UNSTABLE LINEAR DIFFERENTIAL GAMES

by

M. S. Nikol'skiy

USSR

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**AUTHOR**

M. S. Nikol'skiy

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**ABSTRACT**

### KEY WORDS

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Differential Equation  
Linear Differential Equation  
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§1. Suppose the motion of n-dimensional vector $z$, in Euclidean space $\mathbb{R}^n$, is described by a linear vector differential equation

$$z = A(t) z - u + v, \quad (1)$$

where $A(t)$ is a quadratic matrix of order $n$, continually dependent on $t (-\infty < t < +\infty)$; the control parameters $u$ and $v$ belong to the convex compacts $P(t)$, $Q(t)$ respectively, which are embedded in $\mathbb{R}^n$ and change continually over $(-\infty < t < +\infty)$. Parameter $u$ is controlled by the pursuer; parameter $v$ -- by the evader. Suppose a convex closed terminal set $M$ is fixed in $\mathbb{R}^n$. Pursuit begins from point $z_0 \in M$ at moment $t_0$ and is considered completed when $z(t)$ (see (1)) first contacts $M$.

The goal of the pursuer is to bring point $z(t)$ to $M$ as rapidly as possible. It is assumed that the pursuer knows $z(t)$ and $v(t)$ at each moment in time $t$, i.e. pursuit with discrimination of the evader is studied. The evader acts arbitrarily, using measurable control $v(t)$, which follows the requirement $v(t) \in Q(t)$.

We will say that game (1) can be completed from position $(z_0, t_0)$ in a finite time if there is a number $t (z_0, t_0)$ such that with any measurable change $v(t)$ the pursuer, using his information, can construct a measurable change $u(t)$ ($u(t) \in P(t)$), such that point $z(t)$ strikes $M$ not later than moment $t_0 + t(z_0, t_0)$.

One of the most important problems arising in the theory of pursuit is the problem of separation of those points $(z_0, t_0)$ from which the game can be completed in a finite time. Strong results have been produced in this direction for stable games (see [1-7] and others). The most complete results have been produced by L. S. Pontryagin in [4]. They were produced by a direct method with
a wider area of application than that of the first direct method developed in [3].

The present article is dedicated to a generalization of the second direct method of L. S. Pontryagin (see [4]) to the unstable case (see (1)).

§2. In this paragraph, we will introduce certain concepts which will be useful for the future.

A. Let \( U(T) \) be a convex compact belonging to \( \mathbb{R}^n \), continually dependent on \( T \) in sector \([p, q]\), where \( p < q \). Let us study all possible measurable vector functions \( u(\cdot) \) in \([p, q]\), satisfying the condition \( u(\tau) \in U(\tau) \). Let us study the set of vector integrals \( \int_p^q u(\tau) \, d\tau \) and represent it by \( \int_p^q U(\tau) \, d\tau \).

It is not difficult to see that this set is a convex limited set. Using [8], it is not difficult to prove that \( \int_p^q U(\tau) \, d\tau \) is a closed set. Thus, we have the operation of integration of a closed set dependent on a parameter.

B. The geometric difference of two convex sets \( \mathbb{N}_1, \mathbb{N}_2 \), belonging to \( \mathbb{R}^n \) refers (see [3]) to the set \( \mathbb{N}_3 \) which consists of all vectors a translating \( \mathbb{N}_2 \) into \( \mathbb{N}_1 \), i.e. \( a + \mathbb{N}_2 \subseteq \mathbb{N}_1 \). This operation is represented as: \( \mathbb{N}_3 = \mathbb{N}_1 - \mathbb{N}_2 \). It is not difficult to show that closure of \( \mathbb{N}_1 \) indicates closure of \( \mathbb{N}_3 \).

C. Suppose \( \mathbb{N}_1, \mathbb{N}_2 \) are arbitrary sets from \( \mathbb{R}^n \). The algebraic sum of these sets refers to the set \( \mathbb{N}_3 \) of all vectors \( a \) of the form \( a_3 = a_1 + a_2 \), where \( a_1 \in \mathbb{N}_1, a_2 \in \mathbb{N}_2 \), and will be written as \( \mathbb{N}_3 = \mathbb{N}_1 + \mathbb{N}_2 \).

D. In [4], L. S. Pontryagin introduced the concept of the alternative integral from \( U(\tau), V(\tau) \), belonging to \( \mathbb{R}^n \) and changing continually over the sector \([p, q]\) \( (p < q) \) of convex compacts with the initial closed convex set.

B. This integral is represented by the symbol \( \int_p^q [U(\tau) \not\subseteq V(\tau)] \, d\tau \).

We require the altered interval

\[
\int_{L, P}^q [U(\tau) \not\subseteq V(\tau)] \, d\tau,
\]

fixing not the initial integration set, but rather the final integration set. In constructing integral (2), we will base ourselves on rational subdivisions \( \omega \) of sector \([p, q]\) by means of points \( p = \tau_0 < \tau_1 < \ldots < \tau_k = q \), where \( \tau_1, \ldots, \tau_{k-1} \) are rational numbers. This rational subdivision \( \omega \) is compared
with the convex set

\[ \Sigma_\omega = \left( \left( \left( \left( B + \int_{\tau_{k-1}}^{\tau_k} U(\tau) \, d\tau \right) \leq \int_{\tau_{k-1}}^{\tau_k} V(\tau) \, d\tau \right) \left( \left( B + \int_{\tau_{k-2}}^{\tau_{k-1}} U(\tau) \, d\tau \right) \leq \int_{\tau_{k-2}}^{\tau_{k-1}} V(\tau) \, d\tau \right) \ldots \right) \right) \]

which we will call the integral sum.

By integral (2), we refer to the intersection of sets \( E_\omega \) with respect to all rational divisions \( \omega \):

\[ \left[ \left[ U(\tau) \, d\tau \wedge V(\tau) \, d\tau \right] \cap \bigcap_\omega \Sigma_\omega. \]

We note that the existence of integral (2) as a non-empty set requires non-emptiness of all \( E_\omega \). If \( \bigcap_\omega E_\omega \) is not empty, the altered integral (2) is a closed convex set.

Suppose integral (2) is not empty and rational point \( t_1 \in (p, q) \) is selected on sector \([p, q]\). Let us study the rational divisions \( \omega' \) of the form \( p = \tau_0 < \tau_1 = t_1 < \tau_2 < \ldots < \tau_k = q \).

Obviously,

\[ \bigcap_\omega \Sigma_\omega \subseteq \bigcap_\omega \Sigma_{\omega'}. \]

Let us study the rational subdivision \( \omega'' \) of sector \([t_1, q]\), generated by subdivision \( \omega' \). It follows from formula (3) that

\[ \Sigma_{\omega''} \subseteq \left( \Sigma_{\omega'} + \int_{p}^{t_1} U(\tau) \, d\tau \right) \subseteq \int_{p}^{t_1} V(\tau) \, d\tau. \]

Rational subdivisions \( \omega'' \) are always even numbers. They can be renumbered: \( \omega_1'', \omega_2'', \ldots \). Let us represent by \( \mu_1(i = 1, 2, \ldots) \) the rational subdivision produced by combining the points of the subdivisions \( \omega_1'', \ldots, \omega_i''. \) The integral sum (3) corresponding to rational subdivision \( \mu_1 \) will be represented by \( \Sigma_{\mu_1} \).
Work [4] gives the following formulas:

$$(A \pm U) \subseteq V \pm (U \pm V), \quad (A + U) \supseteq (A \pm V) + U,$$

where $A, U, V$ are convex sets in $\mathbb{R}^n$. Using definition $\Sigma$, and these formulas, it is not difficult to show that $\Sigma_1 \supseteq \Sigma_2 \supseteq \ldots$ and that

$$\bigcap_{t=1}^{n} [U(t) dt \pm V(t) dt] = \bigcap_{t=1}^{n} \Sigma_t.$$  \hspace{1cm} (7)

Let us show that

$$\bigcap_{t=1}^{n} \Sigma_t \subseteq \left( \bigcap_{t=1}^{n} \Sigma_t + \int_{t}^{t} U(t) dt \right) \pm \int_{t}^{t} V(t) dt.$$  \hspace{1cm} (8)

Let us study the rational subdivision of sector $[p, q] \omega_1$, generated by point $t_1$ ($t_1$ is a rational number) and subdivisions $\omega_1$. Obviously $\bigcap_{t=1}^{n} \Sigma_t \subseteq \bigcup_{t=1}^{n} \Sigma_t$.

To prove inclusion (8), it is sufficient to prove inclusion

$$\bigcap_{t=1}^{n} \Sigma_t \subseteq \left( \bigcap_{t=1}^{n} \Sigma_t + \int_{t}^{t} U(t) dt \right) \pm \int_{t}^{t} V(t) dt.$$  \hspace{1cm} (9)

Let us study point $\xi_0$, satisfying the condition $\xi_0 \in \bigcap_{t=1}^{n} \Sigma_t$. Due to equation (6), this inclusion indicates the relationship

$$\xi_0 + \int_{t}^{t} V(t) dt \subseteq \Sigma_t - \int_{t}^{t} U(t) dt,$$

which is correct with any $i = 1, 2, \ldots$. It follows from this that for any given measurable vector function $v(\tau), p \leq \tau \leq t_1 (v(\tau) \in V(\tau))$, a measurable vector function $u_1(\tau), p \leq \tau \leq t_1 (u_1(\tau) \in U(\tau))$, can be found such that

$$\xi_0 + \int_{t}^{t} v(\tau) d\tau - \int_{t}^{t} u_1(\tau) d\tau = \eta \in \Sigma_t.$$  \hspace{1cm} (10)
On the strength of the assumed continuity of sets $U(\tau), V(\tau)$ in sector $[p, q]$, the estimate $|n_1| \leq \text{const}$ is correct for $n_1$. Therefore, it can be considered that a certain subsequence of vectors $n_i$, which we will represent by $n_{i_k}$, converges to a certain vector $n^*$. The embeddedness of closed sets $\Sigma_i$ indicates $n^* \in \bigcap_i \Sigma_i$.

We can consider that subsequence $u_{i_k}(\tau)$ converges weakly in sector $[p, t_1]$ to a certain function $u_0(\tau) \in U(\tau))$. Then from equation (10) it follows easily that

$$
\xi_0 + \int_p^{t_1} u(\tau) d\tau - \int_p^{t_1} u_0(\tau) d\tau = n^* \in \bigcap_i \Sigma_i.
$$

From this we get

$$
\xi_0 \in \left( \bigcap_i \Sigma_i \right) + \int_p^{t_1} U(\tau) d\tau \leq \int_p^{t_1} V(\tau) d\tau,
$$

i.e. inclusion (9) is proven and, consequently, inclusion (8) is proven. Using inclusion (5) and equations (4), (7), we produce an important formula:

$$
\int_p^{t_1} |U(\tau) d\tau \pm V(\tau) d\tau| \subset \int_p^{t_1} [U(\tau) d\tau \pm V(\tau) d\tau] + \int_p^{t_1} U(\tau) d\tau \pm \int_p^{t_1} V(\tau) d\tau,
$$

where $t_1$ is either any rational number in $[p, q]$, or a number corresponding with one of the ends of $[p, q]$.

The following will be useful in producing further properties of the altered integral (2).

E. Let $\mathcal{A}_i(s), \mathcal{C}_i(s)$ be convex compacts from $\mathbb{R}^n$, dependent on parameter $s$. Suppose in a certain area of point $s_0$, set $\mathcal{A}_i(s) \subseteq \mathcal{C}_i(s)$ is not empty, while at point $\mathcal{A}_i(s)$, it is upward semicontinuous relative to inclusions (see [8]); $\mathcal{C}_i(s)$ is continuous. We then have the following
Lemma. The set $\mathbb{N}_i(s) \subseteq \mathbb{N}_j(s)$ is upper semicontinuous relative to inclusions at point $s_0$.

The proof of the lemma is simple, and we will not present it.

F. Let us represent integral (2) as a function of the lower limit of $p$ through $B(p)$. Let us assume in formula (11) $- p + t_1 = \varepsilon$. Then inclusion (11) can be rewritten as:

$$B(p) \subseteq \left\{ \int (p - t) : \frac{p - c}{t} \int U(t) dt \leq \frac{p - c}{t} \int V(t) dt, \right\}$$

where $p + \varepsilon$ is either any rational number from sector $[p, q]$ or one of the ends of sector $[p, q]$.

Let us prove that inclusion (12) is correct with any $p + \varepsilon$ belonging to sector $[p, q]$.

Let us study the sequence of numbers $p_i \leq p$ such that $p_i + \varepsilon \in [p, q]$, $p_i + \varepsilon$ is a rational number and $p_i + \varepsilon = p + \varepsilon$, where $p + \varepsilon$ is an arbitrary fixed number from interval $(p, q)$. From inclusion (12)

$$B(p) \subset \frac{p - c}{t} \int V(t) dt \subset B(p_i - \varepsilon) + \frac{p - c}{t} \int U(t) dt.$$

Let us study a certain rational subdivision $\omega$ of sector $[p + \varepsilon, q]$, generated by points $p + \varepsilon = \tau_0 < \tau_1 < \tau_2 < \ldots < \tau_k = q$, and rational subdivisions $\omega_i$ $(i = 1, 2, \ldots)$ of sector $[p_i + \varepsilon, q]$, generated by points $p_i + \varepsilon = \tau_i \subset \tau_1 < \tau_2 < \ldots < \tau_k = q$. Thus, subdivisions $\omega$ and $\omega_i$ differ only in their left point.

Let us represent by $E_{\omega}$ and $E_{\omega_i}$ $(i = 1, 2, \ldots)$ the integral sums (3), corresponding to divisions $\omega, \omega_i$ with finite set $B$. It follows from inclusion (13) that

$$B(p) \subset \frac{p - c}{t} \int V(t) dt \subset \sum_{\omega_i} \frac{p - c}{t} \int U(t) dt, \quad i = 1, 2, \ldots$$

Let us take arbitrary vector $b \in B(p)$. It follows from the inclusion produced...
Using the definition of the operation \( \ast \), formula (3), the even limited nature of sets \( U(\tau), V(\tau) \) in sector \([p, q] \) and inclusion (14), we can prove the inclusion

\[
b + \int_p^{p_1+r} V(\tau) d\tau \subset \Sigma_{\mu_1} + \int_p^{p_{1+\varepsilon}} U(\tau) d\tau.
\]

where \( b \) is a convex compact belonging to \( B \), while \( \Sigma_{\omega_i}(B_1) \) is an integral sum constructed with respect to subdivision \( \omega_i \) but with finite set \( B_1 \). This statement is trivial when \( B \) is limited and interesting when \( B \) is unlimited.

Using the lemma of point "E" and inclusion (15), it is not difficult to produce the relationship

\[
b + \int_p^{p_1+r} V(\tau) d\tau \subset \Sigma_{\omega_i}(B_1) \subset \int_p^{p_1+r} U(\tau) d\tau \subset \sum \int_p^{p_{1+\varepsilon}} U(\tau) d\tau,
\]

where \( \Sigma_{\omega_i}(B_1) \) is an integral sum corresponding to rational subdivision \( \omega \), with finite set \( B_1 \).

Earlier in point "D" we showed that the altered integral (2) can be produced as the intersection of integral sums \( \Sigma_{\mu_i} \) \((i = 1, 2, \ldots)\), forming a sequence of sets embedded in each other \( \Sigma_{\mu_i} \supset \Sigma_{\mu_{i-1}} \supset \cdots \). Taking such a sequence of subdivision \( \mu_i \) \((i = 1, 2, \ldots)\) as the \( \omega \) in inclusion (16), we produce

\[
b + \int_p^{p_1+r} V(\tau) d\tau \subset \Sigma_{\mu_i} + \int_p^{p_{1+\varepsilon}} U(\tau) d\tau.
\]

Using the limited nature of closed set \( \int_p^{p_{1+\varepsilon}} U(\tau) d\tau \), the embeddedness of the closed sets \( \Sigma_{\mu_i} \) and the equation \( B(p + \varepsilon) = \bigcap \Sigma_{\mu_i} \), it is not difficult to prove that

\[
b + \int_p^{p_{1+\varepsilon}} V(\tau) d\tau \subset B(p + \varepsilon) + \int_p^{p_{1+\varepsilon}} U(\tau) d\tau.
\]

Since \( b \) is an arbitrary element from \( B(p) \), inclusion (12) is proven for arbitrary point \( p + \varepsilon \in \mathcal{E}(p, q) \).
§3. Every thing is now prepared for investigation of game (1) using the altered integral. Let us represent by $C(t, \tau) (t \geq \tau)$ the matrixant of the homogeneous system $y = A(t)y$ (for a definition of a matrixant and its properties, see [9]). We recall only that if measurable controls $u(\tau), v(\tau) (u(\tau) \in \mathcal{P}(\tau), v(\tau) \in \mathcal{Q}(\tau))$ are fixed in the sector $[t_0, t]$, then according to the Cauchy formula

$$z(t) = C(t, t_0)z_0 + \int_{t_0}^{t} C(t, \tau)(-u(\tau) \cdot v(\tau))d\tau.$$ 

Let us study the altered integral

$$W(t, t_0) = \int_{t_0}^{t} [C(t, \tau)P(\tau)d\tau \pm C(t, \tau)Q(\tau)d\tau]$$

where $t \geq t_0$. We will assume that the set $W(t, t_0)$ is not empty with all $t, t_0 (t \geq t_0)$. Let us study also vector $C(t, t_0)z_0 (t \geq t_0)$. Two cases are possible: 1) with no $t$ does vector $C(t, t_0)z_0$ belong to $W(t, t_0)$; 2) there is at least one $\bar{t}$ with which the inclusion

$$C(t', t_0)z_0 \in W(t', t_0)$$

is true.

In the first case, we can say nothing concerning the possibility of completion of pursuit from point $(z_0, t_0)$.

Let us study the second case.

Lemma. There is a minimum $\bar{t}$ for which inclusion (18) is fulfilled.

Proof. There are two possibilities: a) there is a finite number of moments $\bar{t}$ at which inclusion (18) is fulfilled; b) there is an infinite number of moments $\bar{t}$, at which inclusion (18) is fulfilled. In case "a" everything is clear. In case "b" we can take the decreasing sequence of numbers $\bar{t}_i$, which converges to the lower bound of all numbers $\bar{t}$ satisfying condition (18). Let us assume that the limit of $\bar{t}_i$ is equal to $t_0 + T(z_0, t_0)$.

For brevity we will write $T$ in place of $T(z_0, t_0)$. Let us take a certain rational subdivision of sector $[t_0, t_0 + T]$. It is generated by the points $\tau_0 = t_0 < \tau_1 < \ldots < \tau_k = t_0 + T$. Let us study also the rational subdivisions $\omega_1$ of sectors $[t_0, t_1]$, produced from subdivision $\omega$ as follows: $\tau_0 = t_0 < \ldots < \tau_k = t_0 + T$. Let us assume that
< \tau_1 < \ldots < \tau_{k-1} < \tau_k = \bar{\tau}_1 (i = 1, \ldots). Thus, they differ from subdivision \omega only in the rightmost point.

Let us represent by \Sigma_{\omega} the integral sum (3) corresponding to subdivision \omega with finite set M and

\[ U(t) = C(t_0 + T, \tau) P(\tau), \quad V(t) = C(t_0 + T, \tau) Q(\tau), \quad t_0 \leq \tau \leq t_0 + T. \]

We represent by \Sigma_{\omega_1} the integrals sum (3), corresponding to the division \omega_i (i = 1, 2, \ldots) with finite set M and

\[ U_i(t) = C(t_0 + \bar{t}_i, \tau) P(\tau), \quad V_i(t) = C(t_0 + \bar{t}_i, \tau) Q(\tau), \quad t_0 \leq \tau \leq t_0 + \bar{t}_i. \]

It follows from the definition of \bar{\tau}_1 that \( C(\bar{\tau}_1, \tau_0) Z_0 \in \Sigma_{\omega_1}. \)

Let us study the curve \( C(t, t_0) Z_0 \) as a function of parameter t in sector \([t_0, \bar{\tau}_1].\) Obviously, there is a sphere D with its center at the coordinate origin so large that this curve will be within it where \( t_0 \leq t \leq \bar{\tau}_1. \) For the following, it is sufficient to study the set \( \Sigma_M \cap D, \Sigma_M \cap D. \)

Using formula (3) for \( \Sigma_{\omega_1} \) and \( \Sigma_{\omega}, \) the even limitation of sets \( P(\tau), Q(\tau) \)
in \([t_0, \bar{\tau}_1]\) and the definition of the operation *, it is not difficult to prove that \( \Sigma_{\omega_1} \) and \( \Sigma_{\omega} \) correspond in sphere D with the integral sums \( \Sigma_{\omega_1}(M_1) \) and \( \Sigma_{\omega}(M_1) \) respectively, constructed on the basis of the subdivisions \( \omega_1 \) and \( \omega \) and the same sets \( U_1(\tau), V_1(\tau), U(\tau), V(\tau), \) as \( \Sigma_{\omega_1}, \Sigma_{\omega}, \) but with finite set \( M_1, \)
where \( M_1 \) is a convex compact, independent of the number i and belonging to M.

This statement is trivial with limited M and interesting for unlimited M. It follows from the above that \( C(t, t_0) Z_0 \in \Sigma_{\omega_1} \). We note that \( \Sigma_{\omega_1}(M_1) \subset \Sigma_M, \Sigma_{\omega}(M_1) \subset \Sigma_M. \)

Let us now study set \( \Sigma_{\omega_1}(M_1) \) as a function of \( \bar{\tau}_1. \) Using the upper semicontinuity of operation * relative to inclusions (see paragraph 2, "E"), it is easy to prove that the fixed \( \epsilon \) can be used to find a number \( N(\epsilon), \) such that \( i > N(\epsilon) \)

\[ C(t, t_0) Z_0 (\Sigma_{\omega_1}(M_1) : S_i \subset \Sigma_M : S_\epsilon. \]
where $S_\varepsilon$ is a sphere of radius $\varepsilon$ with its center at the coordinate origin. From inclusion (19) it is not difficult to see that $C(t_0 + T, t_0) z_0 x \Sigma_w + S_\varepsilon$. Since was an arbitrary rational subdivision of sector $[t_0, t_0 + T]$, while $\varepsilon$ is an arbitrary positive number, it follows from this that

$$C(t_0 + T, t_0) z_0 x W(t_0 + T, t_0),$$

which was to be proven.

Theorem. Pursuit can be completed from point $z_0, t_0$ in time $T(z_0, t_0)$ if the pursuer knows control $v(s)$ of the evader at each moment $t$ on sector $t \leq s \leq t + \varepsilon (\varepsilon > 0$ is arbitrary).

The basis of the reality of this hypothesis of information in the hands of the pursuer is presented in [4].

Proof. According to our assumption, the pursuer knows the control of the evader in sector $[t_0, t_0 + \varepsilon]$; suppose this control is $v(t) (v(t) \in Q(t)$ where $t_0 \leq t \leq t_0 + \varepsilon$).

Without limiting generality, we can consider $\varepsilon \leq T(z_0, t_0)$. Subsequently to simplify our inscription, let us write $T$ in place of $T(z_0, t_0)$. Using inclusion (12) for the altered integral $W(t, t_0)$ (see (17)), we produce

$$W(t + T, t_0) \subseteq W(t + T, t_0 + \varepsilon) + \int_{t_0}^{t + T} C(t_0 + T, t) P(t) dt \subseteq \int_{t_0}^{t + T} C(t_0 + T, t) Q(t) dt,$$

from which, using the definition of the operation $*$ (see paragraph 2, "B"), we easily produce the inclusion

$$W(t_0 + T, t_0) \subseteq W(t_0 + T, t_0 + \varepsilon) + \int_{t_0}^{t + T} C(t_0 + T, t) P(t) dt - \int_{t_0}^{t + T} C(t_0 + T, t) v(t) dt,$$

from which and from formula (20) it follows that a measurable control $u(t) (t \leq t \leq t_0 + \varepsilon, u(t) \in P(t))$ is found such that

- 10 -
\[ C(t_0 + T, t_0) x_0 - \int_{t_0}^{t_1} C(t_0 + T, \tau) u(\tau) \, d\tau + \int_{t_0}^{t_1} C(t_0 + T, \tau) v(\tau) \, d\tau = W(t_0 + T, t_0 + \epsilon). \] (21)

As we know (see [9]), matrixant \( C(t, t_0) \) has the property \( C(t, t_0) = C(t_1, t_0) \) \( C(t, t_1) \), where \( t_0 < t_1 < t \).

Using this property, we produce from formula (21)

\[ C(t_0 + T, t_0 + \epsilon) (C(t_0 + \epsilon, t_0) x_0 - \int_{t_0}^{t_0 + \epsilon} C(t_0 + \epsilon, \tau) u(\tau) \, d\tau + \int_{t_0}^{t_0 + \epsilon} C(t_0 + \epsilon, \tau) v(\tau) \, d\tau) = C(t_0 + T, t_0 + \epsilon) x(\epsilon) \in W(t_0 + T, t_0 + \epsilon). \]

From which it follows that for point \((x(\epsilon), t_0 + \epsilon)\), the inequality \( T(x(\epsilon), t_0 + \epsilon) \leq T - c \) is correct. Thus, we have decreased time \( T(x_0, t_0) \) by at least \( c \) in time \( c \). Performing similar steps further, the pursuer will complete pursuit in a time \( \leq T(x_0, t_0) \) (we note that \( W(t, t) = M \)).

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