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UPPER PERCENTAGE POINTS OF THE INTERMEDIATE ROOTS OF THE MANOVA MATRIX

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OXFORD, OHIO

PROJECT NO. 7071

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FOREWORD

This report was prepared for the Applied Mathematics Research Laboratory, Aerospace Research Laboratories by P. R. Krishnaiah, F. J. Schuurmann and V. B. Waikar. The work of Schuurmann was performed at the Aerospace Research Laboratories while in the capacity of a Technology Incorporated Visiting Research Associate under Contract F 33615-71-C-1463. The work of Waikar was performed at the Aerospace Research Laboratories while in the capacity of an Ohio State University Research Foundation Visiting Research Associate under Contract F 33615-67-C-1758. The present affiliation of Schurrmann and Waikar is Miami University, Oxford, Ohio.

In this report, the authors gave tables for the upper percentage points of the intermediate roots of the matrix $S_1(S_1+S_2)^{-1}$ where S_1 and S_2 are distributed independently as central Wishart matrices.

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13. ABSTRACT

Let S_1 and S_2 be independently distributed as $p \times p$ central Wishart matrices with n_1 and n_2 $(p < n_1, n_2)$ degrees of freedom and let $E(S_1/n_1) = E(S_2/n_2) = \Sigma$. Further, let $\theta_1 < \theta_2 < \ldots < \theta_p$ be the characteristic roots of $S_1 (S_1 + S_2)^{-1}$. Let $r = (n_1 - p - 1)/2$ and $n = (n_2 - p - 1)/2$. In this paper, the authors gave tables for the exact values of the upper 5% and 1% points of the distribution of θ_i, $i = 2, 3, \ldots, p - 1$ for $p = 4,5,6,7$ and of the distribution of θ_2 and θ_7 for $p = 8$ when $r = 0\ (1)\ 5, 7, 10, 15$ and $n = 5\ (1)\ 10\ (2)\ 20\ (5)\ 50$. These tables were constructed by using the exact expression for the c.d.f. of θ_s $(2 \leq s \leq p - 1)$ given by Krishnaiah and Waikar (J. Multivariate Analysis, 1 (1971)).

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ABSTRACT

Let S_1 and S_2 be independently distributed as $p \times p$ central Wishart matrices with n_1 and n_2 $(p < n_1, n_2)$ degrees of freedom and let $E(S_1/n_1) = E(S_2/n_2) = \Sigma$. Further, let $\theta_1 < \theta_2 < \cdots < \theta_p$ be the characteristic roots of $S_1 (S_1 + S_2)^{-1}$. Let $r = (n_1 - p - 1)/2$ and $n = (n_2 - p - 1)/2$. In this paper, the authors gave tables for the exact values of the upper 5% and 1% points of the distribution of θ_i, $i = 2, 3, \cdots, p - 1$ for $p = 4, 5, 6, 7$ and of the distribution of θ_2 and θ_7 for $p = 8$ when $r = 0\ (1)\ 5, 7, 10, 15$ and $n = 5\ (1)\ 10\ (2)\ 20\ (5)\ 50$. These tables were constructed by using the exact expression for the c.d.f. of θ_s $(2 \leq s \leq p - 1)$ given by Krishnaiah and Waikar (J. Multivariate Analysis, 1 (1971)).

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1. INTRODUCTION

The marginal disbributions of the individual roots of random matrices are useful in testing certain statistical hypotheses. Roy [7] derived the cumulative distribution function (c.d.f.) of any single intermediate root of the central MANOVA matrix; the expression obtained by Roy is complicated. Davis [2] showed that the marginal densities of the individual roots of the MANOVA and Wishart matrices satisfy certain differential equations. Also, Davis [3] gave a recurrance relation for these marginal densities. Krishnaiah and Waikar [5] gave expressions for the c.d.f.'s of the intermediate roots of a class of random matrices which includes the MANOVA matrix. These expressions are in terms of the linear combinations of products of double integrals. Using these expressions, the authors have constructed exact values of the upper 5% and 1% points of the distributions of the intermediate roots of the MANOVA matrix.

2. C.D.F. OF AN INTERMEDIATE ROOT

Let S_1 and S_2 be independently distributed as $p \times p$ $(p \leq n_1, n_2)$ central Wishart matrices with n_1 and n_2 degrees of freedom, and let $E(S_1/n_1) = E(S_2/n_2) = \Sigma$. Also let $\theta_1 < \theta_2 < \cdots < \theta_p$ be the latent roots of $S_1(S_1 + S_2)^{-1}$. Then it is well known (see [7]) that the joint probability density of $\theta_1, \cdots, \theta_p$ is

$$f(\theta_1, \cdots, \theta_p) = c(p,r,n) \prod_{i=1}^{p} \{\theta_i^r (1 - \theta_i)^n\} \prod_{i>j}^{p} (\theta_i - \theta_j)$$
$$0 < \theta_1 < \cdots < \theta_p < 1 \qquad (2.1)$$

where

$$c(p,r,n) = \frac{\pi^{p^2/2}\,\Gamma_p(r+n+p+1)}{\{\Gamma_p((2r+p+1)/2)\,\Gamma_p((2n+p+1)/2)\,\Gamma_p(p/2)\}},$$

$$\Gamma_p(a) = \pi^{p(p-1)/4}\prod_{i=1}^{p}\Gamma(a-\tfrac{1}{2}(i-1)),$$

$$r = (n_1 - p - 1)/2 \text{ and } n = (n_2 - p - 1)/2 .$$

Krishnaiah and Waikar [5] gave the following exact expression for the c.d.f. of an intermediate root θ_s $(1 \le s \le p-1)$:

$$P[\theta_s < x] = P[\theta_{s+1} < x] + c(p,r,n)\,\Sigma_1 \pm \rho(\psi;s,\{k_1,\cdots,k_s\},0,x)\cdot \rho(\psi;p-s,\{t_1,\cdots,t_{p-s}\},x,1), \tag{2.2}$$

where $\{k_1,\cdots,k_s\}$ is a subset of the set of integers $\{0,1,\cdots,p-1\}$ such that $k_1 < \cdots < k_s$ and $t_1 < t_2 < \cdots < t_{p-s}$ is the subset complementary to $k_1,\cdots,k_s$ while Σ_1 denotes summation over $\binom{p}{s}$ possible subsets $k_1,\cdots,k_s$. Further, $\psi(y) = y^r(1-y)^n$ and the sign inside Σ_1 is positive or negative according as $s(s+3)/2 + \Sigma\, k_i$ is even or odd. The function $\rho(\cdot)$ is defined by

$$\rho(\psi;p,\{k_1,\cdots,k_p\},L,U) = \Delta(\psi;2m,\{k_1,\cdots,k_{2m}\},L,U) \quad \text{when } p = 2m \tag{2.3}$$

and

$$\rho(\psi;p,\{k_1,\cdots,k_p\},L,U) = \sum_{i=1}^{2m+1}(-1)^{i+1}F_{k_i}(L,U)G_i(\psi;2m+1,\{k_1,\cdots,k_{2m+1}\},L,U) \quad \text{when } p = 2m+1 \tag{2.4}$$

where $L < U$ are arbitrary constants, $k_1,\cdots,k_p$ is a set of nonnegative integers,

$$\Delta(\psi;2m,\{k_1,\cdots,k_{2m}\},L,U) = |(f_{k_i}^{k_j}(L,U))i,j = 1,\cdots,2m|^{1/2},$$

$$G_t(\psi;2m+1,\{k_1,\cdots,k_{2m+1}\},L,U) = |(f_{k_i}^{k_j}(L,U))i,j = 1,\cdots,t-1,t+1,\cdots,2m+1|^{1/2}$$

for $t = 1,2,\cdots,2m+1$ while $G_1(\psi;1,k_1,L,U) \equiv 1$. Further $f_s^t(L,U) = F_s^t(L,U) - F_t^s(L,U)$, $F_s^t(L,U) = \int_L^U F_s(L,\theta)\theta^t\psi(\theta)d\theta$,

$$F_s(L,\theta) = \int_L^\theta x^s\psi(x)dx .$$

Note that Eq. (2.2) is a recurrance formula and to start with, we need to compute the $P[\theta_p < x]$ which can be done easily by using the exact expression for the c.d.f. of the largest root θ_p given in Krishnaiah and Chang [4].

Using the recurrance relation (2.2) we constructed the exact upper 5% and 1% points of the distribution of θ_s, $s = 2,3,\cdots,p-1$ for $p = 4, 5, 6, 7$ and of the distributions of θ_2 and θ_7 for $p = 8$ when $r = 0\ (1)\ 5, 7, 10, 15$ and $n = 5\ (1)\ 10\ (2)\ 20\ (5)\ 50$. Here we note that Pillai and Dotson [6] computed the upper 5% and 1% points of the median root for $p = 3$ and certain values of r and n by using the expressions involving pseudo-determinants and reduction formulas on them. As a check for the accuracy of tables given in this paper, we computed a few values of the percentage points of the median root for $p = 3$ and compared them with the values of Pillai and Dotson [6]. As an additional check, we have also used the program to compute the upper percentage points of the smallest root by using the recurrance relation (2.2) and starting with the probability integral of the largest root. Similarly, the upper percentage points of the largest root are computed for some values of the parameters by using the recurrance relation (2.2) and starting with the probability integral of the smallest root. The resulting values of the percentage points of the extreme roots are compared with the corresponding values given in Chang [1] and Schuurmann and Waikar [8]. The entries given in the table at the end of this paper differ from actual values by at most one unit in the last decimal.

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The entries in the following table are the values of x for different values of r, n, p, s and α where

$$P[\theta_s \leq x] = P[\theta_{s+1} \leq x] + \int \cdots \int_D f(\theta_1,\cdots,\theta_p)d\theta_1\cdots d\theta_p = (1 - \alpha)$$

$$f(\theta_1,\cdots,\theta_p) = c(p,r,n) \prod_{i=1}^{p} \{\theta_i^r(1 - \theta_i)^n\} \prod_{i>j} (\theta_i - \theta_j),$$

$$c(p,r,n) = \frac{\pi^{p^2/2}\, \Gamma_p(r + n + p + 1)}{\Gamma_p((2r + p + 1)/2)\, \Gamma_p((2n + p + 1)/2\, \Gamma_p(p/2)}$$

and the region of integration D is

$$D : 0 < \theta_1 < \cdots < \theta_s < x < \theta_{s+1} < \cdots < \theta_p < 1.$$

TABLE 1

UPPER PERCENTAGE POINTS OF THE INTERMEDIATE ROOTS

P = 4 S = 2 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.2620	.3523	.4217	.4772	.5227	.5608	.6211	.6856	.7550
6	.2350	.3198	.3865	.4407	.4858	.5241	.5856	.6526	.7263
7	.2130	.2927	.3565	.4093	.4537	.4918	.5538	.6225	.6995
8	.1948	.2699	.3309	.3820	.4255	.4632	.5252	.5951	.6746
9	.1794	.2503	.3087	.3582	.4006	.4378	.4994	.5699	.6514
10	.1663	.2334	.2893	.3371	.3785	.4149	.4760	.5467	.6297
12	.1451	.2056	.2569	.3015	.3408	.3756	.4351	.5054	.5902
14	.1287	.1836	.2311	.2727	.3098	.3431	.4007	.4700	.5552
16	.1156	.1660	.2099	.2490	.2840	.3158	.3712	.4391	.5242
18	.1049	.1514	.1923	.2290	.2622	.2924	.3458	.4120	.4964
20	.0960	.1392	.1775	.2120	.2435	.2723	.3237	.3880	.4714
25	.0792	.1158	.1487	.1788	.2066	.2324	.2790	.3387	.4186
30	.0675	.0992	.1279	.1545	.1793	.2026	.2451	.3005	.3764
35	.0587	.0867	.1123	.1361	.1584	.1796	.2185	.2701	.3418
40	.0520	.0770	.1000	.1216	.1419	.1613	.1971	.2452	.3131
45	.0467	.0693	.0902	.1099	.1286	.1463	.1795	.2245	.2889
50	.0423	.0629	.0821	.1002	.1174	.1339	.1649	.2070	.2681

P = 4 S = 2 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.3323	.4218	.4884	.5406	.5828	.6177	.6723	.7300	.7910
6	.2995	.3848	.4497	.5016	.5442	.5798	.6364	.6973	.7631
7	.2727	.3537	.4167	.4678	.5103	.5462	.6040	.6672	.7369
8	.2501	.3272	.3881	.4382	.4802	.5162	.5746	.6395	.7123
9	.2311	.3044	.3632	.4120	.4534	.4891	.5478	.6139	.6892
10	.2147	.2846	.3412	.3887	.4294	.4647	.5234	.5902	.6674
12	.1880	.2517	.3043	.3492	.3882	.4226	.4804	.5477	.6277
14	.1672	.2256	.2746	.3169	.3542	.3873	.4438	.5109	.5922
16	.1506	.2044	.2501	.2901	.3256	.3574	.4124	.4786	.5604
18	.1369	.1869	.2297	.2675	.3012	.3318	.3850	.4501	.5318
20	.1255	.1720	.2123	.2481	.2802	.3096	.3610	.4247	.5060
25	.1039	.1436	.1785	.2099	.2386	.2651	.3124	.3722	.4509
30	.0887	.1232	.1539	.1820	.2078	.2318	.2752	.3311	.4066
35	.0773	.1079	.1353	.1606	.1840	.2059	.2459	.2982	.3702
40	.0685	.0960	.1207	.1437	.1650	.1852	.2222	.2713	.3397
45	.0615	.0864	.1090	.1300	.1497	.1683	.2027	.2487	.3138
50	.0558	.0786	.0994	.1186	.1369	.1542	.1864	.2296	.2916

TABLE 1 (continued)

P = 4 S = 3 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.4810	.5579	.6136	.6563	.6902	.7179	.7605	.8045	.8502
6	.4386	.5145	.5708	.6149	.6504	.6797	.7255	.7736	.8246
7	.4030	.4772	.5334	.5780	.6145	.6449	.6930	.7446	.8002
8	.3725	.4448	.5004	.5451	.5821	.6132	.6631	.7173	.7768
9	.3463	.4164	.4711	.5156	.5527	.5844	.6354	.6918	.7545
10	.3236	.3914	.4449	.4890	.5262	.5579	.6099	.6678	.7333
12	.2858	.3492	.4003	.4431	.4797	.5114	.5641	.6242	.6938
14	.2559	.3152	.3637	.4049	.4406	.4719	.5245	.5856	.6581
16	.2317	.2872	.3333	.3727	.4073	.4379	.4899	.5513	.6255
18	.2117	.2636	.3074	.3453	.3786	.4084	.4594	.5207	.5960
20	.1948	.2437	.2852	.3214	.3537	.3825	.4326	.4932	.5689
25	.1624	.2048	.2416	.2742	.3035	.3301	.3771	.4354	.5107
30	.1392	.1767	.2095	.2389	.2657	.2903	.3341	.3896	.4630
35	.1218	.1553	.1849	.2117	.2363	.2589	.2999	.3525	.4233
40	.1083	.1385	.1655	.1901	.2126	.2337	.2719	.3217	.3899
45	.0974	.1250	.1498	.1724	.1934	.2129	.2488	.2958	.3613
50	.0885	.1139	.1368	.1578	.1773	.1956	.2292	.2739	.3365

P = 4 S = 3 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.5612	.6299	.6787	.7157	.7448	.7683	.8043	.8410	.8788
6	.5157	.5850	.6356	.6746	.7057	.7313	.7708	.8120	.8551
7	.4766	.5457	.5972	.6374	.6700	.6971	.7394	.7842	.8321
8	.4428	.5111	.5628	.6038	.6374	.6655	.7100	.7579	.8099
9	.4135	.4804	.5319	.5733	.6075	.6364	.6826	.7330	.7885
10	.3876	.4531	.5042	.5455	.5801	.6095	.6570	.7094	.7680
12	.3445	.4067	.4562	.4971	.5317	.5616	.6107	.6660	.7294
14	.3099	.3687	.4164	.4563	.4905	.5203	.5700	.6271	.6941
16	.2815	.3372	.3828	.4215	.4551	.4845	.5342	.5923	.6616
18	.2579	.3105	.3542	.3916	.4242	.4532	.5025	.5608	.6318
20	.2379	.2877	.3295	.3655	.3973	.4256	.4742	.5325	.6045
25	.1992	.2431	.2805	.3133	.3427	.3691	.4154	.4724	.5450
30	.1713	.2104	.2441	.2740	.3011	.3257	.3694	.4242	.4957
35	.1502	.1854	.2160	.2435	.2684	.2914	.3325	.3848	.4545
40	.1338	.1657	.1937	.2190	.2422	.2636	.3022	.3520	.4195
45	.1206	.1498	.1756	.1990	.2206	.2406	.2770	.3243	.3894
50	.1098	.1366	.1606	.1823	.2025	.2213	.2556	.3006	.3633

TABLE 1 (continued)

P = 5 S = 2 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.2005	.2812	.3465	.4008	.4467	.4860	.5500	.6207	.6994
6	.1803	.2554	.3174	.3698	.4146	.4534	.5174	.5894	.6712
7	.1638	.2340	.2928	.3432	.3868	.4249	.4884	.5611	.6452
8	.1500	.2159	.2718	.3202	.3625	.3997	.4625	.5355	.6211
9	.1384	.2003	.2536	.3000	.3410	.3774	.4392	.5120	.5987
10	.1285	.1869	.2376	.2823	.3219	.3574	.4182	.4905	.5779
12	.1124	.1649	.2111	.2524	.2895	.3231	.3816	.4526	.5404
14	.0998	.1473	.1899	.2282	.2630	.2949	.3509	.4200	.5074
16	.0897	.1333	.1725	.2083	.2411	.2712	.3249	.3918	.4783
18	.0815	.1216	.1581	.1916	.2224	.2510	.3023	.3672	.4523
20	.0748	.1119	.1458	.1773	.2065	.2336	.2827	.3456	.4290
25	.0618	.0931	.1222	.1495	.1750	.1992	.2433	.3011	.3800
30	.0527	.0798	.1052	.1292	.1520	.1736	.2135	.2667	.3411
35	.0459	.0698	.0923	.1138	.1342	.1538	.1903	.2394	.3094
40	.0407	.0621	.0823	.1017	.1203	.1380	.1715	.2172	.2831
45	.0365	.0558	.0742	.0918	.1088	.1252	.1562	.1988	.2609
50	.0332	.0507	.0675	.0838	.0995	.1146	.1434	.1832	.2419

P = 5 S = 2 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.2569	.3395	.4046	.4575	.5015	.5389	.5987	.6640	.7355
6	.2318	.3097	.3720	.4236	.4672	.5044	.5651	.6324	.7076
7	.2112	.2845	.3443	.3944	.4372	.4741	.5350	.6036	.6817
8	.1939	.2632	.3204	.3689	.4107	.4472	.5079	.5772	.6575
9	.1792	.2448	.2996	.3465	.3873	.4231	.4834	.5531	.6350
10	.1666	.2288	.2813	.3267	.3664	.4015	.4611	.5309	.6139
12	.1461	.2024	.2507	.2931	.3306	.3643	.4221	.4913	.5756
14	.1301	.1814	.2260	.2657	.3012	.3333	.3892	.4572	.5418
16	.1172	.1644	.2058	.2430	.2766	.3072	.3610	.4274	.5117
18	.1066	.1502	.1890	.2239	.2557	.2849	.3367	.4013	.4848
20	.0978	.1383	.1746	.2075	.2377	.2656	.3153	.3783	.4605
25	.0811	.1155	.1468	.1755	.2022	.2271	.2722	.3307	.4092
30	.0692	.0992	.1265	.1520	.1759	.1983	.2395	.2937	.3682
35	.0604	.0868	.1113	.1341	.1556	.1761	.2138	.2641	.3346
40	.0536	.0773	.0992	.1200	.1395	.1582	.1930	.2399	.3066
45	.0481	.0695	.0896	.1085	.1265	.1437	.1760	.2198	.2830
50	.0437	.0633	.0816	.0991	.1157	.1316	.1616	.2028	.2627

P = 5 S = 3 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.3711	.4488	.5084	.5557	.5946	.6271	.6784	.7331	.7920
6	.3374	.4122	.4707	.5181	.5574	.5906	.6438	.7016	.7650
7	.3092	.3810	.4381	.4850	.5244	.5580	.6124	.6726	.7397
8	.2854	.3542	.4097	.4558	.4950	.5287	.5839	.6457	.7158
9	.2649	.3309	.3847	.4300	.4687	.5022	.5577	.6208	.6933
10	.2472	.3104	.3626	.4068	.4449	.4783	.5339	.5976	.6721
12	.2180	.2762	.3251	.3672	.4040	.4364	.4915	.5560	.6332
14	.1950	.2488	.2946	.3345	.3698	.4013	.4553	.5197	.5983
16	.1763	.2262	.2693	.3072	.3410	.3713	.4240	.4878	.5670
18	.1609	.2075	.2480	.2839	.3163	.3456	.3967	.4594	.5388
20	.1480	.1916	.2298	.2640	.2949	.3230	.3726	.4342	.5131
25	.1232	.1608	.1942	.2245	.2522	.2778	.3236	.3817	.4585
30	.1055	.1384	.1681	.1952	.2203	.2436	.2858	.3404	.4142
35	.0923	.1216	.1482	.1727	.1956	.2168	.2560	.3072	.3777
40	.0820	.1083	.1325	.1548	.1757	.1955	.2318	.2798	.3471
45	.0738	.0978	.1198	.1403	.1596	.1779	.2117	.2569	.3210
50	.0670	.0890	.1093	.1283	.1461	.1631	.1949	.2375	.2986

P = 5 S = 3 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.4392	.5130	.5685	.6121	.6474	.6767	.7226	.7709	.8224
6	.4013	.4735	.5289	.5732	.6095	.6400	.6883	.7403	.7966
7	.3694	.4396	.4943	.5387	.5755	.6068	.6569	.7117	.7720
8	.3421	.4100	.4638	.5080	.5451	.5767	.6280	.6850	.7487
9	.3185	.3842	.4368	.4805	.5174	.5493	.6015	.6600	.7265
10	.2979	.3613	.4128	.4558	.4925	.5243	.5769	.6367	.7055
12	.2638	.3229	.3716	.4131	.4490	.4804	.5332	.5945	.6668
14	.2367	.2918	.3379	.3776	.4124	.4431	.4955	.5572	.6317
16	.2146	.2661	.3097	.3477	.3812	.4112	.4627	.5243	.6000
18	.1962	.2446	.2858	.3222	.3545	.3834	.4338	.4950	.5712
20	.1808	.2262	.2654	.3001	.3311	.3592	.4083	.4686	.5450
25	.1510	.1904	.2250	.2561	.2842	.3100	.3559	.4135	.4887
30	.1297	.1645	.1954	.2233	.2490	.2726	.3153	.3698	.4428
35	.1136	.1447	.1725	.1979	.2214	.2433	.2829	.3345	.4046
40	.1011	.1292	.1545	.1778	.1994	.2196	.2567	.3052	.3724
45	.0911	.1166	.1399	.1613	.1813	.2001	.2348	.2806	.3450
50	.0828	.1064	.1278	.1477	.1662	.1838	.2163	.2597	.3213

TABLE 1 (continued)

P = 5 S = 4 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.5699	.6306	.6754	.7102	.7380	.7609	.7962	.8330	.8716
6	.5263	.5878	.6342	.6707	.7004	.7250	.7637	.8046	.8481
7	.4886	.5501	.5973	.6350	.6660	.6919	.7331	.7775	.8254
8	.4558	.5167	.5642	.6026	.6344	.6614	.7046	.7517	.8036
9	.4271	.4870	.5344	.5731	.6055	.6332	.6778	.7273	.7825
10	.4016	.4605	.5074	.5462	.5790	.6070	.6529	.7042	.7623
12	.3587	.4149	.4557	.4991	.5319	.5604	.6078	.6618	.7244
14	.3240	.3775	.4216	.4592	.4917	.5202	.5681	.6237	.6896
16	.2954	.3461	.3886	.4251	.4569	.4851	.5330	.5896	.6578
18	.2713	.3196	.3603	.3957	.4267	.4545	.5020	.5588	.6285
20	.2509	.2967	.3358	.3699	.4002	.4273	.4742	.5309	.6016
25	.2112	.2517	.2869	.3180	.3460	.3715	.4163	.4717	.5429
30	.1822	.2185	.2503	.2789	.3048	.3286	.3708	.4242	.4944
35	.1602	.1930	.2220	.2482	.2722	.2944	.3342	.3853	.4536
40	.1430	.1728	.1994	.2236	.2459	.2667	.3042	.3528	.4190
45	.1291	.1564	.1810	.2035	.2243	.2437	.2791	.3253	.3892
50	.1176	.1429	.1657	.1866	.2061	.2243	.2577	.3018	.3633

P = 5 S = 4 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.6402	.6931	.7317	.7614	.7850	.8042	.8338	.8644	.8961
6	.5953	.6502	.6911	.7231	.7488	.7700	.8031	.8378	.8746
7	.5559	.6118	.6542	.6878	.7151	.7380	.7739	.8122	.8534
8	.5210	.5773	.6206	.6553	.6839	.7080	.7462	.7876	.8327
9	.4902	.5463	.5900	.6255	.6550	.6800	.7201	.7640	.8127
10	.4626	.5182	.5621	.5980	.6281	.6538	.6955	.7416	.7933
12	.4156	.4696	.5131	.5493	.5800	.6066	.6503	.6998	.7565
14	.3771	.4292	.4717	.5076	.5384	.5653	.6101	.6618	.7224
16	.3450	.3950	.4363	.4716	.5021	.5291	.5744	.6274	.6908
18	.3180	.3658	.4058	.4401	.4703	.4969	.5424	.5962	.6616
20	.2947	.3405	.3791	.4126	.4420	.4684	.5136	.5677	.6345
25	.2492	.2902	.3255	.3565	.3842	.4093	.4530	.5068	.5751
30	.2157	.2528	.2850	.3137	.3396	.3632	.4050	.4573	.5254
35	.1903	.2239	.2535	.2800	.3042	.3264	.3661	.4165	.4834
40	.1701	.2009	.2281	.2528	.2754	.2963	.3338	.3821	.4475
45	.1538	.1822	.2075	.2304	.2515	.2712	.3068	.3531	.4163
50	.1404	.1667	.1902	.2116	.2315	.2500	.2838	.3280	.3893

TABLE 1 (continued)

P = 6 S = 2 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.1596	.2308	.2911	.3426	.3874	.4264	.4914	.5654	.6504
6	.1440	.2101	.2669	.3163	.3595	.3976	.4619	.5363	.6233
7	.1312	.1928	.2465	.2937	.3354	.3725	.4358	.5100	.5984
8	.1205	.1782	.2291	.2741	.3143	.3504	.4125	.4863	.5755
9	.1114	.1656	.2138	.2570	.2958	.3308	.3916	.4646	.5542
10	.1036	.1547	.2005	.2418	.2792	.3132	.3726	.4449	.5346
12	.0908	.1367	.1784	.2165	.2512	.2833	.3399	.4101	.4991
14	.0808	.1224	.1606	.1958	.2284	.2584	.3124	.3803	.4682
16	.0729	.1108	.1460	.1789	.2093	.2377	.2890	.3545	.4408
18	.0663	.1012	.1339	.1645	.1932	.2201	.2689	.3321	.4166
20	.0608	.0932	.1237	.1524	.1793	.2048	.2515	.3124	.3948
25	.0504	.0778	.1038	.1286	.1522	.1747	.2164	.2719	.3493
30	.0430	.0667	.0894	.1112	.1321	.1522	.1898	.2408	.3132
35	.0375	.0584	.0786	.0980	.1167	.1348	.1691	.2161	.2838
40	.0333	.0519	.0701	.0875	.1046	.1210	.1525	.1958	.2596
45	.0299	.0467	.0631	.0792	.0947	.1098	.1388	.1792	.2391
50	.0272	.0425	.0576	.0722	.0866	.1005	.1274	.1651	.2216

P = 6 S = 2 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.2057	.2804	.3418	.3932	.4371	.4750	.5373	.6072	.6861
6	.1861	.2561	.3143	.3640	.4069	.4443	.5065	.5773	.6589
7	.1699	.2356	.2910	.3389	.3805	.4172	.4789	.5503	.6338
8	.1564	.2181	.2710	.3169	.3574	.3933	.4542	.5256	.6106
9	.1448	.2031	.2535	.2977	.3369	.3719	.4320	.5032	.5890
10	.1348	.1899	.2380	.2807	.3187	.3528	.4118	.4824	.5688
12	.1184	.1683	.2123	.2518	.2875	.3199	.3766	.4458	.5324
14	.1056	.1510	.1915	.2284	.2619	.2926	.3469	.4144	.5004
16	.0953	.1369	.1745	.2088	.2404	.2696	.3216	.3871	.4720
18	.0868	.1252	.1602	.1925	.2223	.2499	.2998	.3632	.4467
20	.0798	.1154	.1481	.1784	.2066	.2330	.2807	.3420	.4240
25	.0662	.0965	.1246	.1509	.1757	.1991	.2421	.2986	.3761
30	.0566	.0829	.1075	.1308	.1529	.1738	.2128	.2649	.3380
35	.0495	.0726	.0945	.1154	.1353	.1543	.1899	.2381	.3069
40	.0439	.0646	.0844	.1032	.1213	.1387	.1715	.2162	.2810
45	.0394	.0583	.0762	.0934	.1100	.1259	.1563	.1980	.2591
50	.0358	.0530	.0694	.0853	.1006	.1153	.1435	.1826	.2404

TABLE 1 (continued)

P = 6 S = 3 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.2977	.3714	.4308	.4788	.5196	.5543	.6104	.6721	.7405
6	.2708	.3409	.3979	.4455	.4859	.5207	.5778	.6415	.7134
7	.2483	.3150	.3700	.4164	.4564	.4910	.5484	.6135	.6882
8	.2292	.2927	.3457	.3910	.4301	.4645	.5218	.5879	.6647
9	.2128	.2733	.3244	.3684	.4068	.4406	.4977	.5642	.6428
10	.1987	.2564	.3055	.3482	.3858	.4191	.4757	.5423	.6222
12	.1753	.2281	.2738	.3139	.3497	.3817	.4370	.5033	.5846
14	.1570	.2054	.2479	.2857	.3197	.3504	.4041	.4695	.5512
16	.1420	.1869	.2265	.2622	.2945	.3239	.3757	.4399	.5215
18	.1296	.1713	.2085	.2422	.2729	.3011	.3511	.4138	.4947
20	.1193	.1582	.1932	.2250	.2542	.2813	.3295	.3906	.4705
25	.0995	.1328	.1631	.1912	.2172	.2415	.2856	.3425	.4193
30	.0852	.1144	.1412	.1662	.1896	.2116	.2519	.3049	.3781
35	.0746	.1004	.1245	.1469	.1682	.1882	.2253	.2749	.3442
40	.0663	.0895	.1113	.1317	.1511	.1695	.2039	.2500	.3159
45	.0596	.0808	.1006	.1194	.1372	.1542	.1861	.2294	.2918
50	.0542	.0736	.0917	.1091	.1256	.1415	.1712	.2120	.2712

P = 6 S = 3 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.3554	.4279	.4848	.5309	.5692	.6016	.6535	.7099	.7716
6	.3245	.3942	.4499	.4957	.5342	.5671	.6205	.6795	.7452
7	.2985	.3654	.4196	.4648	.5032	.5363	.5905	.6514	.7204
8	.2763	.3405	.3932	.4375	.4755	.5085	.5633	.6255	.6970
9	.2572	.3187	.3697	.4132	.4507	.4835	.5383	.6015	.6751
10	.2405	.2996	.3490	.3914	.4283	.4607	.5155	.5792	.6544
12	.2129	.2673	.3137	.3540	.3895	.4210	.4750	.5391	.6165
14	.1910	.2414	.2848	.3231	.3570	.3876	.4404	.5041	.5826
16	.1732	.2200	.2608	.2970	.3296	.3590	.4104	.4732	.5522
18	.1584	.2021	.2406	.2749	.3060	.3344	.3842	.4460	.5248
20	.1459	.1869	.2232	.2559	.2856	.3129	.3611	.4216	.4999
25	.1219	.1573	.1890	.2180	.2447	.2694	.3140	.3709	.4468
30	.1047	.1358	.1640	.1899	.2140	.2366	.2776	.3310	.4038
35	.0917	.1194	.1447	.1682	.1902	.2109	.2488	.2989	.3683
40	.0816	.1066	.1295	.1510	.1711	.1902	.2254	.2724	.3385
45	.0735	.0963	.1173	.1369	.1555	.1732	.2060	.2502	.3132
50	.0669	.0877	.1071	.1252	.1425	.1590	.1897	.2314	.2914

TABLE 1 (continued)

P = 6 S = 4 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.4608	.5259	.5763	.6165	.6496	.6773	.7212	.7682	.8190
6	.4236	.4878	.5382	.5792	.6132	.6420	.6882	.7385	.7937
7	.3919	.4547	.5047	.5459	.5804	.6099	.6578	.7107	.7698
8	.3646	.4257	.4750	.5161	.5508	.5808	.6298	.6848	.7470
9	.3408	.4001	.4486	.4892	.5240	.5542	.6040	.6605	.7254
10	.3199	.3774	.4249	.4650	.4996	.5298	.5801	.6378	.7049
12	.2849	.3388	.3841	.4230	.4569	.4868	.5374	.5965	.6669
14	.2567	.3074	.3504	.3878	.4208	.4502	.5004	.5601	.6325
16	.2336	.2812	.3221	.3581	.3900	.4186	.4680	.5276	.6014
18	.2143	.2591	.2980	.3325	.3632	.3911	.4395	.4987	.5731
20	.1980	.2403	.2773	.3102	.3399	.3669	.4142	.4727	.5472
25	.1663	.2033	.2361	.2658	.2928	.3177	.3621	.4181	.4915
30	.1432	.1761	.2055	.2325	.2572	.2801	.3215	.3747	.4460
35	.1258	.1553	.1820	.2065	.2292	.2504	.2890	.3393	.4080
40	.1123	.1389	.1632	.1858	.2068	.2264	.2625	.3100	.3760
45	.1012	.1256	.1480	.1688	.1882	.2066	.2405	.2854	.3486
50	.0922	.1147	.1354	.1546	.1728	.1900	.2217	.2644	.3250

P = 6 S = 4 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.5238	.5843	.6302	.6667	.6965	.7211	.7600	.8013	.8455
6	.4840	.5444	.5913	.6289	.6600	.6861	.7277	.7726	.8215
7	.4497	.5095	.5566	.5949	.6269	.6540	.6977	.7455	.7984
8	.4198	.4786	.5256	.5642	.5968	.6245	.6698	.7200	.7764
9	.3935	.4512	.4977	.5364	.5692	.5975	.6438	.6959	.7552
10	.3704	.4267	.4726	.5111	.5440	.5725	.6197	.6733	.7351
12	.3312	.3847	.4290	.4668	.4995	.5281	.5761	.6319	.6975
14	.2995	.3501	.3927	.4294	.4615	.4899	.5381	.5949	.6632
16	.2733	.3212	.3620	.3974	.4288	.4567	.5046	.5618	.6319
18	.2512	.2967	.3357	.3699	.4003	.4276	.4749	.5321	.6033
20	.2325	.2756	.3129	.3458	.3753	.4020	.4485	.5053	.5770
25	.1959	.2340	.2674	.2975	.3246	.3495	.3934	.4485	.5201
30	.1693	.2033	.2335	.2608	.2859	.3090	.3503	.4031	.4731
35	.1489	.1796	.2072	.2322	.2554	.2768	.3157	.3658	.4339
40	.1330	.1609	.1861	.2092	.2307	.2507	.2872	.3348	.4005
45	.1202	.1458	.1690	.1904	.2103	.2291	.2634	.3087	.3718
50	.1096	.1331	.1547	.1746	.1933	.2109	.2433	.2863	.3470

TABLE 1 (continued)

P = 6 S = 5 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.6383	.6866	.7230	.7516	.7747	.7938	.8235	.8548	.8878
6	.5955	.6455	.5840	.7145	.7395	.7604	.7933	.8284	.8662
7	.5577	.6087	.6484	.6804	.7068	.7291	.7646	.8032	.8451
8	.5242	.5755	.6160	.6490	.6765	.7000	.7376	.7789	.8246
9	.4943	.5456	.5864	.6201	.6485	.6726	.7120	.7557	.8047
10	.4676	.5184	.5594	.5935	.6224	.6472	.6880	.7337	.7855
12	.4216	.4712	.5120	.5462	.5757	.6013	.6439	.6926	.7492
14	.3838	.4318	.4716	.5056	.5351	.5610	.6047	.6554	.7156
16	.3521	.3982	.4370	.4704	.4996	.5256	.5697	.6217	.6844
18	.3252	.3694	.4070	.4397	.4685	.4942	.5383	.5910	.6557
20	.3021	.3445	.3808	.4126	.4409	.4662	.5101	.5631	.6290
25	.2564	.2946	.3279	.3575	.3840	.4082	.4506	.5032	.5705
30	.2227	.2573	.2877	.3152	.3401	.3628	.4034	.4545	.5215
35	.1967	.2283	.2563	.2817	.3050	.3264	.3650	.4142	.4801
40	.1762	.2051	.2311	.2546	.2764	.2966	.3332	.3804	.4446
45	.1595	.1863	.2103	.2324	.2528	.2718	.3064	.3516	.4139
50	.1458	.1706	.1930	.2136	.2328	.2507	.2836	.3269	.3872

P = 6 S = 5 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.6996	.7411	.7720	.7962	.8155	.8315	.8563	.8821	.9092
6	.6568	.7009	.7343	.7607	.7822	.8001	.8281	.8578	.8895
7	.6185	.6641	.6994	.7276	.7507	.7701	.8010	.8341	.8700
8	.5840	.6306	.6671	.6966	.7211	.7418	.7750	.8111	.8507
9	.5529	.6000	.6374	.6679	.6933	.7151	.7502	.7889	.8319
10	.5247	.5720	.6099	.6411	.6674	.6899	.7266	.7675	.8135
12	.4759	.5228	.5610	.5930	.6203	.6440	.6830	.7273	.7785
14	.4352	.4811	.5190	.5511	.5789	.6032	.6437	.6906	.7457
16	.4007	.4453	.4826	.5145	.5424	.5669	.6084	.6569	.7151
18	.3711	.4143	.4508	.4823	.5100	.5345	.5764	.6261	.6864
20	.3456	.3874	.4229	.4538	.4811	.5055	.5474	.5977	.6598
25	.2948	.3329	.3659	.3950	.4212	.4447	.4860	.5365	.6009
30	.2569	.2917	.3223	.3496	.3742	.3967	.4365	.4863	.5511
35	.2276	.2596	.2879	.3133	.3366	.3579	.3960	.4444	.5086
40	.2043	.2338	.2601	.2839	.3057	.3260	.3623	.4090	.4720
45	.1852	.2126	.2371	.2594	.2800	.2991	.3338	.3788	.4402
50	.1695	.1949	.2179	.2389	.2583	.2764	.3094	.3526	.4123

TABLE 1 (continued)

P = 7 S = 2 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.1306	.1935	.2485	.2968	.3396	.3777	.4422	.5176	.6067
6	.1183	.1766	.2284	.2744	.3155	.3523	.4156	.4907	.5809
7	.1082	.1625	.2113	.2550	.2946	.3302	.3921	.4665	.5574
8	.0996	.1504	.1965	.2383	.2762	.3108	.3711	.4446	.5357
9	.0922	.1401	.1837	.2236	.2601	.2935	.3523	.4248	.5156
10	.0860	.1310	.1725	.2106	.2458	.2780	.3354	.4066	.4971
12	.0756	.1161	.1537	.1887	.2212	.2515	.3059	.3747	.4638
14	.0675	.1042	.1386	.1709	.2013	.2296	.2812	.3473	.4348
16	.0610	.0945	.1262	.1563	.1846	.2113	.2602	.3238	.4091
18	.0555	.0864	.1159	.1439	.1705	.1957	.2422	.3033	.3865
20	.0510	.0796	.1071	.1333	.1583	.1822	.2265	.2852	.3662
25	.0424	.0666	.0900	.1126	.1345	.1555	.1949	.2482	.3237
30	.0363	.0572	.0777	.0976	.1168	.1356	.1710	.2198	.2901
35	.0317	.0501	.0683	.0860	.1034	.1202	.1524	.1971	.2628
40	.0282	.0446	.0609	.0769	.0926	.1080	.1375	.1788	.2403
45	.0253	.0402	.0550	.0696	.0839	.0980	.1251	.1635	.2213
50	.0230	.0366	.0501	.0635	.0767	.0897	.1149	.1507	.2051

P = 7 S = 2 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.1691	.2362	.2930	.3420	.3848	.4223	.4853	.5576	.6417
6	.1535	.2161	.2699	.3169	.3583	.3949	.4571	.5298	.6156
7	.1405	.1991	.2502	.2952	.3352	.3710	.4321	.5046	.5916
8	.1296	.1847	.2331	.2763	.3149	.3497	.4097	.4817	.5694
9	.1202	.1721	.2183	.2596	.2970	.3308	.3896	.4608	.5489
10	.1121	.1613	.2052	.2449	.2810	.3138	.3713	.4417	.5298
12	.0988	.1431	.1832	.2200	.2536	.2845	.3394	.4079	.4954
14	.0883	.1286	.1655	.1996	.2311	.2603	.3127	.3789	.4652
16	.0798	.1168	.1510	.1827	.2122	.2398	.2898	.3538	.4385
18	.0728	.1070	.1388	.1685	.1963	.2224	.2701	.3318	.4147
20	.0670	.0986	.1284	.1562	.1825	.2073	.2529	.3124	.3934
25	.0558	.0826	.1081	.1323	.1554	.1773	.2182	.2726	.3486
30	.0478	.0711	.0934	.1147	.1352	.1549	.1918	.2418	.3130
35	.0418	.0624	.0822	.1013	.1197	.1374	.1712	.2172	.2841
40	.0371	.0556	.0734	.0906	.1074	.1236	.1545	.1973	.2600
45	.0334	.0501	.0663	.0821	.0974	.1123	.1408	.1806	.2397
50	.0303	.0456	.0605	.0749	.0891	.1028	.1293	.1665	.2224

TABLE 1 (continued)

P = 7 S = 3 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.2453	.3135	.3700	.4177	.4586	.4941	.5526	.6187	.6940
6	.2235	.2879	.3421	.3884	.4286	.4638	.5224	.5897	.6676
7	.2052	.2653	.3182	.3630	.4023	.4370	.4953	.5632	.6432
8	.1898	.2476	.2973	.3408	.3791	.4131	.4709	.5391	.6204
9	.1765	.2314	.2791	.3210	.3583	.3917	.4488	.5169	.5993
10	.1649	.2171	.2629	.3035	.3397	.3723	.4287	.4964	.5795
12	.1458	.1934	.2357	.2736	.3079	.3389	.3934	.4601	.5436
14	.1306	.1744	.2135	.2490	.2814	.3110	.3635	.4287	.5119
16	.1184	.1586	.1952	.2285	.2591	.2874	.3377	.4013	.4837
18	.1082	.1457	.1797	.2111	.2401	.2670	.3155	.3772	.4584
20	.0997	.1345	.1666	.1961	.2238	.2495	.2959	.3558	.4356
25	.0832	.1130	.1408	.1667	.1911	.2141	.2563	.3117	.3875
30	.0713	.0974	.1218	.1450	.1667	.1875	.2259	.2772	.3490
35	.0625	.0856	.1075	.1282	.1479	.1668	.2021	.2497	.3174
40	.0556	.0764	.0961	.1149	.1330	.1502	.1828	.2271	.2912
45	.0500	.0689	.0869	.1042	.1207	.1367	.1667	.2083	.2688
50	.0456	.0628	.0793	.0952	.1105	.1252	.1534	.1923	.2497

P = 7 S = 3 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.2945	.3633	.4190	.4654	.5048	.5387	.5940	.6558	.7254
6	.2692	.3346	.3886	.4341	.4731	.5070	.5631	.6265	.6992
7	.2479	.3103	.3623	.4067	.4452	.4789	.5351	.5996	.6747
8	.2296	.2891	.3394	.3826	.4203	.4536	.5097	.5750	.6519
9	.2139	.2707	.3191	.3611	.3981	.4309	.4866	.5522	.6306
10	.2002	.2544	.3011	.3420	.3781	.4103	.4656	.5311	.6106
12	.1774	.2272	.2706	.3091	.3435	.3745	.4283	.4935	.5741
14	.1593	.2053	.2457	.2820	.3147	.3445	.3966	.4608	.5416
16	.1446	.1871	.2250	.2592	.2904	.3188	.3693	.4321	.5126
18	.1323	.1719	.2075	.2399	.2695	.2968	.3455	.4068	.4866
20	.1220	.1590	.1925	.2232	.2514	.2775	.3245	.3842	.4630
25	.1020	.1339	.1631	.1901	.2153	.2388	.2817	.3375	.4130
30	.0877	.1157	.1415	.1656	.1883	.2096	.2489	.3008	.3727
35	.0769	.1018	.1249	.1467	.1672	.1867	.2229	.2714	.3396
40	.0685	.0908	.1118	.1316	.1504	.1684	.2019	.2471	.3118
45	.0617	.0820	.1012	.1194	.1367	.1533	.1845	.2269	.2882
50	.0561	.0748	.0924	.1092	.1253	.1407	.1698	.2097	.2680

P = 7 S = 4 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.3826	.4474	.4991	.5418	.5775	.6079	.6571	.7113	.7713
6	.3514	.4142	.4651	.5076	.5437	.5746	.6253	.6818	.7456
7	.3249	.3855	.4355	.4775	.5134	.5447	.5962	.6546	.7214
8	.3021	.3606	.4093	.4506	.4864	.5176	.5697	.6294	.6987
9	.2823	.3386	.3860	.4266	.4620	.4931	.5453	.6059	.6773
10	.2649	.3192	.3653	.4050	.4399	.4707	.5229	.5842	.6570
12	.2358	.2863	.3297	.3677	.4014	.4315	.4832	.5448	.6200
14	.2125	.2595	.3004	.3367	.3691	.3983	.4490	.5104	.5867
16	.1933	.2373	.2759	.3104	.3416	.3698	.4192	.4799	.5567
18	.1774	.2186	.2551	.2879	.3178	.3451	.3931	.4529	.5296
20	.1638	.2026	.2373	.2685	.2971	.3234	.3701	.4287	.5050
25	.1375	.1712	.2018	.2297	.2555	.2795	.3227	.3782	.4523
30	.1185	.1483	.1755	.2007	.2242	.2460	.2861	.3382	.4095
35	.1042	.1308	.1553	.1782	.1997	.2198	.2569	.3059	.3740
40	.0929	.1169	.1393	.1602	.1799	.1986	.2331	.2792	.3442
45	.0838	.1058	.1263	.1456	.1637	.1811	.2133	.2567	.3187
50	.0763	.0965	.1155	.1333	.1502	.1664	.1966	.2376	.2967

P = 7 S = 4 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.4384	.5005	.5496	.5894	.6226	.6507	.6958	.7449	.7989
6	.4042	.4651	.5140	.5542	.5880	.6170	.6640	.7159	.7739
7	.3749	.4343	.4826	.5228	.5569	.5864	.6347	.6888	.7502
8	.3495	.4073	.4547	.4947	.5289	.5586	.6077	.6636	.7278
9	.3274	.3834	.4299	.4693	.5035	.5332	.5830	.6400	.7066
10	.3078	.3621	.4075	.4465	.4803	.5100	.5600	.6180	.6865
12	.2749	.3259	.3691	.4067	.4397	.4690	.5190	.5781	.6494
14	.2484	.2962	.3373	.3733	.4054	.4340	.4835	.5428	.6158
16	.2265	.2714	.3105	.3450	.3759	.4039	.4524	.5115	.5854
18	.2081	.2504	.2875	.3206	.3505	.3776	.4250	.4835	.5578
20	.1925	.2325	.2678	.2995	.3282	.3544	.4008	.4584	.5326
25	.1621	.1971	.2285	.2570	.2832	.3073	.3506	.4056	.4785
30	.1400	.1710	.1992	.2250	.2489	.2712	.3115	.3636	.4341
35	.1231	.1511	.1766	.2001	.2221	.2426	.2802	.3294	.3973
40	.1099	.1353	.1586	.1802	.2004	.2195	.2546	.3011	.3662
45	.0993	.1225	.1439	.1638	.1827	.2004	.2333	.2773	.3395
50	.0906	.1119	.1317	.1503	.1677	.1843	.2153	.2570	.3165

TABLE 1 (continued)

P = 7 S = 5 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.5341	.5885	.6310	.6652	.6934	.7172	.7550	.7957	.8399
6	.4957	.5503	.5935	.6288	.6584	.6833	.7236	.7677	.8163
7	.4623	.5165	.5601	.5960	.6264	.6524	.6945	.7413	.7936
8	.4331	.4865	.5300	.5664	.5972	.6238	.6674	.7164	.7720
9	.4072	.4598	.5029	.5393	.5705	.5975	.6422	.6929	.7513
10	.3842	.4358	.4785	.5147	.5459	.5732	.6187	.6708	.7315
12	.3452	.3944	.4358	.4714	.5024	.5298	.5762	.6302	.6947
14	.3133	.3601	.4000	.4347	.4652	.4925	.5389	.5941	.6611
16	.2868	.3312	.3696	.4032	.4331	.4598	.5060	.5616	.6303
18	.2644	.3067	.3434	.3759	.4049	.4312	.4768	.5325	.6021
20	.2452	.2854	.3207	.3520	.3802	.4059	.4507	.5060	.5763
25	.2075	.2433	.2750	.3037	.3298	.3538	.3963	.4500	.5202
30	.1798	.2120	.2408	.2669	.2911	.3134	.3536	.4050	.4737
35	.1586	.1877	.2140	.2381	.2605	.2813	.3191	.3680	.4348
40	.1419	.1685	.1926	.2149	.2357	.2551	.2907	.3373	.4016
45	.1285	.1528	.1751	.1958	.2152	.2333	.2668	.3112	.3732
50	.1172	.1398	.1605	.1798	.1979	.2150	.2466	.2888	.3485

P = 7 S = 5 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.5915	.6411	.6794	.7100	.7351	.7561	.7894	.8251	.8634
6	.5516	.6021	.6418	.6739	.7006	.7231	.7591	.7983	.8410
7	.5165	.5674	.5078	.6409	.6687	.6924	.7306	.7727	.8194
8	.4855	.5362	.5770	.6108	.6394	.6638	.7038	.7484	.7985
9	.4579	.5081	.5490	.5832	.6123	.6374	.6787	.7253	.7785
10	.4331	.4828	.5235	.5578	.5873	.6128	.6552	.7035	.7592
12	.3908	.4387	.4788	.5130	.5426	.5686	.6123	.6630	.7229
14	.3558	.4019	.4409	.4745	.5040	.5301	.5744	.6266	.6895
16	.3266	.3707	.4084	.4414	.4703	.4962	.5407	.5938	.6588
18	.3017	.3440	.3804	.4124	.4408	.4663	.5106	.5641	.6305
20	.2804	.3208	.3559	.3869	.4147	.4398	.4836	.5370	.6044
25	.2382	.2744	.3064	.3351	.3610	.3848	.4268	.4793	.5473
30	.2069	.2397	.2689	.2953	.3196	.3419	.3818	.4325	.4998
35	.1830	.2128	.2396	.2640	.2866	.3075	.3452	.3939	.4597
40	.1640	.1912	.2160	.2387	.2597	.2794	.3151	.3616	.4254
45	.1485	.1737	.1966	.2178	.2375	.2559	.2897	.3341	.3958
50	.1357	.1591	.1804	.2002	.2187	.2361	.2681	.3105	.3700

TABLE 1 (continued)

P = 7 S = 6 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.6918	.7308	.7607	.7844	.8037	.8198	.8451	.8719	.9006
6	.6509	.6920	.7240	.7498	.7710	.7888	.8172	.8476	.8807
7	.6141	.6566	.6901	.7175	.7402	.7594	.7903	.8240	.8609
8	.5808	.6242	.6589	.6874	.7113	.7317	.7647	.8011	.8418
9	.5508	.5946	.6301	.6594	.6842	.7055	.7403	.7791	.8229
10	.5236	.5676	.6034	.6334	.6589	.6809	.7172	.7580	.8047
12	.4762	.5199	.5560	.5867	.6130	.6360	.6745	.7185	.7700
14	.4365	.4793	.5152	.5459	.5727	.5963	.6359	.6823	.7375
16	.4027	.4443	.4797	.5102	.5370	.5608	.6013	.6492	.7072
18	.3738	.4141	.4486	.4788	.5054	.5292	.5700	.6190	.6790
20	.3486	.3876	.4213	.4509	.4772	.5007	.5417	.5911	.6527
25	.2983	.3340	.3653	.3932	.4183	.4412	.4813	.5310	.5946
30	.2606	.2933	.3223	.3485	.3722	.3940	.4328	.4816	.5456
35	.2313	.2614	.2883	.3127	.3352	.3558	.3930	.4404	.5037
40	.2079	.2357	.2608	.2836	.3047	.3244	.3598	.4055	.4676
45	.1887	.2146	.2380	.2595	.2793	.2979	.3317	.3757	.4362
50	.1729	.1969	.2189	.2391	.2579	.2754	.3077	.3500	.4087

P = 7 S = 6 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.7454	.7785	.8036	.8236	.8397	.8531	.8740	.8961	.9196
6	.7054	.7411	.7688	.7908	.8089	.8241	.8481	.8738	.9014
7	.6688	.7064	.7359	.7598	.7795	.7962	.8228	.8518	.8833
8	.6353	.6743	.7052	.7305	.7516	.7696	.7985	.8302	.8653
9	.6047	.6446	.6766	.7030	.7252	.7442	.7750	.8093	.8477
10	.5767	.6172	.6499	.6771	.7003	.7201	.7526	.7890	.8303
12	.5274	.5682	.6018	.6302	.6544	.6755	.7106	.7506	.7969
14	.4855	.5261	.5599	.5888	.6137	.6356	.6724	.7151	.7654
16	.4495	.4895	.5231	.5521	.5774	.5997	.6377	.6822	.7357
18	.4184	.4574	.4907	.5195	.5448	.5675	.6060	.6519	.7079
20	.3912	.4293	.4619	.4903	.5156	.5383	.5771	.6240	.6818
25	.3364	.3718	.4025	.4298	.4542	.4764	.5152	.5629	.6235
30	.2949	.3276	.3564	.3823	.4056	.4270	.4648	.5122	.5738
35	.2625	.2927	.3197	.3441	.3663	.3867	.4232	.4696	.5311
40	.2364	.2646	.2898	.3127	.3337	.3532	.3883	.4334	.4940
45	.2151	.2413	.2649	.2865	.3065	.3250	.3587	.4022	.4617
50	.1972	.2218	.2440	.2644	.2833	.3010	.3332	.3752	.4333

TABLE 1 (continued)

P = 8 S = 2 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.1091	.1649	.2150	.2600	.3005	.3371	.4003	.4758	.5673
6	.0993	.1509	.1980	.2407	.2794	.3148	.3764	.4511	.5430
7	.0911	.1392	.1835	.2241	.2613	.2953	.3552	.4288	.5208
8	.0840	.1292	.1710	.2096	.2453	.2781	.3364	.4088	.5005
9	.0781	.1205	.1601	.1969	.2311	.2627	.3194	.3905	.4816
10	.0729	.1128	.1505	.1857	.2185	.2491	.3042	.3738	.4642
12	.0643	.1002	.1344	.1667	.1970	.2256	.2776	.3442	.4329
14	.0576	.0902	.1214	.1512	.1794	.2062	.2553	.3193	.4057
16	.0521	.0819	.1107	.1383	.1648	.1899	.2364	.2978	.3818
18	.0476	.0750	.1018	.1275	.1523	.1759	.2201	.2790	.3592
20	.0438	.0693	.0942	.1183	.1416	.1639	.2059	.2624	.3416
25	.0365	.0581	.0793	.1001	.1204	.1400	.1773	.2284	.3018
30	.0313	.0500	.0685	.0868	.1047	.1222	.1557	.2022	.2705
35	.0274	.0438	.0603	.0766	.0926	.1084	.1388	.1815	.2451
40	.0244	.0390	.0539	.0686	.0832	.0974	.1252	.1646	.2240
45	.0219	.0352	.0487	.0621	.0753	.0884	.1140	.1506	.2063
50	.0199	.0321	.0444	.0567	.0689	.0810	.1047	.1388	.1912

P = 8 S = 2 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.1418	.2019	.2543	.3005	.3415	.3781	.4405	.5139	.6014
6	.1292	.1852	.2347	.2788	.3182	.3538	.4150	.4881	.5766
7	.1186	.1710	.2179	.2600	.2981	.3324	.3923	.4648	.5538
8	.1096	.1589	.2034	.2436	.2802	.3135	.3720	.4437	.5328
9	.1019	.1484	.1906	.2291	.2644	.2967	.3538	.4244	.5134
10	.0952	.1392	.1794	.2163	.2503	.2816	.3372	.4067	.4954
12	.0842	.1238	.1604	.1945	.2261	.2556	.3084	.3750	.4629
14	.0754	.1115	.1452	.1767	.2063	.2339	.2842	.3486	.4345
16	.0683	.1013	.1325	.1619	.1896	.2157	.2635	.3256	.4067
18	.0624	.0930	.1219	.1494	.1754	.2001	.2456	.3054	.3858
20	.0574	.0858	.1129	.1387	.1633	.1866	.2300	.2875	.3670
25	.0479	.0721	.0953	.1176	.1391	.1597	.1985	.2508	.3251
30	.0412	.0621	.0824	.1021	.1212	.1396	.1746	.2225	.2918
35	.0360	.0545	.0726	.0902	.1073	.1240	.1558	.1999	.2647
40	.0321	.0486	.0649	.0808	.0963	.1115	.1407	.1815	.2423
45	.0289	.0439	.0586	.0731	.0874	.1013	.1283	.1662	.2233
50	.0262	.0400	.0535	.0669	.0800	.0929	.1178	.1533	.2071

TABLE 1 (continued)

P = 8 S = 7 ALPHA = .050

N \ R	0	1	2	3	4	5	7	10	15
5	.7345	.7662	.7910	.8109	.8273	.8410	.8626	.8859	.9109
6	.6958	.7299	.7568	.7786	.7968	.8122	.8367	.8634	.8924
7	.6605	.6962	.7248	.7482	.7679	.7846	.8116	.8412	.8741
8	.6281	.6650	.6948	.7196	.7405	.7583	.7874	.8197	.8560
9	.5986	.6362	.6670	.6927	.7145	.7334	.7643	.7989	.8383
10	.5715	.6097	.6411	.6675	.6901	.7097	.7421	.7788	.8210
12	.5237	.5621	.5943	.6217	.6453	.6661	.7008	.7408	.7877
14	.4829	.5212	.5535	.5812	.6056	.6270	.6633	.7057	.7564
16	.4479	.4855	.5176	.5455	.5700	.5919	.6291	.6734	.7269
18	.4175	.4542	.4859	.5137	.5382	.5603	.5981	.6435	.6995
20	.3909	.4267	.4578	.4852	.5097	.5317	.5698	.6160	.6737
25	.3370	.3704	.3997	.4259	.4496	.4712	.5090	.5560	.6162
30	.2959	.3269	.3544	.3792	.4019	.4227	.4596	.5061	.5671
35	.2638	.2925	.3183	.3417	.3632	.3831	.4187	.4642	.5250
40	.2379	.2647	.2888	.3109	.3312	.3501	.3844	.4286	.4885
45	.2167	.2417	.2642	.2851	.3043	.3224	.3552	.3979	.4566
50	.1989	.2222	.2435	.2632	.2815	.2987	.3300	.3713	.4285

P = 8 S = 7 ALPHA = .010

N \ R	0	1	2	3	4	5	7	10	15
5	.7816	.8083	.8290	.8456	.8591	.8705	.8884	.9075	.9280
6	.7444	.7737	.7967	.8154	.8308	.8438	.8645	.8869	.9112
7	.7098	.7411	.7660	.7864	.8034	.8178	.8410	.8664	.8944
8	.6777	.7106	.7370	.7588	.7771	.7928	.8182	.8463	.8775
9	.6481	.6820	.7097	.7326	.7521	.7687	.7960	.8265	.8609
10	.6207	.6555	.6839	.7078	.7281	.7457	.7746	.8072	.8445
12	.5717	.6074	.6371	.6622	.6839	.7028	.7344	.7705	.8127
14	.5295	.5654	.5956	.6215	.6441	.6640	.6974	.7362	.7824
16	.4928	.5285	.5589	.5852	.6082	.6286	.6633	.7043	.7537
18	.4606	.4959	.5262	.5526	.5758	.5966	.6322	.6746	.7266
20	.4324	.4670	.4969	.5232	.5466	.5674	.6035	.6471	.7011
25	.3745	.4072	.4359	.4614	.4843	.5052	.5416	.5864	.6437
30	.3302	.3608	.3879	.4123	.4345	.4547	.4906	.5356	.5942
35	.2952	.3238	.3493	.3725	.3937	.4133	.4482	.4926	.5514
40	.2668	.2935	.3177	.3396	.3598	.3786	.4123	.4557	.5141
45	.2433	.2685	.2912	.3120	.3313	.3492	.3817	.4239	.4814
50	.2236	.2473	.2687	.2885	.3068	.3240	.3552	.3961	.4524