TECHNICAL REPORT

SUBMERSIBLE MANEUVERING

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Dr. C. K. Taft, Principal Investigator
603-868-5074
University of New Hampshire

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Scientific Officer - Program Director
Ocean Technology Programs, Ocean Science and
Technology Division, ONR
800 N. Quincy Street
Arlington, VA 22217

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SUMMARY

The wake steering shroud is a method of steering marine vessels which uses a shroud surrounding a propeller to accelerate the fluid and cause a low pressure region inside the shroud downstream from the propeller. The shroud shape is such that the wake is attached to the inside of a shroud under these conditions. Control ports are provided in the side of the shroud. When the control ports are open, flow is induced into the shroud from the outside and this causes separation of the wake from the inside of the shroud downstream of the propeller. The resulting separation causes an asymmetrical pressure distribution inside the shroud and provides a radial steering force. The concern of this project is to gain a better understanding of this phenomena and develop the concept to the point where it can be applied to marine vessels for steering and propulsion. The approach is to use existing mathematical models or develop a mathematical model for the relationships between shroud shape, propeller geometry and control port location and radial thrust. At the same time, an experimental system for measuring the radial and axial thrust on a shroud and propeller model as well as the propeller speed, torque and the pressure distribution inside the shroud has been developed. From this information, by use of dimensional analysis, it is proposed to further develop the concept to the point where the maximum amount of radial thrust for a given axial thrust with the control ports open can be produced. In this way, a given shroud would provide the maximum amount of steering.

There are several characteristics of the system which must be optimized; the shape of the shroud, the shape and number of propeller blades, and the control port location. The first phases of this work are being conducted at zero forward velocity so that the shroud will be optimized for low speed.
operation. It has been proposed that the work will be extended to non-zero forward velocities during a later phase of the project. So far, the methods of analysis available have been investigated and there is no mathematical model which describes the behaviour of the shroud with good accuracy. Most models have been developed for shrouds in which separation does not occur, and are only valid for finite forward velocities. Consequently, work is continuing on the modification of existing methods of analysis to provide at least approximate results. Radial force is developed in the shroud by causing the pressure distribution downstream of the control port to approach atmospheric pressure. This means that if the pressure downstream of the port with the ports closed is quite low, then the difference between this pressure and atmospheric pressure will be large and one might expect a large radial thrust. Initial tests on two different shroud shapes have indicated that this is not quite the case and that the radial force results from the combination of shroud shape differences and the region over which the control port has influence downstream of its location as well as the pressure distribution when the ports are closed. This means that an analysis based on closed port operation will not greatly aid in the synthesis of the optimum shroud shape; experimental measurements of pressure distribution should clarify this. Experimentally measured thrust with ports closed on the present design provide a thrust coefficient which is quite comparable to those obtained with conventional shrouds and multibladed propellers. With a control port open, the axial thrust increases slightly and a radial thrust is produced. The ratio of radial to axial thrust is of the order of 0.5 which compares to a ratio of about 0.25 at zero forward velocity for the half shroud proposed by Gordon and Tarpgaard(8) which is the only other shroud-steering concept which has been described in detail in the literature. Work is continuing to investigate the behaviour of the shroud and determine its limitations at zero
forward speed.

The implications of this concept to DOD are many and varied. It appears that the wake steering shroud will produce larger radial to axial thrust ratios than some other steering methods. In addition, it produces an axial thrust at the same time that it produces a radial thrust which is quite different from a conventional rudder which produces an axial drag at the same time that it produces a radial thrust. Consequently, the steering qualities of a vessel with this kind of thruster would be quite different from a conventional vessel with a rudder. This may or may not provide advantages. In submersible or submarine operation, the thruster has the great advantage of providing four quadrant steering. That is, the radial thrust vector can be applied in any direction perpendicular to the propeller axis by suitable location of the control ports. This means that a single thruster has the capability of steering a vehicle or vessel up or down or sideways.
INTRODUCTION

Shrouded propellers offer several advantages over free propellers. An accelerating shroud can accelerate the fluid ahead of the propeller to provide more efficient propeller operation at low forward velocities. Decelerating nozzles or pump jets can cause the fluid velocities to decrease ahead of the propeller and prevent propeller cavitation at high forward velocities. Both of these concepts have been explored both experimentally and analytically.\(^{(1,2,3,4,5,6)}\) It is also possible to design shrouds for a given pressure distribution inside the shroud with a given propeller design at finite forward velocities.\(^{(7)}\)

Shrouds have also been proposed for vessel steering. Gordon and Tarpgaard devised a concept using a partial shroud around a propeller to provide steering.\(^{(8)}\) A partial shroud was located so that it could be rotated about an axis perpendicular to the propeller to accomplish steering. This approach produced a significant turning force together with complete elimination of rudder appendage drag and an increase in propulsion thrust. In addition, this concept eliminates the need to reverse the rudder with reversing engines. The experimental results of tests performed on a model to verify the concept, indicated that radial thrust to axial thrust ratios of approximately one fourth were

*Numbers in parenthesis refer to references at conclusion of proposal.*
obtainable using this approach. As forward velocity increased, the axial thrust decreased, but the radial to axial thrust increased slightly at finite forward velocities. Tests were not conducted at zero forward velocity.

Another approach using the shroud for steering purposes was proposed by Oosterveld. Oosterveld's concept consists of changing the circulation around a shroud surrounding a propeller by injecting fluid at the trailing edge of the shroud. Water would be injected at the exterior surface of the nozzle to locally increase the circulation around the nozzle and cause a radial thrust on the shroud. Suction or injection of water along the interior or external surfaces could accomplish a similar purpose. An experimental verification of this concept has not been published at this time.

The wake steering shroud as proposed by Wozniak uses the notion that an accelerating shroud has a pressure distribution downstream of the propeller which is lower than ambient pressure. This low pressure region will entrain external flow if the shroud is vented to the outside. This flow causes the pressure distribution inside the shroud to become asymmetrical and results in a radial force on the shroud. This approach produces a rather large radial thrust, and in fact, radial thrust to axial thrust ratios up to one half have been obtained. The wake steering shroud approach would use a shroud of the form shown in Figure 1. Ports located around the outside of the shroud just downstream of the propeller are opened and closed by shutters. These shutters allow external fluid to enter the wake downstream of the propeller and cause
separation of the propeller wake from the inside of the shroud. The resulting asymmetrical pressure distribution causes a radial steering force.

The wake steering shroud has several advantages over the half shroud as proposed by Gordon and Tarpgaard in addition to having the advantages that they describe.

1. The shroud, depending on the number and location of the control ports, is capable of providing radial steering forces in any direction perpendicular to the propeller. A large number of ports could be located downstream of the propeller, around the outer surface of the shroud to provide steering forces in almost any direction. Consequently, a single wake steering shroud can perform the functions of a larger number of individual thrusters. For highly maneuverable vehicles, many additional modes of control might be exercised using the wake steering shroud. For example, underwater torpedoes or diver-manned sleds could have additional degrees of freedom not easily supplied by conventional propellers and rudders.

2. In addition, of course, the forces required to steer the shroud are only the forces necessary to move the shutters to open and close the control ports. The pressure difference between the inside and outside of the shroud are small.
Consequently, except for the sliding friction on the shutters, the operational forces are not very large and are considerably smaller than the operational forces on conventional rudders or other steering surfaces or even the half shroud as proposed by Gordon and Tarpgaard.

3. A prime advantage of the wake steering shroud is its ability to provide a steering force at zero forward velocity. This is of considerable advantage in low speed vehicle maneuvering and docking and could provide more precise control for many marine vessels.

There are several possible uses for this concept which may have considerable military significance.

a) Reconnaissance submarines would be simpler, require fewer thrusters and could have greater maneuverability using the wake steering concept.

b) Surface vessels could have much lower steering forces, and much higher maneuverability using this concept.

c) Torpedoes could be controlled much more precisely in depth and direction.

d) A high speed fleet submarine might find this form of steering advantageous because of its different noise properties and high forward efficiency with the control ports closed. It is also possible that this concept would provide a different noise signature when the vessel was steering, and thereby confuse possible detection methods based on propeller noise.
In summary, the wake steering concept appears to be much more effective than previous steering concepts and also provides considerable advantages for military and civilian marine vessels. The work done so far on this grant has been concerned primarily with the zero forward velocity behaviour of the shroud and propeller. If this concept is to replace a rudder as the steering device on a vessel, then it is most important that the concept operate very effectively at zero forward velocity. In addition, it is felt that the zero forward velocity operation is probably the most critical region of operation. If the wake was to separate from the shroud without opening the control ports, this failure would most likely occur at high propeller loading. Analyses by Morgan, Chaplin, Ordway, and Wozniak as well as the program developed by Weetman and Cromack at the University of Massachusetts are being evaluated as possible methods of analytically devising a shroud shape which would provide a maximum ratio of radial to axial thrust. There is no mathematical analysis which is valid when control ports are open. A very simple analysis by Wozniak provided some insight into this behavior, but it is too crude for use in synthesis without further refinement.

At the same time, experimental tests on several models of the shroud-propeller system were conducted to gain insight into system behavior and develop experimental data needed for design optimization.

**EXPERIMENTAL RESULTS**

Several shroud shapes have been fabricated from aluminum and used with a two bladed propeller of 4.45 cm diameter with a pitch to diameter
ratio of 1.8. The propeller is driven by a DC motor through a flexible shaft of speeds of 0 to 83 revolutions per second. The shroud and propeller combination is mounted on a beam that serves as a dynamometer. The beam is instrumented with strain gages. See Figure 2. The gages and associated instruments allow the axial and radial horizontal forces produced by the propeller and shroud to be recorded as functions of time. The moments about the propeller axis, the beam axis and the third mutually perpendicular axis can also be recorded. With these measurements taken for tests with each of the control ports opened, it is possible to measure the magnitude of the axial thrust and the location, magnitude and direction of the radial thrust provided by the propeller and shroud combination. The beam is damped in the horizontal plane by two mechanical dampers constructed using "airpots" manufactured by Airpot Corporation to effectively provide nearly optimum dynamometer damping. The beam and shroud are immersed in a tank .914 meters by 2.44 meters with a water depth of .61 meters. The tank is baffled so as to reduce recirculation eddies in the region of the shroud which would occur due to the finite tank size and the pumping action of the propeller. Tests performed in a large pool verify that the test results in this tank are essentially equivalent to open water tests at zero forward velocity.

In addition, a set of five pressure taps are located axially along the shroud surface at each of six circumferential locations. These are connected to manometers to provide a pressure profile for the inside surface of the shroud. These pressures provide considerable insight into shroud behavior.
For a given propeller and shroud design, it can be shown that

\[
\text{Thrust} = F = f_1(V, n, D, \rho, \mu), \text{ Newton}
\]

Pressure at any point - ambient pressure \( \equiv \Delta p = f_2(V, n, D, \rho, \mu), \text{ nt/m}^2 \)

where

\( V \equiv \text{relative velocity of shroud and water, m/sec.} \)

\( n \equiv \text{propeller speed, rev/sec.} \)

\( D \equiv \text{propeller diameter, m} \)

\( \rho \equiv \text{fluid density, Kg/m}^3 \)

\( \mu \equiv \text{fluid viscosity, nt-sec/m}^2 \)

A dimensional analysis yields

\[
\frac{F}{\rho n^2 D^4} = \phi_1 \left( \frac{V}{n D}, \frac{\rho n D^2}{\mu} \right)
\]

and

\[
\frac{\Delta p}{\rho n^2 D^2} = \phi_2 \left( \frac{V}{n D}, \frac{\rho n D^2}{\mu} \right)
\]

This is one set of dimensionless numbers, others are also possible. (10)

In most submersible and ship designs Reynolds number is very high so that the flow is essentially turbulent and independent of Reynolds number.

Also shown in the tests conducted so far: Reynolds Number based on tip speed \( = \frac{\rho \pi n D^2}{\mu} = 0.75 \text{ to } 3.75 \times 10^5 \) at 100-500 radians/sec. In addition, the Cavitation Number based on tip speed \( = \frac{\rho - np}{\frac{1}{2} \rho (\pi n D)^2} = 1.63 \) minimum which is large enough to insure that propeller cavitation is not occurring in the tests. Hence the dimensionless equations can be simplified
\[
\frac{F}{\rho n^2 D^4} = \phi_1 \left( \frac{V}{nD} \right)
\]
and
\[
\frac{\Delta p}{\rho n^2 D^2} = \phi_2 \left( \frac{V}{nD} \right)
\]
where
\[
\frac{F}{\rho n^2 D^4} \equiv \text{thrust coefficient } K_t
\]
\[
\frac{V}{nD} \equiv \text{advance coefficient } J
\]
\[
\frac{\Delta p}{\rho n^2 D^2} \equiv \text{pressure coefficient } K_p
\]

The primary focus of the present test program is to evaluate performance of the steering system at zero forward velocity. Hence, the advance coefficient is zero. Thus, the present thrust coefficient at zero forward velocity and the pressure coefficient distribution inside the shroud characterize a given propeller and shroud geometry.

With a given shroud and propeller, the pressure distribution inside the shroud is axisymmetrical. Hence, the pressure inside is a function of axial location alone. The solid curve in Figure 3 shows this pressure distribution where pressure coefficient is plotted versus normalized axial location \( z/L \) with all control ports closed.

\[
z = \text{axial location measured from upstream edge of shroud}
\]
\[
L = \text{shroud chord length}
\]

As expected, below ambient pressures occur inside the shroud downstream from the propeller indicating the propeller wake is attached to the inside of the shroud. When a control port is open, the pressure
profile inside the shroud is no longer axisymmetrical. Downstream of the control port a region of higher pressure results. (See Figure 4.) The maximum pressure coefficients in this region are also shown on the dotted curve in Figure 3. The minimum pressure coefficients in the rest of the shroud do not seem to change from those values which were present before the control port was opened. However, there is a variation with angular location between these maximum and minimum values as shown in Figure 4.

The difference between the pressures which occur downstream of the control port and the pressures on the rest of the shroud act on the shroud to produce the radial thrust which will be used for steering. Thus, this difference and the effective area over which it acts should be maximized in the design procedure.

The axial and radial thrusts and the angle of the radial thrust vector were measured as a function of propeller speed for shroud Number 2 shown in Figure 1, which has an inside shape of an NACA-16-021 airfoil section at a zero angle of attack. Another shroud design designated Number 1, was also tested which has an inside shape based on nozzle No. 32 developed by the Netherlands Ship Model Basin, and described in reference 10. (See Figure 5.) The thrust was measured with all ports closed, then a single port was opened and radial and axial thrusts were determined. Results of these tests are shown in Figures 6 and 7. The angle of the radial thrust vector with one port open is shown in Figure 8.

From these data, it is possible to determine a thrust coefficient for the axial and radial thrust with a control port open as given by
\[ K_{ta} = \frac{F_a}{\rho n^2 D^4} \quad \text{where } F_a = \text{axial thrust} \]

\[ K_{tr} = \frac{F_r}{\rho n^2 D^4} \quad \text{where } F_r = \text{radial thrust} \]

The ratio of these two thrust coefficients \( \frac{K_{tr}}{K_{ta}} \) is a measure of the ability of the system to produce steering moments on a vessel. The ratio should be as large as possible. The data in Figures 6 and 7 is summarized below.

<table>
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<tr>
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<th>( K_{ta} )</th>
<th>( K_{tr} )</th>
<th>( \frac{K_{tr}}{K_{ta}} )</th>
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<tr>
<td>Nozzle 1</td>
<td>0.706</td>
<td>0.383</td>
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<tr>
<td>Nozzle 2</td>
<td>0.79</td>
<td>0.281</td>
<td>0.356</td>
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The tests were conducted with a two bladed propeller as described earlier. With three or four bladed propellers, it would be expected that these thrust coefficients would increase somewhat.\(^{(11)}\) Even with a two bladed propeller, the axial thrust coefficients with all ports closed are only slightly lower than the above values which are quite
close to those obtainable with conventional shrouded propellers. This would indicate that the wake steering shroud which has a longer chord to diameter ratio \( L/D = 2.29 \) does not exhibit poorer performance. Tests at nonzero advance coefficient (nonzero shroud velocity) will be necessary to substantiate this conclusion, however.

The thrust coefficient ratios indicate that shroud No. 1 has better performance as a steering device than shroud No. 2. This ratio can serve as a measure of shroud design performance.

The radial thrust vector does not pass through the control port, but makes an angle with the radius vector through the port in the direction of propeller rotation. This is shown in Figure 8. The swirl imparted to the fluid by the propeller causes the high pressure region inside the shroud induced by the control port to take a spiral pattern as was shown in Figure 4. This is the reason an angular displacement of the radial thrust vector occurs. There is some dependency on propeller speed, however, this effect is small. For a given shroud design, the ports could be located to provide thrust vectors in any desired direction with little variation caused by propeller speed.

**ANALYTICAL RESULTS**

Various analytical methods for predicting wake steering shrouded-propeller performance with all control ports closed characteristics have been investigated so far. The analyses of Chaplin,\(^{(4)}\) Morgan,\(^{(2)}\) and two of Ordway,\(^{(3,5)}\) et al, were considered. In addition, discussions, were held with Dr. Cronack and Mr. Wootman of the University of Massachusetts concerning their analytical program. Also, the analytical procedure
developed by Mr. Wozniak (9) was investigated to determine if improvements could be made to permit realistic analytical designs to be made. The respective analyses and their merits and disadvantages are tabulated in Table I and discussed below.

The analysis prepared by John Wozniak has been re-examined to determine its suitability as an analytical design procedure for optimizing a shrouded propeller design. Initially, the boundary conditions are being evaluated at the propeller plane instead of the exit plane. It will be assumed that axial swirl is constant and there is zero vorticity in the wake flow. This permits the flow field changes to depend on the shroud geometry changes. This work is presently continuing and a program is being readied to determine the effect of these changes.

The analysis of Morgan (2) was examined to determine what other approaches are possible in the analysis of shrouded propellers. Morgan's analysis (also tabulated in Table I) is a very detailed analytical procedure, but has limitations since its boundary conditions are linearized which restricts its validity to large advance ratios. This also restricts the applicability to shrouds of nearly cylindrical shape.

Two separate analyses of Ordway (3,5) et. al. were considered. One was valid for high advance ratios and the other was for the low speed or the static case. Both use a three dimensional analysis of induced velocities for the description of the flow field. However, both analyses also use linearized boundary conditions and pressure coefficients which again restrict their use to slightly diverging shrouds. A zeroth and first order approximation are also presented. The static approach
is being considered as an alternate method of obtaining the pressure field for various shaped ducts to help determine an optimum shroud shape based on a minimum pressure distribution with all control ports closed. This analysis is independent of the wall shape for the static or low speed case. Preliminary pressure measurements indicate this is approximately true for the two shrouds that have been tested. Chaplin's\(^{(4)}\) non-linear analysis was also studied. Its major aim was to determine the wake contraction and little information is available on the pressure field within the shroud.

The University of Massachusetts's program is currently being investigated. Sufficient information to evaluate all of its limitations is not available at this time. The information in Table I has been completed from discussions held with Cromack and Weetman. When their program is completed, it may provide another method for determining an optimum shroud shape. It is a non-linear program that uses an actuator disk for the propeller and uses Martensen's\(^{(12)}\) results for describing the shroud shape.

Improvement of the analyses of the shroud with the control ports open as proposed by John Wozniak has not been attempted. This will be investigated when a satisfactory model for the axisymmetrical case has been devised.

**PROPOSED PROGRAM**

During the remainder of the present contract period, the following tasks will be performed:

1. An experimental system is being constructed to allow shrouds to be easily fabricated. Shroud shapes will be fabricated in
three parts from wax supported by a metal shell. A lightweight
air actuated flapper valve will open and close the control ports.
The system is shown in Figure 9.

2. Experiments will be performed to relate shroud shape and control
port geometrics to wake steering system performance. Relationships between control port flow and radial thrust will indicate
the degree of stability of the propeller wake attachment to the
shroud surface. Propeller speed and torque measurements will
also be conducted to evaluate propeller effectiveness.
REFERENCES


<table>
<thead>
<tr>
<th>Assumption (fluid)</th>
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<th>Shroud Model</th>
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**TABLE I: SHROUDED PROPELLER ANALYSIS METHODS**
FIG. 3. PRESSURE DISTRIBUTION INSIDE SHROUD No. 2
FIG. 4. PRESSURE DISTRIBUTION INSIDE SHROUD No.2. WITH ONE CONTROL PORT OPEN
FIG. 5. WAKE STEERING SHROUD DESIGN
No.1.
FIG. 6. PROPELLERED SHROUD No. 2 AXIAL & RADIAL THRUST VS. PROP. SPEED

- PORTS CLOSED
- SMALL PORT OPEN
- AXIAL THRUST
- RADIAL THRUST

$K_{tr} = 0.281$
$K_{ta} = 0.790$
$K_{tr}/K_{ta} = 0.356$

PROPELLER SPEED (RPS)

THRUST (NEWTONS)
FIG. 7. PROPELLER SHROUD NO. 1. AXIAL Vs. RADIAL THRUST Vs. PROPELLER SPEED

- Ports Closed
- Small Port Open
- Large Port Open
- Axial Force
- Radial Force

\[ K_{tr}^{(1)} = 0.383 \]
\[ K_{ta}^{(2)} = 0.706 \]
\[ K_{tr}/K_{ta} = 0.543 \]
FIG. 8. RELATIVE ANGLE OF THE RADIAL THRUST VECTOR MEASURED COUNTER-CLOCKWISE FROM PORT OPENING.
FIG. 9. PROPOSED SHROUD VARIATION SYSTEM