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COMPUTER GRAPHICS METHOD FOR SOLVING TRANSCENDENTAL EQUATIONS

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A COMPUTER GRAPHICS METHOD

FOR SOLVING TRANSCENDENTAL EQUATIONS

by

Carl H. Durney

December 1970 UTEC-CSc-70-109

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Introduction

Finding the roots of an equation $F(x)=0$ when $F(x)$ involves transcendental functions and x is complex usually involves some kind of search method. The efficiency of a search method depends to a certain extent on knowledge of the roots--where they are likely to occur in the x plane, and how many there are. If a root is known to lie in a given region in the x plane, then a search routine can quickly find the root to the desired accuracy. But if no information about the location of the roots is available, a search over a wide area must be conducted, and this can be time consuming and expensive. Consequently, a method for locating the general area of the roots and determining the pattern of the roots is very valuable.

This report describes a simple method for graphically displaying the pattern of roots in the complex plane. The advantages of this method are the same as the advantages of computer graphics in general. It provides a way of visualizing the system, in this case, the transcendental equation; and it allows one to obtain a qualitative feel for the behavior of the system and therefore intuition about the system.

Determining the Root of a Transcendental Equation

There are a number of ways to determine the values of x which satisfy the equation $F(x)=0$. (The case of interest here is that in which F is a complex function of the complex variable x.) One way is to search for values of x which make Real $(F(x))$ and Imaginary $(F(x))$ both zero at the same time where the search is based on sign changes in both quantities. There is a problem in that a sign change occurs across a pole as well as a zero.

The method used here is that the transcendental equation is written in the form

$$f(x) = 1 \qquad (1)$$

and $f(x)$ is written in the form

$$f(x) = R(x)e^{i\phi(x)} \qquad (2)$$

where $R(x)$ is the magnitude of $f(x)$ and $\phi(x)$ is the phase. The roots of Equation 1 are values of x which make

$$R(x) = 1$$
$$\phi(x) = 2m\Pi \quad m = 0, \pm 1, \pm 2, \ldots$$

Let

$$A(x) = 0.5 \text{ if } R(x) \geq 1$$
$$A(x) = 0 \quad \text{if } R(x) \leq 1$$
$$P(x) = \frac{0.5\phi}{2\Pi} \quad 0 \leq \phi < 2\Pi$$
$$B(x) = A(x) + P(x)$$

Now B(x) is to be the brightness on a graphics display. Since A(x) and P(x) each have discontinuities, B(x) will show two kinds of discontinuities. The solutions to Equation 1 occur where the discontinuities of A(x) and P(x) occur simultaneously. These points may be seen on a display of B(x). An example is shown in Figure 1

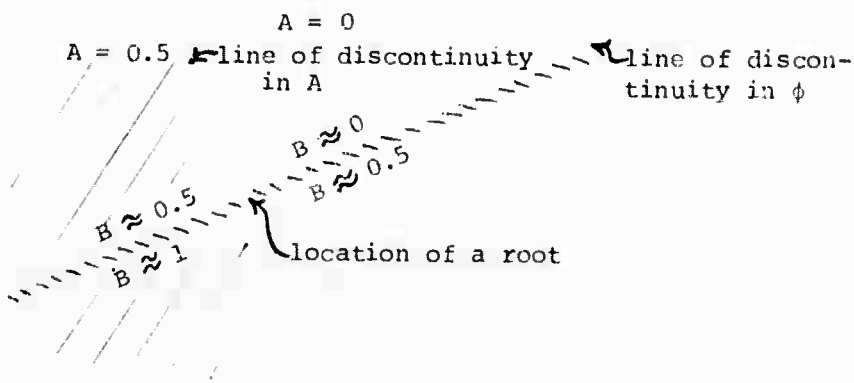


Figure 1: Illustration of a display of B(x).

A computer program can be written to compute and display B(x) for a grid of points in the x plane. The roots of Equation 1 are then immediately obvious, but more important, the pattern of the roots in the complex plane can be seen, and a qualitative feel for the transcendental equation can be obtained.

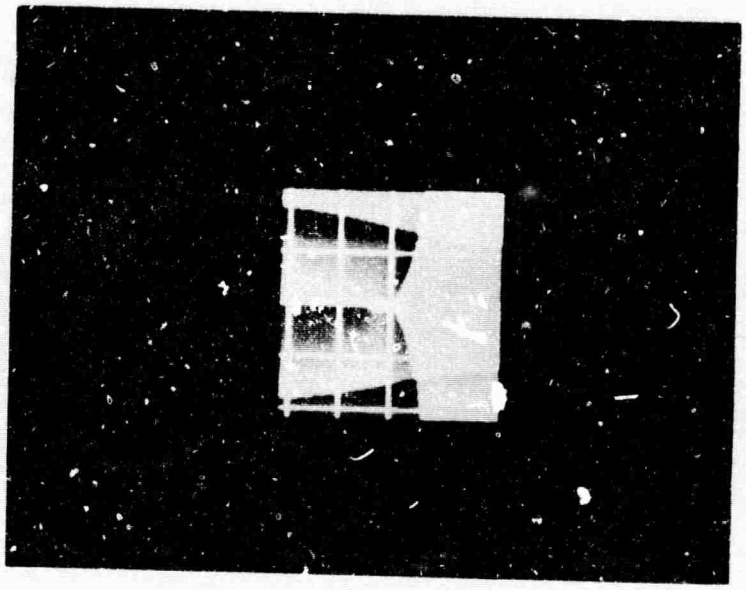
A Graphics Display

The method described above was used to obtain the computer-generated photograph shown in Figure 2. The transcendental equation is

$$\frac{e^{-y}}{x-(1+i0.02)} = 1$$

A computer program (see printout in Appendix A) was written to calculate and display B(x) for 131x131 points in the complex x plane in increments of 0.1. Coordinate lines spaced 3 units in x are also shown. The location of three of the roots is readily apparent. One of them, which is not quite obvious at first glance, is almost on the real axis near 1.3. An infinite number of complex roots occur upwards and downwards on the line of discontinuity in A, which is the line in the general up and down direction. The lines which go across the photograph are discontinuities in P. This photograph shows how the pattern of the roots is readily identifiable. A search routine could be used to find two roots as accurately as desired, and very efficiently, because their approximate location can be easily obtained from the photograph.

The photograph shown in Figure 2 was generated by a computer graphics system of the Computer Science Division at the University of Utah utilizing a multiple point-by-point exposure. A diagram of the system is shown in Figure 3. The approximate Univac 1108 computer time used to make the photograph was 170 seconds.



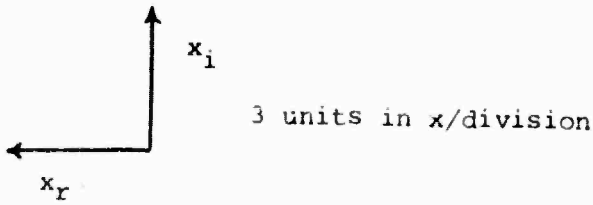


Figure 2: A computer-generated photograph showing B(x) in the complex x plane. There are 131x131 points corresponding to increments in x of 0.1. The roots occur at the intersections of the lines of discontinuity. Three roots are shown in this photograph. One of them is almost on the real axis.

A Poor Man's Graphic Method

All the information contained in Figure 2 is also contained in the printout shown in Figure 3, even though the printout is not as pretty. The direction of real x is reversed from that of the photograph. The same basic method was used to obtain the printout in Figure 3, but the program is much simpler because no graphics system is involved, and the method can be used with any digital computer system. The program listing is given in Appendix B.

In the printout shown in Figure 3 each space represents a complex value of x and the increments used here are 0.1. A slash in a space indicates $R(x) \geq 1$. The digitized value of $\phi(x)$ are indicated as:

Blank	0
Period	1
2	2
Comma	3
4	4
Dash	5
6	6

Since the value of ϕ was calculated in floating point and then converted to fixed point, a 2 means $2.0 \leq \phi < 3.0$. The locus of points where ϕ changes from 2π to 0 are readily apparent, and the other symbols show how ϕ is changing over the plane. Since the program is extremely simple, this method should be a very useful tool, even to those without access to a graphics system. The 1108 computer time used to obtain the printout of Figure 3 was 13 seconds. The method

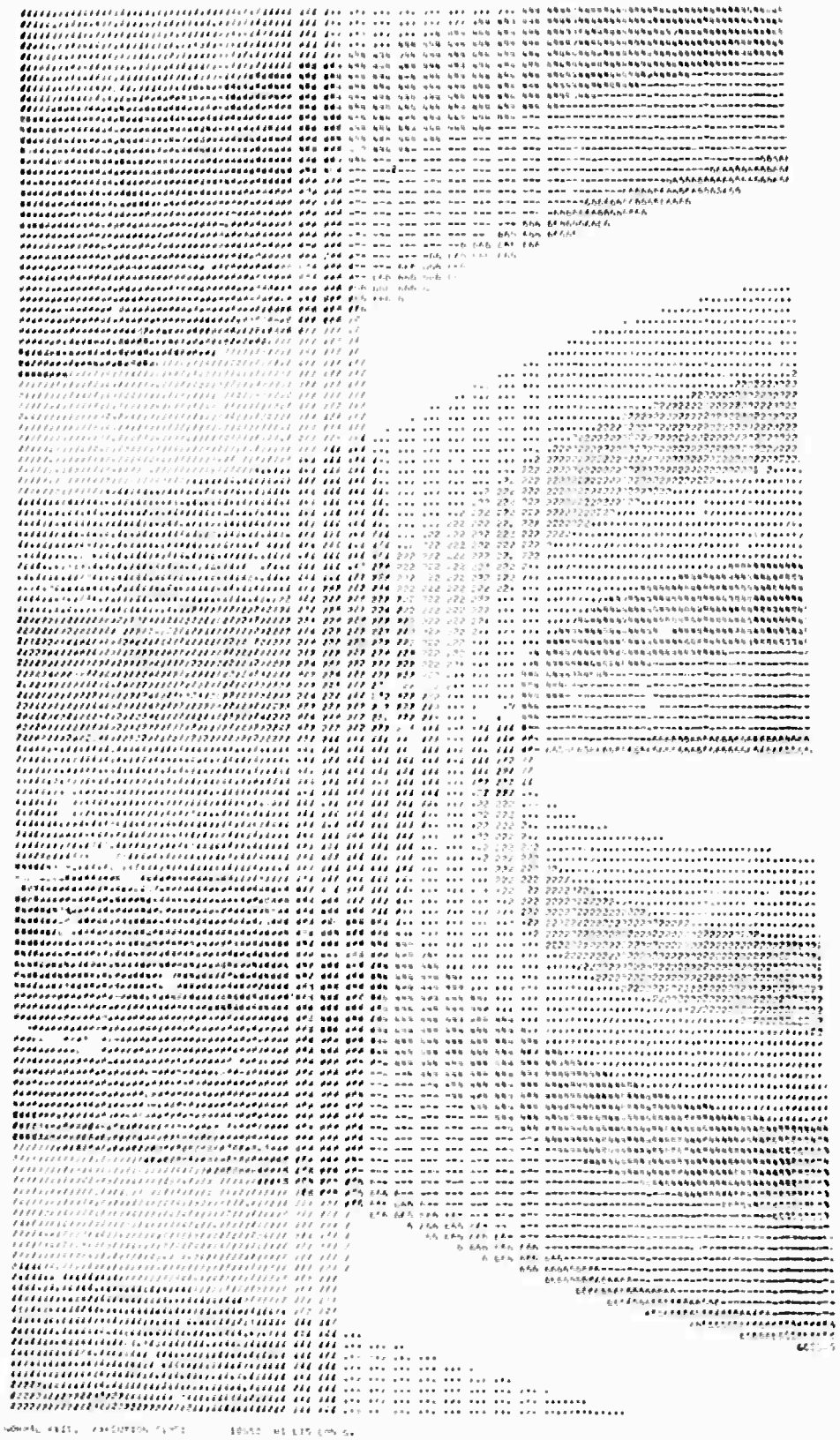


Figure 3: A printout showing the same information as the photograph in Figure 2.

was used here at the University of Utah with good success in solving the dispersion equation describing the double-stream microwave interaction between electrons and holes in semiconductors.[1] A little experimentation with density of characters would result in a more pleasing printout.

Conclusions

The inherent power of computer graphics has been applied to the solving of transcendental equations. The method described here is simple, yet powerful. It provides a way to visualize the characteristics of the transcendental equation and obtain insight into the nature of the roots. With this insight, a search routine could be used very efficiently to obtain the roots to the desired accuracy.

[1] Lewis C. Goodrich. A small-signal field analysis of double-stream interactions in finite semiconductors. Ph.D. thesis, University of Utah, Salt Lake City, Utah, June 1970.

APPENDIX A

This appendix contains the listing of the program used to obtain the photograph in Figure 2.

```
MELT,BIL DORNEY
LET PROCESSOR LEVEL      3
000001      000          COMPLEX X,R
000002      000          INTEGER DF(171)
000003      000          NAMELIST/QUIT/EX
000004      000          CALL RELOAD
000005      000          CALL INOUTM
000006      000          CALL SETLST
000007      000          READ(5,QUIT)
000008      000          CALL JUMPS('QUIT',$200)
000009      000          CALL CHRINT(1,$8)
000010      000          CALL SWPCHR('*')
000011      000          CALL SWAP
000012      000        ; XR=-6.5
000013      000          XI=6.5
000014      000          K=6
000015      000          DO 100 I=1,512
000016      000          IF(I.GE.1.AND.I.LE.190)GO TO 100
000017      000          IB=0
000018      000          IW=1
000019      000          M=6
000020      000          DO 101 J=1,513
000021      000          IF(I.GE.322.AND.I.LE.512)GO TO 41
000022      000          IF(J.GE.1.AND.J.LE.190)GO TO 41
000023      000          IF(J.GE.322.AND.J.LE.513)GO TO 41
000024      000          IF((I-190).EQ.K.OR.(J-190).EQ.M)GO TO 40
000025      000          X=CMPLX(XR,XI)
000026      000          F=1./(CEXP(X)*(X-(1.0,.0200)))
000027      000          AM=CABS(F)
000028      000          R=REAL(F)
000029      000          AI=AIMAG(F)
000030      000          A=0
000031      000          IF(AM.GE.1.)A=50
000032      000          PHI=ATAN2(AI,R)
000033      000          IF(PHI.LT.0.)PHI=PHI+6.2831
000034      000          P=PHI*7.96
000035      000          BB=102.4*EXP(.023*(A+P))
```

```
000036    000          XR=XR+.1
000037    000          GO TO 5
000038    000       40 BB=885.
000039    000          M=M+30
000040    000          XR=XR+.1
000041    000          GO TO 5
000042    000       41 BB=0
000043    000        5 FLD(IB,12,MF(IW))=FLD(24,12,INT(BB))
000044    000          IF(IB.GE.24)GO TO 50
000045    000          IB=IB+12
000046    000             TO 101
000047    000       50     )
000048    000             IW+1
000049    000      101 CONTINUE
000050    000          IF((I-190).EQ.K)K=K+30
000051    000          XR=-6.5
000052    000          XI=XI-.1
000053    000      100 CALL SNDTEX(DF)
000054    000          CALL SWAP
000055    000          CALL IDLE
000056    000          GO TO 1
000057    000      200 CALL EXIT
000058    000        8 CALL TTY
000059    000          CALL INTRET
000060    000          END
```

FIN

APPENDIX B

This appendix contains the listing of the program used to obtain the printout given in Figure 3.

```
00101      1*        COMPLEX X,F
00103      2*        DIMENSION IP(131),IA(131 ,IC 7)
00104      3*        DATA IC/1H , 1H.,  1H2, 1 ,,  H4, 1H-  1H
00106      4*        DATA M2/1H//, M3/1H
00111      5*        CALL NESKIP
00112      6*        XR=-7.
00113      7*        XI=7.
00114      8*        DXI=-.1
00115      9*        DXR=.1
00116     10*        DO 6 J=1,131
00121     11*        DO 1 I=1,131
00124     12*        X=CMPLX(XR,XI)
00125     13*        F=1./(CEXP(X)+(X-(1.,.02 0)))
00126     14*        AM=CABS(F)
00127     15*        IF(AM.GE.1.)IA(I)=M2
00131     16*        IF(AM.LT.1.)IA(I)=M3
00133     17*        R=REAL(F)
00134     18*        AI=AIMAG(F)
00135     19*        PHI=ATAN2(AI,R)
00136     20*        IF(PHI.LT.0.)PHI=PHI + 6 283
00140     21*        K=PHI
00141     22*        IP(I)=IC(K+1)
00142     23*      1 XR=XR + DXR
00144     24*        PRINT 3,IP
00152     25*      3 FORMAT(1Y,131A1)
00153     26*        PRINT 4,IA
00161     27*      4 FORMAT(1H4,131A1)
00162     28*        XR=-7.
00163     29*      6 XI=XI+DXI
00165     30*        STOP
00166     31*        END
```