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Semi-Annual Technical Report Summary
on S-2td Plate Lens Research Program
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and Technical Report
IMAGING THROUGH A CONFOCAL OPTICAL INTERFERENCE FILTER
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J. A. Clark, Principal Investigator

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The problem of spatial and temporal filtering of an image by a spherical mirror type of optical interference filter is formulated. Improvements over current optical filter designs by $10^3$ in the temporal resolution of optical images appear feasible. Applications—including velocity, temperature and turbulence distribution measurements and acousto-optic image conversion—are discussed.
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The attached Technical Report (Enclosure 1) is submitted in accordance with reference (1). The report presents results of the research program being conducted according to the plan proposed in reference (2) and is intended to document technical progress previously reported in references (3) and (4). (The report has been accepted for publication in the Journal of Optics Communications.)
A summary of activities in the research program is given below.

The purpose of this research program is to improve the operation of high resolution sonar systems such as those used by the Navy for ocean bottom searches and navigation. The novel approach being followed in this investigation is the substitution of a solid plate lens for both the conventional transducer array and the electronic signal processors employed in current multi-beam sonar systems.

Two significant results of this substitution are anticipated: (1) the time required to obtain detailed acoustical images of underwater objects or regions of the bottom will be greatly reduced, and (2) the external dimensions of the sonar system required to obtain a given resolution will be decreased, possibly by an order of magnitude. These results are derived in reference (2).

Objectives of this research program are to experimentally verify the new solid plate lens sonar system design principles discussed in reference (2) and to provide design knowledge for incorporating the solid plate lens into shadowgraph sonar systems. Design knowledge is understood to include procedures for manufacturing the plate lens and experimental data on its performance.

Five technical problems associated with the solid plate lens sonar system design are currently under investigation by a staff of seven researchers, and are identified below.

1. **Coupling of Sound and Plate Waves** One component of a solid plate lens is a large flat plate. A part of the plate is immersed in the sea. Sound waves, scattered by objects in the sea, are detected in the solid plate lens sonar system by transverse plate waves which are produced when the waterborne sound waves come into contact ("couple") with the immersed portion of the plate. A theoretical analysis of the coupling mechanism between sound and plate waves is being conducted to determine the conditions of optimum coupling.

2. **Materials Survey** Materials to be used in the construction of the plate lens are being selected by a review of the literature and by experimental tests.

3. **Focusing/Acoustical Processing of Plate Waves** Waves propagating on the plate are focused by changing their velocity of propagation in appropriate regions of the plate to form a (one-dimensional) acoustical image of objects in the sea in much the same way as a (cylinder) lens focuses light waves. (In a sonar system, the second dimension of an image can be obtained by time resolution.) This "acoustical processing" of the wave is equivalent functionally to the electronic processing performed in current sonar systems, but it enables the solid plate lens sonar system to process data significantly faster than currently available electronic processors. (For details see reference (2).) A preliminary laboratory model of a solid plate lens has been constructed and several experimental methods of measuring the motions of the plate associated with plate waves are being employed to investigate the focusing of the waves.

4. **Acousto-Optic Image Conversion** The acoustical image formed on the plate must be converted into an optical image by some method which does not significantly degrade the image resolution. While conducting a theoretical analysis of the
various image conversion methods proposed in reference (2), a new type of acousto-optic image convertor design particularly suited for the application was found. The new design is described in Enclosure (1). A high intensity continuous wave laser and a specially fabricated confocal optical interference filter, which have recently been obtained, are being used to experimentally test the new image convertor. In addition to its application in solid plate lens sonar systems, the new acousto-optic image convertor has advantages over currently employed methods for measuring the vibrations of underwater sound transducers. This second application will be discussed in an invited paper to be presented to the Acoustical Society this fall.

(5) Visual Detection Problem Review The operation of a solid plate lens sonar system is sufficiently different from current high-resolution, multi-beam sonar systems to warrant a review of its function in the total visual detection problem. Such a review will assist in determining the best way to adapt the solid plate lens sonar system to the operational requirements of the Navy and of other groups concerned with undersea exploration and development. The review will also assist in determining the implications of the new system design to the Department of Defense by identifying those applications where its particular advantages have the greatest practical significance. This review of the problem of visual detection of underwater objects is being initiated by an extensive literature search, an experimental comparison of acoustical and optical imaging techniques, and discussions with other investigators actively involved with the problem.

Sincerely,

J. A. Clark, Ph.D.
Principal Investigator

cc: Dr. Frank Andrews
   Electrical Engineering
Imaging Through a Confocal Optical Interference Filter

by

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ABSTRACT

The problem of spatial and temporal filtering of an image by a spherical mirror type of optical interference filter is formulated. Improvements over current optical filter designs by $10^3$ in the temporal resolution of optical images appear feasible. Applications--including velocity, temperature and turbulence distribution measurements and acoustooptic image conversion--are discussed.
INTRODUCTION

A spherical mirror type of optical interference filter is proposed. (An "optical filter" is understood to be a device through which both spatial and temporal information passes, as distinguished from a "spectral filter" which gives only spectral information about a beam of light). It will be shown that a precisely confocal optical interference filter operates spatially as a low pass filter and should achieve at least three orders of magnitude higher temporal resolving power than an optical interference filter of the plane parallel mirror type which is designed to resolve the same number of spatial resolution elements. It will also be shown that by varying the mirror spacing slightly from the confocal distance, the optical interference filter operates spatially as a tuneable bandpass filter and that such operation has the following advantages over low pass operation: (1) permits dark field imaging, (2) substantially increases spatial resolving capacity in cases where images are characterized by a narrow range of spatial frequencies, and (3) selectively enhances the contrast of details in an image according to detail size, in addition to temporally filtering the image.

A spectral filter which employed spherical mirrors in a confocal arrangement as a Fabry-Perot interferometer was first proposed and demonstrated by Connes¹. He showed that the light gathering ability (etendue) and alignment tolerances were significantly greater for the confocal mirror F-P interferometer than for the plane-parallel mirror design. Jackson² employed a confocal F-P interferometer to observe line structure with an atomic beam and obtained spectral resolving powers of 3 x 10⁷. Other theoretical and experimental studies of the operation of confocal spectral interference filters have since been reported³-⁷. However, to the author's knowledge, there has been no
previous discussion in the literature of a confocal mirror interference filter
design which permits the viewing of temporally filtered optical images.

In this paper, a brief formulation of the problem of combined spatial and
temporal filtering of an image will be given for the case of image transmission
through a confocal mirror cavity. Design formulae will be derived, a numerical
example of practical spatial and temporal resolution capacities will be presented,
and several potential applications will be discussed.

**IMAGE PROJECTION**

The principal advantage of the confocal mirror design over plane parallel
mirror designs for forming temporally filtered images is that the images pro-
duced by multiple reflections in the confocal cavity are all accurately super-
posed, while those produced in a plane-parallel cavity are not. This advantage
can be demonstrated by considering the optical interference filter designs
illustrated in Fig. 1. A real image at location $L_0$ in the confocal mirror case
(Fig. 1A) is projected to just four other locations ($L_1$, $L_2$, $L_3$, $L_4$) by an
infinite series of successive reflections at the mirrors. This filter response
is in marked contrast to the plane-parallel case (Fig. 1B) where each successive
reflection of an extended field image produces an image at a new location. The
improvement in optical filtering which results from this important advantage
of the confocal design can be derived from relations between design and image
parameters.

**DESIGN AND IMAGE PARAMETERS**

The mirror design in a confocal optical interference filter (Fig. 2) is
characterized by four parameters: the mirror radius ($r$), the mirror separation
($r + \varepsilon$) which may deviate slightly ($|\varepsilon|/100 \leq \lambda$, the wavelength of light) from
the precise confocal distance ($r$), the partial-mirror reflectivity ($R$) and the
mirror aperture diameter ($A$). The operation of a confocal optical filter
is characterized by six image parameters associated with the intensity distribution of the filtered image or their corresponding Fraunhofer diffraction patterns: the resonant temporal frequencies \( (v_r) \), the temporal resolution bandwidth \( (\Delta v_m) \), the free spectral range \( (\Delta v_f) \), the diameter of the smallest resolvable details \( (D_y) \), the diameter of the field of view \( (D_v) \) and the diameter of the largest resolvable details \( (D_L) \) (which is less than \( D_v \) for the case of spatial bandpass filtering). It is also convenient to define the instrumental finesse, \( F = \Delta v_f / \Delta v_m \) and the spatial resolving capacity (or maximum number of resolvable details), \( N = (D_v / D_y)^2 \).

**TEMPORAL FILTERING CHARACTERISTICS**

Hercher\(^5\) has reviewed both theoretical and experimental means of determining the instrumental finesse \( (F) \) of a spherical mirror Fabry-Perot interferometer and has shown that \( F \) is determined by the reflectivity \( (R) \) if the mirror surface figure is sufficiently small. For high reflectivities the following approximate relation holds\(^5\):

\[
F = \pi / 2 (1 - R) \quad (1)
\]

The transmitted intensity distribution \( (I_t) \) produced at image location \( L_0 \) by a coherent illuminating beam of initial intensity \( I_0 \) at image location \( L_0 \) as shown in Fig. 2A, is found by adding the series of waves transmitted to \( L_1 \) by multiply-reflected waves in the mirror cavity. If absorption at the mirrors is neglected, the sum of this infinite series can be shown to be\(^8\):

\[
I_t = \frac{(1/2) I_0}{1 - \left(\frac{2F}{\pi} \right)^2 \sin^2 \left(\frac{\delta}{2}\right)} \quad (2)
\]

where \( \delta \) is the phase difference between two successive transmitted waves. The initial wave intensity \( I_0 \) is divided by one-half to account for waves transmitted to location \( L_3 \).
The phase difference ($\delta$) is a function of the wavelength ($\lambda$) or frequency ($v$) and the path length difference ($d$) between two successive waves transmitted to $L_1$: 

$$\delta = 2\pi d/\lambda = 2\pi v d/c,$$

where $c$ is the velocity of light. For the case of paraxial rays

$$d = 4(r + \varepsilon)$$

and therefore the phase difference is:

$$\delta = 8\pi v(r + \varepsilon)/c$$

The temporal filtering characteristics ($v_r$, $\Delta v_f$, $\Delta v_m$) are parameters of the normalized transmitted intensity distribution ($I_t/I_0$) which is determined by the mirror parameters ($r$, $\varepsilon$ and $R$ or $F$) through Eqs. 1, 2, and 3. The resonant temporal frequencies ($v_r$) occur whenever Eq. 2 attains its maximum value ($I_t = 1/2 I_0$); i.e., whenever $\delta = 2\pi n$; (n, an integer). Hence from Eq. 3:

$$v_r = nc/4(r + \varepsilon)$$ (4)

The free spectral range ($\Delta v_f$) corresponds to the bandwidth between two successive resonant frequencies. Therefore:

$$\Delta v_f = c/4(r + \varepsilon)$$ (5)

If the frequency $v = v_r$, Eq. 2 can be replaced by the approximate relation:

$$I_t = \frac{(1/2) I_0}{1 + \left(\frac{8F(r + \varepsilon)}{c}\Delta v\right)^2}$$ (6)

where $\Delta v$ is a small frequency shift from one of the resonant frequencies ($v_r$).

The temporal bandwidth ($\Delta v_m$) of the optical filter can be determined by noting that the transmitted intensity as given by Eq. 6 falls to one-half its maximum value if $(8F(r + \varepsilon)\Delta v'/c) = 1$. Therefore:

$$\Delta v_m = 2\Delta v' = c/4F(r + \varepsilon)$$ (7)

SPATIAL FILTERING CHARACTERISTICS

A coherently illuminated object modulates the incident light beam, as shown schematically in Fig. 2B. One effect of this modulation is the formation of a Fraunhofer diffraction pattern of the image at the second mirror plane.
where the point source illumination is focused. The intensity of the diffraction pattern ($I^*$) can be determined as a function of the diffraction angle ($\alpha$) defined in Fig. 2B.

The diffraction pattern is spatially filtered by the confocal optical interference filter and therefore the detail size in the filtered image is limited to some range ($D_2$ to $D_L$). Spatial filtering occurs because spherical aberrations associated with the mirrors produce deviations in the phase difference ($\delta_1$) from the value for paraxial rays ($\delta$) given by Eq. 3. Connes analyzed the phase differences associated with non-paraxial ray propagation through a confocal mirror system. The phase difference for non-paraxial rays can be specified approximately as:

$$\delta_1 = \frac{2\pi}{\lambda} \left( \frac{\rho_1^2}{r^2} + \frac{\rho_2^2}{r^2} \right) + \delta$$

(8)

where $\rho_1$ and $\rho_2$ are the distances of the ray from the optical axis at the first and second mirror planes, respectively. For the special case of rays parallel to the optical axis ($\rho_1 = \rho_2 = \rho$), Eq. 8 can be simplified to:

$$\delta_1 = \frac{2\pi}{\lambda} \left( \frac{\rho^4}{16} + c\alpha^2 \right) + \delta$$

(9)

where $\alpha = 2\rho/r$ in the small angle approximation, as can be noted from Fig. 2B. If the input diffraction pattern intensity ($I^*_{o}$) (that is, the light intensity which would be observed at plane $M_2$ if the mirrors were removed from the optical system) is uniform, the output diffraction pattern intensity ($I^*_{o}(\alpha)$) is:

$$I^*_{o}(\alpha) = \frac{I^*_{o}}{1 - \left(\frac{2F}{\pi}\right)^2 \frac{2\delta_1}{\delta}}$$

(10)

Parameters of the intensity distribution ($I^*_{o}(\alpha)$) can be determined as follows: a central diffraction angle ($\alpha_c$) is defined as the value of $\alpha$ for which
\( \delta_0(\delta_1) / \delta_0 = 0 \). By Fermat's principle, this is the zone of best focusing. It is also the zone where the largest range of detail size is passed by the optical filter. From this relation and Eq. 9 we obtain:

\[
\alpha_c = \pm \sqrt{8e/r}
\]  

(11)

The phrase difference \( \delta_c \) at \( \alpha_c \) is from Eqs. 9 and 11:

\[
\delta_c = (2\pi/\lambda)(d - 4e^2/r)
\]  

(12)

If the intensity \( I(\alpha) \) is a maximum at \( \alpha_c \), then it falls to 1/2 intensity at the cutoff angles \( (\alpha_+, \alpha_-) \) which occur when:

\[
\delta_+ = (2\pi/\lambda)(d - 4e^2/r + \lambda/F)
\]  

(13)

From Eqs. 9 and 13, the cutoff angles are found to occur at:

\[
\pm \alpha_+ = \sqrt{(8e/r)} \pm \sqrt{16\lambda/F}
\]  

(14)

(\( \alpha_+ \) is restricted to real values.) The spatial filtering characteristics \( (D_s, D_L) \) are determined by the cutoff angles. The diameter of the smallest resolvable detail \( (D_s) \) is given by the Airy disk formula:

\[
D_s = 1.22\lambda/2\alpha_+
\]  

(15)

Similarly, the diameter of the largest resolvable detail \( (D_L) \) is:

\[
D_L = 1.22\lambda/2\alpha_-
\]  

(16)

The diameter of the field of view \( (D) \) for images located at \( (L_0 - L_4) \) is:

\[
D = A/2
\]  

(17)

where \( A \) is the diameter of the first mirror aperture as can be shown by drawing the chief ray through the optical system (Fig. 2B). However, the transmitted images will then overlap. To avoid overlapping, either the field of view can be reduced to one-half its area by a light stop as shown in Fig. 2A or, if no polarization information is contained in the image, polarization filters can be inserted as shown in Fig. 2B.
SPATIAL RESOLVING CAPACITY

The spherical mirror optical interference filter operates as a low pass spatial filter (in addition to its bandpass temporal filtering operation) if the mirror spacing is precisely confocal \((e = 0)\). The central diffraction angle \((\alpha_c)\) is then zero (Eq. 11) and the diameter of the first mirror aperture \((A)\) which just admits rays having a diffraction angle equal to or less than \(\alpha_+\) is by Eq. 14:

\[
A = r\alpha_+ = 2(\frac{\lambda r^3}{F})^{1/4}
\]

(18)

The spatial resolving capacity \((N)\) for low pass operation is obtained from Eqs. 15, 17 and 18:

\[
N = \left(\frac{D_V}{D_s}\right)^2 = \frac{16r}{(1.22)^2} \frac{\lambda F}{X_F}
\]

(19)

For comparison, the temporal resolution bandwidth of a plane parallel optical interference filter is \(\Delta \nu_m = c/2dF\), where \(d\) is the mirror separation. The product \(F \cdot D_V\) is typically limited by figure and alignment inaccuracies\(^5\) to \(F \cdot D_V = 1\) meter. It can be shown that the minimum resolvable diameter \((D_s)\) is limited by the successive image separations discussed earlier in this paper to

\[
D_s = 2 \sqrt{\lambda F d}
\]

The spatial resolving capacity of the plane parallel mirror design is therefore:

\[
N = \frac{1}{4\lambda F^3 d}
\]

(20)

Eqs. 19 and 20 demonstrate an important characteristic of the confocal design: for constant finesse, the spatial resolving capacity of a confocal optical interference filter increases as the mirror separation and radii \(r\) (and hence the temporal resolving power \(\nu/\Delta \nu_m\) (see Eq. 7)) are increased. However, the spatial resolving capacity of a plane parallel mirror interferometer decreases as the mirror separation \((d)\) is increased. Connes noted a similar result regarding the entendue of a confocal spectral interference filter\(^1\).
The spatial resolving capacity of a spherical mirror optical interference filter can also be increased by decreasing the mirror spacing and therefore increasing the upper cutoff angle ($\alpha_+$; Eq. 14). The presence of a lower cutoff angle ($\alpha_-$) indicates that the low spatial frequency components of the image spectrum are filtered out, with the resulting effects noted in the introduction.

EXAMPLES AND APPLICATIONS

The spatial resolving capacity of a confocal optical interference filter operating at a wavelength of $5 \times 10^{-6}$ m and having a mirror spacing of $1/4$ m and a finesse of 300 is predicted by Eq. 19 to be 17,900 resolution elements. The temporal resolution bandwidth (Eq. 7) of this filter is 1 MHz and the temporal resolving power is $6 \times 10^8$. The mirror separation of a plane parallel optical interference filter having the same spatial resolving capacity as the confocal design and a typical finesse of 25 is 1.8 mm and therefore the temporal bandwidth is 3,300 MHz and the temporal resolving power is $1.8 \times 10^5$.

The confocal optical interference filter should find applications in the measurement of spectral distributions in the images of luminous bodies and of phenomena related to doppler shifts and doppler broadening such as velocity distributions associated with plate and surface vibrations, temperature distributions, and turbulence distributions. It can also foreseeably be applied to improve optical images degraded by transmission through a scattering medium. It should prove especially useful as an acousto-optic image conversion device when the acoustic image is formed by a vibrating surface because of its high sensitivity, its large data rate capacity (associated with $N$) and its unique ability amongst acousto-optic image conversion devices to both spatially and temporally filter out noise.
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References

Superposed and reflected images. The initial image is transmitted through the system.

(A) CONFOCAL OPTICAL INTERFERENCE FILTER

(B) PLANE-PARALLEL OPTICAL INTERFERENCE FILTER

FIG. 1 SCHEMATIC DIAGRAM OF (A) CONFOCAL AND (B) PLANE-PARALLEL OPTICAL INTERFERENCE FILTERS ILLUSTRATING THE FORMATION OF THE MULTIPLE IMAGES WHICH ARE SUPERPOSED IN (A) BUT SEPARATED IN (B).
FIG. 2 SCHEMATIC DIAGRAM OF A CONFOCAL OPTICAL INTERFERENCE FILTER INCORPORATED INTO A COHERENT IMAGING SYSTEM ILLUSTRATING (A) THE PATH OF AN UNSCATTERED SOURCE BEAM AND (B) THE RAY PATHS ASSOCIATED WITH SCATTERING BY ONE SPATIAL FREQUENCY COMPONENT OF AN ILLUMINATED OBJECT.