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Microwave Physics Branch
Ballistic Research Laboratories
U.S. Army Aberdeen Proving Ground

PROXIMITY EFFECTS FOR PARALLEL RECTANGULAR CONDUCTORS IN NON-TRANSMISSION-LINE MODE

By

Robert W. Burton

February 1972

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In the light of experimental results a first-order theory is developed and verified for analyzing relative proximity effects at the extremes of conductor separation distances.

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Robert W. Burton*

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ABSTRACT

In applications of electrically short antennas such as multiturn loops, variation in ohmic losses due to proximity effects can significantly affect uncorrected estimates of efficiency. This paper investigates experimentally the proximity effects on the current distributions and associated ohmic losses for two identical parallel rectangular cross-section conductors as a function of separation distance and cross-section. The system is operated in a non-transmission-line mode.

In the light of experimental results a first-order theory is developed and verified for analyzing relative proximity effects at the extremes of conductor separation distances.

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I. Introduction

A strong interest exists in determining the radiation efficiency of electrically small or minute antennas. In such cases the ratio of radiated power to ohmic losses is a significant parameter affecting calculations of efficiency. The efficiency of power transfer is given by

$$\text{Eff.} = \frac{1}{1 + L_0/P_r} \tag{1}$$

where

L_0 = system ohmic losses

P_r = radiated power

In general ohmic losses are a function of the operating frequency, the macroscopic properties and geometry of the conducting material, and proximity effects due to the presence of other current-carrying elements. For a given system, ohmic losses are typically calculated by integrating the square of the current distribution of an isolated conductor and multiplying it by an appropriate equivalent resistance. More often than not a uniformly distributed current is assumed, or better a theoretically calculated or experimentally measured current distribution for an isolated conductor is employed.

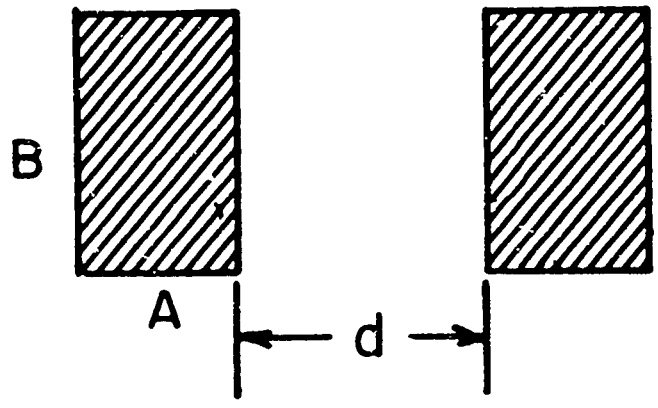
In electrically small multiturn loop antennas, however, proximity effects may dramatically alter the transverse current distribution of the isolated conductors. Recently Smith[1] showed theoretically that the errors in calculating the total power radiated by an electrically small two-turn loop made from a circular cross-section conductor can be as large as 30% if proximity effects are neglected.

For rectangular conductors (Fig. 1) the determination of proximity effects is far from straightforward. However, some insight into the problem is given by Cockcroft[2]. In this paper Cockcroft reviews the approach as suggested by Strutt[3] that the rectangular conductor be approximated by an ellipse of large eccentricity wherein at high frequencies as pointed out by Kelvin[4] the problem of the distribution of the fields becomes analogous to the electrostatic problem. Cockcroft applies this analogy directly to the rectangular conductor using the Schwartz-Christoffel transformation. Briefly stated, Cockcroft showed that the a. c. resistance per unit length, R, in ohms per meter of an isolated rectangular conductor is given by*

$$R = \frac{2f(B/A)}{\pi} \sqrt{\frac{\omega\mu\rho}{8\pi AB}} \qquad (2)$$

where ρ is the resistivity in ohm-meters, ω the angular frequency; $\mu = \mu_r\mu_0$ the absolute permeability with $\mu_0 = 4\pi \times 10^{-7}$ henry/m; A and B the dimensions of the conductor; and $f(B/A)$, a shape factor involving elliptical integrals. More recently King[5] examined the current distribution and impedance per unit length of a very thin isolated strip. The case for analyzing proximity effects between two parallel conductors, however, involves intractable integrals and an exact analytical solution cannot be found although numerical evaluation might be considered a useful substitute.

*Cockcroft's formula has been converted to MKS units.



GEOMETRY OF PARALLEL RECTANGULAR CONDUCTORS

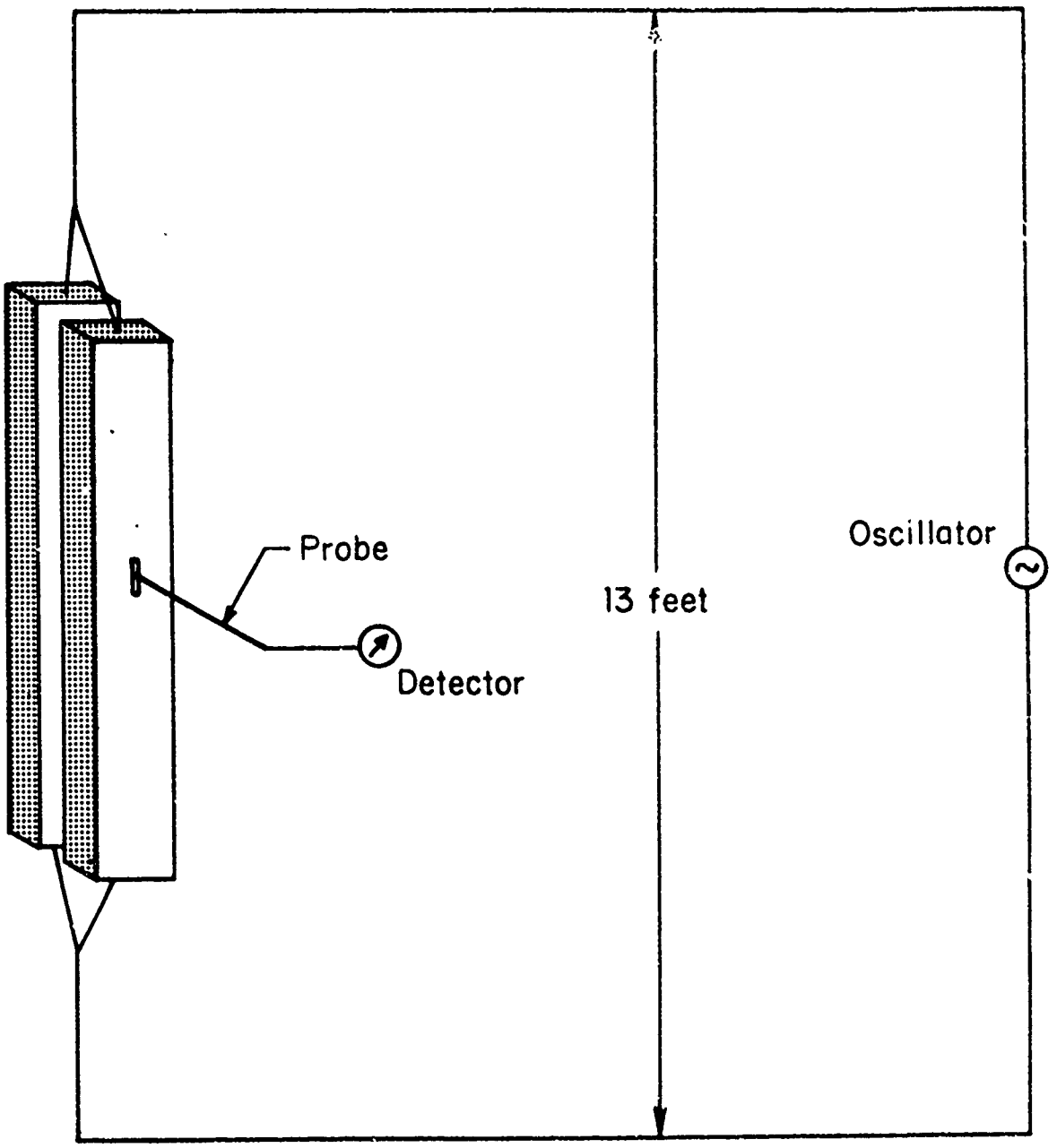
FIG. 1

In this paper a series of experimental measurements of surface current distributions are taken over two identical parallel rectangular conductors carrying equal currents, as a function of separation distance and cross section. Relative ohmic losses due to proximity effects over the range of measurements are calculated and generalized to a significant set of cross-section geometries and separation distances. From these observations a first-order theory, which extends the experimental data to include very small separation distances, is formulated and compared with the analytical solution developed by Cockcroft.

II. Experimental Apparatus and Error Analysis

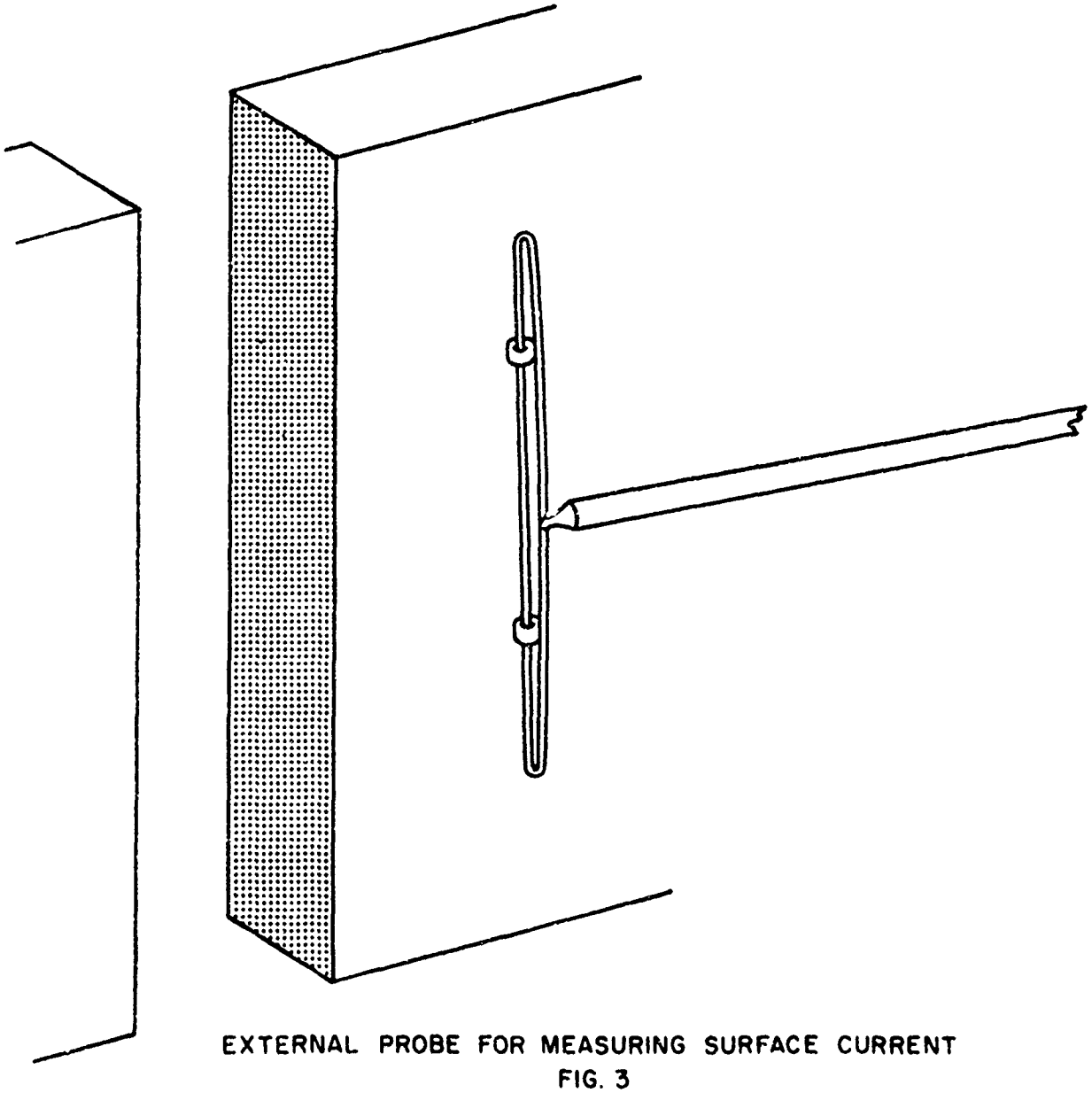
The basic experimental apparatus (Fig. 2) employs a 100 KHz CW oscillator with an external shielded loop probing system. The rectangular conductors are of sufficient length to be considered infinitely long in the sense that field contributions from current-carrying feed cables were kept less than 5% of the average probe reading from opposite sides.

The particular geometry of a rectangular conductor does not lend itself to the conventional approach of field probing using internally mounted probes with the consequence that an externally mounted shielded loop probe system (Fig. 3) was designed and raked across each conducting face. The probe is constructed of 0.021 inch O.D. coaxial cable formed into a 1/16 inch by 1-1/2 inch rectangle. A uniform probe separation was accomplished using a 1/32 inch radius teflon bearing. The measure of isolation between the conductors and the probe system was derived by raking the probe first in one direction and then the other (Fig. 4). Excellent agreement exists except where the probe penetrates the high

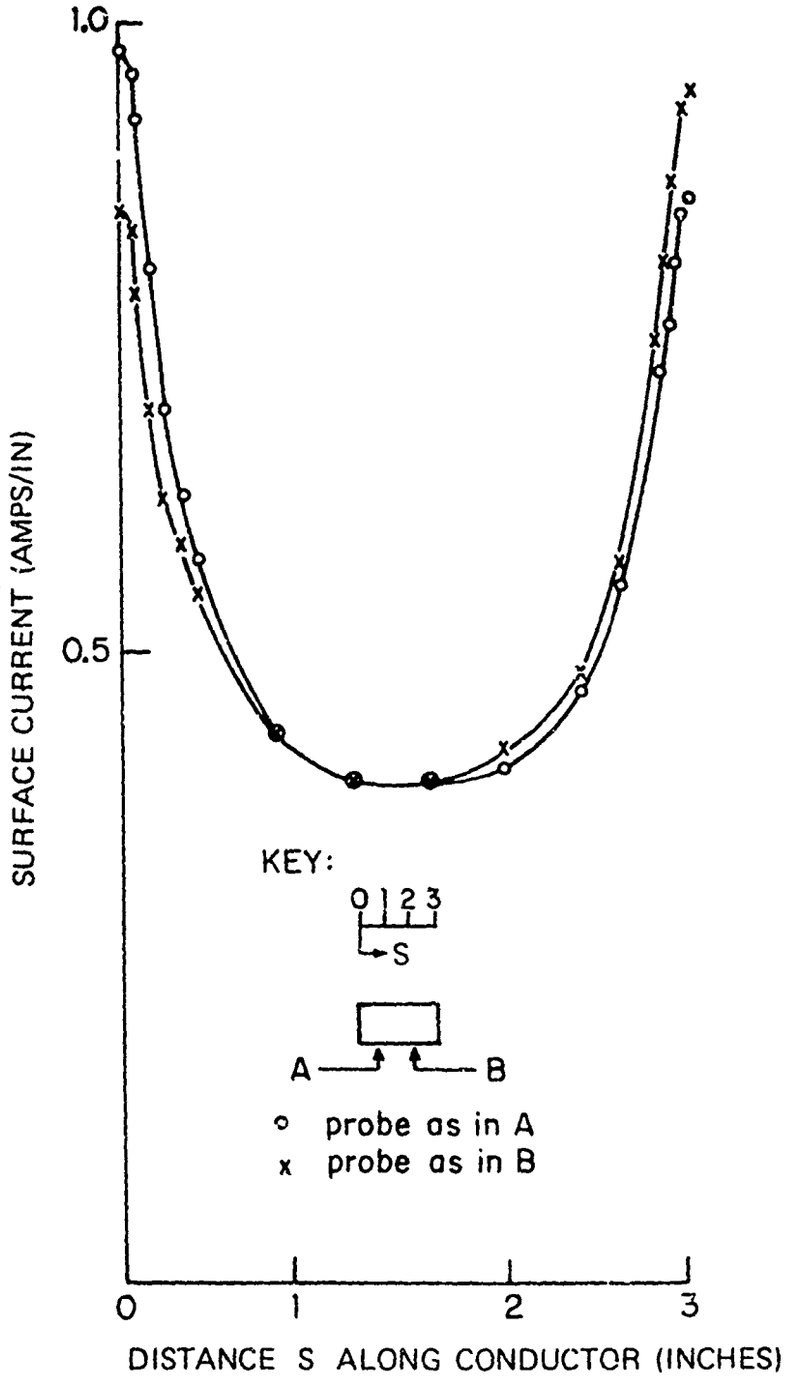


EXPERIMENTAL APPARATUS

FIG. 2



EXTERNAL PROBE FOR MEASURING SURFACE CURRENT
FIG. 3



MEASURE OF PROBE ISOLATION

FIG. 4

field at the edges of the conductor. In this area contributions from the current on the adjacent conductor face asymmetrically distort the probe readings. The overall error from this distortion generates an error of less than 3% in the calculations.

In practice a rectangular loop probe senses field contributions derived from currents (I_s) sampled over an area larger than that directly below the probe. The area of sampled current as a function of probe height, geometry and distance from probe centerline is given by

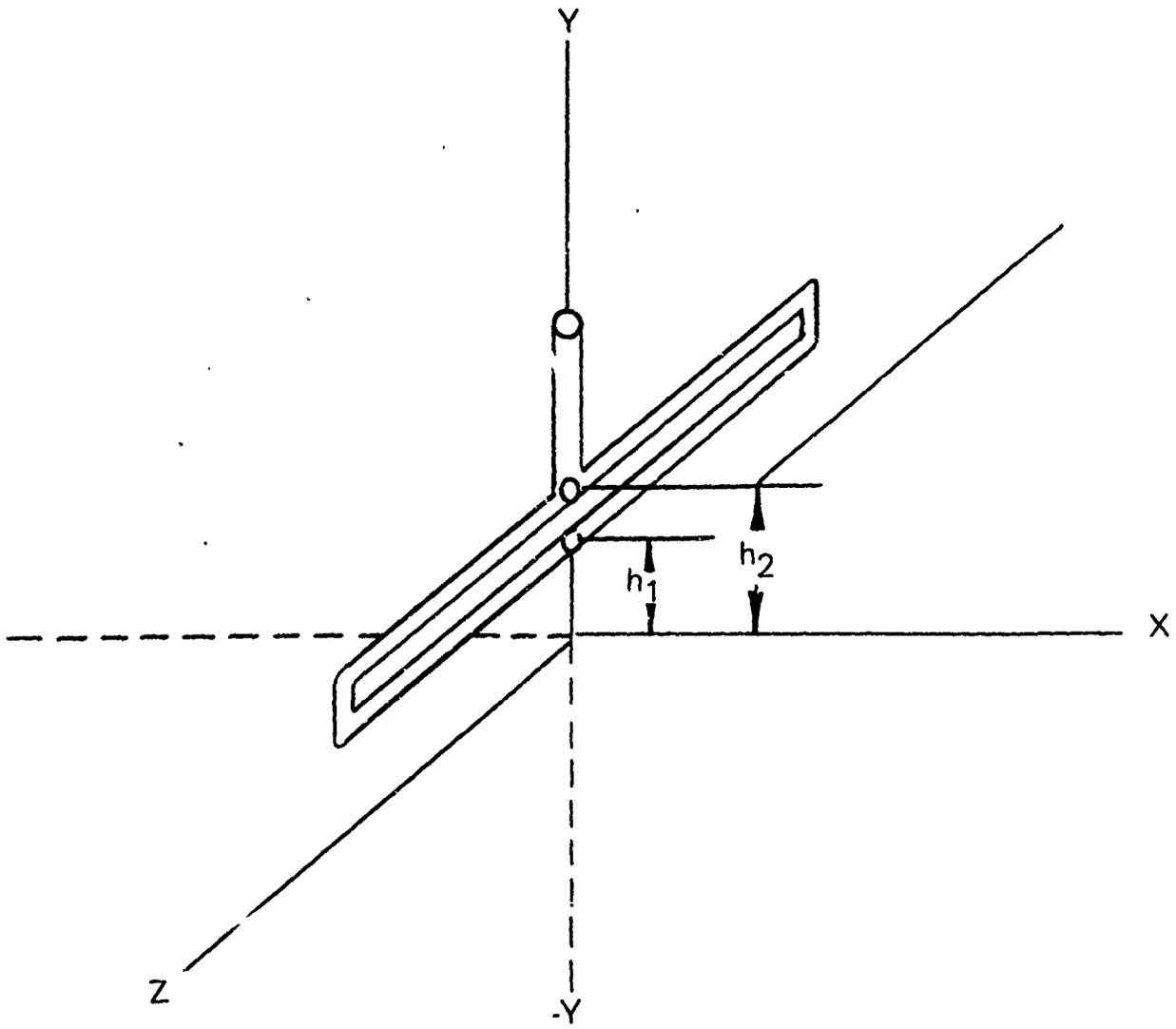
$$I_s(x, h_2 - h_1) \approx \int_{h_1}^{h_2} \int_0^x \frac{I(x)y}{\pi(y^2 + x^2)} \, dx \, dy \qquad (3)$$

with y, h_1, h_2, x defined as in Figure 5.

Assuming $I(x)$ varies at most linearly over the sampled range, fifty percent of the probe (Fig. 3) readings are derived from currents up to 1/16 inch away from the probe centerline; and 85% from currents within 1/4 inch of the centerline[6].

The results of this integration effect in the measurement of surface currents is most significant as the probe approaches the edge. If there were no contribution from he adjoining side, the actual current at the edge would simply be twice the measured value. In such a case with a uniform circumferential current distribution, the correction factor by which the measured values must be multiplied to give the actual surface current would be

$$\text{Correction Factor} \approx 2 - \frac{2}{\pi} \tan^{-1}(s/h) \qquad (4)$$



PROBE GEOMETRY
FIG. 5

as derived from (3) for an infinitely small probe at height, h, and located at a distance s from the edge.

A more accurate representation of the current distribution near the edges would be a rising exponential, the maximum value of which would again be twice the measured value if there were no contribution from the adjoining side and of the same form as the correction factor (4) but weighted by the exponential. Numerical integration of the weighted correction factor shows that the probe reading at the edge is essentially derived (90%) from currents present up to a distance 3 h from the edge.

At the edge, however, the probe does in fact measure currents from the adjoining vertical side according to

$$I_s(y, h_2 - h_1) \approx \int_{h_1}^{h_2} \int_{0}^{-y} \frac{I(y)}{2\pi(h-y)} \, dy \, dy \qquad (5)$$

where I(y) is the current distribution on the adjoining side (the lower half of the y-z planes) and (-y) is the distance measured down from the edge as shown in Fig. 5.

The sum total of the contributions to the measured readings therefore is one half of the sampled current given by (3) plus the sampled current on the adjacent side as given by (5). The cumulative effect of these contributions is such that the measured currents at the edge are not less than 85% of the actual value. This error translates into an error of less than 3% in computing relative ohmic losses as described by (6).

III. Experimental Results

Surface currents were measured for three conductor cross sections (1-1/2 in. by 3 in., 3/4 in. by 3 in., and 1-1/2 in. by 1-1/2 in.), two conductor orientations, and a range of conductor separations from 1/2 inch to infinity. The results are presented in Figs. 6-19 The experimental data for the surface current are normalized to the total current flowing in a single conductor such that the integral of the surface current over the four sides A, B, C and D is unity for each of the five cases investigated. No phase shifts in the current distribution were noted. Because there exists a small contribution from the horizontal current-carrying feed wires, the relative position of the oscillator is noted. It is easy to observe that this contribution adds to the probe reading on the near side and subtracts on the far side. Typically this asymmetry results in a current on side A with an amplitude approximately 10% higher than on side C.

From the experimental data presented in Figs. 6-19, proximity effects on ohmic losses may be determined by integrating the square of the surface currents and comparing the results to the appropriate isolated conductor. In so doing, a normalized relative ohmic loss, L_n, may be defined for two identical parallel rectangular cross-section conductors carrying equal currents in terms of a conductor geometry (B/A), relative separation distance (d/C), and referred to the case of electrically isolated conductors.

$$L_n(B/A, d/C) = \frac{L_0(B/A, d/C)}{L_0(B/A, \infty)} \tag{6}$$

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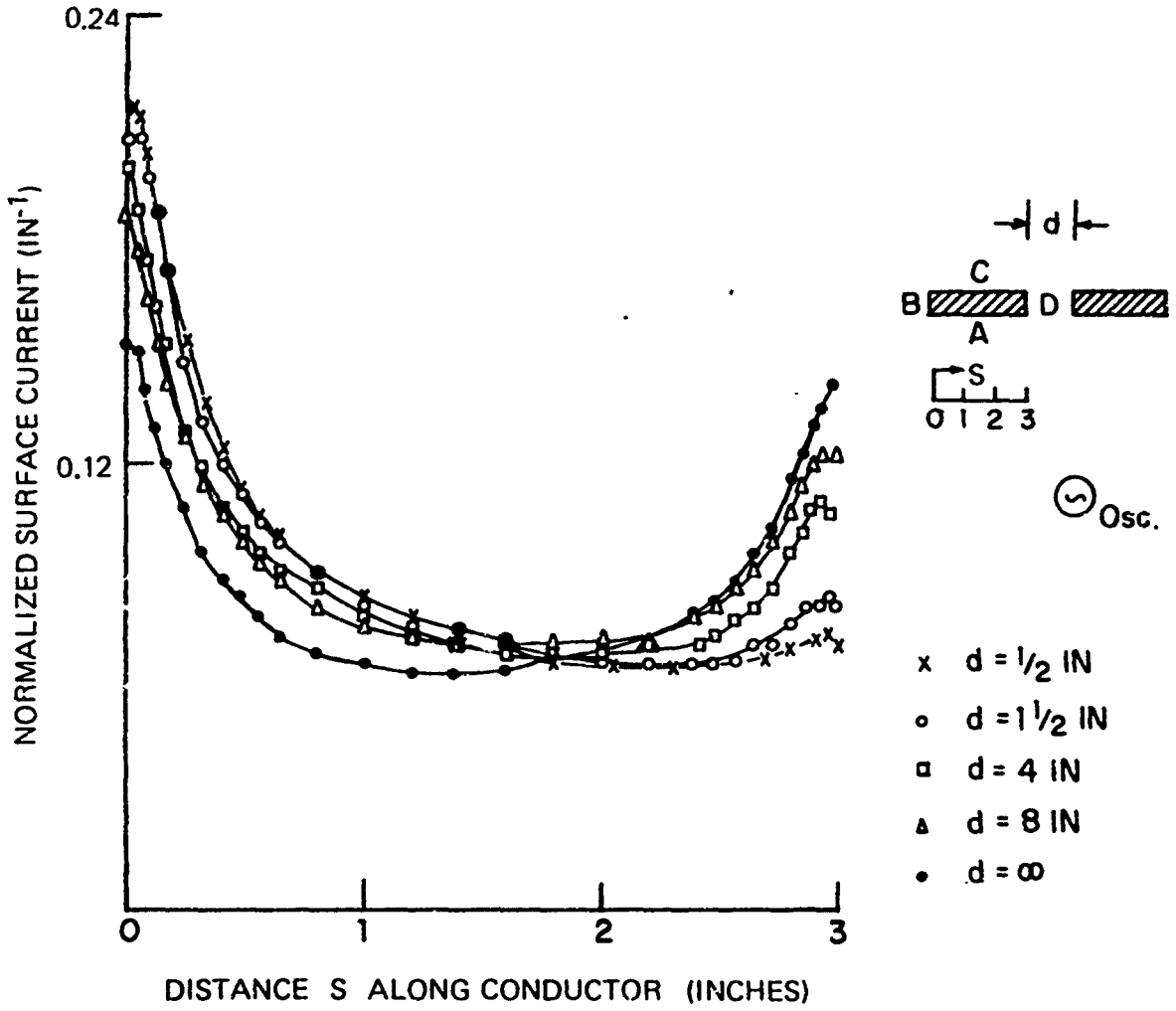
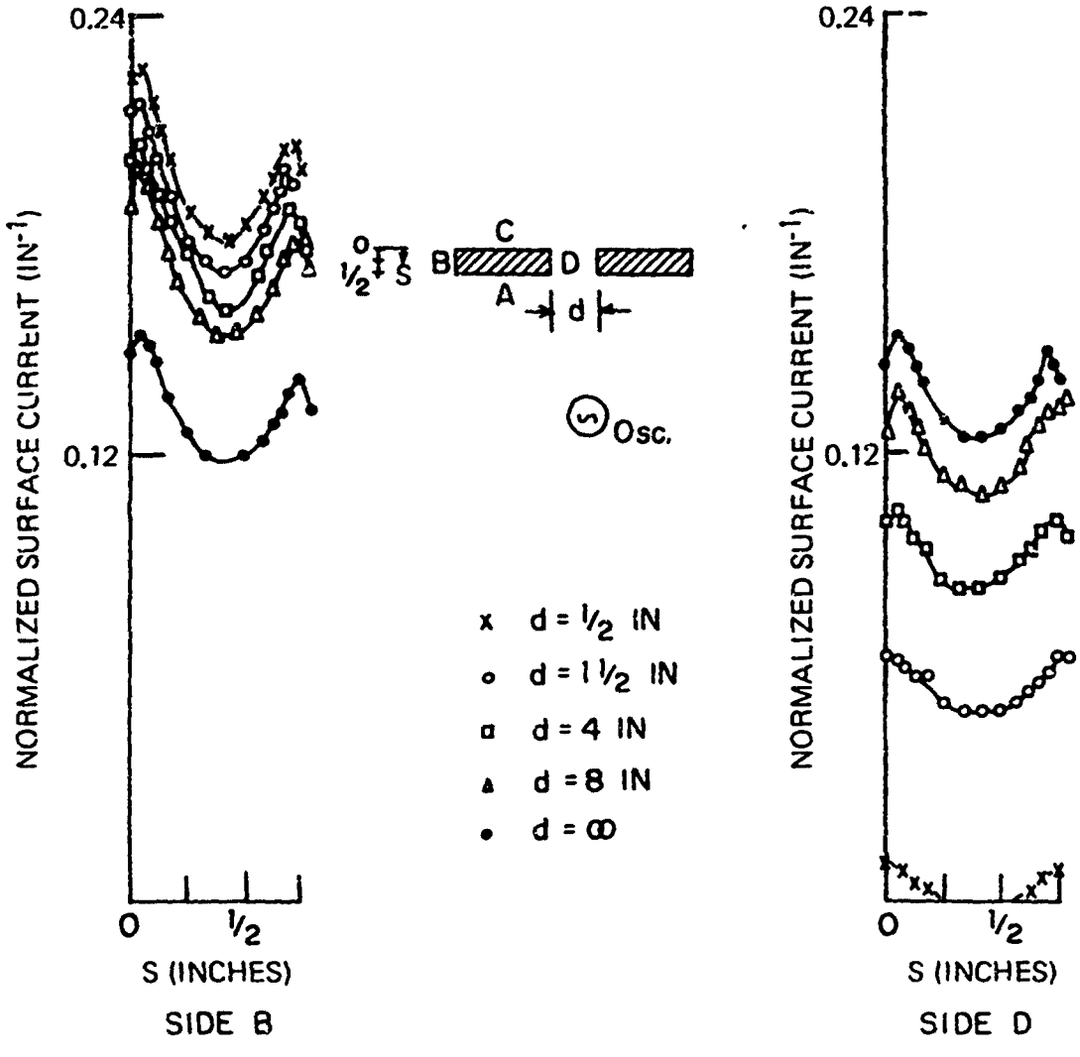


SURFACE CURRENT ON SIDE A WITH THE CONDUCTOR SEPARATION d AS PARAMETER
(¾ INCH × 3 INCH CONDUCTORS WITH ¾ INCH SIDES PARALLEL)

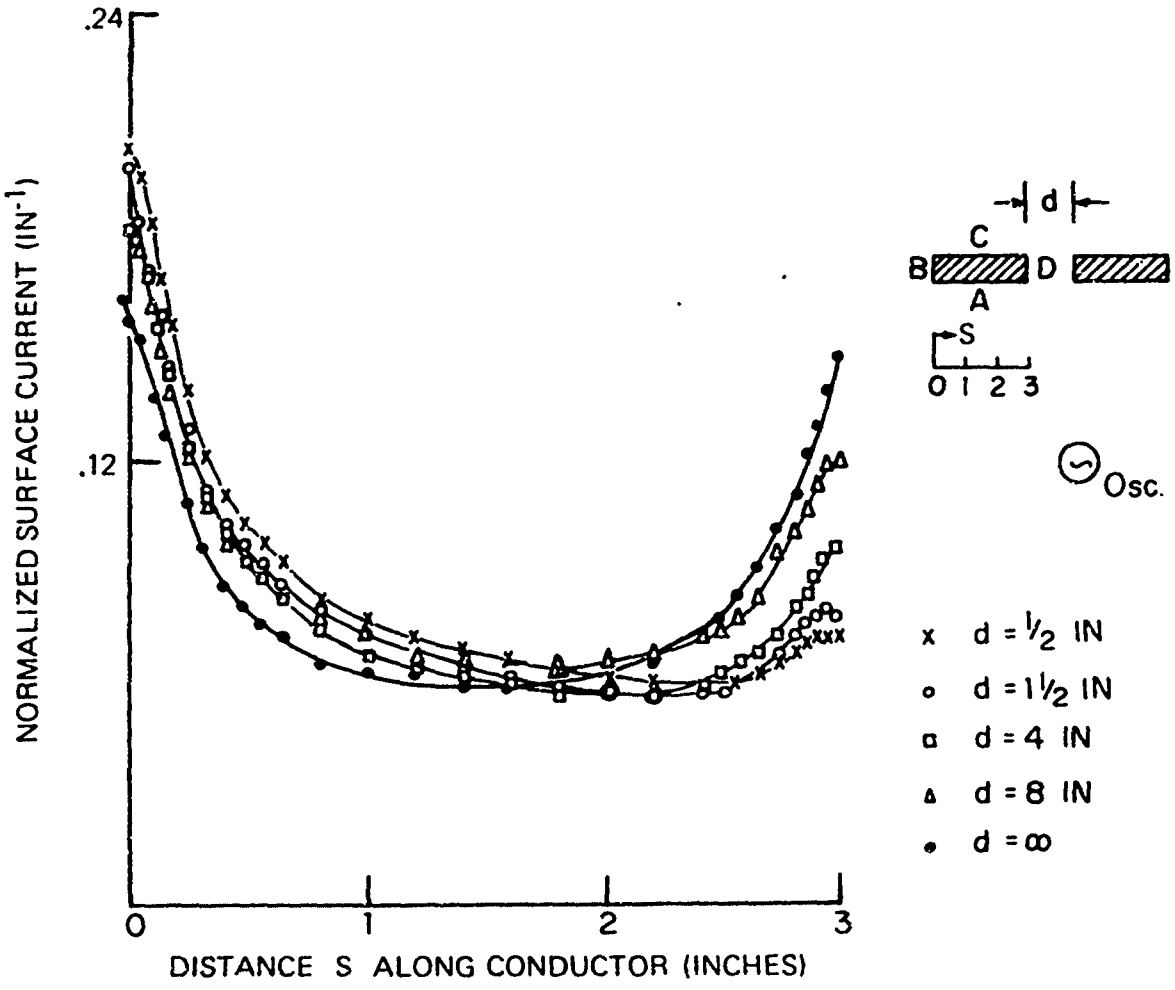
FIG. 6



SURFACE CURRENTS ON SIDES B, D WITH THE CONDUCTOR SEPARATION d AS PARAMETER

(¾ INCH x 3 INCH CONDUCTOR WITH ¾ INCH SIDE PARALLEL)

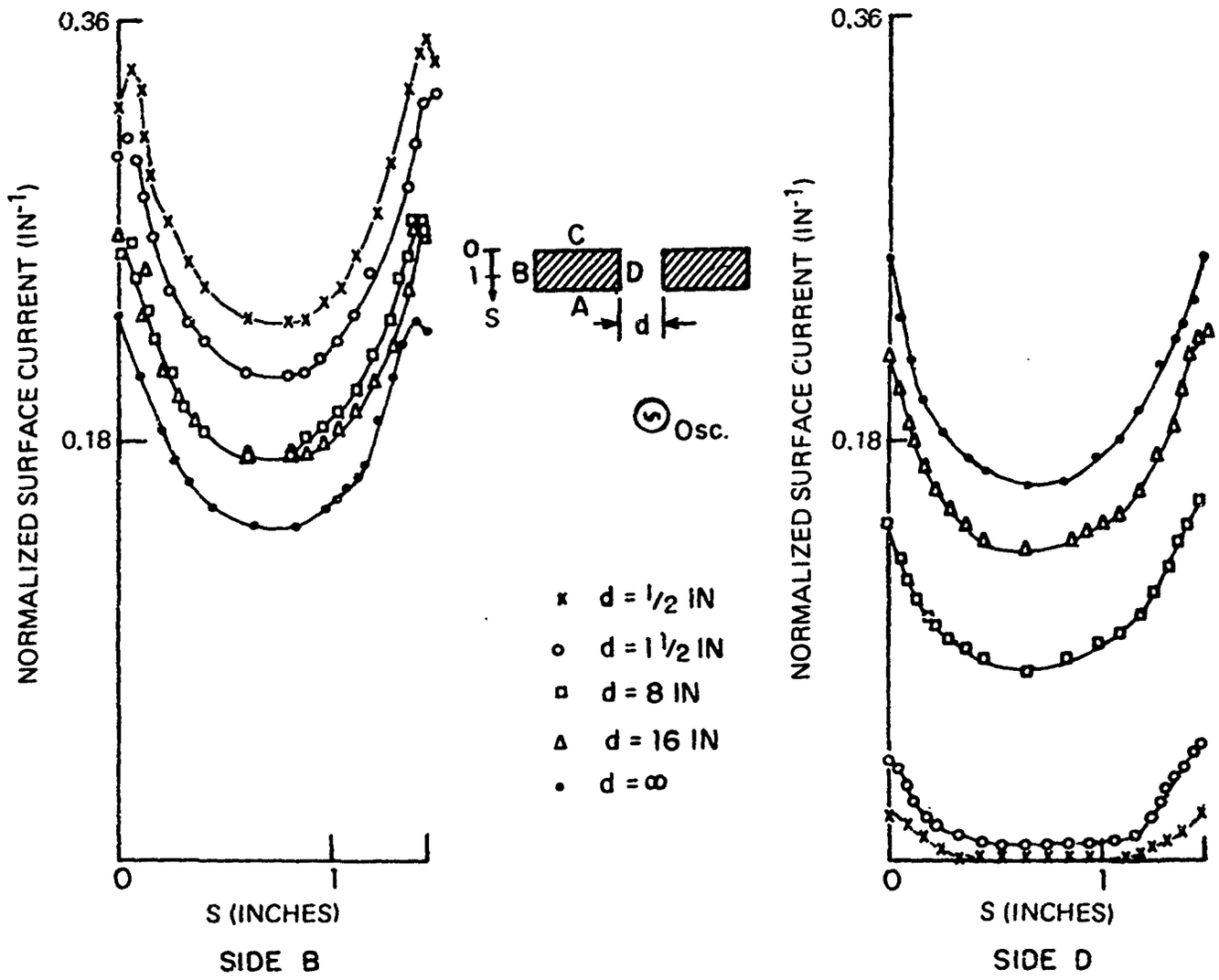
FIG. 7



SURFACE CURRENT ON SIDE C WITH THE CONDUCTOR SEPARATION d AS PARAMETER

(¾ INCH x 3 INCH CONDUCTORS WITH ¾ INCH SIDE PARALLEL)

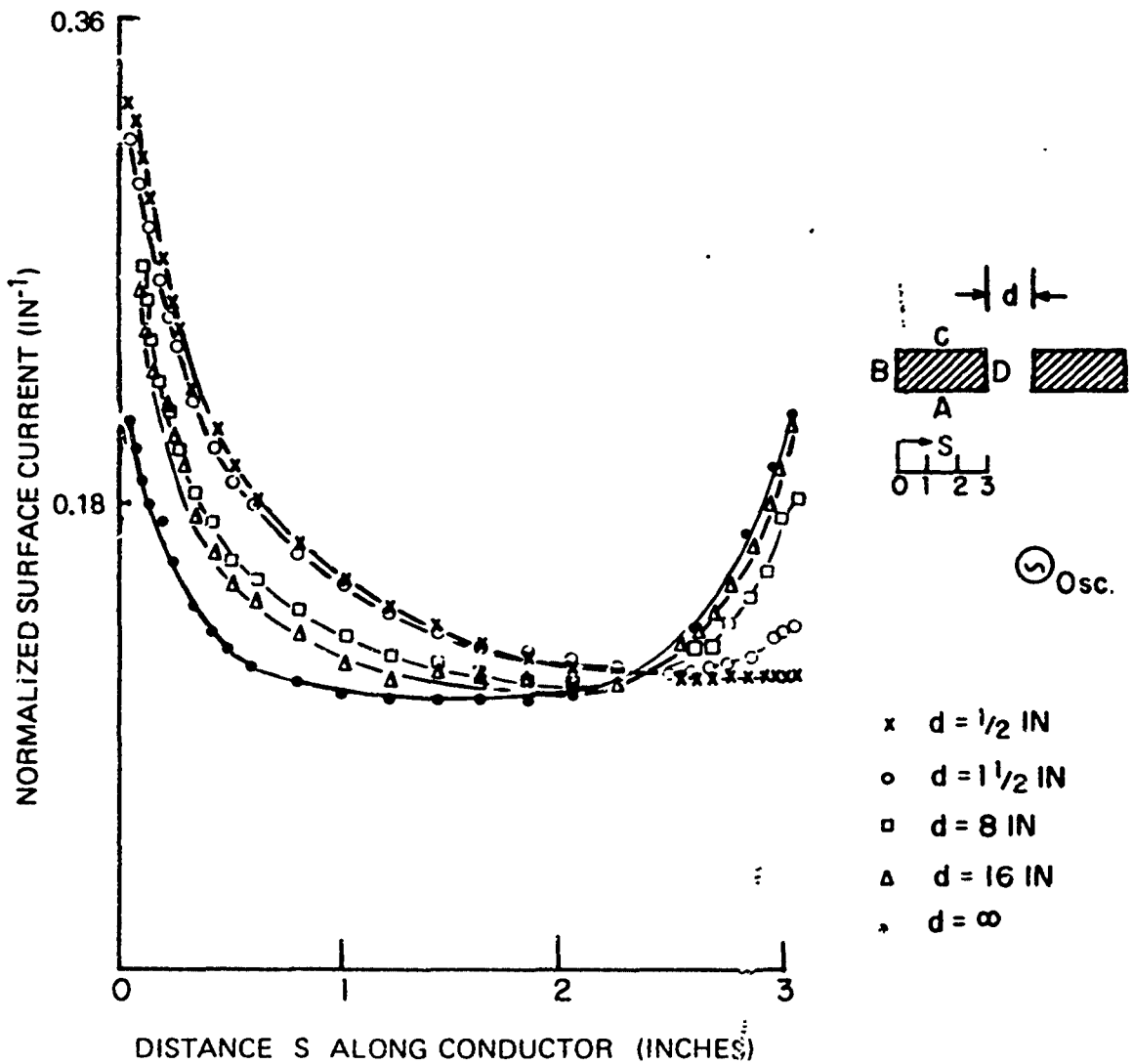
FIG. 8



SURFACE CURRENTS ON SIDES B, D WITH THE CONDUCTOR SEPARATION d AS PARAMETER

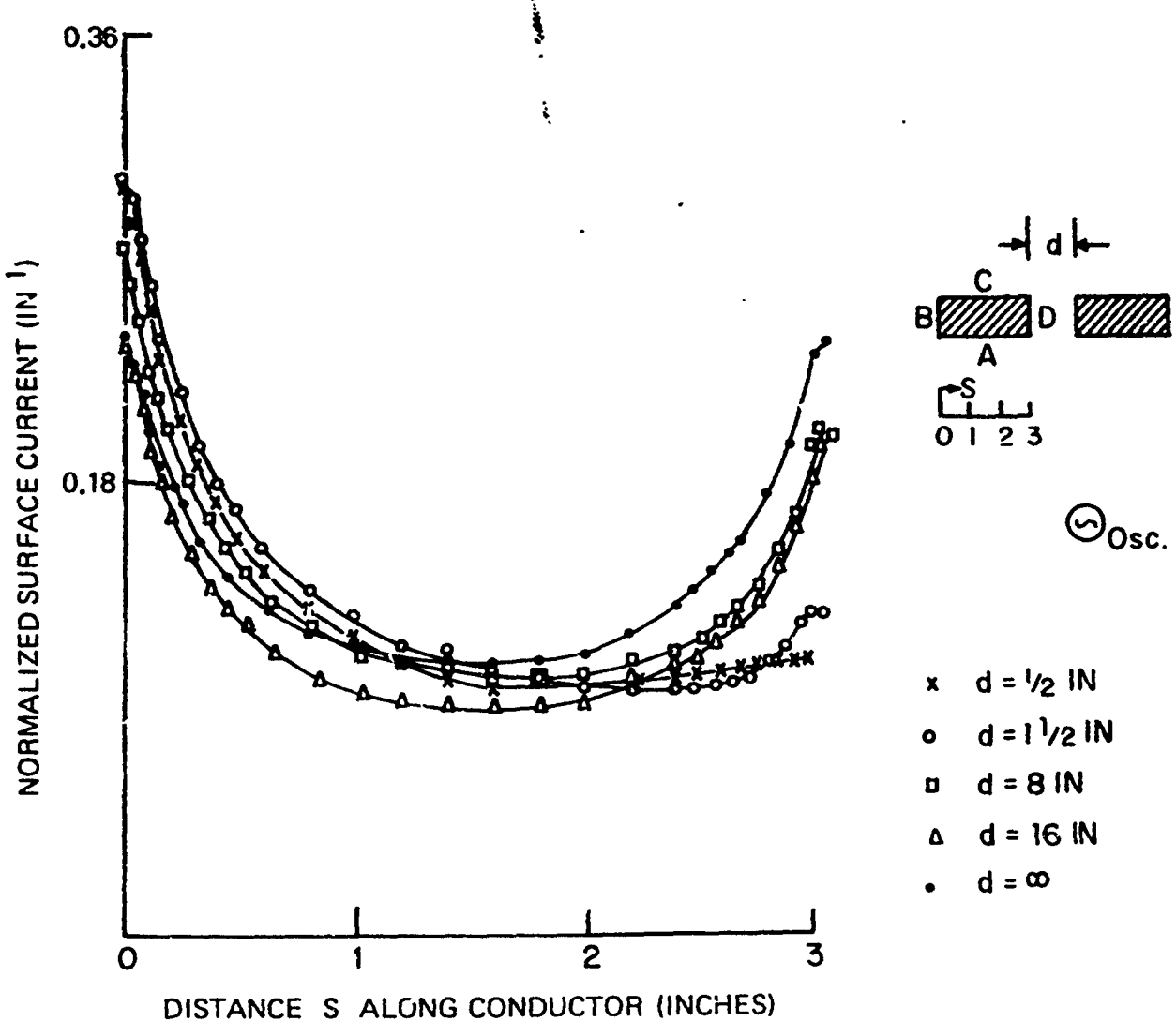
(1½ INCH x 3 INCH CONDUCTOR WITH 1½ INCH SIDE PARALLEL)

FIG. 10



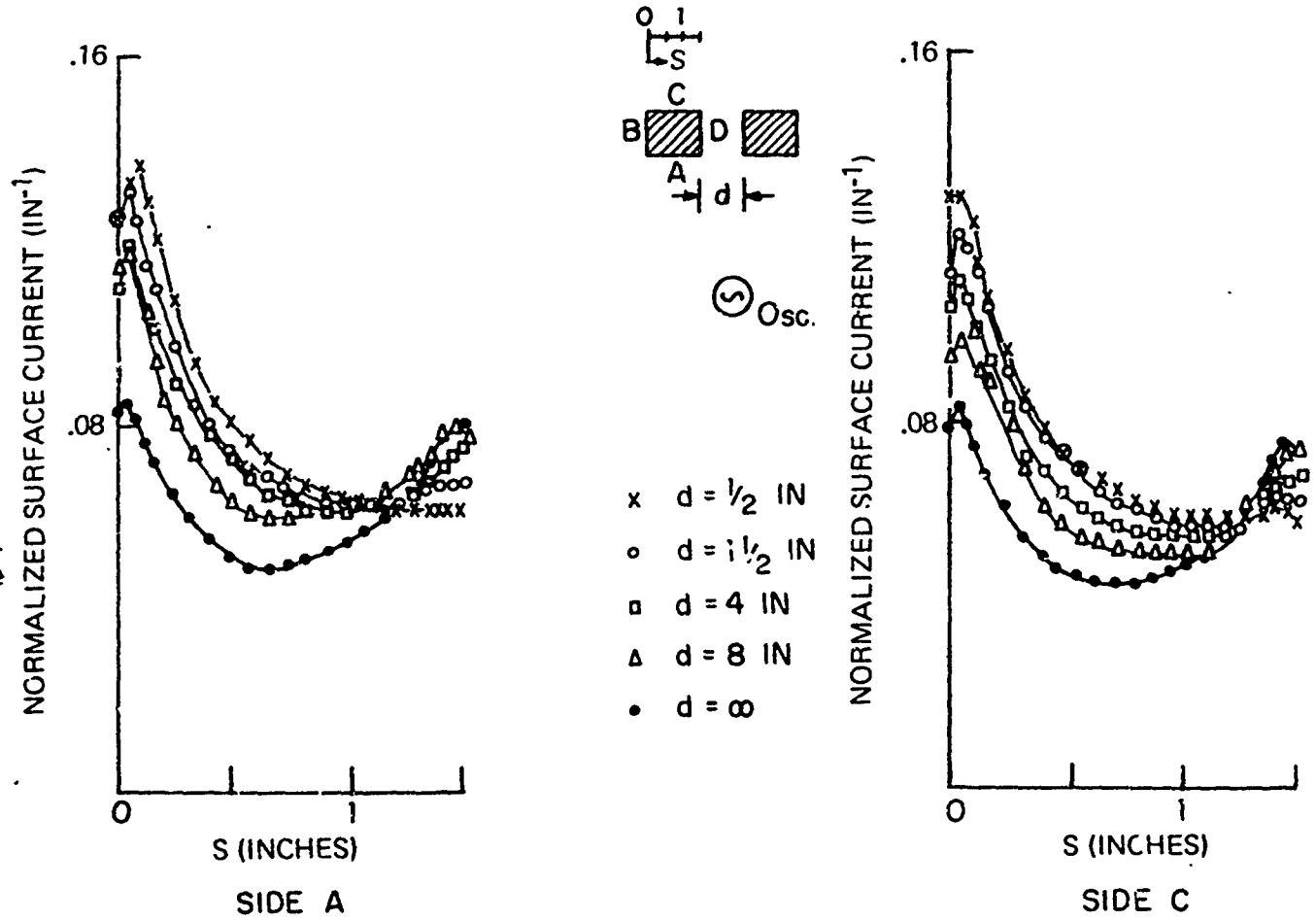
SURFACE CURRENT ON SIDE A WITH THE CONDUCTOR SEPARATION d AS PARAMETER
(1½ INCH x 3 INCH CONDUCTORS WITH 1½ INCH SIDES PARALLEL)

FIG. 9



SURFACE CURRENT ON SIDE C WITH THE CONDUCTOR SEPARATION d AS PARAMETER
(1½ INCH x 3 INCH CONDUCTORS WITH 1½ INCH SIDES PARALLEL)

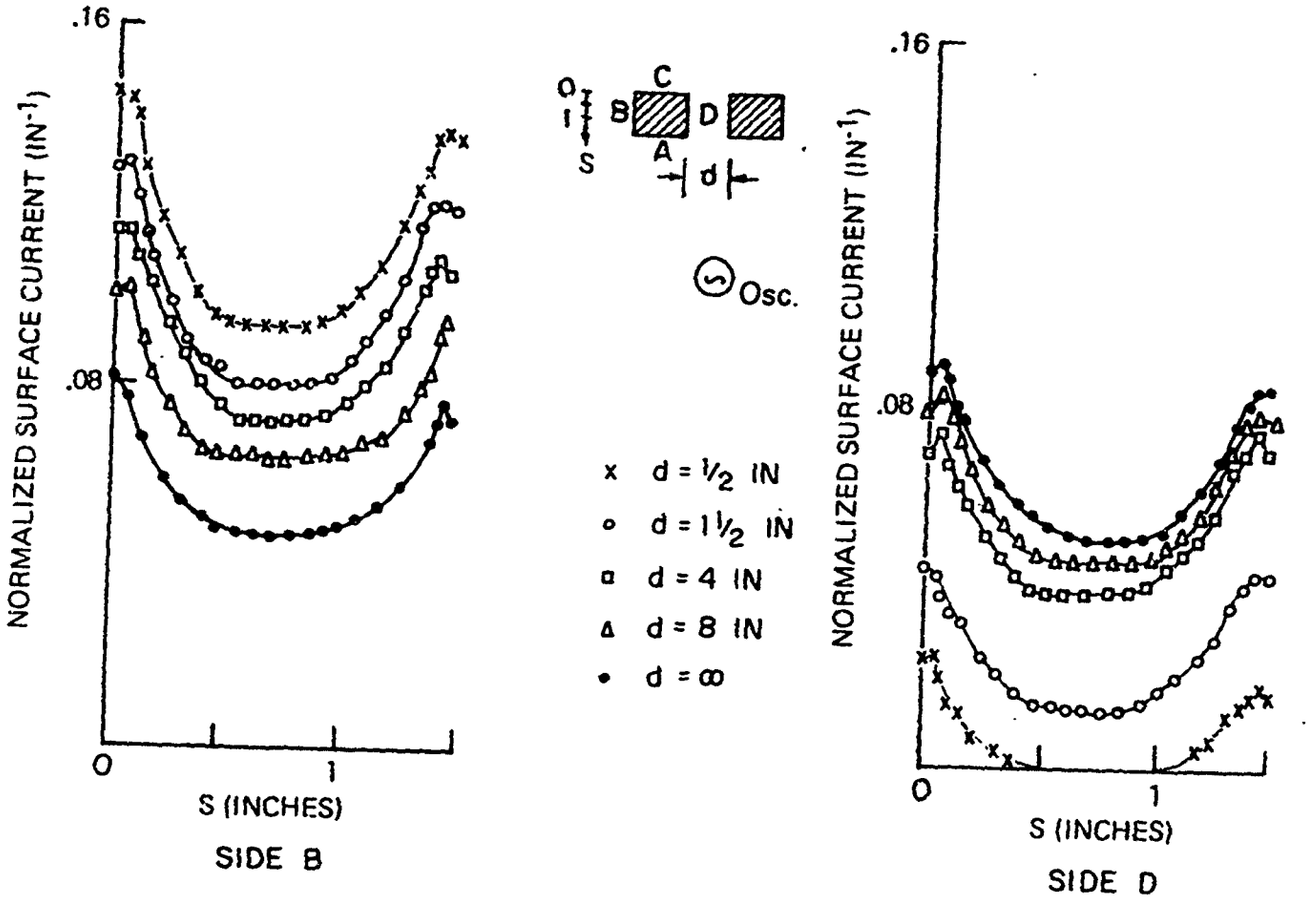
FIG. 11



SURFACE CURRENTS ON SIDES A, C WITH THE CONDUCTOR SEPARATION d AS PARAMETER

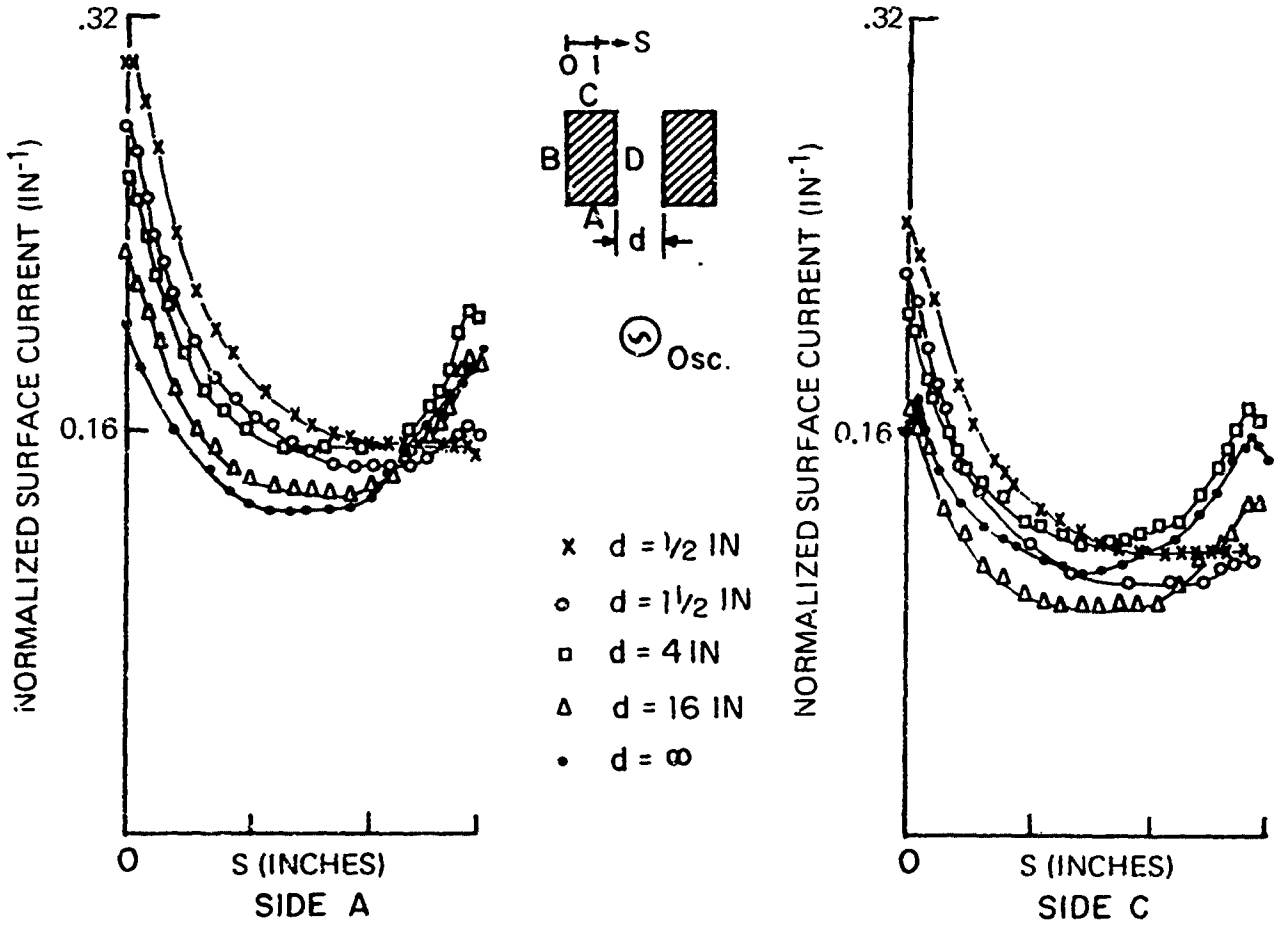
(1½ INCH x 1½ INCH CONDUCTORS)

FIG. 12



SURFACE CURRENTS ON SIDES B, D WITH CONDUCTOR SEPARATION d AS PARAMETER
(1½ INCH x 1½ INCH CONDUCTORS)

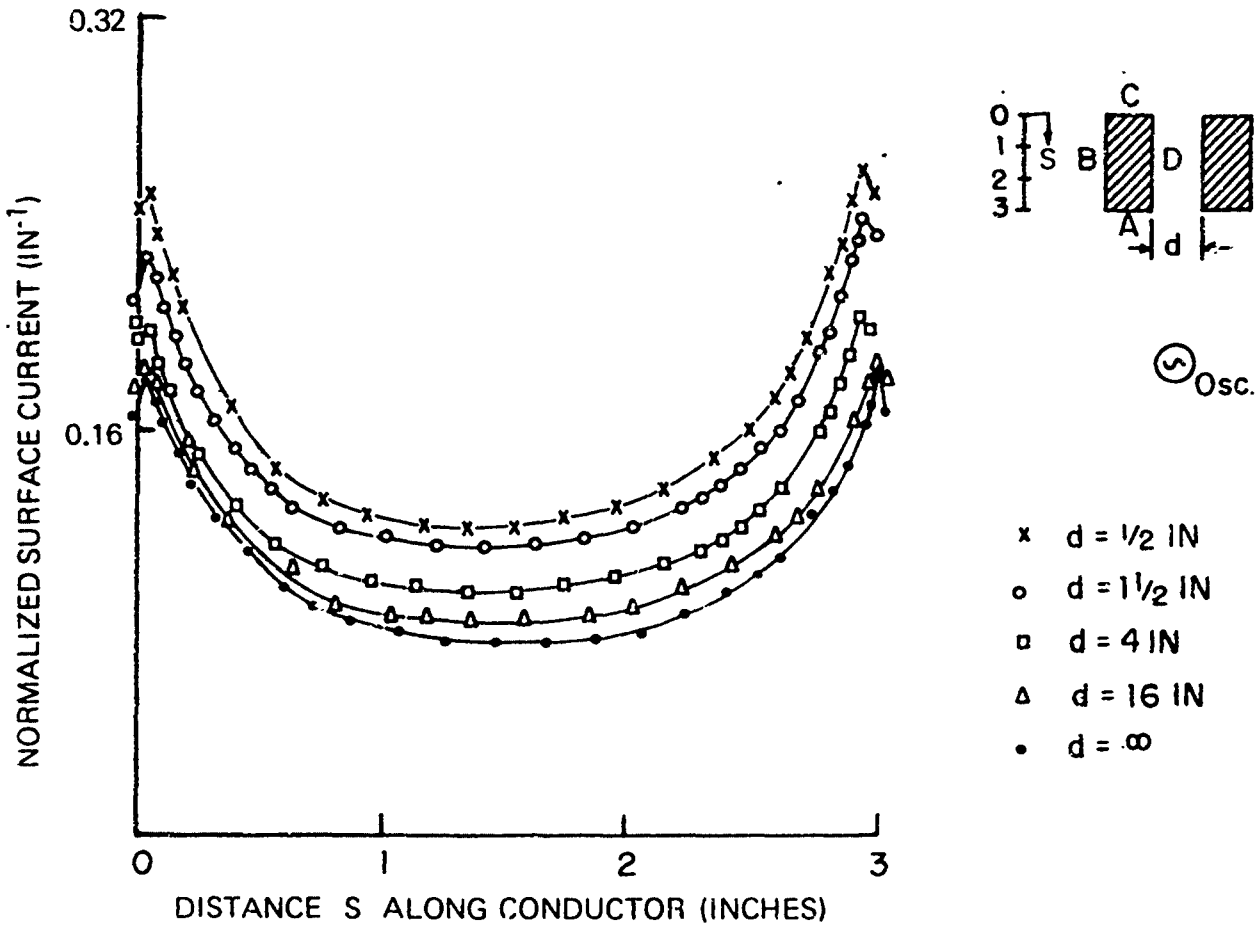
FIG. 13



SURFACE CURRENTS ON SIDES A, C WITH CONDUCTOR SEPARATION d AS PARAMETER

(1½ INCH x 3 INCH CONDUCTOR WITH 3 INCH SIDES PARALLEL)

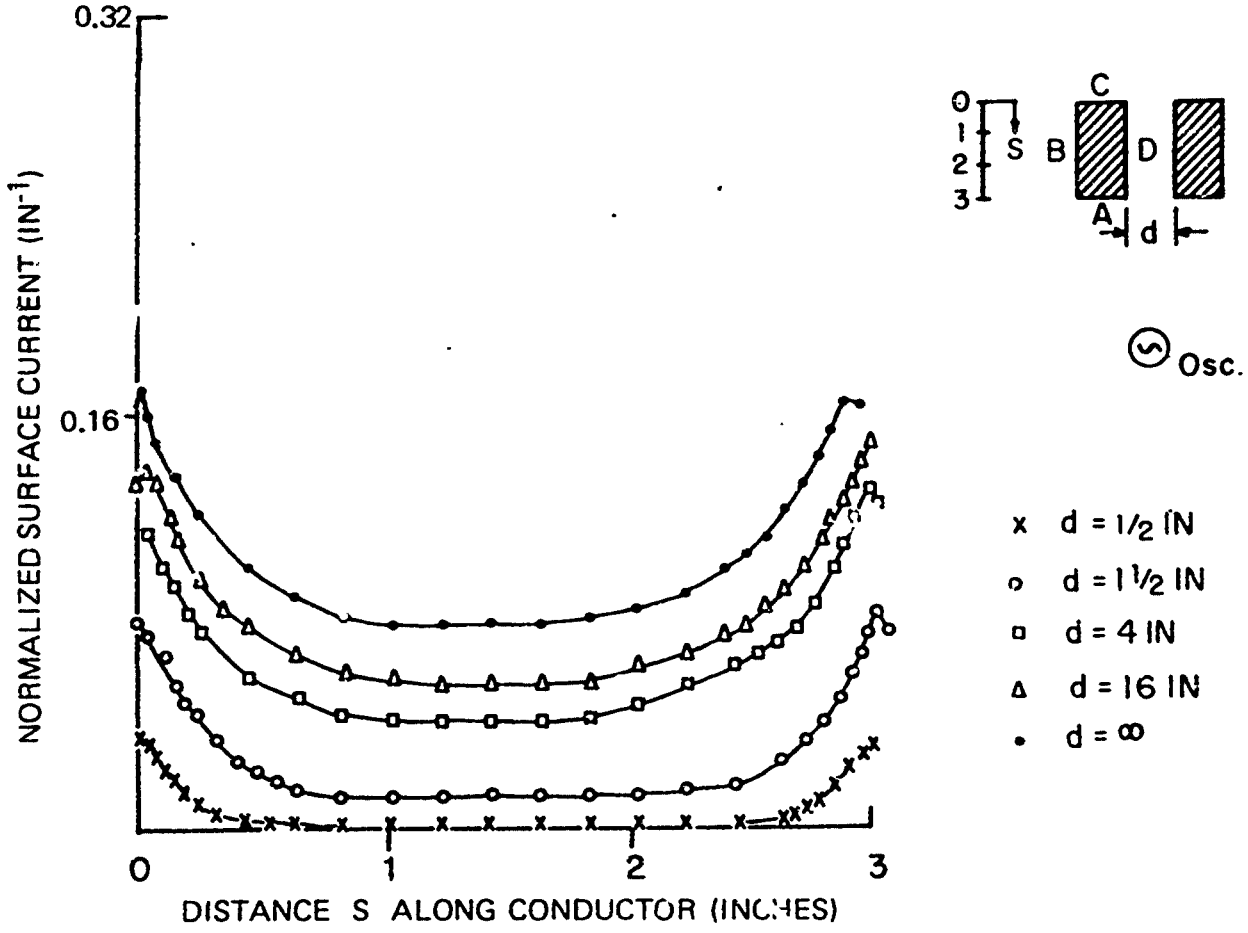
FIG. 14



SURFACE CURRENT ON SIDE B WITH CONDUCTOR SEPARATION d AS PARAMETER

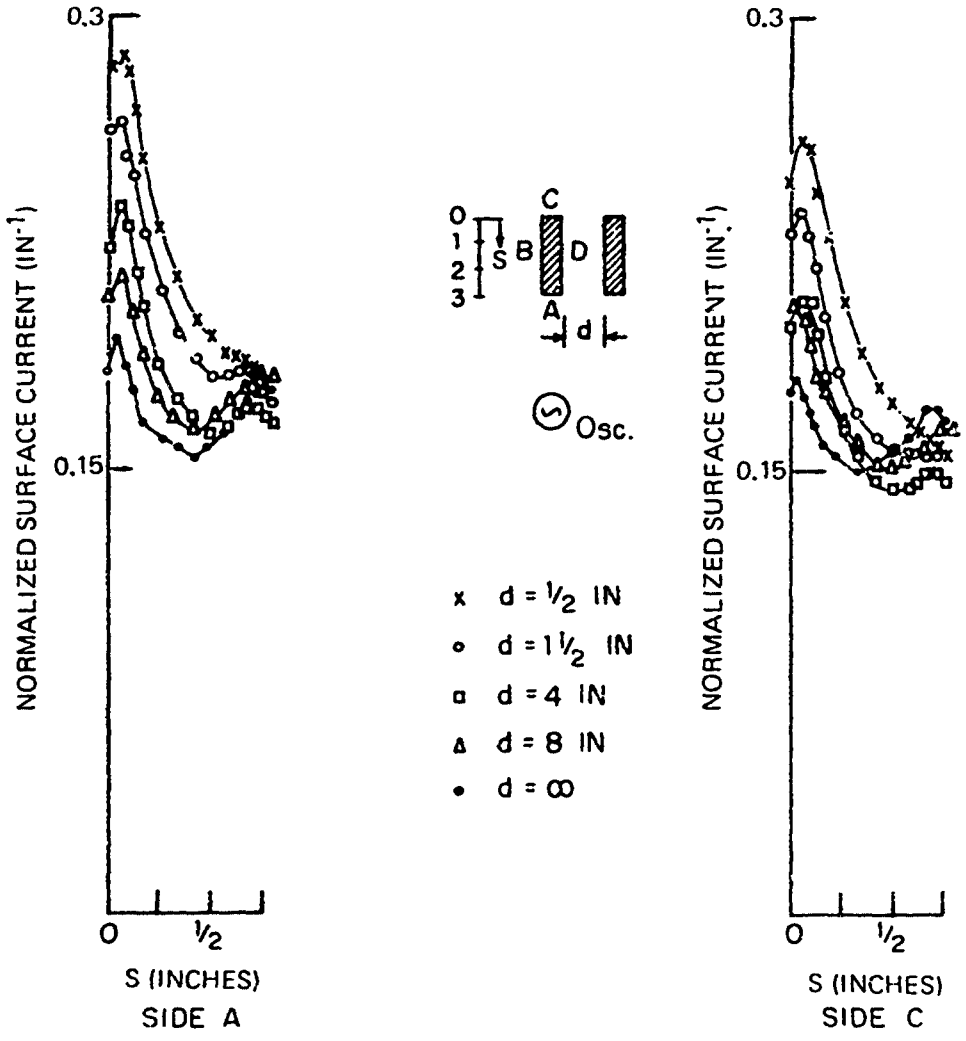
(1½ INCH x 3 INCH CONDUCTOR WITH 3 INCH SIDES PARALLEL)

FIG.15



SURFACE CURRENT ON SIDE D WITH CONDUCTOR SEPARATION d AS PARAMETER
(1½ INCH x 3 INCH CONDUCTOR WITH 3 INCH SIDES PARALLEL)

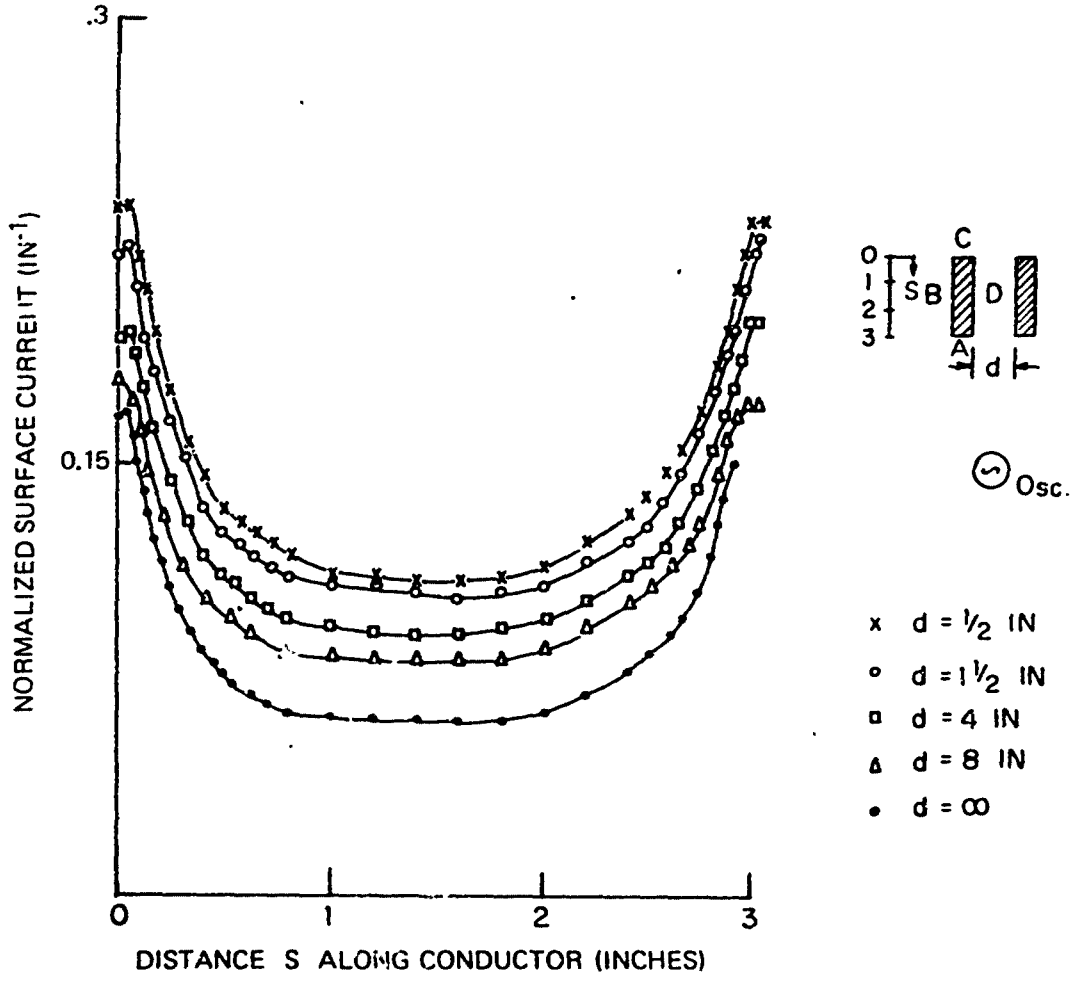
FIG. 16



SURFACE CURRENTS ON SIDES A, C WITH CONDUCTOR SEPARATION d AS PARAMETER

(¼ INCH x 3 INCH CONDUCTOR WITH ¼ INCH SIDES PARALLEL)

FIG. 17



SURFACE CURRENT ON SIDE B WITH CONDUCTOR SEPARATION d AS PARAMETER
(¾ INCH x 3 INCH CONDUCTOR WITH ¾ INCH SIDES PARALLEL)

FIG. 18

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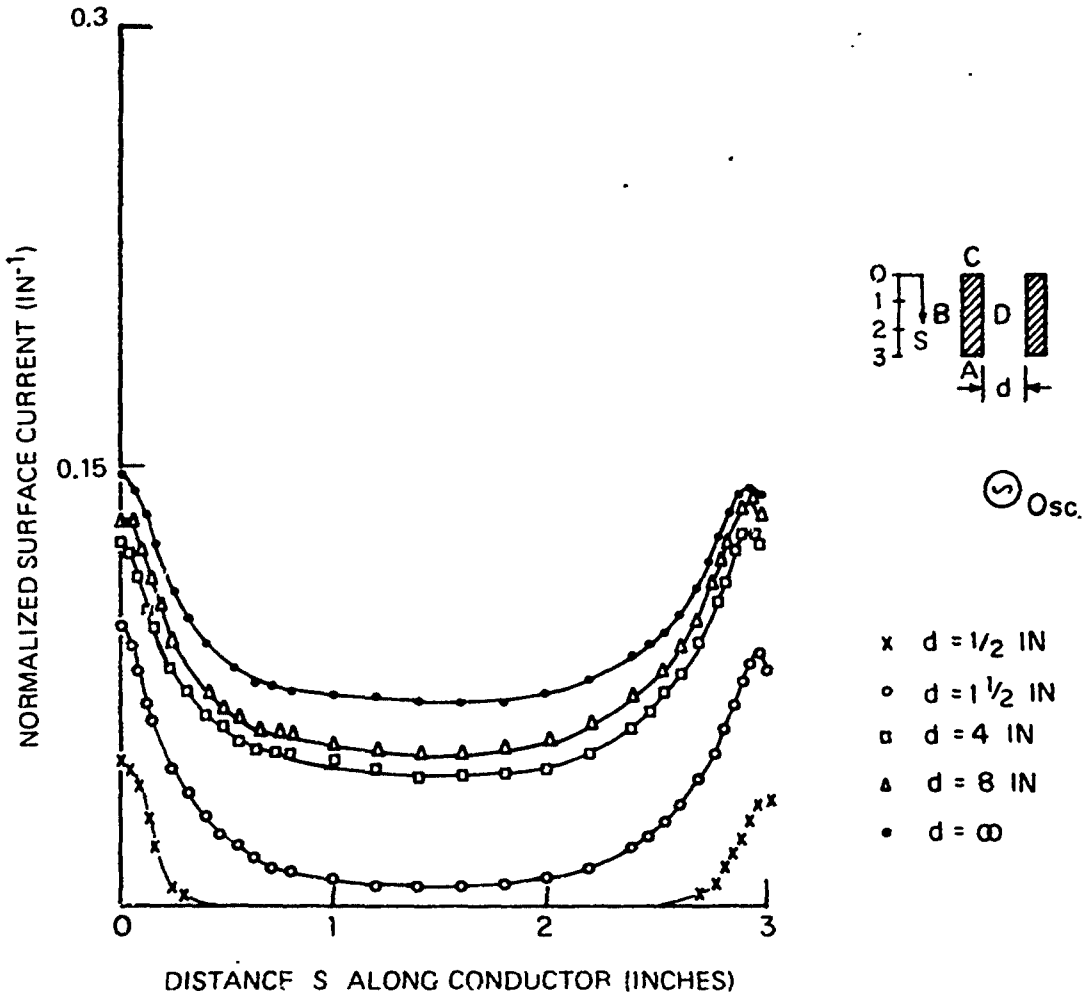


SURFACE CURRENT ON SIDE D WITH CONDUCTOR SEPARATION d AS PARAMETER
(¼ INCH x 3 INCH CONDUCTOR WITH ¼ INCH SIDE PARALLEL)

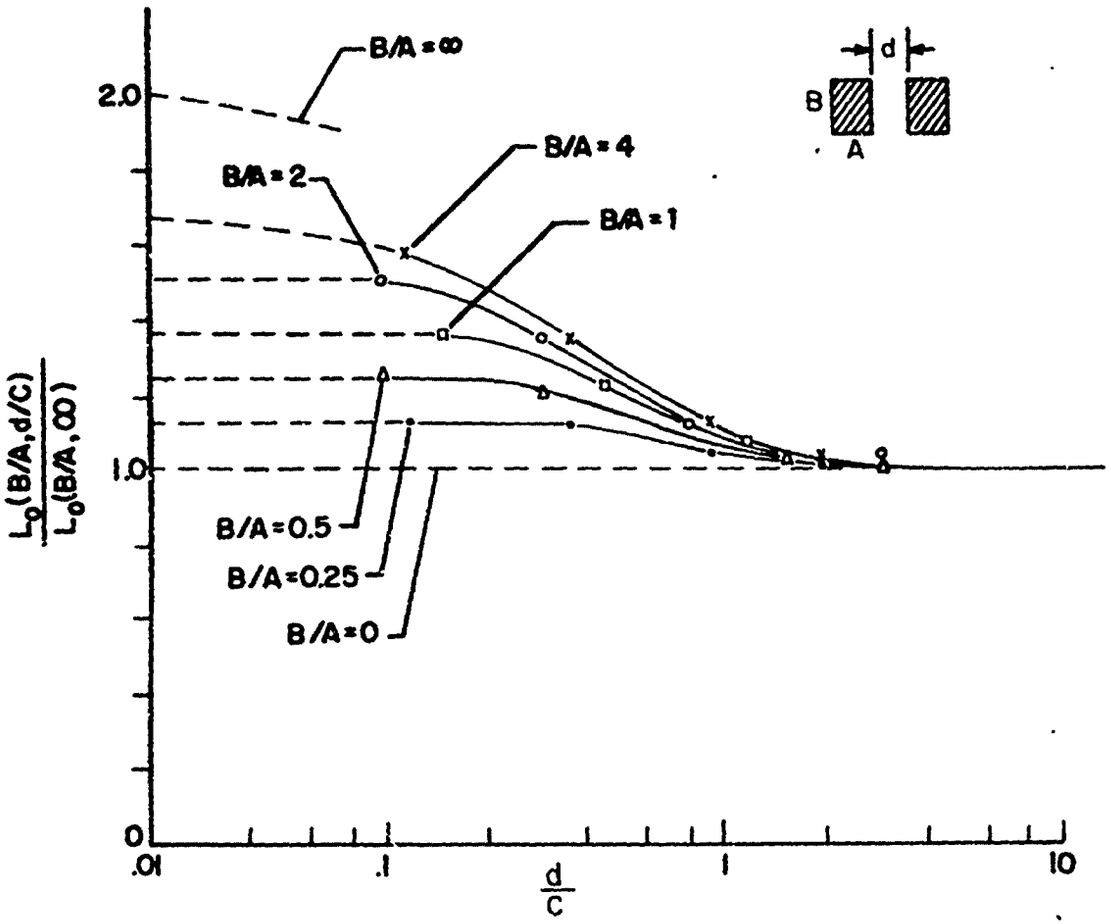
FIG. 19

where C, the circumference of one conductor, is held constant.

Composite results for L_n normalized for constant circumference conductors are presented in Fig. 20 where the dashed lines represent a first-order theory developed in the following section which extends the experimental results to very small separation distances, i.e., $d/C < 0.05$.

IV. First-Order Theory for Determining Relative Ohmic Losses at the Limits of Separation Distance

In analyzing proximity effects for conductors of rectangular cross-section, experimental results show that as the separation distance decreases the current on the inside face (side D) gradually decreases and is distributed over the remaining sides. At d/C of 0.05 (that is, for example, a 1/2 inch separation of 1-1/2 in. by 3 in. conductors) approximately 95% of the current on the inside face has been redistributed to the other faces (see Figs. 9-11 or Figs. 14-16). Furthermore, for decreasing separation distances the current distributions on sides A and C tend towards the form of the current distribution on the left half of a conductor of dimensions 2A by B (see Figs. 9 or 11, for example). Although it is clear that such a redistribution of current is accompanied by changes in skin depth and varying influences of the corner, it is nonetheless useful to construct a simplified model of the process with an assumption that for changes of B/A by a factor of two the relative errors in determining proximity effects which arise from variations in skin depth are minimal and may be neglected.



PROXIMITY EFFECT ON RELATIVE OHMIC POWER LOSS PER UNIT LENGTH OF PARALLEL RECTANGULAR CONDUCTORS OF CONSTANT CIRCUMFERENCE (C) AS A FUNCTION OF SEPARATION DISTANCE (d/C) AND GEOMETRY (B/A)

FIG. 20

In general when an imperfect conductor is excited by a unit alternating current, it exhibits internal power losses commonly referred to as ohmic losses. Ohmic losses generally are a function of the operating frequency, the geometry of the conductor, its macroscopic properties (which also may be frequency-dependent) as well as proximity effects. In the elementary case of an isolated conductor of unit length, area (A), and resistivity (ρ) carrying a unit direct current, the ohmic loss (L_{dc}) is given by

$$L_{dc} = \frac{\rho}{A} \tag{7}$$

The AC case becomes somewhat more involved because the current is no longer uniformly distributed throughout the conductor but decreases from a maximum at the conductor surface to 1/e of that value at the depth δ, the so-called skin depth. For an isolated imperfect conductor with a constant smooth cross-section (i.e., no corners or radii of curvature of the same magnitude as the skin depth) and at least several δ thick, the ohmic loss (L_{ac}) per unit length for a uniformly distributed current with a unit alternating current drive is given approximately by

$$L_{ac} \approx \frac{\rho}{C\delta} \tag{8}$$

where C is the conductor circumference.

Although clearly only an approximation to the actual case, it is greatly simplifying and quite accurate to approximate L_{ac} by (8) when comparing the relative ohmic losses between rectangular conductors of cross-section A by B and 2A by B. This situation is equivalent to a comparison of proximity effects between two rectangular conductors of

dimensions A by B with zero separation distance. In terms of (6) and the above approximation, L_n may be computed at the touching point for two conductors as follows

$$L_n(B/A, 0) = \frac{L_0(B/A, 0)}{L_0(B/A, \infty)} \approx \frac{L_{ac}(B/2A)}{2\,L_{ac}(B/A)} \tag{9}$$

In terms of (8), L_n at the touching point may be rewritten as

$$L_n(B/A, 0) \approx \frac{\dfrac{\rho}{2(2A+B)\delta}}{\dfrac{2\rho}{2(A+B)\delta}} \tag{10}$$

and reduced to

$$L_n(B/A, 0) \approx \frac{1+B/A}{2+B/A} \tag{11}$$

For the values of B/A used, the dashed lines of Fig. 20 represent a first-order theory extending the relative separation distance (d/C) to the limit of 0. For $B/A = \infty$, it is assumed that $B \approx C/2$ and $A \approx 2\delta$; similarly for $B/A = 0$, $A \approx C/2$ and $B \approx 2\delta$. Fig. 21 presents over the range of interest a comparison of the first-order theory with the Cockcroft theory in determining relative ohmic power loss per unit length for parallel rectangular conductors of constant circumference at zero separation distance.

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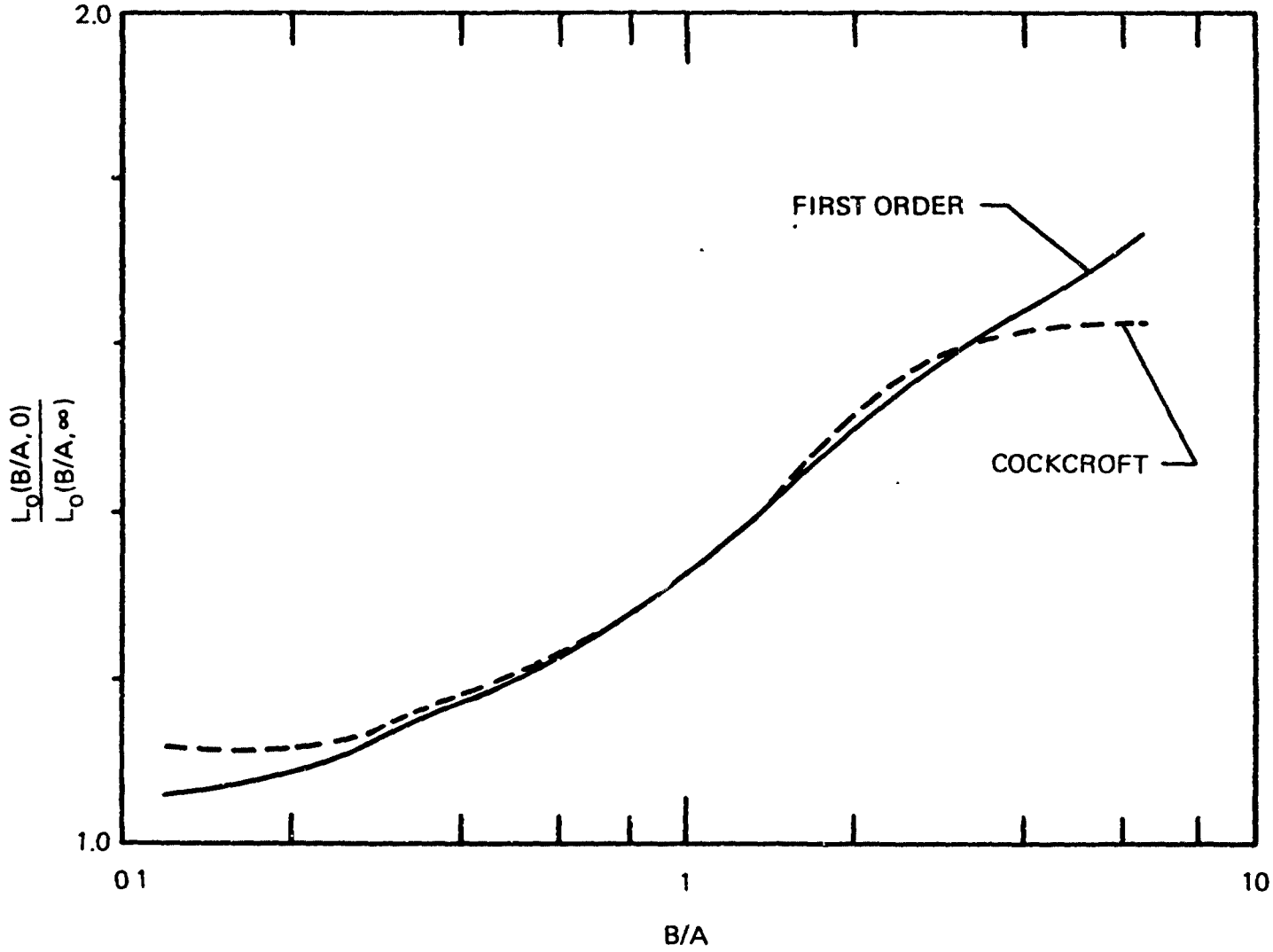

COMPARISON OF FIRST ORDER THEORY TO COCKCROFT THEORY IN DETERMINING RELATIVE OHMIC POWER LOSS PER UNIT LENGTH FOR PARALLEL RECTANGULAR CONDUCTORS OF CONSTANT CIRCUMFERENCE AT ZERO SEPARATION DISTANCE

providing helpful theoretical and experimental insight. All are members of the Division of Engineering and Applied Physics, Gordon McKay Laboratory, Harvard University. This research was also sponsored in part by the F. J. Seiler Research Laboratory, Air Force Systems Command.

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