RESEARCH ON STRUCTURAL DYNAMIC TESTING
BY IMPEDANCE METHODS. VOLUME I.
STRUCTURAL SYSTEM IDENTIFICATION FROM
MULTIPOINT EXCITATION

William G. Flannelly, et al

Kaman Aerospace Corporation

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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS
VOLUME I
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By
William G. Flannelly
Alex Berman
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November 1972
EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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KAMAN AEROSPACE CORPORATION
BLOOMFIELD, CONNECTICUT

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This program was conducted under Contract DAAJ02-70-C-0012 with Kaman Aerospace Corporation.

This report contains the theoretical derivation and the presentation of a methodology for system identification of structures. Computer experiments were run to verify this methodology.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. Arthur J. Gustafson, Technology Applications Division.
RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS

Volume I
Structural System Identification From Multipoint Excitation

Final Report

Kaman Report R-1001-1

By
William G. Flannelly
Alex Berman
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Prepared by
Kaman Aerospace Corporation
Bloomfield, Connecticut

for

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS
VOLUME I - STRUCTURAL SYSTEM IDENTIFICATION FROM MULTIPOINT EXCITATION

The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data and the approximate natural frequency associated with each mode. The natural frequencies are readily available from response plots. Thus, using only impedance-type test data without the use of an intuitive mathematical model, the equations of motion for the structure may be obtained — a process referred to as system identification.

In conjunction with the determination of the aforementioned parameters, the eigenvector or mode shape and generalized mass corresponding to each natural frequency are also calculated.

A digital computer program was generated to numerically test the system identification theory. Computer experiments were conducted to test the sensitivity of the theory to errors in input data.
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FOREWORD

The work presented in this report was performed by Kaman Aerospace Corporation under Contract DAAJ02-70-C-0012 (Task 1F162204AA4301) for the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia. The program was implemented under the technical direction of Mr. Joseph H. McGarvey of the Reliability and Maintainability Division* and Mr. Arthur J. Gustafson of the Structures Division.** The report is presented in four volumes, each describing a separate phase of the basic theory of structural dynamic testing using impedance techniques.

Volume I presents the results of an analytical and numerical investigation of the practicality of system identification using fewer measurement points than there are degrees of freedom. The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data. Volume II describes the method of system identification wherein the necessary impedance data are experimentally determined by applying a force excitation at a single point on the structure. Volume III presents a method of determining the free-body dynamic responses from data obtained on a constrained structure. Volume IV describes a method of obtaining the equations for the combination of measured mobility matrices of a helicopter and its subsystems. The response of the combination of a helicopter and its subsystems is determined from data based on the experimental results of the main system and subsystems separately.

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*Division name changed to Military Operations Technology Division.

**Division name changed to Technology Applications Division.
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<tr>
<td>$[c]$</td>
<td>the damping matrix:</td>
</tr>
<tr>
<td>$[d]$</td>
<td>a damping matrix; $[d] = \omega [c]$; for damping forces which are proportional to displacement</td>
</tr>
<tr>
<td>${f}$</td>
<td>vector of external forces acting along the generalized coordinates</td>
</tr>
<tr>
<td>$\tilde{{f}}$</td>
<td>force phasor, ${f} = \tilde{{f}} e^{i \omega t}$</td>
</tr>
<tr>
<td>$g_i$</td>
<td>the structural damping coefficient of the $i$-th mode</td>
</tr>
<tr>
<td>$i$ or $j$</td>
<td>indices; imaginary operator ($i = \sqrt{-1}$)</td>
</tr>
<tr>
<td>$\chi_i$</td>
<td>the generalized stiffness of the $i$-th mode</td>
</tr>
<tr>
<td>$[k]$</td>
<td>the stiffness matrix</td>
</tr>
<tr>
<td>$\varpi_i$</td>
<td>the generalized mass of the $i$-th mode</td>
</tr>
<tr>
<td>$[m]$</td>
<td>the mass matrix</td>
</tr>
<tr>
<td>$N$ or $n$</td>
<td>the number of degrees of freedom in the structure</td>
</tr>
<tr>
<td>${\dot{y}}$</td>
<td>vector of velocities of the generalized coordinates</td>
</tr>
<tr>
<td>$\tilde{{\dot{y}}}$</td>
<td>velocity phasor, ${\dot{y}} = \tilde{{\dot{y}}} e^{i \omega t}$</td>
</tr>
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<td>$[Y(\omega)]$</td>
<td>matrix of mobilities at forcing frequency $\omega$; $[Y(\omega)] = <a href="%5Comega">\partial \tilde{\dot{y}}_i / \partial f_j</a>$</td>
</tr>
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LIST OF SYMBOLS (Continued)

\( Y_i(\omega) \) \ generalized mobility of the \( i \)-th mode at forcing frequency \( \omega \)

\( [Y] \) \ matrix of acceleration mobilities

\( [\dot{z}(\omega)] \) \ matrix of impedances at forcing frequency \( \omega \);
\( [\dot{z}(\omega)] = [\partial f_i/\partial y_j](\omega) \)

\( \dot{z}_i(\omega) \) \ generalized impedance of the \( i \)-th mode at forcing frequency \( \omega \)

\( \dot{z}_i^{*}(\omega) \) \ complex conjugate of the \( i \)-th mode generalized impedance at forcing frequency \( \omega \)

\( |\dot{z}_i(\omega)| \) \ absolute value of the \( i \)-th mode generalized impedance at forcing frequency \( \omega \)

\( \{y\}_i \) \ the \( i \)-th column of \([\Gamma]\); the gamma vector of the \( i \)-th mode; a left-hand eigenvector of \([k]^{-1}[m]\)

\( [\Gamma] \) \ the left-hand eigenvectors of \([k]^{-1}[m]\); \([\phi]^{-T}\)

\( \delta_i^j \) \ Kronecker's delta

\( \{\phi\}_i \) \ the modal vector of the \( i \)-th mode

\( [\phi] \) \ matrix of modal vectors

\( \omega \) \ forcing frequency

\( \Omega_i \) \ the natural frequency of the \( i \)-th mode
LIST OF SYMBOLS (Continued)

SUPERSCRIPTS

R the real part of a complex quantity

I the imaginary part of a complex quantity

* a generalized parameter associated with a particular mode

T the transpose

-T the inverse transpose

SUBSCRIPTS

(ω) the forcing frequency at which the quantity was measured or calculated

k forcing frequency

A dot over a quantity indicates differentiation with respect to time

BRACKETS

[ ],( ) matrix

[,] diagonal matrix

{ } column or row vector
INTRODUCTION

The success of a helicopter structural design is highly dependent on the ability to predict and control the dynamic response of the fuselage and mechanical components. Conventionally, this involves the formulation of intuitively based equations of motion. Ideally, this process would reduce the physical structure to an analytical mathematical model which would predict accurately the dynamic response characteristics of the actual structure. Obviously, the creation of such an intuitive abstraction of a complicated real structure requires considerable expertise and inherently includes a high degree of uncertainty. Structural dynamic testing is required to substantiate the analytical results. The analysis is modified until successful correlation is obtained between the analytical predictions and the test results. Finally, the mathematical model can be used to incorporate changes to improve the structural integrity of the helicopter.

This report describes the theory of structural dynamic testing using impedance techniques as applied to a mathematical model having fewer degrees of freedom than the structure. Reference 1 describes the method of obtaining a model directly from test measurements for a hypothetical structure which has the same number of degrees of freedom as the mathematical model. In reality, the number of degrees of freedom of a physical structure is infinite; therefore, the usefulness of model identification, necessarily with a finite number of degrees of freedom, using impedance testing techniques depends on the ability to simulate the real structure with a small mathematical model.

The process of deriving the equations of motion from test data is referred to as system identification. The only input information required in this theory is measured mobility data and the approximate natural frequency of each mode. This information can be obtained from impedance testing of the actual structure over the frequency range of interest yielding the second order, structurally damped linear equations of motion.

System identification theories to be of any practical engineering significance must be functional with a reasonable degree of experimental error. In this report, a series of computer experiments incorporating experimental errors was documented. This report presents a modification and extension of the analysis derived in Reference 1 such that an identified model with a finite number of degrees of freedom simulates the actual structure wherein the number of degrees of freedom is infinite.
DERIVATION

The equations of motion in matrix form of a linear system are, as shown in Reference 1,

$$[m]\ddot{y} + [c]\dot{y} + [k]y = f$$

Assume a steady-state solution of the form

$$\dot{y} = \tilde{y}e^{i\omega t} \quad \text{and} \quad f = \tilde{f}e^{i\omega t}$$

Substitute these equations into Equation (1) to give

$$\left(\omega^2 - \frac{1}{\omega}[k]\right)i + [c] \{\tilde{y}\} = \{\tilde{f}\}$$

or

$$i [\tilde{z}^R_\omega] \{\tilde{y}\} \equiv [\tilde{z}^R_\omega] \{\tilde{y}\} = \{\tilde{f}\}$$

where \(\tilde{z}_{ij}(\omega)\) is defined herein as the element velocity impedance measured at \(\omega\).

The element impedance can also be expressed as

$$\dot{z}_{ij}(\omega) = \frac{\partial\tilde{f}_i}{\partial\tilde{y}_j}$$

If Equation (2) is premultiplied by \([\phi]^{-T}[\phi]^T\) and postmultiplied by \([\phi][\phi]^{-1}\) where \([\phi]\) is the matrix of modal vectors, the result is

$$[\phi]^{-T} \left[i([\phi]^T[m][\phi] - \frac{1}{\omega}[\phi]^T[k][\phi]) + [\phi]^T[c][\phi]\right] [\phi]^{-1} = [\dot{z}(\omega)]$$

The diagonal generalized mass is expressed by

$$[\mathcal{M}] = [\phi]^T[m][\phi]$$

The diagonal generalized stiffness is given by

$$[\mathcal{K}] = [\phi]^T[k][\phi]$$
Assume that

\[ \frac{1}{\omega} \{g\phi\} = [\phi]^T [c] [\phi] \tag{6} \]

such as would be expected from structural damping in a lightly damped structure. Substituting Equations (4), (5) and (6) into Equation (3) yields

\[ [\ddot{z} (\omega)] = [\phi]^T \left[ j (\gamma \omega - \frac{\kappa_i}{\omega}) + \frac{g_i}{\omega} \right] [\phi]^{-1} \tag{7} \]

Define the i-th modal impedance as

\[ \hat{z}_i^*(\omega) = j (\gamma_i \omega - \frac{\kappa_i}{\omega}) + \frac{g_i}{\omega} \]

and substitute into Equation (7) to give

\[ [\ddot{z} (\omega)] = [\phi]^T \left[ \hat{z}_i^*(\omega) \right] [\phi]^{-1} \tag{8} \]

The elemental mobility at forcing frequency \( \omega \) is defined as

\[ \hat{Y}_{ij}(\omega) = \frac{\partial \ddot{y}_i}{\partial f_j} \]

and is equal to the ratio of the velocity phasor along the coordinate i to the external force phasor along the coordinate j when no other forces are externally applied. The full mobility matrix is given by

\[ [\hat{Y} (\omega)] = [\partial \ddot{y} / \partial f] (\omega) = [\partial f / \partial \ddot{y}] (\omega)^{-1} \equiv [z (\omega)]^{-1} \tag{9} \]

Therefore, using Equation (8) it is seen that

\[ [\ddot{y} (\omega)] = [\phi] \left[ \frac{1}{\hat{z}_i^*} \right] [\phi]^T = [\phi] \left[ \hat{y}_i^* (\omega) \right] [\phi]^T \tag{10} \]

The modal mobility of the i-th mode measured at \( \omega \) is

\[ \hat{y}_i^*(\omega) = \hat{y}_i^{*R}(\omega) + i \hat{y}_i^{*I}(\omega) = \frac{1}{\hat{z}_i^* (\omega)} = \frac{\hat{z}_i^* (\omega)}{\hat{z}_i^* (\omega)} \]

\[ = \frac{\hat{z}_i^{*R} (\omega) - i \hat{z}_i^{*I} (\omega)}{(\hat{z}_i^{*R})^2 + (\hat{z}_i^{*I})^2} = \frac{\frac{g_i}{\omega} \kappa_i - i (\gamma_i \omega - \frac{\kappa_i}{\omega})}{(\frac{g_i}{\omega})^2 + (\gamma_i \omega - \frac{\kappa_i}{\omega})^2} \]
Dividing numerator and denominator of the previous equation by the generalized mass $m_i$

$$\dot{Y}^*_i(\omega) = \frac{\frac{g_i\kappa_i}{m_i\omega} - i(\omega - \frac{\kappa_i}{m_i\omega})}{m_i(\frac{g_i\kappa_i}{\omega m_i})^2 + m_i(\omega - \frac{\kappa_i}{m_i\omega})}$$

Substituting the natural frequency of the $i$-th mode

$$\Omega_i = \sqrt{\frac{\kappa_i}{m_i}}$$

$$\dot{Y}^*_i(\omega) = \frac{\frac{g_i\Omega_i}{\omega} - i(\omega - \frac{\Omega_i}{\omega})}{m_i(\frac{g_i\Omega_i}{\omega})^2 + m_i(\omega - \frac{\Omega_i}{\omega})}$$

Separating this equation into the real and imaginary components yields

$$\dot{Y}^*_i(\omega) = \frac{1}{\omega m_i \Omega_i^2} \left[ \frac{2}{g_i} - i \left( \frac{\frac{\omega^2}{\Omega_i^2} - 1}{2} \right) \right]$$

Finally, from Equation (10), the real mobility may be written as

$$[\dot{Y}^R(\omega)] = [\phi][\dot{Y}^R(\omega)]T$$

Reference 1 indicated that because the real modal mobilities of modes far removed from the forcing frequency become negligible compared to adjacent modes, the real mobility matrix at any frequency is ordinarily affected only by modes in close proximity to the forcing frequency. The measured real mobility matrix at a particular frequency reflects the influence of only the most dominant modes in that frequency of measurement region. Therefore, it is unrealistic to use the real mobility matrix measured at any specific frequency to determine parameters other than those associated with neighboring modes.
Reference 1 also shows that the imaginary modal mobilities of modes associated with frequencies less than the forcing frequency asymptotically approach a constant. An imaginary mobility matrix contains the effect of all lower modes in proportion to, or greater than, the magnitudes of their generalized masses. Therefore, it is impractical to use imaginary mobility matrices to evaluate properties associated with natural frequencies far above the forcing frequency.

These characteristics of the modal mobility make it impossible to determine the system parameters from the \( n \) equations in \( n \) unknowns obtained from mobility matrices measured at any two forcing frequencies.

Even if the modal mobility were amenable to determination of the system parameters, the precision of measurement which would be required to do this for most systems is impossible to achieve. The modal approach derived below avoids this problem.

**DERIVATION OF THE DOMINANT MODE EIGENVALUE PROBLEM**

Equation (10) may be written

\[
\begin{bmatrix}
\dot{y}_i(\omega)
\end{bmatrix} = \begin{bmatrix}
\Phi
\end{bmatrix} \begin{bmatrix}
\dot{Y}^*(\omega)
\end{bmatrix} \begin{bmatrix}
\dot{\phi}_i
\end{bmatrix}^T = \sum_{i=1}^{N} \dot{Y}^*_i(\omega) \begin{bmatrix}
\phi_i
\end{bmatrix} \begin{bmatrix}
\phi_i
\end{bmatrix}^T
\]

where \( \{\phi\} \) is a column in \( \Phi \) and \( N \) is the order of the matrices. Define \( \Gamma = \Phi^{-T} \), and Equation (8) may be written as

\[
\begin{bmatrix}
\dot{Y}_i(\omega)
\end{bmatrix}^{-1} = \begin{bmatrix}
\dot{Z}_i(\omega)
\end{bmatrix} = \begin{bmatrix}
\Gamma
\end{bmatrix} \begin{bmatrix}
\dot{Z}^*(\omega)
\end{bmatrix} \begin{bmatrix}
\Gamma
\end{bmatrix}^T = \sum_{i=1}^{N} \frac{1}{Y_i(\omega)} \begin{bmatrix}
\gamma_i
\end{bmatrix} \begin{bmatrix}
\gamma_i
\end{bmatrix}^T
\]

where \( \{\gamma\} \) is a column in \( \Gamma \).

Each matrix \( \begin{bmatrix}
\dot{Y}_i(\omega) \begin{bmatrix}
\phi_i
\end{bmatrix} \begin{bmatrix}
\phi_i
\end{bmatrix}^T
\end{bmatrix} \) and \( \begin{bmatrix}
\frac{1}{Y_i(\omega)} \begin{bmatrix}
\gamma_i
\end{bmatrix} \begin{bmatrix}
\gamma_i
\end{bmatrix}^T
\end{bmatrix} \) in Equations (13) and (14) is of rank one, but the summation of as many of these successive modal matrices as the order \( N \) of the matrix is a nonsingular matrix.
Similarly,

\[
\begin{align*}
\bf{\hat{Y}}^R_{(\omega)} &= \sum_{i=1}^{N} \bf{\hat{y}}^R_{i(\omega)} \{\phi\}_i \{\phi\}_i^T \\
\bf{\hat{Y}}^I_{(\omega)} &= \sum_{i=1}^{N} \bf{\hat{y}}^I_{i(\omega)} \{\phi\}_i \{\phi\}_i^T \\
\bf{\hat{Y}}^R_{(\omega)}^{-1} &= \sum_{i=1}^{N} \frac{1}{\bf{\hat{y}}^R_{i(\omega)}} \{\gamma\}_i \{\gamma\}_i^T \\
\bf{\hat{Y}}^I_{(\omega)}^{-1} &= \sum_{i=1}^{N} \frac{1}{\bf{\hat{y}}^I_{i(\omega)}} \{\gamma\}_i \{\gamma\}_i^T
\end{align*}
\]  

(15)

The iteration procedure used to solve the eigenvalue problem in Reference 1 employed the imaginary part of a mobility matrix measured at a frequency just above the N-th natural frequency. The method used to solve the eigenvalue problem in the present report, which was found to give more accurate results, utilizes the sum of the real parts of the mobility matrices measured near each of the natural frequencies associated with the actual model. It has been indicated previously that a measured real mobility matrix reflects the influence of only the most dominant modes in the vicinity of the forcing frequency. Therefore, summation of a discrete set of the real mobility matrices measured at forcing frequencies near the corresponding natural frequencies should contain precisely the information relevant to the model normal modes.

The eigenvalue problem may be formulated as follows. Consider the summation of the real mobility matrices measured at a discrete set of frequencies near the first NR natural frequencies. Take the inverse of this matrix and pre-multiply by a real mobility matrix measured at any frequency \(\omega_k\).
\[
[\hat{Y}^R(\omega_k)]^{\Omega_{NR}} = \sum_{\omega_j=\Omega_1}^{\omega_j=\Omega_{NR}} \hat{Y}^R_i(\omega_j) -1
\]

\[
= \sum_{i=1}^{NR} \hat{Y}^*_R i(\omega_k) \{\phi_i\} \{\phi_i\}^T \left( \sum_{i=1}^{NR} \hat{Y}^*_R i(\omega_j) \{\phi_i\} \{\phi_i\}^T \right)^{-1}
\]

\[
= \left[ \phi \right] \left[ \hat{Y}^*_R i(\omega_k) \right] \left[ \phi \right]^T \left( \sum_{j=1}^{NR} \hat{Y}^*_R i(\omega_j) \right) \left[ \phi \right]^T
\]

\[
= \left[ \phi \right] \left[ \hat{Y}^*_R i(\omega_k) \right] \left[ \phi \right]^T \left( \sum_{j=1}^{NR} \hat{Y}^*_R i(\omega_j) \right)^{-1}
\]

\[
= \left[ \phi \right] \left[ \sum_{j=1}^{NR} \frac{\hat{Y}^*_R i(\omega_j)}{\sum_{j=1}^{NR} \hat{Y}^*_R i(\omega_j)} \right]^{-1}
\]

(16)

If Equation (16) is postmultiplied by \{\phi_i\}, there results

\[
\left[ \hat{Y}^R(\omega_k) \right] \left[ \sum_{\omega_j=\Omega_1}^{\omega_j=\Omega_{NR}} \hat{Y}^R_i(\omega_j) \right]^{-1} \{\phi_i\} = \left[ \phi \right] \left[ \sum_{j=1}^{NR} \frac{\hat{Y}^*_R i(\omega_j)}{\sum_{j=1}^{NR} \hat{Y}^*_R i(\omega_j)} \right]^{-1} \{\phi_i\}
\]

but \[\phi\]^{-1}\{\phi_i\} yields a column matrix comprised of zeroes except for a 1 in the \(i\)-th position. Finally,
The eigenvalue problem is finally formulated as

\[
\begin{pmatrix}
\dot{Y}^R_i(\omega_k) \\
\Sigma_{j=1}^{NR} \dot{Y}^R_i(\omega_j)
\end{pmatrix} \{\phi\}_{i} = \frac{\dot{Y}^R_i(\omega_k)}{\Sigma_{j=1}^{NR} \dot{Y}^R_i(\omega_j)} \{\phi_{i}\}
\]

If the order of multiplication is reversed in Equation (16), an eigenvalue problem is developed in which the eigenvector is the gamma vector of the i-th mode. Consider the same parameters as in Equation (16); only the order of multiplication of the matrices is changed.

\[
[\dot{Y}^R(\omega_k)][\omega_j = \Omega_{1} = \Sigma_{i=1}^{NR} \dot{Y}^R_i(\omega_j)]^{-1} [\dot{Y}^R_i(\omega_k)] = \left( \Sigma_{i=1}^{NR} \dot{Y}^R_i(\omega_j) \{\phi_{i}\}\{\phi_{i}\}^T \right)^{-1} \left( \Sigma_{i=1}^{NR} \dot{Y}^R_i(\omega_k) \{\phi_{i}\}\{\phi_{i}\}^T \right)
\]

\[
= \left( [\dot{Y}^R_i(\omega_j)] \{\phi\}_{i} \{\phi\}_{i}^T \right)^{-1} \left( [\dot{Y}^R_i(\omega_k)] \{\phi\}_{i} \{\phi\}_{i}^T \right)
\]

\[
= \left( \frac{\Sigma_{j=1}^{NR} \dot{Y}^R_i(\omega_j)}{\Sigma_{j=1}^{NR} \dot{Y}^R_i(\omega_j)} \right) \left( \frac{\dot{Y}^R_i(\omega_k)}{\Sigma_{j=1}^{NR} \dot{Y}^R_i(\omega_j)} \right) \{\phi\}_{i} \{\phi\}_{i}^T
\]

\[
= \{\phi\}_{i} \{\phi\}_{i}^T
\]
By definition, $[\Gamma] = \left[\phi\right]^{-T}$ and $[\Gamma]^{-1} = \left[\phi\right]^{T}$; substituting into Equation (18) yields

$$
\begin{bmatrix}
\Omega_{NR} & \hat{Y}_{R}^{*} \\
\hat{Y}_{i}(\omega) & \Omega_{NR}
\end{bmatrix} = \left[\Gamma\right]^{-1}
\begin{bmatrix}
\hat{Y}_{i}(\omega_k) \\
\sum_{j=1}^{NR} \hat{Y}_{i}(\omega_j)
\end{bmatrix}
\tag{19}
$$

If Equation (19) is postmultiplied by $\{\gamma\}_i$, a column of $[\Gamma]$, and the same procedure is followed as was used in obtaining Equation (17), Equation (19) becomes

$$
\begin{bmatrix}
\Omega_{NR} & \hat{Y}_{R}^{*} \\
\hat{Y}_{i}(\omega) & \Omega_{NR}
\end{bmatrix}^{-1} \begin{bmatrix}
\hat{Y}_{R}^{*} \\
\hat{Y}_{i}(\omega)
\end{bmatrix} \{\gamma\}_i = \frac{\hat{Y}_{i}(\omega_k)}{\sum_{j=1}^{NR} \hat{Y}_{i}(\omega_j)} \{\gamma\}_i
\tag{20}
$$

which is an eigenvalue problem with the eigenvector equal to the gamma vector of the $i$-th mode.

IDENTIFICATION OF STRUCTURAL PARAMETERS

A successful identification procedure, using normal mode techniques, should separate the effect of each mode in a mathematical sense, regardless of the number of stations where mobility measurements are taken on the structure. If satisfactory normal mode separation required a certain minimum number of measurement stations greater than the number of degrees of freedom chosen for the model, the most that can be expected is an approximate model, possibly including optimization procedures designed to satisfy all system constraints. This situation is considered in detail in Reference 2 in which a mathematical model is derived from test data such that identification of the structure is obtained closest to any specified analytical approximation.

Satisfactory normal mode separation requires that the values of $\hat{z}^{*R}$ and $\hat{z}^{*I}$ be independent of the number of degrees of freedom of the model. The values of the generalized mass ($m^{*}_i$), the corresponding identified natural frequency ($\Omega_i$), and the generalized stiffness as defined below are then also independent of the number of measurement stations.
\[
\gamma_i = \frac{\omega_k z_i^*(\omega_k) - \omega_j z_i^*(\omega_j)}{(\omega_k^2 - \omega_j^2)} \tag{21}
\]
and
\[
\Omega_i^2 = \omega_j \omega_k \frac{z_i^*(\omega_k) - \omega_k z_i^*(\omega_j)}{z_i^*(\omega_k) - \omega_j z_i^*(\omega_j)} \tag{22}
\]

\[
\kappa_i = \Omega_i^2 \gamma_i \tag{23}
\]

The two forcing frequencies \((\omega_k)\) and \((\omega_j)\) are chosen in the vicinity of the corresponding natural frequency which is available from test data. The generalized impedance of the \(i\)-th mode at forcing frequency \((\omega)\) is obtained from the generalized mobility of the \(i\)-th mode at forcing frequency \((\omega)\). It follows from Equation (13) that the modal mobilities are given by

\[
\begin{bmatrix}
\dot{\gamma}_i
\end{bmatrix} = [\Phi]^{-1} [\dot{\gamma}(\omega)] [\Phi]^{-T}
\]

\[
= [\Gamma]^T [\dot{\gamma}(\omega)] [\Gamma]
\]

and, therefore, the orthogonality condition for gamma vectors is

\[
\{\gamma\}^T [\dot{\gamma}(\omega)] \{\gamma\} = \dot{\gamma}_i^* (\omega) \delta_{ij}
\]

The modal impedance of the \(i\)-th mode at \(\omega_j\) is

\[
\dot{z}_i^*(\omega_j) = \frac{\dot{\gamma}_i^* (\omega_j)}{|\dot{\gamma}_i^* (\omega_j)|^2} = \frac{\dot{\gamma}_i^* (\omega_j) - i\dot{\gamma}_i^* (\omega_j)}{|\dot{\gamma}_i^* (\omega_j)|^2}
\]
It follows that

\[ z_i^*(\omega_j) = -\frac{y_i^*(\omega_j)}{|y_i^*(\omega_j)|^2} \]

and

\[ z_i^*(\omega_j) = \frac{y_i^*(\omega_j)}{|y_i^*(\omega_j)|^2} \]

The damping coefficient for the i-th mode is most readily given by

\[ g_i = \frac{\omega_j z_i^*(\omega_j)}{\kappa_i} \]  

which follows directly from Equation (7). The damping coefficient for the i-th mode may also be obtained by

\[ g_i = \frac{\omega_j^2 z_i^*(\omega_i)}{\Omega_i^2} \]

Using a measurement of real mobility taken precisely at resonance, the damping coefficient may be calculated using Equation (11) as

\[ g_i = \frac{1}{\gamma_i^* y_i(\Omega_i) \Omega_i^2} \]

**PARAMETERS OF THE MATHEMATICAL MODEL**

The elements of the influence coefficient matrix, being a measure of displacement per unit force, are independent of the number of measurement stations defining the order of the matrix. Conversely, the elements of the stiffness and mass matrices assume different values as the number of degrees of freedom of the model is changed. The identification procedure used in Reference 1 calculates both stiffness and mass matrices by summing the effects of each consecutive mode and defining the incomplete matrices as the sum up to and including a particular mode. If the order of the model
degrees of freedom is changed from ND for the structure to NR for the model, the corresponding incomplete mass and stiffness matrices will not be directly comparable, on a modal basis, to the structure mass and stiffness matrices. It is more expedient to identify the influence coefficient matrix \([c]\) and the inverse of the mass matrix \([m]\). Premultiplying Equation (4) by \([\phi]^{-T}\) and postmultiplying \([\phi]^{-1}\) and taking the inverse of the resulting equation yields

\[
[m] = [m]^{-1} = \sum_{i=1}^{NR} \frac{1}{m_i} \phi_i \phi_i^T
\]  

(28)

If the same operations are performed on Equation (5), the result is

\[
[c] = [k]^{-1} = \sum_{i=1}^{NR} \frac{1}{\omega_i m_i} \phi_i \phi_i^T
\]  

(29)

Set \([c] = \frac{1}{\omega} [d]\) and using Equation (6) there results

\[
\frac{1}{\omega} [g] = \frac{1}{\omega} [\phi]^T [d] [\phi]
\]

Solving for the damping matrix yields

\[
[d] = \phi^{-T} [g] \phi^{-1}
\]

Substituting \([\Gamma] = [\phi]^{-T}\), \([\Gamma]^T = [\phi]^{-1}\) and \([k] = [\omega^2 m]\) into the previous equation gives

\[
[d] = [\Gamma] [g \omega^2 m] [\Gamma]^T
\]

The damping matrix can be expressed as

\[
[d] = \sum_{i=1}^{NR} g_i \omega_i^2 m_i \gamma_i \gamma_i^T
\]  

(30)

**ITERATION PROCEDURE**

The calculation of the modal parameters such as generalized mass, stiffness and the corresponding natural frequency requires the generalized impedance at a particular frequency for each mode under consideration. The modal impedance is a function of the generalized mobility for the same mode and forcing frequency. As indicated in Equation (24), the modal mobilities are dependent upon the matrix of gamma vectors and its transpose. The iteration process as originally formulated
in the present work involved iteration on the normal mode vectors with a subsequent inversion operation to determine the gamma vectors. This sequence introduced errors into the system, with the result that the gamma vectors did not resemble the associated gamma vectors obtained from the specimen representing the actual structure. The iterated normal mode vectors obtained from the mathematical model were extremely close, particularly at the lower modes, to the specimen, or exact, normal mode vectors. Nevertheless, any discrepancy between the model iterated modal vectors and the exact values, however small, was magnified in the inversion process, causing the gamma vectors to bear little resemblance to the specimen gamma vectors. Therefore, it was deemed advisable to iterate on the gamma vectors directly and disperse with the intermediate inversion operation.

To equalize the effect of each modal mobility in the matrix iteration Equation (20), several normalization procedures were incorporated into the method. First, each real mobility matrix was normalized on the largest element of the respective matrix. This procedure proved satisfactory except in some situations where the elements of the mobility matrices were approximately of the same magnitude but the largest elements differed in algebraic sign. This resulted in a cancellation effect among the real mobility matrices and an incorrect summation, thereby causing erroneous calculated gamma vectors. A modification to the normalization technique was applied whereby the real mobility matrices at each forcing frequency were divided by the absolute value of the largest element in the respective matrices. As a further refinement on the normalization procedure, the real mobility matrix at each forcing frequency was normalized on the root mean square associated with each respective matrix. Occasionally, these operations also caused problems in the final modal generalized mass and natural frequency calculations. For example, if a mobility matrix calculated at a particular frequency contained one element that dominated the matrix, normalization of the mobility matrix on this element would effectively submerge the influence of the matrix in the summation of the real modal mobilities. Again, the calculated modal parameters would obviously be incorrect. Similarly, if several elements of the mobility matrix measured at a specific forcing frequency were of greater magnitude than the remaining elements, the root mean square value would be affected and normalization by this value would yield a matrix wherein the elements were substantially reduced. Therefore, any such matrix would not be realistically represented in the summation of the real mobility matrices; consequently, the modal generalized mass and natural frequency would be incorrect.
Finally, each mobility matrix was multiplied by the respective forcing frequency yielding an acceleration mobility. These acceleration mobility matrices were substituted for the velocity mobilities appearing in Equation (17) and Equation (20) when iterating for the mode shapes and gamma vectors respectively. This technique was also plagued with similar difficulties that the aforementioned normalization procedures incurred. Fortunately, when the computer experiments were executed incorporating any of the previously discussed normalization methods, the conditions which yielded erroneous results were readily discernible. In these instances, the calculations for the modal generalized mass and natural frequency produced results which were obviously incorrect. For the conditions which were recognized to be in error, the computer experiments were reevaluated substituting a different normalization option. Generally, the results obtained by altering a normalization procedure yielded modal parameters which were correct.

INTERPRETATION OF ELEMENTS IN THE REDUCED MASS MATRIX

In general, it may be expected that the algebraic sum of all the elements of a reduced mass matrix from system identification will approximate the gross weight of the aircraft. Due partly to restraints, the sum of the elements should not exactly equal the gross weight, because masses at elastic restraints do not act as if they were ungrounded. Masses at pinned joints to ground do not even figure in the mass matrix because they do not move.

Individual mass elements cannot be interpreted as reflecting lumped physical weights at their assigned locations. The elements of any reduced mass matrix represent the inertial, as opposed to elastic and damping, dynamic effects of the two (for off-diagonal) degrees of freedom with which they are associated in an actual system having many more degrees of freedom than the model. The off-diagonal terms in a reduced mass matrix will usually be large and sometimes negative. The matrix will usually be fully populated.

The identified mass and stiffness matrices can be used to draw a dynamic circuit of the helicopter and, if any one were interested, it would be possible to construct an actual spring-mass system (utilizing both positive and negative springs and moments of inertia) which would be an exact physical duplication of the identified model, element by element, and would have the same natural frequencies and modal eigenvectors as the helicopter; but it would not "look" like a helicopter. Neither negative spring rates nor negative off-diagonal masses are physically unrealizable; the former are used by
Lockheed in its control system and by all light-switch manufacturers, the latter are the essential part of the dynamic antiresonant vibration isolator.

The objective is not to identify a system which "looks" like a helicopter but one which "performs" like a helicopter under various dynamic loadings. The physical interpretation of the ij-th element of the identified mass matrix, for example, is that the helicopter will generally exhibit a partial derivative of a force at i with respect to a response at j which has an effective* mass component that is the ij-th element of the identified mass matrix (similarly for the stiffness and damping matrices).

It is immaterial in the identification whether there are as many points on the structure as there are degrees of freedom in the model, or if up to three degrees of freedom (in orthogonal directions) occur at any one point. It is important only that elements in the motion vector have the properties of generalized coordinates for the holonomic model considered. An identified reduced model in which some of the displacement elements represent the orthogonal Cartesian or polar coordinates of a given structural point would look much like an identified model of a similar system with parallel coordinates of separate points.

The impedance matrix, of which the mass and stiffness matrices are terms, of a mathematical model of a larger system is a function of the size of the model, and the terms must reflect this. It was found that frequency-independent mass, stiffness and damping matrices as described can accurately reflect the responses of a continuous structure over a finite spectrum by approximating a lambda matrix the inverse of which very closely approximates the mobility. The spectral mobility matrix, even of an order that equals the number of degrees of freedom in the structure, cannot be expressed as a lambda matrix.

*Not to be confused with the formal definition of "Effective Mass" as

\[ ME_{jki} \equiv \frac{\{\phi\}_i^T [m] \{\phi\}_i}{\phi_j^i \phi_k^i} \]
THE REDUCED MASS MATRIX

Consider the actual structure to consist of an infinite number of degrees of freedom of which \( R \) degrees of freedom are retained in the model. The mobility

\[
\begin{bmatrix}
[Y_{RR}] & [Y_{RE}] \\
[Y_{ER}] & [Y_{EE}]
\end{bmatrix}
= \begin{bmatrix}
[Z_{RR}] & [Z_{RE}] \\
[Z_{ER}] & [Z_{EE}]
\end{bmatrix}^{-1}
= \begin{bmatrix}
[K_{RR}] & [K_{RE}] \\
[K_{ER}] & [K_{EE}]
\end{bmatrix}
- \omega^2 \begin{bmatrix}
[M_{RR}] & 0 \\
0 & [M_{EE}]
\end{bmatrix}^{-1}
\]

(31)

The model impedance is defined as the inverse of the mobility matrix in the \( R \) degrees of freedom:

\[
[Z_m] = [Y_{RR}]^{-1} = [Z_{RR}] - [Z_{RE}][Z_{EE}]^{-1}[Z_{ER}] = [K_m] - \omega^2 [M_m]
\]

(32)

The stiffness of the model, \([K_m]\), is the inverse of the \( R \times R \) influence coefficients:

\[
[K_m] \equiv [C_{RR}]^{-1} = [K_{RR}] - [K_{RE}][K_{EE}]^{-1}[K_{ER}]
\]

(33)

From Equation (31),

\[
[Z_{RR}] = [K_{RR}] - \omega^2 [M_{RR}]
\]

\[
[Z_{EE}] = [K_{EE}] - \omega^2 [M_{EE}]
\]

\[
[Z_{RE}] = [K_{RE}]
\]

\[
[Z_{ER}] = [K_{ER}]
\]

Substitute into Equation (32)
\[
\begin{align*}
\mathbf{z}_m &= \mathbf{K}_m - \omega^2 \mathbf{M}_m = \mathbf{K}_{RR} - \omega^2 \mathbf{M}_{RR} - \mathbf{K}_{EE}^\top \left( \mathbf{K}_{EE} \right) \\
- \omega^2 \mathbf{M}_{EE}^{-1} \mathbf{K}_{ER} &= \mathbf{K}_{RR} - \mathbf{K}_{RE} \left( \mathbf{K}_{EE}^{-1} \right)^{-1} \mathbf{K}_{ER} \\
- \omega^2 \mathbf{M}_{RR} - \mathbf{K}_{EE}^\top \left( \mathbf{I} - \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \right)^{-1} \mathbf{K}_{EE}^{-1} \mathbf{K}_{ER} \\
+ \mathbf{K}_{RE} \left( \mathbf{K}_{EE} \right)^{-1} \mathbf{K}_{ER}
\end{align*}
\]

Substitute Equation (33). Then
\[
\begin{align*}
\mathbf{M}_m &= \mathbf{M}_{RR} + \frac{1}{\omega^2} \mathbf{K}_{RE} \left( \left( \mathbf{I} - \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \right)^{-1} \right) \\
- \mathbf{I} - \mathbf{K}_{EE}^{-1} \mathbf{K}_{ER} &= \mathbf{M}_{RR} + \frac{1}{\omega^2} \mathbf{K}_{RE} \left( \left( \mathbf{I} - \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \right)^{-1} \right) \\
- \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \left( \mathbf{I} - \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \right)^{-1} \mathbf{K}_{EE}^{-1} \mathbf{K}_{ER} \\
- \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \left( \mathbf{I} - \omega^2 \mathbf{K}_{EE}^{-1} \mathbf{M}_{EE} \right)^{-1} \mathbf{K}_{EE}^{-1} \mathbf{K}_{ER}
\end{align*}
\]

Equation (34) is the "exact" RxR reduced mass matrix of a system with an infinite number of degrees of freedom. Note that \( \mathbf{M}_m \) is not diagonal and is a function of forcing frequency.

The frequency dependency of the "exact" reduced mass matrix simply reflects the fact that R linear differential equations with constant coefficients cannot contain enough information to exactly reflect the action of an infinite number of degrees of freedom over a spectrum containing R modes. The frequency dependency makes it impractical to use this in a linear engineering mathematical model.

The "Consistent Mass Matrix" (Reference 3), often used in finite-element dynamics work, is also based on a model stiffness matrix \( \mathbf{K}_m \) being the inverse of the RxR influence coefficient matrix:

\[ \mathbf{K}_m = \left( \mathbf{C}_{RR} \right)^{-1} = \mathbf{K}_{RR} - \mathbf{K}_{RE} \left( \mathbf{K}_{EE}^{-1} \right)^{-1} \mathbf{K}_{ER}. \]

The kinetic energy of the structure is set equal to the
kinetic energy of the model:

\[
\begin{bmatrix}
\frac{\dot{y}_R}{T} \\
\frac{\dot{y}_E}{T}
\end{bmatrix}
\begin{bmatrix}
\frac{[M_{RR}]}{0} \\
0 & [M_{EE}]
\end{bmatrix}
\begin{bmatrix}
\frac{\ddot{y}_R}{T} \\
\frac{\ddot{y}_E}{T}
\end{bmatrix}
= \{\ddot{y}_R\}^T [M_{RR}] \{\ddot{y}_R\}
\]

+ \{\ddot{y}_E\}^T [M_{EE}] \{\ddot{y}_E\} = \{\ddot{y}_R\}^T [M_m] \{\ddot{y}_R\}

(35)

It is implicitly assumed, however, that the inertial forces occur only along the R generalized coordinates, giving

\[
\begin{bmatrix}
\frac{[K_{RR}]}{[K_{RE}]} \\
\frac{[K_{ER}]}{[K_{EE}]} \\
\end{bmatrix}
\begin{bmatrix}
\frac{y_R}{T} \\
\frac{y_E}{T}
\end{bmatrix}
= \begin{bmatrix}
\frac{[M_m]}{0} \\
0 & \frac{[M_m]}{0}
\end{bmatrix}
\]

which is clearly not the case but from which it follows that

\(\{\ddot{y}_E\} = -[K_{EE}]^{-1}[K_{ER}]\{\ddot{y}_R\}\)

in sinusoidal vibration. Substituting the above in Equation (35) gives

\[
\{\ddot{y}_R\}^T [M_{RR}] + \left([K_{ER}]^T [K_{EE}]^{-T} [M_{EE}] [K_{EE}]^{-1} [K_{ER}]\right) \{\ddot{y}_R\}
= \{\ddot{y}_R\}^T [M_m] \{\ddot{y}_R\}
\]

(36)

and the "Consistent Mass Matrix" is given by

\[
[M_m] = [M_{RR}] + [K_{ER}]^T [K_{EE}]^{-T} [M_{EE}] [K_{EE}]^{-1} [K_{ER}]
\]

(37)

This matrix is nondiagonal, like the "exact" reduced mass matrix, and has the advantage of being independent of frequency. However, comparison of Equation (37) with Equation (34) shows that the "Consistent Mass Matrix" reduces to the "exact" reduced mass matrix only at zero frequency; that is, in the static condition. As the frequency increases, the "Consistent Mass Matrix" yields increasingly erroneous results.
The reduced mass matrix in system identification is, like the others, nondiagonal and related to a model stiffness matrix which is the inverse of the R x R influence coefficients (as represented by the first R modes, which is accurate beyond direct measurement capability by many orders of magnitude); but the system identification reduced mass matrix also is independent of frequency and is exact at all the natural frequencies of the model, which are the first R natural frequencies of the helicopter. The system identification mass matrix is given by

\[
[M_m] = [M_{RR}] + [\phi_{RR}]^{-T}[\phi_{ER}]^T[M_{EE}][\phi_{ER}][\phi_{RR}]^{-1}
\]  

(38)

or \([M_m] \cong [C_{RR}]^{-1}[\phi_{RR}][\frac{1}{\Omega_r^2}][\phi_{RR}]^{-1}\) very nearly.  

(39)

At the r-th natural frequency,

\[
[C_{RR}]^{-1} \left( [M_{RR}] + [K_{RE}][K_{EE}]^{-1} \left( \frac{1}{\Omega_r^2} [I] 
- [C_{EE}] [M_{EE}] \right) [K_{ER}] [C_{RR}] [M_{RR}] \right) \{\phi_{RR}\} = \{\phi_{RR}\} \frac{1}{\Omega_r^2}
\]  

(40)

exactly. Note in Equation (38) that the reduced system identification mass matrix is expressed in terms of the modal eigenvectors of the first R modes only but includes all the masses of the actual helicopter.

Alterations in masses on the R generalized coordinates which do not affect the modal eigenvectors are, as seen from Equation (38), exactly represented. Such alterations can substantially change natural frequencies and responses. Other types of changes which do alter the modal eigenvectors may or may not be accurately reflected in the model response depending on the degree of eigenvector effects—a limit which has not been algebraically defined for any mathematical model, whether from intuitive analysis or system identification.
That such a limit should somewhere exist is a practical engineering fact. One cannot expect to obtain the equations of a sweet pea on a rubber band, then attach it to the Golden Gate bridge and expect to find the dynamic response of the bridge (the reverse, incidentally, is equally impractical). Prudence marks the boundary between utility and uselessness.

INFORMATION LOSS IN MATRIX INVERSION

It is inevitable that there will be a loss in information in numerically obtaining the response matrix from any mathematical model, or in obtaining the mathematical model from responses, even if no deliberate error is introduced.

The following is a slight modification of a derivation by Rosanoff and Ginsburg (Reference 4). Consider the equation

\[ [A] \{x\} = \{b\} \quad (41) \]

in which \([A]\) is a real symmetric nonsingular matrix. Because we calculate with numbers which have a finite number of digits, we actually solve the equation

\[ ([A] - [E]) \{x + \delta x\} = \{b\} \quad (42) \]

where \([E]\) is an "error" matrix. Premultiplying both sides of Equation (42) by \([A]^{-1}\) and substituting \([A]^{-1}\{b\} = \{x\}\) gives

\[ ([I] - [A]^{-1}[E]) \{x + \delta x\} = \{x\} \quad (43) \]

or

\[ \{\delta x\} = ([I] - [A]^{-1}[E])^{-1} \{x\} \quad (44) \]

Take the norm (see References 5 and 6, for example) of both sides:

\[ ||\{\delta x\}|| = ||([I] - [A]^{-1}[E])^{-1} - [I]||\{x|| \]

But the norm of the product of a matrix and a vector is less than the product of the matrix norm and the consistent vector norm:

\[ ||\{\delta x\}|| \leq ||([I] - [A]^{-1}[E])^{-1} - [I]|| \cdot ||\{x\}|| \quad (45) \]

or

\[ \frac{||\{\delta x\}||}{||\{x\}||} \leq ||([I] - [A]^{-1}[E])^{-1} - [I]|| \]
Assume that $||[A]^{-1}[E]|| < 1$. From Faddeeva (Reference 5), it is well known that

$$|| (I - [A]^{-1}[E])^{-1}[I] + ([A]^{-1}[E] + ([A]^{-1}[E])^2...$$

$$+ ([A]^{-1}[E])^k || \leq \frac{||[A]^{-1}[E]||^{k+1}}{1 - ||[A]^{-1}[E]||} \quad (46)$$

if $||[A]^{-1}[E]|| < 1$. Setting $k = 0$ gives

$$|| (I - [A]^{-1}[E])^{-1} - [I] || \leq \frac{||[A]^{-1}[E]||}{1 - ||[A]^{-1}[E]||} \quad (47)$$

or

$$\frac{||\{\delta x\}||}{||\{x\}||} \leq \frac{||[A]^{-1}[E]||}{1 - ||[A]^{-1}[E]||} \leq \frac{||[A]^{-1}|| \cdot ||[E]||}{1 - ||[A]^{-1}|| \cdot ||[E]||}$$

which is identical to the result obtained by Rosanoff and Ginsburg.

$$\frac{||\{\delta x\}||}{||\{x\}||} \leq \frac{||[A]|| \cdot ||[A]^{-1}|| \cdot ||[E]|| / ||[A]||}{1 - ||[A]|| \cdot ||[A]^{-1}|| \cdot ||[E]|| / ||[A]||} = \frac{k_n \ell}{1 - k_n \ell} \quad (48)$$

where, following Rosanoff et al, $k_n$ is defined as a conditioning number and $\ell$ as a relative error:

$$k_n \equiv ||[A]|| \cdot ||[A]^{-1}||$$

$$\ell \equiv ||[E]|| / ||[A]||$$

Taking the number of digits in the arithmetic as $\log_{10} \frac{1}{k_n}$, the reciprocal of Equation (48) gives an estimate of the number of significant digits $p$.

$$p = \log_{10} \left| \frac{||x||}{||[A]^{-1}[E]||} \right| \geq \log_{10} \left( 1 - k_n \ell \right) + \log_{10} \frac{1}{\ell} - \log_{10} k_n$$

but, assuming $1 \gg k_n \ell$, this estimate may be written
Thus, as shown in Reference 4, the number of information digits $q$ lost in inverting $[A]$ is approximately

$$q = \log_{10}k_n = \log_{10}|[A]||[A]^{-1}|$$

(50)

This is true for any norm. However, the norm of a symmetrical positive definite matrix, subordinate to the Euclidian vector, is the maximum eigenvalue; and the maximum eigenvalue of the inverse is the reciprocal of the minimum eigenvalue of the matrix. Substituting this norm of $[A]$ into Equation (50) gives the lost digits estimate.

$$q = \log_{10}\frac{\lambda(A)_{\text{max}}}{\lambda(A)_{\text{min}}}$$

(51)

To illustrate the immense practical importance of this, consider as an example a matrix having

$$\frac{\lambda(A)_{\text{max}}}{\lambda(A)_{\text{min}}} = 1.72 \times 10^3$$

This is the ratio of natural frequencies in the 20x20 specimen of the helicopter used in this contract. The IBM 360 uses six hexadecimal places resulting in $16^6-1$ or 16777215 as the largest decimal mantissa in single precision. The inversion of the $k^{-1}m$ matrix with single precision on the computer results in an inverse having (estimated) $\log_{10}16777215 - \log_{10}1.72 \times 10^3 = 3.99$ significant digits. In other words, even starting with eight decimal places in floating point, we end up with approximately four decimal places of information in the inverse.

It is absolutely essential when dealing with test data matrices which will be inverted that the ratio of the extreme eigenvalues be minimized. Otherwise, all the physical information in the matrix is likely to be destroyed in the inversion, leaving meaningless numbers. Test data has few enough significant figures of information to begin with.
HOW TO MINIMIZE INFORMATION LOSS

A major step in this system identification process is the determination of the \( \{\gamma\} \) and \( \{\phi\} \) vectors by iteration. The matrix involved is the product of a mobility matrix and the inverse of another mobility matrix. This inverse presents a serious danger of information loss.

To minimize the extraneous information content of modes higher than the order of the matrix, which amounts to noise, and to narrow the spread of modal mobilities, the matrix to be inverted was made the sum of the dissipative (e.g., \([\gamma I]\) matrices measured near each natural frequency.

Each dissipative mobility matrix has a high information content about the dominant mode and very little information about other modes. This minimizes pollution by unwanted modes but results in a very poorly conditioned matrix. For example, the 10x10 imaginary acceleration mobility of a typical helicopter measured at 3 Hz has an extreme modal mobility ratio of \(10^6\). However, the sum of mobility matrices over the frequency range is a matrix having the same modal vectors as a mobility matrix at any one frequency.

\[
[Y_I] \omega^p_i = \sum_{i=1}^{N} Y_i \omega_p \{\phi_i\} \{\phi_i\}^T = [\phi] [Y_I] \omega_p \{\phi\}^T \tag{52}
\]

\[
\sum_{\omega} [Y_I] \omega_i = \sum_{i=1}^{n} \sum_{\omega} Y_i \omega_i \{\phi_i\} \{\phi_i\}^T = [\phi] [\Sigma Y_I] \omega_i \{\phi\}^T \tag{53}
\]

Therefore, Equation (53) can be used as one of the matrices in the modal eigenvector equations

\[
[Y_I] \omega^{-1}_w \Sigma Y_I \{\gamma\}_i = \lambda \{\gamma\}_i \tag{54}
\]

\[
[Y_I] \omega^{-1}_w \Sigma Y_I \{\phi\}_i = \alpha \{\phi\}_i \tag{55}
\]

The range of values from the maximum to the minimum in \(\Sigma Y_i \omega^{-1}_w\) is very small compared to the range of \(Y_i \omega^{-1}_w\).
If $\Sigma[Y^I_\omega]$ or $\Sigma[Y^R_\omega]$ is used in place of $\Sigma[Y^T_\omega]$ it is necessary to normalize each of the matrices in the sum because the displacement and velocity mobilities decrease in magnitude with increased frequency. Normalization on the root mean square of the matrix elements and on the largest element absolute value were both investigated experimentally. Normalization on the RMS gave results about as satisfactory as those from acceleration mobility and is preferred over normalization on the largest element, as the latter is sensitive to errors in one term which could throw off the entire matrix. However, the differences in results, while evident, were not dramatic.

The $\log_{10}$ of the ratio of the maximum $\Sigma[Y^T_\omega]$ to the minimum was generally about .75 for the 5x5 models in these experiments and generally around 1.8 for the 15 x 15 models. The 5 x 5 models performed excellently but the 15 x 15 models performed capriciously.

If the engineer could normalize so that the matrix $[\Sigma[Y^T_\omega]]$ is unity, information would still be lost in the inversion but certainly less information than if the ratio of extreme values is very high. The $\Sigma[Y^T_\omega]$ terms are not the eigenvalues of $\Sigma[Y^I_\omega]$. The only matrix which has a unit eigenvalue matrix is the unit matrix itself; it follows therefore that some information is always lost in the numerical inversion of any matrix other than unity.

The matrix we wish to invert is

$$\Sigma[Y^I_\omega] = [\phi] \begin{bmatrix} \Sigma[Y^*,\omega^*] \\ \Sigma[Y^*,\omega] \end{bmatrix} [\phi]^T$$ (56)

Express the modal vector matrix in terms of its own eigenvectors $[J]$ and its own eigenvalues $\lambda_\phi$ (that is, $[J]$ is the eigenvector matrix of the eigenvector matrix of $[k]^{-1}[m]$).

$$[\phi] = [J] \lambda_\phi [J]^{-1}$$ (57)

Substitute Equation (57) into Equation (56).
The only operation on the eigenvectors $[J]$ between Equation (58) and Equation (59) was to change relative positions; all the inversions were of diagonal matrices. As a diagonal matrix is a matrix of its own eigenvalues, having the unit matrix for eigenvectors, the central term may be treated rigorously as an eigenvalue matrix. The matrix of Equation (58) may be substituted for $[A]$ in Equation (41), and in Equation (50), we can consider $[A]$ as the product of the three matrices of Equation (56).

$$
\Sigma \{Y^I/\omega\} = [A] = [\phi] \cdot [\Sigma Y^I/\omega I] [\phi]^T
$$

It is well known that

$$
||[A]|| \leq ||[\phi]|| \cdot ||[\Sigma Y^I/\omega I] [\phi]^T||
$$

and that

$$
||[\Sigma Y^I/\omega I] [\phi]^T|| \leq ||[\Sigma Y^I/\omega I]|| \cdot ||[\phi]||
$$

Therefore

$$
||[A]|| \leq ||[\phi]|| \cdot ||[\Sigma Y^I/\omega I]|| \cdot ||[\phi]||
$$

At this point we wish to substitute eigenvalues, but $[\phi]$ is not symmetric so $||[\phi]|| \neq |\max \lambda_\phi|$. Rather, $||[\phi]|| \geq |\max \lambda_\phi|$. Consider, therefore, the eigenvalues $\lambda_b$ of $[\phi]^T[\phi]$ which is symmetrical.

$$
[\phi]^T[\phi] = [L] \cdot \lambda_b \cdot [L]^T = [L] \cdot \lambda_b \cdot [L]^T
$$


$$
||[\phi]|| = |\max \sqrt{\lambda_b}|
$$
Substitute Equation (65) into Equation (63).

\[ ||[A]|| \leq |\max \lambda_b| \cdot |\max \Sigma Y_{\omega i}| \quad (66) \]

Using Equation (51), the number of digits lost in inverting \( \Sigma[Y_\omega] \) is approximated by

\[ q \approx \log_{10} \frac{|\max \lambda_b| \cdot |\max \Sigma Y_{\omega i}|}{|\min \lambda_b| \cdot |\min \Sigma Y_{\omega i}|} \]

\[ = \log_{10} |\max \lambda_b| + \log_{10} \frac{|\max \Sigma Y_{\omega i}|}{|\min \Sigma Y_{\omega i}|} \quad (67) \]

\( [\lambda_B] \) would equal a scalar times the unit matrix only if the modal vectors \( \{\phi\} \) were orthogonal (i.e., \( \{\phi_i^T\} \{\phi_j\} = 0 \)), a condition which could occur only in the academic cases of uniform mass: \( [m] = [m][I] \). In this case, the loss of information digits would be indicated by

\[ q \approx \log_{10} \frac{|\max Y_{\omega i}|}{|\min Y_{\omega i}|} \quad (68) \]

and only in this case could zero information loss be achieved by normalizing the matrices such that \( |\max Y_{\omega i}|/|\min Y_{\omega i}| = 1 \).

But the case is trivial, for if it were true, an inversion would be unnecessary as \( \phi \) would be the eigenvector matrix of \( \Sigma[Y_\omega] \).

If the mass distribution is not uniform diagonal but the engineer could so normalize the matrices in the summation so that \( |\max Y_{\omega i}|/|\min Y_{\omega i}| = 1 \), it is seen from Equation (67) that there would still be a loss of information digits approximated by
The ratio $|\max \lambda_b|/|\min \lambda_b|$ increases with the order of the mobility matrix; that is, with the number of degrees of freedom of the model. It follows, therefore, that there is an upper limit to the size of a physically meaningful reduced complete model regardless of normalization of the matrices in the summation.

As a crude "rule of thumb", Figure 1 shows the trend in the reliability of the inversion of $\Sigma[Y^T\omega]$.

![Figure 1: Reliability of the Inversion of $\Sigma[Y^T\omega]$.](image)

It is seen that the reliability of the calculation becomes questionable above 10 or 15 degrees of freedom. This does not mean that accurate identifications cannot be made using the iterative step for modes of, say, 20 degrees of freedom but, rather, that any one calculation has a higher probability of failure.

In passing, it should be noted that the treatment of bounds using matrix norms, as above, opens up some highly promising avenues of research on the reliability of many helicopter
theoretical calculations as well as on the reliability of the
processing of test data in general. Whether, for example,
some of the conventional methods of processing strain gage
data yield physically meaningful results is open to question
in the light of the above method.

WHEN A CALCULATION FAILS

The most common mode of failure of iteration on $\left[ Y_\omega \right] \left( \sum \left[ Y_\omega \right] \right)^{-1}$
is catastrophic, producing such absurd values for one or more
generalized masses as negative numbers or unusually large
numbers. This is signified also by a very large number of
iterations required for convergence on one or more modes.
Failures almost never occur with small number of degrees of
freedom (e.g., five), and an identification which is quite
accurate with one seed may, in the larger models, diverge with
another seed.

This phenomenon results from the fact that the significant
effect of error is not insidious accumulation of inaccuracies
in the generalized masses but, rather, information destruction
in the inversion. Fortunately, it is usually very obvious to
the engineer when an identification fails on the computer, and
corrective measures may be taken without rerunning the test
on the helicopter.

A most obvious and effective corrective measure is to elimi-
nate one or more of the degrees of freedom. This can be done
on the computer, as the program is written so that the system
may be instructed to select any of the available data which
is in digital form on tape. The size of the model and the
number of modes covered are consequently reduced. It is
possible also to eliminate any mode, not just the highest,
if it appears that a certain mode contributes little in-
formation - a local resonance, for example, in which only a
small portion of the helicopter is significantly responding.

The computer experiments included a local resonance in the
form of a mode in which only the most forward station showed
substantial motion. When this station was not included
in the identification, but the local resonance associated
with its movement was included, then, as expected, there
were evidences of failure in generalized mass calculation.
The computer was attempting to identify a natural mode for
which the input mobility data showed a largely nonresponding
helicopter. This situation would be detected from the
mobility plots before committing the data, as it is
very apparent in the dissipative mobility spectra. The
ability to handle local resonances, or dispose of them when required, is important to a practical identification because all real structures have them. In fact, as the number of degrees of freedom of a simulated structure are realistically increased, the modal density usually increases more rapidly than the simple mathematics of uniform chains would lead one to believe. When that degree of freedom which is the predominant motion of a local resonance is eliminated, the mode it causes should be eliminated also; the mobility spectra plots for the included degrees of freedom would indicate this by an insignificant peak.

**THE Γ MATRIX AND MODAL PARAMETERS**

The dominant modal vector at frequency \( \omega_i \), near the i-th natural frequency \( \Omega_i \), is given by Equation (55) and the i-th gamma vector by iteration on the transpose, Equation (54). The modal mobility is obtained from

\[
\{\gamma_i\}^T \begin{bmatrix} \gamma_i \end{bmatrix} = \gamma_i^{*i} \]  

where \( \{\gamma_i\} \) is the vector from iteration (Equation 54). It is impractical to attempt the calculation using \( \{\gamma_i\} \) from \( [\phi]^T \) because of information loss in the inversion, as shown in Equation (59). The dominant mode is the only one used, of course, as there is negligible information content in \( \{\gamma_i\} \) about modes other than the i-th. Therefore,

\[
[Y_i^{\omega_i}] = \gamma_i^{*i} \{\phi_i\}^T \{\phi_i\} = \gamma_i^{*i} \{\phi_i\}^T \{\phi_i\} \]  

and \( \{\gamma_i\}^T \{\phi_i\} = 1 \) is forced.

A peculiar situation often occurred when a calculation diverged: it was noticed that the natural frequency of the "bad" mode was usually identified with great accuracy although the calculated generalized mass was absurd, often negative, and negative calculated values of \( Y_i^{*R} \) often occurred.
The key here is the occurrence of negative values of $\dot{Y}_{i\omega}^R$. Ideally, $\dot{Y}_{i\omega}^R$ is a positive definite matrix and cannot, theoretically, be negative definite on grounds that it represents the dissipation, not a source, of energy. For any positive definite matrix $B$, $(x)^T[B](x)$ is a positive number regardless of the choice of the vector $x$. The fault for negative values of $\dot{Y}_{i\omega}^R$, which are physically impossible, cannot therefore be laid solely to $\{y\}$, and therefore to the loss of information in the inverse of $\sum_\omega [\dot{Y}_{i\omega}^R]$, because

$\{y\}^T[\dot{Y}_{i\omega}^R]\{y\}$ must be positive even for arbitrary $\{y\}$ if $[\dot{Y}_{i\omega}^R]$ is, as it is supposed to be, positive definite. We are forced to conclude that numerical errors can act in such a way as to make $[\dot{Y}_{i\omega}^R]$ not positive definite.

The mobility $[\dot{Y}_{i\omega}^R]$ is very nearly equal to the positive semi-definite matrix $[\dot{Y}_{i\omega}^R\{\phi_i\}\{\phi_i\}^T]$ in which $\dot{Y}_{i\omega}^R$ is necessarily positive. Then

$$\{y\}^T[\dot{Y}_{i\omega}^R\{\phi_i\}\{\phi_i\}^T]\{y\} = \dot{Y}_{i\omega}^R\{\gamma_i\}^T[\dot{Y}_{i\omega}^R]\{\gamma_i\}$$

But $\{y\}$ and $\{\phi_i\}$ are composed of real numbers, as opposed to imaginary or complex numbers, which makes $\{y\}^T[\dot{Y}_{i\omega}^R]\{\phi_i\}$ real and $\{\gamma_i\}^T[\dot{Y}_{i\omega}^R]\{\gamma_i\}$ real and positive even for arbitrary elements in $\{\gamma\}$. The dominance of $[\dot{Y}_{i\omega}^R]$ by one mode is therefore not a cause of calculating negative values of $\dot{Y}_{i\omega}^R$.

The calculation of absurd values of $\dot{Y}_{i\omega}^R$ is nevertheless due mainly to information loss in inverting $[\dot{Y}_{i\omega}^T]$ (or other normalized mobility matrices having similar properties), which results in poor eigenvectors in the iteration. Examination of the computer experiments shows that errors in the $[\dot{Y}_{i\omega}^R]$ or $[\dot{Y}_{i\omega}^T]$ matrices are not sufficient to cause as erratic results as have sometimes been observed if the $\{y\}$ vectors in $\{y\}^T[\dot{Y}_{i\omega}^R]\{y\}$ are accurate. In the "bad" cases, the $\{y\}$ vectors from iteration are invariably very bad. The reason for the occasional negative calculated values of $\dot{Y}_{i\omega}^R$ is, in part,
that errors in \([Y]\) can cause the matrix to not be positive definite. For example, in Computer Experiment 188 a nine-point identification with error yielded good results but the same identification with a different seed (Computer Experiment 184) gave poor results which included negative \(\dot{Y}\) for the seventh mode. The errors by chance happened to act in such a way in Experiment 184 that excessive information was lost in the inverse, as indicated by iterations that failed to converge. The principal minor associated with the eighth and ninth positions in mobilities dominated by the seventh mode was found to be negative in the bad case (Experiment 184), due to a peculiar accumulation of random errors, which, of course, meant that the mobility was no longer positive definite, as in pure theory, and could give negative values of \(\{\gamma\}_i^T\{Y\}_i\{\gamma_i\} \). However, precise \(\{\gamma\}\) vectors would not have caused the negative values of \(\dot{Y}_i\) even with \([Y]\) not being positive semidefinite.

Calculation of physically meaningless values of \(\dot{Y}_i\), and therefore of \(M^*\), is caused primarily by information loss in inversion.

The reason for fairly accurate identifications of natural frequencies even when the generalized mass identifications are poor lies in the fact that \(\omega_j\) and \(\omega_k\) in Equation (22) are taken near \(\Omega_i\); therefore,

\[
\omega_j^* z_i^* \omega_k - \omega_k^* z_i^* \omega_j
\]

\[
\omega_k^* z_i^* \omega_k - \omega_j^* z_i^* \omega_j
\]

\[
(73)
\]

and

\[
\Omega_i^2 = \omega_j^* \omega_k \frac{\omega_j^* z_i^* \omega_k - \omega_k^* z_i^* \omega_j}{\omega_k^* z_i^* \omega_k - \omega_j^* z_i^* \omega_j} \approx \omega_j^* \omega_k = (\Omega_i - \delta \omega_j)(\Omega_i + \delta \omega_k)
\]

\[
\Omega_i^2 = \omega_j^* \omega_k \frac{\omega_j^* z_i^* \omega_k - \omega_k^* z_i^* \omega_j}{\omega_k^* z_i^* \omega_k - \omega_j^* z_i^* \omega_j} \approx \omega_j^* \omega_k = (\Omega_i - \delta \omega_j)(\Omega_i + \delta \omega_k)
\]

\[
\omega_j^* \omega_k = \Omega_i^2 + \Omega_i(\delta \omega_k - \delta \omega_j) - \delta \omega_j \delta \omega_k
\]

But \(\delta \omega_k \ll \delta \omega_j\) so \(\Omega_i^2 \approx \omega_j^* \omega_k\).
IDENTIFIED GENERALIZED MASSES

Typical generalized mass identifications are shown in Tables I through VI. Note in Tables I, III and V that the generalized mass of the first mode identified for a reduced model with no experimental error has always been less than the first mode generalized mass calculated from the modal vector and mass matrix of the specimen. This is not true of other modes.

Tables I and II show results of two different five-point models. No outstanding differences between the models is evident. Model 9A produced acceptable results, as shown in Table III, for different distribution of random error but Model 9B, as shown in Table IV, worked with some seeds and failed with other seeds. The failed experiments of Table IV, Computer Experiments 168 and 184, yielded drastically unrealistic values of generalized mass for most of the modes.

Table V shows a twelve-point model identification which failed only in the eighth mode. Computer Experiment 178 is identical to Computer Experiment 169 except that in the former, the computer was instructed to skip the eighth mode and, instead, operate on tape data for the thirteenth mode which resulted in satisfactory identification.

Using different stations for a twelve-point model, as shown in Table VI, produced proper identification of all models, including the eighth, with various error distributions.

Information loss in the inversion of mobility matrices is the primary cause of such failures, as shown in Computer Experiments 168, 184 and 169. The averaging of mobility test data, properly done, would greatly minimize the chances of such identification failures. Test data averaging is the customary practice. These computer experiments did not take advantage of averaging experiments.
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<td>±1</td>
<td>±1</td>
<td>±1</td>
<td>-</td>
</tr>
<tr>
<td>Seed</td>
<td>287</td>
<td>206</td>
<td>395</td>
<td>619</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stations (In.)</th>
<th>Mode</th>
<th>Generalized Masses (Lb-Sec²/In.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>7.4445 7.5891 7.6797 7.0969 8.5341</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>4.2851 4.4084 23.2234 4.5730 4.5615</td>
</tr>
<tr>
<td>120</td>
<td>3</td>
<td>.4741  .4545  .6876  .4314  .4951</td>
</tr>
<tr>
<td>180</td>
<td>4</td>
<td>1.0194 1.0226 28.5896 1.0968 1.0872</td>
</tr>
<tr>
<td>240</td>
<td>5</td>
<td>.6343  .6740  .5667  -7.9847  .6302</td>
</tr>
<tr>
<td>280</td>
<td>6</td>
<td>.7020  .6987  -8.5143  .5237  .7429</td>
</tr>
<tr>
<td>320</td>
<td>7</td>
<td>1.1877 1.0711  -.0080  .0125  1.1769</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
<td>1.2510 1.7815  .1256  -.2199  1.4683</td>
</tr>
<tr>
<td>460</td>
<td>9</td>
<td>.9347  .9398  -.0159  -.0810  .9836</td>
</tr>
</tbody>
</table>

* Model 9B

** From 20 x 20 Model
### TABLE V. IDENTIFICATION OF GENERALIZED MASSES, 12 X 12 MODEL* OF 20 X 20 SPECIMEN

<table>
<thead>
<tr>
<th>Computer Experiment Number</th>
<th>169</th>
<th>178</th>
<th>1**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Amp Error</td>
<td>±5%</td>
<td>±5%</td>
<td>0</td>
</tr>
<tr>
<td>Bias Amp Error</td>
<td>±5%</td>
<td>±5%</td>
<td>0</td>
</tr>
<tr>
<td>Random Phase Error</td>
<td>±1°</td>
<td>±1°</td>
<td>0</td>
</tr>
<tr>
<td>Seed</td>
<td>492</td>
<td>492</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stations (In.)</th>
<th>Mode</th>
<th>Generalized Masses (Lb-Sec²/In.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>7.4551</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>4.1298</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>0.4587</td>
</tr>
<tr>
<td>120</td>
<td>4</td>
<td>1.0446</td>
</tr>
<tr>
<td>160</td>
<td>5</td>
<td>0.5950</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
<td>0.6869</td>
</tr>
<tr>
<td>240</td>
<td>7</td>
<td>1.2036</td>
</tr>
<tr>
<td>280</td>
<td>8</td>
<td>-7.9616</td>
</tr>
<tr>
<td>320</td>
<td>9</td>
<td>0.9410</td>
</tr>
<tr>
<td>370</td>
<td>10</td>
<td>0.0425</td>
</tr>
<tr>
<td>430</td>
<td>11</td>
<td>0.1718</td>
</tr>
<tr>
<td>460</td>
<td>12</td>
<td>1.0012</td>
</tr>
<tr>
<td>480</td>
<td>13</td>
<td>0.7924</td>
</tr>
</tbody>
</table>

* Model 12D

** From 20 x 20 Model
# TABLE VI. IDENTIFICATION OF GENERALIZED MASSES, 12 X 12 MODEL* OF 20 X 20 SPECIMEN

<table>
<thead>
<tr>
<th>Computer Experiment Number</th>
<th>150</th>
<th>149</th>
<th>155</th>
<th>163</th>
<th>1**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Amp Error</td>
<td>0</td>
<td>+5%</td>
<td>+5%</td>
<td>+5%</td>
<td>0</td>
</tr>
<tr>
<td>Bias Amp Error</td>
<td>0</td>
<td>+5%</td>
<td>+5%</td>
<td>+5%</td>
<td>0</td>
</tr>
<tr>
<td>Random Phase Error</td>
<td>0</td>
<td>+1°</td>
<td>+1°</td>
<td>+1°</td>
<td>0</td>
</tr>
<tr>
<td>Seed</td>
<td>-</td>
<td>23</td>
<td>492</td>
<td>87</td>
<td>-</td>
</tr>
</tbody>
</table>

## Stations (In.) Mode  Generalized Masses (Lb-Sec^2/In.)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>7.9718</th>
<th>7.7160</th>
<th>7.2917</th>
<th>7.4071</th>
<th>8.5342</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2</td>
<td>4.6071</td>
<td>4.5010</td>
<td>4.2722</td>
<td>4.3406</td>
<td>4.4491</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>3</td>
<td>.4941</td>
<td>.4640</td>
<td>.4682</td>
<td>.4611</td>
<td>.4951</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>1.0857</td>
<td>1.0499</td>
<td>1.0625</td>
<td>1.0425</td>
<td>1.0872</td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>5</td>
<td>.6348</td>
<td>.6094</td>
<td>.5958</td>
<td>.5936</td>
<td>.6302</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>6</td>
<td>.7441</td>
<td>.7155</td>
<td>.6930</td>
<td>.7097</td>
<td>.7429</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>7</td>
<td>1.1765</td>
<td>1.1433</td>
<td>1.1101</td>
<td>1.1278</td>
<td>1.1769</td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>8</td>
<td>1.4158</td>
<td>1.3467</td>
<td>1.3225</td>
<td>1.3454</td>
<td>1.4115</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>9</td>
<td>.7808</td>
<td>.7329</td>
<td>.7395</td>
<td>.7362</td>
<td>.7866</td>
<td></td>
</tr>
<tr>
<td>340</td>
<td>10</td>
<td>.0430</td>
<td>.0419</td>
<td>.0422</td>
<td>.0422</td>
<td>.0432</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>11</td>
<td>.1705</td>
<td>.1596</td>
<td>.1665</td>
<td>.1689</td>
<td>.1723</td>
<td></td>
</tr>
<tr>
<td>460</td>
<td>12</td>
<td>.9112</td>
<td>.5712</td>
<td>.8417</td>
<td>1.0946</td>
<td>1.3235</td>
<td></td>
</tr>
</tbody>
</table>

* Model 12B

** From 20 x 20 Model
Figures 2 through 7 portray typical acceleration response obtained from the various models investigated in the present study. In each instance, the exact curve was obtained from the twenty-point structure with zero error. Figure 2 indicates the effect of random number seed for a typical five-point model. Figure 3 presents the results obtained for one of the nine-point models considered in the investigation. Figure 4 portrays the effect of random number seed on the twelve-point model. All the computer experiments which considered error used a ±5% random, 5% bias and a 1° phase error.

Figure 5 presents the effect of model variation on the acceleration response. The models varied in that different spanwise masses were considered. Model 5A utilized stations 0, 120, 220, 340 and 460 (inches) whereas model 5B consisted of stations 0, 100, 200, 320, and 460 (inches). Figure 6 presents the effect of model for the nine-point model. The model 9A consisted of stations 0, 30, 100, 160, 220, 280, 340, 400 and 460 (inches). Model 9B included stations 0, 60, 120, 180, 240, 280, 320, 400 and 460 (inches). The twelve-point model 12B used stations 0, 30, 60, 100, 140, 180, 220, 260, 300, 340, 400 and 460 (inches) whereas model 12E utilized stations 0, 30, 60, 100, 120, 160, 200, 260, 280, 340, 400, 460 (inches). For each model the computer experiments were executed using the same random number seed and the aforementioned errors were incorporated.
Figure 2. Five-Point Model Response Obtained From Equations With Identified Parameters; Driving Point at Hub.
Figure 3. Nine-Point Model Response Obtained From Equations With Identified Parameters, Driving Point at R/t.s.
Figure 4. Twelve-Point Model Response Obtained From Equations With Identified Parameters; Driving Point at Hub.
Figure 5. Five-Point Model Response, Effect of Model; Driving Point at Station 1.
Figure 6. Nine-Point Model Response, Effect of Model; Driving Point at Station 1.
Figure 7. Twelve-Point Model Response, Effect of Model; Driving Point at Station 3.
CONCLUSIONS

1. The equations of motion for a structure may be obtained using only impedance-type test data without the use of an intuitive mathematical model.

2. The method also yields the eigenvector or mode shape and generalized mass corresponding to each natural frequency.

3. The accuracy of the dynamic response of a structure using impedance-type experimental data is not dependent on the accuracy of the test measurements, provided the data is within the state of the measurement art.

4. The mass matrix assumed for an intuitive mathematical model should be fully populated to yield accurate dynamic response results.

5. To insure minimum information loss in the inversion of mobility matrices, the averaging of mobility test data should be used in practice.

6. There is an upper limit to the size of a physically meaningful reduced complete model yielding minimum loss of information digits. The present report indicates the maximum to be a model of approximately 15 degrees of freedom.
LITERATURE CITED


2. Berman, A., STUDY OF INCOMPLETE MODELS OF DYNAMIC STRUCTURES, Kaman Aerospace Corporation; Goddard Space Flight Center, Greenbelt, Maryland, R-826, January 1970.


APPENDIX
COMPUTER PROGRAM DESCRIPTION

Note: All integer variables must be right justified with no decimal point.

Tape, Card Reader and Printer Assignments.

1 Card Reader
3 Printer
9 Contains influence coefficient matrix for use in XACT.
10 Tape assignment in XACT program. Contains mobility data for all degrees of freedom, with no error for specified frequencies for use in INXACT program.
11 Tape assignment in INXACT program. Contains mobility data with reduced stations and error (i.e., simulated test data) for use in program IDENxFRE.

All input data must be in the following units.

Mass - lb-sec^2/in.
Stiffness - lb/in.
Frequencies - Hz
### PROGRAM XACT

<table>
<thead>
<tr>
<th>Card</th>
<th>1</th>
<th>Columns</th>
<th>1</th>
<th>IC</th>
<th>Program Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2-80</td>
<td>HEAD Case Description</td>
</tr>
<tr>
<td>Card</td>
<td>2</td>
<td>Columns</td>
<td>1-10</td>
<td>ND</td>
<td>Number of Degrees of Freedom (&lt; 20)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11-20</td>
<td>G</td>
<td>Structural Damping Coefficient</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>21-30</td>
<td>NC</td>
<td>Number of Modes to be Obtained From Matrix Product [C][M]. If NC = 0, K is not inverted.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>31-40</td>
<td>NK</td>
<td>Number of Modes to be Obtained from Matrix Product [K]^{-1}[M].</td>
</tr>
<tr>
<td>Card(s)</td>
<td>3</td>
<td></td>
<td>M</td>
<td>Mass Matrix. (8E10.0 Format). For full symmetric matrix load lower triangular matrix only starting each row on a new card and ending with the diagonal element. Use as many cards as necessary. For a diagonal mass matrix, load one blank card followed by cards containing diagonal elements in sequence (8E10.0 Format).</td>
<td></td>
</tr>
<tr>
<td>Card(s)</td>
<td>4</td>
<td></td>
<td>K</td>
<td>For direct loading of K matrix from cards, proceed as for M matrix as described above.</td>
<td></td>
</tr>
<tr>
<td>C Matrix</td>
<td>Option</td>
<td>C</td>
<td>To load C matrix from TAPE 9, load one blank card. This will read C matrix from TAPE 9. Unformatted record contains heading (20 words, first character blank); NX (order of matrix). Force deflection influence coefficient matrix.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Card 5 Columns 1-5 NF Number of Frequencies Used (< 100)
Print Control of Data Written on TAPE 10.
IP1 = 0 No Printed Output Except List of Frequencies
IP1 = 1 Print Full Mobility Matrix, Real and Imaginary at Each Frequency
IP1 = 2 Print Only Diagonal Elements and Row of Mobility Matrix, Real and Imaginary at Each Frequency

Columns 11-15 IP2 Control on Printed Output
IP2 = 0 Same as Written on Tape Above, Complex Velocity Mobility Matrix at Each Frequency
IP2 = 1 Print Acceleration Amplitude and Phase Angle

16-20 NRW This is the row to be printed when IP2 = 2.
If NRW = 0 then only diagonal (driving point) elements are printed as output.

Card(s) 6 HZ Frequencies in Hertz. 10 Columns Per Value, 8 Values Per Card (100 Maximum). Format (8F10.2)
Omit if NF = 0

Card 7 Frequency sweep control. This card is the same as Card 5 except that TAPE 10 is not written. To get response data with no tape use a blank card for Card 5 followed by Card 6. To generate TAPE 10 and print no other response data follow Card 5 by one blank card for Card 6. Both options indicated by Card 5 and Card 6 may be used simultaneously.

Card 8 Column For termination of Case Use 1 in Column 1. Blank card indicates another case to follow, beginning with card 1 again.
## PROGRAM INXACT

<table>
<thead>
<tr>
<th>Card</th>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card 1</td>
<td>Column 1</td>
<td>IC Program Control</td>
</tr>
<tr>
<td>Card 2</td>
<td>Columns 1-10</td>
<td>NR Number of Points Tested (Number of Degrees of Freedom of the Model)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11-20 PCT Random Error Applied to Amplitude, Uniform between - and + PCT Element Amplitude.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21-30 PCTB Bias Error Applied to Amplitude. PCTB Element Amplitude.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31-40 PHE Random Error in Degrees Applied to Phase Angle. Uniform between -PHE and +PHE.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71-80 IZ Random Number Seed.</td>
</tr>
<tr>
<td>Card 3</td>
<td>KEEP</td>
<td>Stations to be used in model. Card 3 is included only if NR &lt; ND (From Program XACT). Five columns per value, maximum of 10 values per card (Format 10I5)</td>
</tr>
<tr>
<td>Card 4</td>
<td>Columns 1-5</td>
<td>NFR Number of Frequencies to be Used (From TAPE 10, XACT Program) IF NFR = 0 all frequencies on TAPE 10 are to be used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6-10 IP1 Same Definitions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11-15 IP2 as in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16-20 NRØW XACT Program</td>
</tr>
<tr>
<td>Card(s) 5</td>
<td>INDX</td>
<td>Indices of Frequencies to be Used from TAPE 10 XACT Program. Indices must be in ascending order. Five columns per value, 16 values per card (Format 16I5).</td>
</tr>
<tr>
<td>Card</td>
<td>Columns</td>
<td>IC</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>Card</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Card</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Card(s)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Card(s)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Card(s)</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

From INXACT Program to be Used in Summation of Real Parts of Mobility Elements (NFR Frequencies. Must be in Ascending Order) Five Columns Per Value, 16 Values Per Card (Format 16I5)

Indices of Frequencies to be Used for Phi Iteration (MODE SHAPE). Same Number of Indices as the Number of Degrees of Freedom of the Model. Indices in Ascending Order.

Indices of Frequencies to be Used in Forming $y_{i}^{*}$ and $z_{i}^{*}$ in the Calculation of Generalized Mass and Natural Frequency ($2* Number of Degrees of Freedom of the Model$). Indices in Ascending Order.
<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>1-5</th>
<th>NF</th>
<th>No. of Frequencies at Which Reidentification of Mobility Matrices is Calculated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-10</td>
<td>IP1</td>
<td></td>
<td></td>
<td>Print Control of Mobility Data</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IP1 = 0 No printed output except list of frequencies</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IP1 = 1 Full matrices printed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IP1 = 2 Diagonal elements and row printed</td>
</tr>
<tr>
<td>11-15</td>
<td>IP2</td>
<td></td>
<td></td>
<td>IP2 = 0 Complex velocity mobilities printed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IP2 = 1 Acceleration mobilities printed Amplitude in g's and phase in degrees</td>
</tr>
<tr>
<td></td>
<td>NROW</td>
<td></td>
<td></td>
<td>This is Row to be Printed when IP1 = 2. If Equal to Zero the Only Diagonal (Driving Point) Elements are Printed</td>
</tr>
<tr>
<td>16-20</td>
<td>NN</td>
<td></td>
<td></td>
<td>Controls Type of Damping Used in Reidentification of Mobilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NN = 0 Use Scalar Structural Damping Coefficient *K Matrix</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NN = 1 Use Damping Matrix</td>
</tr>
<tr>
<td>Card(s)</td>
<td>7</td>
<td></td>
<td></td>
<td>HZ Frequencies at Which Reidentification is Calculated Ten Columns Per Value, 8 Values Per Card (Format 8F10.0).</td>
</tr>
<tr>
<td>Card</td>
<td>8</td>
<td>Column 1</td>
<td></td>
<td>A 2 in Column 1 Terminates Program Otherwise Return to Card 1 for Beginning of New Case</td>
</tr>
</tbody>
</table>
COMPUTER PROGRAM FORTRAN LISTING

C       XACT XACT XACT XACT XACT XACT XACT XACT
INTEGER HEAD(20), HEAD1(20), IT(20), ITR(20)
INTEGER HT(7)
REAL M(20,21), K(20,21), C(20,21), A(20,21), B(20,21), PHI(2,21),
A FRE(20), DUM(20), GM(20), MU(20,21), PHI(2,21), FRE(20), G(21)
REAL MZ(100), XR(20,21), YR(20,21), Y1(20,21), DPI(100,20),
A DPI(100,20), TR(100,20), T(100,20)
LOGICAL TORF, TAPE
DATA HT(*,EXACT,*), TA *
READ INPUT
C        100 READ (1,110) IC, HEAD
C        110 FORMAT (11,A3,19A4)
C        IF (IC .NE. 0) GO TO 700
C        READ (1,120) ND, G, NC, NK
C        120 FORMAT (110,F10.0,2110)
C        ND1=ND-1
C        READ (1,130) M(1,1)
C        130 FORMAT (8E10.0)
C        IF (M11 .NE. 0) GO TO 150
C        DIAGONAL MASS
C        DO 140 I=1,ND
C        DO 140 J=1,ND
C        140 M11,J=0
C        READ (1,130) (M11,J), I=1,ND
C        GO TO 170
C        FULL MASS MATRIX
C        150 DO 160 I=2,ND
C        160 READ (1,130) (M11,J), J=1,I
C        CALL SYM (M, ND)
C        170 READ (1,130) K11, I
C        IF (K11 .EQ. 0) GO TO 190
C        K INPUT
C        DO 180 I=2,ND
C        180 READ (1,130) (K11,J), J=1, I
C        CALL SYM (K, ND)
C        GO TO 230
C        FROM TAPE
C        190 READ (9) HEAD1, NX, IC1(J), J=1, NX
C        IF (ND .EQ. NX) GO TO 210
C        WRITE (3,201) HEAD1, HEAD1
C        200 FORMAT (10X,A3,19A4, 'C MATRIX WRRONG SIZE'//1S, 'TAPE HEADING:',
L10X, 'A '10X, '10X, '19A4, '11X)
C        CALL EXIT
C        INVERT AND SYMMETRIZE C
C        210 CALL INVRS (C, ND, K)
C        DO 220 I=1, ND
C        K(I,I)=1
C        DO 220 J=1,ND
C        K(I,J)*=K(J,I) / (K(J,J)+K(I,I))
C        220 K(I,J)*=K(I,J)
C        INVERT K
C        230 IF (NX .EQ. 0) AND (NC .NE. 0) CALL INVRS (K, ND, C)
C        SUM K ROWS
C        DO 240 I=1, ND

DO 380 L=1,ND
380 B(I,L)+B(I,L)-DUM(I)*DUM(L)*CON
390 IF (NC.EQ.0) GO TO 430
C MODAL OUTPUT
WRITE (3,400)
400 FORMAT (17,'/T45,'NORML MODES FROM C MATRIX'/)
CALL MOUT2 (PMI,NO,NC)
WRITE (3,410) (FREQ(J),J=1,NC)
410 FORMAT (7/ T45,'FREQUENCIES - HZ'/(T10,1DF12.6))
WRITE (3,420) (GMK(I),I=1,NK)
420 FORMAT ('/T45,'GENERALIZED MASS'/(T10,1DF12.6))
430 IF (NK.EQ.0) GO TO 450
WRITE (3,440)
440 FORMAT (17,'/T45,'NORML MODES FROM K MATRIX'/)
CALL MOUT2 (PMK,NO,NK)
WRITE (3,410) (FREQ(I),I=1,NK)
WRITE (3,420) (GMK(I),I=1,NK)
C READ TAPE CONTROLS
450 TAPE=.TRUE.,
460 READ (1,470) NF,IP1,IP2,NROW
470 FORMAT (4,1S)
IF (.NOT.TAPE .AND. IP1.EQ.0) GO TO 100
IF (NF.EQ.0) GO TO 640
TORG=NRGOT.O .AND. NROW.LE.NO
READ (1,130) (HZ(I),I=1,NF)
C FORM MOBILITY AND WRITE TAPE
IF (TAPE) WRITE (10) NF,HEAD,NO,(HZ(I),I=1,NF)
DU 570 L=1,NO
CALL MOB (MK,NO,HZ,IL,ZR,ZI,YY,YI)
IF (.NOT.TAPE) WRITE (10) HZIL,(YY(I),YI(I),I=1,NO),J=1,NO
READ (1,130) (HZ(I),I=1,NF)
480 IF(IP2.NE.0) CALL MATAMP (HZIL,YY,YI,NO)
C IF (TAPE) WRITE (3,490)
IF (TAPE) WRITE (3,490)
490 FORMAT ('/T40,'COMPLEX MOBILITY WRITTEN ON TAPE'/)
IF (.NOT.TAPE) WRITE (3,540)
IF(IP2.NE.0) GO TO 510
WRITE (3,550) HZIL
500 FORMAT ('/T40,'REAL MOBILITY, IMAGINARY MOBILITY
FREQ =*F10.2, I=1,48
A * HERTZ//'I)
GU TO 530
510 WRITE (3,520) HZIL
520 FORMAT ('/T40,'ACCELERATION AMPLITUDE IN G'SS, PHASE IN DEG.
FREQ=',I=1,52
A =*F10.2 echoes HERTZ//'I)
530 CALL MOUT2 (YY,NO,NO)
WRITE (3,540)
540 FORMAT (17,'/T40,'/)
CALL MOUT2 (YY,NO,NO)
GO TO 570
550 DO 560 I=1,NO
560 IF(NOT.TORF) GU TO 560
DPII(I)=YK(I,1)
FORMAT (1)='YK(NROW,I)
570 CONTINUE
IF (IP1-1) E 580,690,600  
580 WRITE (3,590) (H2(I,i)=1,NF)  
590 FORMAT (1/4*MOBILITY MATRICES AT THE FOLLOWING FREQUENCIES (HZ)  
A) HAVE BEEN WRITTEN ON TAPE**/(1=10,10F12.6))  
GO TO 690  
600 IF(IP2+NF.1) GO TO 620  
CALL AMP (HZ,DP,R,DP1,ND)  
IF(TORF) CALL AMP (HZ,TR,TI,ND)  
WRITE (3,610)  
610 FORMAT (**V=0.0,DRIVING POINT RESPONSE, AMP IN G**S AND PHASE IN  
DEGREES**/)  
GO TO 640  
620 WRITE (3,630)  
630 FORMAT (**V=0.0,DRIVING POINT MOBILITY, REAL AND IMAGINARY**)  
640 CALL YOUT (HZ,DP,R,ND,0)  
WRITE (3,540)  
CALL YOUT (HZ,DP1,ND,IP2)  
IF(.NOT.TORF) GO TO 690  
IF(IP2+NF.1) GO TO 660  
WRITE (3,650) NROW  
650 FORMAT (**V*T30,TRANSFER RESPONSE, ROW *5,* AMP IN G**S AND PHAS  
AE IN DEG**/)  
660 WRITE (3,670) NROW  
670 FORMAT (**V*T30,TRANSFER MOBILITY, ROW *5,* REAL AND IMAG.**)  
680 CALL YOUT (HZ,TR,TI,ND,0)  
WRITE (3,540)  
CALL YOUT (HZ,TR,TI,ND,IP2)  
690 IF (.NOT.TAPE) GO TO 100  
TAPE = .FALSE.  
GO TO 460  
700 REWIND 10  
CALL EXIT  
END
SUBROUTINE MOUT2 (A,M,N)
REAL A(20,21)
ID=MENDIN,10
WRITE (3,100) (1,i=1,ID)
100 FORMAT (/75,10112)
WRITE (3,100)
DO 110 I=1,M
110 WRITE (3,120) i,(A(I,J),J=1,ID)
120 FORMAT (15,5X,1P10E12.4)
IF (ID-N) 130,150,150
130 WRITE (3,100) (1,i=1,N)
WRITE (3,100)
DO 140 I=1,N
140 WRITE (3,120) i,(A(I,J),J=11,N)
150 RETURN
END
FUNCTION GEN (FUN,A,N)  
GEN = FJNTRANS + A * FIN  

DIMENSION A(20:21), FUN(20)  
DO 10 I = 1, N  
   DUM = 0  
   DO 10 J = 1, N  
      DUM = DUM + J * FUN(J)  
   10   CONTINUE  
   GEN = GEN + DUM * FUN(I)  
   RETURN  
END
SUBROUTINE INVRS (B,N,A)

A = INVERSE OF B
B UNDISTURBED

DIMENSION A(20,21),I20(21),IROW(21),ICOL(21),BI(20,21)

DO 100 I=1,N
DO 100 J=1,N
100 A(I,J)=G(I,J)

N=N+1
DO 110 I=1,N
IROW(I)=I
110 ICOL(I)=I
DO 120 K=1,N
AMAX= A(K,K)
DO 130 I=K,N
DO 130 J=K,N
IF(AABS(A(I,J))>ABS(AMAX))AMAX=A(I,J)
120 AMAX= A(I,J)
IC=I
JC=J
130 CONTINUE
KI=ICOL(KI)
ICOL(KI)=ICOL(IC)
ICOL(IC)=KI
KI=IROW(KI)
IROW(KI)=IROW(JC)
IROW(JC)=KI
IF(AMAX)160,40,160
140 WRITE (3,150)
150 FORMAT(* SOLUTION OF EXISTING MATRIX NOT POSSIBLE*)
GO TO 330
160 DO 170 J=1,N
E=A(K,J)
A(K,J)=A(I,J)
170 A(I,J)=E
DO 180 I=1,N
E=A(I,K)
180 A(I,K)=A(I,J)
DO 210 I=1,N
IF(I-K)200,190,200
190 A(I,M)=I,
GO TO 210
200 A(I,M)=0.
210 CONTINUE
PVT=A(K,K)
DO 220 J=1,M
A(K,J)=A(K,J)/PVT
DO 250 I=1,N
IF(I-K)230,250,230
230 AMULT=A(I,K)
DO 240 J=1,M
240 A(I,J)=A(I,J)-AMULT*A(K,J)
250 CONTINUE
DO 260 I=1,N
260 A(I,K)=A(I,M)
DO 290 I=1,N
SUBROUTINE MMYP (A, B, N1, N2, N3, C)

C

C

C

C

REAL A(20,21), B(20,21), C(20,21)
DO 100 I=1,N1
DO 100 J=1,N3
C(I,J)=0
DO 100 K=1,N2
100 C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END
SUBROUTINE CINV (A,B,N,C,D)

C + I * D = INVERSE OF A + I * B

I = SQRT(-1)

B ASSUMED NON SINGULAR

REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)

CALL INVRS(B,N,C)
CALL MMPP(C,A,N,N,E)
CALL MMPP(A,E,N,N,N,C)

DO 100 I=1,N
100 DO 100 J=1,N
C(I,J) = C(I,J) + B(I,J)

CALL INVRS(C,N,D)
CALL MMPP(E,D,N,N,N,C)

DO 110 I=1,N
110 DO 110 J=1,N
D(I,J) = D(I,J)
RETURN
END
SUBROUTINE MOB (K,G,N,ON.Z1,Y.R.Y1)
CALCULATES COMPLEX IMPEDANCE AND MOBILITY
K IS SQUARE COMPLEX IMPEDANCE MATRIX
G IS SQUARE STRUCTURAL DAMPING MATRIX
N IS ORDER
IMPEDEANCE = ZR + I*Z1
MOBILITY = YR + I*Y1

USES CINV, INVR, NMPY
REAL M(20,21) K(20,21) Z(20,21), Z1(20,21), YR(20,21), Y1(20,21)
ONE=3M2.83185
ONE=4M2.83185
ONE=100 J2IM IN
ONE=100 J2IM IN
ONE=100 J2IM IN
ONE=100 J2IM IN
100 CALL CINV(M,R,K,G,N)
RETURN
END
SUBROUTINE SITER (A, PHI, FRE, N, ITN, PHAX)
REAL A(20,21), PHI(20,21), FRE(22), DUM(20)
K=ND-J+1
ANG=3.14159*K/(ND-1)
AN=3.14159*J/(ND-1)
DO 100 I=1,ND
100 ANG=AN*(I-1)
ANG=ANG*(I-1)
100 PHI(I,J)=SIN(ANG)+SIN(ANGK)+1.0)/3.0
ITN=0
PNO=100.
110 DO 120 I=1,ND
DUM(I)=0.
DO 120 I=1,ND
120 DUM(I)=DUM(I)+A(I,L)*PHI(L,J)
PHAX=0.
DO 130 I=1,ND
130 PHAX=MAX(PHAX,ABS(DUM(I)))
DO 140 I=1,ND
PHI(I,J)=DUM(I)/PHAX
IFABS(PHAX/PHNO.10)-.005)301)160,160,155
150 ITN=ITN+1
PHNO=PHAX
IF (ITN=100) 110,110,140
160 FRE(I,J)=1.0/SQRT(ABS(PHAX))
RETURN
END
SUBROUTINE YOUT (OMH, A, NINC, ND, NAMP)
REAL OMH(100), A(100,20)
J1=1
ID=NINC(ND,10)
100 IF (ID=NINC,50)
   110 WRITE (3,120) (1,i=J1,ID)
   120 FORMAT (I5,*HERTZ*16,9112)
   130 WRITE (3,130)
   140 FORMAT (I1X)
   150 IF (NAMP) 140,140,170
   140 DO 150 I=1,IL
   150 WRITE(3,160) OMH(I), A(I,J),J=J1,ID)
   160 FORMAT (I1X,F9.3,1PI12.4)
   170 GO TO 200
   180 DO 180 I=1,IL
   190 FORMAT (I1X,F9.3,10F12.2)
   200 IF (I=NINC) 210,230,230
   210 WRITE (3,220)
   220 FORMAT (*1+//)
   230 IF (I=NINC) 240,250,250
   240 RETURN
END

1 1
2 2
3 3
4 4
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21 21
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23 23
24 24
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27 27
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29 29
30 30
31 31
SUBROUTINE MATAMP (OMH, A, B, NR)
C
C CONVERTS MOBILITY, A * I*b IN VEL UNITS TO
C AMP (IN A) IN G/S AND PHASE (IN B) IN DEG
C MATRICS ARE AT FREQUENCY OMH IN HERTZ
C
DIMENSION A(20,21), B(20,21)
OM=OMH=0.01626
DO 210 I=1,NR
DO 210 J=1,NR
R=AIM(I,J)
C=AIM(I,J)
A(I,J)=SQR(R*R+C*C)*OM
IF(C) 140,100,140
100 IF(R) 110,120,130
110 B(I,J)=270.
   GO TO 210
120 B(I,J)=0
   GO TO 210
130 B(I,J)=90.
   GO TO 210
140 P=ATAN(ABS(R/C))*57.2958
   IF(C) 150,150,180
150 IF(R) 160,160,170
160 B(I,J)=180.+P
   GO TO 210
170 B(I,J)=180.-P
   GO TO 210
180 IF(R) 190,190,200
190 B(I,J)=360.-P
   GO TO 210
200 B(I,J)=P
210 CONTINUE
RETURN
END
SUBROUTINE AMP (OMH,A,B,NING,NR)
C
CONVERTS A * I*B IN VELOCITY UNITS TO
C
AMP (IN A) IN G'S AND PHASE (IN B) IN DEG
C
EACH ROW IS AT A FREQUENCY OMH(I) IN HERTZ
C
DIMENSION OMH(100),A(100,201),B(100,20)
D0 210 I=1,NING
OMH=OMH(I)*0.01626
D0 210 J=1,NR
R=A(I,J)
C=B(I,J)
A(I,J)=SQRTR(M*R*C*C)*OMH
IF(C) 140,100,140
100 IF(R) 110,120,130
110 B(I,J)=700.
GO TO 210
120 B(I,J)=0
GO TO 210
130 B(I,J)=90.
GO TO 210
140 PRINTABS(R/C)*57.2958
IF(C) 150,150,180
150 IF(R) 160,160,170
160 B(I,J)=180.-P
GO TO 210
170 B(I,J)=180.-P
GO TO 210
180 IF(R) 190,190,200
190 B(I,J)=360.-P
GO TO 210
200 B(I,J)=P
210 CONTINUE
RETURN
END

AMP  1
AMP  2
AMP  3
AMP  4
AMP  5
AMP  6
AMP  7
AMP  8
AMP  9
AMP 10
AMP 11
AMP 12
AMP 13
AMP 14
AMP 15
AMP 16
AMP 17
AMP 18
AMP 19
AMP 20
AMP 21
AMP 22
AMP 23
AMP 24
AMP 25
AMP 26
AMP 27
AMP 28
AMP 29
AMP 30
AMP 31
AMP 32
AMP 33
AMP 34
AMP 35
C
XACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT I
INTEGER HEAD(20),HEAD(20),MT(7),HTM(7),KEEP(20),INDX(100)
REAL HZ(100),YR(20,21),YI(20,21),DFP(100,20),DFP(100,20),
A(100,20),YI(100,20)
LOGICAL IORF
DATA HTM/*INACT SIMULATED TEST DATA*/
C INPUT CARDS AND TAPE
READ (1,10) IC,HEAD
10 FORMAT (11,A3,19A4)
READ (10) HT,HEAD,MT,NO,HTM(1),I=1,MT)
WRITE (3) (11,10) HEAD,MT,HEAD,HEAD,NO,HTM(1),I=1,MT)
A T25,14/A(25,A3,19A4/T25,12,* DEGREES OF FREEDOM/T25,*FREQUENCIES
B(HZ) ON TAPE*/(T10,10F10.2)
READ (1,120) NR,PCT,PCTB,PHE,I2
120 FORMAT (110,3F10.0,30X,110)
I=12*2+1
WRITE (3,130) NR,PCT,PCTB,PHE,I2
130 FORMAT (*11*10,12* POINTS TESTED*/T10,*MAX RAND ERROR */F6.3,*,
A BIAS ERR RUSR *=F6.3,*,OF ELEMENTS, MAX RAND PHASE ERROR */F5.2,*,DE
B. SEED */110*/T10,*STATIONS USED *)
READ (1,150) (KEEP(I),I=1,MR
150 FORMAT (165)
WRITE (3,160) (KEEP(I),I=1,MR)
WRITE (120) (KEEP(I),I=1,MR)
READ (120) (KEEP(I),I=1,MR)
170 DO 180 I=1,NF
180 INDEX(I)=1
READ (1,150) NFR,IP1,IP2,NROW
IF (NFR.GT.0) (INDEX(I),I=1,NFR)
IF (NFR.EQ.0) NFR=NF
WRITE (11) HHM,HEAD,HEAD,NFR,NF,(HZ(INDEX(I)),I=1,NFR)
WRITE (3,190) (HZ(INDEX(I)),I=1,NFR)
190 FORMAT (**11*10,FREQUENCIES USED*/(F10,10F10.2))
TOF = NROW,GT.0.AND.,NROW.1E,MR
INF=1
DO 290 L=1,NF
290 READ (10) FREQ,((YR(I,J),YI(I,J)),I=1,ND,J=1,ND)
IF (L.NE.INDX(IFR)) GO TO 290
IF (NFR.GT.0) CALL RED (YR,YI,MR,KEEP)
IF (PCT,NE.0) OR.PCTB,NE.0.OR.PHE,NE.0) CALL ERR (YR,YI,PCT,PCTB,
A.PHE,MR)
WRITE (11) FREQ,((YR(I,J),YI(I,J)),I=1,MR,J=1,MR)
IF (IP(1)=NFR) 280,290,290
280 IF (IP2.EQ.0) GO TO 290
CALL NAMAP (FREQ,YR,YI,MR)
WRITE (3,210) FREQ
210 FORMAT (**11*10,*ACCELERATION AMPLITUDE IN G**5,PHASE IN DEG.
A FREQ */F10.2,*HZ)**
GO TO 240
220 WRITE (3,230) FREQ
230 FORMAT (**11*10,*REAL MOBILITY, IMAGINARY MOBILITY, FREQ *
A F10.2,*HZ)**
240 CALL MOUT2(YR,MR,MR)
WRITE (3,250)
250 FORMAT (**11*11/*}
CALL MOUT2(Y1, NR, NR)
INF = INF + 1
GO TO 290

260 J = INF
DO 270 I = 1, NR
DPR(I, J) = YR(I, I)
DPI(I, J) = YI(I, I)
IF (.NOT. T0RF) GO TO 270
TR(I, J) = YR(NROW, I)
270 T(I, J) = YI(NROW, I)
280 HZ(INF) = HZ(L)
INF = INF + 1
IF (INF .LE. NFR) GO TO 300
290 CONTINUE
300 IF (IP1 .LE. 2) GO TO 390
IF (IP2 .LE. 1) GO TO 320
ZAMP = (HZ, DPR, DPI, NFR, NR)
IF (T0RF) CALL AMPL (HZ, TR, T1, NFR, NR)
WRITE (3, 310)
310 FORMAT ('11///130, ' 'DRIVING POINT RESPONSE, AMP IN G**S AND PHASE IN
A DEG///)
GO TO 340
320 WRITE (3, 330)
330 FORMAT ('11///130, ' 'DRIVING POINT MOBILITY, REAL AND IMAGINARY///')
340 CALL YOUT (HZ, DPR, DPI, NR, 0)
WRITE (3, 350)
350 FORMAT ('11///130, ' 'TRANSFER RESPONSE, ROW#15, ' ' AMP IN G**S AND PH
A DEG///)
AASE IN DEG///
GO TO 380
360 WRITE (3, 370) NROW
370 FORMAT ('11///130, ' 'TRANSFER MOBILITY, ROW#15, ' ' REAL AND IMAG///')
380 CALL YOUT (HZ, TR, NFR, NR, 0)
WRITE (3, 390)
390 REWIND 10
REWIND 11
CALL EXIT
END
SUBROUTINE ERR (A,B,PCT,PCTB,PHE,N,IX)
C EACH ELEMENT OF A COMPLEX MATRIX, A * [B], IS MODIFIED TO
C INCLUDE A SMALL PHASE ERROR, PHE (DEG), A BIAS ERROR,
C PCTB (RATIO) ON AMPLITUDE, AND A UNIFORM RANDOM ERROR
C HAVING A +/- MAXIMUM OF PCT (RATIO) ON AMPLITUDE.
C THE PHASE ERROR IS ALSO RANDOMLY DISTRIBUTED
C THE RESULTING MATRIX IS SYMMETRIZED
C USES RANDU
C)
DIMENSION A(20,21),B(20,21)
IF (PCT) 120,100,120
100 IF (PCTB) 120,110,120
110 IF (PHE) 120,140,120
120 P=PHE/57.296
DO 130 I=1,N
DO 130 J=1,N
CALL RANDU (IX,IX,YFL)
IX=IX
E=2.0*P*(YFL-0.5)
A(I,J)=A(I,J)-E*B(I,J)
B(I,J)=B(I,J)+E*A(I,J)
A(I,J)=B(I,J)
CALL RANDU (IX,IX,YFL)
IX=IX
E=1.0+2.0*PCTB*(YFL-0.5)+PCTB
A(I,J)=A(I,J)-E*B(I,J)
B(I,J)=B(I,J)+E*A(I,J)
130 B(I,J)=B(I,J)
140 N1=N-1
DO 150 I=1,N1
J=I+1
DO 150 J=J,I,N
A(I,J)=A(I,J)+A(J,I))/2.0
B(I,J)=B(I,J)+B(J,I))/2.0
B(I,J)=B(I,J)
150 A(J,I)=A(I,J)
160 RETURN
END
SUBROUTINE RANDU (IX, IY, YFL)

   THIS SUBROUTINE IS FROM SSP VERS. II

   1Y=IX#65539
   [F(IY) 100. 110, 110
   100 IY=IY+2147483647+1
   110 YFL=IY
   YFL-YFL*.656613E-9
   RETURN
   END
SUBROUTINE MATAMP (OMH, A, B, NR)
C CONVERTS MOBILITY, A + i*B IN VEL UNITS TO
C AMP (IN A) IN G/S AND PHASE (IN B) IN DEG
C MATRICES ARE AT FREQUENCY OMH IN HERTZ
DIMENSION A(200,21), B(20,21)
OM=OMH=0.01626
DO 210 I=1, NR
DO 210 J=1, NR
R=A(I, J)
C=B(I, J)
4(I, J)=SQRT(R*R+C*C)*OM
IF(C) 140, 100, 140
100 IF(R) 110, 120, 130
110 B(I, J)=270.
GO TO 210
120 B(I, J)=0
GO TO 210
130 B(I, J)=90.
GO TO 210
140 P=ATAN(ABS(R/C))*57.2958
IF(C) 150, 150, 180
150 IF(R) 160, 160, 170
160 B(I, J)=180.+P
GO TO 210
170 B(I, J)=180.-P
GO TO 210
180 IF(R) 190, 190, 200
190 B(I, J)=360.-P
GO TO 210
200 B(I, J)=P
210 CONTINUE
RETURN
END
SUBROUTINE AMP (OMH,A,B,NINC,NR)
C
CONVERTS A + i*B IN VELOCITY UNITS TO
C
AMP (IN A) IN G/ S AND PHASE (IN B ) IN DEG
C
EACH ROW IS AT A FREQUENCY OMH(I) IN HERTZ
C
DIMENSION OMH(100),A(100,20),B(100,20)
DO 210 I=1,NINC
OM=OMH(I)*0.01626
DO 210 J=1,NR
R=A(I,J)
C=B(I,J)
A(I,J)=SORT0%*R*C)*OM
IF(C) 140,100,140
100 IF(R) 110,120,130
110 B(I,J)=270.
GO TO 210
120 B(I,J)=0
GO TO 210
130 B(I,J)=-90.
GO TO 210
140 P=ATAN1ABS(B/C))*S/.2958
IF(C) 150,150,180
150 IF(R) 160,160,170
160 B(I,J)=180.+P
GO TO 210
170 B(I,J)=180.-P
GO TO 210
180 IF(R) 190,190,200
190 B(I,J)=360.-P
GO TO 210
200 B(I,J)=P
210 CONTINUE
RETURN
END
SUBROUTINE RED (A, B, NO, NR, KEEP)
INTEGER KEEP (20)
REAL A(20,21), B(20,21)
DO 100 I=1, NR
DO 100 J=1, NR
A(I,J) = A(KEEP(I), KEEP(J))
100 B(I,J) = B(KEEP(I), KEEP(J))
RETURN
END
RED 1
RED 2
RED 3
LRED 4
2RED 5
2RED 6
2RED 7
RED 8
RED 9
SUBROUTINE YOUT (OMH,A,NINC,ND,NAMP)
REAL OMH(100),A(100,20)
Jl=1
ID=MIND(ND,10)
100 IL=MIND(NINC,50)
I=1
110 WRITE (3,120) (I,1=J1,1D)
120 FORMAT (15,9HERTZ*16.912)
WRITE (3,130)
130 FORMAT (1X)
IF (NAMP) J=0,1,40,170
140 DO 150 I=1,1L
150 WRITE(3,160) OMH(I),A(I,1,J=J1,1D)
160 FORMAT (1X,F9.3,1P10E12.4)
GO TO 200
170 DO 180 I=11,1L
180 WRITE(3,190) OMHI1,1A(I,1,J=J1,1D)
190 FORMAT (1X,F9.3,1P10E12.2)
200 IF (IL-NINC) 210,230,230
210 WRITE (3,220)
220 FORMAT (**1//
   
   IL=NINC
   
   GO TO 110
230 IF (ID-ND) 240,250,250
240 J=11
   
   ID=ND
   
   WRITE (3,190)
   
   GO TO 100
250 RETURN
END
SUBROUTINE MOUT2 (A,M,N)
REAL A(20,21)
ID=MIND(N,10)
WRITE (3,100) (1,I=1,10)
100 FORMAT (/TS,10112)
WRITE (3,100)
DO 110 I=1,N
110 WRITE (3,120) I,(A(I,J),J=1,ID)
120 FORMAT ([5,5X,1P10E12.4])
IF (ID-N) 130,150,150
130 WRITE (3,100) (1,I=11,10)
WRITE (3,100)
DO 140 I=1,N
140 WRITE (3,120) I,(A(I,J),J=11,N)
150 RETURN
END
C IDENTRE IDENTRE IDENTRE IDENTRE IDENTRE IDENTRE IDENTRE IDENTRE
INTEGER IEVI(20), HEADN(20), HEAD(20), HNT(1), IINDX(20), ITI(20).
IDN 1
A IDN(20, 20), IFP(20)
REAL HZ(100), YRS(20, 21), YR(20, 21), YI(20, 21), YRNS(20, 21), DUM(20),
IDN 2
A PHI(20, 21), GAMMA(20, 21), OMI(20, 21), YRSTAR(20, 21), YISTAR(20, 21),
IDN 3
B ZRSTAR(20, 21), ZLSTAR(20, 21), GAF(20), OMEGA(20), GAF(20), G(20),
IDN 4
REAL DUM(20), GAF(20, 21), FACT(20)
IDN 5
LOGICAL EXCD
IDN 6
100 READ (1, 110) IC, HEAD
IDN 7
IF (IC.GT.1) CALL EXIT
IDN 8
READ (1, 110) NNR
IDN 9
110 FORMAT (1, 13, 19, 4)
IDN 10
READ (1) HTN, HEAD, HEAN, NFR, NR, (HZ(I), I=1, NFR)
IDN 11
WRITE (3, 120) HEAD, HTN, HEAD, HEAN, NR, (HZ(I), I=1, NFR)
IDN 12
120 FORMAT (1)/T5, 124 IDENTRE '1'/T5, 1944/10, 'TAPE HEADING'/
IDN 13
A T5, 7A4/2(T5, A3, 1944/1, T25, 'ORDER OF MATRICES = 14/T5, 'FREQUENCY
IDN 14
BIES ON TAPE'/T10, 10F10.21)
IDN 15
READ (1, 130) (INDEX(I), I=1, NFR)
IDN 16
WRITE (3, 140) (HZ(INDEX(I)), I=1, NFR)
IDN 17
130 FORMAT (1, 15)
IDN 18
140 FORMAT (1)/T5, 124 'FIRST PASS FREQUENCIES'/T110, 10F10.21)
IDN 19
GO TO (150, 170, 190, 210), NNR
IDN 20
150 WRITE (3, 160) FREQ
IDN 21
160 FORMAT ('1', 'NORMALIZATION OF REAL MOBILITY BY MAX ABS(YR)'
IDN 22
GO TO 210
IDN 23
170 WRITE (3, 180)
IDN 24
180 FORMAT ('1', 'NORMALIZATION OF REAL MOBILITY BY RMS OF YR'
IDN 25
GO TO 210
IDN 26
190 WRITE (3, 200)
IDN 27
200 FORMAT ('1', 'SUMMATION OF ACCELERATION MOBILITIES'
IDN 28
C SUMM REAL PARTS AND INVERT (FIRST PASS)
IDN 29
210 DO 220 I=1, NR
IDN 30
DO 220 J=1, NR
IDN 31
220 YRS(I, J)=0
IDN 32
210 END
IDN 33
C NORMALIZATION OF MATRICES IF NNR=1, 2
IDN 34
IF (NNR.GE.INDX(NFR)) GO TO 310
IDN 35
310 DO 310 I=1, NFR
IDN 36
READ (1) FREQ, (LYRI, J, YI(I, J), I=1, NR, J=1, NR)
IDN 37
NORMALIZATION OF MATRICES IF NNR=1, 2
IDN 38
IF (NNR.LE.INDX(NFR)) GO TO 310
IDN 39
GO TO (230, 250, 270, 290), NNR
IDN 40
230 CALL YNRMN (YR, NR)
IDN 41
WRITE (3, 240) FREQ
IDN 42
240 FORMAT ('1', 'REAL MOB NORMALIZED ON YR (MAX) FREQ=*,F8.3,*HZ'
IDN 43
GO TO 290
IDN 44
250 CALL YNRMS (YR, NR)
IDN 45
WRITE (3, 260) FREQ
IDN 46
260 FORMAT ('1', 'REAL MOB NORMALIZED ON RMS OF YR FREQ=*,F8.3,*HZ'
IDN 47
GO TO 290
IDN 48
270 CALL YRFREQ (YR, FREQ, NR)
IDN 49
WRITE (3, 280) FREQ
IDN 50
280 FORMAT ('1', 'ACCELERATION MOBILITY FREQ=*,F8.3,*HZ'
IDN 51
290 CALL MQUT2 (YR, NR, NR)
IDN 52
DO 300 1=1, NR
IDN 53
DO 300 J=1, NR
IDN 54
300 YRS(I, J)=YRS(I, J)+YR(I, J)
IDN 55
}
INFR=INFR+1
IF (INFR.GT.NFR) GO TO 320
310 CONTINUE
320 CALL INVS (YRS, NR, YRSIN)
REWIND 11
READ (11)
IF (IC.EQ.0) GO TO 350
WRITE (3,330)
330 FORMAT ('*1/'/T30,'SUM OF REAL MOBILITIES'/)
CALL MOUT2 (YRS, NR, NR)
WRITE (3,340)
340 FORMAT ('*1/'/T30,'INVERSE OF SUM OF REAL MOB'/)
CALL MOUT2 (YRSIN, NR, NR)
C
ITERATE FOR PHI (SECOND PASS)
350 READ (1,130) (INDEX(i),I=1,NA)
WRITE (3,360) (INDEX(i),I=1,NA)
360 FORMAT ('*1/'/T25,'SECOND PASS FREQUENCIES'/
& IT10,10F10.2))
INFR=1
DO 380 L=1,NFR
READ (11) FREQ,(YRI,J),YRI(J),I=1,NA),J=1,NA)
IF(L.NE.INDEX(INFR)) GO TO 380
CALL MITER (YR,YRSIN,NA,0.001,99,DUM,VAL,ITF)
ITP(INFR)=ITN
CALL MITER (YR,YRSIN,NA,0.001,99,DUM,VAL,ITN)
IT(INFR)=ITN
DO 370 I=1,NA
GAMI(i)=GUM(i)
370 CONTINUE
DO 390 I=1,NA
390 CONTINUE
400 SUM=0.
DO 410 J=1,NA
SUM=SUM+GAMI(J)
DO 410 J=1,NA
410 GAMI(J)=GUM(J)/SUM
420 CONTINUE
WRITE (3,430)
430 FORMAT ('*1/'/T40,'ITERATED PHI'/)
CALL MOUT2 (PHI, NR, NR)
WRITE (3,440) (ITP(I),I=1,NA)
440 FORMAT ('*1/'/T40,'ITERATIONS'/
& (T5,10I12))
WRITE (3,450)
450 FORMAT ('*1/'/T40,'ITERATED GAMMA'/)
CALL MOUT2 (GAMI, NR, NR)
WRITE (3,460) (IT(I),I=1,NA)
EXCD = .FALSE.
DO 460 I=1,NA
IF (IT(I).GT.99) EXCD = .TRUE.
460 CONTINUE
IF (EXCD) WRITE (3,470)
470 FORMAT ('*1/'/T10,'WARNING - ITERATION NOT CONVERGED***')
DO 480 I=1,NA
DO 480 J=1,NA
CALL INVR3 (Y1,NR,GAMMA)
WRITE (3,490)
490 FORMAT (///T4O,GAMMA = PHI INVERSE TRANSPOSE///)
CALL MOUT3 (GAMMA,NR,NR)
C READ THIRD PASS FREQ
BYPASS RD
RETRN
END

I12 = 1
INFR = 1
DO 550 L = 1, NR
READ (11) FREQ,L(YK(J),YJ(J),I=1,1,NR),J=1,1,NR
IF (L.NE.10M(INFR,112)) GO TO 550
OM(INFR,112) = FREQ
IF (112.EQ.2) GO TO 530
DO 520 I = 1, NR
GAMMA(I,INFR) = GAMMA(I,INFR)
530 YRSTAR(INFR,112) = GENDUM,YR,NR
YSTAR(INFR,112) = GENDUM,YI,NR
IF (112.EQ.2) GO TO 540
I12 = 2
GO TO 550
540 I12 = 1
INFR = INFR+1
IF (INFR.GT.NR) GO TO 560
550 CONTINUE
C FORM Z STAR
DISPLAY Z
DO 570 L = 1, NR
DO 570 LL = 1, 2
COM = YRSTAR(L,LL)**2+YISTAR(L,LL)**2
ZRSTAR(L,LL) = YISTAR(L,LL)/COM
570 ZISTAR(L,LL) = YISTAR(L,LL)/COM
WRITE (3,580)
560 FORMAT (///T4O,*YSTAR USING ITERATED GAMMA///)
WRITE (3,590)
590 FORMAT (///T4Z,ZSTAR (MODE)*TBB, *ZSTAR (MODE)**T3,*MODE JN L
A OM 2 REAL (OM 1) OM 2 IMAG (OM 1) OM 2 REAL (OM 2)**
B OM 1 IMAG (OM 1) OM 2**
WRITE (3,600) I,OM(1,1),YRSTAR(I,1),YSTAR(I,1),ZRSTAR(I,1),
A ZISTAR(I,1),OM(1,2),YRSTAR(I,2),YSTAR(I,2),ZRSTAR(I,2),
B ZISTAR(I,2),I=1,1,NR
600 FORMAT (///I5,OPF10.2,1PE24.4/18,OPF10.2,1PE24.4)
IDENTIFY GEN MASS, NAT FREQ
DO 610 I = 1, NR
OM(I) = OM(1,1) ZISTAR(I,1)-OM(1,2)*ZISTAR(I,2)/OM(1,1)**2-
A OM(1,2)**2/6.283185
OM(1,2) = 6.283185
OMEGA(I) = OMEGA(I)-OM(1,2)*ZISTAR(I,1)+OM(1,1)*ZISTAR(I,2)
A /OM(1,1)*ZISTAR(I,1)-OM(1,2)*ZISTAR(I,2)
OMEGA(I) = OMEGA(I)-39.4784
IF (OMEGA(I).GT.0) OMEGA(I) = SQRT(OMEGA(I))
1ME 159
1DM 160
1DM 161
1DM 162
1ME 163
1ME 164
1ME 165
G(I,J,OM(I,J)) = ZRST(I,J) / (OMEGA(I) *OMEGA(J) *GM(I) *GM(J) *GM(I) *GM(J))

610 CONTINUE

WRITE (3,620) (I,GM(I),OMEGA(I),I=1,NR)

620 FORMAT (*999.999F40,*999.999F40,*999.999F40)

CALL REIDN(NR,GM,OMEGA,PHI)

REWIND 11
GO TO 100
END
SUBROUTINE MITER (A,B,N,TOL,ITMAX,FUN,VAL,IT)
  ITERATES ON A*B FOR DOM'NENT EIGENFUNCTION (FUN)
  AND EIGENVALUE (VAL).
  N IS ORDER
  TOL IS DECIMAL (<= .01 PERCENT) TOLERANCE ON VAL.
  ITMAX IS MAX NO OF ITERATIONS.
  IT IS NUMBER OF ITERATIONS PERFORMED.
  A,B ARE SQUARE OF ORDER N (DIMENSIONED (20,21)).
  USES MNPY (A,B,N1,N2,N3,C)

REAL A(20,21),B(20,21),C(20,21),DUM(201),FUN(20)
CALL MNPY (A,B,N,N,N,N,C)
VAL=100.
IT=1
DO 100 I=1,N
  100 FUN(I)=1,C
  CALL MNPY (C,FUN,N,N,N,DUM)
  VAL=DUM(I)
  DO 120 I=2,N
    120 IF(ABS(VAL)-ABS(DUM(I))) 120,130,130
  120 CONTINUE
  130 IT=IT+1
  IF(IT-ITMAX) 110,110,160
  RETURN
END
SUBROUTINE ROUTZ (A.M.A.

REAL (20.21) 

100 WRITE (5,100) (D1(I),I=1,100)

110 DO 120 I=1,100

120 IF (T(I).LT.0.0) 100,130,150

130 WRITE (5,130)

140 RETURN

150 END
function gen (fun, a, n)
    gen = finitans1 * a * fim
    dimension a(20, 21), fun(20)
    gen = 0
    do 110 i = 1, n
        dum = 0
        do 100 j = 1, n
            dum = dum + a(i, j) * fun(j)
        end do
        gen = gen + dum + fun(i)
    end do
    return
end

SUBROUTINE INVR (B,N,A)
C A = INVERSE OF B  B UNDISTURBED
C
DIMENSION A(20,21),B(20,21)
DO 100 I=1,N
DO 100 J=1,N
100 A(I,J)=B(I,J)
M=N+1
DO 110 I=1,N
IROW(I)=I
110 ICOL(I)=I
DO 120 K=1,N
AMAX= A(K,K)
DO 120 130 I=K,N
DO 120 J=K,N
120 IF(ABS(A(I,J))>AMAX) 130,120,120
130 CONTINUE
KI=ICOL(K)
ICOL(K)=ICOL(K)
ICOL(K)=KI
KI=IROW(K)
IROW(K)=IROW(K)
IROW(K)=KI
IF(AMAX) 160,140,160
140 WRITE (3,150)
150 FORMAT('SOLUTION OF EXISTING MATRIX NOT POSSIBLE')
GO TO 330
160 DO 170 J=1,N
E=A(K,J)
170 A(I,J)=A(I,J)
DO 180 I=1,N
E=A(I,K)
A(I,K)=A(I,J)
180 A(I,J)=E
DO 210 I=1,N
IF(I=K) 200,190,200
190 A(I,M)=1.
GO TO 210
200 A(I,M)=0.
210 CONTINUE
PVT=A(K,K)
DO 220 J=1,N
220 A(K,J)=A(K,J)/PVT
DO 250 I=1,N
IF(I=K) 230,250,230
230 A(I,K)=-AMULT*A(I,K)
DO 240 J=1,N
240 A(I,J)=A(I,J)-AMULT*A(K,J)
250 CONTINUE
DO 260 I=1,N
260 A(I,K)=A(I,M)
SUBROUTINE MPY (A,B,N1,N2,N3,C)

C
C  C = A * B
  A (N1 X N2)  B (N2 X N3)  C (N1 X N3)

REAL A(20,21),B(20,21),C(20,21)
DO 100 I=1,N1
  DO 100 J=1,N3
  C(I,J)=0.
  DO 100 K=1,N2
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END
SUBROUTINE YNRM (YR, NR)
DIMENSION YR(20, 21)
VAL = YR(1, 1)
DO 110 I = 1, NR
DO 110 J = 1, NR
I = I, ABS(VA) - ABS(YR(I, J)) 100, 110, 110
100 VAL = YR(I, J)
110 CONTINUE
DO 120 I = 1, NR
DO 120 J = 1, NR
120 YR(I, J) = YR(I, J) / ABS(VA)
RETURN
END
SUBROUTINE YRMS (YR, NR)

C
YR NORMALIZATION BY RMS OF YR
DIMENSION YR(20, 21)
RMS = 0.
DO 130 I = 1, NR
DO 130 J = 1, NR
130 RMS = YR(I, J) * YR(I, J) + RMS
RMS = SQRT(RMS / NR * NR)
DO 140 I = 1, NR
DO 140 J = 1, NR
140 YR(I, J) = YR(I, J) / RMS
RETURN
END
SUBROUTINE MOB2(M,K,G,N,OM,ZR,ZI,YR,YI,D,IS)

CALCULATES COMPLEX IMPEDANCE AND MOBILITY
M IS SQUARE MASS MATRIX
K IS SQUARE STIFFNESS MATRIX
G IS SCALAR STRUCTURAL DAMPING
D IS SQUARE DAMPING MATRIX
OM IS FREQUENCY IN Hertz
N IS ORDER

EITHER G OR D IS USED
IF IS = 0 ZR = G*K/OMR
IF IS = 1 ZR = D/OMR

IMPEDANCE IS ZR + I*YI (I = SQRT(-1))
MOBILITY = YR + I*YI

ALL SQUARE MATRICES ARE DIMENSIONED (20,21)

USES CINV, INVRS, KMPY

REAL M(20,21),K(20,21),ZR(20,21),ZI(20,21),YR(20,21),YI(20,21)
REAL D(20,21)
OMR=OMR6.283185
IF (IS.EQ.0) CON=K/OMR
DO 110 I=1,N
DO 110 J=1,N
IF (IS.EQ.0) GO TO 100
ZR(I,J)=D(I,J)/OMR
GO TO 110
100 ZR(I,J)=CON*K(I,J)
110 ZI(I,J)=OMR*MI(I,J)-K(I,J)/OMR
CALL CINV(ZR,ZI,N,YR,YI)
RETURN
END
SUBROUTINE CINV (A,B,N,C,D)

C+I D = INVERSE OF A+I B  I=SQRT(-1)

B ASSUMED NON SINGULAR

REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)
CALL INVRSC(N,E)
CALL MHPY(A,E,N,N,E)
DO 100 I=1,N
DO 100 J=1,N
100 C(I,J)=C(I,J)+B(I,J)
CALL INVRSC(N,D)
CALL MHPYE(D,N,N,N,C)
DO 110 I=1,N
DO 110 J=1,N
110 D(I,J)=-D(I,J)
RETURN
END
SUBROUTINE TREFEQ (YR, FREQ, NR)
DIMENSION YR(120, 21)
DO 100 I=1, NR
    IF (YR(I, 1) .GT. 0) THEN
        YR(I, 1) = 0
    ELSE
        YR(I, 1) = 1
    END IF
100 CONTINUE
RETURN
END
SUBROUTINE MATAMP (OMH, A, B, NR)

CONVERTS MOBILITY, A * 10^8 IN VEL UNITS TO
AMP (IN A IN G'S AND PHASE (IN B) IN DEG
MATRICES ARE AT FREQUENCY OMH IN HERTZ

DIMENSION A(20,21), B(20,21)

OM=OMH=0.01626  MAT 1
DO 210 I=1,NR  MAT 2
DO 210 J=1,NR  MAT 3
R=A(I,J)  MAT 4
C=B(I,J)  MAT 5
A(I,J)=SQRT(R*R+C*C)*OM  MAT 6

IF(C).NE.140,100,140  MAT 7
100 IF(R).NE.110,120,130  MAT 8
110 B(I,J)=270.  MAT 9
GO TO 210  MAT 10
120 B(I,J)=0  MAT 11
GO TO 210  MAT 12
130 B(I,J)=90.  MAT 13
GO TO 210  MAT 14
140 P=ATAN(ABS(R/C))*.57.2958  MAT 15
150 IF(R).NE.150,150,160  MAT 16
160 IF(R).NE.160,160,170  MAT 17
170 IF(R).NE.190,190,200  MAT 18
180 B(I,J)=360.-P  MAT 19
GO TO 210  MAT 20
200 B(I,J)=P  MAT 21
210 CONTINUE  MAT 22
RETURN  MAT 23
END  MAT 24
SUBROUTINE AMP (OMH, A, B, INC, NR)

CONVERTS A * I*B IN VELOCITY UNITS TO

AMP (IN A ) IN GFS AND PHASE (IN 0 ) IN DEG

EACH ROW IS AT A FREQUENCY OMH(I) IN Hertz

DIMENSION OMH(100), A(100, 20), B(100, 20)

DO 210 I = 1, INC

OM = OMH(I) *0.01626

DO 210 J = 1, NR

R = A(I, J)

C = B(I, J)

A(I, J) = SQRT(R*R + C*C) * OM

IF(C) 140, 140, 140

100 IF(R) 110, 120, 130

110 B(I, J) = 270.

GO TO 210

120 C(I, J) = 0.

GO TO 210

130 B(I, J) = 90.

GO TO 210

140 P = ATAN(ABS(R/C)) *57.2958

150 IF(P) 160, 160, 170

160 B(I, J) = 180. + P

GO TO 210

170 B(I, J) = 180. - P

GO TO 210

180 IF(P) 190, 190, 200

190 B(I, J) = 360. - P

GO TO 210

200 B(I, J) = P

210 CONTINUE

RETURN

END
SUBROUTINE REIDN (NR,GM,OM,PHI,GAMI,GK,G)  
IDENTIFICATION OF MASS, STIFFNESS, DAMPING MATRICES  
DIMENSION GM(20,20),OM(20),PHI(20,20),AM(20,20),AK(20,20)  
DIMENSION GAMI(20,20),GM(20,20),AD(20,20),DK(20,20)  
DIMENSION ZR(20,20),ZI(20,20),VR(20,20),YR(20,20),NZ(100,20)  
DIMENSION DPF(100,20),DPF(100,20),TR(100,20),TV(100,20),TF(100,20)  

LOGICAL TFRG  
DO 100 I=1,NR  
DO 100 J=1,NR  
AD(I,J)=0.  
ANG(I,J)=0.  
UT(I,J)=0.  
100 CONTINUE  

DO 120 I=1,NR  
GAMAI(I)=I/(GM(I)*OM(I)*OM(I)*39.4784)  
DO 110 K=1,NR  
ANG(K)=-PHI(I,K)  
110 CONTINUE  

CALM=0.  
DO 130 K=1,NR  
ANGK=CM(1,K)*PHI(J,K)  
130 CONTINUE  

ANG=CM(1,J)*CM(1,J)-CM(1,J)*CM(1,J)  
ANG=CM(1,J)*CM(1,J)+CM(1,J)*CM(1,J)  
ANG=CM(1,J)*CM(1,J)*CM(1,J)+CM(1,J)*CM(1,J)  
ANG=CM(1,J)  
120 CONTINUE  

CONTINUE  
CALL INVS (C,NR,AK)  
CALL INVS (U,NR,AN)  
WRITE (3,130)  
130 CONTINUE  

WRITE *  
140 CONTINUE  

WRITE *  
150 CONTINUE  

WRITE *  
160 CONTINUE  

WRITE *  
170 CONTINUE  

WRITE *  
180 CONTINUE  

WRITE *  
190 CONTINUE  

WRITE *  
200 CONTINUE  

WRITE *  
210 CONTINUE  

IF (NMF.EQ.0) GO TO 410  
TFRM=NFOM+1.0+AD, NROW.LE.NR  
READ (1,230) (H(I),I=1,NF)  
DO 300 L=1,NF  
GMF=WZ(L)  
CALL MOD2 (AH,AK,GK,MR,OMF,ZR,VI,YR,AD,NR)  
IF (P1) 220,220,280  
1
220 IF(P2 NE.0) CALL MATAMP (HZ(L),YR,YI,NP)
  IF(P2 NE.0) GO TO 250
  WRITE (3,3200) HZ(L)
  1REI 56
  1REI 57
  1REI 58
230 FORMAT (8F10.0)
240 FORMAT ('*REAL MOBILITY, IMAGINARY MOBILITY  FREQ **F10.2,'REI 60
  A =*F10.2,* HERTZ**/)
  GO TO 270
  1REI 61
  1REI 62
250 WRITE (3,3260) HZ(L)
  1REI 63
260 FORMAT('*ACCELERATION AMPLITUDE IN G**S, PHASE IN DEG.  FREQ
  A =*F10.2,* HERTZ**/)
  1REI 64
270 CALL MOUT2 (YR, NR, NP)
  CALL MOUT2 (YR, NR, NP)
  GO TO 300
  1REI 66
  1REI 67
280 DO 290 L=1,NR
   DPI(L,1)=YR(L,1)
   DPI(L,1)=YR(L,1)
   IF(.NOT. TURF) GO TO 290
   TURF(K)=YR(NROW,1)
   290 TURF(K)=YR(NROW,1)
300 CONTINUE
  1REI 73
  1REI 74
310 IF(P2 NE.1) GO TO 330
   CALL AMP (HZ, DPI, NP, NR)
   IF(TURF) CALL AMP (HZ, TR, TI, NF, NR)
   WRITE (3,3200)
320 FORMAT ('*DRIVING POINT RESPONSE, AMP IN G**S AND PHASE IN
  DEGREES**/)
   GO TO 350
   1REI 76
   1REI 77
330 WRITE (3,3400)
340 FORMAT ('*DRIVING POINT MOBILITY, REAL AND IMAGINARY**/)
350 CALL YOUT (HZ, DPI, NF, NR, 0)
   WRITE (3,3600)
360 FORMAT ('**/)
   1REI 80
380 WRITE (3,3900) NROW
390 FORMAT ('*TRANSFER MOBILITY, REAL AND IMAG**/)
400 CALL YOUT (HZ, TR, NF, NR, 0)
   WRITE (3,3600)
   CALL YOUT (HZ, TI, NF, NR, P2)
   410 RETURN
END
LIST OF FORTRAN SUBROUTINES

AMP    Converts mobility from velocity units to acceleration as amplitude (in g's) and phase angle (in degrees)
CINV   Complex inverse of complex matrix
ERR    Incorporates measurement errors into simulated measurements
GEN    Generalized function of form $f^T Af$ where $f$ is a vector and $A$ is a square matrix
INVRS  Inverse of a matrix
ITER   Matrix iteration for eigenvalues and eigenvectors
MITER  More general iteration on product of two matrices; used for gamma iteration
MMPX   Matrix multiplication
MØB    Calculates complex impedance and mobility
MØUT   Special output for square matrix
RANDU  Random number generator
RED    Removes rows and columns from matrix
YØUT   Special matrix output
SYM    Forms symmetric matrix from lower triangle
MØUT2  Special output for nonsquare matrix
MMPY   Matrix multiplication
SITER  Matrix iteration for eigenvalues and eigenvectors
MATAMP Converts velocity mobility to amplitude (g's) and phase (degrees)
YRNRM  Performs normalization of mobility matrix on absolute value of largest element of mobility matrix
YRRMS  Performs normalization of mobility matrix on root mean square value of mobility matrix

MØB2  Calculates complex impedance and mobility

YRFREQ Multiplies each velocity mobility matrix by its respective frequency to give acceleration mobility

REIDN  Identification of mass stiffness and damping matrices
SAMPLE OUTPUT

INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT INACT
INACT 9 POINT MODEL 20 POINT STRUCTURE 12/11/70

TAPE READING

EXACT DATA SIMULATED TEST
EXACT 20 POINT UE 8/19/70
20 DEGREES OF FREEDOM

FREQUENCIES (H.2) ON TAPE

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121
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