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The research conducted in this contract contributed several results in a number of active areas of study; these can be summarized as follows:

1. Residual deformation of real earth models.
2. Theoretical amplitudes of body waves from dislocation sources in the earth.
3. A unified approach to the representation of seismic waves.
4. Theoretical seismograms.
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TABLE OF CONTENTS

RESEARCH SUMMARY

I. Residual Deformation of Real Earth Models with Application to the Chandler Wobble.

By using Volterra's relation, it is shown that a tangential dislocation in a gravitating radially inhomogeneous sphere can be characterized by discontinuities in the stress and displacement fields across the source surface $r=r_0$. This representation of the source facilitates the numerical evaluation of the displacement field.

It is found that at the free surface of the earth the six simultaneous linear differential equations governing the spheroidal field associated with the Legendre polynomial of the first degree ($\ell=1$) degenerate into five. The two equations corresponding to the toroidal field for $\ell=1$ degenerate into one. Therefore, when dealing with the case $\ell=1$, one must incorporate additional conditions, namely, that the angular momentum of the sphere aobut its center is zero and that the center of mass of the sphere is not displaced.

The changes in the inertia tensor due to an earthquake model of arbitrary depth and orientation are calculated for the cases of neutral and unstable density stratifications in the core. The numerical results are similar in both cases. Comparison with homogeneous, non-gravitating earth model shows that, in general, real earth models render a smaller value for the changes in the inertia tensor. It appears from our results that earthquakes are insufficient to maintain the Chandler wobble. It thus seems that previous theoretical results are unacceptable.

Observations of far-field residual strains caused by the Alaskan earhtquake of March 28, 1964 opened new avenues to studies of earthquake sources. Immediately after this event, Press (1965) computed the residual displacement, strain and tilt fields for vertical, rectangular, strike-slip and dip-slip faults in a semi-infinite medium. Using this theory, Press showed, for the first time, that far-fields from major earthquakes are large enough to be detected by modern seismographs.

Mansinha and Smylie (1967) used Press' displacement functions to calculate the elements of the inertia-change-dyadic in a half-space model.

Ben-Menahem and Singh (1968) derived explicit expressions for the deformation of a uniform, non-gravitating sphere due to an internal Volterra dislocation of arbitrary orientation and depth. These results constitute the theoretical nucleus of a fundamental study by Ben-Menahem et al. (1969) in which the displacements and strains were given everywhere on the surface of a spherical earth model.

Ben-Menahem and Israel (1970) generalized the results of Ben-Menahem and Singh (1968) by calculating the residual displacement field at any point of a homogeneous, isotropic, non-gravitating, elastic sphere. The inertia changes due to earthquake and explosive sources were also calculated. It was shown that a spherical earth model yields higher inertia changes than the corresponding half-space model.

Smylie and Mansinha (1971) have computed the changes in the inertia tensor of a real earth model on the assumption that the Adams-Williamson condition does not hold. On the other hand they assumed that the radial component of the displacement has a jump across the core mantle boundary. Dahlen (1971) computed the inertia changes assuming the Adams and Williamson condition in the core. Although the numerical results of Smylie and Mansinha (1971) and Dahlen (1971) disagree, both agree that a real earth model increases the change in the inertia tensor in comparison to a homogeneous sphere.

In this paper, we first derive the Green's dyadic for the static deformation of the real earth. This dyadic is then used to obtain the explicit expressions for the displacements caused by an arbitrary tangential dislocation. However, these expressions are not convenient for numerical computation. Therefore, we find the jumps in the displacement and the radial stress vectors across the spherical surface $r=r_0$ passing through the source. Knowing these jumps the displacement field is easily computed numerically.

A detailed treatment is given for the case $\ell=1$. This case poses some problems since a sphere of finite radius cannot maintain a static equilibrium under the action of an unbalanced force system. This point was overlooked by Smylie and Mansinha (1971), whereas Dahlen (1971) did not treat the case $\ell=1$.

Assuming the validity of the Adams-Williamson condition, the change in the inertia tensor is calculated. Our results are smaller by an order of magnitude from Dahlen's results. Using the expressions of the displacement given in the present paper for a source located near the Moho discontinuity, it is shown that Dahlen's results are unacceptable.

Recently Pekeris and Accad (1972) have developed an asymptotic theory for the long-period bodily tides in real earth models. They found that in the case of unstable density stratification the stress tends to zero with diminishing frequency throughout the liquid core, except for a boundary layer of diminishing thickness near the core mantle boundary. Within the boundary layer the stress rises steeply from a near-zero value to a finite value. The conditions at the base of the mantle, obtained by these authors, are different from those obtained by Smylie and Mansinha (1971). We show that the jump in the derivative of the gravitational perturbation potential assumed by Smylie and Mansinha is inconsistant with their own assumption about the discontinuity of the radial displacement. Our suggested jump for the derivative of the potential together with the jumps of the radial displacement and stress assumed by Smylie and Mansinha (1971) are shown to be equivalent to the conditions of Pekeris and Accad (1972). The numerical results thus obtained, indicate that the validity of the Adams-Williamson condition does not have a significant effect on the changes in the inertia tensor.

II. Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - I. Core Reflections.

Expressions are obtained for the ray-theoretical spectral amplitudes of body waves induced by a shear dislocation of arbitrary orientation and depth situated in a radially heterogeneous model of the earth. Account is taken of the azimuthal and colatitudinal radiation patterns of the source, the geometrical spreading, and the reflections and refractions at the free surface and at the mantle-core boundary.

Spectral amplitudes are calculated for PcP , PcS , ScP , ScSV and ScSH . The results are presented in the form of tables for a source of strength $U_0 dS=10^{15} cm^3$, where U_0 is the amount of the dislocation and dS is the fault area. Given the slip and dip angles of the source, the amplitudes of the five core reflected phases can be obtained from these tables for all azimuths, for most of the epicentral distances at which a particular phase is observable, and for all the fourteen focal depths included in the Jeffreys-Bullen tables. It is found that the depth of the source has a strong effect on the amplitudes of the body wave signals.

It is the first time that detailed numerical results are given for the core-reflected body wave amplitudes for a realistic source in a realistic model of the earth. In most of the studies made so far, the asymmetry of the radiations from the source was not taken into account.

The results of computation for other important phases and the application of the theoretical amplitudes to source studies will be given in subsequent publications.

The method of spectral amplitude equalization of body waves is an important tool for determining source parameters from spectral analysis of isolated body wave signals. Ben-Menahem et al. (1965) demonstrated that source information can be extracted from the spectrums of P and S waves, in the period range 10-100 s , recorded with a network of stations around the source. Since then, the method has been applied by several investigators for source mechanism studies using P and S wave observations.

The observed amplitudes of the core reflected phases have been used for different purposes during the last few years. Kanamori (1967a) estimated the Q distribution for compressional waves in the mangle through a spectral analysis of P and PcP phases. In a later publication (Kanamori, 1967b), he determined the attenuation of short-period body waves from PcP , PcS , ScP and ScS observations. The core reflected body wave observations have also been utilized to determine the fine structure of the core-mantle boundary (Kanamori, 1967a; Buchbinder, 1968a,b).

The aim of the present paper is to obtain the ray-theoretical spectral body wave amplitudes induced by an arbitrary shear dislocation placed in a real earth model and to report the results of computation for the core reflected phases. Three factors have been taken into account: (i) the saymmetrical radiation pattern of the source, (ii) the reflection or refraction at the free surface and at the mantle-core boundary, and (iii) the geometrical spreading. In most of the theoretical studies made so far, the first factor has been ignored or eliminated (e.g., Gutenberg and Rechter, 1935; Dana, 1945; Martner, 1950; Ergin, 19[illegible]3; Kanamori, 1967a,b; Buchbinder, 1968a).

The radiation pattern function for a shear dislocation is taken from Singh and Ben-Menahem (1969). The unknown reflection coefficients are approximated by the corresponding reflection coefficients for a plane wave incident on a plane interface. These reflection coefficients are calculated from the closed form expressions. The divergence coefficient is calculated from the J-B Tables (Jeffreys and Bullen, 1967) by using a cubic spline interpolation method (Shimshoni and Ben-Menahem, 1970).

The spectral amplitudes of the core reflected phases are calculated for points on the top of the mantle. It is well known (Phinney, 1964; Ben-Menahem et al., 1965) that the response of different crustal structures differ considerably. Therefore, the results of computation at the free surface of the earth would have been of limited use. While comparing the theoretical amplitudes given in the present paper with the observations of ground motions, one can reduce the latter to the base of the crust by using Haskell's formalism (Haskell, 1960, 1962).

III. Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - II. Core Phases.

Expressions are obtained for the ray-theoretical spectral amplitudes of body waves induced by a shear dislocation of arbitrary orientation and depth situated in a radially heterogeneous model of the Earth. Account is taken of the azimuthal and colatitudinal radiation patterns of the source, the geometrical spreading, and the reflections and refractions at the free surface and at the ma le-core boundary.

In this work spectral amplitudes are calculated for PKP , PKS , SKP and SKS . The results are presented in the form of tables for a source of strength $U_0 dS=10^{15}cm^3$, where U_0 is the amount of the dislocation and dS is the fault area. Given the slip and dip angles of the source, the amplitudes of the four core phases can be obtained from these tables for all azimuths, for most of the epicentral distances at which a particular phase is observable, and for all the fourteen focal depths included in the Jeffreys-Bullen Tables. It is found that the depth of the source has a strong effect on the amplitudes of the body wave signals.

In the first paper of this series (Singh et al., 1972, Henceforth referred to as ABW-I), we gave theoretical amplitudes for the core-reflection phases PcP , PcS , ScP , ScSV and ScSH In the present paper we shall treat the various core phases without internal reflection at the core mantle boundary. The phases to be dealt with are PKP , PKS , SKP and SKS . The shear wave in the mantle for the phases treated in this paper is of the SV type, since no conversion between SH and K waves is possible at the core-mantle boundary.

For all the core phases in question there exists more than one branch; each branch has its own amplitude characteristic. In our results we give tables and plots for all branches mentioned in the Seismological Tables of Jeffreys and Bullen (1967) except for the branch PKP-BC which could not be handled reliably. At a few other places where we could not obtain reasonably reliable results for the amplitudes, we preferred to omit the amplitudes rather than give very uncertain values. Below we shall detail the cases where we omitted amplitudes and the reasons for doing so.

As an example of our reasoning let us mention here why the branch PKP-BC has been dropped. As we base all our work on the Jeffreys-Bullen times and the model which goes with them, we have a numerical reason for being unable to deal with this branch satisfactorily. For a surface focus the branch exists only for the short range 143°-147° . In this short interval the first difference drops from 3.0 s/° to 2.5 s/° . We felt that so few points did not give us reliable estimates of the factors $dt/d\theta$ and $d^2t/d\theta^2$, which play the leading role in all of our amplitude computations. On the other hand the uncertainties for this branch may be due to the characteristic of the transient region between the core and the inner core, a region of rapid changes not too well determined in the model on which the J-B Tables are based.

IV. Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - III. Surface Reflections.

Expressions are obtained for the ray-theoretical spectral amplitudes of body waves induced by a shear dislocation of arbitrary orientation and depth situated in a radially heterogeneous model of the Earth. Account is taken of the azimuthal and colatitudinal radiation patterns of the source, the geometrical spreading, and the reflections and refractions at the free surface and at the mantle-core boundary.

In this work spectral amplitudes are calculated for PP , PPP , PS , PSS , SP , SPP , SSH , SSSH , SSV , SSSV and P , SH , SV . The results are presented in the form of tables for a source of strength $U_0 dS = 10^{15} cm^3$, where U_0 is the amount of the dislocation and dS is the fault area. Given the slip and dip angles of the source, the amplitudes of these surface reflections and direct phases can be obtained from these tables for all azimuths, for most of the epicentral distances at which a particular phase is observable, and for all the fourteen focal depths included in the Jeffreys-Bullen Tables. It is found that the depth of the source has a strong effect on the amplitudes of the body wave signals.

In the first two papers of this series (Singh et al., 1972; Shimshoni et al., 1970, henceforth referred to as ABW-I, ABW-II respectively) we gave theoretical amplitudes for the core-reflection phases PcP , PcS , ScP , ScSV and ScSH and for the core phases PKP , PKS , SKP and SKS . In the present paper we shall treat the surface-reflection phases PP , PPP , PS , PSS , SP , SPP , SSH , SSSH , SSV and SSSV , as well as the direct phases P , SH and SV .

For all the phases treated here there exist two branches, due to the 20° discontinuity. Following the seismological tables of Jeffreys and Bullen (1967), the two branches are presented together for each phase. It should be noted that as a consequence of Snell's law, for the phases where P and S appear combined, i.e., PS , PSS , SP and SPP the cusp in the travel time table is a result of the 20° discontinuity of P , and not of S .

Although the phases PPS , PSP and SSP , SPS could have been readily dealt with, these were not included since these phases arrive at the same time for all focal depths, thus being of limited applicability.

V. A Unified Approach to the Representation of Seismic Sources.

An attempt is made to give a unified treatment to the problem of the representation of various sources commonly used in theoretical studies in seismology. Beginning with the Stokes-Love solution for a concentrated force, the displacement field due to a dipolar source in a homogeneous, isotropic, unbounded medium is expressed in terms of the eigenvector solutions of the vector Navier equation. This field is transformed to a spherical coordinate system having its origin at the centre of the Earth. The transformed field is then used to calculate the jumps in the displacements and stresses across the concentric spherical surface passing through the source. These jumps constitute a convenient representation of the source. Since it exhibits the properties of the source and not that of the medium, the above representation is also valid when the medium under consideration is bounded and inhomogeneous. A similar representation is obtained in the case of the circular cylinder coordinate system.

There are two fundamentally identical but practically distinct approaches by which one can represent a source in seismological boundary value problems. In the first approach, one uses the body force equivalent of the source. Consequently, the equation of motion becomes inhomogeneous due to the presence of the source term. If the medium under consideration is isotropic and homogeneous, or if the anisotropy or inhomogeneity is of a 'simple type', one may be able to solve the equation, for example, by using integral transforms. In the second approach, the source is removed from the equation of motion which becomes homogeneous, but appears, instead, as a 'source condition'. This source condition is the jump in the displacement and stress components across a coordinate surface passing through the source. Knowing these jumps, one can use the Thomson-Haskell matrix method (Haskell, 1953), the Pekeris method (Alterman et al., 1959) or any other suitable method to solve the problem under consideration. We shall discuss both approaches.

The plan of the paper is as follows. The next section recapitulates the solution of the Navier equation corresponding to a concentrated force. In the third section, we obtain the source term for various dipolar sources. In the fourth section, we obtain the representation of various sources in terms of the eigenvector solutions of the Navier equation. The fifth section is concerned with the second approach to the source representation. In this section, we obtain the jumps in the displacements and stresses across a spherical surface passing through the source for the spherical system and across a plane surface passing through the source in the case of the cylindrical system.

VI. Theoretical Seismograms by Realistic Sources in Flat Multilayered Earth Models - I. Source Coefficients and Path Integrals.

A variant of the Cagniard-Pekeris inversion technique is extended to non-symmetric earthquake sources of arbitrary multipolar order with a particular emphasis on shear dislocations. Aiming at the generation of theoretical seismograms for a stratified half-space we concentrate in the present work on the infrastructure of the time-domain displacement field in the presence of a single interface. A series of transformations in the complex plane together with a novel decomposition of the total field into three fundamental constituents enables us to separate the real-time contributions due to head waves, geometrical waves and the so-called non least-time arrivals.

Theoretical seismograms could become a powerful tool in the hands of seismologists who wish to study the fine structure of the earth's interior and the nature of earthquake origins. However, the usefulness of this tool depends on ones ability to simulate the conditions under which a real seismogram is formed. A reasonable starting point for a project of this kind is the production of seismograms from dislocation sources in multi-layered flat earth models. But even this modest undertaking is beyond the capability of contemporaty computers and the limited funds of most research groups. For this reason nobody as yet has succeded in generating on the computer anything that even resembles a real complete earthquake recording. Exact methods failed because of oversimplified source and structural models while approximate methods could only reproduce the initial portion of the seismic event. In two articles of which this is the first, we intend to present a new scheme of computation which is not exact yet takes into account relevant features of the medium and the source and is able to reproduce with reasonable speed, the main features of a genuine earthquake record.

VII. Computation of Models of Elastic Dislocations in the Earth.

This review is intended to acquaint physicists and geoscientists with the current state of research in the field of earthquake source mechanism. It will be assumed that the reader is familiar with the fundamentals of linear elasticity and that he also has a fair knowledge of the basic techniques of mathematical physics, such as integral transforms, differential equations, special functions, complex variable theory, tensor analysis and matrix theory.

We shall begin with a brief historical survey of seismology. We could, quite arbitrarily, divide its history into three intervals:

I.	1821 - 1891	The pre-seismograph	era.
II.	1892 - 1950	The pre-computer	era.
III.	1951 - ?	The pre-prediction	era.

The first period is distinguished for the intensive theoretical work done by mathematicians and physicists that laid the foundations to the mathematical theory of infinitesimal elasticity. Louis Navier derived in 1821 the differential equations of static and dynamic elasticity. In 1829, Simeon-Denis Poisson established the existence of longitudinal and transverse elastic waves. George Gebriel Stokes (1849) conceived the first mathematical model of an earthquake point-source. Somigliana (1885) produced a formal solution to Navier's equations for a wide class of sources and boundary conditions. In the same year Rayleigh (1885) predicted the existence of elastic surface waves. Voigt (1892) derived the stress-strain relations for a viscoelastic medium. Other important contributions in this period were done by Kelvin (1848), Lorintz (1861) and Betti (1872).

In 1892 the first seismograph for world-wide use was invented by John Milne and seismological observatories were set up on a global basis to record ground movements.

The second era is characterized by the availability of seismic data which motivated theoretical research. Simple models for earth structure were established and tested against the data. Knott (1899) derived general equations for reflection and refraction of plane seismic waves at plane boundaries. A.E.H. Love (1903, 1911) developed the fundamental theory of point-sources in an infinite elastic space and also gave a theoretical explanation to a new type of horizontally polarized surface waves which now bear his name. Horace Lamb (1904) laid the theoretical foundation for propagation of seismic waves in layered media. Soon after, Vito Volterra (1907) published his theory of dislocations based on Somigliana's work.

Concurrently, mathematicians and theoretical physicists in Europe were striving to discover new methods for tackling problems of radio-wave propagation. Among these were the method of steepest descent (Debye, 1909), integral relations among plane, spherical and cylindrical waves (Sommerfeld, 1909; Weyl, 1919), the Watson transformation (Poincare, 1910; Watson, 1918) and operational methods (Bromwich, 1916). Later, the quest for analytic solutions to problems of quantum mechanics gave birth to new asymptotic solutions of differential equations (Jeffreys, 1923; Langer, 1937), perturbation methods and variational techniques. All these methods were soon used by seismologists to solve problems of wave propagation in the earth.

Jeans (1923) was first to treat propagation of seismic waves in a spherical earth model. His work was continued by Sezawa (1927) and others. On the other hand, Jeffreys (1931), Smirnov and Sobolev (1932), Cagniard (1939), Lapwood (1949), Pekeris (1955) and Garvin (1956) used operational methods to solve problems of wave propagation from point and line sources in half-space configurations.

The third era is marked with two outstanding features: The developement of sensitive long-period seismographs and the increasing influence of the computer both on the choice of the problems and the methods of attack. In anticipation of the increasing role of computers in seismological theory, new methods were introduced to calculate dispersion, spectral amplitudes and theoretical seismograms for realistic source models in realistic earth structures.

This article is divided into two parts. In the first part

(Section II) we portray a summary of current theories of seis-mology. The second part (Sections III-V) include important computational techniques together with some of their applications to seismic-source studies.

VIII. A Note on the Calibration of the Electromagnetic Seismograph.

Explicit expressions are given for the response in the time domain of a zero-coupled seismometer-galvanometer combination for a step of acceleration as initial conditions. The results may be directly applied for computing the system parameters.

Many papers have already been written dealing with the calibration of an electromagnetic seismograph (Mitchell and Landisman, 1969; Espinosa, Sutton and Miller, 1962; Eaton, 1957; Hagiwara, 1958; Chakrabarty, 1949). The earlier methods of calibration had to rely on a single signal coil, and hence the methods of calibration consisted of tapping, seismometer release, seismometer displacement or galvanometer release or impulse.

The advent of the separate calibration coil through which a known current could be driven, and whose motor constant is known, meant that a new formulation was necessary. The method of supplying a sinusoidal known current of given period from an electronic function generator makes it possible to provide the amplitude response in the frequency domain directly. For a daily check on the instruments it is customary to drive a given current suddenly applied which causes a step in acceleration on the seismometer which is then recorded on a photographic drum. Thus, what we actually have is the system response in the time domain. What we would like to know is how to use this information to check on the instrument parameters and to make small corrections. Mitchell and

Landisman used the known relations in the frequency domain, transformed to the time domain and then used a least-squares procedure to determine the parameters of the instrument. Espinosa et al. obtained the parameters by comparing a small number of points in the time domain with a table of known responses.

It was seemed desirable to try to produce the explicit expressions for the time domain response. As will be shown later, if arbitrary coupling is included, it is necessary to use a numerical solution for the roots of a quartic, whereas for zero coupling, it is possible to obtain the time domain response in closed form.

Chakrabarty in his formulation came to the conclusion that the coupling term was actually small and could therefore be neglected and hence we do not feel too bad about doing the same procedure, especially as for example the WWNSS long-period instruments are supposed to be zero-coupled and critically damped.

On additional approach that was tried (Teng and Ben-Menahem, 1965) was to determine the frequency response by taking the step response in the time domain and Fourier analysing it. They found that because of the ω^2 term, inaccuracies were introduced into the high-frequency part of the spectrum, and hence they used the expression for a critically damped, zero-coupled seismograph.

LIST OF PUBLICATIONS PERTINENT TO THIS CONTRACT

(1) Residual Deformation of Real Earth Models with Application to the Chandler Wobble. A. Ben-Menahem, M. Israel and S.J. Singh. Geophys. J. (in press).

(2) Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - I. Core Reflections. A. Ben-Menahem, M. Shimshoni and S.J. Singh. Phys. Earth Planet. Interiors, 5, 231-263.

(3) Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - II. Core Phases. M. Shimshoni, Y. Silman and A. Ben-Menahem. Phys. Earth Planet. Interiors (in press).

(4) Theoretical Amplitudes of Body Waves from a Dislocation Source in the Earth - III. Surface Reflections. Y. Silman, M. Shimshoni and A. Ben-Menahem. Phys. Earth Planet. Interiors (in press).

(5) A Unified Approach to the Representation of Seismic Sources. S.J. Singh, M. Vered and A. Ben-Menahem. Proc. Roy. Soc. London (in press).

(6) Theoretical Seismograms by Realistic Sources in Flat Multilayered Earth Models - I. Source Coefficients and Path Integrals (in press).

(7) Computation of Models of Elastic Dislocations in the Earth. A. Ben-Menahem and S.J. Singh. In Methods of Computational Phys., 12, 1972.

(8) A Note on the Calibration of the Electromagnetic Seismograph. H. Jarosch and A.R. Curtis (in press).

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