AD

AD 748089

REPORT NO. RD-TR-72-7

A STUDY OF THE COST EFFECTIVENESS OF TAPS

by

Nancy R. Rich and Latricha Greene

D D C
RECEIVED
SEP 14 1972
D

June 1972

Approved for public release; distribution unlimited.

U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

17

MSMI.1021, 1 DEC 65 PREVIOUS EDITION IS OBSOLETE

DISPOSITION INSTRUCTIONS

Destroy this report when it is no longer needed. Do not return it to the originator.

ACCESSION for
NTIS
DDC
UNANNOUNCED
JUSTIFICATION
BY
DISTRIBUTION/AVAILABILITY CODES
Dist. AVAIL. and/or SPECIAL

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

TRADE NAMES

Use of trade names or manufacturers in this report does not constitute an official indorsement or approval of the use of such commercial hardware or software.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Aeroballistics Directorate Directorate for Res, Dev, Eng & Msl Sys Lab US Army Missile Command Redstone Arsenal, Alabama 35809	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP NA

3. REPORT TITLE

A STUDY OF THE COST EFFECTIVENESS OF TAPS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Report

5. AUTHOR(S) *(First name, middle initial, last name)*

Nancy R. Rich
Latricha Greene

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 June 1972	12	0

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. AMC Management Structure Code No. 202254.00000 d.	9a. ORIGINATOR'S REPORT NUMBER(S) RD-TR-72-7 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* AD_____

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None.	Same as No. 1

13. ABSTRACT

This report presents an investigation into the use of TAPS (Trajectory Accuracy Prediction System) on the PERSHING missile. The objective of the study was to determine whether the expected number of missiles saved through the use of TAPS would justify its developmental cost.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Trajectory Accuracy Prediction System						
PERSHING missile						
Cost-effectiveness study						

1 June 1972 Report No. RD-TR-72-7

A STUDY OF THE COST EFFECTIVENESS OF TAPS

by
Nancy R. Rich and Letricha Greene

AMC Management Structure Code No. 202254.00000

Approved for public release; distribution unlimited.

Aeroballistics Directorate
Directorate for Research, Development, Engineering
and Missile Systems Laboratory
U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

ABSTRACT

This report presents an investigation into the use of TAPS (Trajectory Accuracy Prediction System) on the PERSHING missile. The objective of the study was to determine whether the expected number of missiles saved through the use of TAPS would justify its developmental cost.

ACKNOWLEDGMENT

The authors wish to acknowledge the able guidance provided by Mr. Carl A. Pinyerd, formerly of the PERSHING Project Office. Appreciation is also given to Mr. Albert S. Graham, Directorate for Research, Development, Engineering and Missile Systems Laboratory, and to Mr. James L. Flinn, of the PERSHING Project Office, for their review of and suggestions for the report.

CONTENTS

		Page
A.	Introduction	1
B.	Assumptions	1
C.	Probability of a Successful Mission	2
D.	Number of Missiles Fired	3
E.	Cost Effectiveness Calculations	5
F.	Summary	8

NOTATIONS

EN	Expected total number of missiles fired
ET	Expected total number of targets killed
M	Number of missiles available for the target
N_M	Number of M-missile targets
P_{fb}	Probability that a flight will be bad, i.e., fail to destroy the target
P_{fg}	Probability that a flight will be good
P_s	Probability of successfully destroying the target
P_{tbg}	Probability that TAPS will call a flight good when it is a bad flight
P_{tgg}	Probability that TAPS will call a flight good given that it is indeed a good flight
X	Probability that TAPS will not call a flight good

A STUDY OF THE COST EFFECTIVENESS OF TAPS

A. INTRODUCTION

The PERSHING Project Office is currently investigating the feasibility of a new telemetry package, the Trajectory Accuracy Prediction System (TAPS). This report gives the results of a cost effectiveness study of the system.

The purpose of TAPS is to supply missile performance information to the launch point. When the position of the missile at the time of warhead separation has been determined, TAPS transmits a signal indicating whether the boost phase was within tolerance. This information is used in the decision of whether or not another missile should be fired. For a given set of targets with the minimum probability of kill specified for each target, the expected number of missiles needed will be smaller with the TAPS system than without it. For TAPS to be economically advantageous, the expected savings in missile costs must be greater than the cost of developing TAPS.

B. ASSUMPTIONS

The following assumptions were made throughout this paper.

1) The probability that any one missile will destroy the target is constant from missile to missile and is not affected by whether or not TAPS is present.

2) The various probabilities associated with TAPS are constant from system to system.

3) The target will be totally destroyed by one successful missile and completely unaffected by a failure.

4) After the research and development on TAPS is completed, the cost of installing the telemetry package on each missile will not significantly change the cost of the missile.

5) The targets are independent.

6) If the boost phase is within tolerance, the missile will be successful.

C. PROBABILITY OF A SUCCESSFUL MISSION

Without TAPS, the total number of missiles needed to meet the minimum kill probability requirements of a target are fired at the target. With TAPS installed, it is assumed the following procedure will be used: one round is fired; if and only if TAPS does not report that the flight was good, another missile is fired; if and only if TAPS does not report that missile was good, another round is fired. This procedure is repeated until either TAPS reports a good missile or all of the rounds available for the target have been fired.

Thus, with TAPS installed, when one missile is available for a target the probability of a successful mission is simply the probability that the one round will be good (P_{fg}). With two missiles available, the probability of a success with TAPS is the sum of the probability that the first flight is successful (P_{fg}) plus the probability that the first flight is a failure (P_{fb}) and TAPS does not say it is good ($1-P_{tbg}$) and the second flight is a success (P_{fg}). When three missiles are available, the probability of killing the target is the probability of killing it with either of the first two missiles plus the probability that the first two missiles will be bad $(P_{fb})^2$ and TAPS will not call them good $(1-P_{tbg})^2$ and the third missile will be good (P_{fg}). Therefore, with M missiles available for a target, the probability of successfully destroying the target with TAPS is

$$P_{s,TAPS,M} = P_{fg} \sum_{i=o}^{M-1} P_{fb}^{i} \left(1-P_{tbg}\right)^{i} \quad , \qquad (1)$$

or

$$P_{s,TAPS,M=1} = P_{fg}$$

$$P_{s,TAPS,M=2} = P_{fg} + P_{fb} \left(1-P_{tbg}\right) P_{fg}$$

$$P_{s,TAPS,M=3} = P_{fg} + P_{fb} \left(1-P_{tbg}\right) P_{fg} + P_{fb}^{2} \left(1-P_{tbg}\right)^{2} P_{fg} \quad .$$

If TAPS has not been installed and M missiles are fired at one target, then the probability of a successful mission is one minus the probability that all M flights will fail, or

$$P_{s,no\ TAPS,M} = 1-P_{fb}^{M} \quad . \qquad (2)$$

With the probability of a successful mission calculated for each target, the expected number of targets killed, ET, for either the with-TAPS or without-TAPS case, may be found by

$$ET = P_{s,M=1} N_1 + P_{s,M=2} N_2 + \dots + P_{s,M=M} N_M \quad ,$$

where $P_{s,M}$ is the probability of a successful mission using Eq. (1) or Eq. (2), respectively.

D. NUMBER OF MISSILES FIRED

For the general case, assume that M missiles are available for a target and TAPS is used. The first missile is always fired, and the second is fired when TAPS does not report the first flight as good. The probability that TAPS reports the first missile as good is $P_{fg}P_{tgg} + P_{fb}P_{tbg}$. Therefore, X, the probability that TAPS does not report the first flight as good, i APS either is silent or reports a bad flight, is

$$X = 1-\left(P_{fg}P_{tgg} + P_{fb}P_{tbg}\right) \quad . \tag{3}$$

Thus X is the probability that the second round is fired. The third round is fired when the first and second rounds are not reported by TAPS as good flights, so the probability of the third round being fired is X^2. Similarly, the probability that the Kth round is fired is X^{K-1}.

If only one missile is available (M=1), one missile will be fired with probability 1.0. For $M > 1$, the probability of firing exactly K missiles with TAPS is

$$\text{Prob fire exactly K missiles of M)} = \begin{cases} X^{K-1} - X^K, & 1 \le K \le M-1 \\ X^{M-1} & , K=M \end{cases} \quad .$$

Then, with TAPS, for a one-missile target

$$\text{Prob(1 missile is used)} = 1.0 \quad ;$$

for a two-missile target

$$\text{Prob(exactly 1 missile is used)} = 1-X \quad ,$$

$$\text{Prob(exactly 2 missiles are used)} = X \quad ;$$

for a three-missile target

$$\text{Prob(exactly 1 missile is used)} = 1-X \quad ,$$

$$\text{Prob(exactly 2 missiles are used)} = X-X^2 \quad ,$$

$$\text{Prob(exactly 3 missiles are used)} = X^2 \quad ;$$

and so on.

For an M-missile target, the expected number of missiles fired, EN_M, with TAPS is

$$EN_{M,TAPS} = 1(1-X) + 2(X-X^2) + \ldots + M\left(X^{M-1}\right) = 1 + X + X^2 + \ldots + X^{M-1} \quad .$$

Where TAPS is not used the expected number of missiles fired is simply the number of missiles available for the target.

For the total target array of N_M M-missile targets, the expected total number of missiles fired, EN, with TAPS, is

$$EN_{TAPS} = N_1\ EN_1 + N_2 EN_2 + \ldots + N_M\ EN_M \quad .$$

For example, if there are N_1 one-missile targets, N_2 two-missile targets, N_3 three-missile targets, and no targets requiring more than three missiles each, the expected total number fired with TAPS is

$$EN_{TAPS} = N_1 + N_2\ (1 + X) + N_3\ (1 + X + X^2) \quad .$$

Without TAPS, the expected total number is exactly the number of missiles available, or

$$EN_{no\ TAPS} = N_1 + 2N_2 + 3N_3 \quad .$$

Therefore, the expected savings in missiles due to the installation of TAPS is a decreasing function of X, the probability that TAPS will not call a missile good.

E. COST EFFECTIVENESS CALCULATIONS

Examples of calculations using the formulas developed in the preceding sections are described below. The following constant values were selected for these particular examples.

1) Probability that a flight will be good, P_{fg}, is 0.90.
2) Probability that TAPS will call a good flight good, P_{tgg} is 0.80.
3) Probability that TAPS will call a bad flight good, P_{tbg}, is 0.05.
4) The unit missile cost, with or without TAPS, is $750,000. The cost of developing TAPS is $10,000,000. Thus the use of TAPS must be expected to save at least 13.33 missiles for TAPS to be economically advantageous.

1. Net Savings

Using the above values, the probability that TAPS will not call a flight good is

$$X = 1-\left(P_{fg}P_{tgg} + P_{fb}P_{tbg}\right) = 0.275 \quad .$$

It is assumed that no more than three missiles will be fired at any one target. The probabilities of a successful mission for one-, two-, or three-missile targets are

$$P_{s,no\ TAPS,M=1} = 0.900$$

$$P_{s,no\ TAPS,M=2} = 0.9900$$

$$P_{s,\text{no TAPS},M=3} = 0.9990$$

$$P_{s,\text{TAPS},M=1} = 0.9000$$

$$P_{s,\text{TAPS},M=2} = 0.9855$$

$$P_{s,\text{TAPS},M=3} = 0.9936 \quad .$$

With N_1 one-missile targets, N_2 two-missile targets, and N_3 three-missile targets, the expected number of targets killed, ET is

$$ET_{\text{no TAPS}} = 0.9000\ N_1 + 0.9900\ N_2 + 0.9990\ N_3 \quad ,$$

or

$$ET_{\text{TAPS}} = 0.9000\ N_1 + 0.9855\ N_2 + 0.9936\ N_3 \quad .$$

The expected number of missiles used is

$$EN_{\text{no TAPS}} = N_1 + 2N_2 + 3N_3 \quad ,$$

or

$$EN_{\text{TAPS}} = N_1 + 1.275\ N_2 + 1.351\ N_3 \quad .$$

TAPS will be cost effective when the expected net savings

$$E(\text{net savings}) = \$543{,}750\ N_2 + \$1{,}237{,}031\ N_3 - \$10{,}000{,}000 \quad ,$$

has a positive value. The region in which TAPS is expected to effect a cost reduction is shown in Fig. 1. The expected net savings as a function of N_2, for various values of N_3, shown in Fig. 2.

To illustrate a net savings calculation, consider an array of 100 targets divided as follows: 60 targets, each with a minimum

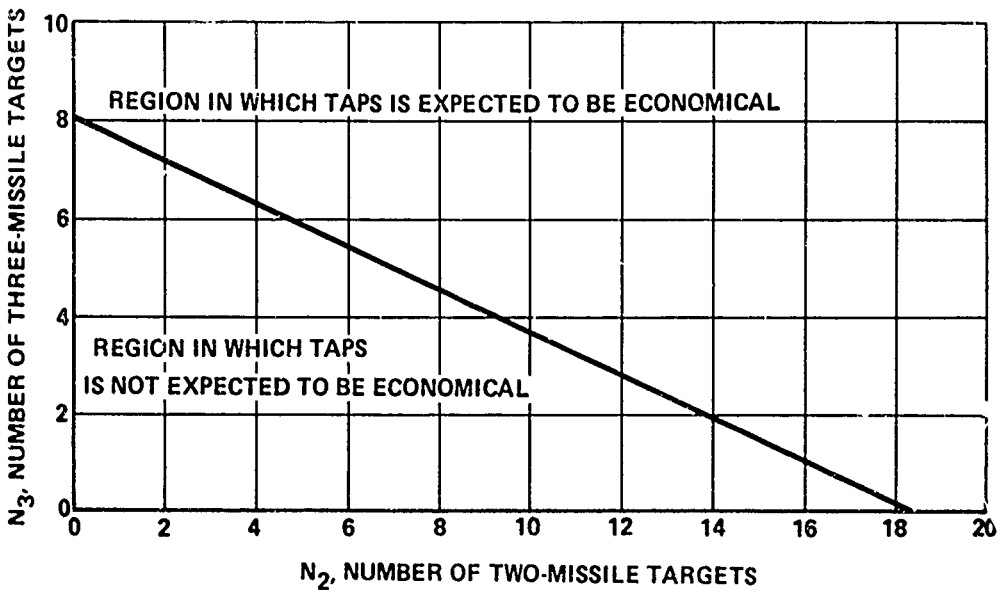

Figure 1. Cost Effectiveness Regions

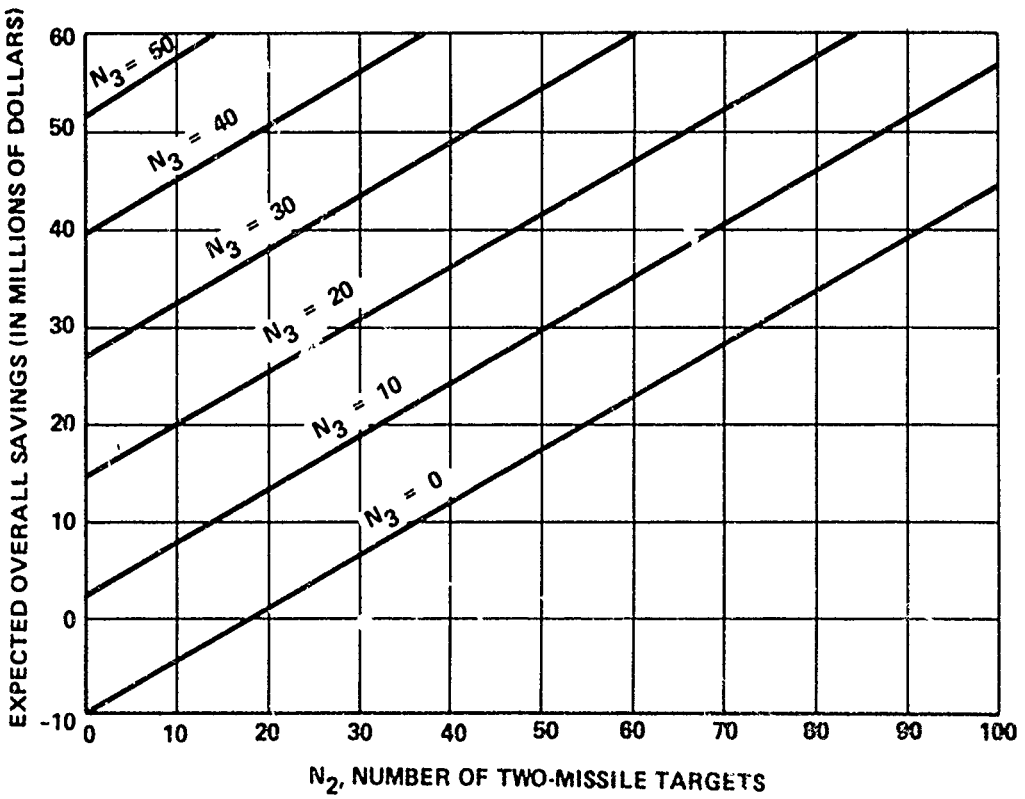

Figure 2. Expected savings from TAPS

probability of kill of 0.90; 30 targets, each with minimum probability of kill of 0.95; and 10 targets, each with a probability of kill of 0.98. For this case there are 60 one-missile targets and 40 two-missile targets. If TAPS is not used, 140 missiles will be fired; with TAPS, the expected number of missiles used is 111, and the expected net savings is $11,750,000.

2. Additional Considerations

It should be noted that the discussion has centered on expected numbers of missiles used and not upon the number that should be manufactured. Due to the randomness of the number of missiles that would be fired and the high risk entailed by a shortage, the number of missiles should be greater than the number expected to be used. The minimum "safe" number of missiles deployed depends upon the arrangement of the launch sites. For instance, if each of the two-missile targets in the above example requires a corresponding launch site, then two missiles must be set up at each site. Along with the 60 missiles for the one-missile targets, this would result in a total of 140 missiles manufactured although only 111 are expected to be used. On the other hand, if all of the missiles are located in one launch site, then the number can be reduced. For the above target array there is a 0.9999 probability that only 123 missiles will be used. Thus, if every missile can be assumed to be in working condition at the time of firing, only 123 missiles need be manufactured.

On the other hand, if a fixed number of missiles are manufactured, the use of TAPS would permit the designation of alternate, secondary targets. Those missiles allocated to but not required for the primary targets could be fired at secondary targets. Thus, for a given number of missiles, the use of TAPS would increase the expected number of targets destroyed.

F. SUMMARY

To determine whether TAPS would be expected to be cost effective for a particular target array, the following steps should be taken:

1. Obtain the best available estimates of the probability of a good flight (P_{fg}), the probability that TAPS will call a good flight good (P_{tgg}), and the probability that TAPS will call a bad flight good (P_{tbg}). Evaluate the cost of a missile and the developmental cost of TAPS.

2. Calculate the probability that TAPS will not call a flight good (X) and the probability of successful missions with and without

TAPS for the necessary values of M ($P_{s,no\ TAPS,M}$), and ($P_{s,TAPS,M}$). From these probabilities determine the number of M-missile targets (N_M).

3. Calculate the numbers of missiles expected to be fired with and without TAPS ($EN_{no\ TAPS}$ and EN_{TAPS}). Multiply the difference between these numbers by the cost of a missile and subtract the developmental cost of TAPS to obtain the expected net savings.