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IMPACT ON COMPLEX MECHANICAL STRUCTURES

Song Fong Jan

Texas University

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IMPACT ON COMPLEX MECHANICAL STRUCTURES

TECHNICAL REPORT

72-49-AD

Contract No. DAAG 17-70-C-012?

by

Song Fong Jon

The University of Texas at Austin

Austin, Texas

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July 1971

UNITED STATES ARMY

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Natick, Massachusetts 01760

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13. ABSTRACT

A complex structure such as a vehicle is represented by a lumped parameter model. Using the finite element method, a set of equations of motion is formulated for this model. Then these equations are solved numerically by the Runge - Kutta method. A model for representing a specific vehicle, namely the M-37 military truck cushioned for an airdrop is used to illustrate the procedure. Some of the physical constants for the model are modified as required to bring the computed results of displacements and accelerations at various points in the model into agreement with measured values. It is found that the response of a structure properly cushioned and subjected to impact loading is not sensitive to the elastic properties of the interconnecting structure. Thus a suitable lumped parameter system for representing a complex mechanical structure can be obtained with a small effort. However, special attention should be focused on important components such as the engine in the vehicle. Experimental results show that more than one fourth of the system energy is dissipated through the structural damping. Obviously damping cannot be neglected in the analysis. Since the forcing function is the most essential factor affecting the dynamic response of the system, it must be represented as exactly as possible. A digital computer program has been developed to solve the airdrop impact problems and to serve as a design tool.

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July 1971

Airdrop Engineering Laboratory
U. S. ARMY NATICK LABORATORIES
Natick, Massachusetts 01760

FOREWORD

This work was performed under US Army Natick Laboratories Contract No. DAAG17-70-C-0127 during the period of 1 Apr 70 to 31 Mar 71, and follows the work reported on in report reference No. 7. The Project No. was 1F162203D195 entitled "Exploratory Development of Airdrop Systems", and the Task was No. 13 entitled "Impact Phenomena". Messrs. Edward J. Giebutowski and Marshall S. Gustin of the Airdrop Engineering Laboratory served as the Project Officers.

The effort is part of a continuing investigation directed toward obtaining a better understanding of the failure mechanism of energy dissipater materials, and the response of airdroppable supplies and equipment to airdrop impact phenomena; and toward obtaining improved airdrop energy dissipater materials and techniques.

This report is concerned primarily with the development of design analyses of the dynamic response of a complex structure to the shock of a vertical and planar airdrop impact. Primary consideration was given to the use of the finite element method of analysis, of a three dimensional lumped mass model of the Army's M-37 truck.

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ABSTRACT

The principal purpose of this study has been to develop a procedure for analyzing the response of complex structures to impact and to provide a computer code for making the necessary computations. Attention has been focused specifically on the displacements, velocities and accelerations produced at various points in military vehicles subjected to ground impact in airdrop operations.

The vehicle is modeled by a lumped parameter (spring-mass) system. Using the finite element method, a set of equations of motion is formulated for this model. Then these equations are solved numerically by the Runge-Kutta method. A model for representing a specific vehicle, namely the M-37 military truck cushioned for an airdrop is used to illustrate the procedure. Some of the physical constants for the model are modified as required to bring the computed displacements and accelerations at various points in the model into agreement with measured values. It is found that the response of a structure properly cushioned and subjected to impact loading is not sensitive to the elastic properties of the interconnecting members. Thus, the development of a suitable lumped parameter model of a given vehicle is simplified. However, special attention should be given to the more important components of the vehicle such as, for example, the engine.

Experimental results show that more than one fourth of the system energy is dissipated through the structural damping. Hence, damping must be included in the equations of motion.

The most essential factor affecting the dynamic response of the system has been found to be the force applied as a result of the impact. In the example used, this force is applied by the cushioning system.

INTRODUCTION

The finite element method appears to be ideally suited to the problem under consideration, namely, the computation of the displacements, velocities and accelerations at selected points in a structure subjected to an impulsive loading. To apply this method of analysis one must first prepare or select a conceptual representation of the continuous structure as an assemblage of structural elements interconnected at nodal points. The idealized structure is assumed to be acted on by external equivalent forces and to possess equivalent inertia properties only at the nodal points. Thus the continuous structure is replaced, for analytical purposes, by a lumped parameter system. The accuracy of the predicted dynamic response of a structure will depend on how well the structure is represented by the selected lumped-parameter model. It has been reported[1] that even for a very simple beam or uniform plate with boundary conditions which can be exactly expressed in mathematics, errors in the predicted responses can easily be as much as 35 percent. Although no investigation of the validity of lumped parameter models for complex structures, such as vehicles, has been reported in the literature, it may be assumed that discrepancies of even more than 35% can be expected for poorly represented structures. The general details of the finite element method have been discussed in the literature.[2] However, the nature of the model which will best represent a structure such as a vehicle, with its varied elements, irregular geometry and discrete masses, is not at all clear. The primary objective in this study will be to select a model and then determine by computation and by experiment how valid the model is. It is expected that some suitable rules can be formulated regarding the representation of vehicles by lumped parameter models. A model for representing a specific vehicle, namely the M-37 military truck, cushioned for an airdrop is used to illustrate the procedure.

In the procedure followed for this study, the lumped parameter model for the M-37 is first developed. Then the equations of motion of the model are formulated following standard finite element methods. These equations, with appropriate initial conditions are then solved using the Runge - Kutta method.* Some important factors such as structural elastic properties, damping, and impulse loading which affect the dynamic response are investigated. Finally, an experimental program of actual truck drop tests is carried out, and the results are compared with computed results. In the analysis of the mathematical model, the concept of linear transformation is extensively used. Linear transformation techniques streamline and simplify considerably the procedures involved in the analysis. It should be mentioned here that the computer program developed for the vehicle is also applicable to other complex structures.

* Algorithms for this method are available in most computational facilities.

MATHEMATICAL MODEL

A vehicle such as the M-37 truck may be represented by the model shown in Fig. 2.1. In this model the engine, transmission, transfer case, differentials and wheels are treated as discrete masses. The mass of the winch is assumed to be distributed uniformly along the two main longitudinal and the remaining transverse members of the truck frame. The adoption of this model is, however, quite arbitrary. Many other arrangements of masses would, no doubt, be equally acceptable.

When a structure such as this vehicle is to be intentionally subjected to an impact, as in an airdrop, cushioning is provided to reduce the severity of the shock produced by the impact. Usually all of the discrete masses shown in Fig. 2.1 would be cushioned independently, if possible. For the M-37 truck the engine is not cushioned independently, partly because it is shock mounted on the frame, and partly because of geometrical and structural problems. The engine is supported on rubber cushions, or shock mounts at three points, one in front and two in the rear. The action of these mounts can be represented by the spring-damper system shown in Fig. 2.2. The stiffness and the damping capacity of a mount depends on many factors such as the hardness of the rubber, the shape, and the age.[3] Rather than try to determine a precise set of values for the M-37 mounts, values of 20,000 lb/in. for k and 60% of critical for the damping were arbitrarily assumed. Later these values were varied to improve the "fit" between experimental observations and computed results.

The transmission is actually attached to the engine but since it can be cushioned independently, it has been assumed to be a mass which is attached to the engine by a very stiff element.

The transfer case is supported at four points on two central cross frame members which act as spring supports for it.

Wheels and differentials are connected by the axles which in this analysis are assumed to be rigid, massless rods attached to leaf springs. Tires absorb considerable energy and this energy is given back in rebound. Also measured relative displacements between the rear axle and truck frame indicate that little energy is dissipated through the four shock absorbers associated with the wheels during the impact. Consequently the shock absorbers are neglected and the vehicle is supported by the leaf springs in Fig. 2.3.

The spring constants K and damping factor C for the tire in Fig. 2.3 must be determined experimentally, or be estimated using whatever guidance is available. These quantities are initially assumed to be 7,000 lb/in. and 20% respectively.

The winch and the other distributed masses of the truck are cushioned with the two main longitudinal frame members. In the model shown in Fig. 2.1, all individual components are interconnected at the nodes numbered from 1 to 39. In the present study only a vertical, planar im-

NOTE : NODES 24, 26, 33, AND 35 ARE DIRECTLY BENEATH THE FRAME

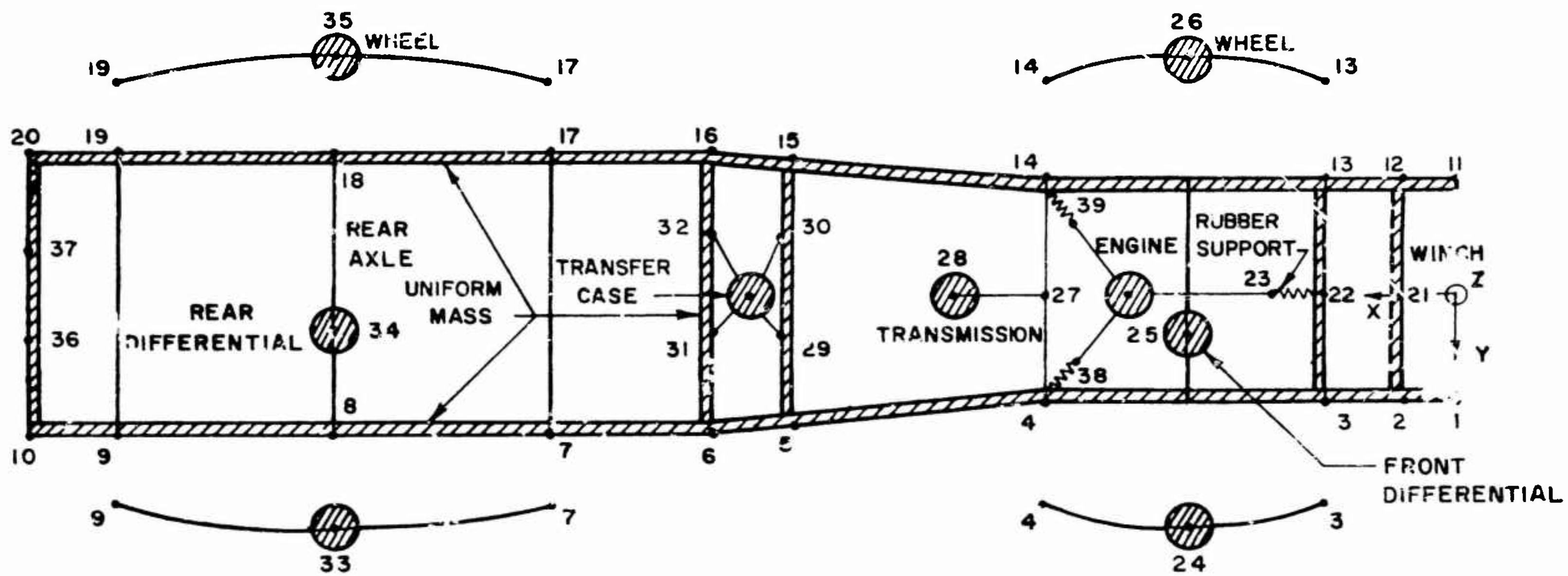


Fig. 2.1 Lumped-Parameter Model - M-37 Truck.

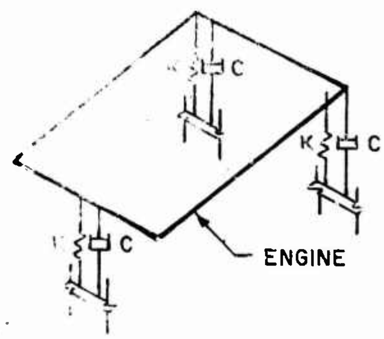


Fig. 2.2 Engine Supports

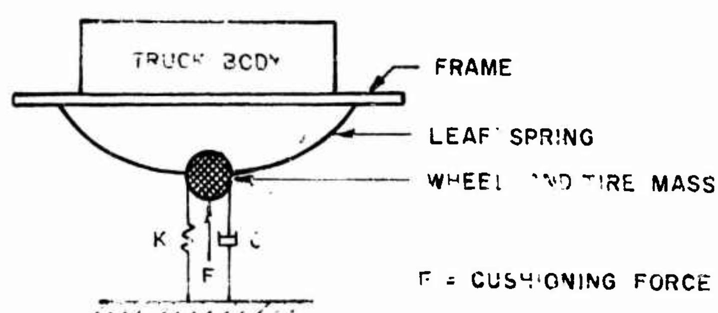


Fig. 2.3 Suspension System of Vehicle

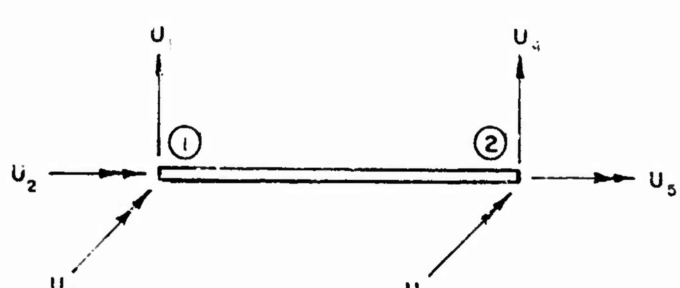


Fig. 2.4 Element Displacements

pact is considered. Consequently each nodal point in the model is to have only 3 degrees of freedom, consisting of one vertical translation and two planar rotations. Three degrees of freedom are associated with each end of all members as shown in Fig. 2.4.

ANALYSIS OF THE MODEL

For the model shown in Fig. 2.1 with assumed damping subject to impact loading, the equation of motion can be written as:

$$\underline{\underline{M}}\ \underline{\ddot{q}} + \underline{\underline{C}}\ \underline{\dot{q}} + \underline{\underline{K}}\ \underline{q} = \underline{F}(t) \tag{3.1}$$

where $\underline{\underline{M}}$, $\underline{\underline{C}}$ and $\underline{\underline{K}}$ are square matrices of inertia, damping and stiffness respectively and $\underline{F}(t)$ is the column matrix of cushion forces. The generalized displacement matrix $\underline{q}$ is numbered in the sequence according to the joint numbers. For example, coordinates q_{3i-2}, q_{3i-1} and q_{3i} are associated with the translatory and two rotational motions about common datum axes, at the ith nodal joint of the model. For example, at Joint 5, the coordinates are q_{15-2} q_{15-1} q_{15-0} or q_{13}, q_{14} and q_{15}.

The stiffness matrix $\underline{\underline{K}}$ of the complete element assembly[4] is obtained by the direct stiffness method. This method consists of first deriving the individual element stiffness as in element coordinates, followed by a coordinate transformation and the subsequent superposition of each element stiffness so that the translational and rotational degrees of freedom of all elements which share a common nodal point are expressed in the same coordinates. The superposition of each transformed stiffness is accomplished by adding its individual terms into the complete stiffness matrix according to the element nodal point numbers. The same method can be employed to obtain the mass matrix $\underline{\underline{M}}$ and the force vector $\underline{F}$. The derivations of mass matrix $\underline{\underline{M}}$, stiffness matrix $\underline{\underline{K}}$, force vector $\underline{F}$ and damping matrix $\underline{\underline{C}}$ are discussed in detail as follows:

1. Element Displacement Functions

To determine the elastic and inertia properties of a structural element the strain and kinetic energies of the actual continuous element are equated to the corresponding quantities for the equivalent discrete model. Consider now a uniform structural element in the horizontal plane as shown in Fig. 3.1. The common datum (X,Y,Z) is established for all structural elements so that all displacements and corresponding forces will be referred to this common coordinate system. The origin of element coordinates (x,y,z) is located at node 1 with the ox axis taken along the length of element and with the oy and oz axes as the principal axes of the element cross section.

The column matrix $\underline{U}$ for this element, as mentioned before, consists of six displacements, two vertical (Z direction) deflections U_1 and U_4 and four rotations, U_2, U_3, U_5, and U_6.

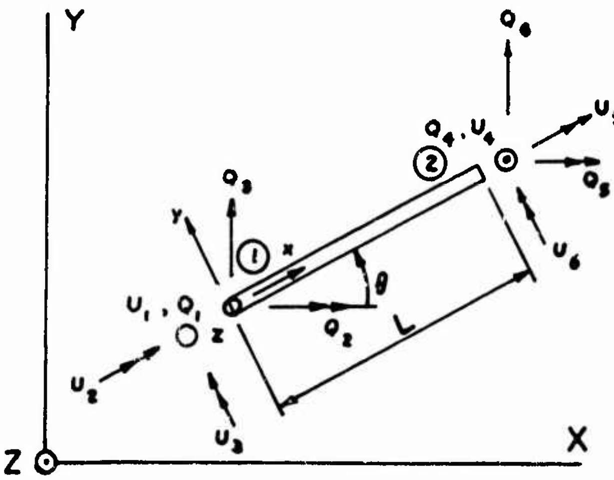


Fig. 3.1 Element Coordinates

$$\underline{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} \qquad (3.2)$$

Denote the displacement functions in element coordinates as:

$$\underline{w} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \qquad (3.3)$$

The displacement function w_z may be assumed as

$$w_z = C_1 + C_2x + C_3x^2 + C_4x^3 + C_5y + C_6xy \qquad (3.4)$$

where C_1 to C_6 are constants to be determined by boundary conditions at both ends of the element.

At node 1: $(x = 0, y = 0)$

$$w_z = U_1 = C_1 \qquad (3.5a)$$

$$\frac{\partial w_z}{\partial x} = -U_3 = C_2 \qquad (3.5b)$$

$$\frac{\partial w_z}{\partial y} = +U_2 = C_5 \qquad (3.5c)$$

At node 2: $(x = L, y = 0)$

$$w_z = U_4 = C_1 + C_2L + C_3L^2 + C_4L^3 \qquad (3/5d)$$

$$\frac{\partial w_z}{\partial x} = -U_6 = C_2 + 2C_3L + 3C_4L^2 \qquad (3.5e)$$

$$\frac{\partial w_z}{\partial y} = +U_5 = C_5 + C_6L \qquad (3.5f)$$

Solving for the C's,

$$\begin{aligned} C_1 &= U_1 \\ C_2 &= -U_3 \\ C_3 &= \frac{1}{L^2}[-3U_1 + 2LU_3 + 3U_4 + LU_6] \\ C_4 &= \frac{1}{L^3}[2U_1 - LU_3 - 2U_4 - LU_6] \\ C_5 &= U_2 \\ C_6 &= \frac{1}{L}[-U_2 + U_5] \end{aligned} \qquad (3.6)$$

Define the nondimensional parameters

$$r = \frac{x}{L} \qquad s = \frac{y}{L} \qquad t = \frac{z}{L} \tag{3.7}$$

Thus the displacement function w_z becomes

$$\begin{aligned} w_z = {} & (1 - 3r^2 + 2r^3)U_1 + Ls(1 - r)U_2 \\ & + L(-r + 2r^2 - r^3)U_3 + (3r^2 - 2r^3)U_4 \\ & + LrsU_5 + L(r^2 - r^3)U_6 \end{aligned} \tag{3.8}$$

The displacement function w_y caused by twisting is

$$w_y = - Lt(1 - r)U_2 - LrtU_5 \tag{3.9}$$

This is a direct geometric relationship, it can be obtained by assuming $w_y = C_1 + C_2x + C_3x^2 + C_4x^3 + C_5z + C_6xz$.

The displacement function w_x is

$$w_x = - z\frac{\partial w_z}{\partial x} - y\frac{\partial w_y}{\partial x} \tag{3.10}$$

Then by combining 3.8, 3.9, and 3.10 obtain

$$\begin{aligned} w_x = {} & 6t(r - r^2)U_1 + Lt(1 - 4r + 3r^2)U_3 \\ & + 6t(-r + r^2)U_4 + Lt(-2r + 3r^2)U_6 \end{aligned} \tag{3.11}$$

Thus the displacement functions $\underline{w}$ can be expressed in terms of the discrete displacements $\underline{U}$ as

$$\underline{w} = \underline{\underline{a}}\,\underline{U} \tag{3.12}$$

where $\underline{\underline{a}}$ is the 3 x 6 matrix whose transpose is given by:

$$\underline{\underline{a}}^T = \begin{bmatrix} 6t(r - r^2) & 0 & 1 - 3r^2 + 2r^3 \\ 0 & - Lt(1 - r) & - Ls(1 - r) \\ Lt(1 - 4r + 3r^2) & 0 & L(-r + 2r^2 - r^3) \\ 6t(-r + r^2) & 0 & 3r^2 - 2r^3 \\ 0 & - Lrt & - Lrs \\ Lt(-2r + 3r^2) & 0 & L(r^2 - r^3) \end{bmatrix} \tag{3.13}$$

2. Element Mass Matrix

To determine the element mass matrix begin with the element displacement functions $\underline{w}$ in terms of discrete displacement $\underline{U}$ as indicated by Eq. 3.12.

The transpose of $\underline{w}$ is:

$$\underline{w}^T = \underline{U}^T\,\underline{\underline{a}}^T \tag{3.14}$$

and the second time derivatives are:

$$\ddot{\underline{w}} = \underline{\underline{a}}\ \ddot{\underline{U}} \tag{3.15}$$

The virtual displacement of $\underline{w}$ is:

$$\delta\underline{w} = \underline{\underline{a}}\delta\underline{U} \tag{3.16}$$

and its transpose is

$$\delta\underline{w}^T = \delta\underline{U}^T\underline{\underline{a}}^T \tag{3.17}$$

Thus the virtual work done by the inertia force of the element is:

$$\delta W_{inertia} = \int_V \delta\underline{w}^T(\rho\ddot{\underline{w}})\,dV$$

$$= \int \delta\underline{U}^T\underline{\underline{a}}^T(\rho\ \underline{\underline{a}}\ \ddot{\underline{U}})\ dV$$

$$= \delta\underline{U}^T\left(\int_V \rho\ \underline{\underline{a}}^T\underline{\underline{a}}\ dV\right)\ddot{\underline{U}}$$

where the integration is performed over the whole volume of the element.

Hence the element mass matrix is defined as:

$$\underline{\underline{m}} = \int_V \rho\ \underline{\underline{a}}^T\underline{\underline{a}}\ dV \tag{3.18}$$

Substituting the matrix $\underline{a}$ [Eq. (3.13)] into Eq. (3.18) and performing the integrations, the resulting 6 x 6 mass matrix, Eq. (3.19) is obtained. In the mass matrix, terms with the moment of inertia I_y represent rotatory inertia and terms with the polar moment of inertia I_x represent the torsional inertia of the element. From the results of numerical computation, the effects of rotatory and torsional inertia of the element during impact are considered negligible.

3. Element Stiffness Matrix

The bending strain in the xz plane of the element shown in Fig. (3.1) is:

$$e_b = \frac{\partial^2 w_z}{\partial x^2} = \frac{1}{L^2}(-6 + 12r)U_1 + \frac{1}{L}(4 - 6r)U_3$$

$$+ \frac{1}{L^2}(6 - 12r)U_4 + \frac{1}{L}(2 - 6r)U_6 \tag{3.20}$$

The twist strain is:

$$e_t = \frac{1}{L}(-U_2 + U_5) \tag{3.21}$$

The total strain $\underline{e}$ in matrix form is:

$$\underline{e} = \underline{\underline{b}}\ \underline{U} \tag{3.22}$$

where $\underline{e} = \begin{bmatrix} e_b \\ e_t \end{bmatrix}$

$$\underline{\underline{m}} = \frac{\rho AL}{420}\left[\begin{array}{cccccc}
156 + 504\frac{I_y}{AL^2} & & & \text{symmetric} & & \\
0 & 140\frac{I_x}{A} & & & & \\
-22L - 42\frac{I_y}{AL} & 0 & 4L^2 + 56\frac{I_y}{A} & & & \\
54 - 504\frac{I_y}{AL^2} & 0 & -13L + 42\frac{I_y}{AL} & 156 + 504\frac{I_y}{AL^2} & & \\
0 & 70\frac{I_x}{A} & 0 & 0 & 140\frac{I_x}{A} & \\
-13L - 42\frac{I_y}{AL} & 0 & -3L^2 - 14\frac{I_y}{A} & 22L + 42\frac{I_y}{AL} & 0 & 4L^2 + 56\frac{I_y}{A}
\end{array}\right]$$

(3.19)

and

$$\underline{\underline{b}} = \begin{bmatrix} \frac{6}{L^2}(-1 + 2r) & 0 & \frac{1}{L}(4 - 6r) & \frac{1}{L^2}(6 - 12r) & 0 & \frac{1}{L}(2 - 6r) \\ 0 & -\frac{1}{L} & 0 & 0 & \frac{1}{L} & 0 \end{bmatrix} \tag{3.23}$$

The bending moment is:

$$S_b = EI_y e_b$$

where E is elasticity modulus. The twisting moment is:

$$S_t = GJe_t$$

where G is the shear modulus and J is the torsional constant for the element.
Thus the stress-strain relationship in matrix form is:

$$\underline{S} = \underline{\underline{A}}\ \underline{e} \tag{3.24a}$$

Where:

$$\underline{S} = \begin{bmatrix} S_b \\ S_t \end{bmatrix} \tag{3.24b}$$

$$\underline{\underline{A}} = \begin{bmatrix} EI_y & 0 \\ 0 & GJ \end{bmatrix} \tag{3.24c}$$

From Eq. (3.22), we have the virtual strain

$$\delta\underline{e} = \underline{\underline{b}}\delta\underline{U}, \tag{3.25}$$

and the transpose of virtual strain

$$\delta\underline{e}^T = \delta\underline{U}^T\underline{\underline{b}}^T. \tag{3.26}$$

Thus the virtual strain energy expression is:

$$\delta V_{strain} = \int_0^L \delta\underline{e}^T\underline{s}\ dx$$

$$= \int_0^L \delta\underline{U}^T\underline{\underline{b}}^T\underline{\underline{A}}\ \underline{\underline{b}}\ \underline{U}\ dx$$

$$= \delta\underline{U}^T\left(\int_0^L \underline{\underline{b}}^T\underline{\underline{A}}\ \underline{\underline{b}}\ dx\right)U \tag{3.27}$$

The element stiffness $\underset{=}{k}$ is then defined as:

$$\underset{=}{k} = \int_0^L \underset{=}{b}^T \underset{=}{A}\, \underset{=}{b}\, dx \tag{3.28}$$

Substituting the matrix $\underset{=}{b}$ [Eq. (3.23)] and $\underset{=}{A}$ [Eq. (3.24c)] into Eq. (3.28) and performing the integrations, the 6 x 6 stiffness matrix k is obtained as follows:

$$\underset{=}{k} = \frac{EI_y}{L^3}\begin{bmatrix} 12 & & & & & \\ 0 & \frac{GJL^2}{EI_y} & & & \text{symmetric} & \\ -6L & 0 & 4L^2 & & & \\ -12 & 0 & 6L & 12 & & \\ 0 & -\frac{GJL^2}{EI_y} & 0 & 0 & \frac{GJL^2}{EI_y} & \\ -6L & 0 & 2L^2 & 6L & 0 & 4L^2 \end{bmatrix} \tag{3.29}$$

4. Element End Forces

In general, the virtual work done by the external forces $\underline{p}(x,t)$ acting along the element at any instant in time through virtual displacements $\delta\underline{w}$ is:

$$\delta w_{external} = \int_S \delta \underline{w}^T \underline{p}\, ds$$

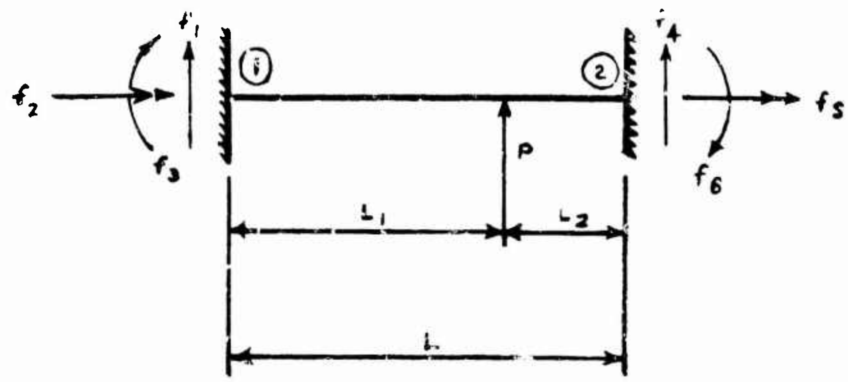
$$= \int_S \delta \underline{U}^T \underset{=}{a}^T \underline{p}\, ds$$

$$= \delta \underline{U}^T \int_S \underset{=}{a}^T \underline{p}\, ds$$

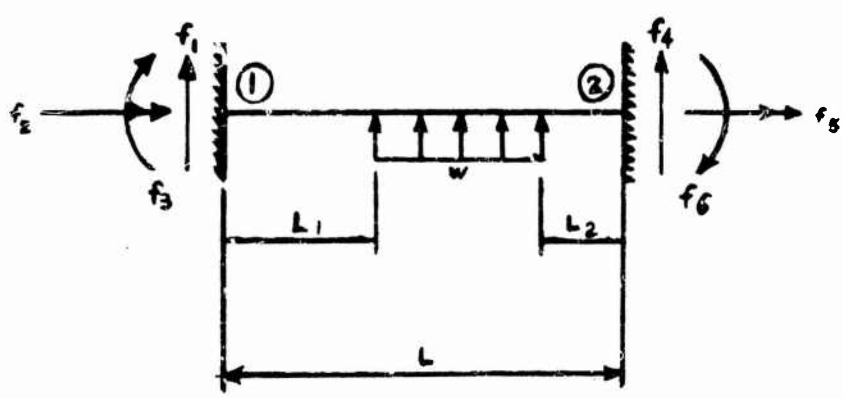
Thus the equivalent concentrated forces $\underline{f}$ at the element ends due to a distributed loading $\underline{p}$ is:

$$\underline{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_6 \end{bmatrix} = \int_S \underset{=}{a}^T \underline{p}\, ds \tag{3.30}$$

However, the forces $\underline{f}$ in Eq. (3.30) are the fixed end forces (with the signs reversed) of the element corresponding to the distributed forces $\underline{p}$ along the element. Two cases of cushioning forces applied on the element as shown in Fig. 3.2 are considered in the analysis.



CASE 1
CONCENTRATED FORCE P



CASE 2
UNIFORM FORCE

Fig. 3.2 Element Forces

The fixed end forces for case 1 are:

$$\underline{f} = \begin{bmatrix} \dfrac{PL_1 + M_1 - M_2}{L} \\ 0 \\ -M_1 \\ \dfrac{PL_1 + M_2 - M_1}{L} \\ 0 \\ M_2 \end{bmatrix} \qquad (3.31)$$

where:

$$M_1 = \frac{PL_1L_2^2}{L^2}$$

$$M_2 = \frac{PL_1^2L_2}{L^2}$$

For case 2:

$$\underline{f} = \begin{bmatrix} [\frac{w}{2}(L - L_1 + L_2)\ (L - L_1 - L_2) + M_3 - M_4]/L \\ 0 \\ -M_3 \\ [\frac{w}{2}(L + L_1 - L_2)\ (L - L_1 - L_2) + M_4 - M_3]/L \\ 0 \\ +M_4 \end{bmatrix}$$

$$(3.32)$$

where:

$$M_3 = \frac{w}{12L^2}[(L - L_1)^3\ (L + 3L_1) - L_2^3\ (4L - 3L_2)]$$

$$M_4 = \frac{w}{12L^2}[(L - L_2)^3\ (L + 3L_2) - L_1^3\ (4L - 3L_1)]$$

5. Linear Transformation

If the set of coordinates $\underline{q}$ with n degrees of freedom in the equation of motion Eq. (3.1) is a linear combination of a different set of coordinates $\underline{u}$ with m degrees of freedom,

$$\underline{q} = \underline{\underline{B}}\ \underline{u} \tag{3.33}$$

then the mass, damping, stiffness and force matrices in u coordinates can be calculated by using the concepts of energy and virtual work. The elements of the n x m matrix $\underline{\underline{B}}$ [Eq. (3.33)] are constants.
In the q coordinate system, the kinetic and potential energy expressions have the matrix forms:

$$T = \frac{1}{2}\ \dot{\underline{q}}^T \underline{\underline{M}}\ \dot{\underline{q}} \tag{3.34}$$

$$V = \frac{1}{2}\ \underline{q}^T \underline{\underline{K}}\ \underline{q} \tag{3.35}$$

where $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are mass and stiffness matrices in q coordinates respectively.
From Eq. (3.33),

$$\dot{\underline{q}} = \underline{\underline{B}}\ \dot{\underline{u}} \tag{3.36}$$

and the transposed matrices

$$\underline{q}^T = \underline{u}^T \underline{\underline{B}}^T \tag{3.37}$$

$$\dot{\underline{q}}^T = \dot{\underline{u}}^T \underline{\underline{B}}^T \tag{3.38}$$

Introducing the linear transformation Eqs. (3.33), (3.36), (3.37), and (3.38) into the kinetic and potential energy expression Eqs. (3.34) and (3.35) results in:

$$\begin{aligned} T &= \frac{1}{2}\ \dot{\underline{q}}^T \underline{\underline{M}}\ \dot{\underline{q}} \\ &= \frac{1}{2}\ \dot{\underline{u}}^T \underline{\underline{B}}\ \underline{\underline{M}}\ \underline{\underline{B}}\ \dot{\underline{u}} \\ &= \frac{1}{2}\ \dot{\underline{u}}^T \underline{\underline{m}}\ \dot{\underline{u}} \end{aligned} \tag{3.39}$$

$$\begin{aligned} V &= \frac{1}{2}\ \underline{q}^T \underline{\underline{K}}\ \underline{q} \\ &= \frac{1}{2}\ \underline{u}^T \underline{\underline{B}}^T \underline{\underline{K}}\ \underline{\underline{B}}\ \underline{u} \\ &= \frac{1}{2}\ \underline{u}^T \underline{\underline{k}}\ \underline{u} \end{aligned} \tag{3.40}$$

where.

$$\underline{\underline{m}} = \underline{\underline{B}}^T \underline{\underline{M}}\ \underline{\underline{B}} \tag{3.41}$$

$$\underline{\underline{k}} = \underline{\underline{B}}^T \underline{\underline{K}}\ \underline{\underline{B}} \tag{3.42}$$

are the corresponding mass and stiffness matrices in the u coordinate system. Since $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are symmetric, it follows that $\underline{\underline{m}}$ and $\underline{\underline{k}}$ are symmetric. The virtual work done by the external forces $\underline{F}$ in q coordinates acting through the virtual displacement $\delta\underline{q}$ is:

$$\delta W = \delta\underline{q}^T\underline{F} \tag{3.43}$$

Since the virtual displacements $\delta\underline{q}$ are related to the virtual displacements $\delta\underline{u}$ by:

$$\delta\underline{q} = \underline{\underline{B}}\delta\underline{u} \tag{3.44}$$

and:

$$\delta\underline{q}^T = \delta\underline{u}^T\underline{\underline{B}}^T \tag{3.45}$$

thus

$$\delta W = \delta\underline{q}^T\underline{F} = \delta\underline{u}^T\underline{\underline{B}}^T\underline{F} = \delta\underline{u}^T\underline{f} \tag{3.46}$$

where

$$\underline{f} = \underline{\underline{B}}^T\underline{F} \tag{3.47}$$

are the external applied forces corresponding to the u coordinates.

Consider now the viscous damping force $\underline{D}$. These internal forces may be expressed as:

$$\underline{D} = -\underline{\underline{C}}\,\dot{\underline{q}} \tag{3.48}$$

where $\underline{\underline{C}}$ is the damping matrix in q coordinates. The virtual work done by the damping forces $\underline{D}$ is:

$$\delta W = \delta\underline{q}^T\underline{D} = -\delta\underline{q}^T\underline{\underline{C}}\,\dot{\underline{q}} = -\delta\underline{u}^T\underline{\underline{B}}^T\underline{\underline{C}}\,\underline{\underline{B}}\,\dot{\underline{u}} = -\delta\underline{u}^T\underline{\underline{c}}\,\dot{\underline{u}} \tag{3.49}$$

where

$$\underline{\underline{c}} = \underline{\underline{B}}^T\underline{\underline{C}}\,\underline{\underline{B}} \tag{3.50}$$

is the damping matrix in u coordinates.

In summary, the mass matrix $\underline{\underline{M}}$, stiffness matrix $\underline{\underline{K}}$, damping matrix $\underline{\underline{C}}$ and force vector $\underline{F}$ in the q coordinate system under the linear transformation $\underline{q} = \underline{\underline{B}}\,\underline{u}$ are transformed into the element mass matrix $\underline{\underline{m}}$, the stiffness matrix $\underline{\underline{k}}$, the damping matrix $\underline{\underline{c}}$, and the force vector $\underline{f}$ in the new u-coordinate system according to the following transformations:

$$\underline{\underline{m}} = \underline{\underline{B}}^T\underline{\underline{M}}\,\underline{\underline{B}} \tag{3.51a}$$

$$\underline{\underline{k}} = \underline{\underline{B}}^T\underline{\underline{K}}\,\underline{\underline{B}} \tag{3.51b}$$

$$\underline{\underline{c}} = \underline{\underline{B}}^T\underline{\underline{C}}\,\underline{\underline{B}} \tag{3.51c}$$

$$\underline{f} = \underline{\underline{B}}^T\underline{F} \tag{3.51d}$$

6. Transformation of Element Coordinates to Datum Coordinates

Since the element mass matrix $\underline{\underline{m}}$, stiffness matrix $\underline{\underline{k}}$, and end force vector $\underline{f}$ are initially calculated in local element coordinates, suitably oriented to minimize the computing effort, it is necessary to introduce transformation matrices changing the frame of reference from a local to a datum coordinate system. Consider again the element shown in Fig. 3.1. U_1 to U_6 are displacements in the direction of local coordinates x,y,z and Q_1 to Q_6 are displacements in the directions of datum coordinates XYZ. Θ is the angle between the y and X axes. The displacements $\underline{U}$ can be related to displacements $\underline{Q}$ as:

$$U_1 = Q_1$$

$$U_2 = Q_2\cos\Theta + Q_3\sin\Theta$$

$$U_3 = -Q_2\sin\Theta + Q_3\cos\Theta$$

$$U_4 = Q_4$$

$$U_5 = Q_5\cos\Theta + Q_6\sin\Theta$$

$$U_6 = -Q_5\sin\Theta + Q_6\cos\Theta$$

or in matrix form

$$\underline{U} = \underline{\underline{B}}\ \underline{Q} \tag{3.52}$$

where

$$\underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & \cos\Theta & \sin\Theta & & & \\ 0 & -\sin\Theta & \cos\Theta & & & \\ & & & 1 & 0 & 0 \\ & 0 & & 0 & \cos\Theta & \sin\Theta \\ & & & & -\sin\Theta & \cos\Theta \end{bmatrix} \tag{3.53}$$

The element mass matrix $\underline{\underline{m}}^*$, stiffness matrix $\underline{\underline{k}}^*$, and the force vector $\underline{f}^*$ in q- displacements can be obtained by using the transformations of Eqs. (3.51a), (3.51b), and (3.51d) as:

$$\underline{\underline{m}}^* = \underline{\underline{B}}^T \underline{\underline{m}}\ \underline{\underline{B}}$$

$$\underline{\underline{k}}^* = \underline{\underline{B}}^T \underline{\underline{k}}\ \underline{\underline{B}} \tag{3.54}$$

$$\underline{f}^* = \underline{\underline{B}}^T \underline{f}$$

7. System of Assembled Structure

Since all element stiffness matrices, mass matrices and force vectors are now referred to the common datum, the stiffness matrix, mass matrix and force vector of the complete element assemblage can be obtained by the direct stiffness method as mentioned previously. The concentrated masses at nodal joints such as transmission, differentials and wheels are simply added to the corresponding diagonal terms in the assembled mass matrix. For example, the mass of the wheel at node no. 24,* which is associated with the coordinate q, can be directly added to the diagonal element $m_{70,70}$ of the mass matrix of the system. The concentrated cushion forces applied at nodal points are also simply added to the corresponding terms in the system force vector. For example, the cushion force applied at the transmission can be added to the element f_{82} of the system force vector $\underline{F}$. However, the discrete masses such as the engine and transfer case which connect several nodal points can not be superimposed on the system directly. A special study of these masses is necessary.

8. Engine and Transfer Case

Since the engine, transfer case and similar masses are all supported at several nodal points, special consideration of these parts is required. Consider now a discrete mass supported by n-springs with coordinates $q_1 \ldots . q_n$. Since three points define a plane, q_1, q_2 and q_3, as generalized coordinates, sufficiently define the motions of mass. Let u_1, u_2, and u_3 be the rotations about the principal axes of mass and the vertical translation respectively. The relationships between coordinates q's and u's can be formed in the following way:

Fig. 3.3 shows the system under consideration:

$\vec{a}_1$, $\vec{a}_2$ = The unit vectors along two principal directions of the mass.

$\vec{i}$, $\vec{j}$, $\vec{k}$ = unit vectors along datum coordinates X, Y, Z.

X_i, Y_i, Z_i = coordinates of supports.

$\vec{d}_i$, $\vec{d}_2$, $\vec{d}_3$ = position vectors of points where the springs attach to the mass, referred to the center of mass.

$\vec{e}_i$ = vectors from support point q_1 to q_i.

For small rotations,

$$q_i = u_1(\vec{a}_1 \times \vec{d}_i \cdot \vec{k}) + u_2(\vec{a}_2 \times \vec{d}_2 \cdot \vec{k}) + u_3$$

$$i = 1,2,3$$

which in matrix form is

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \times \vec{d}_1 \cdot \vec{k} & \vec{a}_2 \times \vec{d}_1 \cdot \vec{k} & 1 \\ \vec{a}_1 \times \vec{d}_2 \cdot \vec{k} & \vec{a}_2 \times \vec{d}_2 \cdot \vec{k} & 1 \\ \vec{a}_1 \times \vec{d}_3 \cdot \vec{k} & \vec{a}_2 \times \vec{d}_3 \cdot \vec{k} & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3.55}$$

or $\underline{q} = \underline{\underline{A}}\underline{U}$

* The q coordinates at node 28 are q_{70}, translation in the Z direction, q_{71}, rotation about the x-axis and q_{72}, rotation about the y-axis.

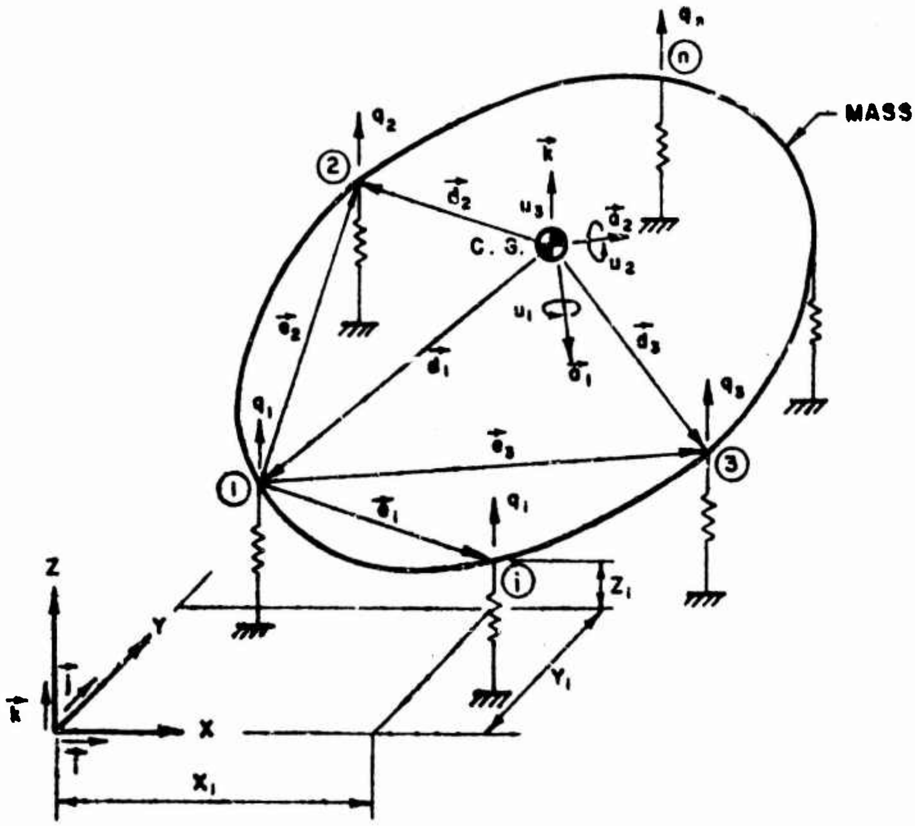


Fig. 3.3 Mass Supported at Several Points.

Thus

$$\underline{u} = \underline{\underline{A}}^{-1}\underline{q} = \underline{\underline{B}}\,\underline{q} \quad (3.56a)$$

where

$$\underline{\underline{B}} = \underline{\underline{A}}^{-1} \quad (3.56b)$$

If m_1, m_2 and m_3 are the mass moments of inertia and the mass, then the matrix of the discrete masses in the u coordinate system can be represented as:

$$\underline{\underline{m}} = \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{bmatrix}$$

According to the linear transformation (3.56a), the mass matrix corresponding to the q coordinate system is:

$$m^* = \underline{\underline{B}}^T \underline{\underline{m}}\,\underline{\underline{B}} \quad (3.57)$$

If there are cushioning forces f_1, f_2 and f_3 directly applied in u_1, u_2, and u_3 directions, then the forces in the q_1, q_2, and q_3 directions are:

$$\begin{bmatrix} f_1^* \\ f_2^* \\ f_3^* \end{bmatrix} = \underline{\underline{B}}^T \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

or

$$f^* = \underline{\underline{B}}^T \underline{f} \quad (3.58)$$

Now the mass matrix $\underline{\underline{m}}^*$ can be added to the mass matrix of the complete element assemblage and the forces f* can be added to the system forces. It must be noted that before taking this step, the constraint coordinates q_4 through q_n should be eliminated from the system. To do this, first find a transformation matrix $\underline{\underline{B}}$ such that:

$$\begin{bmatrix} q_4 \\ \vdots \\ q_n \end{bmatrix} = \underline{\underline{B}} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (3.59)$$

and then by linear transformation get the modified mass, stiffness and force matrix of the whole system.

The transformation matrix $\underline{B}$ in Eq. (3.59) is formulated by finding all the expressions of q_4 through q_n in terms of q_1, q_2, and q_3. Consider now the displacement q_i as indicated in Fig. 3.3. The vectors from support 1 to 2, 3 and i can be expressed as:

$$\vec{e}_2 = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (q_2 - q_1)\vec{k}$$

$$\vec{e}_3 = (x_3 - x_1)\vec{i} + (y_3 - y_1)\vec{j} + (q_3 - q_1)\vec{k}$$

$$\vec{e}_i = (x_i - x_1)\vec{i} + (y_i - y_1)\vec{j} + (q_i - q_1)\vec{k}$$

Since supports 1, 2, 3 and i are in a plane,

$$\vec{e}_2 \times \vec{e}_3 \cdot \vec{e}_i = 0$$

or

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & q_2 - q_1 \\ x_3 - x_1 & y_3 - y_1 & q_3 - q_1 \\ x_i - x_1 & y_i - y_1 & q_i - q_1 \end{vmatrix} = 0 \qquad (3.60)$$

Solving this equation for q_i results in

$$q_i = \frac{E_1 - E_2 + E_3}{E_3} q_1 - E_1 q_2 + E_2 q_3 \qquad (3.61a)$$

where

$$E_1 = \begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ x_i - x_1 & y_i - y_1 \end{vmatrix} \qquad (3.61b)$$

$$E_2 = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_i - x_1 & y_i - y_1 \end{vmatrix} \qquad (3.61c)$$

$$E_3 = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \qquad (3.61d)$$

Thus all displacements q_4 through q_n can be expressed in terms of q_1, q_2, and q_3 simply by replacing the index i in the determinants E_1 and E_2 by the numbers 4 to n.

9. Static Condensation

If an element is not rigidly connected to another element, for example, a hinged connection, the element stiffness matrix, mass matrix and fixed end force vector must be modified. If some coordinates included in the static analysis are excluded from the dynamic analysis, or some coordinates with zero or very small mass must be removed from the equations of motion to avoid unreasonable numerical results (infinite values in computed accelerations), then the system mass matrix, stiffness matrix and system force vector must all be modified. All the modifications can be achieved by a static condensation technique.[2] By using this technique a transformation matrix similar to the matrix $\underline{B}$ in Eq. (3.33) can be formulated. Then the modified matrices can be written in the format of Eq. (3.51).

The first step is to partition the mass matrix $\underline{\underline{M}}$, stiffness matrix $\underline{\underline{K}}$, force matrix $\underline{F}$ and displacement matrix $\underline{U}$ into:

$$\underline{\underline{M}} = \begin{bmatrix} \underline{\underline{M}}_{11} & \underline{\underline{M}}_{12} \\ \underline{\underline{M}}_{21} & \underline{\underline{M}}_{22} \end{bmatrix} \tag{3.62a}$$

$$\underline{\underline{K}} = \begin{bmatrix} \underline{\underline{K}}_{11} & \underline{\underline{K}}_{12} \\ \underline{\underline{K}}_{21} & \underline{\underline{K}}_{22} \end{bmatrix} \tag{3.62b}$$

$$\underline{F} = \begin{bmatrix} \underline{F}_1 \\ \underline{F}_2 \end{bmatrix} \tag{3.62c}$$

$$\underline{U} = \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \tag{3.62d}$$

The column matrix $\underline{U}_1$ refers to all the displacements we wish to retain, while $\underline{U}_2$ denotes all the remaining displacements which will not be employed in formulating the new equivalent matrices. The displacements $\underline{U}_2$ may be determined from the static equilibrium equation $\underline{F} = \underline{\underline{K}}\,\underline{U}$ by assuming that the forces $\underline{F}_2$ corresponding to the displacements $\underline{U}_2$ are all equal to zero. Hence:

$$\underline{U}_2 = -\underline{\underline{K}}_{22}^{-1}\,\underline{\underline{K}}_{21}\,\underline{U}_1 \tag{3.63}$$

thus:

$$\underline{U} = \begin{bmatrix} \underline{\underline{I}} \\ -\underline{\underline{K}}_{22}^{-1}\,\underline{\underline{K}}_{21} \end{bmatrix} \underline{U}_1 = \underline{\underline{B}}^*\,\underline{U}_1 \tag{3.64}$$

where $\underline{\underline{I}}$ is a unit matrix and

$$\underline{\underline{B}}^* = \begin{bmatrix} \underline{\underline{I}} \\ -\underline{\underline{K}}_{22}^{-1}\,\underline{\underline{K}}_{21} \end{bmatrix} \tag{3.64a}$$

Thus the modified mass matrix M*, stiffness matrix K*, and force matrix F* for displacements $\underline{U}_1$ are obtained from Eq. (3.51) as:

$$M^* = \underline{\underline{B}}^{*T}\underline{\underline{M}}\,\underline{\underline{B}}^* \;, \quad K^* = \underline{\underline{B}}^{*T}\underline{\underline{K}}\,\underline{\underline{B}}^* \;, \quad F^* = \underline{\underline{B}}^{*T}\underline{F} \tag{3.65}$$

Eq. (3.65) may be obtained directly from the equations of motion as follows:

$$\underline{\underline{M}}_{11}\ddot{\underline{U}}_1 + \underline{\underline{M}}_{12}\ddot{\underline{U}}_2 + \underline{\underline{K}}_{11}\underline{U}_1 + \underline{\underline{K}}_{12}\underline{U}_2 = \underline{F}_1 \tag{3.66a}$$

$$\underline{\underline{M}}_{21}\ddot{\underline{U}}_1 + \underline{\underline{M}}_{22}\ddot{\underline{U}}_2 + \underline{\underline{K}}_{21}\underline{U}_1 + \underline{\underline{K}}_{22}\underline{U}_2 = \underline{F}_2 \tag{3.66b}$$

Eq. (3.66b) can be rewritten as:

$$\underline{U}_2 = -\underline{\underline{K}}_{22}^{-1}\underline{\underline{K}}_{21}\underline{U}_1 + \underline{\underline{K}}_{22}^{-1}(\underline{F}_2 - \underline{\underline{M}}_{21}\ddot{\underline{U}}_1 - \underline{\underline{M}}_{22}\ddot{\underline{U}}_2) \qquad (3.66c)$$

If we assume $\underline{F}_2$ and $\ddot{\underline{U}}$ to be linear in a small time interval, then the second time derivative of Eq. (3.66c) is

$$\ddot{\underline{U}}_2 = -\underline{\underline{K}}_{22}^{-1}\underline{\underline{K}}_{21}\ddot{\underline{U}}_1 \qquad (3.66d)$$

Substituting Eqs. (3.66c) and (3.66d) into Eq. (3.66a) the mass matrix M*, stiffness matrix K* and force matrix F* are obtained as shown in Eq. (3.65). These are the matrices that will now be used in Eq. 3.1 for actual computations.

10. Structural Damping

Until now the discussion of the damping matrix in the equations of motion Eq. (3.1) has been intentionally avoided. However, earlier experimental studies of the M-37 truck indicate that an appreciable amount of energy may be dissipated through internal friction within the truck body or at joints between frame elements. Structural damping forces $\underline{D}$ are proportional in magnitude to the internal elastic forces and opposite in direction to the velocity of the system. The force may be represented as:

$$\underline{D} = i\, b_1\, \underline{\underline{K}}\, \underline{q} \qquad (3.67)$$

where b_1 is a constant and i is the imaginary number.

This expression for the damping force is not readily amenable to structural analysis. Thus the concept of viscous damping is used in the analysis as indicated by the damping matrix $\underline{C}$ in Eq. (3.1). Furthermore, this damping matrix may be assumed to be proportional to the mass and stiffness matrices as:

$$\underline{\underline{C}} = a\, \underline{\underline{M}} + b\, \underline{\underline{K}} \qquad (3.68)$$

where a and b (b is not the same as b_1) are constants and can be determined so as to give reasonable damping in the system. If Eq. (3.68) is substituted into Eq. (3.1), the modified differential equation

$$\underline{\underline{M}}\, \ddot{\underline{q}} + (a\, \underline{\underline{M}} + b\, \underline{\underline{K}})\, \dot{\underline{q}} + \underline{\underline{K}}\, \underline{q} = \underline{F}(t) \qquad (3.69)$$

is obtained.

Since both $\underline{\underline{M}}$ and $\underline{\underline{K}}$ have non zero off-diagonal coefficients, these differential equations are coupled. By using the modal matrix, obtained from the arrangement of normal modes, as a transformation matrix, the equations of motion can be reduced to a set of uncoupled equations of the type:

$$m_r\ddot{q}_r + (am_r + bk_r)\, \dot{q}_r + k_r q_r = f_r(t) \qquad (3.79)$$

where q_r is the normal coordinate in rth mode of the system and m_r and k_r are the corresponding mass and stiffness in that mode respectively.

Since this equation is the same form as the single degree of freedom equation, the critical damping in the rth mode is:

$$(am_r + bk_r)_{cr} = 2m_r\omega_r \qquad (3.80)$$

where $\omega_r^2 = k_r/m_r$ is the natural frequency of rth mode.

Dividing both sides of Eq. (3.80) by M_r,

$$(a + b\omega_r^2)_{cr} = 2\omega_r$$

or

$$S_r = \frac{a}{2\omega_r} + b\,\frac{\omega r}{2} \tag{3.81}$$

where S_r is the ratio of actual to critical damping.

For given values of a and b the frequency $\bar{\omega}$ which yields a minimum value for the damping ratio S can be found by differentiating Eq. (3.81) with respect to ω_r and then setting the derivative equal to zero. Thus

$$\bar{\omega} = \sqrt{a/b} \tag{3.82a}$$

and

$$\bar{S} = \sqrt{ab} \tag{3.82b}$$

If $\bar{S}$ and $\bar{\omega}$ are given, the damping coefficients a and b are calculated from the following equations:

$$a = \bar{S}\,\bar{\omega} \tag{3.83a}$$

and

$$b = \bar{S}/\bar{\omega} \tag{3.83b}$$

Therefore, if the significant frequency range is established for a structure and the equivalent modal damping is selected, the constants a and b can be calculated from Eqs. (3.83). However, in the numerical analysis of this study, the frequency of the structure is not calculated. The value of $\bar{\omega}$ is estimated as:

$$\bar{\omega} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{k_i}{m_i}} \tag{3.84}$$

where m_i and k_i are the diagonal elements in the n x n mass matrix and stiffness matrix.

It appears that for the M-37 truck model a reasonable value of damping ratio $\bar{S}$ corresponding to the frequency $\bar{\omega}$ in Eq. (3.84) is around 0.01. This number was estimated after a comparison between some computed and some experimental results was made.

NUMERICAL SOLUTION

1. Runge-Kutta Method

The equations of motion Eq. (3.69) can be solved numerically by using the Runge-Kutta method. However, Eq. (3.69) must be decomposed into first order differential equations in order to apply the Runge-Kutta method.

Premultiply all terms in the equations of motion Eq. (3.69) by $\underline{\underline{M}}^{-1}$ and eliminate the coefficient of $\ddot{q}$. Thus

$$\ddot{\underline{q}} + (a\underline{\underline{I}} + b\underline{\underline{M}}^{-1}\underline{\underline{K}})\dot{\underline{q}} + \underline{\underline{M}}^{-1}\underline{\underline{K}}\,\underline{q} = \underline{\underline{M}}^{-1}\underline{F}(t) \tag{4.1}$$

where $\underline{\underline{I}}$ is a unit matrix, and all matrices are modified as discussed in the section on static condensation. By letting

$$u = \dot{\underline{q}} \tag{4.2}$$

from which

$$\underline{\dot{u}} = \underline{\ddot{q}} \tag{4.3}$$

The set of equations (4.1) is decomposed into 2 sets of first order ordinary differential equations:*

$$\underline{\dot{q}} = \underline{u} \tag{4.4a}$$

$$\underline{\dot{u}} = \underline{\underline{M}}^{-1}\underline{F} - \underline{\underline{M}}^{-1}\underline{\underline{K}}\,\underline{q} - (a\underline{\underline{I}} + b\underline{\underline{M}}^{-1}\underline{\underline{K}})\underline{u} \tag{4.4b}$$

For the study of airdropped impact, the initial displacements $\underline{q}_o$ (usually $\underline{q}_o = \underline{0}$) and initial velocity $\underline{\dot{q}}_o$ are known, thus the initial conditions are:

$$\underline{q} = \underline{q}_o \tag{4.5a}$$

$$\underline{u} = \underline{\dot{q}}_o \tag{4.5b}$$

Equation (4.4) with initial conditions as expressed by Eq. (4.5) can be solved numerically by the Runge-Kutta[5] method. Eqs. (4.4a) and (4.4b) may be combined in a compact form as:

$$\underline{\dot{v}} = \underline{g}(t,\underline{v}) \tag{4.6}$$

where t is the independent time variable and

$$\underline{v} = \begin{bmatrix} \underline{q} \\ \\ \underline{u} \end{bmatrix} \tag{4.7}$$

The corresponding initial conditions are:

$$\underline{v}_o = \begin{bmatrix} \underline{q}_o \\ \\ \underline{\dot{q}}_o \end{bmatrix} \tag{4.8}$$

If the system has n degrees of freedom, then the expression of Eq. (4.6) in index notation is:

$$\dot{v}_i = g_i(t,\ v_1\ \ldots.\ v_{2n}) \qquad i = 1,2\ \ldots.2n \tag{4.9}$$

The basic equations in the Runge-Kutta method for solving the set of ordinary first order differential equations of Equations (4.9) are:

$$v_i\ (t + h) = v_i\ (t) + \frac{1}{6}(a_i + 2b_i + 2c_i + d_i)$$
$$i = 1,2\ \ldots.\ 2n \tag{4.10}$$

where

* u should not be confused with a displacement. It is introduced here purely for convenience and is as defined by Eq.(4.2)

h = time step length of integration

$$a_i = h[g_i(t, v_1, \ldots\ v_{2n})]$$

$$b_i = h[g_i(t + \tfrac{1}{2}h,\ v_1 + \tfrac{1}{2}a_1,\ \ldots,\ v_{2n} + \tfrac{1}{2}a_{2n})]$$

$$c_i = h[g_i(t + \tfrac{1}{2}h,\ v_1 + \tfrac{1}{2}b_1,\ \ldots,\ v_{2n} + \tfrac{1}{2}b_{2n})]$$

$$d_i = h[g_i(t + h,\ v_i + c_1,\ \ldots,\ v_{2n} + \tfrac{1}{2}c_{2n})]$$

2. Digital Computer Program

A computer program code has been developed to solve the equations of motion with the initial conditions of the drop impact problem. This program is listed in the appendix. The required input data are nodal point coordinates, mass distribution, cushioning forces, initial displacements and velocities, damping coefficients, and member properties such as cross sectional area, flexural rigidity EI, torsional rigidity GJ, and torsional inertia I_x. The computer program automatically generates the structural mass matrix, stiffness matrix and force matrix and then solves the equations of motion by the Runge-Kutta method. The displacement, velocity and acceleration at any coordinate at any time may be printed out. The printed out displacement is relative to the position where impact begins. From these displacements the relative displacement of any two points can be computed.

This program is primarily developed for either checking the design of an existing vehicle which may be subjected to airdrop impact or as a guide in designing a vehicle which may be destined for delivery by airdrop. This program can be used not only for vehicles but for any complex mechanical structure that can be represented by a grid structure model. It is expected that through the use of this computational procedure the amount of actual experimentation required for developing cushioning systems can be materially reduced. If elements of the vehicle which might undergo excessive deformations during impact can be identified before the vehicle is drop tested, or even built, the computer program will serve as a design tool as well as an aid in cushioning design.

FACTORS AFFECTING THE RESPONSE TO IMPACT

The previous analysis shows that the dynamic response of a structure is dependent upon the nature of the applied forcing function, the elastic and inertia properties of the structure, and the damping characteristics of the system. By study of these factors, information on design of cushioning systems and the design of the structure itself may be obtained. Insight into the appropriateness of the lumped-parameter model should also be provided. In following sections, these factors are discussed in detail. The effects of the tires on the structural response to the impact, and the dynamic behavior of the engine on the rubber supports will also be considered.

1. Shape of Applied Force

In an airdrop of a vehicle it is fastened to a platform which holds the cushioning system in place but provides no support for the vehicle. In order to avoid excessive damage to the vehicle, cushioning materials are placed between it and the platform. According to previous experimental investigations,[6] the force transmitted to the structure through the cushioning material is essentially independent of the degree of crushing. The amplitude and duration of the force are generally dependent upon the arrangement of the cushioning materials and the crushing characteristics of these materials. To study the response of a structure to the force applied by the cushioning, the response of the structure due to the application of a rectangular impulse can be examined. The effect of amplitude and duration of the force on the behavior of the structure is of particular interest. Three rectangular pulses with different amplitudes and durations and one triangular pulse, shown in Fig. 5.1, were chosen for this study. For simplicity, the area of each pulse was kept the same. Thus the momentum imparted to the structure is the same for each pulse. In order to simplify the analysis, the centroid position of the pulse has been chosen as a characteristic parameter. In Fig. 5.1, case A is the impulse for a design acceleration of 17.5g. Case B and C represent the same impulse as A, but with different time duration. Case D is a triangular pulse with the same area and time duration as case A.

Neglecting the damping, the maximum displacements and peak accelerations at all nodal points in Fig. 2.1 for all cases have been calculated. For the purpose of demonstration and discussion, the maximum displacements and peak accelerations at node 18 are shown in Fig. 5.2 and 5.3 respectively. This node is on the frame over the left rear wheel of the truck. There is no cushioning force applied directly at this point. Fig. 5.2 shows that the maximum vertical displacement of this point as a result of the application of the four different pulse shapes to the truck is linearly proportional to the time to the centroid of the pulse shape.

The peak accelerations produced by each of the different impulses are shown. These results suggest that both the displacement and acceleration produced by a given impulse depend essentially on the time to the centroid of the area under the force-time (impulse) curve.

2. Structural Properties

The structure of an M-37 truck is so complicated that simplifications must be made for analytical studies. If the truck is to be represented by a simplified model, the question of how to estimate the stiffness of the structure must be answered. As a part of the attack on this problem the significance of changes in the impact response with variation in the structural properties should be investigated. Consider now the mathematical model of the M-37 truck shown in Fig. 2.1. Three different sets of values of stiffness are assumed:

- Case 1: All member stiffnesses are estimated based on the truck frame only
- Case 2: All member stiffnesses of case 1 are multiplied by five

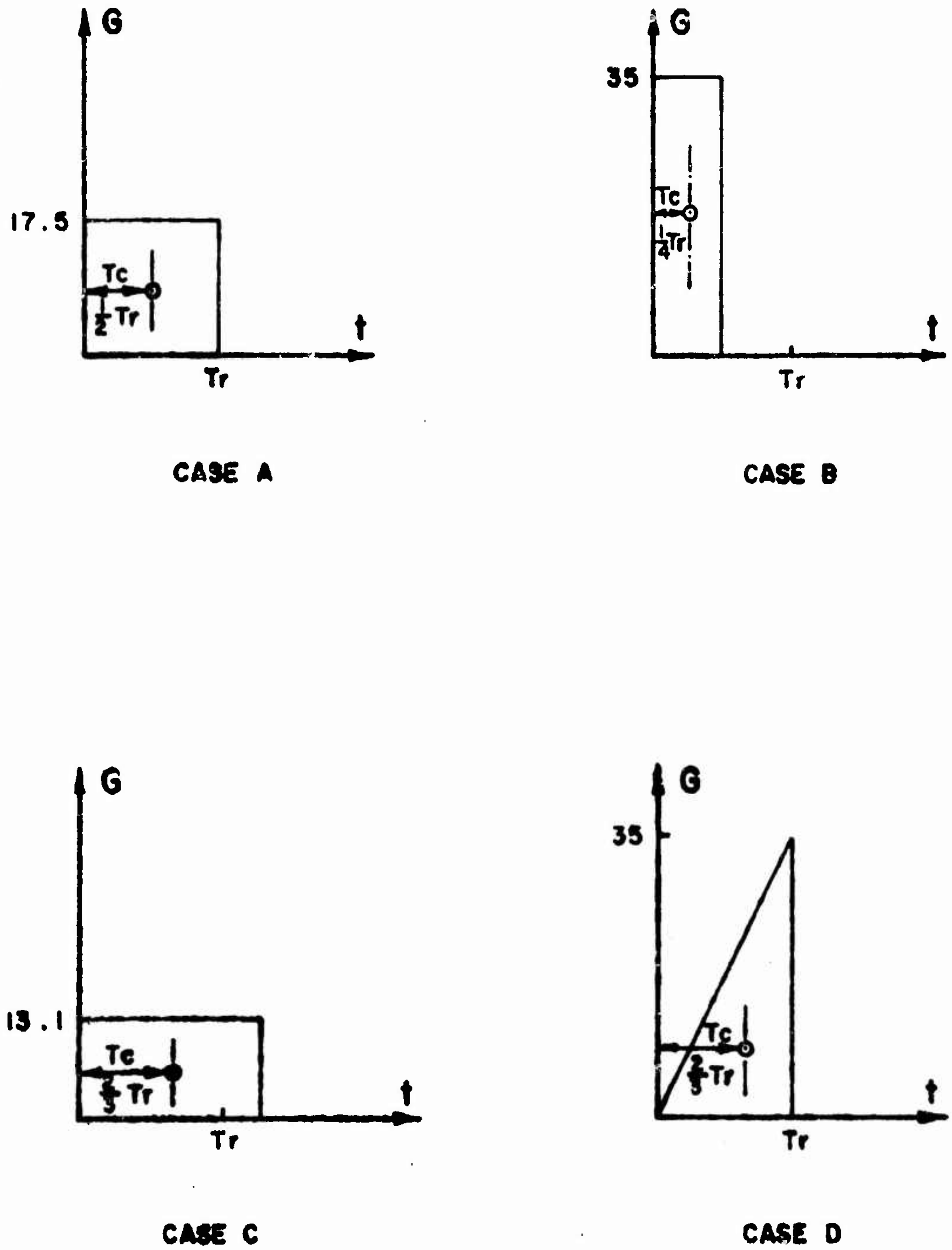


Fig. 5.1 Impulse Shapes.

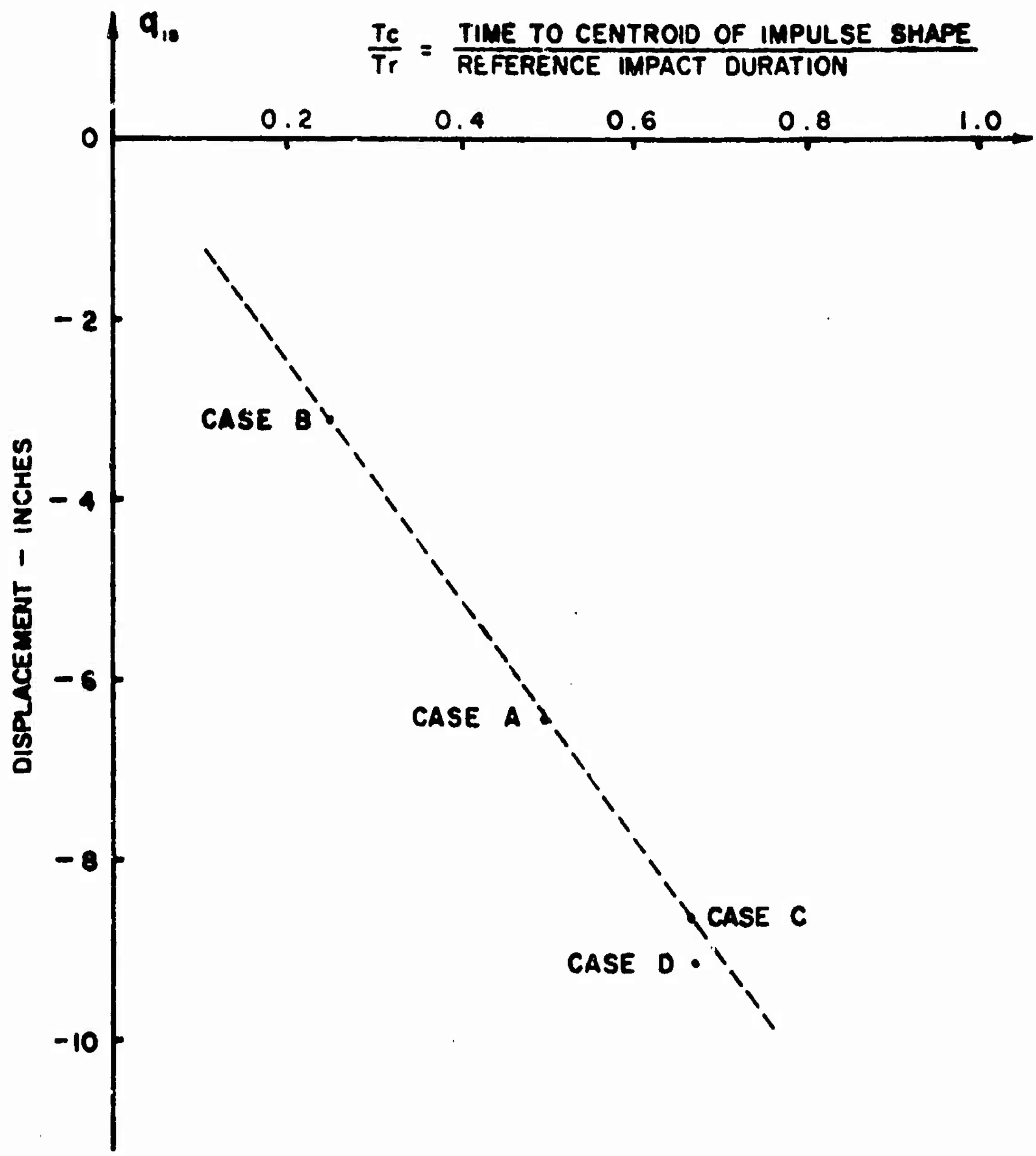


Fig. 5.2 Impulse Shape Effect on Maximum Displacement.

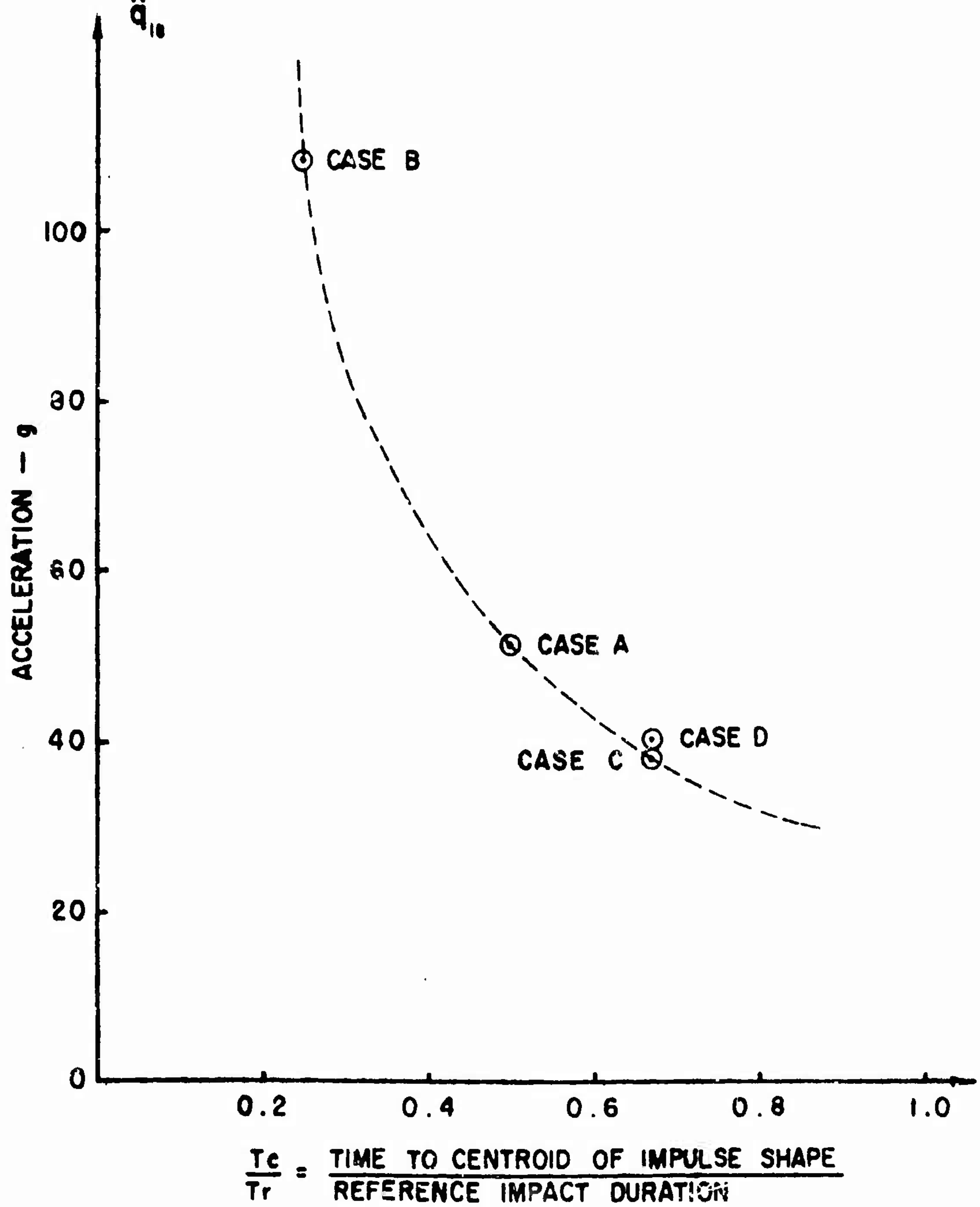


Fig. 5.3 Impulse Shape Effects on Peak Acceleration

Case 3: All member stiffnesses of case 1 are multiplied by ten.

Computed total displacements and peak accelerations along the main frame joint No. 1 through No. 10 for the above three cases are shown in Fig. 5.4 and 5.5. All calculations are assumed to be for a 10 ft. equivalent drop height with a design acceleration level of 17.5g and no damping. The centroid of the honeycomb area is located intentionally 6 inches toward the front from the center of gravity of the loaded truck. This causes a non-uniform crushing of the honeycomb cushioning as shown in Fig. 5.4. The most uniform displacement occurs for case 3, the truck with the greatest stiffness. However, Case 2 deflections differ very little from those of Case 3. The peak acceleration curves shown in Fig. 5.5 indicate that case 3 also has the most uniformity in the acceleration at all points along the truck frame. The differences in peak accelerations among the three cases at a given point are small. All curves are very close together. Thus it appears that a reasonable impact response can be obtained by using very rough approximations to the structural stiffness.

3. Damping

The amount of structural damping in the vehicle must be determined by experimentation. If experimental data or reliable information is not available, no damping should be assumed in the analysis since the omission of damping results in conservative estimates of deflections, and conservative cushioning system designs. Numerical computations for the M-37 model (Fig. 2.1) show that at all points of the truck frame the absolute magnitudes of displacements are increasing as damping decreases but the peak accelerations are affected very little. Fig. 5.6 shows the maximum crushing displacements for a damping ratio range from zero up to 0.015. There is no change in the configuration of the truck for all three cases. The crushing displacement curve with 0.009 as the damping ratio is the one which approximates most closely the experimental M-37 measurements which are shown later in section 6.

4. The Effect of Tires on the Response of Vehicle Body

Since the vehicle body is connected to the wheels through a leaf spring arrangement, the magnitude and shape of forces transmitted to the vehicle body as the result of an impact would be significantly affected by the material properties of the tires. These forces must be considered in computations of the overall response of the vehicle body.

For the purpose of analysis, the wheel and tire are replaced by a mass, a spring K, and dashpot C arrangement as depicted in Fig. 2.3. Using different sets of values of K and C for the tire, the displacement-time curves at point No. 3 (q_3) of the vehicle body, and the wheel at point No. 24 (q_{24}) have been calculated and plotted in Figs. 5.7, 5.8, 5.10, and 5.11. In Figs. 5.9 and 5.12, the maximum displacement and acceleration of nodal point No. 3 and the wheel have been plotted as a function of spring constants and damping ratios.

In general, the larger the spring constants and the damping ratio, the smaller the displacement of the vehicle.

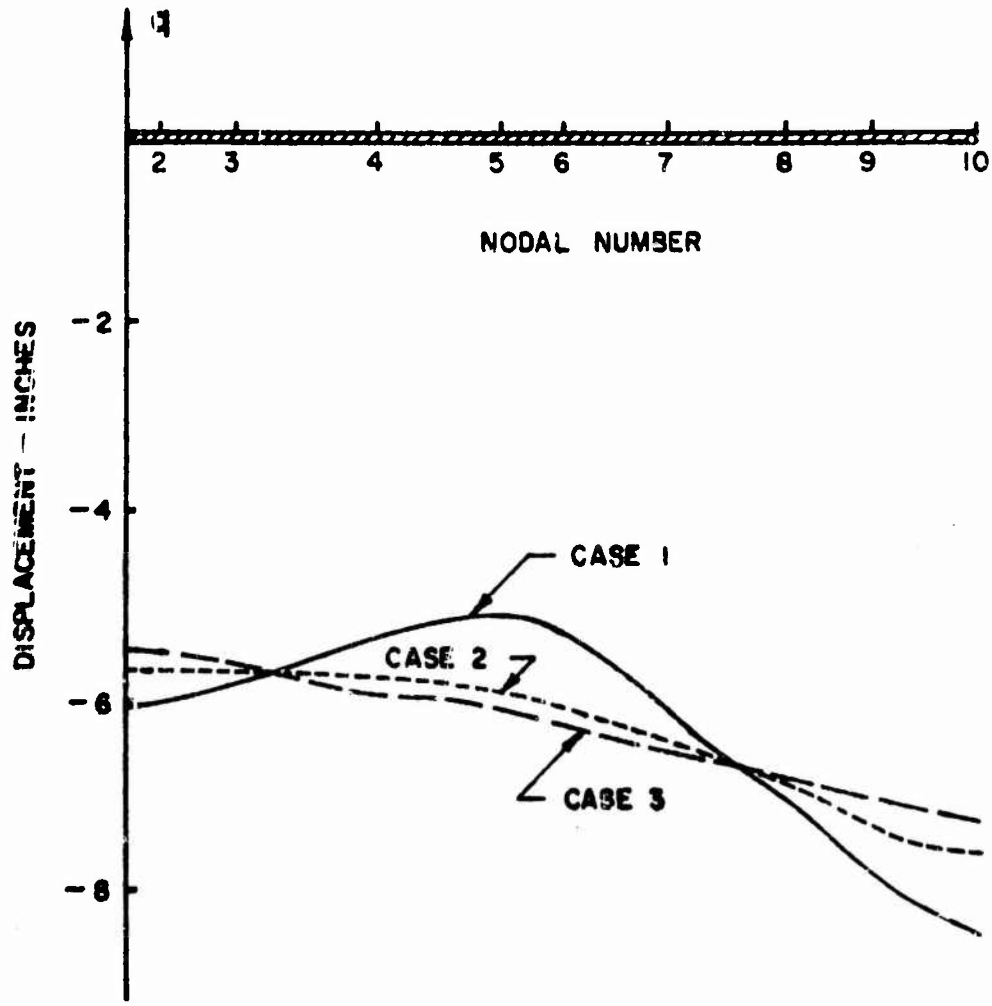


Fig. 5.4 Structural Stiffness Effect on Maximum Displacements.

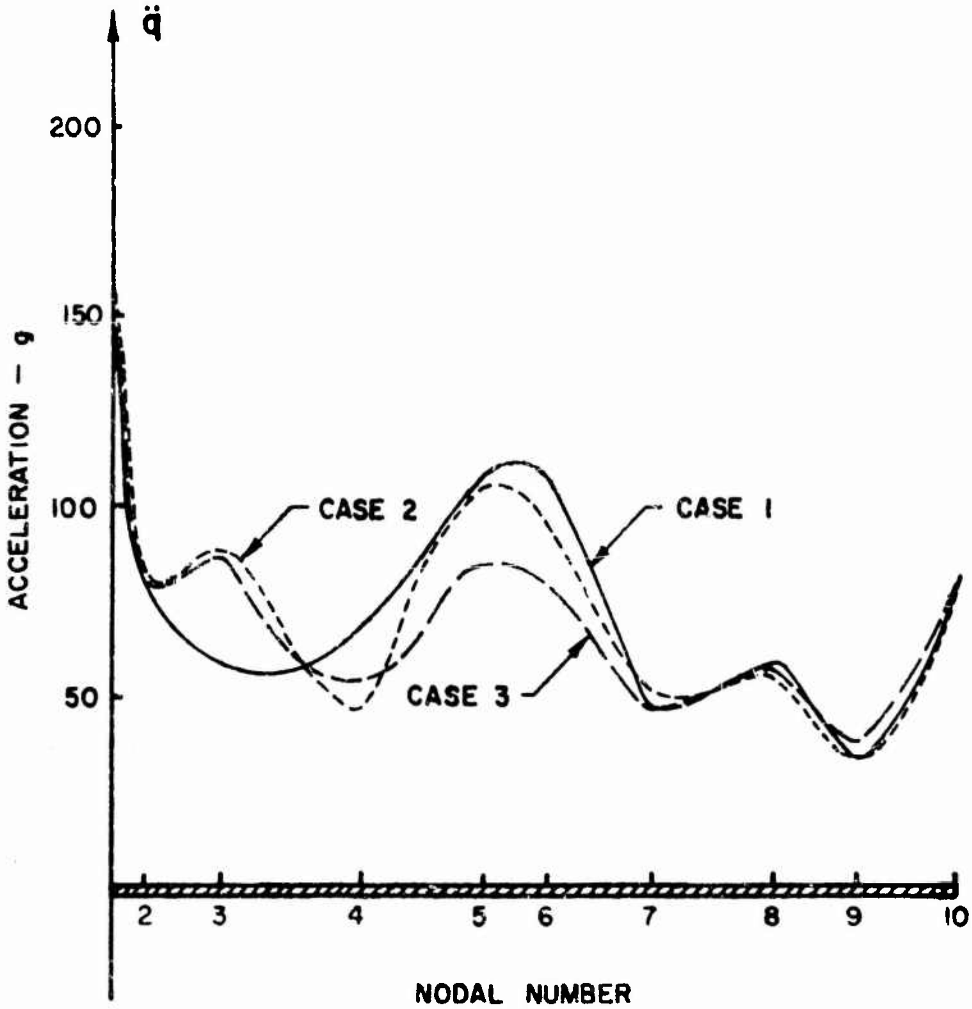


Fig. 5.5 Structural Stiffness Effect on Peak Accelerations.

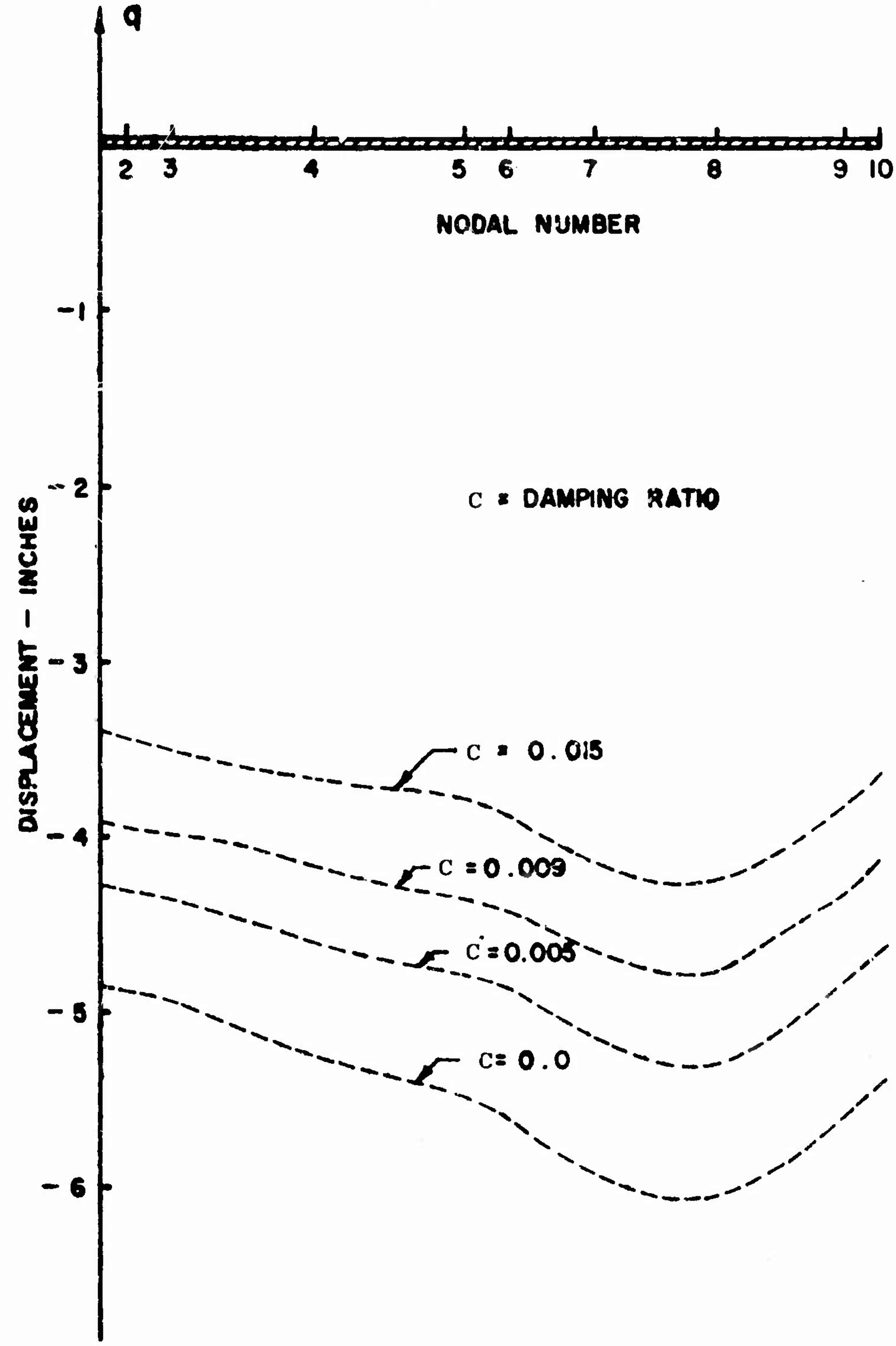


Fig. 5.6 Structural Damping Effect on Maximum Displacements.

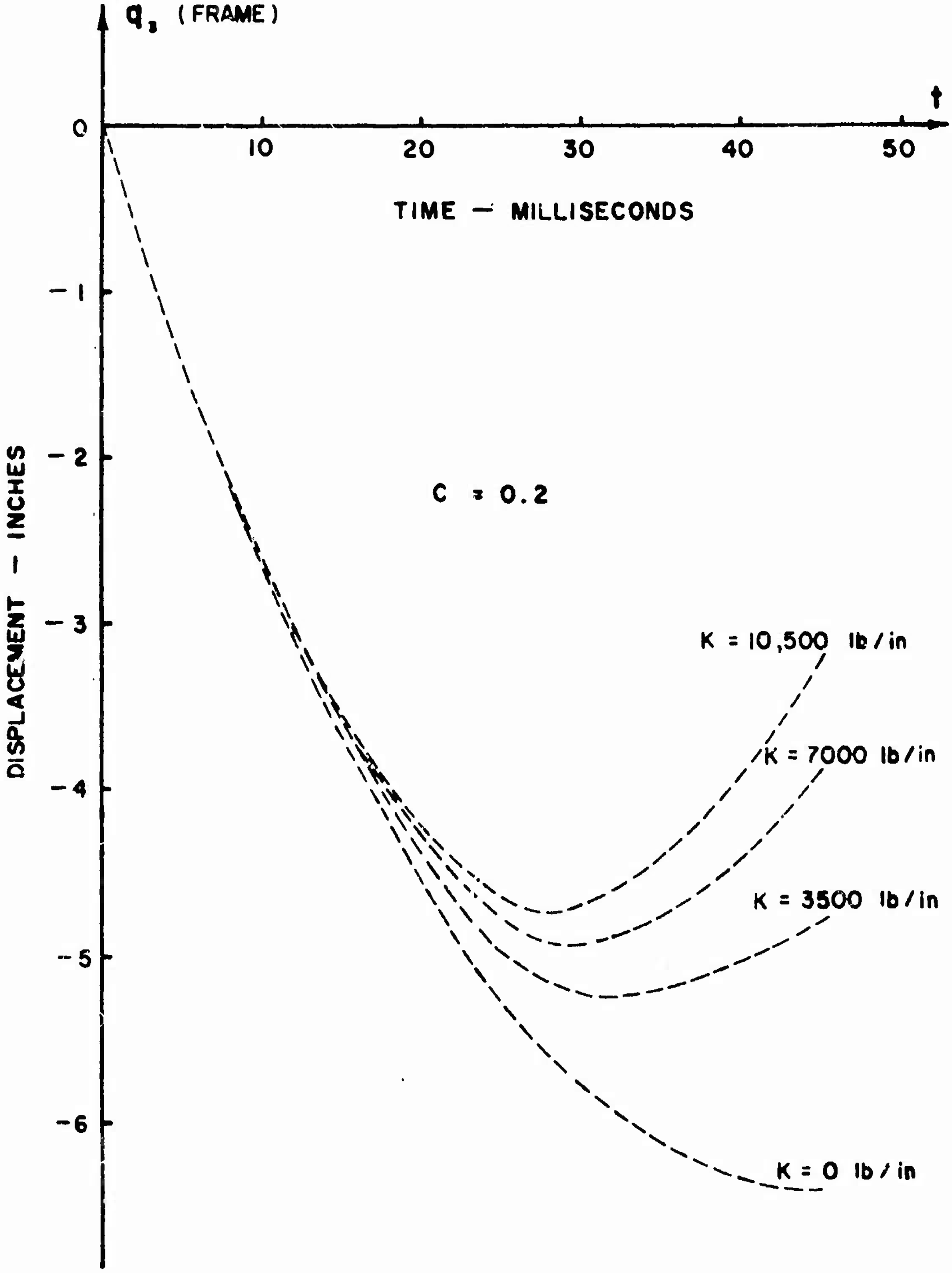


Fig. 5.7 Spring Constant Effect of Tire on Frame Displacement.

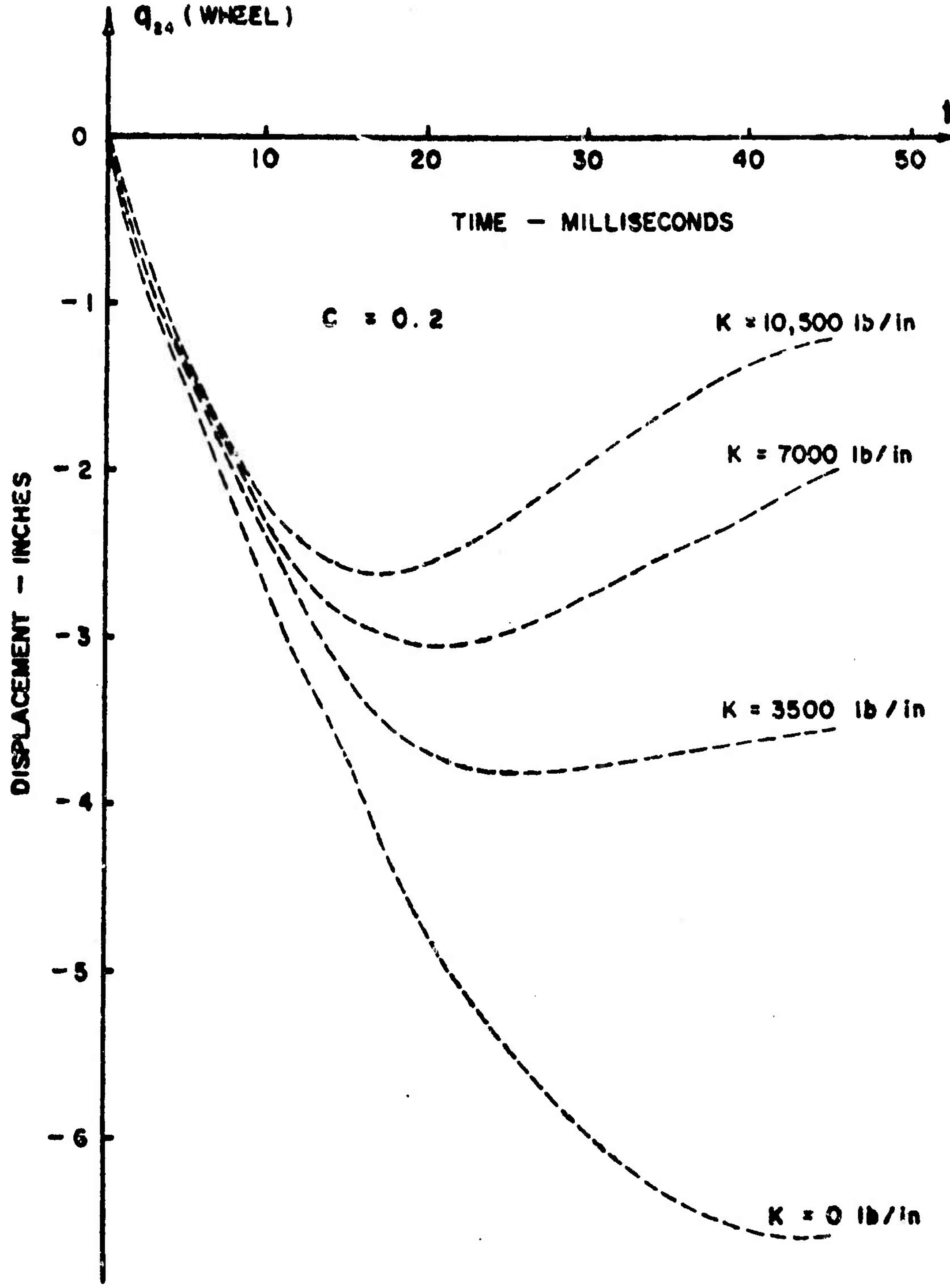


Fig. 5.8 Spring Constant Effect of Tire on Wheel Displacement.

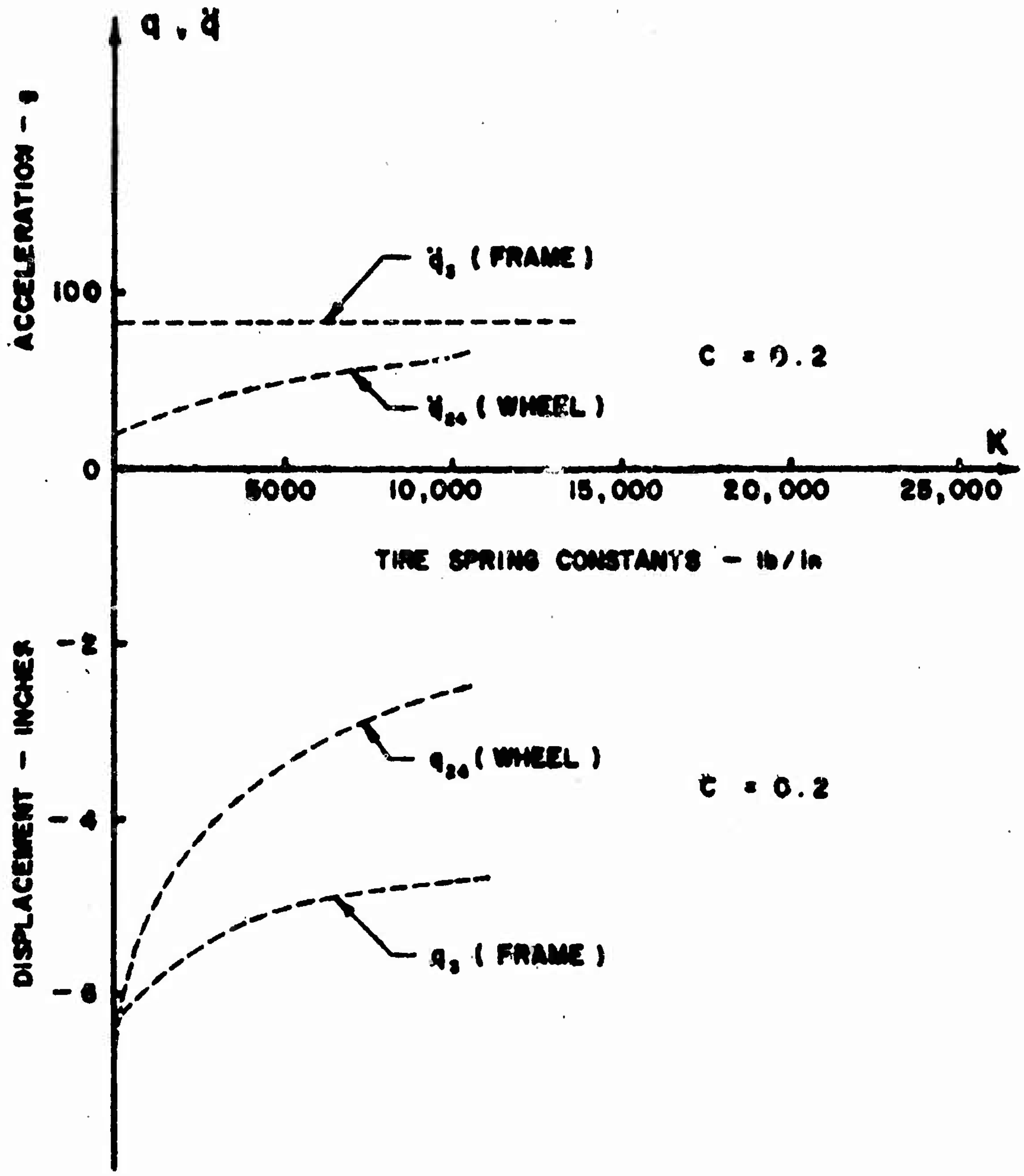


Fig. 5.9 Spring Constant Effect of Tire on Maximum Displacements and Accelerations.

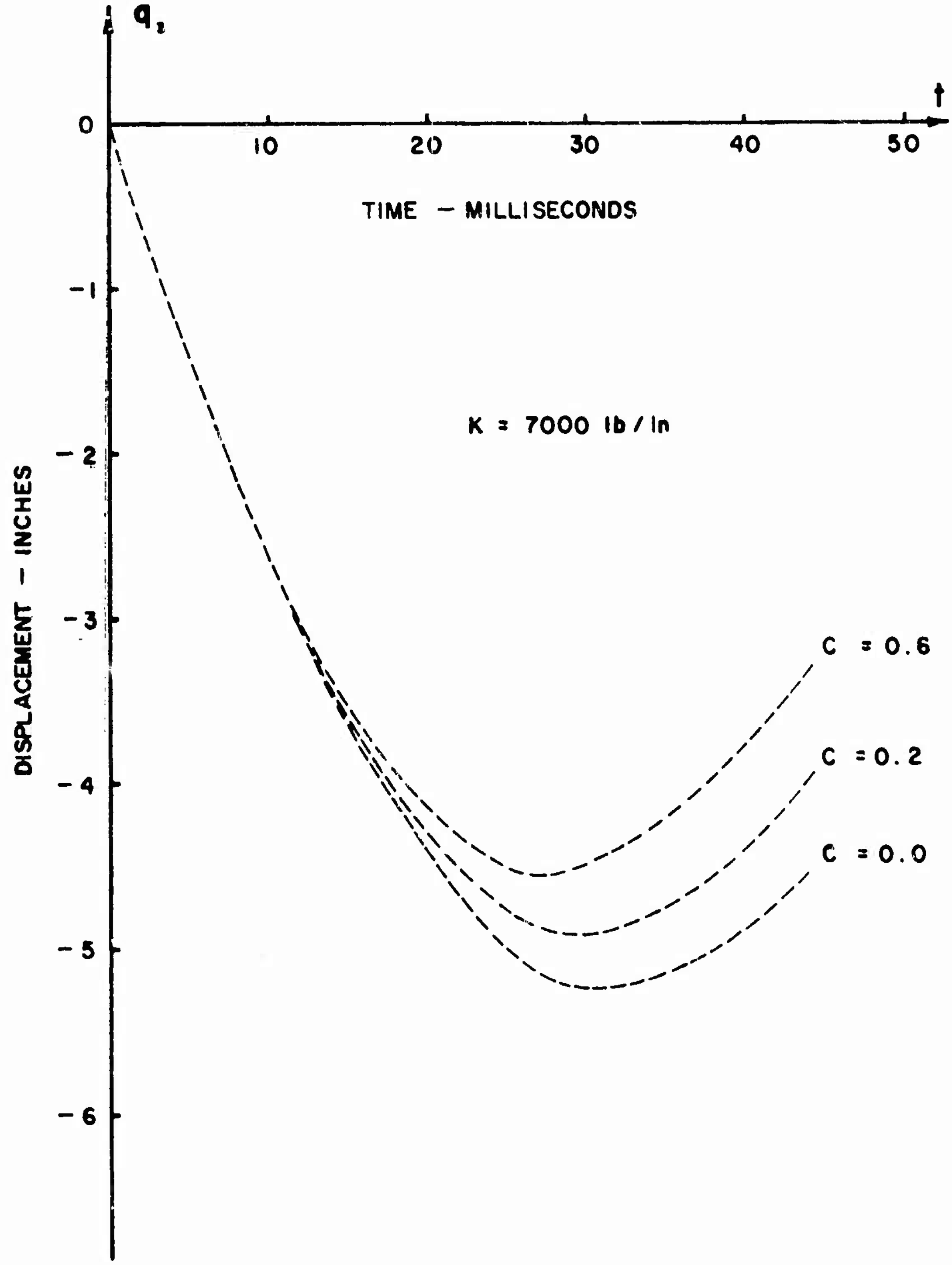


Fig. 5.10 Internal Damping Effect of Tire on Frame Displacement.

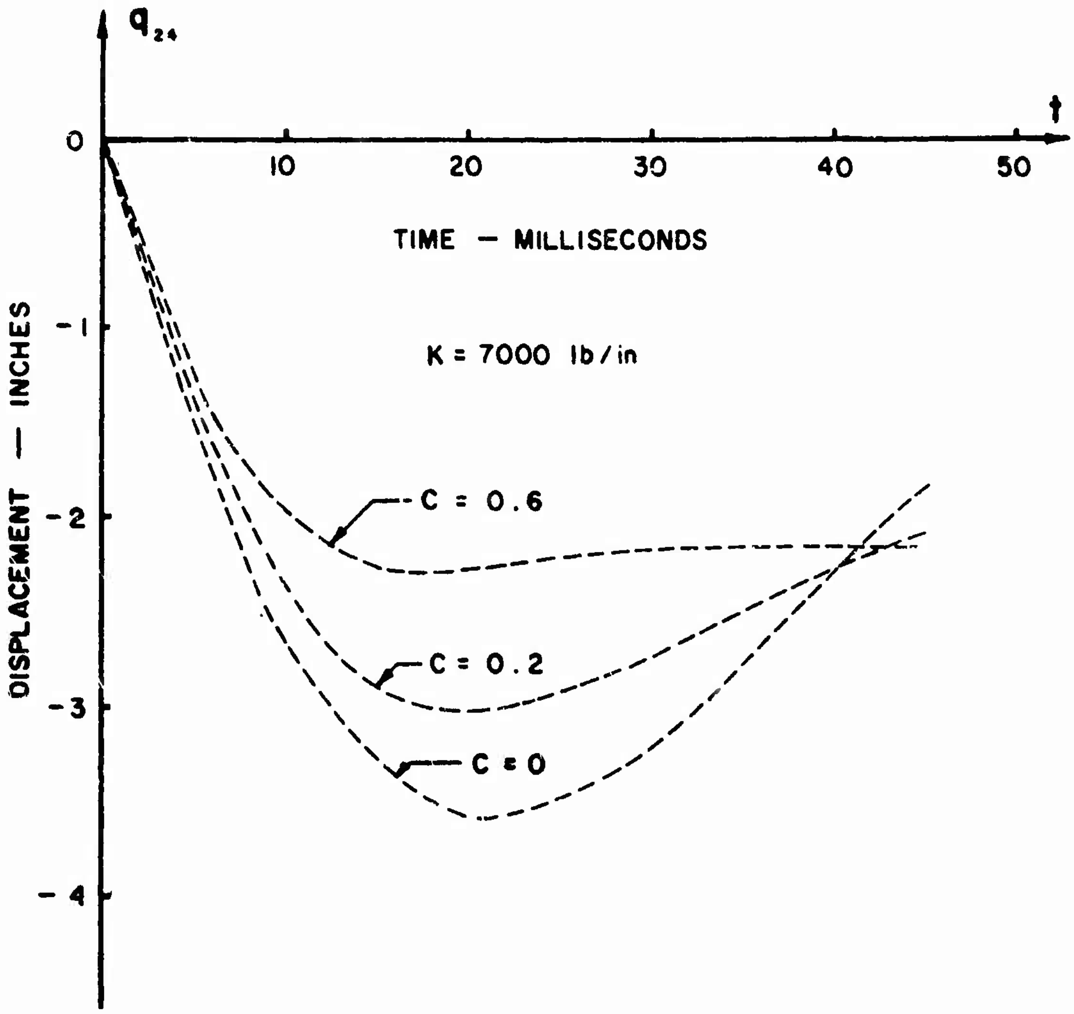


Fig. 5.11 Internal Damping Effect of Tire on Wheel Displacement.

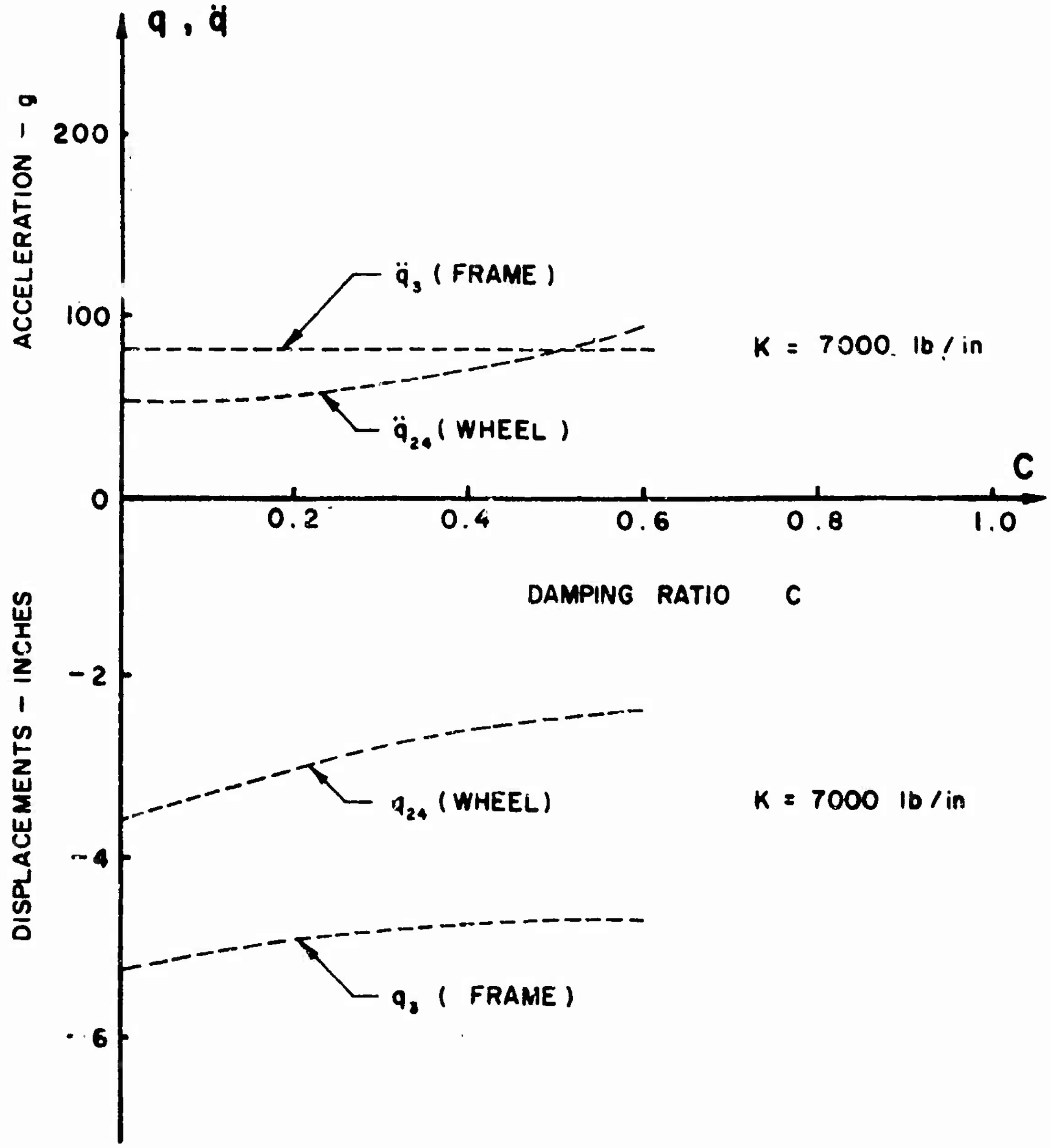


Fig. 5.12 Internal Damping Effect of Tire on Maximum Displacements and Accelerations.

It is also seen in Figs. 5.9 and 5.12 that the wheels have less displacement than the vehicle body. This is the reason that fewer cushion pads are required under the tires. The peak acceleration of the vehicle body does not show any significant variations as the spring constant and the damping ratio of the tires vary in the range of this investigation.

5. The Effect of Engine Supports on the Behavior of Engine

The idealized engine supporting arrangement is shown in Fig. 2.2. Computational results indicate that the values of the spring constant K and damping ratio C in Fig. 2.2 affect neither the peak accelerations nor the maximum displacements at any point in the vehicle except the engine itself during the period of impact. Figs. 5.13 through 5.16 show the effects of the elastic stiffness and internal damping of engine supports on the dynamic responses of the engine. As in the case of the tire, in general, the larger the spring constant and internal damping of the supports, the smaller the engine relative displacement to the truck frame.

The peak acceleration of the engine is not influenced much by the spring constant of the supports but it decreases with an increase in internal damping of the support.

EXPERIMENTAL INVESTIGATION -- M-37 TRUCK DROP TEST

To obtain some experimental data for comparison with the computed results as described in the previous chapters, three drops of the M-37, 3/4 ton truck with a 1500 lb. simulated load of sandbags have been made from a drop height of 10 ft., and at a design acceleration of 17.5 g.

The cushioning system used for all drops is shown in Fig. 6.1. Typical design calculations for such cushioning systems are shown in the Appendix. The cushioning material used throughout this series was 80-0-1/2 paper honeycomb with a characteristic stress-strain curve as shown in Fig. 6.2 and an average crushing stress of 6400 psf. The truck was rigged for drop by attaching lifting plates and shackles to each of the wheels. The entire rigging is shown in Fig. 6.3.

Accelerations are measured with fluid damped resistance type accelerometers. Engine displacements relative to the truck frame are measured with slide-wire type transducers. A special deflection gage shown in Fig. 6.4 is used to measure the displacements at other positions on the vehicle body. Accelerometers and displacement gages were mounted on the vehicle in the following positions:

Drop No. 1 (Series No. M-37-17)

- Accelerometers at
  - Engine front (Joint No. 23)
  - Thin plate above the winch (Joint No. 21)
  - Rear bumper (Joint No. 36)
  - Right rear wheel (Joint No. 33)
  - Right middle frame (Joint No. 6)
- Displacement gages at
  - Engine front - Relative to frame (Joint No. 23 relative to No. 3)
  - Rear frame - Relative to wheel (Joint No. 18 relative to No. 35)

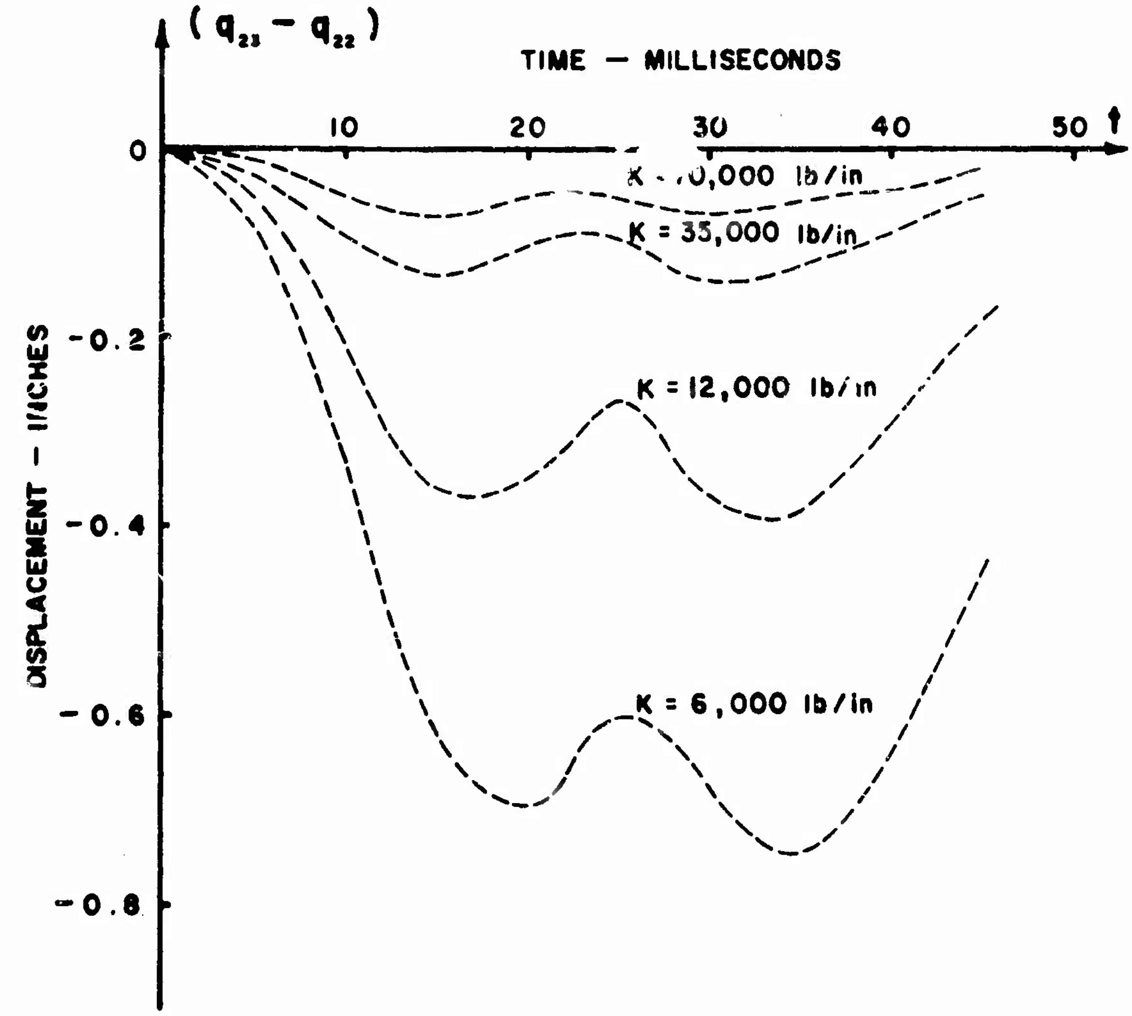


Fig. 5.13 Engine Support Stiffness Effect on Engine Displacement.

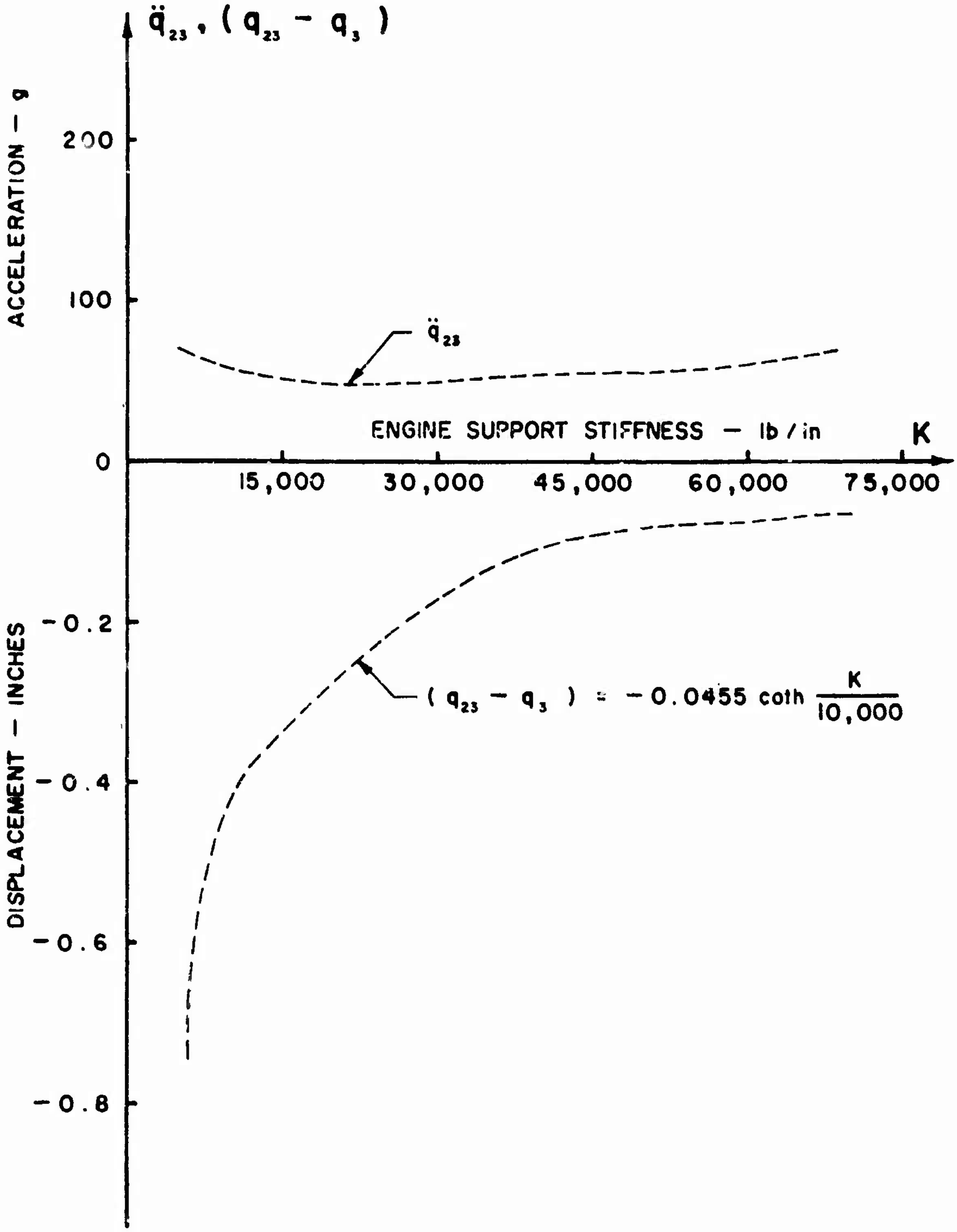


Fig. 5.14 Engine Support Stiffness Effect on Maximum Displacement and Acceleration of Engine.

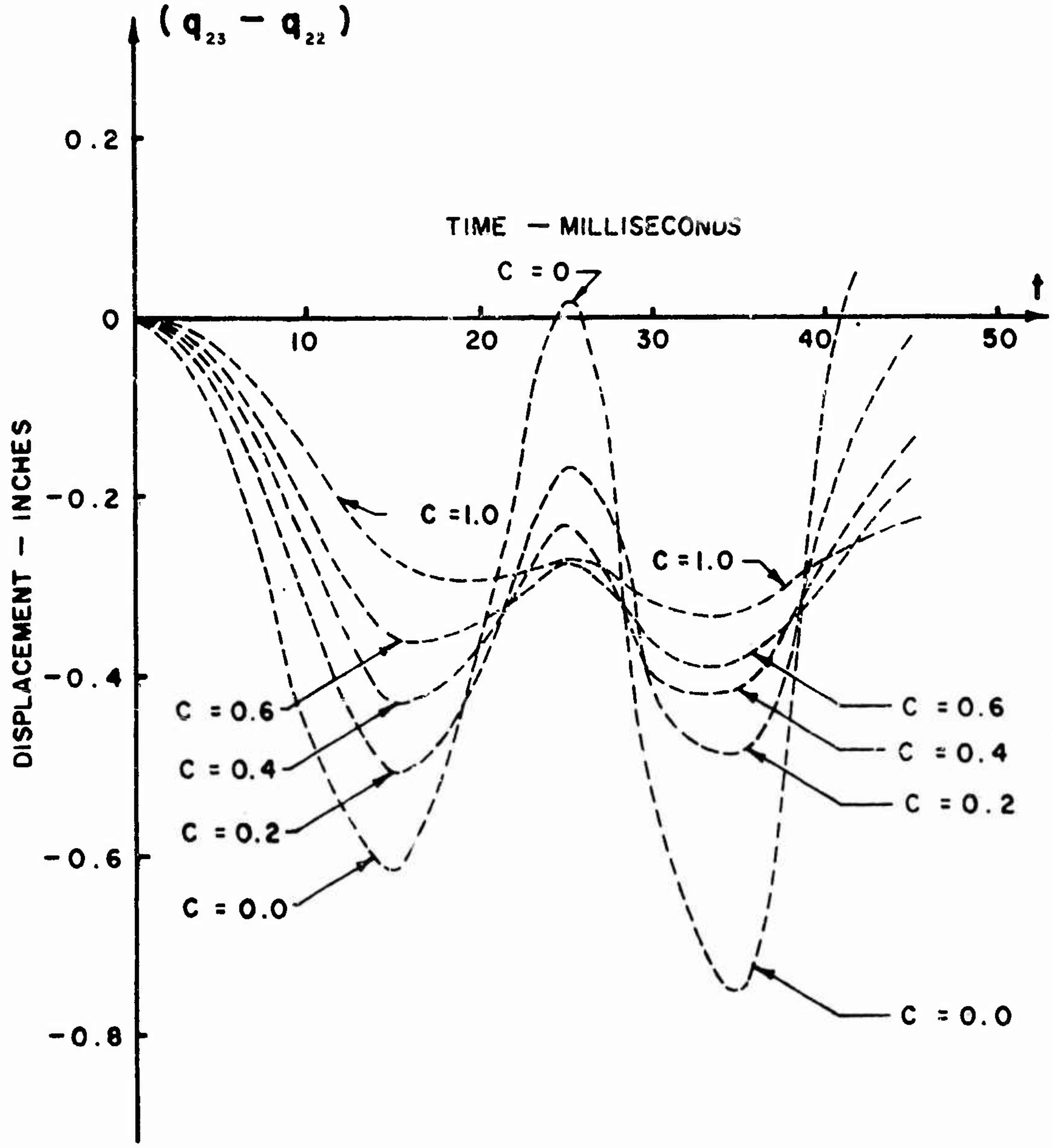


Fig. 5.15 Internal Damping Effect of Engine Support on Displacements of Engine.

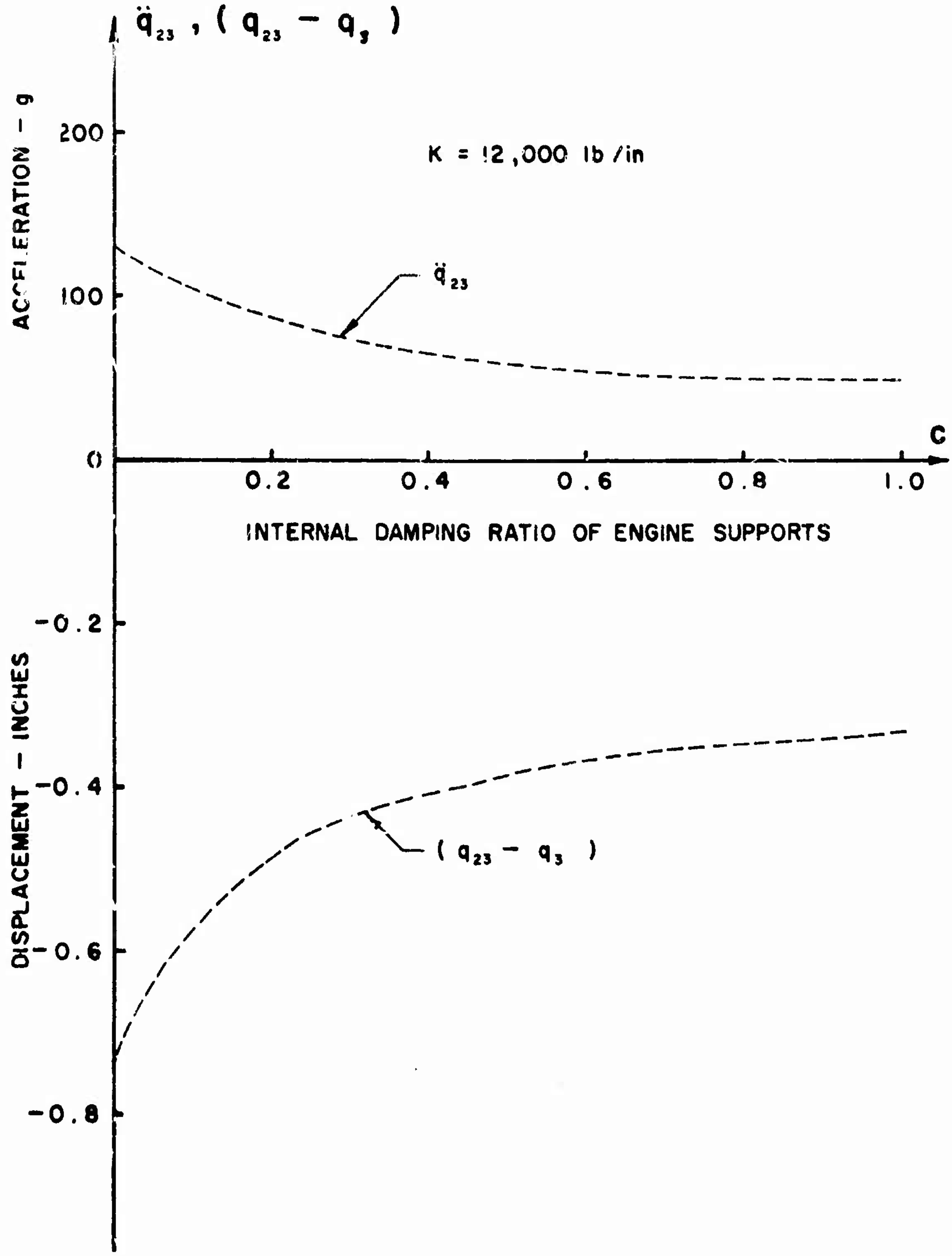


Fig. 5.16 Internal Damping Effect of Engine Support on Maximum Displacement and Acceleration of Engine.

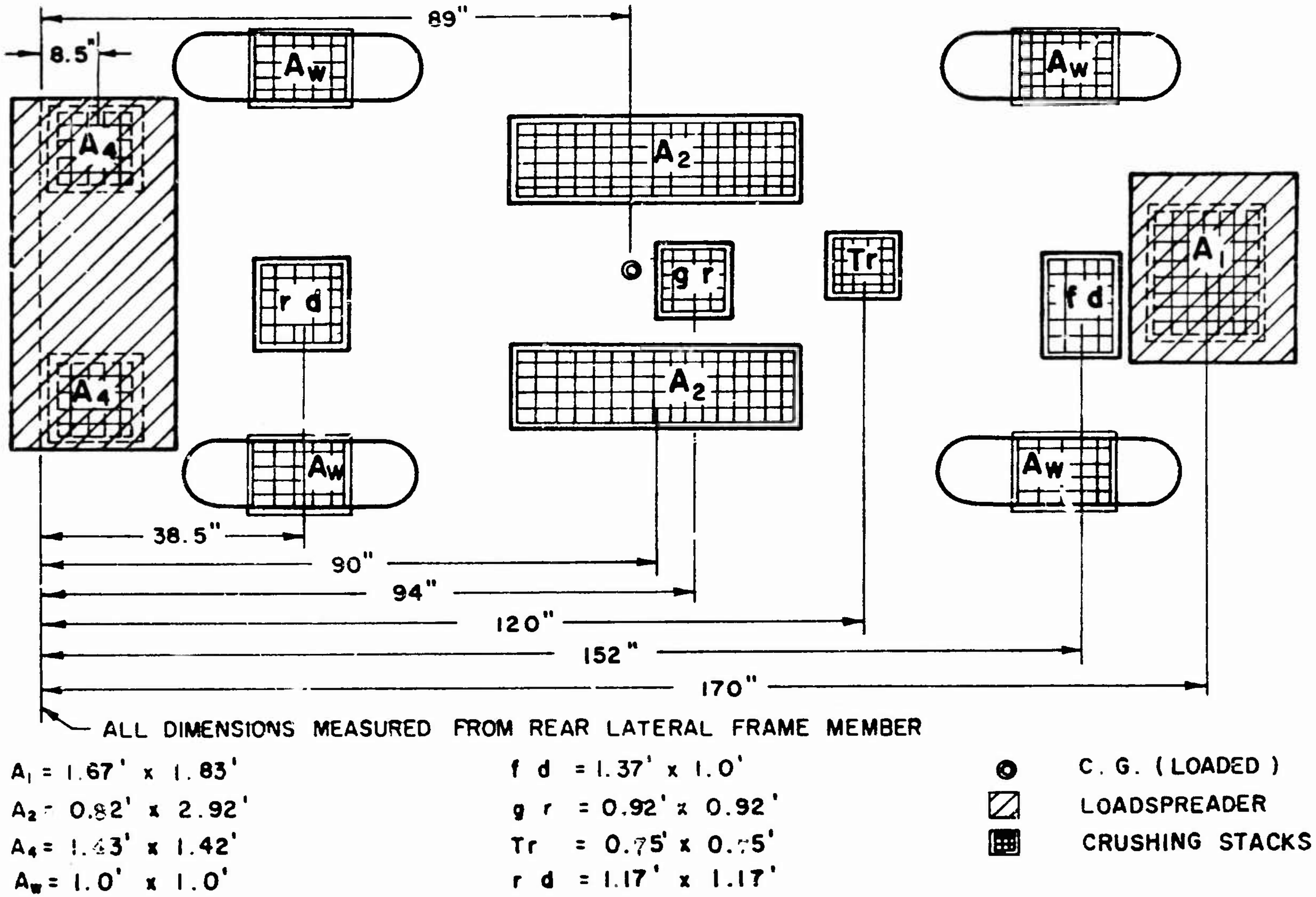


Fig. 6.1 Cushioning Configuration for M-37 Truck.

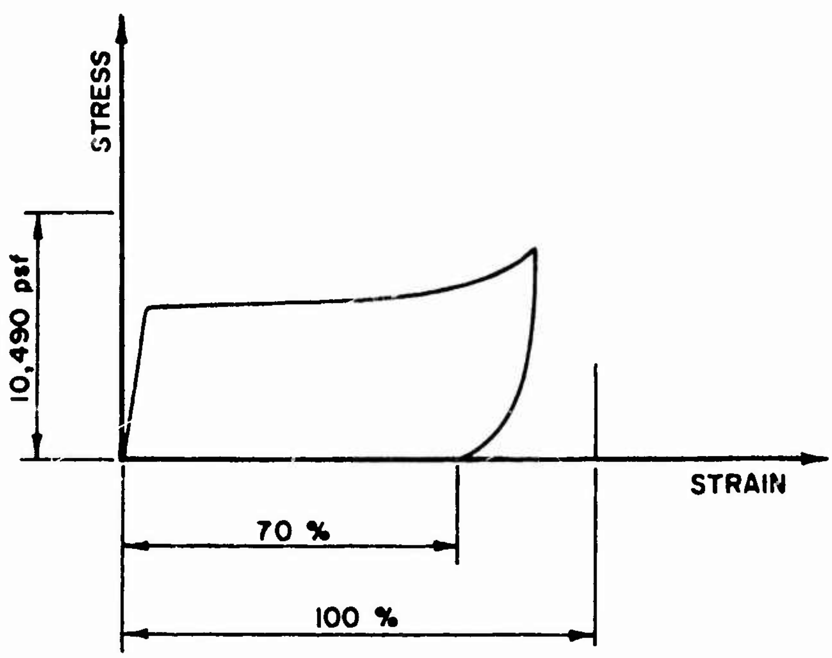


Fig. 6.2 Typical Stress-Strain Record For Paper Honeycomb.

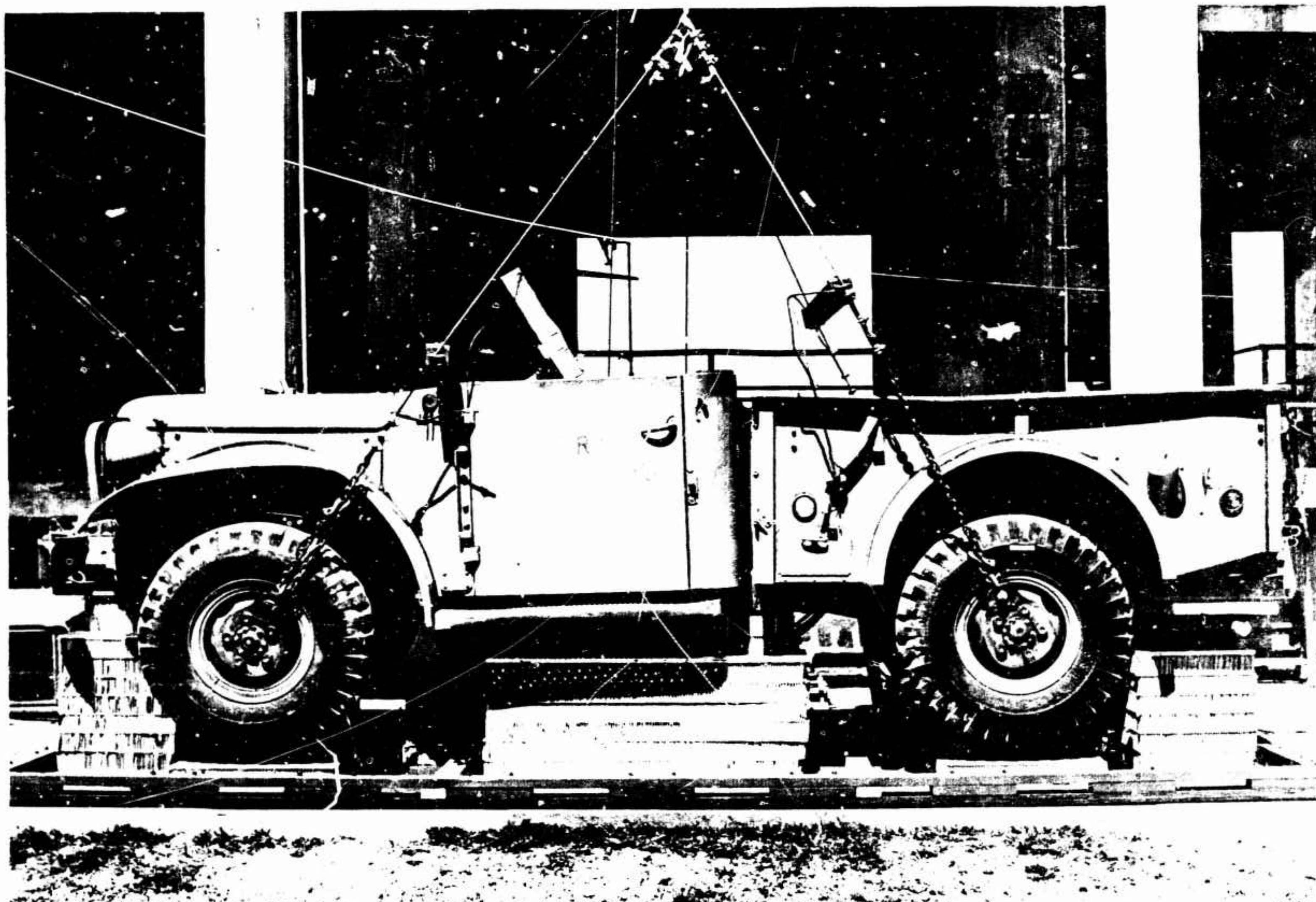


Fig. 6.3 Rigging for Lifting the Vehicle.

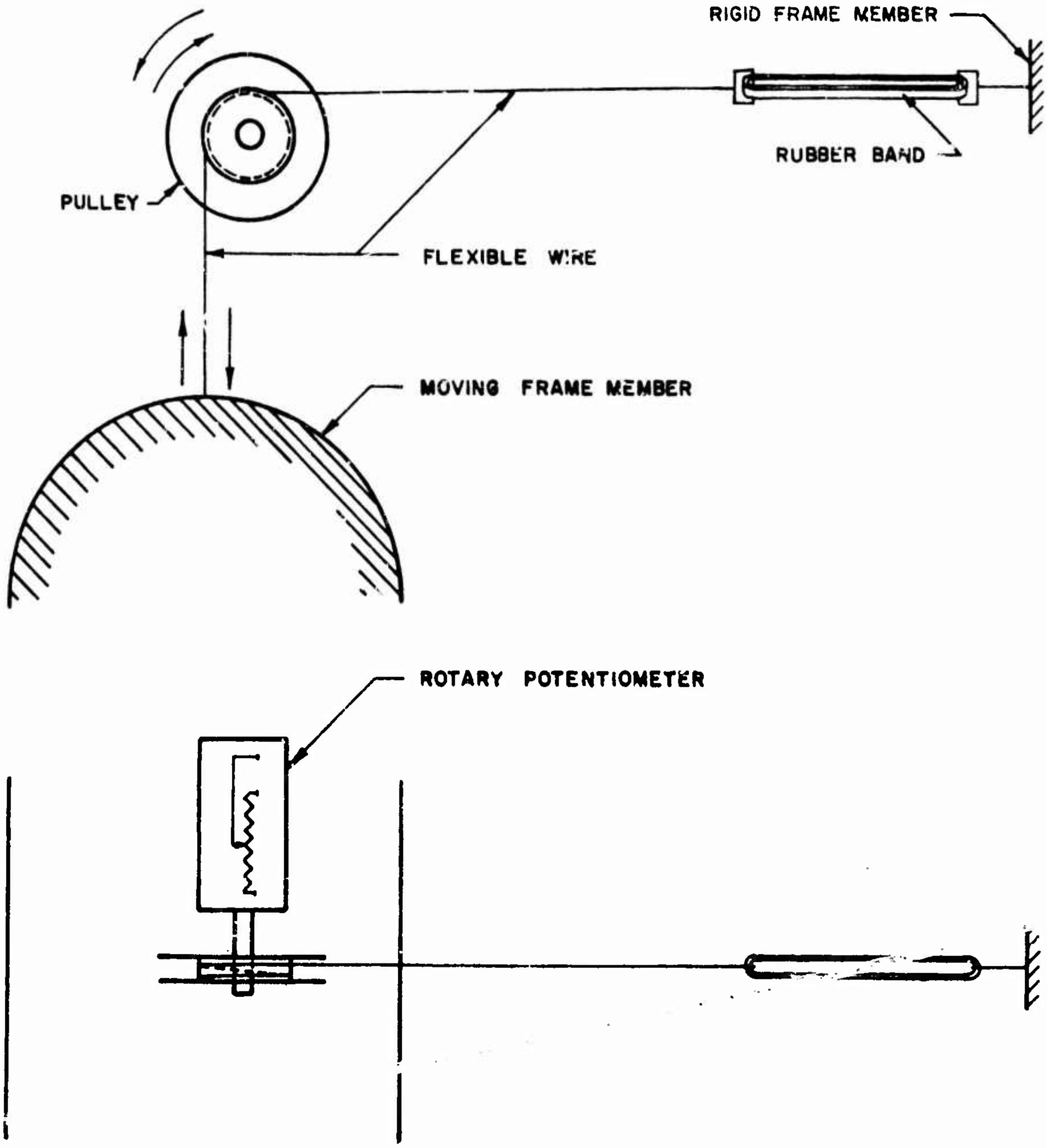


Fig. 6.4 Rotary Displacement Transducer.

Drop No. 2 (Series No. M-37-18)
Accelerometers at
Frame (Joint No. 2)
Transmission (Joint No. 28)
Rear Differential (Joint No. 34)
Displacement gages at
Front frame (Joint No. 3)
Rear frame (Joint No. 18)
Transfer case (Joint No. 30)
Engine rear - Relative to frame (Joint No. 38
Relative to No. 4)
Drop No. 3 (Series No. M-37-19)
Accelerometers at
Engine front and rear (Joints No. 23 and No. 38)
Truck bed
Displacement gages at
Front frame (Joint No. 1)
Middle frame (Joint No. 5)
Left wheel (Joint No. 35)

All acceleration and displacement data were recorded by a magnetic tape system and re-recorded from the tape to a visible record on paper using a Visicorder oscillograph.

In addition to acceleration and displacement records, high-speed motion pictures were taken for all drops. These pictures were studied to determine the gross aspects of vehicle motion during impact.

Numerical values used in the computation for the M-37 truck model are as follows:

Drop height 10 ft. (or impact velocity 302 in/sec.)
Design acceleration level - 17.5 g.
Impact duration = 45.2 milliseconds
Average crushing stress of paper honeycomb = 6400 psf.
Dimensions and arrangement of cushioning pads (see Fig. 6.1)
Applied cushioning forces:
Concentrated forces -
Wheels and tires each $F_w = 350 \times 18.5 = 6480$ lb.
Differentials each $F_{fd} = F_{rd} = 480 \times 18.5 = 8890$ lb.
Transmission $F_{Tr} = 200 \times 18.5 = 3700$ lb.
Gear Reducer $F_{gr} = 300 \times 18.5 = 5550$ lb.

Uniform distributed forces:
At pad A_1, $F_1 = 20600/(2 \times 23) = 448$ lb/in.
At pad A_2, $F_2 = 13900/29 = 475$ lb/in.
At pad A_4, $F_4 = 13100/19 = 697$ lb/in.
(See Appendix B)

The weights of some of the vehicle components were obtained from the M-37 technical manual[7] and others were assumed. These weights are listed as follows:

Total truck weight = 5,390 lb.
Load weight = 1,500 lb.
Discrete masses:

Engine	600 lb.
Wheels and tires (each)	350 lb.
Transmission	200 lb.
Gear Reducer	300 lb.
Differentials (each)	480 lb.

Uniformly distributed masses:

All weights not included in discrete masses distributed as follows:

Longitudinal frame members:

Joint No. 1 to No. 6, and No. 11 to No. 16,
= 570/2 x 97 = 2.94 lb/in.

Joint No. 6 to No. 10, and No. 16 to No. 20
= 2560/2 x 89 = 14.38 lb/in.

Transverse frame members:

Joint No. 2 - No. 12 (Winch)	= 10 lb./in.
Joint No. 3 - No. 13	= 2.3 lb/in.
Joint No. 5 - No. 15	= 1.2 lb/in.
Joint No. 6 - No. 16	= 0.8 lb/in.
Joint No. 10 - No. 20	= 0.8 lb/in.

Except for Joint 2 to Joint 12 these weights were arbitrarily assigned. They are approximately proportioned to the member cross section. All member properties, calculated using truck frame cross-sectional dimensions only, are listed in Table 6.1. The tire spring constants are assumed to be 7000 lb/in and the damping ratio 0.2. The spring constants of the engine supports are 12,000 lb/in at front and 40,000 lb/in at rear and the damping ratio is 0.6. The structural damping ratio is assumed to be 0.009. Joint coordinates in an x-y system with the origin at the front of the truck as shown in Fig. 2.1 are given in Table 6.2.

The measured results (solid line) are plotted with the calculated results (dotted line) in Figs. 6.5 to 6.20. These results are discussed in the next chapter.

Member Start Joint No. - End Joint No.	Cross Section Area A - in^2	Moment of Inertia I_y - in^4	Polar Moment of Inertia I_x - in^4	Torsional Constant J - in^4
1 - 6	1.6	4.29	4.39	0.021
6 - 10	2.0	10.8	11.0	0.027
11 - 16	1.6	4.29	4.39	0.021
16 - 20	2.0	10.8	11.0	0.021
2 - 12	100.0	100.0	20.0	20.0
2 - 21	0.3	0.01	0.5	0.01
21 - 12	0.3	0.01	0.5	0.01
3 - 13	3.4	4.47	12.57	10.6
24 - 26	40.0	30.0	60.0	60.0
33 - 35	40.0	30.0	60.0	60.0
4 - 14	100.0	100.0	100.0	100.0
27 - 28	100.0	100.0	100.0	100.0
5 - 15	1.3	0.383	1.4	0.017
6 - 16	1.3	0.383	1.4	0.017
7 - 17	2.0	6.3	8.0	14.4
9 - 19	1.8	2.5	5.2	0.024
10 - 20	1.8	2.5	5.2	0.024
3 - 4				
13 - 14				
7 - 9	2.0	0.167	0.92	0 917
17 - 19				

Table 6.1 Member Properties

Joint No.	Coordinates			Joint No.	Coordinates	
	X -in.	Y -in.			X -in.	Y -in.
1	0	15.0		21	6.0	0
2	6.0	15.0		22	17.0	0
3	17.0	15.0		23	17.0	0
4	51.0	15.0		24	34.0	15.0
5	87.0	18.6		25	34.0	5.0
6	97.0	19.0		26	34.0	-15.0
7	117.5	19.0		27	51.0	0
8	147.5	19.0		28	66.0	0
9	177.5	19.0		29	87.0	6.0
10	186.0	19.0		30	87.0	-6.0
11	0	-15.0		31	97.0	6.0
12	6.0	-15.0		32	97.0	-6.0
13	17.0	-15.0		33	147.5	19.0
14	51.0	-15.0		34	147.5	5.0
15	87.0	-18.6		35	147.5	-19.0
16	97.0	-19.0		36	186.0	12.0
17	117.5	-19.0		37	186.0	-12.0
18	147.5	-19.0		38	51.0	15.0
19	177.5	-19.0		39	51.0	-15.0
20	186.0	-19.0				

Table 6.2 Joint Coordinates

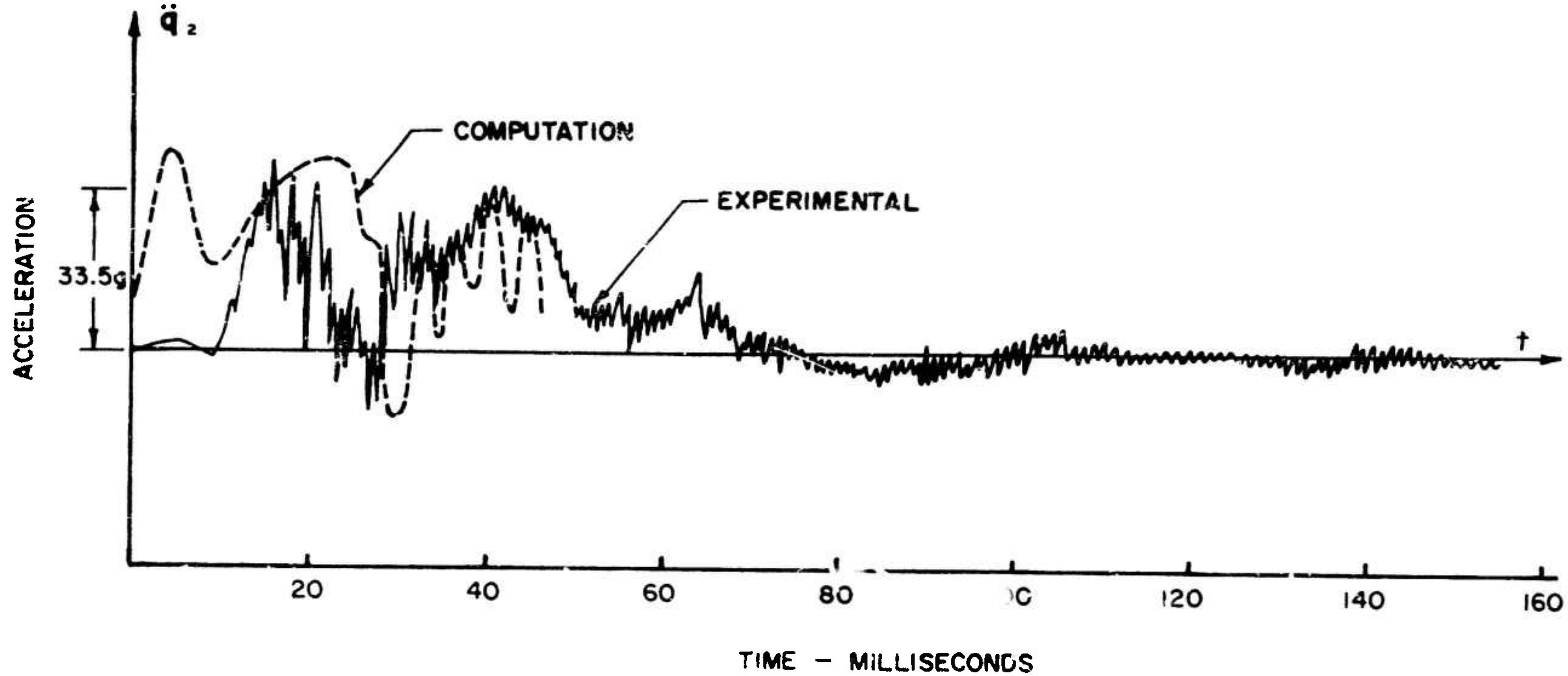


Fig. 6.5 Measured and Computed Acceleration of q_2 - winch.

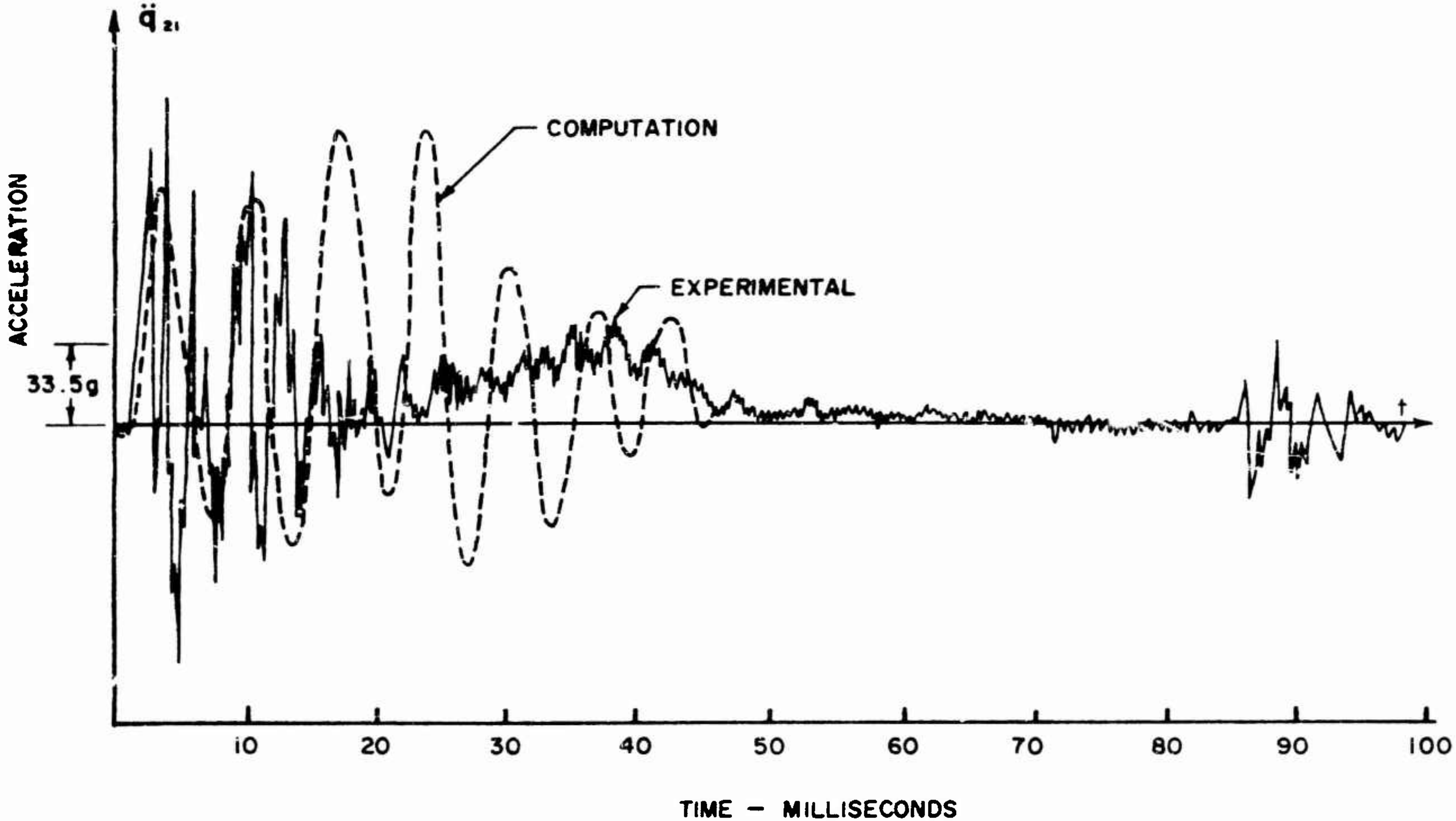


Fig. 6.6 Measured and Computed Acceleration of q_{21} - winch.

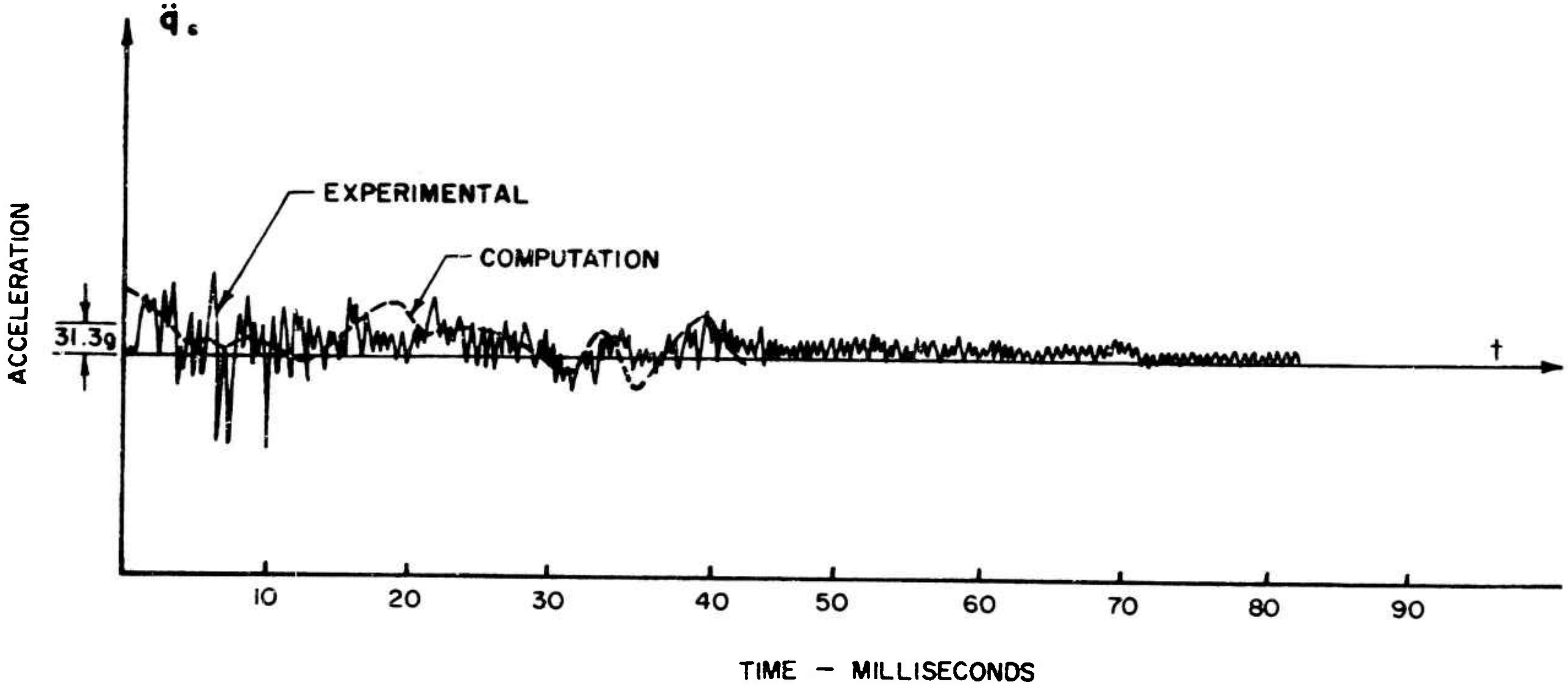


Fig. 6.7 Measured and Computed Acceleration of q_6 - Middle Frame.

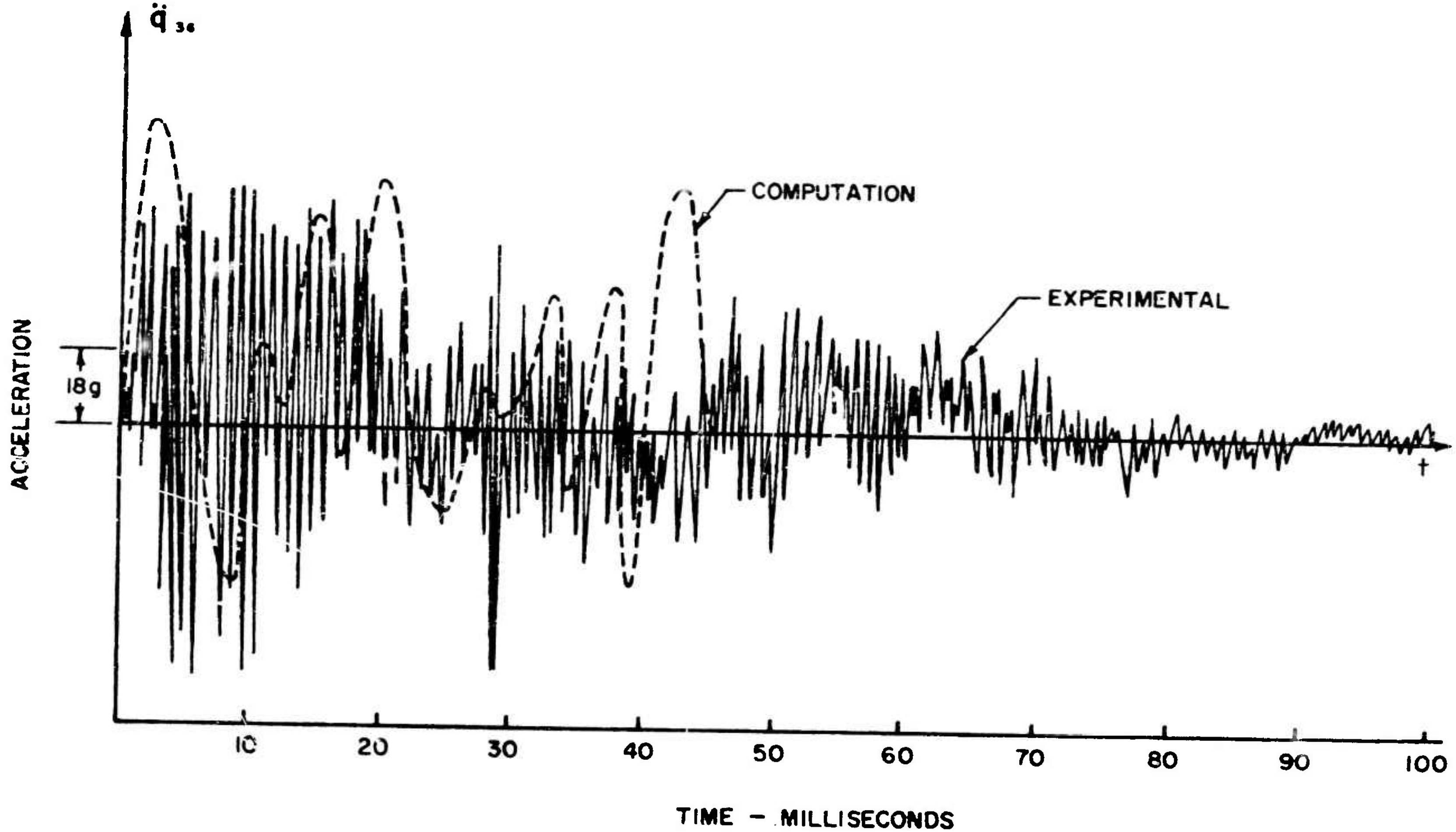


Fig. 6.8 Measured and Computed Acceleration of q_{36} - Rear Bumper.

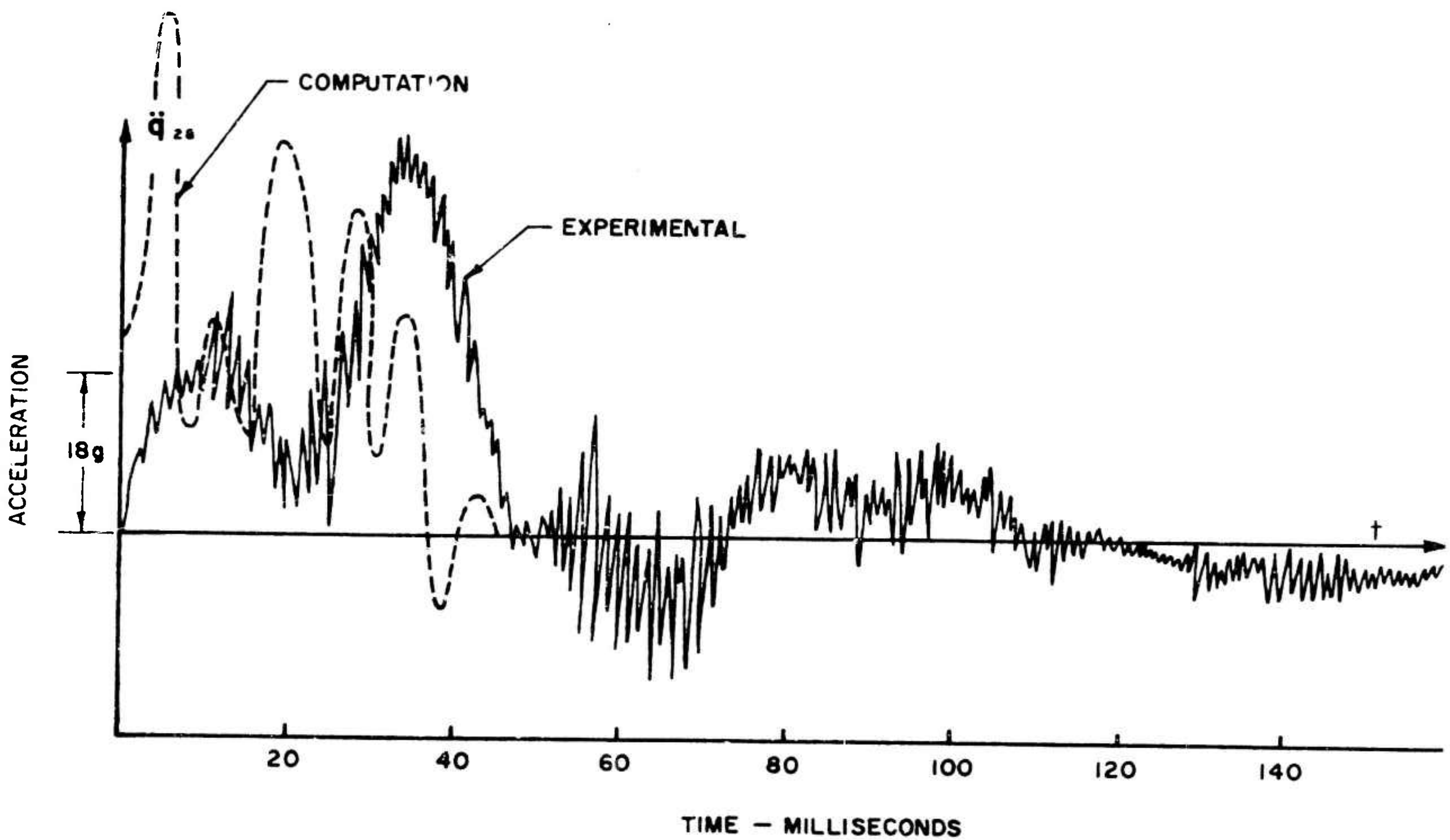


Fig. 6.9 Measured and Computed Acceleration of q_{28} - Transmission.

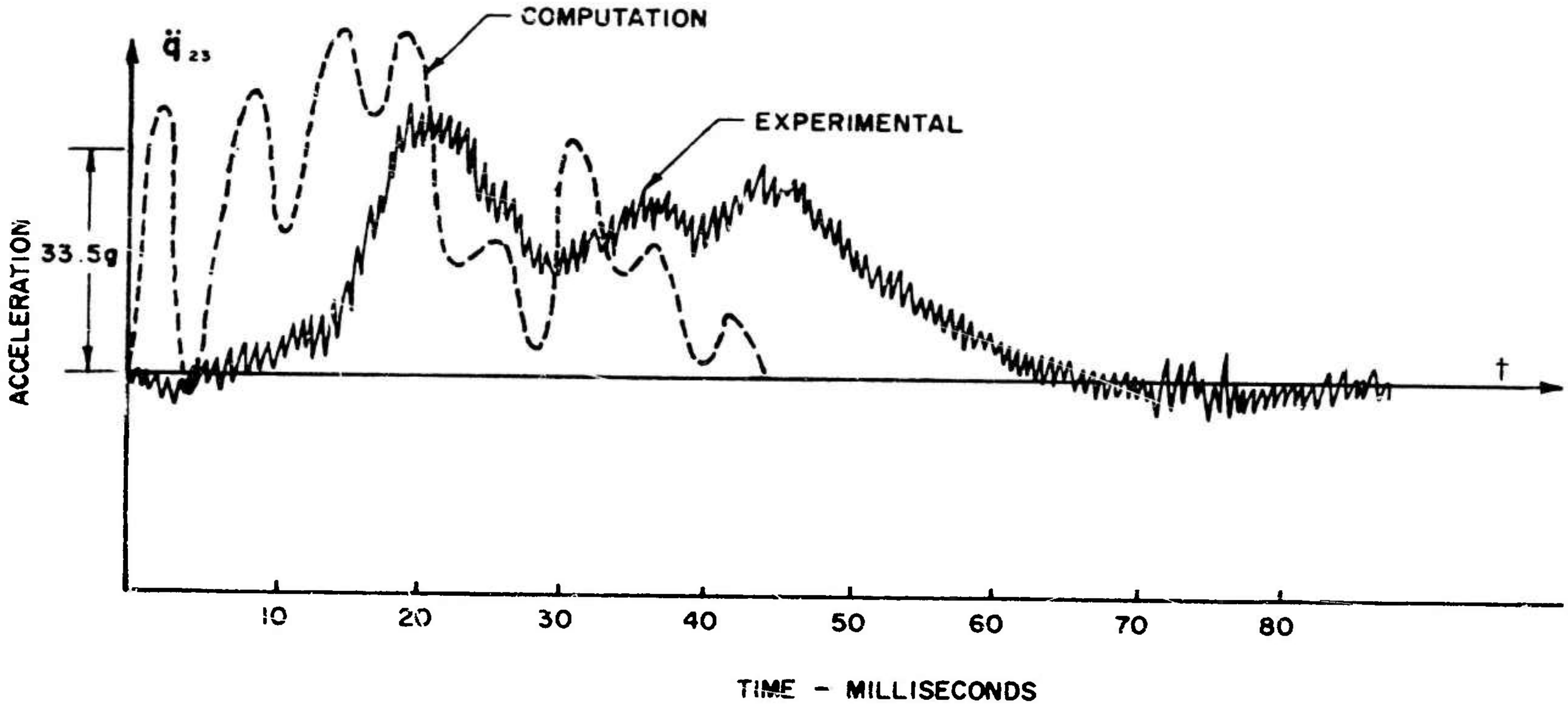


Fig. 6.10 Measured and Computed Acceleration of q_{23} - Engine.

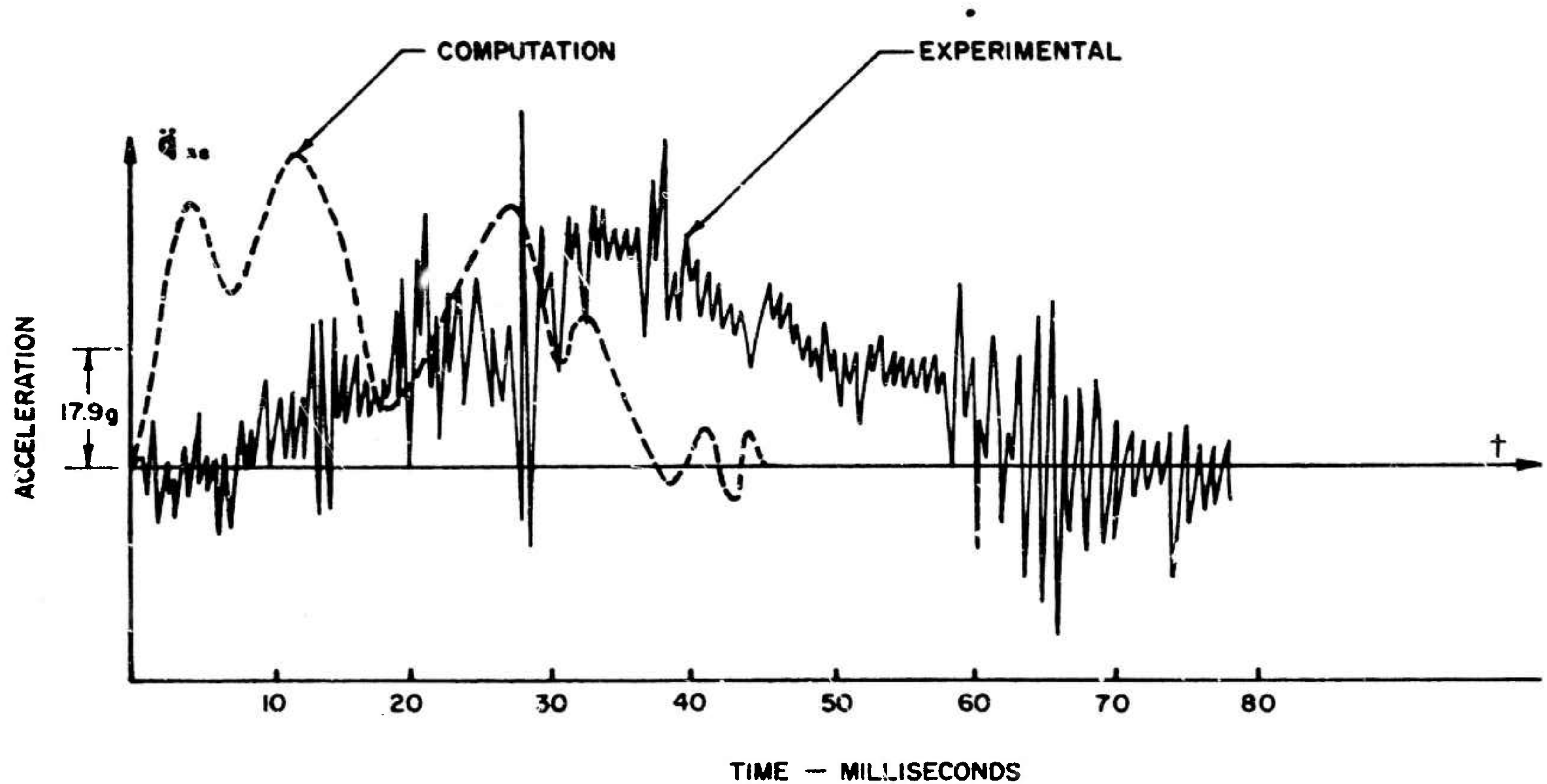


Fig. 6.11 Measured and Computed Acceleration of q_{38} - Engine.

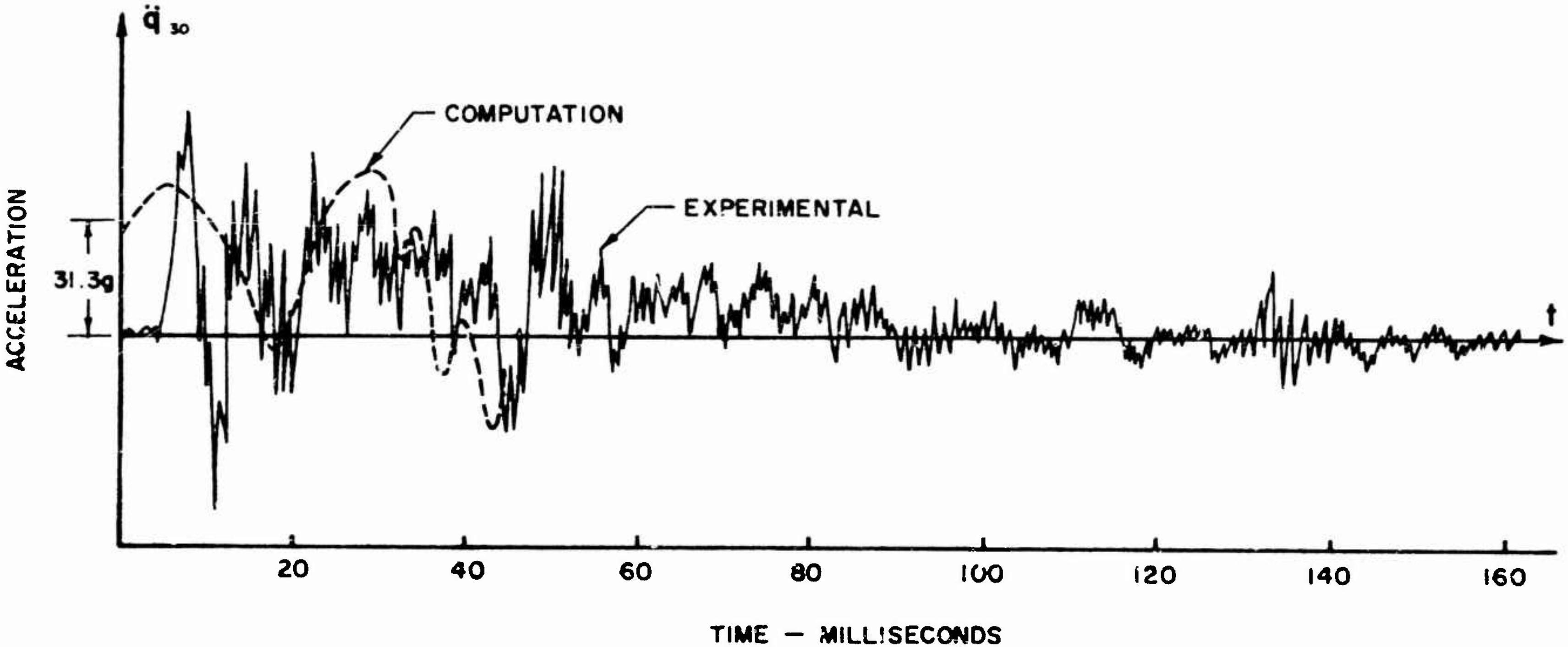


Fig. 6.12 Measured and Computed Acceleration of q_{30} - Transfer Case.

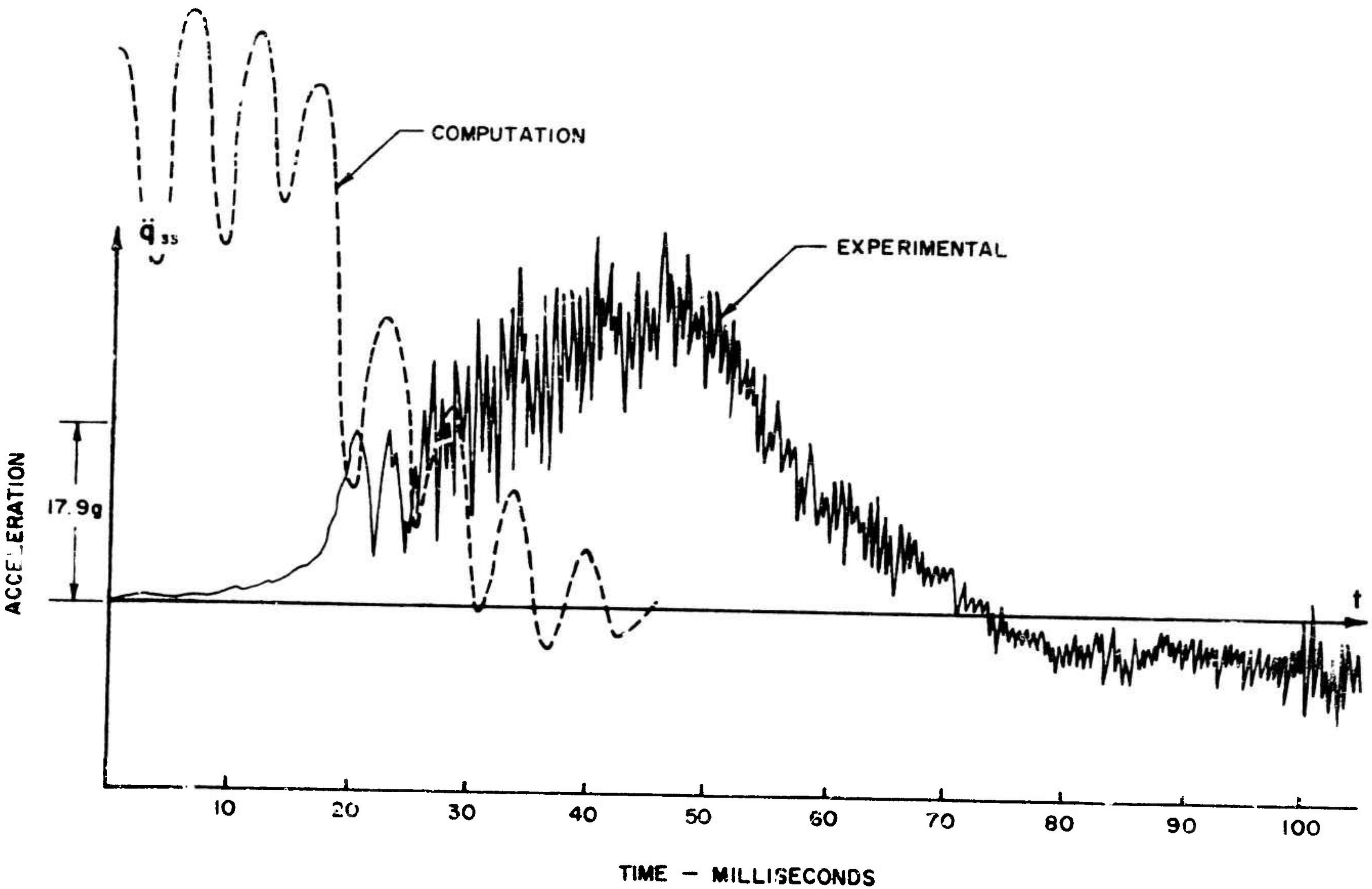


Fig. 6.13 Measured and Computed Acceleration of q_{33} - Rear Wheel.

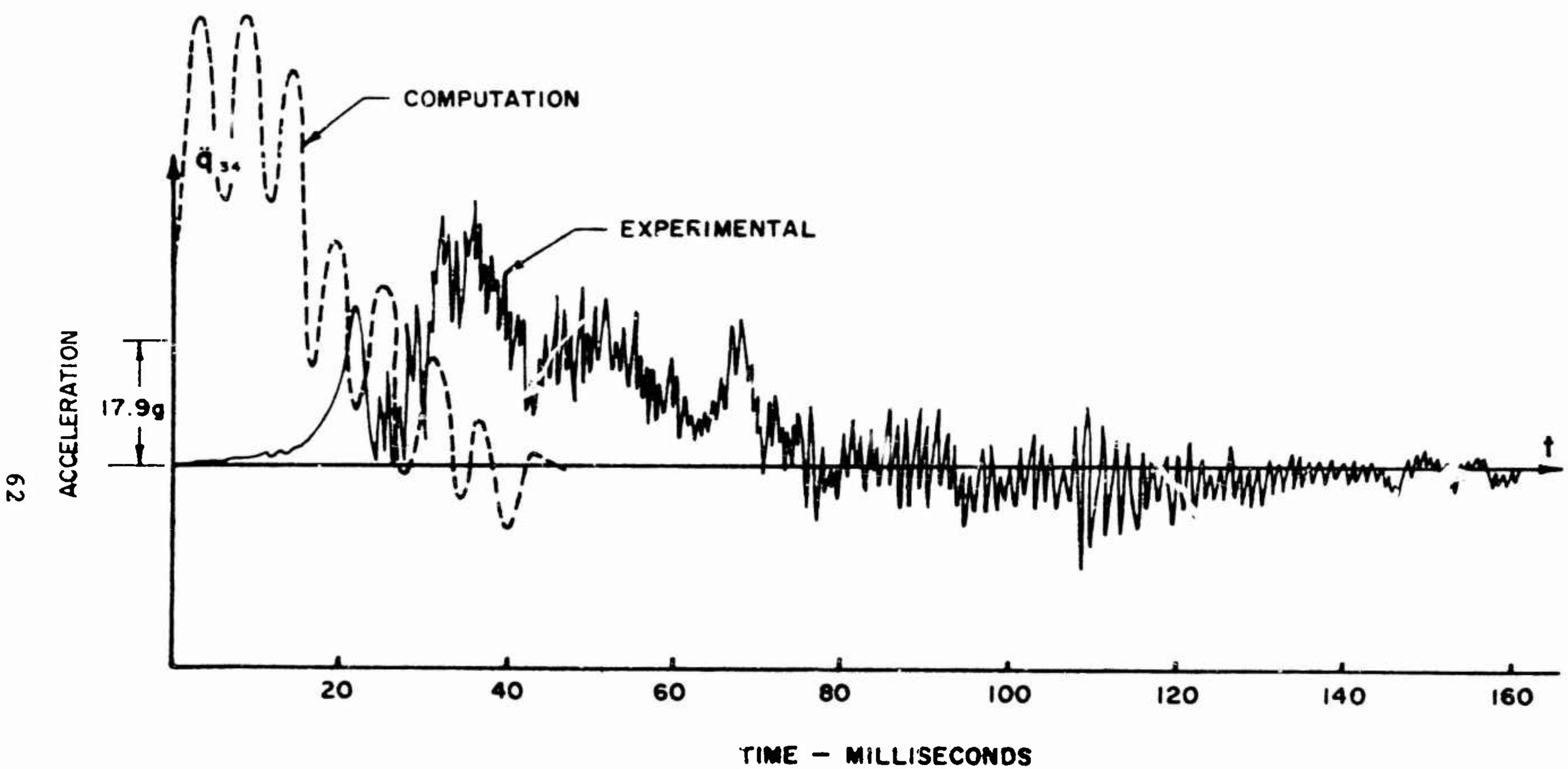


Fig. 6.34 Measured and Computed Acceleration of q_{34} - Rear Differential.

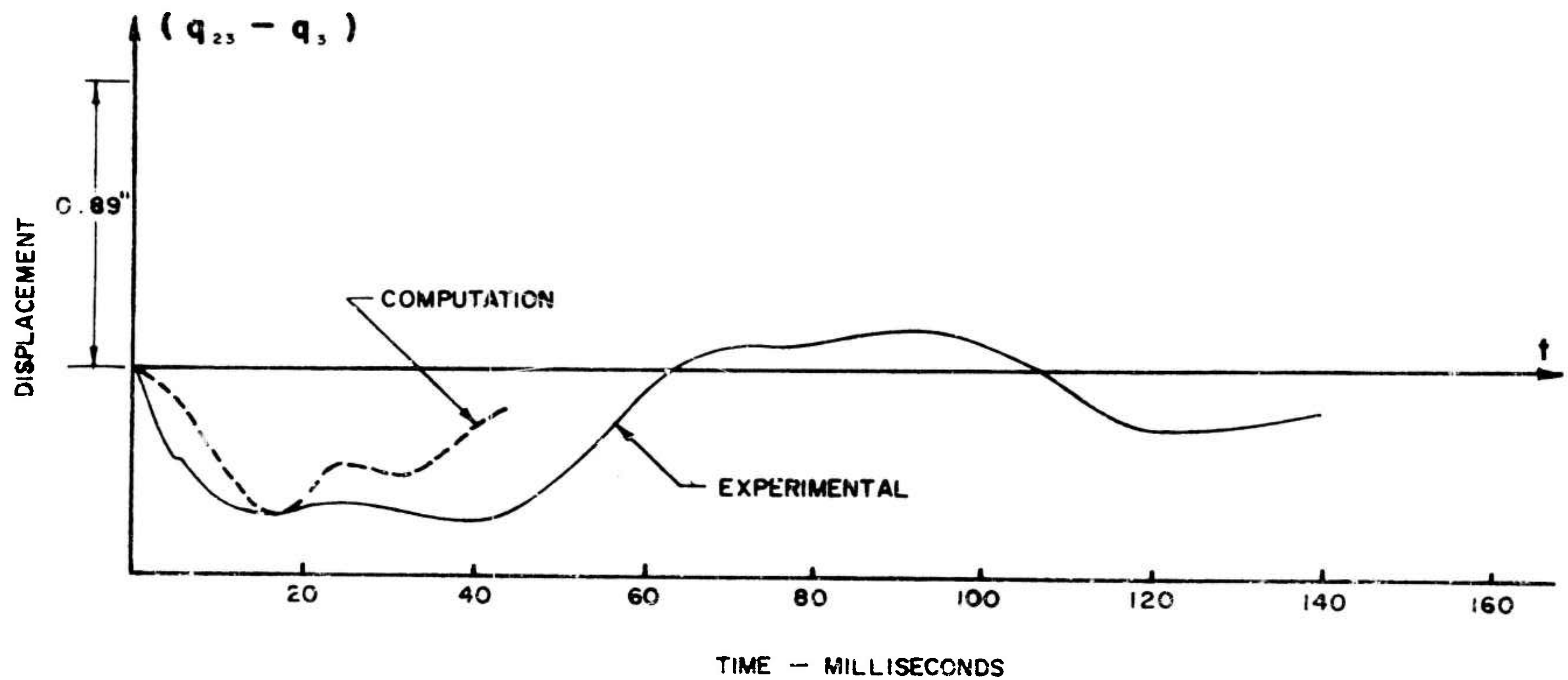


Fig. 6.15 Measured and Computed Relative Displacement of $(q_{23} - q_3)$ - Engine Relative to the Frame.

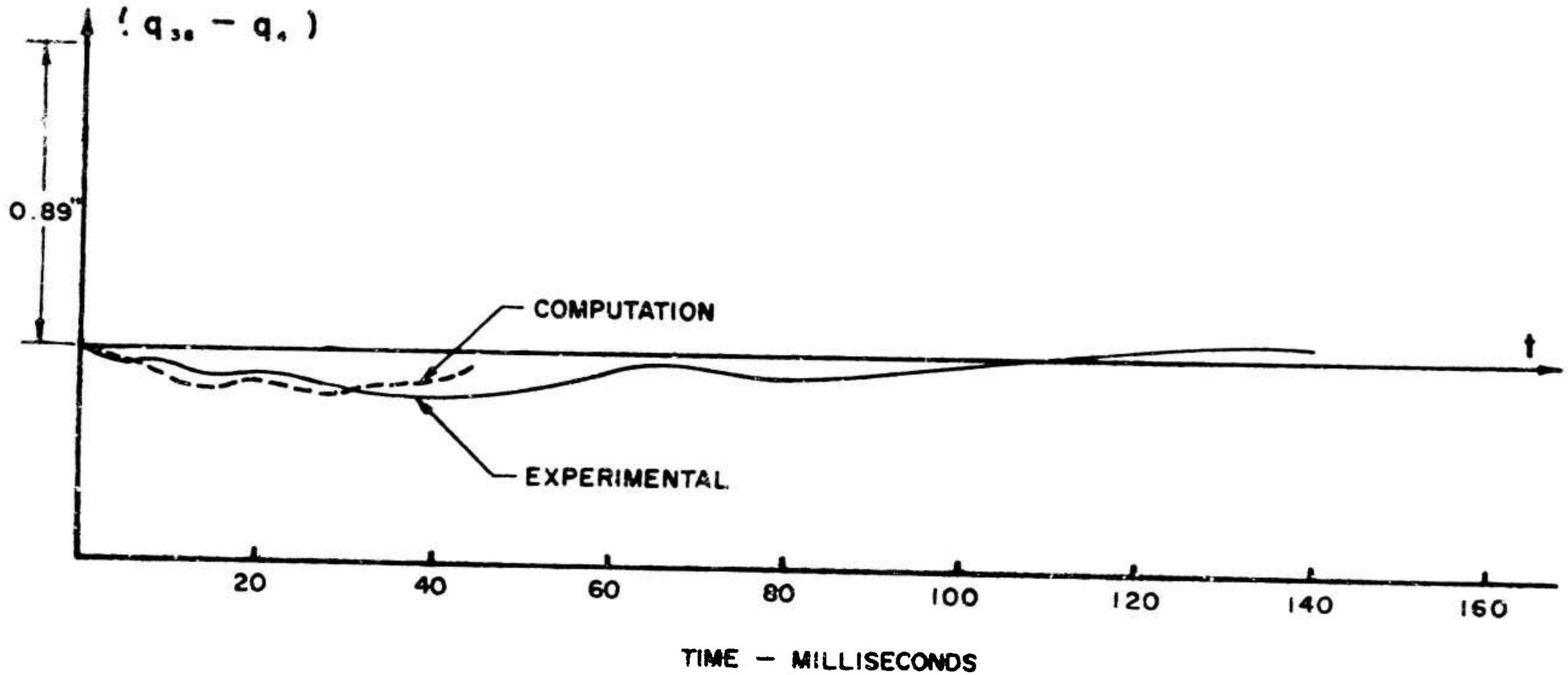


Fig. 6.16 Measured and Computed Relative Displacement of $(q_{38} - q_4)$ - Engine Relative to the Frame.

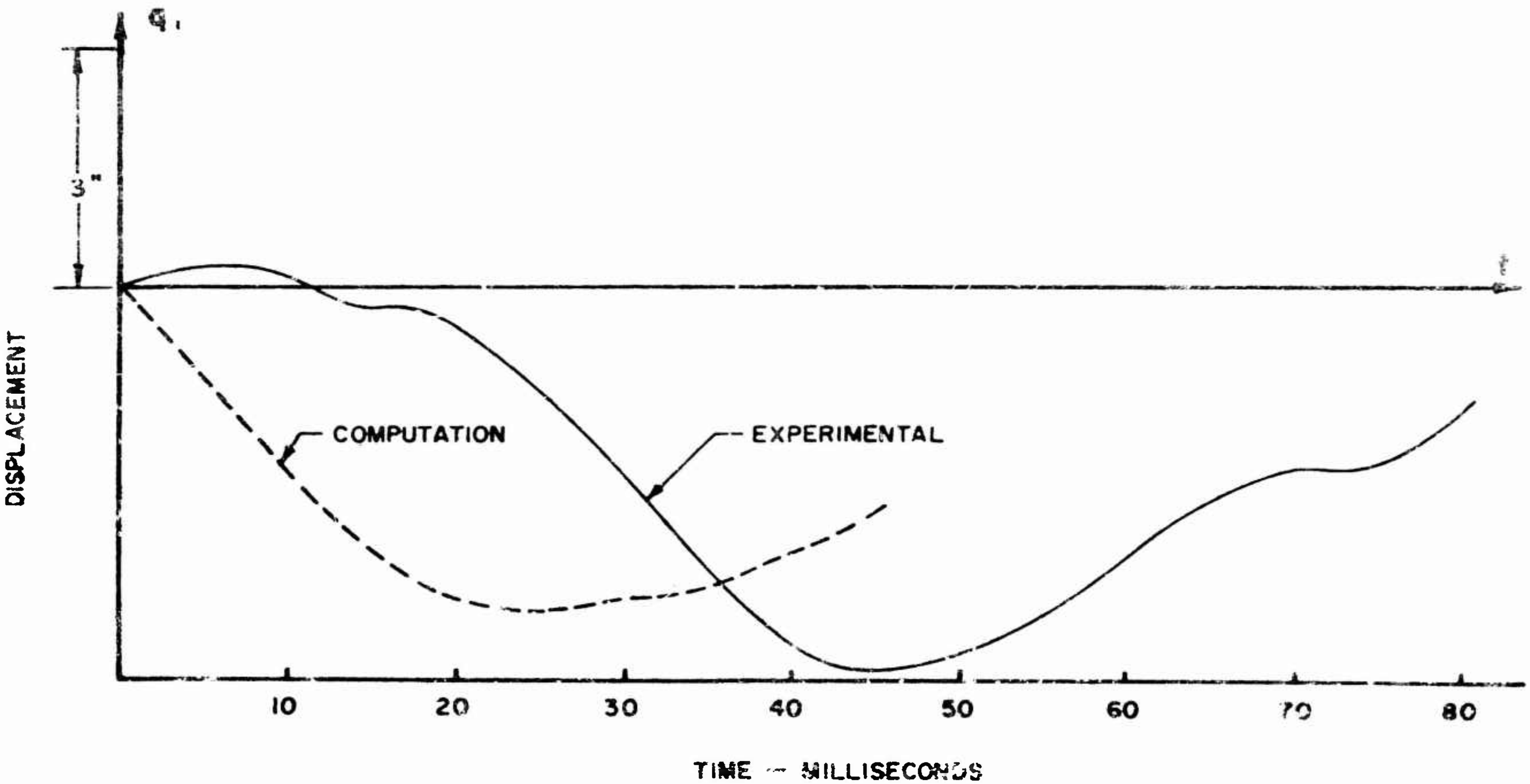


Fig. 6.17 Measured and Computed Displacement of q_1 - Front Frame.

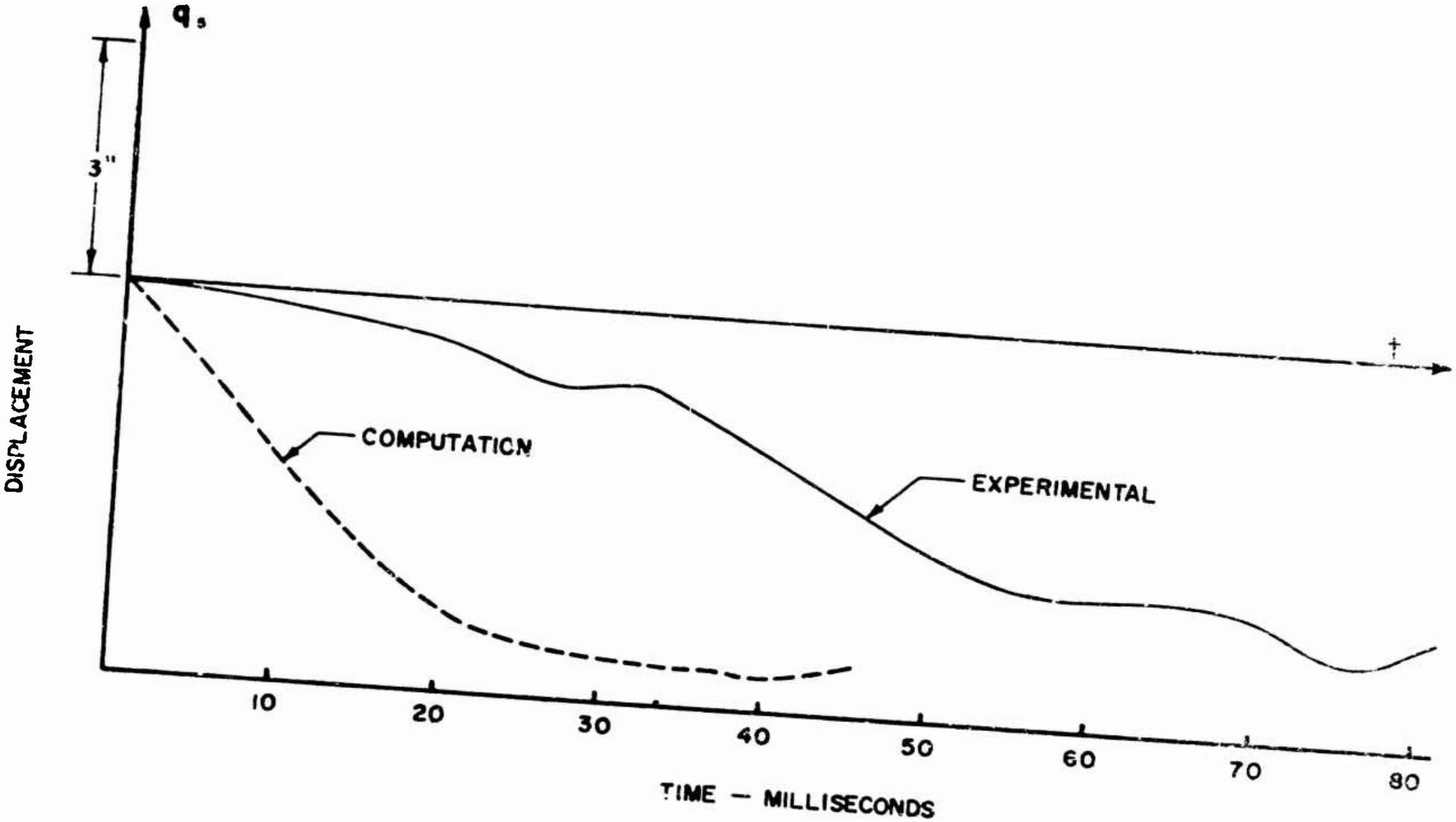


Fig. 6.18 Measured and Computed Displacement of q_5 - Middle Frame.

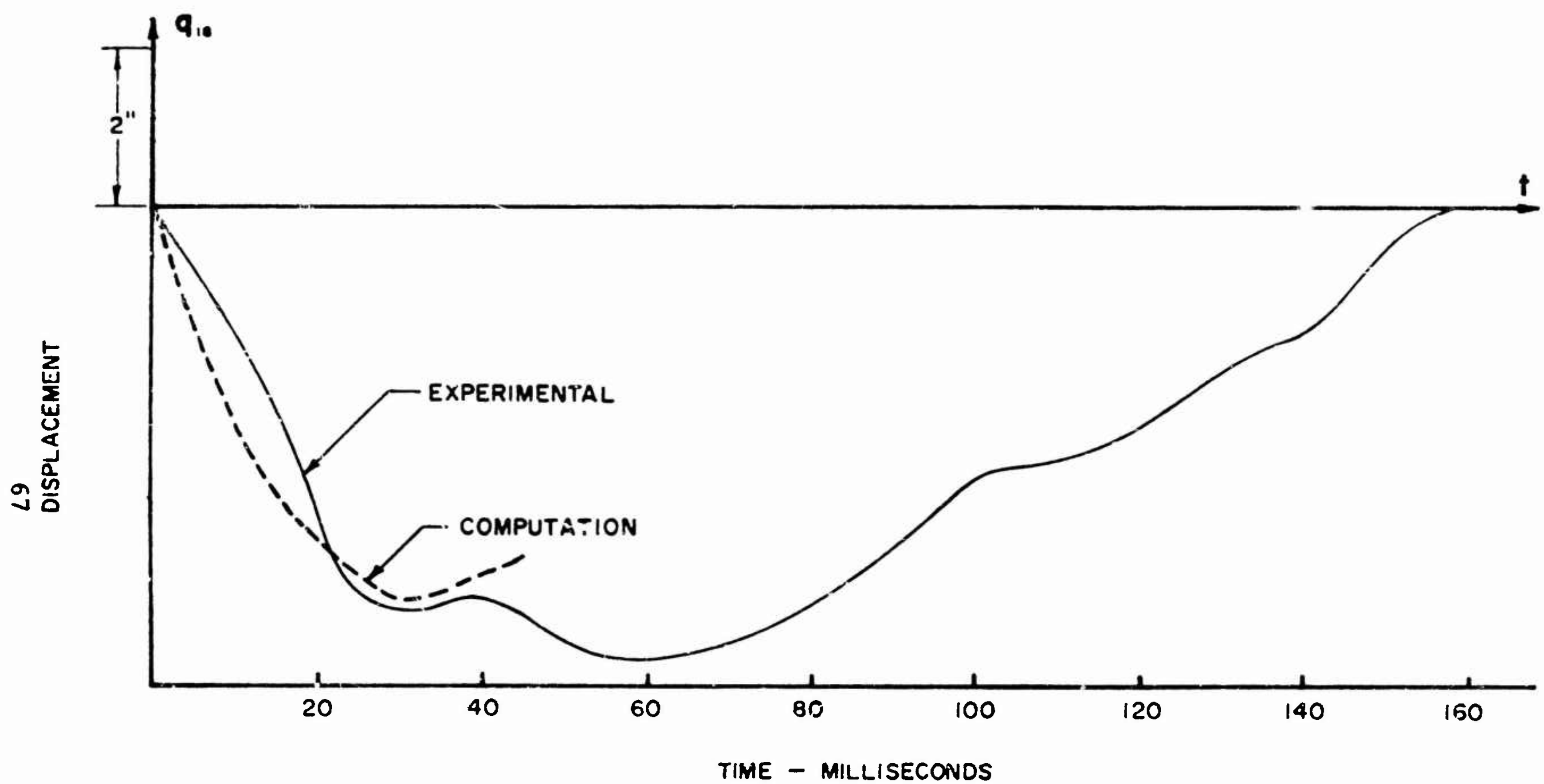


Fig. 6.19 Measured and Computed Displacement of q_{18} - Rear Frame.

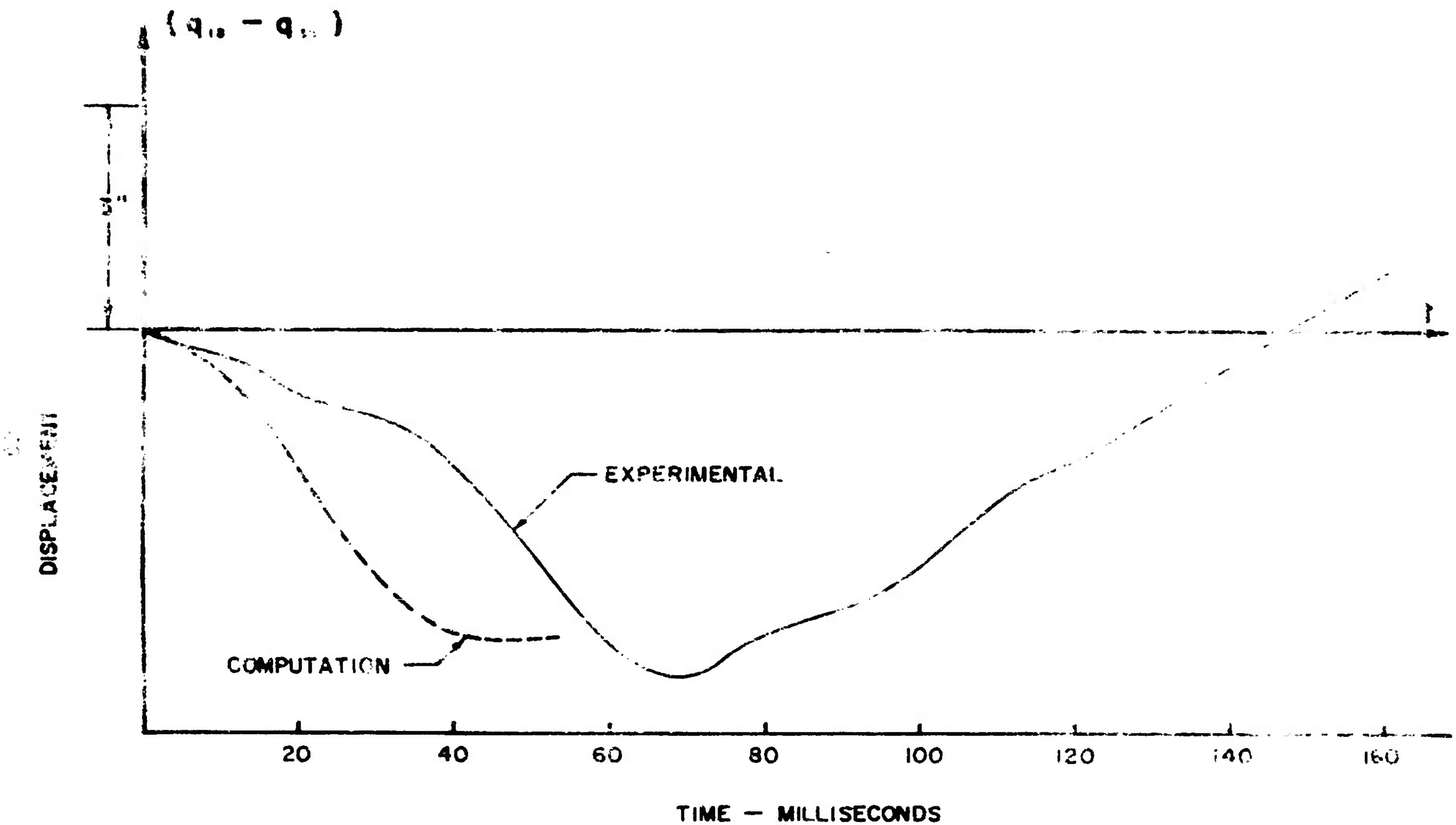


Fig. 6.20 Measured and Computed Relative Displacement of $(q_{18} - q_{35})$ - Rear Frame Relative to the Wheel.

DISCUSSION

L. Mathematical Model

The analysis of the complex structure presented in this work is based on the possibility of modeling the structure by a lumped-parameter system. Modeling of the structure is somewhat arbitrary. In general, however, the following two factors need to be carefully considered.

(a) Select and sort out the components in the structure whose motion is to be studied. It is impractical to try to include all the components of the structure in a model. It is necessary for the designer to select the components whose motions are considered to be most important. For example, in an airdrop of a vehicle, the motion of the engine block, transmission, differentials, and wheels may be the important parameters for the proper design of a cushioning system to protect the vehicle. Consequently the model should be so designed as to represent the motions of these components as realistically as possible. On the other hand, if the task is to design a cushioning system for the protection of a car radio, a completely different model will be needed. The model shown in Fig. 2.1 is developed for the purpose of designing a cushioning system for the protection of the vehicle during airdrop. The weights of the engine, transmission, transfer case, differentials, and wheels are modeled by concentrated masses at proper nodal points. The winch and the frame of the truck were replaced by two longitudinal beams coupled together by eight transverse beams. The weight of the truck frame and the load are further assumed to be uniformly distributed along the two longitudinal beams. The stiffnesses of all the connecting members in Fig. 2.1 are calculated on a static basis using the dimensions of the truck members. It should be noted that Fig. 2.1 is a particular model, many other models may be developed for the same purpose. The suitability of the present model will be discussed in the section where the comparison is made between the measured and computed results.

(b) Estimate the motion of the structure. In modeling a complex structure, information on the environment that the structure is likely to be subjected to is also important. In airdrop of a vehicle, the whole structure is expected to land with an impact velocity of approximately 30 fps, and the rise time of the impulse imparted to the vehicle is of the order of milliseconds. These are important data. A continuous elastic structure, such as a vehicle, has in principle an infinite number of modes of vibration that can be excited. A lumped parameter system can be excited in only as many modes as it has degrees of freedom. Therefore a lumped parameter system can not be expected to accurately represent a continuous structure under conditions in which the higher modes of the structure might be excited. The long rise time of the impulse applied to the vehicle means that the higher modes will not be excited and the lumped parameter model can therefore provide a suitable approximation to the motion of the prototype system. On the other hand, an input force with a very short rise time would tend to excite higher modes of vibration. As a consequence a lumped parameter model would not be able to represent the motion of the prototype structure.

In the present work, the whole truck body is modeled by a lumped parameter system with 39 nodal points. Since there are six degrees of freedom at each nodal point, the total number of degrees of freedom of the system is 234. Only a normal impact (in the vertical direction) is considered here, hence the vertical displacements of the nodal points are of the main concern. The six displacements possible at each nodal point consist of three translational motions and three rotational motions. Two of the translational displacements and one of the rotational displacements can be disregarded. Thus the total number of degrees of freedom are reduced to 117. This number can be further reduced to 39 by eliminating the remaining two rotational components at the nodal points by the static condensation process.[4]

In the numerical computations, the element mass matrix and stiffness matrix are calculated according to equations (3.19) and (3.29) respectively. The element end forces are calculated by equations (3.31) and (3.32). Only concentrated forces and uniformly distributed forces are considered in the analysis. The direct stiffness method is used to formulate the system mass matrix and stiffness matrix. The structural damping matrix is treated as a linear combination of the mass and stiffness matrices of the system. The amount of computer storage required by the program for this analysis is 30,000 words. For the CDC 6600 computer, the formulation of the equations of motion requires three minutes. The numerical integrations by the Runge-Kutta method require 1.5 minutes for an impact duration of 45 milliseconds, using an integration step size of 0.1 milliseconds.

2. Comparison of Measured and Computed Results

From Fig. 6.5 to 6.20, it may be seen that agreement between measured and computed results is, in general, not very good, so far as the shapes of the curves are concerned. However, the agreement in the amplitudes of displacements and accelerations, with the exception of the accelerations of wheels and differential is quite good. The maximum discrepancy between measured and computed displacements occurred at the front frame and is 18%. The maximum discrepancy between measured and computed accelerations at the engine is 34%. For the acceleration of the differential the maximum discrepancy is 70%, and for the wheels the maximum discrepancy is 50%. The displacements are believed to be more significant as indicators of possible damage, than the accelerations. Hence it is encouraging to find, in this example, that the displacements can be computed with the degree of success indicated. The actual shapes of the displacement and acceleration curves are probably of little consequence except for the indications they give of the adequacy of the lumped-parameter model.

The agreement between computed and measured displacements and accelerations may be improved in several ways, namely:

(a) Change the centroid position of the impact forcing function by adjustments in the cushioning areas.
(b) Adjust the structural damping ratio.
(c) Adjust the spring constants and damping ratios for a more accurate representation of rubber properties.
(d) Replace the constant cushioning force applied to the wheel by a linearly varying force.

Finally, it should be noted that computed and measured results should not be expected to agree in all details since the model on which computations are based is only an approximation of the real structure, and the initial conditions used for computations are idealized approximations to the initial conditions that actually occurred in the experiments. In general, in lumped-parameter modeling, the parameters can be adjusted to provide agreement between computed frequencies and corresponding frequencies in the prototype structure, or the adjustment can be made so amplitudes of displacements and accelerations at selected points agree between model and prototype. However, agreement between frequencies, and between amplitudes cannot both be achieved simultaneously. A decision must be made before parameter values are assigned, as to what quantities are to be matched.

CONCLUSIONS

From the previous discussions the following conclusions are believed to be justified.

a. Displacements, velocities and accelerations at various points of a complex mechanical structure can be satisfactorily predicted using a lumped parameter mathematical model and a numerical computation procedure.

b. The dynamic response of a complex structure when properly cushioned and subjected to an impact loading is not sensitive to the elastic properties of the structure. Thus the elastic coupling between masses in the lumped parameter model need not be known to any great degree of precision.

c. Structural damping dissipates a considerable amount of energy, and as a consequence decreases the displacements. However, the peak accelerations at various points in the structure are affected very little by the structural damping.

d. The forcing function is the major factor which affects the dynamic response of the system. It must be represented as exactly as possible. This means that the cushioning characteristics are very important.

e. The procedure for handling the engine and transfer case in the analysis can be applied to any rigid discrete mass which cannot be included in the elastic properties of the structure.

f. The developed computer program can be employed to predict the dynamic response of any complex mechanical structure if the structure can be represented by a grid type model.

This program can be used to determine where large relative deflections may be expected to occur if an existing structure is subjected to an airdrop impact, or to pinpoint trouble points, so far as airdrop is concerned, in vehicles still in the design stage.

RECOMMENDATIONS FOR FURTHER STUDIES

a. Only vertical motion was considered in this study. This is an oversimplification of the real situation because non-vertical and non-planar motion is always possible due to wind drift and system oscillation in the airdrop process. The effects on the dynamic response of a structure due to non-vertical and non-planar impact should be investigated. This can be done by introducing six degrees of freedom at each nodal point of the lumped parameter model in the mathematical analysis. However, shearing properties of cushioning materials will be needed.

b. For the purpose of structural design, the computation of stresses in all members should be included in the computer program. It may be necessary to consider the behavior of elasto-plastic structural elements when subjected to high velocity impact. However, the step-by-step numerical integration method used in this study can be modified to include the yield conditions of the element.

LIST OF SYMBOLS

$\underline{\underline{a}}$	Transformation matrix relating $\underline{W}$ and $\underline{U}$.
$\underline{\underline{A}}$	Matrix relating stress and strain $\underline{S} = \underline{\underline{A}}\ \underline{C}$
a_1, a_2	Unit vectors
$\underline{\underline{b}}$	Matrix relating strain and displacement $\underline{e} = \underline{\underline{b}}\ \underline{U}$
$\underline{\underline{B}}$	Transformation matrix $\underline{q} = \underline{\underline{B}}\ \underline{U}$
$\underline{\underline{c}}$	Damping matrix in u coordinates
$\underline{\underline{C}}$	Damping matrix in q coordinates
c	A damping coefficient
$c_{1,2,3}$	Constants
$\underline{D}$	Damping force matrix $\underline{D} = -\underline{\underline{C}}\ \dot{\underline{q}}$
$\vec{d}_1$, $\vec{d}_2$, d_3	Position vectors
$\underline{e}$	Strain matrix
e_b	Bending strain
e_t	Torsional strain
F	Force
$\underline{F}(t)$	Cushion force matrix
f	Equivalent concentrated forces acting at element ends due to distributed forces.
$\underline{f}^*$	Force vector in global coordinates
G	Shear Modulus
I_x	Torsional inertia
I_y	Rotatory inertia
$\vec{i},\vec{j},\vec{k}$	Unit vectors along datum coordinates
J	Torsional constant (polar moment of inertia)
$\underline{\underline{k}}$	Element stiffness matrix in u coordinates
$\underline{\underline{k}}^*$	Element stiffness matrix transformed to global coordinates
$\underline{\underline{K}}$	Stiffness matrix
K	A spring constant
L	Element length
$\underline{\underline{m}}$	Element mass matrix in u coordinates
$\underline{\underline{m}}^*$	Element mass matrix transformed to global coordinates
$\underline{\underline{M}}$	Mass matrix
M	Moment
$\underline{p}(x\ t)$	External force matrix, acting as an element
q_i	Joint coordinates

Q_i	Displacements in direction of datum coordinates X,Y,Z
$r = X/L$	Nondimensional parameter
$s = Y/L$	Nondimensional parameter
$\underline{S}$	Stress matrix
S_b	Bending moment
S_t	Twisting moment
$t = Z/L$	Nondimensional parameter
T	Kinetic energy
U_i	Element displacements referred to local coordinates x,y,z
$\underline{U}$	Displacement matrix
V	Volume, strain energy
W_i	Displacements referred to element coordinates x,y,z
W	Work

APPENDIX A

Flow Chart and Computer Program

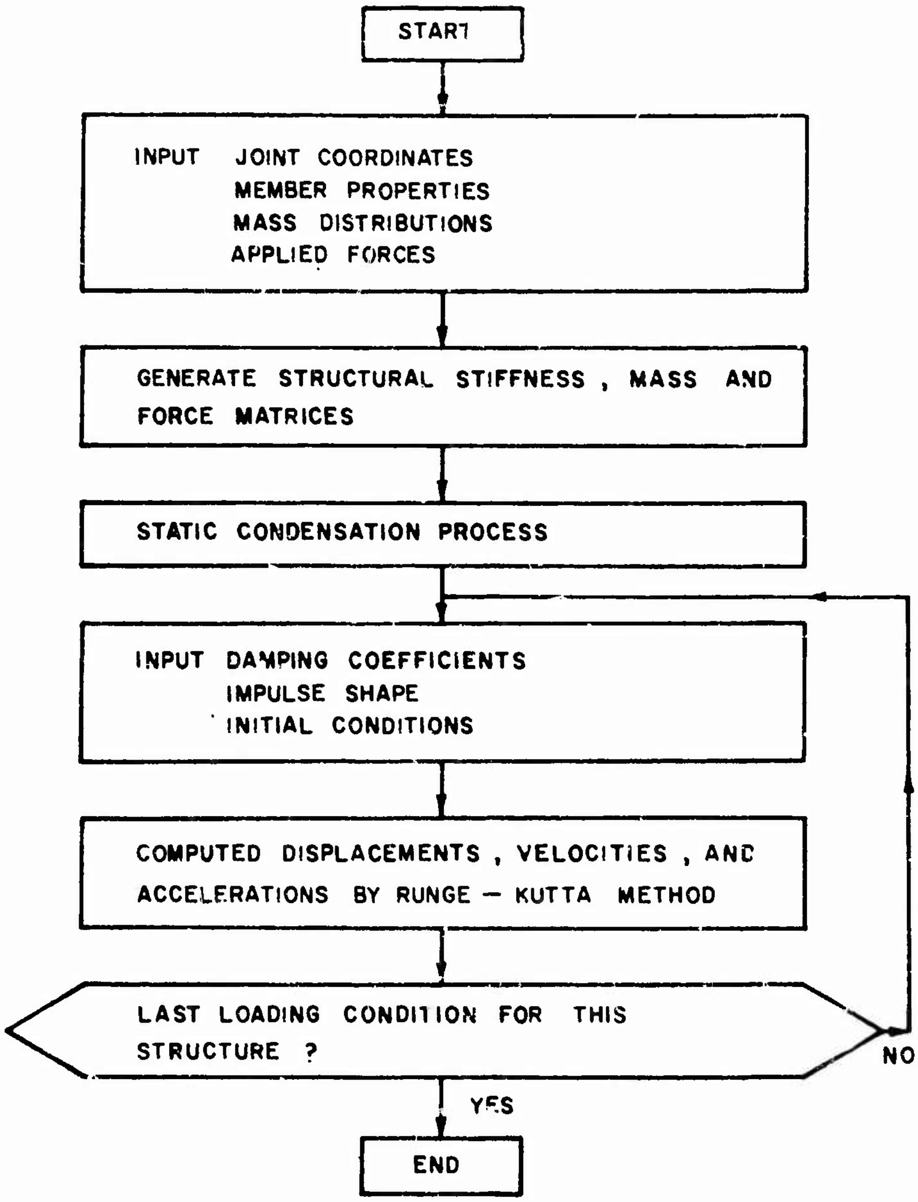


Fig. A.1 Main Program Flow Chart.

```
C      NUMERICAL SOLUTION OF A LUMPED-PARAMETER MODEL FOR
C      AIRDROPPED STRUCTURES BY USING RUNGE-KUTTA METHOD.
C      CDC 6600 , FORTRAN IV, STORAGE REQUIRED 30000 WORDS
       PROGRAM  ARDROP (INPUT,OUTPUT)
       COMMON  N,S(120,120),SMK(40,40)
       COMMON  TM(20),CP(20),SF(120),WG(120),LN1,LN2,SLOP
       COMMON  NDP,JP1(50),JP2(50),DP(50)
       COMMON  GAMA,SK,BETA ,MTF
       DIMENSION  W(6,6),R(6,6),WM(6),WF(6),FD(6),EM(6),AB(6),
     1 DA(6,6),DAT(6,6),ED(6,6),A(6,6),B(6,6),D(6,6),
     2 SK(120),SM(120),X(40),Y(40),MR(120),TR(120),NR(120),
     3 MV(120),MQ(120),NQ(120),YJ(80),DF(5,80) ,SF0(120)
     4 , EF(6),SD(120)
       PRINT 6001
C ---  READ IN   JNT =NO. OF JOINTS
C                MT  =NO. OF ELEMENTS
C                LAGM=NO. OF DISCRETE MASSES SUPPORTED BY
C                     POINTS MORE THAN TWO
C                IOR =-1, ELIMINATING ALL ROTATIONAL COORDINATES
C                    = 0, NO REDUCING IN  COORDINATES
C                    =NO. OF COORDINATES WILL BE ELIMINATED
       READ   5040, JNT,MT,LAGM,IOR
       PRINT  5040, JNT,MT,LAGM,IOR
C --   READ IN   E=MODULUS OF ELASTICITY
C                G=SHEAR MODULUS
       READ  5010,E,G
       PRINT 5010,E,G
       IR=3*JNT
       DO 10 I=1,IR   $   SF(I)=0.   $   WG(I)=0.   $   SK(I)=0.
       SD(I)=0.0   $   SM(I)=0.
       DO 10 J=1,IR   $   S(I,J)=0.
10      CONTINUE
       DO 20 I=1,JNT
C ---  READ IN   J  = JOINT NUMBER
C                X(J),Y(J)=X,Y COORD. OF NODE J
C                WM(1),WM(2),WM(3)=MASS,MASS MOMENT OF INERTIA
C                     ABOUT X AND Y RESPECTIVELY AT NODE J
C                WF(I)=FORCES APPLIED AT NODE J
       READ 5020, J,X(J),Y(J),(WM(K),K=1,3),(WF(K),K=1,3)
       PRINT 5020,J,X(J),Y(J),(WM(K),K=1,3),(WF(K),K=1,3)
       K1=J*3-2  $   WG(K1)=WG(K1)-WM(1)*386.4
       DO 15 K=1,3   $   SF(K1)=SF(K1)+WF(K)   $   SM(K1)=SM(K1)+WM(K)
       K1=K1+1
15      CONTINUE
20      CONTINUE
       DO 220  IMB=1,MT
C ---  READ IN   NJ1,NJ2=JOINT NUMBER AT ELEMENT ENDS
C                MR(I)=1, RELEASE THE MEMBER CONSTRAINT IN  THE
C                         MEMBER COORD. U(I)
C                     =0,NO MEMBER CONSTRAINT IS RELEASED IN U(I)
```

```
C              KF =0, NO MEMBER FORCES
C                 =1,  UNIFORM MEMBER FORCES
C                 =2, CONCENTRATED FORCE AT  THE MEMBER
C              RO= MASS PER UNIT LENGTH OF THE MEMBER
C              AA,AI,XJ,GJ= AREA,MOMENT OF INERTIA, POLAR MOMENT
C                   OF INERTIA AND TORSIONAL CONSTANT OF
C                   THE MEMBER RESPECTIVELY
C              FD = UNIFORM FORCE FD(1)  FROM FD(2) TO FD(3),OR
C
C                   CONCENTRATED FORCE AT FD(2)
      READ 5030,   NJ1,NJ2,(MR(I),I=1,6),KF,AI,GJ,RO,(FD(I),I=1,3)
      READ   5010,XJ,AA
      PRINT 5030,  NJ1,NJ2,(MR(I),I=1,6),KF,AI,GJ,RO,(FD(I),I=1,3)
      PRINT  5070,XJ,AA
      X1=X(NJ2)-X(NJ1)   $   Y1=Y(NJ2)-Y(NJ1)
      SL=SQRT(X1*X1+Y1*Y1)
      IF(SL)  25,25,30
25    K1=NJ1*3-2  $   K2=NJ2*3-2   $   SK(K1 ) =SK(K1 ) +AI
      SK(K2 ) =SK(K2) +AI
      S(K1,K1)=SK(K1)   $   S(K2,K2)=SK(K2)
      IF(K1.GT.K2)  GO TO 26
      S(K1,K2)=S(K1,K2)-AI   $   GO TO 220
26    S(K2,K1)=S(K2,K1)-   AI    $   GO TO 220
30    G1=X1/SL   $   G2=Y1/SL   $   CS=E*AI/SL**3
C     *** TRANSFORMATION MATRIX (DIRECTION COSINES) ***
      DO 11 I=1,6             $ DO 11 J=1,6
11    DA(I,J)=0.0
      DA(1,1)=1.0             $ DA(2,2)=G1            $ DA(2,3)=G2
      DA(3,2)=-G2             $ DA(3,3)=G1           $ DA(4,4)=1.0
      DA(5,5)=G1              $ DA(5,6)=G2           $  DA(6,5)=-G2
      DA(6,6)=G1
C     *** ELEMENT STIFFNESS ***
      W(1,1)=12.0             $ W(2,1)=0.             $ W(3,1)=-6.0*SL
      W(4,1)=-12.             $ W(5,1)=0.            $ W(6,1)=-6.0*SL
      W(2,2)=G*GJ/(SL*CS)   $   W(3,2)=0.
      W(4,2)=0.               $ W(5,2)=-W(2,2)        $ W(6,2)=0.
      W(3,3)=4.*SL**2         $ W(4,3)=6.*SL          $ W(5,3)=0.
      W(6,3)=2.*SL**2   $ W(4,4)=12.   $   W(5,4)=0.
      W(6,4)=6.*SL            $ W(5,5)=W(2,2)         $ W(6,5)=0.
      W(6,6)=4.*SL**2
      DO 40 J=1,6   $   DO 40 I=J,6   $   W(I,J)=CS*W(I,J)
40    W(J,I)=W(I,J)
      IF(RO.LE.0.)    GO TO 70
      CM=RO*SL/420.   $   DO 60 I=1,6   $   DO 60 J=1,6
60    R(I,J)=0.
      WM(1)=RO*386.4   $   WM(2)=0.   $   WM(3)=SL
      CALL ENDFOC (EM, 1,WM,SL)
      A0=AI/AA   $   A1=A0/SL   $   A2=A1/SL   $   AJ=XJ/AA
      S2=SL*SL
```

```
C       *** ELEMENT MASS MATRIX ***
        R(1,1)=156.*504.*A2   $   R(1,3)=-22.*SL-42.*A1
        R(1,4)= 54.-504.*A2   $   R(1,6)=13.*SL-42.*A1
        R(2,2)=140.*AJ   $ R(2,5)=70.*AJ
        R(3,3)=4.*S2+56.*A0   $   R(3,4)=-13.*SL+42.*A1
        R(3,6)=-3.*S2-14.*A0
        R(4,4)=156.+504.*A2   $   R(4,6)=22.*SL+42.*A1
        R(5,5)=140.*AJ   $   R(6,6)=4.*S2+56.*A0
        DO 75 I=1,6   $   DO 75 J=I,6
        R(I,J)=R(I,J)*CM
75      R(J,I)=R(I,J)
70      CONTINUE
        IF(KF.LE.0)  GO TO 80
        CALL ENDFOC(EF,KF,FO,SL)
80      DO  160 I=1,6
        IF(MR(I).LE.0)  GO TO 160  $  M=0
        IF(W(I,I).EQ.0.)  GO TO 160
        DO 110 J=1,6  $  M=M+1  $   IF(J.EQ. I)  GO TO 90
        TR(  M)=-W(I,J)/W(I,I)  $  NR(M)=J  $  GO TO 110
90      M=M-1
110     CONTINUE
        CALL CONM86  (W,6,NR,5,I,TR)
        IF(KF)  130,150,120
120     DO 121 K=1,M  $  K1=NR(K)
121     EF(K1)=EF(K1)+EF(I)*TR(K)
        EF(I)=0.
130     CONTINUE  $  IF(RO.LE.0)  GO TO 160
        CALL CONM86  (R,6,NR,5,I,TR)
        DO 131  K=1,M  $  K1=NR(K)
131     EM(K1)=EM(K1)+EM(I)*TR(K)
        EM(I)=0.
160     CONTINUE
        K1=NJ1*3-3  $  K2=NJ2*3-3  $  DO 170 I=1,3  $  J=I+3
        MR(I)=K1+I  $  MR(J)=K2+I
170     CONTINUE  $  CALL TRANSP(DA,DAT,6,6)
        CALL MMUL(W,DA,ED,6,6,6)  $  CALL MMUL(DAT,ED,W,6,6,6)
        CALL ADDMS(W,6,S,SK,IR,MR,1)
        IF(KF.LE.0)  GO TO 200
        CALL  MUL1(DAT,EF,AB,6)  $  DO 190 I=1,6  $  K=MR(I)
190     SF(K)=SF(K)+AB(I)
200     IF(RO.LE.0)  GO TO 220
        CALL MUL1(DAT,EM,AB,6)
        DO 205  I=1,6  $  K=MR(I)  $  WG(K)=WG(K)-AB(I)
205     CONTINUE
        CALL MMUL(R,DA,ED,6,6,6)  $  CALL MMUL(DAT,ED,R,6,6,6)
        CALL ADDMS(R,6,S,SM,IR,MR,2)
220     CONTINUE
        IJ=0  $  IK=0  $  DO 250 I=1,IR  $  IK=IK+1
        IF( SK(I).GT.0.)  GO TO 240
        IK=IK-1  $  IJ=IJ+1  $  MV(IJ)=I  $  GO TO 250
240     NQ(IK)=I
250     CONTINUE
```

```
      IF(IJ.LE.0) GO TO 272
      CALL REMOMS (S,SK,IR,MV,IJ, 1)
      CALL REMOMS (S,SM,IR,MV,IJ, 2)
      CALL REMOF ( SF,IR,MV,IJ)  $  CALL REMOF (WG,IR,MV,IJ)
      [illegible]-IJ
272   IF([illegible]M.LE.0)   GO TO 385
      DO 282 I=1,LAGM  $  DO 280 J=1,3  $  DO 280 K=1,3
280   R(J,K)=0.
C --- READ IN  LMS=NO. OF SUPPORTS OF  A DISCRETE MASS
C                 ( LMS MUST BE GREATER THAN TWO)
C             MR(J)= NODE NUMBER OF SUPPORT J
C             XC,YC= CENTROID OF MASS
C             XA,YA= ANGLES IN DEGREES BETWEEN THE PRINCIPAL
C                 AXES OF THE MASS AND X-AXIS
C             WF(1),WF(2),WF(3)=  APPLIED MOMENTS ABOUT X,Y AXES
C                 AND FORCE IN Z-DIRECTION RESPECTIVELY
C             R(1,1),R(2,2),R(3,3)=MASS MOMENTS OF INERTIA ABOUT
C                 X,Y AXES AND MASS RESPECTIVELY
      READ    5040,LMS,(MR(J),J=1,LMS)
      PRINT   5040,LMS,(MR(J),J=1,LMS)
      READ    5042,XC,YC,XA,YA,(WF(J),J=1,3),(R(J,J),J=1,3)
      PRINT   5042,XC,YC,XA,YA,(WF(J),J=1,3),(R(J,J),J=1,3)
      XA=XA*3.1416/180.  $  YA=YA*3.1416/180.
      IF(LMS.LE.3)  GO TO 310

      MN=LMS-3                  $  K1=MR(1)  $  K2=MR(2)  $  K3=MR(3)
      A1=X(K2)-X(K1)  $  A2=Y(K2)-Y(K1)  $  B1=X(K3)-X(K1)
      B2=Y(K3)-Y(K1)  $  CC=A1*B2-A2*B1
      DO 285 J=1,3  $  KI=MR(J)*3-2
285   NR(J)=MACH(NO,IR,KI)
      DO 300 J=4,LMS  $  J1=J-3  $  KK=MR(J)  $  C1=X(KK)-X(K1)
      C2=Y(KK)-Y(K1)  $  AA=B1*C2-B2*C1  $  BB=A1*C2-A2*C1
      TR(1)=(AA-BB+CC)/CC  $  TR(2)=-AA/CC  $  TR(3)=BB/CC
      KK=KK*3-2
      MV(J1)=MACH(NO,IR,KK)  $  LMK=MV(J1)
      CALL  CONMS(S,SK,IR,NR,3,LMK,TR,1)
      CALL  CONMS(S,SM,IR,NR,3,LMK,TR,2)
      CALL CONF(SF,IR,NR,3,LMK,TR) $ CALL CONF(WG,IR,NR,3,LMK,TR)
300   CONTINUE
      CALL REMOMS(S,SK,IR,MV,MN,1)
      CALL REMOMS(S,SM,IR,MV,MN,2)
      CALL REMOF(SF,IR,MV,MN)  $  CALL REMOF(WG,IR,MV,MN)
      CALL SEQN(NO,IR,MV,MN)
310   CONTINUE
      A(1,1)=COS(XA)  $  A(1,2)=SIN(XA)  $  A(2,1)=COS(YA)
      A(2,2)=SIN(YA)  $  DO 320  J=1,3  $  K=MR(J)
      D(J,1)=X(K)-XC
320   D(J,2)=Y(K)-YC
      DO 330 J=1,2  $  DO 330  K=1,3
330   B(K,J)=A(J,1)*D(K,2)-A(J,2)*D(K,1)
      DO 340 K=1,3
340   B(K,3)=1.
```

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```
      CALL  MIV(B,ED,3)  $  CALL  MMUL(R,ED,H,3,3,3)
      CALL  TRANSP(  ED,D,3,3)  $  CALL MMUL(D,B,R,3,3,3)
      CALL  MUL1(D,WF,AB,3)
      EF(1)=R(3,3)*386.4  $  EF(2)=0.   $  EF(3)=0.
      CALL MUL1(D,EF,FD,3)
      DO 370 II=1,3  $  DO 350 J=1,IR
      KI=MR(II)*3-2  $  IF(NQ(J).EQ.KI)  GO TO 360
350   CONTINUE  $  GO TO 370
360   MV(II)=J  $  SF(J)=SF(J)+AB(II)
      WG(J)=WG(J)-FD(II)
370   CONTINUE
      CALL ADDMS(R,3,S,SM,IR,MV,2)
380   CONTINUE
385   CONTINUE
      IF(IQR)  400,408,388
388   CONTINUE
C --- IF IQR GREATER THAN ZERO
C     READ IN  MQ(I)=COORD. TO BE ELIMINATED
      READ  5040,  (MQ(I),I=1,IQR)  $        IQ=1
      PRINT 5040,  (MQ(I),I=1,IQR)
      GO TO 408
400    CONTINUE
      IQR=JNT*2  $  DO 405 I=1,JNT  $  I2=3*I  $  I1=I2-1
      K2=2*I  $  K1=K2-1  $  MQ(K1)=I1
405   MQ(K2)=I2
408   IQ=1  $  J1=0
      DO 600 I=1,IR  $  I1=I1+1  $  K=NQ(I)
410   IF(IQ.GT.IQR)  GO TO 450
      IF(MQ(IQ).NE.K )  GO TO 430
      IQ=IQ+1  $  GO TO 470
430   IF(MQ(IQ) .GT.K)  GO TO 450

      IQ=IQ+1  $  GO TO 410
450   IF(SM(I).GT.0.)  GO TO 600
470   J1=J1+1  $  MV(J1)= I
510   M=0
      DO 560 M1=1,IR
      IF(SK(M1).EQ.0.)  GO TO 560
      M=M+1
      IF(M1-I)  520,515,540
515   M=M-1  $  GO TO 560
520   BB=S(M1,I)  $  GO TO 550
540   BB=S(I,M1)
550   TR(M)=-BB/SK(I)  $  MR(M)=M1
560   CONTINUE
      CALL CONMS(S,SK,IR,MR,M, I ,TR,1)
      CALL CONF(SF,IR,MR,M, I ,TR)
      IF(SM(I).EQ.0.)  GO TO 600
      CALL CONMS(S,SM,IR,MR,M, I ,TR,2)
      CALL CONF(WG,IR,MR,M, I ,TR)
600   CONTINUE
```

```
660    M2=A(  (S,I,K,2)
665    IF(I.  I,K)  670,675
670    B1=S (I)  $  GO TO 680
675    B1=  (  S,  ,1)
680    S        +B1*B2
685    C
       SMK(I,J)=SUM
690    CONTINUE
       N=IR*2
       DO 69  I=1,IR
       SFP(I)=SF(I)
       S (  )=  (SD(  /TR(I))
       SUM=0.
       DO 69  I=1,IR
692    SUM=SUM+SK(I)
       SIR=IR  $  AVGW=SUM/SIR
695    CONTINUE
C ---  READ IN  NCASE= LOADING CASE NUMBER
C               NDP= NO. OF VISCOUS DAMPINGS
C               GAMA=STRUCTURAL DAMPING RATIO OF CRITICAL VALUE
       READ   6200,NCASE,NDP,GAMA
       IF(NCASE.LE.0)   GO TO 800
       PRINT  6221
       PRINT  6220,NCASE,NDP,GAMA
       DO 694  I=1,IR
       Y(I)=0.
694    X(I)=0.
       BETA=GAMA/AVGW +1.  $  GAMA=GAMA*AVGW
C ---  READ IN  ALPHA,TEND= BEGINING AND END OF TIME  CALCULATION
C                              RESPECTIVELY
C               DT= TIME INTERVAL OF PRINT OUT
C               V0=IMPACT VELOCITY
C               EM(I)=TIME SIZES OF INTEGRATION IN RUNGE-KUTTA METHOD
```


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```
      READ 5050, ALPHA,TEND,DT,  V0
      PRINT 5050,  ALPHA,TEND,DT,  V0
      NTT=1  $  ALPHA0=ALPHA
      READ   5050, (EM(I),I=1,6)
      PRINT  5050, (EM(I),I=1,6)
C --- READ IN IMPULSE SHAPE
      SF0(I)=SF(I)
C               K=NO. OF TIME STATIONS OF THE PULSE SHAPE
C               TM(J)= TIME FROM IMPACT BEGINS OF STATION J
C               CP(J)=RATIO OF FORCE AT STATION J TO THE
CC                      INPUT FORCE
      READ  5060, K,(TM(J),CP(J),J=1,K)
      PRINT 5060, K,(TM(J),CP(J),J=1,K)
      IF(NDP.LE.0)  GO TO 625  $  DO 622 I=1,NDP
C --- READ IN SPRING CONSTANT DK AND DAMPING RATIO DPC
C        FOR REPRESENTING RUBBER MATERIAL PROPERTIES
C        J1= JOINT NUMBER OF THE ASSOCIATED MASS
C        J2= JOINT NUMBER OF THE SUPPORT, OR
C            RIGID SUPPORT IF J2=0
      READ   6200,J1,J2,DPC,DK
      PRINT  6200,J1,J2,DPC,DK
      J1=J1*3-2  $  JP1(I)=MACH(NQ,IR,J1)  $  J=JP1(I)
      JP2(I)=0
      IF(J2.LE.0 )  GO TO 621
      J2=J2*3-2  $  JP2(I)=MACH(NQ,IR,J2)  $  DK=SD(J)
621   DP(I)=DPC*2.*SQRT(DK/TR(J))
      SK(I)=DK/TR(J)
622   CONTINUE
625   CONTINUE
      JNT3= JNT*3  $  JNT6=JNT*6
696   H=EM(NTT)
      DO 612 I=1,IR  $  MQ(I)=1
612   SF(I)=SF0(I)
      MTF=0
      MNF=1
      DO 700 I=1,JNT6
700   YJ(I)=0.
      DO 710 I=2,JNT6,6
710   YJ(I)=V0
      K=NQ(1)  $  I1=1
      DO 720 I=1,JNT3
      IF(I.NE.K)  GO TO 720
      I2=I1*2-1  $  I3=I2+1  $  K2=K*2-1  $  K3=K2+1
      YJ(I2)=YJ(K2)  $  YJ(I3)=YJ(K3)  $  I1=I1+1

      IF(I1.GT.IR)  GO TO 725  $  K=NQ(I1)
720   CONTINUE
725   CONTINUE
      LN1=0
730   OMEGA=ALPHA+DT
      IF(OMEGA.LE.TEND)  GO TO 727
```

```
      [illegible] 742,I[illegible]
728   [illegible] 5080 [illegible] I1,X[illegible]
      [illegible]
727   CONTINUE
      PRINT 6004, OMEGA
      CALL RGKT (ALPHA,OMEGA,H,YJ,DF)
      DO 740 I=2,N,2  $  I1=I/2  $  I2=I-1   $  I3=NQ(I1)
      DFG=DF(1,I)/386.4
      IF(ABS(DFG).LE.X(I1) )  GO TO 736
      X(I1)=DFG  $  Y(I1)=OMEGA
736   CONTINUE
      I1=(I3+2)/3   $  I3=I3-I1*3+3
      I4=I/2  $  I5=I4+1
      IF(I3.EQ.1)  GO TO 731
      GO TO (740,733,735),MNF
731   MNF=1  $  IF(YJ(I ).LT.0.0 .AND.MQ(I4).EQ.1)  GO TO 740
      MTF=1
      IF(MQ(I4).EQ.1)  732,734
732   MQ(I4)=2  $  MNF=2
733   SF(I4)=0.  $  GO TO 740
734   IF(YJ(I ).GE.0.)  GO TO 732
      MQ(I4)=3  $  MNF=3
735   SF(I4)=3F0(I4)
740   PRINT 6100,I1,I3  ,YJ(I2),YJ(I),DF(1,I) ,DFG
      IF(ABS(YJ(1)).GT.100.)  GO TO 750
      ALPHA=OMEGA  $  GO TO 730
750   IF(NTT .GE.6)  GO TO 800
      ALPHA=ALPHA0
      NTT=NTT+1  $  GO TO 696
5010  FORMAT(8F10.0)
5020  FORMAT(I3,F7.2,7F10.2)
5030  FORMAT(2I5, 6I1.I4,6F10.3)
5040  FORMAT(16I5)
5042  FORMAT(4F10.3,/,8F10.3)
5050  FORMAT(8F10.5)
5060  FORMAT(I10,10X,6F10.3,/,(8F10.3))
5070  FORMAT(20X,6F10.4)
5080  FORMAT(I10,4E12.3)
6001  FORMAT(1H1)
6004  FORMAT( 5X,* T=*,F10.5)
6100  FORMAT(2I5,3E12.3, 2F9.2)
6200  FORMAT(2I5,3F10.3)
7000  FORMAT(12F11.2)
800   STOP  $  END
```

```
      SUBROUTINE  RGKT(A1,A2,A3,YJ,D)
C     *** RUNGE-KUTTA METHOD ***
      COMMON  N
      DIMENSION  D(5,80),YJ(80),W(10),A(10),PHI(80),B(4,3),
     1  X(10),Y(5,80)
C -------  COEFFICIENTS
      W(1)=W(4)=1./6. $ W(2)=W(3)=1./3.      $      KK=4
      A(2)=A(3)=.5  $   A(4)=1.
      B(2,1)=B(3,2)=.5  $  B(3,1)=B(4,1)=B(4,2)=0.  $  B(4,3)=1
      IQ = 4    $    FCT = 19./270.
      ALPHA = A1   $   OMEGA = A2   $   H = A3
      IQ=KK
      IQM1 = IQ-1    $    IQP1 = IQ+1
      ISTP = 0    $    SIGN = 1.
      IF( H.LT.0. )    SIGN = -1.
      X(1) = ALPHA
      DO 3 I=1,N
    3 Y(1,I) = YJ(I)
    4 MM = 1    $     IFLG = 0
    6 M = MM    $     MM = M + 1
      IF( MM.GT.IQP1 )    MM = 1
      X(MM) = X(M) + H
      TEST = OMEGA - X(MM)
      TEST1 = TEST/OMEGA
      IF QUOTIENT OVERFLOW 7,8
    7 TEST1 = TEST
    8 IF( ABS(TEST1).LT.1.0E-10 )    GO TO 12
      IF( SIGN*TEST1 )   9,12,13
    9 TEST2 = OMEGA - X(M)
   11 H = TEST2    $    IFLG = 0
      X(MM) = X(M) + H
   12 ISTP = 1
   13 XJ = X(M)
      DO 14 I=1,N
   14 YJ(I) = Y(M,I)
C -------  RUNGE-KUTTA PROCEDURE
      DO 25 K=1,KK
      IF( K.EQ.1 )    GO TO 22
      XJ = X(M) + H*A(K)
      DO 15 I=1,N
   15 PHI(I) = 0.
      KM1 = K - 1
      DO 19 J=1,KM1
   19 PHI(I) = PHI(I) + H*B(K,J)*D(J,I)
   20 YJ(I) = Y(M,I) + PHI(I)
   22 CALL DERFCN(XJ,YJ,K,D)
   25 CONTINUE
C -------  RKAMSUB ENTRY POINT
```

```
C -------  RKAMSUB EXIT POINT
      A2=X(MM)  $  A3=H
      DO 105 I=1,N
  105 YJ(I) = Y(MM,I)
  500 RETURN
      END
      CALL DERFCN(A1,YJ,1,D)
      DO 27 I=1,N
   27 PHI(I) = 0.
      DO 30 I=1,N
      DO 29 K=1,KK
   29 PHI(I) = PHI(I) + H*W(K)*D(K,I)
   30 Y(MM,I) = Y(M,I) + PHI(I)
      IF( ISTP.EQ.1 )   GO TO 100
      GO TO 6
100   CONTINUE
      SUBROUTINE DERFCN(XJ,YJ,M,DF)
C     *** DIFFERENTIAL EQUATIONS ***
      COMMON  N,S(120,120),SMK(40,40)
      COMMON  TM(20),CP(20),SF(120),WG(120),LN1,LN2,SLOP
      COMMON  NDP,JP1(50),JP2(50),DP(50)
      COMMON  GAMA,SK,BETA ,MTF
      DIMENSION  SQ(40),DF(5,80),YJ(80),SU(40),SK(120)
      N1=N/2  $  IF(LN1.EQ.0)  GO TO 10
      IF(XJ.LE.TM(LN1).AND.MTF.EQ.0)  GO TO 40
      IF(XJ.LE.TM(LN1))  GO TO 15
10    LN1=LN1+1
15    CONTINUE
      DO 20 I=1,N1
20    SQ(I)=WG(I)+CP(LN1)*SF(I)
      DO 30 I=1,N1  $  SU(I)=0. $ DO 30 J=1,N1
30    SU(I)=SU(I)+AIJ(S,I,J,2)*SQ(J)
40    CONTINUE
      DO 60 I=2,N,2  $  K=I-1  $  DF(M,K)=YJ(I)  $  I1=I/2
      SUM1=0.
      DO 50 J=1,N1  $  J1=J*2-1
50    SUM1=SUM1+SMK(I1,J)*YJ(J1) *BETA
      SUM1=SUM1+GAMA*YJ(I)
      DF(M,I)=SU(I1)-SUM1
60    CONTINUE
      IF(NDP.LE.0)  GO TO 80  $  DO 70 I=1,NDP  $  I1=JP1(I)*2
      DV=YJ(I1)*DP(I)  $  I2=JP2(I)*2
      IF(I2.LE.0)  GO TO 65
      DV=(YJ(I1)-YJ(I2))*DP(I)  $  DF(M,I2)=DF(M,I2)+DV
      GO TO 70
65    I3=I1-1  $  DV=DV+SK(1)*YJ(I3)
70    DF(M,I1)=DF(M,I1)-DV
80    CONTINUE
      MTF=0
      RETURN  $  END
```

```
      SUBROUTINE  RGKT(A1,A2,A3,YJ,D)
C     *** RUNGE-KUTTA METHOD ***
      COMMON  N
      DIMENSION  D(5,80),YJ(80),W(10),A(10),PHI(80),B(4,3),
     1  X(10),Y(5,80)
C -------  COEFFICIENTS
      W(1)=W(4)=1./6. $ W(2)=W(3)=1./3.      $      KK=4
      A(2)=A(3)=.5  $  A(4)=1.
      B(2,1)=B(3,2)=.5  $  B(3,1)=B(4,1)=B(4,2)=0.  $  B(4,3)=1
      IQ = 4   $    FCT = 19./270.
      ALPHA = A1  $  OMEGA = A2  $  H = A3
      IQ=KK
      IQM1 = IQ-1   $   IQP1 = IQ+1
      ISTP = 0   $   SIGN = 1.
      IF( H.LT.0. )   SIGN = -1.
      X(1) = ALPHA
      DO 3 I=1,N
    3 Y(1,I) = YJ(I)
    4 MM = 1   $    IFLG = 0
    6 M = MM   $    MM = M + 1
      IF( MM.GT.IQP1 )   MM = 1
      X(MM) = X(M) + H
      TEST = OMEGA - X(MM)
      TEST1 = TEST/OMEGA
      IF QUOTIENT OVERFLOW 7,8
    7 TEST1 = TEST
    8 IF( ABS(TEST1).LT.1.0E-10 )   GO TO 12
      IF( SIGN*TEST1 )   9,12,13
    9 TEST2 = OMEGA - X(M)
   11 H = TEST2   $   IFLG = 0
      X(MM) = X(M) + H
   12 ISTP = 1
   13 XJ = X(M)
      DO 14 I=1,N
   14 YJ(I) = Y(M,I)
C -------  RUNGE-KUTTA PROCEDURE
      DO 25 K=1,KK
      IF( K.EQ.1 )   GO TO 22
      XJ = X(M) + H*A(K)
      DO 15 I=1,N
   15 PHI(I) = 0.
      KM1 = K - 1
      DO 19 J=1,KM1
   19 PHI(I) = PHI(I) + H*B(K,J)*D(J,I)
   20 YJ(I) = Y(M,I) + PHI(I)
   22 CALL DERFCN(XJ,YJ,K,D)
   25 CONTINUE
C -------  RKAMSUB ENTRY POINT
```

```
      SUBROUTINE CONF (A,IR,MR,ID,LMS,T)
      DIMENSION  A(120),MR(120),T(120)
      DO 10 I=1,ID   $   I1=MR(I)
10    A(I1)=A(I1)+A(LMS)*T(I)
      A(LMS)=0.0
      RETURN   $   END

      SUBROUTINE TRANSP (A,B,NR,NC)
      DIMENSION   A(6,6),B(6,6)
      DO 20 I=1,NR   $   DO 20 J=1,NC
20    B(J,I)=A(I,J)
      RETURN   $   END

      SUBROUTINE SEGR(NQ,IR,MV,MN)
      DIMENSION  NQ(120),MV(120),NEW(120)
      I1=0
      DO 20 I=1,IR   $   I1=I1+1
      DO 10 K=1,MN   $   IF(I.EQ.MV(K))  15,10
15    I1=I1-1   $   GO TO 20
10    CONTINUE
            NEW(I1)=NQ(I)
20    CONTINUE
      IR=IR-MN   $   DO 30 I=1,IR
30    NQ(I)=NEW(I)
      RETURN   $   END

      SUBROUTINE  SYMINV(A,M)
      DIMENSION  A(120,120),T(120)
      M1=M-1
      DO 10 I=1,M1   $   T(I)=0.   $   DO 10 J=1,M1
      IF(I.GE.J)  2,4
2     B=A(I,J)   $   GO TO 10
4     B=A(J,I)
10    T(I)=T(I)+B*A(M,J)
      A2=0.   $   DO 20 I=1,M1
20    A2=A2+T(I)*A(M,I)
      A2=A(M,M)-A2   $   A2=1./A2   $   A(M,M)=A2
      DO 30 I=1,M1
30    A(M,I)=-T(I)*A2
      DO 40 I=1,M1   $   DO 40 J=I,M1
40    A(J,I)=A(J,I)-A(M,I)*T(J)
      RETURN   $   END
```

```
      SUBROUTINE  REMOF(A,IR,MV,MN)
      DIMENSION  A(120),MV(120)
      I1=0
      DO 20 I=1,IR   $   I1=I1+1
      DO 10 K=1,MN
      IF(I.EQ.MV(K))  GO TO 15
15    I1=I1-1   $   GO TO 20
10    CONTINUE   $   A(I1)=A(I)
20    CONTINUE
      RETURN   $   END
      SUBROUTINE MMUL (A,B,C,NR,N12,NC)
      DIMENSION   A(6,6),B(6,6),C(6,6)
      DO 20 I=1,NR   $   DO 20 J=1,NC   $   C(I,J)=0.
      DO 20 K=1,N12
20    C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN  $  END

      SUBROUTINE ENDFOC (EF,NGO,DF,SL)
      DIMENSION  EF(6),DF(6)
      EF(2)=0.    $   EF(5)=0.
      GO TO (20,10) ,NGO
10    B=SL-DF(2)
      EF(1)=DF(1)*B **2*(SL+2.*DF(2))/SL**3   $   EF(4)=DF(2)-EF(1)
      WP=DF(1)*DF(2)*B/SL**2   $   EF(3)=-B*WP   $   EF(6)=DF(2)*WP
      GO TO 30
20    IF(DF(3).EQ.0.)  DF(3)=SL
      A=DF(2)  $  B=SL-DF(3)  $   WP= DF(1)/(12.*SL*SL)
      EF(3)=WP*((SL-A)**3*(SL+3.*A)-B**3*(4.*SL-3.*B))
      EF(6)=-WP*((SL-B)**3*(SL+3.*B)-A**3*(4.*SL-3.*A))
      SH=DF(3)-DF(2)   $   WM=SH*(B     +SH/2.)*DF(1)
      EF(1)=(WM+EF(3)+EF(6))/SL   $   EF(4)=SH*DF(1)-EF(1)
      EF(3)=-EF(3)   $   EF(6)=-EF(6)
30    RETURN  $  END

      SUBROUTINE  ADDMS(W,M,S,SD,IR,IJ,IK)
      DIMENSION  W(6,6),S(120,120),SD(120),IJ(120)
      DO 5 I=1,IR
5     S(I,I)=SD(I)
      DO 30 I=1,M   $   I1=IJ(I)
      DO 20 J=I,M   $   J1=IJ(J)
      IF(J1.GE.I1.AND.IK.EQ.1)  GO TO 10
      IF(J1.LE.I1.AND.IK.EQ.2)  GO TO 10
      GO TO 15
10    S(I1,J1)=S(I1,J1)+W(I,J)   $   GO TO 20
15    S(J1,I1)=S(J1,I1)+W(I,J)
20    CONTINUE
      SD(I1)=S(I1,I1)
30    CONTINUE
      RETURN  $  END
```

```
      SUBROUTINE  MUL1(A,B,C,IR)
      DIMENSION   A(6,6),B(6),C(6)
      DO 10 I=1,IR   $   C(I)=0.
      DO 10 J=1,IR
10    C(I)=C(I)+A(I,J)*B(J)
      RETURN   $  END

      FUNCTION  MACH(NQ,IR,KI)
      DIMENSION  NQ(120)
      DO 10 I=1,IR
      IF(NQ(I).NE.KI)  GO TO 10
      MACH=I  $  GO TO 20
10    CONTINUE
20    CONTINUE
      RETURN  $  END

      SUBROUTINE  CONMS6(S,M,MR,ID,LMS,TR)
      DIMENSION  S(6,6),MR(120),TR(120),T1(120),T2(120)
      IK=1
20    DO 30 I=1,M
      IF(I.GE.LMS.AND.IK.EQ.1)  21,22
21    T1(I)=S(LMS,I)   $  GO TO 30
22    T1(I)=S(I,LMS)
30    CONTINUE
      DO 110 I=1,ID   $   I1=MR(I)
      DO 40 J=1,M
      IF(J.GE.I1.AND.IK.EQ.1)   31,32
31    A=S(I1,J)   $  GO TO 40
32    A=S(J,I1)
40    T2(J)=A+TR(I)*T1(J)
      DO 50 J=1,ID   $   J1=MR(J)
50    T2(J1)=T2(J1)+T2(LMS)*TR(J)
      L1=I-1
      DO 100 K=1,M   $   IF(L1.LE.0)   GO TO 70
      DO 60 L=1,L1
      IF(K.EQ.MR(L))   GO TO 100
60    CONTINUE
70    IF(K.GE.I1.AND.IK.EQ.1)  80,90
80    S(I1,K)=T2(K)   $  GO TO 100
90    S(K,I1)=T2(K)
100   CONTINUE
110   CONTINUE
      DO 150 I=1,M
      IF(I.GE.LMS.AND.IK.EQ.1)  120,140
120   S(LMS,I)=0.   $  GO TO 150
140   S(I,LMS)=0.0
150   CONTINUE
      DO 160 I=1,M   $   DO 160 J=I,M
160   S(J,I)=S(I,J)
      RETURN  $  END
```

```
      SUBROUTINE CONMS(S,SD,M,MR,ID,LMS,TR,IK)
      DIMENSION   S(120,120),SD(120),MR(120), TR(120),T1(120),
     1  T2(120)
6     DO 7 I=1,M
7     S(I,I)=SD(I)
20    DO 30 I=1,M
      T1(I)=AIJ(S,LMS,I,IK)
30    CONTINUE
      DO 110 I=1,ID   $   I1=MR(I)
      DO 40 J=1,M
      A=AIJ(S,I1,J,IK)
      T2(J)=A+TR(I)*T1(J)
40    CONTINUE
      DO 50 J=1,ID   $   J1=MR(J)
50    T2(J1)=T2(J1)+T2(LMS)*TR(J)
      L1=I-1
      DO 100 K=1,M   $   IF(L1,LE,0)   GO TO 70
      DO 60 L=1,L1
      IF(K.EQ.MR(L))  GO TO 100
60    CONTINUE
70    IF(K.GE.I1.AND.IK.EQ.1)  GO TO 80
      IF(K.LE.I1.AND.IK.EQ.2)  GO TO 80
      GO TO 90
80    S(I1,K)=T2(K)   $   GO TO 100
90    S(K,I1)=T2(K)
100   CONTINUE
110   CONTINUE
      S(LMS,LMS)=0.
      DO 150 I=1,M   $   SD(I)=S(I,I)
150   CONTINUE
      RETURN   $   END

      SUBROUTINE REMOMS(S,SD, IR,MV,MN,IK)
      DIMENSION  S(120,120),SD(120),MV(120),T(120)
      DO 2 I=1,IR
2     S(I,I)=SD(I)
      I1=0
      DO 50 I=1,IR   $   I1=I1+1
      DO 10 K=1,MN   $ IF(I.NE.MV(K))   GO TO 10
      I1=I1-1   $   GO TO 50
10    CONTINUE
      K=0
      DO 30 J=I,IR   $   K=K+1
      DO 20 L=1,MN   $   IF(J.NE.MV(L))   GO TO 20
      K=K-1   $   GO TO 30
20    CONTINUE
      T(K)=AIJ(S,I,J,IK)
30    CONTINUE
      DO 40 J=1,K   $   L=I1+J-1
      IF(L.GE.I1.AND.IK.EQ.1)   GO TO 32
      IF(L.LE.I1.AND.IK.EQ.2)   GO TO 32
      GO TO 33
```

```
32     S(I1,L)=T(J)  $  GO TO 40
33     S(L,I1)=T(J)
40     CONTINUE
       SD(I1)=S(I1,I1)
50     CONTINUE
       RETURN  $  END

       FUNCTION AIJ(S,I1,I2,IK)
       DIMENSION  S(120,120)
       IF(I2.GE.I1.AND.IK.EQ.1)  GO TO 1
       IF(I2.LE.I1.AND.IK.EQ.2)  GO TO 1
       GO TO 2
1      AIJ=S(I1,I2)  $  GO TO 3
2      AIJ=S(I2,I1)
3      RETURN  $  END

       SUBROUTINE MIV(A,U,NM)
       DIMENSION  A(6,6),U(6,6)
       DO 9001 I=1,NM
       DO 9001 J=1,NM
       U(I,J)=0.
       IF (I.EQ.J) U(I,J)=1.0
9001   CONTINUE
       EPS=0.0000001
       DO 9015 I=1,NM
       K=I
       IF (I-NM) 9021,9007,9021
9021   IF (A(I,I)-EPS) 9005,9006,9007
9005   IF(-A(I,I)-EPS) 9006,9006,9007
9006   K=K+1
       DO 9023 J=1,NM
       U(I,J)=U(I,J)+U(K,J)
9023   A(I,J)=A(I,J)+A(K,J)
       GO TO 9021
9007   DIV=A(I,I)
       DO 9009 J=1,NM
       U(I,J)=U(I,J)/DIV
9009   A(I,J)=A(I,J)/DIV
       DO 9015 MM=1,NM
       DELT=A(MM,I)
       IF (ABS(DELT)-EPS) 9015,9015,9016
9016   IF (MM-I) 9010,9015,9010
9010   DO 9011 J=1,NM
       U(MM,J)=U(MM,J)-U(I,J)*DELT
9011   A(MM,J)=A(MM,J)-A(I,J)*DELT
9015   CONTINUE
       RETURN
       END
```

APPENDIX B

Cushioning System Design

Cushioning System Design

The weight distribution of an M-37 3/4 ton truck with 1500 lb. simulated load of sandbags and the arrangement of cushioning forces are shown in Fig. B.1. In order to have uniformity in the crushing of the honeycomb cushioning, the centroid of the honeycomb area or the center of cushioning forces should be located at the center of gravity of the loaded vehicle. Wheels, differentials, gear reducer and transmission can be cushioned independently and the cushioning forces for those masses are calculated as:

(Design acceleration level 17.5 g)

Wheels and tires (each	$F_w = 18.5 \times 350 = 6480$ lb.
Differentials (each)	$F_{fd} = F_{rd} = 18.5 \times 480$ $= 8890$ lb.
Gear Reducer	$F_{gr} = 18.5 \times 300 = 5550$ lb.
Transmission	$F_{Tr} = 18.5 \times 200 = 3700$ lb.

The remainder of the structure is cushioned by F_1, $2F_2$ and $2F_4$ through the truck frame.

$\Sigma F = 0$ (Balance of cushioned forces with inertial forces)

$$F_1 + 2F_2 + 2F_4 = (1500 + 1060 + 570 + 600 + 300) \times 18.5$$

or

$$F_1 + 2F_2 + 2F_4 = 74{,}500 \qquad (B.1)$$

$\Sigma M_{CG} = 0$ (Sum of moments of cushion forces about CG)

$$2F_4 \times 80.5 + (2F_w + F_{rd}) \times 50.5$$

$$= 2F_2 + F_{Tr} \times 31 + (2F_w + F_{fd}) \times 63$$

$$+ 81 \times F_1 + 5 \times F_{gr}$$

or

$$161F_4 - 2F_2 - 81F_1 = 415{,}325 \qquad (B.2)$$

Since we have three unknowns, F_1, F_2, and F_4 we need one more constraint equation. This can be achieved by introducing an artificial hinge at the internal cushioned point F_2. If we take the moment about this point, we get:

$$2F_4 \times 81.5 = (1500 + 1060) \times 45 \times 18.5 \qquad (B.3)$$

Thus

$F_1 = 20{,}600$ lb.

$F_2 = 13{,}900$ lb.

$F_4 = 13{,}100$ lb.

The sizes of the pads required for the cushioning forces are calculated by:

$$A = \frac{F}{S}$$

where

A = required area of honeycomb under cushion force F.

S = average stress of honeycomb (6400 psf).

From a work energy balance, the stack height z required to provide the volume of honeycomb necessary to cushion the vehicle is determined from:

$$z = \frac{H}{Ge} = \frac{10 \times 12}{17.5 \times 0.7} = 9.7''$$

where

H = equivalent free drop height (10 ft.)

G = design acceleration level (17.5)

e = design strain

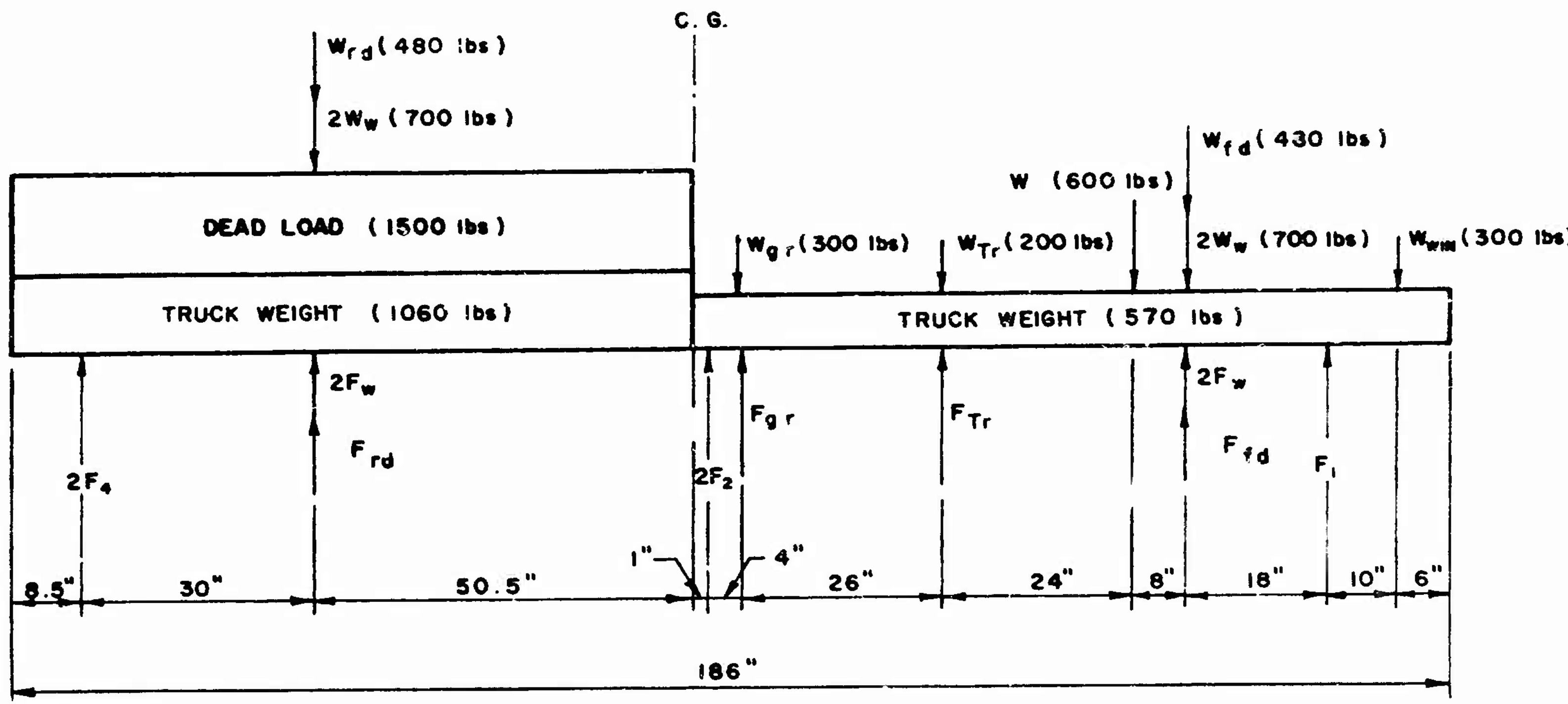

Fig. B.1 Weight Distribution of M-37 Truck.

REFERENCES

1. Wilfred E. Baker, Validity of Mathematical Models of Dynamic Response of Structures to Transient Loads, The Shock and Vibration Bulletin, Part 7, pp 19 - 28, 1970.
2. J. S. Przemieniecki, Theory of Matrix Structural Analysis, New York, McGraw-Hill Book Company, 1968.
3. C. M. Harris and C. E. Crede, Shock and Vibration Handbook, Chapter 35, New York, McGraw-Hill Book Company, 1961.
4. John Robinson, Structural Matrix Analysis for the Engineer, New York, John Wiley and Sons, Inc., 1966.
5. R. L. Ketter and S. P. Prawel, Modern Methods of Engineering Computation, New York, McGraw-Hill Book Company, 1969.
6. B. C. Ellis, E. A. Ripperger, and J. N. Thompson, Design of Cushioning Systems for Air Delivery of Equipment, Structural Mechanics Research Laboratory, The University of Texas at Austin, 1961, Contract DA 19-129-QM-1383, AD 270 241L.
7. Operation and Organizational Maintenance , 3/4 Ton, 4 x 4 Cargo Truck, M-37, TM 9-8030, May, 1955.