

AD-763 330

EXPERIMENT ON RELATIVISTIC RIGIDITY  
OF A ROTATING DISK

Thomas E. Phipps, Jr.

Naval Ordnance Laboratory  
White Oak, Maryland

30 April 1973

DISTRIBUTED BY:

**NTIS**

National Technical Information Service  
U. S. DEPARTMENT OF COMMERCE  
5285 Port Royal Road, Springfield Va. 22151

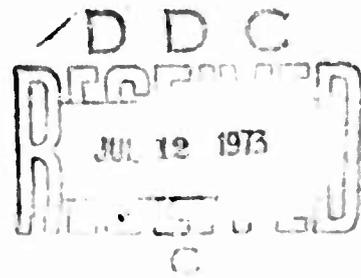
NOLTR 73-9

AD 763330

EXPERIMENT ON RELATIVISTIC RIGIDITY  
OF A ROTATING DISK

By  
Thomas E. Phipps, Jr.

30 APRIL 1973



NOL

NAVAL ORDNANCE LABORATORY, WHITE OAK, SILVER SPRING, MARYLAND

APPROVED FOR PUBLIC RELEASE;  
DISTRIBUTION UNLIMITED

NOLTR 73-9

NATIONAL TECHNICAL  
INFORMATION SERVICE

68

R

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Ordnance Laboratory White Oak, Silver Spring, Maryland		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP NONE	
3. REPORT TITLE EXPERIMENT ON RELATIVISTIC RIGIDITY OF A ROTATING DISK			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Final			
5. AUTHOR(S) (First name, middle initial, last name) Thomas E. Phipps, Jr.			
6. REPORT DATE 30 April 1973	7a. TOTAL NO. OF PAGES 65 68	7b. NO. OF REFS 33	
8a. CONTRACT OR GRANT NO.  b. PROJECT NO  c. MAT-03L-000/ZR00-001-010  d.		9a. ORIGINATOR'S REPORT NUMBER(S) NOLTR 73-9	
		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) -	
10. DISTRIBUTION STATEMENT APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.			
11. SUPPLEMENTARY NOTES -		12. SPONSORING MILITARY ACTIVITY Chief of Naval Material Naval Material Command Headquarters Department of the Navy Washington, D. C. 20360	
13. ABSTRACT Several theories have suggested the possibility that the space-time metric of a rotating disk could be nonstatic as well as non-Euclidean. These, and also a theory of optical aberration based on the Thomas precession of vectors, predict a (real or apparent) decrease in angular velocity with radius, even in an ideally "rigid" disk, with a rim lag time per period of $\alpha v^2/c^2$ , where $v$ is rim speed, $c$ is light speed, and $\alpha$ is a constant between 1 and 1/6, depending on the theory. A spinning-disk experiment was done at the Naval Ordnance Laboratory (NOL) over a four-month period of continuous rotation to look for such effects. Observations on a small stainless steel disk establish that $\alpha = (-1.90 \pm 3.36) \times 10^{-4}$ and that almost surely $ \alpha  < 6 \times 10^{-4}$ . Consequently the existence of either kinematic or optical effects of rim lag of physically (approximately) rigid structures is highly unlikely. Implications for the problem of logical consistency of relativistic extended-structure kinematics are discussed.			

UNCLASSIFIED

Security Classification

ia

**UNCLASSIFIED**

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Relativity Kinematics Rigidity Rotating Disk Thomas Precession Aberration Non-static Metric						

*ii*

EXPERIMENT ON RELATIVISTIC  
RIGIDITY OF A ROTATING DISKPrepared by:  
T. E. Phipps, Jr.

ABSTRACT: Several theories have suggested the possibility that the space-time metric of a rotating disk could be nonstatic as well as non-Euclidean. These, and also a theory of optical aberration based on the Thomas precession of vectors, predict a (real or apparent) decrease in angular velocity with radius, even in an ideally "rigid" disk, with a rim lag time per period of  $\alpha v^2/c^2$ , where  $v$  is rim speed,  $c$  is light speed, and  $\alpha$  is a constant between 1 and 1/6, depending on the theory. A spinning-disk experiment was done at the Naval Ordnance Laboratory (NOL) over a four-month period of continuous rotation to look for such effects. Observations on a small stainless steel disk establish that  $\alpha = (-1.90 \pm 3.36) \times 10^{-4}$  and that almost surely  $|\alpha| < 6 \times 10^{-4}$ . Consequently the existence of either kinematic or optical effects of rim lag of physically (approximately) rigid structures is highly unlikely. Implications for the problem of logical consistency of relativistic extended-structure kinematics are discussed. The most immediately significant outcome of the experiment is that the nonexistence of the optical analogue of the Thomas precession casts doubt on the Thomas effect as physics, and suggests the need for further experimentation. Resolution of this question will be necessary to permit meaningful interpretation of the behavior of a high-precision gyroscope in a long-term earth satellite orbit. As with much basic research, the relevance to Navy interests is to be found in the questions raised, rather than in those directly answered.

NAVAL ORDNANCE LABORATORY  
WHITE OAK, MARYLAND

30 April 1973

**Experiment on Relativistic Rigidity of a Rotating Disk**

The work reported here, supported under Foundational Research Task No. MAT-03L-000/ZR00-001-010, is part of a systematic effort to clarify by experimental observations certain issues arising in the relativity theory of accelerated extended structures. These issues affect, for example, the operation or observation of high-speed gyroscopes that must for purposes of accurate navigation maintain extreme precision over very long times. Whereas the Navy has no current operational requirement for long-term automated precise navigation, a future requirement of this nature is conceivable.

The author wishes to acknowledge the cooperation of many individuals at the Naval Ordnance Laboratory and elsewhere whose assistance enabled the observations to be made expeditiously and at low cost. Specific acknowledgments appear on pages 50-51.

ROBERT WILLIAMSON, II  
Captain, USN  
Commander

  
G. K. HARTMANN  
By direction

CONTENTS

	Page
CHAPTER 1 - THEORY.....	1
INTRODUCTION.....	1
KINEMATIC PRELIMINARIES.....	2
NONSTATIC METRICS.....	8
SPT RIGIDITY.....	13
OPTICAL EFFECT.....	17
WOULD PRIOR OBSERVATIONS HAVE REVEALED AN EFFECT?.....	20
SUMMARY.....	21
CHAPTER 2 - EXPERIMENT.....	23
DISK DESIGN CONSIDERATIONS.....	23
Temporal Resolution.....	24
Spatial Resolution.....	24
Improved Figure of Merit.....	25
Flash Illumination.....	25
Photographic Parameters.....	26
Operating Conditions.....	27
Aerodynamic Drag.....	27
Available Torque.....	28
Disk Radius Optimization.....	29
Rotor Radial Cross-section.....	31
Disk Material.....	33
DESCRIPTION OF EXPERIMENT.....	34
RESULTS.....	38
SOURCES OF ERROR.....	43
LESSONS OF TECHNIQUE.....	43
CHAPTER 3 - IMPLICATIONS.....	45
IMPLICATIONS FOR THEORY.....	45
IMPLICATIONS FOR FURTHER EXPERIMENTATION.....	48
ACKNOWLEDGMENTS.....	50
REFERENCES.....	52

ILLUSTRATIONS

Figure	Title
1	"Signal-To-Noise" Figure of Merit $F_2$ Vs. Rotor Frequency
2	Figure of Merit $F_1$ Vs. Rotor Frequency
3	High-Speed Rotor Cross-Section
4	Schematic of Pressurized Gas Supply to Turbine
5	Sketch of Laser Illumination Arrangement, Approximately to Scale
6	Flash Photograph of Disk Stationary Before Run
7	Flash Photograph of Disk in Motion at 6116 RPS After Four Months of Continuous Rotation
8	General View of Apparatus Near End of Run
9	Close-up of Disk Showing Tachometer Light Reflecting Off its Surface to Dental Mirror; X-Y-Z Positioner Micrometers Visible in Background

TABLES

Table	Title	Page
1	Relativity Effect Constant $\alpha$ For a Rotating Disk, Where (Lag Time/Period) = $\alpha v^2/c^2$ , According to the Tentative and Speculative Nonstatic Metric Theories of Several Authors.....	10
2	Raw Data on Line Separation From Negatives of Figures 6 and 7, as Measured with Traveling-Stage Microscope.....	40

## Chapter 1

## THEORY

INTRODUCTION

1. The present report describes a small physics experiment done recently at NOL in an observationally little-explored field, the relativistic properties of extended structures. Specifically, observations were made on a high-speed rotating disk to determine (a) its kinematic rigidity properties, (b) whether the disk metric was static or nonstatic, and (c) any possible optical aberrational effects at the  $v^2/c^2$  level, related to the Thomas effect accompanying circular motion.

2. The remaining sections of this chapter establish motivation for doing the experiment. In a word the experiment represents an integral part of a systematic attempt to clear up conceptual difficulties concerning the kinematics of extended structures and "rigid bodies," which have plagued special relativity since its inception. These difficulties have never shown any sign of diminishing, despite the steady accumulation of an extensive specialized theoretical literature aimed at that objective. The experiment to be described clears up very cheaply (for less than \$2000, plus the investigator's time) speculations that have lain unresolved in the theoretical literature for over 25 years and continue to form the subject matter of theoretical disagreements.

3. In informally declining to fund this project, the Office of Naval Research questioned its "relevance." Some words on this subject may therefore be in order. It is true that even at the breathtaking speeds promised by Surface Effect or Semi-submersible Ships the operating Navy will not be troubled by kinematic problems arising from the Lorentz contraction. Also the Navy is well aware that even its sturdiest ships possess no fewer than six degrees of freedom, and so will not be impressed by the revelation that relativity theorists have for over sixty years conceded to the "rigid body" no more than three degrees of freedom--even at zero speed. Efforts to clear up such rabbinical pilpul offer no obvious payoff to the Navy. But plans have been made to spend large amounts of government money to put a superprecise gyroscope into a long-term satellite orbit around the earth in order to test hypotheses connected with general relativity--a more fashionable, hence perhaps more relevant, field. In interpreting results, assumptions will be made not only about general relativistic effects but about special relativistic  $v^2/c^2$  effects, such as the Thomas precession of a

gyroscope carried repeatedly around a reentrant trajectory. Anyone who has to do such interpreting would be well advised to take note of the present report and related materials mentioned in paragraphs 46-50. These raise some interpretational questions that can hardly be ignored by people concerned with understanding whatever data may emerge from such a satellite experimentation program. They might even be persuaded that some preliminary laboratory experiments would be cost-effective.

4. Whenever a small experiment exerts leverage on much larger-scale national enterprises (as most really basic inquiries aimed at improving understanding do), it is hardly relevant to question relevance. The proper question is where do the interest and capability reside. In the present case interest is not a trivial prerequisite, since, as mentioned, the issues dealt with have been in the public domain for over a quarter-century without action from the scientific community, several members of which recently declined the writer's invitation to do the experiment in their more prestigious institutions. It makes sense to do experimentation in basic but unpopular research fields in a government in-house laboratory, because in that environment the publish-or-perish pressures, which elsewhere seem to reinforce tendencies to fadism in basic science, are at a minimum. This is a tentative thought that lacks scholarly documentation.

5. Apart from such generalities, there were several specific reasons for doing this experiment specifically at NOL. The experiment depended on laser flash photography, a field in which NOL is a pioneer in developing techniques, with equipment available off the shelf. It also depended on a reliable, sustained source of clean, dry, inert, pressurized gas for driving a high-speed air bearing air turbine (see Chapter 2). This was available from the dry nitrogen high pressure bottle field maintained for the NOL wind tunnel, a facility of a type not found at every research institution. Auxiliary equipment needed for the experiment was also plentifully available at a laboratory of NOL's size, and could be borrowed on a no-cost basis. The combination of these assets made NOL uniquely suited for doing the experiment at minimum cost and effort.

#### KINEMATIC PRELIMINARIES

6. Kinematics is concerned with the motional freedoms available to points and to extended structures. Webster's dictionary defines it as "the science which treats of motions considered in themselves, or apart from their causes." All "objects" in kinematics are therefore massless pure geometrical constructs\*; and a kinematic "rigid body" or "metric standard" is any such object that lacks internal degrees of freedom.

\*Not all authors agree on this terminology. For example Rohrlich, reference (a), admits mass into kinematics. This erodes the distinction between kinematics and dynamics, which we shall rigorously preserve here.

7. Special relativistic kinematics deals successfully with the motions of points, described by single world lines in Minkowski space; reduces properly (with one minor exception discussed in paragraph 49) to Newtonian point particle kinematics in the low-speed limit,  $v^2 \ll c^2$ ; and is in every respect confirmed by observation.

8. The special relativistic kinematics of extended structures, described by correlated groups of world lines, is in much less satisfactory condition. The existing theory of the relationships among particles at space-like separations admits no consistent purely kinematic concept of rigidity; therefore fails to reduce to Newtonian rigid body kinematics in the limit  $v^2 \ll c^2$ ; and is in no respect confirmed by laboratory observations, since it is not feasible to impart relativistic speeds to extended structures. Special relativity thus neatly divides into two parts, one confirmed experimentally and the other unconfirmed.

9. Are there reasons to question the logical links binding the two parts of the theory? Warning signals on this point began to appear early in the history of the subject. The first indication of kinematic problems connected with extended structures was the Ehrenfest paradox (reference (b)). It suggested that a laboratory observer could measure no Lorentz contraction of the rim of a disk set into rotation, contrary to the widespread opinion that all kinematic objects set into relative motion are measured by a Lorentz observer as being contracted in the direction of their motion. This was followed by the remarkable independent deductions of Herglotz (reference (c)) and Noether (reference (d)), who showed that a seemingly natural definition of relativistic "rigidity" due to Born (reference (e)) produced a non-physical result: The Born-rigid body possessed only three degrees of freedom, instead of the six needed to yield Newtonian rigid body kinematics in the limit  $v^2 \ll c^2$ . That is, given the world line of any single point in the body, the world lines of all points were determined. One therefore avoided the Ehrenfest paradox for the "rigid" disk by denying the disk any kinematic capability to change its rotational state.

10. There seem to be three general methods of dealing with the "crisis" created by this unexpected\* failure of relativistic kinematics to yield the right number of degrees of freedom in the nonrelativistic limit. Method I accepts that the failure is unavoidable and that in fact the "rigid body" represents an impermissible idealization in physics. That is, on pain of logical contradiction, it is forbidden conceptually to "freeze out" the internal degrees of freedom of a structure, or to make corrective allowances for what degrees of freedom remain in quasi-rigid physical structures, as is done in classical physics. According to Pauli (reference (f)) and most physicists who have followed him, "rigid bodies" are not allowed, but "rigid motions" are (a baffling deduction, since kinematics is

\*Einstein in 1907 wrote to Ehrenfest that the special theory concerned only idealized rigid bodies.

about nothing but motions). To explain the fact that physical structures exhibit six degrees of freedom, presumably at all speeds, Method I invokes internal degrees of freedom, via Herglotz stresses (reference (g)), later sometimes called Dewan-Beran stresses (reference (h)), material dilatations (reference (i)), etc.

11. The possibility of kinematic rigidity is thus rejected and is replaced by a sort of "kinematic elasticity." This is anomalous because kinematics is not concerned with causes of motion, hence not with forces, stresses, material properties, acoustic characteristics, etc., which normally lie in the province of dynamics or rheology. To restore the missing degrees of freedom, Method I appeals to the larger corpus of physics, and thus admits and accepts that there exists no purely kinematic solution to the kinematic problem. (Reference (i) states that "... the relativistic kinematics for extended bodies is not generally self-consistent. The solution of the [Ehrenfest] paradox ... is intrinsically dynamical ...") In short one is forbidden to study the possibilities of motions "in themselves, or apart from their causes," within a purely geometrical context, as Einstein originally set out to do.

12. The chief objection to Method I is that by depriving relativity theory of "rigid bodies" one has thereby deprived it of extended structure metric standards, and has thus forced increasing reliance on light signals for the mensuration of both space and time. Since one has been coerced in this direction by the threat of logical contradiction, it is nontrivial to observe that logical circularity then results from Einstein's second postulate of lightspeed constancy. Method I therefore puts the theory of extended structures into deep logical troubles, which resonate unpleasantly with the total lack of observational support of this portion of relativity.

13. Worse still, certain experimental observations suggest that as physicists we may have been hasty in discarding "rigid bodies". In Compton scattering from heavy atoms, for instance, the effective mass is that of the atom, not the electron, so the atomic structure as a whole accomplishes the scattering and responds as if "rigid." (I am indebted to Dr. R. G. Newburgh for this observation.) More dramatically in the Moessbauer effect the body mass is effective, so the lattice "as a whole" takes up the gamma-ray recoil momentum. Here a macroscopic structure behaves as if physically "rigid". Thus hints from nature as well as logic cast doubt on the mountain of literature [much of it skillfully reviewed in references (i) and (f)] devoted to Method I.

14. Now, if physical structures exhibit six degrees of freedom at all speeds, it would seem that there thus exists a kinematic attribute independent of speed. Consequently the permissibility of the corresponding kinematic idealizations should also be speed-independent. One should hence not have to invoke forces or other extra-kinematic concepts to formulate or solve the kinematic problem at any speed. Methods II and III pursue this line of optimism. It is not conceded that no purely kinematic solution to Einstein's

original problem of 1905 exists. Instead, it is permitted to look outside the realm of theoretical formulations Einstein examined, subject only to the restraint that the experimentally confirmed portion of the theory -- viz., whatever is describable by the Lorentz transformation as applied to single world lines -- be rigorously kept intact. These methods aim at recovering Newtonian rigid body kinematics in the low-speed limit and at sacrificing none of the idealizations inherent in low-speed kinematics. Being thus deliberately more geometrical and less "physical" in content than Method I, they conform more closely to the spirit of Einstein's original inquiry.

15. In introducing these other methods it should in honesty be said that they are not part of the accepted "art," but are more or less the penchant of the writer -- a fact that counsels caution to the reader. Moreover, the entire subject area tends to be devalued or ignored by most physicists, including relativity authorities. For instance a book on Paradoxes in the Theory of Relativity (reference (j)) appeared in Moscow in 1966 in which the author, a respected Soviet nuclear and cosmic ray physicist, makes no mention of Ehrenfest, Herglotz - Noether, rigidity, or rotation. Similarly, the editors of the Physical Review consider special relativity so cut-and-dried that they have paired it in their subject index with "Acoustics." We shall nevertheless persevere as if the fundamental research interest of the subject transcended acoustics.

16. Method II employs a non-Minkowskian geometrical representation of events, based on "space-proper-time" (SPT). The approach is to try to extend the single-particle theory of reference (k) to the multi-worldline case. The starting point of this theory is the recognition that the Lorentz transformations, which keep invariant the form  $d\tau^2 = dt^2 - \frac{1}{c^2} (dx^2 + dy^2 + dz^2)$ , preserve also the equivalent form

$$dt^2 = d\tau^2 + \frac{1}{c^2} (dx^2 + dy^2 + dz^2).$$

The latter suggests the geometrical interpretation that  $dt$  is an element of distance in a Euclidean 4-space, referred to as SPT, whose axes are labeled by  $(x/c, y/c, z/c, \tau)$ . It is shown in reference (k) that for a single particle the resulting SPT kinematics fully reproduces Minkowski space kinematics and that the two are related by a simple geometrical convention. It is this single-particle portion of the theory, as we have remarked, that is observationally confirmed.

17. In order to extend SPT theory in the simplest way to the description of extended structures it is necessary to make a physically dubious, but not hitherto observationally refuted hypothesis: viz., that there exists such a thing as the proper time of a structure. That is, the structure is presumed to possess an inherent physical integrity or oneness, mathematically expressible through the assignment of a single descriptive proper time parameter  $\tau$  to all of its individual parts. (Such an hypothesis has been applied to the classical finite-sized electron [reference (a)], but not, to the writer's knowledge, to structures in general.)

18. It turns out that this hypothesis has observational consequences, worked out in Section 3 of this chapter. Suffice it to say that kinematic rigidity in SPT implies kinematic elasticity in Minkowski space, and vice versa; so one cannot by purely theoretical considerations conclude that a logical failure of the rigidity concept in Minkowski space necessarily implies a failure of rigidity in SPT. The geometry of event representation is intimately linked with available kinematic freedoms. A rotating disk that is rigid on hyperplanes of constant  $\tau$  in SPT exhibits in Minkowski space a radial dependence of angular velocity — i.e., an "elasticity" on hyperplanes of constant laboratory time  $t$ , such that an initially straight radial line progressively curves backwards (as long as rotation continues) at a rate proportional to  $v^2/c^2$ . This implies an observable "kinematic lag," increasing quadratically with radius and linearly with time. Alternatively, it may be said to imply a non-static disk metric in Minkowski space, though a static one in SPT.

19. Method II thus treats the Ehrenfest problem by the hypothesis that Born rigidity applies in SPT rather than in Minkowski space. It represents a sort of "last stand" for the traditional relativistic parameterization, based on application of the same homogeneous Lorentz transformation to all parts of an extended structure. It merely makes an essentially trivial transfer of the theater of application of this familiar parameterization to a new geometrical 4-space. The game is played by the same rules but in a different space.

20. Method III adopts a much more radical approach. It notes that the problem of getting six degrees of freedom, instead of three, into the description of extended structures in Minkowski space is one of getting extra mathematical parameters into the formalism. There is one obvious, untapped source of extra parameters, compatible with preserving the single-particle theory. This involves exploiting the 10-parameter inhomogeneous Lorentz transformation group, instead of relying — as all current formulations do — on the 6-parameter homogeneous group. To contemplate freeing the "bound degrees of freedom" by a change in the basic parameterization of relativity theory (such that each particle of a structure is assigned its own "private" space-time origin, designated by four new parameters, independently assigned by each Lorentz observer in such a way as to provide for him a kinematically consistent "rigidity") courts several varieties of heresy. (I need mention only that the Lorentz contraction need not occur.) It has drastic implications for our understanding of metric properties.

21. The basic philosophy of Method III is clear: For  $N$  particles at spacelike separations (as von Laue, reference (1), first pointed out) there are  $4N$  degrees of freedom in space-time, and anyone who wants to describe a collective with 6 degrees of freedom can obviously do so by judiciously discarding ("freezing out") all but six, instead of all but three. One falls into the "all but three" trap only by taking the popular short-cut of choosing the same space-time origin for all particles, thus wantonly discarding all the translatory parameters of the full (inhomogeneous) Lorentz group. This approach (of not discarding too many parameters) sounds very attractive, even

inevitable, but has such remarkable consequences that a short exposition is impossible without an appearance of "anti-relativism" — which would incur the wrath both of amateurs and of that rather plentiful brand of experts in whose eyes long familiarity with their territory has transformed even its ugliest features into beauty spots.

22. Method III in effect calls for a "revolution of parameterization." In a sense the 1905 Einstein revolution was also one of parameterization, inasmuch as its most radical feature was the assignment of separate space-time parameters to each inertial frame. If one also needs to assign separate space-time origin parameters to each particle within a frame, the implications of this second-stage revolution could be equally far-reaching.

23. In summary, Method I of dealing with the Ehrenfest paradox and related problems, by abandoning kinematics in favor of dynamics, is popular because it lets the physicist do his thing. But it overlooks the fact that the kinematic problem, though in a sense part of physics, is primarily geometrical and logical in nature. It is in fact truly pre-physical. Hence all abandonments of kinematics leave its logical problems not only unsolved but unaddressed; thus courting circular reasoning in a (metrical) part of the theory that is supported by logic alone, not by observation.

24. Method II, the approach primarily treated in this report, proposes a different (non-Minkowskian) geometrical formulation for the simple description of extended structures. It involves no basic change in parameterization, but merely transfers the scene of "Born rigidity" from Minkowski space to space-proper-time. Its virtue is that it submits itself to experimental judgment.

25. Method III is genuinely revolutionary at the basic parametric level. It involves abandoning the 6-parameter homogeneous Lorentz group for the 10-parameter inhomogeneous group, in order to obtain a consistent relativistic kinematics that properly reduces to Newtonian rigid body kinematics in the low-speed limit.

26. Methods I and III are not observationally distinguished, since I claims to be, and III is, compatible with straight lines on a rotating disk remaining straight to all orders in  $(v/c)$ . Method I encounters severe logical difficulties (insolvable in purely kinematic terms) in explaining how a real disk can thus appear to be "Born rigid," when the physical existence of any Born rigid object is considered to be logically forbidden. Method III neatly clears up this logical difficulty by shifting the ground rules of problem parameterization, but it has other implications that would make it ideologically unpopular with relativity dogmatists. One does not want to launch into Method III lightly.

27. Since Method II is observationally distinguished from the others, clearly the rational first order of business is to dispose of it. This means doing a rotating disk experiment and observing whether

or not straight lines on the disk stay straight. In the following sections we discuss additional motivations for doing this kind of experiment, elaborate on the predictions of Method II, and mention an interesting optical aberrational effect related to the Thomas precession, which might also be observed in the experiment. That will conclude our attempts at motivating experimentation. The second chapter will describe the experiment, as done, and its results. The final chapter will examine the implications with reference to the present discussion.

### NONSTATIC METRICS

28. The notion that the Minkowski space metric of a rotating disk should be non-Euclidean (curved) in its spatial part, which is orthogonal to an uncurved time part, was originally endorsed by Einstein and has become incorporated in the standard lore of relativity. It is consistent with the Method-I picture of a stressed rotating disk, relative to which comoving unconstrained meter sticks contract. There has been a persistent literature, much of it summarized in references (m) and (n), to which more recently reference (o) may be added, compatible with the idea that, if any curvature at all occurs (contrary to the necessary vanishing of the covariant Riemann curvature tensor in all frames of reference in any "flat space" such as that of the laboratory), this curvature ought to occur in the full space-time sense and should therefore include the time dimension. In other words, if the theory is going to contradict itself, it ought to do so in approved space-time symmetrical style.

29. From this arises the hardy perennial idea of the ~~time-dependent~~ metric. Most people who have this idea are quickly embarrassed by it and end up indicating their skepticism (cf. reference (p)). Skepticism is certainly in order about many points in the theory, this one no more than some others. For if in a certain space-time region there were a spatial curvature detectable by some observers and not by others (a truly insane proposition which would have given headaches to Gauss and Riemann), it is at least a consistent brand of insanity to ask that it be a space-time curvature.

30. One cannot leave this topic of the "non-Euclidean nature of the disk geometry" without posing a few questions for those physicists who are convinced that "the space of the disk is curved." When did space start belonging to structures? Where are the limits of this curved space ... one millimeter above the disk surface? Or is it the material of the disk, rather than "space," that is curved? Curved into what, a fourth spatial dimension? The writer has become persuaded that all of the contradictory verbalism about curvature in a flat space originates in bad kinematics. Nevertheless, let us march on as if no doubts existed.

31. We consider typical reasoning that might lead to a nonstatic metric hypothesis. The classic example of a relativity effect involving single particles is the "coming back late" of a charged particle in circular orbit in a high-energy accelerator. If we confine

attention to low speeds,  $v^2 \ll c^2$ , the time late per period is  $(1/2)v^2/c^2$ . Normally, this tardiness is attributed to mass increase of the particle; but the lag time is actually mass-independent and depends only on particle speed. Suppose we therefore think of the effect as kinematic. Then it should not have to do with forces, but should apply to any particle or group of particles, moving at orbital speed  $v$ , whether inter-particle forces are present or not. By this reasoning the particles at radius  $r$  could be replaced by a rotating ring of solid material, each particle of which "comes back late" by the prescribed lag time. One might then consider  $r$  variable and contemplate a series of concentric rings comprising a disk-like structure. Could the rings fuse together and form a solid disk, or would they require separation for mutual slippage? (Recall that  $v^2/c^2$  differs at each radius, so we have trouble imparting a common angular velocity.) It is difficult to answer, without some basis in observation. The question is, which internal degrees of freedom of the would-be "structure" can be legitimately frozen out. The writer does not at the present time know any method to answer this by pure reason. Some other students of the subject acknowledge a similar incapacity. For example in a comparable instance Arzeliès (reference (n), p. 238) states, "Only experimental confirmation can provide a possible justification of these hypotheses, which are effectively definitions of the rotating body." In any case, if the rings are fused together without affecting the kinematic lag at each radius, it is clear that an initially straight radial line on the disk surface will curve backwards quadratically with radius and linearly with time. The resulting effective radial dependence of angular velocity, as measured in the laboratory, is

$$\omega(r) = \omega(0) \left\{ 1 - \frac{1}{2} \frac{r^2 \omega^2(0)}{c^2} + O\left[\left(\frac{r\omega(0)}{c}\right)^4\right] \right\}, \quad r^2 \omega^2(0) \ll c^2. \quad (1.1)$$

The implications of a relation such as (1.1) for the nonstatic nature of the metric will be discussed presently.

32. Various nonstatic metric theories are referenced in Table 1. No attempt will be made to derive these authors' results here, since the reader can consult the indicated references. However, Takeno's result is interesting and an elementary discussion of it will illustrate the type of considerations that can enter.

33. Consider several circular disks in various states of rotation about a common central axis. We might expect their relative angular velocities to possess a group property, just as colinear relative translational velocities do (reference (m), p. 33). In fact this is the case; but the further natural expectation that the group additivity law be independent of disk radius is wrong, if the usual assumption of Born rigidity of the disks,

$$u_i(r) = r\omega_i, \quad \omega_i = \text{constant}, \quad i = 1, 2, \dots, \quad (1.2)$$

is valid, where  $u_i$  is the tangential speed of a point on the  $i^{\text{th}}$  disk at radius  $r$ . For we see at once from the Einstein composition law of tangential speeds at radius  $r$  that

Table 1

RELATIVITY EFFECT CONSTANT  $\alpha$  FOR A ROTATING DISK, WHERE (LAG TIME/  
PERIOD) =  $\alpha v^2/c^2$ , ACCORDING TO THE TENTATIVE AND SPECULATIVE  
NONSTATIC METRIC THEORIES OF SEVERAL AUTHORS

Author	Reference	Effect <sup>1</sup> Constant $\alpha$
Rosen	(p)	1/2
Hill	(q)	1/2
Present Work	This Section [Eq. (1.1)]	1/2
Present Work	Paragraphs 39-41 [Eq. (1.25)]	1/2
Takeno	(r)	1/3
Weinstein	(o)	1/6
Born, Herglotz, and all others	(e), (c), ...	0

<sup>1</sup>A positive value of  $\alpha$  indicates a lag or retardation in the sense opposite to that of disk rotation.

$$u_3(r) = \frac{u_1(r) + u_2(r)}{1 + \frac{u_1(r) \cdot u_2(r)}{c^2}}, \quad (1.3)$$

whence

$$\omega_3 = \frac{\omega_1 + \omega_2}{1 + \frac{r^2 \omega_1 \omega_2}{c^2}}, \quad (1.4)$$

a law that shows an r-dependence of one of the  $\omega$ 's. Therefore, as long as Born rigidity applies, the group additivity law cannot take the simple form

$$F(\omega_3) = F(\omega_1) + F(\omega_2), \quad (1.5)$$

but must take the more restricted form

$$F_R(\omega_3) = F_R(\omega_1) + F_R(\omega_2). \quad (1.6)$$

[It is easily verified that the necessary function of form (1.6) to agree with (1.4) is

$$F_R(\omega_i) = \ln \left[ \frac{1 + \frac{r\omega_i}{c}}{1 - \frac{r\omega_i}{c}} \right], \quad i=1,2,3.$$

This result can be derived, for example, by treating  $\omega_1$  as infinitesimal and integrating the angular velocity analogue of the Fresnel drag law,

$$\omega_3 = \omega_2 + \left(1 - \frac{r^2 \omega_2^2}{c^2}\right) \omega_1 + O(\omega_1^2),$$

which follows from (1.4).]

34. Under Born rigidity it is thus shown that there exists no radius-independent group property of relative angular velocities. Since Born rigidity is known (Herglotz - Noether) to introduce logical contradictions, we might discard (1.2) and ask what law would replace it if we postulated radius-independent group properties, equation (1.5). In this case (1.2) must be generalized to

$$u_i(r) = r \omega_i(r), \quad i = 1, 2, \dots \quad (1.7)$$

We shall confine attention to a special case; viz., that in which (1.5) takes its simplest form,

$$\omega_3 = \omega_1 + \omega_2, \quad (1.8)$$

and in which  $u_i(r)$ , though no longer equal to  $r \omega_i$ , is at least some function of  $r \omega_i$ . From the latter requirement and (1.7) it follows that

$$\omega_i(r) = \omega_i f(r\omega_i), \quad f(0) = 1, \quad \omega_i(0) = \omega_i. \quad (1.9)$$

In the relationship

$$\omega_1 f(r\omega_1) = \frac{\omega_1 f(r\omega_1) + \omega_2 f(r\omega_2)}{1 + \frac{r^2}{c^2} \omega_1 \omega_2 f(r\omega_1) f(r\omega_2)},$$

which follows from (1.3), we treat  $\omega_1$  as infinitesimal and obtain

$$\omega_1 f(r\omega_1) = \omega_2 f(r\omega_2) + \omega_1 f(r\omega_1) \left[ 1 - \frac{r^2}{c^2} \omega_2^2 f^2(r\omega_2) \right] + O(\omega_1^2).$$

Using (1.8) in this, we get

$$\frac{\omega_2 \left[ f(r(\omega_1 + \omega_2)) - f(r\omega_2) \right]}{r\omega_1} = - \frac{f(r(\omega_1 + \omega_2))}{r} + \frac{f(r\omega_1)}{r} \left[ 1 - \frac{r^2}{c^2} \omega_2^2 f^2(r\omega_2) \right] + O(\omega_1^2).$$

Letting  $\omega_1 \rightarrow 0$ , with  $r\omega_2 \equiv x$ , we have

$$\frac{df(x)}{dx} = - \frac{f(x)}{x} + \frac{f(0)}{x} \left[ 1 - \frac{x^2}{c^2} f^2(x) \right].$$

Multiplying by the integrating factor  $f^{-2} x^{-1} dx$ , using  $f(0) = 1$ , and letting  $u = x^{-1} f^{-1}$ , we obtain

$$du = dx \left( u^2 - \frac{1}{c^2} \right),$$

which integrates to yield Takeno's result,

$$\omega_i(r) = \frac{c}{r} \tanh \left( \frac{r\omega_i}{c} \right), \quad i = 1, 2, 3. \tag{1.10}$$

For a real disk  $r\omega_i \ll c$ , and we have approximately

$$\omega_i(r) \approx \omega_i \left\{ 1 - \frac{1}{3} \left( \frac{r\omega_i}{c} \right)^2 + O \left[ \left( \frac{r\omega_i}{c} \right)^4 \right] \right\}, \tag{1.11}$$

from which follows the factor  $\alpha = \frac{1}{3}$  in Table 1.

35. Thus the simple group additivity property (1.8), postulated to apply independently of radius, requires that the disk exhibit a "kinematic elasticity" in Minkowski space, which manifests itself as a radial dependence of angular velocity. The minus sign in (1.11) means that this is a lag increasing quadratically with radius, as in the previous case, equation (1.1).

36. Suppose the radial line to be marked with centimeters. As rotation continues, the line, in the view of a laboratory observer, curves increasingly and stretches out. Hence (most rapidly near the disk rim), the local "centimeter" is changing its definition with time. One therefore terms the metric nonstatic. But what does a nonstatic metric mean? Whose metric is changing? The laboratory observer's? He has a perfectly serviceable static, flat-space metric for everything he could want to measure. The disk rider's? Obviously not. (He is being stretched right along with his metric, so he can detect no time change of his centimeter.) The metric "of the disk"? Though such terminology is used loosely throughout this report, we have already raised questions about it. On reflection, it seems that the "metric" that is changing is one that is in some sense attributed\* by the laboratory observer to the disk-riding observer. If so, it is a rather peculiar hybrid. Perhaps it is not ridiculous to contemplate that such a "metric" could change in time. For it appears to be the metric of an uninhabited world -- inasmuch as no observer's instruments record a time change of his own space's metric properties.

37. Table 1 indicates that a number of theoretical variations have been played on this theme. All predict "kinematic lag" rates proportional to  $v^2/c^2$ . There is one rare saving grace in these theories: they submit themselves to experiment. For whatever "non-static metric" may mean to anybody, an alleged curvature of a radial line means something definite to the laboratory observer: to wit, he should go into his laboratory and observe.

#### SPT RIGIDITY

38. In this section, following the Method II just discussed, we examine implications of the hypothesis that the rotating disk is rigid in space-proper-time (SPT). Our analysis will not be restricted to uniform angular velocity, but will consider arbitrary time variations. We shall not repeat the single-particle SPT analysis of reference (k), which the interested reader can consult. Suffice it to say that SPT is rightly a "private" space of the individual particle, because proper time, used as a coordinate in that space, is unique to the particle. In supposing that a common proper-time parameter can be assigned to all particles of an extended macroscopic structure, we are admittedly making a questionable hypothesis -- but one not explicitly refuted by past observations, to the writer's knowledge.

\*Berenda, reference (s), distinguishes "intrinsic" from "relative" geometries, but to name is not to explain.

39. Suppose the disk's central angular velocity is known as some function  $\omega_0(t) \geq 0$  of laboratory time  $t$  (equivalent to proper time  $\tau$  at the disk center). Consider a radial line  $L$  on the disk that is straight when the disk is at rest in the laboratory. Let the rotation start at time  $t = 0$ , with  $\omega_0(t) = 0$  for  $t < 0$ . At some laboratory time  $t > 0$ , we inquire through what azimuthal angle  $\phi(r, t)$  the line  $L$  has turned in SPT. The answer depends on radius as well as time. At the center,  $L$  has turned through an azimuthal angle

$$\phi(0, t) = \int_0^t \omega_0(t') dt'. \quad (1.12)$$

At radius  $r$  the angle is

$$\phi(r, t) = \int_0^{\tau(t)} \frac{d\phi}{d\tau} d\tau, \quad (1.13)$$

where

$$\tau(t) = \int_0^t dt' \sqrt{1 - \frac{v^2(t')}{c^2}}, \quad (\tau(t) \leq t), \quad (1.14)$$

is the proper-time coordinate value in SPT associated with laboratory time  $t$ . The condition of disk rigidity in SPT requires that the "proper angular velocity,"  $d\phi/d\tau$ , be a constant independent of radius, in order that radial straight lines in SPT remain straight lines on all hyperplanes of given  $\tau$ . The constant angular velocity in question must be equal to  $\omega_0(\tau) = \omega_0(t)$ , the central angular velocity in both SPT and Minkowski space. That is,

$$\frac{d\phi}{d\tau} = \omega_0(\tau) \quad (\text{SPT Rigidity Condition}). \quad (1.15)$$

Using (1.15) in (1.13), we have

$$\phi(r, t) = \int_0^{\tau(t)} \omega_0(\tau) d\tau = \int_0^{\tau(t)} \omega_0(t') dt'. \quad (1.16)$$

Comparing (1.12) with (1.16) and noting (1.14), we see that  $\phi(r, t) \leq \phi(0, t)$ , so the rim lags the center. This is true in the laboratory as well as in SPT, because Minkowski space and SPT are identical in their spatial dimensions and differ only in the orthogonal time dimension. We note that  $L$  is straight in SPT only on hyperplanes of constant  $\tau$ . Our interest centers on what the appearance of  $L$  is at the instant of laboratory time  $t$ , such as might correspond to simultaneous illumination of all parts of  $L$  by a flash of light; so  $L$  curves in SPT on a surface of constant  $t$  -- and by the previous remark curves identically in Minkowski space.

40. The azimuthal angle of lag at radius  $r$  relative to the center at laboratory time  $t$  is

$$\begin{aligned} \Delta\phi(t) &\equiv \phi(o,t) - \phi(r,t) && (1.17) \\ &= \int_0^t \omega_0(t^1) dt^1 - \int_0^{\tau(t)} \omega_0(t^1) dt^1 \\ &= \int_{\tau(t)}^t \omega_0(t^1) dt^1 . \end{aligned}$$

If we define the quantity

$$\Delta t \equiv \int_0^t dt^1 \left[ 1 - \sqrt{1 - \frac{v^2(t^1)}{c^2}} \right] , \quad (1.18)$$

which by (1.14) obeys  $\tau(t) = t - \Delta t$ , then (1.17) can alternatively be written

$$\Delta\phi(t) = \int_{t-\Delta t}^t \omega_0(t^1) dt^1 . \quad (1.19)$$

This is our basic result for the angular lag observable in the laboratory.

41. We now proceed to make a few approximations that will put (1.19) into more usable form without impairing its practical validity. Let us consider  $r$  to be the radius of the disk. If the rim speed  $v(t^1)$  is at all times very small compared to  $c$ , then  $\Delta t \ll t$  and we can neglect fourth and higher powers of  $v/c$ , so

$$\Delta t = \int_0^t dt^1 \left[ 1 - \left( 1 - \frac{1}{2} \frac{v^2(t^1)}{c^2} + \dots \right) \right] \approx \frac{1}{2c^2} \int_0^t v^2(t^1) dt^1 .$$

To the same order of accuracy  $v(t^1) \approx r\omega_0(t^1)$ , and

$$\Delta t \approx \frac{r^2}{2c^2} \int_0^t \omega_0^2(t^1) dt^1 . \quad (1.20)$$

If, during the very short time interval  $(t - \Delta t)$  to  $t$ , the instantaneous central angular velocity  $\omega_0(t^1)$  is essentially constant, (1.19) reduces approximately to

$$\begin{aligned} \Delta\phi(t) &= \omega_0(t) \int_{t-\Delta t}^t dt' = \omega_0(t) \Delta t \\ &\approx \frac{r^2}{2c^2} \omega_0(t) \cdot \int_0^t \omega_0^2(t') dt' . \end{aligned} \quad (1.21)$$

If we define a root mean square angular velocity by

$$\omega_{\text{rms}}(t) \equiv \sqrt{\frac{1}{t} \int_0^t \omega_0^2(t') dt'} , \quad (1.22)$$

then (1.21) may be expressed as

$$\begin{aligned} \Delta\phi(t) &\approx \alpha \frac{r^2}{c^2} \omega_{\text{inst}}(t) \cdot \omega_{\text{rms}}^2(t) \cdot t , \\ \alpha &= \frac{1}{2} , \end{aligned} \quad (1.23)$$

where we have written  $\omega_{\text{inst}}(t)$  for  $\omega_0(t)$  to emphasize the distinction between instantaneous and rms quantities. The lag distance at the rim is  $r\Delta\phi$ , and the time lag there is

$$t_{\text{lag}} = \frac{r\Delta\phi(t)}{v(t)} \approx \frac{r\Delta\phi(t)}{r\omega_{\text{inst}}(t)} = \alpha \frac{r^2}{c^2} \omega_{\text{rms}}^2(t) \cdot t . \quad (1.24)$$

After one period of rotation we have

$$(\text{Lag time/period}) = \alpha \frac{r^2}{c^2} \omega_{\text{rms}}^2(t) , \quad \alpha = \frac{1}{2} , \quad (1.25)$$

and if we consider the rim speed  $v$  as an rms value,  $v(t) = r\omega_{\text{rms}}(t)$ , then  $\alpha = \frac{1}{2}$  is the  $\alpha$ -value listed in Table 1.

42. Equation (1.23) is the fundamental relationship on which the experimental design, discussed in Chapter II, is based. If the angular velocity is roughly constant over the duration  $t$  of the experiment,

$$\omega_{\text{inst}} \approx \omega_{\text{rms}} \approx \omega , \quad (1.26)$$

then

$$\Delta\phi(t) \approx \alpha \frac{r^2}{c^2} \omega^3 t , \quad (1.27)$$

which is also a useful relationship for some purposes.

43. The fact that  $\omega_{inst}(t)$  is a factor in (1.23) means that if the disk stops rotating then  $\Delta\phi \rightarrow 0$ ; i.e., the lag disappears as the rim "catches up" almost instantly. Actually, equation (1.19), the exact relation, shows there is a "catchup time"  $\Delta t$ , but this is of the order of microseconds in a practical experiment. It corresponds to the aging difference between a "space-traveling twin" on the rim, who stays young, and a "stay-at-home twin" at the center, who ages differentially by amount  $\Delta t$  during the unwinding of the curve L. (That is, at the moment the rim stops the traveler's age is  $\tau(t)$ , whereas the center observer's age is  $t = \tau(t) + \Delta t$ . In the laboratory the center stops a time  $\Delta t$  before the rim does.) Thus our description is compatible with the usual differential-aging result of the twin paradox, as it must be according to the analysis of reference (k).

44. Suppose that  $\omega_{inst}$  is constant for a very long time, then for a short time dips down nearly to zero, but not actually to zero, and then is restored to its former value. Equation (1.23) establishes the remarkable result that this temporary speed decrease has almost no effect on the observed curving of L. During the dip in angular velocity, the lag angle  $\Delta\phi$  proportionally decreases. But after the dip the factor  $\omega_{inst}$  returns to its original value, and  $\omega_{rms}$ , as given by (1.22), is little affected because the total rotation duration  $t$  is large and a short-term notch in the integrand alters the value of the integral very little. Hence  $\Delta\phi$ , after the dip, is restored practically to its value just before.

45. In short, as long as rotation continues, the disk displays a kinematic "memory" for what it has "learned" by past rotation. The agent of this memory is the integral in equation (1.22). This observation is of great practical usefulness, because it means that a temporary speed decrease of the disk does not necessitate restarting the experiment.

#### OPTICAL EFFECT

46. An internal NOL publication concerning a possible  $v^2/c^2$  optical effect, termed the "Thomas aberration," deals in detail with the matter to be discussed in this section. This information is available from the present author on request. A short summary follows.

47. Most texts in treating the aberration of light consider only the simplest case of a source in uniform translatory motion. In the case of light reflected from a rotating disk the source (any point on the disk surface) is continuously accelerated; so one would not be surprised to find in addition to the normal first-order ( $v/c$ ) aberration also something appearing at the  $v^2/c^2$  level. What is perhaps surprising is that the  $v^2/c^2$  effect turns out theoretically to be time-cumulative (the surprise may be reduced by observing that

$v^2$  is direction-independent); so that eventually it overpowers the first-order effect and should be detectable even where first-order aberration is completely negligible.

48. The basis for this remarkable possibility is an argument by strict analogy with the derivation (reference (t)) of the Thomas precession of spins. From the group properties of the Lorentz transformation it is known that successive "parallel-axis" Lorentz transformations with nonparallel velocities produce nonparallelism of axes. This means a turning or precession, relative to fixed laboratory axes, of axes judged coparallel by any observer performing circular motion in the laboratory. From this geometrical or kinematic turning of axes, Thomas (reference (u)) in 1926 inferred a precession of physical electron spin vectors in the laboratory, and from this a precession energy needed to correct the imperfect spectroscopic theory of the time. Since then virtually all physicists have considered this physical turning of the spin vector in the laboratory to be observationally confirmed.

49. However, the following reservations are in order:

a. The axis precession is torque-free, hence without visible physical source of energy.

b. The torque-free turning of a physical vector, as distinguished from coordinate axes, violates both Newton's laws (in the Newtonian limit of low-speed circulation for a very long time) and Mach's Principle (according to which the fixed stars, which do not precess, determine local inertial properties, of which spin angular momentum would seem to be one).

c. The Dirac electron theory, which appeared soon after Thomas's work, cleared up all problems of spectroscopic energy without appeal to geometrical pictorializations, such as were involved in Thomas's model of the electron as a vector-bearing object carried continuously around a circuit.

d. The quantum theory of measurement, though hardly a thing of beauty, is probably correct in casting doubt on any physical view of an electron spin as "carried around" a circuit. Rather, the spin direction is considered to be created in or by the act of its measurement, and prior to that act no meaning is conceded to "spin direction."

50. For these and other reasons an agonizing reappraisal of Thomas's "effect," including experimental work to test directly (rather than indirectly by inference from spectroscopic energy evidence) the geometrical aspect of Thomas's claim that spins change their directions in the laboratory under torque-free conditions, seems long overdue. There is no question that the coordinate axes of a disk-riding observer precess relative to the laboratory axes. But from this the great leap to an assertion that physical vectors

similarly precess is fraught with perils that any physicist by definition must recognize. The physical vector has to precess relative either to the laboratory observers or the disk comoving observers. But that it must be relative to the former, and not the latter, is not a deduction from relativity theory but a new physical postulate at variance with all previous postulates. The justification that "it works" (with reference to spectroscopy) may have satisfied Thomas's contemporaries, but it should not satisfy today's sophisticates. For we have the example of Sommerfeld's fine-structure theory to remind us that the Dirac electron made obsolete certain other "relativistic" artifacts that had "worked" in the preceding era.

51. Setting aside such caveats and accepting at face value the Thomas precession of physical vectors, we observe that this precession ought to apply not only to a spin vector but to any physical vector, such as the  $\mathbf{k}$ -vector ( $|\mathbf{k}| = 2\pi/\lambda$ ) of light propagation produced, say, by flash illumination of the disk surface. The  $\mathbf{k}$  vector is not physically "carried around" continuously, anymore than the spin vector is -- but if the same formal component-transformation law is applied a similar precession results. By considering the geometry it is easily shown that, like the total angle of Thomas spin precession, the total angle of turning of the  $\mathbf{k}$ -vector increases linearly with time and quadratically with tangential speed or radius of the radiation source point on the disk surface. The effect of this turning resembles first-order aberration in that it produces an apparent image displacement. But the time- and radius-dependent nature of the effect implies a progressive apparent curving of an initially straight radial line, which is observationally indistinguishable from effects already discussed as possible "kinematic lags." It can be shown that after rotation for time  $t$  at uniform angular velocity  $\omega$  the total apparent azimuthal lag angle of the optical image of a source point at radius  $r$  is

$$\Delta\psi_{\text{opt.}} = \alpha_{\text{opt.}} \frac{r^2}{c^2} \omega^3 t, \quad (1.28)$$

$$\alpha_{\text{opt.}} = \frac{1}{2}.$$

This agrees in functional form and algebraic sign with the kinematic lag effects discussed in the preceding section [compare equation (1.27)], and even agrees in magnitude with several of the theories listed in Table 1.

52. The optical effect, here called "Thomas aberration," described by equation (1.28), differs from the kinematic effects only interpretationally. It is interpreted as an apparent image displacement, as seen by a telescope, eye, or camera that views the disk from along its axis; whereas the kinematic effects are interpreted as "real" in the same sense that Lorentz contraction is real.

Neither, either, or both the optical and kinematic effects may exist. If both exist they will add, and the resulting net value of the effect constant is

$$\alpha_{\text{total}} = \alpha + \alpha_{\text{opt.}} \leq 1. \quad (1.29)$$

All effects are in the direction of a lag (retardation opposite to the sense of disk rotation), so an observed total absence of line curvature will unambiguously refute all curvature theories discussed in this chapter.

WOULD PRIOR OBSERVATIONS HAVE REVEALED AN EFFECT?

53. If a time cumulative  $v^2/c^2$  effect of line curvature existed in nature, would it have been discovered through intentional or unintentional prior observations? Since the effects are kinematic or aberrational in nature, they disappear when motion comes to a halt -- just as the Lorentz contraction does. Hence the observations would have to be of the nature of high-speed photographs taken while the motion was in progress. One can rule out wheelspokes, aircraft propellers, and large-scale rotating machinery as orders of magnitude off from the required speeds and durations. It is shown in Chapter II that small geometries are favorable to large angular lag rates, so the likeliest place to look for such effects would be long-term operations of vacuum ultracentrifuges with small rotors. Professor J. W. Beams has conducted extensive experiments with such devices for many years. He was of the opinion that had such effects existed he would have seen them. However, to show the effect in question, a single mark on a rotor does not suffice. One needs at least two marks at different radii, preferably a radial line. And one needs to observe it by very high-speed stop-action photography, with a camera located somewhere roughly along the axis of the rotor. To look at the disk by eye is of no avail. The writer has not been able to determine from the ultracentrifuge configurations published by Professor Beams and his coworkers how a camera and flash equipment would have been arranged to make the necessary observations. This seems to remain an open question. There is no doubt that Professor Beams's magnetically-suspended rotors are ideal experimental vehicles for performing the necessary observations.

54. Dr. C. W. Sherwin suggested that computer memory disks, which rotate for long periods at high speeds, might by an appearance of bit migration near the rim reveal the effect in question. His calculation shows this to be quite plausible. The fact that no such undesirable data "migration" has ever been reported is strong presumptive evidence against the kinematic hypotheses, though it does not bear on the optical aberration.

55. Dr. D. H. Weinstein, the author of reference (o), performed some months earlier a version of the experiment described in Chapter II. His results were negative, but provided no quantitative bound on  $|\alpha|$ . They have not been published. The present writer did not

consider Dr. Weinstein's results entirely conclusive, even qualitatively, because of adverse factors: (1) the disk diameter of 1.5 inches was somewhat large for best results by the criteria of Chapter II, (2) the rotation (in air) was interrupted by power failure after three months, (3) photographs were taken with a flash duration of 0.5-1 microsecond, which (taken with the above factors) is long.

56. Finally, there is the question of the earth as a rotor. As residents of the earth, we are "disk riders" who would not see either the optical aberrations or the kinematic effects that an external observer might. We would participate unwittingly in any distortion of our metric. Therefore in principle none of our observations would bear on the matter. And even if we became detached external observers who looked down on the earth along the north polar axis and painted a fiducial stripe along a line of longitude, after a few hundred million years, when the appearance of distortion became appreciable (the earth's crust near the equator, for the  $\alpha = \frac{1}{2}$  theories, lags by 1.7 meters per century), the earth's rotational axis would be as likely as not to shift, thus spoiling our Gedanken experiment.

57. It appears that there is a fair amount of presumptive observational evidence against all of the theories discussed here, but nothing conclusive, and certainly nothing to place a quantitative upper bound on  $|\alpha|$ , the magnitude of the effect constant.

#### SUMMARY

58. A short-duration flash photograph of the surface of a spinning disk that has been in high-speed rotation for a long time may, according to certain theories, reveal that an initially straight radial line has developed a backward curvature, particularly pronounced near the rim. Either or both of two distinct causes may produce such a result: (1) a kinematic effect evidencing a "nonstatic metric" to complement the "non-Euclidean geometry of the disk," or (2) an optical analogue of the Thomas effect, manifested as a time-cumulative aberration of light reflected from the disk surface.

59. When probed critically, none of these theories is free from theoretical objections and caveats, but none appears refutable by pure reason; and all are more or less closely allied to tenets of the physicist's abiding faith in never-directly-observed aspects of kinematics, such as the Lorentz contraction and the Thomas precession. The relativistic kinematics of extended structures is itself a dark swamp of inference hitherto unlighted by a single ray of quantitative observational fact.\* Perhaps that alone conveys enough justification

\*One minor exception is the experiment of Ditchburn and Heavens, ref. (v), which tested a theory of kinematic bending-under-rotation quite different from any discussed here, put forward by Gardner and elaborated by Synge, ref. (w). The experiment involved repeating the Michelson-Morley observations with an interferometer tilted at an angle. The result was null.

to motivate an experiment. Needless to say, the type of experiment it is practical to do is dictated by the continuing technological inability to impart relativistic speeds to extended structures in the laboratory. One chooses instead to search for alleged time-cumulative effects, for here one's observations are facilitated by the ready availability of a measuring instrument of supreme reliability and accuracy, a government calendar.

Chapter 2

EXPERIMENT

DISK DESIGN CONSIDERATIONS

1. In this section the main design considerations and rationale for the way the experiment was done will be given. Whether kinematical or optical in origin, the effect sought was a photographable azimuthal angular lag,

$$\Delta\phi = CR^2f^3 \text{ radians/day,} \quad (2.1)$$

where

$$C = 1.5384 \times 10^{-13}\alpha \text{ for } R \text{ in inches, } f \text{ in rps.,}$$

$$= 2.3846 \times 10^{-14}\alpha \text{ for } R \text{ in cm, } f \text{ in rps.}$$

( $\alpha$  = theoretical relativity effect constant =  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  
... etc.)

relative to its central portion, of the rim of a disk of radius R, rotating at constant frequency f, as discussed in Chapter 1. A first approximation to a figure of merit for experimental design is therefore  $\Delta\phi$ , to be maximized, or its reciprocal,

$$F_1 = \text{Days per degree of effect} \quad (2.2)$$

$$= \frac{\pi}{180CR^2f^3} ,$$

to be minimized.

2. Without proceeding further one can observe that, whether the experiment is done in vacuum or in air, a small disk size will be advantageous. In the former case strength of materials, in the latter air drag, is limiting. In either case rim speed v is the significant parameter affected by the limitation; and since  $v = 2\pi fR$ ,

$$\Delta\phi \propto R^2f^3 \propto \frac{v^3}{R} ,$$

it is apparent that in the limiting condition of operation a decrease in R will increase  $\Delta\phi$ . Hence we anticipate small geometries.

3. A better figure of merit than  $F_1$  should bring in the spatial and temporal resolution limitations on observability of the presumed effect, as will now be discussed.

4. Temporal Resolution: The use of Q-switched laser photography of the disk surface was favored by the high intensity and short duration of the flash, which make this type of photography ideal for "freezing" high-speed motion. The mean duration  $t_0$  of the flash results in a "blur angle" contribution

$$\Delta\phi_{b_1} = 2\pi ft_0 \text{ radians.} \quad (2.3)$$

This is the azimuthal angle subtended at the center of the disk by the distance any point on the disk surface travels during time  $t_0$ . (We neglect any possible second-order effect of variation of frequency with radius, since  $\Delta\phi_{b_1}$  is already very small.)

5. Spatial Resolution: If  $N_f$  is the maximum number of resolvable lines per millimeter in the image recorded on the photographic film, then

$$N_f = \left( \frac{1}{N_1} + \frac{1}{N_2} \right)^{-1} \text{ (lines per mm.),} \quad (2.4)$$

where

- $N_1$  = number of lines per mm. resolved by the film as processed, and
- $N_2$  = number of lines per mm. resolved by the lens and optical system at the focus and f-stop employed.

The least distance in the azimuthal direction resolvable on the film in the image of the disk rim is thus approximately

$$\frac{1}{k_1 N_f} = R_f \Delta\phi_{b_2} ,$$

where

$$\begin{aligned} k_1 &= 25.4 \text{ for } R_f \text{ in inches,} \\ &= 10 \text{ for } R_f \text{ in cm.} \end{aligned}$$

Hence

$$\Delta\phi_{b_2} = (k_1 N_f R_f)^{-1} \text{ radians.} \quad (2.5)$$

Here

- $R_f$  = radius of the disk image on the film, and
- $\Delta\phi_{b_2}$  = azimuthal blur angle in radians subtended at the disk center by the least resolvable distance on the film at the position of the rim image.

6. Assuming statistical independence of spatial and temporal uncertainties, we have for the "total blur angle"  $\Delta\phi_b$ , specifying

the uncertainty of angular position of the image of a sharp-edged marker at the disk rim,

$$\Delta\phi_b = \sqrt{\Delta\phi_{b1}^2 + \Delta\phi_{b2}^2} . \quad (2.6)$$

7. Improved Figure of Merit. The actual effect to be observed is one of shape change of a radial line or lines on the disk surface. It depends on appearances at all radii, not merely at the rim. An accurate analysis would therefore require a radial integration. As a crude approximation we can use what happens at the radius of maximum effect (the rim) as a measure of overall effect. Hence we employ as a working figure of merit the rim "signal to noise ratio,"

$$F_2 = \left( \frac{\Delta\phi}{\Delta\phi_b} \right)_{\text{rim}} = \frac{CR^2 f^3}{\sqrt{(2\pi f t_0)^2 + (k_1 N_f R_f)^{-2}} \left[ \frac{\text{Effect angles/day}}{\text{Blur angle}} \right]} \quad (2.7)$$

to be maximized. Both figures of merit,  $F_1$  and  $F_2$ , have the virtue of being independent of the degree of enlargement of the photographic image.

8. Flash Illumination. For laser photography reference (x) gives an empirical formula expressible in the form

$$P = \frac{k_2 A_0 N^2 (1 + M)^2}{t_0 S R_0 T_0 T_p} , \quad (2.8)$$

where

- P = laser peak power (watts),
- $A_0$  = illuminated area,
- N = numerical aperture (f-stop) of optical system,
- $t_0$  = laser pulse duration (sec.),
- S = exposure index (ASA rating) of film for laser wavelength and pulse duration,
- $R_0$  = reflectivity of illuminated subject for laser wavelength,
- $T_0$  = fractional transmission of light intensity by camera and optics,
- $T_p = \exp(-0.01\alpha_m x)$  = fractional transmission of light by medium, where  $\alpha_m$  = medium absorption coefficient for laser wavelength and  $x$  = path length (cm.),

and

$$k_2 = 0.07 \text{ for } A_0 \text{ in ft}^2 \\ = 7.54 \times 10^{-5} \text{ for } A_0 \text{ in cm}^2$$

9. In formula (2.8) we have replaced the  $N$  of reference (x) by an effective value,  $N(1 + M)$ , where

$$M = \frac{R_f}{R} \quad (2.9)$$

is the image magnification ratio, to allow for the effects of bellows extension in closeup photography. For photography in air at short path lengths we can take  $T_0 = 1$ . The illuminated area may be taken equal to the disk surface area,

$$A_0 = \pi R^2, \quad (2.10)$$

any spreading of the illumination being accommodated by a reduced value of  $T_0 < 1$ . The  $f$ -stop value  $N$  may be chosen for best resolution of the lens used, and the other quantities,  $T_0$ ,  $P$ ,  $t_0$ ,  $R_0$ ,  $S$ , may be treated as known constants for the particular photographic strategy and equipment employed. Putting (2.9) and (2.10) into (2.8), we find

$$R_f = C_1 - R, \quad (2.11a)$$

where

$$C_1 = \frac{k}{N} \sqrt{P t_0 S R_0 T_0} \quad (2.11b)$$

and

$$\begin{aligned} k_1 &= 25.59 \text{ for } R \text{ and } R_f \text{ in inches,} \\ &= 65 \text{ for } R \text{ and } R_f \text{ in cm.} \end{aligned}$$

By means of (2.11a)  $R_f$  can be eliminated from equation (2.7) and the figure of merit  $F_2$  can be expressed as a function of two design variables,  $R$  and  $f$ .

10. Photographic Parameters. The best lens available for the experiment for closeup photography (Nikon Micro-Nikkor f3.5, 55mm focal length) is claimed by the manufacturer to have resolution  $N_2 = 150$  lines/mm at its optimum aperture of  $f8$ . The best film for use in ruby laser photography from the standpoint of combined speed and resolution is the red-sensitive Kodak Linagraph Shellburst film, which has effective ASA speed  $S = 400$  [see reference (x)]. The resolution of this film was estimated to be in excess of  $N_1 = 75$  lines/mm. By equation (2.4) the combined resolution of film and lens therefore exceeded  $N_f = 50$  lines/mm. The reflectivity of the disk was guessed to be  $R_0 = 0.5$ , and the fraction of laser light concentrated on the disk by a parabolic reflector was estimated to exceed  $T_0 = 0.1$ . The duration of the Q-switched laser pulse was

approximately 20 nanoseconds. The laser initially available for the experiment had a peak power of 4-7 megawatts, but initial experimental design was based on the more conservative assumption of one megawatt. For these parameter values equation (2.11b) yields  $C_s = 2.02$  inches. For the disk size employed in the experiment ( $R = 0.265$  inch), equation (2.11a) gives  $R_f = 1.8$  inch. The resulting image diameter of 3.6 inches is ideal for the 4" x 5" Shellburst film in NOL stock. The image magnification (equation 2.9))  $M = 1.8/0.265 = 6.8$  is suitable for macrophotography with about 16-inch bellows draw on the available 4" x 5" Calumet view camera. An extension tube/adaptor was fitted to the front of this camera in order to permit the lens to be reversed for best closeup performance, and to get the body of the camera out of the way. The relief distance from front (actually, the original back) of lens to subject was  $2 \frac{5}{8}$  inches, adequate to allow laser illumination to strike the disk at roughly a 45-degree angle. Many alternatives to this combination were examined, including the use of other film sizes and emulsions, other lenses, geometries, and light-concentration schemes, etc. It cannot be claimed that the arrangement chosen was in any sense "best," but it was simple and adequate.

11. Operating Conditions. Primarily for reasons of economy it was decided to do the experiment in air, although the advantages of doing it in vacuum are manifold. The need for long, uninterrupted, effect integration times dictated the use of air bearings, and the small geometry suggested the use of a high-speed dental drill or "grinder," in which the same gas supply used for the bearings powers an air turbine. (Alternatives examined included NOL air-driven fuse spinners and vacuum ultracentrifuges. The larger geometry of the fuse spinners was unfavorable, and no suitable ultracentrifuge was available.) The necessity to work at low rim speeds of the order of  $2.6 \times 10^4$  cm/sec., imposed by operation in air, had the incidental cost-saving benefit that none of the safety precautions was needed that would have been required in the materials strength-limited regime attainable in vacuum.

12. For an air turbine of known speed-torque characteristics, air drag imposes a limit on  $f$  for given  $R$ . This provides a further relationship permitting elimination of either  $R$  or  $f$  from  $F_2$ , with the possibility to optimize design by suitable choice of the noneliminated variable.

13. Aerodynamic Drag. The Reynolds number is

$$R = \frac{2\pi f R^2}{v}, \quad (2.12)$$

where  $v$  is the kinematic viscosity of air, equal to about  $1.564 \times 10^{-4}$  ft<sup>2</sup>/sec., or 0.1453 cm<sup>2</sup>/sec. For realistic values of  $R$  and  $f$  this implies operation in the turbulent flow drag limited regime. For such operation reference (y) gives as the dimensionless drag

coefficient associated with a flat disk "wetted" on both sides

$$C_M = \frac{2M}{\frac{1}{2}\rho(2\pi f)^2 R^5} = \frac{0.146}{R^{1/5}}, \quad (2.13)$$

where  $2M = T_d$  is the drag torque on the disk and  $\rho$  is air density, approximately 0.002378 slug/ft<sup>3</sup> or 0.001226 gm/cm<sup>3</sup>. From (2.12), (2.13) we have

$$\begin{aligned} T_d &= \frac{0.073\rho(2\pi f)^2 R^5}{R^{1/5}} = 0.073\rho v^{1/5} (2\pi f)^{9/5} R^{23/5} \quad (2.14a) \\ &= k R^{4.6} f^{1.8}, \end{aligned}$$

where

$$\begin{aligned} k &\approx 1.07 \times 10^{-7} \text{ for } T_d \text{ in inch-lb., } f \text{ in r.p.s., } R \text{ in inches,} \quad (2.14b) \\ &= 0.00166 \text{ for } T_d \text{ in dyne-cm., } f \text{ in r.p.s., } R \text{ in cm.} \end{aligned}$$

The true drag (actual k-value) is higher than this idealization as a result of (a) departures from flat disk shape, (b) whatever effect the disk markings and surface irregularities have in reducing aerodynamic smoothness.

14. Available Torque. No torque vs. frequency specifications were available for small air-bearing, air-turbine units. For the Westwind Model 115 Pencil Grinder (Federal-Mogul Corporation, Ann Arbor, Michigan) unit selected, the maximum load-free speed at 70 p.s.i. gas pressure was stated to be 500,000 r.p.m.; i.e.,  $f_m = 8333$  r.p.s. The static (stall) torque was not known, but could be estimated from the geometry of the unit and some preliminary observations of disk rotation performance to be of the order of

$$\begin{aligned} T_s &\approx 0.004 \text{ to } 0.006 \text{ inch-lb.} \quad (2.15) \\ &\approx 4500 \text{ to } 6800 \text{ dyne-cm.} \end{aligned}$$

for turbine air pressure of about 78 p.s.i.

15. From promotional literature it was deduced that for air motors in general a crudely valid assumption is that the available torque declines linearly from  $T_s$  at  $f = 0$  to 0 at  $f = f_m$ . That is, the turbine torque  $T_t$  roughly obeys

$$T_t \approx T_s \left(1 - \frac{f}{f_m}\right), \quad 0 \leq f \leq f_m. \quad (2.16)$$

16. This relation applies for constant turbine gas pressure. No attempt was made to treat the latter as a design parameter, since it was observed that for small turbines having equal gas pressures on the bearing and turbine there is an upper limit to the pressure region of stable operation. Beyond about 80 p.s.i. it was found empirically that even the best-balanced rotor could unexpectedly develop instability. A proper turbine design to exploit the availability of higher gas pressures would permit the application of excess (stabilizing) gas pressure to the bearing. No unit employing this differential pressure principle was available commercially in the very high-frequency range needed for the experiment, although two air-bearing, air-turbine circuit board drills (Barden Model 100) with  $f_m \approx 180,000$  r.p.m., which use this principle, were kindly made available for experimental purposes by the IBM Corporation (Endicott, New York). Their low value of  $f_m$  made them noncompetitive with the smaller Westwind unit for this particular application.

17. Disk Radius Optimization. The air-drag limited equilibrium frequency of the disk is the value of  $f$  for which  $T_d = T_t$ ; i.e., from (2.14a) and (2.16),

$$k R^{4.6} f^{1.8} = T_s \left(1 - \frac{f}{f_m}\right).$$

From this we can express  $R$  as a function of  $f$ ,

$$R = \left[ \frac{T_s}{k} f^{-1.8} \left(1 - \frac{f}{f_m}\right) \right]^{\frac{1}{4.6}}. \quad (2.17)$$

That is, for any given frequency in the range 0 to  $f_m$  there is a disk radius, given by (2.17), at which air-drag limited steady operation occurs.

18. When  $R_f$  is eliminated from (2.7) by means of (2.11a), and  $R$  is eliminated from the result by means of (2.17), we obtain an expression for the figure of merit  $F_2$  in terms of  $f$  alone, plus constants known or capable of being estimated; viz.,

$$F_2 = \frac{C f^3 \left[ \frac{T_s}{k} f^{-1.8} \left(1 - \frac{f}{f_m}\right) \right]^{\frac{1}{2.3}}}{\sqrt{(2\pi t_0)^2 f^2 + k_1^2 N^2 f \left( C_1 - \left[ \frac{T_s}{k} f^{-1.8} \left(1 - \frac{f}{f_m}\right) \right]^{\frac{1}{4.6}} \right)^2}}. \quad (2.18)$$

19. The "optimum" frequency  $f = f_0$  is that which maximizes  $F_2$ , as determined from a plot of  $F_2$  vs.  $f$ , such as illustrated by the solid curves in Figure 1. This figure is drawn for typical (estimated) parameter values. The relativity effect constant  $\alpha$ , to which  $F_2$  is proportional, is here given its maximum value of  $1/2$ .

20. If we fix  $R$  in equation (2.7) or (2.18), a curve such as the dashed one of Figure 1 for  $R = 0.265$  inch is obtained. This happens to be the size of disk used in the experiment. The actual mean frequency observed for this rotor was about 6070 rps. The solid and dashed curves refer to an assumed 1 MW peak laser power. The data points in squares correspond to 4 MW, a more realistic value. Although Figure 1 suggests the optimum disk radius to be about 0.25 inches, a slightly larger value was chosen to ease problems of photographic magnification and focus.

21. It is to be noted that two of the least-known parameters,  $T_s$  and  $k$ , enter (2.18) only in their ratio. The previously mentioned fact that  $k$ , given by equation (2.14b), is an underestimate of the true drag means that  $T_s$ , given by (2.15), is an underestimate of the available torque by the same factor.

22. The observations on which equation (2.15) is based were made on disks of diameters 0.5 inch, 0.53 inch, and 0.6 inch, with various degrees of balance and surface roughness. The data are plotted as three triangle-enclosed points on Figure 2, which shows also the figure of merit  $F_1$  for the same parameter values as were used in Figure 1. The solid curves in Figure 2 are obtained by eliminating  $R$  from equation (2.2) by means of (2.17), for three assumed constant values of  $T_s$ . The dashed curve in Figure 2 corresponds to the  $f^{-3}$  relation obtained by taking  $R = \text{constant} = 0.265$  inch in equation (2.2). The triangular data points result from inserting the observed  $f$  and  $R$  values for the three disks into equation (2.17), solving for  $T_s$ , and plotting the corresponding data point on the resulting (interpolated)  $T_s = \text{constant}$  contour in Figure 2 at the observed  $f$ -value. The  $T_s$ -values given in equation (2.15) were obtained in this way, based on the assumed constant  $k$ -value of equation (2.14b). It will be noted that there is good consistency between the data for the (similarly-shaped) disks of radius 0.265 and 0.3 inches. Both correspond to  $T_s = 0.0056$  inch-lb. The 0.25-inch radius disk, on the other hand, had a different shape of its lower support portion, which exposed more wetted area, and it was less well balanced -- facts that probably imply an increased  $k$ -value, rather than the decreased  $T_s$ -value indicated.

23. From Figure 2, which again assumes a maximum value  $\alpha = 1/2$  of the relativity effect constant, it is apparent that with the 0.265-inch radius disk used in the experiment there should have been one azimuthal degree of cumulative relativity effect at the rim for every 14.5 days of continuous rotation. It is instructive to note from Figure 1 that the simple figure of merit  $F_1$ , if we assume  $T_s = 0.0056$  inch-lb, calls for an optimum frequency  $f \approx 7000$  rps and

disk radius  $R \approx 0.223$  inch. The limitations of observability taken into account in  $F_2$  (Figure 1) call for a somewhat larger disk.

24. In summary, all our design considerations to this point boil down to two parameter values: the best practical disk radius appears to be about 0.265 inch, to match the highest r.p.m. air turbine unit commercially available for the experiment, and the corresponding image size fits satisfactorily on 4" x 5" film.

25. Rotor Radial Cross-section. With the radius of the rotor determined, we now have to consider its shape. The questions of optimum rotor cross-section for maximum strength, materials, and other related design criteria have been extensively considered in the literature, particularly by the ultracentrifuge pioneer, J. W. Beams. [See references (z), (aa).] Without any attempt at originality, we shall analyze the matter in a simple way as it bears on the present experiment.

26. Although high rim speeds are not attained, it is sound practice to design for maximum strength. It is well known that this implies a cross-section tapering toward the rim, as indicated in Figure 3, in order to reduce the proportion of mass subject to greatest acceleration. The air turbine was fitted with a collet that required a 1/16-inch diameter shaft of approximately 7/16-inch length. This, the disk, and an intermediate support section were machined from a single piece of metal. The choice of materials will be discussed presently.

27. Let  $r$  be the radial distance from the disk axis, limited to  $r_0 \leq r \leq R$ , where  $r_0 = 1/32$  inch, the shaft radius, and  $R =$  disk radius. Let  $h(r)$  be the cross-sectional height or thickness of the disk at radius  $r$ . The mass element between  $r$  and  $r + dr$  is

$$dm = 2\pi r dr \cdot h(r) \cdot \rho,$$

where  $\rho =$  material density. The centrifugal force on this mass element is

$$dF = r\omega^2 dm,$$

where  $\omega = 2\pi f$  is the angular frequency of disk rotation, assumed constant. The total centrifugal force exerted on material at radius  $r$  due to material beyond radius  $r$  is

$$\begin{aligned} F_{>r} &= \int_r^R dF = \int_r^R r'^2 \omega^2 \cdot 2 r' dr' \cdot h(r') \cdot \rho \\ &= 2\pi\omega^2 \rho \int_r^R (r')^3 h(r') dr' , \quad r > r_0. \end{aligned}$$

At radius  $r$  the azimuthal cross-sectional area is  $A(r) = 2\pi rh(r)$ . We establish as our ideal design criterion the requirement that the total centrifugal force per unit azimuthal cross-sectional area be independent of  $r$ , for all  $r > r_0$ . (For  $r < r_0$ , there is support from the collet, so this case need not be examined.) That is, we require that no disk radius experience greater stress density than any other. In symbols,

$$\frac{F_{>r}}{A(r)} = \text{Constant} = f g e, \quad r_0 \leq r \leq R,$$

where

- $e$  = yield strength of material (gm/cm<sup>2</sup>),
- $g$  = acceleration of gravity (to convert grams to dynes force),
- $f$  = design safety factor representing the fraction of yield strength at which the disk is to operate.

We have

$$2\pi\omega^2\rho \int_r^R (r')^2 h(r') dr' = 2\pi f g e r h(r),$$

$$\int_r^R (r')^2 h(r') dr' = (1/2) r_c^2 r h(r),$$

where

$$r_c \equiv \frac{1}{\omega} \sqrt{\frac{2fge}{\rho}}. \quad (2.19)$$

Differentiating with respect to  $r$ , we find

$$-r^2 h(r) = (1/2) r_c^2 h(r) + (1/2) r_c^2 r \frac{dh(r)}{dr},$$

$$\frac{dh}{h} = -\frac{dr}{r} - \frac{2rdr}{r_c^2}.$$

Integrating,

$$\ln h = -\ln r - \frac{r^2}{r_c^2} + \ln a,$$

$$h(r) = \frac{a}{r} e^{-r^2/r_c^2},$$

where  $a$  is an integration constant. Let  $d = h(R)$  be the outer edge thickness. Then  $a$  is evaluated to yield

$$h(r) = \frac{Rd}{r} \exp \left\{ \left( \frac{R}{r_c} \right)^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \right\}, \quad r_0 \leq r \leq R. \quad (2.20)$$

For  $r_0 \ll R$ , the ratio of thickness at the shaft to thickness at the rim is

$$\frac{h(r_0)}{h(R)} \approx \frac{R}{r_0} e^{\left( \frac{R}{r_c} \right)^2}. \quad (2.21)$$

From this we see that  $r_c$  is a critical parameter. With  $R$  fixed by other design criteria, it is obvious that  $r_c$  cannot be allowed to be too small. Otherwise the exponential factor becomes unrealistically large, so that we need either a very thin disk near the rim or a very thick disk near the shaft. This, of course, is the controlling factor that limits the feasible magnitude of  $\omega$  for a magnetically-supported disk spinning in vacuum.

28. Disk Material. The best material from the safety standpoint is that which maximizes  $r_c$  for given  $\omega$ ; i.e., maximizes (according to equation (2.19)) the yield-strength to density ratio,  $e/\rho$ . This criterion is universally known among designers of high-speed rotary machinery. In the present experiment  $r_c$  is of the order of several inches, while  $R$  is a fraction of an inch, so the exponential term in equation (2.21) is essentially unity, and air drag limits  $\omega$ , so all safety problems are negligible as long as the disk is structurally sound.

29. Disks were machined from 17-4 PH stainless steel (125,000 p.s.i. yield strength) and from ST 7075 (T6) aluminum (73,000 p.s.i. yield strength -- essentially what used to be called "duralumin"). Both have excellent values of  $e/\rho$ . (Probably the best nonexotic material by this criterion is beryllium. I am indebted to NOL for the gift of some titanium, but this proved to be not needed.) Both of these materials were quite satisfactory from the  $e/\rho$  standpoint, so the choice was based on a different consideration: For the purposes of counting spin rate of the disk a tachometer was used that detected light reflected from the disk surface. To improve tachometer signal-to-noise ratio it was desirable to polish a portion of the disk surface. It was found that stainless steel took an incomparably better polish than aluminum (which tended to "ball up" under lapping). Hence stainless steel was chosen. The actual shape of the disk used in the experiment departed from the ideal  $r^{-1}$  curve of equation (2.20), but approximated that curve by two straight-line segments, as shown in the cross-sectional drawing of Figure 3. Stress calculations made for the actual shape indicated very little loss as compared with the ideal shape, and showed a safety factor greater than 10.

30. The stainless steel disk weighed 1.43 grams total with shaft. It was precision balanced by the Schenck Trebel Corporation, Farmingdale, New York, with a residual static imbalance of  $3.3 \times 10^{-5}$  gram-cm. This balancing was essential to stable high-speed operation.

#### DESCRIPTION OF EXPERIMENT

31. A general view of the apparatus is shown in Figure 8. The experiment was done in a basement room of NOL, Building 402, where dry nitrogen from the 10,000 psi bottle storage field for the wind tunnel could be used to power the turbine. A line from an 1800 psi. intermediate storage bottle led to reducer, popoff, and pressure-regulation valves, followed by pressure gauges, filters, and a small silica gel dryer, as indicated in Figure 4.

32. The gas supply appears to have been of exceptionally high quality, since during more than four months of operation and additional experimentation the gel gave no indication of moisture (but was in fact dried by the gas) and the special Wilkerson filter (claimed to stop particles down to less than 0.03-micron diameter) remained as clean as the day it was installed. The pressure, though not subject to short-term fluctuations, was not entirely steady, but showed a tendency to drop off by 1-2 p.s.i. per week. The cause was not investigated, since the problem was easily corrected by occasional valve adjustments.

33. The choice of photographic lens and camera has already been discussed. The camera was mounted vertically on a heavy high-speed camera stand and was pointed down at the disk, the upper surface of which was horizontal, as indicated in Figure 5, drawn approximately to scale. The disk was supported in a homemade (rubber bands and epoxy) version of an x-y-z positioner, anchored to the stand, with a ball joint for leveling adjustments and three micrometers that allowed not-quite-independent horizontal and vertical motions. Micrometer adjustment was necessary, particularly in the vertical, because focus was critical to 0.001 inch. Two factors made focus a major problem that necessitated much cut-and-try photographic work. First, the disk rose up about 0.015 inch when gas pressure was applied to the turbine, so that static and dynamic focal positions differed. Secondly, the focal plane for ruby laser light did not quite coincide with that for broad-spectrum illumination. Of these two effects the first was by far the more serious. Fortunately, once the proper focal position was found it did not shift. An extra brace between the x-y-z positioner and the lens-holding tube helped to maintain this stability. Once set up, the camera itself was not moved and all adjustments were made with the positioner.

34. Disk rotation speed was measured with a Hewlett-Packard Model 506A tachometer head, consisting of an incandescent light source and photo diode detector. This was connected to a Hewlett-Packard Model 500B frequency meter, which supplied standard output pulses to a Beckman Model 7260 counter for recording cumulative

counts and to a Monsanto counter for instantaneous frequency readings. Tachometry was accomplished by a double reflection of the HP-506A light signal from a reflecting half of the disk surface to a nearby small dental mirror and back to the detector. The attempt to maintain an accurate total count record had to be abandoned because of counter failures and repeated burnouts of the tachometer light bulbs, which, it was discovered, were not intended for full-voltage continuous duty. By reducing the voltage to about 85-90 volts this problem was largely overcome, but the reduced intensity contributed to other problems that will be described and that ultimately forced termination of the run. A record was kept of instantaneous frequency readings taken throughout the run. From the statistics of these random samplings it was concluded that (with one exception, to be discussed) during the four-month duration of the experiment the disk speed never varied more than about 1%. Because of this stability of operation it is probably not too serious that an accurate total count record was not kept. (In any case this would have been difficult to do because the tachometer light had to be turned off during photographic operations.) There is no question that tachometry was a major weakness of the experimental design. If the experiment were to be done again a better way should be found, preferably not depending on light, in order to avoid interference with photography. (Possibly localized magnetism might be induced in the disk.)

35. Though tachometry was seemingly the most trivial aspect of the experiment, it gave trouble in another way; viz., in connection with marking the disk. It was desired to put radial straight lines on the disk. These had to be of various depths and widths, because it was not known what their appearance would be in laser flash illumination incident at random orientations. (No attempt was made to synchronize flashes with disk rotation.) This problem was crudely solved (after some unsuccessful attempts) by drawing a variously-weighted steel knife point across the polished disk in an adjustable scribing machine. Failure attended a surprising number of methods and attempts to reduce the reflectance of half the disk surface for purposes of tachometry. These included various chemical approaches, surface treatments, painting, and anodizing (in the case of aluminum disks). It cannot be said that a good solution was found to this simple problem. Finally, attempts to maintain aerodynamic "smoothness" were abandoned and half the (stainless steel) disk surface was sand-blasted, under a mask, with small glass spheres. This killed the reflection on half the disk surface and left it visibly rough, as can be seen in the photographs (Figures 6,7). Very little aerodynamic penalty seems to have resulted. However, the problem of optical contrast, which was a recurrent one in this experiment, provides another reason why nonoptical tachometry would be preferable.

36. About electronic equipment reliability it should be said that most of the equipment was old, all of it was borrowed, and four months of continuous operation is a severe test. It was not surprising that a number of failures occurred. The older tube-type

equipment seemed to perform at least as well as newer types. The only piece of electrical equipment that suffered no failure (aside from a Variac) was the HP 500B frequency meter. The runner-up was the old (all-tube) Beckman 7260 counter, which had one failure, easily repaired. Another more modern Beckman 6030A preset EPUT meter operated so erratically it had to be abandoned entirely, and of two new transistorized Monsanto counters one was erratic and the other operated perfectly until near the end of the run, when it showed some signs of erratic behavior.

37. Laser light was focused on the disk by a deep-dish parabolic reflector of about six-inch diameter, as indicated in Figure 5, after passing through a hole at the back of the reflector and being reradiated from a white diffuse reflector of about 3/4-inch diameter located near the focal point of the parabola. This reflector had an in-and-out adjustment for focusing. The purpose of diffuse reradiation was to destroy as much as possible of the spatial correlation ("twinkle") of the laser light. This equipment is more extensively described in reference (x). The results were excellent beyond expectation. The decorrelation was successful, and the quality of the photographs (Figures 6,7) speaks for itself.

38. Two lasers were used. The first, used during most of the experiment for taking over 50 photographs, is the one described in reference (x). It employs a 1/4-inch diameter ruby rod. In this experiment it was Q-switched by a dye cell (vanadium phthalocyanine in nitrobenzene) and initially gave a measured 0.145 joule output, which corresponds to a peak power of seven megawatts for 20 nanoseconds. The photograph in Figure 6 was taken with this laser before the start of the run. With use of the Q-switch progressive bleaching of the dye occurred. This resulted in eventual onset of laser double pulsing (which produced multiple-image photographs). Attempts to overcome this by raising the pumping power, or possibly other causes, resulted in spalling the glass of the dye cell. Although this was corrected, it was decided to make the final photographs with a more powerful laser, which fortunately became temporarily available.

39. This had a similar flash duration of about 20 nanoseconds at a peak power of approximately 100 megawatts, produced by a Frustrated Total Internal Reflection (FTIR) Q-switch, of the type described in reference (bb). This (Korad K1-Q) laser used a four-inch long, 3/8-inch diameter ruby rod. The photograph in Figure 7 was taken with this light source just before termination of the run after four months of operation. The same parabolic reflector/decorrelator was used as before. Alignment and focusing procedures for the reflector were facilitated by the use of a portable He-Ne laser.

40. Initial attempts to spin aluminum and stainless steel disks at high speeds were frustrated by imbalances. This was corrected by precision balancing, as already described. The final run began on 30 March 1972 and terminated after 172,529 minutes on 28 July 1972. During this time there were no known electrical outages, although a

continuous record of voltage was not kept. Such a record would have been needed only if the attempt had been made to accumulate total counts of the tachometer.

41. There was one severe pressure drop on 14 July 1972 due to a valving oversight during routine maintenance of the nitrogen bottle field. Fortunately, provisions had been made that permitted switching to the "house air" in such an emergency, and this was done for about an hour until nitrogen pressure was restored. The disk speed declined over several hours preceding this measure from its usual average of about 6070 r.p.s. to a minimum of about 2500 r.p.s., but at no time did the disk stop turning. As discussed in Chapter 1, temporary speed variations have no serious influence on the effect being sought, because they alter very little the rms speed averaged over a long time. Had the disk come to a complete stop, this would have destroyed the putative effect and terminated the run. Almost 100 flash photographs were taken at various times during the run. After two weeks it was already apparent that there was no relativity effect of the magnitude theoretically anticipated, and the run was continued primarily to determine just "how zero" the zero effect was, i.e., to lower the upper bound on  $|\alpha|$ .

42. The termination of the run was brought about by an entomologically interesting circumstance. The room in which the experiment was performed was not designed with relativity experimentation in mind, but instead the pumping of sewage. It therefore provided an ecological niche for a prolific species of small fly, of the type most widely noted for its intrepid approach to the environmental hazards of urinals. These little scientists showed an insatiable curiosity about the rotating disk, and, along with any dirt in the air, were continually being sucked down by the low-pressure region in the center of the disk and hurled radially outwards along the boundary layer. Since the disk itself was not aerodynamically smooth, its upper surface gradually became contaminated. This process, imperceptible at first, accelerated after a few months. The more contaminants stuck to the disk the more were able to stick. The dirtying of the surface began to interfere with tachometry, which depended on high surface contrast. This could be corrected only by increasing the voltage on the tachometer light, with decreasing life of the bulb. To avoid deterioration of the quality of the critical final photograph (Figure 7), on which the bounding of  $|\alpha|$  depended, it was decided to end the experiment while the surface markings were still clearly visible.

43. It is easy in hindsight to observe that this problem could have been avoided entirely by placing a simple plastic hood over the critical portion of the apparatus. However, because of the delayed onset and nonlinear nature of the phenomenon, the trouble was not diagnosed until most of the damage had been done. It is interesting, incidentally, that the accumulation of surface dirt plainly visible in Figure 7 (compare with the initial condition, Figure 6) did not decrease the speed nor impair the balance of the disk. Indeed, the

speed seemed to increase slightly toward the end, a fact that suggests a streamlining effect. In Figure 9, a closeup view of the turbine and disk, one of the flies can be seen on the white reflecting card behind the disk.

RESULTS

44. The photographs of Figures 6 and 7 represent the basic data of the experiment. Figure 6 is a flash photograph taken on 23 March 1972, seven days before commencement of the four-month run, with the disk stationary. The negative is Kodak Linagraph Shellburst emulsion on 4" x 5" Estar base film (thickness 0.004 inch). Figure 7 is a flash photograph taken with the more powerful laser on 28 July, with the disk in rotation at approximately 6116 r.p.s., a few minutes before run termination. The negative is a similar emulsion on a 4" x 5" glass plate. It would have been preferable to record both pictures on plates, but the latter were obtainable only on special order, for which it was decided not to delay the start of the experiment.

45. The total elapsed time from commencement of rotation to taking of the final record picture, Figure 7, was  $T = 172,517$  minutes. Acceleration to speed took about 20 minutes at the start of the run. The best estimate of the root mean square speed of rotation during the run is 6071.8 r.p.s. A reasonable minimum estimate of the rms speed is 6050 r.p.s. (best estimate minus 0.36%). The effects of the brief large excursion in speed on 14 July, mentioned in the preceding section, and other smaller variations are included in these estimates. The time-weighted arithmetical average frequency was 6070.6 r.p.s. The total number of disk revolutions during the experiment was roughly  $6.285 \times 10^{10}$ . These estimates are based on 115 Monsanto counter readings of instantaneous disk frequency, which divided the experiment into  $n = 114$  nonequal intervals  $\Delta\tau_i$  (minutes), such that

$$\sum_{i=1}^n \Delta\tau_i = T .$$

To each of these intervals was assigned a mean frequency  $f_i$ , evaluated as the average of the initial and final frequency readings that bounded the  $i^{\text{th}}$  interval  $\Delta\tau_i$ . The average frequency for the run of duration  $T$  is thus

$$f_{av} = T^{-1} \sum_{i=1}^n f_i \Delta\tau_i ,$$

and the rms frequency is

$$f_{rms} = \left[ T^{-1} \sum_{i=1}^n f_i^2 \Delta\tau_i \right]^{1/2} .$$

The averaging involved in determining  $f_i$  reduces maximum excursions and therefore results in a conservative (slightly low) estimate of  $f_{rms}$ .

46. The minimum estimate of 6050 r.p.s. is based on total count data for a selected period of duration 28,827 minutes from 31 May to 20 June 1972, during which it is believed that relatively few counts were lost. In this sample period the Beckman counter recorded  $1.05133 \times 10^{10}$  counts. The resulting average frequency based on total counts is 6078.39 r.p.s., to be compared with  $f_{av} = 6096.34$  r.p.s. and  $f_{rms} = 6096.36$  r.p.s., obtained as described above for the period in question. If this sample period is representative, and if no counts were lost by the Beckman counter, a downward adjustment of  $f_{av}$  by about 0.3% is implied. Applying this to the whole-run average (or rms) frequency of about 6071 r.p.s., and rounding downward to the nearest multiple of 10, we get 6050 r.p.s. as our minimum frequency estimate, in which the distinction between average and rms frequencies is not significant. It will be understood that the discrepancy between 6071 and 6050 is not a "standard deviation" or other statistical measure, but an educated estimate of maximum reasonable deviation based on additional physical data (readings of another counter). [For the sample data period, one has high assurance from the Beckman data source that the  $f_{av}$  value obtained from the Monsanto data source was no more than 0.3% high.]

47. Ideally, one might wish for a complete and exact time record of each of the  $\sim 6 \times 10^{10}$  counts, and a running computer analysis, but resources available for this experiment, which was primarily a qualitative one, were not compatible with such an ambitious plan.

48. The negatives were visually analyzed with a simple traveling-stage microscope. The best corresponding lines on the disk surface, indicated by arrows in Figures 6 and 7, were selected and the plate and film were superposed, emulsion sides up, with the plate below on the stage of the microscope. The two lines were so oriented that they were almost but not quite superposed. The image sizes were approximately the same in the two negatives, since no change in the lens-to-film distance occurred, and the lens-to-subject distance could vary by no more than about 0.001 inch for critical focus. The radius of the disk image was about 1.781 inches. At twenty different radii between 0.544 inch and 1.768 inch, readings of the line separation, transverse to the radius, were made with a micrometer eyepiece at about 30x magnification. These (azimuthal) separation readings, taken between estimated line centers, are tabulated in Table 2.

49. The concern of the experiment being with the shape difference between the two lines, it was natural to use polynomial fitting to the difference (azimuthal separation) data just discussed. At a glance it can be seen that the lines are both nearly straight. Hence it was natural first to investigate a linear fit,

$$d = a_0 + a_1 r \quad (2.22)$$

(d = line separation distance at radius r),

in order to study the "noise" in the data. By use of the NOL library program LSFITW, it was found that the data fit the linear law with

Table 2

RAW DATA ON LINE SEPARATION FROM  
NEGATIVES OF FIGURES 6 AND 7, AS  
MEASURED WITH TRAVELING-STAGE  
MICROSCOPE

<u>Disk Image Radius r (inches)</u>	<u>Separation Between Line Centers d (inches)<sup>1</sup></u>
1.7681	0.013438
1.7376	0.013012
1.7073	0.012787
1.6693	0.012286
1.6318	0.012224
1.5819	0.012057
1.5378	0.012241
1.4855	0.012213
1.4497	0.012299
1.3939	0.012403
1.3329	0.011705
1.2748	0.012314
1.2216	0.011531
1.1464	0.010853
1.0814	0.010356
0.99116	0.010717
0.90907	0.0097953
0.83467	0.0093455
0.74667	0.0092178
0.54392	0.0071077

---

<sup>1</sup>These values are converted from millimeter readings without regard to significant figures. Probably no more than three figures are meaningful.

---

a standard deviation

$$\sigma = 5.07 \times 10^{-4} \text{ inch.} \quad (2.23)$$

This can be used as an upper bound to the standard deviation of "noise" in the data, applicable if line curvature is present -- since any law with curvature will include additional fitting constants  $a_i$ , and thus cannot fail to reduce the standard deviation of fitting error below that for the two-constant (linear) case.

50. Now consider the effect of a relativistic lag. As our theoretical considerations (Chapter 1, paragraph 41) indicated, the lag distance at radius  $r$  for the line in Figure 7 after rotation time  $T$  should be

$$\begin{aligned} \text{Distance} &= r\Delta\phi = \alpha\omega_{\text{inst.}} T \cdot \frac{r^2\omega_{\text{rms}}^2}{c} \quad (2.24a) \\ &= \frac{\alpha}{K} r^3, \end{aligned}$$

where

$$K = \frac{c^2}{(2\pi)^3 f_{\text{inst.}} \cdot f_{\text{rms}}^2 T} \quad (2.24b)$$

In the present experiment the instantaneous frequency at the time the photograph of Figure 7 was taken was  $f_{\text{inst.}} = 6116$  r.p.s., the rms frequency (best estimate) was  $f_{\text{rms}} = 6071.8$  r.p.s., and  $T = 172,517$  min. = 10,351,020 sec. These values imply  $K = 0.24063$  (best estimate); or, for  $f_{\text{rms}} = 6050$  r.p.s.,  $K = 0.24237$  (conservative estimate) for distances in inches.

51. The relativity effect (in terms of azimuthal distance) being therefore cubic in radius, we may consider a law

$$d = a_0 + a_1 r + a_3 r^3 \quad (2.25)$$

as the natural polynomial to investigate. Here the term  $a_0$  measures any constant offset of the centers of the two superposed disk images, the term in  $a_1$  measures the difference in azimuthal angular orientation of the images, and the term  $a_3 r^3$  measures any relativity effect present. Comparing with (2.24a), we have  $\alpha/K = a_3$ , or

$$\alpha = K a_3 \quad (2.26)$$

A least-squares fit of the data led to  $a_3 = -7.824 \times 10^{-4}$  for distances in inches. Using our best-estimate  $K$ -value, we therefore obtain

$$\alpha = 0.24063(-7.824 \times 10^{-4}) = -1.88 \times 10^{-4} \quad (2.27)$$

(best estimate)

This is to be compared with the theoretical values  $\alpha = 1/2, 1/3, 1/6$ , etc., discussed in Chapter 1.

52. A rough estimate of the error in this "best estimate" of  $\alpha$  can be expressed in terms of the error bound in equation (2.23). Considering that part of the separation  $d$  due to a supposed relativistic effect, we have  $d_{rel} = a, r^3 \pm 3\sigma$ , if we take three standard deviations as bounding the error. From (2.26) we have  $Kd_{rel}/r^3 = \alpha \pm 3\sigma K(1/r^3)$ , or, redefining quantities in terms of mean values,

$$\alpha = \alpha_{\text{best est.}} \pm 3\sigma K\left(\frac{1}{r^3}\right) \quad (2.28)$$

In the present case the mean value of the reciprocals of the cubes of the 20 radii listed in Table 2 is  $0.907503 \text{ inch}^{-3}$ . Hence

$$\alpha = (-1.88 \pm 3.32) \times 10^{-4},$$

or, if the more conservative estimates of  $\alpha$  and  $K$  based on  $f_{\text{rms}} = 6050 \text{ r.p.s.}$  are used,

$$\alpha = (-1.90 \pm 3.36) \times 10^{-4} \quad (2.29)$$

53. It will be noted that the algebraic sign of the effect constant (best estimate) is negative. That is, the observed "effect," if any, is in the wrong (leading vice lagging) direction. There is almost certainly no significance to such a result, because small systematic errors of observation in the microscope were definitely present. The physiology and psychology of observation of the "center" of a broad line consisting of mottled blotches would make an interesting separate study unrelated to the present objectives. (Most of the figures in the second column of Table 2 are not significant.) Suffice it to say that a much more elaborate analysis of the negatives could be made, and they are available to anyone with the inclination and equipment to make it.

54. Had there been a  $v^2/c^2$  effect with  $\alpha$  as large as  $(1/2)$ , a lagging curvature opposite to the sense of disk rotation (which is counterclockwise in Figures 6,7) would have been observed, amounting to 8.36 degrees of rim lag subtended at the disk center. This would have been clearly visible, as indicated by the dashed line in Figure 7. Effects of the magnitudes suggested by any of the theories discussed in Chapter 1 are obviously not present. When all error sources (see next section) are considered, we may conservatively conclude from our observations that, if a rim lag per period of  $\alpha v_{\text{rim}}^2/c^2$  is present, then

$$|\alpha| < 6 \times 10^{-4} \quad (2.29)$$

From this it is reasonable to infer the physical absence of any relativity effect of visible line-bending at the  $v^2/c^2$  level.

#### SOURCES OF ERROR

55. The pictures themselves are ideally simple data sources. The only distortions that could occur in them are aberrations due to imperfect correction of the lens and dimensional variations of the negatives due to the effects of humidity, heat, and handling or processing. Although small lens distortions are undoubtedly present, they are believed to alter mainly the radial representation, hence to affect both negatives in about the same way. No estimate of the azimuthal distortion of the Nikkor lens was available, and it was assumed negligible in comparison with other error sources.

56. The plate (Figure 7) is probably quite stable dimensionally. The Estar film base (Figure 6) is much less so. Since reasonable care was exercised in processing, handling, and storage, it is believed that Kodak estimates of the dimensional stability of their film are applicable. These indicate a maximum dimensional change under processing and accelerated aging-shrinkage of 0.04%. In an image of 3.6-inch diameter this means a maximum dimensional change of 0.0015 inch, which is comparable with the blur distance at the rim. Since Kodak states that "the dimensional properties of Estar base films are nearly the same in all directions of the sheet," it seems likely that any distortions due to temperature and humidity are again mainly radial and that any radially-varying azimuthal distortions are down by an order of magnitude, hence quite small, of the order of 1% of the d-values listed in Table 2.

57. The remaining major source of error not subsumed in the standard deviation figure of equation (2.23) is the error in calibrating the transverse separation measurements  $d$  in the micrometer eyepiece. This error was estimated at 12%, and is an order of magnitude larger than the error in measuring the radius  $r$ . These errors are all included in the estimate of equation (2.29).

#### LESSONS OF TECHNIQUE

58. The main lessons of technique learned in this experiment have been mentioned before, but will be recapitulated here.

59. Laser flash photography is now a well-developed, practical method. Decorrelation of the light by diffuse reflection [reference (x)] removes the last obstacle to its general-purpose use wherever an intense flash of duration 20 nanoseconds or less is required.

60. Air turbines that use air bearings need separate gas supplies for the two functions, even in small geometries. (Excess pressure is desirable on the bearing.) Otherwise precision load balancing is essential for high-speed operation and the avoidance of "hunting" (speed instability).

61. "Clean room" conditions should have been established in this experiment to avoid contamination of the disk, or (preferably) the experiment should have been done in a vacuum.

62. Tachometry ought to be based, if possible, on detecting a non-optical signal.

63. For very long-term continuous operation of commercial electronic equipment, high assurance of uninterrupted operability can be secured (if at all) only through careful overall design involving equipment redundancy, probably at considerable expense.

## Chapter 3

## IMPLICATIONS

IMPLICATIONS FOR THEORY

1. The observational null result just discussed implies that a rotating steel disk behaves as if kinematically rigid\* in Minkowski space-time, with a static metric. The disk therefore shows a kinematic "elasticity" (nonstatic metric) in space-proper-time (SPT), where a kinematic lead of  $2\pi v^2/2c^2$  radians per turn occurs. From this it may be inferred that the hypothesis of existence of a proper time of a structure is physically untenable. No such concept of a collective proper time, shared among the parts of a macroscopic structure, can lead to a simple description; and any notion of "proper rigidity" (rigidity on hyperplanes of constant proper time in SPT) is excluded as nonphysical. In effect laboratory "simultaneity" appears to have an absolute meaning for all portions of the disk. As in reference (k), separate SPT's must be assigned to each individual particle; and proper time remains the private property of the particle. Method II of resolving the Ehrenfest paradox and related kinematic inconsistencies (see Chapter 1, paragraphs 38-45) is therefore observationally refuted.

2. The observations also rule out all other nonstatic Minkowski-space metric theories, such as those of Table 1, almost surely at the  $v^2/c^2$  level and presumptively to all orders in  $v/c$ . The observations similarly rule out the optical analogue of the Thomas precession (here termed "Thomas aberration") and thereby cast doubt on the reality of the Thomas precession of other physical vectors, such as spins. There being only energy-related evidence\*\* in favor of the Thomas spin precession--evidence independently accounted for

\* The word "rigid" here may be understood in either the Born [reference (e)] or the classical sense. Classical kinematics is in full agreement with the observations. Since  $(v/c) < 10^{-6}$  in our experiment, no question concerning the applicability of Newtonian kinematics would arise except for the possibilities and logical difficulties concerning "rigidity" (at all speeds) that are generated exclusively within relativity theory. Classical mechanics is both logically self-consistent and, as usual, consistent with observation in the low-speed regime.

\*\* Not all of this evidence is spectroscopic. Inglis and Furry, reference (cc), used a Thomas precession energy to explain inverted multiplets of nuclei. This may merely reflect the present lack of a "Dirac equation" in nuclear physics.

(in spectroscopy) by the Dirac equation without geometrical pictorializations--experimentation to try to observe the geometrical aspect of the Thomas precession would seem to be in order. The doubt just mentioned communicates itself to such widely-held beliefs as that expressed in reference (dd), "Any physical object which can be described as an infinitesimal line segment will precess if accelerated." The statement just quoted is unqualified as to speed and hence is in direct conflict with Newton's laws, if we presume the "accelerated" motion to be torque-free. Such a conflict can hardly be allowed to go unresolved; for, as Synge (reference (æ)) points out, special relativistic mechanics is hatched by a "cuckoo process" (referring to eggs laid in the nest of another bird) that leaves it with no authority to dictate "physics" to Newtonian mechanics in the Newtonian domain.

3. We may remark in passing that some authors [e. g., reference (q)] have maintained that the problem of the disk and the "geometry" of its motion cannot be discussed apart from consideration of the process of generation of the rotation. From the standpoint of correct analysis of world line shapes, which is indispensable, e. g., to an adequate theory of axis calibration by extended-structure metric standards, the point is unassailable. But from the standpoint of the present observations it is implausible that the manner or rate of bringing the disk up to speed (a process that occupied about 0.01% of the run duration) would have affected the qualitative findings of no line curvature and a static Minkowski space metric.

4. Method I, which involves replacing kinematical problems by dynamical or rheological ones, is unaffected in status by the observations here reported. This method rests on two remarkable ratiocinations:

a. Logical problems of kinematics are not to be solved, but are to be abolished by the assertion that rigid bodies do not physically exist. Physics thus relieves us of our obligations to logic.

b. Kinematic theory, reincarnated as physics, then proceeds to "predict" (or infer from the tacit assumption of a static metric or from "common sense") that the disk will physically be rigid--i. e., that all internal degrees of freedom will be frozen out (immobilized) on hyperplanes of constant laboratory time in Minkowski space. In other words, radial straight lines stay straight, as confirmed by laboratory observation.

Our Figure 7, then, is a photograph of something that by premise (a) cannot physically exist, and that by premise (b) confirms the theory of its own nonexistence.

5. We are being ungenerous to Method I in attributing to its status invariance under the present observations. Method I is in fact a universal invariant: nothing whatever can alter its logical

status or its plausibility status. The practitioners of Method I, having rejected the kinematic problem as insoluble, are solving another problem ad hoc having little to do with kinematics. Because of the strongly "physical" (and thoroughly unstructured) character of this other problem, the theory or theories appropriate to Method I are readily adaptable to anything that observation may reveal (unlike Method II, which was refuted by observation). Thus, if the disk line in Figure 7 had formed figure-eight's instead of staying straight, practitioners of Method I would have been quickly on hand with generally covariant gravito-elasto-hydrodynamic equations compatible with such a result, and the "prediction" of straight lines would quietly have been modified. There is nothing nature can do that such victims of their own virtuosity cannot mimic by elastomeric mathematics; hence no way nature can convey to them her displeasure with their premises. The endorsement given to Method I by the present observations is not unlike the endorsement given to Ptolemy's epicycle theory by the discovery of a new planet describable by epicycles.

6. With Method II thus observationally eliminated and Method I self-eliminated as means of removing the logical contradictions of kinematics, there presently remains only Method III. It was doubtless Sherlock Holmes who said that when the probabilities of all alternatives but one have been reduced to zero whatever remains, however improbable, must provide the solution. It is a pity to end this detective story without naming the villain or even clearly identifying the crime. But our subject here is confined to the experiment and its direct implications. The experiment has not revolutionized anything, but it has done its part to shift the odds in favor of Method III, a revolutionary approach to kinematics.

7. Method III seeks to solve the problem of parametric deficiency (loss of degrees of freedom) responsible for the Ehrenfest paradox by introducing extra parameters from the full Lorentz group of inhomogeneous transformations. More specifically, this method requires the Minkowski "rotations" of a particle collective to occur, not around a single, shared space-time origin, but independently for each particle world line around separate space-time origins appropriate to each of the constituent particles. In this it finds compatibility with the present observational evidence that there is no physical meaning to a shared proper time. It has taken us more than sixty years to begin to appreciate the true "apartness" of particles at spacelike separations. Being permanently "elsewhere" from each other, they exist literally in different worlds, and may descriptively share much less than conventional wisdom supposes.

8. That dynamics can exist without the foundation of a logically consistent kinematics is absurd; for any structures or motions that can occur for cause can be described apart from causes--and that description is known as kinematics. Kinematics is foundational (logically preconditional) to physics. Physics should rest on its foundations, not rescue them. The logical order of development of

physics is clear: first get kinematics right, then go on to dynamics. The opposite approach is not physics but mathematical mimicry of nature. From Method III of handling the situation, recommended by the present considerations, may arise other contradictions, as yet unsuspected, requiring still more violently radical reformulations. Many physicists these days seem capable of entertaining revolutionary thoughts only in the social and political spheres. Arise, ye rigid, irrotational prisoners of homogeneous Lorentz transformations! You have nothing to lose but your three degrees of freedom.

#### IMPLICATIONS FOR FURTHER EXPERIMENTATION

9. If anyone should in the future wish to repeat the present observations, the experiment might best be done with magnetic (or other nonmechanical) support of a small disk in vacuum.

10. The main need for further experimentation is in verifying or refuting the Thomas precession of spins. Thomas took a flying leap into the unknown when he "deduced" from the kinematic precession of coordinate axes a precession of physical vectors. The failure of the optical analogue of his effect to manifest itself in the present experiment reinforces the doubts about this "effect" that should have arisen with the advent of the Dirac equation [and that apparently have at least fleetingly arisen in the mind of one author\*--see reference (dd)]. It is most important that this issue be unambiguously resolved: do spin angular momenta, or other vectors, turn with respect to the fixed stars when carried torque-free around circuits in flat space, or do they not? For without a definite answer to this question all long-term earth satellite experiments with accurate gyroscopes will be incapable of meaningful interpretation.

11. In connection with the design of an experiment to test the geometrical aspect of the Thomas precession, Edmund Trownson, NOL, has suggested that use be made of a material resembling an ideal permalloy, wherein magnetization direction is unaffected by relative orientation of the lattice. This might be spun in the laboratory in the form of a disk. The magnetization direction of a locally magnetized portion of the material near the rim might be initially aligned with some fiducial (e. g., radial) direction on the disk. If the magnetization vector, a sum of atomic spin contributions, precesses relative to the fiducial direction during a long period of disk rotation, and the disk is then stopped, the net precession should be

\*Another author, Whitmire [references (ff), (gg)], has recently cast doubt on the Thomas precession of a macroscopic object such as a gyroscope. His reasoning is of the Method-I variety, for it invokes kinematic "shear stresses." He believes in the observability of the Thomas effect in respect to electronic spin precessions, a supposition for which the experiment proposed in the next paragraph should be crucial.

"frozen" and the direction change should be measurable in the stopped disk--unlike the putative kinematic and aberrational effects discussed in this report, which vanish when rotation stops. Alternatively, there exists a phase-comparison method, suggested by Abraham Silverstein, NOL, whereby the magnetization direction can be observed while the disk is in continuous rotation. Thus a few weeks of high-speed rotation of a small disk should suffice to resolve the issue. For the record, the writer offers his prediction, based on acceptance of Newton's laws in the Newtonian limit, that no evidence for the Thomas directional precession of physical vectors will be found by this or any other observation.

ACKNOWLEDGMENTS

The main benefit to the writer of doing the experiment was a renewed and much deepened appreciation of the amazing concentration of engineering, technical, and scientific talent at this Laboratory. It is impossible adequately to acknowledge all debts, but since the personal thanks due to many have not been conveyed, an attempt will be made here to remedy the main deficiencies.

First, if one acknowledgment is due before all others, it is to Eugene Horanoff, whose help at critical junctures in disk machining and marking and in arranging the experimental setup was vital. His willingness, and that of Edmund Trounson, who proposed the method of tachometry and furnished other experimental design guidance, to take seriously the value of the experiment provided much needed moral support. An equal debt is owed to Stanley Richards, who by designing and installing the entire pressurized gas system (Figure 4) did the hardest part of the experiment--with the exception of the laser flash illumination part, which was accomplished entirely by the indispensable efforts and equipment of Allen Erickson.

Thanks are due to Robert Ball and Louis Ungar for photographic advice, film processing assistance and the loan of lenses and a camera stand; to Robert Gastrock for advice on film, for lens and equipment loans, and for the gift of special film; to Robert Leary for encouragement in the exploratory phase, for the loan of fuze-spinner equipment and a tachometer, and for useful information about air bearings; to Wallace Anderson and Al Syeles for spending selfless hours and inexhaustible ingenuity in solving problems of disk marking and air turbine preparation, and to their supervisor, Al Krall for letting them do it; to William Buehler and Dr. Henry Belson for advice on disk materials and related matters; to Eugene Flipppo for his accurate machining of the disks; to George Schulz for timely electronic repairs; to Peter Vial for information on air bearings and rotors; to Nicholas Sarelas, Emmett Hunt, Joseph Fleischman, George ("Gabby") Hayes, Dan Eno, and Robert Amann for advice and help relating to rotor preparation, polishing, and marking; to William Talbert and Paul Koloc for advice and suggestions on optical design; to W. Lawton King, Charles Grover, and Charles Spring, and Dr. S. J. Jacobs for advice on high-speed photography; to Wellman Clark for the loan of laboratory equipment; to Louis Hirschel for the loan of solid-state counters and a sympathetic ear; to Leonard Crogan for the gift of titanium for a rotor (not used); to Lucian Minor and Mrs. Dorval McKean for vital assistance in ordering supplies; to Herman Templin for the loan of a traveling-stage microscope; to Charles Dyer for information on microdensitometry; to Dr. Leon Schindel for essential help in elucidating disk drag aerodynamics and in finding a place to do the experiment; to Dr. R. E. Wilson and Joseph Iandolo

for space authorization (Dr. Wilson's Voltairean tolerance in accepting the need for observation while correctly predicting the null outcome was a decisive factor in enabling the experiment to take place); to Dr. C. M. Schoman for equipment purchase authorizations and for unfailing administrative support, without which the enterprise could not have been contemplated; to Mrs. Helen Bovello for tireless efforts in stub preparation and secretarial support; and to all these and many more for their interest, goodwill and assistance.

Outside NOL, thanks are due to Dr. Louis Maxwell (NOL, ret.) whose encouragement and reference to the work of Professor Hill (reference (q)), were particularly helpful; to J. W. Beams (Emeritus Professor, University of Virginia) whose advice concerning high-speed rotation and reference to Dr. Weinstein's work (reference (o)) were much appreciated; to Dr. D. H. Weinstein (Superior Oil Company, Houston, Texas) for information about his version of the experiment; to Dr. C. W. Sherwin for suggesting alternative observational approaches; and to Dr. R. G. Newburgh (Air Force Cambridge Research Laboratories) for many discussions and idea exchanges that have played an essential part in stimulating the writer's awareness of kinematic problems.

Important contributions were also made by industry representatives, notably R. G. Willing (Federal-Mogul Corporation, Ann Arbor, Michigan) who provided information on air turbine characteristics and maintenance procedures; G. I. Allen (Schenck-Trebel Corporation, Farmingdale, New York) whose help in precision disk balancing was essential; and A. R. Pellicciotti, M. Sabo, H. J. Koerner, and others of the IBM Corporation (Endicott, New York) who generously provided heavier-duty air turbines (circuit board drills) that would have been used if the Westwind unit had failed.

REFERENCES

- (a) F. Rohrlich, Classical Charged Particles, Addison-Wesley, Reading, Mass. (1965)
- (b) P. Ehrenfest, Phys. Zeits. 10, 918 (1909)
- (c) G. Herglotz, Ann. d. Phys. 31, 393 (1910)
- (d) F. Noether, Ann. d. Phys. 31, 919 (1910)
- (e) M. Born, Ann. d. Phys. 30, 840 (1909)
- (f) W. Pauli, Theory of Relativity, Pergamon Press, New York (1958)
- (g) G. Herglotz, Ann. Phys. Lpz. 36, 493 (1911)
- (h) E. Dewan and M. Beran, Am. J. Phys. 27, 517 (1959)
- (i) G. Cavalleri, Nuovo Cim., VIII B, 415 (1968)
- (j) Y. P. Terletskii, Paradoxes in the Theory of Relativity, Plenum Press, New York (1968)
- (k) R. G. Newburgh and T. E. Phipps, Jr., Nuovo Cim. 67B, 84 (1970)
- (l) M. von Laue, Phys. Zeits. 12, 85 (1911)
- (m) O. Costa de Beauregard, Précis of Special Relativity, Academic Press, New York (1966)
- (n) H. Arzeliès, Relativistic Kinematics, Pergamon Press, New York (1966)
- (o) D. H. Weinstein, Nature 232, 548 (1971)
- (p) N. Rosen, Phys. Rev. 70, 93 (1946) and 71, 54 (1947)
- (q) E. L. Hill, Phys. Rev. 69, 488 (1946) and 71, 318 (1947)
- (r) H. Takeno, Progr. Theor. Phys. 7, 367 (1952)
- (s) C. W. Berenda, Phys. Rev. 62, 280 (1942)
- (t) C. Møller, The Theory of Relativity, First Edition, Oxford, London, England (1952)
- (u) L. H. Thomas, Nature 117, 514 (1926)
- (v) R. W. Ditchburn and O. S. Heavens, Nature 170, 705 (1952)
- (w) G. H. F. Gardner, Nature 170, 243 (1952); also J. L. Synge, Nature 170, 244 (1952)
- (x) W. L. King and A. M. Erickson, An Investigation of Laser Applications to Photographic Illumination, NOLTR 69-55, 5 May 1969
- (y) H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1955)
- (z) J. W. Beams and E. G. Pickels, Rev. Sci. Instr. 6, 299 (1935)
- (aa) J. W. Beams, J. Appl. Phys. 8, 795 (1937)
- (bb) A. M. Erickson, Shock-Excited FTIR Q-Switch, NOLTR 71-158, 9 Aug 1971
- (cc) D. R. Inglis, Phys. Rev. 50, 783 (1936); also W. H. Furry, Phys. Rev. 50, 784 (1936)
- (dd) G. P. Fisher, Am. J. Phys. 40, 1772 (1972)
- (ee) J. L. Synge, Relativity: The Special Theory, Second Edition, North-Holland, Amsterdam, and John Wiley, New York (1964)
- (ff) D. P. Whitmire, Nature 235, 175 (1972)
- (gg) D. P. Whitmire, Nature 239, 207 (1972)

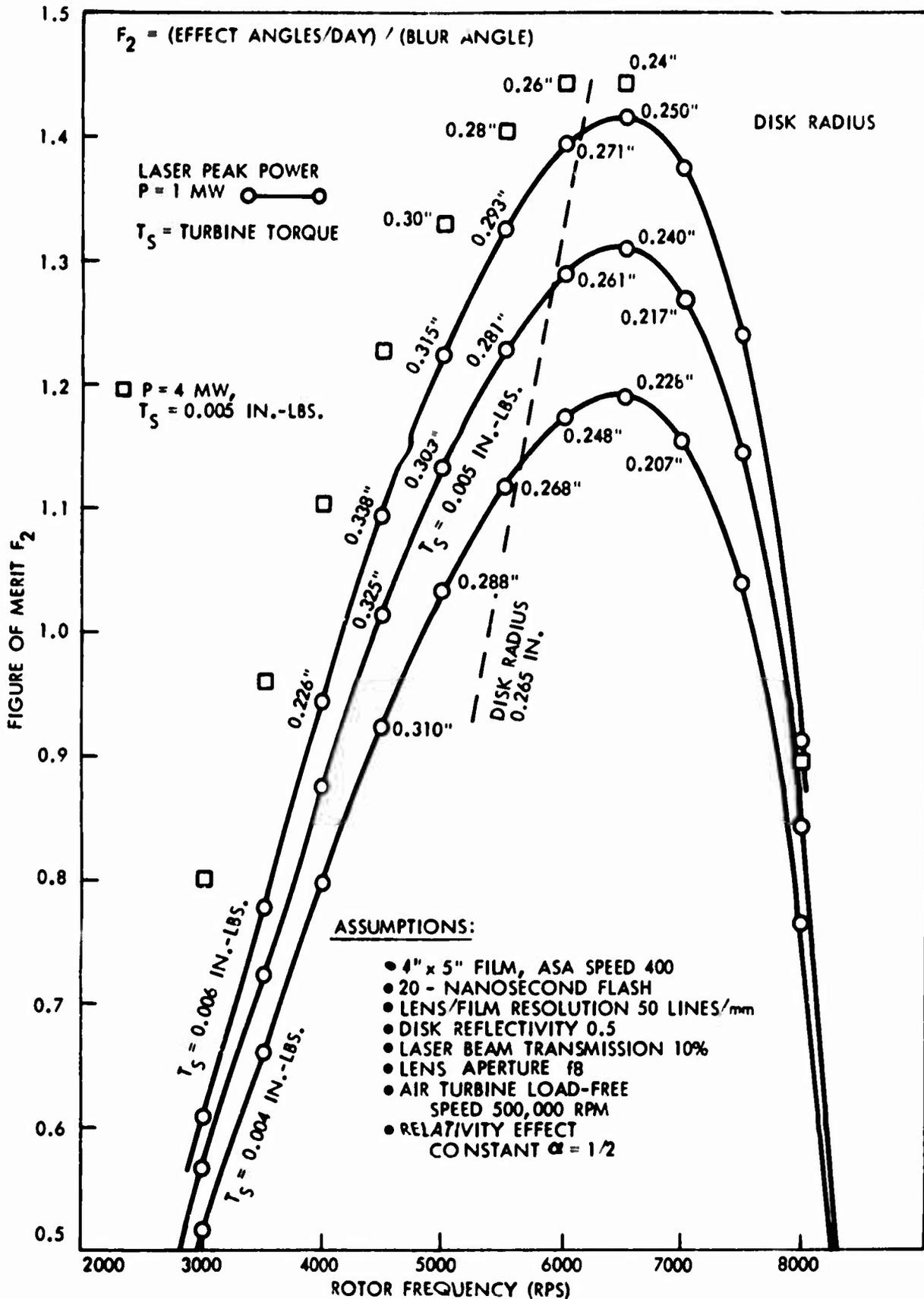


FIG. 1 "SIGNAL - TO - NOISE" FIGURE OF MERIT  $F_2$  VS. ROTOR FREQUENCY

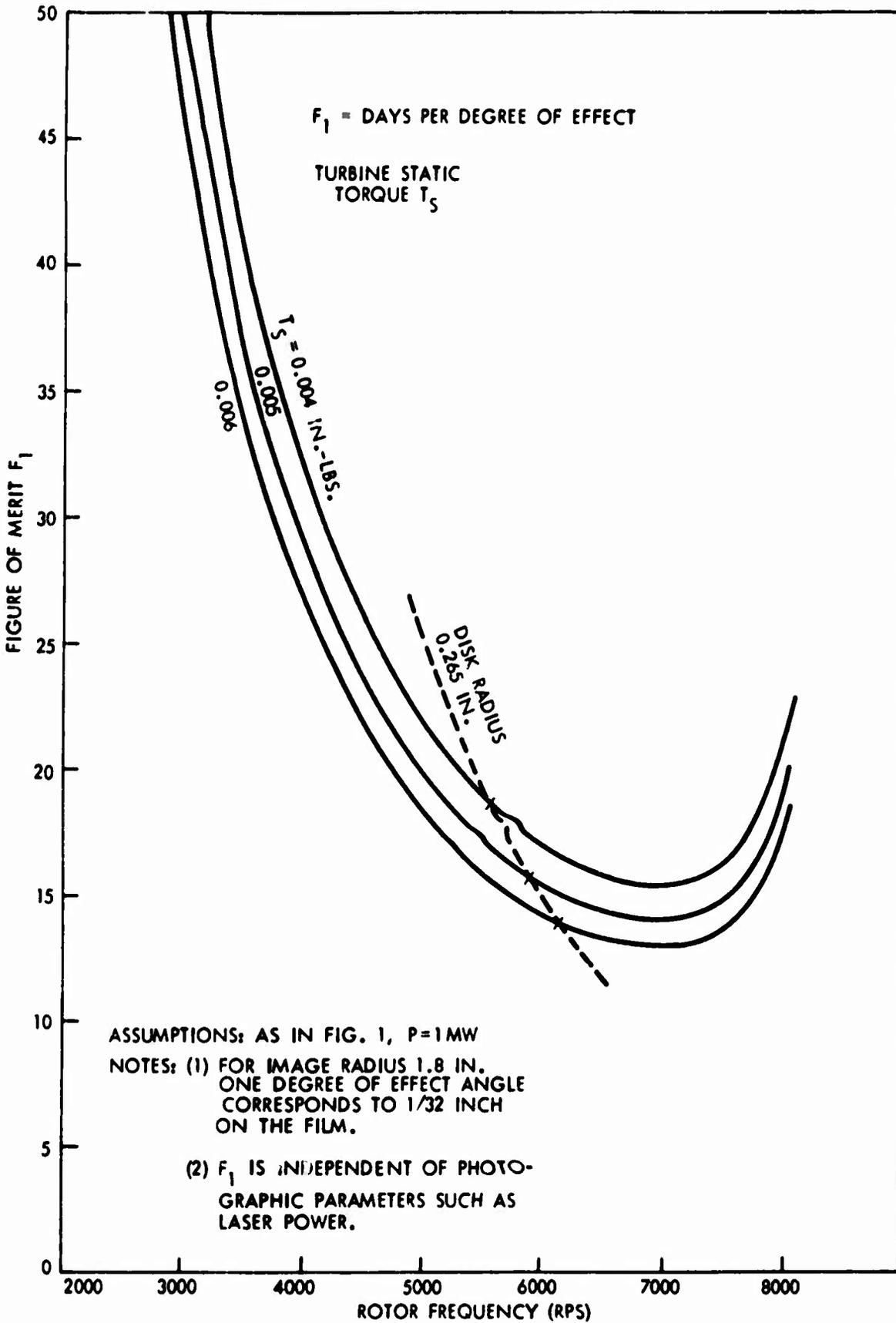


FIG. 2 FIGURE OF MERIT  $F_1$  VS. ROTOR FREQUENCY

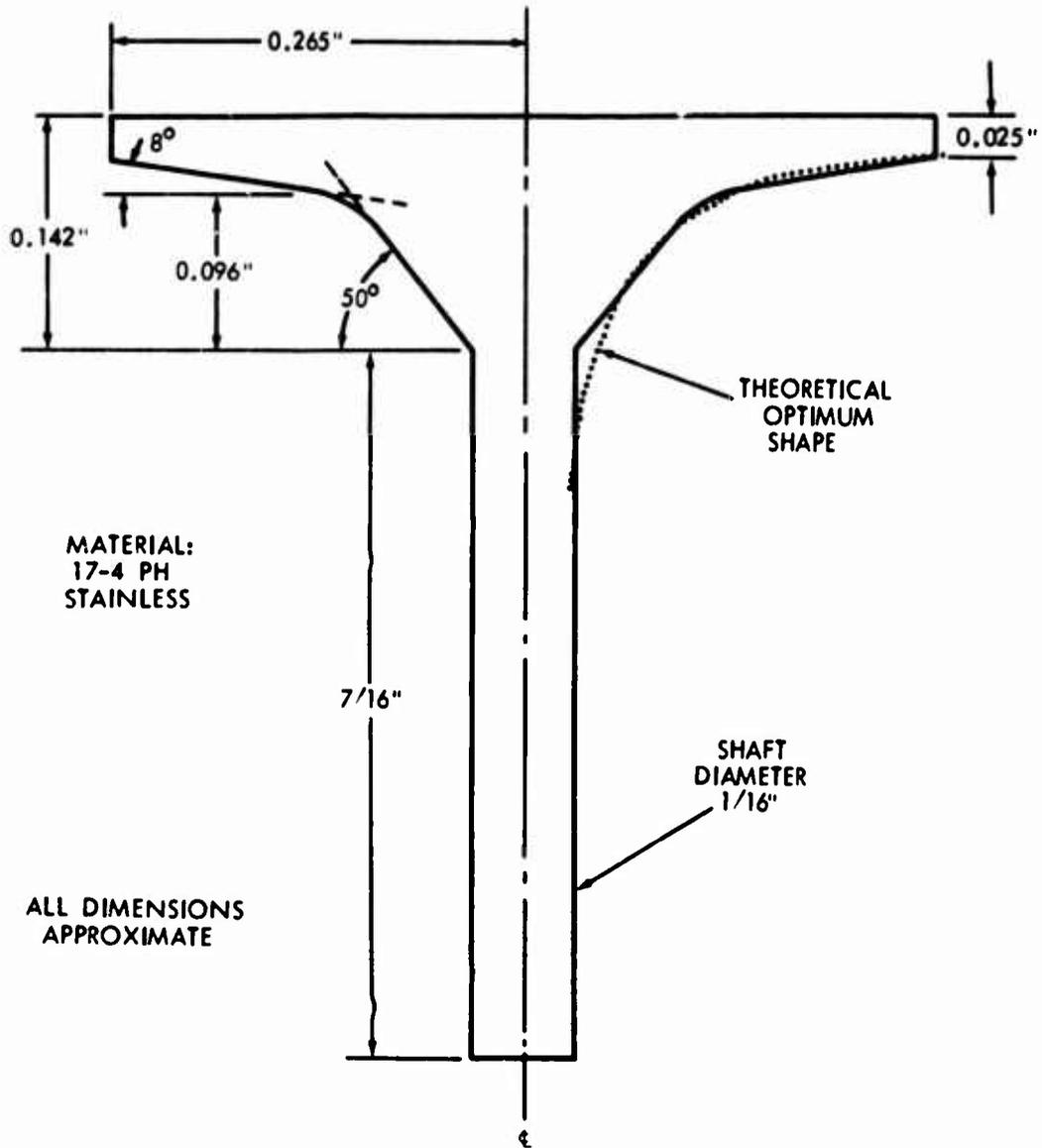


FIG. 3 HIGH-SPEED ROTOR CROSS-SECTION

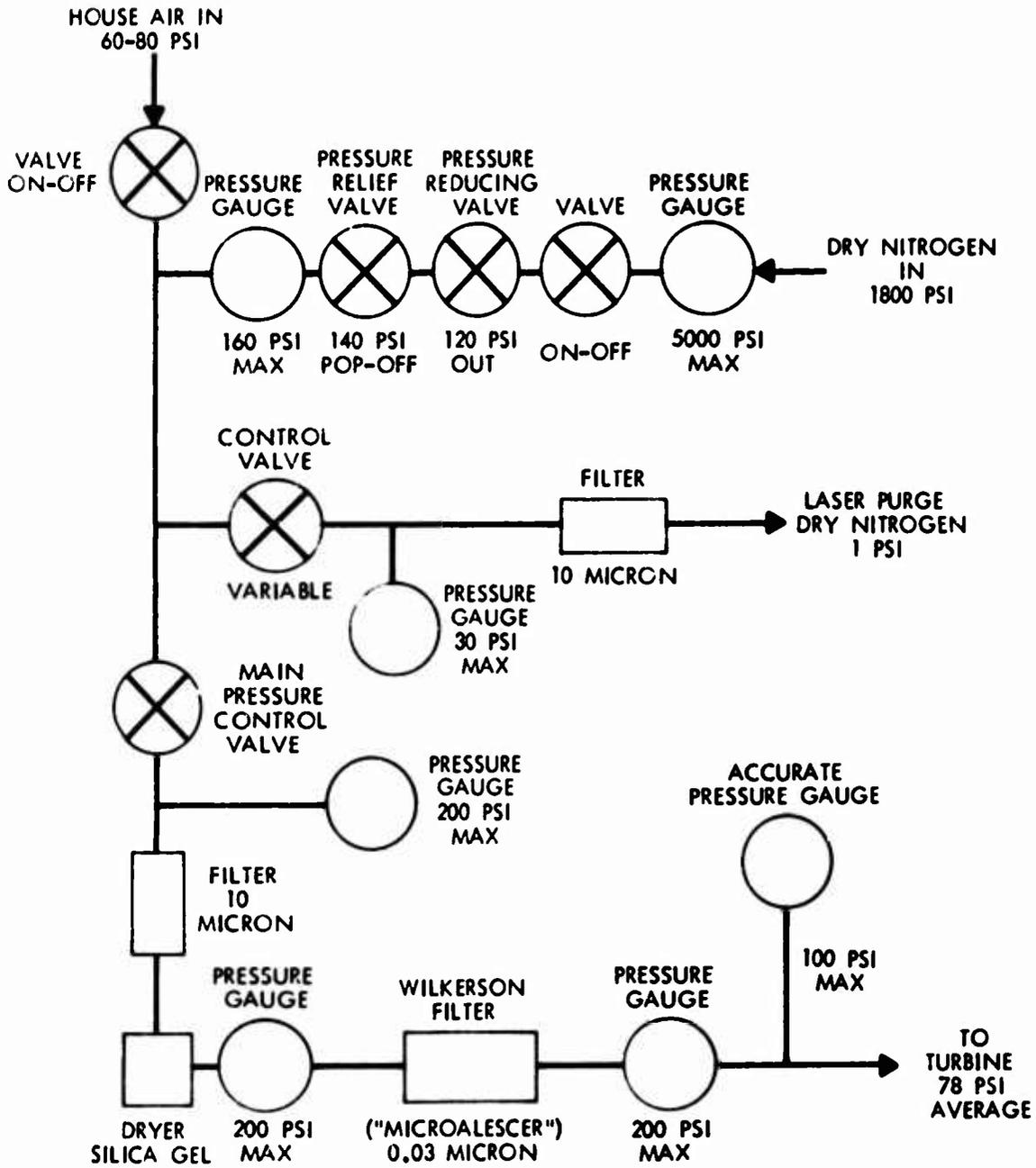


FIG. 4 SCHEMATIC OF PRESSURIZED GAS SUPPLY TO TURBINE

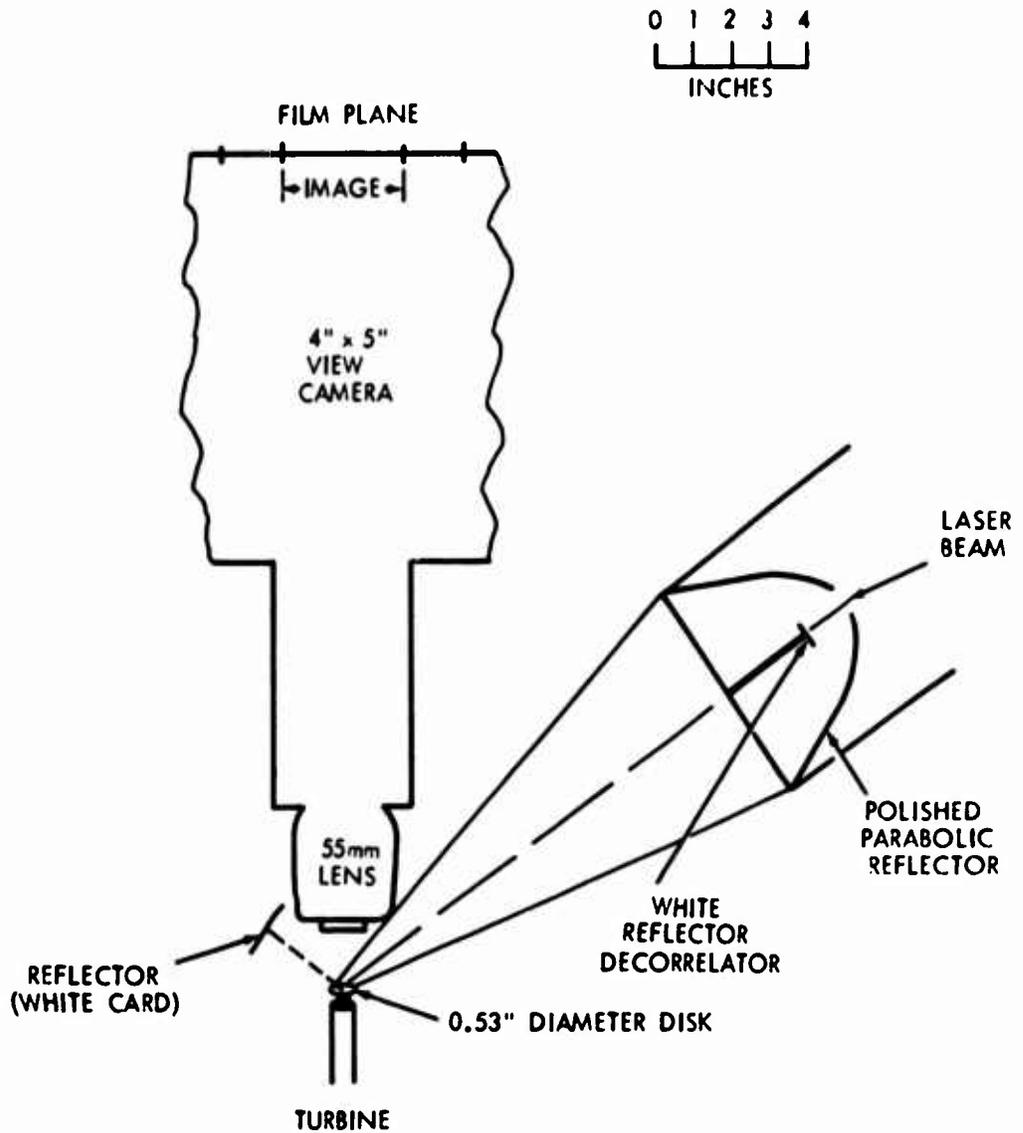


FIG. 5 SKETCH OF LASER ILLUMINATION ARRANGEMENT,  
APPROXIMATELY TO SCALE

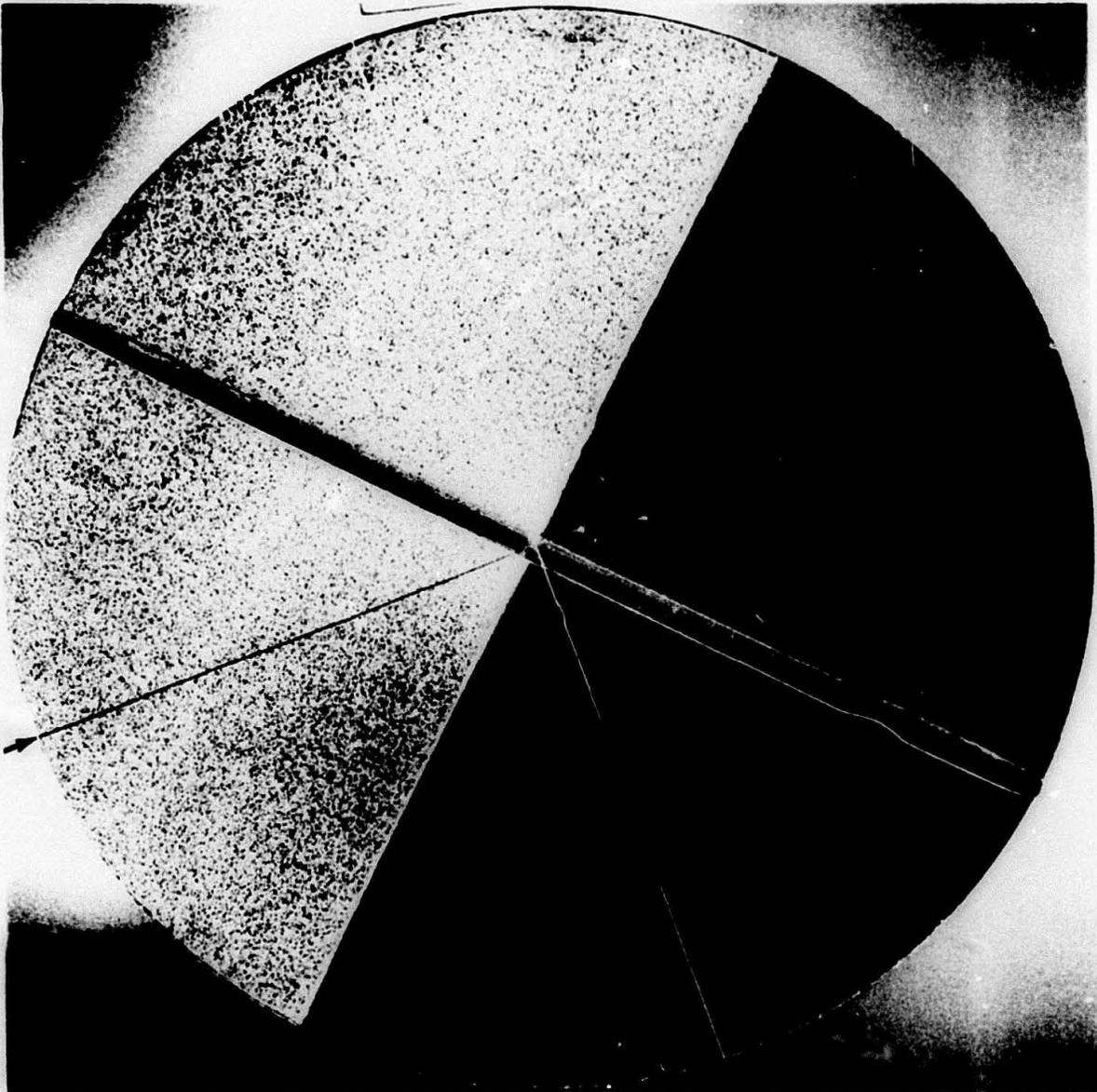


FIG. 6 FLASH PHOTOGRAPH OF DISK STATIONARY BEFORE RUN.  
ARROW INDICATES RADIAL LINE USED IN SUBSEQUENT  
DATA ANALYSIS

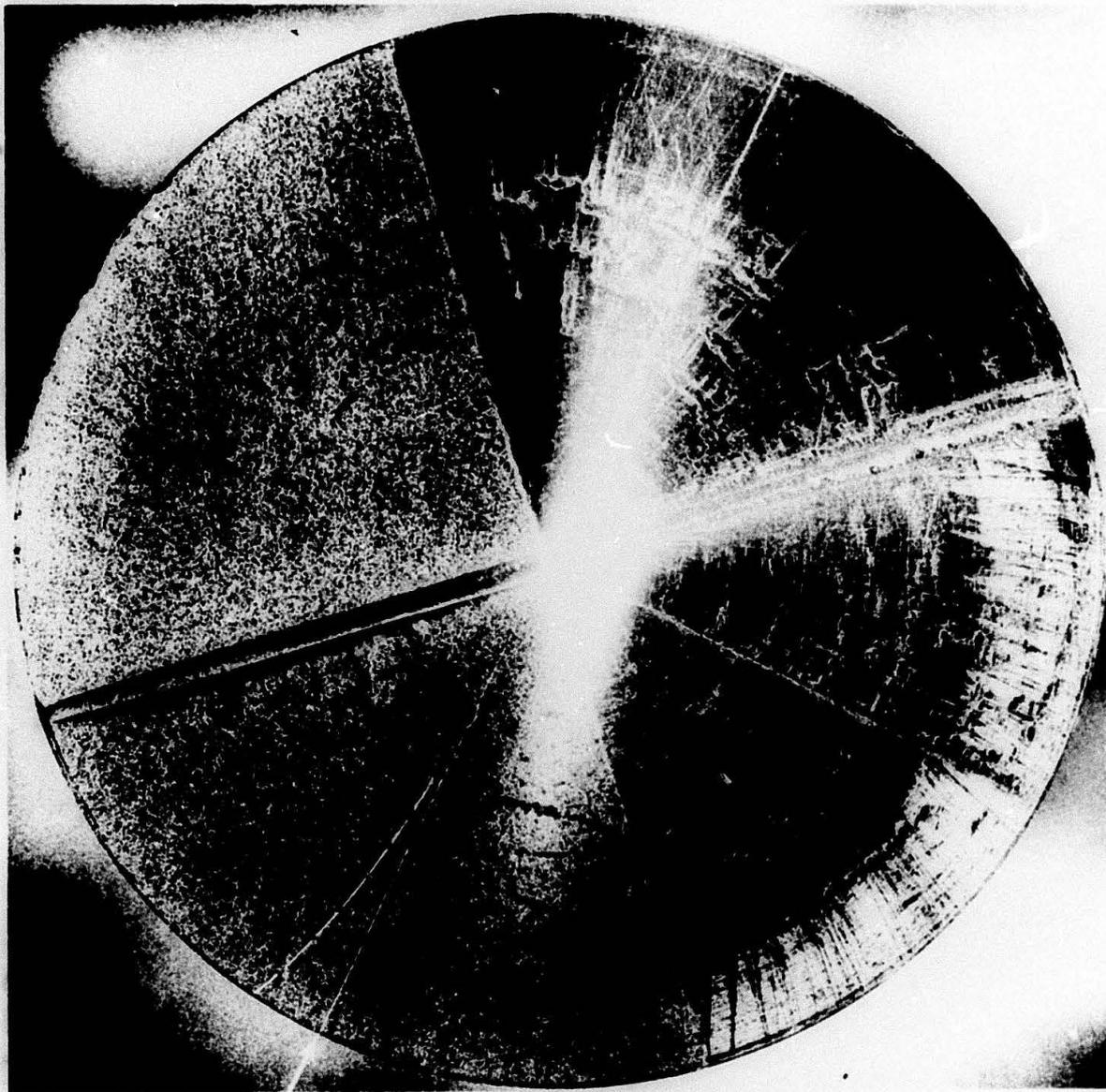


FIG. 7 FLASH PHOTOGRAPH OF DISK IN MOTION AT 6116 RPS AFTER FOUR MONTHS OF CONTINUOUS ROTATION. ARROW INDICATES SAME LINE SO INDICATED IN FIG. 6. DASHED CURVE SHOWS RELATIVITY EFFECT THAT WOULD HAVE BEEN SEEN FOR  $\alpha = 1/2$

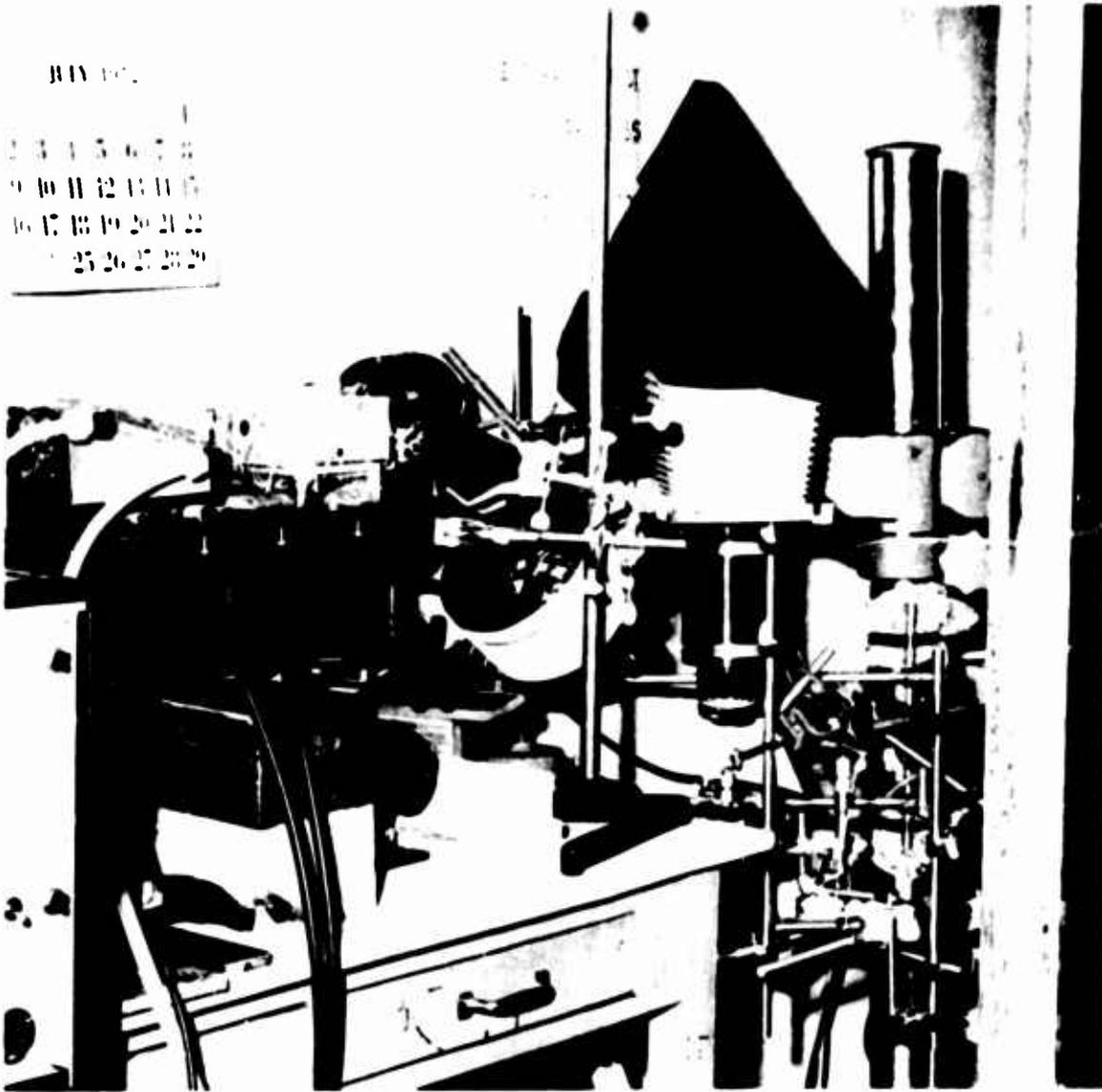


FIG. 8 GENERAL VIEW OF APPARATUS NEAR END OF RUN, KORAD LASER TO LEFT, CAMERA RIGHT WITH DISK JUST BELOW LENS

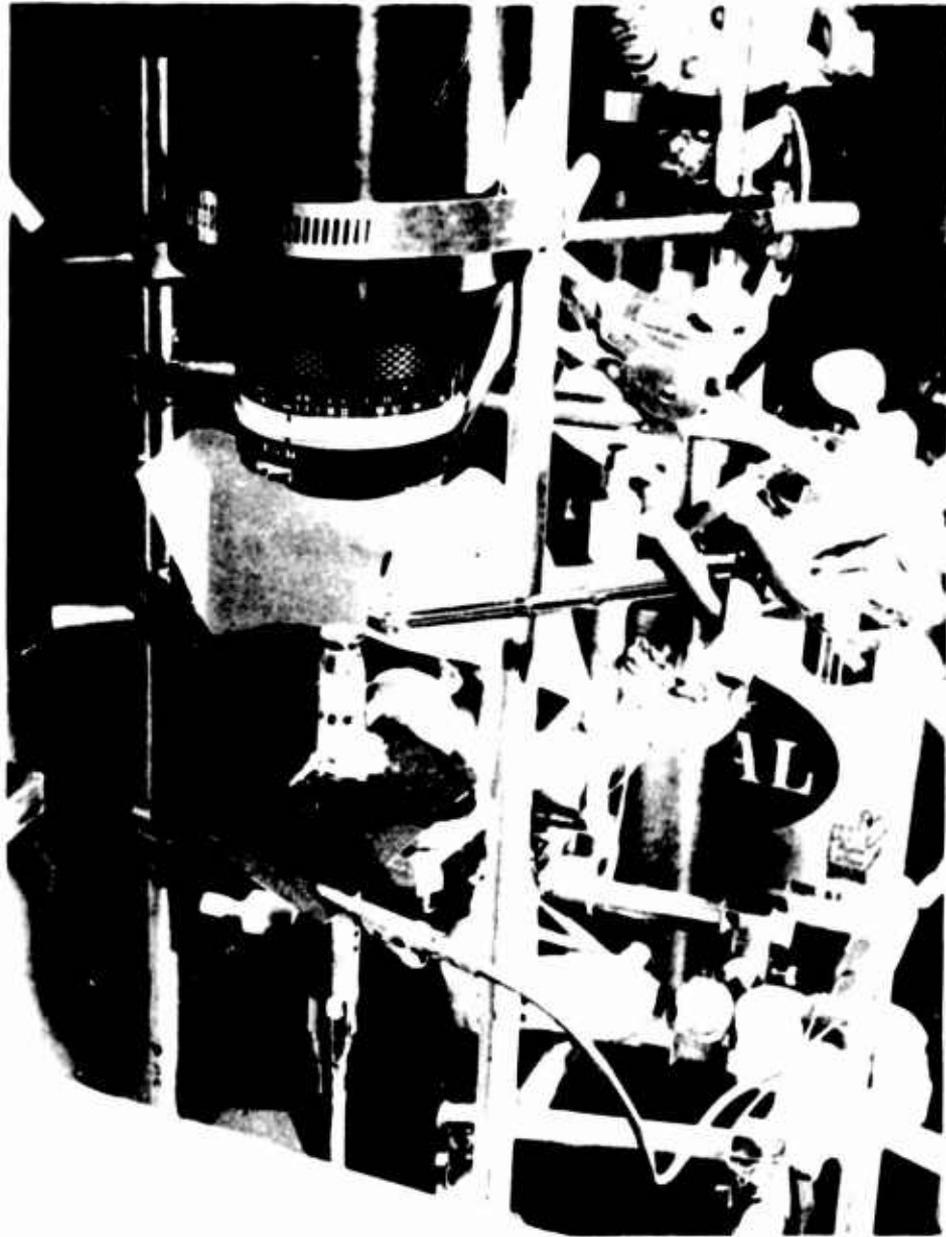


FIG. 9 CLOSE-UP OF DISK SHOWING TACHOMETER LIGHT REFLECTING OFF ITS SURFACE TO DENTAL MIRROR; X-Y-Z- POSITIONER MICROMETERS VISIBLE IN BACKGROUND