AD-750 545

OPTICAL BIREFRINGENCE MEASUREMENT BY MEANS OF A ROTATING ANALYZER WITH APPLICATION TO KERR EFFECT

G. B. Thurston, et al

Texas University

Prepared for:

Air Office of Scientific Research

March 1972

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UNCLASSIFIED
Security Classification

AD 750545

DOCUMENT CONTROL DATA - R&D
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
The University of Texas at Austin
Electronics Research Center
Austin, Texas 78712

2a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

2b. GROUP

3. REPORT TITLE
OPTICAL BIREFRINGENCE MEASUREMENT BY MEANS OF A ROTATING ANALYZER WITH APPLICATION TO KERR EFFECT

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Scientific ---------- Interim

5. AUTHOR(S) (Last name, first name, initial)
Thurston, G. B., and Wilkinson, R. S.

6. REPORT DATE
15 March 1972

7a. TOTAL NO. OF PAGES
30

7b. NO. OF REFS
15

8a. CONTRACT OR GRANT NO.
F44620-71-C-0091

b. PROJECT NO.
4751

c. 61102F

d. 681305

9a. ORIGINATOR'S REPORT NUMBER(S)
Technical Report No.122

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
AFOSR-TR-72-0732

10. AVAILABILITY/LIMITATION NOTICES
Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES
TECH, OTHER

12. SPONSORING MILITARY ACTIVITY JSEP through
AF Office of Scientific Research (NE)
1400 Wilson Blvd., Arlington, VA 22209

13. ABSTRACT

A rotating analyzer method has been developed for rapid measurement of the ellipticity of polarized light as produced by optically birefringent materials. A plane optical analyzer, in the form of a Glan-Thompson prism, is placed in the light path ahead of an electrical photodetector and is continuously rotated on an axis coincident with the light path. For light having either constant or slowly varying ellipticity, this leads sinusoidally time varying electrical signals which bear a simple relationship to the eccentricity and orientation of the ellipticity. In application to the Kerr effect, the ellipticity results from passage of circularly polarized light through the Kerr cell. For sinusoidally time varying electric fields applied to the Kerr cell, the optical retardation of the cell contains both steady and alternating components. These components are separated in the frequency structure of the electrical photoresponse. In this application the method may be used to discriminate against effects of light intensity fluctuations and small residual birefringence in windows. This permits measurement of the Kerr effect in materials having very small Kerr constants.

Ia

DD FORM 1473

OPTICAL BIREFRINGENCE MEASUREMENT BY MEANS OF A ROTATING ANALYZER WITH APPLICATION TO KERR EFFECT

By

G.B. Thurston and R. S. Wilkinson
Departments of Mechanical Engineering and Physics

Technical Report No. 122
March 15, 1972

BIOMEDICAL ELECTRONICS RESEARCH LABORATORY

ELECTRONICS RESEARCH CENTER

THE UNIVERSITY OF TEXAS AT AUSTIN

Austin, Texas 78712

*Research sponsored in part by the Joint Services Electronics Program under Research Contract F44620-71-C-0091, and in part by National Science Foundation Grant No. 22797.

Approved for public release; distribution unlimited.

Ib

ABSTRACT

A rotating analyzer method has been developed for rapid measurement of the ellipticity of polarized light as produced by optically birefringent materials. A plane optical analyzer, in the form of a Glan-Thompson prism, is placed in the light path ahead of an electrical photodetector and is continuously rotated on an axis coincident with the light path. For light having either constant or slowly varying ellipticity, this leads to sinusoidally time varying electrical signals which bear a simple relationship to the eccentricity and orientation of the ellipticity. In application to the Kerr effect, the ellipticity results from passage of circularly polarized light through the Kerr cell. For sinusoidally time varying electric fields applied to the Kerr cell, the optical retardation of the cell contains both steady and alternating components. These components are separated in the frequency structure of the electrical photoresponse. In this application the method may be used to discriminate against effects of light intensity fluctuations and small residual birefringence in windows. This permits measurement of the Kerr effect in materials having very small Kerr constants.

TABLE OF CONTENTS

	Page
ABSTRACT	ii
LIST OF FIGURES	iii
INTRODUCTION	1
PRINCIPLE OF MEASUREMENT	2
THEORY OF APPLICATION TO KERR ELECTRO-OPTIC SYSTEM	8
APPARATUS DESCRIPTION AND KERR EFFECT MEASUREMENTS	13
CONCLUSIONS	25
REFERENCES	27

Introduction

The optical birefringence of a material sample is usually characterized by the orientation angle of the optical axes and the magnitude of the difference between the optical indices of refraction for these axes. If the birefringence is constant in time, then measurements of these properties may be accomplished by a variety of schemes in which visual null indications or intensity matching methods are employed [1,2,3,4]. These methods generally require precise mechanical orientation settings of polarizing and analyzing elements for each measurement. While high precision may be achieved, such methods are not suitable for rapid measurement as required for time-dependent, dynamic birefringence.

Time-dependent optical birefringence occurs in systems designed for analysis of kinetic processes. These include the streaming birefringence in a Couette system when flow is suddenly stopped [5], dynamic birefringence of a thin fluid layer is oscillatory and stopped flow [6], strain induced birefringence in solids [7] and electro-optical systems for measurement of the Kerr effect for oscillatory fields [8,9]. Some methods which are suitable for dynamic birefringence have been described [10] for the special case of fixed optical axis orientation. The need then remains for a method of simultaneous and continuous measurement of both magnitude of birefringence and optical axis orientation for dynamic conditions.

A new method of measurement of optical birefringence has now been developed which utilizes a continuously rotating plane optical analyzer followed by a suitable electro-optical detector. The resulting electrical signal may be rapidly analyzed by standard techniques of electrical measurement to give instantaneous indications of both the magnitude of birefringence and optical axis orientation. This method also gives a high degree of descrimination against complications due to variations in background optical intensity. These features have been examined in depth as they apply to measurements in a Kerr system with oscillatory electric fields.

Principle of Measurement

The principle of measurement is illustrated in Figure 1. Monochromatic circularly polarized light of wavelength λ is transmitted through the birefringence to be measured. The orientation of the optical axes m and n with respect to a reference direction is specified by the angle Ψ. Right circular light becomes elliptically polarized on transmission through the birefringent plate, of thickness L. The elliptically polarized light then passes through a plane analyzer which is mechanically rotating at constant angular velocity, Ω, and is finally received by an electro-optic photodetector such as a photomultiplier tube. In the event the birefringence is zero, the voltage out of the photomultiplier is constant as the rotating analyzer revolves over the contour of the circular light. With birefringence

and elliptical light the photovoltage becomes sinusoidally time varying, producing two cycles for each revolution of the analyzer. As the analyzer direction coincides with one of the optical axes the photovoltage is instantaneously at the level for circular light. At orientations 45 degrees to the optical axes the photovoltage reaches its maximum or minimum value.

An optical system of this general configuration has been previously analyzed [10] for right circular light incident upon a plane analyzer having a fixed orientation angle θ_A. If this analyzer is rotating, $\theta_A = \Omega t$, and the photodetector output voltage is

$$e = e_o [1 - \sin\delta \sin 2(\theta_A - \psi)] \tag{1a}$$

$$= e_o [1 + \sin\delta \sin 2(\Omega t + \psi)] \tag{1b}$$

where

$$\delta = (2\pi/\lambda)\, \Delta n\, L \tag{2}$$

where δ is the optical retardation and Δn is the difference between the optical indices in the n and m directions,

$$\Delta n = n_n - n_m. \tag{3}$$

It is seen that if δ is a constant, then this represents a signal at a radian frequency 2Ω, phase 2ψ, and of amplitude $e_{max} = e_o \sin\delta$ superimposed upon a constant voltage e_o. For small retardations the approximation $\sin\delta \cong \delta$ holds and the birefringence is given by

$$\Delta n \approx (\lambda/2\pi L)(e_{max}/e_o). \qquad (4)$$

Thus, from measurement of e_{max} and e_o the $(\sin \delta = e_{max}/e_o)$ may be determined, and for small retardations, the birefringence, Δn, is given by (4). The angle Ψ specifying the orientation of the optical axes is given by the phasing of the alternating component, the time reference for this being established by the crossing of the analyzer axis with the reference direction as shown in Figure 1.

The principle of measurement is illustrated in Figure 2. The figure is derived from a cathode ray oscillograph presentation using apparatus to be described subsequently. In this case a quarter-waveplate, $\delta = \pi/2$ radians, was inserted in the light path and the sine wave recorded. Also shown is a simultaneously recorded pulse, event r, which marks the coincidence of the analyzer direction with the reference direction. The horizontal trace at the level e_o is the background for circular light as obtained with the waveplate removed. The events n and m correspond to coincidences of the analyzer direction with the optical axes n and m respectively. The angle Ψ is the angle between the event r for which $\Omega t = 0$ and the event n for which $\Omega t = -\Psi$. A waveplate having accurately known positions of fast and slow axes is convenient for use in instrument calibration of both Ψ and the sign of the birefringence.

The features of the voltage traces of Figure 2, e_{max} and Ψ may be continuously monitored to follow changes in retardation δ and optical axis orientation versus time. The time resolution

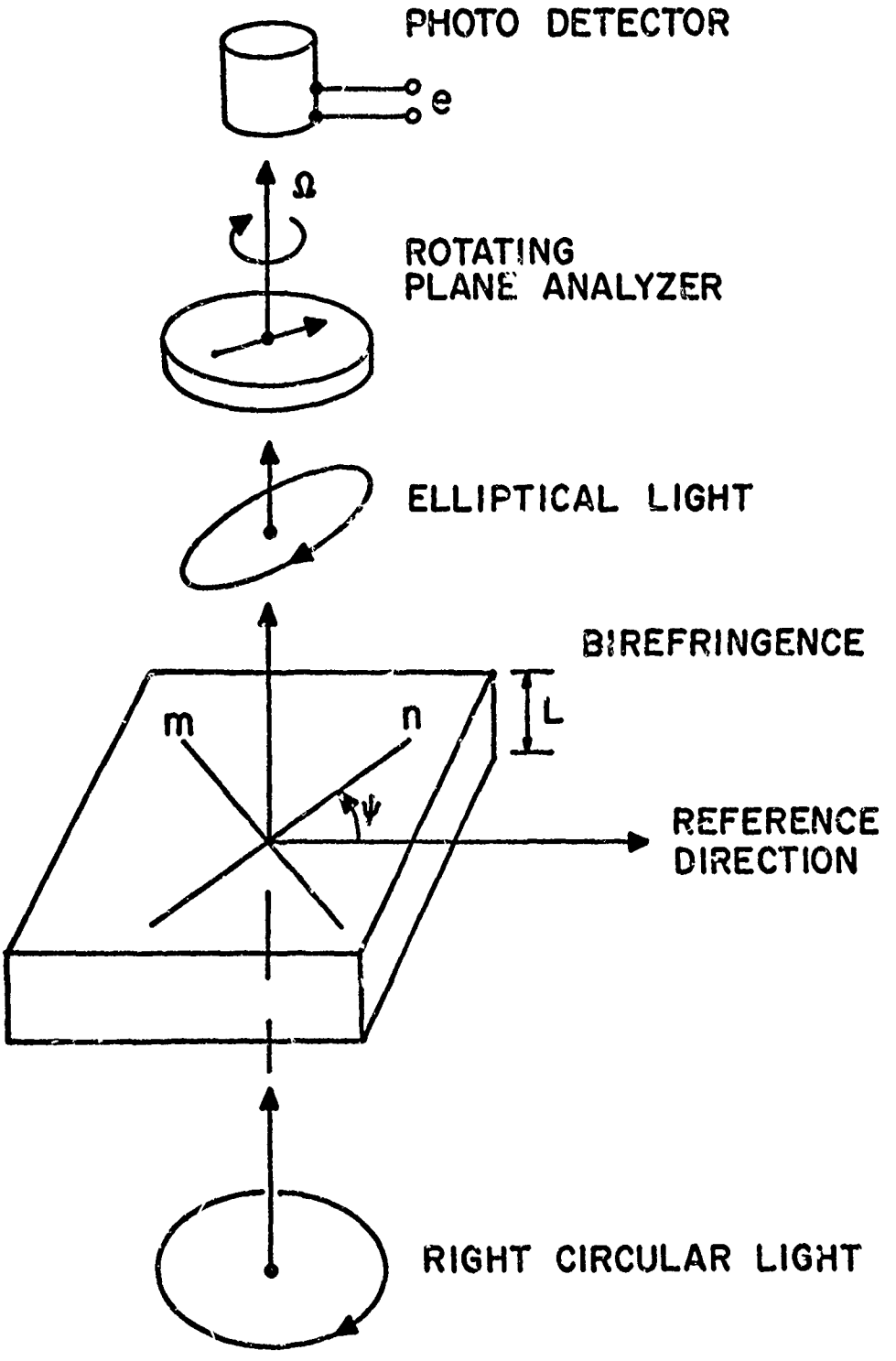

Figure 1: Optical system on which the principle of measurement is based.

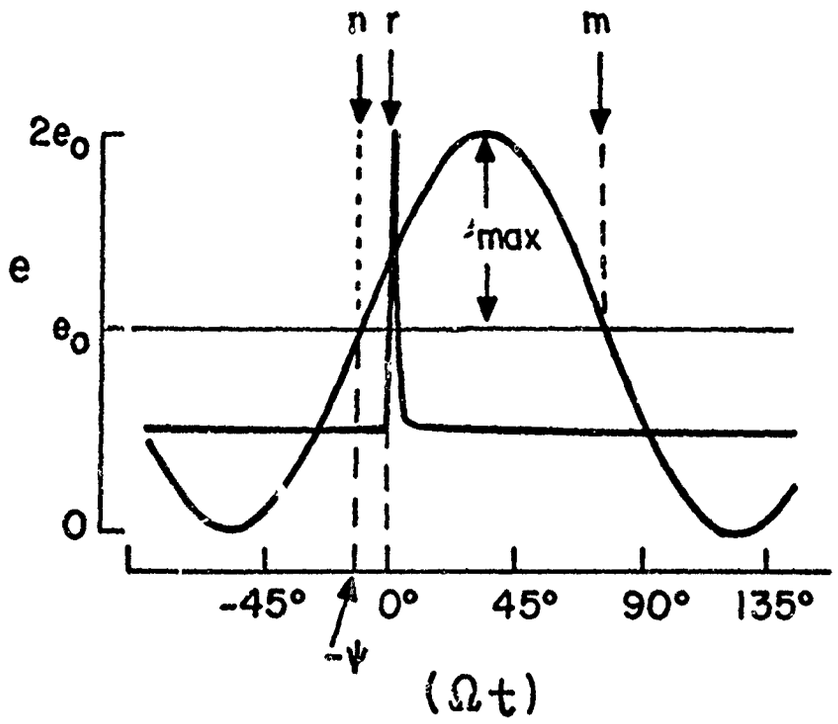

Figure 2: Photovoltage e versus analyzer rotation angle, (Ωt) for a quarter waveplate inserted as the birefringence. Events n, r, and m mark the coincidence of the analyzer direction with the n optical axis, the reference direction, and the m optical axis respectively.

attainable is dependent upon the period of rotation of the analyzer and the response time characteristics of the instrumentation used. With more commonly used analog instruments which require several cycles of signal to develop a reading, the response time may be of the order of one second. Correspondingly this limits the measurement to slowly varying birefringences.

The rotating analyzer method is useful in its tendency to minimize the effects of drift in background light level upon the measurement of retardation. With a conventional fixed analyzer, the photoresponse is given by (1a), letting $(\theta_A - \Psi) = \pi/4$. In this case a background fluctuation, in amount Δe_o, appears as an additive to the light intensity change due to birefringence, $e_o \sin \delta$. With the rotating analyzer for which (1b) applies, measurement of the alternating component of the photoresponse at the frequency 2Ω has an amplitude $e_o \sin \delta$ with an additional component due to background drift, $\Delta e_o \sin \delta$. For small retardations, $\sin \delta$ is small, and thus the influence of the background drift on the measurement is small while with the fixed analyzer the relative effect of drift becomes enhanced.

Theory of Application to Kerr Electro-optic System

The rotating analyzer system is useful for analysis of birefringence due to a Kerr electro-optic cell for sinusoidally time-varying electric fields. By this method the sensitivity and precision of measurement may be improved and some effects due to optical absorption in the cell may be delineated.

The Kerr cell occupies the position of the birefringence shown in Figure 1, the cell path length being L. Let the electric field be directed along the n axis. The form of the optical birefringence of the cell has been previously described [8,9] as containing a steady component plus an alternating component at twice the frequency of the electric field. Using eq. (2), this birefringence gives an optical retardation

$$\delta = \delta_{st} + \delta_{alt} \cos(2\omega t - \theta) \tag{5}$$

where δ_{st} is the steady component of the retardation, δ_{alt} is the magnitude of the alternating component, and θ is the phase, these factors being characteristic of the material filling the Kerr cell. The radian frequency of the electric field is ω.

The photovoltage response for the retardation [5] is found by substitution into (1). This gives

$$e = e_o + e_c \sin[\delta_{st} + \delta_{alt} \cos(2\omega t - \theta)] \sin(2\Omega t + 2\psi). \tag{6}$$

The character of the Kerr effect is that δ is generally proportional to the square of the electric field in the material for moderate

field strengths and it vanishes as the field vanishes in which case the photoresponse becomes e_o. With $\delta \neq 0$, the photoresponse is seen in (6) to contain the sinusoidal term at the radian frequency 2Ω, but with amplitude being also time dependent at the frequency 2ω. This leads to a spectrum of frequency components. The exact analysis of these components is not carried out here since the primary concern is with smaller retardations.

Equation (6) may be rewritten as

$$e = e_o + e_o\{\sin\delta_{st}\cos[\delta_{alt}\cos(2\omega t - \theta)] + \cos\delta_{st}\sin[\delta_{alt}\cos(2\omega t - \theta)]\}\sin(2\Omega t + 2\psi). \tag{7}$$

The approximation to smaller retardation here centers upon the δ_{alt}, which is generally less than δ_{st} [8,9]. As a first step in the approximation the square and cube terms in the Taylor series expansion of the cosine and sine functions of δ_{alt} are employed. In this case, (7) becomes for $\delta_{alt} \lesssim 0.5$ rad.

$$\begin{aligned} e \cong e_o + e_o\{&\sin\delta_{st}(1 - \delta_{alt}^2/4) \\ &+ \cos\delta_{st}(\delta_{alt} - \delta_{alt}^3/8)\cos(2\omega t - \theta) \\ &- (1/4)\sin\delta_{st}\,\delta_{alt}^2\cos(4\omega t - 2\theta) \\ &- (1/24)\cos\delta_{st}\,\delta_{alt}^3\cos(6\omega t - 3\theta)\}\sin(2\Omega t + 2\psi). \end{aligned} \tag{8}$$

Further expansion of (8) leads to seven frequency components plus the constant term e_o. Each of the components may be expressed in the form $A \sin(\nu t + \phi)$. Table I gives the resulting component descrptions.

Table I

Amplitude A and phase ϕ of alternating components of the photoresponse for $\delta_{alt} \lesssim 0.5$ rad.

ν rad/sec	A	ϕ
2Ω	$e_o \sin \delta_{st}(1 - \delta_{alt}^2/4)$	2Ψ
$2\Omega \pm 2\omega$	$(1/2)e_o \cos \delta_{st}(\delta_{alt} - \delta_{alt}^3/8)$	$2\Psi \mp \theta$
$2\Omega \pm 4\omega$	$(1/8)e_o \sin \delta_{st}\, \delta_{alt}^2$	$2\Psi \mp 2\theta + \pi$
$2\Omega \pm 6\omega$	$(1/48)e_o \cos \delta_{st}\, \delta_{alt}^3$	$2\Psi \mp 3\theta + \pi$

A still more stringent approximation to small retardations is the case where $\sin \delta \approx \delta$ where δ is as defined in (5). Equation (6) then leads immediately to the photoresponse for $\delta \lesssim 0.2$ rad,

$$\begin{aligned} e \cong e_o &+ e_o \delta_{st} \sin[2\Omega t + 2\psi] \\ &+ (1/2)\, e_o \delta_{alt} \{\sin[2(\Omega + \omega)t + 2\psi - \theta] \\ &+ \sin[2(\Omega - \omega)t + 2\Omega + \theta]\}. \end{aligned} \tag{9}$$

In this case the photoresponse is seen to contain only the constant component e_o plus three alternating components at the radian frequencies 2Ω, $2(\Omega+\omega)$, and $2(\Omega-\omega)$. The amplitude of the

2Ω component is determined by δ_{st} while the $2(\Omega \pm \omega)$ components are determined by δ_{alt}. Thus in this limiting case the effects of the retardations δ_{st} and δ_{alt} are simply separated in the frequency domain.

It is found that the background illumination of circularly polarized light transmitted by a macromolecular solution in a Kerr cell may change on application of the electric field. This is observed with the optical analyzer removed from the apparatus and thus is not due to birefringence or dichroism. The background change generally consist of a small reduction of the steady component of the intensity with an additional modulation at twice the frequency of the electric field. In some cases a smaller component at the field frequency is also observed. Similar observations have been reported for colloids [11] and liquid crystals [12].

To a first approximation the influence of these light modulation effects on the photoresponse may be described by replacing the constant e_o by a field dependent e_o', where

$$e_o' = e_o + \tilde{e} \cos(2\omega t + \chi) \tag{10}$$

With the electric field zero, $\tilde{e}$ becomes zero, and e_b becomes e_o.

The influence of this modulation on the photoresponse with the rotating analyzer system may now be determined by substitution of (10) into any of the foregoing equations for e. The simplest case is that of (9) for small total retardation. This leads to contributions to the photoresponse at six frequencies. These are given in Table II.

Table II

Amplitude A and phase ϕ of alternating components at the radian frequency ν to the photoresponse for the Kerr effect at small retardations (9) and with light modulation (10).

(rad/sec)	A	ϕ
2ω	$\tilde{e}$	χ
2Ω	$e_b\ \delta_{st}$	2Ψ
	$\tilde{e}\ \delta_{alt}/4$	$2\Psi-\theta-\chi$
	$\tilde{e}\ \delta_{alt}/4$	$2\Psi+\theta+\chi$
$2(\Omega+\omega)$	$e_b\ \delta_{alt}/2$	$2\Psi-\theta$
	$\tilde{e}\ \delta_{st}/2$	$2\Psi+\chi$
$2(\Omega-\omega)$	$e_b\ \delta_{alt}/2$	$2\Psi+\theta$
	$\tilde{e}\ \delta_{st}/2$	$2\Psi-\chi$
$2(\Omega+2\omega)$	$\tilde{e}\ \delta_{alt}/4$	$2\Psi-\theta+\chi$
$2(\Omega-2\omega)$	$\tilde{e}\ \delta_{alt}/4$	$2\Psi+\theta-\chi$

Comparing these frequency components with those due to finite retardations given in Table I shows that both effects contribute to some of the same components. In the solution tested in the Kerr cell as to be described subsequently, it is found that $\tilde{e}$, (e_o-e_b), δ_{alt}, and δ_{st} all are dominently proportional to the square of the electric field strength. However, the contributions

due to finite retardations are an order of magnitude more dominent than the modulation contributions. Hence, this modulation is best measured by means other than the rotating analyzer, for example by removing the analyzer from the light path.

Mention should be made of the possibility of dichroism being induced in the Kerr cell on application of the electric field. With the rotating analyzer system as described above, this could lead to an additional signal which is not accounted for in the analysis. The presence of dichroism can be detected by use of plane polarized light [11,14] oriented alternately parallel and then perpendicular to the electric field, or by use of a continuously rotating plane polarized light [15].

Apparatus Description and Kerr Effect Measurements

The schematic arrangement of elements of the optical system for production of circularly polarized light and subsequent analysis for ellipticity by the rotating optical analyzer is shown in Fig. 3. Collimated, monochromatic light is passed through a fixed Glan-Thompson polarizing prism and two quarter waveplates which are mounted to permit precise rotational adjustment. The perfection of the circularity of the light must be much better than the ellipticity to be introduced by the birefringence to be measured. It can be shown that by proper adjustment of the orientation of the two waveplates, circularly polarized light is obtained [13]. The perfected right circular light then passes through the birefringent element to be measured,

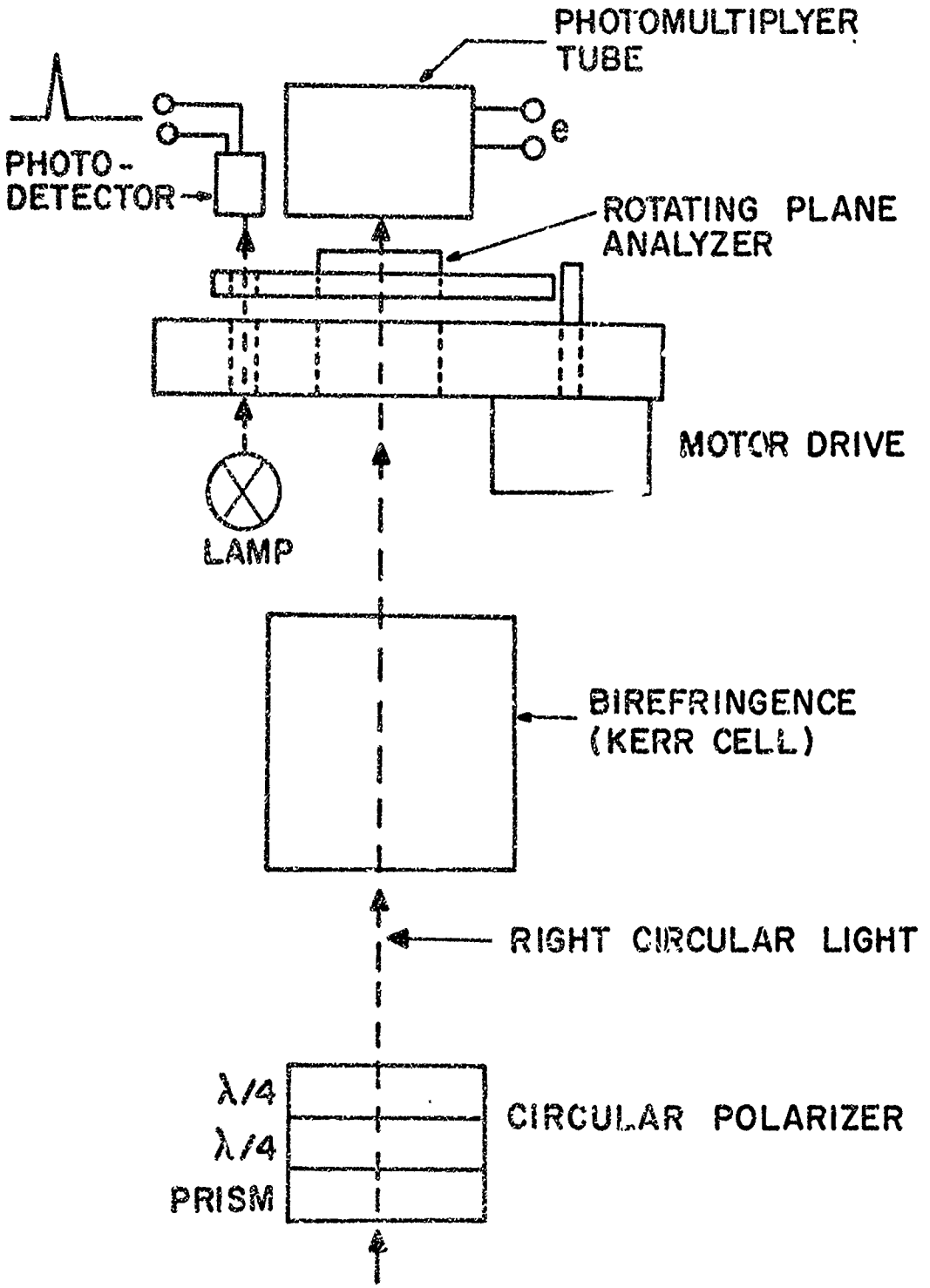

Figure 3: Schematic arrangement for the production of circularly polarized light, and analysis of polarization ellipticity produced by a birefringence. Analysis is by the rotating optical analyzer.

perhaps a Kerr cell, and then through a rotating optical analyzer, a Glan-Thompson prism. The prism is rotated by a motor drive at 41 revolutions/sec. A small a.c. induction motor is coupled by a rubber belt to the precision ball bearing mounted prism. A slotted disk which rotates with the analyzer is used to chop a separate auxiliary light beam. This beam is sensed by a small solid state detector. The resulting pulse serves to mark the angular position of the analyzer. The principal light beam is finally detected by a photomultiplier after transmission through the rotating optical analyzer. Using the auxiliary light pulse as a phase reference, the photovoltage from the photomultiplier may be analyzed.

As the system is applied to Kerr effect measurements, other features of the apparatus are of note. The light source used is a low voltage tungsten filament bulb with internal reflector. This is focused on a small aperture, approximately 0.7 mm square, the image of which is formed in the optical region of the Kerr cell by a converging lens. An optical interference filter, 5790 Å, is inserted in the light beam just ahead of the photodetector. Aperture stops are also located at the position of the converging lens and between the Kerr cell and the rotating analyzer. These stops are adjusted to restrict the light reaching the photomultiplier to that passing through the Kerr cell without interference with the cell walls. The optical path in the cell is 8.0 cm and the cell cross section is 0.32 cm square.

The preparation of the system for measurements consists of adjustment of the waveplates with the birefringence removed to achieve perfect circularity of the light as indicated by a null photovoltage at the radian frequency 2Ω. In this case the frequency is 82 Hz. For Kerr measurements this is carried out with the Kerr cell in place but with no applied electric field. A small adjustment of the position of the 3/16 in. diameter aperture ahead of the rotating analyzer helps to perfect this balance. It is found that this null is very sensitive to the voltage at the tungsten source as this may affect its spectral emission characteristics. This voltage is regulated to limit variations to approximately 1 mv in the 6 v.d.c. applied to the lamp.

The character of the background null was examined using a Hewlett Packard Model 302 wavemeter coupled to an automatic frequency sweep ranging from 15 Hz to 180 Hz. The analyzer output was simultaneously graphed on an x y pen recorder. The photomultiplier output was fed directly into the wavemeter. This was done with a Kerr cell in place, with no applied voltage. The material filling the cell was an aqueous solution of bentonite at a concentration of 3.55 mg/ml. After tracing the background, an electric field of 1.24 V/cm rms at a frequency $\omega/2\pi$ = 15 Hz is applied and the photovoltage spectrum is again traced. These two superimposed traces are shown in Fig. 4. The background trace (dashed curve) shows small residual components at 41 Hz, the rotation frequency of the analyzer, and at the ac power line

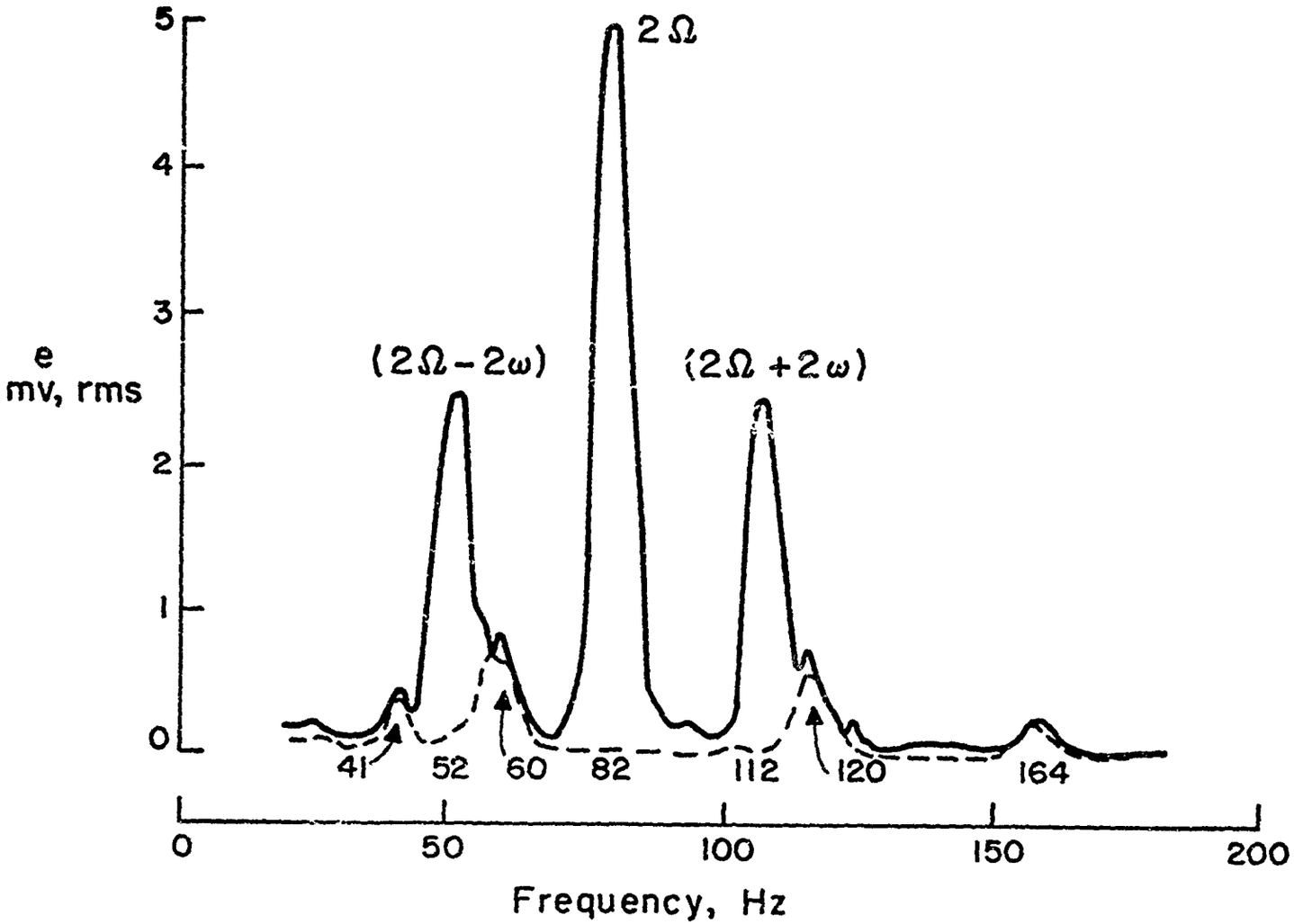

Figure 4: Frequency spectrum for the Kerr effect of a bentonite solution. The background (dashed curve) is for the field off in the Kerr cell while the solid curve is for an electric field of 1.24 v/cm rms at 15 Hz.

dependent frequencies 60 Hz and 120 Hz. With the applied field three prominent peaks appear which correspond to the radian frequencies 2Ω and $2\Omega \pm 2\omega$. The frequencies are thus at 82 Hz, 112 Hz and 52 Hz. The instrument sensitivity is such that insertion of a quarter waveplate gives a photovoltage of 620 mv rms at 82 Hz. For these small retardations equation (9) is applicable and the central peak is of amplitude $e_o\delta_{st}$ (peak volts) and the two other peaks are $(1/2)e_o\ \delta_{alt}$, where $e_o = 620 \times 1.414 = 877$ mv. So, here $\delta_{st} = .0082$ rad. and $\delta_{alt} = .0074$ rad.

If the field strength is increased larger retardation effects become evident and other frequency components are found as in Table I. The results for a field strength of 35 v/cm rms at 30 Hz is shown in Fig. 5. Here it is seen that in addition to the three components of Fig. 3, other components now begin to appear at 38 and 202 Hz corresponding to $(2\Omega \pm 4\omega)$ and at 98 Hz corresponding to $(2\Omega - 6\omega)$. Note that in determining the frequency components a negative frequency simply indicates a phase shift.

In application to measurement of the steady component of the retardation due to the Kerr effect, attention is placed upon the analysis of the 2Ω component at 82 Hz. For this measurement, best results have been obtained using a phase sensitive detector (Princeton Applied Research Model HR8). For this, the disk rotating with the analyzer is designed to give an 82 Hz square wave using the auxiliary light source and photodector. This signal then serves to activate the reference

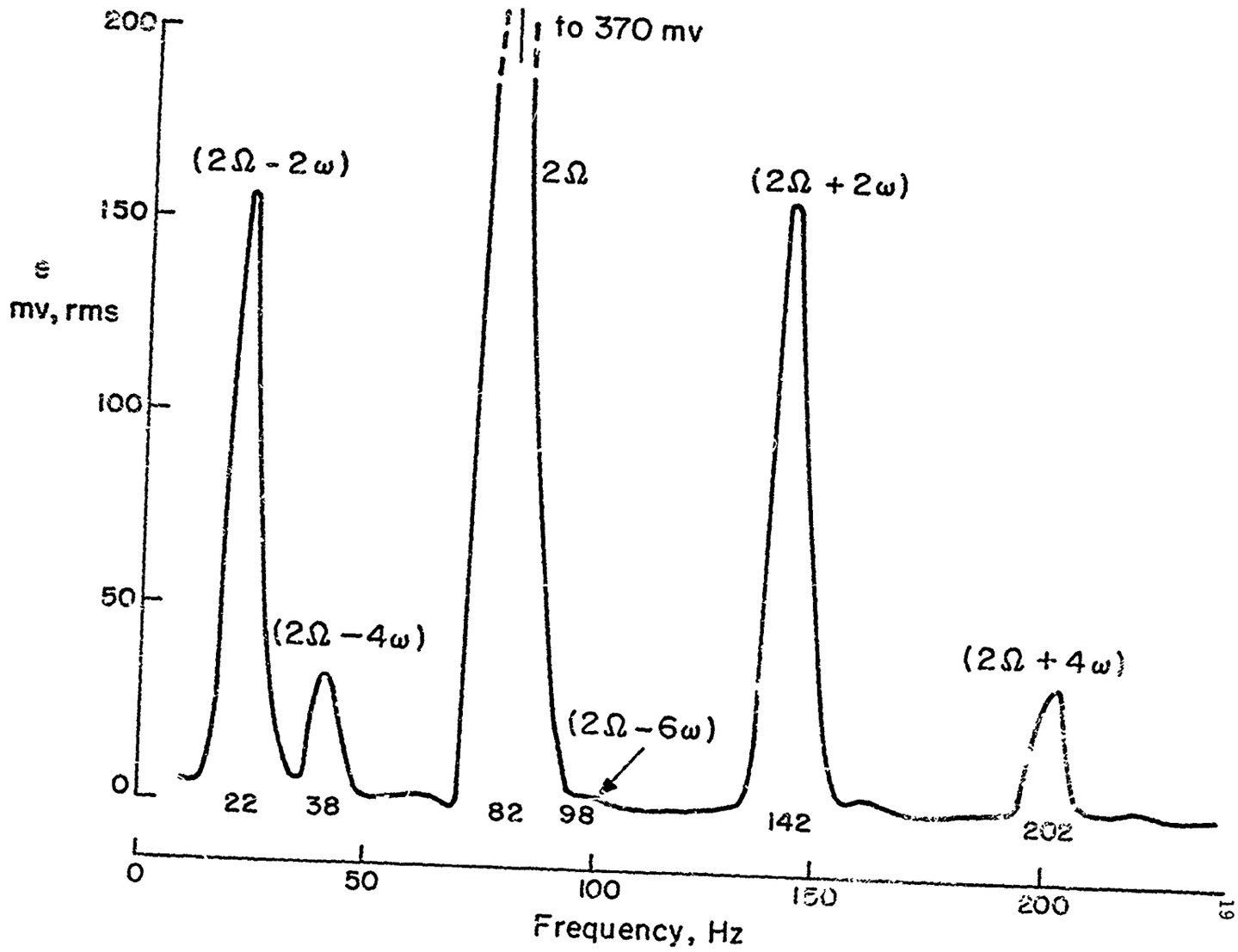

Figure 5: Frequency spectrum for the bentonite solution of Fig. 3. The field strength is 35 v/cm rms at 30 Hz.

channel of the phase sensitive detector. The system is calibrated by insertion of a quarter waveplate in the optical path at the position of the Kerr cell and being oriented with optical axes coincident with those of the Kerr cell. The resulting 82 Hz signal is adjusted in phase by the lock-in system and the magnitude measured. In removing the waveplate the system is finally adjusted with the Kerr cell in position as previously described and a null is obtained at 82 Hz. Then, on application of the electric field, the δ_{st} produces an 82 Hz component which is measured. In this procedure, the background signal at 82 Hz is held to less than 10 microvolts rms while a quarter wavelength retardation gives a 620 millivolt rms signal. This background is equivalent to a retardation of 1.62×10^{-5} rad. For the previously described Kerr cell and optical wavelength, this is equivalent to a birefringence Δn of 1.86×10^{-11}.

The question arises as to the influence of the birefringence of the windows of the Kerr cell on measurements done in the above described manner. If the exit window has no birefringence, then this procedure will establish that circularly polarized light passes through the Kerr cell when no electric field is present. With a small birefringence in the exit window, and adjustment of the quarter waveplates of Figure 3 will result in circular light at the rotating analyzer position, but with the light in the Kerr cell being slightly elliptical. On

establishing the electric field in the Kerr cell the ellipticity of the light becomes slightly changed and a measurable 2Ω component, 82 Hz, is obtained. The question is then whether or not this signal can be interpreted as being the $e_o\ \delta_{st}$ which is sought. An exact theoretical analysis of this optical system was carried out and calculations were made of the error due to the birefringence of the exit window. The results show that the exit window does modify the photoresponse slightly. For example, for a retardation of 1 degree in the exit window, the maximum error introduced in the measured Kerr retardation, δ_{st}, is 0.015 percent for δ_{st} ranging from 1 to 40 degrees. Thus, for windows of small retardation, the measurement error is negligible, even for cases where the retardation to be measured is comparable to that of the window.

It has been noted that the rotating analyzer method is effective in diminishing undesirable effects of background level changes in e_o. To illustrate this effect the Kerr cell was filled with an aqueous bentonite solution having a concentration of 175 μg/ml. The solution resistivity was adjusted to 1506 ohm-cm at 1 kilo-Hz by addition of NaCl. Then an electric field at 20 Hz and of intensity 22. v/cm rms was applied for approximately 33 seconds. The optical response was measured by three methods, the results of which are shown in Fig. 6. In all cases right circular light is incident upon the Kerr cell. In the first

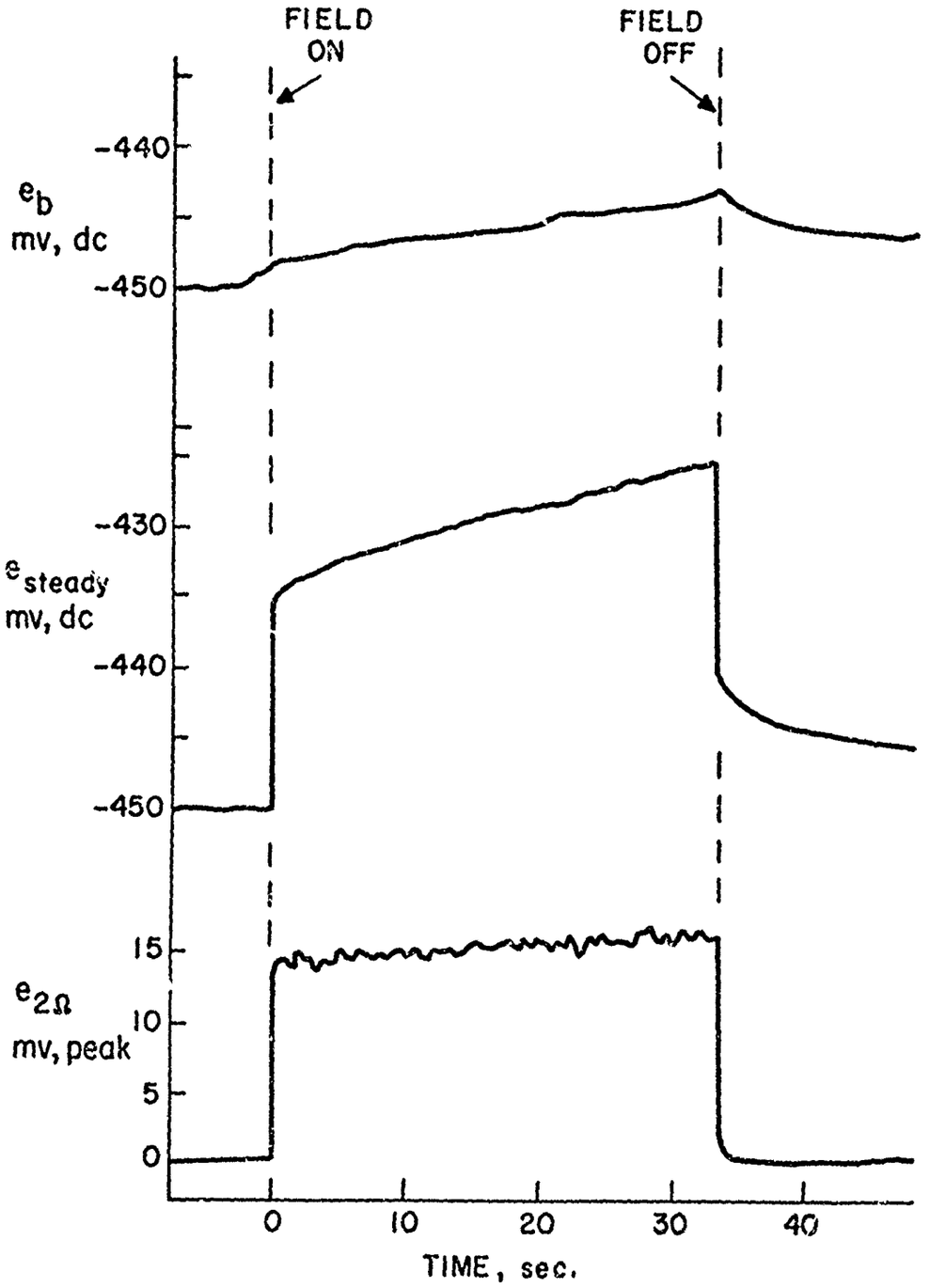

Figure 6: Photoresponses for a 175 μg/ml bentonite solution for a 33 second burst of a 20 Hz field of 22. v/cm in the Kerr cell. Trace A is the background with the analyzer removed; B is the steady response due to birefringence with a fixed optical analyzer in position; C is the 2 Ω component when the analyzer rotates at the frequency Ω.

case, the steady component of the background intensity was recorded on application and cessation of the field (trace A). The photomultiplier output was adjusted to the approximate -450 mv level. This is a trace of e_b as it enters into (10). The small, rapid change in e_b is here obscured by the much slower superimposed drift. Next, a fixed plane optical analyzer is placed after the Kerr cell, the analyzer direction being oriented at 45° to the electric field, and the steady component of the photoresponse is recorded. Figure 6 (trace B) shows the rapid upward step on application of the electric field due to the birefringence. This is followed by the drift in background which has an adverse effect on precise measurement of the birefringence. Finally, the analyzer is set into rotation at the frequency Ω and the 2Ω component resolved by the wave analyzer as previously described. Figure 6 (trace C) shows the response by this method. It is seen that the drift of the traces A and B has been relatively suppressed since the response is proportional to e_b in which the drift is only a minor fraction. Thus it is seen that the rotating analyzer method tends to suppress the significance of these drift problems in measurement of the steady component of the retardation introduced by the Kerr cell.

The character of the results of measurement of the frequency dependence of the steady component of the retardation as determined from the 2Ω component of the photoresponse with the rotating analyzer is shown in Fig. 7. These are results for

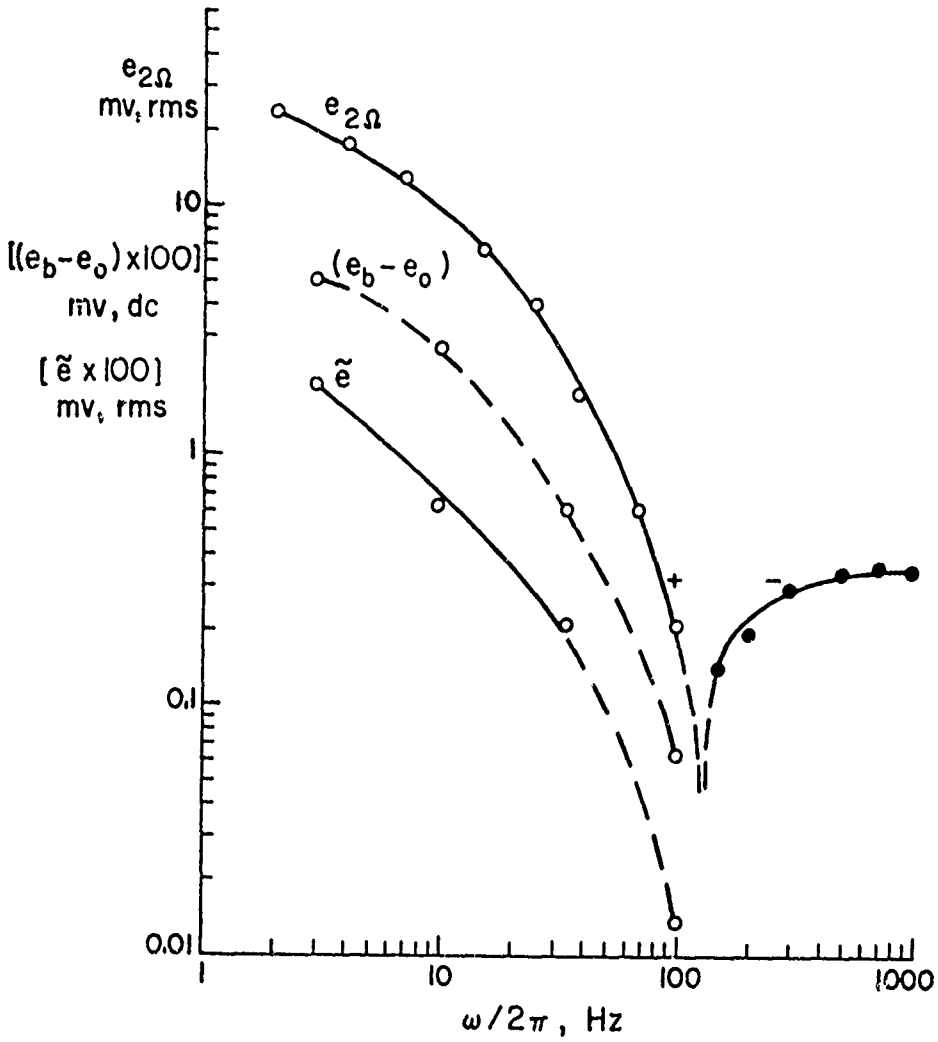

Figure 7: Steady component of the Kerr response, as represented by $e_{2\Omega}$ versus field frequency. The solution is bentonite in water at a concentration of 3.55 mg/ml. The electric field strength is 1 v/cm rms. Note that the sensitivity is such that a quarter wavelength retardation will give a signal of 620 mv rms. The dashed curves are the steady component (e_b-e_o) and alternating component e of the rapid change in the light intensity on application of the field.

an aqueous bentonite solution of concentration 3.55 mg/ml and having a resistivity of 1230 ohm-cm at 1 kilo Hz. Plotted is the signal voltage $e_{2\Omega}$ in mv, rms versus the frequency of the electric field, $\omega/2\pi$. This is the response for a field strength of 1 volt/cm, rms. The retardation undergoes a change of sign near 120 Hz, going from negative to positive with increasing frequency. Also shown as dashed lines are the steady component (e_b-e_o) and alternating (2ω) component $\tilde{e}$ of the background light intensity. In this case these rapid intensity changes are small compared to the signal due to the birefringence. It is to be noted that the (e_b-e_o) represents a reduction of light intensity on application of the field.

Conclusions

By the introduction of the rotating plane analyzer in front of a photodetector and in the path of elliptically polarized light, a photoresponse is obtained which consists of a periodic electrical signal. If the ellipticity of the light is constant, then the principal axes of the ellipse and its eccentricity are directly related to the phase and amplitude of the sinusoidal photoresponse and measurement of the ellipticity is thus reduced to an electrical measurement. When the ellipticity of the light is due to passage of circularly polarized light through a Kerr electro-optic cell which is activated by a sinusoidally time-varying electric field, then the rotating analyzer method leads to photoresponses which are simply related to both the steady and

alternating components of the retardation of the cell. These two components are separated in the frequency domain of the photoresponse. A measurement procedure utilizing light having a polarization which is very nearly circular almost eliminates error due to birefringence of the Kerr cell windows. In general, the rotating analyzer method discriminates against the adverse effects of changes of overall light intensity.

References

[1] H. Janeschitz-Kriegl, Adv. Polymer Sci. 6, p. 170 (1969).

[2] V. N. Tsvetkov, "Flow Birefringence," Newer Methods of Polymer Characterization, Edited by B. Ke, Interscience Publishers, 1964.

[3] H. G. Jerrard, J. Optical Soc. Amer. 46, p. 259 (1956).

[4] J. Leray and G. Scheibling, Compt. Rend. 251, p. 349 (1960).

[5] D. S. Thompson and S. J. Gill, J. Chem. Phys. 47, 5008 (1967).

[6] G. B. Thurston and J. L. Schrag, Trans. Soc. Rheol. (U.S.A.) 6, 325 (1962).

[7] D. G. LeGrand and P. F. Erhardt, Trans. Soc. Rheol. (U.S.A.) 6, 301 (1962).

[8] H. Benoit, Ann. Phys. (Paris) 12, 36 (1951); J. Chem. Phys. 49, 517 (1952).

[9] G. B. Thurston and D. I. Bowling, J. Colloid and Interface Sci. 30, 34 (1969).

[10] G. B. Thurston, Applied Optics 3, 755 (1964).

[11] G. A. Dvorkin and V. I. Krinskii, Doklady Akad. Nauk, S.S.S.R. 140, 942 (1961).

[12] L. K. Vistin and A. P. Kapustin, Opt. Spektrosk, 24, 650 (1968).

[13] W. A. Shurcliff, Polarized Light, Harvard University Press, 1962 (see p. 103).

[14] G. A. Dvorkin, Doklady Akademii Nauk, S.S.S.R., 135, 737 (1960).

[15] J. M. Brown, A. D. Buckingham, and D. A. Ramsay, Canadian Journal of Physics, 49, 914 (1971).