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REMOTE DETERMINATION OF THE PROFILES OF THE ATMOSPHERIC STRUCTURE CONSTANTS AND WIND VELOCITY ALONG A LINE-OF-SIGHT PATH BY A STATISTICAL INVERSION PROCEDURE

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Remote Determination of the Profiles of the Atmospheric Structure Constants and Wind Velocity Along a Line-of-Sight Path by a Statistical Inversion Procedure

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by

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13. ABSTRACT

A method is developed for remotely determining the average transverse wind velocity and the atmospheric structure constant (strength of turbulence) at N points along a line-of-sight path. The technique avoids the basic instability problem that was encountered in previous work, limiting the calculations to one or two points. Linear integral equations relate the data, the amplitude correlation function and the amplitude and phase structure functions, with the unknown structure constant and wind velocity. The standard inversion method leads to large variations in the unknown for small data errors; thus, the problem is ill posed. To counteract this, a statistical inversion procedure is developed that is dependent upon a priori knowledge of the statistics of the unknowns. The error in the final solution can also be predicted by computer simulation. For example, with an input error of one percent, the RMS error in the unknown will be on the order of ten percent. This is an increase in accuracy of ten orders of magnitude over the standard inverse moment method.

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I. Introduction

It is proposed that the atmospheric structure constant and the average transverse wind velocity be determined remotely through line of slight microwave or optical scattering measurements [1]. The method will yield values of the unknown at any number of points, along the path, between the source and detector. This is opposed to previous methods where the atmospheric parameters are predicted for only one or two points [2,3]. The method has application in the detection of clear air turbulence, the study of the basic atmospheric turbulence properties and in the everyday measurement of common meteorological parameters. The measurement system is shown in Fig. 1. The source which is located at the origin produces a wave propagation in the x-direction. The beam can take the form of a plane, spherical or beam wave. The detecting array is located at some distance L from the source. It consists of a set of point receivers in a horizontal array, perpendicular to the x-axis. The signal incident on the different array elements can be correlated spatially or temporally to obtain the statistical properties of the scattered signal. In the general case, the statistics are expressed in terms of the structure function, i.e.,

$$D_f(\vec{r}_1,\vec{r}_2,t_1,t_2) = E\{|f(\vec{r}_1,t_1) - f(\vec{r}_2,t_2)|^2\} \tag{1}$$

where $\vec{r}$ and t denote the spatial and temporal coordinates, and E denotes the expected value and f is the random quantity. This function is useful in that it is expressed only in terms of $(\vec{r}_1-\vec{r}_2)$ and (t_1-t_2) when the atmosphere is assumed to be locally stationary (locally homogeneous), the usual case. The more common correlation function expressed as

$$B_f(\vec{r}_1-\vec{r}_2,t_1-t_2) = E\{f(\vec{r}_1,t_1)\ f^*(\vec{r}_2,t_2)\} \tag{2}$$

is also used. It is a function of $(\vec{r}_1-\vec{r}_2)$ and (t_1-t_2), and is realted to the structure function by Eq. (3) if the atmosphere is assumed to be strictly stationary and the incident beam takes the form of a plane or spherical wave.

$$D_f(\vec{r}_1-\vec{r}_2,t_1-t_2) = 2\ B_f(0,0) - 2\ B_f(\vec{r}_1-\vec{r}_2,t_1-t_2) \quad . \tag{3}$$

The statistics of the scattered beam can be related to three basic atmospheric parameters. They are the atmospheric structure constant, $C_n^2(x)$, the atmospheric turbulence spectrum, $\Phi_n^{(o)}(\kappa)$, and the average transverse wind velocity, $V(x)$. C_n^2 is actually the square of the structure constant; however, for convenience it is simply referred to as the structure constant. $C_n^2(x)$, which represents the strength of turbulence along the path and $V(x)$ are both considered to be smoothly varying along the transmission path; they are the quantities to be determined by the inversion method. $\Phi_n^{(o)}(\kappa)$ is assumed to take the form of the Kolmogorov spectrum; i.e.,

$$\Phi_n^{(o)} = \kappa^{-11/3} . \tag{4}$$

This simple spectral form is required in order to obtain integral equations in closed form. The turbulence parameters came about through the assumption that the atmospheric index of refraction has a slight random variation about its mean value. This can be expressed as

$$n(\vec{r},t) = 1 + n_1(\vec{r},t) \tag{5}$$

where n is the index of refraction with average one and n_1 the random variation. It is assumed that

$$|n_1| \ll 1 . \tag{6}$$

This and the "frozen-in" hypothesis are used in the derivation of the integral equations that relate the atmospheric parameters to the beam parameters.

II. Integral Equations

The relationship between the beam parameters and the atmospheric parameters for plane and spherical waves was originally developed by Tatarski [4] and extended to include the case of an incident beam wave by Ishimaru [5]. The method commences with the development of the wave equation where the index of refraction is in the form of Eq. (5). Rytov's method of small perturbations

is used to separate the incident from the scattered field. The time dependence of the index of refraction is removed through the "frozen-in" hypothesis, and the solution is obtained by use of a spectral technique. For the general case of an incident beam wave, the correlation functions and the structure functions become

$$\left.\begin{matrix} B_A \\ B_S \end{matrix}\right\} = \pi^2\, 0.033 \int_0^L d\eta\, C_n^2(\eta) \int_0^\infty \kappa d\kappa \left\{ [J_o(\kappa P) + J_o(\kappa P^*)]\, |H|^2 \pm J_o(\kappa Q) H \pm J_o(\kappa Q^*) H^* \right\} \Phi_n^{(o)}(\kappa) \tag{7}$$

$$\left.\begin{matrix} D_A \\ D_S \end{matrix}\right\} = 2\pi^2\, 0.033 \int_0^L d\eta\, C_n^2(\eta) \int_0^\infty \kappa d\kappa \left\{ [J_o(i2\gamma_i \kappa \rho_1) + J_o(i2\gamma_i \kappa \rho^2) - J_o(\kappa P) - J_o(\kappa P^*)]\, |H|^2 \pm [1 - J_o(\kappa Q)]\, H^2 \pm [1 - J_o(\kappa Q^*)]\, H^{*2} \right\} \Phi_n^{(o)}(\kappa) \tag{8}$$

where the upper sign is for amplitude data and the lower for phase data. The variables in Eqs.(7) and (8) are

$$P = (\gamma_r y_d + i\gamma_i y_c + V_y \tau)^2 + (\gamma_r z_d + i\gamma_i z_c + V_z \tau)^2 \tag{9}$$

$$Q^2 = (\gamma y_d + V_y \tau)^2 + (\gamma z_d + V_z \tau)^2$$

$$y_d = y_1 - y_2 \qquad y_c = y_1 + y_2$$

$$z_d = z_1 - z_2 \qquad z_c = z_1 + z_2$$

$$\rho_1 = (y_1^2 + y_2^2)^{\frac{1}{2}} \qquad \rho_2 = (z_1^2 + z_2^2)^{\frac{1}{2}}$$

$$|H|^2 = k^2 \exp\left(\gamma_i \frac{L-\eta}{k} \kappa^2\right) \qquad H^2 = -k \exp\left(i\gamma \frac{L-\eta}{k} \kappa^2\right)$$

$$\gamma = \frac{1 + i\alpha\eta}{1 + i\alpha L} = \gamma_r + i\gamma_i$$

$$\gamma_r = \frac{1 - \alpha_2 L + [(\alpha_1^2 + \alpha_2^2)\, L - \alpha_2]\eta}{(1 - \alpha_2 L)^2 + (\alpha_1 L)^2}$$

$$\gamma_i = - \frac{\alpha_1 (L-\eta)}{(1 - \alpha_2 L)^2 + (\alpha_1 L)^2}$$

$$\alpha_1 = \frac{\lambda}{\pi W_o^{\,2}} \qquad\qquad \alpha_2 = \frac{1}{R_o}$$

V_y and V_z are the components of the average wind velocity, the y's and z's are the receiver coordinates, κ is the atmospheric wave number, k is the electromagnetic wave number, W_o is the radius of the transmitting aperture, and R_o is the position of the beam focus.

If $\Phi_n^{(o)}(\kappa)$, the atmospheric spectrum, is taken as in Eq. (4), the following integral may be used:

$$\int_0^\infty \kappa^\mu \exp(-\alpha\kappa^2)\, J_o(\beta\kappa)\, d\kappa = \frac{\Gamma(\frac{\mu + 1}{2})}{2\,\alpha^{\frac{\mu + 1}{2}}}\; {}_1F_1\left(\frac{\mu + 1}{2};1;-\frac{\beta^2}{4\alpha}\right) \tag{10}$$

where

$$\mathrm{Re}(\alpha) > 0 \qquad \text{and} \qquad \mathrm{Re}(\mu) > -1.$$

Notice that while the equations do not satisfy the condition $\mathrm{Re}(\mu) > -1$, the integrands approach zero for small κ. (The phase correlation function is an exception, and will not converge under this assumption.) Further observation reveals that the integrand can be analytically continued to $\mu = -8/3$. After completing the integration over the atmospheric spectrum the correlation and structure functions become

$$B_A(\vec{r}_1,\vec{r}_2,\tau) = 0.033\, k^2\pi^2\Gamma(-5/6)L \;\mathrm{Re} \int_0^1 dx\; C_n^{\,2}(x) \left\{ {}_1F_1\left[-5/6\,;1\,;\frac{P^2}{4\delta_i \frac{L}{k}(1-x)}\right] \right.$$

$$\left. \left[-\delta_i \frac{L}{k}(1-x)\right]^{5/6} - {}_1F_1\left[-5/6\,;1\,;-\frac{Q^2}{4i\delta \frac{L}{k}(1-x)}\right] \left[i\delta \frac{L}{k}(1-x)\right]^{5/6} \right\} \tag{11}$$

$$\begin{matrix} D_A \\ D_S \end{matrix}(\vec{r}_1,\vec{r}_2,\tau) = 0.033\, k^2\pi^2\Gamma(-5/6)L \;\mathrm{Re} \int_0^1 dx\; C_n^{\,2}(x) \left[\left\{ {}_1F_1\left[-5/6\,;1\,;-\frac{\delta_i\rho_1^{\,2}}{\frac{L}{k}(1-x)}\right] \right.\right.$$

$$\left. + {}_1F_1\left[-5/6\,;1\,;-\frac{\delta_i\rho_2^{\,2}}{\frac{L}{k}(1-x)}\right] - 2\,{}_1F_1\left[-5/6\,;1\,;\frac{P^2}{4\delta_i \frac{L}{k}(1-x)}\right] \right\} \left[-\delta_i \frac{L}{k}(1-x)\right]^{5/6}$$

$$\left. \mp 2\left\{1 - {}_1F_1\left[-5/6\,;1\,;-\frac{Q^2}{4i\delta \frac{L}{k}(1-x)}\right]\right\} \left[i\delta \frac{L}{k}(1-x)\right]^{5/6} \right] \tag{12}$$

where the path length has been normalized to vary from zero to one, and

$$\delta = \frac{1 + i\alpha L x}{1 + i\alpha L}\,. \tag{13}$$

As can be seen from Eq. (11) and (12) the unknown structure constant and wind velocity are contained within integral equations; thus, the solution calls for an inversion procedure. Upon closer inspection it is seen that this is a formidable task. Both unknowns are involved in the integral equations, the structure constant, $C_n^{\,2}$, in a linear fashion and the wind velocity, V, in a non-linear one. To simplify the equations it would be desirable to find them in terms of one unknown or the other, and both in a linear form. This can be done by noting that the wind velocity always occurs in conjunction with the time delay variable, τ; hence the structure constant can be found independent of the wind velocity by taking τ to be zero. This results in a change in only the P and Q variables in Eq. (9). They become P_o and Q_o respectively.

The linearization of the wind velocity is accomplished through a method proposed by Shen [6]. First differentiate Eqs. (7) and (8) with respect to τ. The equation is then linearized by eliminating the V(x) term from the kernel. This is done by equating τ to zero; the result is a linear equation in $C_n^2(x)\ V(x)$.

$$\frac{\partial}{\partial\tau}\begin{matrix}D_A\\D_S\end{matrix}(\vec{r}_1,\vec{r}_2,\tau)\Big|_{\tau=0} = 0.033\ k^2\pi^2\Gamma\ (1/6)\ L \tag{14}$$

$$\mathrm{Re}\int_0^1 C_n^2(x)\ \vec{V}(x)\cdot\Big\{{}_1F_1\Big[\frac{1}{6};2;\frac{P_o^2}{4\delta_i\frac{L}{k}(1-x)}\Big]\Big[-\delta_i\frac{L}{k}(1-x)\Big]^{-1/6}\ [\delta_r\vec{r}_d + i\delta_i\vec{r}_c]$$

$$\mp\ {}_1F_1\Big[\frac{1}{6};2;-\frac{Q_o^2}{4i\delta\frac{L}{k}(1-x)}\Big]\Big[i\delta\frac{L}{k}(1-x)\Big]^{-1/6}\ [\delta\vec{r}_d]\Big\}$$

where

$$r_d = (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k} \tag{15}$$

$$r_c = (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k}$$

$$P_o^2 = (\gamma_r y_d + i\gamma_i y_c)^2 + (\gamma_r z_d + i\gamma_i z_c)^2$$

$$Q_o^2 = (\gamma y_d)^2 + (\gamma y_c)^2$$

While $C_n^2(x)$ and V(x) cannot be obtained independently in this case, the wind velocity can be determined since C_n^2 is already known from the previous equations. In this derivative format, it is found that the correlation function and the structure function are related in a very simple way, i.e.,

$$\frac{\partial}{\partial\tau} D_A(\vec{r}_1,\vec{r}_2,\tau)\Big|_{\tau=0} = -\ 2\ \frac{\partial}{\partial\tau} B_A(\vec{r}_1,\vec{r}_2,\tau)\Big|_{\tau=0}\ . \tag{16}$$

Thus, either one can be used in the solution for the wind velocity.

III. The Inversion Method

The equations describing the scattering of waves in the atmosphere, as

derived in the last section, are found to take the form of a Fredholm integral equation of the first kind. The general form of this integral equation is

$$g(y) = \int_{\alpha}^{\beta} K(x,y)\, f(x)\, dx, \quad \alpha' \leq y \leq \beta'. \tag{17}$$

$g(y)$ is a known function or data, $K(x,y)$ is the kernel of the integral equation, $f(x)$ is the unknown and α, β, α', β' are fixed constants. The equation can be solved analytically if the kernel is very simple or if it can be expressed as a complete set of orthogonal functions. When the kernel is more complicated the use of numerical methods is usually necessary, and the moment method is commonly employed. This method is developed by expanding the integral equation into N simultaneous equations in N unknowns, and contracting into matrix form, as shown.

$$g = Af \tag{18}$$

where

$$g = [g(y_i)] \tag{19}$$

$$f = [f(x_i)]$$

and

$$A = [W(x_i)\, K(x_i, y_i)]$$

where the braces enclose the elements of the g, f, and A matrices. $W(x_i)$ is a weighting function dependent upon the quadrature expansion of the integral. In subsequent steps $W(x_i)$ will not be shown since it can be carried with the kernel. The x_i's and y_i's are discrete values in the range

$$\alpha \leq x_i \leq \beta \tag{20}$$

and

$$\alpha' \leq y_i \leq \beta'.$$

The solution of Eq. (18) can be easily obtained by numerically inverting the A matrix; however, it is soon discovered that the results are highly unstable, and do not represent the unknown by any stretch of the imagination. The problems arise from the errors associated with the data and those introduced in the quadrature expansion and the inversion process. This can be described mathematically

by the following uniqueness argument. Suppose that two sets of data, g_1 and g_2, correspond to two sets of unknowns, f_1 and f_2, where $f_2 = f_1 + W \sin(\omega x)$. In integral equation form

$$g_1(y) = \int_\alpha^\beta K(x,y)\, f_1(x)\, dx \tag{21}$$

and

$$g_2(y) = \int_\alpha^\beta K(x,y)\, f_2(x)\, dx = \int_\alpha^\beta K(x,y)\, [f_1(x) + W \sin(\omega x)]\, dx\ . \tag{22}$$

Equation (22) can be expanded to

$$g_2(y) = \int_\alpha^\beta K(x,y)\, f_1(x)\, dx + \int_\alpha^\beta K(x,y)\, W \sin(\omega x)\, dx \tag{23}$$

or

$$g_2(y) = g_1(y) + \int_\alpha^\beta K(x,y)\, W \sin(\omega x)\, dx\ . \tag{24}$$

For any constant W, ω can be chosen large enough so that the integral of the kernel and the rapidly varying sine term average to an arbitrarily small constant, or

$$g_2(y) = g_1(y) + \varepsilon\ . \tag{25}$$

Thus, for two sets of data that vary by only some small experimental error, the values of the unknowns can differ by $W \sin(\omega x)$, a highly oscillatory function of great magnitude. This indicates that the solutions of the Fredholm integral equation of the first kind are not unique when experimental errors are taken into account.

To compensate for this problem one must account for the errors in the data and the unknown. This can be done by modifying Eq. (18), i.e.,

$$g + \varepsilon = A(f + \xi). \tag{26}$$

ε can be considered the experimental error in the data and ξ the resultant errors

in the unknown. It is sometimes convenient to denote $g + \varepsilon$ and $f + \xi$ as

$$\tilde{g} = g + \varepsilon \tag{27}$$

$$\tilde{f} = f + \xi \ . \tag{28}$$

$\tilde{g}$ is the data actually used in the determination of the unknown since the true data and its error are inseparable. By the same token, $\tilde{f}$ represents the solution that is obtained from the inversion process. Of course it is desired that $\tilde{f}$ approach f as closely as possible and this condition is attained by minimizing ξ.

The minimization of ξ is accomplished by noting that the A matrix in Eq.(26) is a linear operator; hence its inverse must also be linear, and

$$f + \xi = B(g + \varepsilon) \tag{29}$$

or

$$\xi = -f + B(g + \varepsilon). \tag{30}$$

The B matrix is an unknown, linear operator that is to be determined by the minimization of ξ. Multiplying the vector ξ by its transpose, a square matrix is obtained of the form

$$\begin{pmatrix} \xi_1 \xi_1 & \xi_1 \xi_2 & \cdots & \xi_1 \xi_N \\ \xi_2 \xi_1 & \xi_2 \xi_2 & \cdots & \xi_2 \xi_N \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \xi_N \xi_1 & \xi_N \xi_2 & \cdots & \xi_N \xi_N \end{pmatrix} \tag{31}$$

Through the minimization of the diagonal terms, the B can be found, the solution amounts to a minimum squared error method. This is analogous to results obtained by Franklin and others [7,8]. (Previously, a deterministic approach had been taken by many people [9,10,11,12,13].) This specific form of the solution is found convenient in the later determination of the wind velocity. Expanding Eq. (30) as indicated above, an equation in terms of square matrices is obtained.

$$\xi\xi^T = ff^T - f(Bg)^T - (Bg)f^T - f(B\varepsilon)^T - (B\varepsilon)f^T$$
$$+ (Bg)(B\varepsilon)^T + (B\varepsilon)(Bg)^T + (Bg)(Bg)^T + (B\varepsilon)(B\varepsilon)^T \tag{32}$$

with diagonal terms

$$\xi_i\xi_i = f_if_i - 2f_i \sum_k B_{ik}g_k - 2f_i \sum_k B_{ik}\varepsilon_k$$
$$+ 2 \sum_k B_{ik}g_k \sum_\ell B_{i\ell}\varepsilon_\ell + \sum_k B_{ik}g_k \sum_\ell B_{i\ell}g_\ell + \sum_k B_{ik}\varepsilon_k \sum_\ell B_{i\ell}\varepsilon_\ell \ . \tag{33}$$

To minimize this equation, it is differentiated with respect to each element of the B matrix, B_{mn}, obtaining

$$\frac{\partial \xi_i\xi_i}{\partial B_{\ell n}} = 0 = 0 - 2f_i \sum_k \delta_{kn}g_k - 2f_i \sum_k \delta_{kn}\varepsilon_k$$
$$+ 2 \sum_k \delta_{kn}g_k \sum_\ell B_{i\ell}\varepsilon_\ell + 2 \sum_k B_{ik}g_k \sum_\ell \delta_{\ell n}\varepsilon_\ell \tag{34}$$
$$+ 2 \sum_k \delta_{kn}g_k \sum_\ell B_{i\ell}g_\ell + 2 \sum_k \delta_{kn}\varepsilon_k \sum_\ell B_{i\ell}\varepsilon_\ell .$$

δ_{kn} is a matrix with all terms zero except the k, nth which is one. Notice that all equations developed from $\frac{\partial \xi_i\xi_i}{\partial b_{mn}}$ with $i \neq m$ are zero. Since both i and n range from one to N, a set of $N \times N$ simultaneous equations are obtained for the solution of the B matrix. Doing the summations over the delta functions Eq. (34) becomes

$$0 = -f_ig_n - f_i\varepsilon_n + g_n \sum_\ell B_{i\ell}\varepsilon_\ell + \varepsilon_n \sum_k B_{ik}g_k + g_n \sum_\ell B_{i\ell}g_\ell + \varepsilon_n \sum_\ell B_{i\ell}\varepsilon_\ell \tag{35}$$

which can be contracted back into matrix form to obtain

$$0 = -fg^T - f\varepsilon^T + Bg\varepsilon^T + B\varepsilon g^T + Bgg^T + B\varepsilon\varepsilon^T. \tag{36}$$

This equation relates the B matrix to the unknown, the data, and the error in the data. Since it is desired to find B in terms of the statistics of these quantities it is necessary to take an expected value to obtain equations in terms of the covariances matricies.

$$0 = -R_{fg} - R_{f\varepsilon} + B(R_{g\varepsilon} + R_{\varepsilon g} + R_{gg} + R_{\varepsilon\varepsilon}) \tag{37}$$

$$B_{\alpha\beta} = E(\alpha\beta^T) \tag{38}$$

To further simplify Eq. (37) the propagation theorem is used (i.e., if $\alpha = A\beta$ then $R_{\alpha\alpha} = AR_{\beta\beta}A^T$ and $R_{\beta\alpha} = R_{\beta\beta}A^T$) obtaining

$$0 = -R_{ff}A^T - R_{f\varepsilon} + B(AR_{fc} + R_{f\varepsilon}A^T + AR_{ff}A^T + R_{\varepsilon\varepsilon}) \ . \tag{39}$$

Finally one solves for B and subsitutes back into Eq. (29) with the result:

$$\tilde{f} = (R_{ff}A^T + R_{f\varepsilon})\ (AR_{ff}A^T + AR_{f\varepsilon} + R_{f\varepsilon}A^T + R_{\varepsilon\varepsilon})^{-1}\ \tilde{g} \ . \tag{40}$$

This is identical to the form presented by Franklin. In practical situations the unknown is independent of the data error, requiring

$$R_{f\varepsilon} = R_{\varepsilon f} = 0 \tag{41}$$

and

$$\tilde{f} = R_{ff}A^T\ (AR_{ff}A^T + R_{\varepsilon\varepsilon})^{-1}\ \tilde{g} \tag{42}$$

which is the usual form of the inversion equation. As a special case assume that the errors in the system are zero. This implies $R_{\varepsilon\varepsilon} = 0$, $\tilde{f} \to f$, and $\tilde{g} \to g$; resulting in

$$f = A^{-1}\ g \ , \tag{43}$$

the original integral equation when no errors are present.

The predicted accuracy of the inversion method can be found by taking the expected value of Eq. (32). By using the matrix propagation theorem and combining terms, the following is obtained.

$$R_{\xi\xi} = R_{ff} - (R_{ff}A^T + R_{f\varepsilon})\ (AR_{ff}A^T + R_{\varepsilon\varepsilon} + R_{\varepsilon f}A^T + AR_{f\varepsilon})^{-1}\ \ (AR_{ff} + R_{\varepsilon f}) \tag{44}$$

$R_{\xi\xi}$ is the covariance matrix of the error term; the other terms have been defined previously. The RMS error of the predicted value of the unknown is then

$$H = [\frac{1}{N} T_r(R_{\xi\xi})]^{\frac{1}{2}} = [\frac{1}{N} \sum_{i=1} \xi_i^2]^{\frac{1}{2}} . \tag{45}$$

Another method for error evaluation has been provided by Franklin. Δ is defined as

$$\Delta = \frac{\dfrac{||\xi||}{||f + \xi||}}{\dfrac{||\varepsilon||}{||g + \varepsilon||}} = \frac{\text{normalized error in the unknown}}{\text{normalized error in the data}} \tag{46}$$

where $||h||$ is the norm of h. If delta is on the order of one then the error generated in the unknown, by the inversion method, is about the same as the error in the data. Clearly, if delta is large, then the unknown error is much greater than the data error and the inversion method is not satisfactory. Since the norm is simply a number, Eq. (46) can be modified to

$$\Delta = \frac{||\xi||}{||\varepsilon||} \; \frac{||g + \varepsilon||}{||f + \xi||} . \tag{47}$$

This relation can be simplified with the equations

$$g + \varepsilon = A(f + \xi) \tag{48}$$

and

$$f + \xi = B(g + \varepsilon). \tag{49}$$

If it is assumed that the error in the unknown is strictly due to the errors in the data, then the second equation will reduce to

$$\xi = B\varepsilon \tag{50}$$

for no input. In this case Eq.(47) becomes

$$\Delta = \frac{||B\varepsilon||}{||\varepsilon||} \; \frac{||A(f + \xi)||}{||f + \xi||} . \tag{51}$$

For the worst case

$$\Delta_{max} = \max \left\{\frac{||B\varepsilon||}{||\varepsilon||}\right\} \max \left\{\frac{||A(f + \xi)||}{||f + \xi||}\right\} . \tag{52}$$

By use of the Schwartz inequality, i.e.,

$$\frac{||\alpha A||}{||\alpha||} \leq \frac{||\alpha|| \quad ||A||}{||\alpha||} = ||A|| = \max \left\{\frac{||\alpha A||}{||\alpha||}\right\}; \tag{53}$$

Eq.(51) can be simplified further. The final form is

$$\Delta_{max} = ||A|| \quad ||B||. \tag{54}$$

By defining $\Delta\varepsilon$ and $\Delta\xi$ to be the average error in the data and unknown respectively and using the definition of Δ, Eq.(46), it can be seen that

$$\Delta\xi = ||A|| \quad ||B|| \; \Delta\varepsilon \; . \tag{55}$$

These equations lead to the solutions of the atmospheric structure constant at several points along the path. An estimate of the error in the solution is also obtained.

When the solution of the wind velocity is desired Eq. (14) must be solved. Unlike the previous case, the unknown wind velocity is associated with another variable, the structure constant. Consequently, the solution of the equation will not yield the wind velocity profiles directly. Since C_n^2 is known from the previous developments it should be possible to evaluate the wind velocity itself. It was found that the most accurate method of solution was to associate C_n^2 with the kernel matrix obtaining

$$g' = A'V \tag{56}$$

where g' is the derivative data, V is the unknown and A' contains the kernel, the weighting function and the structure constant, i.e.,

$$A' = [W(x_j) \; C_n^2(y_j) \; K'(\rho_i, x_j)]. \tag{57}$$

In the implementation of the solution it is again necessary to acount for the errors in the data and the unknown; thus,

$$g' + \varepsilon' = A'(V + \zeta), \tag{58}$$

where ε' is the error in the derivatives and ζ the errors in the wind velocity. It should also be remembered that the value of C_n^2 used in the kernel is not exact but contains some error; this implies that the A matrix is actually of the form

$$A' = [(C_n^2 j + \xi_j)\, K'_{ij}] \,. \tag{59}$$

($W(x_j)$ is contained in the K' term.)

While this presents more complications it is assumed that the solution is in terms of a linear operator, thus

$$V + \zeta = B'(g' + \varepsilon') \,. \tag{60}$$

The coefficient matrix, B', is found by minimizing ζ, thereby obtaining

$$Vg'^T = B'g'g'^T + B\varepsilon'\varepsilon'^T. \tag{61}$$

The procedure is identical to the last section where the errors in g are assumed to be independent of V. With the proper substitutions g can be eliminated obtaining an equation of the form

$$VV^TA'^T = B'A'VV^TA'^T + B'\varepsilon'\varepsilon'^T \tag{62}$$

with substitution of Eq.(59) one obtains the following:

$$VV^T[(C_{nj}^2 + \xi_j)\, K'_{ij}]^T = B'[(C_{nj}^2 + \xi_j)\, K'_{ij}]\, VV^T[(C_{nj}^2 + \xi_j)\, K'_{ij}]^T + B'\varepsilon'\varepsilon'^T. \tag{63}$$

The expected value can now be taken with the reasonable assumption that ε, the error in the structure constant is independent of the wind velocity.

$$\begin{aligned} R_{VV}[C_{nj}^2\, K'_{ij}]^T + R_{VV}[E(\xi_j)K'_{ij}]^T &= B'[C_{nj}^2\, K'_{ij}]\, R_{VV}[C_{nj}^2\, K'_{ij}]^T \\ &+ B'[C_{nj}^2\, K'_{ij}]\, R_{VV}[E(\xi_j)\, K'_{ij}]^T + B'[E(\xi_j)\, K'_{ij}]\, R_{VV}[C_n^2\, K'_{ij}]^T \\ &+ B'R_{\varepsilon'\varepsilon'} + B'E[(\xi_j K'_{ij})\, VV^T(\xi_j K'_{ij})^T] \end{aligned} \tag{64}$$

It can be shown that $E(\xi_j) = 0$ if the expected value of the error in the structure function is zero. It is also assumed that the errors in the structure constant are related only to the errors in the structure function. With this and with simplification of the last term in Eq. (64) the equation becomes

$$R_{VV}(C^2_{nj}K'_{ij})^T = B'(C^2_{nj}K'_{ij})\, R_{VV}(C^2_{nj}K'_{ij})^T + B'(R_{\xi\xi}) \cdot (K'R_{VV}K'^T) + B'R_{\varepsilon'\varepsilon'} \tag{65}$$

where the dot denotes the matrix operation defined by

$$\alpha \cdot \beta = (\alpha_{ij}\, \beta_{ij}) \qquad i,j = 1,2, \cdots, N. \tag{66}$$

Solving for B' and substituting into Eq. (60) $\tilde{V}$ can be found

$$\tilde{V} = R_{VV}(C^2_{nj}K'_{ij})^T\, [(C^2_{nj}K'_{ij})\, R_{VV}(C^2_{nj}K'_{ij})^T$$

$$+ (R_{\xi\xi}) \cdot (K'R_{VV}K'^T) + R_{\varepsilon'\varepsilon'}]^{-1}\, [\frac{\partial}{\partial\tau} D(\rho_i,\tau)\Big|_{\tau=o}] \tag{67}$$

This expression is similar to that derived by Franklin's method. The present inversion method introduces an additional term to account for the errors in C_n^2.

The preceding solution gives the best mean squared estimate of the unknown structure constant and average transverse wind velocity. The solutions are dependent upon the input data, the correlation and structure functions of the scattered beam that propagates through the medium to be remotely sensed. The solutions are also dependent upon the statistics of the unknowns and the error. These are represented by the covariance matricies R_{ff} and $R_{\varepsilon\varepsilon}$. It is next necessary to determine the form of these matricies.

IV. Statistical Quantities

In the development of the unknown covariance matricies it is convenient to represent the unknown functions in terms of a random Fourier series.

$$f(x) = \hat{f} + \sum_{n=o}^{N} b_n[a_n \cos(n\pi x) + a'_n \sin(n\pi x)] \tag{68}$$

where $\hat{f}$ is the mean value of $f(x)$, the b_n's are fixed constants relating the magnitude of the variations to the mean value, a_n and a_n' are independent random coefficients with zero mean and variance one giving the necessary variability to the unknown, and N limits the rate at which the fluctuations occur. The function also has the advantage of being easily generated in computer simulation schemes.

To find the covariance matrix one employes Eq. (38) obtaining

$$R_{ff} = \hat{f}^2 + \sum_{n=o}^{N} b_n^2 \cos n\pi(x_i - x_j) \tag{69}$$

where

$$E(a_n a_m) = \delta_{mn} \tag{70}$$

and

$$E(a_n) = 0 \tag{71}$$

as stated previously. The $\hat{f}$ and b_n coefficients are chosen from experience and certain realizability conditions, the specifics of which be given in the next section.

The covariance matricies of the error terms are developed with the assumption that the ε_i's are independent of each other yielding

$$R_{\varepsilon\varepsilon} = (\sigma_\varepsilon^2 \, \delta_{ij}). \tag{72}$$

The covariance matrix of the error in the derivative function is developed in a slightly more general way. Since

$$\tilde{g}_i = g_i + \varepsilon_i \tag{73}$$

the derivative is represented by

$$\frac{\Delta \tilde{g}_i}{\Delta x_i} = \frac{g_{i+k} - g_{i-k}}{x_{i+k} - x_{i-k}} + \frac{\varepsilon_{i+k} - \varepsilon_{i-k}}{x_{i+k} - x_{i-k}} ; \tag{74}$$

thus, ε_i' corresponds to the last term of Eq. (74); i.e.,

$$\varepsilon_i' = \frac{\varepsilon_{i+k} - \varepsilon_{i-k}}{x_{i+k} - x_{i-k}} . \tag{75}$$

The covariance matrix of the derivative error becomes

$$R_{\varepsilon'\varepsilon'} = \frac{\sigma_\varepsilon^2}{4R^2h^2}(2\delta_{ij} - \delta_{i+k,i-k} - \delta_{i-k,i+k}) \tag{76}$$

where h represents the increment between the x_i's, the i and j subscripts represent the i,jth matrix element. The k represents the increment over which the derivatives of the data are taken. If k is taken very small the errors in the structure functions will produce very large errors in the derivative data. If k is too large the numerical derivatives will not represent the slope of the function. k is finally chosen as a compromise value that best fits the particular numerical solution.

V. Numerical Evaluation

The numerical evaluation of the theory is accomplished through both a computer simulation and the evaluation of experimental results. For convenience, the parameters chosen are modeled after an operational system under the direction of A. T. Waterman at Stanford University. The system consists of a transmitter located on the east side of San Francisco Bay at an elevation of 300 meters. To implement the theory, the transmitter is assumed to be located at the origin of a coordinate system with the beam propagating along the x-axis. The beam shape approximates that of a spherical wave. A cross section of the transmission path is shown in Fig. 2. It is 28 km in length, traversing San Francisco Bay, with the terminus on the west side of the bay at an elevation of 120 meters. The transmission path is perpendicular to the longitudinal axis of the bay, thus it is likely that any wind will blow transverse to the path of the beam. The wavelength of the incident beam is 8.6 mm; the transmitting antenna is a 1.5 m diameter paraboloid with a 0.4 degree beam. Using these conditions one can find a in the beam wave equation. It is found to be 4.86×10^{-3}. This determines the point at which the incident wave makes the transition from a plane wave to a spherical wave; it is about 206 meters from the transmitter. Since the path is 28 km in length, it is seen that the incident field behaves as a spherical wave over virtually the entire path. The receiving system consists of an array of eight point receivers located near the axis of the beam. The array is perpendicular to the path and positioned horizontally above the ground,

with element spacings of 400 wavelengths. See Fig. 3. If the first element is assumed to be on the x-axis, the kernel of Eq. (11) takes the form

$$K(x,\rho) = \pi^2\ 0.033\ k^2\ \Gamma(-5/6)\ L$$

$$\mathrm{Re}\ {}_1F_1\left\{-5/6\ ;\ 1\ ;\ \frac{P^2}{4\delta_i \frac{L}{k}(1-x)}\right\} \left\{-\delta_i \frac{L}{k}(1-x)\right\}^{5/6} \tag{77}$$

$$-\ {}_1F_1\left\{-5/6\ ;\ 1\ ;\ \frac{-Q^2}{4i\delta \frac{L}{k}(1-x)}\right\} \left\{i\delta \frac{L}{k}(1-x)\right\}^{5/6}$$

where

$$P^2 = Q^2 = \delta\ 0.086 \cdot 400\ (j-1) \qquad j = 1,2, \cdots 8$$

$$L = 28 \times 10^3 \tag{78}$$

$$\alpha = 4.86 \times 10^{-3}$$

$$k = 2\pi/0.086$$

j corresponds to the position of the jth element in the receiving array. The kernel was evaluated numerically to an accuracy of 8 decimal places. Both ascending and asymptotic series are used to evaluate the hypergeometric function, the choice depending upon the magnitude of the argument.

After evaluation of the kernel, the atmospheric structure constant, C_n^2, can be determined from the matrix equation

$$[B_A(\rho_i)] = [A(\rho_i,x_j)]\ [C_n^2(x_j)] \tag{79}$$

through use of the statistical inversion method. The A matrix is the combination of the kernel and the weighting function. It will be shown that when this inversion procedure is used, the predicted errors in the unknown are about ten times the error in the data. This is referred to as the sensitivity of the inversion method. For comparison, it would be interesting to know the sensitivity if a standard matrix inversion were used to determine C_n^2. This can be found through the product of the matrix norms, as in Eq. (55). In this case, the B matrix is simply A^{-1}, since in the standard matrix inversion, it is assumed that

$$f = A^{-1} g. \tag{80}$$

The result of the computation is $||A|| \; ||A^{-1}|| = 7.44 \times 10^{11}$ indicating that for a one percent data error, the error in the unknown will be on the order of 10^{11} percent. This certainly leaves some doubt as to the existence of any method that could counteract such large instabilities. Consequently, to confirm the usefulness of the inversion method, a computer simulation was implemented.

To reasonably evaluate the inversion method, the true value of the unknown structure constant must be known at many points. This true value was generated through the use of an equation similar to Eq. (68); i.e.,

$$C_n^2(x) = 10^{-14} + 0.4 \times 10^{-14} \sum_{n=0}^{1} [a_n \cos(n\pi x) + a_n' \sin(n\pi x)] \tag{81}$$

where x, the normalized path length, varies from zero to one. The a_n and a_n' are computer generated, Gaussian random numbers with zero mean and variance one. The 10^{-14} represents a typical average value of C_n^2 as derived from atmospheric experiments. After C_n^2 is generated at eighteen equispaced points along the path, the true data, $B_A(\rho)$, is calculated by matrix methods from Eq. (79). To model the errors that are inherently present in any real measurement device, the data are perturbed by adding a certain amount of error to them. The magnitude of the error is defined as the standard deviation of the error deviated by the peak of the correlation curve. It should be evident that the errors between 0.1 and 1.0 percent would be typical in experimental cases; this leads to errors on the order of tens of percents in the tail of the curve. The perturbed data, which approximates the true data, are used in the inversion method to obtain a prediction for C_n^2; this prediction is denoted by $\tilde{C}_n^2$. The accuracy of the prediction is found by comparing C_n^2 with $\tilde{C}_n^2$. The computer simulation scheme is diagramed in Fig. 4. An example of the results of the simulation, for the atmospheric structure constant is shown in Figs. 5 and 6. The errors involved in each case are indicated in the figures. For data errors between 0.1 and 1.0 percent the structure constant is predicted quite accurately. Even for large errors in the correlation function, the points denoted by the squares, the predicted value of the structure constant corresponds to the mean value of the true curve.

The sensitivity of the inversion can be represented in another way as shown in Fig. 7. The average percent error in the unknown is plotted versus the average percent error in the data. Three curves are shown: a) the error as predicted by the product of the matrix norms, b) the errors as predicted by the covariance matrix of $R_{\eta\eta}$, and c) the actual errors as derived from the computer simulation. When the data error is between 0.1 and 1.0 percent, the unknown error lies between 1.0 and 10.0 for the worst case (highest curve). This is a remarkable improvement over the errors generated by the standard matrix inversion method, as indicated earlier in this section. An interesting feature occurs for the larger data errors; the unknown errors seem to be limiting. This indicates that the even for large data errors, the statistical inversion method is useful for predicting the average value of the unknown.

The structure constant can be found from data other than the correlation function; namely, the amplitude and phase structure functions. In theory, the phase correlation function could also be used; however, due to the foregoing form of the Kolmogorov spectrum, it does not converge. An error analysis has been done for data in the form of the amplitude and phase structure functions. Figure 8 shows the results, comparing them with the correlation function case. It is found that the unknown errors predicted in the case of the amplitude structure function are slightly higher than those from the correlation function expansion. On the other hand, the error produced by the phase data is much too large to be of any use.

The other parameter to be measured through the inversion process is the average transverse wind velocity. To evaluate the accuracy of the inversion, an error analysis similar to that of the last section was studied. Since the three methods of error prediction (matrix norm, covariance matrix, and computer simulation) have been shown to agree quite well, only one will be used to evaluate the wind velocity equations. The simplest of the three is the procedure employing the product of the matrix norms. The norm of the A matrix is obtained from the derivative form of the kernel matrix; the norm of the B matrix is calculated from Eq. (67). Since this equation is rather complicated, it is simplified by assuming C_n^2 constant with zero error. The results of the error analysis are shown in Fig. 9. Curves are shown for both amplitude and phase data. The errors predicted for the amplitude data are quite low; those pre-

dicted in the case of the phase are quite high. This reinforces the conclusion that the phase data are not suitable for the inversion method.

Data obtained from an experiment conducted by J. C. Harp [14] at the Stanford Electronics Laboratories will now be evaluated to predict the wind velocity and structure constant, at several points between the transmitter and the receiver. The data consists of a set of correlation curves. To adapt them to the problem one observes the value of the correlation function and its derivative at the point $\tau=0$. In Harp's paper data points are found for seven receiver separations. These data are interpolated and shown in Figs. 10 and 11. Notice that the correlation curves are normalized to one. This being the case, it is impossible to determine the average value of the structure constant in the two cases to be studied. This limits one to examining the shape and relative magnitudes of the structure constant along the path. The average value of the wind velocity is not lost through the normalization process. To denormalize the correlation curves, the peak of each is assumed to have a typical value of 1.0×10^{-2}. The other parameters necessary for the inversion are the statistics of the unknowns and the assumed values for the average data error. The covariance matricies modeling the structure constant and the wind velocity are shown below.

$$S_{CC} = 10^{-28} \left\{1.0 + (0.04)^2 \sum_{n=o}^{10} \cos[n\pi(x_i - x_j)]\right\} \tag{82}$$

$$S_{VV} = 25\left\{1.0 + (0.1)^2 \sum_{n=1}^{6} \cos[n\pi(x_i - x_j)]\right\} \tag{83}$$

The average value of C_n^2 is assumed to be 10^{-14}; the $(0.04)^2$ in Eq. (82) is developed from the constraint that C_n^2 is greater than or equal to zero. The average value of the wind velocity was taken to be zero, since its direction can vary; its standard deviation was taken as 5 meters/sec. The magnitude of the error in the correlation function was taken to be 5 percent; this is reasonable considering the amount of interpolation that is necessary.

Using the above parameters, the atmospheric structure constant was evaluated using Eq. (42). The results are shown in Figs. (12a) and (13a). Equation (67) was used to determine the average transverse wind velocity. These results are

shown in Figs. (12b) and (13b). The structure constant curves can be interpreted physically by observing the topography over which the turbulence was formed. A cross section of the transmission system was shown in Fig. (2). It represents San Francisco Bay bordered on both sides by hills. If a wind were blowing up or down the bay, a velocity gradient would be formed from the difference in the velocity in the center of the bay and the smaller velocity that would occur near the hills. This gives rise to higher turbulence near the sides of the valley than in the center. At the same time, the turbulence very near the sides would be reduced because the larger eddies could not exist at that location. These facts are reflected in the curves representing the structure constants. The curves also compare reasonably well with the results developed by Harp. The structure constant as determined by Harp was found at three points along the path: near the transmitter, in the center, and near the receiver. The curves were assumed to be constant in these regions. As can be seen, the minimums and maximums of the structure constant curves, as derived from the inversion method agree with those obtained by Harp. One should remember that, due to the normalization of the correlation curves, the plots cannot be compared in absolute magnitude.

The wind velocity, as predicted by the statistical inversion method, can be closely compared with that found by Harp. The velocity curves are not affected by the normalization of the correlation function, and the curves presented reflect both the general shape and the absolute magnitude of the wind velocity. Before a comparison of the curves, it should be mentioned that Harp's wind velocity data were inferred by predicting the velocity at only two points near the ends of the path, and assuming a smooth variation between the two values. Thus, the velocity plots will not necessarily compare near the center of the path. As seen by inspection of Figs. (12b) and (13b), the velocity plots are in a very close agreement. Variations do occur near the center of the propagation path, and in some cases at other points; however, Harp's curves seem to indicate the average value that would be obtained from the more general statistical inversion curves.

VI. Conclusion

The value of the statistical inversion method for predicting the atmospheric structure constant and the average transverse wind velocity at several points along the path has been demonstrated by computer simulation and by application to data taken under normal atmospheric conditions. In both cases it was found that the predicted value of the unknown was within ten percent of the true value for reasonable data errors. It was also determined that for larger errors the predicted solutions correspond to the average value of the true curve. In addition, the error analysis has shown that the use of amplitude data in the inversion method leads to solutions that are ten times more accurate than those obtained from phase fluctuation data. On this basis, phase data are deemed inappropriate for use in the inversion method. From these results it can be concluded that the statistical inversion method has great potential in the remote determination of two atmospheric parameters, the structure constant and the average transverse wind velocity.

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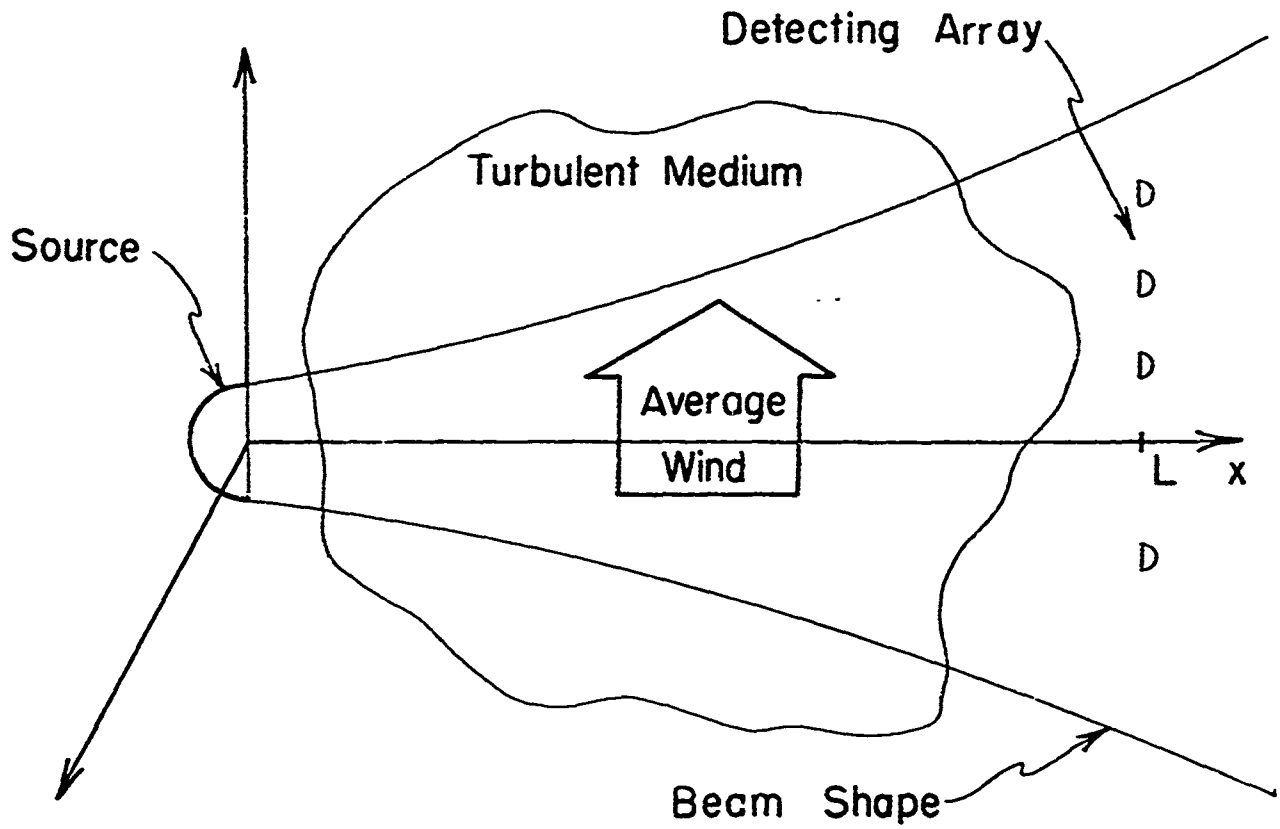

Detecting Array
Turbulent Medium
Source
D
D
D
Average
Wind
L
x
D
Beam Shape

Figure 1

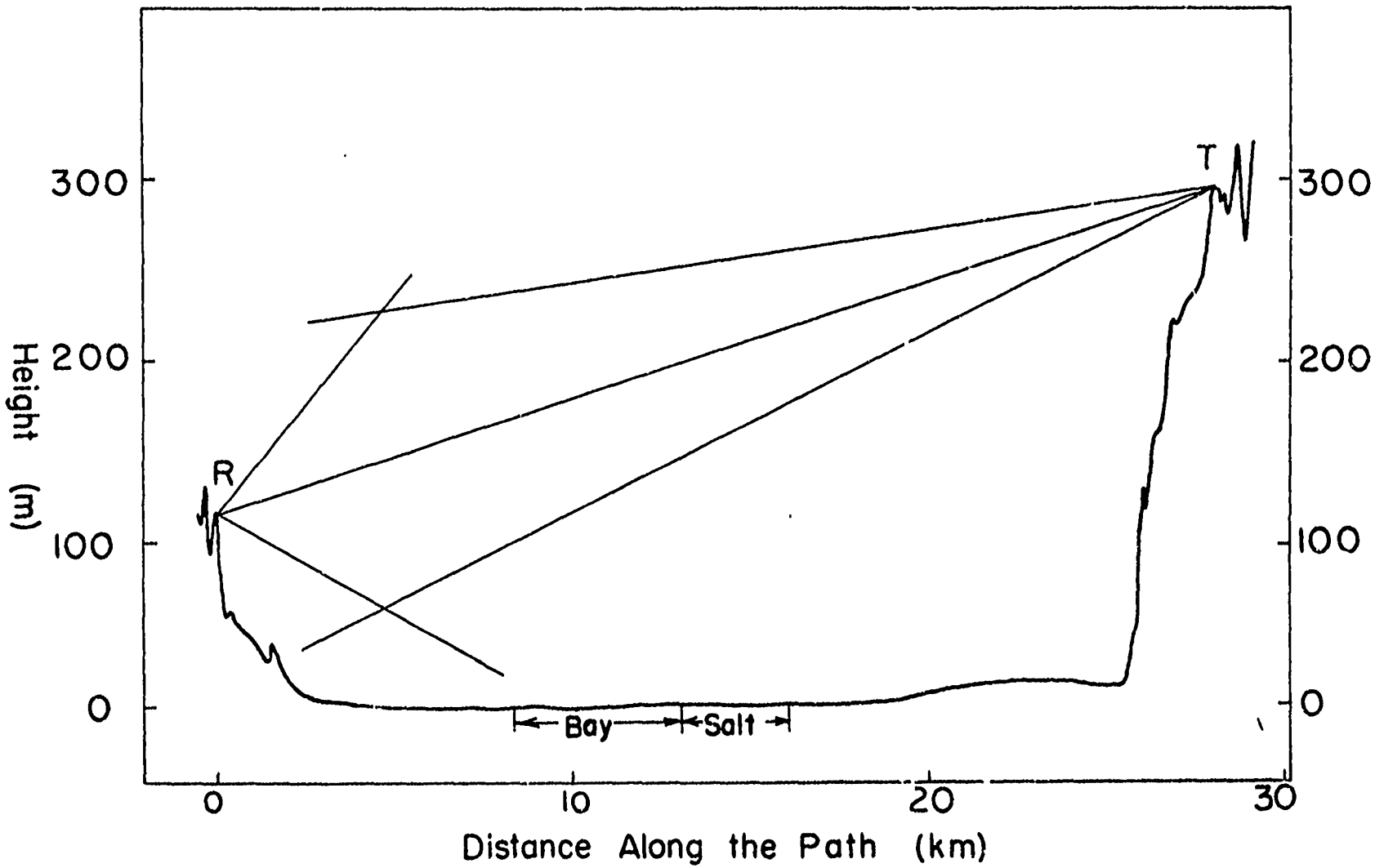
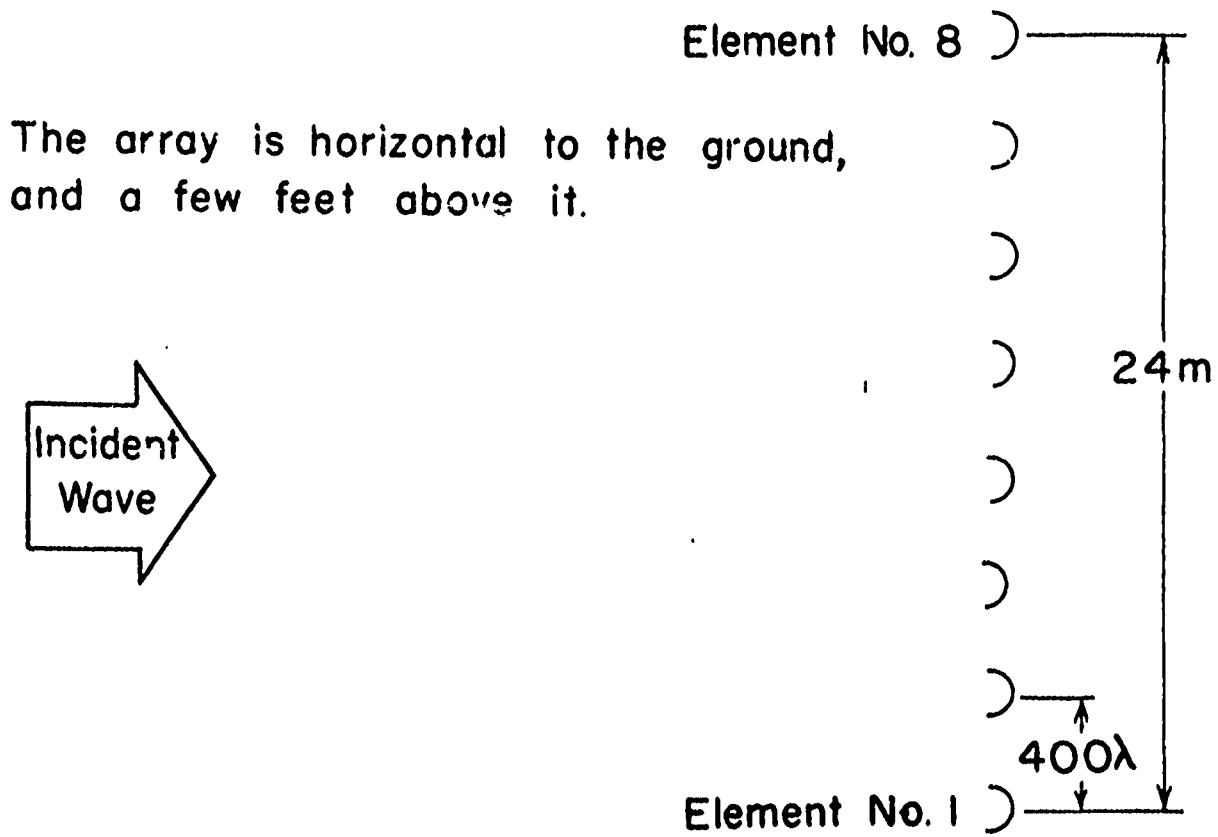

300
200
100
0
Height (m)
R
T
Bay
Salt
0
10
20
30
Distance Along the Path (km)

Figure 2

Figure 3


Element No. 8
The array is horizontal to the ground,
and a few feet above it.
24m
Incident
Wave
400λ
Element No. 1

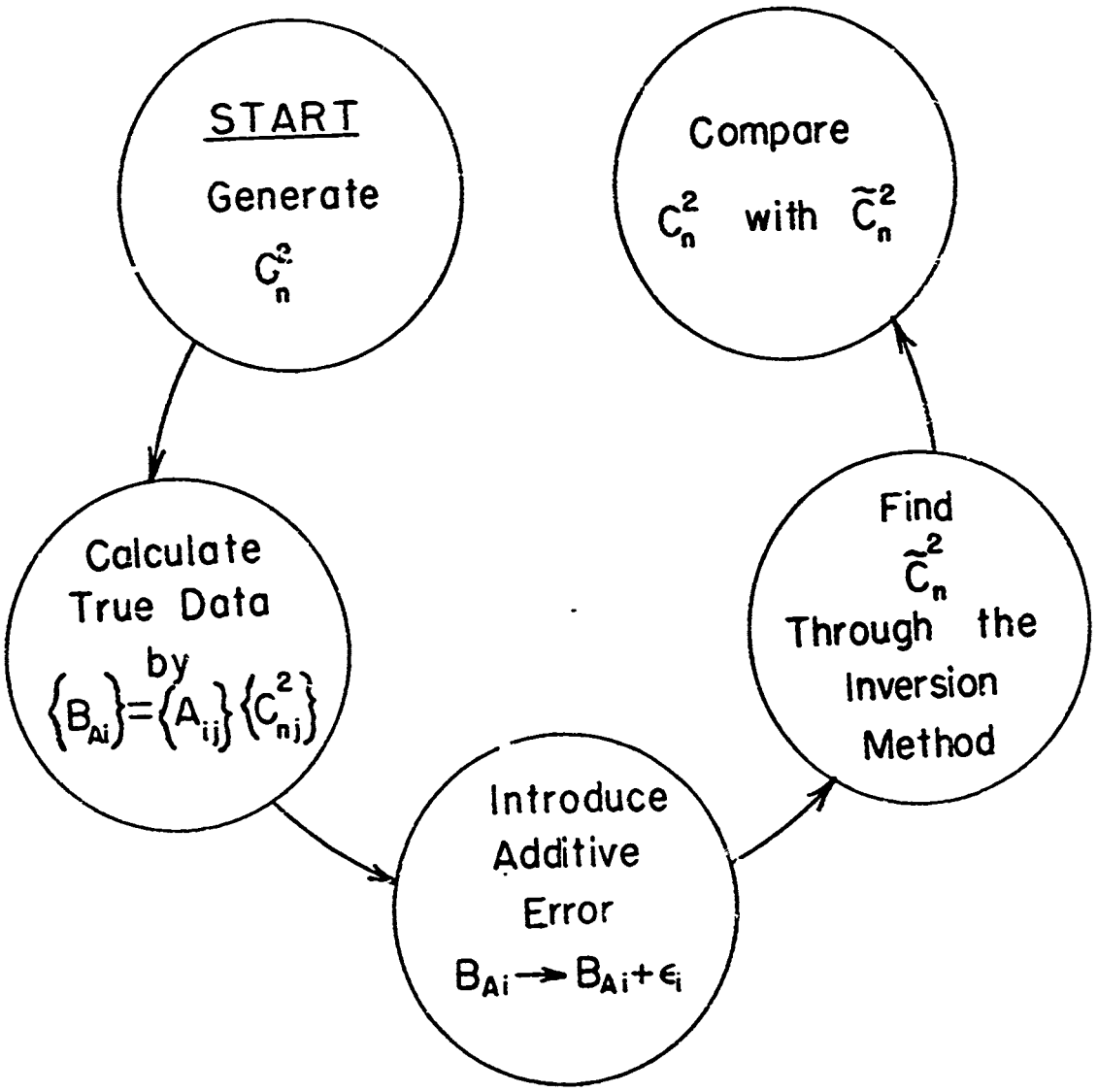
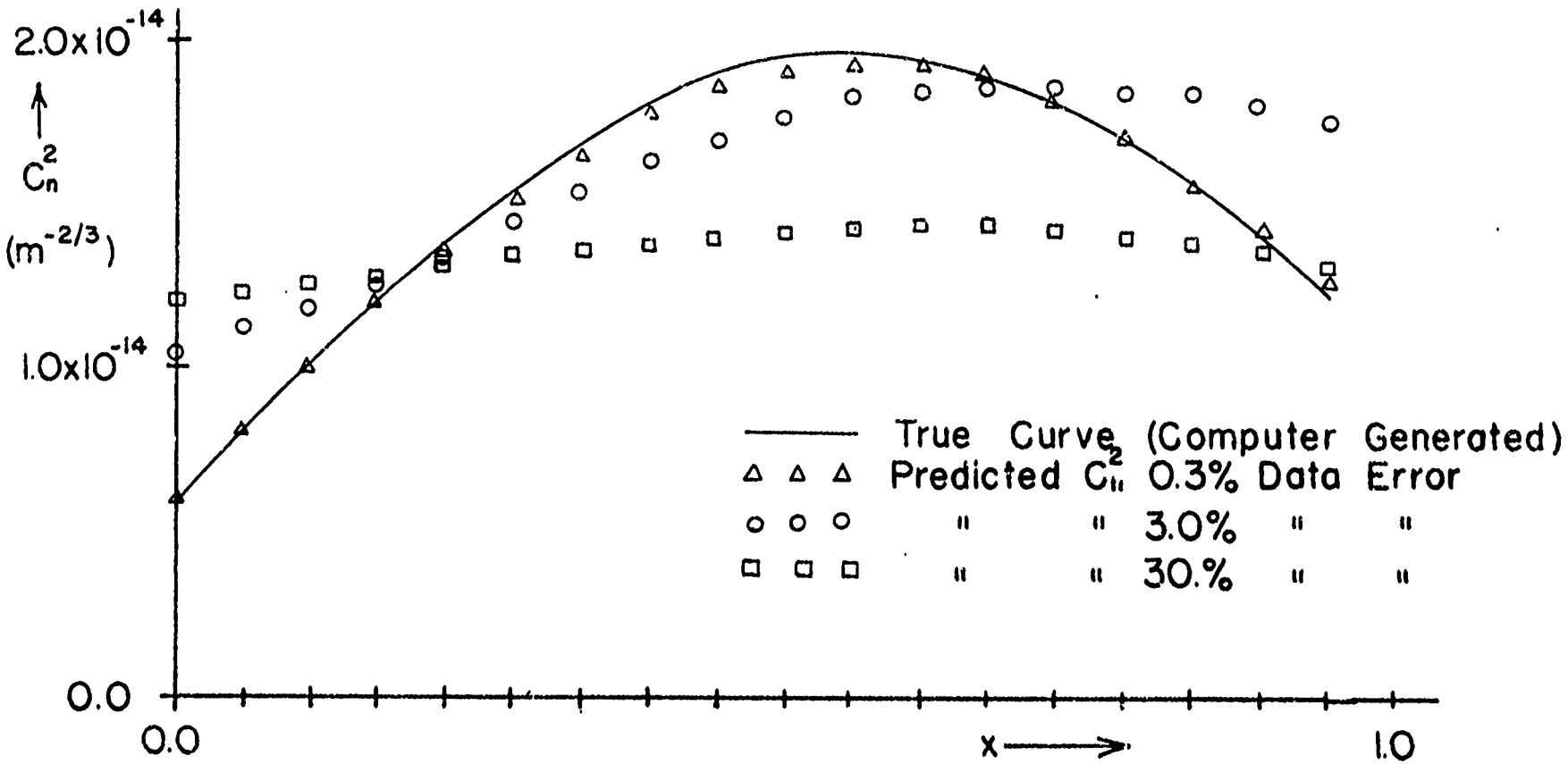

START
Generate
C_n^2
Calculate
True Data
by
$\{B_{Ai}\}=\{A_{ij}\}\{C_{nj}^2\}$
Introduce
Additive
Error
$B_{Ai} \rightarrow B_{Ai}+\epsilon_i$
Find
$\tilde{C}_n^2$
Through the
Inversion
Method
Compare
C_n^2 with $\tilde{C}_n^2$

Figure 4

Figure 5


2.0x10^-14
1.0x10^-14
0.0
C_n^2
(m^-2/3)
0.0
1.0
x
True Curve (Computer Generated)
Predicted C_n^2 0.3% Data Error
" " 3.0% " "
" " 30.% " "

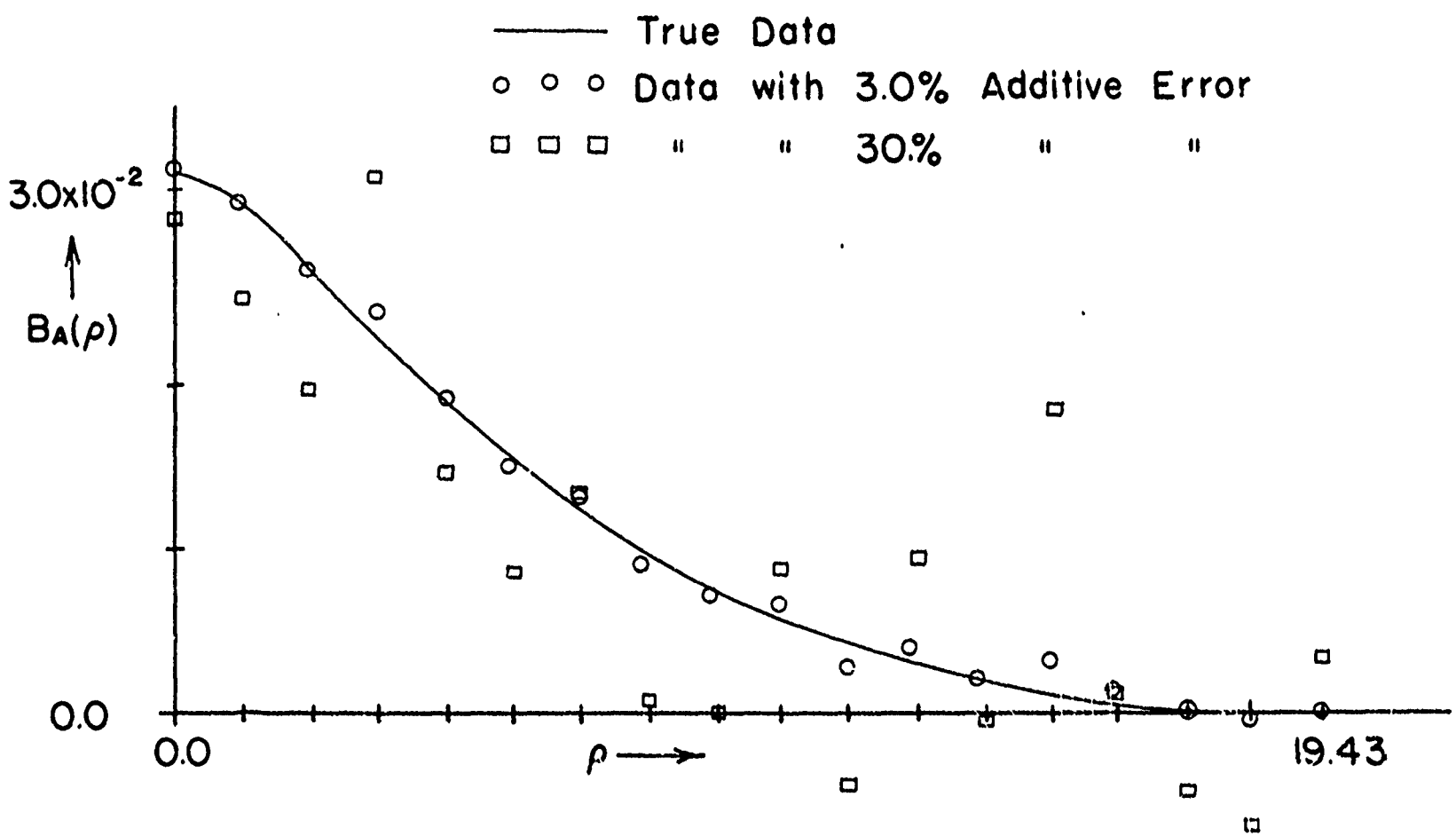
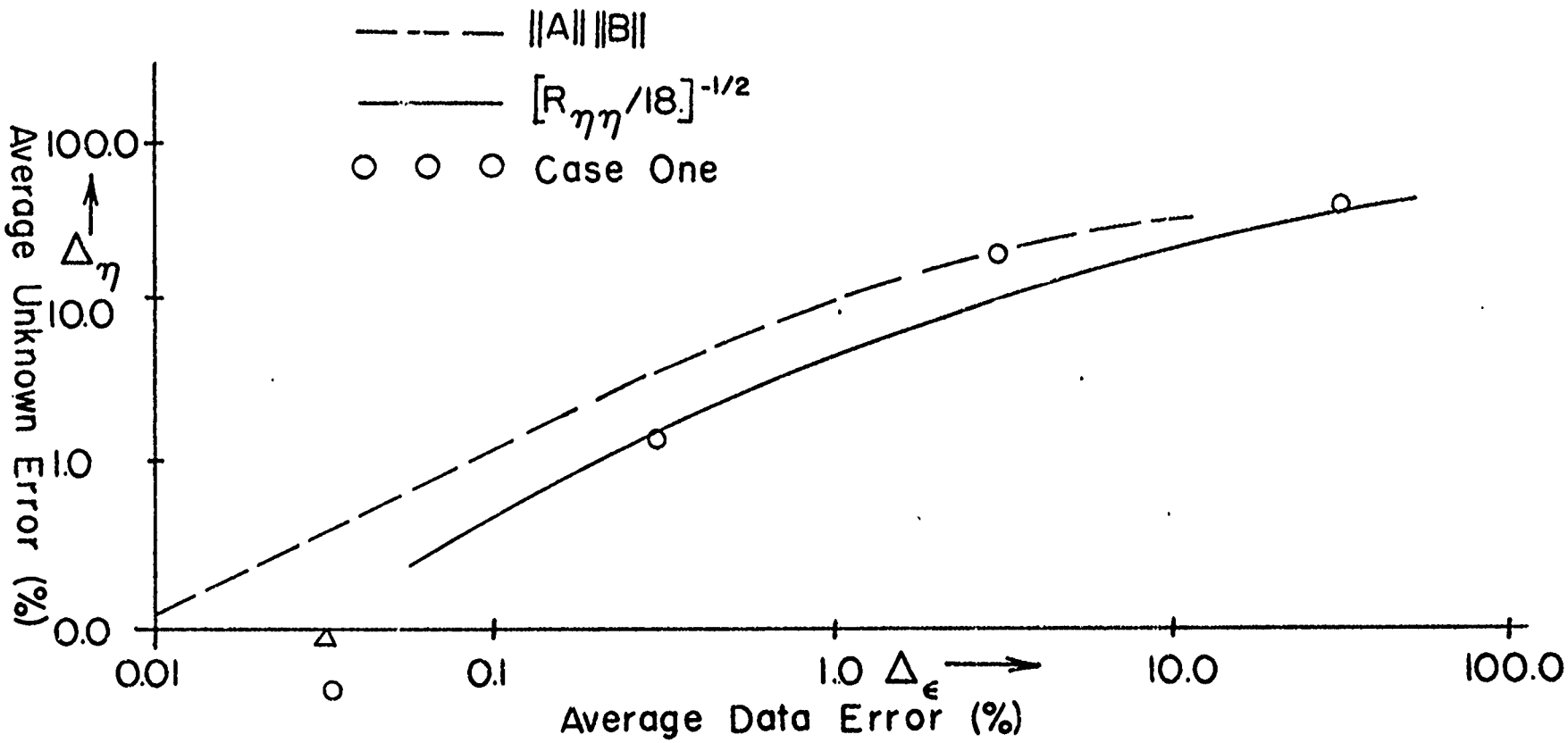

True Data
Data with 3.0% Additive Error
" " 30% " "
3.0x10^-2
B_A(ρ)
0.0
0.0
ρ
19.43

Figure 6

Figure 7


||A|| ||B||
[R_ηη/18.]^-1/2
Case One
Average Unknown Error (%)
Δ_η
100.0
10.0
1.0
0.0
0.01
0.1
1.0
10.0
100.0
Δ_ε
Average Data Error (%)

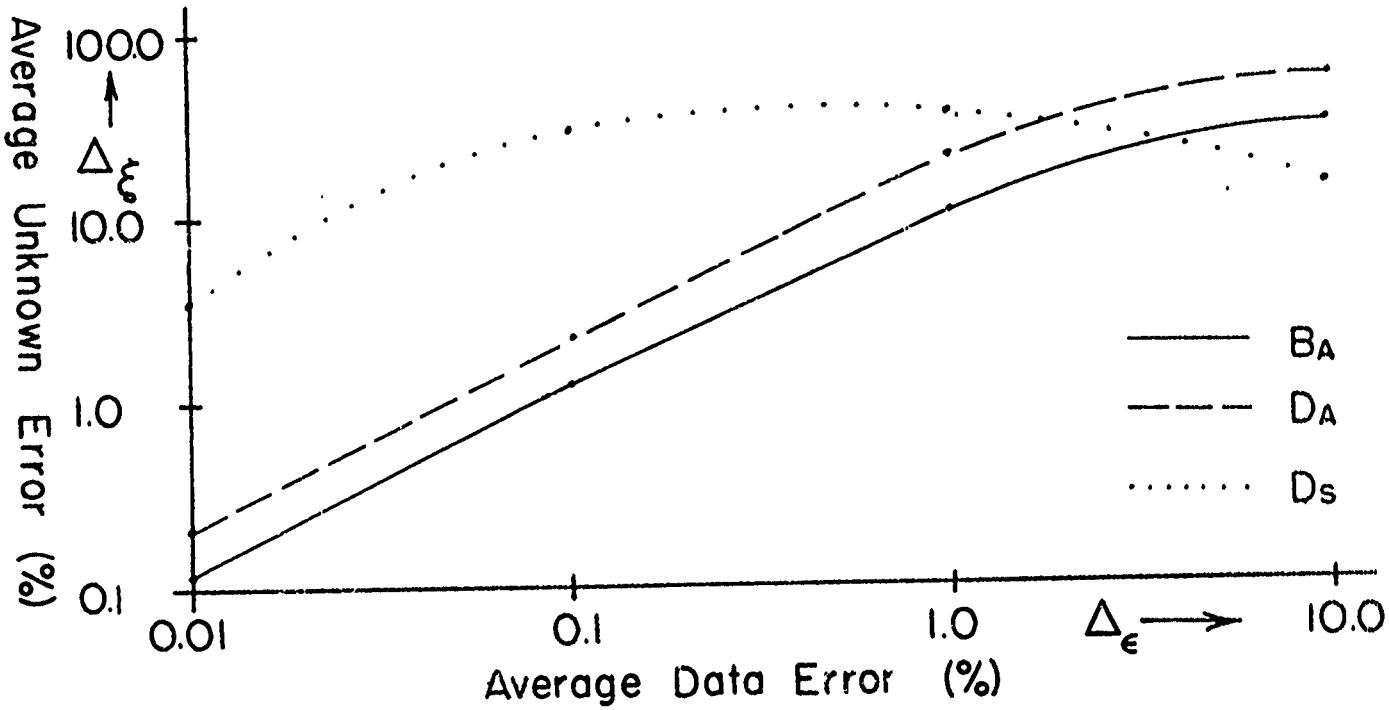
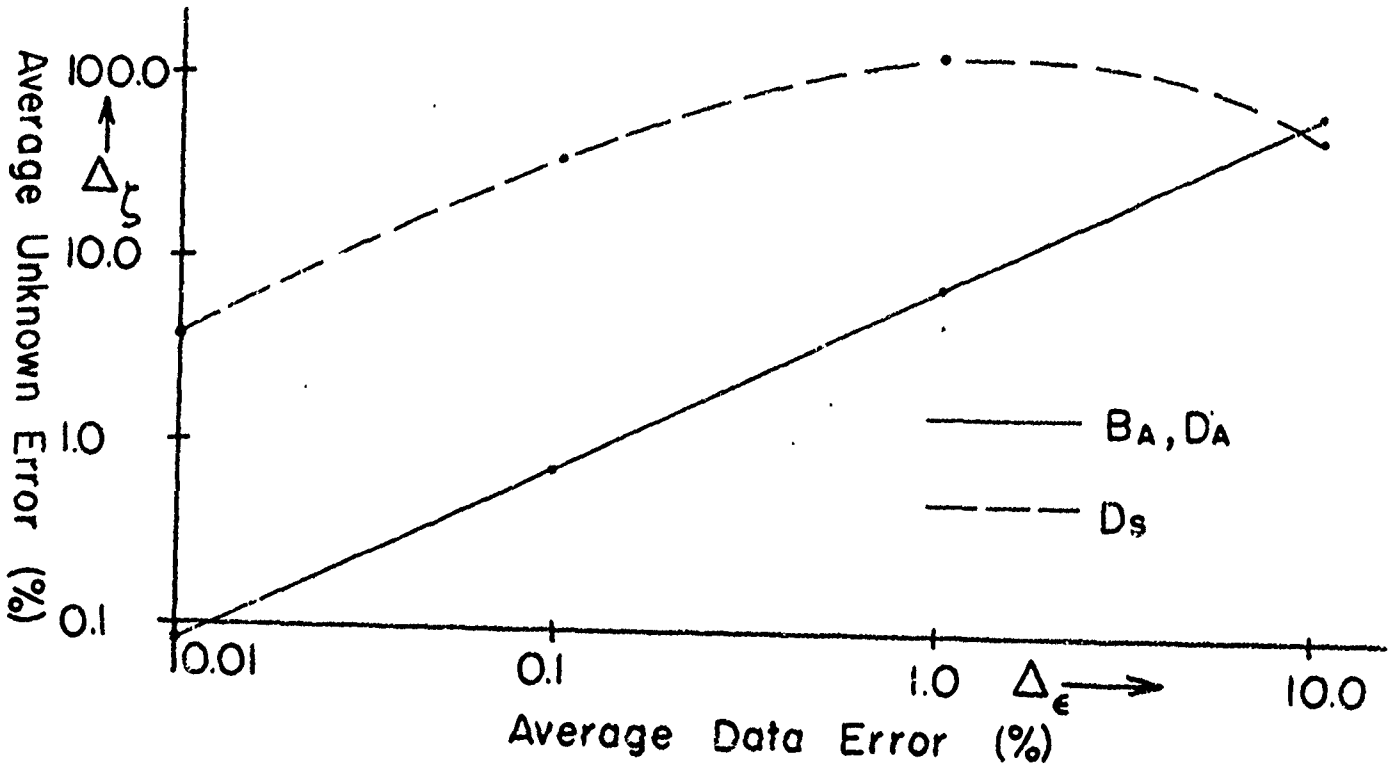

Average Unknown Error (%)
100.0
10.0
1.0
0.1
Δ_ξ
0.01
0.1
1.0
10.0
Δ_ϵ
Average Data Error (%)
B_A
D_A
D_S

Figure 8

Figure 9


Average Unknown Error (%)
100.0
10.0
1.0
0.1
Δ_ζ
0.01
0.1
1.0
10.0
Δ_ϵ
Average Data Error (%)
B_A, D_A
D_S

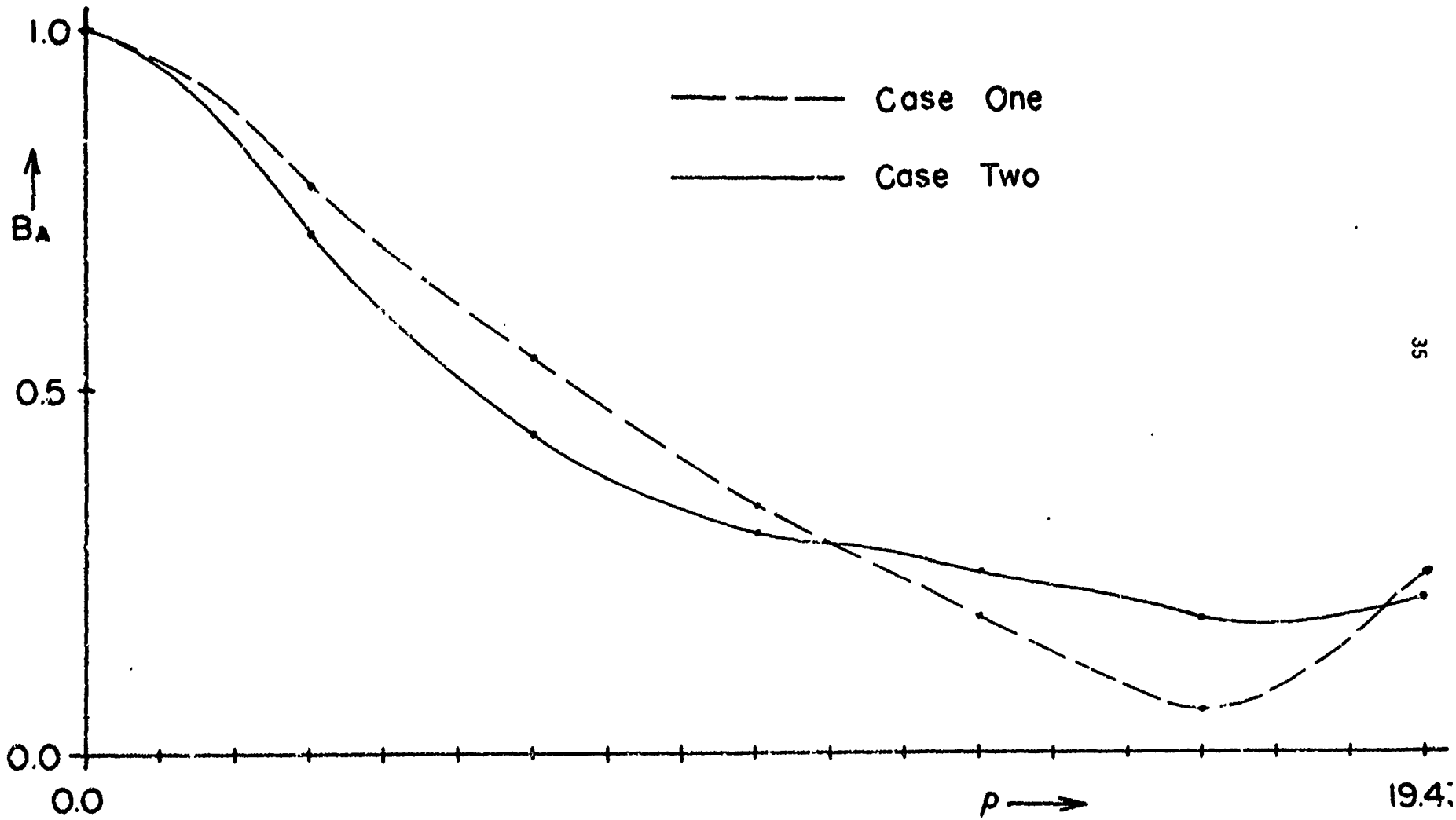
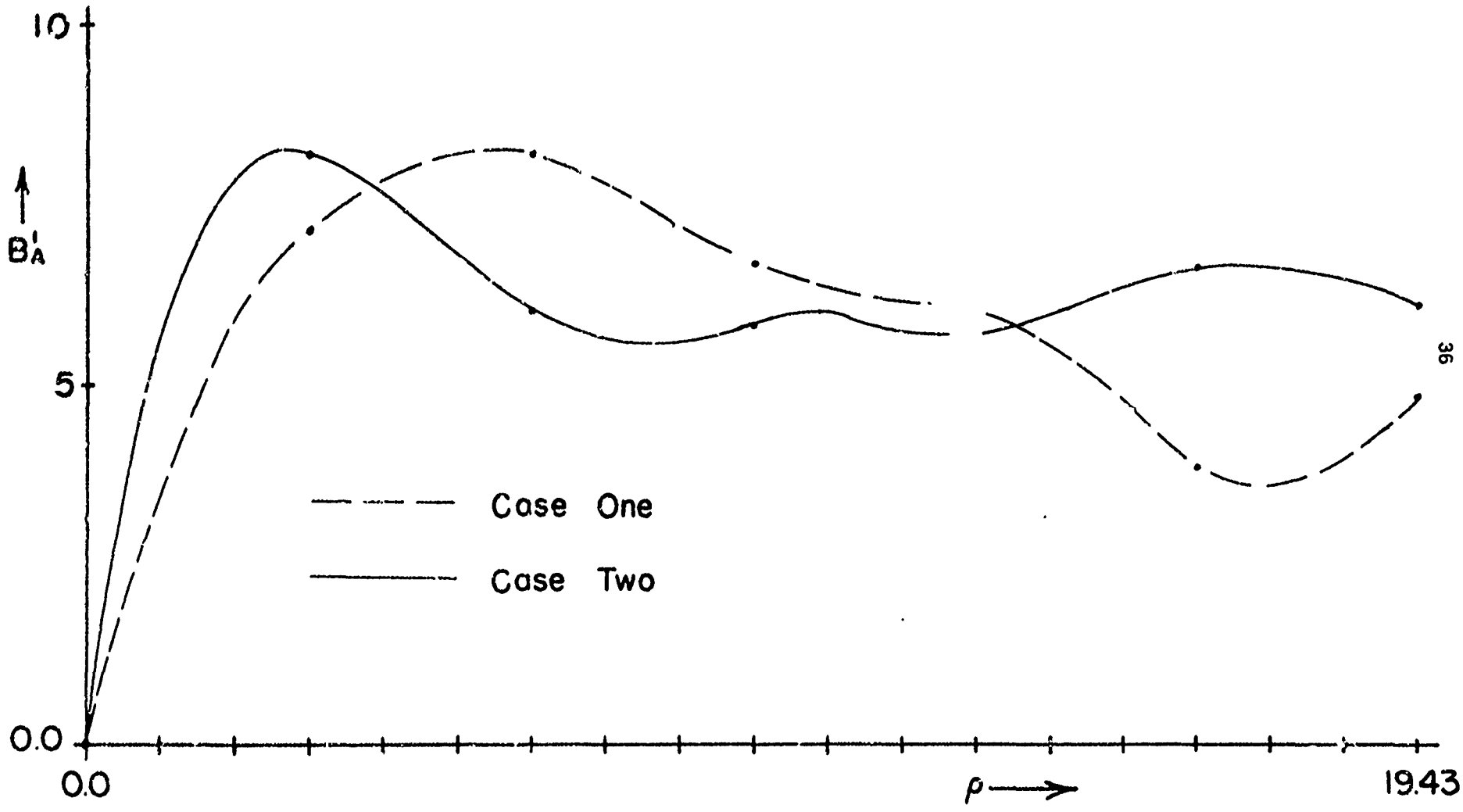


Figure 10

Figure 11



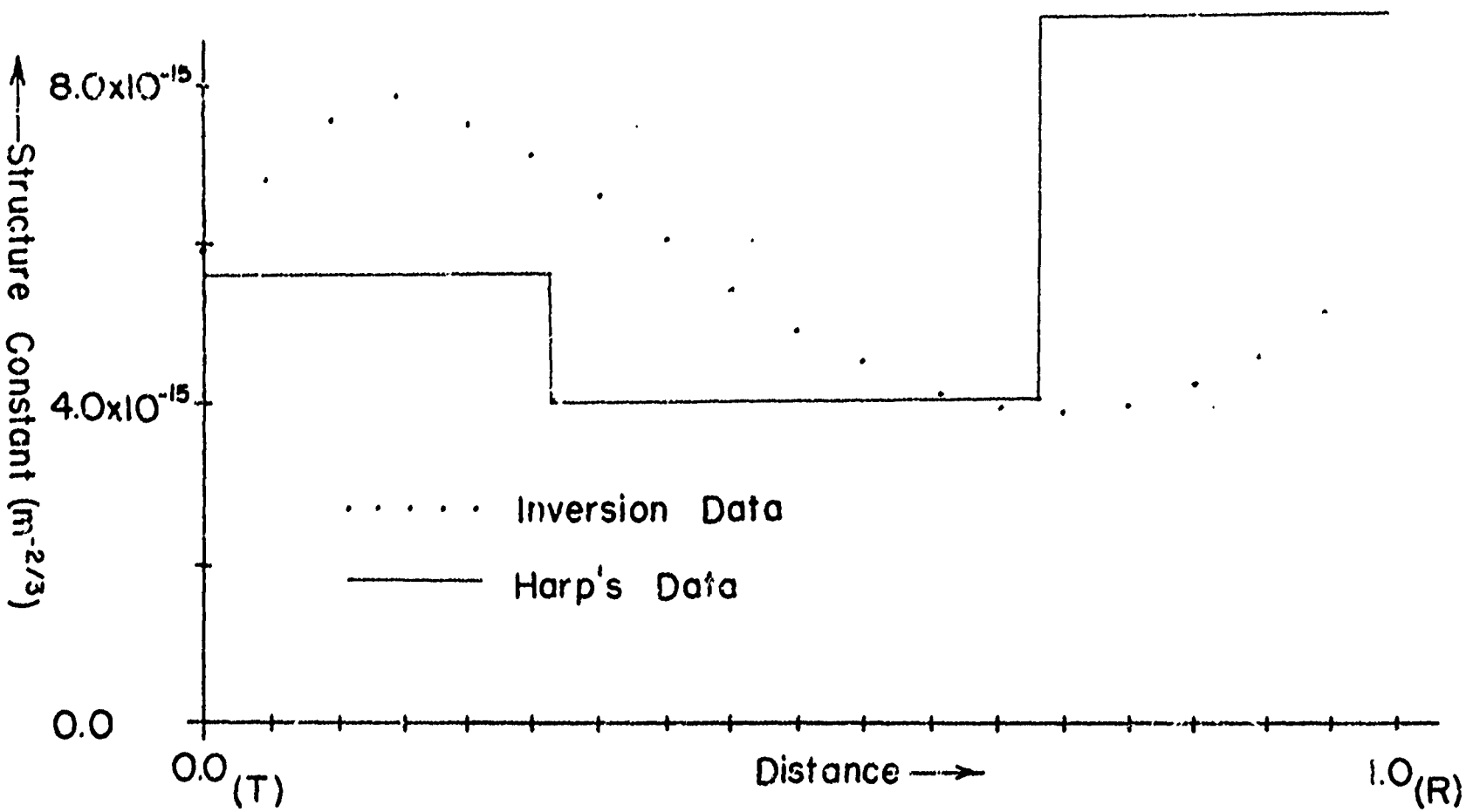
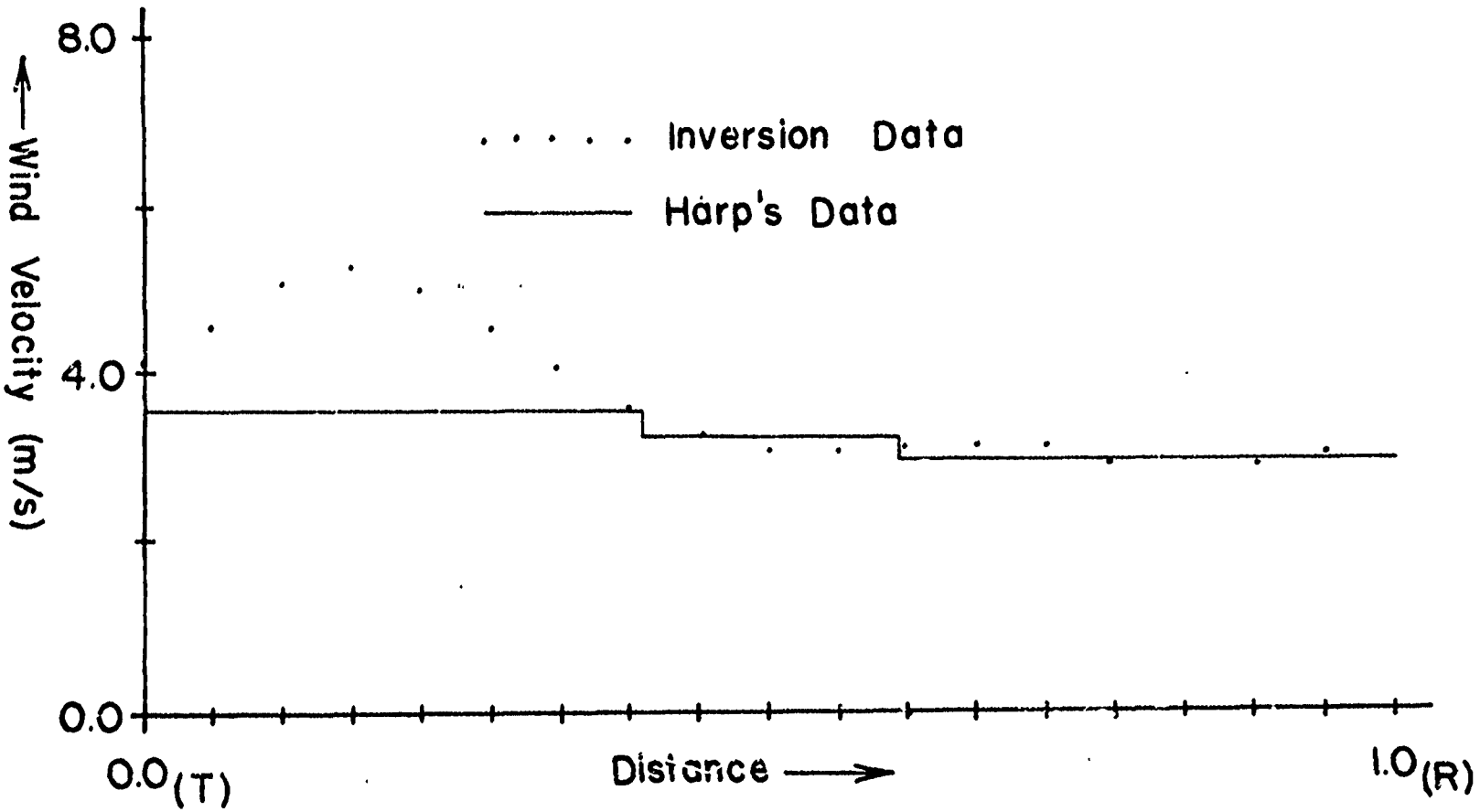

8.0x10^-15
4.0x10^-15
0.0
Structure Constant (m^-2/3)
Inversion Data
Harp's Data
0.0 (T)
Distance
1.0 (R)

Figure 12(a)

Figure 12(b)


8.0
4.0
0.0
Wind Velocity (m/s)
Inversion Data
Harp's Data
0.0 (T)
Distance
1.0 (R)

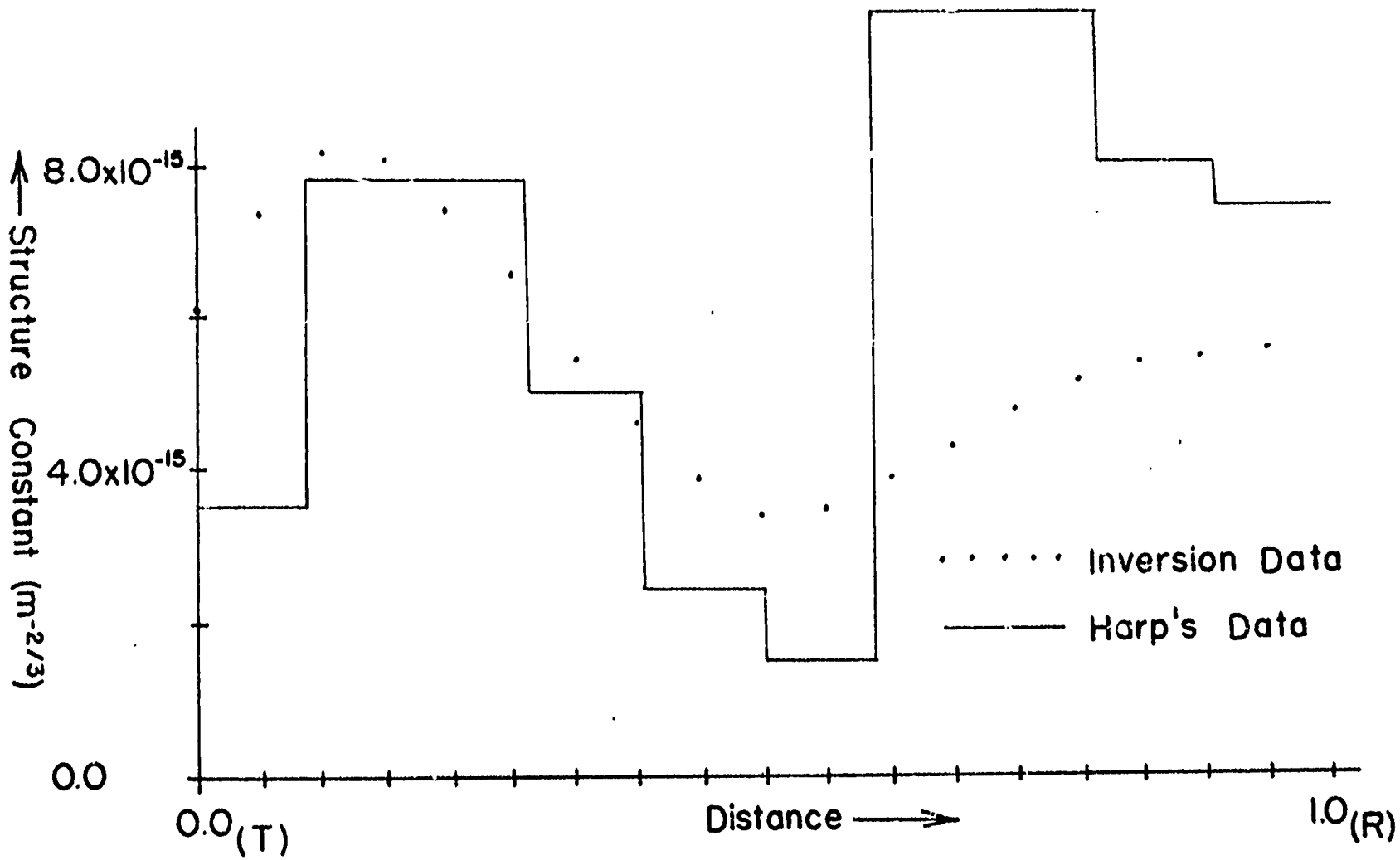
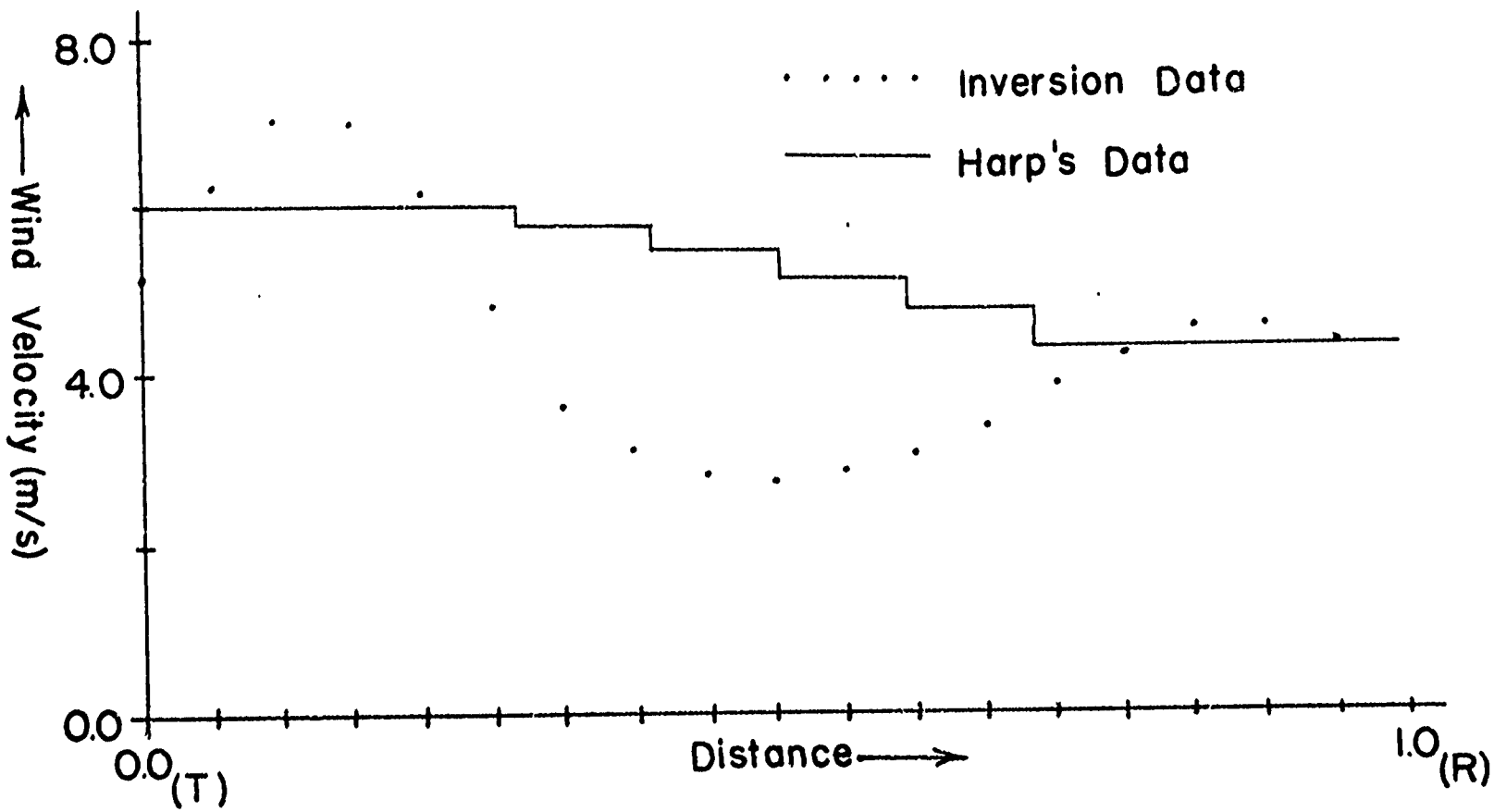

8.0x10^-15
4.0x10^-15
0.0
Structure Constant (m^-2/3)
0.0 (T)
Distance
1.0 (R)
Inversion Data
Harp's Data

Figure 13(a)

Figure 13(b)


8.0
4.0
0.0
Wind Velocity (m/s)
Inversion Data
Harp's Data
0.0 (T)
Distance
1.0 (R)