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# A BRIEF REVIEW OF THE AIRCRAFT TRAILING VORTEX PROBLEM

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13. ABSTRACT

In this paper, an attempt will be made to review the simplest theory of the wake, as well as to bring to attention a more detailed description of vortex roll-up that has been available for many years. This more complete description of the wake is in far better agreement with experimental results than the conventional description and may be used to illustrate several problems that face the experimentalist in the interpretation of his data. Once the roll-up problem has been discussed, the ways in which trailing vortex systems may be dissipated are reviewed. Finally, a system for evaluating the danger potential of aircraft wakes is discussed and the wake characteristics of a representative selection of present-day aircraft are presented in terms of this method of classifying aircraft wakes.

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| AIRCRAFT VORTICES | | | | | | |
| AIRCRAFT WAKES | | | | | | |
| VORTICES | | | | | | |
| VORTEX ROLL-UP | | | | | | |

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AFOSR Scientific Report
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# A BRIEF REVIEW OF THE AIRCRAFT TRAILING VORTEX PROBLEM

by
Coleman duP. Donaldson

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# I. INTRODUCTION

For the past few years, there has been a great deal of discussion of the hazard that may exist for other aircraft as a result of the trailing vortex systems of the larger civilian and military aircraft, i.e., the 707-320, C141, C5, and the 747. There are three separate questions that must be considered in dealing with this problem, First, one must adopt some method for defining, in a simple way, some measure of the danger associated with the wake of any aircraft, should one be so unfortunate as to penetrate the center of a trailing vortex system. Second, one must have some method of estimating the persistence of any given aircraft wake. Third, one must develop a viable rationale for computing the probability that one aircraft will actually experience a dangerous encounter with the wake of another. When an answer to each of the three problems set forth above is available, it will be possible to make a rational specification of aircraft separation in any given situation in such a way as to make optimum use of available air space.

To answer each of the three problems set forth above in detail is an exceedingly complex task. The apparent complexity of the problem has been enhanced in the past by the fact that the simplest and most commonly referred to description of the roll-up of vortices behind conventional wings gives results that are at odds with experimental measurements near the high speed cores of the trailing vortices.

In this brief paper, an attempt will be made to review the simplest theory of the wake, as well as to bring to attention a more detailed description of vortex roll-up that has been available for many years. This more complete description of the wake is in far better agreement with experimental results than the conventional description and may be used to illustrate several problems that face the experimentalist in the interpretation of his data.

Once the roll-up problem has been discussed, the ways in which trailing vortex systems may be dissipated are reviewed.

Finally, a system for evaluating the danger potential of aircraft wakes is discussed and the wake characteristics of a representative selection of present-day aircraft are presented in terms of this method of classifying aircraft wakes.

# II. THEORIES OF VORTEX WAKES

The general features of aircraft trailing vortex systems have been described for many years by an overly simplified theory that gives a good description of the aircraft wake well outside the vortex cores (Refs. 1 and 2). The simplicity of the theory makes it extremely useful for sorting wake categories. It is, however, not powerful enough when more detailed information about the wake in the vicinity of the vortex cores is needed.

Vortex roll-up can be treated in more detail by an inviscid theory (Ref. 3) which gives a fairly realistic description of the wake, not only well outside the vortex cores but close to the centers of rolled-up vorticity as well. In what follows, a very brief outline of the main features of these two descriptions of the wake are given.

## Uniformly distributed vorticity method

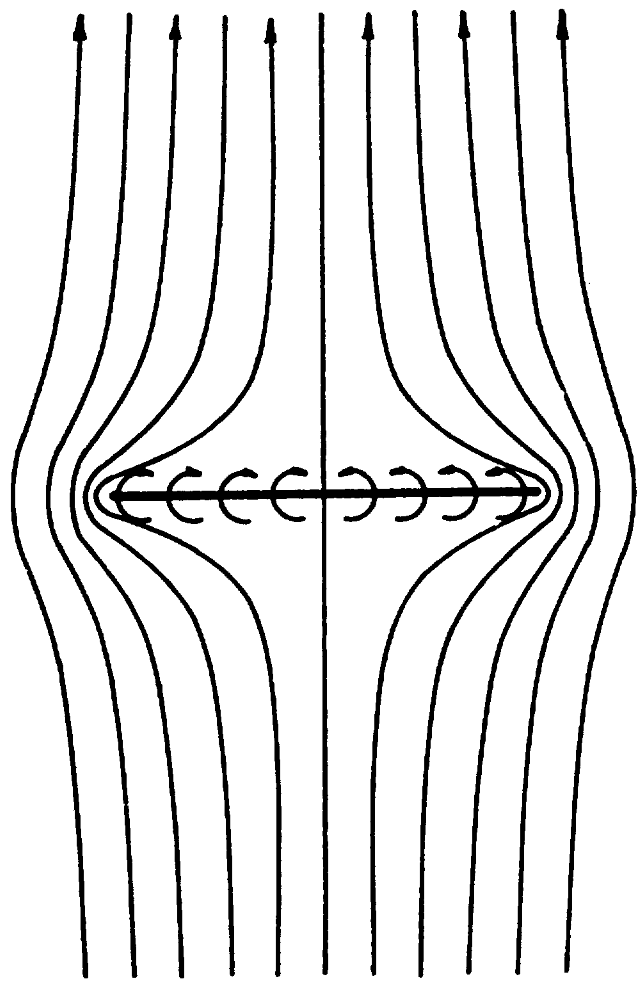
In the simplest analysis of a wing at angle of attack, one can imagine that the wing sees an upward flow towards its bottom surface of $U \sin \alpha$. Here $U$ is the forward velocity and $\alpha$ is the angle of attack. At small angles of attack, this cross flow velocity is approximately equal to $U\alpha$ as shown in Figure 2.1. This flow cannot pass through the wing but must pass around it, as shown in the figure. The flow may be diverted around the wing if a series of vortices trail from the wing in pairs as shown. The strength of each vortex pair leaving the airfoil is determined by the requirement that the normal component of velocity $U\alpha$ be cancelled at the wing surface.

If a solution to the problem just posed is developed, one will find that the wing possesses an elliptic loading specified by

$$\Gamma = \Gamma_o \sqrt{1 - (2y/b)^2} \qquad (2\text{-}1)$$

Here $b$ is the wing span and $\Gamma_o$ the strength of the bound vortex at the center of the wing. This is related to the lift of the wing, as follows:

$$L = \frac{\pi}{4} \rho U \Gamma_o b \qquad (2\text{-}2)$$



$w = U \sin \alpha \cong U\alpha$

Figure 2.1

The downward velocity at the wing $U\alpha$ may be expressed in terms of the vortex strength $\Gamma_o$. It is

$$w_o = \frac{\Gamma_o}{2b} = U\alpha \tag{2-3}$$

The state of affairs at the wing is shown to the left-hand side of Figure 2.2. This situation does not last long. The individual trailing vortices tend to roll themselves up into two trailing vortices of opposite sign, each having strength $\Gamma_o$. These two vortices are separated by a distance that is slightly less than the wing span $b$. This distance may be found by noting that a wing of constant loading and, hence, a single pair of trailing vortices would generate a lift of

$$L = \rho U \Gamma_o b' \tag{2-4}$$

where $b'$ is the trailing vortex separation. If these two lifts are to be equal,

$$b' = \frac{\pi}{4} b \tag{2-5}$$

The flow after the roll up of the individual trailing vortices is shown on the right-hand side of Figure 2.2. Much larger downward velocities are now in evidence than at the wing and, in general, the whole scale of the region of high velocities is increased greatly.

How high are the velocities that are encountered and over what scale are these velocities large? The velocity induced in the far field by a vortex of strength $\Gamma_o$ is

$$w = \frac{\Gamma_o}{2\pi r} \tag{2-6}$$

where $r$ is the distance from the vortex to the point of interest. As the distance from the vortex $r$ decreases, the velocity gets larger and would approach infinity for $r$ approaching zero. What then limits the rotational velocities in the wake of an aircraft according to this theory? To answer this question, it is observed that the energy in the vortex pattern of the wake must have been produced by the work done by the inviscid or induced drag of the

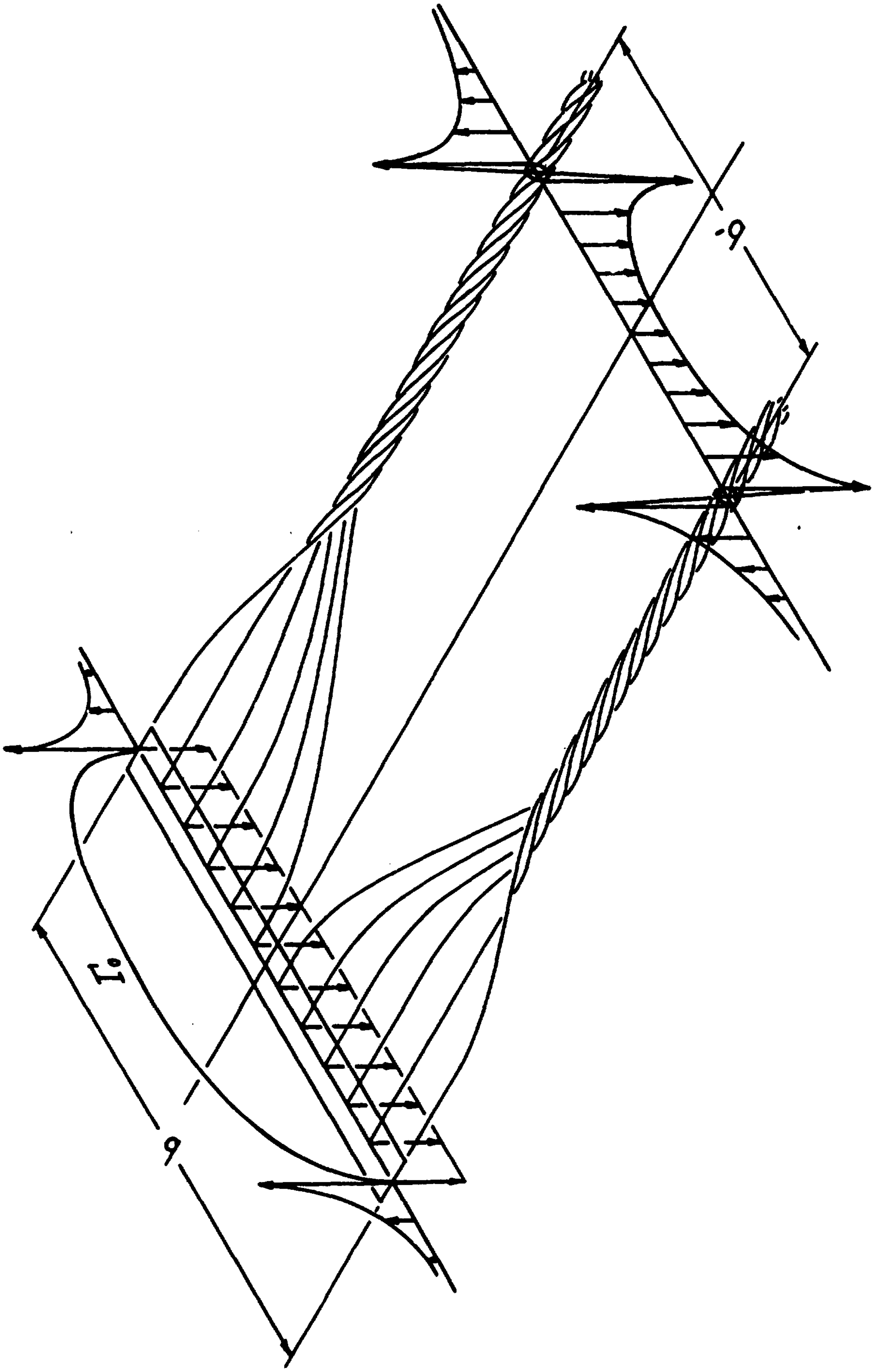


Figure 2.2

wing. Thus the work done by the induced drag in moving a distance $U\,dt$, namely, $D_{ind}U\,dt$, must show up as kinetic energy in the wake in a length also equal to $U\,dt$. Thus we write

$$D_{ind}U\,dt = U\,dt \iint_{-\infty}^{+\infty} \frac{\rho q^2}{2}\,dy\,dz \tag{2-7}$$

where $q$ is the total velocity in the wake cross plane defined by axes $y$ and $z$. A calculation of the kinetic energy in the vortex wake results in

$$\iint_{-\infty}^{+\infty} \frac{\rho q^2}{2}\,dy\,dz = \frac{\rho\Gamma_o^2}{2\pi}\,\ell n\,\frac{b'}{a} \tag{2-8}$$

In this formula, $a$ is the radius of a small circle drawn about the center of each vortex. The energy considered in Eq. (2-8) is the energy in the wake external to the two circles of radius $a$ drawn around the two rolled-up vortices. It is seen immediately that if $a$ were allowed to go to zero, the energy in the wake would be infinite. To get around the difficulty, the assumption is made that the vorticity is distributed uniformly over the region defined by the two circles so that solid body rotation occurs inside these circles. If this is done, the energy in the wake is (see Ref. 1)

$$E_w = \frac{\rho\Gamma_o^2}{2\pi}\left(\frac{1}{4} + \ell n\,\frac{b'}{a}\right) \tag{2-9}$$

If we equate this to the induced drag which is

$$D_{ind} = \frac{Lw_o}{U} = \frac{\pi}{8}\,\rho\Gamma_o^2 \tag{2-10}$$

we find that

$$\frac{b'}{a} = 9.1 \tag{2-11}$$

or

$$\frac{a}{b} = \frac{a}{b'}\cdot\frac{b'}{b} = 0.086 \tag{2-12}$$

Thus the radius of the center of the vortex rolled up behind a wing of elliptical loading is close to $1/10^{th}$ the span. One can compute the velocity at this point. It is

$$w_{max} \cong \frac{\Gamma_o}{2\pi r} = \frac{\Gamma_o}{0.172\pi b} \qquad (2\text{-}13)$$

Expressing $\Gamma_o$ in terms of lift or weight by means of Eq. (2-2), we obtain

$$w_{max} = 2.35 \frac{L}{\rho U b^2} = 2.35 \frac{W}{\rho U b^2} \qquad (2\text{-}14)$$

This equation shows the nature of the problem. The severity of the vortices (as given by the maximum velocity near the cores) increases with the weight of the aircraft involved. For given velocity, it is inversely proportional to the square of the span.

If we use Eq. (2-14) together with the result that the diameters of the regions of intense velocity are of the order of a little more than 17 percent of the span of the producing aircraft, we have, according to the simple theory, defined the nature of the problem. We should note that the trailing vortices would descend, in a neutrally stable atmosphere and if there were no changes of density in the formation of the vortices, at a rate given by

$$\frac{\Gamma_o}{2\pi b'}$$

If we view this motion from a position moving downward with the disturbances, we see a flow similar to that shown in Figure 2.3. This is the equivalent of the cross-plane flow at the wing given in Figure 2.1 and represents the change in the nature of this cross-plane flow as a result of the removal of the wing and the subsequent vortex roll-up.

If we compare the rolled-up wake as described by the method just presented, we find to our dismay that the result seems very much at odds with experimental data near the center of the trailing vortex. This is shown in Figure 2.4 where measurements made by McCormick, Tangler, and Sherrieb (Ref. 4) of the tangential velocity in the vortex wake of an Army O-1 aircraft are compared with the simple theory. It is apparent that the size of the vortex core

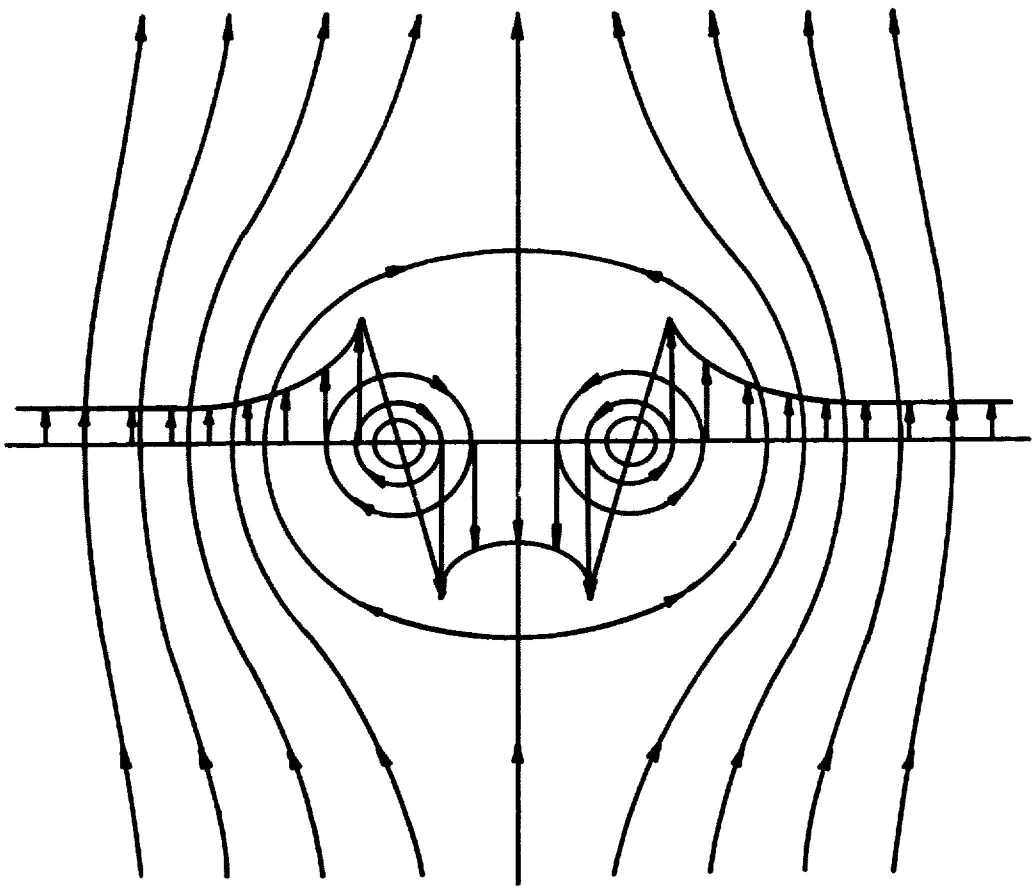


Figure 2.3

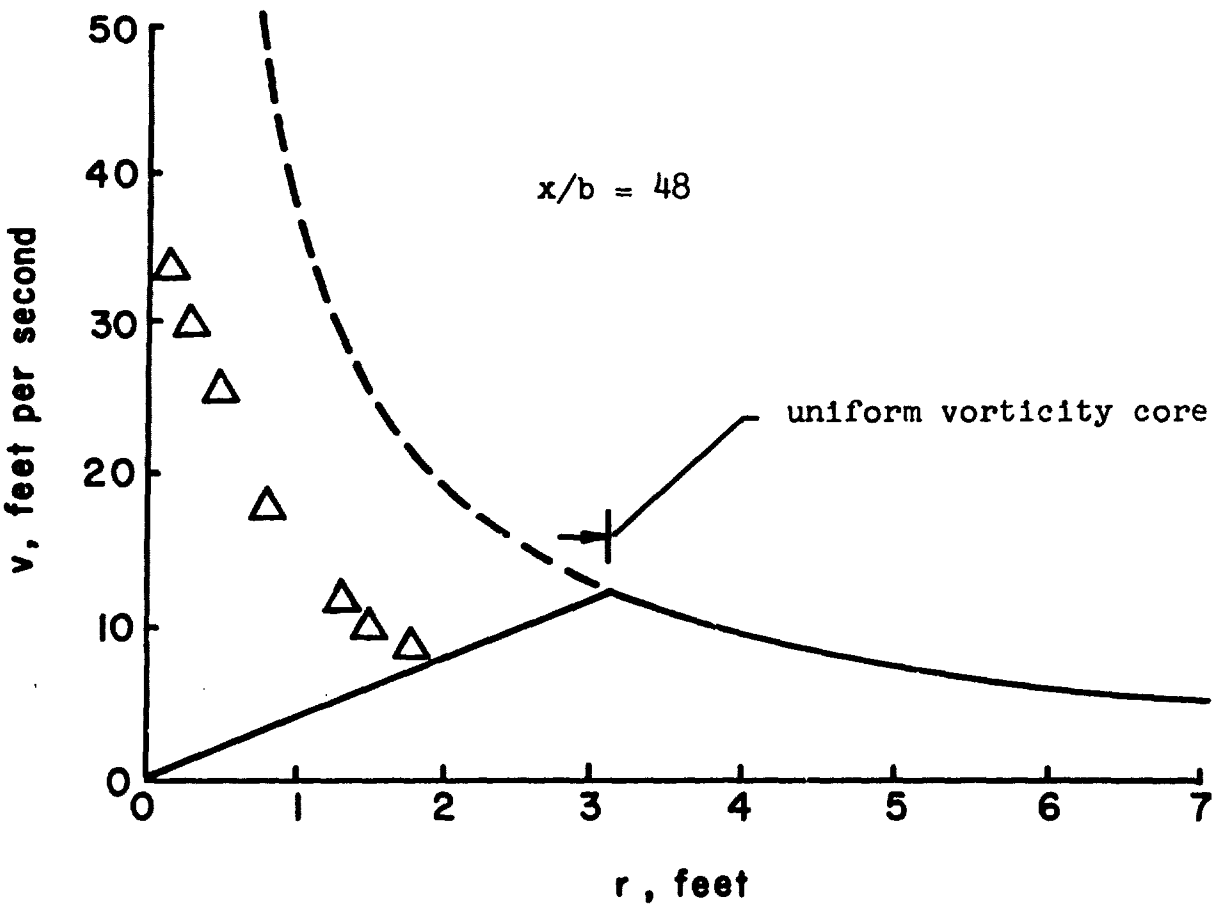

50
40
30
20
10
0
v, feet per second
x/b = 48
uniform vorticity core
0
1
2
3
4
5
6
7
r, feet

Figure 2.4

as measured is an order of magnitude less than that predicted by the theory. This is a general shortcoming of the simple theory. One also finds that the vorticity rolled up inside the radius $r = 0.086b$ is far less than the total that is computed to be shed by the appropriate side of the wing as a result of the lift it is generating.

To explain these shortcomings, it is necessary to look in somewhat more detail at the roll-up process and do away with the assumption of a uniform distribution of vorticity in the core region.

Distributed vorticity method

An analysis of the wake which deals in detail with the roll-up of the vorticity shed by a wing having elliptic loading has been given by Betz (Ref. 3). In this method, use is made of the following basic facts:

1. All the vorticity shed by each half of the wing is found rolled up in the trailing vortex behind the appropriate half of the wing;
2. The "center of gravity" of the vorticity distribution remains at a constant distance from the plane of lateral symmetry;
3. The "moment of inertia" of the vorticity shed by each half of the wing about its "center of gravity" is a constant.

Expressed analytically for a wing having an elliptic loading,

$$\Gamma = \Gamma_o \sqrt{1 - (2y/b)^2}$$

The basic facts listed above take the following form.

$$\int_0^{b/2} \frac{d\Gamma}{dy}\, dy = \text{const} = \Gamma_o = \int_0^{b/2} \frac{d\Gamma}{dr}\, dr \qquad (2\text{-}15)$$

$$\int_0^{b/2} y\, \frac{d\Gamma}{dy}\, dy = \text{const} = \bar{y}\Gamma_o = \frac{\pi b}{8}\, \Gamma_o \qquad (2\text{-}16)$$

$$\int_0^{b/2} (y - \bar{y})^2 \frac{d\Gamma}{dy} dy = \text{const} = \int_0^{b/2} r^2 \frac{d\Gamma}{dr} dr \qquad (2\text{-}17)$$

In these equations, $\bar{y}$ is the spanwise position of the rolled-up trailing vortex and $r$ is the radial distance from the center of the rolled-up vortex.

Betz assumes that in the roll-up process

$$\Gamma(r) = \Gamma^*(y_1) = \int_{y_1}^{b/2} \frac{d\Gamma}{dy} dy \qquad (2\text{-}18)$$

and

$$\int_0^{r} r^2 \frac{d\Gamma(r)}{dr} dr = \int_{y_1}^{b/2} y^2 \frac{d\Gamma}{dy} dy - \Gamma^*(y_1)\bar{y}_1^2 \qquad (2\text{-}19)$$

where

$$\bar{y}_1 = \frac{1}{\Gamma^*(y_1)} \int_{y_1}^{b/2} y \frac{d\Gamma}{dy} dy \qquad (2\text{-}20)$$

to deduce the distribution of $\Gamma$ in the rolled-up wake. This distribution of circulation is shown in Figure 2.5. For the calculations we will make in this paper, a very useful approximation to the precise distribution that was given by Betz is used. It is

$$\begin{aligned} \frac{\Gamma}{\Gamma_o} &= \left[6 \frac{r}{b} - 9\left(\frac{r}{b}\right)^2\right]^{1/2} \quad &; \; 0 < \frac{r}{b} \leq \frac{1}{3} \\ \frac{\Gamma}{\Gamma_o} &= 1 &; \; \frac{r}{b} > \frac{1}{3} \end{aligned} \qquad (2\text{-}21)$$

This distribution is compared with the precise calculation in Figure 2.5. As observed by Betz, since the simple result given in Eq. (2-21) agrees fairly well with the exact calculation for the region in which 90 percent of $\Gamma_o$ is contained, this simple formula is of the greatest usefulness.

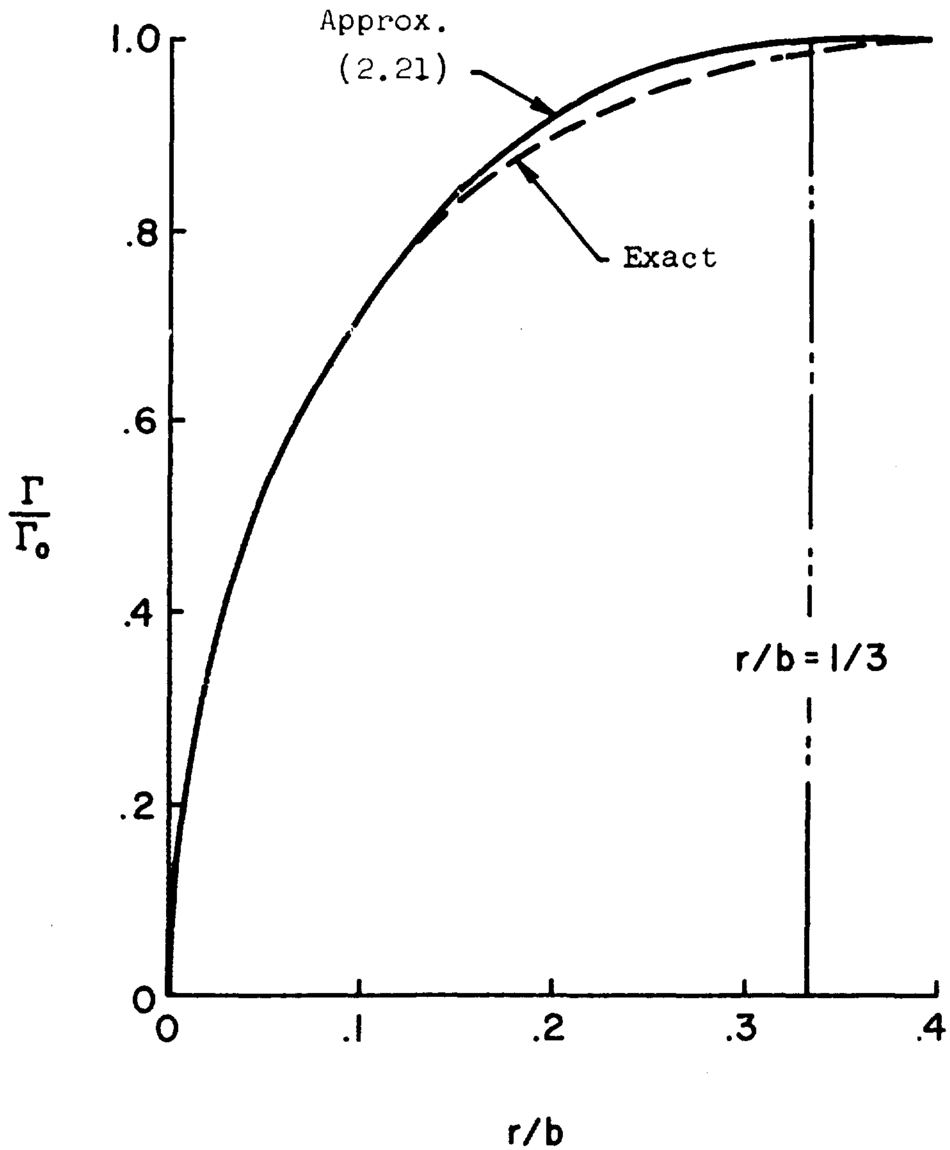



Figure 2.5

If we use Eq. (2-21) for the description of aircraft trailing vortices, we obtain the following results for the distributions of tangential velocity and vorticity:

$$v_T = \frac{\Gamma}{2\pi r} = \frac{\Gamma_o}{2\pi r}\left[6\,\frac{r}{b} - 9\left(\frac{r}{b}\right)^2\right]^{1/2} \; ; \quad 0 < \frac{r}{b} \leq \frac{1}{3}$$

$$v_T = \frac{\Gamma_o}{2\pi r} \; ; \qquad \frac{r}{b} > \frac{1}{3} \tag{2-22}$$

and

$$\zeta = \frac{\partial v_T}{\partial r} + \frac{v_T}{r} = \frac{3}{2\pi}\,\frac{\Gamma_o}{b^2}\left[\frac{b}{r}\left(1 - \frac{3r}{b}\right)\left(\frac{6r}{b} - 9\left(\frac{r}{b}\right)^2\right)^{-1/2}\right]; \quad 0 < \frac{r}{b} \leq \frac{1}{3}$$

$$\zeta = 0 \; ; \qquad \frac{r}{b} > \frac{1}{3} \tag{2-23}$$

If we compare the results of computations using the more detailed description of vortices just presented, we find a very acceptable agreement between theoretical results and experiment. This is shown in Figures 2.6 and 2.7 where we have used Eq. (2-8) to compute the tangential velocity in the trailing vortex for both the light aircraft previously considered in Figure 2.4 and for a Convair 880 aircraft. The latter measurements are those of Garodz (Ref. 5).* It is seen that the agreement of the more complete roll-up theory with experimental data is most gratifying.

We must now discuss an experimental observation that appears to have created some misunderstanding in the past. This is the fact remarked on by a number of investigators that the integrated rolled-up vorticity in the wake as measured does not seem to add up to the shed vorticity (Refs. 6 and 7). We may discuss this problem in the light of the results shown in Figure 2.8. Here we plot the wake vorticity distributions given by the two wake models we have discussed above. In both cases, the integral of the vorticity is $\Gamma_o$, but it is seen that the vorticity is actually

*In these figures, the tangential velocity plotted is that due to both trailing vortices.

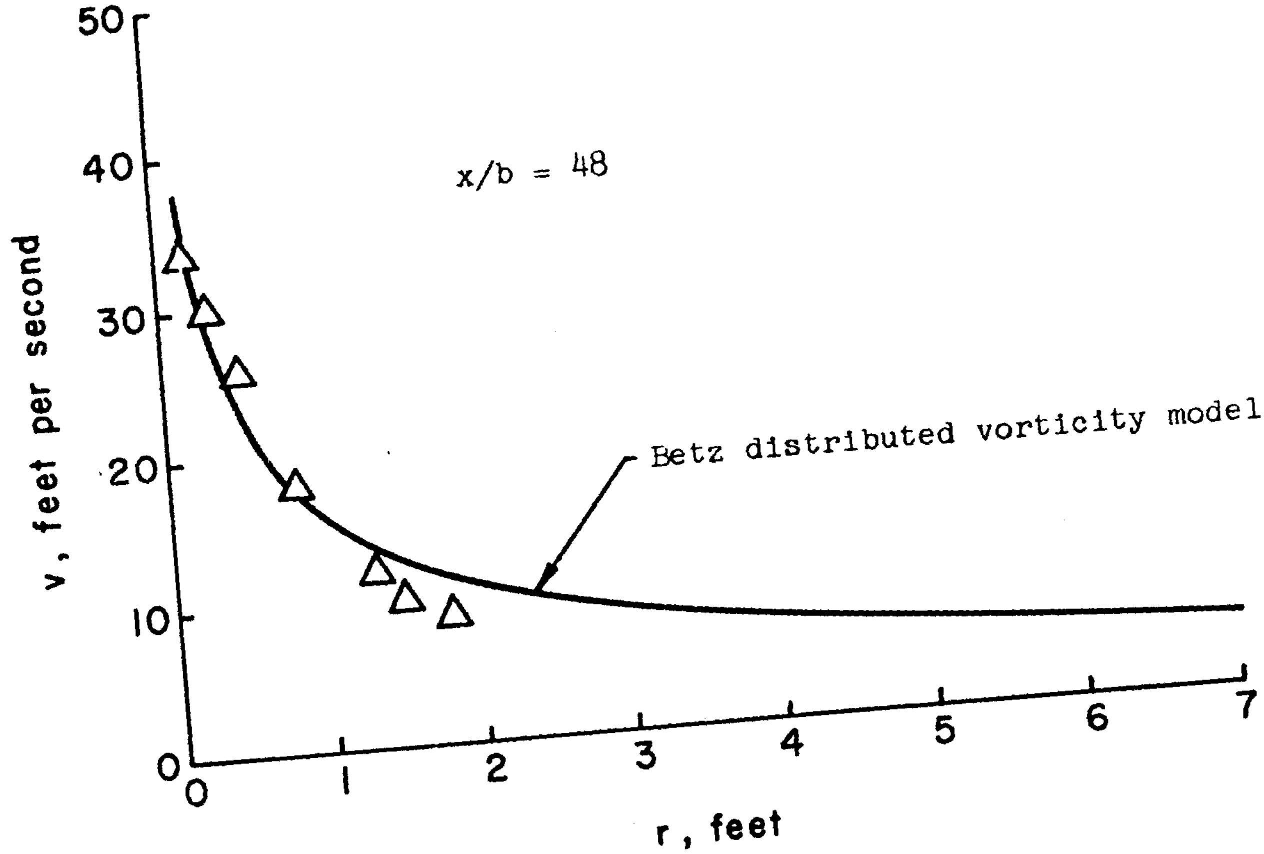

50
40
30
20
10
0
v, feet per second
x/b = 48
Betz distributed vorticity model
0
1
2
3
4
5
6
7
r, feet

Figure 2.6

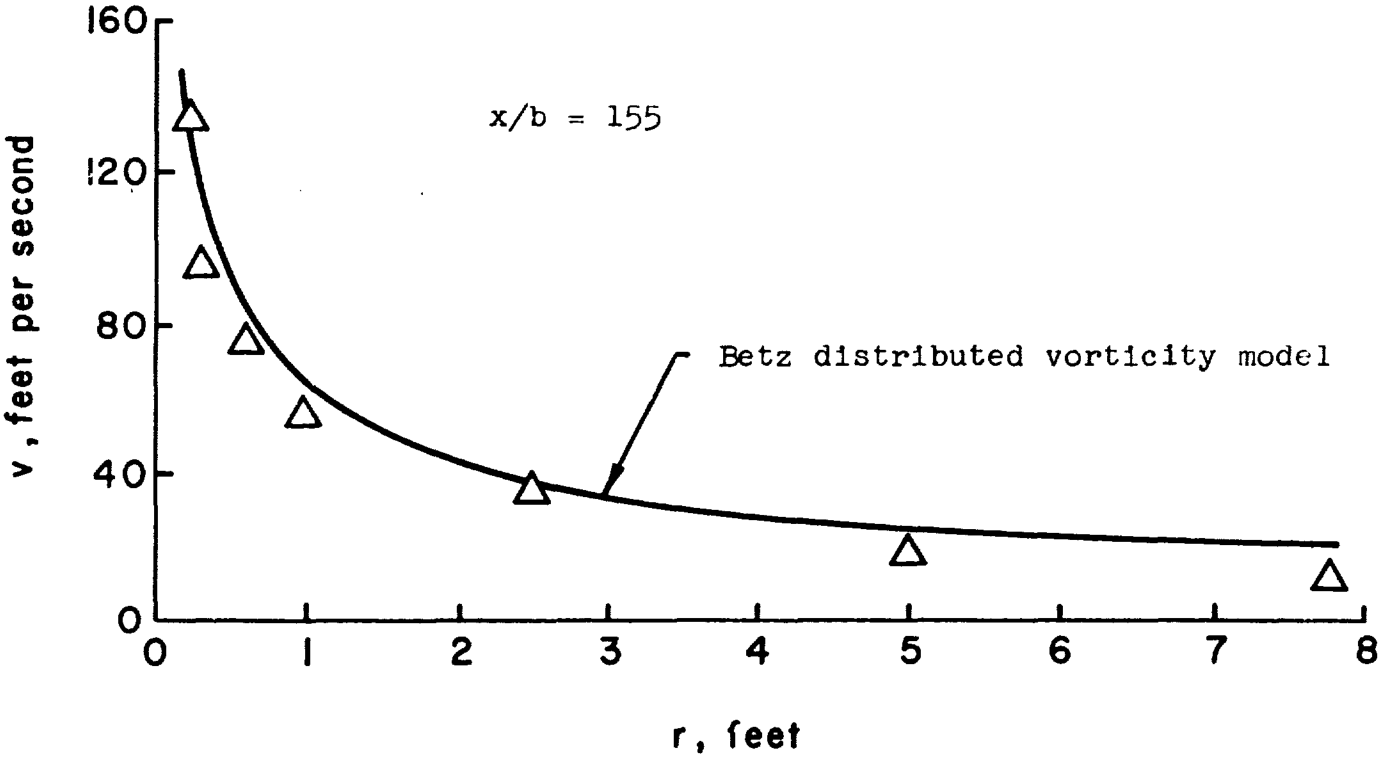

160
120
80
40
0
v, feet per second
x/b = 155
Betz distributed vorticity model
0
1
2
3
4
5
6
7
8
r, feet

Figure 2.7

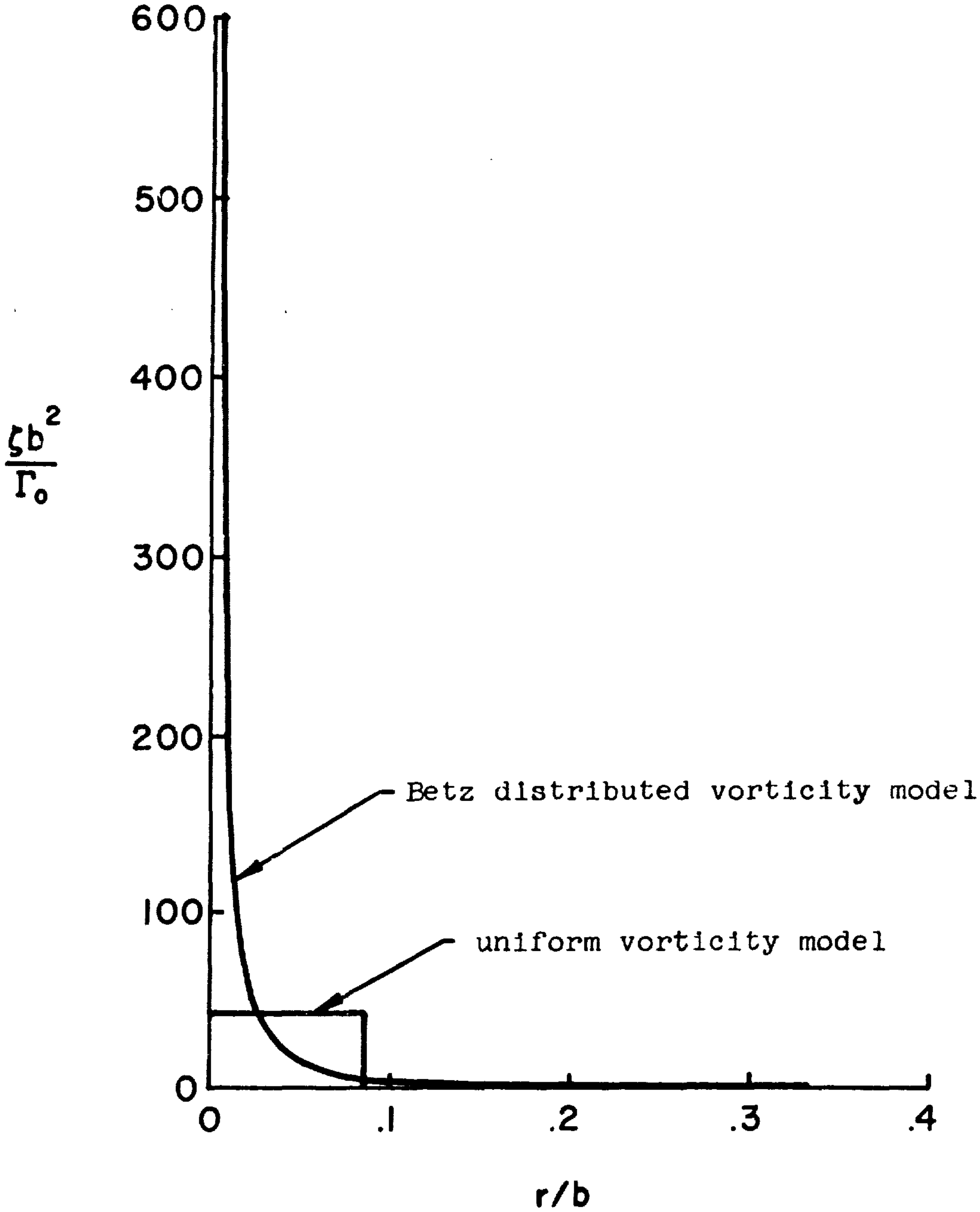

600
500
400
$\frac{\zeta b^2}{\Gamma_0}$
300
200
Betz distributed vorticity model
100
uniform vorticity model
0
0
.1
.2
.3
.4
r/b

Figure 2.8

distributed in a manner that is very far from uniform. The actual vorticity distribution follows very closely that given by the more detailed theory except that, due to viscous diffusion, the infinite vorticity associated with the integrable singularity at $r = 0$ does not exist in nature. Note that the detailed theory gives a very large spike of vorticity for $r/b < 0.05$ and a long "tail" of weak vorticity for $0.05 < r/b \leq 0.333$. It is an unfortunate fact for the experimentalist that only 54% of the total circulation $\Gamma_0$ is contained within the vorticity spike inside $r/b \leq 0.05$ and the other half of the vorticity is distributed at a very low level between $r/b = 0.05$ and $r/b = 1/3$. It is for this reason, in all probability, that several investigators have failed to account for a total circulation close to $\Gamma_0$ when integrating measured vorticity distributions. Indeed, it can be demonstrated from the roll-up theory that the further the lift distribution on the wing is from uniform loading the more widely distributed will be the vorticity (Eq. 2-3).

# III. WAKE VORTEX DECAY

It would appear from what has been presented in Section II that the general features of the formation of a wake behind a simple aircraft are reasonably well understood. There are a number of fine points in this picture which are not as well understood as one would like but, by and large, the techniques available for describing the nature of the inviscid wake are adequate for most engineering purposes. When one considers the state of knowledge concerning the decay of wakes, the situation is not so well in hand. Very little is known about the decay of aircraft wakes under general atmospheric conditions, and some of what has been published concerning vortex decay in the laboratory and its scaling to the case of large aircraft in the earth's atmosphere needs further and careful study.

What do we know? While not completely aware of all the details involved in the various modes of vortex decay, we do know that we can classify the nature of several decay mechanisms that have been identified. The four basic modes of decay that are presently known to exist are:

(a) Viscous dissipation - we include here any dissipation caused by small-scale turbulence that interacts with the velocity gradients in the vortex to produce local Reynolds stresses.

(b) Vortex instability - here we include any instability that may cause the rolled-up trailing vortex system to break up into a motion of smaller scale.

(c) Vortex breakdown - this is a phenomenon that might be considered to be an instability but is listed separately because the phenomenon has been studied for some time as a distinctive problem that can be associated with a single vortex and is not necessarily related to a vortex system. Vortex breakdown is said to occur when the vortex motion makes a sudden transition from a motion with tightly centered angular momentum and an associated axial momentum profile to a second motion in which the angular momentum is not tightly centered and the axial momentum profile takes on an appropriately different form. This transition from one possible motion that satisfies the flow invariants to another which satisfied these same invariants has been likened to a shock wave or a hydraulic jump because all these transitions satisfy a set of motion invariants that are appropriate to the particular flow under consideration.

(d) Atmospheric interaction - this category is a catch-all for the many effects due to large-scale atmospheric patterns as they interact with and have an effect on the breakdown of the organized motion represented by the inviscid wake.

Of the above-listed processes by which a trailing vortex system can be broken down, only the first two categories, viscous decay and vortex instability, have been studied in any great detail analytically. The recent work of Crow which has described an instability by which a trailing vortex system can break down into a series of loop vortices is of real significance (Ref. 9). It is most important that work be done to understand the input of atmospheric turbulence to this mechanism of instability.

For some time past, it has been assumed by many that the "normal" viscous decay of a vortex might be adequately described by some sort of turbulent process governed by eddy viscosity that was general enough in form so that one experiment could be scaled to another successfully. Alas, this concept does not appear to be an adequate tool for the trailing vortex problem. Scaling of results has not been possible and, indeed, the whole notion of the appropriateness of using an eddy viscosity concept has been questioned. During the past year, as a result of research at A.R.A.P. sponsored by the Air Force, a method has been developed which permits one to analyze the behavior of turbulent vortices. This work (Ref. 8) has been reported elsewhere and time does not permit a detailed discussion of this work at this time. This research has already shown that the use of a simple eddy viscosity model for the behavior of trailing vortices for distances close behind a generating aircraft of conventional type can lead to serious errors. Much remains to be done in this area, but it is felt that a start has been made in the right direction. Many questions remain to be answered by our continuing research efforts, including the effect of the turbulence level introduced into the vortex by the producing aircraft and by the atmosphere, the effect of exhaust gases entrained in the vortex, the effect of the axial momentum profile, and the effect of a turbulent atmospheric shear in which the vortex system may be embedded.

At this point, it should be pointed out that when hot gases from the engines are trapped in the descending vortex system shown in Figure 2.3, buoyancy forces play a major role in modifying the way in which the vortex system decays. It is also obvious that when considering the descent of such a system that the lapse rate in the atmosphere, i.e., the atmospheric stability in the meteorological sense, must be considered.

Although vortex breakdown has been studied for a number of years, it is still not well enough understood to make predictions concerning the breakdown of aircraft vortex systems. Intimately connected with the problem of vortex breakdown is the nature of the initial vortex roll-up and the behavior of the axial components of the aircraft wake. One can find from the results given in the previous section a parameter which, in a gross way, gives a measure of the degree to which the tangential and axial velocity profiles in the trailing vortex system interact. This parameter is a wake pressure coefficient defined in the following manner. One of the rolled-up vortices behind the aircraft is taken out of its environment and considered as a single isolated vortex. It is then a simple matter to compute the pressure rise $\Delta p$ from the center of the vortex to ambient conditions. This rise is given by

$$\Delta p = \int_0^\infty \frac{\rho v^2}{r}\, dr \tag{3-1}$$

where $v$ is the local tangential velocity. This may be broken into the sum of an integral in the core and an integral external to the core*

$$\Delta p = \int_0^a \frac{\rho v^2}{r}\, dr + \int_a^\infty \frac{\rho v^2}{r} \tag{3-2}$$

*For the purposes of this paper, we will use the uniformly distributed model of the vortex system to compute $\Delta p$ since this model does not need the application of a viscous core to arrive at a finite $\Delta p$ for wake classification purposes. This is legitimate here since we are not so much interested in calculating the actual pressure drop $\Delta p$ as we are in determining a number indicative of the character of the vortex wake.

If the integrals indicated in the above equations are carried out, one obtains

$$\Delta p = \rho \left( \frac{\Gamma_o}{2\pi a} \right)^2 \doteq \rho w_{max}^2 \qquad (3\text{-}3)$$

where $w_{max}$ is the velocity at the edge of the core as defined by Eq. (2-14). We note that the pressure difference given by Eq. (3-3) is a measure of the rise in pressure that will be seen by any axial component of velocity rolled up in the vortex behind an aircraft as the vortex is dissipated. Thus, if we make the $\Delta p$ we have derived nondimensional by dividing by the free stream axial dynamic pressure $\rho U^2/2$, we will have a measure of the effect the decay of the vortex may exert on the axial components of velocity in any given situation. Thus we define the pressure coefficient of the trailing vortex system of an aircraft as

$$C_p = \frac{2\Delta p}{\rho U^2} = 2 \left( \frac{\Gamma_o}{2\pi a U} \right)^2 = 2 \left( \frac{w_{max}}{U} \right)^2 \qquad (3.4)$$

In general, we would expect the effect of vortex decay to have a large effect on the axial components of velocity if $C_p$ were of the order of 1 or larger and a small effect if $C_p$ were of the order of much less than 1.

It will be seen in what follows that the variation of $C_p$ for operational aircraft varies from 0.02 to the neighborhood of one. From this we would infer that we might expect very different mechanisms of decay to be predominant at these extreme values of $C_p$. Indeed, the possibility of vortex breakdown, although it depends on more than just $C_p$, can in all probability be related to the general level of $C_p$ in many instances. This is an area of investigation that has been overlooked to date. We will return to a discussion of this point in the following section.

The last category of wake dissipation that we consider is that due to atmospheric interaction. Here we must report that even less is known at the present time than about the other three categories, if that is possible.

# IV. CLASSIFICATION OF AIRCRAFT WAKES

An attempt is made here to define, in terms of the simple parameters set forth in a previous section, the danger potential of the wake of a given aircraft. To do this in any complete way is a very complicated problem involving in a very detailed way the characteristics of the aircraft encountering a given wake. If one keeps in mind that the results given here are of a very general character and intended to serve only as a guide for more detailed consideration, then it will be seen that a rational method can be presented for ordering the danger potential of a given wake.

To order the wake severity of a given aircraft, we will calculate, with the formulas already given, the two most important wake parameters for any aircraft. To do this on some common basis, we shall, in the present instance, calculate these wake parameters for each aircraft traveling at a reference velocity and a reference density. We choose for these two reference quantities the standard sea-level density (0.00238 slugs/ft$^3$) and the velocity of 180 knots. Any other conditions might be chosen and, as will be seen, the ordering of the danger potential will not change. The only changes will be the numerical values of these parameters.

There are two things that are important when an aircraft penetrates a wake. First, one wishes to know what rotational velocities might be encountered and, second, one wants to know whether these velocities extend over a significant portion of the wing of the penetrating aircraft. For example, if there were maximum rotational velocities of the order of 60 ft/sec in a vortex but the extent of this vortex was so small that no rotational velocities larger than 5 ft/sec existed outside a region some four feet in diameter, the vortex would not pose a hazard to any flying objects save, perhaps, small drones and model aircraft. For these aircraft, the vortex would most definitely present a hazard.

To compute in a rough way the diameters for which extreme danger exists for penetrating aircraft, we need to know the diameter of that region for which the rotational velocities exceed the wing tip velocities that can be achieved by a typical airplane with full aileron deflection. From this we see that the danger area will be larger for aircraft with poor roll power and smaller for aircraft with large roll power. For the present study, we will assume that the penetrating aircraft can, at full aileron deflection, develop a rolling velocity $p$ given by the expression

$$\frac{pb}{2U} = 0.06 \tag{4-1}$$

The tip velocity at maximum roll will be

$$\frac{pb}{2} = 0.06U \tag{4-2}$$

For this study we will consider the penetrating aircraft to also be flying at 180 knots (304 ft/sec) so that a tip velocity of 18.24 ft/sec can be achieved.

To compute an area of a wake that may be dangerous, we must compute the diameter of that region surrounding the core of a vortex for which the rotational velocity exceeds 18.24 ft/sec. It should be pointed out here that while this selection is somewhat arbitrary, the results for any other velocity or $pb/2U$ that might be chosen can be scaled for the results obtained using 18.24 ft/sec.

This computation may be carried out with the aid of the distributed vorticity (Betz) model of a trailing vortex discussed in section II. For this model of a trailing vortex, we have

$$\begin{aligned} v_T &= \frac{\Gamma_o}{2\pi r}\left[6\left(\frac{r}{b}\right) - 9\left(\frac{r}{b}\right)^2\right]^{1/2} \quad ; \quad 0 < \frac{r}{b} \leq \frac{1}{3} \\ v_T &= \frac{\Gamma_o}{2\pi r} \quad ; \quad \frac{r}{b} > \frac{1}{3} \end{aligned} \tag{4-3}$$

If we define the critical or danger radius as $r_d$ , we have

$$\left(\frac{pb}{2U}\right)U = 0.06U = \frac{\Gamma_o}{2\pi b}\frac{b}{r_d}\left[6\left(\frac{r_d}{b}\right) - 9\left(\frac{r_d}{b}\right)^2\right]^{1/2} \tag{4-4}$$

for $0 < r/b \leq 1/3$ and

$$\left(\frac{pb}{2U}\right)U = 0.06U = \frac{\Gamma_o}{2\pi b}\frac{b}{r_d} \tag{4-5}$$

for $r_d/b > 1/3$ .

These equations define a radius about the center of the rolled-up vorticity in the wake for which the tangential velocities exceed the maximum wing tip roll-rate velocity that can be achieved by an airplane that might penetrate a wake at velocity U. The radius so defined can, for the case of generating aircraft and encountering aircraft traveling at the same speed, be put in the following forms:

$$\frac{r_d}{b} = \frac{6(C_L/A)^2_{gen}}{9(C_L/A)^2_{gen} + \pi^4(pb/2U)^2_{enc}} \quad ; \quad 0 < \frac{r_d}{b} \leq \frac{1}{3} \tag{4-6}$$

and

$$\frac{r_d}{b} = \frac{(C_L/A)_{gen}}{\pi^2(pb/2U)_{enc}} \quad ; \quad \frac{r_d}{b} > \frac{1}{3} \tag{4-7}$$

In these expressions, $C_L$ is the lift coefficient and A is the aspect ratio of the generating aircraft. The general behavior of $r_d/b$ as obtained from these formulas is shown in Figure 4.1.

Since the radius $r_d$ is one for which the tangential velocities in a wake exceed the roll-rate capabilities of most aircraft, we may use this radius to define an area behind any aircraft which is proportional to the area behind that aircraft for which a following aircraft having a span less than some critical span will be endangered by the wake. This danger area we define, quite arbitrarily, as

$$A_d = 2\pi r_d^2 \tag{4-8}$$

We will also define a critical span for the wake of the generating aircraft. This we define as

$$b_c = 4r_d \tag{4-9}$$

This last formula states that the critical span defined for the wake of an aircraft is twice the diameter of danger of either of the trailing vortices. One would suspect that aircraft with spans larger than $b_c$ would not encounter serious wake upsets due to lack of roll power while aircraft having spans small compared to $b_c$ could experience severe roll-control problems during wake encounter.

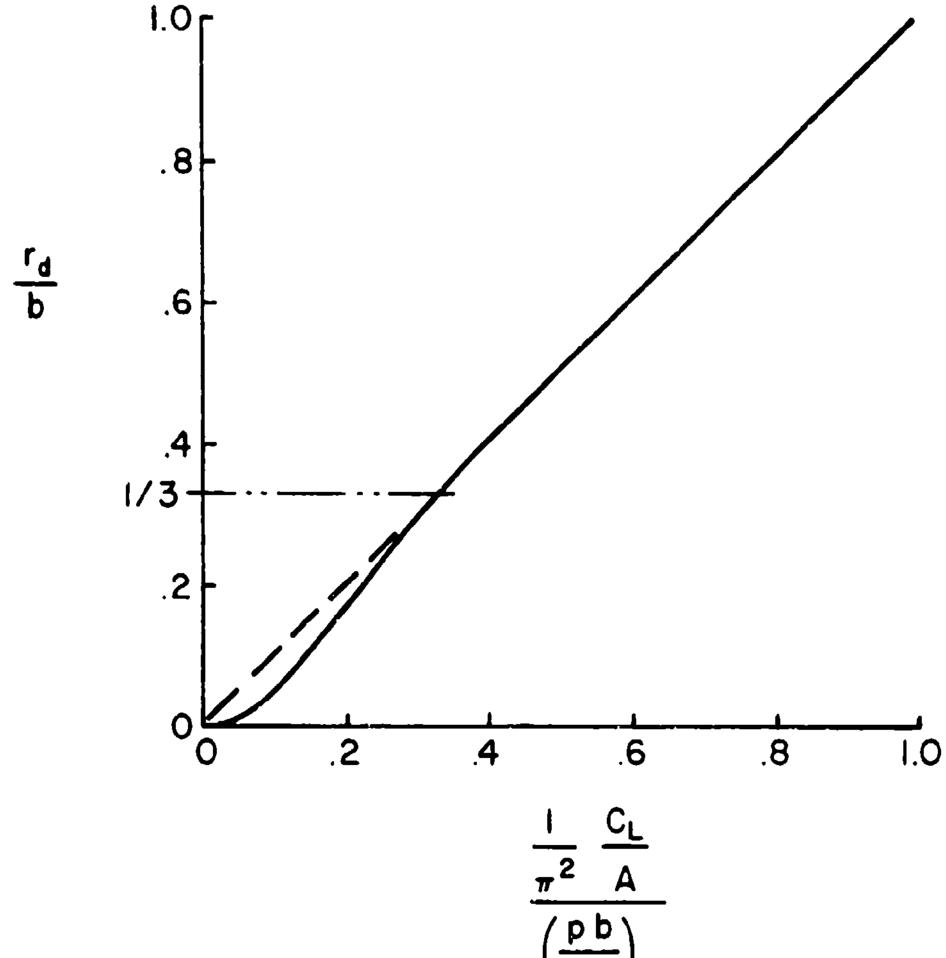

1.0
.8
.6
.4
1/3
.2
0
$\frac{r_d}{b}$
0
.2
.4
.6
.8
1.0
$\frac{\frac{1}{\pi^2}\frac{C_L}{A}}{\left(\frac{pb}{2U}\right)}$

Figure 4.1

The ideas put forward above are illustrated in Table 4.1. In this table, a large variety of aircraft have been arranged in order of decreasing wake danger potential as defined by the magnitude of the danger areas that the aircraft would develop at 180 knots. The table also presents the critical span for each wake. Listed in the table for reference are the takeoff weights of the aircraft involved and the span of each aircraft. In addition to the quantities just mentioned, we also have listed the pressure coefficient $C_p$ associated with each wake, believing, as discussed in a previous section, that this quantity is a gross measure of the class of wake decay we shall expect.

The table provides a very nice overview of the potential wake danger of a spectrum of civil and military aircraft. Several very interesting conclusions may be drawn from the results that are presented.

First, it should be noted that the late XB-70 would win the potential danger derby by a large margin over all competitors. The next most striking thing about the data is the high-danger potential ratings that are exhibited by military aircraft even though their weights are much less than civilian or cargo aircraft of equivalent rating. When we consider these military aircraft, we see that they all represent a real hazard to themselves for, in general, their spans are considerably less than the critical span defined for their wake. This may be critical to military operations

Of great interest also is the fact that the 747, the DC-10, and the supersonic transports are the first commercial or cargo aircraft which have wakes that present a possible hazard to themselves, for these are the only such aircraft whose spans are less than their wake critical spans at 180 knots.

There is another large difference between the wakes of military aircraft and the wakes of present civilian or cargo aircraft that shows up in the table. This difference is associated with the very different values of $C_p$ for the high-performance military aircraft and conventional civilian aircraft. Note in particular the very high $C_p$ values for the XB-70, the Concorde, the B-58, and the Dessault Mirage. Do we really expect the wake

| AIRPLANE | WEIGHT $W(lbs)$ | SPAN $b(ft)$ | DANGER AREA $A_d(ft^2)$ | CRITICAL SPAN $b_c(ft)$ | PRESSURE COEFF. $C_p$ |
|---|---|---|---|---|---|
| XB-70 | 530,000 | 105 | 37,600 | 310 | .53 |
| Concorde | 385,000 | 84 | 31,100 | 282 | .69 |
| SST-2000 | 480,000 | 116 | 25,500 | 255 | .29 |
| Boeing 747C | 775,000 | 196 | 23,500 | 245 | .09 |
| DC 10-20, 30 | 530,000 | 155 | 17,350 | 210 | .11 |
| C5 B | 764,000 | 223 | 16,000 | 202 | .05 |
| C5 A | 728,000 | 223 | 13,600 | 186 | .05 |
| B58-A | 160,000 | 57 | 11,700 | 173 | .56 |
| L 1011 | 409,000 | 155 | 8,700 | 149 | .07 |
| DC8-63 | 350,000 | 148 | 7,800 | 141 | .06 |
| Boeing 707-320C | 332,000 | 146 | 7,300 | 136 | .06 |
| F-111A (w.closed) | 70,000 | 32 | 7,200 | 135 | 1.06 |
| Dassault M.IV-A | 69,700 | 39 | 4,800 | 110 | .49 |
| C 141 | 316,600 | 160 | 4,500 | 107 | .03 |
| Boeing 727 | 160,000 | 108 | 2,800 | 84 | .04 |
| Douglass C-133B | 300,000 | 180 | 2,300 | 76 | .02 |
| McD. F-4M | 46,000 | 38 | 2,100 | 73 | .22 |
| Electra Or. 185 | 127,000 | 100 | 1,900 | 70 | .04 |
| F-111A (w. open) | 70,000 | 63 | 1,800 | 68 | .07 |
| Boeing 737-200 | 93,500 | 89 | 1,200 | 55 | .03 |
| DC 9 | 90,800 | 87 | 1,200 | 65 | .03 |
| Hercules C 130E | 155,000 | 133 | 1,000 | 51 | .02 |
| LTV F-8C | 28,000 | 36 | 900 | 48 | .11 |
| X-15 | 17,100 | 22 | 900 | 48 | .29 |
| Lockheed Jet Star | 40,900 | 54 | 730 | 43 | .04 |
| XV-6A (STOL) | 15,500 | 23 | 670 | 41 | .17 |
| Gulfstream II | 54,000 | 69 | 660 | 41 | .03 |
| Northrop T-38 | 11,800 | 25 | 330 | 29 | .08 |
| Dassault Falcon 20 | 25,500 | 51 | 230 | 24 | .02 |
| Sabreliner T-39 | 17,700 | 44 | 130 | 18 | .02 |
| Lear Jet 23 | 12,500 | 36 | 110 | 17 | .02 |
| N.A. OV-10A | 7,600 | 30 | 50 | 11 | .02 |

Table 4.1 Characteristics of the Wakes of Representative Aircraft at Sea Level at 180 Knots

of the Concorde with a $C_p$ of 0.69 to behave in the same general way as the wake of the C-141 with a $C_p$ of 0.03? For some reason, this particular parameter of wake decay has not, to this date, been part of any systematic investigation that the author knows of. This is most unfortunate. Indeed, for military aircraft if, say, the wake tended to dissipate faster if the $C_p$ of a wake was increased from 0.5 to 0.6, one might wish, for safety's sake, to slow aircraft down on approach so as to allow them to have a $C_p$ above some critical value. In this way, more aircraft might be able to use a given airstrip per unit time. This same possibility holds true for supersonic bombers and transport aircraft.

In connection with the above comments concerning the effect of $C_p$, the F-111A is an interesting aircraft. Note that by changing the wing sweep configuration of this aircraft $C_p$'s ranging from 0.07 to the order of 1 can be obtained. This aircraft would seem an almost ideal vehicle to use in connection with a preliminary study of the general way in which the character of wake decay is related to the pressure coefficient we have defined.

It is also interesting to note that the B-58 represents a fairly good model of the Concorde from the point of view of wake characteristics and might be used to study the expected wake characteristics of the Concorde before that aircraft is available to us for wake testing.

# V. USE OF METHOD OF CLASSIFICATION

The author must point out here that the numbers presented in Table 4.1 are not to be taken as exact. They are intended for purposes of classification only. They could be made more exact by more detailed calculation but that is not the point. What is important is that the wakes of aircraft cannot be defined unless, at the very least, three parameters are defined: a danger area, a critical span, and a pressure coefficient. It is certain that the details of wake decay will depend on many other parameters, as has been pointed out in a previous section. The behavior of wake decay is most important from the point of view of aircraft operations, for a knowledge of wake decay in conjunction with a knowledge of wake danger area determines a volume of dangerous air behind any given aircraft. It is this volume of dangerous air that is directly related to the probability of a severe wake encounter.

While discussing the applicability of the results given in Table 4.1, it should be emphasized that this table only considers the problem of loss of roll control during wake encounter. It should be obvious that a similar table could be constructed for gust loading encounters due to perpendicular wake penetration.

# VI. SUMMARY AND RECOMMENDATIONS

In the present paper, the author has reviewed very briefly the applicability of existing wake theories to the calculation of wake roll-up. The shortcomings of the uniformly distributed vorticity model were discussed and the ability of a more detailed (and apparently not well-known) model of wake roll-up due to Betz to predict the general characteristics of wakes was demonstrated. Three suggestions are in order in view of the general success of the Betz method; they are

1. The model should be extended to the case of nonelliptic load distributions.
2. A method of putting viscous diffusion into this model should be developed so that finite core velocities can be computed.
3. Once such a method is developed, a more accurate computation of wake pressure coefficients than is presently possible should be carried out.

In a prior section of this paper a method of classifying aircraft wakes with regard to the roll control problem was suggested. This classification shows that a minimum of three parameters are needed to define the danger of a given aircraft wake. The parameters are

1. a potential danger area;
2. a critical span;
3. a pressure coefficient.

Since the effect of wake pressure coefficient on wake decay has not yet been carefully evaluated, it is suggested that tests be run using an aircraft of variable sweep (F-111) to establish the general effect of wake pressure coefficient on the nature of wake decay.

Finally, it was pointed out that the operational hazard of aircraft wakes depends on a volume of dangerous air behind each aircraft. The length that, in conjunction with the danger area defined in this paper, determines this volume is the vortex decay length. Research must continue on the nature of wake decay in order to determine this length. Of particular importance here is the effect of atmospheric shear and stability on wake decay.

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