EXTENDED ARRAY EVALUATION PROGRAM.
SPECIAL REPORT NO. 1: INDIRECT ESTIMATES
OF SURFACE WAVE DETECTION PROBABILITIES

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INDIRECT ESTIMATES OF SURFACE WAVE DETECTION PROBABILITIES

SPECIAL REPORT NO. 1
EXTENDED ARRAY EVALUATION PROGRAM

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Previously reported techniques for basing indirect estimates of the surface wave detection probability are shown to yield results inconsistent with direct estimates when applied to data from the Alaskan Long Period Array (ALPA). A modification to these techniques is proposed. It is demonstrated that this modified procedure yields indirect estimates very close to those observed directly.
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SECTION I
INTRODUCTION

An item of primary interest in the monitoring of underground nuclear tests is the surface wave detectability of a given array or station. In the context of this report detection levels are set sufficiently high so that for every detected signal it is possible to make a reasonable estimate of the surface wave magnitude $M_s$ from the Rayleigh wave appearing on a vertical component long-period seismometer. The results reported, however, are equally applicable to measurements obtained with a horizontal component seismometer. The most accurate method for estimating detectability is the so-called direct method. This involves processing a large population of events and plotting the percentage of detections as a function of body wave magnitude $m_b$. The accumulation of a statistically meaningful population, in practice, necessitates data collection over an extended period of time.

An indirect method for estimating detectability has also been used (Lacoss, 1969 and Harley, 1971). The estimates are based on observed noise levels at the array or station in question. This technique is advantageous in that a shorter data collection period is required to provide reliable results. Perhaps more importantly, if one has some information about the noise levels at a proposed array location, this technique allows projection of the performance of the proposed array. This permits evaluation of the cost effectiveness of the proposed array.

This report concerns observed inaccuracies in the indirect method as compared to the direct method. In particular it has been noted that direct estimates of the 90% incremental detection threshold of the nineteen-element Alaskan Long-Period Array (ALPA) for Eurasian events
are near $m_b = 4.5$ for events in Central Asia and slightly lower for Kuriles-Kamchatka events (Heiting et al., 1972). Indirect estimates of the 90% level are at $m_b = 4.1$ for $\Delta = 60^\circ$ and at $m_b = 3.7$ for $\Delta = 35^\circ$. It is believed that these errors are a result of the manner in which estimates made as a function of $M_s$ are converted to estimates as a function of $m_b$. In the past a fixed relationship such as $M_s = 1.59m_b - 3.97$ has been used to make this conversion. Alternatively fixed relationships more appropriate to the array and magnitude range of interest have been used. In either case this type of conversion overlooks the fact that at any given value of $m_b$ a substantial scattering of $M_s$ values are observed. It will be demonstrated that more consistent results are obtained if the statistical nature of the $M_s : m_b$ relationship is taken into account. A technique identical in concept to that discussed here has been suggested by at least one other independent investigator (Lacoss, 1971).
SECTION II
DESCRIPTION OF TECHNIQUE AND RESULTS

The indirect method as it has been applied previously will be described briefly. Then the technique for incorporating the statistical nature of the $M_s : m_b$ relationship will be presented and its effects discussed.

A population of noise samples, believed to be representative of the ambient noise at the array or station, is accumulated. In the case of an array the noise samples are beamsteered for the area of interest and bandpass filtered with a filter chosen to enhance signals from the region of interest. At ALPA a beam direction of 340 degrees and an 0.025 to 0.055 Hz filter are used. The largest noise peak in each one-half hour noise sample is then measured. It is assumed that if a signal had been present with a peak-to-peak value at least three dB greater than that of the largest noise peak, it would have been detected. This then provides an ensemble of detectable signal amplitudes for the time period of the noise samples. This is converted to an ensemble of detectable surface wave magnitudes using the formula:

$$M_s = \log A / T + 1.66 \log \Delta$$

where:
- $A$ is the peak-to-peak value of the smallest detectable signal in millimicrons
- $T$ is the period - nominally assumed to be 25 seconds at ALPA
- $\Delta$ is the epicentral distance in degrees at which detectability is to be estimated.

The percentage of times that a given $M_s$ could be detected is then plotted
versus $M_s$ as shown in Figure II-1. These data were obtained from a population of 22 noise samples at ALPA covering the time period August, 1971 to March, 1972 and assume an epicentral distance of sixty degrees. The figure is interpreted as an estimate of the incremental surface wave detection probability expressed as a function of $M_s$.

It is customary, however, to express this probability as a function of $m_b$. To do so in the past, fixed $M_s : m_b$ relationships have been used such as $M_s = 1.59 m_b - 3.97$ (Lacoss, 1969). This relationship does not seem to fit the ALPA data very well. In the magnitude range $m_b = 4.5$ to 5.2 a better fit for the ALPA data is provided by the formula $M_s = 1.0 m_b - 1.1$. The 90% detection probability of Figure II-1 is at $M_s = 3.0$. Using the ALPA conversion, this corresponds to an $m_b$ of 4.1. Direct estimates of the ALPA 90% detection probability place it at about $m_b = 4.5$ for events in Central Asia (Heiting et al, 1972). This overly optimistic view provided by the indirect method when compared with the direct method has also been observed with Norwegian Seismic Array (NORSAR) data.

It is believed that this discrepancy is caused at least in part by the fact that use of the fixed $M_s : m_b$ conversion fails to account for the statistical nature of $M_s$ values at any given $m_b$. Observed values of $M_s$ at a given $m_b$ may lie anywhere within a range at least two $M_s$ units wide. In using the fixed conversion one is in effect saying, for example, that the curve of Figure II-1 indicates that events with $M_s = 2.9$ can be detected 75 percent of the time, that the fixed conversion $M_s = 1.0 m_b - 1.1$ shows that all events with $M_s = 2.9$ have $m_b = 4.0$, and that consequently events with $m_b = 4.0$ can be detected 75% of the time. In reality, however, if we assume that events with $m_b = 4.0$ are normally distributed about a mean value of $M_s = 2.9$, then those events which happen to have a larger $M_s$ will be more detectable while those that happen to have a smaller $M_s$ will be less detectable. Since the detection probability decreases more rapidly as $M_s$ decreases...
Assumes: 3 dB needed for detection
\[ \Delta = 60^\circ \]

**FIGURE II-1**
ALPA SURFACE WAVE DETECTION PROBABILITY
ESTIMATED FROM 22 NOISE SAMPLES
below 2.9 than it increases as $M_s$ increases above 2.9, the net result is that the true detection probability for $m_b = 4.0$ is less than 0.75. By the same argument detection probabilities for $m_b$ values corresponding to $M_s$ values below the point of inflection of Figure II-1 will be biased low if the fixed conversion is used. It is clear then that if the distribution of $M_s$ values is taken into account in converting to $m_b$, the detection probability curve will tend to flatten and the 90% detection level will occur at higher values of $m_b$.

At a given value of $m_b$ the probability that $M_s$ will lie within some small range is independent of the probability that an event with that $M_s$ can be detected. Therefore the joint probability that the event will fall in a particular range and that an event with that $M_s$ can be detected is simply the product of the two individual probabilities. The correct procedure is to sum up these joint probabilities over the range of conceivable $M_s$ values corresponding to the given value of $m_b$ as described below.

It is assumed that at a given $m_b = m$ the $M_s$ values are distributed normally about a mean value determined by the relationship $M_s = 1.0 m^{-1.1}$. Then the probability that the value of $M_s$ lies within a given range about a particular value $M_i$, given that $m_b$ has the value $m$, $P(M_i - \epsilon \leq M_s \leq M_i + \epsilon \mid m_b = m)$, is the difference between the cumulative normal distribution evaluated at the end points of this range. The probability that an $M_s$ lying in this range can be detected $P(M_i$ can be detected) is obtained from the incremental detection probability curve of Figure II-1. Finally, the probability than an event with $m_b = m$ can be detected is obtained as follows

$$P(m \text{ can be detected}) = \sum_i P(M_i - \epsilon \leq M_s \leq M_i + \epsilon \mid m_b = m) \cdot P(M_i \text{ can be detected})$$
where the range of $i$ is chosen to include all reasonable values of $M_s$.

This quantity is computed using $\epsilon = 0.025 M_s$ units and for discrete values of $m_b$ spaced at increments of $0.05 m_b$ units. Smooth curves are plotted through the computed points as seen in Figure II-2. The variable parameter in this plot $\sigma$ is the standard deviation of the assumed normal distribution of $M_s$.

A population of 106 ALPA events with $4.5 \leq m_b \leq 5.2$ have been partitioned into magnitude bins $0.1 m_b$ units wide. Within each bin the variance of $M_s$ was computed. As a result of the relatively small number of events within each bin, the variance differs considerably from bin to bin. To get a more stable estimate of the true variance in this range the variances within the bins were averaged giving $\sigma^2 = 0.18$ or a standard deviation of 0.43. Thus it appears that the curve of Figure II-2 most appropriate to ALPA is the $\sigma = 0.4$ curve. This curve indicates that at $m_b = 4.5$ the probability of detection is 0.9. This is in excellent agreement with the results of the direct method for events in Central Asia (Heiting et al, 1972).

Since the assumed $M_s : m_b$ relationship has a slope of one, the curves of Figure II-2 can be used for distances other than sixty degrees. One simply shifts them to the right by $1.66 \log (\Delta/60)$ units. This places the 90% detection probability for events 35 degrees from ALPA at $m_b = 4.1$. Direct estimates for the Kamchatka-Kuriles area place it at about $m_b = 4.2$. The events used for this estimate are 25 to 40 degrees distant from ALPA. It may be noted that the $340^\circ$ beam direction used for beamsteering these noise samples is more appropriate for Central Asia than for the Kamchatka-Kuriles area. In practice, however, little difference in output noise levels is observed between Central Asia and Kamchatka-Kuriles beams. Thus it is probably not necessary to form new beams for the Kamchatka-Kuriles estimate.
FIGURE II-2
INDIRECT SURFACE WAVE DETECTION PROBABILITY FOR EVENTS SIXTY DEGREES FROM ALPA
SECTION III
CONCLUSIONS

Indirect estimates of detection probability are made by inferring incremental surface wave detection probabilities from observed noise levels at an array or station. In the past fixed $M_s : m_b$ relationships have been used to express the probability as a function of $m_b$. It has been observed that the use of this procedure at ALPA and NORSAR leads to estimates of the 90% detection level which are about 0.5 $m_b$ units lower than directly obtained estimates.

The use of a fixed $M_s : m_b$ conversion overlooks the fact that at any given value of $m_b$, a wide range of $M_s$ values are normally observed. Thus it is necessary to use available information about the statistical nature of the $M_s : m_b$ relationship to express surface wave detection probabilities as a function of $m_b$. When this is done the indirect estimates are found to be in very close agreement with direct estimates. For events from Central Asia and from the Kuriles-Kamchatka area as observed at ALPA, the direct and indirect estimates of the 90% detection level agree within 0.1 $m_b$ units.

The availability of a reliable indirect method is a valuable asset. Noise measurements from 20 or 30 noise samples appear to provide sufficient information to yield reliable indirect estimates. Reliable direct estimates seem to require 100 or more events for a given region. Thus it is possible to obtain indirect estimates for a new or proposed array or station much more economically and faster than in the case of direct estimates. This presupposes that the distribution of $M_s$ values at a specific $m_b$ are not strongly dependent on station location so that the 0.4 standard deviation reported here can be validly applied at new locations.
SECTION IV
REFERENCES


Lacoss, R. T., 1971, NORSAR Conference, Oslo, Norway, November.