SURVEY AND COMMENTS ON METHODS FOR MEASURING THE SPECTRA OF OCEAN SURFACE SHORT WAVELENGTH GRAVITY-CAPILLARY WAVES

E. Y. T. Kuo, et al

Ocean and Atmospheric Science, Incorporated

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Short Wavelength Gravity-Capillary Waves

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Dr. E. Y. T. Kuo and Dr. Chester Grosch

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Abstract

A review was made of short (less than 10 cm) ocean surface wave measurement techniques. Two distinct systems require development to satisfy this measurement problem: a standardizing system and an in-situ system. The best standardizing system is probably the direct wave-wire instrumentation, although this system has fish line problems in the short wavelength region. The best in-situ system is probably an airborne laser profilometer although stereographic techniques are another possibility.
1.0 Introduction

This report is a preliminary survey of the available methods for determining the spectrum of ocean surface gravity waves. More specifically, emphasis is placed on the short wavelength, non-directional spectrum.

There is no known method which can obtain data over the complete surface wave spectrum and most instrumentation has been designed to measure the larger more active waves. Thus, this report stresses the inherent limitations and accuracies of each method. Typical engineering data (Wiegel, 1964) extend down to periods of about 1 sec and measurements at shorter periods seem to have been restricted to the laboratory.

The deployment method has been left unspecified until a set of technically satisfactory capabilities have been well established. Final preference of a particular measurement method will depend on the following considerations, not necessarily mutually exclusive:

1. Cost
2. Operational requirements, e.g., mobility, time duration, weather conditions, etc.
3. Inherent accuracy of the principle, the required platform, and the required data processing.
4. Availability of dependable calibration.
5. Maintainability.

-1-
The general approach was to classify the measurement techniques according to whether the physical quantity being measured is directly sensed (height, pressure . . . .) or remotely sensed (light, acoustic, electromagnetic . . . .). The direct measurement techniques are then subclassified according to the sensed quantities. The remote measurement techniques are first subclassified according to the physical principle involved, followed by further subclassification according to the kind of sensor employed.

In general, the direct measurement techniques are inherently more accurate but they are either operationally cumbersome or immobile. It is felt that these direct techniques, although cumbersome, should be developed for calibrating the remote techniques that are operationally more desirable. A calibration technique is essential for estimating the effect of the anomalies that are inherent in indirect methods.

The fact that there is not as yet a well-established method cannot be overstressed. There have been scattered attempts to measure small gravity waves. All apparently stopped at satisfying themselves either with the fact that spectrum shapes are consistent with the theoretical equilibrium concept, or with the fact that their extended spectra seemed consistent with other measurement methods at lower wave numbers. There has been no independent qualitative verification of the measured values in this wavelength range.

The gravity waves of interest have wavelengths below 10 cm. The parameters corresponding to this range are listed below.

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The maximum wave height range is not as yet clearly established. It is rather tempting to estimate this range by using the saturation (or equilibrium) principle of Phillips (1958). Accordingly, the range is of the order of 0.3 cm to 0.425 cm. These numbers are based on the saturation levels within the equilibrium spectrum. Measurements in a wind-water tunnel at a mean period of 0.1 sec (Sibul, 1955) yielded a significant wave height, $H_{1/3}$ - the average height of the largest 1/3 of all wave heights - was approximately 0.3 cm.

Small waves can not only be of the narrowband type, but also of the spatially local type. The limiting heights are controlled by the balance between energy inputs and energy dissipations (especially through interactions with background waves) and are difficult to estimate. It should be noted that the intensity of capillary waves varies sporadically over the ocean surface in response to wind turbulence and other local conditions.
2.0 Direct Measurements Near the Surface

Techniques are classified here according to whether the physical quantity being measured is probed locally or remotely sensed by its effect on radiation. The distinction between local and remote sensing is not always a distinct one. Direct measurements will refer to the particular ones done by probing a moving surface to determine displacements or accelerations but it will also refer to pressure measurements made beneath the surface.

2.1 Direct Wave Height Measurement Techniques

Direct wave height measurement techniques record changes of an electronic circuit element such as resistance or capacitance resulting from changes due to immersion. Usually thin wires are employed and the changes in resistance or capacitance between the wires is monitored. Because salt water is such a good conductor, the resistance of a wire of small diameter leading into it is determined in large measure by the length of wire not immersed. That resistance fluctuates with the movement of the surface. Another procedure uses an insulated wire of high conductivity. A thin layer of dielectric insulator introduces a capacitance between the conducting water and wire core which varies with the height of water.

When small amplitudes are to be measured, important errors are produced by:
1. Sporadic wetting and various chemical actions, including contamination, which alter the resistance of the wire portion exposed to the air.

2. A meniscus which varies both hysteretically and erratically as the water moves.

3. Vibrations of the wire and associated strain-induced resistance changes.

4. The effects of the wire on the water surface itself including the generation of waves and the production of local eddying.

5. Motions of the wire supporting buoy or platform.

The wires used in such measurements may be of the order of a mm in diameter. Wetting hysteresis effects at sea can be expected to introduce displacement errors amounting to a fair fraction of a mm along such wires. As noted earlier, the total surface displacements expected are of the order of a few mm at wavelengths of 10 cm and shorter. Fortunately, the maximum hysteresis effect may be estimated experimentally by oscillating the wire up and down in still water.

A characteristic of wires not noted in small scale laboratory measurements is the ability of the wires to generate capillary waves or to vibrate and shed eddies at the Strouhal frequency. The latter frequency is (Goldstein, 1938)

\[ f = 0.2 \frac{u_0}{d} \left( 1 - \frac{19.7v}{u_0 d} \right) \]
where \( U_0 \) is the relative speed of the flow past the wire, \( d \) is the wire diameter and \( \nu \) is the kinematic viscosity. If the mean velocity \( U_0 \) has a value of 25 cm sec\(^{-1}\) past a wire of 1 mm diameter, then

\[
f = 0.2 \cdot \frac{25}{0.1} \left(1 - \frac{17.7 \cdot 10^{-2}}{25 \cdot 0.1}\right) \approx 50 \text{ Hz}
\]

This frequency is most likely high enough to ignore but the lower frequency "strumming" may not be as easily overlooked.

A more fundamental problem has to do with the wake created by the wire as the water moves past. The important wavelengths contained in such a wake disturbance are found to cluster about the critical wavelength 1.7 cm. However, the effect may spread to the range of small wavelength gravity waves and, for this reason, Munk (1952) suggested avoiding puncturing the water surface in the measurement of waves in the capillary-gravity region.

The generation of capillary waves becomes important when the water moves relative to the wire at a speed exceeding the minimum phase velocity, \( c_m \), of the surface gravity-capillary waves (Lamb, 1945). No such generation takes place when \( c < c_m \). As the speed increases above \( c_m \) waves going upstream acquire shorter wavelengths and tend increasingly to bend and trail behind. The trailing waves are of relatively long length. As the speed increases, water piles up higher at the leading edge of the wire while the surface is depressed at the trailing edge. The pileup is sensitive to the speed and so errors of magnitude comparable to the
diameter of the wire are expected as water speeds fluctuate above \( c_m \).

Well below the surface there is a pressure excess of \( \frac{1}{2}U^2 \) at the leading edge of the wire and a similar deficiency at the downstream surface of the wire. For \( U = 25 \text{ cm sec}^{-1} \), that pressure is 312 dynes cm\(^{-2}\) or 3 mm of water. These pressure alterations are released as the surface is approached but some pressure difference is maintained when speeds are such that short capillary waves are generated. As the speed is increased further, a value will be reached where instabilities occur and a large pocket of air is entrained behind the wire. It is doubtful that such drastic action will occur in typical seas.

The analysis presented in Lamb pertains to a steady stream velocity and is therefore not directly applicable to the oscillating velocity in a wave. However, it is clear that there will be no probe-generated waves if the instantaneous speed of the fluid in the wave motion is always less than the critical wave speed. If this ratio (instantaneous fluid speed/critical wave speed) is greater than one, some probe-generated waves will appear.\(^1\)

For any wave (wavelength = \( \lambda \)) there is a critical amplitude, \( \alpha_\lambda \), such that if the amplitude of the wave is less than \( \alpha_\lambda \), the fluid speed at

\(^1\) It is easy to write down the time-dependent Green's function for this problem and thus to reduce it to the evaluation of a rather difficult integral. The solution of this problem is rather straightforward, and has probably already been found.
Fig. 1. The critical wave amplitude, $a_*$, and the root mean square surface elevation, $\langle \eta^2 \rangle^{1/2}$, in the equilibrium spectrum as a function of wavelength.
appearance of the spurious capillaries will be intermittent and thus there will be harmonics present in the spurious signal.

Since we are interested in waves with frequencies greater than 4 Hz, it should be possible to filter the fundamental of the spurious signal. If the energy in the unfiltered harmonics of the spurious signal is sufficiently small compared to the true signal, there will be no problem. However, it is not obvious that this will always be the case. Therefore there may well be some spurious signal appearing in the frequency range of interest.

This problem could be considerably minimized if the wave wire platform is designed to move with the waves of lengths say from about 20 cm to several meters and be stationary for waves of lengths less than 10 cm.

The third problem is that of spurious signals generated by the platform vibrations. These vibrations could be caused by wave slamming and flow instabilities.

In addition to these problems, there are others that more closely relate to the specific design of the direct wave height sensor. As examples, uncertainties can be caused by variations in the wetting and sticking of the water. The size of the wire to use is a compromise between the minimum wave length measured and the rigidity of the probe. Wind effects can be minimized by decreasing the length of the probe above the water, which, of course, reduces the dynamic range. Sticking bubbles can be avoided by using alternating rather than direct current. Probe vibrations in response to the wave action must be minimized, particularly resonances in the
frequency range of measurement. Finally there are uncertainties resulting
from foreign material gathering on the probe surface. These are just a
few of the general problems.

Some of the differing approaches to this type of measurement are
exemplified in the following four descriptions. It should be noted that these
measurements were done in laboratory conditions where the deteriorating
effects of weather and high seas were absent.

Example 1.

Stalder (1957) developed a system employing immersed
capacity probes wrapped with thin strips of absorbent
paper tissue. It is said to be capable of measuring one
to 60 Hz waves of amplitude over the range of 0.0127 to
1.9 cm. Average linearity variation is of the order of
± 15% or less. Repeatability errors are of the order of
± 20% or less and phase variability is of the order of 5%
or less. It has a reasonably flat frequency response.
The above data represents static calibration conditions;
i.e., a moving probe in still water.

Example 2.

Crews and Earth (1963) developed a system shown in
Fig. 2. For a probe spacing of 0.6 cm (compared to
10 cm maximum wavelength of interest), the system
responds linearly over the wave height range of 0.64
to 8.9 cm (upper limit is not reached as yet). A wave
period variation of less than one second was indicated
as achievable. One key problem is that corrosion or
electrolysis affects sensitivity. An operational period
of the order of one hour seemed necessary to make
this effect relatively invariant. The hysteresis effect
was found to cause effective wave height variations of
only 0.01 cm.
1/16" dia. stainless steel welding rods mounted in parallel in a lucite holder.

Fig. 2 Crews et al system
Example 3.

Ayers (1962) developed a resistance wire water level measurement system that measured waves in the frequency range from 1 Hz to 1 cycle per day to an accuracy of a fraction of an inch (actual figure not given). Polyethylene jacketed wire rope of suitable physical strength was wound in a continuous spiral with a Nichrome resistance wire. The resistance wire is lightly embedded in the polyethylene giving a near smooth surface offering minimum interference with water runoff.

Example 4.

Kingsbury (1970) used a capacity probe made from a piece of no. 14 copper (1/16" = 0.0625 cm) which had an insulating Formvar coating. In the laboratory, wind waves were dominated by a 0.2 sec period. Static uncertainty due to hysteresis was estimated to be 1/32" which was expected to be much less in wind. Actual measurement errors were not reported. Before starting the measurements, it was found necessary to wipe the probe and immerse it for a while.
2.2 Direct Pressure Measurement Methods

Direct pressure measurement methods are based on using the pressure variations in the water to calculate the water depth variations above the pressure sensor. When a pressure transducer is kept at a fixed depth it will record the pressure variations associated with waves of every frequency. In addition it will sense spurious "dynamic pressures" caused by motion of the water. Much of the latter may be removed by careful design of the sensor housing. The dispersion relation is used to determine the wavenumber appropriate to each frequency; the surface amplitude can then be determined after cognizance is made of the exponential drop of pressure with depth, $e^{-kz}$. Since the surface wave pressure diminishes to one hundredth of the surface value at depths of only 77% surface wave length, measurements in the wavelength range less than 10 cm require the pressure sensors to be within the depth range of 7 cm, i.e., a depth less than one wavelength.

The pressure sensor itself can operate on several principles: pneumatic, diaphragm or strain gage. As the water surface is not punctured, surface disturbances are in principle kept to a minimum. However, there is the design problem of the platform that holds the pressure sensors near the water surface. Besides, the pressure sensor size has to be appreciably smaller than the wave length of interest in order to sense force. This requirement will limit the sensor dimension to the order of 2 to 5 mm in diameter.
Floats, instrumented with inertial gyros, have been used to sense the accelerations accompanying the ocean surface waves. As in inertial guidance systems, integration in time can be used to obtain velocity, and a second integration can be used to obtain height.

Ships themselves have been used as platforms for measuring long waves and a pressure gage on the hull is used to correct for the relative displacement between the water surface and the accelerating hull.

It is noteworthy that the accelerating force is almost constant per octave of spectrum. In the gravitational range, the rms displacement, \( z \), varies as \( \omega^{-2} \) but the acceleration is measured by \( 3 \omega^2 \). Measurements of the acceleration may then give a less contaminated description than is obtained from amplitude determinations. The input data are "white".

The main problem is that the large size and mass of the float mechanically filters the ocean surface wave spectrum. While this technique has been applied to longer gravity waves measurement, the required size and mass of the float would have to be quite small to be effective at very short wavelengths. To be effective, the size should be considerably smaller than the wavelength of interest and thus is of the order of several millimeters.
Methods of measuring wave height based on illuminating the surface are termed "remote methods" in this report. The irradiation may be reflected, refracted or scattered by the ocean surface. The altered radiant energy is then collected by a suitable sensor. Any analysis of the resultant intensities requires an understanding of the complicated interaction between the illumination and the ocean surface. Although this interaction may not be completely understood, some uncertainty in the ultimate interpretation may be justified when the advantages of radiation schemes are considered. They are mobile, they can scan rapidly over large areas and utilize new signal processing techniques and they usually do not influence the surface wave motions.

The remote measurement systems considered here are divided into reflection or refraction systems and scattering systems in the sense that ray theory is applicable to the former but wave theory is required in the latter because the water wavelengths may not be large compared to the wavelength of illumination.

3.1 Remote Methods Based on the Ray Principle

In remote methods based on the ray principle, the illuminating energy is considered to travel in rays\(^1\) from a source to the ocean surface.

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1. The application of principles in this subsection is not limited to just straight lines or rays. They can be bent according to Snell's law in a stratified medium. This subsection, however, is limited to cases where most energy of a selected physical parameter is transported along a ray that follows classical ray theory.
where it is reflected and/or refracted before it reaches a sensor. This sensor records intensity and sometimes activates another instrument to register time of arrival simultaneously.

3.1.1 Skylight Luminance Systems Using the Optical Fourier Transform

In skylight luminance systems the light is specularly reflected by the sea surface facets and then recorded on a photographic transparency as a picture. When this transparency is placed in one focal place of a lens the Fourier transform of the variations appears as light amplitude in the other focal plane. In the transformed plane the intensity of light varies with distance from the axis and with angle around that axis. The distance from the axis is a measure of wavenumber magnitude while angle around the axis measures the direction of the wave vector. The intensity measures the density of the slope spectrum (to first order). This information along with the height and aspect angle of the camera is used to determine the wave spectrum.

The basic principle of operation assumes:

1. small wave slopes (such that squared slope is much less than the slope) to allow for linearization.

2. daylight under clear sky to have monotonic variation of sky luminance with zenith angle.

3. camera height is much greater than the area photographed.

1. Similar restriction to that of acoustic wave scattering method presented in Section 3.2.
4. cotangent of the depression angle is small.

5. no shadowing is present (reflecting angle around Brewster’s angle is desirable).

This method has a 180° ambiguity in wave number direction and requires a clean surface. Oil slicks or any other localized reflective materials on the surface alter the reflectivity of the surface in a discontinuous manner and thereby introduce spectral components not actually existing in the wave structure. Although experimental data exist to indicate its use in the measurement of small gravity waves, comparison with other measurements seems to exist only for periods greater than about 1.4 seconds.

3.1.2 Stereophotographic Method

In this method a pair of stereophotographs is made. These stereophotographs contain the information on the relative wave height. This method has not been critically evaluated as yet.

An effort was made by Dobson (1970) to measure fine-scale wave structure using such stereophotographs. Measurements were made 20 ft above the sea from a ship bow. Two stereo cameras covered 20 ft x 20 ft with 60% overlap. These photographs were read to provide profiles of height versus distance along a line (read at 4 mm intervals) on the surface. Vertical resolution was claimed to be 0.3". Non-stationary effects of low frequency (less than about 1 Hz) waves were removed by performing a
Fourier expansion on each profile, utilizing the first ten harmonics. The residue from the expansion and the original data were then used to compute the spectrum in the wave length range of 0.6 cm to 60 cm.

The results were shown to be consistent both with the saturated spectrum and the lower wave number spectrum (wave length longer than 180 cm) obtained by simultaneous wave gauge measurements. The method is considered plausible. The remaining tasks are to either calculate probable errors or compare this method with other reliable methods for the desired frequency range.

Two obvious extensions are:
1. telestereophotographing from aircraft
2. underwater stereophotographing.

3.1.3 Profiler

Wave height can be obtained from the very simple principle that total differential travel time of a ray is proportional to the wave height.

Resolution is mainly dependent on:
1. the ray beam width
2. sensor distance from the wave
3. decay rates in the propagation medium
4. platform motions
5. medium anomalies
Of these factors, the first three are very much dependent on the choice of energy used and specifics are required for further discussion. Platform motion can be divided into low and high frequency effects. Low frequency platform motion may be removed by high-pass filters; high frequency platform motion requires monitoring by, perhaps, accelerometers. Effects caused by medium anomalies are usually hard to assess and the subject is a major field itself. Some specific systems are described below.

Schule et al (1971) studied fetch-limited wave spectra using an airborne laser mounted in a small aircraft flown at low altitude and low speeds. The one-dimensional wave spectrum was measured in the range of 120 m to 15 m for an illuminated ocean surface strip of 0.91 cm x 4.6 m.

Barnett et al (1967), on the other hand, used the radar altimeters of a large aircraft flying at an altitude of 152 m. The electromagnetic radiation was at 4300 MHz, and the antenna beamwidth was 1.72°. Wave height was measured from 0.6 to 15 m, ± 10%, while wave length resolution was in the range of 9 to 600 m.

The most pertinent example of this method was presented by Olsen et al (1970). The aircraft was flown at an altitude of 60 m and speed of 240 km/hr. The illumination was done with a continuous laser beam transmitter (amplitude modulating frequency of 49.17 MHz and 18 mm beam diameter) and the receiver was a 3-inch telescope. The profilometer operated by comparing the phase of the received signal with that of the transmitted signal. The modulating frequency of 49.17 MHz was
chosen so that each 360° phase shift represented a range change of 3.05 m due to wave height. With the aircraft at 60 m, the illuminated spot was estimated to be 3.4 mm in diameter. The system is claimed to be able to measure ripples having wavelengths greater than or about equal to 2.5 cm and having a wave height of the order of a few millimeters.

Another popular technique to perform profiling is the inverted fathometer in which acoustic rays are directed upward to the ocean surface. For measuring the small wavelengths of interest here, one necessary requirement of an inverted fathometer is that the sonified surface dimension be much less than 10 cm. One way to accomplish this is to use a focusing method in which the aperture is large compared to the wavelength of the radiation. This implies high frequencies so that a reasonable size aperture can produce a very narrow beam. There are two basic problems in using high frequency acoustics. First, there is significant acoustic energy absorption within the water which can severely limit range. Second, the acoustic wave length becomes comparable to the dimensions of sea water volumetric anomalies (e.g., microorganisms, temperature variations, microbubbles) which are very effective scatterers. Thus, the high frequency energy is again lost. Of course, one may use a large array (with or without shading) of transducers to obtain a narrow beam at lower frequencies, but this can be costly and cumbersome.

Kronengold et al (1965) used acoustic energy at 200 kc and a 2.5° beamwidth. The total travel time was, of course, measured. Nearby anomalies caused early arrivals. These were avoided by gating at the
receiver for 3.5 ms which corresponds to about 8 feet. Anomalies at
greater distances from the transducer produced weaker reflections which
were not picked up by the receiver. The shortest wave period measured
was 2 seconds.

In another example, McIlwraith et al (1963) used a parabolic
reflector-echo sounder combination to focus the sonic beam and claimed
that the resulting amplitude resolution was 1 cm and surface spatial reso-
lution was 60 cm. The depth of the instrument was located at 6 to 67 m
and sonar frequency was 200 kc.

3.1.4 Other Systems

Some rather unique systems have been developed for use in water
tunnels. For example, Cox (1958) utilized a photocell-inky water wedge
combination to measure slopes of high frequency wind waves in a water
tunnel. The surface of the water was illuminated from below with the
light passing through an inky wedge. The wedge is so constructed that its
transparency varies linearly from almost nil at the thick end to almost
unity at the thin end. The optical rays passing through the wedge and the
water surface depend on the slope at the water surface. These optical
rays are sensed by a photocell behind a telescope. The region of origin
of the rays on the "inky wedge" depended primarily on the tilt of the water
surface at this focused spot.
Limitations are:

1. The brightness registered by the phototube is affected by the water surface elevation (e.g., caused by low frequency waves) whose error ranges from \( \pm 20\% \) to \( \pm 5\% \) in the wind speed range of 12 m/s to 8 m/s.

2. The sensed brightness is not linearly related to slope greater than 40\(^\circ\) whose error is of the order 5\%.

3. Surface tilt is allowed only for one component (e.g., up/down wind direction).

Other techniques have been developed to measure parameters of the sea surface other than spectrum. For example, the image of the sun reflected by the sea surface (sun glitter) may be used to determine the distribution of sea surface slopes (Cox and Munk, 1954), based on a theoretical relationship. Other theories, using short wave approximations of Kirchoff formulation (Marsh and Kuo, 1965) can be used to relate the scattering coefficient with various mean square properties of the sea surface motion. However, for the purposes of determining spectra, the above theories are of little use because the scattering coefficient does not provide the proper information.
3.2 Remote Measurement Systems Based on Wave Principles

An important difference between acoustic and electromagnetic waves is that acoustic waves are based on scalar quantities, while electromagnetic waves are described by vector quantities. This means that polarization effects will occur when electromagnetic waves are reflected off the ocean surface (Beckmann et al., 1963). The usual assumption is that this depolarization is not a problem for the backscattering case and hence the electromagnetic backscattering phenomenon can be described by the same scalar wave equation used for acoustic waves.

The actual extent of the depolarization depends on the conductivities of media in contact, local angle of incidence, and scattering angle, as well as on the polarization characteristics of the incident wave. It also seems that depolarization is unavoidable for rough surfaces where local radii of curvature is not larger than the radar wavelength. To be specific, in the following discussion, acoustic waves are described as well as electromagnetic waves, provided there are no polarization effects.

In one measurement configuration, a transceiver is used to project a single frequency acoustic wave onto the ocean surface and to receive the backscattered wave (see Fig. 3). A measurable quantity, backscattering strength \( \gamma'' \), can be shown to be proportional to the surface wave amplitude spectrum (Kuo, 1964) assuming small wave height compared to acoustic wavelength.
This means that a measurement of backscattering can directly yield surface wave spectrum at different wave numbers $k = 2K \cdot 2K (\alpha, \beta)$ determined by the acoustic wave number $K$ and the incident acoustic wave direction $(\alpha, \beta)$. For an isotropic case, surface amplitude spectrum is a function only of the wave number magnitude and the selected wave number becomes

$$\frac{2\pi}{\lambda} = |k| = |2K| = 2K \sin \Theta$$

where $\lambda$ is the surface wave length.

The above analysis assumed that a surface wave is stationary within the acoustic wave time scale. This condition can be relaxed to show (Kuo, 1965) that the backscattering spectral level $M''$ (i.e., frequency spectrum of the scattering strength) is proportional to surface
motion spectrum in both wave number and frequency space, \( \Phi(2k, \omega = \omega_i - \omega_r) \) where \( \omega \) and \( \omega_r \) are the incident and reflected acoustic wave frequencies respectively. Accordingly, frequency analysis of the backscattering data gives the surface wave spectrum in both wave number and frequency space. As expected, a surface wave frequency \( \omega \) causes a Doppler shift \( \omega \) in the acoustic wave frequency. The backscattering process does not necessarily select a particular frequency in general. It does select a particular frequency corresponding to the selected wave number if the surface wave motion is dispersive in nature. There is evidence that forced surface waves may not be dispersive.

The first order wave scattering theory described above seems to have the same sort of restriction and simplicity as using the optical transform method. The main difference, from an applications point of view, is that the spatial transform is automatic in the acoustic case, while the optical data still have to be optically processed by an optical transformer (lens). On the other hand, in the acoustic case, one must scan in frequency or depression angle.

Experimental verifications of the acoustic scatter have been mostly in the low frequency range (Marsh, 1963). Even then, acoustic scattering experiments have been developed primarily for the purpose of determining underwater sound propagation loss due to surface bouncing. As the surface wave motions are usually not precisely known, surface loss is predicted usually from scattering theory and the estimated surface wave spectrum (either empirical or theoretical). Thus, the experiments have not been
critically designed for the test of the theory, that is, to relate precisely the scattering strength and the high frequency wave spectrum. According to a recent paper by Jelonek (1970), experimental data on imperfect reflectors confirm the above theory. Unfortunately, these data are not as yet published. Another experiment by Liebermann (1963) proved the correctness of the above theory through the Doppler shift measurements of the acoustic backscattering.
4.0 Conclusions and Recommendations

Although a number of techniques for measuring small gravity waves have been tried, there is no general purpose method in existence today. The methods that are available can be summarized in terms of calibration and measurement systems.

Direct measurement methods are quite necessary from the point of view of establishing a common base of calibration, and under some circumstances it may be possible to use these methods for extensive measurements as well.

The second group of reliable methods consists of profilers and stereophotographs; these are less contaminated with theoretical simplifying assumptions than transform methods. For surveillance, either from aircraft or satellites, radar or laser beams and stereophotographs may be used. In underwater surveillance systems, sound or laser beams may be used. Operationally, these methods seem more appropriate than the direct methods at the moment. It should be realized that profilometers in the present state-of-the-art are only one-dimensional, but two-dimensional scanning techniques could be devised.

The third group of methods obtains results on the basis of various assumptions. The main assumptions are small surface wave slope and small wave height for the optical transform and acoustic scattering methods, respectively. These methods appear to be inherently faster and cheaper than the other methods but they require development and comparison with
the results of direct measurements at short wavelengths.

On the basis of this discussion it can be concluded that for measuring short gravity and capillary waves:

1. Direct measurement techniques such as wave wires should serve as the primary calibration method and may have limited field use. These methods have a number of possible sources of error associated with their use such as platform motion, flow past the wire, spray and surface contamination. The magnitude of these possible errors cannot be determined a priori but must be assessed for a particular experiment. Careful design can minimize these errors. Although these techniques require skillful application, they can be extremely valuable in carefully controlled field experiments as well as in the laboratory. They do not appear to be suitable for routine use and widespread data gathering.

2. The laser profilometer appears to be the most promising technique for rapid, widespread routine data gathering. The spatial resolution of the beam should be decreased to about 1 mm and the amplitude resolution of the system should be improved to less than a millimeter. It would also appear to be desirable to develop a linear array for two-dimensional coverage as well as steering capability. It appears that these improvements require only straightforward engineering development.

3. The optical transform method appears to be applicable only in daylight under clear skies with relatively smooth and clear water surface. Unlike the profilometer, this technique does not appear to lend itself to real time data processing.
Appendix A

It is desired to estimate the critical wave amplitude for the generation of waves by a wave wire. For a wave on the surface of a deep ocean, the potential is (in a coordinate system with $X$ in the horizontal direction and $Z$ vertical and the origin at the surface)

$$\phi = \left(\frac{ag}{\omega}\right) e^{kZ} \cos(kx - \omega t) \quad (a-1)$$

where $a$ is the wave amplitude

$\omega$ is the wave frequency

$k$ is the wave number

$g$ is the acceleration of gravity

The surface elevation is

$$\eta = a \sin(kx - \omega t) \quad (a-2)$$

The horizontal component of the fluid velocity is

$$u = u_0 e^{kZ} \sin(kx - \omega t) \quad (a-3)$$

$$u_0 = \frac{agk}{\omega} = \frac{ag}{c} \quad (a-4)$$

With the phase velocity, given by

$$c^2 = \frac{g}{k} \quad (a-5)$$
The critical value of the fluid velocity is the minimum in the
phase speed, \( c_m \approx 23.2 \text{ cm/sec} \). If \( \alpha \) is sufficiently large and \( u_o > c_m \)
the wave wire will generate waves. Let \( \alpha_o \) be the critical amplitude,
such that
\[
\frac{\alpha_o g}{c} = u_o = c_m \quad (a-6)
\]
Then,
\[
\alpha_o = c_m c/g \quad (a-7)
\]
For given \( \lambda \left( k = \frac{2\pi}{\lambda} \right) \) calculate \( c^2 \) from (a-5), then with \( c_m = 23.2 \text{ cm/sec} \), calculate \( \alpha_o \) from (a-7). The results of these calculations
are plotted in Fig. 1.

Next, it is desired to estimate the amplitude of the spectral compo-
nents in a fully developed sea. The equilibrium spectrum (Phillips, 1966) is
\[
\Phi(\omega) = \beta g^2 \omega^{-5} \quad (a-8)
\]
with \( \beta \) a numerical constant,
\[
\beta \approx 1.33 \times 10^{-2} \quad (a-9)
\]
The mean square surface elevation \( \langle \eta^2 \rangle \) having minimum frequency \( \omega \) is
\[
\langle \eta^2 \rangle = \frac{i}{4} \omega \Phi(\omega) = \frac{i}{4} \beta g^2 \omega^{-4} \quad (a-10)
\]
Then the root mean square surface elevation

\[ \langle \eta^2 \rangle^{\frac{1}{2}} = \frac{1}{2} \beta^{\frac{1}{2}} g \omega^{-2} \]  (a-11)

has been calculated and plotted in Fig. 1.
References


