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INVESTIGATION OF LASER PROPAGATION PHENOMENA

S. A. Collins, Jr.

Ohio State University

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INVESTIGATION OF LASER PROPAGATION PHENOMENA

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The Ohio State University

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INVESTIGATION OF LASER PROPAGATION PHENOMENA

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RADC Project Engineer

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INVESTIGATION OF LASER PROPAGATION PHENOMENA

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This report deals with one aspect of restoration of atmospherically degraded images; the problem of defining isoplanatic patch size. The general restoration situation is considered to indicate the situation where the concept of isoplanaticism is appropriate. Then a model is considered and some problems of the Fourier transform technique of defining isoplanatic patch are pointed out. If the degraded reference function is not identical with the point response function then singularities in the spectrum of the intensity of the restored image can give rise to negative image functions. A measure for indexing this effect is indicated.

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INTRODUCTION

This is the third quarterly technical report for Contract No. F30602-72-C-0305. The object of the program is to provide theoretical studies of selected topics associated with light-beam propagation through a turbulent atmosphere. The present problem of interest has to do with atmospheric imaging and image restoration. The problem is to define and estimate the size of the isoplanatic patch for atmospheric image restoration schemes. This problem is of particular importance to those interested in taking detailed photographs of earth satellites with ground-based telescopes. It may also be of interest to those doing long distance imaging of terrestrial objects.

Image restoration schemes are based on the concept that if the degradation for a single point were known it would give sufficient information to reverse the degradation process because the image is basically a group of points, (the object convolved with the point response function), and it is assumed that all points are degraded identically. That assumption would be completely valid if light from every point on the object and from the reference source came through exactly the same atmospheric index fluctuations. Since the object is a cluster of points, the question is then how far can the points on the object be separated from the reference source before the atmosphere affects the various degraded point images sufficiently differently that no appreciable image restoration can be accomplished. The linear dimension of such an area at the range of the object is the isoplanatic patch size. It provides a limit on the size of the object that can be restored with a single reference source.

This report discusses some of the schemes for image restoration in general and then presents preliminary calculations using a scheme capable of estimating the isoplanatic patch size for a simple situation. In the first section various restoration schemes are indicated, the concept of isoplanatic patch is discussed and restrictions to the applicability of isoplanaticism are indicated. In Section II a particular approach is introduced. It is based on a truncated series representation of the phase of the incoming electromagnetic field. Several other possible representations are also indicated. Section III contains derivations of general expressions for atmospherically degraded images and for images restored using the Fourier transform technique. These expressions are then discussed in the light of the question of isoplanatic patch size, and of certain restoration problems.

I. DISCUSSION OF ISOPLANATIC PATCH

In this section we consider situations in which the concept of isoplanatic patch is useful. Various points are made indicating restrictions useful in its definition. Several schemes have been suggested for the restoration of atmospherically degraded images (Woods Hole, 1966). Among them are the techniques of least squares fitting (Helstrom, 1967), entropy maximization (Frieden, 1972), Fourier transform deconvolution (Harris, 1966), (McGlammery, 1967), holographic phase correction (Goodman, 1966), (Gaskill, 1968), speckle interferometry (Gezari, 1972) and point by point wavefront preprocessing (Vyce, 1973). Image processing in general has also been discussed (Huang, 1971), (Andrews, 1972). Of these the Fourier transform, holographic and preprocessing schemes are of interest because they obtain their information from an additional point reference source.

There are several points in common with these procedures that help to define the isoplanatic patch study. The first is that the isoplanatic patch basically refers to instantaneous single recordings where it is hoped that the reference source contains sufficient information so as to allow complete image restoration. An ensemble average is not employed. Indeed if the ensemble average image and reference fields were to be used they would end up being independent of position because of assumed atmospheric homogeniety. Short exposures are required because temporal averaging effectively brings into action too many ensemble members with similar results.

The second point, although well known, needs documenting. It concerns the telescope aperture size requirements. If a telescope aperture is much less than Fried's coherence length, r_0, then the resolution element size is large and motionless. As the aperture size is increased the individual image elements move about, but maintain shape. At this time the image would appear distorted so that a grid of sharp straight lines would appear as a grid of sharp but wavy lines. In such a situation image detail remains but relative distances must be inferred. As the telescope aperture becomes still larger the airy disc pattern becomes degraded, i.e., broken up. It seems reasonable that in the medium aperture situation with the wavy lines that a fair amount of detail is still available. Further, much information about relative dimensions can be inferred by trying to identify lines that were obviously straight, such as construction lines or edges.

The more difficult problem occurs when the point response function has been broken up so that detail is lost. In such a situation the restoration will cause the most gain in information about the object.

If the distortion of straight lines is the limiting criterion then possibly the isoplanatic patch could be estimated from angle of arrival correlations. The requirement is basically that light from the reference source and from all points on the object exhibit the same tilt fluctuations so that all image points move simultaneously. The typical limiting case would occur when light from two point sources had arrival angle structure functions comparable to the diffraction limited uncertainty indexed by airy disc size. This concept has not yet been pursued.

Another point concerns the definition of the isoplanatic patch. It should be closely related to factors describing image quality. Two of these have already been suggested (Barry, 1971). They are the Rayleigh resolution of a restored point spread function and the Strehl ratio. One measures the resolution of the restored image, and the other, using the relative height of the point response function, gives an indication of contrast improvement. Another indicator for resolution might be the percentage decrease in integral scale width under the operation of restoration. There is also a fidelity criterion, the normalized mean square difference between object and image intensities. It might be helpful if these quantities could be calculated by integrations either in the image plane or in the aperture plane, aperture plane analysis being often convenient. Thus we see that isoplanatic patch applies to instantaneous records (photographs or holograms) where a point reference is used to supply information on the degradation. The concept is most useful when the image is not only distorted, but also blurred. Finally the definition of isoplanatic patch should be basically couched in terms of how the image is degraded.

II. RESTORATION MODEL

It was decided to investigate the restoration process based on a simple model. Since the process is in essence an instantaneous one an approach was chosen that deals with the instantaneous field. It was also decided to work with an extremely simple situation, that is one degraded image being restored with a degraded point reference source. Further it was decided to center the work initially on the Fourier transform restoration scheme. The last two items are in harmony with experiments already in progress at RADC.

The procedure is to use a polynomial expansion that could be truncated to afford maximum simplification. Among the polynomial representations considered were Fourier series representations of either the aperture plane or image plane fields or intensities along with the corresponding sampling function representation of the associated transform plane. The Gaussian-Hermite or prolate spheroidal function representations of the aperture plane or image plane

field or intensities were also considered. Further a Karhunen-Loève series based on the field correlation was contemplated. A series expansion of the aperture plane phase was considered and chosen because it has some background (Fried, 1965), because the series appears to converge after a very small number of terms, and because the series gives a chance to isolate the effects of wavefront tilt from those of higher order distortions. This representation assumes negligible amplitude fluctuations; obviously an approximation. However it is hoped that the ensuing simplicity will speed the results. Further, the situation approaches that for 10.6 micron radiation at the RADC propagation range.

III. CALCULATION

We now proceed to examine on a somewhat preliminary basis the process of restoration, assuming the incoming field to have the form

(1) $$E = E_0 \exp(j\phi).$$

In Eq. (1)

$$\phi = \sum a_n F_n(x,y)$$

where the polynomials $F_n(x,y)$ are equivalent to those defined by Fried (Fried, 1965), but normalized for a square aperture of side a.

(2a) $$F_1 = \frac{1}{a}$$

(2b) $$F_2 = \frac{\sqrt{3}}{a} \frac{x}{a/2}$$

(2c) $$F_3 = \frac{\sqrt{3}}{a} \frac{y}{a/2}$$

(2d) $$F_4 = \frac{1}{a} \frac{3}{2} \sqrt{\frac{5}{2}} \; (x^2 + y^2 - (a^2/6))/(a/2)^2$$

(2e) $$F_5 = \frac{1}{a} (2\sqrt{5/2}/2)(x^2 - y^2)/(a/2)^2$$

(2f) $$F_6 = (3/a)(xy)/(a/2)^2.$$

These polynomials are chosen orthonormal so that

$$(3) \qquad \int F_n(\bar{r})F_m(\bar{r})w(\bar{r})d\bar{r} = \delta_{nm}$$

where

$$w(\bar{r}) = \begin{cases} 1 & -\frac{a}{2} < x < \frac{a}{2} \\ & -\frac{a}{2} < y < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases} .$$

The two-dimensional spatial spectrum $W(\bar{\kappa})$ of the $I(\bar{\rho})$ image-plane intensity is

$$(4) \qquad W(\bar{\kappa}) = \frac{1}{(2\pi)^2} \iint I(\bar{\rho})\, e^{j\bar{\kappa}\cdot\bar{\rho}}\, d\bar{\rho}$$

$$= \frac{1}{(2\pi)^2} \iint d\bar{\eta}\, E(\bar{\eta} + \frac{d\bar{\kappa}}{2k})E^*(\bar{\eta} - \frac{d\bar{\kappa}}{2k})w(\bar{\eta} - \frac{d\bar{\kappa}}{2k})w(\bar{\eta} + \frac{d\bar{\kappa}}{2k}),$$

where the aperture plane coordinates r_1 and r_2 have been replaced by the standard sum and difference variables

$$\bar{\zeta} = \bar{r}_2 - \bar{r}_1$$

and

$$\bar{\eta} = \frac{\bar{r}_2 + \bar{r}_1}{2} \quad ,$$

and $\bar{\zeta}$ has been identified with $\bar{\kappa}d/k$ (d is the image plane distance). In order to evaluate the expression for $W(\bar{\kappa})$ we first consider the expression $E(\bar{\eta} - \frac{d\bar{\kappa}}{2k})\, E^*(\bar{\eta} + \frac{d\bar{\kappa}}{2k})$. Upon inserting the expressions for the $F_i(\bar{r})$ from Eqs. (2) into Eq. (1) and simplifying, we obtain

$$(5) \qquad E(\bar{\eta} - \frac{d\bar{\kappa}}{2k})E^*(\bar{\eta} + \frac{d\bar{\kappa}}{2k}) -$$

$$E_0^2 \exp\left\{ j \sum_{n=1}^{6} a_n (F_n(\bar{\eta} + \frac{d\bar{\kappa}}{2k}) - F_n(\bar{\eta} - \frac{d\bar{\kappa}}{2k})) \right\}$$

$$= E_0^2 \exp j\left\{ \phi_0 + (\eta_x B_{xx}\kappa_x + \eta_x B_{xy}\kappa_y + \eta_y B_{yx}\kappa_x + \eta_y B_{yy}\kappa_y)\frac{k}{d} \right\}$$

where

(6a) $$\phi_o = 2\sqrt{3}(d/a^2k)(a_2\kappa_x + a_3\kappa_y)$$

(6b) $$B_{xx} = 6\sqrt{5/2}\,(a_4 + a_5)\,(d/2k)^2/(a/2)^3$$

(6c) $$B_{xy} = B_{yx} = 6a_6(d/2k)^2/(a/2)^3$$

(6d) $$B_{yy} = 6\sqrt{5/2}\,(a_4 - a_5)(d/2k)^2/(a/2)^3.$$

Inserting the expression in Eq. (5) into Eq. (4) and performing the $\bar{\eta}$ integration gives

(7) $$W(\bar{\kappa}) = 4E_o^2(d/2\pi k)^2\, e^{j\phi_o}\, \frac{\sin[((ka/d)-|\kappa_x|)(B_{xx}\kappa_x+B_{xy}\kappa_y)]}{k(B_{xx}\kappa_x+B_{xy}\kappa_y)} \times \frac{\sin[((ka/d)-|\kappa_y|)(B_{yy}\kappa_y+B_{xy}\kappa_x)]}{k(B_{yy}\kappa_y+B_{xy}\kappa_x)} .$$

Equation (7) is the basic expression for the instantaneous intensity spectrum.

There are several points worth noting. First, if the wavefront aberration coefficients (a_n in Eq. (1)) go to zero, then the expression reduces to the triangular function as expected. Second the wavefront tilt terms a_2 and a_3 act only to shift the image and not to distort the shape, also as expected. Third, $W(\bar{r})$ does not factor into a function of κ_x times a function of κ_y. This arises because of the term B_{xy} which is proportional to the distortion coefficient a_6. Indeed this is expected because $F_6(\bar{r})$ is the only term that mixes x and y.

We now use the expression for the degraded image spectrum to give an indication of the restored image spectrum. Assume that two point sources are imaged, and that both have intensity spectra of the form in Eq. (7). Then the restored image spectrum $W_o(\bar{\kappa})$ will have the form

$$(8)\qquad W_0(\bar{\kappa}) = \frac{W(\bar{\kappa})}{W'(\bar{\kappa})} \cdot e^{j(\phi_0 - \phi_0')} \times \frac{(B'_{xx}\kappa_x + B'_{xy}\kappa_y)(B'_{yy}\kappa_y + B'_{xy}\kappa_x)}{(B_{xx}\kappa_x + B_{xy}\kappa_y)(B_{yy}\kappa_y + B_{xy}\kappa_x)}$$

$$\times \frac{\sin\left[((ka/d)-|\kappa_x|)(B_{xx}\kappa_x + B_{xy}\kappa_y)\right] \sin\left[((ka/d)-|\kappa_y|)(B_{yy}\kappa_y + B_{xy}\kappa_x)\right]}{\sin\left[((ka/d)-|\kappa_x|)(B'_{xx}\kappa_x + B'_{xy}\kappa_y)\right] \sin\left[((ka/d)-|\kappa_y|)(B'_{yy}\kappa_y + B_{xy}\kappa_x)\right]}$$

$$-\frac{ka}{d} < \kappa_x < +\frac{ka}{d}$$

$$-\frac{ka}{d} < \kappa_y < +\frac{ka}{d} \quad .$$

The primes indicate that the wavefront distortion coefficients in the B_{ij}'s have different values from those in the B'_{ij}'s, due to possibly different atmospheric paths.

Nothing has been said about the definition of $W_0(\bar{\kappa})$ outside the range $-(ka/d) < \kappa_x, \kappa_y < +(ka/d)$. It, in principle, could be chosen by analytically continuing the two spectra $W(\bar{\kappa})$ and $W'(\bar{\kappa})$ and then using the quotient of the continued parts (Harris, 1971). For a less laborious approach, it could arbitrarily be chosen as zero. In the situation where perfect restoration is obtained $W_0(\bar{\kappa}) = 1$ for $|\kappa_x|$, $|\kappa_y| \leq ak/d$. This would imply a (sinx)/x function for the restored image intensity, a contradiction since the intensity can not be negative. One might define $W_0(\bar{\kappa})$ to be unity everywhere outside the region $|\kappa_x| \leq ak/d$, $|\kappa_y| \leq ak/d$, giving an impulse function for the perfectly restored image. However for imperfectly restored images, it would give an impulse function plus a second function no matter how poor the restoration, which seems unreasonable. Possibly a Gaussian function exponential or a Hanning type roundoff in both the x and y directions would provide a reasonable first compromise.

The function $W_0(\bar{\kappa})$ has some interesting properties. If the distortion coefficients $a_4 \cdots a_6$ and $a'_4 \cdots a'_6$ are respectively equal, then $W_0(\bar{\kappa}) = 1$ indicating perfect restoration. If $a_4 \cdots a_6$ do not equal the respective $a'_4 \cdots a'_6$ then $W_0(\bar{\kappa})$ will have singularities where $W'(\bar{\kappa}) = 0$. Presumably the singularity is very close to a zero if the distortion coefficients do not differ by much. Further the effect of the singularities goes to zero as the corresponding zeros approach. However singularities must still be reckoned with.

The effect of the singularities can be semi-qualitatively considered by examining a somewhat similar case. Consider the function

$$(9)\qquad W_3(\kappa) = e^{-\left(\frac{\kappa}{\kappa_0}\right)^2} \sum_n \frac{\kappa - (\kappa_n + \delta_n)}{\kappa - \kappa_n}$$

then

$$(10)\quad F(W_3) = \frac{1}{2\kappa}\int_{-\infty}^{\infty} e^{-j\kappa x}\, W_3(\kappa)\,d\kappa$$

$$= \frac{e^{-\frac{\kappa_0 x^2}{4}}}{2\kappa_0} - \frac{\delta}{2}\, e^{j\kappa_0 x\left(1+\frac{\delta}{\kappa_n}\right)} \int dx'\, e^{-jx'\left(1+\frac{\delta}{\kappa_n}\right)} \operatorname{sgn}(x-x')\, e^{-\frac{\kappa_0 x'^2}{4}}$$

where $\operatorname{sgn}(x) = x/|x|$. The "restored image" function $F(W_3)$ in Eq. (10) has one desired property, namely that if the two functions are identical, so that the roots of both the numerator and denominator in Eq. (9) are at identical locations, then $\delta = 0$ and a perfect restoration is achieved. If such is not the case, the "restored image" is imperfect: it has negative intensity regions. These are bad features for well-behaved intensity functions. Equation (10) has imaginary parts but these cancel for the function $W_0(\kappa)$ because the poles are symmetrically located.

Although the results of Eq. (10) are based on the replacement function in Eq. (9), there is no reason to believe that the same results will not be obtained using the restored spectrum in Eq. (8)

Equation (10) describes image plane behavior and is therefore in a form to be used for the definition of isoplanatic patch. For example, the magnitude of the second term in Eq. (10) is positive and goes to zero for perfect restoration. The ratio of that to the average value of the first term would give an indication of the relative effect of the lack of isoplanaticity. The coefficients in the δ_n are proportional to the wavefront distortion coefficients (see Eqs. (6b) and (8)), and might be estimated by using the mean-square values of the distortion coefficients derived using turbulent atmospheric fluctuations. This definition will not be recommended particularly at this time; it is only used as an example.

The discussion presented, although preliminary, does present an approach to the problem of defining isoplanatic patch in the presence of some problems in using the Fourier transform approach to restoration.

IV. SUMMARY

This report has presented a preliminary discussion of some concepts and problems associated with the derivation of an expression for isoplanatic patch size. First the situation was considered in general defining the case where isoplanatic patch might be a useful concept and how it should best be defined. Then a particular model for an atmospherically distorted wavefront was chosen and based upon that model a possible approach to defining isoplanatic patch was indicated.

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