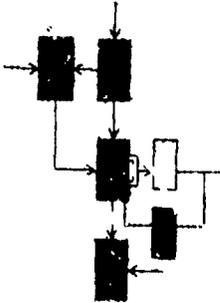


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CONTROL THEORY FOR LARGE SCALE AND UNCERTAIN
SYSTEMS APPLICABLE TO AEROSPACE SYSTEMS
(FINAL REPORT)

Research supported by
Air Force Office of Scientific Research (AFOSR)
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Covering the Time Period
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by

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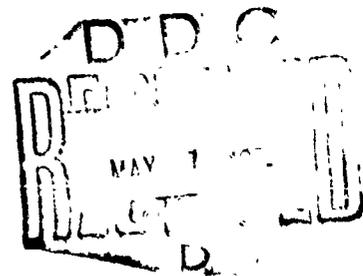
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1. CONTROL AND DECISION MAKING UNDER UNCERTAINTY

1.1 Background for Research

In most control problems uncertainty is present in different forms. Indeed, the primary control objective is to control the system in a desirable way in the presence of these uncertainties. The uncertainties may be present in the form of input and measurement noise or the parameters of the system may be known imperfectly.

Several problems arise in controlling a system in the presence of uncertainty. Some of these are:

- (i) modeling of uncertainty (for example, choice between probabilistic or unknown but bounded disturbances; choice of covariance matrices, etc.);
- (ii) state and parameter estimation;
- (iii) optimum choice of control and measurement strategies;
- (iv) adaptive stochastic control.

Research has been conducted on the above and other aspects of control in the presence of uncertainty.

1.2 Specific Studies

1.2.1 Estimation Methods

The problem of estimating the state variables and/or unknown parameters of dynamical systems based on noisy measurements is of fundamental and practical importance. Progress has been made along several fronts in this area.

A persistent problem that is associated with filtering is that, due to errors in modeling, the state estimates as generated by the Kalman filter may contain bias errors.

Some interesting results on the elimination of mean state errors have been obtained by Athans and reported at the 1971 Conference on Nonlinear Estimation at San Diego [1]. These results were obtained for the case of linear multivariable constant systems for which one wishes to use the constant parameter steady state Kalman-Bucy filter. Errors in the system parameters (modeling errors) result in a "mismatching" between the plant and filter dynamics; this mismatching results in steady state bias errors in the estimates of the state and output variables generated by the Kalman-Bucy filter. A simple practical filter has been obtained by Professor Athans, which can be used to eliminate these steady state bias errors. This filter is called a Compensated Kalman Filter and it consists of the introduction of a dynamic compensator between the residuals and the steady-state Kalman-Bucy filter. The dynamic compensator includes feedforward integration of the residuals and lags. Considerable effort is underway to carry out simulations and to extend these ideas to the extended Kalman filter case.

1.2.2 Analytical Studies in Nonlinear Filtering

Professor Mitter with J. Galdos (teaching assistant in the Electrical Engineering Department) has begun a detailed study of certain fundamental questions in nonlinear filtering.

This investigation seeks to apply the methods of probabilistic functional analysis and stochastic Liapunov theory (more precisely abstract

valued martingale theory) to the areas of filter performance evaluation and algorithm development for the nonlinear filtering problem.

More specifically, this study comprises three areas as follows:

- (a) Comparative study of the nominal trajectory and extended Kalman filters and approximation of the latter by the former;
- (b) Based on (a), determination of a recursive scheme to implement the extended Kalman filter and of some variation of this or another filtering scheme which would yield an algorithm that converges to the Kushner-Stratonovich equations specifying the solution to the nonlinear filtering problem;
- (c) Determination of the role of controllability, observability, and stability in the nonlinear filtering problem.

Some results have been obtained in area (a), while preliminary studies in the other two problems have been made.

Conditions under which the extended Kalman filter can be approximated by serial application of the nominal trajectory Kalman filter have been developed by deriving conditions under which a sequence of solutions to nominal trajectory Kalman filtering problems approximate a second sequence, which in turn converges to the solution of the extended Kalman filtering problem.

Work remains to be done in all three areas. In particular in area (a) it is desirable to derive conditions which are more verifiable and bounds on the difference of performance of the filters. In area (b), just as stability of real valued stochastic processes can be studied via real valued martingale convergence theorems (which comprise Liapunov stability

theory) it will be investigated how abstract valued martingale theory can be used together with random fixed point theorems to investigate the convergence of sequences of functions to the solutions of the problems under consideration. In all areas the theory will be developed first for a simple case (such as examples arising in the area of radar detection and then generalized.

1.2.3 Optimization of Control and Measurement Strategies

The problem of controlling the choice of measurements has been investigated by Professor Athans. The class of problems that are considered deal with the case where several possible noisy measurements of the state of a dynamical system are available. However, at each instant of time one is constrained to make a single set of measurements. In addition, there is an inherent per-unit-time cost associated with each type of measurement. The performance criterion is the optimization of a weighted combination of prediction accuracy and measurement cost. It was found that in the case of linear Gaussian systems the stochastic measurement optimization problem can be transformed into a nonlinear deterministic problem, whose solution yields off-line the optimal measurement strategy. These results are contained in references [2] and [3].

Work has continued by L. C. Kramer and Professor Athans in the area of simultaneous optimization of dynamic control and measurement strategies for discrete-time dynamic systems. This class of problems represents situations in which a stochastic dynamic system must be controlled using information obtained by taking noisy measurements, the "quality" (in some sense) of which is also under instantaneous control. A typical problem of

this type might be the on-line allocation of telemetry resources among several spacecraft subsystems which are being controlled from the ground.

Several results have been obtained in this area. These are reported in the Ph.D. thesis of L. C. Kramer [4]. This report considers in detail the correct methods of solving such problems using the tools of dynamic programming and the Maximum Principle of Pontryagin. In addition it provides conditions under which one-way and two-way separation principles exist in such combined measurement and dynamic optimization problems. References [5] to [7] contain additional details for this class of problems.

1.2.4 Suboptimal Stochastic Control Methods

Mr. Raymond Kwong and Professor Athans have been considering various aspects of stochastic control system design. The first problem he considered deals with the control of weakly coupled large scale linear systems. Examples of such systems can be found in process control applications [8], automated ground transportation systems [9], [10], air transportation systems [11], [12], etc. In these applications, one essentially has a set of distinct dynamical systems, each operating in an almost autonomous manner but each being also weakly coupled with the rest of the systems. If we completely ignore the couplings, we lose some of the system characteristics. If we take all couplings into account and have a central agency collect all measurements and administer all controls, then we will have a mathematically optimal design but we will also have to make many on-line operations and decisions. In the study conducted, the effects of coupling are taken into account by approximating them as equivalent "fake" plant white noises. This is done to communicate to the mathematics that there

is additional uncertainty in the assumed dynamics. The covariance of this fake plant noise is then determined quantitatively as a function of the type and degree of intersystem coupling by solving off-line matrix-valued two-point boundary value problems. The main tool of analysis has been the matrix minimum principle [13], and the implications of the results for sufficiently small coupling has been considered. In this situation, the results obtained using the above approach are the same as those obtained using the mathematically optimal approach of complete centralization. Furthermore, if we are not interested in very slight improvements, we can consider a class of white noise covariances as equivalent to the optimal choice. In that case, we do not have to solve the two-point boundary value problems. The above study has resulted in a joint paper [14].

The next problem now being investigated concerns the stochastic control of mildly nonlinear systems. For strongly nonlinear and noisy systems, there are very few practical results [15]. By analyzing a mildly nonlinear system, hopefully one can get approximately optimal control policies by a perturbational approach. This in turn will hopefully yield some insight into the general nonlinear problem. Preliminary results have indicated that by using a perturbational approach one can, by augmenting the state of the system, obtain approximately optimal control policies. The procedure is as follows: first of all, we try to approximate the mildly nonlinear system by a set of linear systems; then by viewing the set of linear systems as one large linear system we can apply linear stochastic control theory and solve for the optimal control for the new system. The success of this method, therefore, hinges on the accuracy of

the approximation. Considerable efforts are now being devoted to the study of the errors committed and the question of convergence if we use a larger and larger set of linear systems for approximation.

1.2.5 Adaptive Stochastic Control

The study of adaptive stochastic control of linear systems with unknown parameters is being completed by Mr. R. Ku and Prof. M. Athans. For the system under consideration, the system transition matrix $A(k)$ and the input gain vector $b(k)$ are assumed unknown. The adaptive control problem consists of identifying the unknown parameters and minimizing a quadratic performance index. Since the matrix $A(k)$ is incompletely known, a nonlinear estimation problem has to be solved. The extended Kalman filter is used to obtain approximations to the conditional means and covariance matrices [16]. The research extends the previous work by Tse and Athans [17], [18] on the control of linear systems with unknown gains. Their results showed that the open-loop feedback optimal control technique led to a computationally feasible algorithm for on-line applications. The open-loop feedback (O.L.F.O.) and the feedback control methods have the same dependence on the actual outcomes of the process and they differ only with respect to their dependence on the a priori probability distributions. However, the approach reduces the stochastic optimal control problem to a deterministic open-loop optimal control problem in which the Discrete Minimum Principle [19] or Dynamic Programming can be applied. Since the control depends on the estimates of the parameters, the standard Separation Theorem does not hold [20]. Hence the open-loop feedback optimal control

derived for the system is suboptimal. The control input will be used for both identification and control purposes.

The asymptotic behavior of the identifier is shown to converge to the true parameters. The overall behavior of the open-loop feedback control system will also converge to the truly optimal system when the parameters are known. Quantitative results on the convergence rate are given by the simulation results. From the Monte Carlo simulations, the expected value of the cost function is computed. This cost is compared with that of the truly optimal stochastic control system and will indicate the loss in performance when combined identification and control is necessary. The convergence rate and performance of the open-loop feedback optimal control are compared with the ad-hoc control scheme in which separation is enforced. Simulation results show that if the system is stable the O.L.F.O. performed better on the average than the ad-hoc scheme. For unstable systems, the limited simulation runs showed that the O.L.F.O. will incur on the average greater costs than the ad-hoc scheme. The results will be documented in a future report [21].

1.2.6 Feedback Control of Uncertain Systems

The problem of optimal feedback control of uncertain discrete-time dynamic systems was considered by D. P. Bertsekas, Professor Rhodes (now at Washington University, St. Louis, Missouri), and Professor Mitter, where the uncertain quantities do not have a stochastic description, but instead they are known to belong to given sets. The problem was converted to a sequential minimax problem and dynamic programming suggested as a general method for its solution. The notion of a sufficiently informative

function which parallels the notion of a sufficient statistic of stochastic optimal control was introduced, and the possible decomposition of the optimal controller into an estimator and an actuator is demonstrated.

Some special cases involving a linear system were further examined. A problem involving a convex cost functional and perfect state information for the controller was considered in detail. Particular attention was given to a special case, the problem of reachability of a target tube, and an ellipsoidal approximation algorithm is obtained which leads to linear control laws. State estimation problems were also examined, and some algorithms were derived which offer distinct advantages over existing estimation schemes. These algorithms were subsequently used in the solution of some reachability problems with imperfect state information for the controller.

The technical details can be found in the Ph.D. thesis by Bertsekas [22] and in three additional publications [23], [24], [25].

1.2.7 Design of Proportional-Integral-Derivative-Controllers

In industrial applications, control system designs utilizing some form of integral feedback are common. Integral feedback is used to eliminate steady-state (static) error in the face of plant parameter variations by converting a type zero system to a type one system.

In contrast to the designs used in industrial applications, designs for linear control systems by optimal control theory have called for a memoryless feedback of state variables (analogous to the proportional-derivative feedback of classical control theory). Motivated by C. D.

Johnson's disturbance absorbing controller and M. Athans's recent report [26], we set out to reduce this gap between theory and practice.

Our research in this area carried out by N. R. Sandell and M. Athans was directed toward the design of a compensator for an m -input, n -output linear system, when the system output vector is required to track with zero steady-state error a reference vector of step functions. We have obtained an optimal system which consists of a gain channel that operates on the output error vector (proportional), an integrating channel (integral), and plant state variable feedback (derivative). The constant gain matrices in these channels are deduced by solution of matrix algebraic equations; their numerical evaluation requires the solution of a standard matrix Riccati algebraic equation. The design is asymptotically stable in the large and tracks all constant reference input vectors with zero steady state error, regardless of plant parameter variations.

Simulation results have been obtained by use of a standard computer package for the solution of the linear regulator problem of optimal control. By use of such a package, it is possible to calculate the gain matrices of the design and to simulate the resulting compensated system in a single computer run. This feature greatly reduces the amount of work involved in utilizing the design procedure.

As a byproduct of the effort to apply modern control theory to the integral compensation problem, a generalization of the classical type-1 system concept was developed. The definition is given in terms of the system transfer function matrix, and an alternative characterization in the time domain is given. The principal result in this direction is a

generalization of the classical result that a stable, unity feedback, type- ℓ system tracks all polynomial type inputs of degree less than ℓ with zero steady-state error.

Essentially all the results described above may be found in the S.M. thesis of N. R. Sandell [27]. The design procedure along with simulation results is described in Sandell and Athans [28]. Results pertaining to the generalized type- ℓ system concept are found in Sandell and Athans [29].

2. STUDIES IN LARGE-SCALE SYSTEMS

2.1 Background for Research

Most large-scale systems consist of many interconnected sub-systems. For such systems, it is intuitively felt that it would be advantageous to recognize explicitly the sub-system structure and study it as such rather than as a single unit. Control of such systems leads to many new theoretical and computational problems. Our efforts have been directed towards developing a theory for such systems as well as study certain specific problems related to large-scale systems.

2.2 Specific Studies

2.2.1 Basic Theoretical Studies

Mr. C. Y. Chong and M. Athans have continued with the study of control methods for large-scale systems. In such systems, because of the amount of communication and computation involved, a centralized scheme of control with all the information processed and all the control commands dispatched by one central agency is generally impossible or uneconomical. A decentralized scheme of control with each control agent acting more or less autonomously is therefore preferred. On the other hand, a completely decentralized scheme will not be able to achieve the overall objective of the system. On-line coordination of the different controllers should improve the performance. Consideration has been given to the control of two coupled linear systems. The control objective is regulation, keeping the states as close to zero as possible without expenditure of too much control energy. One approach to this problem is to assume the two systems

are uncoupled. The couplings between the systems are replaced by fake white noises driving each system. This approach is appealing since the resulting control laws are extremely simple. On the other hand, the coordination is completely off-line. A second approach would be the use of a two-level control scheme. The lower level controllers have direct control of the two coupled linear systems and choose their controls optimally assuming the two systems are uncoupled. Optimality is defined with respect to the subgoals dispatched by the higher-level coordination. The job of the coordinator is to dispatch subgoals and corrections to the model structures used by the lower-level controllers so that the overall objective of the whole system is achieved. Thus instead of a single large-scale control problem, three smaller problems result. A key question here is the amount of information the coordinator needs to retain a certain degree of optimality. It is reasonable to assume that he will not need to know all the information available to the lower-level controllers since his objective is coordination rather than actual control of the system. Similarly, the actual model of the system should not be required in the decision making of the coordinator. A simplified or aggregated model should be sufficient. The problem of finding this aggregated model is still under investigation.

Research has also been started on the general properties of large-scale systems. It is felt that a two-level approach can be adopted to describe a large-scale system; namely the structural level and the dynamics level. The structural level describes how the various component systems of the large system are interconnected with each other. A qualitative

picture of the global behavior of the system can be derived from this structural description. It should also aid in designing the information structure of the system. The dynamics level gives the dynamic behavior of each subsystem and is useful when actual control laws are required. Notions like aggregation also play a very important part here. The results here have not yet been documented.

Information structures play a crucial role in large-scale systems. Professor Mitter and Mr. E. Hnyilicza have begun a systematic study of information and information structures.

In studying information structures, it must be kept in mind that they may originate from or be implemented by a sequence of coupled processes:

- observation
- communication
- computation
- decision-making

Recent work has been aimed at studying the properties of value and utility of information in the formulation of system strategies for large scale problems. A basic feature is that it is not satisfactory to talk about "information" in a general form. For instance, within a system, it is possible to visualize the following kinds of information:

- (i) Information being recorded by sensing devices.
- (ii) Information being encoded at one end of a communication link.
- (iii) Information being transmitted through a communication channel.
- (iv) Information being processed in a computing unit.
- (v) Information being delivered to a decision-making agent.

- (vi) Information being delivered by a decision-making agent to physical actuators.

It must be emphasized that the sensitivity of the overall performance index to each of the above - and therefore the value or utility of the information - may be radically different. In addition it must be kept in mind that a universal quantitative measure of information (such as that proposed by Shannon's theory for the purposes of communication systems) is grossly inadequate. Variables such as the semantic content of information, noisiness, accuracy, degradation through time delay, computability and volume from the standpoint of storage must be necessarily taken into account in a comprehensive theory.

Current efforts are being made and preliminary results have been obtained on the following:

Use of the entropy function to characterize the propagation of uncertainty through dynamical systems. Modeling of sensing devices as communication channels is straightforward and physically meaningful. Further work needs to be done in the problem of computing the mutual information (in the information-theoretic sense) between input and output of a dynamical system. Preliminary studies involving capital accumulation models with exponential cost functionals seem to be encouraging, since the role of subjective entropy appears to be closely connected to the notion of adaptivity.

Professor Mitter and Mr. E. Hnyilicza are studying decentralized control problems involving multiple controllers and a set-theoretic modeling of uncertainty will be studied.

Consider the linear system:

$$x_{k+1} = A_k x_k + \sum_{j=1}^m B_{kj} u_{kj} + w_k$$

$$y_{kj} = H_k x_k + v_{kj}, v_{kj} \in \Omega_{kj}$$

$$u_{kj} = \gamma_{kj}(y_{kj}) \in \Lambda_{kj}$$

$$w_k \in \Gamma_k$$

Within this framework, the concepts of observability and reachability for uncertain decentralized problems can be precisely formulated. The control objective will be one of guaranteed performance - or "satisfaction approach" - rather than a strictly optimal or minimax criterion. For set descriptions of the uncertainty, questions regarding the least amount of data which need be exchanged between controllers, and their relative impact on overall performance can be precisely formulated since the sets involved (ellipsoids or polyhedra) can be described by a finite set of real numbers. In order to define an appropriate "size" of the sets, the notion of ϵ -entropy in metric spaces should be helpful. The connections with the problem of coordinating a hierarchical decentralized system with variable information structure, should also become apparent.

2.2.2 Communication of Information Problems in Large Scale Systems

The study of large scale systems reveals the need to single out two aspects as being particularly important: hierarchy and uncertainty. All large scale systems exhibit a hierarchical structure in that they can be divided up into coupled subsystems. Some of the more important types of

hierarchy are discussed in [30], which uses bulk electric power systems as a prime example. In addition, the very size of large scale systems precludes any real possibility of obtaining satisfactory results without imposing some form of uncertainty on system models.

An initial study of the consequences of modeling a system as a pair of coupled systems, each with a separate controller, with stochastic (probabilistic) uncertainty was made in [31], [32] and [33]. The key assumption to be made is that each controller has a different information set, and it is this property that allows decentralized control.

The mathematical results obtained so far by C. Carpenter and M. Athans demonstrate the great complexity met in dealing with this problem. In order to obtain a better feel for the relevant factors in large scale system design and control, a similar model (two coupled stochastic systems) was analyzed in a discrete time framework. The results of that study led to a digital computer program which solves for the optimal control strategies of the two controllers. Specific numerical results will be available soon.

As soon as the hierarchical nature of large systems is admitted, we are led naturally to the introduction of more than one controller and then to a consideration of the effects on system design of allowing controllers to communicate part or all of their information sets to one another. This introduction of a communication network into the model of a large scale system seems to be an absolutely essential requirement if we are to use the results of these studies to understand, improve and redesign current engineering, economic and sociological systems.

Within the latter framework the problem of the division of control between the dynamics and communications is of particular interest. Initial results in this direction are aimed at determining under what circumstances intersystem communication is important and also when may we dispense with all or part of a communication system.

Specifically, two ways of modeling the communication link between two controllers have been proposed. A gain model, in which a noisy channel is given and each controller can set the signal to noise ratio of measurements he sends. The optimal control and gain determination problem which this model leads to is nonlinear and has no "nice" analytical solution. A somewhat easier model to work with is obtained if we again model the channel as noisy, but let each controller determine communication accuracy by controlling the covariance of the additive channel noise. In either case, a penalty is imposed for communication which is quadratic in information quality (proportional to gain, inversely proportional to covariance).

It is hoped that through this work a better and more realistic understanding of the requirements of large scale system administration and design can be obtained which will lead to a lessening of the problems which many such systems now face.

2.2.3 Optimal Control with Non-Scalar-Valued Criteria

This research was carried out by H. P. Geering and M. Athans. We have been able to find an infimum principle which is analogous to the well-known Pontryagin minimum principle in optimal control theory for scalar-valued cost functionals. As an interesting application, these results were used to rederive the Kalman-Bucy filter with infimal error covariance

matrix at the final time. These results can be found in the paper presented at the 1971 Princeton Conference by Geering and Athans [34], the doctoral thesis of Geering [35], and [36].

Optimality usually refers to a performance criterion which is a scalar-valued function of the parameters (controls, states, gains, etc.) of the optimal control or estimation problem, and which has to be minimized. It is often difficult to choose a suitable scalar-valued optimality criterion, because several things may simultaneously be of interest, such as energy consumption, transfer time, and integral squared error in optimal control problems, or the mean error vector and the whole covariance matrix in optimal estimation problems. It then is natural to consider non-scalar-valued performance criteria, e.g., vectors, matrices, sets, etc. The meaning of "better than" is not the complement of "worse than." Thus, a superior solution (or infimum) is "better than" all other solutions, whereas a non-inferior solution (or minimum) is not "worse than" any other solution.

The reported research efforts concentrate on extending the theory of optimal control from vector-valued performance criteria to more general non-scalar-valued performance criteria. Presently, the obtained theory can be applied to arbitrary finite-dimensional cost spaces which are partially ordered by a closed positive cone with non-empty interior. It is felt that this theory can be extended to cover appropriate infinite-dimensional cost spaces as well. This would allow convex set-valued performance criteria (partially ordered by set inclusion) via the analysis of the support functionals. Furthermore, the reported research efforts

concentrate on finding necessary conditions in the form of an infimum principle for a control to be superior rather than non-inferior. The resulting necessary conditions of the infimum principle are more restrictive than those of the Pontryagin minimum principle, because superiority is a more restrictive type of optimum than non-inferiority or optimality for scalar-valued criteria. Nevertheless, the necessary conditions of the infimum principle are rather analogous to those of the Pontryagin minimum principle: The costate now is a linear map from the (extended) state space into the cost space. The Hamiltonian attains its values in the cost space and has to be infimized by the superior control for all times along the optimal state trajectory and all costate maps whose component of the final time boundary condition mapping the cost space into itself is a positive map.

In addition the issues of existence of superior solutions and sufficiency of these conditions have been resolved. Another topic that has been analyzed is the cross-connection between this optimization theory for non-scalar-valued performance criteria and the theory of differential games.

2.2.4 Stochastic Differential Games

Game theory is relevant to control theory when the system under consideration is affected by two or more controllers. Unless the controllers have (1) different goals concerning the system or (2) different information about the system, the game degenerates to an ordinary control problem. The stochastic differential game satisfies both these conditions.

There is presently almost a total lack of results in the area of stochastic differential games. Various special cases and suboptimal schemes for the linear-quadratic-Gaussian (LQG) differential game have been obtained by Y. C. Ho and his coworkers and I. B. Rhodes. In the general LQG case, it is known that equilibrium strategies cannot be obtained by use of finite-dimensional filters.

Workers in stochastic differential games to date have been motivated by the results in game theory which depend on the normal form of a game. A recent research effort has been initiated by N. R. Sandell and M. Athans based upon the belief that the extensive form of a game should be the proper motivation for stochastic differential games, and hence the recent work of Auman questioning the appropriateness of the process of normalizing an extensive game is quite relevant. We have obtained encouraging preliminary results by following this philosophy and hope to clear this up in the near future.

3. CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

3.1 Background for Research

Recently, increasing attention has been paid to the control of distributed parameter systems. Besides being of intrinsic theoretical interest, there are a host of applications which would benefit from a better understanding of distributed parameter systems. These applications include:

- (i) Control and Stabilization of Aircraft in the Presence of Severe Gust Load Disturbances
- (ii) Control of Plasma and Hydrodynamic Instabilities
- (iii) Control of Heat Exchanges and Distillation Columns
- (iv) Meteorological Forecasting

3.2 Specific Studies

Professor Mitter with Mr. H. F. Vandevenne has worked on various aspects of distributed parameter systems.

3.2.1 Controllability of Wave-type Systems

In the control of linear ordinary differential equations one of the fundamental results is concerned with the relationship between controllability, observability, stabilizability and the infinite-time quadratic cost problem. Moreover, it is known that if a system described by a linear ordinary differential equation is controllable then it is controllable in an arbitrarily small time. These results turn out to be far more complicated for distributed parameter systems. It is generally not true that if

a system is controllable (in an appropriate sense) then it is controllable in an arbitrarily small time. The results turn out to be true for parabolic (diffusion) systems but not true for second-order hyperbolic (wave-type) systems. A detailed investigation of this phenomenon has been made and reported in the thesis of H. F. Vandevenne [37].

3.2.2 Controllability, Stabilizability and the Infinite-time Quadratic Cost Problem

The relationship between controllability and stabilizability for wave-type systems is still not completely clear. If the system is stabilizable, then it can be shown that the operator Riccati equation characterizing the feedback gain has a solution. Professor Mitter is continuing these investigations further.

4. CONTROL OF HEREDITARY DIFFERENTIAL SYSTEMS

4.1 Background for Research

There are many control problems where significant time delays or hereditary effects are present. A rigorous theory of control of such systems has hitherto not been available.

4.2 Specific Studies

4.2.1 Controllability, Observability, and Feedback Control of Hereditary Differential Systems

Professor Mitter in collaboration with Dr. M. C. Delfour of Centre de Recherches Mathématiques, Université de Montréal, Montreal, Canada (Dr. Delfour was not supported by the grant) has developed a theory of controllability, observability and the optimal feedback control of linear hereditary differential systems. Before a theory of control could be developed it was necessary to develop a theory of hereditary differential systems for non-continuous initial data. The work on differential equations will be published in [38], [39]. The work on control will be published in [40], [41], [42], [43].

We have obtained the following results:

- (i) Conditions for pointwise and functional controllability of linear hereditary differential systems have been obtained.
- (ii) Conditions for pointwise and functional observability of linear hereditary differential systems have been given.
- (iii) A duality theory relating controllability and observability has been developed.

- (iv) The relationship between controllability and stabilizability has been investigated and rather complete results have been obtained [44].
- (v) Detailed results on the optimal feedback control of linear hereditary differential systems with a quadratic cost has been obtained. The optimal feedback gain can be characterized as the solution of an operational differential equation of Riccati type. When the equation is interpreted appropriately, a solution to the equation can be shown to exist.
- (vi) A study of the infinite-time quadratic cost control problem has been made. It can be shown that the resulting constant optimal feedback gain satisfies an operator Riccati equation and that the corresponding closed-loop system is exponentially stable.
- (vii) In very recent work we have obtained preliminary results on the approximation of control problems for hereditary differential systems by control problems for ordinary differential equations. This work will be further developed in the thesis of Mr. C. McCalla (Mr. McCalla is a teaching assistant in the Mathematics Department and is not supported by the grant).

5. AIR TRAFFIC CONTROL

A. H. Sarris and M. Athans continued during the past year research on Air Traffic Control (ATC) in the near terminal area (NTA). Motivation for this research in ATC was provided by the fact that automation seems to be the only way to reach the utilization limits of present airport facilities. Terminal automation proposed thus far deals mainly with information exchange between airborne traffic and ground controllers. Our aim has been to design a fully automated terminal ATC system where the nominal aircraft trajectories are derived in an open loop fashion by a ground computer, while the human traffic controller closes the feedback loop and corrects via voice commands the deviations of the aircraft from the nominal paths.

Some ideas have already been proposed by Athans, Porter and others [45]-[49]. Invariably the fully or semi-automated schemes they proposed assumed that aircraft in the terminal area fly on a plane with constant speed. Our last year's research has dealt with designing a system that does not rely on the above assumptions.

The result was an "area navigation" method for automatic control of aircraft arriving in a random fashion from the en-route centers to the NTA. The main ingredients of the method are the following.

The NTA is supposed to be cylindrical region centered in the outer marker. It is divided in three annular regions also centered in the outer marker. Traffic in the outermost region, called the buffer zone, and the intermediate one, called the outer merging space, is restricted to fly on

any of a finite number of altitude levels, each level carrying traffic of constant speed. Higher levels carry higher speed traffic. The third and innermost region, called the inner merging space, is used for descent of the aircraft to the level of the outer marker, and deceleration to the final landing speed.

Aircraft can enter the NTA from a discrete number of gates which represent the points at which the different fixed air routes outside the NTA intersect the NTA cylinder. Since aircraft can enter the NTA at random times, sequencing, scheduling, and delay assignment are crucial in order to avoid near misses in the final descent stage. Also the choice of the radii of the different annular regions as well as the altitudes of the levels are crucial. These geometrical magnitudes must be chosen as small as possible to minimize the volume of the NTA and consequently the number of aircraft under control at any particular time.

These problems have been solved. Algorithms for sequencing, scheduling, and delay assignment have been derived. The results are presented in the S.M. thesis by Sarris [50] and in an as yet unpublished paper by Sarris and Athans [51].

PUBLICATIONS

The following publications have appeared during the time period of this grant. They have been supported fully or in part by AFOSR 70-1941 or the preceding grants (AFOSR 69-1724 and 69-1724(A)).

- 1) M. Athans, "On the Design of P-I-D Controllers Using Optimal Linear Regulator Theory," M.I.T. Electronic Systems Laboratory Report ESL-P-432, Cambridge, Mass., July 1970 (DDC-AD710300), also Automatica, Vol. 7, pp. 643-647, September 1971.
- 2) M. Athans, "The Compensated Kalman Filter," Proc. Symposium on Estimation Theory, San Diego, Calif., September 1971.
- 3) M. Athans, "On Optimal Allocation and Guidance Laws for Linear Interception and Rendezvous Problems," IEEE Trans. on Aerospace Electronic Systems, Vol. AES-7, No. 5, pp. 842-853, September 1971.
- 4) M. Athans, "The Role and Use of the Linear Quadratic Gaussian Problem in Control System Design," IEEE Trans. on Automatic Control, Vol. AC-16, No. 6, pp. 529-552, December 1971.
- 5) M. Athans, "On the Determination of Optimal Costly Measurement Strategies for Linear Stochastic Systems," Proc. 5th IFAC Congress, Paris, France, June 1972 (accepted).
- 6) M. Athans and L. W. Porter, "An Approach to Semiautomated Optimal Scheduling and Holding Strategies for Air Traffic Control," Proc. 1971 Joint Automatic Control Conference, St. Louis, Missouri, pp. 536-545, August 1971.

- 7) C. Y. Chong and M. Athans, "On the Stochastic Control of Linear Systems with Different Information Sets," IEEE Trans. on Automatic Control, Vol. AC-16, pp. 423-430, October 1971.
- 8) M. C. Delfour and S. K. Mitter, "Hereditary Differential Systems with Constant Delays I: General Case," to be published in Journal of Differential Equations.
- 9) M. C. Delfour and S. K. Mitter, "Hereditary Differential Systems with Constant Delays II: A Class of Affine Systems and the Adjoint Problems," to be published in Journal of Differential Equations.
- 10) M. C. Delfour and S. K. Mitter, "Controllability and Observability of Infinite Dimensional Systems," to be published in SIAM J. on Control, May 1972.
- 11) M. C. Delfour and S. K. Mitter, "Controllability, Observability and Optimal Feedback Control of Affine Hereditary Differential Systems," to be published in SIAM J. on Control, May 1972.
- 12) M. C. Delfour and S. K. Mitter, "Control of Hereditary Differential Systems," to be presented at 5th IFAC World Congress, Paris, France, June 1972.
- 13) M. C. Delfour and S. K. Mitter, " L^2 -Stability, Stabilizability, Infinite Time Quadratic Cost Control Problem and Operator Riccati Equation for Linear Autonomous Hereditary Differential Systems," submitted to SIAM J. on Control.
- 14) H. P. Geering, "Optimal Control Theory for Non-Scalar Valued Performance Criteria," Ph.D. Thesis, Dept. of Electrical Engineering, M.I.T., August 1971, also report ESL-R-445, September 1971.

- 15) H. P. Geering and M. Athans, "Optimal Control Theory for Non-Scalar Valued Criteria," Proc. Fifth Annual Princeton Conference on Information Sciences and Systems, Princeton, New Jersey, pp. 479-483, March 1971.
- 16) H. P. Geering and M. Athans, "Necessary and Sufficient Conditions for Differentiable Non-Scalar-Valued Functions to Attain Extrema" submitted to IEEE Trans. on Automatic Control.
- 17) T. L. Johnson, M. Athans, and G. B. Skelton, "Optimal Control Surface Locations for Aircraft Flexure Controllers," IEEE Trans. on Automatic Control, Vol. AC-16, No. 4, pp. 320-331, August 1971.
- 18) L. C. Kramer, "On Stochastic Control and Optimal Measurement Strategies," Ph.D. Thesis, Dept. of Electrical Engineering, M.I.T., October 1971, also Report ESL-R-463, November 1971.
- 19) L. C. Kramer and M. Athans, "On Simultaneous Choice of Dynamic Control and Measurement Strategies for Stochastic Systems," submitted to 1972 JACC.
- 20) L. C. Kramer and M. Athans, "On Linear Stochastic Systems with Coupled Measurement and Dynamic Control," Proc. 1972 Princeton Conference on Circuits and Systems, March 1972.
- 21) L. C. Kramer and M. Athans, "On the Application of Deterministic Optimization Methods to Stochastic Control Problems," submitted to 1972 JACC.
- 22) R. Kwong, C. Y. Chong, and M. Athans, "On Stochastic Control System Design Methods for Weakly Coupled Large Scale Linear Systems," submitted to the 1972 Joint Automatic Control Conference and to the IEEE Trans. on Automatic Control.

- 23) W. S. Levine, T. L. Johnson and M. Athans, "Optimal Limited State Variable Feedback Controllers for Linear Systems," IEEE Trans. on Automatic Control, Vol. AC-16, No. 6, pp. 785-793, December 1971.
- 24) S. K. Mitter, "Some Remarks on Dynamic Team Decision Problems," presented at 4th IFIP Colloquium on Optimization Techniques, Los Angeles, Calif., October 1971.
- 25) N. R. Sandell, "Optimal Linear Tracking Systems," M.I.T. Electronic Systems Laboratory Report ESL-R-456, Cambridge, Mass., September 1971.
- 26) N. R. Sandell and M. Athans, "On 'Type-2' Multivariable Linear Systems," M.I.T. Electronic Systems Laboratory Report ESL-P-465, Cambridge, Mass., November 1971 (submitted to Automatica).
- 27) N. R. Sandell and M. Athans, "Design of a Generalized P-I-D Controller by Optimal Linear Control Theory," M.I.T. Electronic Systems Laboratory Report ESL-P-467, Cambridge, Mass. (submitted to Automatica).
- 28) A. H. Sarris, "On Optimal Scheduling and Air Traffic Control in the Near Terminal Area," M.I.T. Electronic Systems Laboratory Report ESL-R-459, Cambridge, Mass., September 1971.
- 29) A. H. Sarris and M. Athans, "An Algorithm for Terminal Air Traffic Control," submitted to the IEEE Trans. on Automatic Control.
- 30) E. Tse and M. Athans, "Adaptive Stochastic Control for a Class of Linear Systems," IEEE Trans. on Automatic Control, Vol. AC-17, February 1972.

- 31) H. F. Vandevenne, "Qualitative Properties of a Class of Infinite Dimensional Systems," Ph.D. Thesis, Dept. of Electrical Engineering, M.I.T., January 1972, also to be published as Electronic Systems Laboratory report.
- 32) J. C. Willems and S. K. Mitter, "Controllability, Observability, Pole Allocation, and State Reconstruction," IEEE Trans. on Automatic Control, Vol. AC-16, No. 6, pp. 582-595, December 1971.

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