AD-757 623

NUMERICAL APPROXIMATIONS OF SELECTED METEOROLOGICAL PARAMETERS RELATED TO CLOUD PHYSICS

Walter S. Nordquist, Jr.

Army Electronics Command
White Sands Missile Range, New Mexico

March 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AD 757698

AD

Reports Control Symbol
OSD-1366

RESEARCH AND DEVELOPMENT TECHNICAL REPORT
ECOM-5475

NUMERICAL APPROXIMATIONS OF SELECTED METEOROLOGICAL PARAMETERS RELATED TO CLOUD PHYSICS

By

Walter S. Nordquist, Jr.

March 1973

Approved for public release, distribution unlimited.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE

ECOM

UNITED STATES ARMY ELECTRONICS COMMAND - FORT MONMOUTH, NEW JERSEY

NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The citation of trade names and names of manufacturers in this report is not to be construed as official Government indorsement or approval of commercial products or services referenced herein.

Disposition

Destroy this report when it is no longer needed. Do not return it to the originator.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Atmospheric Sciences Laboratory White Sands Missile Range, New Mexico	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

NUMERICAL APPROXIMATIONS OF SELECTED METEOROLOGICAL PARAMETERS RELATED TO CLOUD PHYSICS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*

Walter S. Nordquist, Jr.

6. REPORT DATE February 1973	7a. TOTAL NO. OF PAGES 36	7b. NO. OF REFS 13
8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. DA Task No. 1T061102B53A-20 d.	9a. ORIGINATOR'S REPORT NUMBER(S) ECOM-5475 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY US Army Electronics Command Fort Monmouth, New Jersey

13. ABSTRACT

Methods of computation and error estimates are presented for numerical approximation of selected meteorological parameters used in the solution of cloud physics problems. These parameters are latent heat, saturation vapor pressure, dew point and wet bulb temperatures, specific heat of liquid water, temperature of the lifting condensation level, change in mass of freely falling water drops, and saturation-adiabatic temperatures. The FORTRAN programming language listings of the approximations are provided in the Appendix.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED

Security Classification

I

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
1. Numerical Approximations						
2. Cloud Physics						
3. Meteorological Parameters						

UNCLASSIFIED

Security Classification

II

Reports Control Symbol
OSD-1366

Technical Report ECOM-5475

NUMERICAL APPROXIMATIONS OF SELECTED METEOROLOGICAL PARAMETERS RELATED TO CLOUD PHYSICS

By

Walter S. Nordquist, Jr.

Atmospheric Sciences Laboratory
White Sands Missile Range, New Mexico

March 1973

DA Task No. 1T061102B53A-20

Approved for public release; distribution unlimited.

US Army Electronics Command

Fort Monmouth, New Jersey

III

ABSTRACT

Methods of computation and error estimates are presented for numerical approximation of selected meteorological parameters used in the solution of cloud physics problems. These parameters are latent heat, saturation vapor pressure, dew point and wet bulb temperatures, specific heat of liquid water, temperature of the lifting condensation level, change in mass of freely falling water drops, and saturation-adiabatic temperatures. The FORTRAN programming language listings of the approximations are provided in the Appendix.

CONTENTS

	Page
INTRODUCTION	1
METEOROLOGICAL PARAMETERS	2
Latent Heats	2
Saturation Vapor Pressures	4
Dew Point Temperature	7
Wet Bulb Temperature	10
Specific Heat of Liquid Water	12
METEOROLOGICAL PROCESS PARAMETERS	12
Temperature of the LCL	12
Change in Mass of Freely Falling Water Drops	14
Saturation-Adiabatic Temperatures	17
CONCLUDING REMARKS	24
LITERATURE CITED	25
APPENDIX -- COMPUTER LANGUAGE LISTINGS OF FUNCTIONS AND SUBROUTINES	27

LIST OF TABLES

		Page
I.	TEMPERATURE UNIT CONVERSIONS	3
II.	Tcal g^{-1} UNIT CONVERSIONS	3
III.	PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR THE CALCULATION OF LATENT HEATS	5
IV.	PRESSURE UNIT CONVERSIONS	6
V.	PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR THE CALCULATION OF SATURATION VAPOR PRESSURES AT SELECTED TEMPERATURES	6
VI.	PERCENTAGE ERROR OF THE DEW POINT AND WET BULB TEMPERATURE ITERATION TECHNIQUES.	8
VII.	PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR THE CALCULATION OF THE SPECIFIC HEAT OF LIQUID WATER .	13
VIII.	ERROR (C) OF THE LIFTING CONDENSATION LEVEL TEMPERATURE FUNCTION	15
IX.	PERCENTAGE ERROR IN THE NUMERICAL APPROXIMATIONS FOR THE CALCULATION OF dM/dt AT SELECTED CONDITIONS. . .	16
X.	COMPARISON OF THE RESULTS OF THE NUMERICAL METHOD FOR SOLVING A DIFFERENTIAL EQUATION APPLIED TO $dx/dy = 0.5y^{-0.5} + 1$	21
XI.	COMPARISONS OF SATURATION-ADIABATIC TEMPERATURES . .	22

LIST OF SYMBOLS

Symbol	Subscript	Item	Units Used
c		Specific Heat	$lTcal\ g^{-1}K^{-1}$
	r	air at constart pressure	
	w	liquid water	
e		Water Drcp Diameter	m
3		Partial Pressure of Water Vapor	mb
	a	actual	
	s	saturation	
	s_i	saturation over solid water	
	s_w	saturation over liquid water	
	w	saturation at the wet bulb temperature	
log		Common Logarithm (base 10)	
L		Latent Heat of water	$lTcal\ g^{-1}$
	f	fusion	
	s	sublimation	
	v	vaporization	
M		Mass of Water Drop	g
P		Pressure	mb
	a	atmosphere (with no water)	
	t	atmosphere total	
R		Dry Air Gas Constant	$lTcal\ g^{-1}K^{-1}$
rh		Relative Humidity	Percent
t		Time	Sec
T		Temperature	K
	a	dry bulb	
	d	dew point	
	LCL	lifting condensation level	
	m	psuedo-adiabatic	
	s	saturation-adiabatic	
	w	wet bulb	
	x	approximation of saturation-adiabatic	
X		Mixing Ratio	$g\ g^{-1}$
	s	water vapor at saturation	
	w	liquid water	
	z	initial value of X_s	
ε		Ratio of molecuiar weights of water vapor and dry air	--

INTRODUCTION

The application of techniques for numerical simulation to various meteorological processes requires that many meteorological parameter descriptions be subject to individual approximation. Thus, the accuracy and adequacy of the descriptions are dependent on the requirements of the individual "simulator." In many cases the importance of the meteorological parameter description is subordinate to some broader scale problem; as such, if the description is deemed adequate, it is normally used without explanation or comment. As a consequence there are many methods available for the description of meteorological parameters, but there is a paucity of information on how well they describe the parameters. If the accuracy must be known, then the investigator must either construct his own approximation or ascertain the accuracy of some available approximation, either of which can detract from the solution of a larger problem.

This report contains descriptions of numerical approximating techniques for meteorological parameters required by the author and his associates in conjunction with numerical modeling efforts concerning clouds and fogs [1], [2], [3], [4]. This presentation is made so that there might be a ready reference to a set of approximating methods and the accuracies associated with the calculated parameter values.

The numerical approximations are divided into two groups: (1) descriptions of meteorological parameters, per se; and (2) descriptions of meteorological parameters associated with a meteorological process. The first group covers the latent heats of water, the saturation vapor pressures of water, the dew point and wet bulb temperatures, and the specific heat of liquid water. The second group covers the temperature of the lifting condensation level, the change of mass of freely falling water drops, and the temperatures resulting from saturated adiabatic processes. The method of approximation and the source of comparative values, as well as the error as determined relative to the comparative values, is provided for each parameter. The units for the comparisons are those of the comparative base, where applicable. In those cases where comparative values are not available, the basis of the accuracy given is discussed. When percentage errors are given the value presented is calculated from

$$\text{Percentage Error} = 100 \frac{\text{Calculated value-Reference value}}{\text{Reference value}} \quad (1)$$

The FORTRAN programming language listing of the approximations is provided in the Appendix in either FUNCTION STATEMENT or SUBROUTINE form as appropriate.

METEOROLOGICAL PARAMETERS

The meteorological parameters described in this section are associated with the state of the atmosphere and are, therefore, essentially definitions of the value of the parameter for the specified state. The parameters covered are those associated with the following: the heat requirements for the change of phase of water; the partial pressure of water vapor over liquid and solid plane water surfaces; the temperature at saturation of air cooled at constant pressure with no change in water vapor content; the temperature of air cooled to saturation at constant pressure; and the specific heat of liquid water.

Latent Heats

The latent heats are associated with the vapor-liquid, vapor-solid, and liquid-solid phase changes of water. The latent heats can be expressed as functions of temperature, T, with the techniques given here using degrees Kelvin (conversion to other temperature units is provided in Table I) and the latent heat expressed in 1Tcal g^{-1} (conversion to other units is provided in Table II).

The latent heat associated with the liquid-vapor/vapor-liquid phase changes is the latent heat of vaporization, L_v. It can be approximated from

$$L_v = 754.817 - 0.575\ T \quad (2)$$

The latent heat associated with the solid-vapor/vapor-solid phase changes is the latent heat of sublimation, L_s. It can be approximated from

TABLE I[1]

TEMPERATURE UNIT CONVERSIONS

Unit	Symbol	Unit	Symbol	Conversion
Kelvin	K	Centigrade	C	K - 273.16
		Fahrenheit	F	1.8 (K - 273.16) + 32
		Rankine		1.8 (K - 273.16) + 491.69

TABLE II[2]

ITcal g^{-1} UNIT CONVERSIONS

1.0 International Steam Tables Calorie (ITcal) = 1.00032 cal_{15}

$$1.0 \text{ ITcal } g^{-1} = 4.18684 \times 10^{7} \text{ cm}^{2} \text{ sec}^{-2}$$

$$4.18684 \times 10^{3} \text{ m}^{2} \text{ sec}^{-2}$$

$$4.5057 \times 10^{4} \text{ ft}^{2} \text{ sec}^{-2}$$

[1]The conversions given here are based on the values provided by List [5, p. 17].

[2]The conversions given here are based on the values provided by List [5, p. 14].

$$L_s = 98.777 + [335566.953 - |T - 238|^2]^{\frac{1}{2}} \quad (3)$$

The latent heat associated with the solid-liquid/liquid-solid phase changes is the latent heat of fusion, L_f. It can be approximated from

$$L_f = (1.7757 - 0.0023245T)T - 0.012\,|T-243| - 231.494 \quad (4)$$

The accuracy of L_v, L_s, and L_f is given in Table III.

Saturation Vapor Pressures

In meteorological parlance, saturation vapor pressure refers to the partial pressure of water vapor for saturated conditions over a plane surface of either liquid or solid water. The saturation vapor pressure, expressed in millibars (conversion to other pressure units is provided in Table IV), is a function of temperature only.

The saturation vapor pressure associated with a plane liquid water surface, e_{s_w}, is approximated by[3]

$$e_{s_w} = 10^{(a+b+c+d+e)} \quad (5)$$

where
$$a = 23.83224$$
$$b = -5.02808 \log_3 T$$
$$c = 8.1328 \times 10^{-3} \times 10^{(3.49149 - 1302.8844/T)}$$
$$d = -1.3816 \times 10^{-7} \times 10^{(11.344 - 0.0303998\,T)}$$
$$e = -2949.076/T$$

and the saturation vapor pressure associated with a plane solid water surface, e_{s_i}, is approximated by

[3] Equations 5 and 6 are restatements of those given by List [5, p. 350].

TABLE III[4]

PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR CALCULATION OF LATENT HEATS

Temperature (C)	Percentage Errors in Latent Heats		
	Vaporization	Sublimation	Fusion
60	0.009		
55	0.004		
50	0.000		
45	-0.004		
40	0.009		
35	0.004		
30	0.017		
25	0.030		
20	0.043		
15	0.058		
10	0.051		
5	0.0[illegible]		
0	0.074	0.000	0.063
-10	0.083	0.003	0.108
-20	0.057	-0.005	-0.080
-30	0.000	0.005	-0.246
-40	-0.153	0.005	-0.462
-50	-0.445	-0.005	0.350
-60		0.003	
-70		0.000	
-80		0.001	
-90		0.006	
-100		0.000	

[4]The comparative base for construction of this Table was that provided by List [5, p. 343].

TABLE IV[5]

PRESSURE CONVERSIONS

1.0 millibars (mb) =	10^3 g cm^{-1} sec^{-2}
	1.4504×10^{-2} lb in^{-2}
	2.08854 lb ft^{-2}

TABLE V[6]

PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR CALCULATION OF SATURATION VAPOR PRESSURE AT SELECTED TEMPERATURES

Temperature (C)	Percentage Errors	
	e_{s_w}	e_{s_i}
-100		0.003
-80		-0.015
-60		0.031
-40	0.017	0.008
-20	-0.003	-0.036
0	-0.004	-0.008
20	-0.004	
40	-0.003	
60	-0.004	
80	-0.005	
100	-0.004	

[5] The conversions given here are based on the values provided by List [5, p. 13].

[6] The comparative base for the construction of this table was that provided by List [5, pp. 351-353 and 360-361].

$$e_{s_i} = 10^{(a+b+c+d)} \qquad (6)$$

where $a = 2.0702$
$b = -2484.99/T$
$c = 3.56654 \log T$
and $d = -0.0032098\ T$

The percentage accuracies of the functions of e_{s_w} and e_{s_i} are given in Table V[7].

Dew Point Temperature

The dew point temperature, T_d, is that which unsaturated air, cooled at constant pressure with no change in the water vapor mixing ratio, would have at saturation. The process for approximating this temperature is an iterative one, with the basis of closure being the difference between the actual vapor pressure determined from the dry bulb temperature and relative humidity and the calculated vapor pressure of the dew point temperature. Assuming that the dry bulb temperature and the relative humidity are known, the calculations are accomplished as follows:

1. Select the accuracy desired in the vapor pressure differences. The resulting number of iterations required for various accuracies are provided in Table VI.

2. Calculate the saturation vapor pressure of the dry bulb temperature, e_s, using Eq. 5.

3. Calculate the desired vapor pressure at the dew point temperature, e_a, from the relationship

$$e_a = \frac{e_s\ rh}{100} \qquad (7)$$

4. Make an initial estimate of the dew point temperature. The initial estimate in this report is found from a restatement of an approximation given by Hewson and Longley [6, p. 226]:

[7] Applying the function statements stated here to all the values provided by List [5, pp. 351-353; 360-361] the maximum percentage error found was 0.04 percent.

TABLE VI[8]

PERCENTAGE ERROR OF THE DEW POINT AND WET BULB TEMPERATURE ITERATION TECHNIQUE[9]

* * DEW POINT TEMPERATURE * *					* * WET BULB TEMPERATURE * *				
Average Error	Computational RMS	Number of Iterations Average	Number of Iterations RMS	Preset Allowable Error (vapor pressure)	Average Error	Computational RMS	Number of Iterations Average	Number of Iterations RMS	Pressure (mb)
0.15	0.59	3	1	1.0	-0.04	0.53	6	3	1000
-0.0025	0.053	8	3	0.1	-0.002	0.052	11	4	
-0.00033	0.0052	13	3	0.01	-0.00037	0.0053	16	4	
0.00003	0.00053	18	3	0.001	0.00001	0.00058	21	5	
-0.00000	0.00005	23	3	0.0001	-0.00000	0.00005	26	5	
				1.0	-0.03	0.54	6	3	600
				0.1	-0.0011	0.053	10	4	
				0.01	-0.0002	0.0052	16	4	
				0.001	-0.00001	0.00099	20	4	
				0.0001	-0.00000	0.00006	25	4	

[8] This table was constructed on the basis of 1150 values for each preset allowable percentage error in the final vapor pressure. Temperatures used ranged from -10C to +100C at steps of 5C combined with relative humidities ranging from 1 percent to 99 percent at steps of 2 percent.

[9] Accuracy is expressed in terms of vapor pressure percentage difference between that resulting from the iterative process and that from the relative humidity (see Eqs. (10) and (15)).

$$T_d = \frac{2340}{8.573 - \log(0.16367e_a)} \qquad (8)$$

5. Establish an initial increment for temperature iteration. The initial increment of temperature, ΔT, used in this report is determined from

$$\Delta T = 7.7 - 0.026T_d \qquad (9)$$

with the added condition that $|\Delta T| > 0.1$.

6. Calculate the saturation vapor pressure corresponding to the estimated T_d, $e_s(T_d)$, using Eq. 5 of Table V.

7. Find the error in this vapor pressure from

$$\text{error} = \frac{100\,[e_a - e_s(T_d)]}{e_a} \qquad (10)$$

If this error is smaller than the value specified in Step 1, STOP: the T_d currently being investigated is acceptable as an estimate of the dew point temperature.

8. If the sign of the error has changed from the previous one and the error is larger than the value specified in Step 1, change Δt to $\frac{1}{2}\Delta T$.

9. Adjust the estimated value, T_d, according to

$$T_d = T_d + \frac{e_a - e_s(T_d)}{|e_a - e_s(T_d)|}\,\Delta T \qquad (11)$$

and then RETURN TO STEP 6.

The accuracy of the dew point temperature is dependent on the accuracy of Eq. 5 and the temperature of the dew point. With a closure accuracy requirement of 0.001 percent, the maximum error to be expected would range from 0.005 C at -50 C to 0.002 C at +100 C.

The percentage accuracies, exclusive of consideration of the vapor pressure function contributions, are provided in Table VI.

Wet Bulb Temperature

The wet bulb temperature, T_w, is that which unsaturated air, cooled by the adiabatic evaporation of water at constant pressure with no external source of heat, would have at saturation. Like the dew point temperature of the preceding section, the process for approximating this temperature is an iterative one, with the basis of closure being the difference in vapor pressures. Assuming that the dry bulb temperature and relative humidity are known, the calculations are accomplished as follows:

1. Select the accuracy desired in the vapor pressure differences. The resulting number of iterations required for various accuracies are provided in Table VI.

2. Calculate the saturation vapor pressure of the dry bulb temperature, e_s, using Eq. 5.

3. Calculate the vapor pressure at the dew point temperature, e_a, from the relationship stated in Eq. 7.

4. Make an initial estimate of the wet bulb temperature. The initial estimate in this report is found from a consideration of the dew point temperature, T_d, as estimated in Eq. 8, in the form

$$T_w = \frac{e_a T_a + e_s T_d}{e_a + e_s} \qquad (12)$$

5. Make an initial estimate of the saturation vapor pressure of the dew point temperature, e_w, using the expression from List [5, p. 366] in the form

$$e_w = e_a + 0.00066\, P(1 + 0.0011\, T_w)(T_a - T_w) \qquad (13)$$

6. Establish an initial increment for temperature iteration. The initial increment of temperature, ΔT, used in this report is determined from

$$\Delta T = [e_s(T_w) - e_w]\,(3.3 - 0.0089 T_a) \qquad (14)$$

where $e_s(T_w)$ is found from Eq. 5, with the added condition that $|dT|>0.2$.

7. Calculate the saturation vapor pressure corresponding to the estimated T_w, $e_s(T_w)$, using Eq. 5.

8. Calculate the vapor pressure, e_w, using Eq. 13.

9. Find the error between the two vapor pressures (from Steps 7 and 8) from

$$\text{error} = \frac{[e_s(T_w) - e_w]\ 100}{e_w} \qquad (15)$$

If this error is smaller than the value specified in Step 1, STOP: the T_w currently being investigated is acceptable as an estimate of the wet bulb temperature.

10. If the sign of the error has changed from the previous one and the error is larger than the value specified in Step 1, change ΔT to $\frac{1}{2}\Delta T$.

11. Adjust the estimated value, T_w, according to

$$T_w = T_w + \Delta T \qquad (16)$$

where ΔT is positive if $e_s(T_w) < e_w$ and negative if $e_s(T_w) > e_w$. RETURN TO STEP 7.

As for the dew point temperature, the accuracy of the wet bulb temperature is mainly dependent on the accuracy of Eq. 5 and the temperature of the wet bulb.

Specific Heat of Liquid Water

The amount of heat required to raise one gram of liquid water one degree K is the specific heat of water, c_w. The function for estimating the value of c_w results from a curve fitting of the data given by List [5, p. 343]. The function is

$$c_w = 0.9979 + \frac{0.3021}{2^{(8.5-0.1\,|T-308|)}} \qquad (17)$$

The accuracy of the application of this function with the values from List [5, p. 343] is summarized in Table VII.

METEOROLOGICAL PROCESS PARAMETERS

The meteorological parameters described in this section are associated with meteorological processes. The parameters covered are: (1) the temperature air would have after it has been lifted (dry adiabatically with no change in water vapor mixing ratio) from an initial level until saturation is reached, which corresponds to the temperature of the lifting condensation level (LCL); (2) the time rate of change of the mass of freely falling water drops; and (3) the temperature resulting from moving air from one pressure level to another in a maintained saturated state, which is the psuedo-adiabatic temperature.

Temperature of the LCL

The temperature of the LCL, T_{LCL}, is computed from the dry bulb temperature, T, and the dew point temperature, T_d, of the initial level in a restatement of the formulation provided by Inman [7], namely

$$T_{LCL} = T_d + (0.098 - 0.001571\,T_d + 0.000436\,T_a)\,(T_a - T_d) \qquad (18)$$

TABLE VII

PERCENTAGE ERROR IN THE NUMERICAL APPROXIMATIONS FOR THE CALCULATION OF THE SPECIFIC HEAT OF LIQUID WATER

Temperature (C)	Percentage Error of c_w
-50	0.000
-40	0.785
-30	-0.609
-20	-0.417
-10	-0.316
0	-0.006
5	0.087
10	0.132
15	0.144
20	0.146
25	0.127
30	0.108
35	0.084
40	0.108
45	0.137
50	0.176
55	0.234
60	0.322

The accuracy of the approximation is given in Table VIII, where the comparative base is that provided by Nordquist [8]. These values were computed using an iterative scheme, which had a closure of 0.001 percent in determination of the LCL water vapor mixing ratio.

Change of Mass of Freely Falling Water Drops

The construction of numerical models which contain consideration of liquid water require that some method be established for considering the mechanism of evaporation. The parameterization used here is based on the method proposed by Kessler [9, pp. 29-30] using the data provided by Kinzer and Gunn [10]. The basis of the approximation is that the change in mass due to evaporation can be expressed in the form

$$\frac{dM}{dT} = A\,(e_s - e_a)\,d^B \qquad (19)$$

The two approximations provided here are based on "best" fits of the Kinzer and Gunn data to Eq. (19), one as the equation stands and the other including a temperature dependence. The first approximation is

$$\left.\frac{dM}{dT}\right)_1 = 0.0152\,(e_s - e_a)\,d^{\,1.615} \qquad (20)$$

while the second is

$$\left.\frac{dM}{dT}\right)_2 = \left[1. - 0.025(T_a - 290)\right]\left.\frac{dM}{dT}\right)_1 \qquad (21)$$

The accuracy of these approximations for selected conditions is given in Table IX, with the Kinzer and Gunn data as the comparative base.

TABLE VIII

ERROR (C) OF THE LIFTING CONDENSATION LEVEL TEMPERATURE FUNCTION

Initial Temperature (C)	Initial Relative Humidity (%)				
	90	70	50	30	10
-20	-0.015	-0.015	-0.013	0.008	0.045
0	0.007	0.007	0.005	-0.002	0.009
+20	0.013	0.014	0.011	-0.004	-0.038
+40	0.021	0.029	0.030	0.004	-0.072

TABLE IX[10]

PERCENTAGE ERROR OF THE dM/dt APPROXIMATIONS FOR FREELY FALLING WATER DROPS FOR SELECTED CONDITIONS

Drop Diameter (cm)	Temperature				Relative Humidity (%)
	0 C		40 C		
	$(dM/dt)_1$	$(dM/dt)_2$	$(dM/dt)_1$	$(dM/dt)_2$	
0.02	50	29	-43	40	10
	45	29	-84	38	50
	44	28	-90	36	90
0.08	33	4	-96	17	10
	33	5	-102	15	50
	32	3	-109	12	90
0.20	32	3	-106	13	10
	32	4	-112	11	50
	31	2	-119	8	90
0.30	43	18	-84	22	10
	43	19	-89	20	50
	42	17	-96	17	90
0.36	50	29	-64	31	10
	50	29	-63	29	50
	49	28	-74	27	90

[10] While these percentage errors are quite large, they relate to rather small numbers, and are adequate for those problems where the cloud physics are treated in a gross sense.

Saturation-Adiabatic Temperatures

Air moved adiabatically from one pressure level to another in a saturated state can take on one of two different values, depending on the disposition of the water content of the air. If the total water content remains constant, the process is reversible and the resulting temperature is the saturation-adiabatic temperature, T_s. If the total water content decreases by the amount of liquid water present so that the process is irreversible, the resulting temperature is the psuedo-adiabatic temperature, T_m.

The technique for determining these temperatures requires use of the statement of the First Law of Thermodynamics, based on the definitions provided by Beers [11, p. 359], in the form

$$c_p dT - \frac{RT}{(P_t - e_s)} \quad d(P_t - e_s) \quad + \quad T\, d\left(\frac{L_v X_s}{T}\right) \quad + \quad c_w (X_s + X_w) dT = 0 \qquad (22)$$

where

$$X_s = \frac{\xi\, e_s}{(P_t - e_s)} \qquad (23)$$

and the Clausius-Clapeyron Equation is

$$de_s = \frac{\xi L_v e_s}{RT^2}\, dT \qquad (24)$$

Differentiation of Equations (2) and (23), and substitution along with Equation (24) into Equation (22), yields after rearrangement

$$\frac{dT}{dP} = \frac{RT + L_v X_s}{(P_t - e_s)\left(c_p + c_w(X_s + X_w) - 0.575X_s + \frac{X_s^2 L_v^2 P_t}{RT^2 e_s}\right)} \tag{25}$$

To establish a value for the saturation-adiabatic temperature, it is assumed that no hydrometeors are within the air at the start of the process, so that $(X_s + X_w) = X_s(T_o, P_o) = X_z$. Thus

$$\frac{dT_s}{dP_t} = \frac{RT_s + L_v X_s}{(P_t - e_s)\left(c_p + c_w X_z - 0.575X_s + \frac{X_s^2 L_v^2 P_t}{RT^2 e_s}\right)} \tag{26}$$

For the psuedo-adiabatic temperature, it is assumed that all the liquid water is removed from the air, so that $X_w = 0$. Thus

$$\frac{dT_m}{dP_t} = \frac{RT_m + L_v X_s}{(P_t - e_s)\left(c_p + c_w X_s - 0.575X_s + \frac{X_s^2 L_v^2 P_t}{RT^2 e_s}\right)} \tag{27}$$

For many meteorological problems it is sufficient to assume that the partial pressure exerted by the air is much larger than that exerted by the water vapor included in the air, that the relative mass of the water vapor is small compared to the mass of the air, and that the latent heat of vaporization is constant. With these assumptions, Eq. (22) can be expressed as

$$c_p dT - \frac{RT}{P_t} dP_t + L_v dX_s = 0 \tag{28}$$

and Eq. (23) can be expressed as

$$x_s = \frac{\xi e_s}{P_t} \tag{29}$$

Here T_x is used to designate this meteorological approximation to the psuedo-adiabatic temperature

$$\frac{dT_x}{dP_t} = \frac{RT_x + L_e x_s}{P_t\left(c_p + \frac{x_s^2 L_v^2 P_t}{RT^2 e_s}\right)} \tag{30}$$

where for each Equation (26), (27), and (30), e_s and L_v are assumed to be functions of T_s, T_m, or T_x, as appropriate.

The solution of these equations is sought by using a second-order predictor-corrector method [12, pp. 385-386][11], where the initial step is accomplished using

$$T_1 = T_o + \left.\frac{dT}{dP}\right)_o \Delta P \tag{31}$$

Thereafter, a first approximation "predictor" is made using

$$T_{i+1} = T_{i-1} + 2\left.\frac{dT}{dP}\right)_i \Delta P \tag{32}$$

and the "corrector" is

[11] A systems sub-program for the Univac 1108 computer is available for accomplishing this integration in a more general manner [18, sec. 11, p.1].

$$T^{n}_{i+1} = T_i + \frac{1}{2}\left[\left.\frac{dT}{dP}\right|_i + \left.\frac{dT}{dP}\right|_{i+1}\right]\Delta P \qquad (33)$$

The "corrector" is repeated until

$$\frac{(T^{n}_{i+1} - T^{n-1}_{i+1})}{T^{n}_{i+1}} \leq \text{specified closure} \qquad (34)$$

To provide some indication of the validity of this integration technique, the expression

$$\frac{dx}{dy} = 0.5\, y^{-0.5} + 1 \qquad (35)$$

was integrated from y=1 to y=30, with a Δy of 0.01 and a closure requirement of 10^{-3}. The results are given in Table X. While an accuracy analysis of the saturation- and psuedo-adiabatic temperatures is not possible due to a lack of a suitable comparative base, it is possible to compare the results with those obtained by another approximation method [5, p. 318]. The comparisons are given in Table XI and include the values provided by List [5, pp. 320-321] for psuedo-adiabatic temperatures with initial conditions of 1096.7 mb and 14C, program requirements of Δp = 2 mb, and a closure requirement of 10^{-3}.

TABLE X

COMPARISON OF THE RESULTS OF THE NUMERICAL METHOD FOR SOLVING A DIFFERENTIAL EQUATION APPLIED TO $dx/dy = \frac{1}{2} y^{-\frac{1}{2}} + 1$

Value of y	Initial Value of y	Value of x		Difference of Values of X
		Integration	$x = y^{\frac{1}{2}} + y$	Percentage
2	1	3.41421	3.41421	-0.00004
4	1	6.00000	6.00000	-0.00006
6	1	8.44949	8.44949	-0.00006
8	1	10.82843	10.82843	-0.00006
16	1	20.00000	20.00001	-0.00005
25	1	30.00000	30.00001	-0.00004
30	1	35.47723	35.47723	-0.00002

TABLE XI

COMPARISONS OF SATURATION-ADIABATIC TEMPERATURES (C)

PRESSURE	PSUEDO-ADIABATIC		SATURATED-ADIABATIC	
	LIST	CALCULATED	CALCULATED	MET APPROX
1096.7	14.0	14.0	14.0	14.0
1046.7	12.0	12.0	12.0	12.0
1000.0	10.0	10.1	10.0	10.0
957.0	8.0	8.1	8.1	8.1
915.0	6.0	6.1	6.1	6.1
876.0	4.0	4.2	4.1	4.1
840.0	2.0	2.2	2.2	2.1
805.0	.0	.2	.1	.1
773.0	-2.0	-1.7	-1.8	-1.9
744.0	-4.0	-3.6	-3.7	-3.8
715.0	-6.0	-5.6	-5.7	-5.8
688.0	-8.0	-7.5	-7.7	-7.8
663.0	-10.0	-9.4	-9.6	-9.7
638.0	-12.0	-11.4	-11.7	-11.8
616.0	-14.0	-13.3	-13.6	-13.7
595.0	-16.0	-15.2	-15.5	-15.6
573.0	-18.0	-17.2	-17.6	-17.8
554.0	-20.0	-19.1	-19.6	-19.7
536.0	-22.0	-20.9	-21.5	-21.6
518.0	-24.0	-22.9	-23.5	-23.6
502.0	-26.0	-24.7	-25.3	-25.5
485.0	-28.0	-26.7	-27.4	-27.6
470.0	-30.0	-28.5	-29.3	-29.5
454.0	-32.0	-30.6	-31.4	-31.6
440.0	-34.0	-32.4	-33.4	-33.5
426.0	-36.0	-34.4	-35.4	-35.5
413.0	-38.0	-36.2	-37.3	-37.5
400.0	-40.0	-38.1	-39.3	-39.4
387.5	-42.0	-40.0	-41.3	-41.4

TABLE XI (con)

COMPARISONS OF SATURATION-ADIABATIC TEMPERATURES (C)

PRESSURE	PSUEDO-ADIABATIC		SATURATED-ADIABATIC	
	LIST	CALCULATED	CALCULATED	MET APPROX
375.3	-44.0	-42.0	-43.3	-43.4
363.5	-46.0	-43.9	-45.3	-45.4
352.1	-48.0	-45.8	-47.2	-47.4
341.0	-50.0	-47.7	-49.2	-49.4
330.2	-52.0	-49.6	-51.2	-51.4
319.7	-54.0	-51.5	-53.2	-53.4
309.5	-56.0	-53.4	-55.2	-55.4
299.6	-58.0	-55.3	-57.2	-57.3
289.9	-60.0	-57.2	-59.2	-59.3
280.5	-62.0	-59.1	-61.2	-61.3
271.3	-64.0	-61.0	-63.1	-63.3
262.3	-66.0	-62.9	-65.1	-65.3
253.6	-68.0	-64.7	-67.1	-67.3
245.1	-70.0	-66.6	-69.1	-69.3
236.8	-72.0	-68.5	-71.1	-71.3
228.7	-74.0	-70.3	-73.1	-73.2
220.8	-76.0	-72.2	-75.1	-75.2
213.2	-78.0	-74.0	-77.1	-77.2
205.7	-80.0	-75.8	-79.1	-79.2
198.4	-82.0	-77.7	-81.1	-81.2
191.3	-84.0	-79.5	-83.0	-83.2
184.3	-86.0	-81.3	-85.0	-85.2
177.6	-88.0	-83.0	-87.0	-87.2
171.0	-90.0	-84.8	-89.0	-89.2
164.6	-92.0	-86.5	-91.0	-91.2
158.4	-94.0	-88.3	-93.0	-93.2
152.4	-95.0	-89.9	-95.0	-95.1

CONCLUDING REMARKS

This report has presented numerical techniques for approximating the values of several meteorological parameters along with the accuracies of the results of the approximations. This has been done so that (1) the methods might be documented for use by others, (2) the representativeness of the results of applying the techniques might be made available for analysis for other purposes, and (3) the comparison of these techniques with others can be readily accomplished.

LITERATURE CITED

1. Nordquist, W. S., and N. L. Johnson, 1970, "One-Dimensional Quasi-Time-Dependent Numerical Model of Cumulus Cloud Activity," ECOM-5350, US Army Electronics Command, Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, 163 pp. (AD 722 216)

2. Doswell, C. A., 1972, "A Two-Dimensional Short-Range Fog Forecast Model," ECOM-5443, US Army Electronics Command, Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, 65 pp.

3. Doswell, C. A., 1972, "An Iterative Method for Saturation Adjustment," ECOM-5444, US Army Electronics Command, Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, 16 pp.

4. Nordquist, W. S., "Fog Clearing Using Helicopter Downdraft: A Numerical Model," in preparation.

5. List, R. J., ed., 1958, Smithsonian Meteorological Tables, Sixth Revised Edition, Washington, DC, Smithsonian Institution.

6. Hewson, E. W., and R. W. Longley, 1954, Meteorology: Theoretical and Applied, John Wiley and Sons, Inc., New York.

7. Inman, R. L., 1969, "Computation of Temperature at the Lifted Condensation Level," Journal of Applied Meteorology, 8, 155-158.

8. Nordquist, W. S., 1969, "Determination of the Temperature and Pressure of the Lifting Condensation Level," ECOM-5282, US Army Electronics Command, Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, 18 pp. (AD 700 953)

9. Kessler, E., 1969, "On the Distribution and Continuity of Water Substance in Atmospheric Circulations," Meteorological Monographs, 10(32), 29-30 (Boston, American Meteorological Society).

10. Kinzer, G. D., and R. Gunn, 1951, "The Evaporation, Temperature and Thermal Relaxation-Time of Freely Falling Waterdrops," Journal of Meteorology, 8, 71-83.

11. Beers, N. R., 1945, "Meteorological Thermodynamics and Atmospheric Statics," Handbook of Meteorology, F. A. Berry, E. Bollay, and N. R. Beers, ed., Mc-Graw-Hill Book Company, Inc., New York.

12. Ritger, P. D., and N. J. Rose, 1968, Differential Equations with Applications, McGraw-Hill Book Company, Inc., New York.

13. Anonymous, 1970, Large Scale Systems MATH-PACK Abstracts, Sperry Rand Corp., UNIVAC Division.

APPENDIX

COMPUTER LANGUAGE LISTINGS OF FUNCTIONS AND SUBROUTINES

NUMERICAL APPROXIMATIONS

+++ FUNCTION STATEMENTS +++

TX = DRY BULB TEMPERATURE (K)
TZ = DEW POINT TEMPERATURE(K)
ES SATURATION VAPOR PRESSURE (MB)
EA ACTUAL VAPOR PRESSURE (MB)
DIA WATER DROP DIAMETER (M)

```
      .....LATENT HEAT OF VAPORIZATION.....

HEATV(TX)= 754.817 - 0.575*TX                                          1

      .....LATENT HEAT OF SUBLIMATION.....

HEATS(TX)= 98.777 +SQRT(335566.953 - ABS(TX - 238.)**2.)               2

      .....LATENT HEAT OF FUSION.....

HEATF(TX)=TX*(1.7757-0.0023245*TW)-0.012*ABS(TX-243.)-231.494          3

      .....SATURATION VAPOR PRESSURE OVER WATER.....

SATVP(TX)=10.**(23.83224-2949.076/TX -5.02808*ALOG10(TX)               4
1     -1.3816E-7*10.**(11.3440 - 0.0303998*TX)                         5
2     +8.1328E-3*10.**(3.49149 - 1302.8844/TX))                        6

      .....SATURATION VAPOR PRESSURE OVER ICE.....

SATVI(TX)=10.**(2.0702-2484.991/TX+3.56654*ALOG10(TX)-.0032098*TX)     7
```

NUMERICAL APPROXIMATIONS

```
.....TEMPERATURE OF THE LIFTING CONDENSATION LEVEL.....
TLCL(TX,TZ)=TZ+(0.098-0.001571*TZ+0.000436*TX)*(TX-TZ)                    8
.....SPECIFIC HEAT OF LIQUID WATER.....
SPH2O(TX)=0.9979+0.3021/(2.**(8.5-0.1*ABS(TX-308.16)))                   9
.....DM/DT FOR FREELY FALLING WATER DROPS.....

                    1.  TEMPERATURE INDEPENDENT
DMDT(ES,EA,DIA)=0.0152*(ES-EA)*DIA**1.615                                10

                    2.  TEMPERATURE DEPENDENT
DMDTT(ES,EA,DIA,TX)=DMDT(ES,EA,DIA)*(1.-0.025*(TX-290.))                 11
```

```
      SUBROUTINE DEWPT(T,RH,ET,TD)
                    ESTIMATING DEW POINT TEMPERATURES
              E          ACTUAL VAPOR PRESSURE -ESTIMATE          MB
              EA         ACTUAL VAPOR PRESSURE                    MB
              ES         SATURATION VAPOR PRESSURE                MB
              ET         ALLOWABLE PERCENTAGE ERROR          PERCENT
              DEL        TEMPERATURE INCREMENT                     K
              RH         RELATIVE HUMIDITY                   PERCENT
              T          TEMPERATURE - DRY BULB                    K
              TD         TEMPERATURE - DEW POINT                   K

      SATVP(TX)=10.**(23.83224 -2949.076/TX
     1                -5.02808*ALOG10(TX)
     2                -1.3816E-7*10.**(11.344-.0303998*TX)
     3                +8.1328E-3*10.**(3.49149-1302.8844/TX))

      ES=SATVP(T)
      EA=RH*ES/100.
      C =0
      TD=2340./(8.573-ALOG10(0.16367*EA))
       IF (RH.GE.100.) TD=T
      DEL=7.7-.026*TD
      IF(ABS(DEL).LT.0.1) DEL=0.1
    1 E=SATVP(TD)
      D=(E-EA)*100./EA
      IF(ABS(D).LT.ET)  RETURN
      G=C*D
      IF(G.LT.0.) DEL=DEL*0.5
        R=100.*E/ES
      TD=TD+(RH-R)*ABS(DEL/(RH-R))
      C=D
      GO TO 1
      END
```

```
      SUBROUTINE WETBLB(T,P,RH,ET,TW)
```

ESTIMATING WET BULB TEMPERATURE

DEL	TEMPERATURE INCREMENT	K
E	ACTUAL VAPOR PRESSURE -ESTIMATE	MB
EA	ACTUAL VAPOR PRESSURE -WET BULB	MB
EB	ACTUAL VAPOR PRESSURE -DEW POINT	MB
ES	SATURATION VAPOR PRESSURE	MB
ET	ALLOWABLE PERCENTAGE ERROR	PERCENT
P	PRESSURE	MB
RH	RELATIVE HUMIDITY	PERCENT
T	TEMPERATURE - DRY BULB	K
TW	TEMPERATURE - WET BULB	K

```
      SATVP(TX)=10.**(23.83224 -2949.076/TX
     1                -5.02808*ALOG10(TX)
     2                -1.3816E-7*10.**(11.344-.0303998*TX)
     3                +8.1328E-3*10.**(3.49149-1302.8844/TX))
      ALIST(PX,DX,TX)=0.00066*(1.+.00115*TX)*PX*DX

      ES=SATVP(T)
      EA=RH*EX/100.
      EB=EA
      TW=2340./(8.573-ALOG10(0.16367*EA))
      TW=(TW*ES+T*EA)/(EA+ES)
       IF(RH.GE.100)  TW=T
      DE=T-TW
      EK=EA+ALIST(P,DE,TW)
      DEL=(SATVP(TW)-EK)*(3.3-.009*T)
      IF(ABS(DEL).LT.0.2) DEL=0.2
    1 E=SATVP(TW)
      DE=T-TW
      EA=EB+ALIST(P,DE,TW)
      D=(E-EA)*100./EA
```

```
      IF(ABS(D).LT.ET)     RETURN
      G=C*D
      DEL=ABS(DEL)
      IF(G.LT.0.) DEL=DEL*.5
      IF(EA.LT.E) DEL=DEL
      TW=TW+DEL
      C=D
      GO TO 1
      END
```

NUMERICAL APPROXIMATIONS

```
      SUBROUTINE ADIAB(T1,P1,P2,L,TSAT)
                                                                        63
          TO CALCULATE SATURATION-ADIABATIC TEMPERATURE FROM
              TEMPERATURE T1 AT PRESSURE P1 TO PRESSURE P2
              FOR L=1  PSUEDO-ADIABATIC PROCESS              .
              .     2  SATURATION ADIABATIC PROCESS          .
              .     3  MET APPROXIMATION                     .


                    REFERENCE INTEGRATION TECHNIQUE
                            RITGER AND ROSE
                 DIFFERENTIAL EQUATIONS WITH APPLICATIONS
                            (MCGRAW-HILL,1968)
                                PP. 385-386


                    .....FUNCTION REQUIREMENTS.....
        EXTERNAL DTDP
      SATVP(TX)=10.**(23.83224-5.02808*ALOG10(TX)-1.3816E-7*10.**(11.344   64
     1-.0303998*TX)+8.1328E-3*10.**(3.49149-1302.8844/TX)-2949.076/TX)     65
                                                                           66
                    .....STORAGE REQUIREMENTS.....
      COMMON/AA/XA1
      DIMENSION TX(3),PX(3),TZ(20)                                         67
                                                                           68
                                INITIALIZE
        ERRORT=1.E-5
      I=1                                                                  69
      IX=1                                                                 70
      KI=1                                                                 71
                                                                           72
                            INITIAL CONDITIONS
      XA1=SATVP(T1)
      XA1=.62197*XA1/(P1-XA1)                                              73
      PX(1)=P1                                                             74
      TX(1)=T1                                                             75
      DELDP=P2-P1                                                          76
                                                                           77
                        NEW INITIAL CONDITIONS
```

```
      DP= ABS(DELDP)*1.E-6/DELDP
      PX(2)=PX(1)+DP
       TX(2)=TX(1)+DP*DTDP(TX(1),PX(1),L)
      PXREF=PX(2)
       IP=ABS(DELDP)*0.5
       IF(IP.LT.1)  IP=1
                        SUCCEEDING CONDITIONS
    1 DO 5 K=IX,IP
      DP=IP
      DP=(P2  -PXREF)/DP
      IF(ABS(DP).LT.1.E-12)  GO TO 7.
      PX(I+2)=PX(I+1)+DP
       DF=DTDP(TX(I+1),PX(I+1),L)
      TX(I+2)=TX(I)+2.*DP*DF
      TZ(1)=TX(I+2)
      LL=L
                              CLOSURE ITERATIONS
      DO 2 M=2,20
       KI=KI+1
      MM=M
       DTDP2=DTDP(TZ(M-1),PX(I+2),L)
      TZ(M)=TX(I+1)+0.5*DP*(DF+DTDP2)
      TZR=ABS((TZ(M)-TZ(M-1))/TZ(M))
      IF(TZR.LT.ERRORT)  GO TO 3
    2 CONTINUE
      WRITE(6,998)  TX(I+1),TZ(M-1),TZ(M),TZR,I,K,M
  998 FORMAT(1H 4F10.5,3I6)
                            DID NOT CONVERGE HALVE DP
      IX=LL
      IP=IP*2
      GO TO 6
    3 TX(I+2)=TZ(MM)
      DO  4  N=2,3
      TX(N-1)=TX(N)
    4 PX(N-1)=PX(N)
```

```
  5  CONTINUE
     TSAT=TX(I+2)
      WRITE(6,999) KI
999  FORMAT(1H+90XI9)
     RETURN
  6  GO TO 1
  7  WRITE(6,8) PX(I+2),TX(I+2),KI,DP,T1,P1,P2

  8  FORMAT(1H /29H THIS DID NOT CONVERGE AT P =F8.1,8H AND T =F9.2/1X
 1     3HON I5,24H ITERATIONS OF DELTA P =1PE12.5/1X
 2  13HGOING FROM T=OPF8.1, 3H P=F8.1,  6H TO P=F8.1)

    RETURN
    END
```

NUMERICAL APPROXIMATIONS

```
      FUNCTION DTDP(T,P,IX)
                                                                          122
 FUNCTION FOR THE CALCULATION OF THE CHANGE OF SATURATED-ADIABATIC
   TEMPERATURE WITH PRESSURE UNDER THREE BASIC SETS OF CONDITIONS

                    .....STORAGE REQUIREMENTS.....
      COMMON/AA/XI
                                                                          123
                              FUNCTION REQUIREMENTS
      HEATV(TX)=754.817-.575*TX
      SATVP(TX)=10.**(23.83224-5.02808*ALOG10(TX)-1.3816E-7*10.**(11.344   124
     1-.0303998*TX)+8.1328E-3*10.**(3.49149-1302.8844/TX)-2949.076/TX)     125
                                                                          126

      A=.068557*T
      E=SATVP(T)                                                          127
      H=HEATV(T)                                                          128
      XA=XI                                                               129
      CX=-.575                                                            130
      TXX=ABS(T-380.16)                                                   131
       CW=0.9979+0.3021/(2.**(8.5-0.1*TXX))                               132
      PB=P-E                                                              133
      IF(IX.EQ.3) PB=P                                                    134
      X=0.62197*E/PB                                                      135
      CP=.2399+.4409*X                                                    136
      IF(IX.EQ.2)  XA=X                                                   137
      IF(IX.EQ.3)  XA=0.                                                  138
      IF(IX.EQ.3)CX=0.                                                    139
      AI=(A+H*X)/PB                                                       140
      BL=X*X*H*H*P/(A*T*E)                                                141
                                                                          142
      DTDP=AI/(CP+CW*XA+CX*X+BI)
                                                                          143
      RETURN
      END                                                                 144
                                                                          145
```

ATMOSPHERIC SCIENCES RESEARCH PAPERS

1. Miers, B. T., and J. E. Morris, Mesospheric Winds Over Ascension Island in January, July 1970, ECOM-5312, AD 711851.
2. Webb, W. L., Electrical Structure of the D- and E-Region, July 1970, ECOM-5313, AD 714365.
3. Campbell, G. S., F. V. Hansen and R. A. Dise, Turbulence Data Derived from Measurements on the 32-Meter Tower Facility, White Sands Missile Range, New Mexico, July 1970, ECOM-5314, AD 711852.
4. Pries, T. H., Strong Surface Wind Gusts at Holloman AFB (March-May), July 1970, ECOM-5315, AD 711853.
5. D'Arcy, E. M., and B. F. Engebos, Wind Effects on Unguided Rockets Fired Near Maximum Range, July 1970, ECOM-5317, AD 711854.
6. Matonis, K., Evaluation of Tower Antenna Pedestal for Weather Radar Set AN/TPS-41, July 1970, ECOM-3317, AD 711520.
7. Monahan, H. H., and M. Armendariz, Gust Factor Variations with Height and Atmospheric Stability, August 1970, ECOM-5320, AD 711855.
8. Stenmark, E. B., and L. D. Drury, Micrometeorological Field Data from Davis, California; 1966-67 Runs Under Non-Advection Conditions, August 1970, ECOM-6051, AD 726390.
9. Stenmark, E. B., and L. D. Drury, Micrometeorological Field Data from Davis, California; 1966-67 Runs Under Advection Conditions, August 1970, ECOM-6052, AD 724612.
10. Stenmark, E. B., and L. D. Drury, Micrometeorological Field Data from Davis, California; 1967 Cooperative Field Experiment Runs, August 1970, ECOM-6053, AD 724613.
11. Rider, L. J., and M. Armendariz, Nocturnal Maximum Winds in the Planetary Boundary Layer at WSMR, August 1970, ECOM-5321, AD 712325.
12. Hansen, F. V., A Technique for Determining Vertical Gradients of Wind and Temperature for the Surface Boundary Layer, August 1970, ECOM-5324, AD 714366.
13. Hansen, F. V., An Examination of the Exponential Power Law in the Surface Boundary Layer, September 1970, ECOM-5326, AD 715349.
14. Miller, W. B., A. J. Blanco and L. E. Traylor, Impact Deflection Estimators from Single Wind Measurements, September 1970, ECOM-5328, AD 716993.
15. Duncan, L. D., and R. K. Walters, Editing Radiosonde Angular Data, September 1970, ECOM-5330, AD 715351.
16. Duncan, L. D., and W. J. Vechione, Vacuum Tube Launchers and Boosters, September 1970, ECOM-5331, AD 715350.
17. Stenmark, E. B., A Computer Method for Retrieving Information on Articles, Reports and Presentations, September 1970, ECOM-6050, AD 724611.
18. Hudlow, M., Weather Radar Investigation on the BOMEX, September 1970, ECOM-3329, AD 714191.
19. Combs, A., Analysis of Low-Level Winds Over Vietnam, September 1970, ECOM-3346, AD 876935.
20. Rinehart, G. S., Humidity Generating Apparatus and Microscope Chamber for Use with Flowing Gas Atmospheres, October 1970, ECOM-5332, AD 716994.
21. Miers, B. T., R. O. Olsen, and E. P. Avara, Short Time Period Atmospheric Density Variations and a Determination of Density Errors from Selected Rocketsonde Sensors, October 1970, ECOM-5335.
22. Rinehart, G. S., Sulfates and Other Water Solubles Larger than 0.15μ Radius in a Continental Nonurban Atmosphere, October 1970, ECOM-5336, AD 716999.
23. Lindberg, J. D., The Uncertainty Principle: A Limitation on Meteor Trail Radar Wind Measurements, October 1970, ECOM-5337, AD 716996.
24. Randhawa, J. S., Technical Data Package for Rocket-Borne Ozone-Temperature Sensor, October 1970, ECOM-5338, AD 716997.

25. Devine, J. C., The Fort Huachuca Climate Calendar, October 1970, ECOM-6054.
26. Allen, J. T., Meteorological Support to US Army RDT&E Activities, Fiscal Year 1970 Annual Report, November 1970, ECOM-6055.
27. Shinn, J. H., An Introduction to the Hyperbolic Diffusion Equation, November 1970, ECOM-5341, AD 718616.
28. Avara, E. P., and M. Kays., Some Aspects of the Harmonic Analysis of Irregularly Spaced Data, November 1970, ECOM-5344, AD 720198.
29. Fabrici, J., Inv. of Isotopic Emitter for Nuclear Barometer, November 1970, ECOM-3349, AD 876461.
30. Levine, J. R., Summer Mesoscale Wind Study in the Republic of Vietnam, December 1970, ECOM-3375, AD 721585.
31. Petriw, A., Directional Ion Anemometer, December 1970, ECOM-3379, AD 720573.
32. Randhawa, J. S., B. H. Williams, and M. D. Kays, Meteorological Influence of a Solar Eclipse on the Stratosphere, December 1970, ECOM-5345, AD 720199.
33. Nordquist, Walter S., Jr., and N. L. Johnson, One-Dimensional Quasi-Time-Dependent Numerical Model of Cumulus Cloud Activity, December 1970, ECOM-5350, AD 722216.
34. Avara, E. P., The Analysis of Variance of Time Series Data Part I: One-Way Layout, January 1971, ECOM-5352, AD 721594.
35. Avara, E. P., The Analysis of Variance of Time Series Data Part II: Two-Way Layout, January 1971, ECOM-5353.
36. Avara, E. P., and M. Kays., The Effect of Interpolation of Data Upon the Harmonic Coefficients, January 1971, ECOM-5354, AD 721593.
37. Randhawa, J. S., Stratopause Diurnal Ozone Variation, January 1971, ECOM-5355, AD 721309.
38. Low, R. D. H., A Comprehensive Report on Nineteen Condensation Nuclei (Part II), January 1971, ECOM-5358.
39. Armendariz, M., L. J. Rider, G. Campbell, D. Favier and J. Serna, Turbulence Measurements from a T-Array of Sensors, February 1971, ECOM-5362, AD 726390.
40. Maynard, H., A Radix-2 Fourier Transform Program, February 1971, ECOM-5363, AD 726389.
41. Devine, J. C., Snowfalls at Fort Huachuca, Arizona, February 1971, ECOM-6056.
42. Devine, J. C., The Fort Huachuca, Arizona 15 Year Base Climate Calendar (1956-1970), February 1971, ECOM-6057.
43. Levine, J. R., Reduced Ceilings and Visibilities in Korea and Southeast Asia, March 1971, ECOM-3403, AD 722735.
44. Gerber, H., et al., Some Size Distribution Measurements of AgI Nuclei with an Aerosol Spectrometer, March 1971, ECOM-3414, AD 729331.
45. Engebos, B. F., and L. J. Rider, Vertical Wind Effects on the 2.75-inch Rocket, March 1971, ECOM-5365, AD 726321.
46. Rinehart, G. S., Evidence for Sulfate as a Major Condensation Nucleus Constituent in Nonurban Fog, March 1971, ECOM-5366.
47. Kennedy, B. W., E. P. Avara, and B. T. Miers, Data Reduction Program for Rocketsonde Temperatures, March 1971, ECOM-5367.
48. Hatch, W. H., A Study of Cloud Dynamics Utilizing Stereoscopic Photogrammetry, March 1971, ECOM-5368.
49. Williamson, L. E., Project Gun Probe Captive Impact Test Range, March 1971, ECOM-5369.
50. Henley, D. C., and G. B. Hoidale, Attenuation and Dispersion of Acoustic Energy by Atmospheric Dust, March 1971, ECOM-5370, AD 728103.
51. Cionco, R. M., Application of the Ideal Canopy Flow Concept to Natural and Artificial Roughness Elements, April 1971, ECOM-5372, AD 730638.
52. Randhawa, J. S., The Vertical Distribution of Ozone Near the Equator, April 1971, ECOM 5373
53. Ethridge, G. A., A Method for Evaluating Model Parameters by Numerical Inversion, April 1971, ECOM 5374.

54. Collett, E., Stokes Parameters for Quantum Systems, April 1971, ECOM-3415, AD 729347.

55. Shinn, J. H., Steady-State Two-Dimensional Air Flow in Forests and the Disturbance of Surface Layer Flow by a Forest Wall, May 1971, ECOM-5383, AD 730681.

56. Miller, W. B., On Approximation of Mean and Variance-Covariance Matrices of Transformations of Joint Random Variables, May 1971, ECOM-5384, AD 730302.

57. Duncan, L. D., A Statistical Model for Estimation of Variability Variances from Noisy Data, May 1971, ECOM-5385.

58. Pries, T. H., and G. S. Campbell, Spectral Analyses of High-Frequency Atmospheric Temperature Fluctuations, May 1971, ECOM-5387.

59. Miller, W. B., A. J. Blanco, and L. E. Traylor, A Least-Squares Weighted-Layer Technique for Prediction of Upper Wind Effects on Unguided Rockets, June 1971, ECOM-5388, AD 729792.

60. Rubio, R., J. Smith and D. Maxwell, A Capacitance Electron Density Probe, June 1971, ECOM-5390.

61. Duncan, L. D., Redundant Measurements in Atmospheric Variability Experiments, June 1971, ECOM-5391.

62. Engebos, B. F., Comparisons of Coordinate Systems and Transformations for Trajectory Simulations, July 1971, ECOM-5397.

63. Hudlow, M. D., Weather Radar Investigations on an Artillery Test Conducted in the Panama Canal Zone, July 1971, ECOM-5411.

64. White, K. O., E. H. Holt, S. A. Schleusener, and R. F. Calfee, Erbium Laser Propagation in Simulated Atmospheres II. High Resolution Measurement Method, August 1971, ECOM-5398.

65. Waite, R., Field Comparison Between Sling Psychrometer and Meteorological Measuring Set AN/TMQ-22, August 1971, ECOM-5399.

66. Duncan, L. D., Time Series Editing By Generalized Differences, August 1971, ECOM-5400.

67. Reynolds, R. D., Ozone: A Synopsis of its Measurements and Use as an Aemospheric Tracer, August 1971, ECOM-5401.

68. Avara, E. P., and B. T. Miers, Noise Characteristics of Selected Wind and Temperature Data from 30-65 km, August 1971, ECOM-5402.

69. Avara, E. P., and B. T. Miers, Comparison of Linear Trends in Time Series Data Using Regression Analysis, August 1971, ECOM-5403.

70. Miller, W. B., Contributions of Mathematical Structure to the Error Behavior of Rawinsonde Measurements, August 1971, ECOM-5404.

71. Collett, E., Mueiler Stokes Matrix Formulation of Fresnel's Equations, August 1971, ECOM-3480.

72. Armendariz, M., and L. J. Rider, Time and Space Correlation and Coherence in the Surface Boundary Layer, September 1971, ECOM-5407.

73. Avara, E. P., Some Effects of Randomization in Hypothesis Testing with Correlated Data, October 1971, ECOM-5408.

74. Randhawa, J. S., Ozone and Temperature Change in the Winter Stratosphere, November 1971, ECOM-5414.

75. Miller, W. B., On Approximation of Mean and Variance-Covariance Matrices of Transformations of Multivariate Random Variables, November 1971, ECOM-5413.

76. Horn, J. D., G. S. Campbell, A. L. Wallis (Capt., USAF), and R. G. McIntyre, Wind Tunnel Simulation and Prototype Studies of Barrier Flow Phenomena, December 1971, ECOM-5416.

77. Dickson, David H., and James R. Oden, Fog Dissipation Techniques for Emergency Use, January 1972, ECOM-5420.

78. Ballard, H. N., N. J. Beyers, B. T. Miers, M. Izquierdo, and J. Whitacre, Atmospheric Tidal Measurements at 50 km from a Constant-Altitude Balloon, December 1971, ECOM-5417.

79. Miller, Walter B., On Calculation of Dynamic Error Parameters for the Rawinsonde and Related Systems, January 1972, ECOM-5422.

80. Richter, Thomas J., Rawin Radar Targets, February 1972, ECOM-5424.

81. Pena, Ricardo, L. J. Rider, and Manuel Armendariz, Turbulence Characteristics at Heights of 1.5, 4.0, and 16.0 Meters at White Sands Missile Range, New Mexico, January 1972, ECOM-5421.

82. Blanco, Abel J., and L. E. Traylor, Statistical Prediction of Impact Displacement due to the Wind Effect on an Unguided Artillery Rocket During Powered Flight, March 1972, ECOM-5427.

83. Williams, B. H., R. O. Olsen, and M. D. Kays, Stratospheric-Ionospheric Interaction During the Movement of a Planetary Wave in January 1967, March 1972, ECOM-5428.

84. Schleusener, Stuart A., and Kenneth O. White, Applications of Dual Parameter Analyzers in Solid-State Laser Tests, April 1972, ECOM-5432.

85. Pries, Thomas H., Jack Smith, and Marvin Hamiter, Some Observations of Meteorological Effects on Optical Wave Propagation, April 1972, ECOM-5434.

86. Dickson, D. H., Fogwash I An Experiment Using Helicopter Downwash, April 1972, ECOM-5431.

87. Mason, J. B., and J. D. Lindberg, Laser Beam Behavior on a Long High Path, April 1972, ECOM-5430.

88. Smith, Jack, Thomas H. Pries, Kenneth J. Skipka, and Marvin Hamiter, Optical Filter Function for a Folded Laser Path, April 1972, ECOM-5433.

89. Lee, Robert P., Artillery Sound Ranging Computer Simulations, May 1972, ECOM-5441.

90. Lowenthal, Marvin J., The Accuracy of Ballistic Density Departure Tables 1934-1972, April 1972, ECOM-5436.

91. Cantor, Israel, Survey of Studies of Atmospheric Transmission from a 4π Light Source to a 2π Receiver, April 1972, ECOM-5435.

92. Barr, William C., Accuracy Requirements for the Measurement of Meteorological Parameters Which Affect Artillery Fire, April 1972, ECOM-5437.

93. Duchon, C. E., F. V. Brock, M. Armendariz, and J. D. Horn, UVW Anemometer Dynamic Performance Study, May 1972, ECOM-5440.

94. Gomez, R. B., Atmospheric Effects for Ground Target Signature Modeling I. Atmospheric Transmission at 1.06 Micrometers, June 1972, ECOM-5445.

95. Bonner, R. S., A Technical Manual on the Characteristics and Operation of a Cloud Condensation Nuclei Collection/Detection/Recording Instrument, June 1972, ECOM-5447.

96. Horn, J. D., R. D. Reynolds, and T. H. Vonder Haar, Survey of Techniques Used in Display of Sequential Images Received from Geostationary Satellites, June 1972, ECOM-5450.

97. Bonner, R. S., and H. M. White, Microphysical Observations of Fog in Redwood Valley near Arcata-Eureka, California, July 1972, ECOM-5455.

98. Waite, R. W., Reliability Test of Electronics Module of Meteorological Measuring Set AN/TMQ-22(XE-4), June 1972, ECOM-5448.

99. Doswell, C. A., III, An Iterative Method for Saturation Adjustment, June 1972, ECOM-5444.

100. Doswell, C. A., III, A Two-Dimensional Short-Range Fog Forecast Model, May 1972, ECOM-5443.

101. Seagraves, Mary Ann B., A General-Purpose Meteorological Rocket Data Reduction Program, August 1972, ECOM-5463.

102. Loveland. Loveland, R. B., J. L. Johnson, and B. D. Hinds, Differential Magnetic Measurements Near Cumulus Clouds, August 1972, ECOM-5463.

103. Cantor, Israel, and Michael Hudlow, Rainfall Effects on Satellite Communications in the K, X, and C Bands, July 1972, ECOM-5459.

104. Randhawa, J. S., Variations in Stratospheric Circulation and Ozone During Selected Periods, August 1972, ECOM-5460.

105. Rider, L. J., Armendariz, Manuel, Mean Horizontal Wind Speed and Direction Variability at Heights of 1.5 and 4.0 Meters Above Ground Level at WSMR, New Mexico, October 1972, ECOM-5466.

106. Nordquist, Walter S., Jr., and Dickson, David H., Helicopter Downwash Applied to Fog Clearing: A Status Summary, October 1972, ECOM-5465.

107. Engebos, Bernard F., Effects of Vertical Wind on Tactical Rockets and Artillery Shells, November 1972, ECOM-5467.

108. Armendariz, M., and James R. Scoggins, Characteristics of the Turbulent Diffusion Parameters as Related to Stability, November 1972, ECOM-5468.

109. White, Kenneth O., James B. Gillespie, Robert Armstrong, and Larry E. Traylor, State-of-the-Art Survey of Meteorological Instrumentation Required to Determine Atmospheric Effects on Airborne Laser Tests, November 1972, ECOM-5469.

110. Duncan, Louis D., and Barbara J. Richart, Mesoscale Variation of Spectral Radiance Near 15 Micrometers, December 1972, ECOM-5470.

111. Schleusener, Stuart A., and Kenneth O. White, Solid-State Laser Multiwavelength Identification and Display System, January 1973, ECOM-5473.

112. Nordquist, Walter S., Jr., Numerical Approximations of Selected Meteorological Parameters Related to Cloud Physics, March 1973, ECOM-5475.