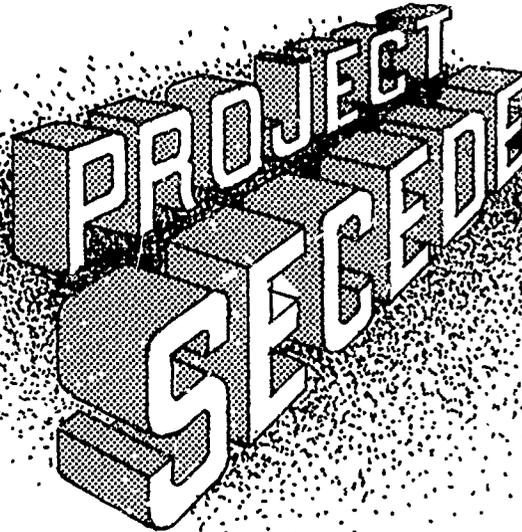


RADC-TR-72-147  
Technical Report  
April 1972



Prepared By  
Rome Air Development Center  
Air Force Systems Command  
Griffiss Air Force Base, New York 13440

AD 746060

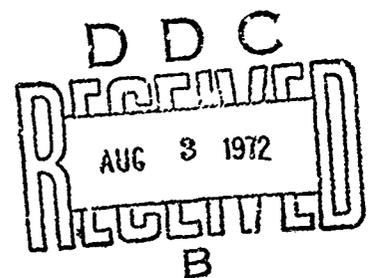


GENERALIZED SLAB MODEL AND  
FINITE LARMOR RADIUS EFFECTS

University of Rochester

Sponsored by  
Defense Advanced Research Projects Agency  
ARPA Order No. 1057

Approved for public release;  
distribution unlimited.



The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the U. S. Government.

ACCESSION for	
RTIS	White Section <input checked="" type="checkbox"/>
DSC	Bulk Section <input type="checkbox"/>
UNA: OFFICES	<input type="checkbox"/>
JUSTIFICATION .....	
BY .....	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	GEN. and/or SPECIAL
A	

Do not return this copy. Retain or destroy.

## DOCUMENT CONTROL DATA - R &amp; D

*Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified*

1. ORIGINATING ACTIVITY (Corporate author) University of Rochester River Campus Rochester, New York 14627		2a. REPORT SECURITY CLASSIFICATION <b>Unclassified</b>	
		2b. GROUP	
3. REPORT TITLE GENERALIZED SLAB MODEL AND FINITE LARMOR RADIUS EFFECTS			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Semi-Annual Technical Report 15 September 1971-14 March 1972			
5. AUTHOR(S) (First name, middle initial, last name) Jeng-Nan Shiau Albert Simon			
6. REPORT DATE APRIL 1971		7a. TOTAL NO. OF PAGES 15	7b. NO. OF REFS 9
8a. CONTRACT OR GRANT NO. F30602-70-C-0001		9a. ORIGINATOR'S REPORT NUMBER(S)	
b. ARPA Order No. 1057			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		RADC-TR-72-147	
10. DISTRIBUTION STATEMENT  Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES Monitored By Leonard Strauss (OCSE) RADC/GAFB, NY 13440		12. SPONSORING MILITARY ACTIVITY Advanced Research Projects Agency 1400 Wilson Bldg. Arlington VA 22209	
13. ABSTRACT  An equilibrium slab model of a barium cloud is derived. This model has finite length in the magnetic field direction and allows one to study the stability of configurations with various degrees of ionospheric short-circuiting. It reduces to previous models for the case of no short-circuiting but is quite different in the more general case.  The combined influence of finite Larmor radius corrections and inertia on the stability of the infinite slab model is studied. A simple dispersion relation is obtained.			

I

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Striations 2. E x B Instability 3. Cross Field Instability 4. Gradient Drift Instability 5. Barium Clouds						

*IL*

GENERALIZED SLAB MODEL AND  
FINITE LARMOR RADIUS EFFECTS

Jeng-Nan Shiau  
Albert Simon

Contractor: University of Rochester  
Contract Number: F30602-70-C-0001  
Effective Date of Contract: 1 May 1969  
Contract Expiration Date: 14 October 1972  
Amount of Contract: \$105,063.00  
Program Code Number: 1L

Principal Investigator: Prof. Albert Simon  
Phone: 716 275-4073

Project Engineer: Vincent J. Coyne  
Phone: 315 330-3107

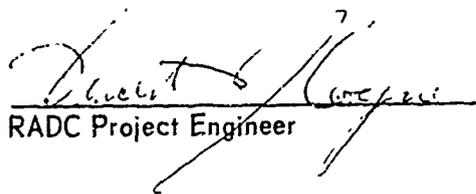
Contract Engineer: Leonard Strauss  
Phone: 315 330-3451

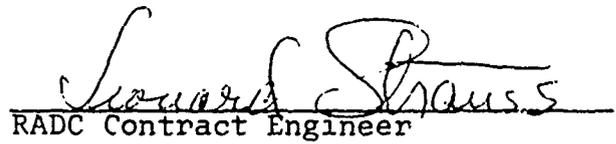
Approved for public release;  
distribution unlimited.

This research was supported by the  
Defense Advanced Research Projects  
Agency of the Department of Defense  
and was monitored by Leonard Strauss  
RADC (OCSE), GAFB, NY 13440.

PUBLICATION REVIEW

This technical report has been reviewed and is approved.

  
RADC Project Engineer

  
RADC Contract Engineer

## ABSTRACT

An equilibrium slab model of a barium cloud is derived. This model has finite length in the magnetic field direction and allows one to study the stability of configurations with various degrees of ionospheric short-circuiting. It reduces to previous models for the case of no short-circuiting but is quite different in the more general case.

The combined influence of finite Larmor radius corrections and inertia on the stability of the infinite slab model is studied. A simple dispersion relation is obtained.

TABLE OF CONTENTS

	Page
Abstract.....	iii
List of Illustrations.....	vii
I. INTRODUCTION.....	1
II. SLAB MODEL IN A FINITE CONDUCTING BACKGROUND..	4
III. ION INERTIA AND FINITE LARMOR RADIUS EFFECTS..	10
References.....	15

Preceding page blank

LIST OF ILLUSTRATIONS

Fig. 1    Finite Slab in a Magnetic Field.

Preceding page blank

## I. INTRODUCTION

In previous semi-annual technical reports, we have developed a three-dimensional model of the equilibrium (i.e. pre-striation) growth of a barium cloud in the presence of a conducting background ionosphere. Early studies<sup>(1,2)</sup> considered the case of a highly conducting background with complete short-circuiting. In this case there is no polarization of the cloud and a diffusing elliptical gaussian density results. Later work<sup>(3,4)</sup> extended these results to include finite background conductivity with polarization effects and steepening of the backside.

Stability calculations<sup>(5)</sup> have been carried out (in the case of the highly conducting background) by studying the growth of small perturbations about the equilibrium developed above. These numerical calculations<sup>(4)</sup> have shown that the fastest growing eigenmode is concentrated on the rear side of the cloud. Computational difficulties have not yet enabled one to establish the stability threshold (or whether one actually exists) nor has it yet yielded a grid-independent shape of the eigenmode. Further work on this point is in progress.

Meanwhile, some information can be gleaned from slab models, although there are troublesome questions concerning the validity of application to real three-dimensional clouds. The E X B instability was first derived in 1963 using such a model by Simon<sup>(6)</sup> and by Hoh<sup>(7)</sup> with  $\vec{E}$  parallel to the density gradient. This analysis yielded a threshold value of  $E$  and a critical wavelength which was the first mode to go unstable (and the fastest growing at any later "linear" stage). Linson and Workman extended this analysis to include a component of the field perpendicular to the density gradient. In their treatment, they take the limit of collisionless electrons ab initio with the result that there is no threshold for instability onset and all wavelengths grow at about the same rate. This result is disturbing because it seems at variance with observed time-delay for striation onset and because the analysis gives no guide for predicting the size of observed striations.

There are a number of effects which could restore a threshold to the slab models above. These include effects of ion inertia and finite larmor radius which are discussed in Sect. III below. Another effect is the internal equilibrium electric field that develops in the plasma. In

the infinite slab model, such as that of Linson and Workman, this necessarily points towards the center of the slab in order to keep ions from diffusing out faster than electrons. This field direction is destabilizing on the basis of the Simon-Hoh model. In Sect. II, we develop a model of a slab with finite extent along the magnetic field and allow for a background of arbitrary conductivity. The resulting field can either point in or point out depending on the degree of short-circuiting by the background. A stability analysis of this more general slab model is now in progress.

## II. SLAB MODEL IN A FINITE CONDUCTING BACKGROUND

The model we consider is sketched in Figure 1. The cloud is represented by a slab of finite extent along the magnetic field. We shall assume no variation of the equilibrium quantities in the  $y$  direction,  $\nabla \times \underline{\underline{E}} = 0$  then implies  $E_y = \text{const.} = E_{0y}$

As shown in a previous report (3), if we expand the current divergence equation

$$\nabla \cdot \underline{\underline{J}} = 0 \quad (1)$$

in the small parameter  $D_{||}^+ / D_{||}^-$  we obtain in the zeroth order approximation, the electric field

$$\underline{\underline{E}}^{(0)} = -A \frac{\nabla N}{N} - \nabla g(r_{\perp}) \quad (2)$$

where  $N$  is the equilibrium density.

To determine  $g(r_{\perp})$  we shall integrate eqn. (1) along a magnetic field line from one boundary of the ionosphere to the other boundary. Using the condition that the current vanishes at the boundaries, we have

$$\int_{-L}^L \nabla_{\perp} \cdot \underline{\underline{J}}_{\perp} dz = 0 \quad (3)$$

In the large  $(\Omega\tau)_{\perp}$  limit

$$\frac{1}{e} \nabla_{\perp} \cdot \underline{\underline{J}}_{\perp} = -D_{\perp}^+ \nabla_{\perp}^2 N + \mu_{\perp} \nabla_{\perp} (N \underline{\underline{E}}_{\perp}) - \mu_{\perp}^2 (N \Omega_{\perp})' \underline{\underline{b}} \cdot \nabla N \times \underline{\underline{E}} \quad (4)$$

where the symbols all have the same meaning as in previous reports.

If we substitute eqn. (2) into eqn. (4) and integrate the equation along a field line, we find

$$\begin{aligned}
 & - (D_I^+ + A\mu_I^+) \frac{\partial^2 \tilde{N}_c}{\partial x^2} - \mu_I^+ \frac{\partial}{\partial x} \left( \tilde{N}_c \frac{\partial \varphi}{\partial x} \right) - \mu_I^+ (\Omega T)_+^{-1} E_{oy} \frac{\partial \tilde{N}_c}{\partial x} \\
 & - \frac{1}{e} \sum_A^P \frac{\partial^2 \varphi}{\partial x^2} = 0
 \end{aligned} \tag{5}$$

In arriving at eqn. (5) we set  $N = N_c + N_A$  where  $N_A$  is the ambient density, and have defined

$$\begin{aligned}
 \tilde{N}_c &= \int_{-L/2}^{L/2} N_c dz \\
 \sum_A^P &= e \int_{-L}^L \mu_I^+ N_A dz
 \end{aligned}$$

We also assume that  $(\Omega T)_+$  is independent of  $z$  within the length of the cloud.

Integrating eqn. (5), we find

$$\begin{aligned}
 & - (D_I^+ + A\mu_I^+) \frac{\partial \tilde{N}_c}{\partial x} - \left( \mu_I^+ \tilde{N}_c + \frac{\sum_A^P}{e} \right) \frac{\partial \varphi}{\partial x} \\
 & - \mu_I^+ (\Omega T)_+^{-1} E_{oy} \tilde{N}_c = \text{const.}
 \end{aligned}$$

To evaluate the constant, we consider a field line far away from the cloud. Where  $\tilde{N}_c = 0$ ,  $N = N_A$  and  $\frac{\partial \varphi}{\partial x} = -E_{cx}$  we find

$$\text{const.} = \frac{1}{e} \sum_A^P E_{cx}$$

We have then

$$\frac{\partial \phi}{\partial x} = - \left( \frac{D_{\perp}^{\dagger}}{\mu_{\perp}^{\dagger}} + A \right) (1-\alpha) \frac{1}{\tilde{N}_c} \frac{\partial \tilde{N}_c}{\partial x} - \alpha E_{0x} - (1-\alpha) \frac{E_{0y}}{(\Omega\tau)_{+}} \quad (6)$$

where

$$\alpha(x) = \frac{\sum_A^P}{L \left[ \mu_{\perp}^{\dagger} \tilde{N}_c(x) + \sum_A^P \right]} \quad (7)$$

is the ratio of Pederson conductivity integrated along a magnetic field line not passing through the cloud to that integrated along a field line through the cloud.

It follows from eqns. (2) and (6) that

$$E_x^{(0')} = -A \frac{1}{N} \frac{\partial N}{\partial x} + \left[ \frac{D_{\perp}^{\dagger}}{\mu_{\perp}^{\dagger}} + A \right] (1-\alpha) \frac{1}{\tilde{N}_c} \frac{\partial \tilde{N}_c}{\partial x} + \alpha E_{0x} + (1-\alpha) \frac{E_{0y}}{(\Omega\tau)_{+}} \quad (8)$$

If the background ionosphere is assumed to be infinitely conducting, then  $\alpha \rightarrow 1$  and we have

$$E_x^{(0')} = -A \frac{1}{N} \frac{\partial N}{\partial x} + E_{0x}$$

in agreement with the electric field used in earlier reports (cf. eqn. (9) of Ref. 3).

If the cloud's contribution dominates in the expression for the integrated conductivity, then  $\alpha \rightarrow 0$ . As the background density is negligible we have  $\frac{1}{N} \frac{\partial N}{\partial x} = \frac{1}{\tilde{N}_c} \frac{\partial \tilde{N}_c}{\partial x}$

and then

$$E_x^{(0')} = \frac{D_{\perp}^{\dagger}}{\mu_{\perp}^{\dagger}} \frac{1}{N} \frac{\partial N}{\partial x} + \frac{E_{0y}}{(\Omega\tau)_{+}}$$

This is the electric field used in the slab model of Linson and Workman(8).

The expression for the electric field to the next order in the expansion in the small parameter  $D_{II}^+ / D_{II}^-$  is obtained by iteration from eqns. (1) and (2). We find

$$\frac{\partial}{\partial z}(NE_z^{(1)}) = C \frac{\partial^2 N}{\partial x^2} + K \frac{\partial}{\partial x} \left( N \frac{\partial \phi}{\partial x} \right) - L E_{0y} \frac{\partial N}{\partial x} \quad (9)$$

where

$$C = [D_I^+ - D_I^- + A(\mu_I^+ + \mu_I^-)] / (\mu_{II}^+ + \mu_{II}^-)$$

$$K = (\mu_I^+ + \mu_I^-) / (\mu_{II}^+ + \mu_{II}^-)$$

$$L = [(\alpha T)_+ \mu_I^+ - (\alpha T)_- \mu_I^-] / (\mu_{II}^+ + \mu_{II}^-)$$

To find the equilibrium density, we substitute eqn. (2) into the ion continuity eqn. and obtain

$$\begin{aligned} \frac{\partial N}{\partial t} - (D_I^+ + A\mu_I^+) \frac{\partial^2 N}{\partial x^2} - (D_{II}^+ + A\mu_{II}^+) \frac{\partial^2 N}{\partial z^2} \\ - \mu_I^+ \frac{\partial}{\partial x} \left( N \frac{\partial \phi}{\partial x} \right) + \mu_I^+ (\alpha T)_+ E_{0y} \frac{\partial N}{\partial x} = 0 \end{aligned} \quad (10)$$

Making use of eqn. (6) we have

$$\begin{aligned} \frac{\partial N}{\partial t} - (D_{II}^+ + A\mu_{II}^+) \frac{\partial^2 N}{\partial z^2} - (D_I^+ + A\mu_I^+) \frac{\partial^2 N}{\partial x^2} \\ + (D_I^+ + A\mu_I^+) \frac{\partial}{\partial x} \left[ (1-\alpha) \frac{N}{N_c} \frac{\partial N_c}{\partial x} \right] \\ + \mu_I^+ \left[ E_{0x} - \frac{E_{0y}}{(\alpha T)_+} \right] \frac{\partial}{\partial x} (\alpha N) + \frac{C}{B} E_{0y} \frac{\partial N}{\partial x} = 0 \end{aligned} \quad (11)$$

If the ambient density is negligible inside the cloud, then

$N \doteq N_c$ . We obtain the solution for  $N_c$

$$N_c = \tilde{N}_c \frac{1}{\sqrt{4\pi D_{11} t}} \exp(-z^2/4D_{11} t) \quad (12)$$

where  $D_{11} = D_{11}^+ + A\mu_{11}^+$

and  $\tilde{N}_c$  is a solution of

$$\begin{aligned} \frac{\partial \tilde{N}_c}{\partial t} - (D_{11}^+ + A\mu_{11}^+) \frac{\partial}{\partial x} \left( \alpha \frac{\partial \tilde{N}_c}{\partial x} \right) \\ + \mu_{11}^+ \left[ E_{0x} - \frac{E_{0y}}{(\Omega T)_+} \right] \frac{\partial}{\partial x} (\alpha \tilde{N}_c) + \frac{C}{B} E_y^0 \frac{\partial \tilde{N}_c}{\partial x} = 0 \end{aligned} \quad (13)$$

In the case of an infinitely conducting background,

$\alpha = 1$ , we have

$$\frac{\partial \tilde{N}_c}{\partial t} - (D_{11}^+ + A\mu_{11}^+) \frac{\partial^2 \tilde{N}_c}{\partial x^2} + \mu_{11}^+ E_{0x} \frac{\partial \tilde{N}_c}{\partial x} + \mu_{11}^+ (\Omega T)_+ E_{0y} \frac{\partial \tilde{N}_c}{\partial x} = 0$$

The solution for an instantaneous line source is

$$\tilde{N}_c = \frac{S_0}{\sqrt{4\pi D_L t}} \exp[-(x - v_c t)^2 / 4D_L t] \quad (14)$$

where

$$D_L = D_{11}^+ + A\mu_{11}^+$$

$$v_c = \mu_{11}^+ [E_{0x} + (\Omega T)_+ E_{0y}]$$

We see that the strong short-circuiting will allow the cloud to diffuse at the ion rate and the cloud as a whole moves with the ion drift velocity in the ambient electric field.

In the case of dominant cloud conductivity

Eqn. (13) becomes

$$\frac{\partial \tilde{N}_c}{\partial t} + \frac{C}{B} E_{0y} \frac{\partial \tilde{N}_c}{\partial x} = 0$$

and then

$$\tilde{N}_c = \tilde{N}_c \Big|_{t=0} (x - U_0 t) \quad (15)$$

The cloud as a whole moves with the velocity  $U_0 = cE_{0y}/B$ .

Had we included electron collisions, the density profile would also diffuse at the electron rate.

### III. ION INERTIA AND FINITE LARMOR RADIUS EFFECT

The results in both references 1 and 8 show that there is no threshold for instability in the limit of collisionless electrons and with no variation of the perturbation in the magnetic field direction. This cannot be generally correct in the limit of collisionless ions as well, since in that case one can remove the constant external electric field by a uniform frame change with no alteration of the plasma dynamics. It is well known that the inclusion of previously omitted inertial terms in the equations of motion is one resolution of this paradox. However, the quantitative threshold for instability onset is too low to account for cloud observations.

Another mechanism with a similar effect is inclusion of finite Larmor radius correction in the fluid equations of motion. For completeness, and because some of the techniques are unfamiliar, we have calculated the linear dispersion relation with both included.

Roberts and Taylor<sup>(9)</sup> have shown that one can include the finite Larmor radius effect by proper modification of the ion pressure tensor in the magnetohydrodynamic equation. The ion momentum equation including inertia and finite Lar-

mor radius takes the form

$$nM \frac{dV^+}{dt} = -KT \nabla n + ne \tilde{E} + ne \frac{V_x^+ B}{c} - \frac{nM V^+}{\tau} + \nu \lambda \quad (16)$$

where

$$\nu = \frac{1}{4} a^2 \Omega$$

$$\lambda_x = \frac{\partial}{\partial x} \left[ nM \left( \frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y} \right) \right] - \frac{\partial}{\partial y} \left[ nM \left( \frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y} \right) \right]$$

$$\lambda_y = -\frac{\partial}{\partial y} \left[ nM \left( \frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y} \right) \right] - \frac{\partial}{\partial x} \left[ nM \left( \frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y} \right) \right]$$

where  $a$  is the ion Larmor radius.

The ratio of the finite Larmor radius term to the collisional term is  $\Omega \tau (a^2/l^2)$  where  $l$  is some characteristic length. We shall limit our discussion to a slab model with no short-circuiting ( $\alpha=0$ ) and uniform in the  $z$  direction. The equilibrium ion velocities are

$$\begin{aligned} V_x^+ &= -D_{\perp}^+ \frac{1}{N} \frac{\partial N}{\partial x} + \mu_{\perp}^+ E_x + (n\tau)_+ \mu_{\perp}^+ E_y \\ &= \frac{c E_{0y}}{B} = U_0 \end{aligned}$$

$$V_y^+ = \mu_{\perp}^+ E_y - (n\tau)_+ \left( -D_{\perp}^+ \frac{1}{N} \frac{\partial N}{\partial x} + \mu_{\perp}^+ E_x \right) = 0$$

where we have inserted the equilibrium electric field obtained in the previous section. Note that the equilibrium velocities are constant in time and uniform in space. It is, therefore, consistent to omit the inertial and the finite Larmor radius terms in the equilibrium momentum equation.

The linearized momentum equation, in the frame moving with velocity  $U_0$ , and with  $\chi = \frac{n_1}{N}$  is:

$$-i\omega v_1^+ = -\frac{kT}{M} ik_x \chi - ik_x \frac{e}{M} \phi + \frac{e}{Mc} v_{1x}^+ B - \frac{v_1^+}{\tau} + \nu \delta \quad (17)$$

Here  $\delta_x = -\frac{1}{d} ik_y v_{1x}^+ - (k^2 + \frac{1}{d} ik_x) v_{1y}^+$

$$\delta_y = (k^2 + ik_x \frac{1}{d}) v_{1x}^+ - ik_y \frac{1}{d} v_{1y}^+$$

In arriving at eqn. (17), we assumed that  $\frac{1}{d} = -\frac{1}{N} \frac{\partial N}{\partial x}$  is constant and Fourier analyzed the perturbed quantities. It is not difficult to solve for  $v_{1x}^+$  and  $v_{1y}^+$  in terms of  $\chi$  and  $\phi$  and substitute the expressions into the linearized ion continuity equation in the primed coordinates. We find

$$\left\{ -i\omega + \left[ \omega \left( \frac{k_x}{d} - ik^2 \right) + \frac{1}{d} \left( \frac{ik_x}{\tau} + ik_y \Omega \right) + \frac{k^2}{\tau} \right] \frac{kT}{M} \frac{1}{D} \right\} \chi + \left[ \omega \left( \frac{k_x}{d} - ik^2 \right) + \frac{k^2}{\tau} + \frac{1}{d} \left( \frac{ik_x}{\tau} + ik_y \Omega \right) \right] \frac{e}{M} \frac{1}{D} \phi = 0 \quad (18)$$

here  $D = \left[ -i\omega + \frac{1}{\tau} + ik_y \frac{\nu}{d} \right]^2 + \left[ \Omega - \nu \left( k^2 + \frac{ik_x}{d} \right) \right]^2$

The inertia term and finite Larmor radius effect is negligible in the electron equation of motion. The linearized electron equation is easily seen to be

$$\left\{ -i\omega - ik_y \frac{c}{B} \frac{E_{0y}}{\Omega \tau} + ik_y \frac{kT}{e} \frac{c}{B} \frac{1}{d} \right\} \chi + ik_y \frac{1}{d} \frac{c}{B} \phi = 0 \quad (19)$$

From eqns. (18) and (19) we obtain the dispersion relation

$$\begin{aligned} \omega \left[ \frac{k_y}{\Omega d} \left\{ \left( \frac{1}{\tau} - i\omega + ik_y \frac{v}{d} \right)^2 + \left[ \Omega - \nu \left( k^2 + \frac{ik_x}{d} \right) \right]^2 - \Omega^2 \right\} \right. \\ \left. + \left( ik^2 - \frac{k_x}{d} \right) \left( \frac{1}{\tau} - i\omega \right) \right] \\ + k_y \frac{U_0}{\Omega \tau} \left[ \left( ik^2 - \frac{k_x}{d} \right) \left( \frac{1}{\tau} - i\omega \right) - \frac{k_y \Omega}{d} \right] = 0 \end{aligned} \quad (20)$$

In the limit of vanishing Larmor radius  $a=0$  (hence  $\nu=0$ ) and with the assumption  $|\omega|\tau \ll 1$ , eqn. (20) becomes:

$$\begin{aligned} \omega \left[ \frac{k_y}{\Omega d \tau} + \frac{1}{\tau} \left( ik^2 - \frac{k_x}{d} \right) \right] \\ + k_y \frac{U_0}{\Omega \tau} \left[ \frac{1}{\tau} \left( ik^2 - \frac{k_x}{d} \right) - \frac{k_y \Omega}{d} \right] = 0 \end{aligned}$$

Therefore

$$\omega = k_y U_0 \frac{\Omega \tau k_y + k_x - ik^2 d}{k_y - k_x \Omega \tau + ik^2 d \Omega \tau} \quad (21)$$

In this limit we have the result of Linson and Workman<sup>(8)</sup> when proper Doppler shift in  $\omega$  is allowed for between the moving frame and the neutral rest frame.

If we take the collisionless limit of eqn. (20) we find

$$\begin{aligned} \omega \left\{ \frac{k_y}{\Omega d} \left[ - \left( k_y \frac{v}{d} - \omega \right)^2 - 2 \Omega \nu \left( k^2 + \frac{ik_x}{d} \right) + \nu^2 \left( k^2 + \frac{ik_x}{d} \right)^2 \right] \right. \\ \left. - i\omega \left( ik^2 - \frac{k_x}{d} \right) \right\} = 0 \end{aligned}$$

Note that the dependence on  $U_0$  vanishes. If we neglect small terms of order  $(ak)^2$  and  $(a/d)^2$  or higher, the above equation divided by  $k_y^2$  reduces to

$$\omega \left\{ \omega \left[ \frac{1}{k_y^2} \left( k^2 + \frac{ik_x}{d} \right) - \frac{\omega}{k_y d \Omega} \right] - \frac{2\nu}{k_y d} \left( k^2 + \frac{ik_x}{d} \right) \right\} = 0 \quad (22)$$

There are two low frequency modes ( $\omega \ll \Omega$ ). One has  $\omega = 0$  and the other is a purely oscillating mode with  $\omega \cong 2\nu k_y / d$ . The third root has a high frequency of order  $k_y d \Omega$  and is not relevant to the present discussion.

If we define the equivalent gravitational force  $g = -U_0 / \tau$  and then take the collisionless limit of eqn. (20), neglecting small terms of order  $(ak)^2$ ,  $(a/d)^2$  and  $\omega/\Omega$  we have the dispersion relation for waves propagation perpendicular to the density gradient ( $k_x = 0$ )

$$\omega^2 - \left( 2\nu k_y \frac{1}{d} - \frac{g k_y}{\Omega} \right) \omega + \frac{g}{d} = 0 \quad (23)$$

This also agrees with that obtained by Roberts and Taylor<sup>(9)</sup> when the difference in the sign of  $\omega$  is allowed for. Threshold estimates are now being investigated.

## REFERENCES

1. A. Simon, J. Geophys. Res. 75, 6287 (1970).
2. A. Simon, RADC-TR-70-27, Feb. 1970.
3. A. Simon and A. M. Sleeper, RADC-TR-71-106, March, 1971.
4. A. Simon and A. M. Sleeper, RADC-TR-71-292, Sept. 1971, also J. Geophys. Res. to appear shortly.
5. A. Simon, RADC-TR-70-208, Sept. 1970.
6. A. Simon, Phys. Fluids 6, 382 (1963).
7. F. C. Hoh, Phys. Fluids 6, 1184 (1963).
8. L. M. Linson and F. B. Workman, J. Geophys. Res. 75, 3211 (1970).
9. K. V. Roberts and J. B. Taylor, Phys. Rev. Letters 8, 197 (1962).

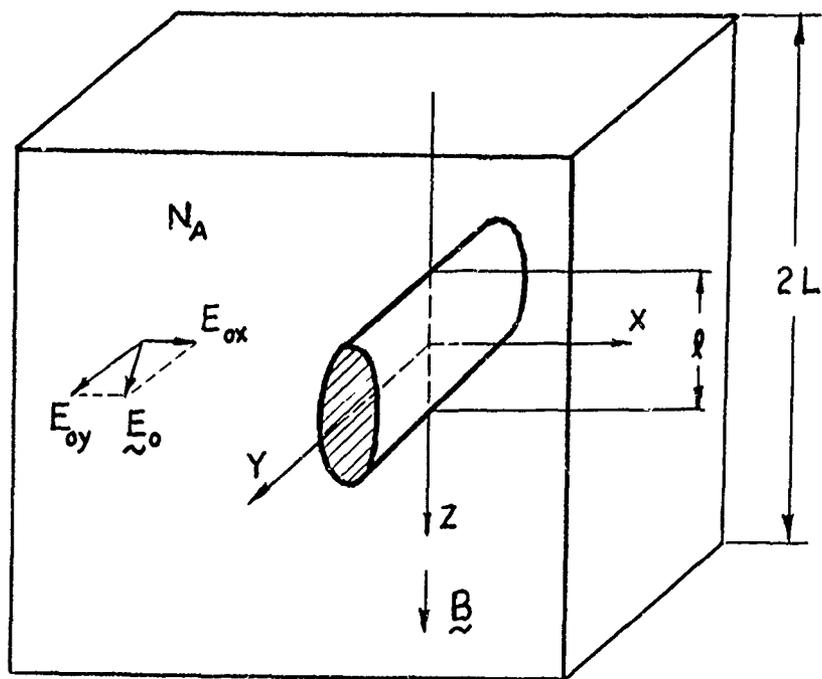


FIG. 1 FINITE SLAB IN A MAGNETIC FIELD