DEVELOPMENT OF HIGH SPEED TAPERED ROLLER BEARING

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Air Force Aero Propulsion Laboratory
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio
FOREWORD

This is an interim report covering the work conducted by the Physical Laboratories of The Timken Company, Canton, Ohio to fulfill the requirements of U.S.A.F. Contract No. F33615-72-C-1890, Project No. 304806. This work was administered under the direction of the Air Force Aero Propulsion Laboratory, with Mr. John Jenkins (AFAPL/SPL) acting as project engineer.

This report covers work conducted from 1 July 1972 to 1 July 1973.

Publication of this report does not constitute Air Force Approval of the reports findings or conclusions. It is published only for the exchange and stimulation of ideas.

Howard F. Jones, Chief
Lubrication Branch
Fuels and Lubrication Division
This report presents the analytical work conducted to evaluate the effects of various material, design and lubrication parameters on the performance of a high speed tapered roller bearing. The program objective is to operate the proposed bearing to 3.5 million DN (107.95 mm bore x 32422 RPM) under 5000 pounds thrust load.

The analysis includes the following: bearing geometry and design, effects of fit, kinematics, dynamics, load distribution, stresses, EHD and friction forces, analysis of the cone rib lubrication system.
ACKNOWLEDGEMENT

The authors acknowledge the contribution of P. M. Ku and Dr. H. J. Carper of Southwest Research Institute of San Antonio, Texas. They consulted on the subjects of lubricant properties, EHD and frictional analysis.
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NOMENCLATURE

a  Semi-major axis of roller-end/rib contact ellipse, in.
A  Defined in Section 5.5.
A_c  Area of cone duct, in.\(^2\).
A_m  Area of manifold trap, in.\(^2\).
A_o  Area of manifold, in.\(^2\).
A_s  Area of shaft duct, in\(^2\).

b  Semi-minor axis of roller-end/rib contact ellipse, in.
B  Defined in Section 5.5.

\(c_p\)  Specific heat of lubricant, BTU/lb.°F.

\(\cos(T)\)  Geometry factor, defined in Section 7.3.

C_1  Normal load, roller-body/cone-race due to \(C_F\) only, lb.
C_2  Normal load, roller-body/cup-race due to \(C_F\) only, lb.
C_3  Normal load, roller-end/rib due to \(C_F\) only, lb.
C_F  Centrifugal force per roller, lb.

C_g  Clearance between cage-pilot/race-guide, in.
C_p  Clearance between roller-body/cage-pocket, in.

d  Mean roller diameter, in.

d_o  Shaft I.D., in.

d_1  Roller large end diameter, in.

d_2  Roller small end diameter, in.

\(d_3\)  Diameter at roller-end/rib contact center measured perpendicular to bearing centerline, in.

D  Defined in Section 5.5.

D_o  Bearing housing O.D., in.
D_1  Cone bore, in.

D_2  Mean Cone O.D., in.
NOMENCLATURE

D3  Cone base diameter, in.
D4  Cup O.D., in.
D5  Mean cup I.D., in.
D11, D12, D13 Defined in Section 5.5.
Dg  Diameter of cage pilot (large end) in.
Ds  Diameter of cage pilot (small end), in.
Dcd Diameter of cone duct, in.
DN  Cone bore x shaft speed, mm.-rev./min.
E   Modulus of elasticity for bearing, shaft and housing material, \(30 \times 10^6\) psi.
E'  Reduced modulus of elasticity, psi.
E C % cone rib circumference supplied with oil.
E(X) Complete elliptic integral of the second kind.
e f  Surface finish, in.
f   Flow friction factor.
f1, f3 Numerical functions for thermal reduction.
f c  Cage friction factor.
f'  Intermediate flow friction factor.
F g  Friction force, cage-pilot/race-guide, lb.
F p  Friction force, roller-body/cage-pocket, lb.
F1, F2, F3 Friction force, lb.
g  Acceleration due to gravity, 386 in./sec.²
G   Rheological constant for lubricant, \(\alpha_o E\)
G*  Dimensionless EHD parameter
NOMENCLATURE

$G_h$ Defined in Section 7.3.

$G_1, G_2, G_3$ Dimensionless friction parameters, defined in Section 11.2.

$h$ Height of roller-end/rib contact center, measured perpendicular to cone-race, in.

$h_1$ EHD film thickness, roller-body/cone-race, micro-in.

$h_2$ EHD film thickness, roller-end/rib, micro-in.

$h_3$ EHD film thickness, roller-end/rib, micro-in.

$h_{cg}$ Height of roller c.g. measured perpendicular to the plane of roller large end, in.

$H$ Headloss, in.$^2$/sec.$^2$

$H_1, H_2$ Heights of right circular cones. (Fig. 9), in.

$H_{min}$ Dimensionless minimum EHD film thickness in general.

$I_x$ Mass moment of inertia of roller about roller centerline perpendicular to the plane of roller large end, lb.-in.$^2$.

$I_y$ Mass moment of inertia of roller about an axis through the center of gravity of roller and perpendicular to roller centerline, lb.-in.$^2$.

$I_{x1}, I_{y1}, I_{x2}$ Mass moments of inertia (Fig. 9) lb.-in.$^2$

$I_{y2}, I_{x3}, I_{y3}$

$J_R$ Sjovall's radial integral.

$J_T$ Sjovall's thrust integral.

$k$ Thermal conductivity of lubricant, BTU/hr.ft.$^\circ$F.

$K$ K-factor of bearing.

$K(X)$ Complete elliptical integral of the first kind.
 NOMENCLATURE

\( l \)  Effective roller length, in.
\( l_p \)  Axial length of contact, roller-body/cage-pocket, in.
\( L_1 \)  Roller slant length, in.
\( L_2 \)  Roller apex length, in.
\( L_g \)  Axial length of cage pilot guide, in.
\( L_{cd} \)  Length cone duct, in.
\( L_{a1} \)  Arc length of roller-body pocket surface conjunction, in.
\( M_c \)  Drag or drive couple applied to cage due to race-pilot, in.-lb.
\( M_j \)  Gyroscopic couple per roller, in.-lb.
\( n_d \)  Number of cone ducts.
\( N_{sd} \)  Number of shaft ducts.
\( N_c \)  RPM of cage.
\( N_r \)  RPM of roller about its own center.
\( p \)  Hertz contact stress, psi.
\( P_1, P_2, P_3 \)  Pressure at points designated by subscripts, psi.
\( P_i \)  Static contact pressure due to interference fit, cone/shaft, psi.
\( P_0 \)  Static contact pressure due to interference fit, cup/housing, psi.
\( P_e \)  Effective contact pressure, cone/shaft, psi.
\( P \)  Lubricant pressure, psi
\( P_1 \)  Normal load, roller-body/cone-race, due to external bearing load and considering \( C_F \), lb.
\( P_2 \)  Normal load, roller-body/cup-race, due to external bearing load and considering \( C_F \), lb.
\( P_3 \)  Normal load, roller-end/rib, due to external bearing load and considering \( C_F \), lb.
NOMENCLATURE

\( P_{atm} \) Atmospheric pressure, psi.

\( P_{max} \) Normal load roller-body/cone-race for max. loaded

\( q_1 \) Load per unit length, roller-body/cup-race adjacent
small end of roller considering \( C_p \) and \( M_g \) only, lb./in.

\( q_2 \) Load per unit length, roller-body/cup-race adjacent
large end of roller considering \( C_p \) and \( M_g \) only. lb./in.

\( Q^* \) Dimensionless EHD pressure force.

\( Q_1, Q_2 \) EHD pressure force, lb.

\( r \) Roller large end spherical radius, in.

\( r_x \) Radial distance measured from bearing centerline, in.

\( r' \) Radius of curvature of rib, in.

\( r_{cg} \) Radial distance of center of roller c.g. measured
perpendicular to bearing centerline, in.

\( r_0, r_1, r_2, r_3, r_4 \) Radial distances, in.

\( R \) External bearing radial load, lb.

\( Re \) Reynold's number, cone rib lubrication system.

\( Re_p \) Reynold's number, roller-body/cage-pocket.

\( re_g \) Reynold's number, cage pilot.

\( R_1, R_2, R_3 \) Equivalent radii roller-body/cone-race, roller-body/
cup-race, roller-end/rib.

\( s \) Slip ratio

\( S \) Cone RPM

\( t \) Time, sec.

\( t_e \) Elapsed time, sec.

\( T \) External bearing thrust load, lb.

\( T_i \) Induced bearing thrust load due to roller centrifugal
force, lb.

\( T_0 \) Oil inlet temperature, °F.
**NOMENCLATURE**

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<td>Tangential velocity of the point on the roller at roller-end rib contact center with respect to roller centerline, in./sec.</td>
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<td>Tangential velocity at mean cone O.D. with respect to bearing centerline, in./sec.</td>
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<td>$V_4$</td>
<td>Tangential velocity of cage-pilot relative to race-guide, in./sec.</td>
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<td>$V_5$</td>
<td>Sliding velocity at roller-end/rib contact center, in./sec.</td>
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<td>$\nu_{\text{rib}}$</td>
<td>Rib velocity i.e. tangential velocity at the point on the rib at roller-end/rib contact center, in./sec.</td>
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<td>$W^*$</td>
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<td>$Y_1, Y_2$</td>
<td>Relative height, in.</td>
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<td>$z$</td>
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<td>$\alpha$</td>
<td>Half included cup angle, deg.</td>
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<td>$\alpha_0$</td>
<td>Pressure-viscosity coefficient at atmospheric pressure, in.$^2$/lb.</td>
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<td>$\alpha^*$</td>
<td>Thermal reduction parameter. (Section 9.5).</td>
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$\beta$  Half included cone angle, deg.

$\beta_0$  Temperature-viscosity coefficient, $(^\circ F)^{-1}$ (used in Section 11.2).

$\beta'_0$  Temperature-viscosity coefficient, $^\circ R$ (used in Section 9.5).

$\gamma$  Half included centerline angle, deg.

$\gamma_{rib}$  Large rib angle, deg.

$\delta_1$  Increase in mean cone O.D. due to static cone/shaft interference fit, in.

$\delta_2$  Decrease in mean cup I.D. due to cup/housing interference fit, in.

$\delta_i$  Initial diametral interference fit, cone/shaft, in.

$\delta_o$  Diametral interference fit, cup/housing, in.

$\delta_{loss}$  Loss of diametral interference fit due to rotation, cone/shaft, in.

$\Delta_P$  Pressure difference, psi

$\Delta_R$  Infinitesimal length, in.

$\Delta_x$  Distance of resultant normal load, roller-body/cup-race, due to roller centrifugal force ($C_p$) and gyroscopic couple ($M_g$) only and measured from roller-body mid-point along cup-race, in.

$\epsilon$  Load zone parameters

$\theta$  Rib-race angle, deg.

$\theta_H$  Cone duct angle relative to backface, deg.

$\mu$  Poisson's ratio, 0.3.

$\mu_o$  Lubricant viscosity at atmospheric pressure, lb.sec./in.$^2$ (Reyn).

$\mu_o/100$  Lubricant viscosity at 100°F, lb.-sec./in.$^2$

$\mu_o/210$  Lubricant viscosity at 210°F, lb.-sec./in.$^2$
NOMENCLATURE

$\mu_h$  Hertzian parameter, roller-end/rib contact ellipse.

$\nu$  Half included roller angle, deg.

$\xi$  Section 7.3.

$\nu_h$  Hertzian parameter, roller-end/rib contact ellipse.

$\nu_o$  Lubricant kinematic viscosity at atmospheric pressure, centistokes.

$\rho$  Density of steel, assumed same for bearing, shaft and housing, .283 lb./in.$^3$.

$\rho_o$  Density of lubricant at atmospheric pressure, gm/cc.

$\rho_o'$  Density of lubricant at atmospheric pressure, lb.in.$^3$.

$\sigma$  Contact stress, psi

$\sigma_1$  Contact stress at center of roller-body/cone-race contact, ksi.

$\sigma_2$  Contact stress at center of roller-body/cup-race contact, ksi.

$\sigma_3$  Contact stress at center of roller-end/rib contact, ksi.

$\sigma_4$  Maximum radial stress in cone due to rotation, psi.

$\tau$  Tangential stress in cone due to rotation, psi

$\tau_i$  Maximum tangential stress in cone due to rotation, psi.

$\tau_l$  Tangential stress at cone bore due to rotation and effective diametral fit ($\delta_{eff}$), psi.

$\tau_o$  Lubricant shear rate, psi.

$\phi$  Angle, defined in Section 2.1, deg.

$\phi_t$  Thermal reduction factor.

$\phi_s$  Ellipticity factor.

$\psi$  Load zone, deg.

$\psi_o$  Angle, defined in Figure 7, deg.

$\psi_n$  Angular position of any roller measured from maximum loaded roller, deg.
NOMENCLATURE

$\omega$  Angular speed of shaft, radians/sec.

$\omega_c$  Angular speed of cage, radians/sec.

$\omega_r$  Angular speed of roller about its own center, radians/sec.
1. INTRODUCTION

1.1 Objective
The objective of this program is to develop a high speed tapered roller bearing capable of operating at a speed of 3.5 million DN under 5000 pounds thrust load. For the 4.25" (107.95 mm) bore bearing to be tested, this is a shaft speed of 32,422 RPM. It is the first program to determine the feasibility of using a tapered roller bearing in a turbine engine environment of high speed and temperature combined with moderate thrust and negligible radial loading.

1.2 Current Technology
Development of high speed tapered roller bearings commenced approximately four years ago (1969) at The Timken Company in conjunction with Boeing-Vertol. The program goals were to develop spiral bevel gear support bearings for use in helicopter transmissions. The bearings ranged in bore sizes from 3.5" (88.9 mm) to 5.11" (125.8 mm). These were tested successfully to 1.4 and 1.8 million DN, respectively.

The problems encountered during development testing were not of the usual nature of high speed ball and cylindrical roller bearings. Review of the open literature indicated that the minimization or control of roller/cage slip
(deviation from epicyclic motion) was the prime objective in ball and cylindrical roller bearing development whereas, the tapered roller bearings did not exhibit any detectable slip. The prime problem encountered was maintaining a supply of lubricant to the cone thrust rib. Various devices were used in an effort to supply the oil to the rib. Some of these are documented in Ref. 1. The most successful technique was to provide a second source of lubricant supply at the cone large rib. This system design will be incorporated in the test program.

1.3 Analytical Approach
This report represents the analytical studies conducted prior to hardware tests. Existing and new technology have been combined to form a comprehensive study of bearing parameters under high speed operation. Note: Throughout the remaining text of this report the term "bearings" will refer to tapered roller bearings.

In Section 2, bearing nomenclature and geometry are defined. Two cage designs, roller and race guided, are presented. Whether the standard stamped roller guided cage has sufficient strength to withstand inertia stresses will be determined in initial testing.
Section 3 covers material requirements and selection for the test bearings. Effects of interference fit and inertia stresses are described analytically in Section 4.

Sections 5 through 11 represent the formulation of speed, load, stress, deformation, EHD and frictional effects.

The analysis of the cone large rib lubrication system (second source of lubricant) from a design and effectiveness standpoint is given in Section 12.

Discussion and results of analytical studies conducted on the candidate test bearing for this current program are presented in Section 13.
2. BEARING GEOMETRY

2.1 Geometrical Relationships

Based on basic principles of plane geometry we can derive the following useful formulae:

Roller apex length \( L_2 = \frac{d_1}{2 \sin \nu} \) (Fig. 1) \( (2.01) \)

Cone base diameter \( D_3 = 2L_2 \sin \beta \) (Fig. 2) \( (2.02) \)

Roller slant length \( L_1 = \frac{d_1 - d_2}{2 \sin \nu} \) (Fig. 3) \( (2.03) \)

Mean cone OD \( D_2 = \frac{d \sin \beta}{\sin \nu} \) (Fig. 2) \( (2.04) \)

Mean cup ID \( D_5 = \frac{d \sin \alpha}{\sin \nu} \) (Fig. 2) \( (2.05) \)

Height of roller-end/rib contact center, measured perpendicular to cone-race = \( h = r (\sin \phi + \cos \theta) \) \( (2.06) \)

where, \( \phi = \left[ \sin^{-1} \left( \frac{d_1}{2r} \right) - \nu \right] \) \( (2.07) \)

and \( \theta = \) rib-race angle

Diameter of roller-end rib contact center measured perpendicular to bearing centerline

\( d_3 = D_3 + 2h \left( \frac{\sin (\theta - \beta)}{\sin \beta} \right) \)
Figure 1 - Typical Tapered Roller Bearing
Figure 2 - Bearing Symbols
Figure 3 - Typical Tapered Roller
Height of roller c.g. measured perpendicular to the plane of roller large end = $h_{cg}$

$$h_{cg} = \frac{L_1 \cos \nu}{4} \left[ \frac{d_1^2 + 2d_1d_2 + 3d_2^2}{d_1^2 + d_1d_2 + d_2^2} \right]$$ (2.09)

Radial distance of roller c.g. measured perpendicular to bearing center = $r_{cg}$ = $(L_2 \cos \nu - h_{cg}) \sin \gamma$ (2.10)

2.2 Cage

Two types of cage design are considered for this bearing. These are the standard stamped design that guide on the roller bodies and the race guided type, which is piloted to either/or both races and the shaft and/or housing. The sole function of both of these cage designs is to maintain roller spacing. Throughout the remaining text these will be referred to as roller or race guided.

Figure 4 shows the roller guided cage. Unless cage breakage occurs, this will be the design used exclusively in the test program. Estimates of the tensile hoop stress at 3.5 million DN indicate the cage will be operating at 20,000 psi, which is near the tensile yield stress for the material. For calculations of fluid drag, the contact arc is considered to be projection of the wing length of the roller body.
Figure 4 - Nomenclature and Geometry of Typical Roller Guided Cage
Figure 5 is a drawing of the race guided cage. The design shown is piloted on the cone large rib O.D. and a spacer abutting the cup backface. This "Z" configuration allows oil jets to be directed at the small end of the roller and eliminates excessive lubricant churning at the large end. Under operating conditions the clearance between the pilots and guide surfaces range from .002" to .008". Fluid drag forces are calculated considering the contact arc to be a projection of the stock thickness on the roller body.
Figure 5 - Nomenclature and Geometry of Typical Race Guided Cage
3. MATERIAL SELECTION

3.1 Cone, Cup and Roller

The material used in a high-speed tapered roller bearing must satisfy three requirements. Under operating conditions, it must permit the bearing to maintain its structural integrity through resistance to cracking, provide adequate temper resistance and hot hardness, and exhibit resistance to fatigue damage. From a metallurgical standpoint, consumable electrode vacuum melted (CEVM) Timken CBS-1000M carburizing grade of steel meets the above requirements.

The bearings will operate with a lubricant supply temperature of 300°F (constant), and lubricant outlet temperature will be in excess of 400°F. Considering these conditions it is reasonable to assume a continuous localized temperature at the rolling or sliding conjunctions in excess of 500°F. The Chemical Analyses and Hot Hardness Characteristics of CBS-600, CBS-1000 and M-50 are shown in Table I and Figure 6, respectively. M-50 is included because it has been the most common temper resistant steel used in high speed ball and straight roller bearings. Timken CBS-600 is recommended for continuous service to 500°F with intermittent service to 600°F.

Refer back to any of the figures showing bearing sections and it is readily apparent that the cone creates unique problems
<table>
<thead>
<tr>
<th>Steel Type</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>Cu</th>
<th>Al</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBS-600*</td>
<td>.16/.22</td>
<td>.50/.70</td>
<td>.025</td>
<td>.025</td>
<td>.90/1.25</td>
<td>1.25/1.65</td>
<td>---</td>
<td>.90/1.10</td>
<td>--</td>
<td>--</td>
<td>----</td>
</tr>
<tr>
<td>CBS-1000*</td>
<td>.18/.23</td>
<td>.40/.60</td>
<td>.025</td>
<td>.025</td>
<td>.40/.60</td>
<td>.90/1.20</td>
<td>---</td>
<td>4.75/5.25</td>
<td>--</td>
<td>--</td>
<td>.75/1.00</td>
</tr>
<tr>
<td>M-50**</td>
<td>.77/.85</td>
<td>.35</td>
<td>.025</td>
<td>.025</td>
<td>.25</td>
<td>3.75/4.25</td>
<td>.10</td>
<td>4.00/4.50</td>
<td>--</td>
<td>--</td>
<td>.90/1.10</td>
</tr>
</tbody>
</table>

* Case-Carburized Timken Specification

** Through-Hardened
Figure 6 - Hot Hardness Characteristics of Various Bearing Steels
for materials. The standard profile with the large rib and undercut presents a narrow section that is susceptible to cracking and fracture, when produced from through hardened material. The manifold and lubrication ducts add even more stress risers. Other considerations of residual stress distribution and unstable crack extension indicate that a low carbon, case-carburized steel offers the best properties for this application.

The third point is in regard to fatigue resistance. It has been repeatedly demonstrated that CEVM steel will improve bearing life when applied for the express purpose of eliminating nonmetallic inclusion origin fatigue damage. The improvement factor varies a great deal in individual tests but a generally accepted average factor ranges from two to five times improvement over conventional vacuum degassed air melted steel.

A fourth item, not previously mentioned, is that CBS-1000M apparently exhibits greater resistance to scuffing than conventional bearing steels. Admittedly, these test results are limited, however, at higher speeds and boundary lubrication conditions this would be an added advantage. Table II summarizes these test results.

3.2 Cage

Calculations show that tensile hoop stresses due to rotation could reach 20,000 psi at 3.5 million DN with the conventional roller guided stamped steel cages. Provided the effects of
TABLE II

Scuffing Resistance Tests of SAE 4620 Case-Carburized Versus Timken CBS-1000M Case-Carburized Steel.

TEST APPARATUS
Timken Extreme Pressure Tester as described in ASTM Standard D2782. Test cups (1.938" O.D.) in all tests were SAE 4620 case-carburized steel. The test blocks consisted of three (3) SAE 4620 case-carburized specimens and four (4) CBS-1000M case-carburized specimens.

PROCEDURE
The procedure consisted of applying 1 pound/minute dead weight to the load lever for ten minutes with the cup rotating at 1165 RPM. Ten pounds lever weight is equivalent to a normal force of 116.6 pounds or 36,000 psi compressive stress between the cup and test block. Throughout the load-up cycle and a subsequent 10 minute run-in period, the sliding conjunction was lubricated with a stream of MIL-L-23699 oil at 100°F. At this time, the supply of oil was shut off and any excess was blown away with shop air. Running time prior to scuffing a scoring was recorded.

RESULTS

<table>
<thead>
<tr>
<th>Quantity of Test Blocks</th>
<th>Material</th>
<th>Running Time Prior to Scoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4620 Case-Carburized</td>
<td>&lt; 10 sec.</td>
</tr>
<tr>
<td>1</td>
<td>CBS-1000M Case-Carburized</td>
<td>2.75 min.</td>
</tr>
<tr>
<td>2</td>
<td>CBS-1000M Case-Carburized</td>
<td>4.38 min.</td>
</tr>
<tr>
<td>1</td>
<td>CBS-1000M Case-Carburized</td>
<td>7.97 min.</td>
</tr>
</tbody>
</table>
unbalance can be minimized and the pocket corners are carefully formed, an SAE 1010 steel should suffice for this application. If cage breakage occurs, added strength can be obtained from heat treatment and/or surface preparation.

For the machined, race guided cage, a higher carbon steel with greater strength would be used.
4. INTERFERENCE FIT AND INERTIA STRESSES

In the following sections 4.1 through 4.6 the varying sections of cone and cup have been approximated to sleeves of uniform section thickness. The mean cone O.D. and the mean cup I.D. are used in the formulae. The physical properties such as Young's modulus of elasticity, Poisson's ratio and density for shaft, bearing, and housing material are assumed identical and the same as those of steel. In addition, it is assumed that all deformations take place within the proportional limit of the material.

4.1 Contact Pressure (Ref. 2)

Based on standard ring formulae the contact pressures due to interference fit are given below:

Contact pressure, cone/shaft,

\[ P_i = \frac{E(D_1^2 - d_o^2)(D_2^2 - d_1^2)}{2D_1^3(D_2^2 - d_o^2)} \delta_i \]  \hspace{1cm} (4.01)

Contact pressure, cup/housing,

\[ P_o = \frac{E(D_4^2 - D_5^2)(D_0^2 - D_4^2)}{2D_4^3(D_0^2 - D_5^2)} \delta_o \]  \hspace{1cm} (4.02)

Where; \( E = \text{Young's modulus} = 30 \times 10^6 \text{ psi} \)
4.2 Mean Cone O.D. Increase (Ref. 2)

When the cone is mounted on a shaft with an interference fit, the shaft and cone deform. The increase in cone O.D. can be calculated using the following formula:

\[
\delta_1 = \frac{D_2(D_1^2 - d_o^2)}{D_1(D_2^2 - d_o^2)} \delta_i
\]  

(4.03)

4.3 Mean Cup I.D. Decrease (Ref. 2)

Similar to increase in cone O.D. due to interference fit, the decrease in cup I.D. is:

\[
\delta_2 = \frac{D_5(D_0^2 - d_4^2)}{D_4(D_0^2 - d_5^2)} \delta_o
\]  

(4.04)

4.4 Inertia Stresses in Cone (Ref. 3)

At high speed, the cone is subjected to centrifugal force which induces radial and tangential (hoop) stress in the cone. The maximum radial stress in the average section of cone occurs at a radial distance = \(0.5(D_1D_2)^{0.5}\) and is:

\[
\sigma_4 = 8.29 \times 10^{-7} S^2 (D_2-D_1)^2
\]  

(4.05)

where; \(8.29 \times 10^{-7} = \frac{(3+\mu)\rho}{g} \left(\frac{\pi}{60}\right)^2\)

and \(\mu = \) Poisson's ratio = .3

\(\rho = .283 \text{ lb/in.}^3\)

\(g = 386.088 \text{ in./sec}^2\)

Generally, the radial stress is much smaller compared to tangential stress due to rotation and may be neglected for approximate calculations.
The tangential tensile stress due to rotation at any point situated at a radial distance of \( r_x \) measured from the bearing centerline is given by the following formula:

\[
\tau = 1.005 \times 10^{-6} S^2 \left[ \frac{3+\mu}{4} \left( D_2^2 + D_1^2 + \frac{D_1^2 D_2^2}{4r_x^2} \right) - (1+3\mu)r_x^2 \right]
\]  

(4.06)

where: \( 1.005 \times 10^{-6} = \rho \left( \frac{\pi}{30} \right)^2 \frac{1}{8g} \)

The maximum tangential tensile stress in the average section of cone occurs at the perimeter of the cone bore and is given by:

\[
\tau_i = 5.024 \times 10^{-7} S^2 \left[ (3+\mu)D_2^2 + (1-\mu)D_1^2 \right]
\]  

(4.07)

where: \( 5.024 \times 10^{-7} = \frac{\rho}{16} \left( \frac{\pi}{30} \right)^2 \frac{1}{g} \)

4.5 Loss of Interference Fit Due to Inertia Stress (Ref. 4)

If the press fitted cone/shaft assembly is rotated at high speeds, both the cone bore and shaft O.D. increase due to inertia stresses. However, the increase in shaft O.D. is less than the increase in cone bore. Consequently, the fit is reduced and the loss of diametral interference fit can be estimated using the formula as given below:

\[
\delta_{\text{loss}} = 5.526 \times 10^{-14} S^2 D_1 (D_2^2 - d_0^2)
\]  

(4.08)

where: \( 5.526 \times 10^{-14} = \left( \frac{\pi}{60} \right)^2 \frac{\rho(3+\mu)}{4gE} \)
4.6 Combined Effects of Interference Fit and Inertia Stresses

The following two formulae assume that the loss of fit ($\delta_{\text{loss}}$) is less than the initial interference fit ($\delta_1$) and the effect of thermal expansions is negligible.

**Effective Contact Pressure, Cone/Shaft**

$$P_e = \frac{E}{2} \left( \frac{D_1^2 - D_0^2}{D_1^2 - D_0^2} \right) \left( \frac{D_2^2 - D_1^2}{D_1^2 - D_0^2} \right) \left( \delta_1 - \delta_{\text{loss}} \right)$$

(4.09)

**Maximum Tangential Stress in the Average Section of Cone which Occurs at Cone Bore**

$$\tau_1 = \tau_i + P_e \left( \frac{D_2^2 + D_1^2}{D_2^2 - D_1^2} \right)$$

(4.10)

where $\tau_i$ is defined in Section 4.4.
5. KINEMATICS AND DYNAMICS

5.1 Epicyclic Speeds

The derivations for RPM of the cage \((N_c)\) and RPM of the roller about its own center \((N_r)\) for the rotating cone and stationary cup condition assume rigid bodies, i.e. the bearing components will maintain exact geometrical shape under operation, and that pure rolling exists at roller-body/cone-race and roller-body/cup-race contacts. In addition, the bearing apex is assumed to be on the bearing centerline.

Under no slip condition, considering roller-body/cone-race contact, the linear tangential velocities for the mean cone O.D. and mean roller diameter can be equated as follows:

\[
\pi S D_2 = \pi N_r d + \pi N_c D_2 \quad \text{(5.01)}
\]

Similarly from roller-body/cup-race contact we can write

\[
\pi N_r d = \pi N_c D_5 \quad \text{(5.02)}
\]

Substitute for \(D_2 = \frac{d \sin \beta}{\sin \gamma} \); \(D_5 = \frac{d \sin \alpha}{\sin \gamma}\)

(5.01) and (5.02) can be solved for \(N_c\) and \(N_r\).

\[
N_c = S \cdot \frac{\sin \beta}{\sin \alpha + \sin \beta} \quad \text{(5.03)}
\]

\[
N_r = S \cdot \frac{\sin \alpha \sin \beta}{\sin \gamma (\sin \alpha + \sin \beta)} \quad \text{(5.04)}
\]
5.2 Surface Velocities

Based on $S$, $N_r$ and $N_c$ various surface velocities can be written as follows:

Tangential velocity of roller-body mid-point with respect to roller centerline,

$$V_1 = \frac{\pi}{60} N_r d$$

Tangential velocity of the point on the roller at roller-end/rib contact center with respect to roller centerline,

$$V_2 = \frac{\pi}{30} N_r (0.5d_1 - h \cos \nu)$$

where, $h$ is defined in Section 2.1.

Tangential velocity at mean cone O.D. with respect to bearing centerline,

$$V_3 = \frac{\pi}{60} S D_2$$

Rib velocity, i.e. tangential velocity of the point on the rib at roller-end/rib contact center,

$$V_{rib} = \frac{\pi}{60} S d_3$$

where, $d_3$ is defined in Section 2.1.
5.3 Sliding Velocity

At roller-end/rib contact center, the rib velocity \( V_{rib} \) is greater than the velocity (about bearing center) of the corresponding mating point on the roller-end and the difference between them is called sliding velocity.

\[
V_s = V_{rib} - \left( V_2 + \frac{\pi}{60} N_C d_3 \right)
\]

this can be simplified to

\[
V_s = \frac{\pi}{30} \text{Sh} \left( \frac{\sin \alpha}{\sin 2\alpha} \right) \tag{5.09}
\]

5.4 Roller Centrifugal Force (Ref. 5)

Approximating the roller to a frustrum of right circular cone the roller weight can be estimated using:

\[
w = \frac{\pi}{12} \rho L_1 \cos \nu \left( d_1^2 + d_1d_2 + d_2^2 \right) \tag{5.10}
\]

Centrifugal force per roller which acts through the center of gravity can be computed as:

\[
C_F = \frac{w}{g} \left( \frac{\pi}{30} N_C \right)^2 r_{cg} \tag{5.11}
\]

where, \( r_{cg} \) is defined in Section 2.1 and represents the radial distance of roller c.g. from bearing centerline.

\( N_C \) is RPM of cage and given in Section 5.1.
5.5 Normal Load Components Due to Roller Centrifugal Force

When a bearing is rotated at a constant high speed the roller centrifugal force becomes quite significant and as a consequence the cone and cup tend to separate from each other. However, if an external thrust equal to induced thrust due to roller centrifugal force (defined in Section 5.6) is applied to the bearing, the three normal load components acting on the roller can be estimated using three equations of equilibrium, namely \[ \Sigma F_x = 0, \Sigma F_y = 0 \text{ and } \Sigma M = 0. \]

Some of the assumptions are as follows:

1. All bearing components behave like a rigid body.

2. The normal loads, roller-body/cone-race and roller-body/cup-race, are uniformly distributed along the entire effective contact length.

3. The normal load, roller-end/rib passes through the center of spherical radius at the roller-end.

4. There is no bearing misalignment.

Figure 7 shows a schematic representation of three normal loads due to roller centrifugal force.
Figure 7 - Normal Loads Due to Roller Centrifugal Force
Based on the principals of plane geometry the following equations may be written:

\[ \begin{align*}
\mathbf{E} \mathbf{P} &= L_2 \cos \nu \\
\mathbf{E} \mathbf{H} &= (L_2 - L_2) \cos \nu \\
\mathbf{H} \mathbf{I} &= H \mathbf{J} \\
\mathbf{G} \mathbf{J} &= G \mathbf{I} = H \mathbf{I} / \tan \psi \\
&\quad \text{let } \mathbf{G} \mathbf{J} = A \\
\mathbf{F} \mathbf{G} &= r \cos \psi_0 - h_{cg} \\
&\quad \text{where } \psi_0 = \sin^{-1} \left( \frac{d_1}{2r} \right)
\end{align*} \]

By referring to Figure 8 it can be seen that

\[ \angle \mathbf{G} \mathbf{F} \mathbf{S} = \nu - (\pi/2 - \theta) \]
\[ = \theta + \nu - \pi/2 \]

Hence, \( \mathbf{G} \mathbf{S} = \mathbf{F} \mathbf{G} \sin (\theta + \nu - \pi/2) \)
\[ \text{let } \mathbf{G} \mathbf{S} = B \]

Now, considering the vertical components of all forces on the roller, we can write:

\[ C_2 \cos \alpha + C_3 \cos (\theta - \beta) - C_1 \cos \beta = C_f \quad (5.12) \]
Similarly, summation of all horizontal components gives:

\[ C_2 \sin\alpha - C_3 \sin(\theta - \beta) - C_1 \sin\beta = 0 \quad (5.13) \]

Finally, taking moment about c.g. of the roller we get

\[ C_2 \sin\beta - C_3 \sin(\theta - \beta) - C_1 \sin\alpha = 0 \]

i.e. \[ C_2 - C_3 \frac{B}{A} - C_1 = 0 \quad (5.14) \]

Equations (5.12), (5.13) and (5.14) can be solved simultaneously for \( C_1, C_2 \) and \( C_3 \) as follows:

\[
D = \begin{vmatrix}
\cos\alpha & \cos(\theta - \beta) & -\cos\beta \\
\sin\alpha & -\sin(\theta - \beta) & -\sin\beta \\
1 & -\frac{B}{A} & -1
\end{vmatrix}
\]

\[
= \cos\alpha (\sin(\theta - \beta) - \frac{B}{A}\sin\beta) - \sin\alpha (-\cos(\theta - \beta) - \frac{B}{A}\cos\beta)
+ (-\sin\beta \cos(\theta - \beta) - \cos\beta \sin(\theta - \beta))
\]

\[
= \cos\alpha (\sin(\theta - \beta) - \frac{B}{A}\sin\beta) + \sin\alpha (\cos(\theta - \beta) + \frac{B}{A}\cos\beta)
- \sin\alpha
\]

\[
= \sin(\alpha + \theta - \beta) + \frac{B}{A}\sin(\alpha - \beta) - \sin\alpha
\]

(\( \alpha - \beta = 2\nu \))

\[
= \sin(\theta + 2\nu) + \frac{B}{A}\sin(2\nu) - \sin\alpha
\]

(5.15)

\[
D_{11} = \begin{vmatrix}
\cos\alpha & \cos(\theta - \beta) & C_F \\
\sin\alpha & -\sin(\theta - \beta) & 0 \\
1 & -\frac{B}{A} & 0
\end{vmatrix}
\]

\[
= C_F \left[ \sin(\theta - \beta) - \frac{B}{A}\sin\alpha \right]
\]

(5.16)
\[
D_{12} = \begin{bmatrix}
C_F & \cos(\theta-\beta) & -\cos\beta \\
0 & -\sin(\theta-\beta) & -\sin\beta \\
0 & -B/A & -1
\end{bmatrix}
\]

\[
= C_F \left[ \sin(\theta-\beta) - (B/A)\sin\beta \right]
\]

(5.17)

\[
D_{13} = C_F (\sin\alpha - \sin\beta)
\]

(5.18)

If \( D = 0 \), we can say

\[
C_1 = \frac{D_{11}}{D}
\]

(5.19)

\[
C_2 = \frac{D_{12}}{D}
\]

(5.20)

\[
C_3 = \frac{D_{13}}{D}
\]

(5.21)

Evaluation of the above equations results in a negative normal load \( C_1 \) at the roller-body/cone-race contact. This cannot physically exist and equations (5.12) and (5.13) may be rewritten by dropping the terms containing \( C_1 \).

Therefore,

\[
C_2 \cos\alpha + C_3 \cos(\theta-\beta) = C_F
\]

(5.22)

\[
C_2 \sin\alpha - C_3 \sin(\theta-\beta) = 0
\]

(5.23)

i.e.

\[
C_2 \left[ \frac{\cos\alpha}{\cos(\theta-\beta)} + \frac{\sin\alpha}{\sin(\theta-\beta)} \right] = \frac{C_F}{\cos(\theta-\beta)}
\]

\[
C_2 \left[ \frac{\sin(\theta-\beta) \cos\alpha + \cos(\theta-\beta) \sin\alpha}{\sin(\theta-\beta) \cos(\theta-\beta)} \right] = \frac{C_F}{\cos(\theta-\beta)}
\]
\[ C_2 \left[ \frac{\sin (\theta - \delta + \alpha)}{\sin (\theta - \beta)} \right] = C_F \]

\[ C_2 = C_F \left[ \frac{\sin (\theta - \beta)}{\sin (\theta + 2\nu)} \right] \quad (5.24) \]

\[ C_3 = C_F \left[ \frac{\sin \alpha}{\sin (\theta + 2\nu)} \right] \quad (5.25) \]

### 5.6 Induced Thrust Due to Roller Centrifugal Force

The summation of axial component of normal load, roller-body/cup-race, for all rollers may be called induced thrust due to roller centrifugal force. Using the equation (5.20) or (5.24) for \( C_2 \) the induced thrust is given by the following formula:

\[ T_i = z C_2 \sin \alpha \quad (5.26) \]

where: \( z = \) number of rollers

### 5.7 Moments of Inertia of Roller (Ref. 5 and 6), (Fig. 9)

Based on the standard formulae for moments of inertia for a solid right circular cone and using the parallel-axis theorem, we can write the moments of inertia for a tapered roller about axes through the center of gravity as follows:

\[ I_x = \frac{3w}{40} \left( \frac{d_1^5 - d_2^5}{d_1^3 - d_2^3} \right) \quad (5.27) \]

\[ I_x = \frac{I_x}{2} + \frac{w}{I_0} (L_1 \cos \nu)^2 \left[ \frac{\left( d_2^2 + 3d_1d_2 + 6d_1^2 \right)}{d_1^2 + d_1^2 + d_2^2} - \frac{5}{8} \left( \frac{d_2^2 + 2d_1d_2 + 3d_1^2}{d_1^2 + d_1^2 + d_2^2} \right)^2 \right] \quad (5.28) \]

### 5.8 Gyroscopic Couple (Ref. 7)

The geometrical configuration of the tapered roller bearing gives rise to an inertia couple called gyroscopic couple which influences the distribution of the normal load at the roller-body/race contact. The direction of this couple is such that it tends to make the large end of the roller dig into the
\[ I_{x_1} = \frac{3}{40} w_1 d_1^2 \]
\[ I_{y_1} = I_{x_1} + \frac{3w_1H_1^2}{5} \]

\[ w_1 = \text{WEIGHT OF CONE} \]

\[ I_{x_2} = \frac{3}{40} w_2 d_2^2 \]
\[ I_{y_2} = I_{x_2} + \frac{3w_2H_2^2}{5} \]

\[ w_2 = \text{WEIGHT OF CONE} \]

\[ I_{x_3} = I_{x_1} - I_{x_2} \]
\[ I_{x_3} = I_{y_1} - I_{y_2} \]

\[ L_1 \cos \gamma = H_1 - H_2 \]

\[ I_x = \text{EQUATION (5.27)} \]
\[ I_y = \text{EQUATION (5.28)} \]

*Figure 9 - Mass Moments of Inertia*
cup-race and the small end into the cone-race. The magnitude of the couple is:

$$M_{g} = 2.84 \times 10^{-5} \left[ \frac{(I_{y} - I_{x})}{2} N_{c}^2 \sin 2\gamma + I_{x} N_{c} \sin \gamma \right]$$  \hspace{1cm} (5.29)

where, $$2.84 \times 10^{-5} = \left(\frac{\pi}{30}\right) \cdot \frac{1}{g}$$

5.9 Combined Effect of Roller Centrifugal Force and Gyroscopic Couple

In Section 5.5 we stated that for most of standard tapered roller bearing designs the cone-race would not support roller centrifugal force and equations (5.24) and (5.25) for normal loads, roller-body/cup-race ($C_{2}$) and roller-end/rib ($C_{3}$), were derived using $\Sigma F_{y} = \Sigma F_{x} = 0$. Now, based on this assumption the effects of gyroscopic couple may be superimposed on $C_{2}$ and load intensities $q_{1}$ and $q_{2}$ may be determined as follows:

Referring to Figures 7, 10 and 11 and taking moments about roller c.g. we get

$$C_{2} \left( \mathbf{G} \mathbf{J} + \Delta X \right) + M_{g} = C_{3} \mathbf{G} \mathbf{S}$$

Rearranging the terms,

$$\Delta X = \frac{(C_{3} \mathbf{G} \mathbf{S} - M_{g})}{C_{2}} - \mathbf{G} \mathbf{J}$$  \hspace{1cm} (5.30)

let $\mathbf{G} \mathbf{S} = B$ and $\mathbf{G} \mathbf{J} = A$ as defined in section 5.5

$$\Delta X = \frac{(C_{3}B - M_{g})}{C_{2}} - A$$  \hspace{1cm} (5.31)
Figure 10 - Roller-Body/Cup-Race Load Due to Centrifugal Force

Figure 11 - Roller Loads Due to Both Centrifugal Force and Gyroscopic Couple
Assuming that normal load $C_2$ varies along the effective contact length ($l$) linearly and using the formula for center of gravity of a plane trapezoid would yield the following two relationships

\[ \Delta X = \frac{l}{2} - \frac{l(q_1 + q_2)}{3(q_1 + q_2)} = \frac{l}{2} \left[ \frac{q_1 - q_2}{3(q_1 + q_2)} \right] \]  

(5.32)

and $(q_1 + q_2) l = 2C_2$  

(5.33)

Substituting for $\Delta X$ from equation (5.32) in equation (5.31) we obtain,

\[ \frac{l}{2} \left[ \frac{q_1 - q_2}{3(q_1 + q_2)} \right] = \frac{(C_3B-M')}{C_2} - A \]  

(5.34)

Now, equations (5.33) and (5.34) can be solved simultaneously for $q_1$ and $q_2$.

From equation (5.33)

\[ q_1 = \frac{2C_2}{l} - q_2 \]  

(5.35)

Substituting this in equation (5.34) we get

\[ \frac{l}{2} \left[ \frac{(C_2/q_2 - q_2)}{3C_2/l} \right] = \frac{C_3B-M'}{C_2} - A \]

i.e. \[ \frac{l}{6} - \frac{q_2 l^2}{6C_2} = \frac{C_3B-M'}{C_2} - A \]

i.e. \[ q_2 = \frac{6C_2}{l^2} \left[ \frac{l}{6} - \frac{C_3B-M'}{C_2} + A \right] \]

(5.36)

i.e. \[ q_2 = \frac{C_2}{l} \left[ 1 - \frac{6}{l} \frac{C_3B-C_2A}{C_2} \right] + \frac{6 M g}{l^2} \]
Substituting for $q_2$ in equation (5.35) gives

$$q_1 = \frac{2C_2}{k} - \frac{C_2}{k} \left[ 1 - \frac{6}{k} \left( \frac{C_3B-C_2A}{C_2} \right) \right] - \frac{6 M g}{k^2}$$

i.e.

$$q_1 = \frac{C_2}{k} \left[ 1 + \frac{6}{k} \left( \frac{C_3B-C_2A}{C_2} \right) \right] - \frac{6 M g}{k^2} \quad \text{(5.37)}$$
6. LOAD DISTRIBUTION UNDER EXTERNAL LOAD

6.1 Thrust Load
If the bearing is subjected to a thrust load only, all rollers are equally loaded under ideal conditions of operation. From conditions of equilibrium and considering the roller centrifugal force, the three normal load components can be calculated as follows:

Normal Load, roller-body/cone-race = \( P_1 = C_1 + \frac{T - T_1}{z \sin \alpha} \cos 2\nu \) (6.01)

Normal Load, roller-body/cup-race = \( P_2 = \frac{T}{z \sin \alpha} \) (6.02)

Normal Load, roller-end/rib = \( P_3 = C_3 + \frac{(T - T_1)}{z \sin \alpha} \sin 2\nu \) (6.03)

where \( T = \) external bearing thrust
\( C_1, C_3, T_1 \) are defined in Sections 5.5 and 5.6.

Above equations are based on the assumption that rib-race angle is, \( \theta = 90^\circ \).

6.2 Combined Radial and Thrust Load (Ref. 8)
When a bearing is subjected to a radial and a centric thrust load, the cone and cup will remain parallel and will be relatively displaced in both axial and radial directions.
Under such conditions, the bearing is essentially of statically indeterminate design and the load distribution within the bearing may be estimated using Sjovall's integrals and load zone parameter $\varepsilon$. The step-wise procedure is as follows:

1. For a given speed, calculate the normal loads $(C_1, C_2, C_3)$ and induced thrust $(T_i)$ due to roller centrifugal force using the formulae given in Sections 5.5 and 5.6.

2. Corresponding to the given radial load and reduced thrust $(T - T_i)$, calculate $\frac{R \tan \alpha}{T - T_i}$ and using the following Table III determine $J_R$ and $\varepsilon$.

3. Approximating the roller-body/race contacts to a line contact for any roller situated in an angle of $\psi_n$ (measured from the maximum loaded roller) the three normal loads may be estimated using the following formulae:

Normal load roller-body/cone-race,

$$ P_1 = C_1 + P_{\text{max}} \cos2\psi \left[ 1 - \frac{1}{2\varepsilon} (1-\cos\psi_n) \right]^{1.1} \quad (6.04) $$

Normal load roller-body/cup-race,

$$ P_2 = C_2 + P_{\text{max}} \left[ 1 - \frac{1}{2\varepsilon} (1-\cos\psi_n) \right]^{1.1} \quad (6.05) $$

Normal load roller-end/rib,

$$ P_3 = C_3 + P_{\text{max}} \sin2\psi \left[ 1 - \frac{1}{2\varepsilon} (1-\cos\psi_n) \right]^{1.1} \quad (6.06) $$

In the above equations $P_{\text{max}} = \frac{R}{2J_R \cos \alpha} \quad (6.07)$
TABLE III

$J_R$, $J_T$, $\varepsilon$ vs. $\frac{R \tan \alpha}{T-T_i}$ for Line Contact

<table>
<thead>
<tr>
<th>$\frac{R \tan \alpha}{T-T_i}$</th>
<th>$J_R$</th>
<th>$J_T$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/z</td>
<td>1/z</td>
<td>0</td>
</tr>
<tr>
<td>0.9215</td>
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<td>0.1885</td>
<td>0.2</td>
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<td>0.4</td>
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<td>0.7939</td>
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<td>0.5</td>
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<td>0.6</td>
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<td>0.4817</td>
<td>1.0</td>
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<td>0</td>
<td>1</td>
<td></td>
</tr>
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</table>
7. CONTACT STRESSES

The contact stresses caused by the normal loads at all three conjunctions in the bearing can be calculated using Hertz theory. Some of the basic assumptions are that the material is homogeneous, isotropic, and elastic in accordance with Hooke's law. The formulae for contact stresses for roller-body/cone-race and roller-body/cup-race given in the following sections are based on infinitely long cylinders, hence, the effects of end-stress are not considered. The formulae (7.01) and (7.02) are essentially the same as given in Ref. 3 and the derivations for equivalent radii are included in Section 9.2. In all three following sections dry contact between the bodies was assumed, i.e., effects of lubricant film at contacts were neglected.

7.1 Roller-body/Cone-Race

Contact stress at center of roller-body/cone-race contact.

\[
\sigma_1 = 0.591 \left[ \frac{30P_1 \sin \gamma}{l \cdot d \cdot \sin \beta} \right]^{0.5}
\]  

Assumptions: Modulus of Elasticity = 30 x 10^6 psi
Poisson's Ratio = 0.3

7.2 Roller-Body/Cup-Race

Contact stress at center of roller-body/cup-race contact

\[
\sigma_2 = 0.591 \left[ \frac{30P_2 \sin \gamma}{l \cdot d \cdot \sin \alpha} \right]^{0.5}
\]  

(7.02)
7.3 Roller-End/Cone Rib (Ref. 9)

The roller-end/rib contact can be represented by two general solid bodies in point contact and the Hertz equations can be used to estimate maximum contact stress at the center of ellipse. An important assumption is made that the ellipse is not truncated and without going into minute details, the procedure can be presented as follows:

Maximum contact stress at the center of roller-end/rib contact

\[ \sigma_3 = \frac{1.5 \times 10^{-3} P_3}{\pi b a} \]  

(7.03)

where,

- \( a \) = semi-major axis of ellipse = \( \nu h \) \( G_h \)
- \( b \) = semi-minor axis of ellipse = \( \nu h \) \( G_h \)

where, \( G_h = \left[ \frac{3(1-\mu^2)P_3}{E(2/r+1/r')} \right]^{1/3} \)

(7.04)

\( E = 30 \times 10^6 \) psi

\( \mu = 0.3 \)

\( r' \) = radius of curvature of rib defined by equation (9.15)

\( \nu h \) and \( \nu h \) are Hertzian parameters related as follows:

\[ \nu h = \frac{\nu h}{\cos \xi} \]  

(7.05)

\[ \nu h = \left[ \frac{2}{\pi} E(\xi) \cos \xi \right]^{1/3} \]  

(7.06)
Figure 12 - Roller-End/Rib Contact
The value of \( \xi \) should be such that it satisfies the following equation:

\[
\frac{\cos(t) - 1}{2} + \frac{K(\xi) - E(\xi)}{E(\xi)} \times \cot^2 \xi = 0 \tag{7.07}
\]

where, \( \cos(t) = \frac{1}{2/r - 1/r'} \) \( \tag{7.08} \)

\( K(\xi) \) = complete elliptic integral of the first kind

\[
= \frac{\pi}{2} \int_{0}^{\pi/2} \left[ 1 - \sin^2(\xi) \sin^2x \right]^{-1/2} \, dx \tag{7.09}
\]

\( E(\xi) \) = complete elliptic integral of the second kind

\[
= \frac{\pi}{2} \int_{0}^{\pi/2} \left[ 1 - \sin^2(\xi) \sin^2x \right]^{1/2} \, dx \tag{7.10}
\]

The two integrals defined above may be approximated by the following polynomials as given in Ref. 10.

\[
K(\xi) = \xi^4 \left[ Y(5) + Y(10) \ln (1/\xi) \right] + \xi^3 \left[ Y(4) + Y(9) \ln (1/\xi) \right] + \xi^2 \left[ Y(3) + Y(18) \ln (1/\xi) \right] + \xi \left[ Y(2) + Y(7) \ln (1/\xi) \right] + Y(1) + Y(6) \ln (1/\xi) \tag{7.11}
\]

\[
E(\xi) = \xi^4 \left[ Y(14) + Y(18) \ln (1/\xi) \right] + \xi^3 \left[ Y(13) + Y(17) \ln (1/\xi) \right] + \xi^2 \left[ Y(12) + Y(16) \ln (1/\xi) \right] + \xi \left[ Y(11) + Y(15) \ln (1/\xi) \right] + 1 \tag{7.12}
\]
where, \( Y(1) \), \( Y(2) \) ... \( Y(18) \) are numerical constants as given below:

\[
\begin{align*}
Y(1) &= 1.38629436 & Y(10) &= 0.0441787012 \\
Y(2) &= 0.0966634426 & Y(11) &= 0.443251415 \\
Y(3) &= 0.0359009238 & Y(12) &= 0.0626060122 \\
Y(4) &= 0.0374256371 & Y(13) &= 0.0475738355 \\
Y(5) &= 0.0145119621 & Y(14) &= 0.0173650645 \\
Y(6) &= 0.5 & Y(15) &= 0.249983683 \\
Y(7) &= 0.124985936 & Y(16) &= 0.0920018004 \\
Y(8) &= 0.0688024858 & Y(17) &= 0.0406969753 \\
Y(9) &= 0.032835535 & Y(18) &= 0.00526449639
\end{align*}
\]

After substituting for \( \cos(\alpha) \), \( K(\xi) \) and \( E(\xi) \) from equations (7.08), (7.11) and (7.12) respectively in equation (7.07), it can be solved for \( \xi \) by Newton's iteration method.
8. LUBRICANT (MIL-L-7808G) PROPERTIES

In various EHD and related thermal computations, data are required on the variations of the rheological and thermal properties of the lubricants with temperature, pressure, rate of shear, etc. The following formulae used to describe the controlling properties of MIL-L-7808G are the recommendations of Southwest Research Institute based on the experimental data available on this type of fluid. The formulae are only representative of 7808G fluids. Different 7808G fluids may have slightly different formulae.

Density at atmospheric pressure

\[ \rho_o = 0.9531 - 0.000394 \text{ (°F-60)} \]  

(8.01)

Kinematic viscosity at atmospheric pressure \((v_o)\) is given by,

\[ \log \log (v_o + 0.6) = 11.75543 - 4.24477 \log (\text{°F} + 460) \]  

(8.02)

Absolute viscosity

Centi-poise = \(\rho_o v_o\)  

Reyn, \(\mu_o = 1.45 \times 10^{-7} \rho_o v_o\)  

(8.04)

Pressure-viscosity coefficient

\[ \alpha_o = 12.717 \times 10^{-4} \text{ (°F)}^{-0.566} \]  

(8.05)
Temperature-viscosity coefficient
\[ \beta_0 = 0.0045 + 1.25 \times 10^{-7} p \]  
\[ (8.06) \]

\[ \beta'_0 = \frac{3400}{(^{\circ}F+460)} \ln \left( \frac{\mu_0/100}{\mu_0/210} \right) \]  
\[ (8.07) \]

Specific Heat
\[ c_p = 0.16 \left( ^{\circ}F \right).215 \]  
\[ (8.08) \]

Thermal Conductivity
\[ k = 0.082 - 1.355 \times 10^{-5} \times ^{\circ}F \]  
\[ (8.09) \]

The following table was generated using the equations (8.01) through (8.05).
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<th>Density gm/cc</th>
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**TABLE V**

Specific Heat and Thermal Conductivity

for MIL-L-7808G as a Function of Temperature

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<tr>
<th>Temperature °F</th>
<th>Specific Heat BTU/lb.°F</th>
<th>Thermal Conductivity BTU/hr.ft.°F</th>
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9. EHD FILM THICKNESS (Ref. 11)

The classical theory of elasto-hydrodynamic lubrication is rather well known and its application to study the performance of lubricated concentrated contacts has been in existence for some time. Without going into details of the basic fundamentals and assumptions, the following sections briefly describe how we can estimate the operating film thickness at three major conjunctions using some empirical formulae.

9.1 Sum Velocities

The sum velocity for two bodies in contact is defined as a sum of the peripheral velocities of the surfaces relative to the conjunction. Based on the assumption of pure rolling at the race contacts the formulae are:

\[
\begin{bmatrix}
\text{The sum velocity at} \\
\text{roller-body/cone-race} \\
\text{contact (} U_1 \text{)}
\end{bmatrix} = \begin{bmatrix}
\text{The sum velocity at} \\
\text{roller-body/cup-race} \\
\text{contact (} U_2 \text{)}
\end{bmatrix}
\]

\[
U_1 = U_2 = 2V_1 = \frac{\pi}{30} N r d
\]

(9.01)

where, \( V_1 \) is defined by equation (5.05)

The sum velocity at roller-end/rib contact

\[
U_3 = 2V_2 + V_s
\]

(9.02)

where, \( V_2 \) and \( V_s \) are given by equations (5.06) and (5.09) respectively.
9.2 Equivalent Radii

Two bodies forming a highly stressed contact can be represented mathematically by an equivalent cylinder near a plane for a rectangular conjunction, or by an equivalent sphere near a plane for an elliptical conjunction. Then, by definition the radius of the cylinder or the sphere is called equivalent radius. Neglecting the crown radius on roller-body for roller-body/race contacts and by referring to Figure 13 the equivalent radii at the center of contact can be derived as follows:

a. Equivalent radius for roller-body/cone-race contact (Fig. 13).

\[ R_1 = \frac{1}{1/(A \cdot D) + 1/(A \cdot K)} \]  \hspace{1cm} (9.03)

where

\[ A \cdot D = A \cdot C / \cos \nu \]  \hspace{1cm} (9.04)

\[ = \frac{d}{2 \cos \nu} \]

\[ A \cdot K = A \cdot I / \cos \beta \]  \hspace{1cm} (9.05)

\[ = \frac{D_2}{2 \cos \beta} \]

Substituting for \( A \cdot D \) and \( A \cdot K \) in equation (9.03) we obtain,

\[ R_1 = \frac{0.5}{\frac{\cos \nu}{d} + \frac{\cos \beta}{D_2}} \]  \hspace{1cm} (9.06)

and we know, \( D_2 = \) mean cone O.D. = \( \frac{d \sin \beta}{\sin \nu} \)

substituting this in (9.06) we get
Roller-Body/Cone-Race Contact

Roller-Body/Cup-Race Contact

Roller-End/Rib Contact

Figure 13 - Equivalent Radii
\[ R_1 = \frac{0.5}{d} \cos \nu + \frac{\sin \nu \cos \beta}{d \sin \beta} \]

rearranging terms and using the trigonometric identity
\[ \sin \beta \cos \beta + \cos \beta \sin \nu = \sin (\beta + \nu) \]
and by definition \( \beta + \nu = \gamma \)
\[ R_1 = \frac{0.5d \sin \beta}{\sin \gamma} \]  \hspace{1cm} (9.07)

b. Equivalent radius for roller-body/cup-race contact

(Fig. 13)
\[ R_2 = \frac{1}{1/B \cdot L - 1/B \cdot E} \]  \hspace{1cm} (9.08)

where \( B \cdot L = d/2 \cos \nu \) \hspace{1cm} (9.09)
\[ B \cdot E = \frac{D_5}{2 \cos \alpha} \]  \hspace{1cm} (9.10)
\[ D_5 = \frac{d \sin \alpha}{\sin \nu} \]  \hspace{1cm} (9.11)

Using equations (9.09), (9.10) and (9.11) and similar procedure as above equation (9.08) can be reduced to,
\[ R_2 = \frac{0.5d \sin \alpha}{\sin \gamma} \]  \hspace{1cm} (9.12)

c. Equivalent radius for roller-end/rib contact (Fig. 13)
\[ R_3 = \frac{1}{1/r - 1/F \cdot H} \]  \hspace{1cm} (9.13)

where, \( F \cdot H = F \cdot G / \cos \gamma_{rib} = r' \) \hspace{1cm} (9.14)
and \( F \cdot G = d_3/2 \) (defined in section 2.1)

therefore,
\[ R_3 = \frac{rd_3}{1 - 2 \cos \gamma_{rib}} \]  \hspace{1cm} (9.15)
9.3 Roller-Body/Race Contacts:  (Ref. 11)

The general expression for minimum EHD - film thickness for a rectangular conjunction as given by Dowson in Ref. 11 is:

\[
H_{\text{min}}^* = \frac{2.65 \ G^{0.54} \ U^{0.7}}{W^{.13}}
\]  

(9.16)

where, \(H_{\text{min}}^*, G^*, U^*, W^*\) are dimensionless parameters.

a. Roller-Body/Cone-race contact

The dimensionless parameters in equation (9.16) are as follows:

\[
H_{\text{min}}^* = \frac{h_1}{R_1} \quad h_1 = \text{minimum film thickness at center of contact}
\]

(9.17)

\[
G^* = E' \ \alpha_o
\]

(9.18)

\[E' = \frac{E}{1-\mu^2} = 33 \times 10^6 \text{ psi}\]

\[
\alpha_o = \text{lubricant pressure viscosity coefficient}
\]

\[
U^* = \frac{\mu_o \ U_1}{E' R_1} \quad U_1 = \text{sum velocity, equation (9.01)}
\]

(9.19)

\[
W^* = \frac{P_1}{\xi E' R_1}
\]

(9.20)

After substituting for dimensionless parameters in equation (9.16) it can be reduced to the following form:

\[
h_1 = 0.68816 \ R_1^{0.43} (\alpha_o \times 10^4) (\mu_o \times 10^6 \cdot U_1) (\xi/P_1)^{0.7/0.13}
\]

(9.21)

b. Roller-Body/Cup-Race contact

\[
h_2 = 0.68816 \ R_2^{0.43} (\alpha_o \times 10^4) (\mu_o \times 10^6 \cdot U_2) (\xi/P_2)^{0.7/0.13}
\]

(9.22)
9.4 Roller-End/Rib Contact

The roller-end/rib contact is a critical area in high speed tapered roller bearings. In view of the spherical roller-end and a flat conical shaped rib the contact area is a point under no load and an ellipse under load. Several empirical formulae are available to calculate the minimum EHD film thickness for an elliptical contact. More recently Thorp and Gohar (Ref. 12) have presented some experimental data in terms of a dimensionless conforming groove with a .555 inch radius. The similarity in their test conditions and the roller-end/rib contact is that the major axis of the contact ellipse is parallel to the direction of lubricant flow. From Figure 14 of Reference 12, the relation between ln \((H^*/W^*G^*)\) and ln \((U^*/W^*1.5G^*0.5)\) is relatively linear and an expression that fits the data reasonably well is,

\[ \frac{H^*}{W^*G^*} = 0.811 \left[ \frac{U^*}{W^*1.5G^*0.5} \right]^{0.769} \]

or

\[ H^* = 0.811 \frac{G^*0.616U^*0.769}{W^*0.152} \]

The above equation can be reduced to the following form in terms of dimensional variables:

\[ h_3 = 0.06634 R_3^{0.535} (\sigma_o \times 10^4)^{0.616} (\mu_o \times 10^6 x U_3)^{0.769} (P_3)^{-0.152} \]

\[ (9.25) \]
9.5 Thermal Reduction Factor

The EHD film thickness equations given in the preceding sections all assume an isothermal flow process. This assumption implies that: (a) the motion is pure rolling so that the temperature distribution across the lubricant film can be taken as essentially uniform, (b) the rolling velocity and/or lubricant viscosity are low enough so that the temperature rise due to shearing of the lubricant film in the inlet region can be neglected, and (c) the lubricant film thickness in the conjunction is determined essentially by what happens in the inlet region so that lubricant shear within the conjunction has a negligible effect on the film thickness. These conditions are not satisfied in this analysis. Estimates of the effects are accounted for in the Thermal Reduction Factor. Cheng (Ref. 13) originated the numerical technique which was later modified by McGrew, et al (Ref. 14). The later version used in this analysis consists of first determining the viscous heating parameter, $Q_m$

$$Q_m = \frac{\mu_o (U_{sum})^2}{2k T_o}$$  \hspace{1cm} (9.26)

(In the above equation $U_{sum}$ represent sum velocities for a given conjunction, defining sum velocities at either the cone, cup or rib.)
Two lubricant parameters are computed

\[ \alpha^* = \alpha_o \frac{\pi}{2} \times 10^5 \]  \hspace{1cm} (9.27)

\[ \beta_o' = 3400 \ln \left[ \frac{\mu_o/100}{\mu_o/210} \right] \frac{T_o}{T_o + 460} \]  \hspace{1cm} (9.28)

With these three parameters and Tables XIII and XIV of Reference 14, the coefficients \( f_1 \) and \( f_3 \) are determined.

For values different from those shown on the Tables, linear interpolation is used within the range and linear extrapolation outside the range. Thermal reduction factor is then computed by the following equation:

\[ \phi_T = f_1 (1 - 0.1s) \left( 1 + f_3 \frac{\sigma}{E} \right) \]  \hspace{1cm} (9.29)

The slip ratio is 's'. Multiplying the isothermal film thickness by the factor \( \phi_T \) gives the nonisothermal film thickness.
10. CAGE FRICTIONAL FORCES

10.1 Roller Body/Cage Pocket Friction

The friction developed between the roller-body and the cage pocket is due to shearing of the lubricant or rubbing of the surfaces. In general, the roller does not operate concentrically to the cage pocket due to weight, unbalance, whirl and dimensional variations. However, if the cage is relatively light weight and well balanced, weight and unbalance effects can be minimized. Accurate manufacturing and quality control reduces dimensional effects. Also, previous experience indicates that any region of metal to metal contact will wear only at initial run-in. No further wear occurs with subsequent operation.

The final condition assumed is that the cavity between the roller-body and cage pocket is completely filled with oil. This conjunction for both cage designs (roller and race guided) is shown on Figure 14.

With the above assumptions the friction is calculated as in a journal bearing operating at no-load using the experimental data of Smith and Fuller (Ref. 15). Their friction data are for a full 360° journal bearing operating in the laminar, transition and fully turbulent regimes. Figure 15 is a plot of their friction coefficient versus Reynolds number. The Reynolds Number for the roller-body/cage pocket conjunction is

\[ \text{Re}_p = \frac{V_1C_p}{\mu_0} \]  

(10.01)
Figure 14 - Section Views Through Roller Mean Diameter for Roller and Race Guided Cases
Figure 15 - Friction As a Function of Reynolds Number
for Unloaded Journal Bearing
Empirical formulas for the friction coefficient in the three (3) regimes are tabulated below.

**TABLE VI**

**Empirical Formulas for Friction Coefficient**

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<tr>
<th>Regime</th>
<th>Reynolds Number</th>
<th>Formulas</th>
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<td>Laminar</td>
<td>$\log \left( \frac{Re}{p} \right) \leq 2.87$</td>
<td>$f_p = \frac{2}{Re_p}$</td>
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<td>Transition</td>
<td>$2.87 &lt; \log(Re_p) \leq 3.04$</td>
<td>$f_p = 0.00251$</td>
</tr>
<tr>
<td></td>
<td>$3.04 &lt; \log(Re_p) \leq 3.16$</td>
<td>$f_p = \frac{5.19 Re_p^{1.212}}{10^7}$</td>
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<tr>
<td>Turbulent</td>
<td>$\log(Re_p) &gt; 3.16$</td>
<td>$f_p = \frac{0.078}{Re_p^{0.43}}$</td>
</tr>
</tbody>
</table>

Shear stress for the journal bearing is defined as

$$T_o = \frac{f_p \rho \nu^2}{2g}$$  \hspace{1cm} (10.02)

To apply the friction data to the roller-body/cage pocket "bearing", the assumption is that the shear stress, in the two partial-arc "bearings" is the same as it would be in a full journal bearing with a journal diameter equal to the roller diameter and with the same radial clearance as exists between the roller body and cage pocket. The friction force in each fluid film is calculated by:

$$F_p = T_o \cdot \text{arc} \cdot \ell_p$$  \hspace{1cm} (10.03)
The total friction force then would be twice this amount, since there are two partial-arc "bearings" formed by each roller and cage pocket.

This same technique has been recently used by Astridge and Smith (Ref. 16) to estimate the heat generation in high speed cylindrical roller bearings.

10.2 Cage-Pilot/Race-Guide Friction

In the race-guided cage design, there is an addition to the friction developed at the roller body/cage pocket, a frictional force between the cage pilot and race guide. Depending on which surface the cage is piloted, this friction force may either drive or impose a drag on the cage. See Figure 16. A pilot relative to the higher speed cone or shaft drives the cage. A pilot relative to the stationary cup or housing apply a resistance to rotation.

To apply the friction data given in Reference 15, the assumptions are as follows:

1. Cage Weight, unbalance and whirl effects are neglected.

2. The cavity between the cage pilot and race guide is filled with oil.
Figure 16 - Method of Piloting
Race Guided Cage
3. The shear stress in the narrow journal bearing formed by the cage pilot and race guide is the same as it would be in an infinitely long journal bearing operating with the same journal diameter, clearance and speed.

As done in the previous section, the friction coefficient is calculated by empirical formulas of the Reynolds Number fit to the data of Smith and Fuller. These formulas have been tabulated in the previous section. Reynolds Number for the cage pilot/race guide is

\[
\text{Re}_g = \frac{V_4 C_g}{\mu_o}
\]

(10.04)

The shear stress in the fluid film is

\[
\tau_o = \frac{f}{g} \rho_o \frac{V_4^2}{2g}
\]

(10.05)

The total force required to shear the fluid film is

\[
F_g = \tau_o \pi D g L g
\]

(10.06)

Since this is a resistance to rotation acting over the full circumference, it can better be defined as a couple

\[
M_c = \frac{F_g D g}{2}
\]

(10.07)
11. EHD FRICTION AND PRESSURE FORCES

The objective of this program is to operate a bearing at 3.5 million DN. Assuming epicyclic motion, this results in a roller-body/race conjunction sum velocity in excess of 8000 inches per second. Further, the bearing is likely to be operating in a starved boundary lubrication condition. Considering this, there is currently no analytical techniques available to compute friction.

In order to estimate friction and pressure forces several assumptions are needed.

First, assume the bearing is operating under full EHD conditions, where a continuous and intact lubricant film separates the surfaces. Second, assume there is sufficient supply of lubricant at each of the race and rib conjunctions so that the effects of starvation can be neglected. And finally, assume the bearing is operating at or near the epicyclic mode for the rolling element contacts.

This latter assumption has been verified at speeds to 1.8 million DN. Up to this speed, there was no measurable cage slip.

Location and orientation of the EHD friction and pressure forces are shown on Figure 17.
SECTION THROUGH
MEAN ROLLER DIAMETER

SECTION THROUGH
LARGE END ROLLER DIAMETER

Figure 17 - EHD Friction and Pressure Forces
63
11.1 Roller-Body/Race Friction and Pressure Forces

Based largely on the success of the predictive schemes employed by Harris (Ref. 17) and Poplawski (Ref. 18) the estimating procedure for nearly pure rolling is to use the following equations first developed by Dowson and Higginson (Ref. 19) and later modified by Harris. In dimensionless terms and for the present, neglecting sliding

\[ F_1^* = -9.2G^{-0.3}U_1^{*0.7} \]  
\[ F_2^* = -9.2G^{-0.3}U_2^{*0.7} \]  

where, \( U_1^* = \frac{\mu_0 U_1}{E'R_1} \) and \( U_2^* = \frac{\mu_0 U_2}{E'R_2} \)

The forces shown on the preceding Figure 17, \( Q_1 \) and \( Q_2 \) are components of the hydrodynamic pressure forces which result from the pressure buildup in the inlet region of the conjunctions. See Ref. 18 and 19. In dimensionless terms these are

\[ Q_1^* = 18.4 \left[ 1 - \frac{2d}{D_2^2 + D_5^2} \right] G^{-0.3} U_1^{*0.7} \]  
\[ Q_2^* = 18.4 \left[ 1 + \frac{2d}{D_2^2 + D_5^2} \right] G^{-0.3} U_2^{*0.7} \]

The force components (\( Q_1 \) and \( Q_2 \)) act through the center of the roller and are colinear with the direction of roller motion. These forces are considered in the translation of the roller but do not enter into a summation of moments about the roller center.
To convert the dimensionless equations (11.01) through (11.04) to a dimensional form in pounds, divide by \((\ell E'R_x)\) where \(R_x\) is the "equivalent radius" for the conjunction (cone or cup).

### 11.2 Roller-End/Rib Friction

Roller end/rib friction, where sliding is predominant, is determined empirically based on experimental data of Johnson and Cameron (Ref. 20). The method was first developed by McGrew, et al (Ref. 14). It involves first calculating three dimensionless parameters \(G_1\), \(G_2\) and \(G_3\) at 86° F, which are

\[
\begin{align*}
G_1 &= \frac{\mu_0 V_s}{\sigma_3 h_3} \\
G_2 &= \frac{\beta_0 \mu_0 V_s^2}{8k} \\
G_3 &= \alpha_0 \sigma_3
\end{align*}
\]

The parameters measure the effect of shear rate, thermal heating and pressure-viscosity, respectively.

Reference 14 has a family of curves for the friction coefficient as a function of the three parameters \(G_1\), \(G_2\) and \(G_3\). The curves are all based on a lubricant inlet temperature of 86° F. A method for adjusting for lubricant inlet temperatures other than 80° F is given.
The procedure used in this analysis differs from McGrew's in that the dimensionless parameters are computed using the actual lubricant supply temperature, then interpolating or extrapolating from the curves directly. Symbolically, the friction coefficient for the roller end/rib conjunction, \( f_{\text{rib}} \), is

\[
f_{\text{rib}} = f(G_1, G_2, G_3, T_0)
\]  

(11.08)

where \( T_0 \) is actual lubricant supply temperature. For further discussion regarding modifications to procedures of Reference 14, refer to Appendix A.
12. CONE RIB LUBRICATION SYSTEM

12.1 System Pumping
The prime function of the cone rib lubrication system is to maintain a sufficient supply of lubricant to the roller-end/rib conjunction. Centrifugal force pumps the lubricant from the shaft I.D. to the rib. In order to estimate the relative effectiveness of a design it is necessary to know the trajectory of the lubricant as it is travels from the cone and wets the rib face.

Schematic section views of the system are shown on Figures 18 and 18a. Figure 18 defines the various pertinent radial distances from the bearing centerline. Figure 18a shows the path of flow internally through the system.

The first assumption is that flow through the shaft, manifold and cone is much less than the capacity of any single component and there is no accumulation of lubricant at either the shaft I.D. or manifold.

It will be shown later that this assumption is reasonable for the flow rates encountered in these bearings. The effects of gravity and Coriolis acceleration are neglected.
Figure 18 - Cone Large Rib Lubrication System
(Section View Parallel to Shaft)
Figure 18a - Cone Large Rib Lubrication System
(Section View Perpendicular to Shaft)
Shaft Duct

First, consider a fluid element in the rotating shaft duct. Refer to Figure 19. The origin of the coordinate system is at the bearing centerline which coincides with the z-axis. This fluid element with cross-sectional area, $A_s$, accelerates from a velocity approximately equal to zero at $r_o (v_o \approx 0)$ to $v_1$ at the outlet $r_1$. The mass of the fluid element is $d_m$ and the radial component of acceleration is $a_r$. For the force balance assume the flow is frictionless. Later, frictional losses will be introduced. The force balance yields

$$ p A_s - (p + dp) A_s = a_r dm $$

(12.01)

From kinematics the radial components of acceleration of the rotating duct is

$$ a_r = \frac{d^2 r}{dt^2} - rw^2 $$

(12.02)

and the mass of the fluid element is

$$ d_m = \frac{p^1}{g} A_s dr $$

(12.03)

Substituting equations (12.02) and (12.03) into (12.01) and rearranging terms

$$ dp + \frac{p^1}{g} \frac{d^2 r}{dt^2} dr - \frac{p^1}{g} \omega^2 r dr = 0 $$

(12.04)
Figure 19 - Segment of Rotating Shaft Duct
but since \( V_r = \frac{dr}{dt} \)

\[
dp + \frac{\rho_o'}{g} \frac{dv_r}{dt} dr - \frac{\rho_o'}{g} \omega^2 r dr = 0 \tag{12.05}
\]

now

\[
\frac{dv_r}{dt} = \frac{dv_r}{dr} \frac{dr}{dt} = \frac{dv_r}{dr} v_r
\]

equation 12.05 can be written

\[
dp + \frac{\rho_o'}{g} v_r \frac{dv_r}{dr} - \frac{\rho_o'}{g} \omega^2 r dr = 0 \tag{12.06}
\]

Integrating equation 12.06 and dividing by the mass density term \( \frac{\rho_o'}{g} \)

\[
\frac{p}{\rho_o' g} + \frac{v_r^2}{2} - \frac{\omega^2 r^2}{2} = \text{Constant} \tag{12.07}
\]

Incorporating a head loss to account for friction between the inlet and outlet denoted by subscripts 0 and 1, respectively, yields

\[
\frac{p_0}{\rho_o' g} + \frac{v_o^2}{2} - \frac{\omega^2 r_o^2}{2} = \frac{p_1}{\rho_o' g} + \frac{v_1^2}{2} - \frac{\omega^2 r_1^2}{2} + H \tag{12.08}
\]
To further simplify this equation assume the flow is steady, i.e., compressible and the pressures adjacent the inlet and outlet are equal to atmospheric \( (P_o \approx P_1 \approx P_{atm}) \)

\[
v_o^2 - \omega r_o^2 = v_1^2 - \omega r_1^2 + 2H \tag{12.09}
\]

Now, from the previous assumption, \( v_o \) is approximately equal to zero. The lubricant exit velocity can be determined by

\[
v_1 = \left[ \frac{r_1^2 - r_o^2}{\omega} - 2H \right]^{1/2} \tag{12.10}
\]

Some of the previous assumptions are not compatible with the continuity equation and the lubricant is probably pumped as an air-oil mixture. Regardless, equation 12.10 should reasonably estimate the exit velocity.

To calculate the flow losses due to friction in the ducts, the headloss from Reference 21 is:

\[
H = \frac{\bar{V}_s}{2} \frac{L_{sd}}{D_{sd}} f \tag{12.11}
\]

where \( \bar{V}_s \) is the mean velocity of the lubricant flowing through the shaft duct and \( f \) is the flow friction factor.
Figure 20 taken from Reference 21 is "Nikuradse Data for Artificially Roughened Pipe Flows". This series of curves gives the friction factor as a function of Reynolds number and roughness to diameter ratio. The three (3) flow regimes are laminar, frictional transition and turbulent. The empirical equations of Blasius, van Karman, Prandtl, et al are used to compute f and are listed in Table VII.
Figure 20 - Nikuradse's Data From Artificially
Roughened Pipe Flows
TABLE VII
Empirical Formulas for Friction Factor

<table>
<thead>
<tr>
<th>Regime</th>
<th>Reynolds Number</th>
<th>Friction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar</td>
<td>$R_e \leq 2300$</td>
<td>$f = 64/R_e$</td>
</tr>
<tr>
<td>Frictional Transition</td>
<td>$2300 &lt; R_e &lt; (\frac{316}{f'})^4$</td>
<td>$f = \frac{316}{R_e^{.25}}$</td>
</tr>
<tr>
<td>(Smooth Pipe Curve)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>where $f' = \frac{1}{D_{sd}^{.2}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(2 \log (\frac{2sef}{D_{sd}}) + 1.74)^2$</td>
<td></td>
</tr>
<tr>
<td>Turbulent</td>
<td>$R_e &gt; (\frac{316}{f'})^4$</td>
<td>$f = f'$</td>
</tr>
</tbody>
</table>

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Equation (12.10) is an implicit function of bearing angular velocity, duct geometry and frictional flow losses. To solve for \( v_1 \) an iterative technique is used. First \( v_1 \) is calculated with \( H = 0 \), then a mean time velocity, next a friction factor and finally, a headloss. With this headloss a new \( v_1 \) is computed. This procedure is repeated until a satisfactory degree of convergence is obtained in \( v_1 \).

Having computed the shaft duct exit velocity, the volumetric flow rate is

\[
Q_{sd} = A_s \times v_1 \tag{12.12}
\]

**Cone Ducts**

The assumptions and boundary conditions used in the analysis of the shaft ducts can be applied to the cone ducts. As these two segments of the system are analogous then the equations defining lubricant velocity and volumetric flow rates are similar.

Substituting the corresponding variables into equations 12.10, 12.11 and 12.12

\[
v_3 = \left[ \omega^2 (r_3^2 - r_2^2) - 2H \right]^{1/2} \tag{12.13}
\]

\[
H = \frac{\mu_{cd} V_{cd}^2}{2L_{cd}} \frac{L_{cd}}{D_{cd}} f \tag{12.14}
\]

\[
Q_{cd} = A_{cd} \times v_3 \tag{12.15}
\]

Friction factor \( f \) is determined using Table VII.
12.2 Manifold Flow

The analyses of lubricant flow through the manifold presents a complex problem. Efforts will be limited to developing some estimates of manifold cross-sectional area and flow capacity. A segment of the manifold is shown on Figure 21. The control volume shown will be used later in the analysis.

The singular function of the manifold is to collect lubricant pumped through the shaft ducts and distribute it equally to each of the cone ducts. The lubricant flowing through one shaft duct is assumed to be divided equally at the manifold. Therefore, manifold capacity is not only dependent on cross-sectional area and velocity, but also, the number of shaft ducts. Let the $m$ subscript denote manifold conditions, then this relationship can be written

$$Q_m = n_{sd}A_m v_m$$  \hspace{1cm} (12.16)

A segment of the manifold adjacent to a shaft duct is shown on Figure 22. The mass of the element of lubricant is $dm$ with an area $dA$. From kinematics, acceleration in the radial direction is

$$a_r = -r \omega^2 + \frac{d^2r}{dt^2}$$  \hspace{1cm} (12.17)

At this segment of the manifold neglect flow in the radial direction so that $\frac{d^2r}{dt^2} = 0$.
Figure 21 - Control Volume For Analysis of Manifold
Figure 22 - Manifold Section Adjacent Shaft Duct
A force balance yields

\[ pdA - (p + dp) dA = a_r dm \]  (12.18)

For the element of lubricant

\[ dm = \frac{\rho '}{g} \omega^2 r dr \]  (12.19)

Substituting equations 12.17 and 12.19 into 12.18 and rearranging terms

\[ dp = \frac{\rho '}{g} \omega^2 r dr \]  (12.20)

This can be integrated directly to determine the pressure differences between any two radial distances.

Referring back to Figure 21, let the m subscript denote manifold conditions just prior to a cone duct and the subscript 0 to represent manifold conditions adjacent the shaft duct. As lubricant accumulates at the shaft duct, it forces oil circumferentially around the manifold. Further simplifying this, assume that the cross-sectional area of the manifold being used at the junction of the shaft duct is considerably larger than the area adjacent the cone duct. \( A_o \) is bounded by \( r_1 \) and \( r_4 \) and \( A_m \) is bounded by \( r_2 \) and \( r_4 \). Applying the "Bernoulli Equation" to this control volume and letting \( v_o \) equal zero and neglecting effects of gravity and friction

\[ \frac{V_m^2}{2} = \frac{p_o g}{\rho_o} - \frac{p_m g}{\rho'_o} = \frac{g}{\rho'_o} (\Delta P) \]  (12.21)
Pressure difference across the manifold control volume from equation 12.20

\[ \Delta P = \frac{\rho_i \omega^2}{2g} (r_2^2 - r_1^2) \]  

(12.22)

and \[ V_m = \omega (r_2^2 - r_1^2)^{1/2} \]

Then the manifold flow rate can be estimated by

\[ Q_m = 2n_{sd} \times A_m \times \omega (r_2^2 - r_1^2)^{1/2} \]  

(12.23)

12.3 Effectiveness of System

The analysis of the previous two sections developed equations to predict lubricant velocities and flow rates. This section will determine the trajectory of the lubricant as it leaves the cone ducts and quantitatively define the effectiveness of distribution around the circumference of the rib.

In this section, effectiveness is defined as the percentage of rib circumference covered by the stream of lubricant. The effects of roller-end/rib starvation and surface wetting are beyond the scope of this analysis. Furthermore, the lubricant stream is considered useful only between the diameters of the cone direct outlet (2r_3) and the center of the roller-end/rib contact (2r_5). Once the lubricant leaves the cone duct, its motion is independent of the cone and atmospheric drag is neglected.
This coordinate system is shown on Figure 23. The components of velocity in the X and Y direction are $\dot{X}$ and $\dot{Y}$.

The angle $\gamma$ for a single cone duct is formed with the bearing center and is opposite the segment of the rib transversed by a particle of lubricant as it leaves the cone duct and intersects the center of the roller-end/rib conjunction.

System effectiveness is defined as

$$E_s = 100 \frac{n_{cd}\lambda}{2\pi}$$

(12.24)

To determine $\lambda$, first the Cartesian coordinates of the point where the path of the lubricant particle intersects the roller-end/rib radius $r_5$ is determined. Using the equation of a circle with the origin at zero.

$$x^2 + y^2 = r_5^2$$

(12.25)

$X$ and $Y$ as a function of time are

$$X = \omega r_3 T$$

(12.26)

$$Y = r_3 + v_3 \cos(\Theta_H) T$$

(12.27)
Figure 23 - Coordinate System to Calculate Effectiveness of Lubrication System
Substituting equations 12.26 and 12.27 into 12.25 and solving for the elapsed time $t_e$

\[
t_e = \sqrt{\frac{(r_3^2 v_3 \cos \theta_H)^2 - (\omega^2 r_3^2 + v_3^2 \cos \theta_H^2)(r_3^2 - r_5^2)}{(2 r_3^2 + v_3^2 \cos \theta_H^2)}}^{1/2} - r_3 v_3 \cos \theta_H
\]  

(12.28)

Now knowing $t_e$, $X_e$ and $Y_e$ can be calculated and

\[
\lambda = \tan^{-1} \left( \frac{X_e}{Y_e} \right)
\]

(12.29)
13. DISCUSSION AND RECOMMENDATIONS

A Timken developed, Fortran language computer program has been written which includes all of the foregoing analyses. Appendix B is the printout of this program entitled, "High Speed Tapered Roller Bearing Analysis". The data shown is for the candidate test bearing XCL933CE-XCL933CD operating at $3.5 \times 10^6$ DN, maximum program thrust load of 5000 pounds, lubricant supply (MIL-L-7808G) temperature of $300^\circ F$ and with an assumed conjunction inlet temperature of $350^\circ F$.

Under pure thrust loading each roller is assumed to be similarly loaded. If radial load was considered, all the parameters such as load, stress, film thickness, friction, etc. would be computed throughout the load zone.

A parametric study was conducted to determine whether this candidate bearing would be suitable for high speed development testing. Assembly drawings of the XCL933 series bearing assembly with the standard roller guided cage and alternate race guided cage are shown on Figures 24 and 25 respectively.

The two study bearings were chosen with the following design considerations:

a. Keeping the inside and outside diameters constant with slight variation in overall bearing width.
Figure 24 - Proposed Test Bearing With Roller Guided Cage

THE CONE, CUP, AND ROLLERS ARE TO BE MADE FROM CBS 1000M STEEL.

THE TIMKEN COMPANY
CANTON, OHIO, U.S.A.

A-39898
Figure 25 - Proposed Test Bearing With Race Guided Cage
b. Reduced roller diameters to minimize effects of centrifugal force.

c. Steeper and shallower contact angles to determine geometry effects.

A summary of the pertinent results are shown on Table VIII. Overall, these studies indicated that changing the existing geometry (new tooling requirements and increased costs) would not be warranted based on these computed parameters.

In both study bearings the roller diameter was reduced by 8.33%. With the reduced roller size and a slightly increased pitch diameter an additional five rollers could be added to the bearing. For the steeper angle study No. 1, the race stresses were reduced by approximately 5% with a negligible change in rib stress and a slight reduction of lubricant film thickness. The shallower angle study No. 2 developed increased race stresses (3.4% at cone), reduced rib loading with 5% thicker film at the rib conjunction.

The initial studies were conducted using a roller spherical end radius equal to 77% of the apex length. Past development tests have shown this to be the optimum radius for high speed operation. Figure 26 is a plot of the calculated lubricant film thickness (central region) at the roller-end/cone rib conjunction as a function of roller spherical end radius.
**TABLE VIII**

**Effects of Bearing Geometry on Operating Parameters**

*Note: Dimensions in inches, force and weight in pounds and stresses in KSI.

Study conditions: \( R = 0; \ T = 5000\#; \ S = 32422 \text{ RPM}; \ T_1, T_2, T_3 = 350^\circ\text{F} \)

<table>
<thead>
<tr>
<th></th>
<th>XC1933CE/</th>
<th>Study No. 1</th>
<th>Study No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XC1933DC</td>
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<td></td>
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</tbody>
</table>

*Geometry*

<table>
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<th>XC1933CE/</th>
<th>Study No. 1</th>
<th>Study No. 2</th>
</tr>
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<tbody>
<tr>
<td>Included Cone Angle (28)</td>
<td>26° 8'</td>
<td>27° 54'</td>
<td>24° 54'</td>
</tr>
<tr>
<td>Included Cup Angle (2(\alpha))</td>
<td>29° 20'</td>
<td>31° 0'</td>
<td>27° 40'</td>
</tr>
<tr>
<td>Roller Diameter ((d_1))</td>
<td>.300</td>
<td>.2755</td>
<td>.2747</td>
</tr>
<tr>
<td>Roller End Radius ((r))</td>
<td>8.2723</td>
<td>7.8418</td>
<td>8.7610</td>
</tr>
<tr>
<td>Mean Cone O.D. ((D_2))</td>
<td>4.7218</td>
<td>4.7659</td>
<td>4.7764</td>
</tr>
<tr>
<td>Cone Base Diameter ((D_3))</td>
<td>4.8578</td>
<td>4.9103</td>
<td>4.9059</td>
</tr>
<tr>
<td>Mean Cup I.D. ((D_5))</td>
<td>5.2880</td>
<td>5.2832</td>
<td>5.2973</td>
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<tr>
<td>Number of Rollers</td>
<td>39</td>
<td>44</td>
<td>44</td>
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</table>

*Computed Parameters*

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<th>XC1933CE/</th>
<th>Study No. 1</th>
<th>Study No. 2</th>
</tr>
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<tbody>
<tr>
<td>Weight/Roller</td>
<td>.0114</td>
<td>.0095</td>
<td>.0096</td>
</tr>
<tr>
<td>Centrifugal Force/Roller</td>
<td>189</td>
<td>161</td>
<td>162</td>
</tr>
<tr>
<td>Roller RPM (No Slip)</td>
<td>277347</td>
<td>303804</td>
<td>304484</td>
</tr>
<tr>
<td>Cage RPM (No Slip)</td>
<td>15294</td>
<td>15377</td>
<td>15373</td>
</tr>
<tr>
<td>Net Bearing Induced Thrust</td>
<td>1812</td>
<td>1832</td>
<td>1658</td>
</tr>
<tr>
<td>Due to (C_f)</td>
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<td>Normal Loads/Stresses</td>
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<td>At Conjunctions:</td>
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<tr>
<td>Cone</td>
<td>322/149.7</td>
<td>269/142.6</td>
<td>318/154.8</td>
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<tr>
<td>Cup</td>
<td>506/177.4</td>
<td>425/170.1</td>
<td>475/179.8</td>
</tr>
<tr>
<td>Rib</td>
<td>56.9/23.0</td>
<td>50.2/22.9</td>
<td>46.3/20.7</td>
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<tr>
<td>EHD Minimum Film</td>
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<tr>
<td>Thickness Without</td>
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<tr>
<td>Thermal Reduction</td>
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<tr>
<td>(Microinches)</td>
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<tr>
<td>Roller-Body/Cone</td>
<td>7.6</td>
<td>7.5</td>
<td>7.4</td>
</tr>
<tr>
<td>Roller-Body/Cup</td>
<td>7.5</td>
<td>7.4</td>
<td>7.3</td>
</tr>
<tr>
<td>Roller-End/Rib</td>
<td>14.2</td>
<td>13.9</td>
<td>14.9</td>
</tr>
</tbody>
</table>

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Note: Central Film Thickness at Rib/Roller End Conjunction Computed Using: Archard and Cowking Formula, References 22 & 23

Proposed Test Bearing - XC1933
Lubricant - MIL-L-7808G
At Inlet Temperature 350°F

Figure 26 - Film Thickness Versus % Apex Roller Spherical End Radius
The maximum film thickness of the central region is developed with a roller spherical end radius of 75% to 80% of the apex length. Previous development testing have verified these results.

At the present time, it is not anticipated that roller slip will be a problem. The tapered roller bearing (unlike ball or cylindrical roller bearings) has a rib which tends to drive the roller at a speed greater than the epicyclic speed. (Refer to Figure 27.) This sketch illustrates the forces that drive the roller. A summation of roller moment loading Table IX indicates that there is a positive driving moment over the full range of speed. In addition to this positive moment, it is reasonable to assume that there is asperity contact between mating surfaces which would also tend to keep the roller rotating at its epicyclic speed.

The selection of 48 ducts for the cone rib lubrication system is based on empirical data of previous tests. The two parameters used for estimating the number of cone ducts are peripheral rib speed and the percent of the rib circumference covered by a stream of oil. Refer to Figure 28 to compare these test conditions with previous work.
\[ E_m = \text{SUMMATION OF MOMENTS ABOUT ROLLER CENTER} \]
\[ \text{(CCW - POSITIVE)} \]
\[ E_m = \left( \frac{d}{2} - h \right) \times F_3 - d \times F_p - \left( \frac{d}{2} \right) \times F_1 - \left( \frac{d}{2} \right) \times F_2 \]
TABLE IX

Summation of Moments About Roller Center

Calculations based on the following conditions:

- Bearing: XC1933CE-XC1933DC
- Lubricant: MIL-L-7808G
- Temperature: Supply - 300°F, Conjunction Inlet - 350°F
- Loading: Thrust - 5000 Pounds, Radial - 0
- Roller-Body/Cage Pocket Clearance: .0001"

<table>
<thead>
<tr>
<th>Speed (DN)</th>
<th>RPM</th>
<th>Friction Forces (Pounds)</th>
<th>Summation Of Moments About Roller Center (Inch-Pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5x10^6</td>
<td>13895</td>
<td>1.1796 1.1858 0.8929 0.713</td>
<td>+.0009</td>
</tr>
<tr>
<td>2.5x10^6</td>
<td>23159</td>
<td>0.2569 0.2657 1.3262 0.1188</td>
<td>+.0006</td>
</tr>
<tr>
<td>3.5x10^6</td>
<td>32422</td>
<td>0.3251 0.3363 1.9048 0.1663</td>
<td>+.0151</td>
</tr>
</tbody>
</table>

*Plus Sign indicates positive moment in direction of rolling."
Peripheral Speed of Rib at Rib/Roller-End Conjunction (Ft./Min.)

Figure 28 - % Lubricant Coverage Versus Rib Speed
RECOMMENDATIONS

It is recommended that the XCl933 series bearing assembly incorporating the following design be tested to 3.5 million DN:

a. Roller spherical and radius equal to 77% of apex length.
b. Standard stamped roller guided cage.
c. A cone rib lubrication system with 48 - .040" diameter ducts.
APPENDIX A

Friction Analysis for High Sliding Combined With Rolling (Roller End/Rib)

For selected friction data (supplied by Southwest Research Institute) in studies with disks in rolling/sliding contact, the three parameters $G_1$, $G_2$ and $G_3$ were calculated and friction coefficient was plotted against $G_1$, for various values of $G_2$ and $G_3$. These friction data are for the full EHD regime. Oil and test conditions are as follows:

Oil: SwRI Oil B, a straight mineral oil

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$T_O = 140^\circ F$</th>
<th>$T_O = 190^\circ F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$ (cp)</td>
<td>29.73</td>
<td>11.55</td>
</tr>
<tr>
<td>$\alpha_0$ (psi$^{-1}$)</td>
<td>$1.38 \times 10^{-4}$</td>
<td>$1.20 \times 10^{-4}$</td>
</tr>
<tr>
<td>$k$ (BTU/hr./ft./$^\circ F$)</td>
<td>.0075</td>
<td>.0075</td>
</tr>
</tbody>
</table>

$\beta = .0272 \, ^\circ R^{-1} @ 150,000$ psi

$v_s = 210$ and $350$ in./sec.

sum velocity = $1050$ in./sec.

$\sigma_m = 140,000$ to $300,000$ psi
These conditions give three values of $G_2$: $38.6 \times 10^{-3}$, $42.8 \times 10^{-3}$ and $107 \times 10^{-3}$, and values of $G_3$ between 16 and 32. These values of $G_2$ more closely approximate the conditions at the roller end/rib conjunction than those plotted in Reference 14. Figure 29 shows the data for the two lower values of $G_2$, which are approximately the same and can be averaged to give a nominal $G_2 = 41 \times 10^{-3}$, this represents the lowest $G_2$ for which SwRI data are available for full EHD regime. Figure 30 presents SwRI data for an even larger $G_2 = 107 \times 10^{-3}$. Figure 31 is a plot of the "ceiling" friction coefficient versus $G_2$. The data points shown in Figure 31 are from Reference 14, and the curve shown has been linearly extrapolated to higher values of $G_2$. From this extrapolated curve, the ceiling friction coefficients at $G_2 = 41 \times 10^{-3}$ and $107 \times 10^{-3}$ were determined and each is shown as the dashed line in Figures 29 and 30. As can be seen in those figures, the "ceiling" friction coefficients look reasonable in comparison with SwRI experimental data.

In comparing Figures 92 through 98 in Reference 14, it will be seen that as $G_2$ increases, the family of curves of constant $G_3$ shift to the right. Also, in the Reference and SwRI data, the "ceiling" friction coefficient decreases.
CEILING BASED ON EXTRAPOLATION
OF DATA FROM REF. 14

G_2 (NOM.) = \frac{\beta \mu \nu s^2}{8k} = 41 \times 10^{-3}

T_0 = 140^\circ F, 190^\circ F

G_1 = \frac{\mu \nu s}{\sigma_m h_m}

Figure 29 - Friction Coefficient Behavior
For G_2 = 41 \times 10^{-3}
Figure 30 - Friction Coefficient Behavior
For $G_2 = 107 \times 10^{-3}$
Figure 31 - Behavior Of Ceiling Friction Coefficient With $G_2$

$G_2 = \frac{\beta \mu \lambda}{\sigma_m \lambda_m}$
as $G_2$ increases. For a given $G_2$, as $G_3$ decreases the curves of constant $G_3$ shift to the right, and enter the "ceiling" asymptote more sharply. SwRI data in Figures 29 and 30 also exhibit all these characteristics, and are in line quantitatively with the data in Reference 14.

The procedure recommended in Reference 14 consisted of calculating $G_1$, $G_2$ and $G_3$ at the reference supply temperature of 86°F, than compensating for the supply temperature difference.

The reason given for this extrapolation technique is that Plint (Ref. 24) and Johnson and Cameron (Ref. 20) found the friction coefficient varies with supply temperature. The application of this extrapolation technique as a generalized procedure is subject to question, since it is doubtful that the relationship between supply temperature and friction coefficient would be the same for all test rigs and mechanisms such as bearings and gears. Moreover, if $G_1$, $G_2$ and $G_3$ do represent the parameters necessary to describe the friction coefficient, then they should apply for any oil supply temperature $T_o$ as long as operation is in full EHD regime.

The effect of the variation of temperature would be taken into account in the $\mu_0$, $k$, $h_3$ and $a_0$ variables. This appears to be the case, since the data furnished by SwRI in Figure 31 are for two supply temperatures, and the viscosity at the lower temperature is nearly three times that at the higher temperature. The sliding velocity is different for the two
supply temperatures with the results that $G_2$ is approximately the same for both sets of data. Refer to Figures 29 and 30. When plotted on the same graph, the friction coefficient data (SwRI) behaves in the same way as they do for all the data in Reference 14, which are for a single temperature.

Friction coefficient is estimated by first calculating the $G$ parameters at the actual lubricant supply temperature. Then using the same numerical techniques as in Reference 14 to extrapolate for $G$ values outside the range of data.
APPENDIX B

Sample Computer Printouts
** Cone Data **

- **Included Angle**: 26-°-0-0
- **Mean Race C.D.**: 4.7213
- **Large Rib C.D.**: 5.0500

** Cup Data **

- **Included Angle**: 29-20-0
- **Mean I.D.**: 5.2380

** Roller Data **

- **No. of Rollers**: 39
- **Slant Length**: 0.0016
- **Weight (Lbs.)**: 0.0114

** Cage Data **

- **Type of Guide Roller**: Stock Thickness 0.0620
- **L.E. Land Depth**: 0.0310

---

APPENDIX B

** Roller-Guided Cage **

- **Included Angle**: 29-20-0
- **Mean I.D.**: 5.2380
- **Mean E.D.**: 6.7500
- **Large Rib C.D.**: 5.0500
- **Small Rib O.D.**: 4.6900
- **RIB Face Flat**: 0.0620

- **Bore**: 4.2500
- **Width**: 0.8438

- **Roller Race Angle**: 89-49-30
- **RIB race Angle**: 99-49-30

- **Bore**: 4.2500
- **Width**: 0.8438

---

- **Roller Race Angle**: 89-49-30
- **RIB Race Angle**: 99-49-30

- **Mean Race C.D.**: 4.7213
- **Mean E.D.**: 6.7500
- **Large Rib C.D.**: 5.0500
- **Small Rib O.D.**: 4.6900
- **RIB Face Flat**: 0.0620

---

- **Included Angle**: 26-°-0-0

---

** Appendix B **

- **RIB Race Angle**: 139-49-30
- **Bore**: 4.8578
- **Small E.D.**: 4.6900
- **Eff. Length**: 0.5475
- **Contact Ht.**: 0.0598

- **Mean I.D.**: 5.2380
- **Mean E.D.**: 6.7500

- **Weight (Lbs.)**: 0.0114
- **Contact Ht.**: 0.0598
- **Spherical End Radius**: 8.2724 (77.02)

- **Mass-Moment of Inertia (Lb. Inch-Sq.)**: I(X) = 0.00012  I(Y) = 0.00040

** Cage Data **

- **Type of Guide Roller**: Stock Thickness 0.0620
- **Pocket Clearance**: 0.0001
- **L.E. Land Depth**: 0.0310

- **Wing Angle**: 14-48-0
- **Wing Length**: 0.0352

- **S.E. Land Depth**: 0.0250

** Cage C.D. to Roller C-L**: 0.0845

** % Roller Projection**: 21.02

---

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**ENGRG PHL022-01**

**HIGH SPEED TAPERED ROLLER BEARING ANALYSIS**

**CONE NO.** XC1933CE  **CUP NO.** XC1933DC  **DATE** 18 JULY 1973  **PAGE** RUN 500

**LARGE FIB LUBRICATION SYSTEM DATA**

<table>
<thead>
<tr>
<th></th>
<th>cone Hole</th>
<th></th>
<th>cone Hole</th>
<th></th>
<th>Hole Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Dia.</td>
<td>4.3600</td>
<td></td>
<td>Outlet Dia.</td>
<td>4.8580</td>
<td>10-0-0</td>
</tr>
<tr>
<td>No. of Shaft Holes</td>
<td>2</td>
<td></td>
<td>Inside Dia.</td>
<td>0.1250</td>
<td>0-0-0</td>
</tr>
<tr>
<td>No. of Cone Holes</td>
<td>48</td>
<td></td>
<td>Inside Dia.</td>
<td>0.0400</td>
<td>0-0-0</td>
</tr>
<tr>
<td>Manifold Trap Width</td>
<td>0.0920</td>
<td></td>
<td>Manifold Dia.</td>
<td>4.4800</td>
<td></td>
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</tbody>
</table>

**MOUNTING DATA**

<table>
<thead>
<tr>
<th></th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft I. D.</td>
<td>3.6250</td>
</tr>
<tr>
<td>Housing O. D.</td>
<td>9.0000</td>
</tr>
</tbody>
</table>
operation data

external radial load (lbs) 0

external thrust load (lbs) 5000

shaft speed (rpm) 32422

bulk inlet temperature (deg.f) 300

conjunction temperature (deg.f) 350

lubricant - mil-l-7808g

<table>
<thead>
<tr>
<th></th>
<th>at</th>
<th>at</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bulk inlet</td>
<td>conjunction</td>
</tr>
<tr>
<td>kinematic viscosity (centistokes)</td>
<td>1.570</td>
<td>1.20e</td>
</tr>
<tr>
<td>absolute viscosity (centipoises)</td>
<td>1.346</td>
<td>1.012</td>
</tr>
<tr>
<td>absolute viscosity (micro-geyns)</td>
<td>0.195</td>
<td>0.147</td>
</tr>
<tr>
<td>density (gams/cc)</td>
<td>0.0565</td>
<td>0.8386</td>
</tr>
<tr>
<td>pressure-viscosity coeff. (sq.inch/lb*10**4)</td>
<td>0.504</td>
<td>0.452</td>
</tr>
<tr>
<td>specific heat (btu / lb-deg.f)</td>
<td>0.5384</td>
<td>0.5508</td>
</tr>
<tr>
<td>thermal conductivity (btu / hr-ft-deg.f)</td>
<td>0.0841</td>
<td>0.0835</td>
</tr>
</tbody>
</table>

coefficient of thermal expansion (cc / deg.f) = 0.000394

temperature-viscosity coefficient (deg.f ** (-1))

(.0045 + (1.25*10**(-7)) * (hertz stress))
HIGH SPEED TAPERED ROLLER BEARING ANALYSIS

CONE NO. XC1933CE CUP NO. XC1933DC

RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

------ INTERFERENCE FITS ------

STATIC EFFECTS

DIAMETRICAL FIT  .001  .005  .009

CONTACT PRESS. CONE/SHAFT (PSI)  444  2223  4002

CONTACT PRESS. CUP/HSG (PSI)  362  1818  3273

INCREASE IN MEAN CONE O.D.  .000597  .002987  .005376

DECREASE IN MEAN CUP I.D.  .000831  .004156  .007481

HOOP STRESS CONE I.D. (PSI)  4239  21198  38157

HOOP STRESS CONE O.D. (PSI)  3795  18975  34155

THERMAL EFFECTS

REDUCTION OF DIAMETRICAL FIT PER 10 DEG.F (CONE/SHAFT) = 0.000255 (IN)

DYNAMIC EFFECTS

LOCATION  HOOP STRESS (PSI)

CONE I.D.  45531 (MAX)

CONE O.D.  39719

COMBINED EFFECTS STATIC & DYNAMIC (CONE / SHAFT)

STATIC DIAMETRICAL FIT  .001  .005  .009

CONTACT PRESS. (PSI)  0  1218  2997

HOOP STRESS CONE I.D. (PSI)  45531  57148  74107

HOOP STRESS CONE O.D. (PSI)  39719  50118  65298

LOSS OF DIAMETRICAL FIT DUE TO ROTATION = 0.002260
CONE NO. XC1933CE  CUP NO. XC1933DC  RUN  500
RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

** BEARING KINEMATICS  EPICYCLIC  DN = 3.500 (10**6)

-- ALL VELOCITIES IN INCH / SECOND - UNLESS NOTED --

CONE

ANGULAR VELOCITY (RPM) = 32422  (RADIAN/SEC) = 3395
MEAN O.D. TANGENTIAL VELOCITY ABOUT OWN CENTER  8015
RIB / ROLLER CONTACT ABOUT CENTER (FT/MIN)  42221

ROLLER

ANGULAR VELOCITY ABOUT OWN CENTER (RPM)  277347
MEAN O.D. TANGENTIAL VELOCITY ABOUT OWN CENTER  4234
RIB / ROLLER CONJUNCTION ABOUT ROLLER CENTER  2620
RIB / ROLLER CONJUNCTION ABOUT CONE CENTER  6603
RIB / ROLLER RUBBING VELOCITY  1640

CAGE

ANGULAR VELOCITY (RPM) = 15294  (RADIAN/SEC) = 1601

SUM VELOCITIES OF CONJUNCTIONS (EHD)

CONE RACE / ROLLER BODY  8469
CUP RACE / ROLLER BODY  8469
CONE-RIB / ROLLER END  7081
**BEARING DYNAMIC LOADING DUE TO ROTATION ONLY**

--- ALL FORCES IN POUND -- COUPLE IN POUND-INCH ---

**CENTRIFUGAL FORCE & GYROSCOPIC COUPLE**

**CENTRIFUGAL FORCE PER ROLLER**

189.2

**TOTAL BEARING INDUCED THRUST DUE TO CF**

1819.

<table>
<thead>
<tr>
<th>INTERFACE</th>
<th>COMPONENT Radial</th>
<th>COMPONENT Axial</th>
<th>RESULTANT Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONE RACE / ROLLER BODY</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>CUP RACE / ROLLER BODY</td>
<td>178.2</td>
<td>46.6</td>
<td>184.2</td>
</tr>
<tr>
<td>CONE-RIB / ROLLER END</td>
<td>11.0</td>
<td>46.6</td>
<td>47.9</td>
</tr>
</tbody>
</table>

**DISPLACEMENT OF CUP RACE / ROLLER BODY RESULTANT FOR STATIC MOMENT EQUILIBRIUM = 0.0189**

**CUP LOAD PER UNIT LENGTH ADJACENT ROLLER (LB/IN)**

<table>
<thead>
<tr>
<th>L.E.D.</th>
<th>S.E.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>266.8</td>
<td>406.1</td>
</tr>
</tbody>
</table>

**GYROSCOPIC COUPLE PER ROLLER = 3.931**

**COMBINED EFFECTS OF CENTRIFUGAL FORCE & GYROSCOPIC COUPLE**

**CUP LOAD PER UNIT LENGTH ADJACENT ROLLER (LB/IN)**

<table>
<thead>
<tr>
<th>L.E.D.</th>
<th>S.E.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>345.5</td>
<td>327.4</td>
</tr>
</tbody>
</table>
HIGH SPEED TAPERED ROLLER BEARING ANALYSIS

DATE 18 JULY 1973

CONE NO. XC1933CE  CUP NO. XC1933DC  RUN 500

RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

LOAD DISTRIBUTION FACTORS (LINE CONTACT)

<table>
<thead>
<tr>
<th>R*TAN(A)/T</th>
<th>SJOVALL’S INTEGRALS</th>
<th>PARAMETER</th>
<th>LOAD ZONE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RADIAL</td>
<td>EPSILON</td>
<td>DEGREES</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.0000</td>
<td>INFINITY</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>360</td>
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</tbody>
</table>

STRESSES AT RACE CONTACTS FOR MAX LOADED ROLLER

<table>
<thead>
<tr>
<th></th>
<th>CONE/ROLLER</th>
<th>CUP/ROLLER</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL LOAD (LBS)</td>
<td>322.0</td>
<td>506.3</td>
</tr>
<tr>
<td>CONTACT STRESS AT CENTER (KSI)</td>
<td>149.7</td>
<td>177.4</td>
</tr>
<tr>
<td>CONTACT WIDTH</td>
<td>0.0050</td>
<td>0.0066</td>
</tr>
<tr>
<td>CONTACT LENGTH</td>
<td>0.5475</td>
<td>0.5475</td>
</tr>
<tr>
<td>MAX SHEAR STRESS (KSI)</td>
<td>44.9</td>
<td>53.3</td>
</tr>
<tr>
<td>DEPTH OF MAX SHEAR STRESS</td>
<td>0.0020</td>
<td>0.0026</td>
</tr>
<tr>
<td>MAX SHEAR STRESS AT SURFACE (KSI)</td>
<td>29.9</td>
<td>35.5</td>
</tr>
<tr>
<td>MAX RANGE (+,−) OF ORTHOGONAL (KSI)</td>
<td>76.6</td>
<td>90.8</td>
</tr>
<tr>
<td>DEPTH OF MAX ORTHOGONAL SHEAR</td>
<td>0.00125</td>
<td>0.00166</td>
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</table>
**HIGH SPEED TAPERED ROLLER BEARING ANALYSIS**

**CONE NO. XC1933CE**

**CUP NO. XC1933DC**

**RUN 500**

**RPM = 3242**

**F = 0**

**T = 5000**

**INLET/CONJ. TEMP = 300/350 F.**

-------------------

**LOAD **

**STRESS **

**LUBRICANT FILM**

-------------------

<table>
<thead>
<tr>
<th>AZIMUTH (DEG)</th>
<th>RADIAL (LBS)</th>
<th>AXIAL (LBS)</th>
<th>NORMAL (LBS)</th>
<th>STRESS (KSI)</th>
<th>FACTOR</th>
<th>FILM (MU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>313.7</td>
<td>72.8</td>
<td>322.0</td>
<td>149.7</td>
<td>0.6841</td>
<td>5.2</td>
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</table>

*** ALL ROLLERS IDENTICALLY LOADED ***
**HIGH SPEED TAPERED ROLLER BEARING ANALYSIS**

**CGNE NO. XC1933CE**
**CUP NO. XC1933DC**
**RPM = 32422**
**R = 0**
**T = 5000**
**INLET/CONJ. TEMP = 300/350 F.**

*************** LOAD ** STRESS ** LUBRICANT FILM ***************

CUP RACE / ROLLER BODY

<table>
<thead>
<tr>
<th>AZIMUTH (DEG)</th>
<th>RADIAL (LBS)</th>
<th>AXIAL (LBS)</th>
<th>NORMAL (LBS)</th>
<th>STRESS (KSI)</th>
<th>FACTOR</th>
<th>FILM (MU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>489.9</td>
<td>128.2</td>
<td>506.3</td>
<td>177.4</td>
<td>0.6856</td>
<td>7.8</td>
</tr>
</tbody>
</table>

*** ALL ROLLERS IDENTICALLY LOADED ***
**HIGH SPEED TAPERED ROLLER BEARING ANALYSIS**

**CONE NO. XC1933CE**
**CUP NO. XC1933DC**
**RPM = 32422**
**R = 0**
**T = 5000**
**INLET/COND. TEMP. = 300/350 F.**

**LOAD ** STRESS ** LUBRICANT FILM **

**CONE RIB / ROLLER END**
**B/A = 0.38742**

<table>
<thead>
<tr>
<th>AZIMUTH (deg)</th>
<th>RADIAL (lbs)</th>
<th>AXIAL (lbs)</th>
<th>NORMAL STRESS (KSI)</th>
<th>STRESS FACTOR</th>
<th>FILM (MU)</th>
<th>CONTACT ELLIPSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>17.0</td>
<td>55.4</td>
<td>56.5</td>
<td>23.1</td>
<td>0.7-10</td>
<td>10.7 0.0427</td>
</tr>
</tbody>
</table>

***ALL ROLLERS IDENTICALLY LOADED***
CONE NO. XC1933CE  CUP NO. XC1933DC  RUN 500

RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

**CAGE ANALYSIS**

**ROLLER BODY / CAGE POCKET CONJUNCTION**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>INCLUDED ANGLE (RADIANS)</td>
<td>0.2418</td>
</tr>
<tr>
<td>ARC LENGTH</td>
<td>0.0353</td>
</tr>
<tr>
<td>REYNOLDS NUMBER</td>
<td>173</td>
</tr>
<tr>
<td>FRICTION FACTOR</td>
<td>0.01150</td>
</tr>
<tr>
<td>SHEAR STRESS (PSI)</td>
<td>8.253</td>
</tr>
<tr>
<td>DRAG FORCE (LBS)</td>
<td>0.1663</td>
</tr>
</tbody>
</table>
HIGH SPEED TAPERED ROLLER BEARING ANALYSIS

CONE NO. XC1933CE
CUP NO. XC1933DC
RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

TRACTION

<table>
<thead>
<tr>
<th>AZIMUTH (DEG)</th>
<th>ROLLER ANGULAR VELOCITY (RAD/SEC)</th>
<th>TRACTIVE LOADING &amp; FLUID PRESSURE FORCES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CUP</td>
</tr>
<tr>
<td>0.0</td>
<td>29043</td>
<td>-0.33630</td>
</tr>
</tbody>
</table>
PHL ENGINE PHL022-01 HIGH SPEED TAPERED ROLLER BEARING ANALYSIS DATE PAGE
CONE NO. XC1933CE CUP NO. XC1933DC RUN 425
18 JULY 1973

** ALL DIMENSIONS IN INCHES - ANGLES IN DEG-MIN-SEC - UNLESS NOTED **

APPENDIX B

Race Guided Gage

CONE DATA

INCLUDED ANGLE RIB-RACE ANGLE BORE
26- 8- 0 89-49-30 4.2500

MEAN RACE O.D. BASE DIA. WIDTH
4.7218 4.6578 0.8438

LARGE RIB O.D. SMALL RIB O.D. RIB FACE FLAT
5.0500 4.6900 0.0620

CUP DATA

INCLUDED ANGLE MEAN I. D. O. D.
29-29- 0 5.2880 5.7500

ROLLER DATA

NC. OF ROLLERS L. E. DIA. S. E. DIA.
39 0.3000 0.2832

SLANT LENGTH EFF. LENGTH APEX LENGTH
0.6016 0.5475 10.7433

WEIGHT (LBS.) CONTACT HT. SPHERICAL END RADIUS
0.0114 0.0558 8.2724 (77.0%) 

MASS-MOMENT OF INERTIA (LB. INCH-SQ.) I(X) = 0.00012 I(Y) = 0.00040

CAGE DATA

TYPE OF GUIDE STOCK THICKNESS POCKET CLEARANCE
RACE 0.0900 0.0020

L.E. PILOT SURFACE DIA. CLEARANCE RELIEF
SHAFT OR CONE 5.1000 0.0030 0.0

S.E. PILOT SURFACE DIA. CLEARANCE RELIEF
MOUSING OR CUP 5.0950 0.0030 0.0
HIGH SPEED TAPERED ROLLER BEARING ANALYSIS

CCNE NC. XC1933CE  CUP NO. XC1933DC

RPM = 32422  R = 0  T = 5000  INLET/CONJ. TEMP = 300/350 F.

CAGE ANALYSIS

<table>
<thead>
<tr>
<th>ROLLER BODY / CAGE POCKET CONJUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCLUDED ANGLE (RADIANS)</td>
</tr>
<tr>
<td>REYNOLDS NUMBER</td>
</tr>
<tr>
<td>SHEAR STRESS (PSI)</td>
</tr>
<tr>
<td>ARC LENGTH</td>
</tr>
<tr>
<td>FRICTION FACTOR</td>
</tr>
<tr>
<td>CAGE LOAD DIRECTION</td>
</tr>
</tbody>
</table>

CAGE / PILOT INTERFACE

<table>
<thead>
<tr>
<th>LARGE END PILOT</th>
<th>SMALL END PILOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRIVE</td>
<td>CRAG</td>
</tr>
<tr>
<td>REYNOLDS NUMBER</td>
<td>5624</td>
</tr>
<tr>
<td>FRICTION FACTOR</td>
<td>0.00190</td>
</tr>
<tr>
<td>SHEAR STRESS (PSI)</td>
<td>1.5871</td>
</tr>
<tr>
<td>MOMENT (LB-IN)</td>
<td>4.047</td>
</tr>
</tbody>
</table>
**Cone Rib Lubrication**

**Calculations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Cone Ducts</th>
<th>Cone Ducts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction Factor</td>
<td>0.0202</td>
<td>0.0273</td>
</tr>
<tr>
<td>Headloss (in/sec)<strong>2</strong></td>
<td>88334.</td>
<td>253199.</td>
</tr>
<tr>
<td>Reynolds Number</td>
<td>98719.</td>
<td>29505.</td>
</tr>
<tr>
<td>Exit Velocity (in/sec)</td>
<td>3742.6743</td>
<td>3495.6187</td>
</tr>
</tbody>
</table>

**Maximum Manifold Pressure (psi)** = 102.20

**Tangential Velocity of Lubricant @ Outlet (in/sec)** = 8247.00

**Angle of Exit Relative to Outlet (Degrees)** = 22.657

**Effective Exposure Time (Sec)** = 0.000017

**% Coverage of Rib** = 42.85

**Flow (pt/min)**

<table>
<thead>
<tr>
<th></th>
<th>Cone</th>
<th>Manifold</th>
<th>Shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>438.13</td>
<td>68.8^0</td>
<td>190.8^0</td>
</tr>
</tbody>
</table>
References


