AD-769 337

AN OPTIMUM INTERFERENCE DETECTOR FOR DABS (DISCRETE ADDRESS BEACON SYSTEM) MONOPULSE DATA EDITING

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Prepared for:

Federal Aviation Administration
Electronic Systems Division

26 September 1973

DISTRIBUTED BY:

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TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. ESD-TR-73-253	2. Government Accession No.	3. Recipient's Catalog No. AD 769 337
4. Title and Subtitle An Optimum Interference Detector for DABS Monopulse Data Editing		5. Report Date 26 September 1973
		6. Performing Organization Code
7. Author(s) R.J. McAulay, T.P. McGarty		8. Performing Organization Report No. Technical Note 1973-48
9. Performing Organization Name and Address Massachusetts Institute of Technology Lincoln Laboratory P.O. Box 73 Lexington, Massachusetts 02173		10. Work Unit No. Proj. No. 034-241-012
		11. Contract or Grant No. IAG-DOT-FA72 WAI-261
12. Sponsoring Agency Name and Address Department of Transportation Federal Aviation Administration Systems Research and Development Service Washington, D.C. 20591		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology under Air Force Contract F19628-73-C-0002.

16 Abstract

In the application of the Discrete Address Beacon System (DABS) concept to Air Traffic Control (ATC) surveillance, estimates of aircraft position must be made using as few replies as possible, preferably one. This requires the use of monopulse techniques. Since the beacon system provides high signal-to-noise ratios (SNR), the fundamental limitation to direction finding (DF) performance is due to externally generated interference from multipath signals and from the present Air Traffic Control Radar Beacon System (ATCRBS). Since there are many bits in any one DABS reply it should be possible to generate an accurate azimuth estimate if those that bear interference could be detected and deleted from the sample. In this report, the generalized likelihood ratio test is used to derive an optimum interference statistic. The detector performance is then analyzed in detail with respect to its dependence on SNR, interference-to-signal ratio (ISR) and on the relative phase between the target and interfering signals. It is shown that good detection performance can be obtained if the phase difference between the target and interference signals are either in- or out-of-phase.

17. Key Words		18. Distribution Statement
air traffic control	transponder	Availability is unlimited. Document may be released to the National Technical Information Service, Springfield, Virginia 22151, for sale to the public.
surveillance	ATCRBS	
communications	DABS	
data link	IPC	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	40	3.00 HC 1.45 MF

Form DOT F 1700.7 (8-69)

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I. INTRODUCTION

The ability of a monopulse processor to determine the angular direction of an incident signal is limited not only by the inherent front-end receiver noise but also by the effect of interfering signals. If there are other time coincident signals present that interfere with the signal whose direction is to be estimated then McAulay [1] has shown that bias effects occur that can seriously degrade the quality of the estimate. It becomes important therefore, to know when such interference is present so that low confidence can be assigned to the associated azimuth estimate.

Sherman [2] has observed that when interfering signals are present the outputs of the monopulse sum and difference beams become incoherent. He proposes to use the quadrature information to resolve the target and interference signals. Our approach is to develop an interference flag that indicates when more than one signal is present in the receiver channels. Depending on the application, the flag would be used to assign a low confidence to the associated angle estimates or to delete the angle estimate altogether. In Section II, we formulate the test for interference as a hypothesis test. Using the Generalized Likelihood Ratio test [3] and following the analysis of Hofstetter and DeLong [4], we obtain the optimum interference detection statistic.

The performance of the detector is analyzed in Section III where it is first shown that the interference statistic has the Rician distribution. Exact evaluation of the false alarm and detection probabilities becomes intractable and use is made of the Gaussian approximation to the Rician variate. Numerical results for some typical cases of interest are given and it is shown that the results depend strongly on the relative phase between the target and interfering signals, but that good overall performance can be obtained. Conditions under which the Gaussian approximation is valid are given and are shown to hold for the cases studied.

II. PROBLEM FORMULATION AND SOLUTION

We shall restrict our attention to amplitude-comparison monopulse processing that is performed on a sampled-data basis. Assuming mixer preamplifiers at the output of each antenna beam channel, the received signal samples in the absence of interference are modeled by

$$y_i = A_S e^{j\varphi_S} G_i(\theta_S, \alpha_S) + n_i \qquad i = 1,2,\ldots,m \tag{1}$$

where y_i refer to the complex output of i^{th} antenna beam channel; $A_S, \varphi_S, \theta_S, \alpha_S$ are the amplitude, phase, azimuth, and elevation of the target signal; $G_i(\cdot)$ is the antenna patterns of the i^{th} antenna beam which may be complex in general; n_i represents zero mean Gaussian noise samples due to the mixer preamplifiers whose real and imaginary parts have variance σ^2. As shown by Hofstetter and DeLong [4], this model arises when the received signal is preprocessed by a matched filter. It can also be used to describe the case in which a simple on-off pulse is transmitted and preprocessed by a filter whose bandwidth is at least equal to the reciprocal of the rise time.

If interference is present that overlaps the target signal return, the received signal samples can be written as

$$y_i = A_S e^{j\varphi_S} G_i(\theta_S, \alpha_S) + A_I e^{j\varphi_I} G_i(\theta_I, \alpha_I) + n_i \qquad i = 1,2,\ldots,m \tag{2}$$

where now A_I, φ_I, θ_I, α_I represent the amplitude, phase, azimuth, and elevation angles of the interference signal. In addition to describing the effects of multipath in an L-band radar the model also arises in the design of the Discrete Address Beacon System (DABS) that is to be used to perform the surveillance function in the next generation Air Traffic Control (ATC) system [5]. During the transition from the present Air Traffic Control Radar Beacon System (ATCRBS) to a completely DABS operation, ATCRBS will represent a source of interference to DABS. Since direction finding is performed at L-band using simple on-off pulses, the received signal samples will be described by (2). We note that in this case, however, that the downlink carrier frequency is known only to within $\pm$ 3 MHz. Therefore, the preprossing filter cannot be exactly matched to the downlink signal and a slight reduction in signal-to-noise ratio (SNR) must be tolerated.

Using (1) and (2) we can formulate the following hypothesis test for the detection of interference:

H_0: interference absent

$$y_i = A_S\, e^{j\varphi_S}\, G_i(\theta_S,\alpha_S) + n_i, \qquad i=1,\ldots m \qquad (3)$$

H_1: interference present

$$y_i = A_S\, e^{j\varphi_S}\, G_i(\theta_S,\alpha_S) + A_I\, e^{j\varphi_I}\, G_i(\theta_I,\alpha_I) + n_i \quad . \; i=1,\ldots m \qquad (4)$$

The solution to this hypothesis testing problem can be obtained from application of the Generalized Likelihood Ratio criterion [3]. First, we form the ratio

$$\Lambda(\underline{y}) = \frac{\max\limits_{\underline{\beta}_S, \underline{\beta}_I} p(\underline{y}|H_1, \underline{\beta}_S, \underline{\beta}_I)}{\max\limits_{\underline{\beta}_S} p(\underline{y}|H_0, \underline{\beta}_S)} \tag{5}$$

where we have used the notation $\underline{y} = (y_1, y_2, \ldots, y_m)$ to denote the complex data vector and $\underline{\beta} = (A, \varphi, \theta, \alpha)$ to denote the unknown parameter vector, and where $p(\underline{y}|H_k)$ denotes the probability density function of $\underline{y}$ under either hypothesis H_0 or H_1. Once $\Lambda(\underline{y})$ is computed, interference is declared present if and only if $\Lambda(\underline{y}) > \lambda'$.

Since the noise components are Gaussian random variables, then under the null hypothesis, we can write

$$\max_{\underline{\beta}_S} p(\underline{y}|H_0, \underline{\beta}_S) = \max_{A_S, \varphi_S, \theta_S, \alpha_S} \left\{ (2\pi\sigma^2)^{-m/2} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{m} |y_i - A_S\, e^{j\varphi_S} G_i(\theta_S, \alpha_S)|^2 \right] \right\} \tag{6}$$

Hofstetter and DeLong [4] have shown that if both θ_S and α_S are unknown then for reasonable antenna patterns, $G_i(\theta_S, \alpha_S)$, the maximization in (6) can be solved if there are 3 linearly independent antenna beams. If only θ_S is unknown or if $G_i(\theta_S)$ is a fan beam in elevation, then they shown that (6) can be solved if there are 2 linearly independent antenna beams. With these restrictions on m, namely m=2 or 3, depending on whether θ_S and/or α_S are unknown, Hofstetter and DeLong show that (6) is maximized at the parameter values $\hat{A}, \hat{\varphi}, \hat{\theta}$ and/or $\hat{\alpha}$ where

$$\hat{A} = \frac{\sum\limits_{i=1}^{m} G_i(\hat{\theta}, \hat{\alpha})\, \mathrm{Re}(y_i e^{-j\hat{\varphi}})}{\sum\limits_{i=1}^{m} G_i^2(\hat{\theta}, \hat{\alpha})} \tag{7}$$

$$\hat{\varphi} = \frac{1}{2} \arg\left(\sum_{i=1}^{m} y_i^2\right) \tag{8}$$

$$\frac{G_1(\hat{\theta},\hat{\alpha})}{G_2(\hat{\theta},\hat{\alpha})} = \frac{\mathrm{Re}(y_1\, e^{-j\hat{\varphi}})}{\mathrm{Re}(y_2\, e^{-j\hat{\varphi}})} \quad , \quad \frac{G_2(\hat{\theta},\hat{\alpha})}{G_3(\hat{\theta},\hat{\alpha})} = \frac{\mathrm{Re}(y_2\, e^{-j\hat{\varphi}})}{\mathrm{Re}(y_3\, e^{-j\hat{\varphi}})} \quad . \tag{9}$$

and hence $\hat{A},\hat{\varphi},\hat{\theta},\hat{\alpha}$ denote the maximum likelihood estimates of A,φ,θ,α. It is also shown that the maximum value of the density function is

$$\max_{\underline{\beta}_S} p(\underline{y}|H_0,\underline{\beta}_S) = (2\pi\, \sigma^2)^{-m/2} \exp\left[-\frac{1}{4\,\sigma^2}\left(\sum_{i=1}^{m} |y_i|^2 - \left|\sum_{i=1}^{m} y_i^2\right|\right)\right] \quad . \tag{10}$$

Under the alternate hypothesis the density function is

$$p(\underline{y}|H_1,\underline{\beta}_S,\underline{\beta}_I) = (2\pi\, \sigma^2)^{-m/2} \exp\left[-\frac{1}{2\,\sigma^2} \sum_{i=1}^{m} \left|y_i - A_S\, e^{j\varphi_S}\, G_i(\theta_S,\alpha_S) - A_I\, e^{j\varphi_I}\, G_i(\theta_I,\alpha_I)\right|^2\right] \quad . \tag{11}$$

When both θ_S and α_S are unknown, we require $m = 3$ and note that there are 6 measurements and 6 unknown parameters. If only θ_S is unknown, we require $m = 2$ and then there are 4 measurements and 4 unknown parameters. In either case, the maximum value of the density is $(2\pi\, \sigma^2)^{-m/2}$ and it is achieved by picking the parameter estimates to solve the equations

$$\hat{A}_S e^{j\hat{\varphi}_S} G_i(\hat{\theta}_S,\hat{\alpha}_S) + \hat{A}_I e^{j\hat{\varphi}_I} G_i(\hat{\theta}_I,\hat{\alpha}_I) = y_i \qquad i = 1,2, \text{ or } 3 \quad . \tag{12}$$

Using these facts, the Generalized Likelihood Ratio, (5), becomes

$$\Lambda(\underline{y}) = \exp\left[\frac{1}{4\sigma^2}\left(\sum_{i=1}^{m} |y_i|^2 - \left|\sum_{i=1}^{m} y_i^2\right|\right)\right] \tag{13}$$

hence, we declare interference present (hypothesis H_1) if and only if

$$\sum_{i=1}^{m} |y_i|^2 - \left|\sum_{i=1}^{m} y_i^2\right| > \lambda \quad . \tag{14}$$

In the application to the ATC problem only the aircraft azimuth can be estimated since elevation fan beams are used to provide surveillance for high and low altitude aircraft. In this case, two antenna beams having azimuth directionality would be used that would take the form of a sum (even) and difference (odd) monopulse configuration. Letting y_1, y_2 denote the outputs of the sum and difference beams respectively, then interference will be declared present if and only if

$$|y_1|^2 + |y_2|^2 - |y_1^2 + y_2^2| > \lambda' \quad . \tag{15}$$

Following [4], we recognize that an equivalent test to (15) is to declare interference absent if, and only if,

$$-\lambda < |y_1 + j\, y_2| - |y_1 - j\, y_2| < \lambda \quad . \tag{16}$$

It is interesting to note that in the test for signal plus noise versus noise alone, which was the problem that was considered in [4], the two quantities in (16) are added rather than subtracted. Furthermore, the maximum likelihood azimuth estimate is related to the phase difference between the signals $y_1 \pm j\, y_2$. Since these signals are readily formed at RF, it is possible to obtain the detection statistic, interference statistic, and azimuth estimate from the same RF hardware configuration. In the remainder of this paper we shall limit our attention only to the case of a two beam monopulse radar, hence we assume that the antenna patterns depend only on one angular variable. For convenience we shall deal with the estimate of azimuth, but it is obvious that the results apply directly to a monopulse system that tracks in elevation.

III. DERIVATION OF THE EXACT PDF

Since our basic motivation for undertaking this problem lies in the ATC context, we can assume that m = 2 and focus our attention on (16) and attempt to analyze the performance of the detector by computing the false alarm and detection probabilities. Therefore, we shall all attempt to compute the probability density function (pdf) of the detection statistic

$$\ell = |y_1 + j\, y_2| - |y_1 - j\, y_2| \quad . \tag{17}$$

We begin the analysis by defining new random variables

$$\begin{aligned} y_{\pm} &= y_1 \pm j\, y_2 \\ &= A_S[G_1(\theta_S) \pm j\, G_2(\theta_S)]\, e^{j\varphi_S} + A_I[G_1(\theta_I) \pm j\, G_2(\theta_I)\, e^{j\varphi_I}] + n_1 \pm j\, n_2 \\ &= \left[A_S\, e^{j\varphi_S}\, G_1(\theta_S) + A_I\, e^{j\varphi_I}\, G_1(\theta_I)\right] \pm j\left[A_S\, e^{j\varphi_S}\, G_2(\theta_S) + A_I\, e^{j\varphi_I}\, G_2(\theta_I)\right] + \xi_{\pm} \end{aligned} \tag{18}$$

where we have defined new noise variables

$$\xi_{\pm} = n_1 \pm j\, n_2 \tag{19}$$

which we note are zero mean, independent complex Gaussian with variance $4\sigma^2$. Therefore, $y_\pm$ are also independent complex Gaussian random variables with variance $4\sigma^2$ and means

$$\mu_\pm = \left[A_S\, e^{j\varphi_S}\, G_1(\theta_S) + A_I\, e^{j\varphi_I}\, G_1(\theta_I)\right] \pm j\left[A_S\, e^{j\varphi_S}\, G_2(\theta_S) + A_I\, e^{j\varphi_I}\, G_2(\theta_I)\right] \quad . \tag{20}$$

Therefore the random variables

$$\ell_\pm = |y_\pm| \tag{21}$$

are independent Rician random variables of order 2, [6], and their probability density functions (pdf) are therefore

$$p_{\ell_\pm}(u) = \frac{u}{2\,\sigma^2} \exp\left(-\frac{u^2 + d_\pm^2}{4\,\sigma^2}\right) I_0\left(\frac{u\, d_\pm}{2\,\sigma^2}\right) \tag{22}$$

where $I_0(\cdot)$ is the modified Bessel Function of the first kind of order zero, and where

$$d_\pm^2 = |\mu_\pm|^2 \quad . \tag{23}$$

Using the definitions in (17), (18), and (21) we see that the detection statistic is given by

$$\ell = |y_+| - |y_-|$$
$$= \ell_+ - \ell_- \tag{24}$$

hence its pdf is

$$p_\ell(u) = \int_{-\infty}^{\infty} p_{\ell_+}(u - v)\, p_{\ell_-}(v)\, dv \quad . \tag{25}$$

Then using (22) in (25) it can be shown that the exact form for this pdf is

$$p_\ell(u) = \frac{1}{\sqrt{2}\sigma} \exp\left(- \frac{u^2}{2\sigma^2}\right) \int_{u/\sqrt{2}\sigma}^{\infty} f(x, u/\sqrt{2}\sigma)\, dx \tag{26}$$

where

$$f(x,y) = (x^2 - y^2) \exp\left[-\left(x^2 + \frac{d_+^2 + d_-^2}{4\sigma^2}\right)\right] I_0[d_+(x + y)]\, I_0[d_-(x - y)] \quad . \tag{27}$$

Since it is difficult, if not impossible, to evaluate (26) analytically or numerically, we have found it more appropriate to approximate the Rician variate by the Gaussian density. In Fig. 1 we have shown curves obtained from [7] that show the Rician pdf for several values of the parameter $d_\pm^2/2\sigma^2$. From the figure it appears that for values of $d_\pm/\sqrt{2}\sigma$ greater than 3 the Gaussian approximation is quite good. In the Appendix we perform a detailed error analysis that shows that except for 2.5% of the area under the lower tail, the Rician pdf is well approximated by the Gaussian density provided

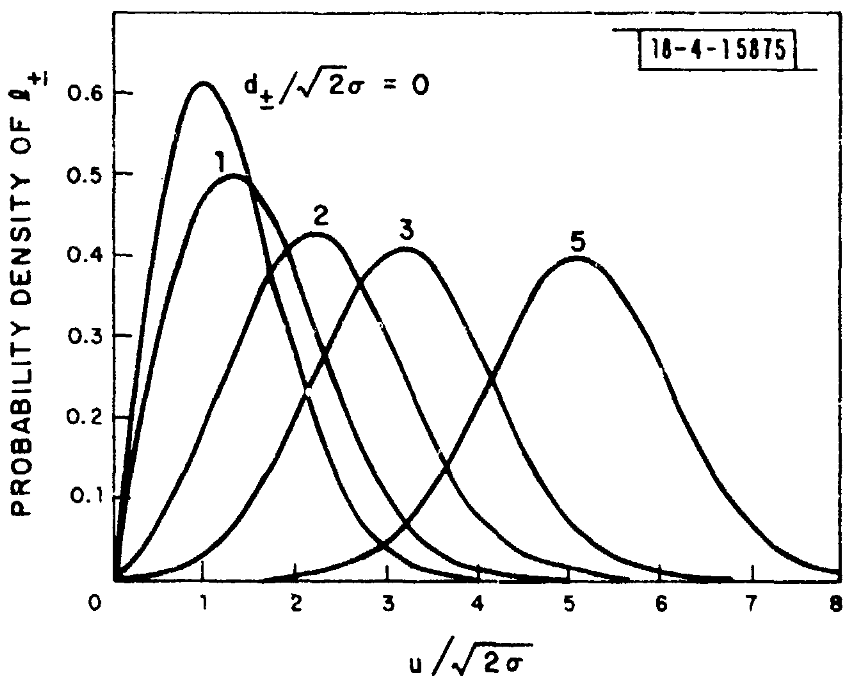


Fig. 1. Probability density function of a rician variate.

$$\left(d_{\pm}/\sqrt{2\sigma}\right)^2 \geqq 10 \quad . \tag{28}$$

Therefore we can write (22) as

$$p_{\ell_{\pm}}(u) \approx \frac{1}{\sqrt{4\pi\sigma^2}} \exp\left[-\frac{(u - d_{\pm})^2}{4\sigma^2}\right] \quad . \tag{29}$$

From (24) the detection statistic is $\ell = \ell_+ - \ell_-$, so that ℓ is also Gaussian but with mean $d_+ - d_-$ and variance $4\sigma^2$. Therefore

$$p_{\ell}(u) = \frac{1}{\sqrt{8\pi\sigma^2}} \exp\left\{-\frac{[u - (d_+ - d_-)]^2}{8\sigma^2}\right\} \tag{30}$$

where $d_{\pm}$ are given by (23) and (20).

In the next section we shall compute the false alarm and detection probabilities and hence develop the criteria needed to evaluate the performance of the detector.

IV. FALSE ALARM AND DETECTION PROBABILITIES

Since the detection statistic can be well approximated by a Gaussian random variable, it is a straightforward problem to calculate the false alarm and detection probabilities.

a) False Alarm Probability

In this case, the interfering signal is absent hence $A_I = 0$. Furthermore, the target of interest lies within the mainbeam of the antenna and it can therefore be assumed that $G_i(\theta_S)$ is real. Using these facts in (20), (23) becomes

$$d_{\pm}^2 = A_S^2 \left| G_1^2(\theta_S) + G_2^2(\theta_S) \right| \quad . \tag{31}$$

Therefore the mean value of the detection statistic is

$$\bar{\ell} = d_+ - d_- = 0 \quad . \tag{32}$$

A false alarm is made whenever $|\ell| > \lambda$, hence the false alarm probability is

$$P_{FA} = 1 - \int_{-\lambda}^{\lambda} \frac{1}{\sqrt{8\pi\,\sigma^2}} \exp\left(-\frac{u^2}{8\,\sigma^2}\right) du$$

$$= 2\,\mathrm{erf}\left(\frac{\lambda}{2\sigma}\right) \tag{33}$$

where

$$\operatorname{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt \quad . \tag{34}$$

This shows that the detection threshold can be set once the level of the background noise is known.

To verify that the Gaussian approximation is indeed valid in this case, we see from (28) that it is sufficient to have

$$\frac{A_S^2 \left| G_1^2(\theta_S) + G_2^2(\theta_S) \right|}{2\,\sigma^2} \geq 10 \quad . \tag{35}$$

The detection signal-to-noise ratio is $A_S^2\, G_1^2(\theta_S)/2\sigma^2$ and since this quantity is at least 20 dB in the ATC context we see that (35) will easily be satisfied.

b) Detection Probability

In this case, we detect interference when $|\ell| > \lambda$, hence if the Gaussian approximation is valid

$$P_D = 1 - \int_{-\lambda}^{\lambda} \frac{1}{\sqrt{8\pi\,\sigma^2}} \exp\left\{ - \frac{[u - (d_+ - d_-)]^2}{8\,\sigma^2} \right\} du$$

$$= \operatorname{erf}\left[\frac{\lambda - (d_+ - d_-)}{2\sigma} \right] + \operatorname{erf}\left[\frac{\lambda + (d_+ - d_-)}{2\sigma} \right] \quad . \tag{36}$$

If we desire a false alarm probability P_{FA}, then we solve

$$P_{FA} = 2\,\operatorname{erf}(\lambda_{FA}) \tag{37}$$

for λ_{FA}. Then setting the threshold according to

$$\lambda = 2\sigma \lambda_{FA} \tag{38}$$

we see that the detection probability is

$$P_D = \operatorname{erf}\left(\lambda_{FA} - \frac{d_+ - d_-}{2\sigma}\right) + \operatorname{erf}\left(\lambda_{FA} + \frac{d_+ - d_-}{2\sigma}\right) \quad . \tag{39}$$

We have yet to verify the validity of the Gaussian approximation in this case, but this requires the evaluation of (23). We shall consider this point in detail in the next section. At that time, we shall consider specific interference cases of interest and carry out the evaluation of the receiver performance in detail.

V. RECEIVER PERFORMANCE

In many cases of practical interest the antenna patterns will be complex at least in a region beyond the near-in sidelobes. Since the interference can originate from any azimuth, we make the complex dependence clear by writing

$$G_i(\theta_I) = A_i(\theta_I) \exp[j\, \psi_i(\theta_I)] \quad . \tag{40}$$

Substituting (40) into (20) and using the fact that $G_i(\theta_S)$ is real we can shown after tedious but straightforward manipulations that

$$\frac{d_\pm^2}{A_S^2} = \left|G_1^2(\theta_S) + G_2^2(\theta_S)\right| + \rho^2\left|A_1^2 \pm 2\, A_1\, A_2 \sin(\psi_1 - \psi_2) + A_2^2\right|$$
$$+ 2\rho\left|G_1^2(\theta_S) + G_2^2(\theta_S)\right|^{1/2} \left|A_1^2 \pm 2\, A_1\, A_2 \sin(\psi_1 - \psi_2) + A_2^2\right|^{1/2}$$
$$\cos(\varphi \mp \alpha + \psi_\pm) \tag{41}$$

where

$$\rho = A_I/A_S \quad , \tag{42a}$$

$$\alpha = \tan^{-1}[G_2(\theta_S)/G_1(\theta_S)] \quad , \tag{42b}$$

$$\psi_\pm = \tan^{-1}\left(\frac{A_1 \sin\psi_1 \pm A_2 \cos\psi_2}{A_1 \cos\psi_1 \mp \sin\psi_2}\right) \quad . \tag{42c}$$

Unfortunately, there are too many parameters involved in these equations to obtain any physical insight into the performance of the detector in the general case. We have had to resort to using a simulation to evaluate this general case. We can obtain some useful analytical results by limiting our interest to real antenna patterns. This will be valid for mainbeam interference and also for interference located at the near-in sidelobes. Therefore, we assume that $G_i(\theta_I)$ are real. In this case, (41) becomes

$$\frac{d_\pm^2}{A_S^2} = \left|G_1^2(\theta_S) + G_2^2(\theta_S)\right| + \rho^2\left|G_1^2(\theta_I) + G_2^2(\theta_I)\right|$$

$$+ 2\rho\left|G_1^2(\theta_S) + G_2^2(\theta_S)\right|^{1/2} \left|G_1^2(\theta_I) + G_2^2(\theta_I)\right|^{1/2} \cos(\varphi \mp \beta) \qquad (43)$$

where

$$\beta = \tan^{-1}\left[\frac{\dfrac{G_1(\theta_S)}{G_2(\theta_S)} - \dfrac{G_1(\theta_I)}{G_2(\theta_I)}}{1 + \dfrac{G_1(\theta_S)}{G_2(\theta_S)} \dfrac{G_1(\theta_I)}{G_2(\theta_I)}}\right] . \qquad (44)$$

It will be useful to use the notion of the monopulse function which is defined as

$$E(\theta) = \frac{G_2(\theta)}{G_1(\theta)} . \qquad (45)$$

In most practical monopulse systems, the monopulse function is linear over the extent of the main beam. Since the target of interest will always be within the mainbeam then

$$E(\theta_S) = k \frac{\theta_S}{\theta_B} \tag{46}$$

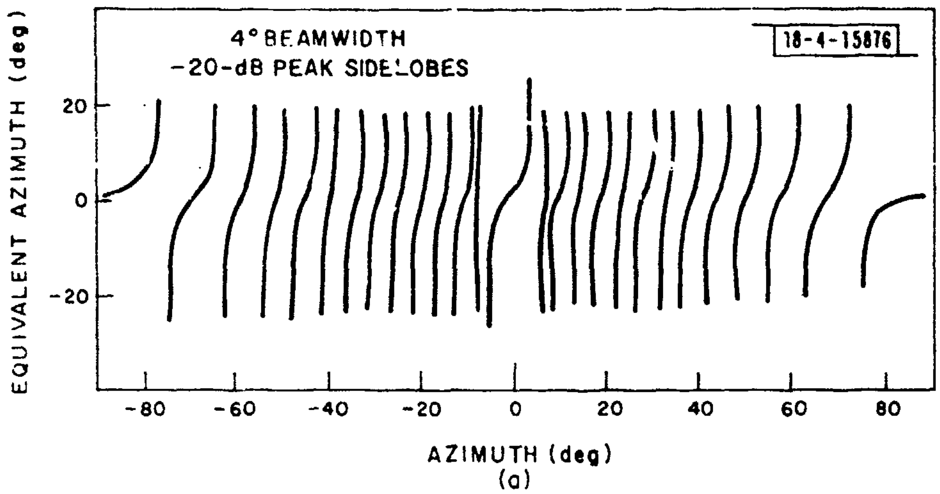
where θ_B is the 3 dB beamwidth and k is a standard parameter that arises in the analysis of monopulse systems. Typically, $1 \leq k \leq 2$ with $k \sim 1.5$ being a reasonable value [8]. Whereas, the target can always be assumed to lie within the mainbeam, the interference signal can originate from the mainbeam or from a sidelobe. However, it is convenient to define an equivalent interference azimuth, $\tilde{\theta}_I$ as

$$\frac{\tilde{\theta}_I}{\theta_B} = E(\theta_I)/k \quad . \tag{47}$$

We note that $\tilde{\theta} = \theta_I$ when θ_I is within the mainbeam of the antenna. In Fig. 2a we have plotted $\tilde{\theta}_I$ vs θ_I using a typical monopulse function derived for a 4° beamwidth antenna with -20 dB sidelobes, where errors in the amplitude and phase tapers render the antenna pattern complex. Therefore, the results can be expected to give some indication of performance even in the more general case. Assuming θ_I is uniformly distributed in $(-\pi,\pi)$, Fig. 2b gives the probability distribution function of $\tilde{\theta}_I$. This shows that $\tilde{\theta}_I$ will lie within ± 2 beamwidths most of the time and hence the cases of sidelobe and mainbeam interference can be treated simultaneously.

Using (46) we can write $E(\theta_I) = k\, \tilde{\theta}_I/\theta_B$, and then absorb θ_B into our definition of θ so that all of the azimuth variables can be expressed in 3 dB beamwidths. Using these relations (42) and (43) can be written as

$$d_{\pm}^2 = A_S^2\, G_1^2(\theta_S) \left(1 + k^2\, \theta_S^2\right) \left[1 + \rho_0^2 \left(\frac{1 + k^2\, \tilde{\theta}_I^2}{1 + k^2\, \theta_S^2}\right) + 2\rho_0 \left(\frac{1 + k^2\, \tilde{\theta}_I^2}{1 + k^2\, \theta_S^2}\right)^{1/2} \cos(\varphi \mp \beta)\right] \tag{48}$$



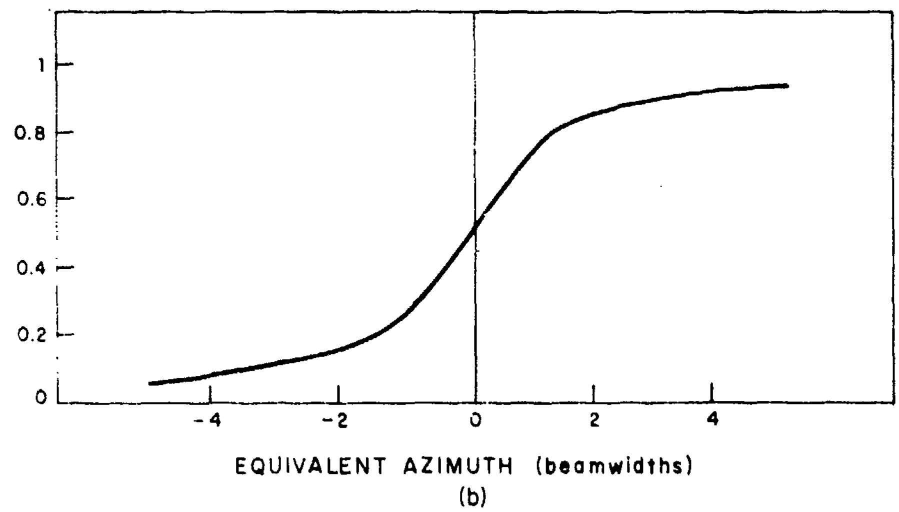


Fig. 2 (a) Equivalent azimuth using the monopulse function (b) Probability distribution function of equivalent azimuth.

where

$$\beta = \tan^{-1}\left[\frac{k(\theta_S - \tilde{\theta}_I)}{1 + k^2\, \theta_S\, \tilde{\theta}_I}\right] \tag{49}$$

$$\rho_0 = \frac{A_I\, G_1(\theta_I)}{A_S\, G_1(\theta_S)} \quad . \tag{50}$$

The quantity ρ_0 represents the interference-to-signal ratio as measured at the output of the antenna terminals. We note that by using (47) we are able to derive an expression which describes the case of mainbeam and sidelobe interference simultaneously.

a) Basic Properties of the Detector

Using the above expression we can obtain some interesting properties of the interference detector. First we note that if the relative phase between the two signals, φ, is 0 or π, then $d_+ = d_-$ and from (39) we see that the detection probability reduces to the false alarm probability. This is an unfortunate property since McAulay [1] has shown that interference causes the worst azimuth errors at the out-of-phase condition. Secondly, we note that if $\theta_S = \theta_I = \tilde{\theta}_I$ then $\beta = 0$ and $d_+ = d_-$. Again, the detection probability is negligible. This is a reasonable behavior for the detector to exhibit since if targets are at the same azimuth, they will not cause any azimuthal error except that due to fading [1].

Another interesting analytical result can be obtained in those cases where the interference has been completely overpowered by the target. In this case, $\rho_0 << 1$ and we can neglect the second term in (48) and use the Binomial Expansion to take the square root. This gives

$$d_{\pm} \approx A_S\, G_1(\theta_S)\, (1 + k^2\, \theta_S^2)^{1/2} \left[1 + \rho_0 \left(\frac{1 + k^2\, \tilde{\theta}_I^2}{1 + k^2\, \theta_S^2}\right)^{1/2} \cos(\varphi \mp \beta)\right] . \tag{51}$$

Using (49) it can then be shown that

$$\frac{d_+ - d_-}{2\sigma} = \frac{A_I\, G_1(\tilde{\theta}_I)}{\sigma}\, \frac{k(\theta_S - \tilde{\theta}_I)}{(1 + k^2\, \theta_S^2)^{1/2}}\, \sin\varphi \quad . \tag{52}$$

This result shows that when the target completely overpowers the interferer, the ability of the receiver to detect interference depends on the interference-to-noise ratio (INR). At first glance this is a somewhat puzzling result since McAulay [1] has shown that the azimuth accuracy depends on the signal-to-interference ratio (SIR) such that if the SIR is large, the azimuth estimate is unaffected by interference. Equation (52) indicates that if the INR is also large then the interference detector would ring. From a data editing point of view this would not be a desirable property. However, the detector is not really testing for the presence of interference since there is no inherent distinction between interference and target. Rather it is testing for the presence of more than one signal. If the SIR is very small, then (48) reduces to

$$\frac{d_+ - d_-}{2\sigma} = \frac{A_S\, G_1(\theta_S)}{\sigma}\, \frac{k(\tilde{\theta}_I - \theta_S)}{(1 + k^2\, \tilde{\theta}_I^2)^{1/2}}\, \sin\varphi \quad . \tag{53}$$

In this case, a large azimuth error would result since the azimuth of the strong interferer would be estimated. However, this situation would be flagged by the detector provided the target SNR were large enough. Therefore the detector performance in these extreme cases is intuitively satisfying, although the results indicate that some provision may have to be made for reducing the detections due to low level interference. We will discuss this point further in a later section.

To obtain a more complete understanding of the detector performance we need to evaluate (48) and (49) and use these with (39) and (37) to obtain the detection and false alarm probabilities. The results we obtain are based on the Gaussian approximation to the Rician densities. From (28) we see that this will be a reasonable assumption provided $d_{\pm}^2/2\sigma^2 \geqq 10$. From (48) we see that the smallest value of $d_{\pm}^2$ occurs when $\varphi \mp \beta = \pi$. Then

$$\frac{d_{\pm}^2}{2\,\sigma^2} \geqq \frac{A_S^2\ G_1^2(\theta_S)}{2\,\sigma^2}\,(1 + k^2\,\theta_S^2)\left[1 - \rho_o\left(\frac{1 + k^2\,\tilde{\theta}_I^2}{1 + k^2\,\theta_S^2}\right)^{1/2}\right] \geqq 10 \qquad (54)$$

By requiring that the second inequality hold, we have a conservative but sufficient condition to guarantee that (28) will hold. Therefore, for a post-detection SNR greater than 20 dB, this inequality will be satisfied for all signal-to-interference ratios except those in the region from -3 dB to + 3 dB. Since this is a conservative assumption and since we would not expect the detection probability to depend on the tails of the pdf, it is reasonable to expect that the Gaussian approximation will adequately describe the performance over an even smaller range of SIR values.

Over the duration of any one DABS reply, the parameters A_S, A_I, θ_S, θ_I are fixed. For mainbeam multipath, φ will also be constant from chip to chip. For the case of ATCRBS interference φ will change randomly from chip to chip, and in fact may also change between samples within a chip due to the frequency offset that can be expected between the DABS and ATCRBS transponders. In addition, we have found that the dependence on φ is crucial to the understanding of the data editing concept. Therefore, we have chosen to evaluate the detection probability as a function of φ.

b) Performance Based on a Single Sample

We have evaluated the performance of the receiver for some cases of interest. Our basic parameter values were taken to be the following:

$$\begin{aligned}
\theta_S &= 0 \text{ (target on boresight)} \quad , \\
SNR &= \frac{A_S^2 \, G_1^2(0)}{2\,\sigma^2} = 20 \text{ dB} \quad , \\
\theta_I &= 0.5 \text{ (interference at 3 dB point)} \quad , \qquad (55) \\
k &= 1.5 \quad , \\
G_1(\theta) &= 1 - 1.17\,\theta^2 \quad .
\end{aligned}$$

All we need do now is specify a false alarm probability, compute λ_{FA} from (37), compute $d_{\pm}/2\sigma$ from (48), (29) and use this to compute the detection probability in (39). In Figure 3 we have plotted the normalized mean value of the detection statistic $(d_+ - d_-)/2\sigma$ for several values of the ISR.

Since in the DABS direction finding (DF) problem there will be many bits available for generating the azimuth estimate, a higher false alarm rate can be tolerated in order to correctly detect interference samples. Therefore, we allowed a false alarm rate of 2 samples in 100, $P_{FA} = 0.02$, and then computed the detection probability for various values of the ISR.

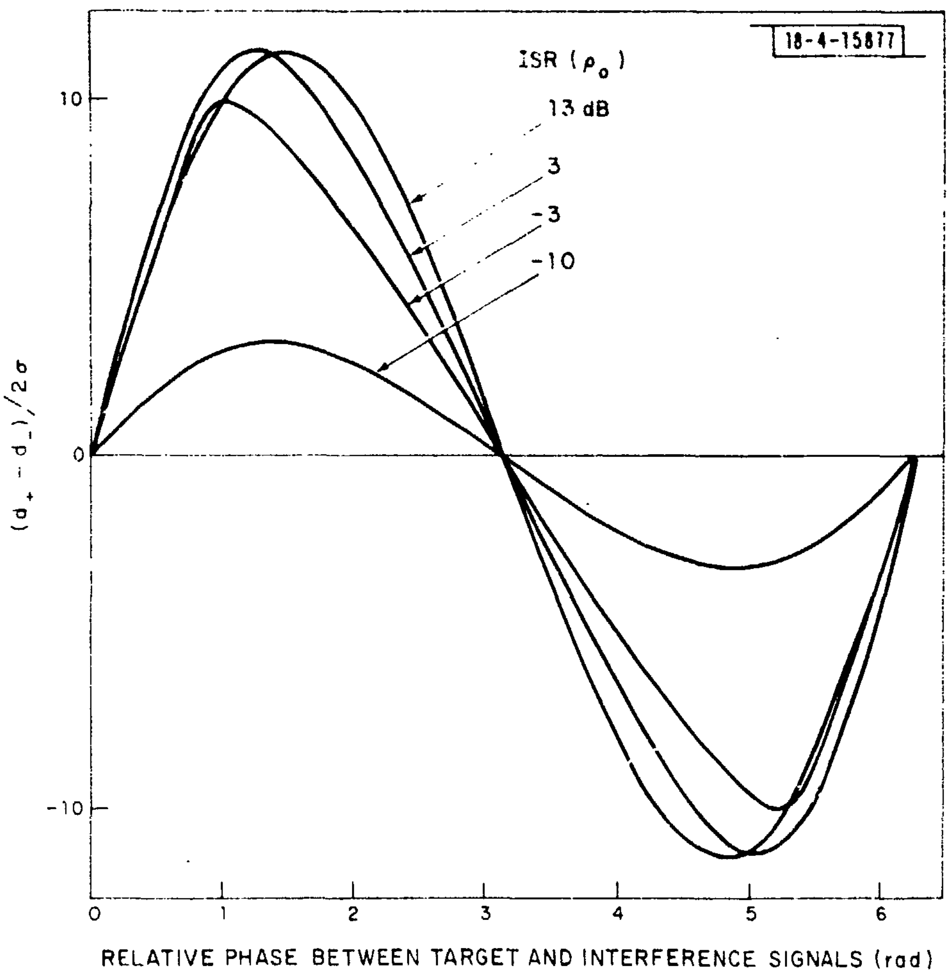


Fig. 3. Mean value of the detection statistic.

In Figure 4 we have plotted the miss probability $P_M(\varphi) = 1 - P_D(\varphi)$ as a function of the relative phase. As we expected, the detector misses interference with probability one when the in-phase and out-of-phase conditions exist. In the case of ATCRBS interference the relative phase is independent and uniformly distributed from bit to bit. Then a measure of performance of the detector is the average miss probability

$$\bar{P}_M = \frac{1}{2\pi}\int_0^{2\pi} P_M(\varphi)\, d\varphi \quad . \tag{56}$$

In Figure 4 we have indicated the average miss probability as a function of ISR for the 0.02 false alarm probability case.

c) The Effect of Frequency Offset

The preceding results give the detection performance when a decision must be made on the basis of a single sample. In the DABS context, there will be several samples available per chip for interference detection and direction finding. Unfortunately, if the interference is multipath, the phase will not change significantly from sample to sample, or even from reply to reply, hence the detector's performance will be essentially the same as that described in the last section. Hence there may be situations when azimuthal multipath will be present, but will not be detected by the interference flag.

In the case of ATCRBS interference, however, there will be a frequency offset between the DABS and ATCRBS transponders that can cause the instantaneous phase to change from sample to sample. For example, if Δf denotes the frequency difference between the two transponders and if f_s represents the rate at which the DABS waveform is to be sampled, then from sample to sample the phase increases

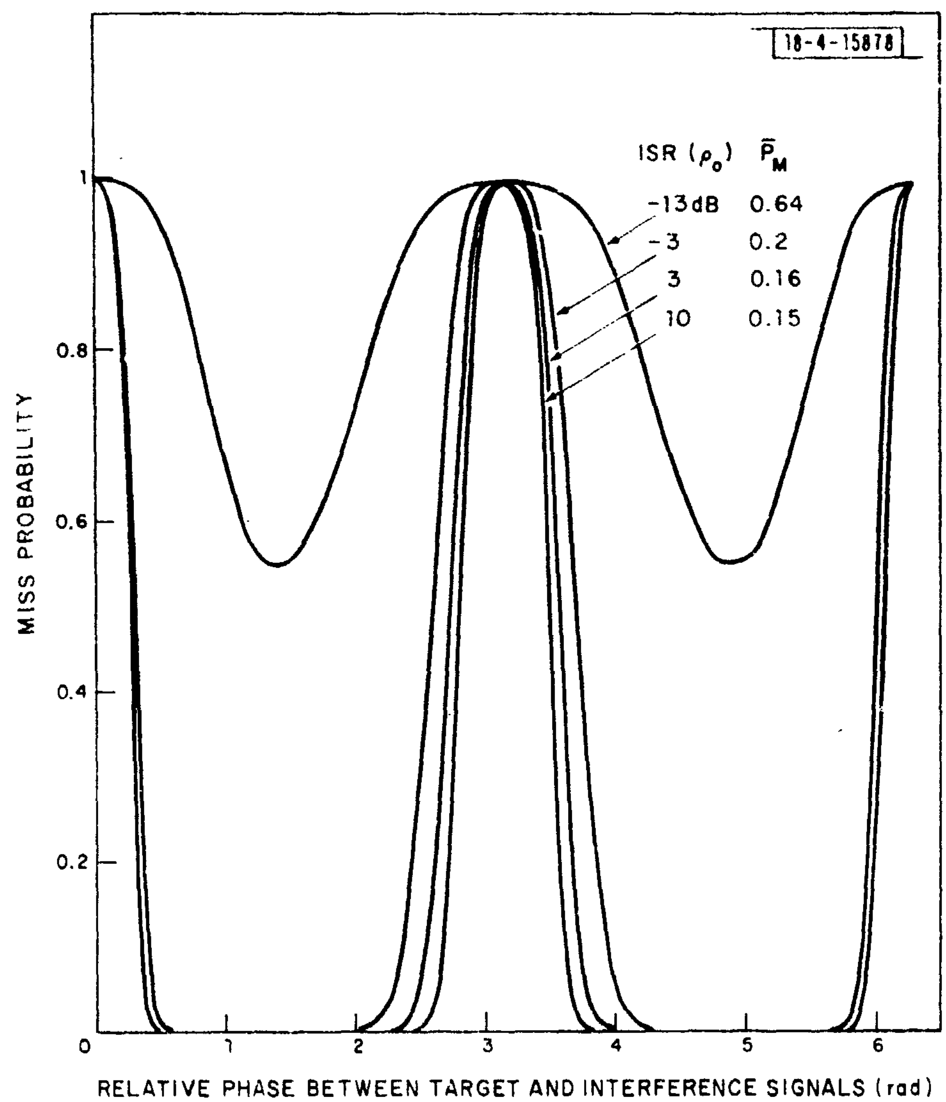


Fig. 4. Probability interference is not detected.

by $2\pi \Delta f/f_s$. Since the sampling is to be done at a 10 MHz rate, then a 0.5 MHz frequency offset* would lead to a 0.31 radian phase shift. From Figure 4 we see that the detector misses the interference when the phase is within a 1 radian interval about 0 or π. Therefore, with two or three additional samples we can expect the detector's performance to improve significantly. We can make these statements quantitative by considering the detection of interference using N samples per chip. Our strategy is to declare interference present if the detection threshold is crossed for at least one of the N samples. Then to yield a miss, the detector must fail to detect on every sample. If φ denotes the relative phase at the time of the first sample, then at the n^{th} sample it is $\varphi + (n-1)\,\Delta\varphi$. The single sample miss probability at this phase is denoted $P_{M_1}[\varphi + (n-1)\,\Delta\varphi]$. Then the miss probability after N samples is

$$P_{M_N}(\varphi) = \prod_{n=1}^{N} P_{M_1}[\varphi + (n-1)\,\Delta\varphi] \quad . \tag{57}$$

The product rule applies because the noise samples are independent from sample to sample. If we let T denote the chip width and f_s the sampling rate, then the number of samples per chip is $N = f_s T$, and the phase shift is $\Delta\varphi = 2\pi\,\Delta f/f_s$. For the DABS application we could expect $T = 0.483\ \mu sec$ and $f_s = 10$ MHz. Therefore 3 samples per chip represents a conservative evaluation. Using this value for N, we plot P_{M_N} as a function of frequency offset for several values of the ISR and an 0.02 false alarm rate. This is shown in Figure 5 and demonstrates the significant improvement in detection performance that can be expected as a result

*A distribution of frequency offset has been measured by G. Colby and E. Crocker and is documented in reference [9].

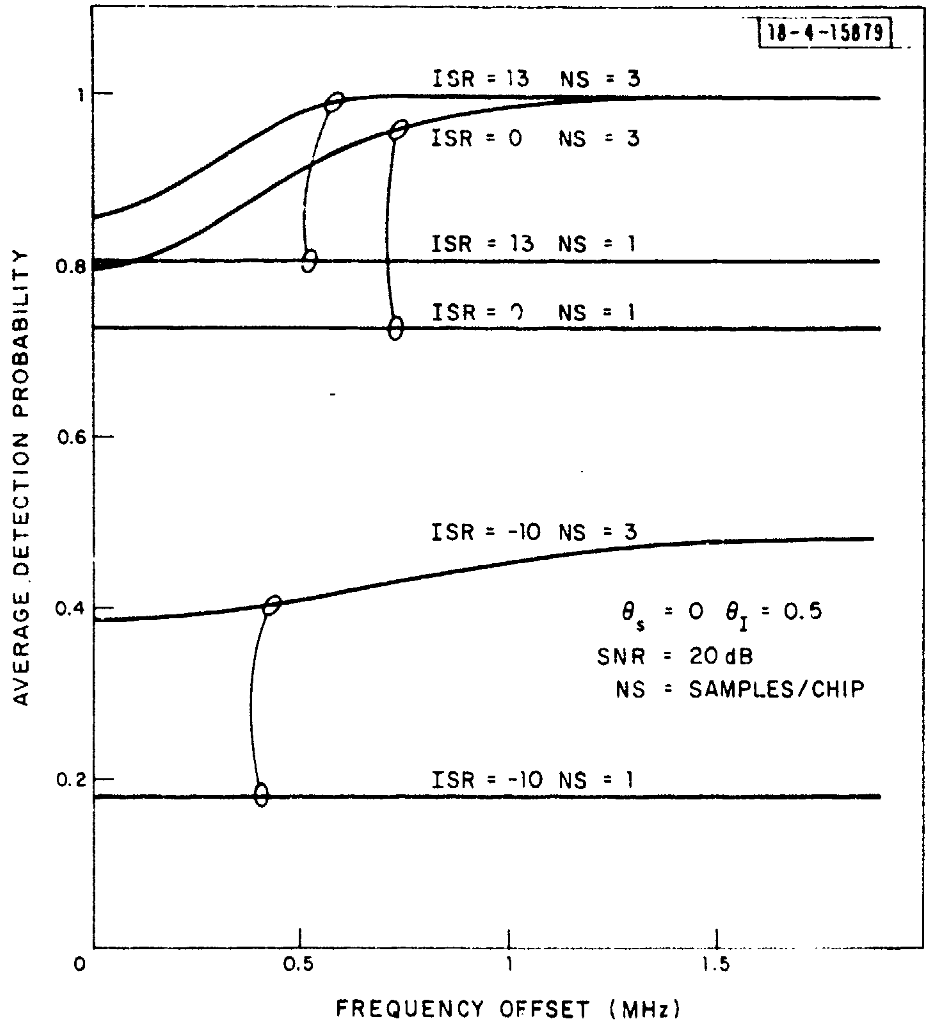


Fig. 5. Effect of frequency offset.

of the instantaneous phase change from sample to sample that results from the frequency offsets between a DABS and an ATCRBS transponder.

VI. CONCLUSIONS

The analysis of the performance of the maximum likelihood interference detector was greatly simplified by recognizing that the interference statistic could be well-approximated by a Gaussian random variable. A conservative condition on when the approximation was valid was found. For example, at a 20 dB SNR, the SIR would have to be within $\pm$ 2 dB for the approximation not to be valid.

False alarm and detection probabilities were calculated in detail for the cases in which the antenna patterns were real. Although formulae for the complex antenna pattern case were derived, it was not possible to obtain simple analytically useful results from them. We then restricted our attention to the case of real antenna patterns and obtained expressions from which it was possible to draw some useful conclusions. It was found that when the relative phase was 0 or π, the receiver would fail to detect the interference with probability one. It was also noted that the detectability improved as the azimuthal separation of the two signal sources increased. Furthermore, it was found that detectability depended on the signal-to-noise ratio of the weaker of the two signals. If this SNR was large enough, good detection was obtained no matter how large the other signal became.

Specific results for a mainbeam interferer were given and it was found that if only a single sample were used for detection that the average detection probability was approximately 0.8 for an 0.02 false alarm rate. This poor performance was due to the fact that misses were guaranteed when the phase differences were 0 or π.

In practice there will be several samples per chip available for interference detection which can result in meaningful improvements in performance provided there is an instantaneous phase change from sample to sample. Unfortunately, if the interference is due to multipath, there will be no significant change in phase for several seconds duration, hence multiple samples cannot be expected to improve the performance in this case. When the interference is due to ATCRBS, however, the probable frequency offset between the DABS and ATCRBS transponders will cause the phase to change from sample to sample. We have examined this case in detail and found that significant improvements in target detection performance can be expected.

APPENDIX

DERIVATION OF THE APPROXIMATE PDF

Let us focus our attention on the Rician pdf

$$p(u) = \frac{u}{\sigma^2} \exp\left(- \frac{u^2 + d^2}{2\sigma^2}\right) I_o\left(\frac{u\,d}{\sigma^2}\right) \quad . \qquad \text{(A-1)}$$

The well-known asymptotic approximation for the $I_o(\cdot)$ Bessel Function is

$$I_o\left(\frac{u\,d}{\sigma^2}\right) \sim \quad 2\pi\left(\frac{u\,d}{\sigma^2}\right)^{-1/2} \exp\left(\frac{u\,d}{\sigma^2}\right) \quad , \quad \frac{u\,d}{\sigma^2} \gg 1 \quad . \qquad \text{(A-2)}$$

This is quite an accurate approximation for $ud/\sigma^2 \geq 3$. Then the pdf becomes

$$p(u) \sim \left(\frac{u}{d}\right)^{1/2} \frac{1}{\sqrt{2\pi\,\sigma^2}} \exp\left[- \frac{(u - d)^2}{2\sigma^2}\right] \quad . \qquad \text{(A-3)}$$

We are most interested in the values of u about d. Let us write

$$u = d + \delta\, u \qquad \text{(A-4)}$$

then

$$p(u) \sim \left(1 + \frac{\delta\,u}{d}\right)^{1/2} \frac{1}{\sqrt{2\pi\,\sigma^2}} \exp\left(- \frac{\delta\,u^2}{2\sigma^2}\right)$$

$$= \left(1 + \frac{1}{2}\,\frac{\delta\,u}{d} + \ldots\right) \frac{1}{\sqrt{2\pi\,\sigma^2}} \exp\left(- \frac{\delta\,u^2}{2\,\sigma^2}\right) \quad . \qquad \text{(A-5)}$$

Therefore, u is well approximated by a Gaussian random variable with mean d and variance $2\sigma^2$ provided $ud/\sigma^2 \geqq 3$ and $\frac{1}{2} \frac{|\delta u|}{d} << 1$. When the Gaussian approximation is valid, 95% of the pdf lies between the 2-sigma limits. Therefore, if we require that

$$\frac{1}{2} \cdot \frac{2\sigma}{d} < 1 \quad , \tag{A-6}$$

then $\frac{1}{2} \frac{|\delta u|}{d} << 1$ "most of the time." As a practical matter we take

$$\frac{\sigma}{d} \leqq \frac{1}{\sqrt{10}} \tag{A-7}$$

as the criterion for which we can neglect first and higher order terms of δu that appear in (A-5). We must yet determine whether or not (A-7) is sufficient to validate the $I_0(\cdot)$ approximation that led to (A-5) in the first place. However, $u \geqq d - 2\sigma$ at least 97.5% of the time, hence

$$\frac{u\,d}{\sigma^2} + \frac{d^2}{\sigma^2}\left(1 - \frac{2\sigma}{d}\right) \geqq 10\left(1 - \frac{2}{\sqrt{10}}\right) = 2.7 \quad . \tag{A-8}$$

Therefore, the condition

$$\frac{d^2}{\sigma^2} \geqq 10 \tag{A-9}$$

is sufficient to guarantee that the Rician pdf for u will be well approximated by the Gaussian density of mean d and variance σ^2.

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