (R_1^2 - R_N^2)} \right\} & \text{if } R_1 > R_{LR} \text{ [for 0] and } R_1/\sqrt{2} + R_{LR} \text{ [for } R_1/\sqrt{2} \text{] > } R_1 \\
\min \left\{ \frac{A_L \text{ [for } R_1/\sqrt{2} \text{]}}{\pi R_1^2}, \frac{A_{LAN} \text{ [for } (R_1 + R_N)/2 \text{]}}{\pi (R_1^2 - R_N^2)} \right\} & \text{otherwise} \end{cases} \quad (26)$$

It may be noted in the above formula that the weapon placement may be at distances from the centroid of 0 (when $P_k = 1$), $R_1 - R_{LR}$ [for $R_1/\sqrt{2}$], $(R_1 + R_N)/2$, or $R_1/\sqrt{2}$. This will be of concern when more than one aircraft type is considered in the model.

It should be noted that these formulas for $P_k$ are inaccurate when the distribution of aircraft is far from uniform over an area, as might be the case if a number of aircraft had not left the base by the time of a weapon arrival. This situation will not occur if the aircraft beddown is a rational one, intended to prevent mass kills by single weapons. Because of the assumption of uniform distribution of aircraft, QUANTO will underestimate the aircraft kills in these situations. However, a simulation program may be used to discover if such conditions exist and to estimate the resultant kills.

If the aircraft were actually uniformly distributed over the areas assumed, the actual $P_k$ values (and, hence, $S_{ij}$ values) realized by the allocated weapons would not agree exactly with those computed by formulas (23) and (24). This is due to the impossibility of determining the realized lethal area sizes before determining the number of weapons (and, therefore, the precise placement of weapons) on each target. The computed weapon allocation is optimal for the $P_k$ values computed. However, the plus and minus errors between realized and
computed $P_k$'s for individual weapons tend to balance out to a small overall error when summed over all the weapons. This is because the computations of $P_k$'s are based on an average placement of each weapon on each target.

It has been assumed that the survivabilities $S_{ij}$ are independent. Thus, no weapon on target $i$ can cause collateral damage on aircraft from another base, and the area purged of aircraft by a detonation can become populated to an equal aircraft density by other aircraft before the next weapon arrives. Detailed base-by-base simulation of the attacks produced by QUANTO has shown that the actual resultant kill (output from the simulator) is not significantly different from QUANTO's predicted kill.
SECTION IV

THE COMPUTER PROGRAM

Figure 11 shows, in general terms, the operation of QUANTO. After the data for the problem is input, areas of lethality and the survivabilities $S_{ij}$ are computed. The optimal missile laydown $n_{ij}$ is then determined. QUANTO will then relocate a submarine to a better position, if the user has requested submarine optimization, and recompute the optimal $n_{ij}$ for the new positions of the submarines. After submarine optimization is completed, aircraft may be relocated to improve the number surviving, with $n_{ij}$ recomputed following each shift of aircraft. When beddown has been optimized, the optimal nonintegral $V_i$ and $n_{ij}$ are integerized and final output is produced.

Figure 12 indicates several additional details of QUANTO. The criteria for terminating submarine optimization and beddown optimization are indicated in test blocks. A mode parameter, input on the first data card of a problem deck, controls from whence input is taken and how much of the program is executed. Table II described the mode options. These options permit the user to observe partial computations for validity without risking a large expenditure of computer time.

The principal subroutines of QUANTO (QUANTO being the name of the main program) and their functions are listed in table III.
Figure 12. Detailed QUANTO Flow Chart
Figure 12. Detailed QUANTO Flow Chart (cont'd)
Table II
MODE OPTIONS

<table>
<thead>
<tr>
<th>Mode</th>
<th>QUANTO Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Input all data except convergence parameters and initial allocation $n_{ij}$ from cards. Terminate problem after computing nuclear effects and constructing flight profiles.</td>
</tr>
<tr>
<td>1</td>
<td>Input all data from cards. Terminate after computing survivabilities $S_{ij}$ and kills resulting from initial allocation. Write information for automatic program restart on tape.</td>
</tr>
<tr>
<td>2</td>
<td>Input all data (except parameters on first card) from restart tape. Terminate problem computations prior to completion only if the time limit for processing is reached, at which time a restart tape will be written.</td>
</tr>
<tr>
<td>3</td>
<td>Input all data from cards. Terminate problem as for mode = 2.</td>
</tr>
</tbody>
</table>
Table III
PRINCIPAL ROUTINES AND THEIR FUNCTIONS

<table>
<thead>
<tr>
<th>Routine Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUANTO</td>
<td>Main Program. QUANTO reads the input data, either from cards or from a restart tape, as controlled by the mode parameter on the first input card of each problem deck. The data describing the problem are printed, and if the data are read from cards, a summary of the input data is also printed. The aircraft profile and parameters affecting nuclear effects are not read by QUANTO, but by PROCESS, called by QUANTO. Computations of survivabilities $S_{ij}$ are performed mainly in QUANTO. DETAREA provides QUANTO the necessary lethal areas, but QUANTO computes flight times and distances and the resultant set of $P_k$ and $S_{ij}$ values, with the help of interpolation, look-up, and distance computational routines. After input of the initial allocation, QUANTO controls the sequencing of operations in the iterative procedure for optimizing the missile laydown. When provided $\Delta n$ by ADJLAM, QUANTO recomputes $n_{ij}$, $S_{ij}$, $\lambda_{ij}$, and $f(n_{ij})$. Control of the iteration cutoff and intermediate output is accomplished in QUANTO. Re-location of aircraft is completely performed in QUANTO, but QUANTO calls other routines for submarine optimization and integerization of laydown and beddown.</td>
</tr>
</tbody>
</table>
Table III (cont'd)

<table>
<thead>
<tr>
<th>Routine Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROCESS</td>
<td>Called by QUANTO. PROCESS reads aircraft profile data and nuclear effects parameters from cards for each type of aircraft. PROCESS generates distance/time coordinates for each aircraft for the specific altitude of level-off. The input data, as well as the generated distance/time coordinates, are output. If the lethal overpressure and/or thermal radius and the time of shock arrival are not present in the input, PROCESS computes these values for the yield of each type of missile. The nuclear effects information is summarized in the output from PROCESS.</td>
</tr>
<tr>
<td>DETAREA</td>
<td>Called by QUANTO. From given aircraft profiles and geometry of flyout and detonation, DETAREA computes the lethal area with respect to the moving aircraft, i.e., the circle/petal area describing the thermal/overpressure kill region.</td>
</tr>
<tr>
<td>ALOUT</td>
<td>Called by QUANTO. ALOUT produces a list of the allocation $n_{ij}$ in two formats. First, by target: the missiles allocated to that target are listed in order by submarine number and salvo number within the submarine. Second, by submarine: the missiles are listed in order by salvo, together with the targets to which they are allocated.</td>
</tr>
</tbody>
</table>
Table III (cont'd)

<table>
<thead>
<tr>
<th>Routine Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALINT</td>
<td>Called by QUANTO. ALINT integerizes the allocation matrix ( n_{ij} ). This process is not a simple rounding of the nonintegral ( n_{ij} ) values, but an assignment of integral values to the highest fractional parts so as to make ( \sum_{i=1}^{M} n_{ij} = N_j ) for each ( j ).</td>
</tr>
<tr>
<td>VINT</td>
<td>Called by QUANTO. VINT integerizes the beddown values ( V_i ) in a manner similar to ALINT.</td>
</tr>
<tr>
<td>TGTKIL</td>
<td>Called by QUANTO. TGTKIL computes the survivability products ( \prod_{j=1}^{S} \frac{n_{ij}}{S_{ij}} ) (for each aircraft type), the ( \lambda_{ij} ) values, the number of aircraft killed at each base, and a rough idea (obtained by rounding ( n_{ij} ) values) of the number of weapons allocated to each base.</td>
</tr>
<tr>
<td>ADJLAM</td>
<td>Called by QUANTO. ADJLAM finds ( \Delta n ) by first finding ( \lambda_{kz} = \min_{i} { \lambda_{i,z} \text{ such that } n_{i,z} \geq 0.0001 } ) ( i = 1, 2, \ldots, M ), and ( \lambda_{mz} = \max_{i} { \lambda_{i,z} } ) ( i = 1, 2, \ldots, M ), such that ( \lambda_{kz} &lt; \lambda_{mz} - \varepsilon ) for some weapon group ( z ) and some tolerance ( \varepsilon ). Then ADJLAM either computes the proper allocation adjustment ( \Delta n ) to force the value of ( \lambda_{kz} ) toward the value of ( \lambda_{mz} ) (for the single type aircraft model) or calls the function XNEWT to compute ( \Delta n ) (for the mixed force model). Tallies of weapons allocated to each base are updated after the change of ( n_{ij} ) by the ( \Delta n ) adjustments.</td>
</tr>
</tbody>
</table>
Table III (cont'd)

<table>
<thead>
<tr>
<th>Routine Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUBADJ</td>
<td>Called by QUANTO. SUBADJ locates the least effective submarine and the submarine location with the most potential, as described in the test on submarine optimization. The submarine is relocated to a better position and its missiles are allocated to bases having high $\lambda_{ij}$ values, as described in the text.</td>
</tr>
<tr>
<td>XAREA</td>
<td>Called by QUANTO. XAREA computes the area of intersection of a circle and an annulus, under all conditions of annulus radii, circle radius, and offset of circle center.</td>
</tr>
</tbody>
</table>
SECTION V
ASSUMPTIONS

In this section, the principal assumptions in QUANTO are described and are briefly discussed.

Assumption: All input SLBMs are used against aircraft, i.e., the attacking force decides what portion of its SLBMs to use against the flushing aircraft force prior to running a problem and the SLBMs in QUANTO represent that portion. Of course, for each submarine, only a partial load of missiles need be input for a problem.

Assumption: The survival probabilities, $S_{ij}$, are independent. No collateral damage may affect an aircraft departing one base as a result of a detonation of an SLBM allocated to another base. Furthermore, the effectiveness of a weapon in group $j$ on base $i$ is measured by $S_{ij}$, a numerical quantity which is independent of the number of weapons which have previously arrived or will subsequently arrive. Stated differently, the area in which aircraft may be located at the time of a later weapon arrival is not considered to contain voids left by previously arriving weapons. Detailed Monte Carlo simulation of QUANTO-produced attacks shows that QUANTO's predicted kill (using such $S_{ij}$ values) is close to the actual kill resulting from the simulation.

Assumption: Thermal and overpressure effects have lethality according to a cookie-cutter criterion. In other words, an aircraft with hardness of $x$ psi and $y$ cal/cm$^2$ is killed if it experiences either of these levels or higher, but is safe from $(x-\epsilon)$ psi and $(y-\epsilon)$ cal/cm$^2$ for any $\epsilon > 0$, no matter how small.

Assumption: At all times, aircraft are uniformly distributed within a maximum circle, defined by the first aircraft's range, the area of which is continually increasing with time. Thus, the survivabilities $S_{ij}$ are computed assuming the attacker will pattern his weapons for uniform coverage of the maximum circle of aircraft. In some cases (few aircraft at early weapon arrival times), this assumption is modified to allow computation of $S_{ij}$ by assuming instead that the aircraft are uniformly distributed throughout an annulus. In this way, a lower
computed $S_{ij}$ results and more realistic expected kills result. Of course, if the aircraft do not disperse in a circular pattern, and the attacker were to be granted advance knowledge of these flyout tactics, a greater expected kill would result since the weapons have a smaller area to attack.

**Assumption:** The aircraft radially emanate from a point called the centroid of the aircraft which, for a given flyout profile and turn geometry, may be determined. The distance of the centroid from brake release point is a parameter which may be input for each base, it is based on the distance the aircraft flies without turning while raising its gear, climbing to turn altitude, etc.

**Assumption:** In the computation of $S_{ij}$, the detonation point of the weapon is at one of several places, as described in another portion of this report. The assumption of detonation point is such as to be in agreement with the uniform-attack-of-the-aircraft-area assumption, with a modification of the location (1) when protrusion of the lethal area beyond the maximum circle occurs, (2) when an annular $P_k$ computation yields a better estimate of $S_{ij}$, or (3) when a weapon on the centroid kills all aircraft of a single type. In this way, some pains are taken to compute $S_{ij}$ based on a reasonable estimate of the weapon location, without knowledge of where other weapons are allocated.

**Assumption:** When multiple aircraft types are included in the model, all aircraft radially emanate from a single centroid and weapons are patterned to attack uniformly the area of all aircraft types. Point values of aircraft of different types may make some bases more attractive than others; but on those bases, the attack is assumed to be uniform.
SECTION VI
LAGRANGE MULTIPLIERS IN THE MIXED FORCE ALLOCATION PROBLEM

When more than one type of aircraft may be leaving each base, the problem of determining the optimal missile laydown is more complicated than the previously described model. For purposes of explanation, the following will assume two aircraft types, bombers and tankers, indicated by B and T subscripts, respectively. However, the procedures are general for any number of aircraft types. The objective to be maximized in the mixed force allocation problem is

$$f(n_{ij}) = \sum_{i=1}^{M} V_{iB} \left[ 1 - \prod_{j=1}^{L} S_{ijB}^{n_{ij}} \right] + \sum_{i=1}^{M} V_{iT} \left[ 1 - \prod_{j=1}^{L} S_{ijT}^{n_{ij}} \right]$$ (27)

with the same stockpile constraints

$$\sum_{j=1}^{M} n_{ij} = N_j, \quad i = 1, 2, \ldots, L$$

The function $f(n_{ij})$ is now the expected kill value of both bombers and tankers with a sum over the target index $i$ for each type of aircraft. The values of the bombers and tankers, respectively, leaving base $i$ are $V_{iB}$ and $V_{iT}$, where each type aircraft may be worth a different amount of value per aircraft. The survivabilities $S_{ijB}$ and $S_{ijT}$ of the bombers and tankers, respectively, of target $i$ from a weapon in group $j$, must be computed slightly differently than the previous $S_{ij}$.

Since the types of aircraft may have different thermal and overpressure hardness levels, there is a circle/petal combination for each aircraft type at a single weapon detonation point. Using the same notations as before, with the additional B (bomber) and T (tankers) subscripts (following the slashes), the following formulas for $P_{k/B}$ and $P_{k/T}$ are used for the computation of bomber and tanker $P_k$'s.

In the mixed force allocation problem, it will be assumed that the SLBMs are aimed uniformly at the entire area of all aircraft (of all types). When the geometry is such that the tankers are within the bombers, as shown in figure 13,
the \( P_{k/T} \) may be approximated by the product of (1) the probability that a random placement within the bomber circle lands within the tanker circle, and (2) the probability of kill of a tanker given that the weapon detonates within the tanker circle. If \( A_{L/T} \) represents the lethal area of the SLBM against tankers, the product is

\[
P_{k/T} = \frac{\pi R_T^2}{\pi R_B^2} \times \frac{A_{L/T}}{\pi R_T^2} = \frac{A_{L/T}}{\pi R_B^2}
\]

(28)

where overlap of the lethal area over the circle of radius \( R_T \) has been ignored. Thus, for these assumptions and approximations, \( P_{k/T} \) is independent of \( R_T \).

Should the bombers be within the tankers,

\[
P_{k/B} = \frac{A_{L/B}}{\pi R_T^2}
\]

(29)

so the maximum radius always appears in the denominator.
AFWL-TR-73-242

For the mixed aircraft model, let \( R_1 \) and \( R_N \) represent the radial distances of the most and least distant aircraft, respectively, from the single centroid (the same point for all aircraft types) at the time of weapon arrival. The subscript MAX means the largest for all aircraft types, and other notations are analogous to those used in the discussion of the single type aircraft model.

A. If

\[ R_1 \leq R_{LR/\text{MAX}} \text{ [for 0]} \]  

then

\[ P_{k/T} = \min \left\{ 1, \frac{\pi (R_{LR/T}^2 - R_N^2)}{\pi (R_1^2 - R_N^2)} \right\} \]  

and

\[ P_{k/B} = \min \left\{ 1, \frac{\pi (R_{LR/B}^2 - R_N^2)}{\pi (R_1^2 - R_N^2)} \right\} \]  

assuming that the weapon detonates at the centroid.

B. If

\[ R_1 > R_{LR/\text{MAX}} \text{ [for 0]} \]

and

\[ R_1/\sqrt{2} + R_{LR/\text{MAX}} \text{ [for } R_1/\sqrt{2} \text{]} \leq R_1 \]

so that neither circle/petal lethal area protrudes beyond \( R_1 \), then

\[ (1) \quad P_{k/T} = \min \left\{ 1, \frac{A_{L/T} [\text{for } R_1/\sqrt{2}]}{\pi R_1^2} \right\} \]  

and

\[ P_{k/B} = \min \left\{ 1, \frac{A_{L/B} [\text{for } R_1/\sqrt{2}]}{\pi R_1^2} \right\} \]  

45
if either

\[
\frac{A_{LAN/T} \left[\text{for } (R_1 + R_N)/2\right]}{\pi \left(R_1^2 - R_N^2\right)} > \frac{A_{L/T} \left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^2}
\]  \quad (36)

or

\[
\frac{A_{LAN/B} \left[\text{for } (R_1 + R_N)/2\right]}{\pi \left(R_1^2 - R_N^2\right)} > \frac{A_{L/B} \left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^2}
\]  \quad (37)

Otherwise,

\[
P_{k/T} = \frac{A_{LAN/T} \left[\text{for } (R_1 + R_N)/2\right]}{\pi \left(R_1^2 - R_N^2\right)}
\]  \quad (38)

and

\[
P_{k/B} = \frac{A_{LAN/B} \left[\text{for } (R_1 + R_N)/2\right]}{\pi \left(R_1^2 - R_N^2\right)}
\]  \quad (39)

In words, if the annulus P_k computations in equation (38) above are both smaller than their corresponding circular P_k computations in equation (34), then the annulus P_k formulas are used for all aircraft types. Otherwise, the formulas in equation (34) are used.

C. Finally, if either circle/petal protrudes when positioned at R_1/\sqrt{2}, the weapon is moved toward the centroid as before and

\[
P_{k/T} = \frac{A_{L/T} \left[\text{for } R_1 - R_{LR/\text{MAX}} \left[\text{for } R_1/\sqrt{2}\right]\right]}{\pi R_1^2}
\]  \quad (40)

and

\[
P_{k/B} = \frac{A_{L/B} \left[\text{for } R_1 - R_{LR/\text{MAX}} \left[\text{for } R_1/\sqrt{2}\right]\right]}{\pi R_1^2}
\]  \quad (41)

if either
\[
\frac{A_{LAN/T} \left[ \text{for } \frac{R_1 + R_N}{2} \right]}{\pi (R_1^2 - R_N^2)} > \frac{A_{L/T} \left[ \text{for } R_1 - \frac{R_{LR/\text{MAX}}}{2} \right]}{\pi R_1^2}
\] (42)

or

\[
\frac{A_{LAN/B} \left[ \text{for } \frac{R_1 + R_N}{2} \right]}{\pi (R_1^2 - R_N^2)} > \frac{A_{L/T} \left[ \text{for } R_1 - \frac{R_{LR/\text{MAX}}}{2} \right]}{\pi R_1^2}
\] (43)

Otherwise,

\[
P_{k/T} = \frac{A_{LAN/T} \left[ \text{for } \frac{R_1 + R_N}{2} \right]}{\pi (R_1^2 - R_N^2)}
\] (44)

and

\[
P_{k/B} = \frac{A_{LAN/B} \left[ \text{for } \frac{R_1 + R_N}{2} \right]}{\pi (R_1^2 - R_N^2)}
\] (45)

It should be noted that these formulas have been written in a slightly different form to ensure that the \(P_k\) computations for different aircraft types are all based on the same placement of the weapon.

The survivabilities are simply

\[
S_{ijB} = 1 - P_{k/B} \times \text{(reliability of weapon in group j)}
\]

\[
S_{ijT} = 1 - P_{k/T} \times \text{(reliability of weapon in group j)}
\]

The technique for solution of the constrained maximization problem for the mixed aircraft force is very similar to the techniques for the previous model. The Lagrangian function for the new objective function \(f(n_{ij})\) is

\[
h(n_{ij}, \lambda_j) = f(n_{ij}) + \sum_{j=1}^{L} \lambda_j \left[ \sum_{i=1}^{M} n_{ij} - N_j \right]
\] (46)

Setting the partial derivatives equal to zero, as before, yields

\[
\frac{\partial h}{\partial n_{k\ell}} = \lambda_{k\ell B} + \lambda_{k\ell T} + \lambda_{\ell} = 0
\]

\[
k = 1, 2, \ldots, M; \ell = 1, 2, \ldots, L
\] (47)
where
\[ \lambda_{k=l} = -V_{kB} \left( \sum_{j=1}^{L} S_{kjB} \right) \prod_{j=1}^{n} S_{kjB} \]  
(48)

and
\[ \lambda_{k=l} = -V_{kT} \left( \sum_{j=1}^{L} S_{kjT} \right) \prod_{j=1}^{n} S_{kjT} \]  
(49)

For the mixed aircraft model, the new definition of \( \lambda_{k=l} \) is
\[ \lambda_{k=l} = \lambda_{k=l}^{B} + \lambda_{k=l}^{T} \]  
(50)

or in the case of more than two aircraft types, \( \lambda_{k=l} \) would be the sum of the lambdas corresponding to each aircraft type. Fixing \( \epsilon \) and letting \( k \) vary results in the system of equations
\[ \lambda_{k=l}^{B} = -\lambda_{l}^{B}, \quad k = 1, 2, ..., M \]

which has the same appearance as in the single aircraft model, although \( \lambda_{k,l} \) is differently defined.

The iterative procedure is again based on finding \( \lambda_{k,l} < \lambda_{l}^{B} \) with \( n_{k,l} \geq 0.0001 \), and choosing \( \Delta n \) so that the new values of \( \lambda_{k,l} \) and \( \lambda_{m,l}^{B} \), say, \( \lambda_{k,l}^{B} \) and \( \lambda_{m,l}^{B} \), become equal. This value of \( \Delta n \) is the root of

\[
g(\Delta n) = \lambda_{k,l}^{B} - \lambda_{m,l}^{B}
\]

\[= \sum_{k} \lambda_{k=l}^{B} - \lambda_{m,l}^{B}
\]

\[= S_{k,l}^{B} - \Delta n \lambda_{k,l}^{B} + S_{k,l}^{T} - \Delta n \lambda_{k,l}^{T} - S_{m,l}^{B} + \Delta n \lambda_{m,l}^{B} - S_{m,l}^{T} + \Delta n \lambda_{m,l}^{T}
\]

(51)

The root of \( \Delta n^{*} \) of \( g(\Delta n) \) is found by Newton's successive approximation method, where

\[ \Delta n_{i+1} = \Delta n_{i} - \frac{g(\Delta n_{i})}{g'(\Delta n_{i})}, \quad i = 1, 2, .... \]  
(52)
In QUANTO, when $|\Delta n_{i+1} - \Delta n_i| < \epsilon$ (the same tolerance used in the test for convergence of $\lambda_{k \ell}$ and $\lambda_{m \ell}$), $\Delta n^*$ is set equal to $\Delta n_{i+1}$. This iterative formula may diverge (as tested by $|\Delta n_{i+1}| > 200$ in QUANTO) for a given selection of $\Delta n_0$. Convergence is attempted for $\Delta n_0$ selections of $n_{k \ell}^*$, $0$, and $n_{k \ell}^*/2$, successively, until the iteration successively converges. If the root $\Delta n^* > n_{k \ell}^*$, $\Delta n$ is chosen as $n_{k \ell}^*$, i.e.,

$$\Delta n = \min \{\Delta n^*, n_{k \ell}^*\}$$

(53)

so as to keep $(n_{k \ell}^* - \Delta n)$ nonnegative.

This choice of $\Delta n$ results in the maximum increase in the expected kill. The proof of this statement is quite similar to the analogous proof in the single aircraft case. If the kill contribution to the objective function $f(n_{ij})$ from targets $k$ and $m$ after $\Delta n$ weapons are moved from target $k$ to target $m$ is indicated as $r(\Delta n)$, then the unconstrained maximum of $r(\Delta n)$ occurs where $r'(\Delta n) = 0$. But this is at $\Delta n^*$, the root of $y(\Delta n)$, since

$$r'(\Delta n) = -g(\Delta n)$$

(54)

The maximum of $r(\Delta n)$, constrained by $0 \leq \Delta n < n_{k \ell}^*$, occurs at $\Delta n = n_{k \ell}^*$ if $\Delta n^* > n_{k \ell}^*$, since

$$r'(0) = \hat{s}_{m \ell} - \hat{s}_{k \ell} > 0$$

and

$$r'' = -g'(\Delta n) \leq 0$$

The iterative procedure for the mixed aircraft model is exactly the same as that for the single aircraft model, with the exception of the new definition of $\lambda_{k \ell}$ and the Newton procedure for finding $\Delta n^*$. The submarine relocation procedure is also based on the new $\lambda_{k \ell} = \lambda_{k \ell B} + \lambda_{k \ell T}$.

The beddown optimization procedure is slightly changed in that each beddown change simultaneously moves some of each type of aircraft. Thus, for bombers, the value

$$\Delta V_B = V_{kB}^* \left( \prod_{j=1}^{L} s_{mjB}^{n_{mj}} - \prod_{j=1}^{L} s_{kjB}^{n_{kj}} \right)$$

(55)
is shifted from base $k$ to base $m$ where these bases have, respectively, the lowest and highest survivability products

$$\prod_{j=1}^{L} S_{ij}^{B}$$

but only those bases $k$ for which $V_{kB} > 0.0001$ compete for the lowest product. The value of tankers shifted, $\Delta V_T$, (computed like $\Delta V_B$, replacing "B" subscripts with "T" subscripts) depends on the tanker survivabilities $S_{ijT}$ and values $V_{iT}$, and thus, bomber and tanker relocations may involve different pairs of bases. The beddown optimization stops when the total number of aircraft (of all types) to be moved in a single beddown change is less than 0.05.
SECTION VII
BENEFITS AND FUTURE USES OF QUANTO

QUANTO was developed to investigate the sensitivity of total bomber force survivability to variations in thermal and overpressure hardness. In the process of this development, numerous other parameters have been included as variables in the model. Consequently, the model is useful for evaluating the sensitivity of surviving aircraft to changes in aircraft beddown and flight profiles, numbers and types of submarines or missiles, SLBM performance characteristics, and reaction times, as well as aircraft hardness. Alternative missile laydowns, submarine locations, and aircraft beddowns may be compared and evaluated using QUANTO to compute expected kills. Contractual studies may be evaluated for validity and contrasted with QUANTO to aid in understanding their results. In-house and intra-AF investigations are facilitated by the availability of QUANTO.

QUANTO has a great deal of flexibility. It is relatively fast and easy to use compared to other flush models. An optimal laydown may be computed in 1 to 3 minutes of computer time, submarines may be optimized in about 5 minutes, and optimal beddowns require up to 10 minutes, where these times are largely dependent on the quality of the selection of initial laydowns, submarine positions, and beddowns. Each weapon is considered a separate entity, not as a member of one of a fixed set of predefined patterns. In addition, QUANTO permits multiple types of aircraft and SLBMs, each with its own performance characteristics. The modular construction of QUANTO permits investigation of selective changes in the assumptions upon which the model is based, with selective program changes.

The projected future uses of QUANTO include the evaluation of other models and results of flush studies, studies of the effects of parametric variations on the survivability of a mixed force, and in-house experimentation and sensitivity analysis.
APPENDIX I
APPLICATIONS OF LAGRANGE MULTIPLIERS
The Basic Problem

The computer program called QUANTO uses the Lagrange multiplier method to optimize the allocation of weapons. This appendix is provided to introduce the reader to the basic fundamentals of the technique.

The problem:
Maximize \( f(x_1, x_2, \ldots, x_n) \)
Subject to \( g_j(x_1, x_2, \ldots, x_n) = b_j, \ j = 1, 2, \ldots, m; \ m < n \)

Lagrange method:

Form the function:

\[
\begin{align*}
&h(x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots, \lambda_m) = f(x_1, x_2, \ldots, x_n) \\
& \quad + \sum_{j=1}^{m} \lambda_j \{g_j(x_1, x_2, \ldots, x_n) - b_j\}
\end{align*}
\]

where the \( \lambda_j \) are constants (known as Lagrange multipliers) as yet to be determined in value. Note that when the constraints are satisfied, \( h \) is formed merely by adding multipliers of zeros to \( f \). Now treat \( x_i, \ i = 1, 2, \ldots, n \), as independent variables, and write down the conditions

\[
\begin{align*}
\frac{\partial h}{\partial x_1} &= 0 \\
\frac{\partial h}{\partial x_2} &= 0 \\
\vdots & \quad \vdots \\
\frac{\partial h}{\partial x_n} &= 0
\end{align*}
\]
Solving the \((n + m)\) equations for the \(x_i\) and \(\lambda_j\) will yield the critical points of \(f\).

**EXAMPLE 1:**

Minimize \(f(x,y,z) = x^2 + y^2 + z^2\)

subject to the condition that \((x,y,z)\) is on the plane

\[ S = \{ (x,y,z): 2x + 3y - z - 1 = 0 \} \]

First introduce the new variable \(\lambda\) to form

\[ F(x,y,z,\lambda) = (x^2 + y^2 + z^2) + \lambda(2x + 3y - z - 1) \]

Now compute \(F_x, F_y, F_z,\) and \(F_\lambda:\)

\[ F_x = 2x + 2\lambda = 0 \]
\[ F_y = 2y + 3\lambda = 0 \]
\[ F_z = 2z - \lambda = 0 \]
\[ F_\lambda = 2x + 3y - z - 1 = 0 \]

These equations yield

\( x = \frac{1}{7},\ y = \frac{3}{14},\ z = -\frac{1}{14},\ \lambda = -\frac{1}{7} \)

The solution satisfies \(F_\lambda = 0\) and is, therefore, on the plane

\[ 2x + 3y - z - 1 + 0 \]
EXAMPLE 2:
Maximize \( f(A, B) = 6A + 2B + AB - A^2 - 2B^2 + 5 \)
subject to \( p(A, B) = 2A - B = 8 \)
Solve by finding the (local) optimum of
\[
h(A, B, \lambda) = f(A, B) + \lambda[p(A, B) - 8]
\]
\[
= 6A + 2B + AB - A^2 - 2B^2 + 5 + \lambda(2A - B - 8)
\]
Set partial derivatives to zero.
\[
\frac{\partial h}{\partial A} = 6 + B - 2A + 2\lambda = 0
\]
\[
\frac{\partial h}{\partial B} = 2 + A - 4B - \lambda = 0
\]
\[
\frac{\partial h}{\partial \lambda} = 2A - B - 8 = 0
\]
Solving yields
\[
A = \frac{33}{7}
\]
\[
B = \frac{10}{7}
\]
\[
\lambda = 1
\]
Thus
\[
f\left(\frac{33}{7}, \frac{10}{7}\right) = 16.5714
\]
Writing the constraint equation as \( 2A - B = \delta \) gives
\[
\frac{\partial h}{\partial \delta} = -\lambda = -1
\]
One can then see that increasing \( \delta \) has the effect of decreasing \( h \) (and, therefore, \( f \)) at the rate of -1 unit of \( f \) per unit of \( \delta \). Indeed, regardless of what \( h \) looks like, if the constraint is written as \( p = \delta \), then \( \frac{\partial h}{\partial \delta} \) will always equal \( -\lambda \).

Examples one and two are both performed in the same manner even though one is a maximum and the other is a minimization problem. The manner in which one would differentiate between which has occurred is by calculating the Hessian matrix.
APPENDIX II
QUANTO'S ITERATIVE PROCEDURE
Three Examples

This appendix is provided for the reader to become acquainted with the types of problems solved by QUANTO. The three examples serve to illustrate the three basic options available to the using organization. The type problems addressed are

1. Optimize \( n_{ij} \)
   Given: three targets
   three weapon groups

2. Optimize aircraft beddown
   Given: bomber/tanker mix
   ten tankers
   seven bombers

3. Optimize submarine locations

In cases 1 and 3 the optimal \( n_{ij} \) is found prior to the optimization of the aircraft beddown or submarine locations.

EXAMPLE 1

Suppose \( L = 3, M = 3 \), i.e., there are three targets and three weapon groups.
Let the number of weapons in each group be \( N_1 = 4, N_2 = 3, N_3 = 7 \) and suppose

\[
S_{ij} = \begin{bmatrix}
0.8 & 0.7 & 0.9 \\
0.6 & 0.5 & 0.7 \\
0.2 & 0.1 & 0.3
\end{bmatrix}
\]

This matrix represents the survival probabilities of target \( i \) from one weapon in group \( j \), e.g., the survival probability of target 2 from a single weapon in group 2 is 0.5. Let \( V_1, V_2, V_3 = 10, 5, 2 \), respectively. As the first step, an arbitrary allocation is formed, and suppose we choose
where, for example, it was decided to send three weapons from the 7 in group 3 to target 1. The $\lambda$ matrix for this $(n_{ij})$ is

$$
\begin{pmatrix}
-\ell n S_{11} & 3 & n_{1j} \\
\Pi_{j=1} S_{1j} & -\ell n S_{12} & 3 & n_{1j} \\
\Pi_{j=1} S_{1j} & -\ell n S_{13} & 3 & n_{1j}
\end{pmatrix}
$$

and the numbers compute to be

$$
(\lambda_{ij})_1 = \begin{pmatrix}
0.51 & 0.82 & 0.24 \\
0.98 & 0.51 & 0.2 \\
0.06 & 0.08 & 0.04
\end{pmatrix}
$$

The smaller $\lambda$'s associated with target 3 indicate that weapons have been over allocated there. Starting with column 1, i.e., the weapons in group one,

$$
\lambda_{31} < \lambda_{21}
$$

so that

$$
\Delta n = \frac{\ln \left( \frac{0.06}{0.38} \right)}{\ln (0.6)(0.2)} = 0.88
$$

This adjustment of weapons from target 3 to target 2 will increase the objective function, and will equate $\lambda_{31}$ and $\lambda_{21}$. The new $(n_{ij})$ is
\[
\begin{bmatrix}
2.00 & 2 & 3 \\
1.88 & 1 & 2 \\
0.12 & 0 & 2 \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
0.51 & 0.82 & 0.24 \\
0.24 & 0.32 & 0.17 \\
0.24 & 0.34 & 0.18 \\
\end{bmatrix}
\]

Since

\[
\lambda_{22} < \lambda_{12}
\]

\[
\Delta n = \frac{\ln 0.32}{\ln [(0.7)(0.5)]} = 0.88
\]

and

\[
\begin{bmatrix}
2.00 & 2.88 & 3 \\
1.88 & 0.12 & 2 \\
0.12 & 0.00 & 2 \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
0.37 & 0.60 & 0.17 \\
0.44 & 0.60 & 0.31 \\
0.24 & 0.34 & 0.18 \\
\end{bmatrix}
\]

Continuing in this manner, the Lagrange multipliers \( \lambda_{ij} \) corresponding to positive \( n_{ij} \) will converge to the unique \( \lambda_{ij} \) for each weapon group \( \lambda \). The final Lagrange multiplier matrix is

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which arises from the optimal allocation

\[\begin{bmatrix}
4 & 3 & 0 \\
0 & 0 & 5 \\
0 & 0 & 2 \\
\end{bmatrix}\]

which will give an expected target value return of 14.6 out of 17, the largest possible. This procedure is easily programmable. For very large matrixes (on the order of several thousand targets) there are more efficient procedures to adjust the multipliers by examining the convergence rates. For smaller cases on the order of a few hundred targets and weapons, the method above should not involve excessive computer time.

**EXAMPLE 2**

Suppose there are ten tankers and seven bombers, with a bomber twice the value of a tankers, bedded down as follows:

\[V_{iB} = \begin{bmatrix} 8 \\ 4 \\ 2 \end{bmatrix} \quad V_{iT} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}\]

and suppose further that \(N_1 = 4, N_2 = 3, N_3 = 7\) with

\[S_{ijB} = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.6 & 0.5 & 0.7 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}, \quad S_{ijT} = \begin{bmatrix} 0.7 & 0.6 & 0.8 \\ 0.5 & 0.4 & 0.6 \\ 0.2 & 0.1 & 0.3 \end{bmatrix}\]

where the \(S_{ij}\)'s are determined by SLBM yield, reliability, trajectory, and aircraft vulnerability, takeoff profile and sequence. We start by making an initial guess at the allocation of SLBMs to bases as follows:
Recalling that
\[ \lambda_{ijB} = -V_{iB} \left( \sum_{j} s_{ijB} \right) \prod_{j} s_{ijB}^{n_{ij}} \]
and
\[ \lambda_{ijT} = -V_{iT} \left( \sum_{j} s_{ijT} \right) \prod_{j} s_{ijT}^{n_{ij}} \]

\[ \lambda_{ij} = \lambda_{ijB} + \lambda_{ijT} \]

\[
\begin{bmatrix}
0.408 & 0.652 & 0.193 \\
0.300 & 0.407 & 0.210 \\
0.116 & 0.155 & 0.088
\end{bmatrix}
\]

\[ \lambda_{ijT} = \begin{bmatrix}
0.161 & 0.231 & 0.101 \\
0.200 & 0.264 & 0.147 \\
0.029 & 0.041 & 0.022
\end{bmatrix} \]

\[ \lambda_{ij} = \begin{bmatrix}
0.569 & 0.883 & 0.294 \\
0.500 & 0.671 & 0.357 \\
0.145 & 0.196 & 0.110
\end{bmatrix} \]

We operate on this matrix column by column, choosing first the column having the largest difference in \( \lambda \)'s. In column 2, \( \lambda_{12} - \lambda_{32} \) represents this largest difference. The procedure requires a certain part of the weapons in group 2 to be moved from target 3 to target 1, since \( \lambda_{32} < \lambda_{12} \). Since, however, there are no weapons in group 2 allocated to target 3 by our first \( (n_{ij}) \) guess, we need to look further. The next largest difference is \( \lambda_{11} - \lambda_{31} \), and we move \( \Delta n \) weapons in group 1 from target 3 to target 1, where \( \Delta n \) is a root of
This equation is solved by use of the Newton successive approximation method of root finding, where

\[ \Delta n_{k+1} = \Delta n_k - \frac{g(\Delta n_k)}{g'(\Delta n_k)} \]

and in

\[ (0.8)^{-\Delta n}(0.408)+(0.7)^{-\Delta n}(0.161)-(0.3)^{\Delta n}(0.116)-(0.2)^{\Delta n}(0.029) = 0 \]

Since \( g'(\Delta n) > 0 \) for \( \Delta n \geq 0 \), \( g(\Delta n) \) has a unique solution. Therefore, our new allocation is

\[ \left( n_{ij} \right) = \begin{bmatrix} 2.88 & 2 & 3 \\ 1.00 & 1 & 2 \\ 0.12 & 0 & 2 \end{bmatrix} \]

and the new \( \lambda_{ij} \) matrix is

\[ \lambda_{ij} = \begin{bmatrix} 0.453 & 0.705 & 0.232 \\ 0.500 & 0.671 & 0.357 \\ 0.453 & 0.617 & 0.343 \end{bmatrix} \]

Notice that our choice of \( \Delta n \) forces \( \lambda_{11} = \lambda_{31} \), which in turn increases the value killed by the SLBM attack. This procedure is repeated until the differences in the \( \lambda \)'s become very small, or it becomes impossible to increase by shifting weapons. After six iterations, the final allocation is

\[ \left( n_{ij} \right) = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix} \]
and the value destroyed is about 21.3 out of the total 24. The bombers killed turn out to be 5.9 out of 7 and tankers 9.4 out of 10. An important question is: can the bombers and tankers be bedded down so as to decrease the number of kills to a minimum? As shown before, the minimum damage that can be inflicted by the SI BM attack occurs when the

\[ \prod_{j=1}^{L} S_{ijB}^{n_{ij}}, \ i = 1, \ldots, N \]

are equal and when

\[ \prod_{j=1}^{L} S_{ijT}^{n_{ij}}, \ i = 1, \ldots, N \]

are equal. In the above example,

\[ \prod_{j} c_{ijB}^{n_{ij}} \]

are 0.17, 0.13, 0.12 and

\[ \prod_{j} S_{ijT}^{n_{ij}} \]

are 0.07, 0.05, 0.06. These numbers indicate that the beddown is already a good one, and could only be slightly improved. Once the given \( n_{ij} \) matrix is in its final form, in this case with the \( \lambda_{ij} \) matrix converged to a tolerance of \( .001 \), the bombers and tankers can be shifted according to the formula

\[ \Delta V = \left( \prod_{j=1}^{L} S_{mj}^{n_{mj}} - \prod_{k=1}^{L} S_{kj}^{n_{kj}} \right) v_k \]

applied to bomber and tanker values independently. With the given initial bomber and tanker values and a converged \( n_{ij} \) matrix (after 17 iterations)

\[
\begin{bmatrix}
3.2649 & 3.0 & 0.0 \\
0.7351 & 0.0 & 4.6912 \\
0.0 & 0.0 & 2.3088
\end{bmatrix}
\]
the products

\[
\prod_{j=1}^{n} S_{ij}
\]

can be formed for each aircraft. Since there are three targets in this example, there are three resultant products. These are

\[
\begin{align*}
(1) & \begin{bmatrix} 0.1655 \\ 0.0674 \end{bmatrix} \\
(2) & \begin{bmatrix} 0.1289 \\ 0.0547 \end{bmatrix} \\
(3) & \begin{bmatrix} 0.1206 \\ 0.0621 \end{bmatrix}
\end{align*}
\]

Bombers  Tankers

In computing the values to shift, \( \Delta V \), for each aircraft, the smallest product (where a value is present) is subtracted from the largest one and the difference is multiplied by the value on the target corresponding to the smallest product. This is the portion of value to be subtracted from the total value corresponding to the largest product. For example, for bombers, the difference in maximum and minimum product values is

\[
0.1655 - 0.1206 = 0.0449
\]

which when multiplied by the value corresponding to the lower product gives

\[
\Delta V = 0.0449 \times 2 = 0.0898
\]

Thus, the new bomber value matrix becomes

\[
V_{iB} = \begin{bmatrix} 8.0898 \\ 4.0000 \\ 1.9102 \end{bmatrix}
\]

By a similar process, the new tanker matrix becomes

\[
V_{iT} = \begin{bmatrix} 5.0508 \\ 3.9492 \\ 1.0000 \end{bmatrix}
\]
With these values, the laydown is again optimized. By moving the appropriate \( \Delta V \) values iteratively from lower to higher \( S_{ij} \), we obtain the following beddown:

\[
V_i^D = \begin{bmatrix} 12 \\ 2 \\ 0 \end{bmatrix} \quad V_i^T = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}
\]

The new allocation of SLBMs changes only slightly as follows:

\[
n_{ij} = \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix}
\]

and similarly the number of kills decreases only slightly to 5.9 out of 7 bombers and 9.3 out of 10 tankers, for a total value destroyed of 21.2. Note that although the SLBM allocation and total SLBM strike effectiveness changed very little, the beddown, in comparison, changed considerably. Hence, within the context of random events (i.e., SLBM reliabilities, CEPs, etc.) precipitating uncertainties as to exact numbers of bombers killed, the beddown problem does not appear to lend itself to a unique solution. Also, note that the bomber to tanker value ratio of 2:1 was taken as fixed for all bases; however, this value ratio can be varied from base to base. Also, note that by inserting zeros in the appropriate locations of the \( S_{ij} \) matrixes, we can effectively enforce the constraints of not permitting tankers or bombers to be based at particular locations.

**EXAMPLE 3**

This example demonstrates the capabilities of QUANTO on a small hypothetical problem which resembles those actually run with QUANTO. The input deck for this problem is listed in figure 14, in the format described in the user documentation of QUANTO. The discussion presented here will be in the form of a guide to reading the output, most of which is shown in appendix III.

The first two lines of output serve to uniquely identify the run and give basic problem data. Both the beddown and the submarine locations appear on the first page of output. Targets 1 to 4 are located at points in Colorado, North
| 1.0 | 4.5 | 1.5 | 60 |
| 0.25 | 9 | 95 | 300.0 | 2000.0 | 1500.0 |

**20 PHANTOM PROBLEM PROFILE**

| 0.0 | 0.0 | 0.0 |
| 2000.0 | 20.0 | 9.0 | 1.406 |
| 3900.0 | 30.0 | 30.0 | 1.406 |
| 6200.0 | 40.0 | 92.0 | 1.946 |
| 9500.0 | 50.0 | 360.0 | 2.538 |
| 13400.0 | 60.0 | 500.0 | 3.154 |
| 15600.0 | 66.0 | 500.01 | 3.549 |
| 17300.0 | 68.0 | 500.02 | 3.672 |
| 17900.0 | 69.0 | 500.03 | 3.734 |
| 18200.0 | 71.0 | 500.04 | 3.863 |
| 22750.0 | 78.0 | 500.05 | 4.355 |
| 26800.0 | 86.0 | 500.06 | 4.620 |
| 33000.0 | 94.0 | 500.07 | 5.636 |
| 37000.0 | 101.0 | 500.08 | 5.955 |
| 44000.0 | 109.0 | 500.09 | 6.379 |
| 49750.0 | 117.0 | 500.10 | 6.617 |
| 55900.0 | 125.0 | 500.11 | 6.865 |
| 61000.0 | 142.0 | 1425.0 | 7.037 |
| 68000.0 | 140.0 | 3950.0 | 7.275 |
| 75500.0 | 148.0 | 5000.0 | 7.312 |
| 76000.0 | 148.0 | 5000.0 | 7.312 |

Figure 14. Sample Input Deck
Dakota, Iowa, and Tennessee and the submarine locations are about 100 nm off the coasts of Virginia, Louisiana, and Oregon. The targets are distinguished not only by their distances from the submarines but by the number of aircraft and the distances (in nm) to the centroid of the aircraft flyout pattern (a function of the numbers of runways). Note that aircraft can take off with small intervals from bases with dual runways. The missile parameters and trajectory data are given on pages 71 and 72 of appendix III. Page 73 lists the input aircraft flyout profile data and pages 74, 75, and 76 show the profile generated by QUANTO with the aircraft leveled-off at 5000 feet. The bottoms of pages 77 and 78 show the lethal radii and time of (overpressure) shock arrival for the aircraft of hardness indicated on page 71, along with other standard output from the nuclear effects routines.

A table of lethal areas (as a function of the distance Q of the detonation from the centroid) is then built as each change of distance to centroid (DSPT) is encountered in the target list. When the aircraft has not reached its terminal altitude (at distance DISMIN), new lethal radii are obtained based on the actual aircraft altitude after it has traveled a distance (Q + DSPT) from the centroid, as shown for two such values of Q on pages 79 to 82 and 85 to 88. Occasionally, a value of Q results in a geometry of lethal area in which the boundaries of the overpressure and thermal kill regions have multiple intersections. When this occurs, approximately two pages of indicators are output to enable a detailed study of this geometry. This output may be ignored on production runs. Lethal area tables appear on pages 83 and 89 of appendix III.

The lethal areas of SLBMs are based on assumed detonation points on each target, dependent on where the aircraft are at time of weapon arrival. Distances and missile flight times from submarines to targets and samples of the computed aircraft locations appear on pages 84 and 90 to 92. Note that weapons are numbered sequentially through the salvos of each submarine location, but only the first SLBM of each submarine's two salvos appears in this output in order to reduce the quantity of printout while providing enough information to indicate when each SLBM arrives on each target. Note also that the annulus \( P_k \) line is only printed occasionally; this is because it is not computed in instances in which the program knows beforehand that the circular \( P_k \) will be the smaller of the two \( P_k \)s. The computed \( S_{ij} \) values are listed on page 93 where \( i \) is the target number and \( j \) is the weapon number, but one row of output contains only the \( S_{ij} \) corresponding to SLBMs from a single submarine location.
The chart at the top of page 94 lists the base-by-base kill resulting from the input laydown (prior to optimization). Since each aircraft is worth one point of value, the number of aircraft and the value are equal. The convergence to the optimal laydown (n_{ij} values) produces the output on pages 94 to 109, where the long form of the output has been requested to show each \( \Delta n \) value, the expected kill after each shift of \( \Delta n \), and the allocation and multipliers \( \lambda_{ij} \) after each convergence to the tolerances \( \varepsilon = 0.1, 0.01, 0.001, \) and 0.0001. The best nonintegral laydown appears on page 107, listed by target and then by submarine and salvo. The expected kill increased from 14.9765 to 19.7147 (out of a total of 50) aircraft during the convergence. Target-by-target kills and weapons allocated (if \( n_{ij} \) are rounded off) appear on page 108.

Optimization of submarine positioning has been requested in this example, and occurs prior to the requested beddown optimization. The first submarine move is indicated on page 110 and the resultant positions of all submarines appear on page 111 following the initial allocation (prior to convergence again). The convergence to the best laydown with the submarines in their new positions follows with the resultant kill shown on page 134. The first submarine move improved the kill (after convergence) from 19.7147 to 20.8742. The second submarine move is shown on pages 136 and 137. A summary of the submarine optimization appears below.

<table>
<thead>
<tr>
<th>After Submarine Move No.</th>
<th>Converged Expected Kill</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19.7147</td>
<td>108</td>
</tr>
<tr>
<td>1</td>
<td>20.8742</td>
<td>134</td>
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<td>2</td>
<td>21.2415</td>
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<tr>
<td>3</td>
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<td>144</td>
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</table>

After the third submarine move, the expected kill decreased slightly so the submarines are fixed (2 at point 1, 2 at point 2, and 1 at point 3) and beddown optimization begins with the first shift of aircraft on pages 145 and 146. Convergence to the optimal laydown follows each beddown change and is summarized on the following page.
After Value Converged
Shift No. Expected Kill Page
0 20.8742 144
1 18.7694 148
2 16.8932 152
3 16.8171 156
4 16.5884 160
5 16.5863 164

Since the sixth value shift was to be a shift of less than 0.05 aircraft (see page 165 of appendix III), the beddown optimization was terminated. The problem is terminated by integerizing both the laydown and the beddown. The expected kill tends to decrease due to the laydown integerization and increase due to the beddown integerization. The resultant expected kill following both integerizations is 16.4428. The integral laydown appears on page 166 and the base-by-base kills and integral beddown appear on page 167.

In analyzing the output, a table of distances from submarine locations to targets is useful. These distances, to the nearest nautical mile, appear below and on pages 84 and 90 to 92.

<table>
<thead>
<tr>
<th>Submarine Location</th>
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<th>2</th>
<th>3</th>
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<tr>
<td>Target 1</td>
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<td>795</td>
<td>1107</td>
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<td>1241</td>
<td>1228</td>
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<tr>
<td>4</td>
<td>580</td>
<td>589</td>
<td>1902</td>
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</table>

The optimal laydown from the initial positioning of submarines appears on pages 107 and 108. It is interesting to note that the submarine at location 1 allocated its missiles to target 3, leaving target 4 to the submarines at location 2. Target 2 drew the most weapons even though it could not be hit as soon as the other targets. This was probably because target two's 15 aircraft departed from a single runway, and, therefore, were dispersing from a point 5.5 nm, in this case, from the brake release point. Dual runways permit more immediate dispersal since the aircraft can take off in opposite directions.
The optimization of the submarine positions resulted in only one submarine at location 3, even though this location had the best shot at target 2.

During the beddown optimization, the proximity of target 4 to the submarines at locations 1 and 2 made target 4 unattractive for bedding down aircraft (see page 167 of appendix III). The greater distance from the coast outweighed the advantages of dual runways (i.e., immediate dispersal due to aircraft taking off in opposite directions) making target 2 the most attractive, although targets 1 and 3 also drew a substantial number of aircraft. In this case, many aircraft would be left on the base when SLBMs arrive, as is clear from page 91 of appendix III which shows a zero inner annulus radius when weapons arrive, even when target 2 had only 15 aircraft. Consequently, it appears that too many aircraft are present to justify the assumption of uniform aircraft distribution, and the kills on target 2 could be underestimated by QUANTO.
APPENDIX III

QUANTO'S OUTPUT
### Aircraft Take-Off Intervals

<table>
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<tr>
<th>Runways</th>
<th>Type</th>
<th>Minutes</th>
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### Sub Locations (Degrees)

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<tr>
<td>1</td>
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### Missiles Launch Reliabilities

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<th>Type</th>
<th>Launch</th>
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<th>Warhead</th>
<th>Range (Min)</th>
<th>Range (Max)</th>
<th>Yield</th>
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**SUBROUTINE PROCESS  CASE 1.  NORE 7**

** AIRCRAFT TYPE 1 OF 1  TYPE(s) **

** VULNERABILITY CRITERIA FOR AIRCRAFT TYPE 1 **

- 1.50 PSI
- 40 CAL/CM**2

** DATA INPUT FOR AIRCRAFT TYPE 1 **

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<th>FLIGHT TIME</th>
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**INITIAL ALTITUDE (IN CLIMBING) OF MAXIMUM MACH**

7400 FFFT
DATA COMPUTED FOR AIRCRAFT TYPE 1 WITH RESPECT TO A TERMINAL ALTITUDE OF 5000 FEET

VELOCITY OF SOUND
1097.11 FEET/SECOND

ACCELERATION COMPONENT
6.200 FEET/SECOND/SECOND

MACH NUMBERS

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VELOCITY

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MISSILE TYPE 1 OF 1 TYPE(S) AGAINST AIRCRAFT TYPE 1

YIELD OF MISSILE TYPE 1
1500 KT

SUBROUTINE SARECH

DATA INPUT TO SARECH
1500 PSI 1ST OVERPRESSURE
1500 KT YIELD
0 FEET TEMPAIN HEIGHT
2500 FEET BURST HEIGHT
5000 FEET AIRCRAFT ALTITUDE

SARECHM OUTPUT FOLLOWS

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<th>SRE</th>
<th>MDF</th>
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YIELD CORRECTION FACTOR IS EQUAL TO ONE
A SUMMARY OF DATA OUTPUT FROM SARECHM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
LETHAL OVERPRESSURE RADIUS = 5.84505857E+00 NAUTICAL MILES
= 3.39379561E+04 FEET
TIME OF SHOCK ARRIVAL = 4.7903E+00 MINUTES
= 2.58340236E+01 SECONDS
### Subroutine SNAPTCM

**Data Input to SNAPTCM**
- **60 CAL/CH** = Thermal Energy
- **1500 KT** = Yield
- **0 FEET** = Terrain Height
- **2500 FEET** = Burst Height
- **5000 FEET** = Aircraft Altitude
- **10000 FEET** = Haze Layer Height
- **5.0 MM HG** = Water Vapor Pressure
- **10.0 MILES** = Visibility (U.S. Statute Miles)
- **3000 (ALHEDO)** = Ground Reflectance
- **1.0 (ACTIND)** = Should be 1.0

**SNAPTC Output Follows**

#### Panel Data

- **CRAFT** = 0.0
- **ALPHAL** = 0.0
- **TMPL** = 0.0
- **WTL** = 0.0
- **ATUL** = 0.0
- **XEL** = 0.0

#### Receiver Parameters

- **FTSEC** = 0.0
- **1STHR** = 2.42013E+04
- **1STALT** = 5.000000E+03
- **DALT** = 0.0
- **MAXALT** = 0.0
- **TILT** = 0.0
- **RETAID** = 1.0000000E+00
- **CAL** = 6.0000000E+01

#### Source Parameters

- **YIELD** = 1.5000000E+03
- **BURST** = 2.5000000E+03
- **TARGET** = 0.0
- **TM EFF** = 4.3224397E-01
- **OBURST** = 0.0
- **WBURST** = 0.0

#### Atmospheric Parameters

- **ATM** = 0.0
- **DAY** = 0.0
- **ALHEDO** = 3.0000000E-01
- **VAPOR** = 5.0000000E+00
- **HAZE** = 1.0000000E+04

#### Receiver Altitude

<table>
<thead>
<tr>
<th>(FT)</th>
<th>(KM)</th>
</tr>
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<tbody>
<tr>
<td><strong>AZ</strong> = 5.00000E+03</td>
<td><strong>AZ</strong> = 1.52393E+00</td>
</tr>
<tr>
<td><strong>HBL</strong> = 2.50000E+03</td>
<td><strong>HBL</strong> = 7.61963E-01</td>
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#### Calculated Horizontal Range

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<th>(FT)</th>
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<tr>
<td><strong>SZ</strong> = 2.28094E+04</td>
<td><strong>SZ</strong> = 6.95096E+00</td>
</tr>
<tr>
<td><strong>AZ</strong> = 3.74620E+00</td>
<td><strong>AZ</strong> = 6.00000E+01</td>
</tr>
<tr>
<td><strong>AZ</strong> = 1.05826E+01</td>
<td><strong>AZ</strong> = 7.97458E+01</td>
</tr>
</tbody>
</table>

#### Unattenuated Energy in Lower Phase (CAL/CH)**2**

- **CL** = 7.97458E+01

#### Unattenuated Energy in Upper Phase (CAL/CH)**2**

- **CU** = 1.05826E+01

#### Attenuated Energy in Lower Phase (CAL/CH)**2**

- **QEL** = 5.10866E+01
- **QELD** = 5.08345E+01
- **QELR** = 9.72130E-01

#### Attenuated Energy in Upper Phase (CAL/CH)**2**

- **QEU** = 6.18799E+00
- **QEUR** = 6.83732E+00
- **QEUR** = 1.35067E+00

#### Total Free Field Energy at Critical Panel (CAL/CH)**2**

- **OA** = 6.00000E+01

#### Total Iterated Energy at Critical Panel (CAL/CH)**2**

- **QE** = 5.99946E+01

A Summary of Data Output from SNAPTCM Used by Subroutine DETAREA in Computing Lethal Area

**Lethal Thermal Radius (FT) = 7.75100000E+00 Nautical Miles**

**2.28061092E+04 FEET**
SUBROUTINE DETAREA NUCLEAR LOOKUP - (N²DSPT) = ( 0.00 + 0.00) = 0.00 NM, WHERE DMIN = 12.42 NM
MISSILE TYPE 1 OF 1 TYPE(S) AGAINST AIRCRAFT TYPE 1
YIELD OF MISSILE TYPE 1
1500 KT

SUBROUTINE SABERCM
DATA INPUT TO SABERCM
1500 WSI BLAST OVERPRESSURE
1500 KT YIELD
0 FEET TERRAIN HEIGHT
2500 FEET BURST HEIGHT
0 FEET AIRCRAFT ALTITUDE
SABERCM OUTPUT FOLLOWS

\[
\begin{align*}
W & = 15.0000E+02 \\
DEL & = 15.0000E-01 \\
M7 & = 0.0 \\
HG & = 0.0 \\
HB & = 25.0000E+02 \\
SH & = 36.56196E+00 \\
RRAR & = 27.9403E-01 \\
SDELP & = 15.60227E-01 \\
FR & = 16.0000E-01 \\
HOFR & = 38.48083E+03 \\
HORN & = 63.10886E+01 \\
TSA & = 26.32797E+00 \\
SFV & = 11.6421E+02 \\
Q & = 53.93939E+03 \\
PMV & = 78.05405E+00 \\
POD & = 25.47701E-04 \\
RHOS & = 23.76900E-04 \\
PDDP & = 45.27825E-01 \\
PDV & = 53.41786E-01 \\
PDOD & = 43.98499E-01 \\
SSZ & = 11.16457E+02 \\
R & = 27.02741E-01 \\
ALFA & = 56.79004E-02 \\
\end{align*}
\]

YIELD CORRECTION FACTOR IS EQUAL TO ONE
A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
LETHAL OVERPRESSURE RADIUS = 6.32908416E+00 NAUTICAL MILES
= 3.84888317E+04 FEET
TIME OF SHOCK ARRIVAL = 4.3799471E+01 MINUTES
= 2.63279682E+01 SECONDS
**SYNOPSIS**

**DATA INPUT TO SNAPTCM**

- **60 CAL/CM**^**2** THERMAL ENERGY
- **1500 KT** YIELD
- **2500 FEET** TERRAIN HEIGHT
- **0 FFET** BURST HEIGHT
- **0 FFET** AIRCRAFT ALTITUDE
- **10000 FEET** HAZE LAYER HEIGHT
- **5.0 MMHG** WATER VAPOR PRESSURE
- **10.0 MILES** VISIBILITY (U.S., STATUTE MILES)
- **.36 (ALBEDO)** GROUND REFLECTANCE
- **1.00 (BEYOND)** SHOULD BE 1.0

**SNAPTCM OUTPUT FOLLOWS**

### PANEL DATA
- **CRAFT = 0.**
- **ALPHAL = 0.**
- **TMPL = 0.**
- **WTL = 0.**
- **RTUL = 0.**
- **XLEL = 0.**

### RECEIVER PARAMETERS
- **FTSEC = 0.**
- **ISTHR = 2.6281362E+04**
- **ISTALT = 0.**
- **DALT = 0.**
- **NAXALT = 0.**
- **TILT = 0.**
- **DETAQ = 1.0000000E+00**
- **CAL = 6.0000000E+01**

### SOURCE PARAMETERS
- **YIELD = 1.5000000E+03**
- **BURST = 2.5000000E+03**
- **TARGET = 0.**
- **TH EFF = 4.3274397E-01**
- **OBURST = 0.**
- **MBURST = 0.**

### RECEIVED ALTITUDE
- **(FT) AZ = 0.**
- **(KM) AZ = 0.**

### CALCULATED HORIZONTAL RANGE
- **(FT) SZ = 2.14475E+04**
- **(KM) SZ = 5.53649E+00**
- **(NM) SZ = 3.51740E+00**

### UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2**) CL = 8.21672E+01
### UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2**) CU = 9.97587E+00

### ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2**) QEL = 5.20776E+01
### ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2**) QEU = 7.91678E+00

### TOTAL FREE FIELD ENERGY AT CRITICAL PANEL (CAL/CM**2**) QA = 6.00000E+01
### TOTAL ITENATT ENERGY AT CRITICAL PANEL (CAL/CM**2**) QE = 5.99944E+01

### ATSUMMARY OUTPUT FROM SNAPTCM USED BY SUBROUTINE DFAREA IN COMPUTING LETHAL AREA
- **LETHAL TERRAIN RADIUS = 3.52755671F+00**
- **NAUTICAL MILES = 2.14475448F+04 FEET**
SUHRoutine DETAREA NUCLEAR LOOKUP - (0+0SP) = ( 1.00 + 0.00 ) = 1.00 NM, WHERE DISMIN = 12.42 NM

MISSILE TYPE 1 OF 1 TYPE(S) AGAINST AIRCRAFT TYPE 1

YIELD OF MISSILE TYPE 1
1500 KT

SUHRoutine SABERCl

DATA INPUT TO SABERCl
1500 PSI BLAST OVERPRESSURE
1500 KT YIELD
0 FEET TERRAIN HEIGHT
2500 FEET BURST HEIGHT
87 FEET AIRCRAFT ALTITUDE

SABERCl OUTPUT SOLUTION

RANGE SOLUTION

W 15.00000E+02 DELP 15.00000E-01 HZ 36.93474E+00 MG 0. MB 25.00000E+02
SR 38.59907E+09 WARP 27.90206E-01 5DELP 15.63016E-01 FR 16.00000E-01
HORF 39.43339E+03 HORN 43.03074E-01
TSA 29.33194E+00 SFV 11.96071E+02 0 54.06113E-03 PMY 78.26641E+00
POD 25.61754E-04 RST 23.70050E-04 PDCP 45.31965E-01 PMY 53.47822E-01
POND 44.01166E+01 S57 11.16126E+02 'R 27.02741E-01 ALFA 56.78879E-02

YIELD CORRECTION FACTOR IS EQUAL TO ONE
A SUMMARY OF DATA OUTPUT FROM SABERCl USED BY SUHRoutine DETAREA IN COMPUTING LETHAL AREA
LETHAL OVERPRESSURE RADIUS = 6.32198175E+00 NAUTICAL MILES
= 3.64339390E+04 FEET
TIME OF SHOCK ARRIVAL = 4.38662289E-01 MINUTES
= 2.6315374E+01 SECONDS
### DATA INPUT TO SNAPCM

- **CAL/CM**
  - 60
- **K/T**
  - 1500
- **FEET**
  - 0
- **FEET**
  - 2500
- **FEET**
  - 67
- **FEET**
  - 10000
- **MM HG**
  - 5.0
- **MILES**
  - 10.0
- **ALBEDO**
  - .30
- **FEET**
  - 1.00

### SNAPCM OUTPUT FOLLOWS

#### PANEL DATA
- **CRAFT** = 0.0
- **ALPHA** = 0.0
- **TMPL** = 0.0
- **WTL** = 0.0
- **RTUL** = 0.0
- **XLEL** = 0.0

#### RECEIVER PARAMETERS
- **FSTFC** = 0.0
- **ISTBA** = 2.8289487E+04
- **ISTALT** = 8.6934736E+01
- **DALT** = 0.0
- **MAKALT** = 0.0
- **VIS** = 1.0

#### SOURCE PARAMETERS
- **YIELD** = 1.5000000E+03
- **BURST** = 2.5000000E+03
- **TARGET** = 0.0
- **TH EFF** = 4.3224397E-01
- **DRUSS** = 0.0
- **HRUSS** = 0.0

#### ATMOSPHERIC PARAMETERS
- **ATM** = 0.0
- **DA** = 0.0
- **ALBEDO** = 3.0000000E-01
- **HAZE** = 1.0000000E+04

#### RECEPTOR ALTITUDE
- **(FT)** **AZ** = 8.6934736E+01
- **(KM)** **AZ** = 2.84964E-02

#### CALIBRATED HORIZONTAL RANGE
- **(FT)** **SZ** = 2.14800E+04
- **(KM)** **SZ** = 6.54675E+00

#### UNATTENIINTED ENERGY IN LOWER PHASE (CAL/CM**2**)
- **CL** = 8.21441E+01

#### UNATTENIINTED ENERGY IN UPPER PHASE (CAL/CM**2**)
- **CU** = 9.90064E+00

#### ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2**)
- **DEL** = 5.20842E+01
- **OEUD** = 6.24514E+00

#### ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2**)
- **OEU** = 7.91033E+00

#### TOTAL EFFECTIVE ENERGY AT CRITICAL PANEL (CAL/CM**2**)
- **QD** = 6.0000000E+01

#### TOTAL ITERATIVE ENERGY AT CRITICAL PANEL (CAL/CM**2**)
- **OE** = 5.999945E+01

---

**A SUMMARY OF DATA OUTPUT FROM SNAPCM USED BY SURROUTINE DETAREA IN COMPUTING LETHAL AREA**

- **Lethal Thermal Radius** = 3.532891068E+00
- **Nautical Miles** = 2.14799831E+04
FOR DISTANCE OF 100.00 NM TO CENTROID FROM START OF TAKE-OFF ROLL

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Missile Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cuando Detonation is 0.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
</tr>
<tr>
<td>Quando Detonation is 1.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
</tr>
<tr>
<td>Quando Detonation is 2.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
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<tr>
<td>Quando Detonation is 3.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Quando Detonation is 4.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
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<tr>
<td>Quando Detonation is 5.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
</tr>
<tr>
<td>Quando Detonation is 6.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
<td></td>
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<td>Quando Detonation is 7.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Quando Detonation is 8.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Quando Detonation is 9.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Quando Detonation is 10.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Quando Detonation is 11.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Cuando Detonation is 50.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<td>Cuando Detonation is 100.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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<tr>
<td>Cuando Detonation is 105.00 NM from Centroid, Lethal Area = 3.56 NM Further From Centroid.</td>
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</tbody>
</table>
TARGET 1 1HR AF DEPART TIMES
AIRCRAFT TYPE = 1 STARTS AT 4.50 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.54 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.47 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.74 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.96 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.00 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.08 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.17 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.25 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.33 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.42 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.50 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.47 MINUTES.

DISTANCE TO TARGET 1 FROM SUB LOCATION 1 IS 1391.25 NM, FLIGHT TIME (MISSILE TYPE 1) = 8.37354084E+00 MIN.
WEAPON 1 ARRIVES ON TARGET 1 WHEN FIRST AIRCRAFT IS 25.13 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .07721, CIRCULAR PK = .0471, WEAPON 1 VS. AIRCRAFT TYPE 1. ANNUAR RADII ARE 25.1281 AND 14.4294

DISTANCE TO TARGET 2 FROM SUB LOCATION 2 IS 795.40 NM, FLIGHT TIME (MISSILE TYPE 1) = 6.34162586E+00 MIN.
WEAPON 3 ARRIVES ON TARGET 1 WHEN FIRST AIRCRAFT IS 7.42 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .27721, CIRCULAR PK = .28734, WEAPON 3 VS. AIRCRAFT TYPE 1. ANNUAR RADII ARE 7.4213 AND 1.0447
ANNULUS PK = .2113, CIRCULAR PK = .2176, WEAPON 4 VS. AIRCRAFT TYPE 1. ANNUAR RADII ARE 9.2527 AND 1.9156

DISTANCE TO TARGET 3 FROM SUB LOCATION 3 IS 1106.51 NM, FLIGHT TIME (MISSILE TYPE 1) = 7.46331120E+00 MIN.
WEAPON 5 ARRIVES ON TARGET 1 WHEN FIRST AIRCRAFT IS 16.75 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .1130, CIRCULAR PK = .0943, WEAPON 5 VS. AIRCRAFT TYPE 1. ANNUAR RADII ARE 16.7542 AND 7.0651
SUBROUTINE DETAPCA NUCLARE LENGTH = (N+USRT) = ( 0.00 + 5.50) = 5.50 NM, WHERE DISMIN = 12.42 NM

MISSILE TYPE 1 OF 1 TYPE(S) AGAINST AIRCRAFT TYPE 1

YIELD OF MISSILE TYPE 1
1500 KT

SUBROUTINE SABERCM

DATA INPUT TO SABERCM

1.50 PSI BLAST OVERPRESSURE
1500 KT YIELD
2500 FEET TERRAIN HEIGHT
500 FEET HURST HEIGHT
500 FEET AIRCRAFT ALTITUDE

SABERCM OUTPUT FOLLOWS

RANGE SOLUTION

W 15.00000E+02 DREP 15.00000E+01 HZ 50.00711E+01 MG 0.00000E+00 HB 25.00000E+02
SR 36.250AM6E+00 RAR 27.72061E+01 SDELP 15.76362E+01 FR 16.00000E+01
MORF 38.20633E+03 HORN 62.65839E+01
RZA2 26.73776E+00 SFV 11.43122E+02 0 54.46410E+00 PMV 79.28493E+00
POO2 25.13658E+04 RH02 23.42313E-04 PD00 45.51721E-01 PO0V 53.76627E-01
POON 44.75440E-01 SSZ 11.1455A2E+02 P 27.02741E+01 ALFA 56.78251E-02

YIELD CORRECTION FACTOR IS EQUAL TO ONE

A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAPCA IN COMPUTING LETHAL AREA
LETHAL OVERPRESSURE RADIUS = 6.24393601E+00 NAUTICAL MILES
= 3.42043309E+04 FEET
TIME OF SHOCK ARRIVAL = 4.37835969E-01 MINUTES
= 2.62737591E+01 SECONDS
SUBROUTINE SNIPTCM

DATA INPUT TO SNIPTCM

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<thead>
<tr>
<th>NO</th>
<th>CAL./CM**2</th>
<th>THERMAL ENERGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>KT</td>
<td>YIELD</td>
</tr>
<tr>
<td>2500</td>
<td>FEET</td>
<td>FRESH AIR HEIGHT</td>
</tr>
<tr>
<td>500</td>
<td>FEET</td>
<td>AIRCRAFT ALTITUDE</td>
</tr>
<tr>
<td>10000</td>
<td>FEET</td>
<td>HAZE LAYER HEIGHT</td>
</tr>
<tr>
<td>5.0</td>
<td>MM HG</td>
<td>WATER VAPOR PRESSURE</td>
</tr>
<tr>
<td>10.0</td>
<td>MILES</td>
<td>VISIBILITY(U.S. STATUTE MILES)</td>
</tr>
<tr>
<td>20</td>
<td>ALBEDO</td>
<td>GROUND REFLECTANCE</td>
</tr>
<tr>
<td>1.00</td>
<td>(HIT/HU)</td>
<td>SHOULD BE 1.0</td>
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</tbody>
</table>

SNIPTCM OUTPUT FOLLOWS

<table>
<thead>
<tr>
<th>PANEL DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRAFT = 0.</td>
</tr>
<tr>
<td>ALPHAL = 0.</td>
</tr>
<tr>
<td>TMPL = 0.</td>
</tr>
<tr>
<td>WIL = 0.</td>
</tr>
<tr>
<td>RTUL = 0.</td>
</tr>
<tr>
<td>XLEL = 0.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RECEIVER PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTSEC = 0.</td>
</tr>
<tr>
<td>MAXALT = 0.</td>
</tr>
<tr>
<td>TILT = 0.</td>
</tr>
<tr>
<td>ISTHR = 2.6324139E+04</td>
</tr>
<tr>
<td>BETAID = 1.0000000E+00</td>
</tr>
<tr>
<td>ISTALT = 5.0007108E+02</td>
</tr>
<tr>
<td>CAL = 6.0000000E+01</td>
</tr>
<tr>
<td>DALT = 0.</td>
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</table>

<table>
<thead>
<tr>
<th>SOURCE PARAMETERS</th>
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</thead>
<tbody>
<tr>
<td>YIELD = 1.5000000E+03</td>
</tr>
<tr>
<td>RURST = 2.5000000E+03</td>
</tr>
<tr>
<td>TARGET = 0.</td>
</tr>
<tr>
<td>TH EFF = 4.3224397E+01</td>
</tr>
<tr>
<td>DBJST = 0.</td>
</tr>
<tr>
<td>MDBURST = 0.</td>
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</table>

<table>
<thead>
<tr>
<th>ATMOSPHERIC PARAMETERS</th>
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</thead>
<tbody>
<tr>
<td>ATN = 0.</td>
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<td>DAY = 0.</td>
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<tr>
<td>VISIBLE = 1.0000000E+01</td>
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<tr>
<td>DAY = 0.</td>
</tr>
<tr>
<td>VAPOR = 5.0000000E+00</td>
</tr>
<tr>
<td>ALBEDO = 3.0000000E-01</td>
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<td>HAZE = 1.0000000E+04</td>
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<table>
<thead>
<tr>
<th>RECEIVED ALTITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(FT) AZ = 5.00071E+02</td>
</tr>
<tr>
<td>(KM) AZ = 1.59414E+01</td>
</tr>
<tr>
<td>RURST ALTITUDE (FT) HBL. = 2.50000E+03</td>
</tr>
<tr>
<td>(KM) HBL = 7.61963E-01</td>
</tr>
<tr>
<td>ANGLE BETWEEN LOCAL HORIZONTAL AND CRITICAL PANEL (RADIANS) BETA = 1.65831E+00</td>
</tr>
<tr>
<td>(FT) SZ = 2.10291E+04</td>
</tr>
<tr>
<td>(KM) SZ = 6.59223E+00</td>
</tr>
<tr>
<td>(NM) SZ = 3.54718E+00</td>
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<table>
<thead>
<tr>
<th>CALCULATED HORIZONTAL RANGE</th>
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<tbody>
<tr>
<td>UNATTENUATED ENERGY IN LOWER PHASE (CAL./CM**2) CL = 8.20459E+01</td>
</tr>
<tr>
<td>UNATTENUATED ENERGY IN UPPER PHASE (CAL./CM**2) CU = 1.00591E+01</td>
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</table>

<table>
<thead>
<tr>
<th>ATTENUATED ENERGY IN LOWER PHASE (CAL./CM**2)</th>
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</thead>
<tbody>
<tr>
<td>OEL = 5.21035E+01</td>
</tr>
<tr>
<td>OELU = 5.11639E+01</td>
</tr>
<tr>
<td>OELR = 9.44659E-01</td>
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</table>

<table>
<thead>
<tr>
<th>ATTENUATED ENERGY IN UPPER PHASE (CAL./CM**2)</th>
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</thead>
<tbody>
<tr>
<td>OEU = 7.88639E+00</td>
</tr>
<tr>
<td>REUD = 6.30641E+00</td>
</tr>
<tr>
<td>OEUR = 1.57993E+00</td>
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<table>
<thead>
<tr>
<th>TOTAL FREE FIELD ENERGY AT CRITICAL PANEL (CAL./CM**2)</th>
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</thead>
<tbody>
<tr>
<td>OA = 6.000000E+01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TOTAL ATTENUATED ENERGY AT CRITICAL PANEL (CAL./CM**2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OE = 5.99949E+01</td>
</tr>
</tbody>
</table>

A SUMMARY OF DATA OUTPUT FROM SNIPTCM USED BY SUBROUTINE DETEAHA IN COMPUTING LETHAL AREA

LETHAL THERMAL RADIUS = 3.55742070F+00 NAUTICAL MILES
                  = 2.14741179F+04 FT
SUBROUTINE DETAREA NUCLEAR DETON - (D+USPT) = ( 1.00 + 5.00 ) = 6.50 NM, WHERE DISM = 12.42 NM

MISSILE TYPE 1 OF 1 TYPE(S) AGAINST AIRCRAFT TYPE 1

YIELD OF MISSILE TYPE 1

1500 KT

SUBROUTINE SABERCM

DATA INPUT TO SABERCM

1.50 PSI PLASTIC OVERPRESSURE

1500 KT YIELD

0 FFET TERRAIN HEIGHT

2500 FEET BURST HEIGHT

900 FEET AIRCRAFT ALTITUDE

SABERCM OUTPUT follows

RANGE SOLUTION

W 15.00000E+02 DELP 15.00000E01 Hz 50.00000E01 MG 0.00000E00 MB 25.00000E02
SN 38.25432E+00 PRAR 27.72061E+01 SDELP 15.76363E01 FR 16.00000E01
MRD 38.20632E+00 MDRN 62.65237E-01
NDS 26.27376E+00 SFV 11.63120E+02 0 54.86404E-03 PMN 79.28491E00
PRN 25.13657E-04 RHOZ 23.42312E-04 PNOOP 45.51722E-01 PDMN 53.56628E-01
PD00 44.25441E-01 SS7 11.16536E+02 'W 27.02741E01 ALFA 56.78251E02

YIELD CORRECTION FACTOR IS EQUAL TO ONE

A SUMMARY OF DATA OUTPUT FROM SABERCM USED IN COMPUTING LETHAL AREA

LETHAL OVERPRESSURE RADIUS = 6.28393442E+00 NAUTICAL MILES

= 3.92063237E+04 FEET

TIME OF SHOCK ARRIVAL = 4.37895454E-01 MINUTES

= 2.62737567E+01 SECONDS
### SURROUNTP: SNAPCH

**DATA INPUT TO SNAPCH**

- **60 CAL/CM**
- **1500 KT**
- **0 FEET**
- **2500 FT**
- **500 FEET**
- **10000 FT**
- **5.0 NM**
- **10.0 MILES**
- **30 AIRFOIL**
- **1.00 (RETIND)**

**SNAPCH OUTPUT FOLLOWS**

**PANEL DATA**

- CRAFT = 0
- ALPHAL = 0
- TMPL = 0
- WTL = 0
- RTUL = 0
- XEL = 0

**SOURCE PARAMETERS**

- YIELD = 1.5900000E+03
- BURST = 2.5000000E+03
- TARGET = 0
- TM FFF = 4.322439E-01
- DBURST = 0
- MBURST = 0

**RECEIVER PARAMETERS**

- FTSEC = 0
- ISTHR = 2.432410E+04
- ISTALT = 5.0008416E+02
- DALT = 0
- MAXALT = 0
- TILT = 0
- BETAI = 1.0000000E+00
- CAL = 6.0000000E+01

**ATMOSPHERIC PARAMETERS**

- ATM = 0
- VISIBLE = 1.0000000E+01
- VAPOR = 5.0000000E+00
- MAZE = 1.0000000E+04

**RECEIVER ALTITUDE**

- (FT) AZ = 5.000000E+02
- (KM) AZ = 1.59418E-01

**CALCULATED HORIZONTAL PANEF**

- (FT) SZ = 2.16291E+04
- (KM) SZ = 6.59223E+00
- (MW) SZ = 3.54718E+00

**UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**

- **CL = 8.29459E+01**

**UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**

- **CU = 1.08591E+01**

**ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**

- **QEL = 5.21085E+01**
- **QELD = 5.11639E+01**
- **QELR = 9.44627E+01**

**ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**

- **GEU = 7.86635E+00**
- **GEO = 6.30641E+00**
- **GEUR = 1.57993E+00**

**TOTAL FRESH FIELD ENERGY AT CRITICAL PANEL (CAL/CM**

- **GA = 6.000000E+01**
- **QE = 5.99949E+01**

**A SUMMARY OF DATA OUTPUT FROM SNAPCH USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA**

- LETHAL THERMAL RADIUS = 3.55712146E+00
- NAUTICAL MILES = 7.16281275E+04
<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>1 versus Missile Type</th>
<th>For Distance of 5.50 NM to Centroid from Start of Take-Off Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mariners</td>
<td>9.00 NM from Centroid</td>
<td>Lethal area extends 3.56 NM further from Centroid</td>
</tr>
<tr>
<td>Mariners</td>
<td>9.00 NM from Centroid</td>
<td>Lethal area extends 3.56 NM further from Centroid</td>
</tr>
<tr>
<td>Mariners</td>
<td>10.00 NM from Centroid</td>
<td>Lethal area extends 3.75 NM further from Centroid</td>
</tr>
<tr>
<td>Mariners</td>
<td>15.00 NM from Centroid</td>
<td>Lethal area extends 3.75 NM further from Centroid</td>
</tr>
<tr>
<td>Mariners</td>
<td>25.00 NM from Centroid</td>
<td>Lethal area extends 3.75 NM further from Centroid</td>
</tr>
<tr>
<td>Mariners</td>
<td>50.00 NM from Centroid</td>
<td>Lethal area extends 3.75 NM further from Centroid</td>
</tr>
</tbody>
</table>
TARGET B  WRECK U'LEAFHI TIMES
AIRCRAFT TYPE = 1 STARTS AT 4.50 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.67 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.43 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.00 MINUTES.
AIRCRAFT TYPE = 2 STARTS AT 5.17 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.33 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.50 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.67 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.00 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.17 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.33 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.50 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.67 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 6.83 MINUTES.

DISTANCE TO TARGET 2 FROM SUR LOCATION 1 IS 1241.38 NM. FLIGHT TIME (MISSILE TYPE 1) = 7.89453438E+00 MIN.
WEAPON 1 ARRIVES ON TARGET 2 WHEN FIRST AIRCRAFT IS 15.21 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .0867, CIRCULAR PK = .1085, WEAPON 1 VS. AIRCRAFT TYPE 1.
ANNULAR RADII ARE 15.2146 AND 0.0000

DISTANCE TO TARGET 2 FROM SUR LOCATION 2 IS 1228.12 NM. FLIGHT TIME (MISSILE TYPE 1) = 7.85267974E+00 MIN.
WEAPON 3 ARRIVES ON TARGET 2 WHEN FIRST AIRCRAFT IS 14.83 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .0919, CIRCULAR PK = .1127, WEAPON 3 VS. AIRCRAFT TYPE 1.
ANNULAR RADII ARE 14.8294 AND 0.0000

DISTANCE TO TARGET 2 FROM SUR LOCATION 3 IS 1174.33 NM. FLIGHT TIME (MISSILE TYPE 1) = 7.68270606E+00 MIN.
WEAPON 5 ARRIVES ON TARGET 2 WHEN FIRST AIRCRAFT IS 13.27 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK = .1070, CIRCULAR PK = .1329, WEAPON 5 VS. AIRCRAFT TYPE 1.
ANNULAR RADII ARE 13.2665 AND 0.0000

ANNULUS PK = .0960, CIRCULAR PK = .1048, WEAPON 6 VS. AIRCRAFT TYPE 1.
ANNULAR RADII ARE 15.5661 AND 0.0000
## TARGET 3 WAKE RELEASE TIMES

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Starts At</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.50 min</td>
</tr>
<tr>
<td>2</td>
<td>4.50 min</td>
</tr>
<tr>
<td>3</td>
<td>4.37 min</td>
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<tr>
<td>4</td>
<td>4.04 min</td>
</tr>
<tr>
<td>5</td>
<td>4.13 min</td>
</tr>
<tr>
<td>6</td>
<td>4.92 min</td>
</tr>
<tr>
<td>7</td>
<td>5.00 min</td>
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<tr>
<td>8</td>
<td>5.09 min</td>
</tr>
<tr>
<td>9</td>
<td>5.17 min</td>
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<tr>
<td>10</td>
<td>5.29 min</td>
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<td>11</td>
<td>5.33 min</td>
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<tr>
<td>12</td>
<td>5.42 min</td>
</tr>
<tr>
<td>13</td>
<td>5.50 min</td>
</tr>
<tr>
<td>14</td>
<td>5.54 min</td>
</tr>
<tr>
<td>15</td>
<td>5.67 min</td>
</tr>
</tbody>
</table>

**Distance to Target 3 from Sub Location 1** is 880.57 NM. Flight Time (Missile Type 1) = 6.68409605E+00 min.

**Weapon 1 Arrives on Target 3 When First Aircraft is 9.91 Nautical Miles Beyond Centroid.**


Annulus PK = .1544, Circular PK = .1579, Weapon 2 vs. Aircraft Type 1. Annular Radii Are 12.1340 and 3.4901

**Distance to Target 3 from Sub Location 2** is 894.71 NM. Flight Time (Missile Type 1) = 6.73436818E+00 min.

**Weapon 3 Arrives on Target 3 When First Aircraft is 10.31 Nautical Miles Beyond Centroid.**

Annulus PK = .1696, Circular PK = .1927, Weapon 3 vs. Aircraft Type 1. Annular Radii Are 10.3146 and 2.3903

**Distance to Target 3 from Sub Location 3** is 1488.83 NM. Flight Time (Missile Type 1) = 9.70338106E+00 min.

**Weapon 5 Arrives on Target 3 When First Aircraft is 28.16 Nautical Miles Beyond Centroid.**
TARGET 4 WAKE RELEASE TIMES
AIRCRAFT TYPE = 1 STARTS AT 4.90 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.97 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 4.97 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.00 MINUTES.
AIRCRAFT TYPE = 1 STARTS AT 5.17 MINUTES.

DISTANCE TO TARGET 4 FROM SUR LOCATION 1 IS 579.79 NM. FLIGHT TIME (MISSILE TYPE 1) = 5.566297467E+00 MIN.
WEAPON 1 ARRIVES ON TARGET 4 WHEN FIRST AIRCRAFT IS 0.00 NAUTICAL MILES BEYOND CENTROID.

DISTANCE TO TARGET 4 FROM SUR LOCATION 2 IS 599.22 NM. FLIGHT TIME (MISSILE TYPE 1) = 5.64020240E+01 MIN.
WEAPON 3 ARRIVES ON TARGET 4 WHEN FIRST AIRCRAFT IS 0.00 NAUTICAL MILES BEYOND CENTROID.

DISTANCE TO TARGET 4 FROM SUR LOCATION 3 IS 1981.39 NM. FLIGHT TIME (MISSILE TYPE 1) = 1.02896268E+01 MIN.
WEAPON 5 ARRIVES ON TARGET 4 WHEN FIRST AIRCRAFT IS 37.24 NAUTICAL MILES BEYOND CENTROID.
ANNULAR PK = .0517; CIRCULAR PK = .0237; WEAPON 5 VS. AIRCRAFT TYPE 1. ANNULAR RADII ARE 37.2406 AND 31.114.
<table>
<thead>
<tr>
<th>TARGET</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td></td>
<td>.9617</td>
<td>.9470</td>
<td>.8781</td>
<td>.9234</td>
</tr>
<tr>
<td></td>
<td>.9274</td>
<td>.9405</td>
<td>.9254</td>
<td>.9131</td>
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<tr>
<td></td>
<td>.9403</td>
<td>.8719</td>
<td>.8468</td>
<td>.9684</td>
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<td>.8178</td>
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<td>.9807</td>
<td>.9828</td>
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</table>

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST 0.000000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>DEFAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>15.0000</td>
<td>3.3141</td>
<td>15.0000</td>
<td>3.3141</td>
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<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>15.0000</td>
<td>4.5631</td>
<td>15.0000</td>
<td>4.5631</td>
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<tr>
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<td>3</td>
<td>1</td>
<td>15.0000</td>
<td>3.0380</td>
<td>15.0000</td>
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<td>1</td>
<td>5.0000</td>
<td>4.0612</td>
<td>5.0000</td>
<td>4.0612</td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td></td>
<td></td>
<td><strong>50.0000</strong></td>
<td><strong>14.9765</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**EXPECTED VALUE KILLED = 14.9765**

**ITERATION NUMBER** 1

**DELTA N IN COLUMN 1 FROM ROW 2 TO ROW 3 IS 1.000**

**EXPECTED VALUE KILLED = 16.0766**

**ITERATION NUMBER** 2

**DELTA N IN COLUMN 2 FROM ROW 2 TO ROW 4 IS 0.475**

**EXPECTED VALUE KILLED = 16.2549**

**ITERATION NUMBER** 3

**DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS 1.000**

**EXPECTED VALUE KILLED = 17.0229**

**ITERATION NUMBER** 4

**DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.474**

**EXPECTED VALUE KILLED = 17.2970**
<table>
<thead>
<tr>
<th>ITERATION NUMBER</th>
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</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>5 FROM ROW 3 TO ROW 2 IS 1.000</td>
</tr>
<tr>
<td>EXPECTED VALUE KILLED</td>
<td>17.9057</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ITERATION NUMBER</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>6 FROM ROW 3 TO ROW 2 IS 1.000</td>
</tr>
<tr>
<td>EXPECTED VALUE KILLED</td>
<td>14.2968</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>2 FROM ROW 2 TO ROW 3 IS 0.525</td>
</tr>
<tr>
<td>EXPECTED VALUE KILLED</td>
<td>18.8510</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ITERATION NUMBER</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>4 FROM ROW 2 TO ROW 1 IS 1.000</td>
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<tr>
<td>EXPECTED VALUE KILLED</td>
<td>19.6168</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>ITERATION NUMBER</th>
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</tr>
</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>3 FROM ROW 1 TO ROW 3 IS 0.298</td>
</tr>
<tr>
<td>EXPECTED VALUE KILLED</td>
<td>19.6509</td>
</tr>
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</table>

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>4 FROM ROW 1 TO ROW 4 IS 0.061</td>
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<tr>
<td>EXPECTED VALUE KILLED</td>
<td>19.6559</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ITERATION NUMBER</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELTA N IN COLUMN</td>
<td>2 FROM ROW 4 TO ROW 3 IS 0.043</td>
</tr>
<tr>
<td>EXPECTED VALUE KILLED</td>
<td>19.6594</td>
</tr>
</tbody>
</table>
ITERATION NUMBER 12

DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS .039

EXPECTED VALUE KILLED = 19.6605

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .010000000 TO CAUSE ITERATION.
ALLOCATION

ITERATION NUMBER 13

TARGET 1
1.7016 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3).
1.1740 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4).

TARGET 2
2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 3 SALVO 1 (WEAPON 5).
2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 3 SALVO 2 (WEAPON 6).

TARGET 3
1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION 1 SALVO 1 (WEAPON 7).
2.9834 MISSILES FROM 1 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 8).

TARGET 4
4.3166 MISSILES FROM 1 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 9).
6.2660 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 10).

SUB LOCATION 1
1.0000 MISSILES FROM SALVO 1 TO TARGET 3.
5.5884 MISSILES FROM SALVO 2 TO TARGET 3.
4.3166 MISSILES FROM SALVO 2 TO TARGET 4.

SUB LOCATION 2
1.7016 MISSILES FROM SALVO 1 TO TARGET 1.
2.9834 MISSILES FROM SALVO 1 TO TARGET 3.
1.3740 MISSILES FROM SALVO 2 TO TARGET 1.
6.2660 MISSILES FROM SALVO 2 TO TARGET 4.

SUB LOCATION 3
2.0000 MISSILES FROM SALVO 1 TO TARGET 2.
2.0000 MISSILES FROM SALVO 2 TO TARGET 2.
MULTIPLIER MATRIX

TARGET 1
0.2956 0.2544
1.3905 1.4260
6.37 4802

TARGET 2
0.8093 0.6633
0.8390 0.6047
4.35 7838

TARGET 3
1.9308 1.5213
1.8444 1.4400
3562 3104

TARGET 4
1.4260 1.4260
1.4260 1.4260
0.0166 0.0148

ITERATION NUMBER 14
DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS 0.036
EXPECTED VALUE KILLED = 9.6422

ITERATION NUMBER 15
DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS 0.071
EXPECTED VALUE KILLED = 19.6641

ITERATION NUMBER 16
DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS 0.042
EXPECTED VALUE KILLED = 19.6665
ITERATION NUMBER  17
DELTA N IN COLUMN  2 FROM ROW  4 TO ROW  3 IS  .045
EXPECTED VALUE KILLED = 19.6692

ITERATION NUMBER  18
DELTA N IN COLUMN  3 FROM ROW  3 TO ROW  1 IS  .036
EXPECTED VALUE KILLED = 19.6696

ITERATION NUMBER  19
DELTA N IN COLUMN  4 FROM ROW  1 TO ROW  4 IS  .045
EXPECTED VALUE KILLED = 19.6724

ITERATION NUMBER  20
DELTA N IN COLUMN  2 FROM ROW  4 TO ROW  3 IS  .045
EXPECTED VALUE KILLED = 19.6751

ITERATION NUMBER  21
DELTA N IN COLUMN  3 FROM ROW  3 TO ROW  1 IS  .035
EXPECTED VALUE KILLED = 19.6795

ITERATION NUMBER  22
DELTA N IN COLUMN  4 FROM ROW  1 TO ROW  4 IS  .045
EXPECTED VALUE KILLED = 19.6743

ITERATION NUMBER  23
DELTA N IN COLUMN  2 FROM ROW  4 TO ROW  3 IS  .045
EXPECTED VALUE KILLED = 19.6809
ITERATION NUMBER 24
DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS .035
EXPECTED VALUE KILLED = 19.6814

ITERATION NUMBER 25
DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS .045
EXPECTED VALUE KILLED = 19.6841

ITERATION NUMBER 26
DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .045
EXPECTED VALUE KILLED = 19.6466

ITERATION NUMBER 27
DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS .035
EXPECTED VALUE KILLED = 19.6973

ITERATION NUMBER 28
DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS .045
EXPECTED VALUE KILLED = 19.6900

ITERATION NUMBER 29
DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .045
EXPECTED VALUE KILLED = 19.6927

ITERATION NUMBER 30
DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS .035
EXPECTED VALUE KILLED = 19.6932
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<td>To Row</td>
<td>Expected Value Killed</td>
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<tr>
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Lagrange multipliers must differ by at least 0.0010000000 to cause iteration.
<table>
<thead>
<tr>
<th>ALLOCATION</th>
<th>ITERATION NUMBER</th>
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</thead>
</table>

| TARGET 1    | 2,0000 MISSILES FROM 2 SHRS AT SUB LOCATION | 2, SALVO 1 (WEAPON 3)* |
| G4460 MISSILES FROM 2 SHRS AT SUB LOCATION | 2, SALVO 2 (WEAPON 4)* |

| TARGET 2    | 2,0000 MISSILES FROM 2 SHRS AT SUB LOCATION | 3, SALVO 1 (WEAPON 5)* |
| 1,0000 MISSILES FROM 2 SHRS AT SUB LOCATION | 3, SALVO 2 (WEAPON 6)* |

| TARGET 3    | 1,0540 MISSILES FROM 2 SHRS AT SUB LOCATION | 2, SALVO 2 (WEAPON 4)* |

| SUB LOCATION 1 | 1,0000 MISSILES FROM SALVO 1 TO TARGET 3 |
| 1,0000 MISSILES FROM SALVO 2 TO TARGET 3 |

| SUB LOCATION 2 | 2,0000 MISSILES FROM SALVO 1 TO TARGET 1 |
| 0,460 MISSILES FROM SALVO 2 TO TARGET 1 |
| 1,0540 MISSILES FROM SALVO 2 TO TARGET 4 |

| S'UR LOCATION 3 | 2,0000 MISSILES FROM SALVO 1 TO TARGET 2 |
| 2,0000 MISSILES FROM SALVO 2 TO TARGET 2 |
### Multiplier Matrix

<table>
<thead>
<tr>
<th>Target</th>
<th>1</th>
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<th>4</th>
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<td>1.9093</td>
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**Lagrange Multipliers Must Differ By At Least** 0.000100000 To Cause Iteration.
<table>
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<td>2,0000 MISSILES FROM</td>
<td>2 SUBS AT SUR LOCATION</td>
<td>2, SALVO 1 (WEAPON 3)</td>
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<tr>
<td>.9460 MISSILES FROM</td>
<td>2 SUBS AT SUR LOCATION</td>
<td>2, SALVO 2 (WEAPON 4)</td>
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<td>TARGET 2</td>
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<td>2,0000 MISSILES FROM</td>
<td>2 SUBS AT SUR LOCATION</td>
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<td>TARGET 3</td>
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<td>1,0000 MISSILES FROM SALVO</td>
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<tr>
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<tr>
<td>SUB LOCATION 3</td>
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<tr>
<td>2,0000 MISSILES FROM SALVO</td>
<td>2 TO TARGET 2</td>
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</table>
MULTIPLIER MATRIX

TARGET 1
0.2974  0.2560
1.9021  1.4347
0.6074  0.4831

TARGET 2
0.8093  0.6633
0.8393  0.6847
0.9833  0.7638

TARGET 3
1.9124  1.5068
1.6268  1.4263
0.3528  0.3074

TARGET 4
1.4347  1.4347
1.4347  1.4347
0.0167  0.0149

MULTIPLIER MATRIX CONVERGED WITHIN TOLERANCE OF 0.00000000
CURRENT DELTA LAMDA IS 0.0001000000
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<th>AIRCRAFT KILLED</th>
<th>VALUE</th>
<th>TOTAL VALUE</th>
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EXCEPTION VALUE KILLED: 19.7147

TOTALS

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</table>
FOR MISSILE TYPE 1
SUM OF LOWEST LAMBDAS WITH WEAPONS = 1.7671
SUM OF HIGHEST LAMBDAS = 3.4193
SUB MOVED FROM LOCATION 3 TO LOCATION 1
THIS IS MOVE NUMBER 1 OF A SUB.
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<td><strong>TARGET 1</strong></td>
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<tr>
<td>2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION</td>
<td>2x SALVO 1 (WEAPON 3)</td>
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<tr>
<td>.9660 MISSILES FROM 2 SUBS AT SUB LOCATION</td>
<td>2x SALVO 2 (WEAPON 4)</td>
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<tr>
<td><strong>TARGET 2</strong></td>
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</tr>
<tr>
<td>1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION</td>
<td>3x SALVO 1 (WEAPON 5)</td>
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<tr>
<td>1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION</td>
<td>3x SALVO 2 (WEAPON 6)</td>
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<tr>
<td><strong>TARGET 3</strong></td>
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<td>2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION</td>
<td>1x SALVO 1 (WEAPON 7)</td>
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<td>2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION</td>
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<td><strong>TARGET 4</strong></td>
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<td>2x SALVO 2 (WEAPON 9)</td>
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<table>
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<tr>
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<table>
<thead>
<tr>
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LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .100000000000 TO CAUSE ITERATION.
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EXPECTED VALUE KILLED = 20.7135

ITERATION NUMBER: 47

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .145

EXPECTED VALUE KILLED = 20.7566

ITERATION NUMBER: 48

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .130

EXPECTED VALUE KILLED = 20.7773

ITERATION NUMBER: 49

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .120

EXPECTED VALUE KILLED = 20.7933

ITERATION NUMBER: 50

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .108

EXPECTED VALUE KILLED = 20.8074
ITERATION NUMBER  41
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  .100
EXPECTED VALUE KILLED = 20.8195

ITERATION NUMBER  52
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  .090
EXPECTED VALUE KILLED = 20.8292

ITERATION NUMBER  53
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  .083
EXPECTED VALUE KILLED = 20.8398

ITERATION NUMBER  54
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  .075
EXPECTED VALUE KILLED = 20.8495

ITERATION NUMBER  55
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  .069
EXPECTED VALUE KILLED = 20.8477

ITERATION NUMBER  56
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  .062
EXPECTED VALUE KILLED = 20.8523

ITERATION NUMBER  57
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  .057
EXPECTED VALUE KILLED = 20.8560
ITERATION NUMBER 96

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.051

EXPECTED VALUE KILLED = 20.8491

ITERATION NUMBER 99

DELTA N IN COLUMN 7 FROM ROW 7 TO ROW 4 IS 0.048

EXPECTED VALUE KILLED = 20.8414

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST 0.00000000 TO CAUSE ITERATION.
TARGET 1
2.0000 MISSILES FROM 2 SHS AT SUR LOCATION 2, SALVO 1 (WEAPON 3).
1.4472 MISSILES FROM 2 SHS AT SUB LOCATION 2, SALVO 2 (WEAPON 4).

TARGET 2
1.0000 MISSILES FROM 1 SHS AT SUR LOCATION 3, SALVO 1 (WEAPON 5).
1.0000 MISSILES FROM 1 SHS AT SUR LOCATION 3, SALVO 2 (WEAPON 6).

TARGET 3
2.0000 MISSILES FROM 2 SHS AT SUR LOCATION 1, SALVO 1 (WEAPON 1).
1.3781 MISSILES FROM 2 SHS AT SUR LOCATION 1, SALVO 2 (WEAPON 2).

TARGET 4
.6219 MISSILES FROM 2 SHS AT SUR LOCATION 1, SALVO 2 (WEAPON 2).
.5378 MISSILES FROM 2 SHS AT SUR LOCATION 2, SALVO 2 (WEAPON 4).

SUB LOCATION 1
2.0000 MISSLES FROM SALVO 1 TO TARGET 3
1.3781 MISSILES FROM SALVO 2 TO TARGET 3
.6219 MISSILES FROM SALVO 2 TO TARGET 4

SUB LOCATION 2
2.0000 MISSILES FROM SALVO 1 TO TARGET 1
.6622 MISSILES FROM SALVO 2 TO TARGET 1
.5378 MISSILES FROM SALVO 2 TO TARGET 4

SUB LOCATION 3
1.0000 MISSILES FROM SALVO 1 TO TARGET 2
1.0000 MISSILES FROM SALVO 2 TO TARGET 2
# Multiplier Matrix

<table>
<thead>
<tr>
<th>TARGET</th>
<th>1</th>
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**Iteration Number 61**

Delta N in Column 4 from Row 4 to Row 1 is .043

Expected Value Killed = 20.8638

**Iteration Number 62**

Delta N in Column 2 from Row 3 to Row 4 is .040

Expected Value Killed = 20.8655

**Iteration Number 63**

Delta N in Column 4 from Row 4 to Row 1 is .036

Expected Value Killed = 20.8670
ITERATION NUMBER  64
DELTA N IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.033
EXPECTED VALUE KILLED =  20.8662

ITERATION NUMBER  65
DELTA N IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.030
EXPECTED VALUE KILLED =  20.8692

ITERATION NUMBER  66
DELTA N IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.027
EXPECTED VALUE KILLED =  20.8701

ITERATION NUMBER  67
DELTA N IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.025
EXPECTED VALUE KILLED =  20.8708

ITERATION NUMBER  68
DELTA N IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.023
EXPECTED VALUE KILLED =  20.8713

ITERATION NUMBER  69
DELTA N IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.020
EXPECTED VALUE KILLED =  20.8718

ITERATION NUMBER  70
DELTA N IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.019
EXPECTED VALUE KILLED =  20.8722
ITERATION NUMBER 71
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .017
EXPECTED VALUE KILLED = 20.8725

ITERATION NUMBER 72
DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 4 IS .016
EXPECTED VALUE KILLED = 20.8728

ITERATION NUMBER 73
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .014
EXPECTED VALUE KILLED = 20.8730

ITERATION NUMBER 74
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .013
EXPECTED VALUE KILLED = 20.8732

ITERATION NUMBER 75
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .012
EXPECTED VALUE KILLED = 20.8734

ITERATION NUMBER 76
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .011
EXPECTED VALUE KILLED = 20.8735

ITERATION NUMBER 77
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .010
EXPECTED VALUE KILLED = 20.8734
ITERATION NUMBER 74
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .009
EXPECTED VALUE KILLED = 20.4777

ITERATION NUMBER 79
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .009
EXPECTED VALUE KILLED = 20.4738

ITERATION NUMBER 80
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .007
EXPECTED VALUE KILLED = 20.4719

ITERATION NUMBER 81
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .007
EXPECTED VALUE KILLED = 20.4719

ITERATION NUMBER 82
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .006
EXPECTED VALUE KILLED = 20.4740

ITERATION NUMBER 83
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .006
EXPECTED VALUE KILLED = 20.4740

ITERATION NUMBER 84
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .005
EXPECTED VALUE KILLED = 20.4740
ITERATION NUMBER: 46
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .005
EXPECTED VALUE KILLED = 20.8749
LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .001000000 TO CAUSE ITERATION.
<table>
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**TARGET 1**
- 2.0000 MISSILES FROM
- 1.6922 MISSILES FROM
- 2 SUBS AT SUR LOCATION
- 2 SALVO 1 (WEAPON 3)*
- 2 SALVO 2 (WEAPON 4)*

**TARGET 2**
- 1.0000 MISSILES FROM
- 1.0000 MISSILES FROM
- 1 SUBS AT SUR LOCATION
- 3 SALVO 1 (WEAPON 5)*
- 3 SALVO 2 (WEAPON 6)*

**TARGET 3**
- 2.0000 MISSILES FROM
- 1.1698 MISSILES FROM
- 2 SUBS AT SUR LOCATION
- 1 SALVO 1 (WEAPON 1)*
- 1 SALVO 2 (WEAPON 2)*

**TARGET 4**
- .8302 MISSILES FROM
- .307A MISSILES FROM
- 2 SUBS AT SUR LOCATION
- 1 SALVO 1 (WEAPON 1)*
- 2 SALVO 2 (WEAPON 4)*

**SUB LOCATION 1**
- 2.0000 MISSILES FROM SALVO
- 1.1698 MISSILES FROM SALVO
- 2 TO TARGET 3
- 2 TO TARGET 3

**SUB LOCATION 2**
- 2.0000 MISSILES FROM SALVO
- 1.6922 MISSILES FROM SALVO
- 2 TO TARGET 1
- 2 TO TARGET 1

**SUB LOCATION 3**
- 1.0000 MISSILES FROM SALVO
- 1 TO TARGET 2
- 1 TO TARGET 2
**Multiplier Matrix**

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**Iteration Number**

- **A7**
  - Delta N in Column 2 from Row 3 to Row 4 is 0.004
  - Expected Value Killed = 20.8741

- **A8**
  - Delta N in Column 4 from Row 4 to Row 1 is 0.004
  - Expected Value Killed = 20.8741

- **A9**
  - Delta N in Column 2 from Row 3 to Row 4 is 0.004
  - Expected Value Killed = 20.8741
ITERATION NUMBER 90
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .003
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 91
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .003
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 92
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .003
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 93
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .002
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 94
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .002
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 95
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .002
EXPECTED VALUE KILLED = 20.8741

ITERATION NUMBER 96
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .002
EXPECTED VALUE KILLED = 20.8741
ITERATION NUMBER  97
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .002
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER  98
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .002
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER  99
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 100
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER  101
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER  102
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .004
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER  103
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742
ITERATION NUMBER 104
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 105
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 106
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 107
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 108
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 109
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .001
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 110
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .000
EXPECTED VALUE KILLED = 20.8742
ITERATION NUMBER 111

DFLTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .000

EXPECTED VALUE KILLED = 20.8742

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .000100000 TO CAUSE ITERATION.
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<td>1.7124 MISSILES FROM</td>
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<td>1.1468 MISSILES FROM</td>
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MULTIPLIER MATRIX

TARGET  1
  0.2575  0.2216
  1.6464  1.2419
  0.5258  0.4182

TARGET  2
  0.9529  0.7810
  0.9878  0.8062
  1.1578  0.9228

TARGET  3
  1.5749  1.2409
  1.5044  1.1746
  0.2905  0.2532

TARGET  4
  1.2409  1.2409
  1.2409  1.2409
  0.0144  0.0129

ITERATION NUMBER  113
DELTA N IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  .000
EXPECTED VALUE KILLED =  20.8742

ITERATION NUMBER  114
DELTA N IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  .000
EXPECTED VALUE KILLED =  20.8742

ITERATION NUMBER  115
DELTA N IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  .000
EXPECTED VALUE KILLED =  20.8742
ITERATION NUMBER    114
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER    117
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER    118
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER    119
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.000
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ITERATION NUMBER    120
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.000
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ITERATION NUMBER    121
DELTAN IN COLUMN  4 FROM ROW  4 TO ROW  1 IS  0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER    122
DELTAN IN COLUMN  2 FROM ROW  3 TO ROW  4 IS  0.000
EXPECTED VALUE KILLED = 20.8742
ITERATION NUMBER 123
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 124
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 125
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 126
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 127
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 128
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.000
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 129
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.000
EXPECTED VALUE KILLED = 20.8742
ITERATION NUMBER 110
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 111
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 112
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 113
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 114
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 115
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS 0.00
EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 116
DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS 0.00
EXPECTED VALUE KILLED = 20.8742
ITERATION NUMBER 137
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .000
EXPECTED VALUE KILLED = 20.8742
MULTIPLIER MATRIX CONVERGED WITHIN TOLERANCE OF .0001000000
CURRENT DELTA LAMBDAS IS .0001000000
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**TARGET 1**
- 2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION
- 1.7147 MISSILES FROM 2 SUBS AT SUB LOCATION
- 2 SALVO 1 (WEAPON 3)
- 2 SALVO 2 (WEAPON 4)

**TARGET 2**
- 1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION
- 1 SALVO 1 (WEAPON 5)
- 1 SALVO 2 (WEAPON 6)

**TARGET 3**
- 2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION
- 1.1448 MISSILES FROM 2 SUBS AT SUB LOCATION
- 1 SALVO 1 (WEAPON 1)
- 1 SALVO 2 (WEAPON 2)

**TARGET 4**
- .0552 MISSILES FROM 2 SUBS AT SUB LOCATION
- .2853 MISSILES FROM 2 SUBS AT SUB LOCATION
- 2 SALVO 2 (WEAPON 4)

**SUB LOCATION 1**
- 2.0000 MISSILES FROM SALVO 1 TO TARGET 3
- 1.1448 MISSILES FROM SALVO 2 TO TARGET 3

**SUB LOCATION 2**
- 2.0000 MISSILES FROM SALVO 1 TO TARGET 1
- 1.7147 MISSILES FROM SALVO 2 TO TARGET 1
- .2853 MISSILES FROM SALVO 2 TO TARGET 4

**SUB LOCATION 3**
- 1.0000 MISSILES FROM SALVO 1 TO TARGET 2
- 1.0000 MISSILES FROM SALVO 2 TO TARGET 2
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<td>SALVO 1 TO TARGET 3</td>
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<td>1.7147 MISSILES FROM</td>
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LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .1000000000 TO CAUSE ITERATION. |
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ALLOCATION  ITERATION NUMBER  155

TARGET 1
3.0000 MISSILES FROM 3 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3).
1.4800 MISSILES FROM 3 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4).

TARGET 2
2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 1 (WEAPON 1).
2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2).
.2562 MISSILES FROM 3 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4).

TARGET 4
1.2634 MISSILES FROM 3 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4).

SUB LOCATION 1
2.0000 MISSILES FROM SALVO 1 TO TARGET 1
2.0000 MISSILES FROM SALVO 2 TO TARGET 3

SUB LOCATION 2
3.0000 MISSILES FROM SALVO 1 TO TARGET 1
1.4800 MISSILES FROM SALVO 2 TO TARGET 1
.2562 MISSILES FROM SALVO 2 TO TARGET 3
1.2634 MISSILES FROM SALVO 2 TO TARGET 4

SUB LOCATION 3
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>AIRKILL</th>
<th>KILLED AIRCRAFT</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
<td>16,0000</td>
<td>9,6330</td>
<td>15,000</td>
<td>9,6330</td>
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<tr>
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<td>1</td>
<td>15,0000</td>
<td>0,0000</td>
<td>15,000</td>
<td>0,0000</td>
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<tr>
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<tr>
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<td>5,0000</td>
<td>4,3957</td>
<td>5,000</td>
<td>4,3957</td>
</tr>
<tr>
<td>TOTALS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21,2415</td>
</tr>
</tbody>
</table>

EXPECTED VALUE KILLED = 21,2415
<table>
<thead>
<tr>
<th>MULTIPLIER MATRIX</th>
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</thead>
<tbody>
<tr>
<td>TARGET 1</td>
</tr>
<tr>
<td>1.2095 1.0107</td>
</tr>
<tr>
<td>1.6321 1.0086</td>
</tr>
<tr>
<td>1.2452 1.0126</td>
</tr>
<tr>
<td>1.2452 1.0107</td>
</tr>
<tr>
<td>TARGET 2</td>
</tr>
<tr>
<td>1.219  1.0196</td>
</tr>
<tr>
<td>1.3631 1.0066</td>
</tr>
<tr>
<td>1.0107 1.0107</td>
</tr>
<tr>
<td>1.0107 1.0098</td>
</tr>
<tr>
<td>TARGET 3</td>
</tr>
<tr>
<td>1.1631 1.0093</td>
</tr>
<tr>
<td>1.3566 1.0066</td>
</tr>
<tr>
<td>1.2506 1.0107</td>
</tr>
<tr>
<td>1.0107 1.0107</td>
</tr>
<tr>
<td>TARGET 4</td>
</tr>
<tr>
<td>1.0107 1.0107</td>
</tr>
<tr>
<td>1.0107 1.0106</td>
</tr>
<tr>
<td>1.0107 1.0105</td>
</tr>
<tr>
<td>1.0107 1.0098</td>
</tr>
</tbody>
</table>
FOR MISSILE TYPE 1
SUM OF LOWEST LAHRAS = 2.3507
SUM OF HIGHEST LAHRAS = 2.4999
SUB MOVED FROM LOCATION 2 TO LOCATION 3
THIS IS MOVE NUMBER 3 OF 8 SUB.
<table>
<thead>
<tr>
<th>ALLOCATION</th>
<th>ITERATION NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

**TARGET 1**

- 2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3)
- .4894 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4)

**TARGET 2**

- 1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 1 (WEAPON 5)
- 1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 2 (WEAPON 6)

**TARGET 3**

- 2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 1 (WEAPON 1)
- 2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2)
- .2562 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4)

**TARGET 4**

- 1.2634 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4)

**SUB LOCATION 1**

- 2.0000 MISSILES FROM SALVO 1 TO TARGET 3
- 2.0000 MISSILES FROM SALVO 2 TO TARGET 3

**SUB LOCATION 2**

- 2.0000 MISSILES FROM SALVO 1 TO TARGET 1
- .4894 MISSILES FROM SALVO 2 TO TARGET 1
- .2562 MISSILES FROM SALVO 2 TO TARGET 3
- 1.2634 MISSILES FROM SALVO 2 TO TARGET 4

**SUB LOCATION 3**

- 1.0000 MISSILES FROM SALVO 1 TO TARGET 2
- 1.0000 MISSILES FROM SALVO 2 TO TARGET 2

<table>
<thead>
<tr>
<th>SUB POINT NUMBER</th>
<th>NUMBER OF SUBS</th>
<th>SUB TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .1000000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>15,000</td>
<td>3.4083</td>
<td>15,000</td>
<td>3.4083</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>15,000</td>
<td>2.2602</td>
<td>15,000</td>
<td>2.2602</td>
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<tr>
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<td>3</td>
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<tr>
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<td>1</td>
<td>1</td>
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<td>4.2579</td>
<td>5,000</td>
<td>4.2579</td>
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<tr>
<td><strong>TOTALS</strong></td>
<td></td>
<td><strong>1</strong></td>
<td><strong>50,000</strong></td>
<td><strong>20.8742</strong></td>
<td><strong>50,000</strong></td>
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</table>

EXPECTED VALUE KILLED = 20.8742
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<tr>
<th>SURVIVABILITY PRODUCT FOR TARGET</th>
<th>AIRCRAFT TYPE</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. AIRCRAFT TYPE</td>
<td></td>
<td>.4394</td>
<td>.8493</td>
</tr>
<tr>
<td>B. AIRCRAFT TYPE</td>
<td></td>
<td>.6035</td>
<td>.1484</td>
</tr>
<tr>
<td>3,504 AIRCRAFT OF TYPE 1 MOVED FROM TARGET 4 TO TARGET 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

THIS IS VALUE SHIFT NUMBER 1.

LAGRANGIAN MULTIPLIERS MUST DIFFER BY AT LEAST .1000000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>15,0000</td>
<td>8,4083</td>
<td>15,0000</td>
<td>8,4083</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>16,5045</td>
<td>2,7882</td>
<td>18,5045</td>
<td>2,7882</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
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<td>5,9478</td>
<td>15,0000</td>
<td>5,9478</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1,4955</td>
<td>1,2736</td>
<td>2,7695</td>
<td>1,2736</td>
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<tr>
<td>TOTALS</td>
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<td></td>
<td></td>
<td></td>
<td>50,0000</td>
<td>18,4179</td>
</tr>
</tbody>
</table>

EXPECTED VALUE KILLED = 18.4179

ITERATION NUMBER 250

DELTAN IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .667

EXPECTED VALUE KILLED = 18.7577

ITERATION NUMBER 251

DELTAN IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .049

EXPECTED VALUE KILLED = 18.7564

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .010000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>ALLOCATION</th>
<th>ITERATION NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>298</td>
</tr>
</tbody>
</table>

**TARGET 1**
- 2,0000 MISSILES FROM 2 SUBS AT SUB LOCATION
- 2,0000 MISSILES FROM 2 SUBS AT SUB LOCATION
- SALVO 1 (WEAPON 3)
- SALVO 2 (WEAPON 4)

**TARGET 2**
- 1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION
- 1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION
- SALVO 1 (WEAPON 5)
- SALVO 2 (WEAPON 6)

**TARGET 3**
- 2,0000 MISSILES FROM 2 SUBS AT SUB LOCATION
- 1,5400 MISSILES FROM 2 SUBS AT SUB LOCATION
- SALVO 1 (WEAPON 1)
- SALVO 2 (WEAPON 2)

**TARGET 4**
- 4520 MISSILES FROM 2 SUBS AT SUB LOCATION
- SALVO 1 (WEAPON 2)

**SUB LOCATION 1**
- 2,0000 MISSILES FROM SALVO 1 TO TARGET
- 1,5400 MISSILES FROM SALVO 2 TO TARGET
- 4520 MISSILES FROM SALVO 2 TO TARGET

**SUB LOCATION 2**
- 2,0000 MISSILES FROM SALVO 1 TO TARGET
- 2,0000 MISSILES FROM SALVO 2 TO TARGET

**SUR LOCATION 3**
- 1,0000 MISSILES FROM SALVO 1 TO TARGET
- 1,0000 MISSILES FROM SALVO 2 TO TARGET
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
<td>15.0000</td>
<td>8.7532</td>
<td>15.0000</td>
<td>8.7532</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>18.5045</td>
<td>2.7882</td>
<td>18.5045</td>
<td>2.7882</td>
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<td>1</td>
<td>15.0000</td>
<td>6.4347</td>
<td>15.0000</td>
<td>6.4347</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>14.955</td>
<td>0.7934</td>
<td>14.955</td>
<td>0.7934</td>
</tr>
<tr>
<td>TOTALS</td>
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<td>1</td>
<td>50.0000</td>
<td>18.7694</td>
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<td></td>
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</tbody>
</table>

EXPECTED VALUE KILLED = 18.7694
<table>
<thead>
<tr>
<th>SURVIVABILITY PRODUCT FOR TARGET</th>
<th>AIRCRAFT TYPE</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1°</td>
<td>1</td>
<td>0.4165</td>
</tr>
<tr>
<td>2°</td>
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<tr>
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<td>1</td>
<td>0.4645</td>
</tr>
<tr>
<td>5°</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

6 Aircraft of type 1 moved from Target 1 to Target 2.

This is value shift number 2.

Lagrang Multipliers must differ by at least 0.000000000 to cause iteration.
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>AIRMEN ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>8.5070</td>
<td>4.9642</td>
<td>8.5070</td>
<td>4.9642</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>24.9975</td>
<td>3.7666</td>
<td>24.9975</td>
<td>3.7666</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
<td>15.0000</td>
<td>6.4347</td>
<td>15.0000</td>
<td>6.4347</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1.4955</td>
<td>0.7934</td>
<td>1.4955</td>
<td>0.7934</td>
</tr>
<tr>
<td>TOTALS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50.0000</td>
<td>15.9588</td>
</tr>
</tbody>
</table>

EXPECTED VALUE KILLED = 15.9588

ITERATION NUMBER 299

DELTA N IN COLUMN 3 FROM ROW 1 TO ROW 2 IS 1.898

EXPECTED VALUE KILLED = 16.7147

ITERATION NUMBER 300

DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS 0.049

EXPECTED VALUE KILLED = 16.7192

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST 0.010000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>ALLOCATION</th>
<th>ITFRACTION NUMBER</th>
<th>401</th>
</tr>
</thead>
</table>

| TARGET 1   |                   |     |
| 1.8170 MISSILES FROM | 2 SUBS AT SUB LOCATION | 2 SALVO 1 (WEAPON 3) |

| TARGET 2   |                   |     |
| 1.8170 MISSILES FROM | 2 SUBS AT SUB LOCATION | 2 SALVO 1 (WEAPON 3) |
| 1.8170 MISSILES FROM | 2 SUBS AT SUB LOCATION | 2 SALVO 2 (WEAPON 4) |
| 1.8170 MISSILES FROM | 2 SUBS AT SUB LOCATION | 3 SALVO 1 (WEAPON 5) |
| 1.8170 MISSILES FROM | 2 SUBS AT SUB LOCATION | 3 SALVO 2 (WEAPON 6) |

| TARGET 3   |                   |     |
| 2.0000 MISSILES FROM | 2 SUBS AT SUB LOCATION | 1 SALVO 1 (WEAPON 1) |
| 1.6543 MISSILES FROM | 2 SUBS AT SUB LOCATION | 1 SALVO 1 (WEAPON 2) |

| TARGET 4   |                   |     |
| 1.6543 MISSILES FROM | 2 SUBS AT SUB LOCATION | 1 SALVO 2 (WEAPON 2) |
| 1.6543 MISSILES FROM | 2 SUBS AT SUB LOCATION | 2 SALVO 1 (WEAPON 2) |
| 1.6543 MISSILES FROM | 2 SUBS AT SUB LOCATION | 2 SALVO 1 (WEAPON 2) |

| SUB LOCATION 1 |                   |     |
| 2.0000 MISSILES FROM SALVO | 1 TO TARGET | 3 |
| 1.6543 MISSILES FROM SALVO | 2 TO TARGET | 3 |
| 1.6543 MISSILES FROM SALVO | 2 TO TARGET | 4 |

| SUB LOCATION 2 |                   |     |
| 1.8170 MISSILES FROM SALVO | 1 TO TARGET | 1 |
| 1.8170 MISSILES FROM SALVO | 1 TO TARGET | 2 |
| 1.8170 MISSILES FROM SALVO | 2 TO TARGET | 2 |
| 1.8170 MISSILES FROM SALVO | 2 TO TARGET | 4 |

<p>| SUB LOCATION 3 |                   |     |
| 1.0000 MISSILES FROM SALVO | 1 TO TARGET | 2 |
| 1.0000 MISSILES FROM SALVO | 2 TO TARGET | 2 |</p>
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
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<td>8.5070</td>
<td>2.8257</td>
</tr>
<tr>
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<td>4</td>
<td>1</td>
<td>24.9975</td>
<td>6.7053</td>
<td>24.9975</td>
<td>6.7053</td>
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<td>1</td>
<td>15.0000</td>
<td>6.5586</td>
<td>15.0000</td>
<td>6.5586</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1.4955</td>
<td>0.8035</td>
<td>1.4955</td>
<td>0.8035</td>
</tr>
<tr>
<td>TOTALS</td>
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<td>1</td>
<td></td>
<td>16.8932</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

EXPECTED VALUE KILLED = 16.8932
SURVIVABILITY PRODUCT FOR TARGET 1, AIRCRAFT TYPE 1 = .667
SURVIVABILITY PRODUCT FOR TARGET 2, AIRCRAFT TYPE 1 = .7318
SURVIVABILITY PRODUCT FOR TARGET 3, AIRCRAFT TYPE 1 = .5627
SURVIVABILITY PRODUCT FOR TARGET 4, AIRCRAFT TYPE 1 = .4627
46024 AIRCRAFT OF TYPE 1 MOVED FROM TARGET 4 TO TARGET 2

THIS IS VALUE SHIFT NUMBER 7

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .100000000 TO CAUSE ITERATION.
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
<th>AIRCRAFT TYPE</th>
<th>TOTAL VALUE</th>
<th>KILLED VALUE</th>
<th>TOTAL AIRCRAFT</th>
<th>KILLED AIRCRAFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
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<td>2.8257</td>
<td>8.5070</td>
<td>2.8257</td>
</tr>
<tr>
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<td>4</td>
<td>1</td>
<td>25.3998</td>
<td>6.8133</td>
<td>25.3998</td>
<td>6.8133</td>
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<tr>
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<td>50.0000</td>
<td>16.7849</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

EXPECTED VALUE KILLED = 16.7849

ITERATION NUMBER 402
DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 2 IS .115
EXPECTED VALUE KILLED = 16.8124

ITERATION NUMBER 403
DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .067
EXPECTED VALUE KILLED = 16.8169

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.
ALLOCATION  ITERATION NUMBER  408

TARGET  1
  1.5906 MISSILES FROM 2 SUBS AT SUB LOCATION 2* SALVO 1 (WEAPON 3).

TARGET  2
  4.094 MISSILES FROM 2 SUBS AT SUB LOCATION 2* SALVO 1 (WEAPON 3).
  2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 2* SALVO 2 (WEAPON 4).
  1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3* SALVO 1 (WEAPON 5).
  1.0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3* SALVO 2 (WEAPON 6).

TARGET  3
  2.0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1* SALVO 1 (WEAPON 1).
  1.7212 MISSILES FROM 2 SUBS AT SUB LOCATION 1* SALVO 2 (WEAPON 2).

TARGET  4
  .2788 MISSILES FROM 2 SUBS AT SUB LOCATION 1* SALVO 2 (WEAPON 2).

SUB LOCATION  1
  2.0000 MISSILES FROM SALVO  1 TO TARGET 3
  1.7212 MISSILES FROM SALVO  2 TO TARGET 3
  .2788 MISSILES FROM SALVO  2 TO TARGET 4

SUB LOCATION  2
  1.5906 MISSILES FROM SALVO  1 TO TARGET 1
  4.094 MISSILES FROM SALVO  1 TO TARGET 2
  2.0000 MISSILES FROM SALVO  2 TO TARGET 2

SUB LOCATION  3
  1.0000 MISSILES FROM SALVO  1 TO TARGET 2
  1.0000 MISSILES FROM SALVO  2 TO TARGET 2
<table>
<thead>
<tr>
<th>TARGET NUMBER</th>
<th>CANNONS ALLOCATED</th>
<th>TOTALS</th>
<th>EXPECTED VALUE KILLED</th>
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</thead>
<tbody>
<tr>
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<tr>
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</tr>
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156
SURVIVABILITY PRODUCT FOR TARGET 1: AIRCRAFT TYPE 1 = 6.67
SURVIVABILITY PRODUCT FOR TARGET 2: AIRCRAFT TYPE 1 = 5.57
SURVIVABILITY PRODUCT FOR TARGET 3: AIRCRAFT TYPE 2 = 6.27

LAGRANGE MULTIPLES MUST DIFFER BY AT LEAST 100000000 TO CAUSE EQUATION

157
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<th>TARGET NUMBER</th>
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TOTALS

EXPECTED VALUE KILLED = 16.3970

ITERATION NUMBER = 409

DELTA N IN COLUMN 1 FROM ROW 3 TO ROW 2 IS 0.892

EXPECTED VALUE KILLED = 16.5333

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST 0.0100000000 TO CAUSE ITERATION.
TARGET 1
1,5460 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3).

TARGET 2
1,2985 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2).
.4540 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3).
2,0030 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 2 (WEAPON 4).
1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 1 (WEAPON 5).
1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 2 (WEAPON 6).

TARGET 3
2,0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 1 (WEAPON 1).
.4198 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2).

TARGET 4
.2817 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2).

SUB LOCATION 1
2,0000 MISSILES FROM SALVO 1 TO TARGET 3.
1,2985 MISSILES FROM SALVO 2 TO TARGET 2.
.4198 MISSILES FROM SALVO 2 TO TARGET 3.
.2817 MISSILES FROM SALVO 2 TO TARGET 4.

SUB LOCATION 2
1,5460 MISSILES FROM SALVO 1 TO TARGET 1.
.4540 MISSILES FROM SALVO 1 TO TARGET 2.
2,0000 MISSILES FROM SALVO 2 TO TARGET 2.

SUB LOCATION 3
1,0000 MISSILES FROM SALVO 1 TO TARGET 2.
1,0000 MISSILES FROM SALVO 2 TO TARGET 2.
<table>
<thead>
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<th>TARGET NUMBER</th>
<th>WEAPONS ALLOCATED</th>
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</table>

EXPECTED VALUE KILLED = 16.5884
SURVIVABILITY PRODUCTION FOR TARGET 1: AIRCRAFT TYPE 1 = .6799
SURVIVABILITY PRODUCTION FOR TARGET 2: AIRCRAFT TYPE 1 = .6672
SURVIVABILITY PRODUCTION FOR TARGET 3: AIRCRAFT TYPE 1 = .6666
SURVIVABILITY PRODUCTION FOR TARGET 4: AIRCRAFT TYPE 1 = .6243
4 AIRCRAFT OF TYPE 1 MOVED FROM TARGET 4 TO TARGET 1

THIS IS VALU OF SHIFT NUMBER 9

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .100000000 TO CAUSE ITERATION.
<table>
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EXPECTED VALUE KILLED = 16.5851

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .000000000 TO CAUSE ITERATION.
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<td>4.921 MISSILES FROM</td>
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<tr>
<td>TARGET 3</td>
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<td>2 SUBS AT SUB LOCATION</td>
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<td>2 SUBS AT SUB LOCATION</td>
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<td>TARGET 4</td>
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<td>2.0000 MISSILES FROM SALVO</td>
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Expected value killed = 16.5863
SURVIVABILITY PRODUCT FOR TARGET 1: AIRCRAFT TYPE 1 = 6751
SURVIVABILITY PRODUCT FOR TARGET 2: AIRCRAFT TYPE 1 = 6672
SURVIVABILITY PRODUCT FOR TARGET 3: AIRCRAFT TYPE 1 = 6666
SURVIVABILITY PRODUCT FOR TARGET 4: AIRCRAFT TYPE 1 = 6610
40145 AIRCRAFT OF TYPE 1 MOVED FROM TARGET 4 TO TARGET 1

THIS IS VALUE SHIFT NUMBER 6
ALLOCATION INTEGRIZED
REDDOWN INTEGRIZED
<table>
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**TARGET 1**
- 2,0000 MISSILES FROM 2 SUBS AT SUB LOCATION 2 SALVO 1 (WEAPON 3).

**TARGET 2**
- 1,0000 MISSILES FROM 2 SUBS AT SUR LOCATION 1 SALVO 2 (WEAPON 2).
- 2,0000 MISSILES FROM 2 SUBS AT SUR LOCATION 2 SALVO 2 (WEAPON 4).
- 1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 1 (WEAPON 5).
- 1,0000 MISSILES FROM 1 SUBS AT SUB LOCATION 3 SALVO 2 (WEAPON 6).

**TARGET 3**
- 2,0000 MISSILES FROM 2 SUBS AT SUR LOCATION 1 SALVO 1 (WEAPON 1).
- 1,0000 MISSILES FROM 2 SUBS AT SUB LOCATION 1 SALVO 2 (WEAPON 2).

**TARGET 4**

**SUB LOCATION 1**
- 2,0000 MISSILES FROM SALVO 1 TO TARGET 3
- 1,0000 MISSILES FROM SALVO 2 TO TARGET 2
- 1,0000 MISSILES FROM SALVO 2 TO TARGET 3

**SUB LOCATION 2**
- 2,0000 MISSILES FROM SALVO 1 TO TARGET 1
- 2,0000 MISSILES FROM SALVO 2 TO TARGET 2

**SUB LOCATION 3**
- 1,0000 MISSILES FROM SALVO 1 TO TARGET 2
- 1,0000 MISSILES FROM SALVO 2 TO TARGET 2
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EXPECTED VALUE KILLED = 16.4428

EOJ
REFERENCES


