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THE USE OF STREAM THRUST CONCEPTS FOR THE APPROXIMATE EVALUATION OF HYPERSONIC RAMJET ENGINE PERFORMANCE

Edward T. Curran, et al

Air Force Aero Propulsion Laboratory
Wright-Patterson Air Force Base, Ohio

July 1973

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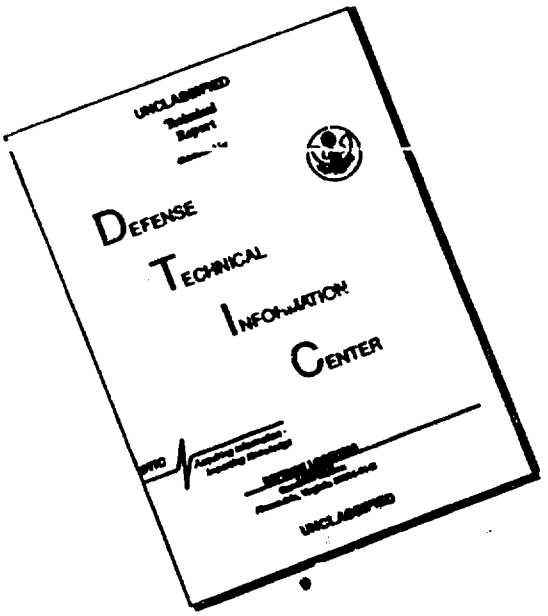

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FOREWORD

This effort was accomplished in the Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio. The report was prepared by the Ramjet Technology Branch (RJT) under Project 3012 and Task 301212. The work covered the time period of August through September 1963.

This report was submitted by the authors April 1973.

The technical report has been reviewed and is approved.

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ABSTRACT

A simplified method of estimating the performance of supersonic combustion ramjet engines is presented. The method utilizes stream thrust concepts and enables valid performance estimates to be made without the aid of a computer program; only a few simple graphs are required. A new "reference" stream thrust quantity is defined and shown to be of value in estimating engine flight performance. The data given in this report enable performance estimates to be made for hydrogen fueled engines operating stoichiometrically for speeds in excess of about Mach 8.

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FIGURE

24. Comparison of Exact and Simplified Calculations

25. Variation of T_3/m_3 with Combustor Enthalpy

26. Combustor Enthalpy - Inlet Velocity Ratio

27. Factor for Computing Combustor Enthalpy vs Inlet Velocity Ratio

SYMBOLS

PAGE		
37	A	Flow Area
45	A_r	Area Ratio
47	A_R	Reference Area
	c	Speed of Sound
48	C_{D_i}	Inlet-Drag Coefficient
	C^*	Characteristic Velocity
	C_p	Specific Heat At Constant Pressure
	F	Internal Thrust
	G	Universal Gas Constant
	g	Gravitional Constant
	h	Static Enthalpy
	h_o'	See Figure 19
	H_t	Total Enthalpy
	H_t°	Assigned Enthalpy of Fuel
	I_{sp}	Fuel Specific Impulse
	J	Mechanical Equivalent of Heat
	k	Ratio of Exit Area to Inlet Area (A_E/A_o)
	K_D	Process Efficiency
	K	Defined in Appendix I
	$\dot{m}$	Mass Flow
	$\dot{m}_f$	Fuel Mass Flow
	$\dot{m}_o$	Air Mass Flow
	M	Mach Number
	$\mathcal{M}$	Molecular Weight

SYMBOLS (CONTD)

$\bar{F}$	Stream Thrust
P	Static Pressure
P_R	Reference Pressure
p_r	Relative Pressure Function (See Reference 3)
q	Fuel/Air Ratio
R	Specific Gas Constant
s	Entropy
Sa	Air Specific Stream Thrust
Sa_*	Critical Air Specific Stream Thrust
Sa_R	Reference Air Specific Stream Thrust
Sa_{max}	Maximum Air Specific Stream Thrust
$[Sa]_{H_2}$	Air Specific Stream Thrust Contribution by Fuel Injection
T	Static Temperature
T_f	Fuel Static Temperature
T_R	Reference Temperature
V	Velocity
V_f	Fuel Injection Velocity
V_R	Velocity at Reference Station
γ	Specific Heat Ratio
η_{KE}	Kinetic Energy Efficiency of Inlet
η_c	Combustion Efficiency (Enthalpy Rise Basis)
η_s	Nozzle Stream Thrust Efficiency
η_f	Nozzle Gauge Thrust Efficiency
ρ	Density

Subscripts

Planes 0, 2, (*), 3, E, R are shown in either Figure 1 or 3.

SYMBOLS (CONTD)

Superscript

$^{\circ}$	Stagnation Conditions

SECTION I

INTRODUCTION

This report is concerned with a simple approximate analysis of the performance of scramjet engines operating at high flight speeds. The method of analysis was conceived in 1961 (Reference 1) when suitable computer programs for calculating performance were not available, but it was not published[1] at this time and it was overtaken by the development of the program described in Reference 2. Subsequently, the simplified method became of interest when it was desired to study the effect of perturbations of certain component efficiencies on engine performance and yet avoid a major production run on the computer. This method of estimating performance is based on the study of the changes in stream thrust of the engine gas flow as it passes through the engine. Consequently, current methods of defining component performance are easily introduced into the analysis. The heart of the method lies in the fact that at very high speeds the area ratio of the exhaust nozzle is large and the state condition of the exit flow is normally located in the non-dissociation region of the Mollier diagram. This fact enables a relatively simple approach to the estimation of the stream thrust of the engine gas flow at the nozzle exit to be made. Using this simple approach, engine performance at speeds of about Mach 15 have been estimated to an accuracy of one to two percent in comparison with more sophisticated computerized calculations.

This report is submitted in the hope that it will stimulate the application of stream thrust concepts to the analysis of engine and component performance.

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[1]This report is a more general version of Reference 1.

SECTION II

EXPLANATION OF METHOD

The object of this method is to evaluate the specific thrust and fuel specific impulse of a supersonic combustion ramjet engine operating at a given flight condition. In evaluating these performance parameters, extensive use is made of the concept of air specific stream thrust.

Referring to Figure 1 the internal thrust of a ramjet engine, expressed in terms of the stream thrusts of the inlet and exit gas streams, is given by the conventional expression:

$$F = \left\{\text{Stream Thrust}\right\}_E - \left\{\text{Stream Thrust}\right\}_0 - P_0 (A_E - A_0)$$

$$= \left\{\frac{\dot{m}V}{g} + PA\right\}_E - \left\{\frac{\dot{m}V}{g} + PA\right\}_0 - P_0 (A_E - A_0)$$

The specific thrust of the engine may be written as

$$\frac{F}{\dot{m}_0} = Sa_E - Sa_0 - \frac{P_0 A_0}{\dot{m}_0} \left\{\frac{A_E}{A_0} - 1\right\}$$

where generally

$$Sa = \frac{1}{\dot{m}_0} \left\{\frac{\dot{m}V}{g} + PA\right\}$$

The foregoing expression for the specific thrust of an engine may also be written as

$$\frac{F}{\dot{m}_0} = \left\{Sa_E - \frac{V_0}{g}\right\} - \frac{A_E}{A_0} \left\{Sa_0 - \frac{V_0}{g}\right\} \qquad (1)$$

If attention is now restricted to engines in which the ratio $\frac{A_E}{A_0}$ is held constant then of course one has

$$\frac{F}{\dot{m}_0} = \left\{Sa_E - \frac{V_0}{g}\right\} - k \left\{Sa_0 - \frac{V_0}{g}\right\}$$

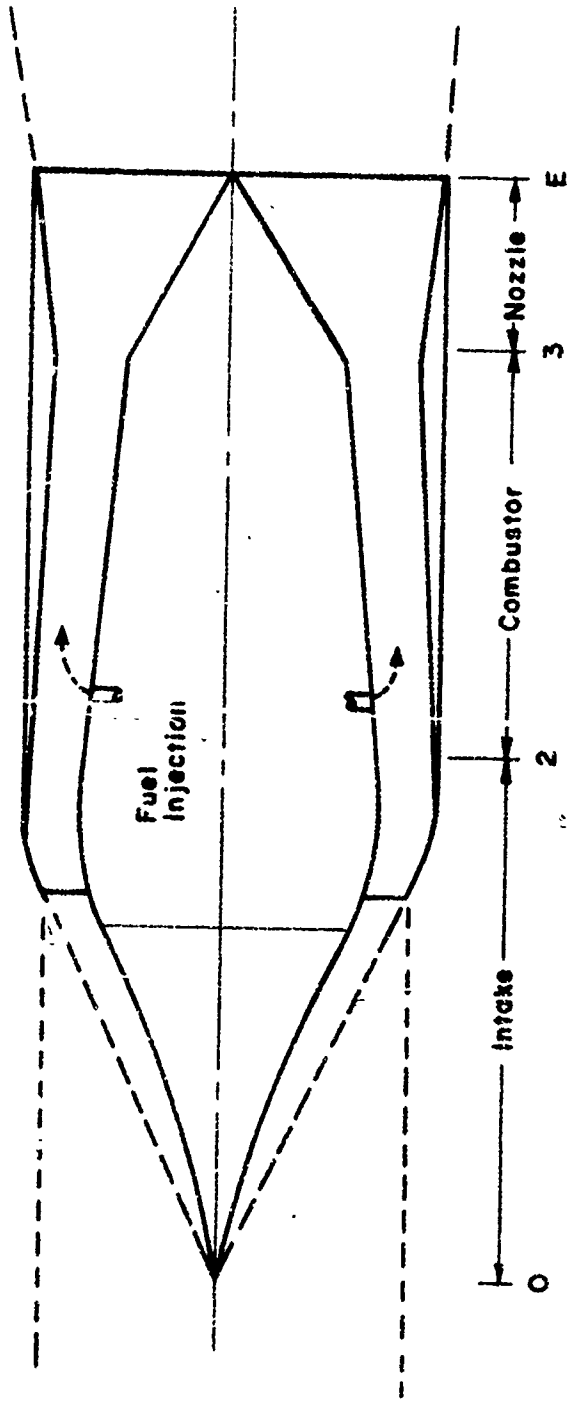

Figure 1. Reference Planes for Performance Analysis

The corresponding fuel specific impulse of the engine is defined by

$$I_{sp} = \frac{F}{\dot{m}f} = \frac{\dot{m}_o}{\dot{m}f} \cdot \left\{\frac{F}{\dot{m}_o}\right\} \qquad (2)$$

On inspection of Equations (1) and (2), it is apparent that for a given engine, operating at a specified flight condition and fuel/air ratio, the only unknown quantity is Sa_E. Once this quantity is determined, the engine performance can be easily evaluated. The unique feature of this simplified method of estimating performance is that it permits a relatively accurate determination of Sa_E without resorting to complex and lengthy calculations.

Consider an engine operating at a given condition and assume for the moment that the flow through the exit nozzle is an equilibrium isentropic process. The actual expansion through the nozzle can be considered as a segment of a hypothetical isentropic expansion which starts from the stagnation condition following combustion, and proceeds until the gas temperature approaches absolute zero. At any intermediate state condition, the corresponding air specific stream thrust can be calculated from

$$Sa = (1 + q) \left\{\frac{V}{g} + \frac{RT}{V}\right\}$$

where q, T, and R are known and V can be obtained from the Equation

$$V^2 = 2gJ\,(H_t - h)$$

For any given expansion process it is thus possible to plot the variation in Sa as a function of a suitable reference area ratio A/A_R or a reference pressure ratio P/P_R. In gas dynamic work, it is, of course, conventional to use the sonic condition as a suitable reference state.

Two reference values of specific thrust have often been used in ramjet performance analysis, namely Sa_* corresponding to the sonic condition in the expansion and Sa_{max} which is the maximum specific stream thrust obtained by expansion to an absolute temperature of zero. These values are indicated on Figure 2 which illustrates a typical variation of Sa/Sa_* with A/A_*. In the performance analysis of subsonic combustion ramjet engines, this type of chart was of the utmost value. This was because the reference area A_* corresponded to the geometrical area of the choked throat of the exhaust nozzle. Thus for a given divergent area ratio A_E/A_*, it was possible to read off directly the value of Sa_E from the chart. However, in the case of the supersonic combustion engine, this type of chart cannot be directly utilized since the sonic reference area does not correspond to any specific geometrical area, except in the limiting case where sufficient heat is added to reduce the exit velocity, say from a constant area combustor, to the sonic value. It is nevertheless informative to consider the application of this chart to the scramjet engine. Referring to Figure 3, in which a scramjet with a constant area combustor is considered, it is apparent that the specific stream thrust at plane 3 is determined by the flow conditions at the diffuser discharge, the stream thrust contributed by the fuel injection system, and the frictional losses in the combustor. If, for the time being, the latter two effects are neglected, we have

$$Sa_3 = Sa_2 = \frac{V_2}{g} + \frac{R_2 T_2}{V_2}$$

and thus for this particular example the specific stream thrust after

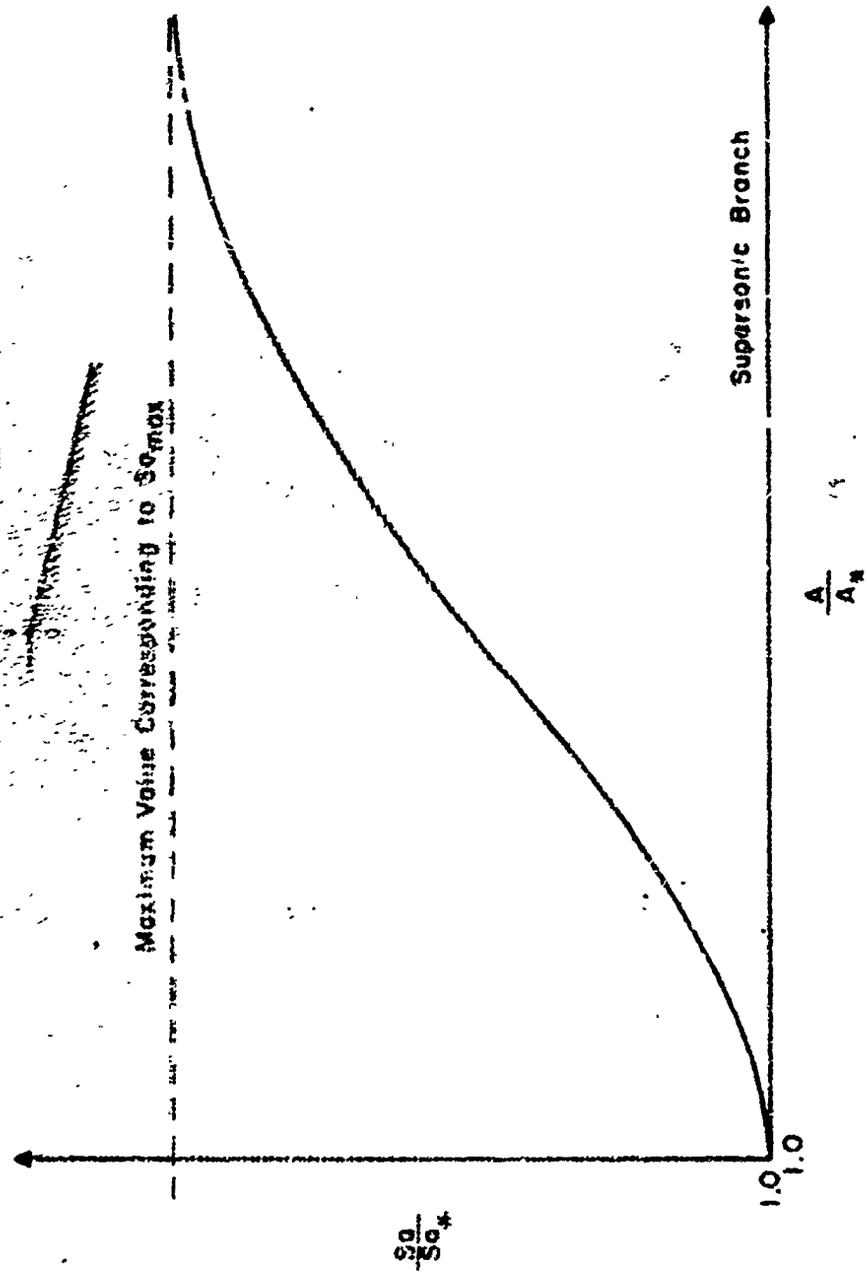

Figure 2. Variation of Impulse Ratio with Area Ratio

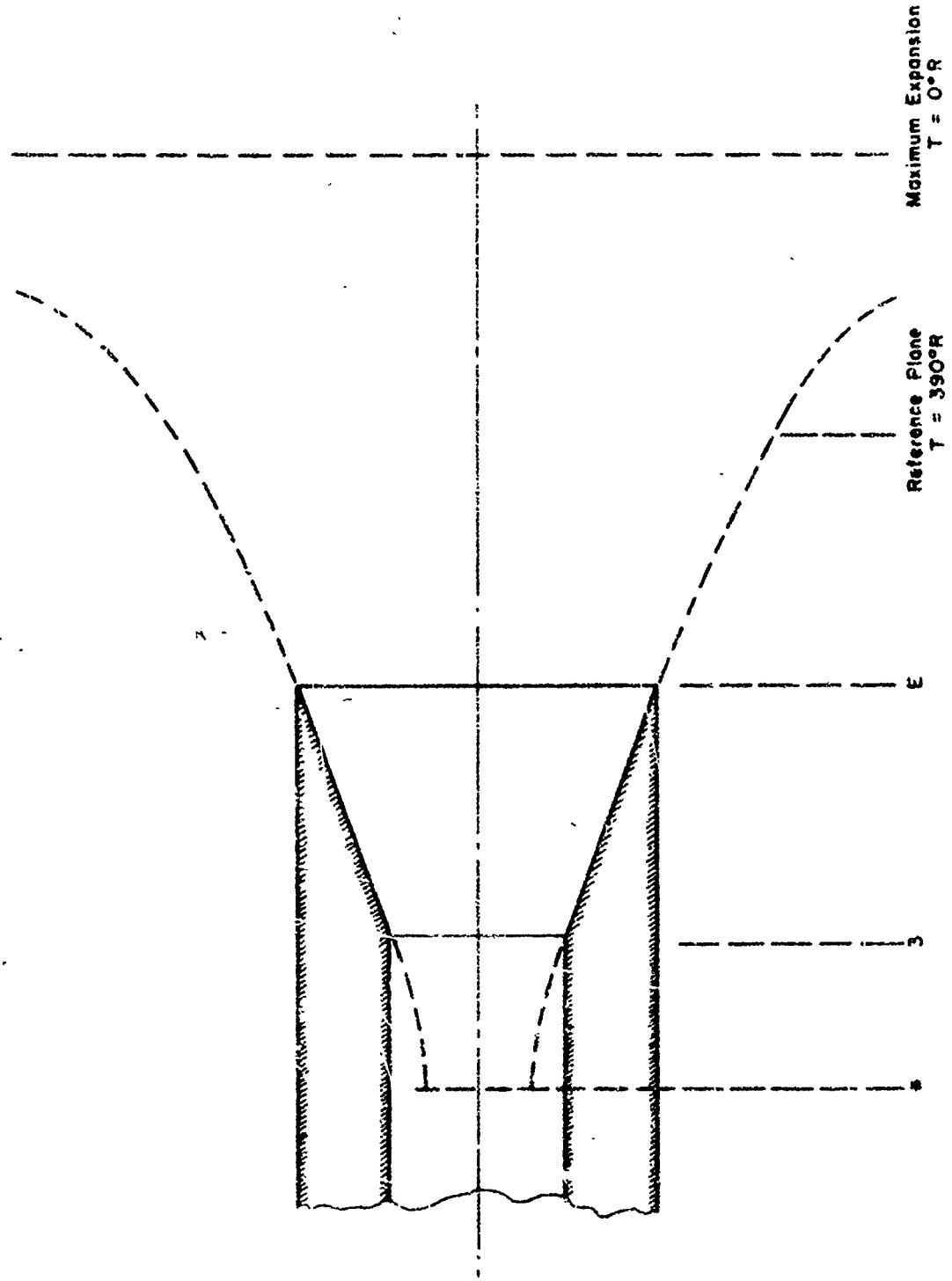

Figure 3. Hypothetical Expansion Process

combustion is uniquely determined once the operating condition of the intake is known. Thus for a specific flight condition and adiabatic inlet flow, Sa_3 is dependent only on the amount of diffusion, V_2/V_0, performed by the intake and is independent of the efficiency of the intake (real gas and viscous effects being neglected). It must, however, be noted that, for a fixed value of A_E/A_0, the nozzle area ratio is given by $\frac{A_E}{A_3} = \frac{A_0}{A_2} \cdot \frac{A_E}{A_0}$, thus for a given amount of diffusion the effect of reducing inlet efficiency is to decrease the available area ratio of the nozzle.

At flight speeds below about Mach 7.5, it is possible to utilize charts of the type illustrated in Figure 2 to determine the performance of scramjet engines. In this case the chart is entered at the value of Sa_3/Sa_*, where Sa_3 corresponds to the specified inlet condition, and a value of A_3/A_* is obtained. The chart is then re-entered at an area ratio

$$\frac{A_E}{A_*} = \frac{A_E}{A_3} \cdot \frac{A_3}{A_*}$$

and the corresponding value of Sa_E obtained. By this procedure it is possible to utilize charts which are already available for subsonic combustion engines for calculating the performance of scramjets. However, such charts are not readily available for speeds in excess of about Mach 7.5 and considerable labor is required to produce the necessary charts to cover all probable conditions of engine operation. Furthermore, as higher flight speeds are considered, it becomes increasingly difficult to calculate A_* and P_* since the sonic state points are located in the high-enthalpy regions closely corresponding to the stagnation condition of the flow. Indeed a condition is soon reached when the

sonic state points correspond to enthalpies which exceed the range of available Mollier diagrams. Additionally this procedure for determining performance is physically unattractive since the flow within the engine never approaches the sonic reference condition. In fact, the state points corresponding to the actual flow through the engine are usually located in the lower regions of the Mollier diagram. It is also relevant to note that at very high speeds the area ratio of the exhaust nozzle becomes very large and the state condition of an exit flow which results from an equilibrium expansion is normally located in the non-dissociation region of the Mollier diagram. This latter consideration permits the use of a simple approximate approach to the estimation of Sa_E.

The basis of this new method is to use a reference specific stream thrust Sa_R which in concept corresponds more closely to Sa_{max} rather than Sa_*; one then works back upstream to determine Sa_E rather than downstream from the value of Sa_* as previously discussed. Of course, Sa_{max} itself cannot be used as a reference condition since it corresponds to an infinite expansion area ratio.

In this report the reference specific stream thrust is arbitrarily defined as the value corresponding to an expansion to a static temperature of 390°R (216.5°K); this temperature corresponds, of course, to the standard tropopause value and is a rational "sink" temperature. The reference specific stream thrust is easily calculated; thus for an adiabatic equilibrium expansion from post-combustion conditions

$$H_{t_3} = h_3 + \frac{V_3^2}{2gJ} = h_R + \frac{V_R^2}{2gJ}$$

so that

$$V_R^2 = 2gJ \cdot (H_{t_3} - h_R)$$

where

$$H_{t_3} = \eta_c \left\{\frac{q}{1+q}\right\} H_f^\circ + \frac{1}{1+q} \cdot \left\{h_0 + \frac{V_0^2}{2gJ}\right\}$$

and thus

$$Sa = (1+q)\left\{\frac{V_R}{g} + \frac{RT_R}{V_R}\right\}$$

The value of Sa_{max} may be calculated in a similar fashion. Computed values of Sa_R, expressed for accuracy as $Sa_R - V_0/g$, for stoichiometric hydrogen/air combustion, are given in Figure 4 for various combustion efficiencies. Additionally, values of Sa_{max} and Sa_R are compared in Figure 5 for stoichiometric products and $\eta_c = 1.00$.

For any given stagnation condition it is a simple matter of calculation to obtain the variation of Sa with area for a simple adiabatic equilibrium expansion. Thus at any state point in the expansion, P, T, and $R = \frac{G}{\dot{m}}$ are known, and the corresponding flow velocity can be calculated from the enthalpy change. Thus

$$Sa = (1+q)\left\{\frac{V}{g} + \frac{RT}{V}\right\}$$

and the flow area per unit flow can be calculated from

$$\frac{A}{(1+q)\,\dot{m}_0} = \frac{RT}{VP}$$

It is thus possible to produce curves of Sa/Sa_R versus A/A_R for any initial stagnation condition and these curves are similar to those of Sa/Sa_* versus A/A_R illustrated in Figure 2. For a given equivalence ratio the Sa/Sa_R - A/A_R curves are only dependent on the total enthalpy of the inlet stream and the initial pressure; however, as the expansion proceeds away from stagnation conditions the dependence of these curves

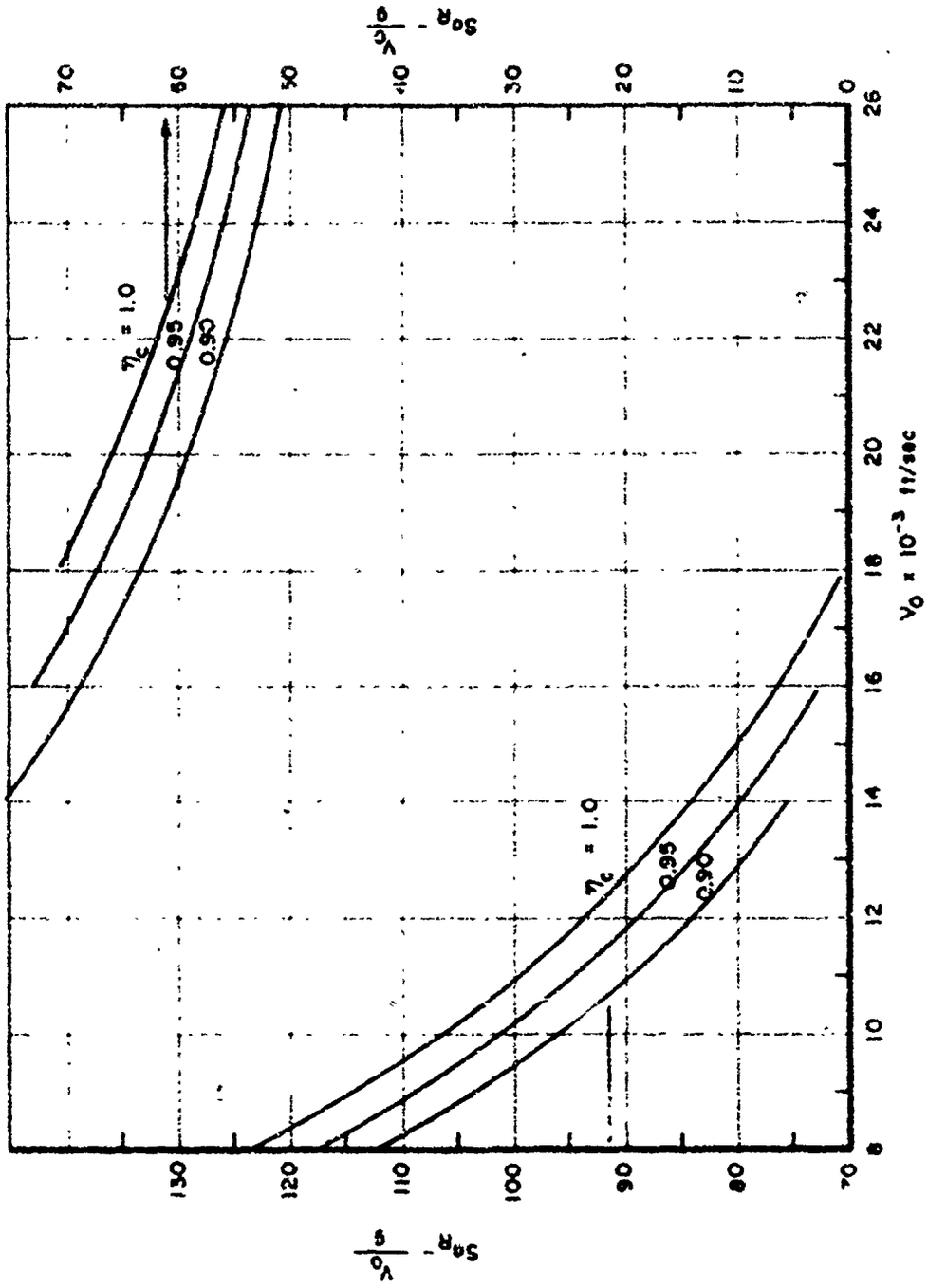

Figure 4. Variation of $(S\eta_R - V_0/g)$ with V_0

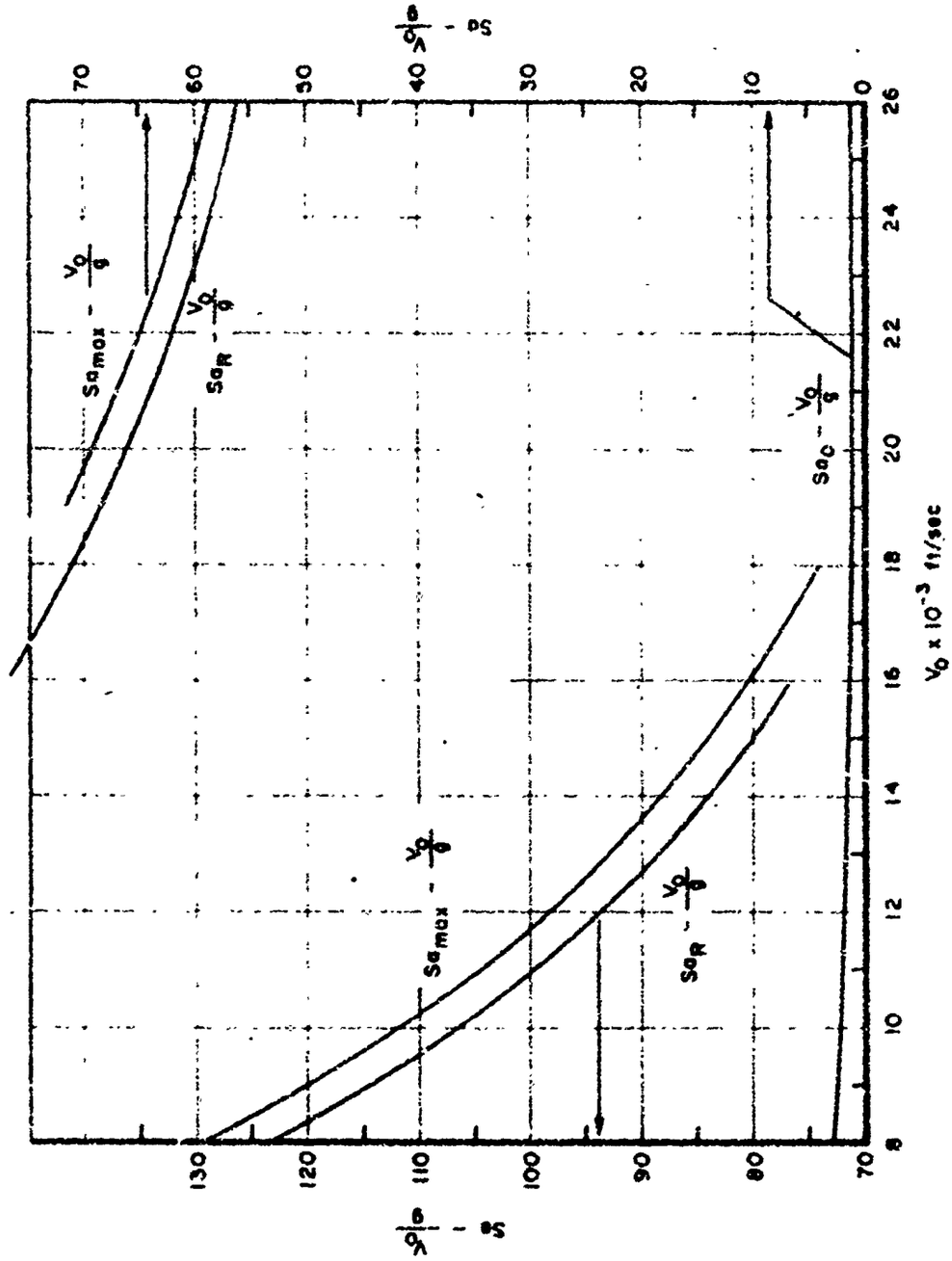

Figure 5. Comparison of Sa_{max} and Sa_R

on the absolute pressure level disappears. Graphs showing the actual variation of Sa/Sa_R with A/A_R are shown in Figures 6 to 18; these graphs are for the stoichiometric combustion of hydrogen air mixtures at initial conditions of 1 atm. pressure. Superimposed on these curves are values of the corresponding expansion pressure ratios.

The above curves can be utilized to calculate the specific stream thrust at the exit of a scramjet engine nozzle. For example, consider an engine with a simple constant area combustor operating at an equivalence ratio of 1.0.

From Figure 4, Sa_R may be obtained from the value of $Sa_R - \frac{V_0}{g}$ corresponding to the flight condition. The quantity Sa_3 will be known for a given intake operating point (i.e., V_2/V_0 given) and correspondingly A_E/A_3 will be known for a given engine geometry, thus $A_E/A_3 = A_E/A_0 \cdot A_0/A_3$.

The exit stream thrust may now be found; thus entering the charts in Figures 6 etc., at a value of $Sa/Sa_R = Sa_3/Sa_R$ corresponding to the flight condition one obtains A_3/A_R; this latter value may be used to calculate $A_E/A_R = A_E/A_3 \cdot A_3/A_R$. The chart is now re-entered at $A/A_R = A_E/A_R$ and Sa_E/Sa_R can be read off, and Sa_E found.

The above process is extremely simple when constant area combustors are considered and it is relatively easy to study the effects of varying such parameters as inlet efficiency, fuel injection conditions, and combustor friction losses on engine performance. Non-equilibrium and other nozzle losses can also be introduced into the analysis provided such losses are expressed in terms of stream thrust changes. The method is very accurate for engines operating at high flight speeds ($M > 12$) with correspondingly high expansion ratios in the nozzle. However, before discussing the validity and limitations of this method of approach, it is considered desirable to illustrate the method further by discussing sample performance calculations.

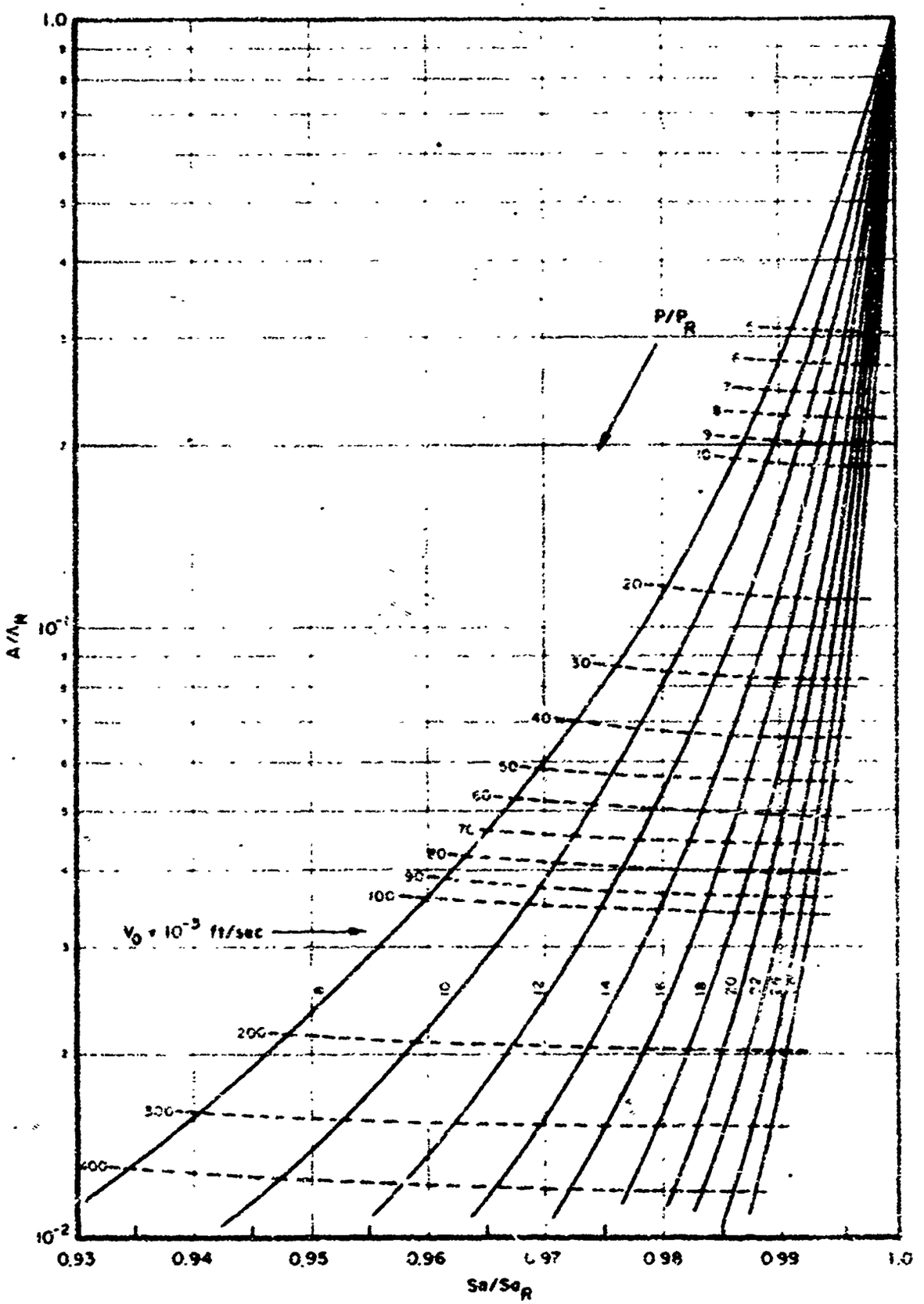

Figure 6. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

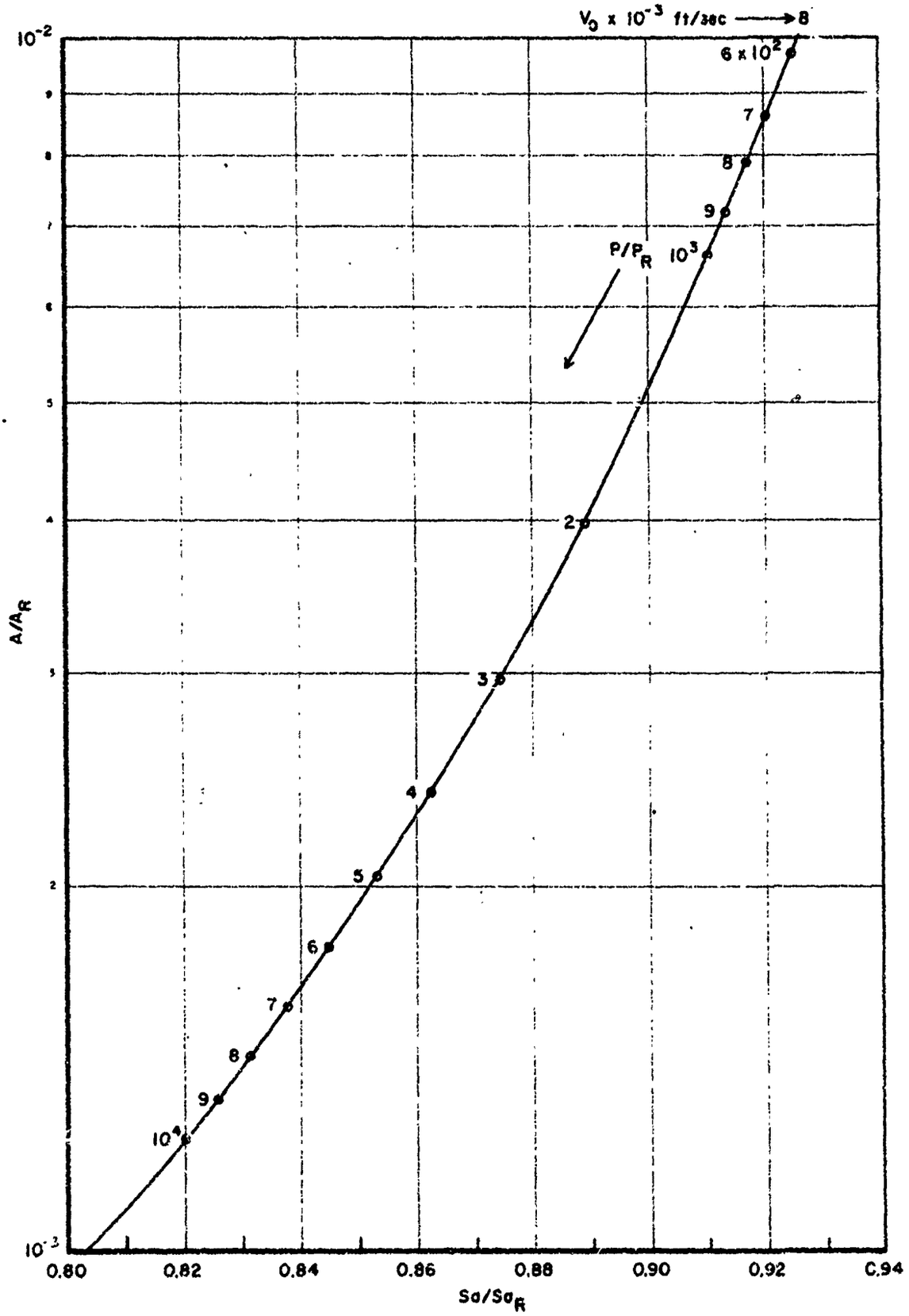

Figure 7. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

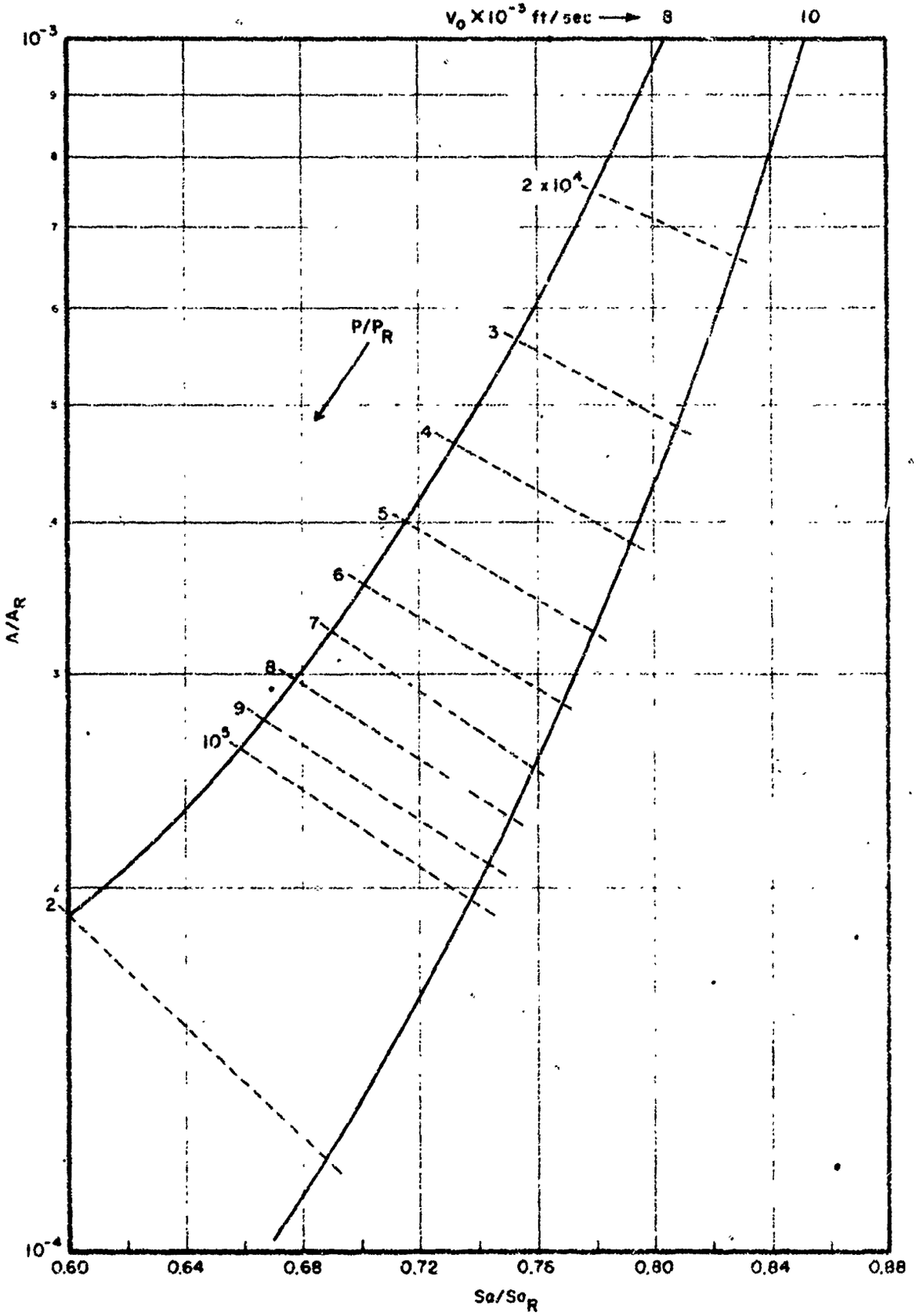

Figure 8. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

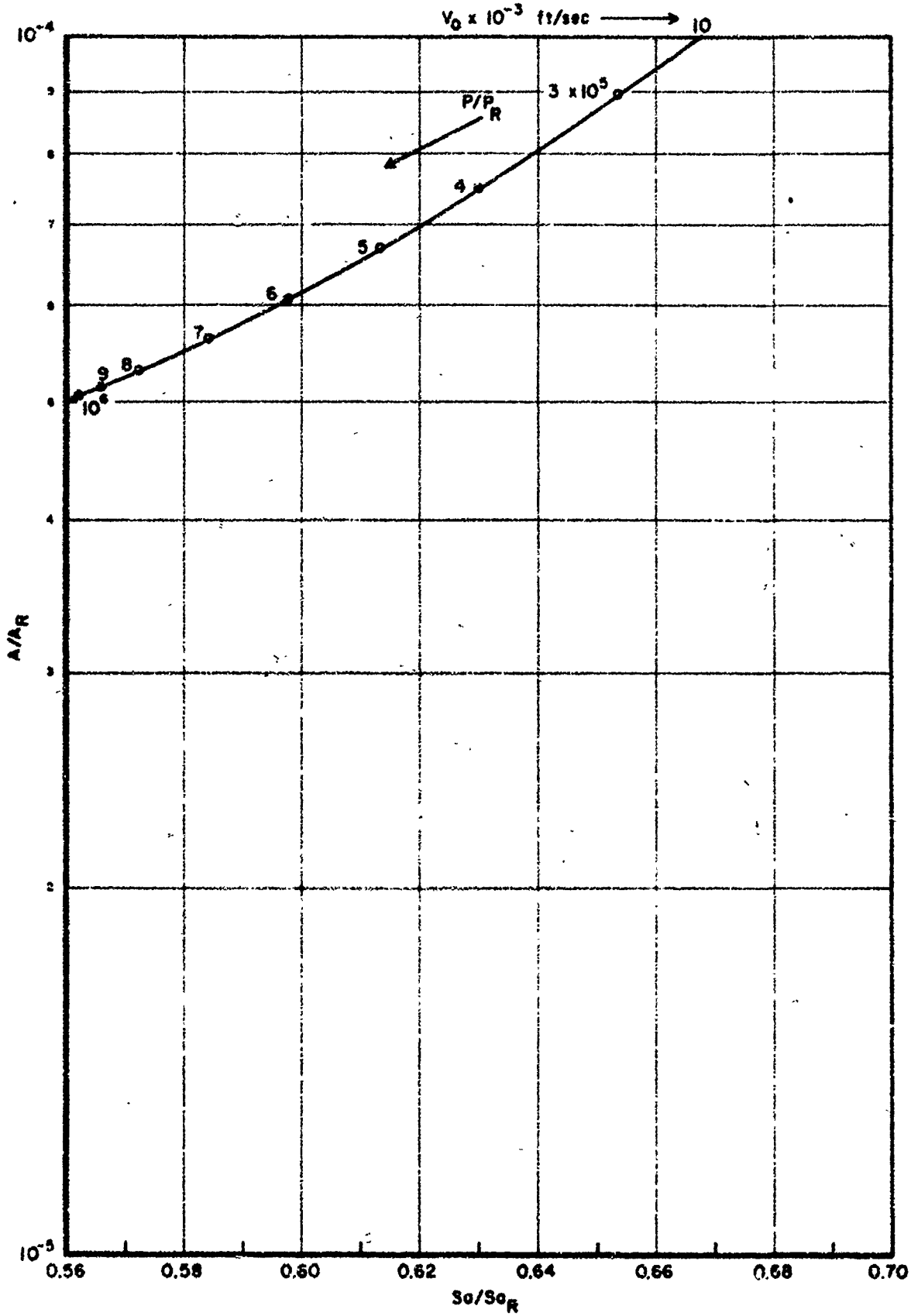

Figure 9. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

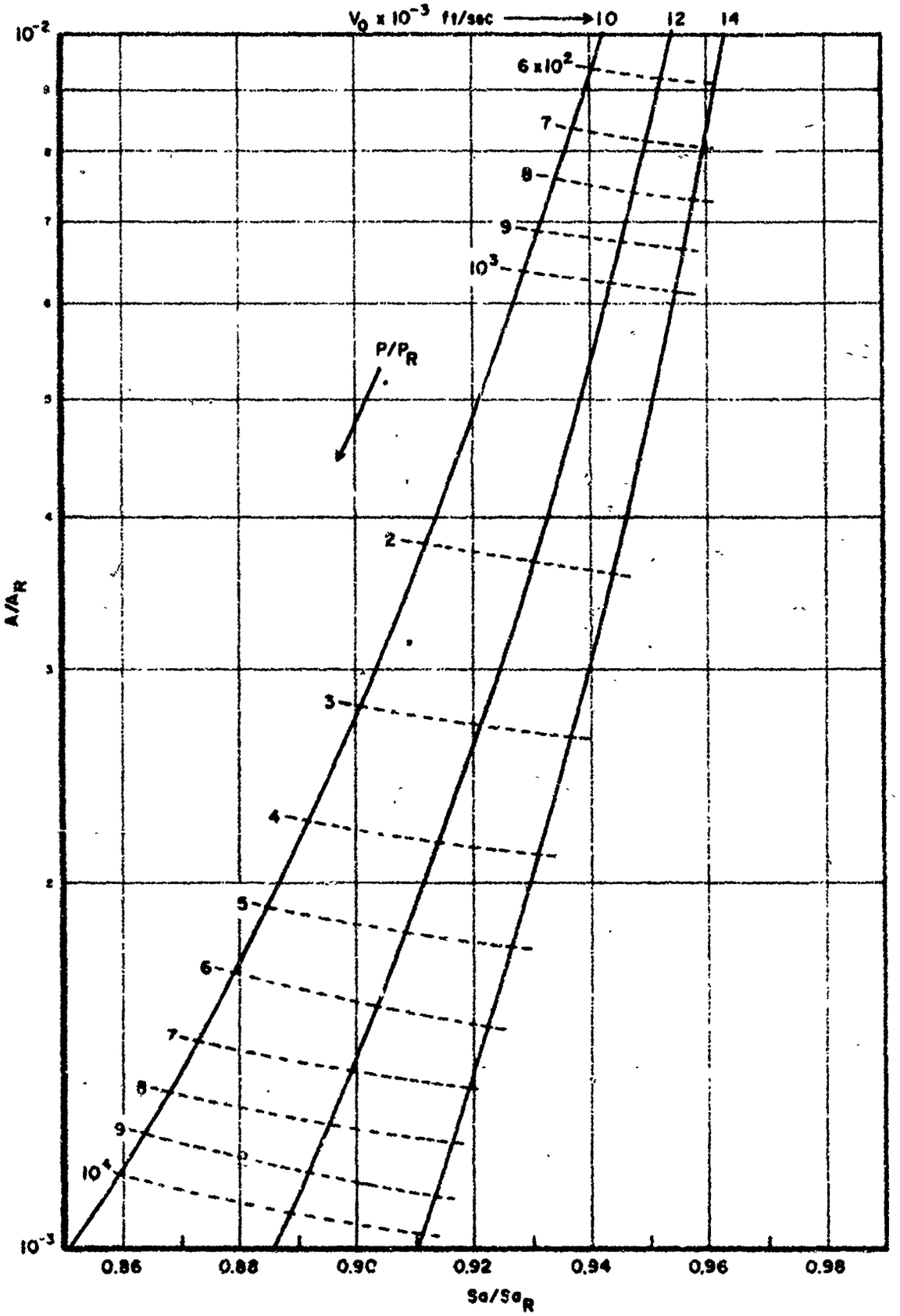

Figure 10. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

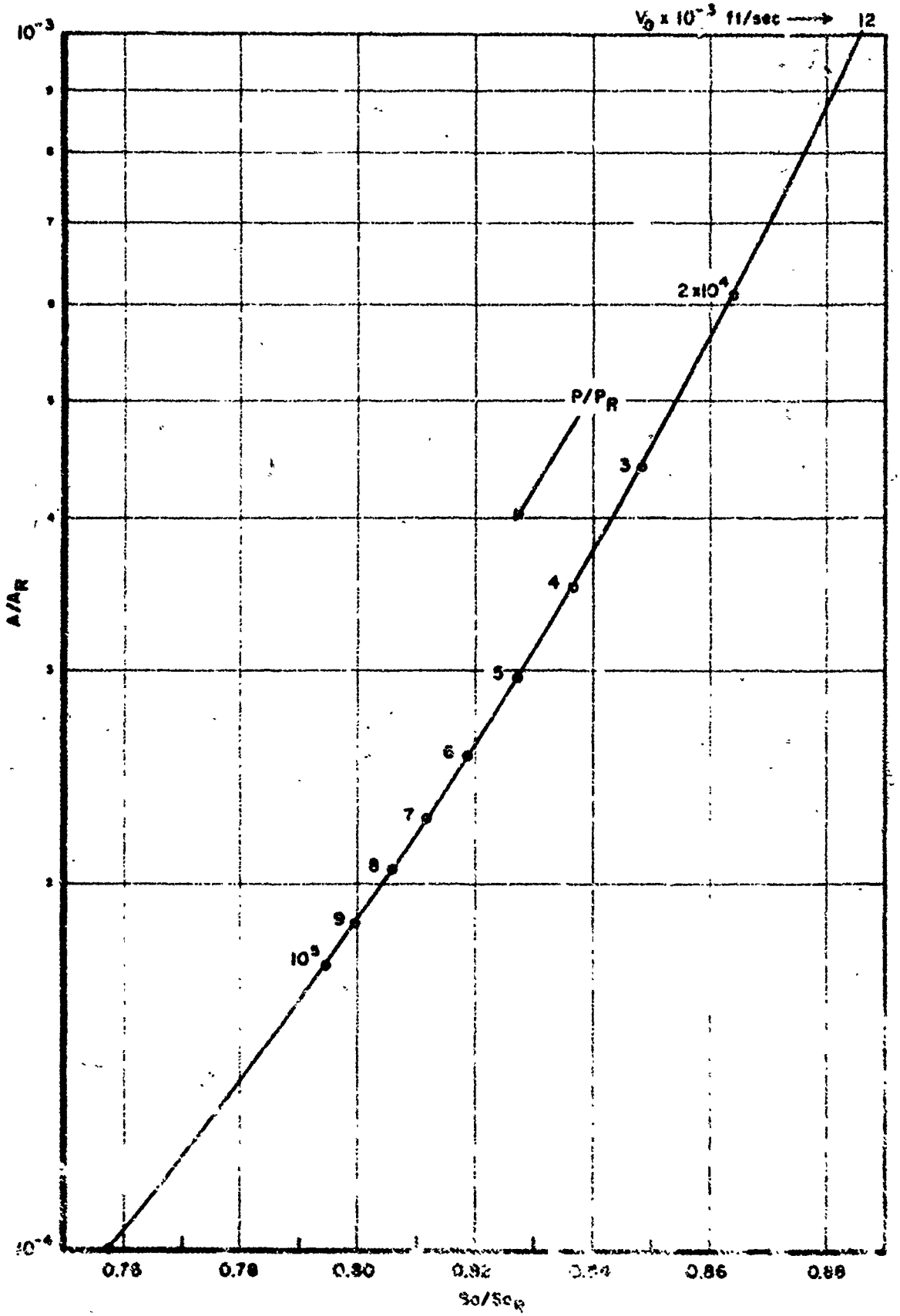

Figure 11. Variation of A/A_R with S_0/S_{0_R} for Sherson flow

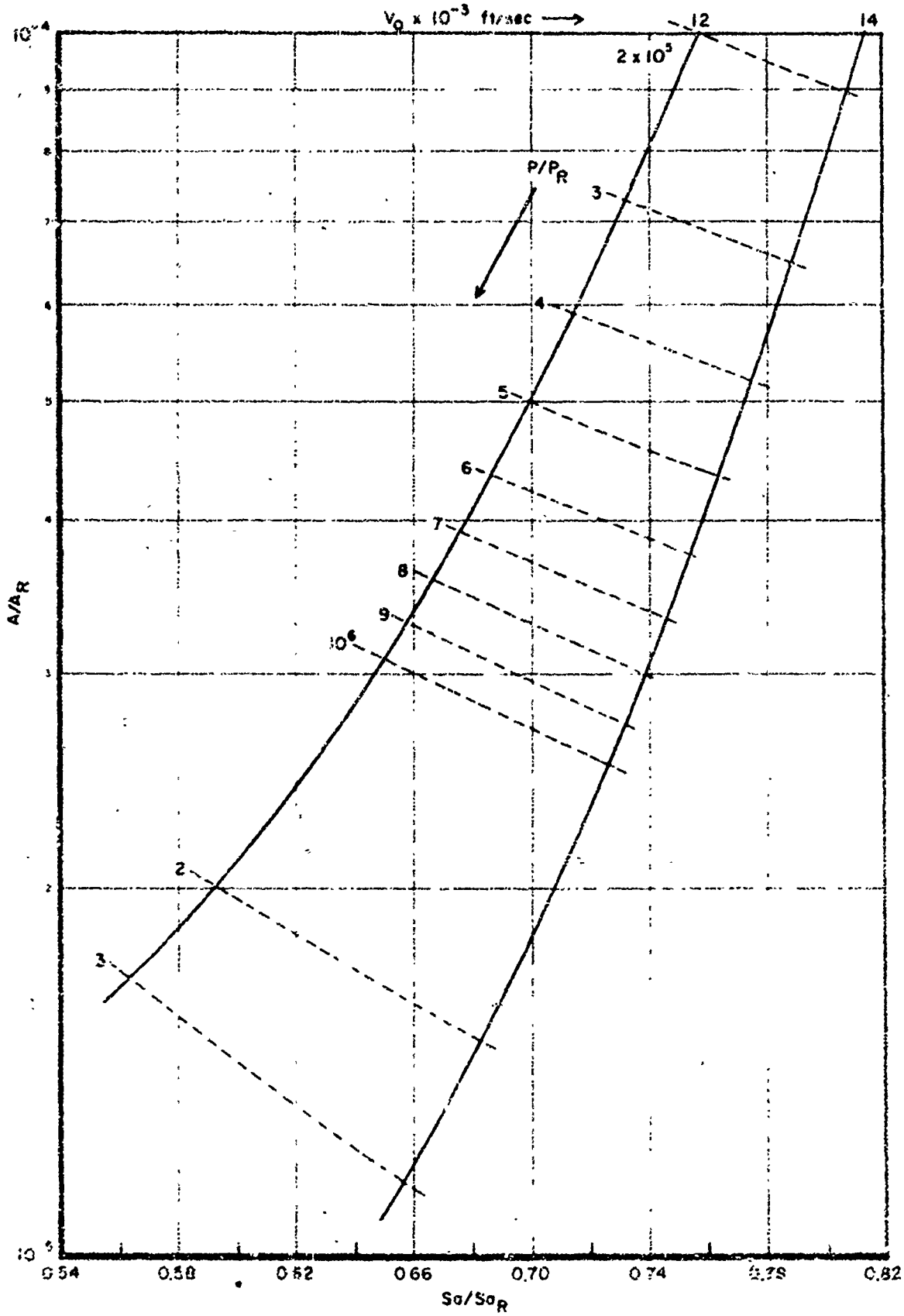

Figure 12. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

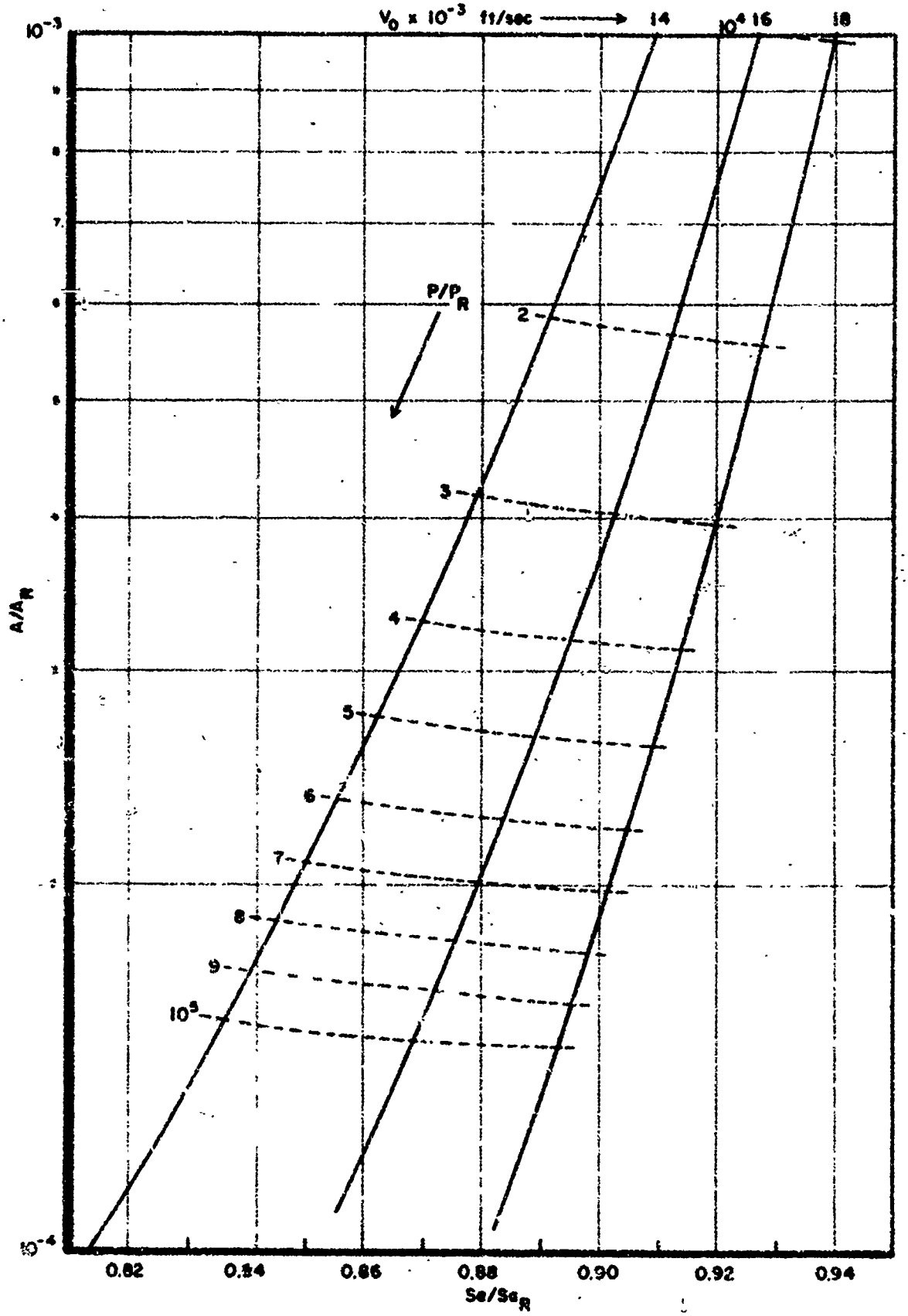

Figure 3. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

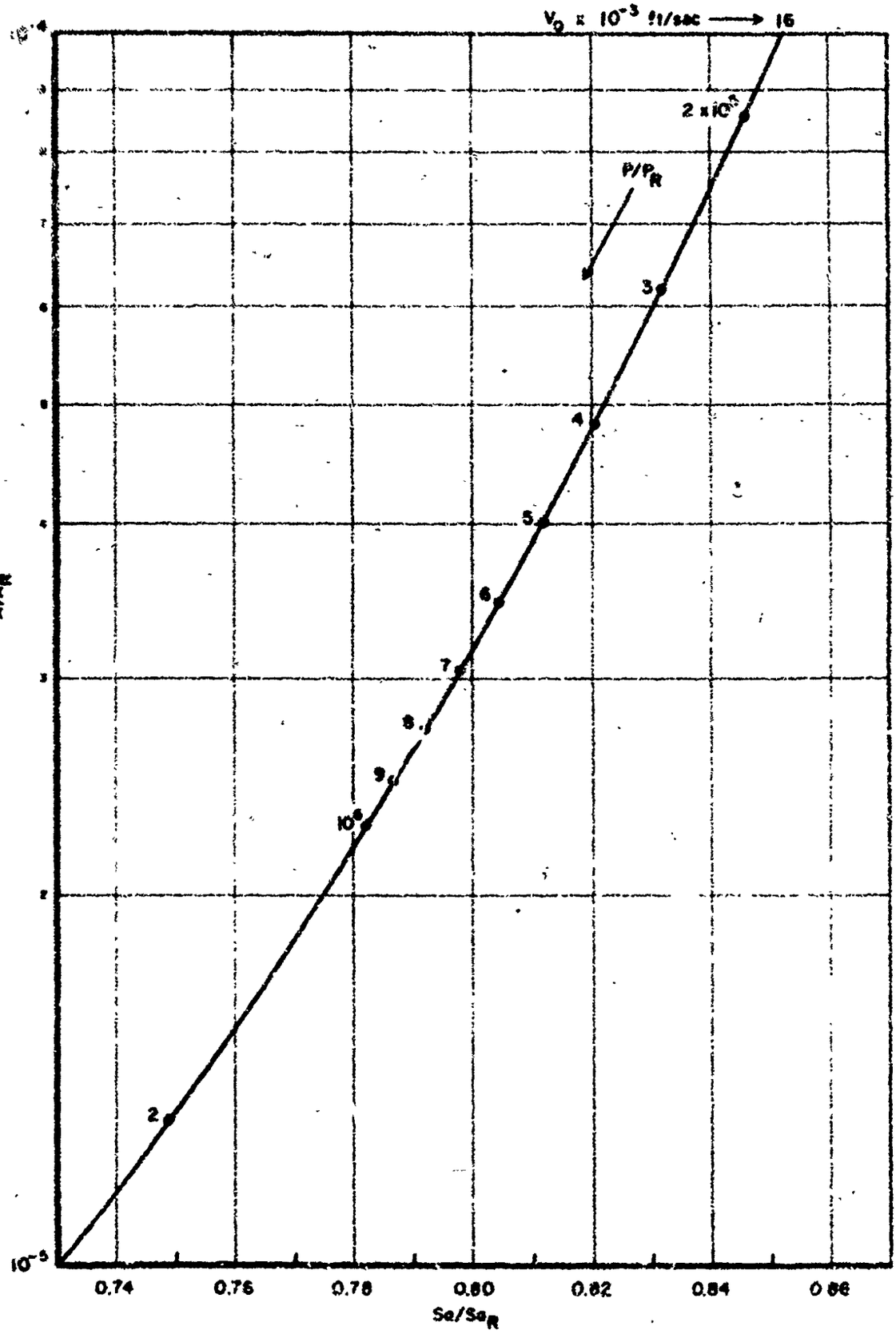

Figure 14. Variation of A/A_R with Se/Se_R for Supersonic Flow

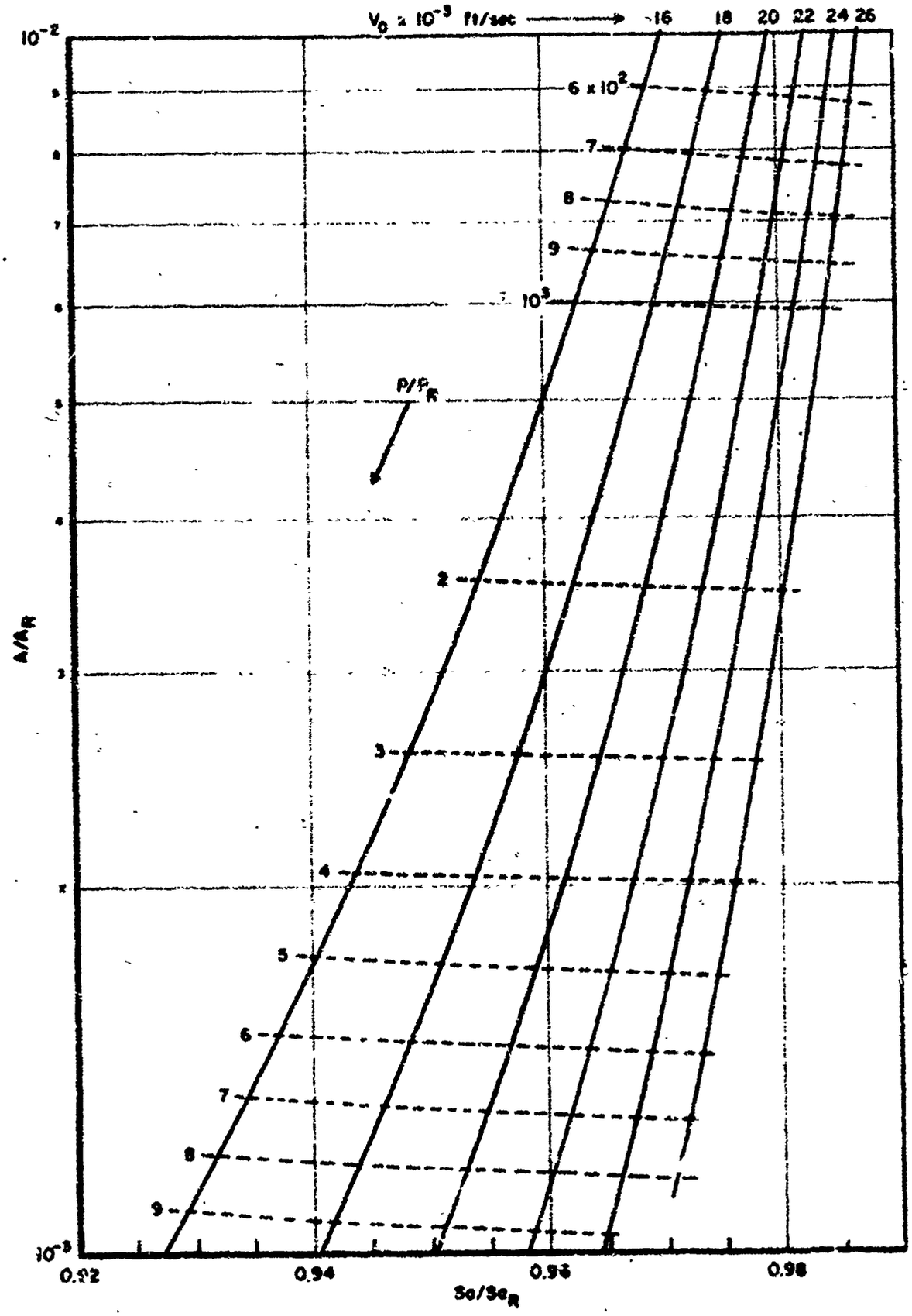

Figure 15. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

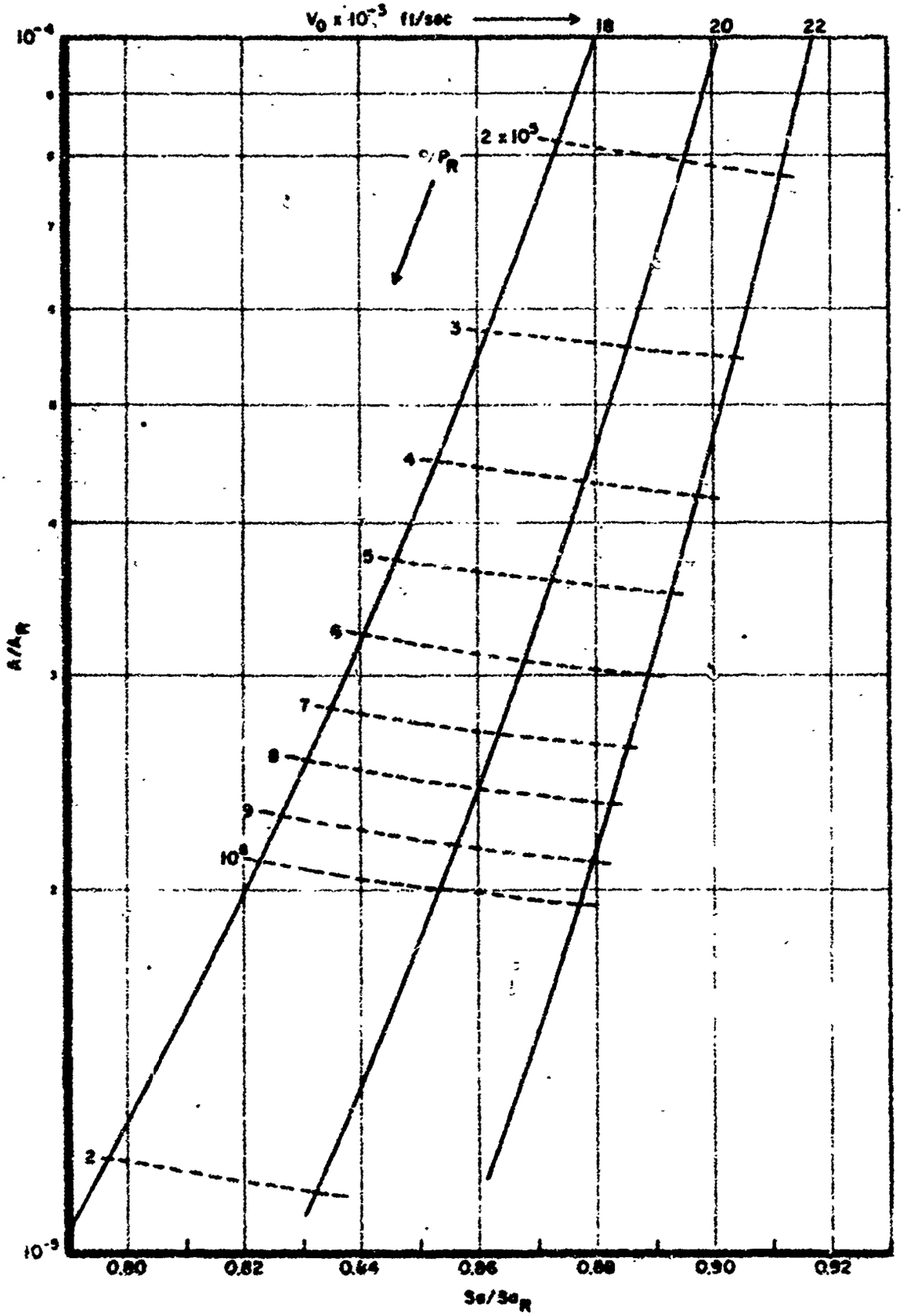

Figure 16. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

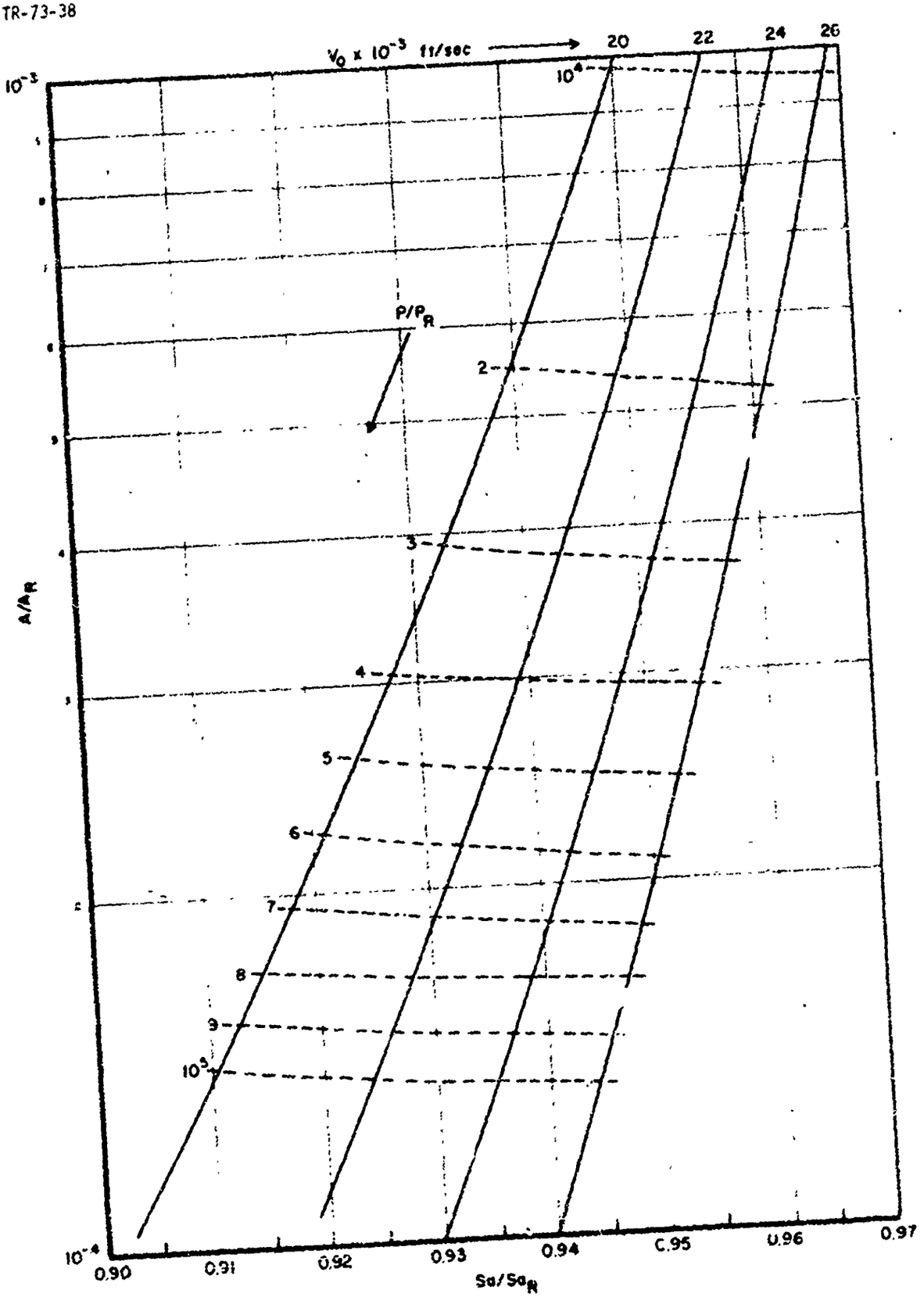

Figure 17. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

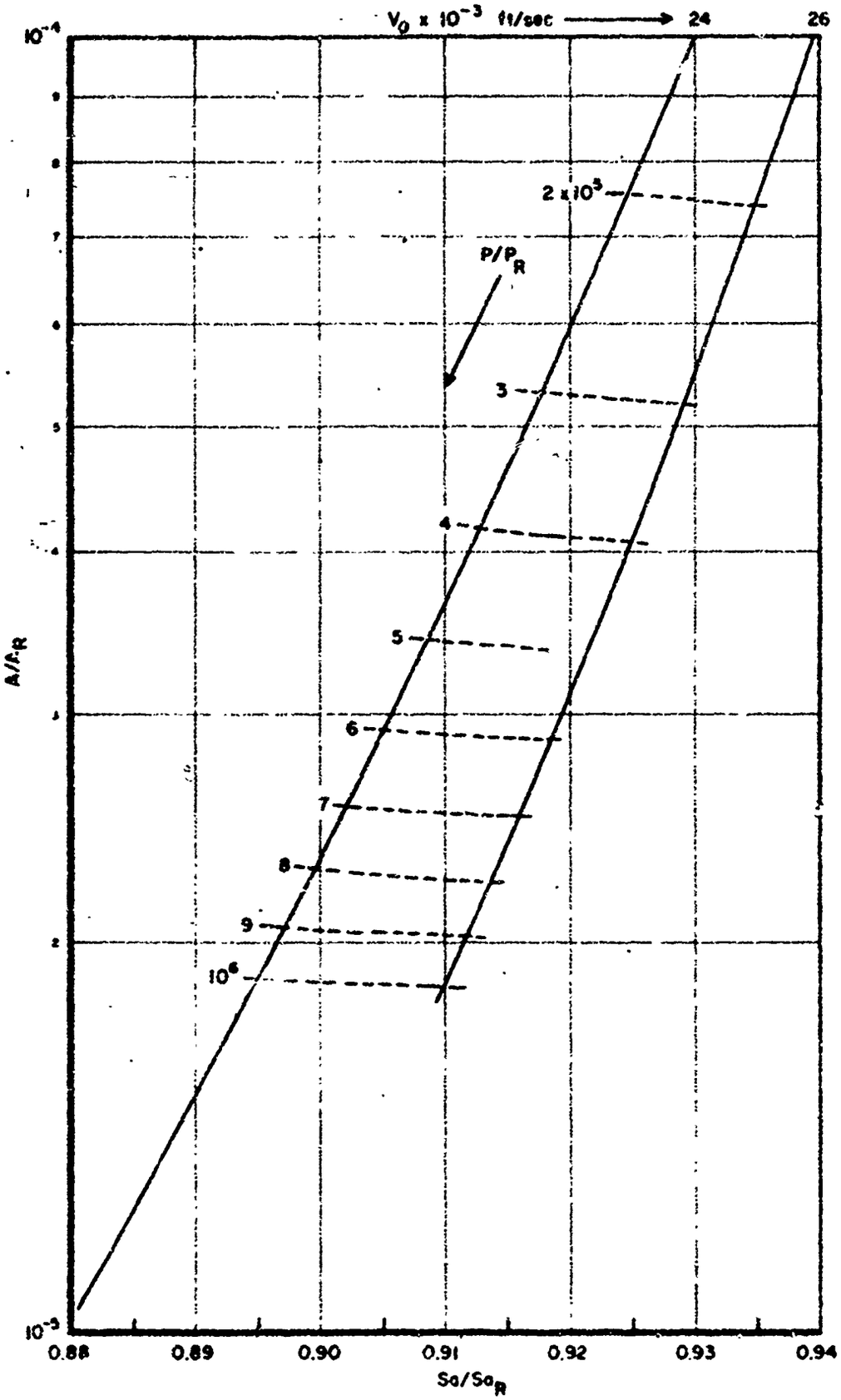

Figure 18. Variation of A/A_R with Sa/Sa_R for Supersonic Flow

SECTION III

PERFORMANCE CALCULATIONS

The key item to be calculated in assessing performance is the value of Sa_E which in turn is dependent on Sa_3 and the available expansion area ratio A_E/A_3. To determine these latter two quantities it is necessary to calculate the intake area ratio and the variation of Sa through the intake and combustor.

1. Calculation of Intake Area Ratio and Exit Specific Stream Thrust

As noted in the Introduction, for a fixed value of A_E/A_0 the area ratio of the nozzle is dependent on the area ratio of the intake. The area ratio of the intake is easily calculated once the degree of diffusion and the associated efficiency of diffusion are known. For a simple adibatic compression process from velocity V_0 to velocity V_2

$$h_0 + \frac{V_0^2}{2gJ} = h_2 + \frac{V_2^2}{2gJ}$$

So that

$$h_2 = h_0 + \frac{V_0^2}{2gJ}\left\{1 - \left(\frac{V_2}{V_0}\right)^2\right\}$$

For assumed values of $\frac{V_2}{V_0}$, h_2 is easily calculated for given flight condition using the tables in Reference 3. The intake efficiency may be defined in various ways (See Reference 4) but if the kinetic energy definition is chosen then from Figure 19, one has

$$\eta_{KE}^{+} = \frac{H_t - h_0'}{H_t - h_0}$$

+Note that conventionally η_{KE} is often expressed as

$$\eta_{KE} = K_D + (1 - K_D)\left(\frac{V_2}{V_0}\right)^2$$

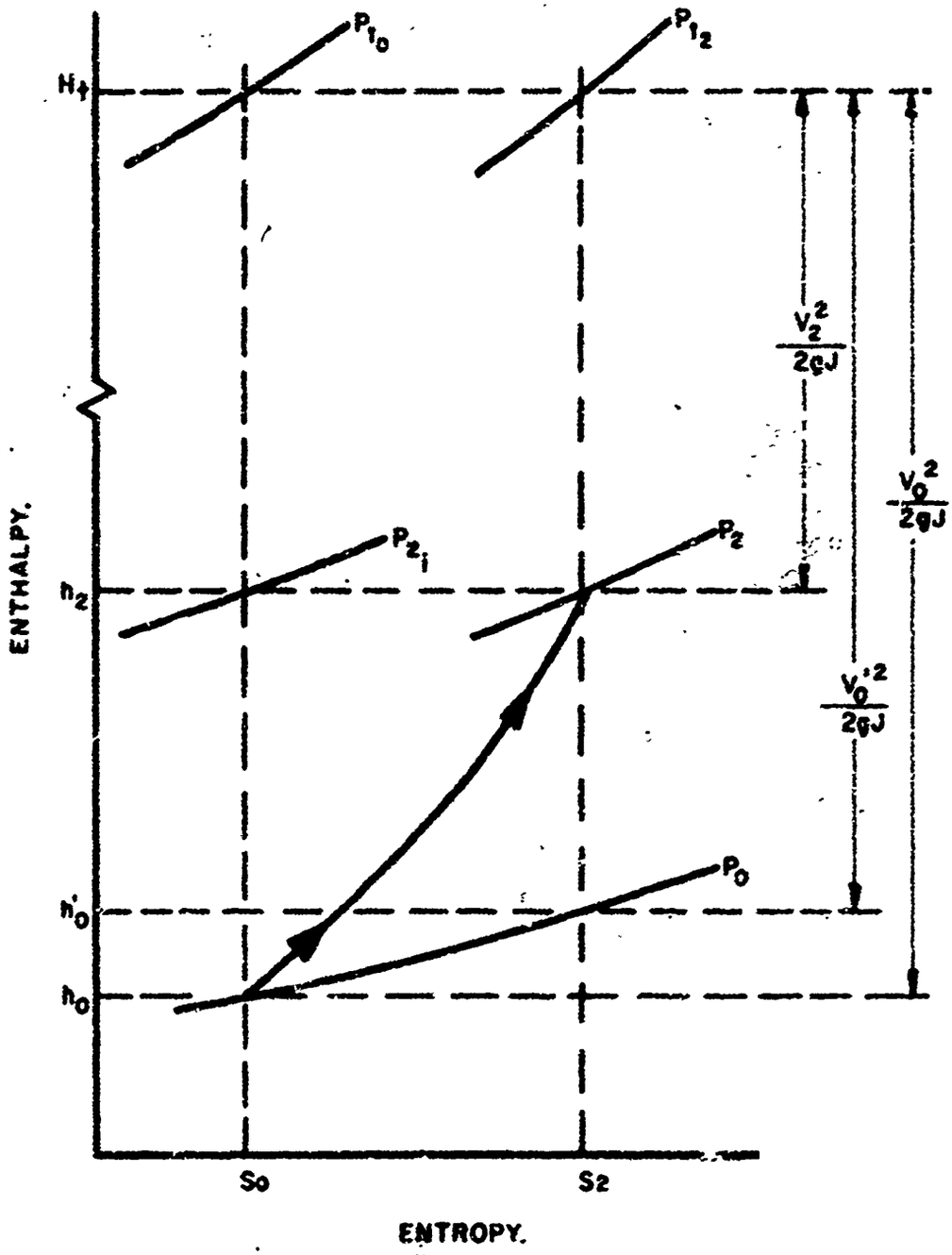

Figure 19. Definition of Intake Efficiency

Where

$$H_t = h_0 + \frac{V_0^2}{2gJ}$$

So that

$$h_0' = h_0 + (1 - \eta_{KE}) \frac{V_0^2}{2gJ}$$

The area ratio corresponding to the compression process is

$$\frac{A_0}{A_2} = \frac{P_2}{P_0} \cdot \frac{T_0}{T_2} \cdot \frac{V_2}{V_0}$$

where $\frac{P_2}{P_0}$ is the only unknown. Along the isentrope corresponding to S_2 (Figure 19) the ratio of the pressures $\frac{P_2 \quad (T_2)}{P_0 \quad (T_0)}$ can be found from the tables of Reference 3 using the relative pressure function. These tables will normally cover all practical conditions of intake operation.

The value of Sa_2 can be calculated from

$$Sa_2 = \left(\frac{V_2}{V_0}\right) \frac{V_0}{g} + \frac{R_2 T_2}{V_0} \left(\frac{V_0}{V_2}\right)$$

2. Variation of Stream Thrust Through Combustor

For the present, attention is restricted to the constant area combustor; thus any change in the stream thrust will arise due to fuel injection or frictional effects. Considering first fuel injection, possible geometries are sketched in Figure 20. If wall slot injectors are used then the area ratio available for expansion in the nozzle is reduced, but of course there is a contribution to the stream thrust at exit from the combustor. For fully axial injection of fuel at velocity V_f and temperature T_f, the corresponding fuel stream thrust is

$$F_f = \frac{\dot{m}_f V_f}{g} + P_f A_f$$

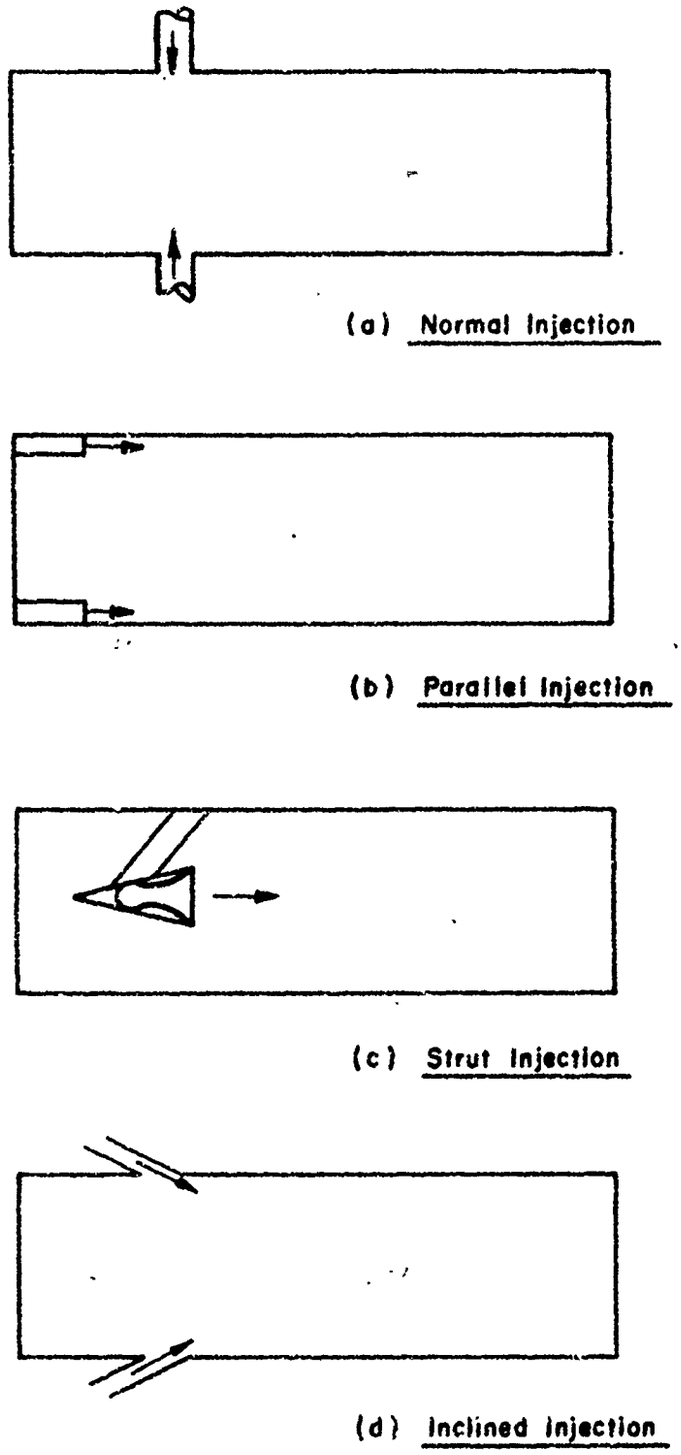

Figure 20. Fuel Injector Configurations

and the corresponding air specific stream thrust of the injected fuel is

$$\left[Sa\right]_{H_2} = q\left\{\frac{V_f}{g} + \frac{P_f A_f}{\dot{m}_f}\right\} = q\left\{\frac{V_f}{g} + \frac{R_f T_f}{V_f}\right\}$$

In Figure 21 the variation of V_f and T_f for various fuel total temperatures is shown and in Figure 22 the variation in $\left[Sa\right]_{H_2}$ with V_f is given. The area required for fuel injection can be calculated from the expression

$$\frac{A_f}{A_2} = q \cdot \frac{R_f}{R_2} \cdot \frac{T_f}{T_2} \cdot \frac{P_2}{P_f} \cdot \frac{V_2}{V_f}$$

If

$$P_f = P_2$$

$$\frac{A_f}{A_2} = q \; \frac{R_f}{R_2} \; \frac{T_f}{T_2} \; \frac{V_2}{V_f}$$

The specific stream thrust at Plane 3 is thus given by

$$Sa_3 = Sa_2 + \left[Sa\right]_{H_2}$$

For the strut injection configuration shown in Figure 20 there is no increase in the combustor area from Plane 2 to Plane 3. However, it is necessary to account for the drag force on the injector when calculating the exit stream thrust.

Although friction losses can be appreciable, no attempt will be made here to give a generalized method of calculating such losses since the computation of friction losses depends significantly on the configuration of the combustor-injector combination and on the precise nature of the boundary layer flow. However, friction force losses can, of course, be treated in a parametric study.

The extension of this analysis to include the constant pressure combustor is given in Appendix I.

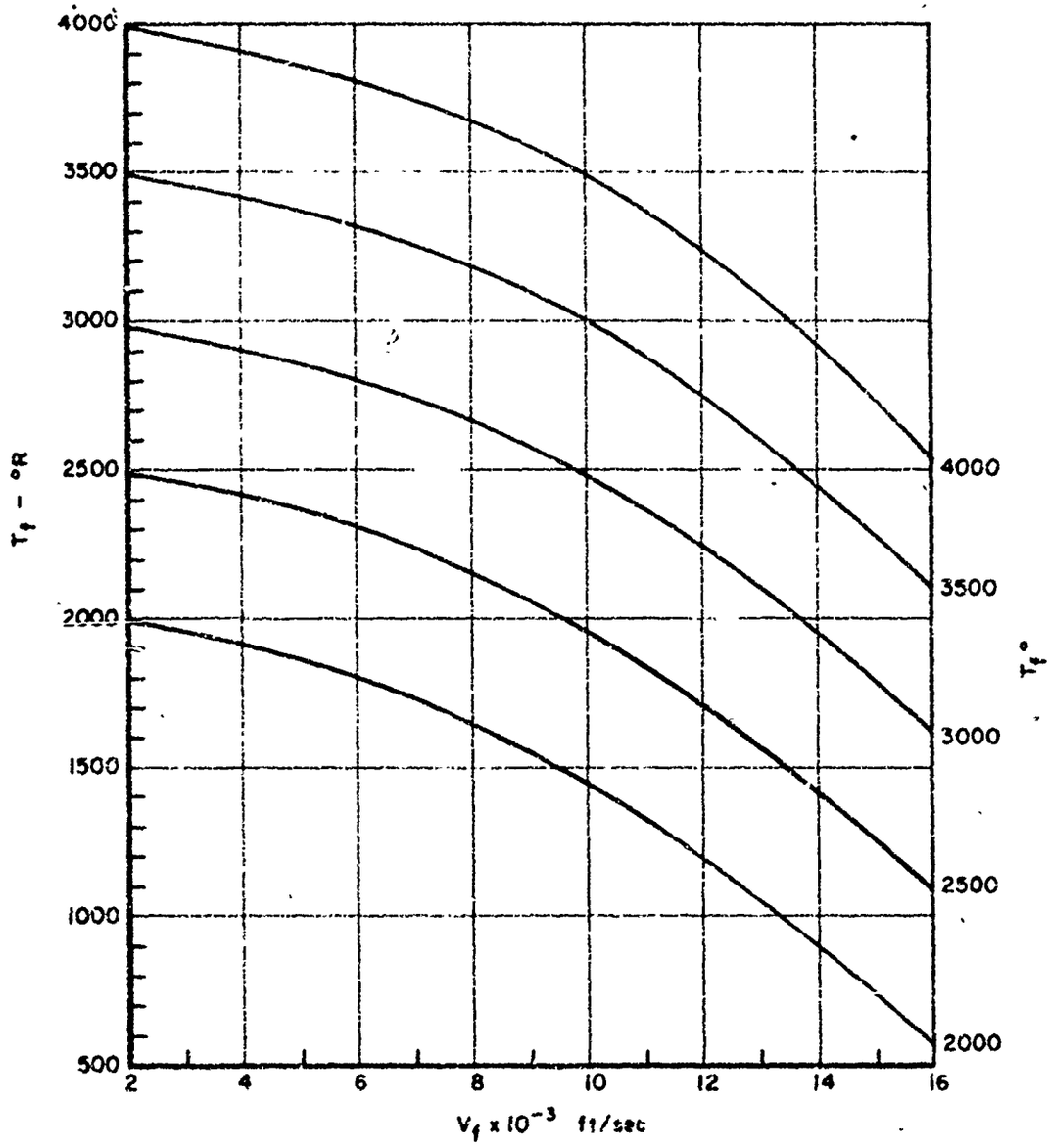

Figure 21. Fuel Static Temperature vs. Fuel Velocity

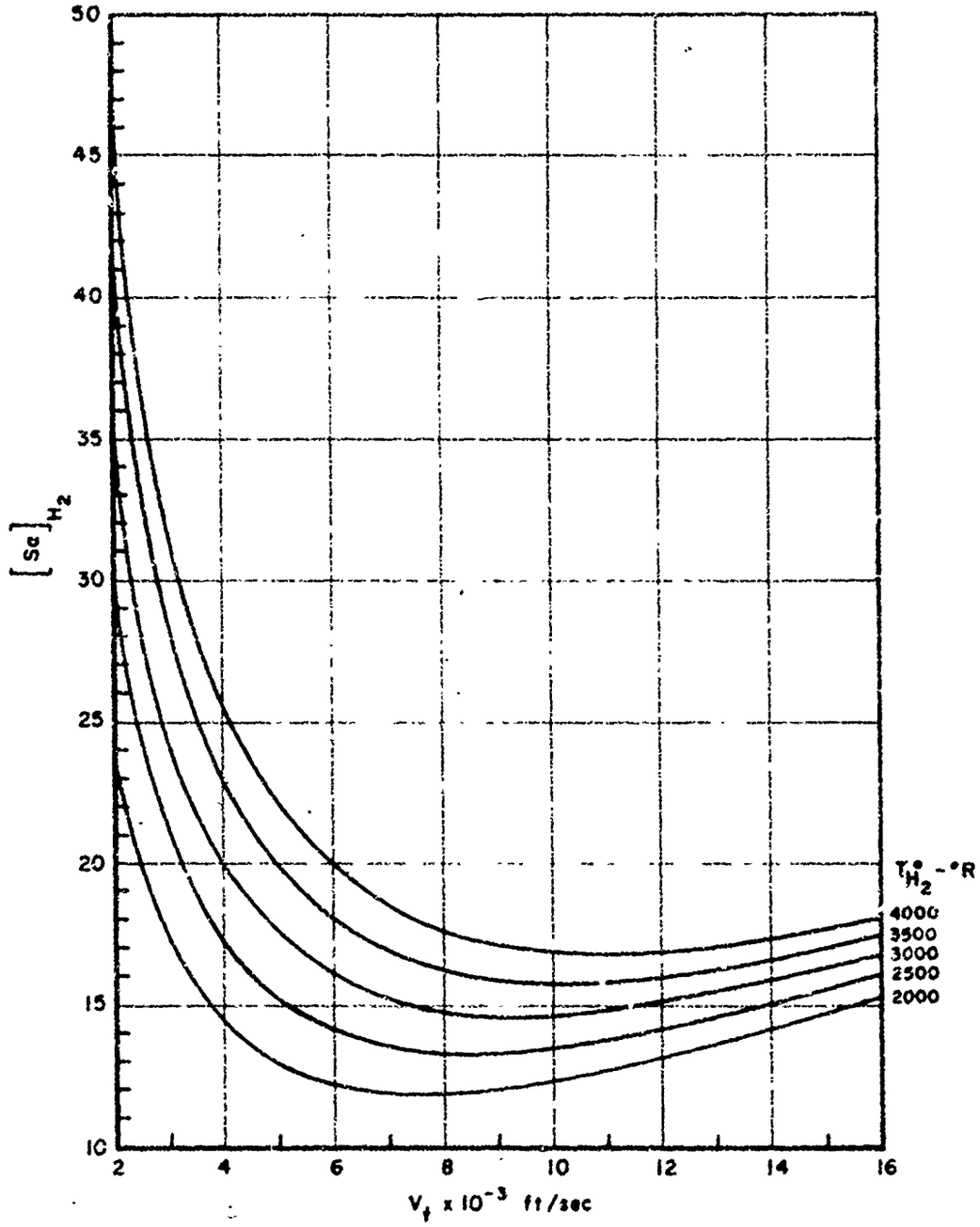

Figure 22. Air Specific Impulse for Hydrogen Fuel vs Fuel Velocity

3. Calculation of the Performance of a Non-Ideal Nozzle

The value of Sa_E given in the charts in Figures 6 to 18 corresponds to an isentropic equilibrium expansion through a given area ratio. The performance of a real nozzle is reduced relative to the ideal nozzle because of non-equilibrium effects, friction losses, divergence losses, and other sources of entropy production. Usually such losses are accounted for by coefficients applied to the ideal results. A large number of coefficients have been defined for nozzle analysis but for the purposes of this paper only those based on, or directly related to, the thrust produced by the exit gas stream are considered. These coefficients are very easily introduced into the analysis. For example, one common coefficient is defined as:

η_s = The ratio of the actual nozzle vacuum thrust to the ideal vacuum thrust of an isentropic nozzle of the same area ratio, the mass flow and applied pressure ratio being the same in both cases.

Thus

$$\eta_s = \frac{(\bar{P}_E)_{actual}}{(\bar{P}_E)_{ideal}} = \frac{(Sa_E)_{actual}}{(Sa_E)_{ideal}}$$

Another coefficient frequently encountered is similar to η_s but it is based on the nozzle gauge thrust. Thus:

η_f = The ratio of the gauge thrust of the actual nozzle to the gauge thrust of an ideal nozzle of the same area ratio, the mass flow and applied pressure ratio being the same in both cases.

$$\eta_f = \frac{\left\{\frac{\dot{m}_E V_E}{g} + A_E (P_E - P_0)\right\}_{actual}}{\left\{\frac{\dot{m}_E V_E}{g} + A_E (P_E - P_0)\right\}_{ideal}} = \frac{\left\{Sa_E - \frac{A_E P_0}{\dot{m}_0}\right\}_{actual}}{\left\{Sa_E - \frac{A_E P_0}{\dot{m}_0}\right\}_{ideal}}$$

From the ideal value of Sa_E one can calculate

$$\left\{Sa_E - \frac{A_E P_0}{\dot{m}_0}\right\}_{ideal} = \left\{Sa_E - \frac{A_E}{A_0}\frac{R_0 T_0}{V_0}\right\}_{ideal}$$

and given η_f the value of $Sa_{E_{actual}}$ can be found.

4. Sample Calculations

Some sample calculations were made to compare the accuracy of this approximate method with the more detailed computer program calculations given in Reference 2, which were basically one-dimensional, equilibrium flow calculations.

The results are shown in Figure 23. Four sets of calculations were made for speeds of 12,000, 14,000, 16,000, and 18,000 ft/sec for a stoichiometric, hydrogen-fueled engine. An intake performance of $K_D = 0.95$ was assumed and an isentropic nozzle expansion with $\eta_s = 1.0$. The altitude corresponded to a dynamic pressure of 500 lb/sq ft.

The agreement between the approximate and the more detailed calculations is seen to be very satisfactory and the agreement appears to improve with increasing flight speed.

A lower speed check case was also calculated for a Mach 9 engine with $\eta_{KE} = 0.975$, $\eta_c = 0.95$, and $A_E/A_0 = 1.5$. The results are shown in Figure 24 and once again the agreement between the approximate and the detailed method is good.

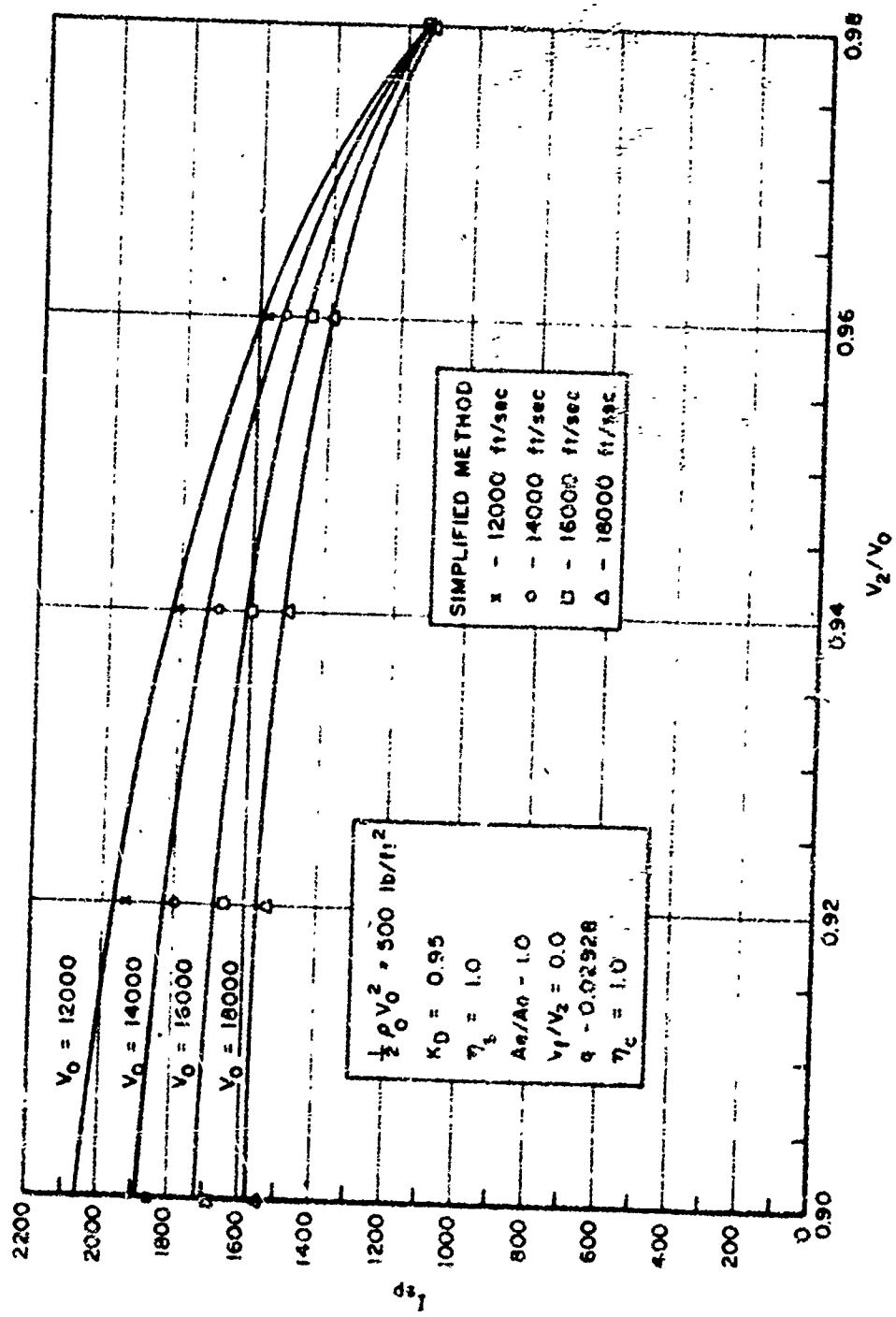

Figure 23. Comparison of Exact and Simplified Calculations

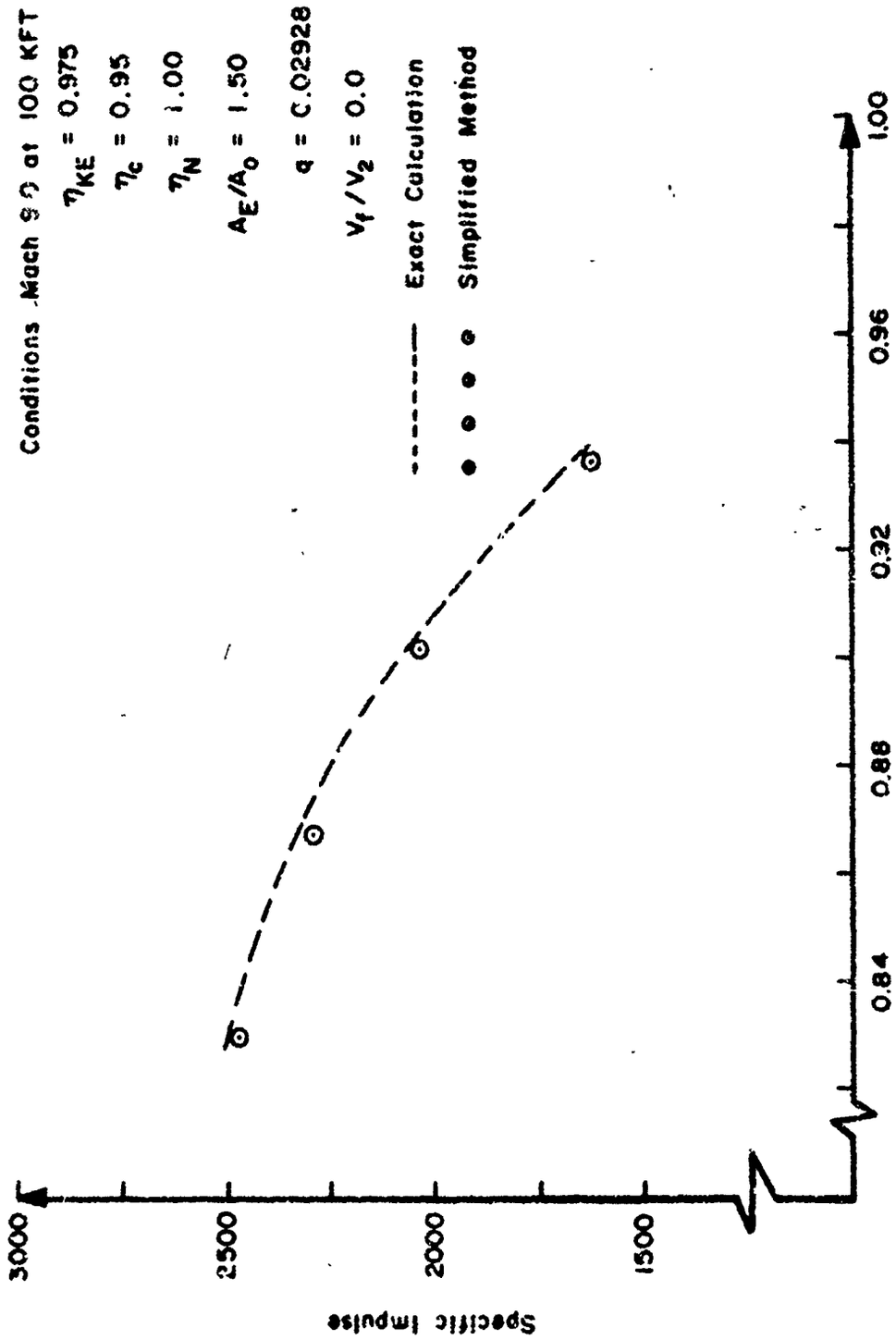

Figure 24. Comparision of Exact and Simplified Calculations

A specific example of a simple slide rule calculation which is intended to illustrate the use of the performance curves is given in Appendix II.

SECTION IV

INCORPORATION AND ANALYSIS OF EXPERIMENTAL RESULTS

Another use of the analysis presented earlier is the interpretation of the results of component tests in terms of overall engine performance. For example, if an experimental combustor is considered, then it may be desirable to "fit" a hypothetical intake and nozzle to the combustor to determine the effect of combustor performance variation on engine performance at a given flight condition.

In assessing the performance of components it is evident that the important qualities to establish are the actual thrust/drag forces on the components. Although the basic thermodynamic efficiencies of components were initially of interest in development and in cycle analysis, the trend of assessment has been toward direct measurement of the forces acting on components. Thus the experience gained in previous years, both in the testing of ramjet combustors and of freejet rigs, has been applied to the measurement of scramjet inlet performance (see References 5 and 6). Similarly, one may expect such techniques to be applied to both combustor and combustor-nozzle development. Apart from the desirability of direct force measurements, this trend has been forced by the inadequacy of, and anomalies associated with, the use of point located instruments to assess average flow properties.

It will thus be apparent that the determination of forces acting on components, by means of force-balance test-rigs allied with mass flow measurements, permits direct calculation of the changes in air specific

stream thrust associated with a component. The main point to be made here is that whether the stream thrust change is estimated theoretically or measured experimentally, it is a relatively easy matter to introduce specific component performance into this analysis or alternatively to use the analysis as a means to express the impact of component performance on flight performance.

SECTION V

CONCLUSION

A simple method of calculating the performance of supersonic combustion engines operating at high flight speeds has been devised. The method has been shown to yield acceptable results for speeds in excess of 9,000 fps but no attempt has been made to assess the validity at lower flight speeds. Charts are only given for stoichiometric hydrogen/air combustion since hydrogen fuel is the prime candidate for high speed engines and operation at lower equivalence ratios is not desirable at high flight speeds.

The concept of a "reference" station corresponding to adiabatic expansion from post combustion stagnation conditions to a reference temperature condition may find further application in other engine cycle studies.

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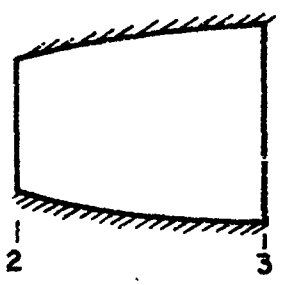
APPENDIX I

STREAM THRUST ANALYSIS OF CONSTANT PRESSURE COMBUSTOR

The calculation of the stream thrust at exit from a constant pressure combustor is relatively straightforward if fuel injection momentum and skin friction effects are neglected. Using these assumptions, the method of approach is as follows. One can write for the constant pressure combustor



$$\bar{P}_3 = \bar{P}_2 + P_2 A_2 \left(\frac{A_3}{A_2} - 1\right)$$

or

$$Sa_3 = Sa_2 + \frac{P_2 A_2}{\dot{m}_0}\left(\frac{A_3}{A_2} - 1\right) \qquad \text{I-1}$$

For given inlet conditions, the only unknown in the above equation is the combustor area ratio A_3/A_2. This ratio can be expressed from the continuity, momentum, and energy equations. Continuity yields

$$(1 + q)\, \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$

or

$$\frac{A_3}{A_2} = (1 + q)\frac{V_2}{V_3}\left(\frac{\mathcal{M}_2}{T_2}\right)\left(\frac{T_3}{\mathcal{M}_3}\right)$$

The momentum equation yields

$$(1 + q)\, V_3 = V_2$$

thus

$$\frac{V_2}{V_3} = (1 + q) \qquad \text{I-2}$$

From the energy equation one has

$$h_3 = H_{t_3} - \frac{V_3^2}{2gJ}$$

where

$$H_{t_3} = \left\{\frac{\eta_c q}{1 + q}\right\} H_t^\circ + \frac{H_{t_0}}{1 + q}$$

Thus for given assumptions, H_{t_3} is easily computed as a function of fuel conditions and flight speed and h_3 can be computed for given combustor entry conditions by using Equation I-2. A relationship between h_3 and $T_3/\mathcal{M}_3$ is shown in Figure 25. (This relationship is exact for a pressure of 1 atmosphere; minor error is involved for use at other pressures.) This graph can now be used to calculate A_3/A_2. Finally, Sa_3 can be obtained from Equation I-1.

The above approach can be extended to cases where the axial fuel momentum term is significant. To speed up such calculations, the following analysis is used to derive some useful charts. Again let us utilize the energy equation

$$h_3 = H_{t_3} - \frac{V_3^2}{2gJ}$$

whence

$$H_{t_3} = \frac{\eta_c q H_t^\circ}{1 + q} + \frac{H_{t_0}}{1 + q} = \frac{\eta_c q H_t^\circ + h_0 + V_0^2/2gJ}{1 + q}$$

For an assumed fuel temperature of 20.39°K and stoichiometric combustion, one has, where all enthalpies are now based on Reference 7,

$$H_t^\circ = 60{,}309 \text{ BTU/lb.}$$

$$q = 0.02928 \text{ lb-fuel/lb-air}$$

$$T_0 = 390°R, \quad h_0 = 203.6$$

then, assuming $\eta_c = 1.00$

$$H_{t_3} = 1913.4 + \frac{V_0^2}{2gJ(1 + q)}$$

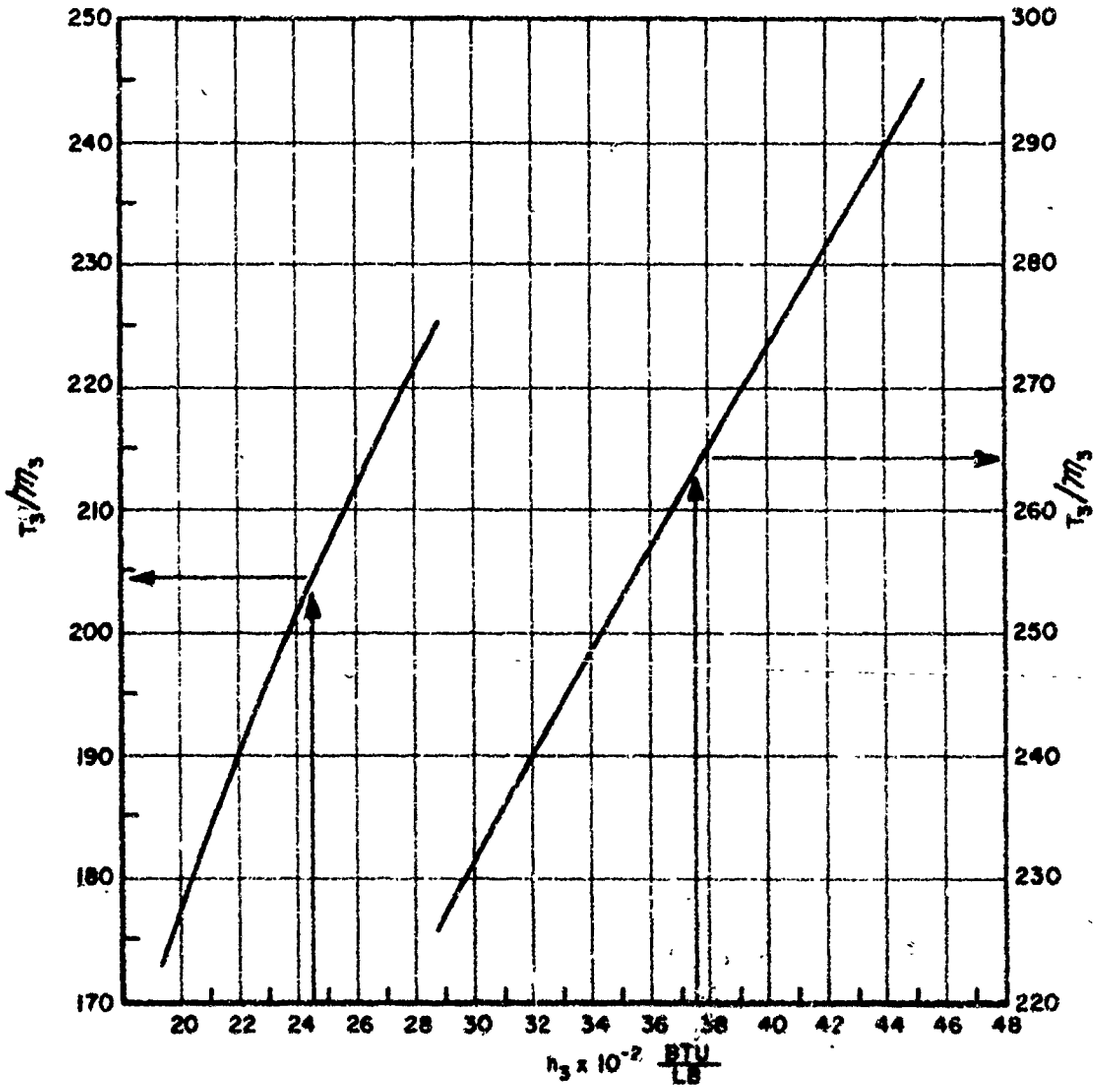


Figure 25. Variation of T_3/m_3 with Combustor Enthalpy

and

$$h_3 = 1913.4 + \frac{V_0^2}{2gJ\,(1+q)} - \frac{V_3^2}{2gJ}$$

From the momentum equation

$$V_2 + q\left(\frac{V_f}{V_2}\right) V_2 = (1+q)\,V_3$$

so that

$$V_3 = \frac{V_2 + q\left(\frac{V_f}{V_2}\right) V_2}{1+q}$$

one then obtains

$$h_3 = 1913.4 + \frac{V_0^2}{2gJ\,(1+q)} - \left\{\frac{\frac{V_2 + q\,\frac{V_f}{V_2}\,V_2}{1+q}}{2gJ}\right\}^2$$

$$h_3 = 1913.4 + \frac{V_0^2}{2gJ\,(1+q)}\left\{1 - \frac{\left(\frac{V_2}{V_0}\right)^2 \left(1 + q\frac{V_f}{V_2}\right)^2}{1+q}\right\}$$

Writing

$$K = \left\{\frac{1 - \frac{\frac{V_2^2}{V_0}\left(1 + q\frac{V_f}{V_2}\right)^2}{1+q}}{1+q}\right\}$$

one has

$$h_3 = 1913.4 + K\frac{V_0^2}{2gJ}$$

Figure 26 shows the variation of h_3 with V_2/V_0 for various free stream velocities and constant pressure combustion with no fuel injection. Figure 27 is a plot of K versus V_2/V_0 for various values of fuel injection velocity ratio. Figure 25 has already shown the variation of $T_3\mathcal{M}_3$ versus h_3.

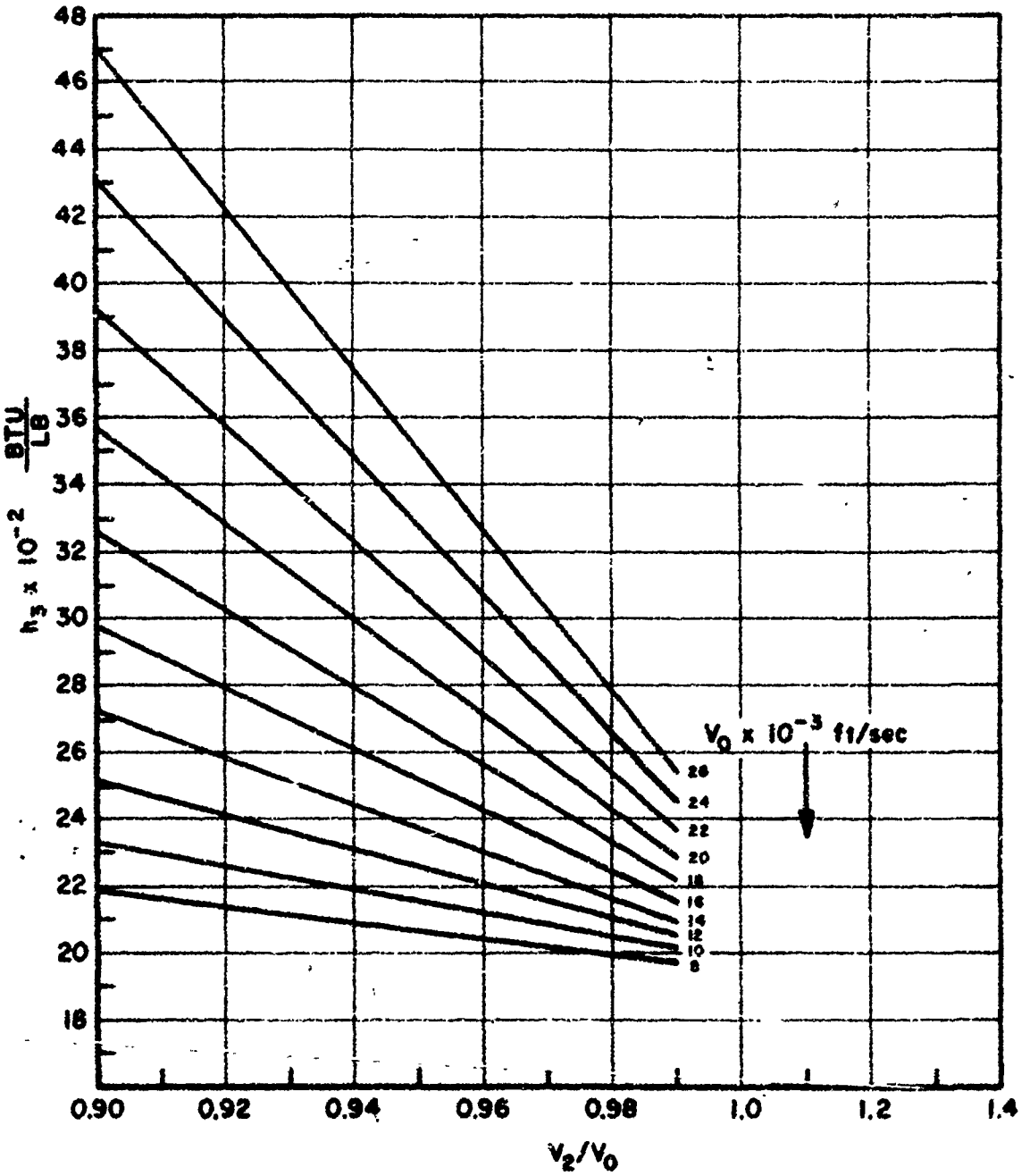


Figure 26. Combustor Enthalpy - Inlet Velocity Ratio

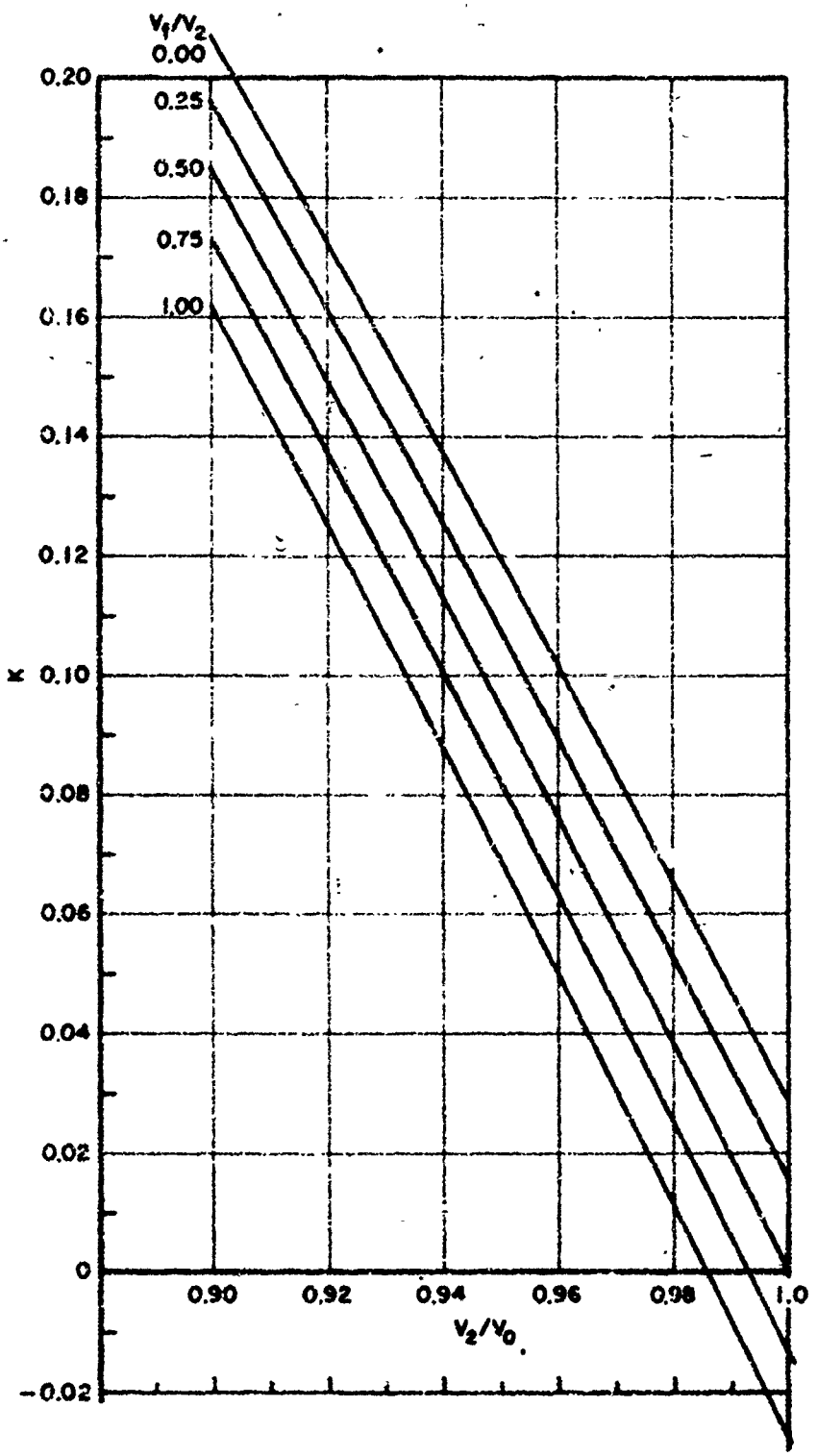


Figure 27. Factor for Computing Combustor Enthalpy vs Inlet Velocity Ratio

The next quantity required is the area ratio of the combustor. Using the continuity equation we have

$$\frac{A_3}{A_2} = (1 + q)\frac{\rho_2 V_2}{\rho_3 V_3}$$

and making use of the equation of state

$$\frac{A_3}{A_2} = (1 + q)\left(\frac{P_2 \mathcal{M}_2 V_2}{T_2}\right)\left(\frac{T_3}{P_3 \mathcal{M}_3 V_3}\right)$$

Expressing V_3 in terms of V_f, V_2, and q, one has

$$\frac{A_3}{A_2} = (1 + q)^2 \frac{\mathcal{M}_2}{T_2}\frac{T_3}{\mathcal{M}_3}\left\{\frac{1}{1 + q\frac{V_f}{V_2}}\right\}$$

which is the required expression.

One must now determine the specific stream thrust at station 3.

This can be accomplished by once again making use of the momentum equation and the definition of air specific stream thrust.

The momentum equation applied to the combustor yields

$$P_f A_f + P_2 A_2 - P_3 A_3 = \dot{m}_3 \frac{V_3}{g} = \dot{m}_0 \frac{V_0}{g} + \dot{m}_f \frac{V_f}{g}$$

Now from the definition of air specific stream thrust, and the fact that

$$P_2 = P_3 = P_f ,$$

$$Sa_3 = Sa_2 + [Sa]_{H_2} + \frac{P_2}{\dot{m}_0}\left\{A_3 - (A_2 + A_f)\right\}$$

and utilizing the continuity equation

$$Sa_3 = Sa_2 + [Sa]_{H_2} + \frac{R_2 T_2}{V_2}\left\{\frac{A_3}{A_2} - \left(1 + \frac{A_f}{A_2}\right)\right\}$$

finally

$$Sa_3 = Sa_2 + [Sa]_{H_2} + \left(Sa_2 - \frac{V_2}{g}\right) \cdot \left\{\frac{A_3}{A_2} - \left(1 + \frac{A_f}{A_2}\right)\right\}$$

Thus with a knowledge of A_3/A_2, it is possible to calculate Sa_3 and then to proceed to the calculation of Sa_E. Finally, the overall engine performance can be determined.

APPENDIX II

EXAMPLE OF THE USE OF PERFORMANCE CURVES

Consider a hydrogen fueled engine operating with stoichiometric fuel/air ratio, $\eta_c = 1.0$, at a flight speed of 16,000 fps and ambient temperature 500°R. Assume $\eta_{KE} = 0.98$, $\eta_N = 1.00$, and fuel injection normal to main airstream. A constant area combustor is assumed $(A_2 = A_3)$ and also we assume $A_0 = A_E$. We shall calculate the engine performance for an intake velocity ratio $V_2/V_0 = 0.95$.

Initial Conditions

Given

$$V_0 = 16{,}000 \text{ fps} \qquad T_0 = 500°R$$

$$\frac{V_0}{g} = 497.3 \qquad \frac{V_0^2}{2gJ} = 5112.5$$

$$Sa_0 = \frac{V_0}{g} + \frac{RT_0}{V_0} = 499$$

From Figure 5,

$$Sa_R - \frac{V_0}{g} = 76.6 , \qquad Sa_R = 573.9$$

From Figure 6,

$$Sa_{max} - \frac{V_0}{g} = 80.2 , \qquad Sa_{max} = 577.5$$

Intake Analysis

$$h_0' = h_0 + (1 - \eta_{KE}) \frac{V_0^2}{2gJ}$$

From Reference 3

$$h_0' = 119.5 + 0.02 \frac{V_0^2}{2gJ}$$

$$= 221.75$$

and relative pressure function

$$p'_{r_0} = 9.173 .$$

To determine Sa_2, we require T_2.

Now, for adiabatic compression

$$H_{t_0} = h_0 + \frac{V_0^2}{2gJ} = h_2 + \frac{V_2^2}{2gJ}$$

$$h_2 = h_0 + \frac{V_0^2}{2gJ}\left\{1 - \left(\frac{V_2}{V_0}\right)^2\right\}$$

$$= 119.5 + 498.5 = 618$$

which yields from Reference 3

$$T_2 = 2403°R \text{ and } p_{r_2} = 369.5$$

$$Sa_2 = \left(\frac{V_2}{V_0}\right)\cdot\frac{V_0}{g} + \frac{R_2T_2}{V_0}\left(\frac{V_0}{V_2}\right)$$

$$= 472.4 + 8.00 = 480.4$$

The corresponding inlet area-ratio is

$$\frac{A_0}{A_2} = \frac{V_2}{V_0}\,\frac{p_{r_2}}{p'_{r_0}}\,\frac{T_0}{T_2}$$

$$= 7.96$$

Combustor Analysis

Note that so far we have simply used the Air Tables in Reference 3; now we proceed to use the performance method outlined in the report with no fuel injection momentum, and ignoring combustor skin friction.

$$Sa_3 = Sa_2 = 480.4$$

now

$$Sa_R = 573.9$$

so that

$$Sa_3/Sa_R = 0.837$$

From Figure 14,

$$A_3/A_R = 7.0 \times 10^{-5}$$

Assuming

$$\frac{A_E}{A_3} = \frac{A_0}{A_2} = 7.96$$

then

$$\frac{A_E}{A_R} = \frac{A_0}{A_2}\frac{A_3}{A_R} = 5.57 \times 10^{-4}$$

From Figure 13,

$$\frac{Sa_E}{Sa_R} = 0.912$$

whence

$$Sa_E = 523.4$$

Performance

For the engine geometry assumed, where $A_4 = A_0$, we have from Equation (2)

$$I_{sp} = \frac{1}{g}\left(Sa_E - Sa_0\right)$$

$$= \frac{24.4}{0.02928}$$

$$= 834.5 \text{ secs.}$$

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