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OPTIMAL SCHEDULING FOR AN OUTPATIENT CLINIC

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13. ABSTRACT

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OPTIMAL SCHEDULING FOR AN OUTPATIENT CLINIC

by

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Abstract

In this paper a problem of an outpatient clinic is described. This problem is then formulated as a queuing problem with some special properties. Using a few results in queuing theory we discuss the existence of some parameters of this model. In order to find explicitly the optimal schedule of patients in a clinic we use a trade-off between patient waiting time and doctor idle time. A simulation program was coded and run in order to find the expected waiting time and the total relevant costs of the clinic.

1. The Outpatient Clinic Model

It is very important that an operating system which gives any kind of service, should try to satisfy its customers as much as possible. Customer satisfaction is often difficult to measure and may be complicated by existing along many different dimensions. For example, in the area of health care delivery, the quality of medical treatment that a patient receives is not the only component which contributes to his satisfaction. The efficiency with which the medical treatment is provided also seems to be a major and important criteria for the patient's satisfaction.

First, it was assumed that the doctor's time is much more valuable than the patients' time. Hence, in order for an outpatient clinic to operate efficiently the idle time of the doctor, i.e. the time that the doctor is idle and waits for his patients, should be as small as possible. However, the doctor's time is not infinitely more valuable than the patients' time, and the cost associated with the time the patient has to wait from his arrival to the clinic until he is accepted by the doctor should not be neglected.

Hence the study of patients waiting time and its relationship to the doctors idle time in an outpatient clinic is an important study of the efficiency with which the medical care is provided, and can be discussed from various points of view.

It seems that the main reason for adopting any formal appointment system instead of the old, first-come, first-served method to determine the order in which patients will be seen by their doctor, was to increase the efficiency of this system, or in other words, to decrease the waiting time of the patients. Intuitively, a formal appointment system permits a patient to show up exactly at the time he is to enter service and thus, incur no waiting time at all.

However, it is a matter of fact that a patient who is scheduled to arrive at some fixed time, will not necessarily be on time. He might come early or even late; see for example [2], [5]. This less of accuracy may increase the idle time of the doctor and thus will decrease the utilization of the out-patient clinic.

If we will try to increase the utilization of the clinic, by assigning the appointments closer to each other, we may decrease the idle time of the doctors but on the other hand increase the waiting time of the patients. This alternative may sometimes be more expensive, especially when the patients' time is quite valuable. Thus, it is important to find a proper appointment schedule, which will be acceptable for the patients and will still remain efficient to the doctor. This can be done by achieving some balance between the patients' waiting time and the doctors' idle time.

The problem of scheduling patients in a clinic is very complicated because while dealing with such a problem, one must consider many random phenomena which are inherited in any real world situation; consider as examples: a patient may come early, late or on time; the doctors may have to leave the office during their working hours; patients without appointments may just "walk in" to the clinic; or patients who were scheduled to come in some day may not appear. Most of those phenomena described above are not under control. It seems that the only controllable parameters for a clinic which operates according to some given policy, such as for example: (1) "walk-ins" are not accepted by the doctor; (2) the doctor is available 8 hours per day each day in the week; etc., is the time between successive appointments of the patients.

In the following sections we try to find the best schedule while assuming that the outpatient clinic operates in the following manner (this defines our policy in the clinic we are dealing with):

(1) Every doctor provides medical treatment for his patients alone; hence, it suffices to find the optimal scheduling appointment for one doctor.

(2) The doctor's service time is a random variable distributed according to a given distribution function.

(3) The patients may arrive before, after or at their appointed times according to some given distribution.

(4) All the scheduled patients for a fixed day will arrive and that there are no "walk-ins".

(5) The doctor doesn't leave his office during the office hours.

Based on the above assumptions we will formulate the model of an outpatient clinic, as a queuing problem. We show that this queuing system satisfies some special properties which will enable us to use only a few results that appear in the literature concerning the existence of some parameters of this model.

We then conclude that in order to find explicitly the optimal schedule, we have to apply a simulation technique to find the expected waiting time in the system. Computational results are given in 3.

2. A Queuing Formulation for an Outpatient Clinic

In this section we will formulate the model of an outpatient clinic described in section 1 as a queuing problem.

We will assume that there is only one doctor who provides medical service to the patients, or when there are more than one, each doctor has his own patients.

The patients who arrive at the clinic will be numbered by i, $i = 1, 2, \ldots$. We will assume that the i^{th} patient, when attended, experiences a service time v_i, which is a random variable (r. v.) with a given distribution function, and that $\{v_i, \ i \geq 1\}$ are independent and identically distributed (i. i. d). Also $E(v_i) < \infty$.

The patients are scheduled to arrive to the clinic "a" units of time apart, but as was mentioned before they usually do not come exactly on time. It is plausible to assume that the i^{th} patient's arrival time to the clinic, which will be denoted by $i \cdot a + \delta_i$ is a r. v., whose density function is symmetric with respect to his scheduled appointment. We will further assume that the random variable $i \cdot a + \delta_i$ attains values with positive probability in the interval $[i \cdot a - b/2, \ i \cdot a + b/2]$ where $b \leq a$, and that $\{\delta_i, \ i \geq 1\}$ are i. i .d.

For example, one way to describe schematically the arrival time of the patients at the clinic is:

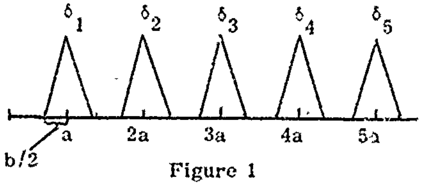

Figure 1

Remark

The triangular density functions where chosen in Figure 1 only for illustration. The sequel's discussion is valid for any density function satisfying the above assumptions.

The interarrival times of the patients to the clinic form a sequence of identically distributed r.v.s $\{a + \delta_{i+1} - \delta_i, \ i \geq 1\}$ which are not independent. This last fact "contributes" to the complexity of the system. It might be very difficult to express analytically some parameters of the model, especially for a general distribution function of δ_i which satisfy the above assumptions. See for example [6] and [7].

In our discussion we will consider the relevant costs of an outpatient clinic to be composed of two parts: (1), the idle time of the doctor, and (2) the waiting time of the patients. As was mentioned in section 1, the only parameter that can be controlled, for a given policy, is "a" - the time between any two appointments. The purpose of this paper is to find an optimal value a* for which the total relevant costs of the system attains its minimum.

In order to find the optimal scheduling of appointments for the outpatient clinic, we will investigate some properties of this system.

Theorem 1:

$\{\delta_{i+1} - \delta_i, \ i \geq 1\}$ is a stationary sequence of random variables.

Proof: For even n

(1) $P\{\delta_2 - \delta_1 \le x_1, \delta_3 - \delta_2 \le x_2, \delta_4 - \delta_3 \le x_3, \ldots, \delta_n - \delta_{n-1} \le x_{n-1}\}$

$$= \int_{y_2, \ldots, y_n} P\{\delta_2 - \delta_1 \le x_1, \delta_3 - \delta_2, \ldots, \delta_n - \delta_{n-1} \le x_{n-1} \mid \delta_2 = y_2, \delta_4 = y_4, \ldots, \delta_n = y_n\} \cdot dP\{\delta_2 = y_2, \ldots, \delta_n = y_n\}$$

and since the δ_i's are independent

$$= \int_{y_1, \ldots, y_n} P\{\delta_1 \ge y_2 - x_1\} P\{y_4 - x_3 \le \delta_3 \le x_2 + y_2\} \ldots P\{y_n - x_{n-1} \le \delta_{n-1} \le x_{n-2} + y_{n-2}\} \cdot dP\{\delta_2 = y_2\} dP\{\delta_4 = v_4\} \ldots dP\{\delta_n = y_n\}$$

which is sufficient for stationarity since the δ_i's are identically distributed r.v.'s. For odd n, the proof is similar.

Let us denote by w_i the r.v. designating the waiting time of the i^{th} patient, and let u_i,

(2) $u_i = v_i - [a + \delta_{i+1} - \delta_i] \qquad i = 1, 2, \ldots, n$

It was shown first by Lindley in [3] that

(3) $w_{i+1} = \max \{w_i + u_i, 0\} \quad i = 1, 2, \ldots, n$

Further, Loynes [4] proved that if both the interarrival time, and the service time form a stationary sequence of non-independent r.v.'s then

(4) $w_i \xrightarrow[i \to \infty]{} w$ almost everywhere

where $P\{w \le x\}$ is the distribution function (d.f.) of the waiting time for a customer in the steady state.

Theorem 2 [4]:

If $E(v_1) < a$ then w is an "honest" random variable, i.e. w is finite almost everywhere. Moreover, $P\{w = 0\} > 0$.

If $E(v_1) = a$ then w is almost everywhere infinite, and for any $y > 0$, $P\{w > y\}$ 1.

Proof: See Theorems 2, and 3 in [4].

Moreover, as Loynes mentioned in [4], Theorem 2 is valid for any value of the waiting time of the first customer, and thus w_1, need not necessarily be equal to 0. In fact, Theorem 2 holds even if the waiting time of the first customer is a r.v.

Theorems 1 and 2 above assure us that for a distance "a" between two successive appointments, such that

$$a < E(v_1)$$

eventually the queue builds up, never again to disappear. Whereas for $a > E(v_1)$, w is finite almost everywhere. Moreover, as is pointed out in Loynes [4], the event that a patient who arrives at the clinic will find an idle doctor will occur infinitely often.

Theorem 3:

The proportion of time that the server is busy tends to

(5) $\min(1, E(v_1)/a)$ almost everywhere

in all circumstances, provided $E(v_1) < \infty$.

Proof: See Loynes [4] corollary to Theorem 6.

Theorem 3 tells actually the proportion of time that the doctor is busy when the queuing system reaches its steady state. Thus, choosing $a > E(v_1)$ in our case, Theorem 2 assures us that the expected waiting time of a customer in the outpatient clinic, in the steady state, is finite, and Theorem 3 then reveals that the proportion of time that the doctor is busy is equal to

$$E(v_1)/a < 1.$$

Let us denote by C_d the estimated value of an hour of work of a doctor, and by C_p the estimated value of an hour for an average patient who arrives at the clinic. Then, when the system reaches the steady state, the cost of an idle doctor and of the waiting time of the patients for a given value of "a" will be

(6) $\quad C_d\left[1 - \frac{E(v_1)}{a}\right] \quad$ and $\quad C_p \frac{E(w)}{a}$, respectively.

According to Theorems 2 and 3 in the case where $a < E(v_1)$, $E(w) = +\infty$ and hence the relevant costs of the system in steady state are equal to

(7) $\quad C_p \frac{E(w)}{a} = +\infty.$

Thus, since our purpose is to minimize the total relevant costs of the outpatient clinic we rule out this alternative.

The relevant costs of the outpatient clinic in the steady state as a function of "a", are shown schematically in Figure 2.

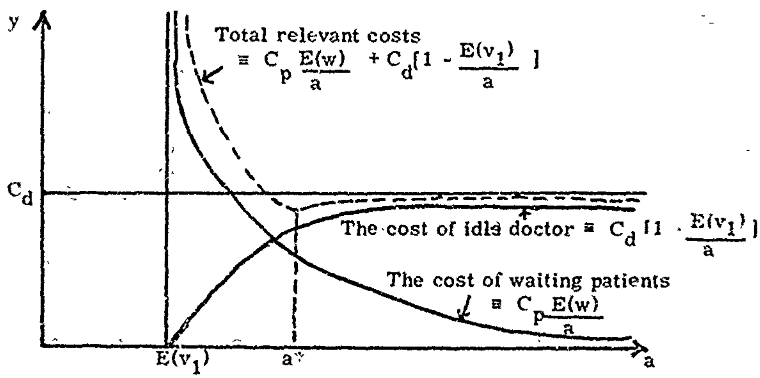

Figure 2

As can be seen in Figure 2, the total relevant costs of the system attains its minimum at $a = a^*$. Such a point a^* always exists and is finite, as can be seen from Lemma 1.

<u>Lemma 1:</u>[*] The function

$$g(a) = C_p \frac{E(w)}{a} + C_d [1 - \frac{E(v_1)}{a}]$$

attains its minimum at a finite point a^*. Moreover $g(a^*) < C_d$.

<u>Proof:</u> Since $E(w)$ satisfies

(8) $E(w) \longrightarrow 0$ as $a \longrightarrow \infty$

(9) $E(w) \longrightarrow \infty$ as $a \longrightarrow E(v_1)$

There exists a point a_1 such that

(10) $$g(a_1) = C_p \frac{E(w/a_1)}{a_1} + C_d [1 - \frac{E(v_1)}{a_1}] = C_d$$

where $E(w/a_1)$ is the value of $E(w)$ at the point a_1, or in other words

(11) $C_p \; E(w/a_1) = C_d \; E(v_1)$

Since $\frac{C_d \; E(v_1)}{C_p}$ is constant and $E(w) \longrightarrow 0$ as $a \longrightarrow \infty$, a_1 is a finite point.

Since $y = g(a)$ is assymptotic to $y = C_d$ there exists a finite point a_1^*, $a_1 \leq a^* < \infty$ for which g attains its minimum and $g(a^*) < C_d$, which completes the proof.

In order to find a^* practically, we have to know $E(w)$. To the best of the authors' knowlege one can not find an analytic form to express $E(w)$. This is mainly due to the fact that the interarrival time of patients to the

Note: The reason for adding this lemma was pointed out by Dr. Schoeman.

clinic form a dependent stationary sequence of r.v.'s.

In order to circumvent this difficulty and to find E(w) (and thus to find a^{*}) we have to simulate the system. Theorem 2 assures us that eventually the expected waiting time of patients in the clinic will converge to E(w) the expected waiting time in the steady state.

3. Computation Results

In order to calculate the expected waiting time of a patient, the total relevant costs, and the optimal scheduling distance "a" for an out-patient clinic, a simulation program was coded.

In the program the random variable δ_i (see 2) possesses a triangular density function, symmetric with respect to the scheduled appointment time $i \cdot a$ (see figure 1), and the service time of the doctor is an exponential r.v. However, the simulation program can easily be altered in order to calculate the above parameters for any other density functions which satisfy the assumptions in 2.

In order to study the sensitivity of the system to changes in various parameters, the program was run for two different values of E(v) - the expected service time, three different triangular density functions and three different sets of C_p and C_d.

The computational results are summarized in figures 2-3 and tables 1-6.

Figure 2 represents the total relevant costs as a function of "a" for $E(v) = 1$, $b=1$, and figure 3 for the case where $E(v) = 5$, $b=5$, b is the length of the basis in the triangular density function (see figure 1).

Tables 1-6 summarize some of the simulation results. Those results reveal the remarkable fact that the optimal scheduling distance a^* is almost invariant of b. For example, according to table 1, the optimal value of a^* for $C_p=1$, $C_d=5$ and $E(v)=1$ is $a^*=1.40$, for the three different values 1, 0.4 and 0.01 for b. In other words, the optimal scheduling distance in the case where the patient comes almost exactly at his fixed appointment time ($b=0.01$), is equal to the optimal scheduling distance a when the patients are quite inaccurate ($b=1.00$).

Observe too, that the total relevant costs of the outpatient clinic are only very slightly changed with the changes in b. For example, the minimum total relevant costs for $C_p=1$, $C_d=5$, $b=1$, $E(v)=1$ are 2.1028 while for b equal to 0.01 and all the other parameters remain the same, the minimum total relevant costs are 2.0889, a decrease of less than 0.6%.

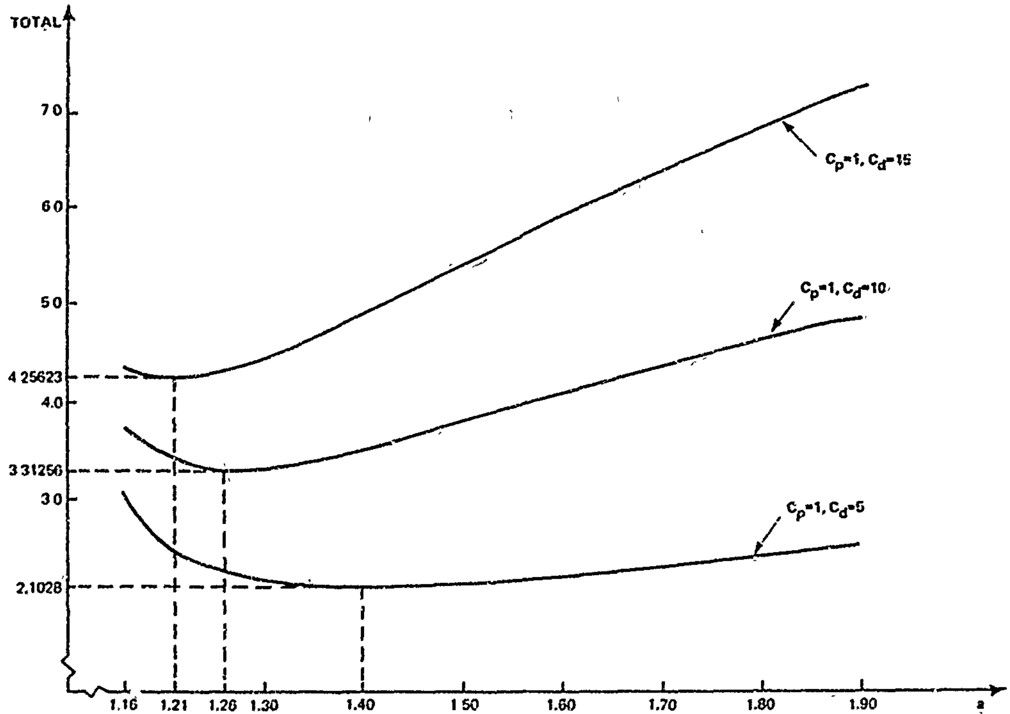

Figure 2 Total relevant costs as a function of the scheduling distance a

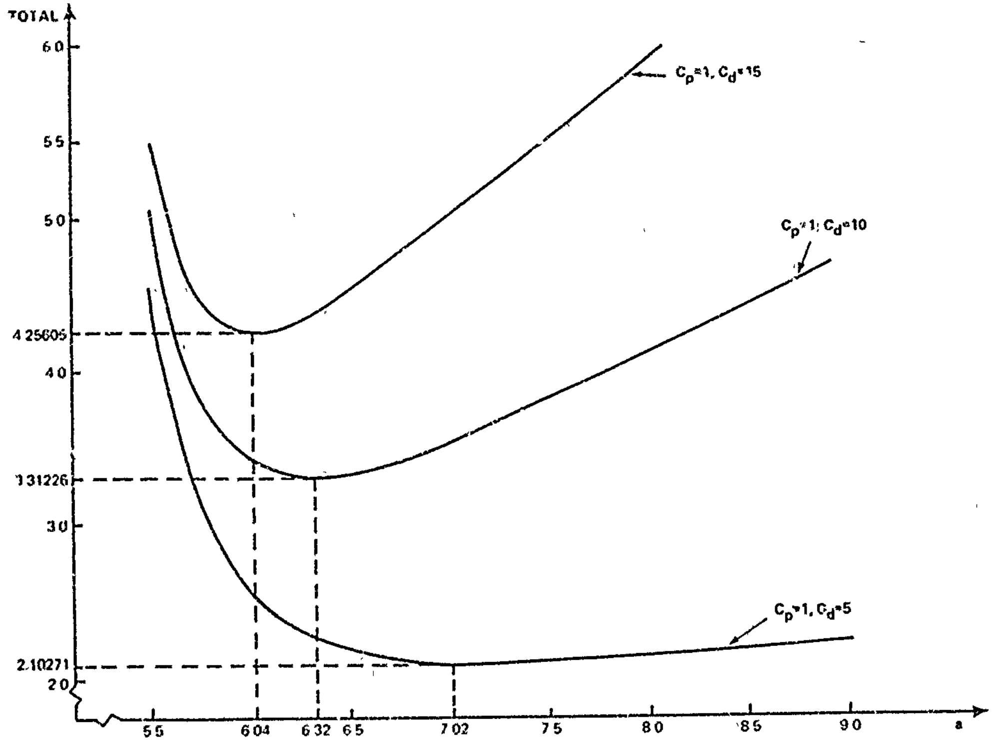

Figure 3 Total relevant costs as a function of the scheduling distance a

							$C_p = 1$	$C_d = 5$							
b \ a	1.16	1.25	1.33	1.35	1.37	1.38	1.39	1.40	1.41	1.42	1.43	1.44	1.45	1.60	1.90
1.00	3.00	2.31	2.1365	2.1196	2.1090	2.1057	2.1037	2.1028	2.1030	2.1041	2.1063	2.1093	2.1128	2.22	2.52
0.40	2.98	2.30	2.1224	2.1063	2.0953	2.0934	2.0917	2.0910	2.0915	2.0931	2.0956	2.0988	2.1025	2.21	2.52
0.01	2.97	2.29	2.1199	2.1038	2.0940	2.0912	2.0896	2.0889	2.0893	2.0910	2.0936	2.0969	2.1007	2.21	2.52

Table 1 Total Relevant Costs for E(v) = 1

							$C_p = 1$	$C_d = 10$							
b \ a	1.16	1.18	1.20	1.21	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.60	1.90
1.00	3.69	3.53	3.4256	3.3884	3.3602	3.3396	3.3251	3.3168	3.3125	3.3127	3.3157	3.3224	3.3327	4.09	4.89
0.40	3.67	3.51	3.4065	3.3701	3.3423	3.3223	3.3086	3.3004	3.2965	3.2970	3.3004	3.3075	3.3180	4.08	4.89
0.01	3.66	3.50	3.4031	3.3668	3.3391	3.3194	3.3059	3.2976	3.2936	3.2940	3.2975	3.3045	3.3152	4.08	4.89

Table 2: Total Relevant Costs for E(v) = 1

							$C_p = 1$	$C_d = 15$							
b \ a	1.16	1.18	1.20	1.21	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.60	1.90
1.00	4.38	4.29	4.2589	4.2562	4.2618	4.2745	4.2929	4.3168	4.3443	4.3757	4.4095	4.4466	4.4866	5.97	7.26
0.40	4.36	4.27	4.2399	4.2378	4.2439	4.2572	4.2763	4.3004	4.3282	4.3600	4.3942	4.4315	4.4718	5.96	7.26
0.01	4.36	4.27	4.2364	4.2346	4.2408	4.2544	4.2736	4.2976	4.3253	4.3570	4.3912	4.4286	4.4691	5.96	7.25

Table 3: Total Relevant Costs for E(v) = 1

b \ a	5.50	6.00	6.98	6.99	7.00	7.01	7.02	7.03	7.04	7.05	7.06	7.07	7.08	8.00	9.00
	C_p = 1 C_d = 5														
5.00	4.59	2.59	2.1030	2.1029	2.1028	2.10273	2.10271	2.1025	2.1028	2.1029	2.1031	2.1033	2.1035	2.22	2.42
1.00	4.57	2.57	2.0896	2.0895	2.08947	2.08945	2.08918	2.0895	2.0897	2.0899	2.0901	2.0904	2.0908	2.21	2.41
0.10	4.57	2.57	2.0891	2.0890	2.08886	2.08896	2.08888	2.0890	2.0891	2.0893	2.0896	2.0899	2.0902	2.21	2.41

Table 4: Total Relevant Costs for E(v) = 5

b \ a	5.50	6.00	6.29	6.30	6.31	6.32	6.33	6.34	6.35	6.36	6.37	6.38	7.00	8.00	9.00
	C_p = 1 C_d = 10														
5.00	5.04	3.43	3.3130	3.3125	3.3123	3.31226	3.31229	3.3124	3.3127	3.3131	3.3135	3.3141	3.53	4.09	4.64
1.00	5.02	3.40	3.2945	3.2942	3.2941	3.2941	3.2941	3.2943	3.2947	3.2951	3.2957	3.2964	3.52	4.08	4.64
0.01	5.02	3.40	3.2939	3.2936	3.29345	3.29342	3.2935	3.2937	3.2940	3.2945	3.2951	3.2957	3.52	4.08	4.64

Table 5: Total Relevant Costs for E(v) = 5

b \ a	5.50	6.00	6.01	6.02	6.03	6.04	6.05	6.06	6.07	6.08	6.09	6.10	7.00	8.00	9.00
	C_p = 1 C_d = 15														
5.00	5.50	4.2589	4.2576	4.2567	4.2561	4.2560	4.2562	4.2566	4.2574	4.2586	4.2600	4.2618	4.96	5.97	6.86
1.00	5.48	4.2372	4.2361	4.2355	4.2351	4.2350	4.2353	4.2359	4.2367	4.2380	4.2395	4.2414	4.95	5.96	6.86
0.01	5.48	4.2364	4.2353	4.2346	4.2343	4.2344	4.2346	4.2351	4.2360	4.2373	4.2389	4.2408	4.95	5.96	6.86

Table 6: Total Relevant Costs for E(v) = 5

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