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DIGITAL REDESIGN OF PERSHING ATTITUDE CONTROL SYSTEM

by

Sherman M. Seltzer

November 1971

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U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

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In this report the revised technique for designing a tactical rocket attitude control system has been extended and applied to the redesign of a continuous control system. The original PERSHING weapon system attitude control system was implemented by an analogue flight computer which has been replaced by a digital flight computer. This report presents a design technique for designing a digital attitude control system for the PERSHING weapon system. The design goal is to match the digital system response to that of the original analogue system.

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ILLUSTRATIONS

1. Introduction

A technique under development by Professor Benjamin C. Kuo of the University of Illinois was applied to the design of a tactical rocket attitude control system.[1] The applicability of the technique has been increased and the analysis used in Reference 1 has been extended accordingly.[2]

In this report the revised technique has been extended and applied to the redesign of a continuous control system. The original PERSHING weapon system attitude control system was implemented by an analogue flight computer. This computer has been replaced by a digital flight computer. This report presents a design technique for designing a digital attitude control system for the PERSHING weapon system. The design goal is to match the digital system response to that of the original analogue system.

2. Design Technique

The theoretical basis is presented in References 1 and 2. Only the basic design technique will be described here.

Assume that a linear continuous system (Figure 1) is described by the state variable equations,

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{m}(t) \tag{1}$$

and

$$\underline{m}(t) = E(0)\,\underline{r}(t) - G(0)\,\underline{x}(t) \quad , \tag{2}$$

where $\underline{x}(t)$ is a vector representing the state of the system, and A, B, G(0), and E(0) represent time-invariant matrices. The design goal is to replace this system with a digital version (Figure 2) whose error signal

[1]Seltzer, S. M., Digital Redesign of a Rocket Attitude Control System, U. S. Army Missile Command, Redstone Arsenal, Alabama, January 1971, Report No. RG-TN-71-1.

[2]Seltzer, S. M., Digital Redesign of a Rocket Attitude Control System. Part II, U.S. Army Missile Command, Redstone Arsenal, Alabama, November 1971, Report No. RG-TN-71-5.

$\tilde{\underline{m}}(t)$ is identical to the continuous system error $\underline{m}(t)$. The discrete data system model that is used to replace the continuous model may be represented analytically by

$$\dot{\tilde{\underline{x}}}(t) = A\,\tilde{\underline{x}}(t) + B\,\tilde{\underline{m}}(t) \tag{3}$$

and

$$\tilde{\underline{m}}(t) = E(T)\,\underline{r}(t) - G(T)\,\tilde{\underline{x}}(t) \quad , \tag{4}$$

where G(T) and E(T) represent time-invariant matrices. As indicated in References 1 and 2, matrices G(T) and E(T) are found by matching corresponding terms of truncated Taylor's series expansions of $\underline{x}(t)$ and $\tilde{\underline{x}}(t)$ yielding:

$$G(T) \approx G(0) + T \left.\frac{\partial G(T)}{\partial T}\right]_{T=0} \tag{5}$$

and

$$E(T) \approx E(0) + T \left.\frac{\partial E(T)}{\partial T}\right]_{T=0} \quad , \tag{6}$$

where

$$\left.\frac{\partial G(T)}{\partial T}\right]_{T=0} = \frac{1}{2}\,G(0)\,[A - B\,G(0)] \quad , \tag{7}$$

and

$$\left.\frac{\partial E(T)}{\partial T}\right]_{T=0} = -\frac{1}{2}\,G(0)\,BE(0) \quad . \tag{8}$$

If the plant described by Equations (3) and (4) contains integration operations, they must further be replaced by numerical integration approximations.

3. Design Problem

The original PERSHING weapon system attitude control system implemented in the on-board analogue computer may be represented in block diagram form (Figure 3), where ϕ, ϕ_p, and $\dot{\eta}$ represent vehicle pitch attitude, desired pitch attitude, and slant altitude velocity, respectively. Symbols r and $\dot{\eta}_\epsilon$ represent pitch attitude and slant altitude velocity errors, respectively, where

$$r(t) = \phi_p(t) - \phi(t) \tag{9}$$

and

$$\dot{\eta}_\epsilon(t) \approx \dot{\eta}(t) \quad . \tag{10}$$

In this report only the pitch attitude control system will be redesigned. Specifically, it is desired to match the continuous system signal y(t) with a digital control system $\tilde{y}(t)$ for any given input signal, r(t). Let

$$y(t) = a_0\, r(t) + x_2(t) \quad , \tag{11}$$

where

$$x_2(t) = \mathcal{L}^{-1}\left\{\frac{a_1 s k(s)}{(\tau_1 s + 1)(\tau_2 s + 1)}\right\} \quad . \tag{12}$$

Because of the linear characteristics of the existing continuous system and the digital system to be proposed, it is sufficient to match the digital system variable $\tilde{x}_2(t)$ to the continuous system variable $x_2(t)$. The continuous system is described in Figure 4, where

$$\alpha_1 \overset{d.}{=} \frac{1}{\tau_1 \tau_2} \tag{13}$$

and

$$\alpha_2 \overset{d.}{=} \frac{\tau_1 + \tau_2}{\tau_1 \tau_2} \quad . \tag{14}$$

In the state space formulation of Equations (1) and (2),

$$\underline{x} = [x_1, x_2]^T \quad , \tag{15}$$

$$\underline{r} = [0, r]^T \quad , \tag{16}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \tag{17}$$

$$B = [0, 1] \quad , \tag{18}$$

$$G(0) = [\alpha_1, \alpha_2] \quad , \tag{19}$$

and

$$E(0) = a_1\alpha_1 \quad . \tag{20}$$

The equivalent digital system with an analogue "plant" (Figure 5) is found by using Equations (5) through (8), yielding

$$G(T) = \left[\tilde{\alpha}_1, \tilde{\alpha}_2\right] \tag{21}$$

and

$$E(T) = a_1\tilde{\alpha}_1 \quad , \tag{22}$$

where

$$\tilde{\alpha}_1 \stackrel{d.}{=} \alpha_1 \left(1 - \frac{1}{2} T \alpha_2\right) \tag{23}$$

and

$$\tilde{\alpha}_2 \stackrel{d.}{=} \alpha_2 + \frac{1}{2} T\left(\alpha_1 - \alpha_2^2\right) \quad . \tag{24}$$

If the integrators of the analogue plant are replaced by digital numerical approximations, D_z, the all-digital control system is of the form shown in Figure 6 and may be written in transfer function form,

$$\frac{\tilde{\tilde{x}}_2(z)}{R(z)} = \frac{E(T)\, D_z}{1 + D_z\left(\tilde{\alpha}_2 + D_z\tilde{\alpha}_1\right)} \quad , \tag{25}$$

where $\tilde{\tilde{x}}_2(z)$ is the z-transform of the output $\tilde{\tilde{x}}_2(t)$ of the all-digital system. The stability considerations brought out in Reference 1 must be adhered to, i.e., in this case both $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ should be positive numbers.

There are numerous numerical approximations (D_z) of integrators[3]. As might be expected, the simplest formulations give the poorest results. Three possible numerical integration operations are:

$$D_z = \frac{\frac{1}{2}\, T\left(1 + z^{-1}\right)}{1 - z^{-1}} \qquad \text{(Trapezoidal Rule)} \tag{26}$$

$$D_z = \frac{\frac{1}{3}\, T\left(1 + 4z^{-1} + z^{-2}\right)}{1 - z^{-2}} \qquad \text{(Simpson's 1/3 Rule)} \tag{27}$$

$$D_z = \frac{0.3\, T\left(1 + 5z^{-1} + z^{-2} + 6z^{-3} + z^{-4} + 5z^{-5} + z^{-6}\right)}{1 - z^{-6}} \qquad \text{(Weddle's Rule).} \tag{28}$$

All three rules have essentially ideal phase characteristics, so the quality of the approximations can be determined primarily by consideration of their amplitude characteristics. This leads to the ranking of

[3]Salzer, J. M., "Frequency Analysis of Digital Computer Operating in Real Time," Proc. IRE, Vol. 42, February 1954, p. 463.

Weddle's rule as best, Simpson's 1/3 rule as good, and the trapezoidal rule as poor. However the trapezoidal rule is simpler to program and introduces attenuation at higher frequencies.[4]

4. Design Problem — Numerical Example

Assume that representative numerical values for the original PERSHING control system are:

$$a_0 = -6, \quad a_1 = -2.4 \quad . \tag{29}$$

If a step input of amplitude R,

$$\begin{aligned} r(t) &= R, \ t \geq 0 \\ &= 0, \ t < 0 \quad , \end{aligned} \tag{30}$$

is applied to the continuous system, the $x_2(t)$ response is formed from Equation (12):

$$x_2(t) = \mathcal{L}^{-1}\left\{\frac{a_1\alpha_1 R}{s^2 + \alpha_2 s + \alpha_1}\right\} = \frac{a_1\alpha_1 R e^{-\frac{1}{2}\alpha_2 t} \sin\sqrt{\alpha_1 - \frac{1}{4}\alpha_2^2}\ t}{\sqrt{\alpha_1 - \frac{1}{4}\alpha_2^2}} \ . \tag{31}$$

Applying the final value theorem, the steady state value $x_2(t)$ is zero. If it is desired to obtain values of ζ and ω_n of 0.707 and 84 radians/second, respectively, α_1 and α_2 are calculated to be

$$\alpha_1 = 7056, \quad \alpha_2 = 118.8 \quad . \tag{32}$$

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[4] Tou, J. T., Digital and Sampled-Data Control Systems, McGraw-Hill Book Company, Inc., New York, 1959 pp. 206-207.

This leads to a time response of

$$x_2(t) = -287.4\ Re^{-59.4\,t} \sin 58.92\,t \tag{33}$$

assuming zero initial conditions, which is plotted in Figure 7.

Similarly, if a ramp input of slope R,

$$\begin{aligned} r(t) &= Rt, \quad t \geq 0 \\ &= 0, \quad t < 0 \end{aligned} \tag{34}$$

is applied to the continuous system, the $x_2(t)$ response is

$$x_2(t) = \mathcal{L}^{-1}\left\{\frac{a_1\alpha_1 R}{s\left(s^2 + \alpha_2 s + \alpha_1\right)}\right\}$$

$$= a_1 R\left[1 - e^{-\frac{1}{2}\alpha_2 t} \sec\theta \cos\left(\sqrt{\alpha_1 - \frac{1}{4}\alpha_2^2}\ t + \theta\right)\right], \tag{35}$$

$$\theta \overset{d}{=} \tan^{-1}\left(\frac{\alpha_2}{2\sqrt{\alpha_1 - \frac{1}{4}\alpha_2^2}}\right) .$$

The steady state value of $x_2(t)$ is seen to be

$$\lim_{t \to \infty} x_2(t) = a_1 R \quad . \tag{36}$$

Again using the representative numerical values of Equations (29) and (32), the ramp response of $x_2(t)$ is found to be

$$\vec{x}_2(t) = -2.4\ R\left[1 + 1.420\ e^{-59.4\,t} \cos(58.92\,t - 0.7894)\right] \tag{37}$$

which is plotted on Figure 8 for R equal to unity.

The proposed digital control system responses to step and ramp inputs will now be found and compared to the continuous system responses of Equations (33) and (37). For the sake of computational simplicity, Equation (26) (trapezoidal rule) is selected for the numerical approximation of integration. Application of Equation (26) to Equation (25) leads to a relation for obtaining values of $\tilde{\tilde{x}}_2(nT)$ at sampling instants:

$$\tilde{\tilde{x}}_2(nT) = Z^{-1}\left\{\frac{\frac{1}{2}T\,F(T)\left(z^2-1\right)R(z)}{\left(1+\frac{1}{2}T\tilde{\alpha}_2+\frac{1}{4}T^2\tilde{\alpha}_1\right)z^2-2\left(1-\frac{1}{4}T^2\tilde{\alpha}_1\right)z+\left(1-\frac{1}{2}T\tilde{\alpha}_2+\frac{1}{4}T^2\tilde{\alpha}_1\right)}\right\} \tag{38}$$

where

$$R(z) = Z\{r(t)\} \quad .$$

If the final value theorem is applied to Equation (38), steady state values of $\tilde{\tilde{x}}_2$ for step and ramp inputs represented by Equations (30) and (34), respectively, are found to be identical to the original continuous system steady state values of zero and a_1R, respectively.

If the PERSHING sampling period of T = 1/122 second and Equations (29) and (32) are applied to Equation (38), the values of $\tilde{\tilde{x}}_2(nT)$ may be found from

$$\tilde{\tilde{x}}_2(nT) = -24.93\left[r_{nT} - r_{(n-2)T}\right] + 1.314\,\tilde{\tilde{x}}_2\Big]_{(n-1)T} - 0.4845\,\tilde{\tilde{x}}_2\Big]_{(n-2)T} \quad , \tag{39}$$

where $r_{(n-k)T}$ and $\tilde{\tilde{x}}_2\Big]_{(n-k)T}$ denote the values of r(t) and $\tilde{\tilde{x}}_2$, respectively, at the $(n-k)^{th}$ sampling instant. The responses to unit step and unity slope ramp inputs are calculated from Equation (39) and plotted in Figures 7 and 8 so they may be compared directly to the continuous system responses to the same inputs.

To show the effect of the numerical approximation for the integration operation, a digital formulation of the control system using ideal integrators is analyzed (Figure 5). The resulting response $\tilde{x}_2(nT)$ that is to be matched to the original $x_2(t)$ is found directly from Equations (3), (4), and (21) through (24), yielding

$$\tilde{x}_2(nT) = Z^{-1}\left\{\frac{T(z-1)\,E(T)\,R(z)}{z^2 - \left(2 - \tilde{\alpha}_2 T - \frac{1}{2}\tilde{\alpha}_1 T^2\right)z + \left(1 - \tilde{\alpha}_2 T + \frac{1}{2}\tilde{\alpha}_1 T^2\right)}\right\} \quad . \qquad (40)$$

Application of the final value theorem to Equation (40) yields the same steady state values for $\tilde{x}_2$ as for x_2 and $\tilde{\tilde{x}}_2$. Again using $T = 1/122$ second and applying Equations (29) and (32) to Equation (40) yields the equation

$$\tilde{x}_2(nT) = -71.25\left[r_{(n-1)T} - r_{(n-2)T}\right] + 1.1415\,\tilde{x}_2\Big]_{(n-1)T} - 0.3848\,\tilde{x}_2\Big]_{(n-2)T} \quad . \qquad (41)$$

The responses to unit step and unity slope ramp inputs are calculated from Equation (41) and plotted in Figures 7 and 8 for comparison with continuous system and all-digital system responses to the same inputs.

5. Conclusions

The step and ramp responses of the original continuous analogue computer control system and the proposed all-digital control system are shown in Figures 7 and 8 for ease of comparison. It is seen that the digital system response $\tilde{\tilde{x}}_2$ follows the continuous system response x_2 with a time lag and lower peak magnitudes. The digital system response can be made to better match the continuous system response if a more accurate numerical integration routine is selected, such as Equation (27) or (28). The best that can be attained is indicated by the response x_2 of the digital system with perfect integrators (Figures 7 and 8). If a better match of responses is needed using the technique of this report, the sampling period will have to be decreased.

The technique used in this report is simple and easy to apply and appears to yield a suitable design tool for designing an all-digital control system from the original PERSHING continuous analogue computer control system. This statement is based on the analytically obtained results presented in this report. It is strongly recommended that the proposed digital implementation be checked out on a computer simulation of the PERSHING attitude control system to confirm the results reported here.

The area of digital redesign of a continuous control system appears to be fruitful for further applied research and poorly understood. To merely add a digital system and retain the same numerical gains that were used for the continuous system will not provide an adequate response.

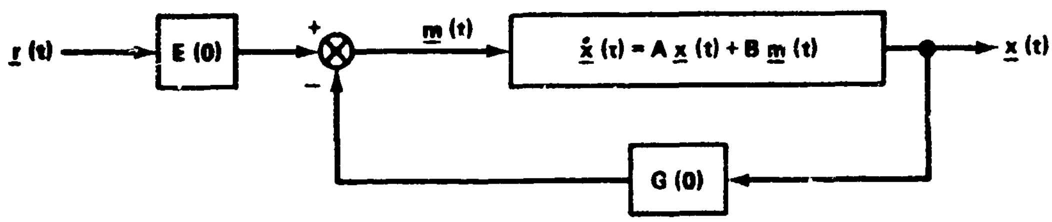

Figure 1. Continuous Model

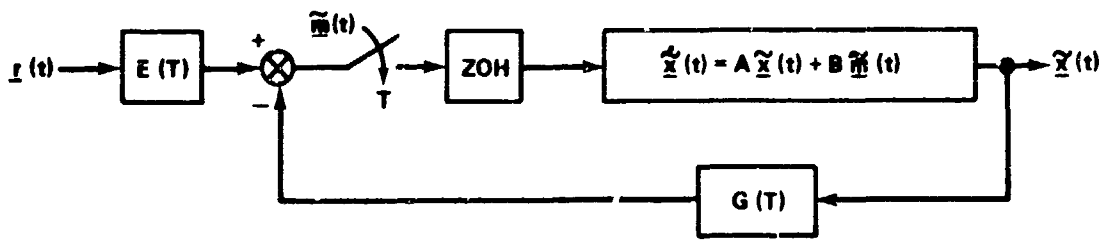

Figure 2. Sampled-Data Model

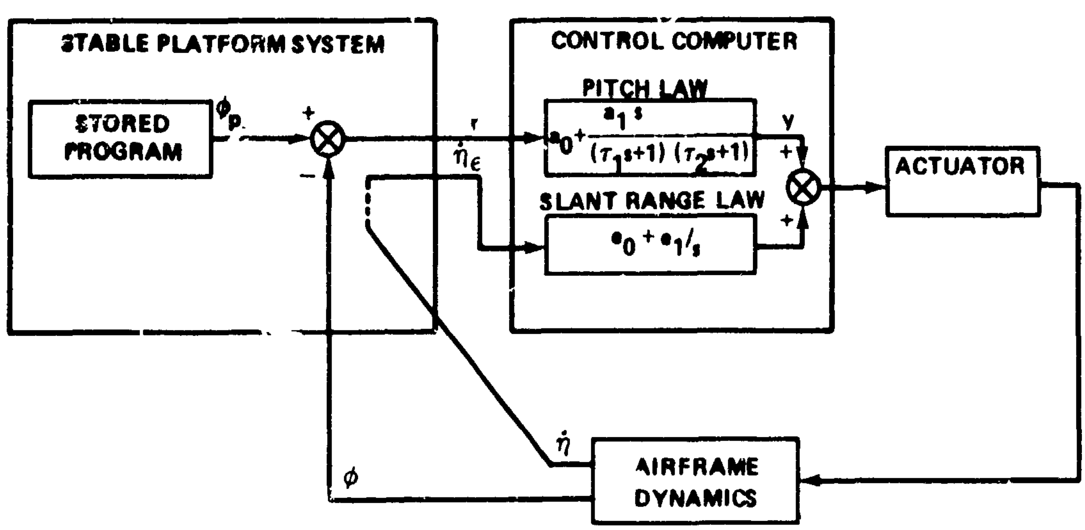

Figure 3. Simplified Block Diagram of PERSHING Analogue Control System

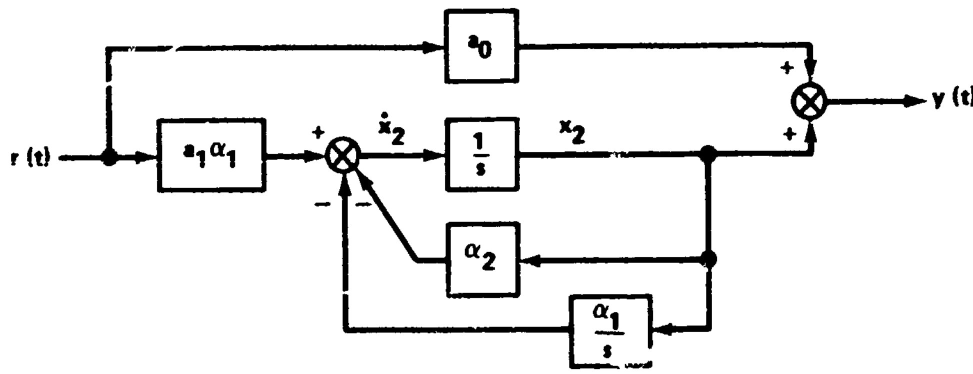

Figure 4. Continuous Pitch Attitude Control System

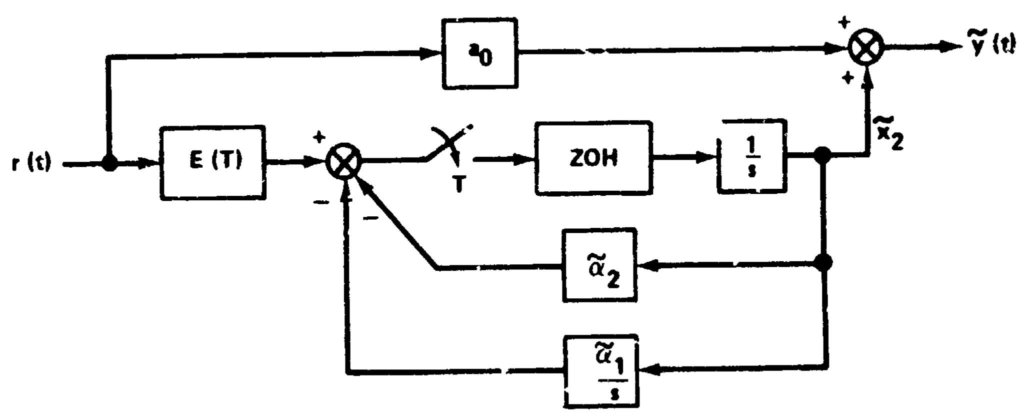

Figure 5. Sampled-Data Model with Analogue Plant

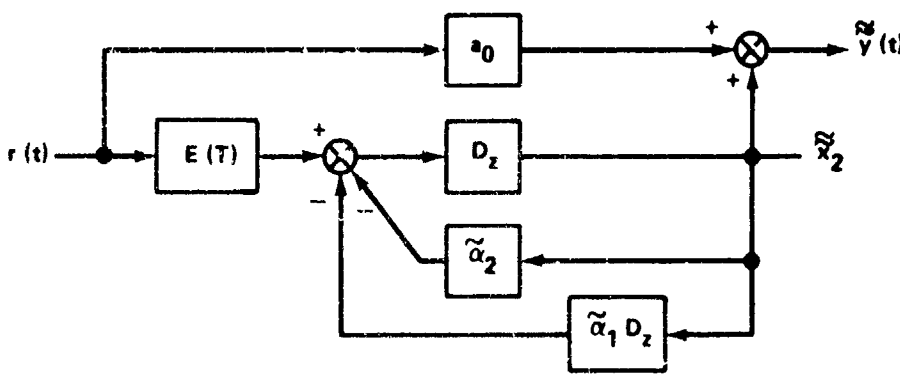

Figure 6. All-Digital Model

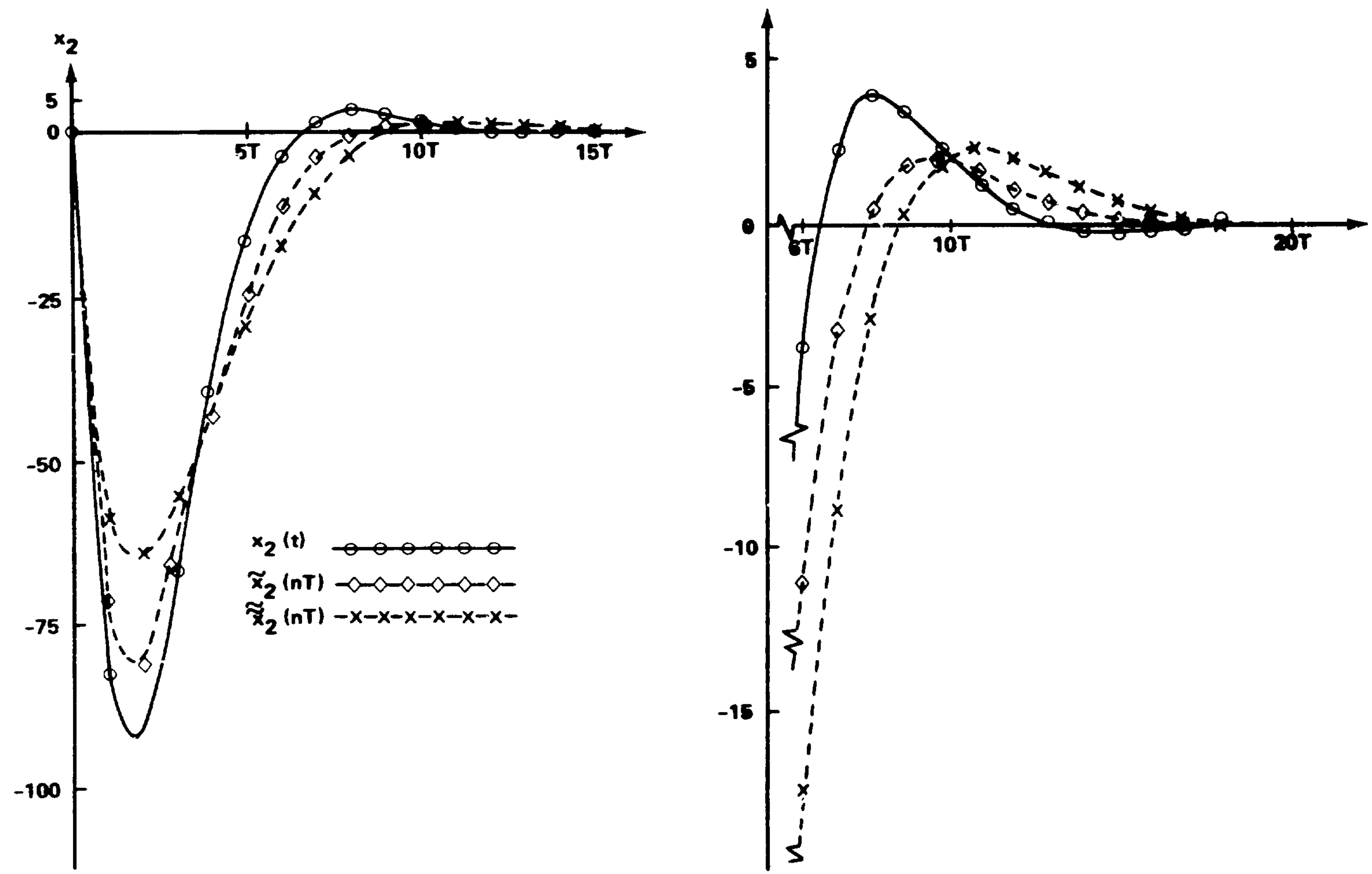

Figure 7. Unit Step Response (T = 1/122)

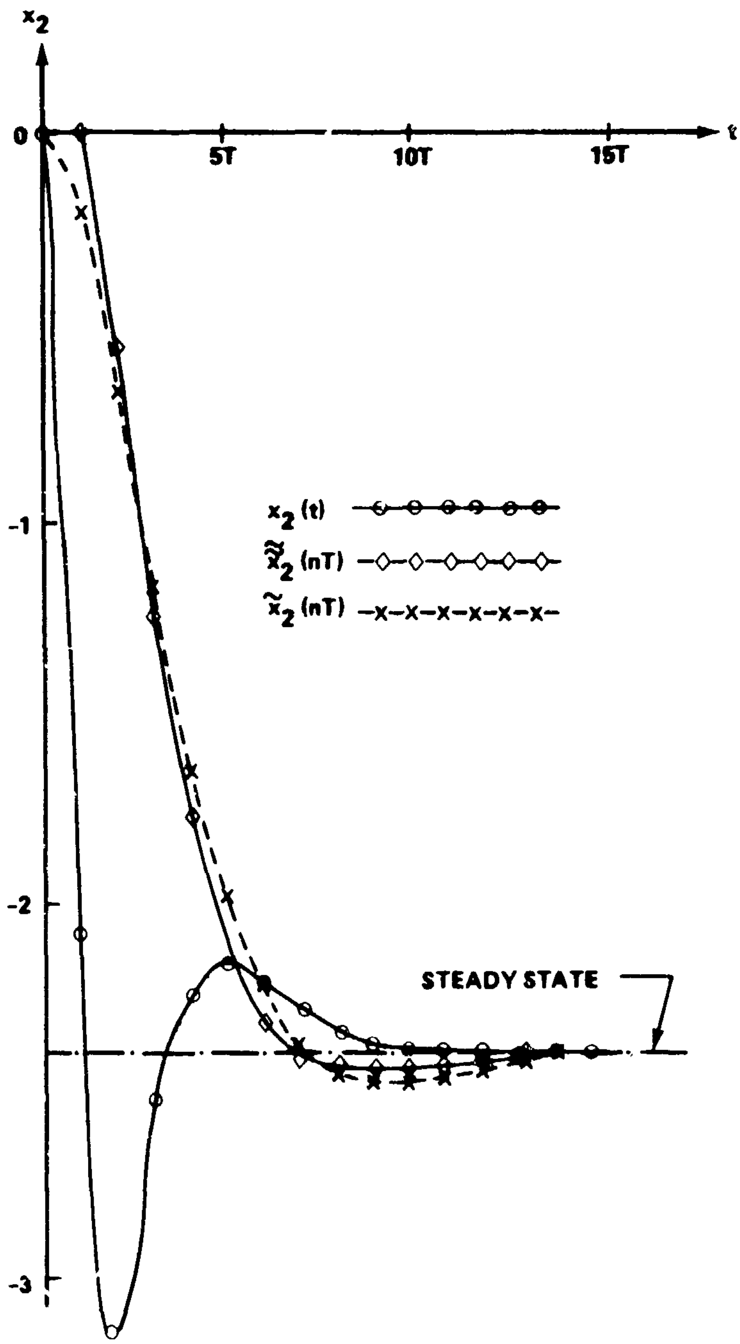

Figure 8. Ramp Response (Unit Slope, T = 1/122)