DETERMINATION OF AERODYNAMIC DRAG FROM RADAR DATA

Robert F. Lieske, et al

Ballistic Research Laboratories

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by

Robert F. Lieske
Antoinette M. MacKenzie

August 1972

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ABERDEEN PROVING GROUND, MARYLAND
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ABERDEEN PROVING GROUND, MARYLAND

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LIST OF SYMBOLS

- \( d \): reference diameter of projectile [m]
- \( g \): acceleration vector due to gravity [m/sec\(^2\)]
- \( g_0 \): acceleration due to gravity (surface) [m/sec\(^2\)]
- \( m \): weight of projectile [kg]
- \( r \): distance between center of earth and projectile [m]
- \( u \): velocity of projectile with respect to ground [m/sec]
- \( a_u \): acceleration of projectile with respect to ground [m/sec\(^2\)]
- \( v \): velocity of projectile with respect to air [m/sec]
- \( w \): velocity of air with respect to ground [m/sec]
- \( \text{Az} \): azimuth of line of fire (clockwise from north) [deg]
- \( C_D \): effective drag force coefficient
- \( L \): latitude of launch point [deg]
- \( R \): radius of earth [m]
- \( \mathbf{X} \): position of projectile with respect to ground fixed coordinate system [m]
- \( \Omega \): angular velocity of the earth [radians/sec]
- \( \mathbf{\Lambda} \): Coriolis acceleration due to rotation of the earth [m/sec\(^2\)]
- \( \rho \): air density as a function of height [kg/m\(^3\)]
- \( QE \): quadrant elevation [mils]
I. INTRODUCTION

The differential equations for the motion of a projectile are the primary tools used in the preparation of Firing Tables. These equations require two basic kinds of input data: aerodynamic coefficients (determined from aerodynamic spark ranges or wind tunnels) and ballistic data (obtained from large scale range firings). The possibility of obtaining both the aerodynamic coefficients and the ballistic data from a single large scale range firing has been discussed by those who prepare Firing Tables ever since the availability of tracking radars on many of our national firing ranges.

Of importance in utilizing radar data to obtain information for the preparation of Firing Tables are the determination of suitable radars and the development of suitable analysis methods. The Ballistic Research Laboratories has been investigating these areas using data supplied by the following radars at Wallops Island, Virginia: Spandar, FPS 16, MPS 19 and MOD II. For detailed descriptions of these radars see Reference 1.

This report describes a method of utilizing point position radar data in the determination of aerodynamic drag, the most important of the aerodynamic coefficients used in the preparation of Firing Tables. Also included are the results obtained when this method was applied to FPS 16 radar data from a firing of the 155mm M107 projectile.

II. TEST PROGRAM

A test program totaling 28 rounds, distributed as shown, was fired at Wallops Island to provide radar tracking data for a series of trajectories which included a typical sample of launch angles and initial

*References appear on page 31.
velocities. Two projectiles were fired at each of nine charge and quadrant elevation combinations. One projectile at each combination was fired on one day while the second projectile at each combination was fired on another day to obtain weapon, atmospheric and radar day to day variations. In addition, a single group of 10 rounds was fired, as rapidly as possible, at charge 7 and 700 mils quadrant elevation to provide a large amount of data on a single occasion under similar meteorological and test conditions.

### Distribution of Rounds Fired

<table>
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<th>700</th>
<th>1120</th>
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<td>Charge (No.) Muzzle Velocity (mps)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>235</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>372</td>
<td>2</td>
<td>2</td>
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<tr>
<td>7</td>
<td>564</td>
<td>2</td>
<td>2, 10</td>
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Four radars, the FPS 16, MPS 19, Spandar and MOD II, were used to track the test rounds, producing time-space histories of each projectile's flight. Additional data recorded for each round were weight and muzzle velocity, the latter being obtained by smear camera. Meteorological measurements, comprising winds and atmospheric conditions, were taken throughout the time of the firing.

### III. REDUCTION OF FLIGHT TEST DATA

At Wallops Island, radar data consisting of slant range, elevation angle and azimuth angle are recorded on digital tape. The first step in the actual radar reduction program is the application of the appropriate trigonometric transformations to convert the data to range, height and deflection referenced to the gun coordinates and firing azimuth. These values are then smoothed, employing for the purpose a moving arc
technique that uses the method of least squares to fit a second-degree polynomial to the data in every 7.5 seconds span along the trajectory. The polynomial is evaluated at the midpoint of each section, and the first and second derivatives are computed at this time to obtain the velocity ($\vec{u}$) and acceleration ($\vec{u}'$) vectors.

IV. COMPUTATION OF DRAG COEFFICIENT ($C_D$)

The frame of reference for all vectors to be presented is a ground-fixed, right-handed Cartesian coordinate system with unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$). Define the $\vec{j}$ axis to be parallel to the vector $\vec{g}$ at the origin and in the opposite direction.

The mass of the projectile, the atmospheric conditions existing at the time of the firing and reduced radar data for the round provide the necessary inputs to the equation for determination of the drag coefficient ($C_D$). $C_D$ is obtained by an inverse solution of the point-mass differential equations of motion.

$$C_D = -\left[ (u_1 - w_1)(\dot{u}_1 - g_1 - \Lambda_1) + u_2 (\dot{u}_2 - g_2 - \Lambda_2) + (u_3 - w_3) \right. \\
\left. (\dot{u}_3 - g_3 - \Lambda_3) \right] \frac{8m/\pi \rho d^2 v^3}{m/\pi \rho d^2 v^3}
$$

The acceleration due to gravity ($\vec{g}$) is

$$\vec{g} = (-g, \frac{R^2}{r^3}) = \left[ \begin{array}{c} X_1 \\ X_2 + R \\ X_3 \end{array} \right]
$$

where $r = \sqrt{X_1^2 + (X_2 + R)^2 + X_3^2}$.  

The Coriolis acceleration ($\vec{\Lambda}$) due to rotation of the earth is expressed by

$$\vec{\Lambda} = \begin{bmatrix} -\lambda_1 u_2 - \lambda_2 v_3 \\ \lambda_1 u_1 + \lambda_3 u_3 \\ \lambda_2 u_1 - \lambda_3 u_2 \end{bmatrix}
$$
For the northern hemisphere the $\lambda$'s are defined by the following equations (for the southern hemisphere replace $L$ by $-L$).

$$\begin{align*}
\lambda_1 &= 2 \Omega \cos L \sin \text{Az} \\
\lambda_2 &= 2 \Omega \sin L \\
\lambda_3 &= 2 \Omega \cos L \cos \text{Az}
\end{align*}$$

V. RESULTS

For the purpose of this report the FPS 16 was the only radar that produced acceptable results over the Mach number region of interest. Spandar locked-on very late in the flights it tracked, yielding too few data for a complete analysis. The MPS 19 and MOD II which are the least precise radars gave inadequate results using this method of analysis.

All of the data collected by the FPS 16 radar were reduced and used in the computation of aerodynamic drag. The resultant drag coefficients have been grouped according to various combinations of quadrant elevation and charge and are presented in graphical form as a function of Mach number. Values lying outside the limits of any one graph are plotted along the edges and bear the following identification. Figure 1 is a plot of the drag coefficients obtained for all charges and quadrant elevations. As shown by the graph, the curve formed by these computed values closely follows the aerodynamic spark range curve representing the expected performance of the M107 round.

The increase in drag noticeable in the lower Mach number region of the plot is probably due either to a combination of the yaw of repose and the precessional and nutational yawing motion or to the yaw of repose alone. Figure 2 shows the subsonic values of the drag coefficient for Mach numbers less than 0.8. The drag coefficient is plotted against
computed values of the yaw of repose squared as defined in the modified point mass trajectory model. To determine the change in drag due to yaw a least squares procedure was used to fit these data as a linear function of yaw squared. The resulting curve also shows good agreement with the aerodynamic spark range curve.

Figures 3, 4 and 5 are plots of the aerodynamic drag coefficients found at each of the quadrant elevations fired, irrespective of charge. Figures 3 and 4 appear to indicate that at the lower elevations there is little if any increase in drag due to the yaw of repose, while at the highest elevation, Figure 5, the increase is considerable. The remaining Figures 6 through 12 present the drag coefficients at each elevation and charge. Figures 6 and 7 are plots for a quadrant elevation of 300 mils at charges 5 and 7. In Figures 8, 9 and 10, quadrant elevation is 700 mils at charges 2, 5 and 7. Finally, Figures 11 and 12 have a quadrant elevation of 1120 mils at charges 5 and 7. The three plots at charge 7 are in good agreement, and the three plots at charge 5 with only the highest quadrant elevation having a large amount of data also indicate good agreement.

Figures 13 and 14 are plots of the drag coefficients calculated for all charges and quadrant elevations for the MPS 19 and MOD II radars, respectively. Both graphs indicate that meaningful results cannot be obtained from these radar data by the method discussed in this report.
VI. CONCLUSION

On the basis of the above study it has been established that it is feasible by a polynomial smoothing and differentiation method to obtain aerodynamic drag force values from flight test data recorded by a radar with the precision of the FPS 16. Assuming a computed yaw of repose as defined in the modified point mass trajectory model, the aerodynamic change in drag due to yaw can also be determined. Also, it can be concluded that, on the average, the drag results obtained from radar data agree well with the drag results from ballistic range firings.
Figure 2. Subsonic Drag Coefficient ($C_D$) vs Yaw Squared, for All Charges and QE's.
Figure 4. Aerodynamic Drag Coefficient ($C_D$) vs Mach No., for Charges 2, 5 and 7, OE = 700 miles
Figure 5. Aerodynamic Drag Coefficient ($C_D$) vs Mach No.,
for Charges 5 and 7, $QE = 1120$ mls
Figure 8. Aerodynamic Drag Coefficient ($C_D$) vs Mach No., for Charge 2. QE = 700 mls
Figure 9. Aerodynamic Drag Coefficient ($C_D$) vs Mach No.,
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Figure 11. Aerodynamic Drag Coefficient ($C_D$) vs Mach No.,
for Charge 5, QE = 1120 mils
Figure 12. Aerodynamic Drag Coefficient ($C_D$) vs Mach No.,
for Charge 7, QE = 1120 mils
REFERENCES

1. Wallops Station, *Radar Information Handbook*, NASA.
