ENERGY LOSS IN SURFACE WAVE SPECTRA DUE TO DATA WINDOWING

by

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TABLE OF CONTENTS

ABSTRACT

INTRODUCTION 2

A NUMERICAL CONFIRMATION

CONCLUSIONS 7

REFERENCES 8

List of Figures

1. Sample Sea-Wave Record Before and After Cosine Windowing 3
2. Wave Spectra of the Signals in Fig. 1 without and with the 8/3 Correction Factor 4
3. Histogram of 101 Empirical Correction Factors obtained from Sea-Wave Records 6
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ABSTRACT

Windowing or tapering of surface-wave time-series is often performed prior to spectral analysis. The loss of variance due to the windowing is theoretically a factor of 8/3 (for cosine windows), but this factor may vary in practice due to the non-white nature of wave spectra. A numerical experiment on 101 wave spectra has shown that although 8/3 is a good correction factor in the mean, corrections on individual records may vary from less than 2 to almost 4.3. Since rarely in geophysical studies are sufficient records available to allow one to approach some theoretical value in the mean, i.e. 8/3, correction of individual records is recommended.
INTRODUCTION

Figure 1 shows a typical geophysical stochastic process, namely a record of sea surface-wave displacement, in its original form (upper) and with a cosine window applied (lower). The reasons for windowing are discussed for example in Ref. 1 and, especially in relation to discrete calculations, in Ref. 2.

It is clear from Fig. 1 that the windowed record has suffered a loss of variance.

The energy spectra of the two records are shown in Fig. 2. In the upper part of the figure the spectra are presented exactly as they are calculated, but in the lower part the spectrum from the windowed record was multiplied by the factor 8/3 to increase the total variance up to about the level of that of the unwindowed record. Heuristic confirmation of the 8/3 factor comes from the observation that the average level of a squared cosine window is precisely 3/8.

With the help of some additional manipulations, the detailed reason for the factor 8/3 is rigorously but implicitly developed in Ref. 2. Essentially, the factor obtains as the normalization of the filter in the frequency domain that corresponds to the cosine window in the time domain. That is, multiplication of the time series by a cosine window is equivalent to convolution in the frequency domain of the Fourier transform of the time series with the three-point smoothing filter $-\frac{1}{4}, +\frac{3}{4}, -\frac{1}{4}$. The normalization factor is the reciprocal of the square root of the sums of the squares of the coefficients of the smoothing function, i.e. \(1/(6/16)^{\frac{1}{2}}\). However, as the energy spectrum is proportional to the square of the filtered (smoothed) Fourier transform, the desired factor is 8/3.
Interestingly, the smoothing function $+\frac{1}{4}, +\frac{1}{2}, +\frac{3}{4}$ (called Hanning smoothing) is often applied to a raw spectral estimate to reduce the variability of the estimate; this same function could be applied to the Fourier transform as well with exactly the same reduction of energy as in the previous example. In fact, the time series is still undergoing a cosine window but one that is unity at the two ends of the record and zero in the middle, thus giving no useful effect from the windowing.

**Fig. 1** Sample sea-wave record before and after cosine windowing
FIG. 2 WAVE SPECTRA OF THE SIGNALS IN FIG. 1 WITHOUT AND WITH THE 8/3 CORRECTION FACTOR
A NUMERICAL CONFIRMATION

That the use of the factor 8/3 is appropriate in the mean, at least for sea wave data, consider Fig. 3. To prepare this histogram, the variance for each of 101 different wave records was calculated directly from the time series, compared with the variance of the windowed record and the set of obtained empirical correction factors plotted. The mean value of the distribution lies at 2.74 and the standard deviation is about 0.53. A Student's-t test (valid for normal distributions but hoped to be usefully valid here as well) at the 0.05 significance level shows that the mean of the empirical correction factors is (with 95% confidence) not different from the theoretical factor of 8/3. The distribution is slightly bimodal and positive skew.

There was no apparent correlation of empirical correction factor with record variance (unwindowed), but it seems reasonable to suppose that the skewness of Fig. 3 toward values larger than 8/3 is because of the non-white, peaky nature of wave spectra.

* Each record was 204.8 seconds long and was based on 10 Hz sampling of a Waverider buoy signal. The variances of the 101 records varied from about 100 cm² to more than 3000 cm².
FIG. 3 HISTOGRAM OF 101 EMPIRICAL CORRECTION FACTORS OBTAINED FROM SEA-WAVE RECORDS
CONCLUSIONS

If non-rectangular windowing of a time series occurs, there is a need to correct the variance of the record. For cosine data windows, corresponding to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ smoothing in the frequency domain, the theoretical correction factor is $8/3$.

A numerical experiment on 101 sea surface-wave records has yielded empirical correction factors (for cosine windows) from 1.98 to 4.28 with a mean value of 2.74, statistically indistinguishable from the $8/3$ theoretical value. The distribution is, however, positive skew and somewhat bimodal, so empirical correction of individual records is recommended if high accuracy is important.

I expect that this recommendation for correction of individual records will be less important for time series whose spectra are more "white" than those of sea waves, and more important for records shorter than those used here, i.e. about 60 times the modal value of the spectral peak.

The variability in the correction factor may account for some of the variability in certain wave parameters, such as the Phillip's equilibrium constant.
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