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ANTENNA PATTERN AVERAGING

Joseph H. Halberstein

Advanced Systems Department

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13. ABSTRACT

Much of the Antenna description, coupling prediction and specification work done for EMC relies on the mean and/or median of the gain values taken in dB of the Antenna patterns concerned. Often the measured Pattern Distribution Function is approximated by a cumulative normal probability function of the dB-gain-values and its median or mean and standard deviation is taken as indication of the EMC-performance of the Antenna. The question has been raised, whether a better figure of merit for the operational EMC-performance of antennas can be found. If such a figure of merit is more closely related to the straight average of gain rather than the average of dB-values of the gain, the second question is: What is the difference between the straight average of gain and the average in dB?

The present technical report provides a formula as answer to the second question for the case of Antenna patterns with normal pattern distribution functions. It is shown that the difference between the average or median dB-gain and the straight average of the gain is typically 5-20dB but may be even higher. The difference varies strongly with the standard deviation of the pattern distribution function.

FOREWORD

This report covers work performed under the NAVORDSYSCOM Compatibility program, ORDTASK No. 451-005-090-103-533-401 and is part of a continuing advanced development effort in the antenna technology area.

This report was given technical review by Dr. John F. Cavanagh.

Released by:

C. Bernard

C. W. BERNARD
Head, Advanced Systems Department

ABSTRACT

Much of the Antenna description, coupling prediction and specification work done for EMC relies on the mean and/or median of the gain values taken in dB of the Antenna patterns concerned. Often the measured Pattern Distribution Function is approximated by a cumulative normal probability function of the dB-gain-values and its median or mean and standard deviation is taken as indication of the EMC-performance of the Antenna. The question has been raised, whether a better figure of merit for the operational EMC-performance of antennas can be found. If such a figure of merit is more closely related to the straight average of gain rather than the average of dB-values of the gain, the second question is: What is the difference between the straight average of gain and the average in dB?

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CONTENTS

I. INTRODUCTION

1. Much of the Antenna description, coupling prediction and specification work in the EMC - community relies on P.D.F 's (Pattern Distribution Functions), the median and/or mean and the standard deviation derived from the P.D.F.[1,2,3,4,5,6,7,8] The Pattern Distribution Function P(g) is the cumulative probability that the gain is smaller than a given gain g. The gain is always given in dB.

2. The experimental work performed on high gain antennas showed that P(g) can be approximated by the cumulative normal function of mean gain $\bar{g}$ and standard deviation σ:

$$P(g) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{g} \exp\left[\frac{-1}{2\sigma^2}\left(g - \bar{g}\right)^2\right] dg \qquad (1)$$

This form for P(g) was found to be a good experimental approximation, provided g is the gain *in dB*. Assuming the cumulative normal form equ. (1) for P(g) the mean and median are identical. The determination of the mean via the median becomes particularly simple. This procedure has found wide usage in the EMC - Community. Obviously, it is considerably more convenient to average data points which have been measured in dB without first converting the dB - values of gain (g) to straight gain (G), where g is related to G by g = 10 log G.

3. The question has been raised, whether this practice of using averages of gain taken in dB is admissible. The second question which arises is: What is the error committed when the average in dB is substituted for the straight average? An answer to the first question will be discussed in a paper to be delivered by the author at the 1972 IEEE Symposium on Electromagnetic Compatibility.

II. SOLUTION AND DISCUSSION

In order to shed light on these questions we have calculated the straight average of a pattern distribution function P(g) of the form:

$$P(g) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{g} \exp\left[-\frac{1}{2\sigma^2}\left(g - \bar{g}\right)^2\right] dg$$

The symbols used throughout this report are:

G = straight gain as opposed to gain taken in dB.

g = gain, taken in dB.

$\overline{G}$ = average of G.

$\bar{g}$ = average of g.

σ = standard deviation of g.

$g(\overline{G}) = 10 \log \overline{G}$.

$P(g)$ = cumulative probability that gain is $\leq g$.

$p(g)dg$ = probability that the gain g lies between g and $g + dg$.

The expressions for $\bar{g}$ and $\overline{G}$ are:

$$\bar{g} = \frac{\int_{-\infty}^{\infty} gp(g)dg}{\int_{-\infty}^{\infty} p(g)dg} = \frac{\int_{-\infty}^{\infty} g \exp\left[-\frac{1}{2\sigma^2}\left(g - \bar{g}\right)^2\right]dg}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left(g - \bar{g}\right)^2\right]dg} \tag{2}$$

Since[9] $p(G) = p(g)\frac{dg}{dG}$ and $p(G)dG = p(g)dg$

$$\overline{G} = \frac{\int_0^{\infty} Gp(G)dG}{\int_0^{\infty} p(G)dG} = \frac{\int_{-\infty}^{\infty} G \exp\left[-\frac{1}{2\sigma^2}\left(g-\bar{g}\right)^2\right]dg}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left(g-\bar{g}\right)^2\right]dg} \tag{3}$$

It is shown in Appendix A that

$$\overline{G}(\bar{g},\sigma) = 10^{\frac{\bar{g}}{10} + \frac{\sigma^2 \ln 10}{200}} \tag{4}$$

and finally converting the equation to dB

$$g(\overline{G}) = \bar{g} + \frac{\sigma^2 \ln 10}{20} \tag{5}$$

and

$$g(\overline{G}) - \bar{g} = \frac{\sigma^2 \ln 10}{20} \tag{6}$$

We note that the difference $g(\overline{G}) - \bar{g}$ of the straight average converted to dB and the average of the dB-values depends on σ. The two averages are equal only if $\sigma = 0$. Table I shows the dependence of $g(\overline{G}) - \bar{g}$ on σ:

TABLE I

σ_{dB}	5	10	15	20
$[g(\overline{G}) - \bar{g}]_{dB}$	2.9	11.5	25.9	46

We note that the difference $g(\overline{G}) - \bar{g}$ is always positive and increases with the square of σ. It will be strongly influenced by any error in the estimation of σ, in particular if $\sigma = 10$ or larger.

Note that it is possible to calculate the straight average $g(\overline{G})$ from $\overline{g}$ and σ, using (5). In the case of P.D.F's which are well represented by cumulative normal probability functions it is possible to dispense with the conversion of the gain from the dB-values to straight gain when only $\overline{G}$ or $g(\overline{G})$, i.e., the straight average is required. This possibility will be discussed by the author in a paper to be delivered at the 1972 IEEE Symposium on Electromagnetic Compatibility.

III. CONCLUSIONS AND RECOMMENDATIONS

It has been shown that the average obtained from dB - values of Antenna gain is different from the average of straight gain values. They are connected by the formula

$$g(\overline{G}) = \bar{g} + \frac{\sigma^2 \ln 10}{20}$$

(see the glossary, Appendix B-1 for explanation of symbols).

In cases, when the straight average is needed for normal P.D.F.'s, the formula permits its computation from $\bar{g}$ and σ without converting all the data from dB's to straight gain values.

It is recommended that EMC-Engineers involved in the description, specification, design, prediction, measurement and evaluation of antenna EMC-performance use the straight average of gain rather than the dB-average or median. When the dB-average or median and the standard deviation is known for P.D.F.'s represented by cumulative normal distributions, the straight average should be calculated from the above formula for $g(\overline{G})$. Errors of 20 dB or more may result if dB-averages or the median are substituted for the straight average.

APPENDIX A

CALCULATION OF THE FORMULA FOR $\overline{G}$

Let

$$\overline{G} = \frac{I_N}{I_D}$$

where, from Page 2, Equation 3, I_N and I_D are:

$$I_N = \int_{-\infty}^{\infty} G \exp\left[-\frac{1}{2\sigma^2}(g - \overline{g})^2\right] dg \qquad \text{(A-1)}$$

$$I_D = \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}(g - \overline{g})^2\right] dg = \sigma\sqrt{2\pi} \qquad \text{(A-2)}$$

from the definition $g = 10 \log G$ and

$$G = 10^{\frac{g}{10}} = \exp\left(\frac{g \ln 10}{10}\right)$$

Hence, from (A-1)

$$I_N = \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}(g - \overline{g})^2 + \frac{g \ln 10}{10}\right] dg$$

$$= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left([g - \overline{g}]^2 - \frac{2 g\sigma^2 \ln 10}{10}\right)\right] dg$$

completing the square in the round brackets we have:

$$[g-\bar{g}]^2 - \frac{2g\sigma^2 \ln 10}{10} = \left(g-\bar{g}-\frac{\sigma^2 \ln 10}{10}\right)^2 + \bar{g}^2 - \left(\bar{g}+\frac{\sigma^2 \ln 10}{10}\right)^2 \tag{A-3}$$

$$I_N = \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left(g-\left[\bar{g}+\frac{\sigma^2 \ln 10}{10}\right]\right)^2\right]\exp\left[\frac{-1}{2\sigma^2}\left(\bar{g}^2-\left[\bar{g}+\frac{\sigma^2 \ln 10}{10}\right]^2\right)\right]dg \tag{A-4}$$

Taking the constant factor $\exp\left[\frac{-1}{2\sigma^2}\left(\bar{g}^2-\left[\bar{g}+\frac{\sigma^2 \ln 10}{10}\right]^2\right)\right]$ out of the integral and simplying it we have:

$$I_N = \exp\left(\frac{\bar{g}\ln 10}{10}+\frac{\sigma^2 \ln^2 10}{200}\right)\int_{-\infty}^{\infty} \exp\left[\frac{-1}{2\sigma^2}\left(g-\left[\bar{g}+\frac{\sigma^2 \ln 10}{10}\right]\right)^2\right]dg \tag{A-5}$$

The integral in the expression for I_N is well known and gives:

$$\int_{-\infty}^{\infty} \exp\left[\frac{-1}{2\sigma^2}\left(g-\left[\bar{g}+\frac{\sigma^2 \ln 10}{10}\right]\right)^2\right]dg = \sigma\sqrt{2\pi}$$

From Equation (A-2) we note that this integral cancels the demoninator I_D in the expression for $\overline{G}$ and

$$\overline{G} = \exp\left(\frac{\bar{g}\ln 10}{10}+\frac{\sigma^2 \ln^2 10}{200}\right) = 10^{\frac{\bar{g}}{10}+\frac{\sigma^2 \ln 10}{200}} \tag{A-6}$$

APPENDIX B

GLOSSARY

G	straight gain as opposed to gain taken in dB
g	gain, taken in dB
$\bar{G} = \frac{I_N}{I_D} =$	average of G
$\bar{g}$	average of g
σ	standard deviation of g
$g(\bar{G})$	$10 \log \bar{G}$
$P(g)$	cumulative probability that the gain is $\leq g$
$p(g)dg$	probability that the gain lies between g and $g + dg$.
P.D.F.	Pattern Distribution Function
I_N	Numerator of $\bar{G}$
I_D	Denominator of $\bar{G}$

APPENDIX C

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