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Practical Methods

for

OBSERVING AND FORECASTING OCEAN WAVES

by means of

Wave Spectra and Statistics

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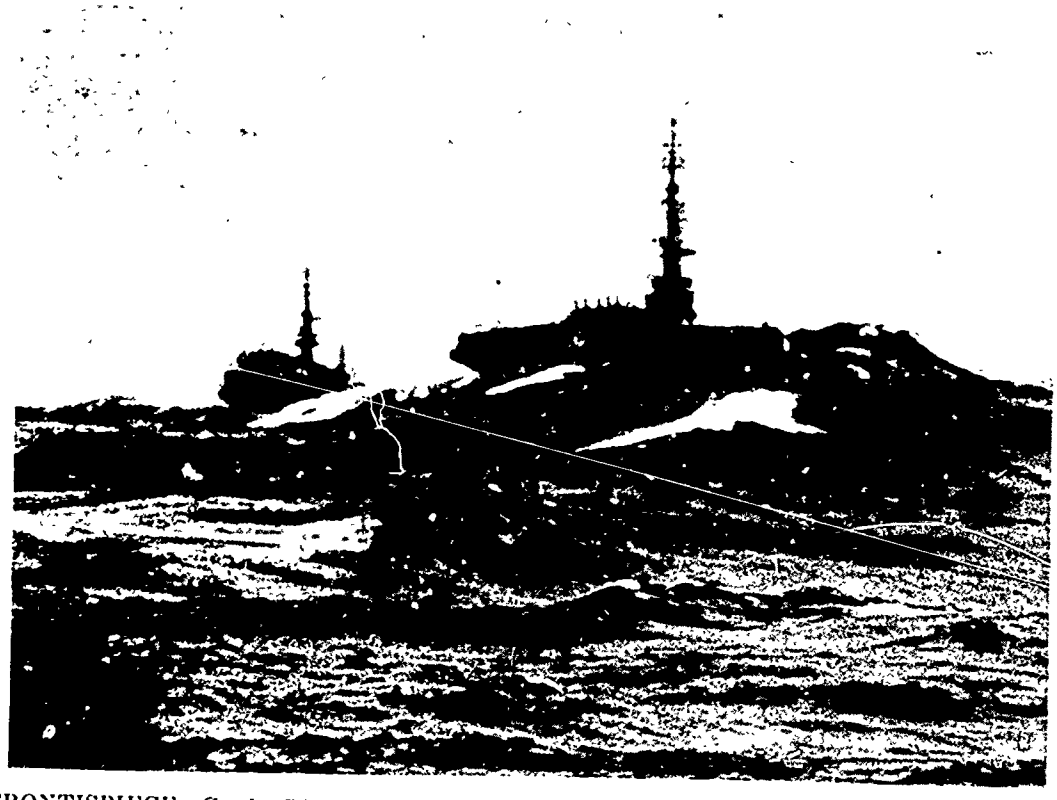

FRONTISPIECE—Carrier Division 27 successfully weathering China Sea typhoon (Ships shown are U. S. S. *Block Island* (CVE–106) and U. S. S. *Gilbert Island* (CVE–107),

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Practical Methods for OBSERVING AND FORECASTING OCEAN WAVES by means of Wave Spectra and Statistics

by
WILLARD J. PIERSON, JR.
GERHARD NEUMANN
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Foreword

Advances in wave forecasting techniques have resulted in ar increasing demand by various operational activities for wave forecasts. With an accurate advanced knowledge of wave conditions, it is possible to conduct such activities as naval operations, merchant vessel routing, seaplane landings, offshore drilling, nearshore construction, and commercial fishing more efficiently and safely.

The Pierson-Neumann-James theory of forecasting ocean waves, based on the mathematical concepts of statistics, is completely new and represents a more mathematically realistic approach to the problem of wave forecasting than existed heretofore. Basically, the Pierson-Neumann-James method describes the sea surface as the result of the combination of an infinite number of infinitely small sine waves of various amplitudes, frequencies, and directions. The distribution of the wave characteristics formed in a summation of these sine waves is described by a Gaussian function. The amplitudes summed over the frequencies and directions can be represented by a spectrum; this method may be called the Wave Spectrum Method. In practice, the wave spectrum is computed from the predicted wind field, and the individual wave characteristics determined by use of graphic methods or simple formulas.

Until the development of the Pierson-Neumann-James technique, several wave forecasting methods were utilized, foremost among which was the method devised by Sverdrup and Munk (H. O. Pub. Nos. 601 and 604). The Sverdrup-Munk wave forecasting method, based on classical wave theory, was adapted to practical use by solving the basic wave equations utilizing empirical data. Thus, the system is partly theoretical and partly empirical. The final wave equations were calculated by determining the combined effect of the tangential and normal stress of the wind on the sea surface. To solve these theoretical equations, Sverdrup-Munk resorted to empiricism by employing the nondimensional factors of wave age and wave steepness.

The Pierson-Neumann-James and Sverdrup-Munk theories for forecasting sea conditions are similar in that the basic equations were deduced by analyzing a great number of observations by graphical methods using known parameters of wave characteristics. On the

other hand, the two theories are dissimilar when dealing with swell. The former technique relies strictly on the theoretical considerations of angular spreading and dispersion of the various components of the spectrum, whereas the latter technique is partly theoretical and partly empirical. Using the Law of Conservation of Energy and an empirically derived premise that wave period increases with time, Sverdrup-Munk assumes that part of the wave energy is used to increase the wave speed during decay, which theoretically accounts for the increase in period.

There are some operational advantages and disadvantages of each method. The most obvious advantage of the Pierson-Neumann-James method is that it allows for a more complete description of the sea surface. Since the displacements are assumed to be distributed normally, this method provides direct forecasting of the percentage occurrence of heights, lengths, and periods of all waves in the spectrum. The major operational disadvantage is the time consuming and somewhat cumbersome techniques employed. This is especially true in swell forecasting and may prove too difficult for the average forecaster to use under operational conditions. On the other hand, the Sverdrup-Munk method is very easy to use by an inexperienced forecaster but lacks some of the refinements of the Pierson-Neumann-James method. For instance, the technique does not provide direct forecasting of periods and wave lengths of all waves in the given spectrum. The prediction graphs refer only to the characteristics of the significant wave; however, by applying various constants developed by Putz [1] to the significant wave height, one is able to predict the average, the highest 10 percent, and the maximum height of all waves in the spectrum.

A complete evaluation of the Sverdrup-Munk wave forecasting method as compared with the Pierson-Neumann-James method is currently in progress at the Experimental Oceanographic Forecast Central of the Hydrographic Office. Preliminary indications show that Pierson-Neumann-James computations of wave heights are too low for low wind velocities and short fetches and too high for the high winds and long fetches. In addition, the U. S. Corps of Engineers, Beach Erosion Board, has found that the forecast time of swell arrival at a given point is later than that actually observed. This tentative analysis, however, is subject to change after a complete study has been made.

The computations and numerical work of chapters 1 and 2 were accomplished by digital computing machines to insure a high degree of accuracy. Chapters 3 through 8 are based on slide-rule computa-

[1] Putz, R. R., "Wave Height Variability: Prediction of Distributed Function." Institute of Engineering Research, University of California, Series 3, Issue 318, Berkeley, Calif., December 1950.

tions only because of the time and accuracy required in operational forecasting.

Standard Navy date-time format is used in all forecasting examples. This is the 24-hour clock and 2-digit date group. Thus, May 1 at 1:15 p. m. GMT becomes 011315Z May. That is, 12 is added to the time if after noon, and all punctuation is omitted. The letter designator, Z, refers to the Zulu time zone (Greenwich). Midnight is indicated by 0000 of the following day.

The standard time system, in use by the U. S. Navy, is based on the division of the surface of the globe into 24 standard time zones, each of 15° of longitude or one hour of time. The initial zone keeping GMT is centered on the meridian of Greenwich (7½° E to 7½° W); it is designated the zero (Zulu) zone since the difference between the standard time of this zone and Greenwich Mean Time is zero. Each other zone is assigned a letter designator (N to Y in West longitude and A to M in East longitude) which is used as the suffix to date-time groups to identify the time zone of the entry. Thus, Eastern Standard Time, in the fifth zone west of Greenwich, is designated as R. The 12th zone, centered on the 180th meridian, is divided into West and East longitude, designated Y and M, respectively, and the time difference is +12 westward and −12 eastward; thus, the hours are the same, but the dates are a day apart.

Washington, D. C.
1955

J. B. COCHRAN
Captain, U. S. Navy
Hydrographer

Preface

Wave research has made great strides during the past 10 years. The spectrum of ocean waves was first studied in Britain by G. E. R. Deacon, N. F. Barber, and F. Ursell. The study of wave spectra was continued in the United States by A. A. Klebba, G. Birkhoff, and many others. The irregularities and the statistical properties of waves were also being studied, and an attempt was being made to fit the various pieces together in a consistent and logical pattern.

During 1949, the members of the staff of the Department of Meteorology and Oceanography at New York University began the study of ocean waves under a contract with the Beach Erosion Board. The problem was to determine the effects of waves on the beaches of the east coast of the United States. Wave refraction theory and wave spectrum theory were studied in connection with these contracts in order to find newer and better techniques for describing the waves.

As this research progressed, the Office of Naval Research became interested and supported work on wave generation, wave spectra, and wave propagation in deep water.

The results of the research conducted for the Beach Erosion Board and the Office of Naval Research provide the basis for this manual on ocean waves. The work done in 1949 and 1950 for the Beach Erosion Board was particularly important in the preparation of this manual since it helped to formulate the problems to be solved and provided information on the questions which really needed answers in the problem of wave forecasting.

The Bibliography lists the papers which were studied and used in the preparation of this manual. In addition to the papers referenced explicitly in the text, reading the papers by Barber and Ursell (1948), Cox and Munk (1954), James (1954), Longuet-Higgins (1952), Neumann (especially BEB Tech. Memo. No. 43, 1954), Pierson and Marks (1954), St. Denis and Pierson (1954), Rice (1945), and Watters (1953) will provide the person interested in the theory with an understanding of the foundation on which this manual is based.

The derivation of the average "wave length", $\tilde{\tilde{L}}$, is not given in any of the references listed above; however, it has appeared in the transactions of the American Geophysical Union (Pierson, 1954).

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This manual is the result of many years of work by many people. It is as up-to-date and as correct as it is possible to make it. However, as in any science, newer and more up-to-date results are continuously being obtained. Also some baffling theoretical problems, especially those connected with the effects of viscosity, still need to be solved. Those who use this manual should therefore not hesitate to apply new knowledge gained by experience and by trial and error to the procedures given.

Many people gave suggestions, knowledge, and skill in the preparation of this manual. Help was asked of the Coast Guard, the Hydrographic Office, the Navy, the Beach Erosion Board, and the Weather Bureau, and it was freely given. We would like to thank all concerned for their help and cooperation.

Lt. Comdr. Donald R. Jones, USN served as the liaison officer between New York University and Project AROWA. The many conferences which we have had with him have helped very much in the preparation of this manual.

The Atlantic Weather Patrol under the supervision, at the New York Port, of Mr. Charles Nelson took wave observations according to the methods described in Chapter IV and sent many sheets of wave data to us. These data were analyzed and studied in order to make it possible to write Chapter IV. The suggestions which Mr. Nelson, Mr. Quintman, and Mr. Kirkman gave us as to the difficulties encountered in making the observations helped to clarify the presentation given in Chapter IV.

During the preparation of this manual, the Search and Rescue Section of the U. S. Coast Guard based in New York City was extremely helpful with comments and advice. We would like to thank Chief Aerographer's Mates Black, Boerner, and Bridenstine, all of whose suggestions were used in the preparation of this manual.

In the beginning days of the preparation of this manual, a visit was made to Elizabeth City, N. C., and Capt. D. B. MacDiarmid, USCG talked with us about the problem of landing seaplanes on the open ocean. He showed motion pictures of the tests which were made and described his experiences in this connection. He also gave us the two wave photographs which were used in Chapter I. His help and suggestions are greatly appreciated.

The staff of the Division of Oceanography at the Hydrographic Office has read this manual and tested the methods presented. Their comments on the original manuscript were used in revising it for final form. The suggestion that the synoptic wave chart procedure, which has been developed and studied at the Hydrographic Office, be incorporated as part of the practical procedures was made by the authors of this manual. The Hydrographic Office willingly gave

permission for this method to be described herein. It shows great promise as a method of preparing wave forecasts in a simple and straightforward manner.

The Beach Erosion Board, in connection with other contracts at New York University, has kept us supplied with wave data. These data were used in checking the forecasting methods and in the preparation of Chapter I and Chapter VIII.

Dr. M. S. Longuet-Higgins visited New York University in 1952 and discussed wave theory with us. Dr. Longuet-Higgins read the original manual and made suggestions as to clarifications in the text. These were used in the preparation of this revised version.

Correspondence with Robin A. Wooding and N. F. Barber of New Zealand provided additional checks of theory and observation. Their interesting letters were greatly appreciated.

Conversations with Mr. S. O. Rice and Prof. J. W. Tukey of Bell Telephone Laboratories were also most helpful.

The original forecasting manual was prepared in 1953 under a contract sponsored by Bureau of Aeronautics, Project AROWA (contract No. N189s-86743). Major changes made in Chapter I, Chapter II, Chapter III, Chapter VII, and Chapter VIII were carried out in 1954 under a contract sponsored by the Office of Naval Research (Nonr-285 (03).

New York University
March 1954

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The purpose of this publication is to describe the basic concept and techniques of the first major attempt to apply recent research in wave spectra and statistics to wave forecasting. It should be pointed out that the theoretical spectrum in terms of wind fetch and duration used in designing this method is that developed by Neumann, and is only one of many such spectra prepared. Revision in this spectrum as more theoretical knowledge is acquired may therefore be expected. For this reason, this volume is not intended as the definite work on wave forecasting, but is published to provide the meteorologist and oceanographer with the means of testing and evaluating spectral and statistical techniques. In the interest of early publication, forecast examples and tables have been worked out to operational accuracy only.

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Chapter I

THE PROPERTIES OF OCEAN WAVES

Sea and Swell

Preliminary Definition of Sea and Swell

WAVES IN A STORM AREA GENERATED BY THE LOCAL WINDS OF THE STORM ARE CALLED "SEA." WAVES THAT HAVE TRAVELED OUT OF THE GENERATING AREA CHANGE INTO "SWELL." The reason for this change is very interesting and important in the problem of forecasting ocean waves. However, it is first necessary to study the differences between sea waves and swell waves.

Figure 1.1 is an aerial photograph of sea waves taken at a considerable altitude. Figure 1.2 is a similar photograph of swell. There are a number of differences between the waves in the two photographs. Heavy sea is different from swell in a great many ways besides the fact that seas are sometimes higher. In fact, sea waves having a certain average height are quite different from swell waves having the same average height.

Properties of Sea

The important properties of "sea" waves are shown in figure 1.1. The individual waves are shaped like mountains with sharp angular tops. The crests are not very long and can be followed a distance of only 2 or 3 times the distance between the crests. There are many small waves superimposed on the larger waves. Looking from one wave crest to the one immediately following, one can see that the heights are not regular. The individual crests are not all lined up in the same direction. Some appear to be at angles of 20 or 30 degrees to the others. There are places where the waves appear to be lined up and regular, but in other parts of the picture the waves are broken up and confused. It is not possible to follow one system of wave crests from one edge of the picture to the other. There are some parts of the picture where the waves are quite low. There are also some parts where the waves are quite high.

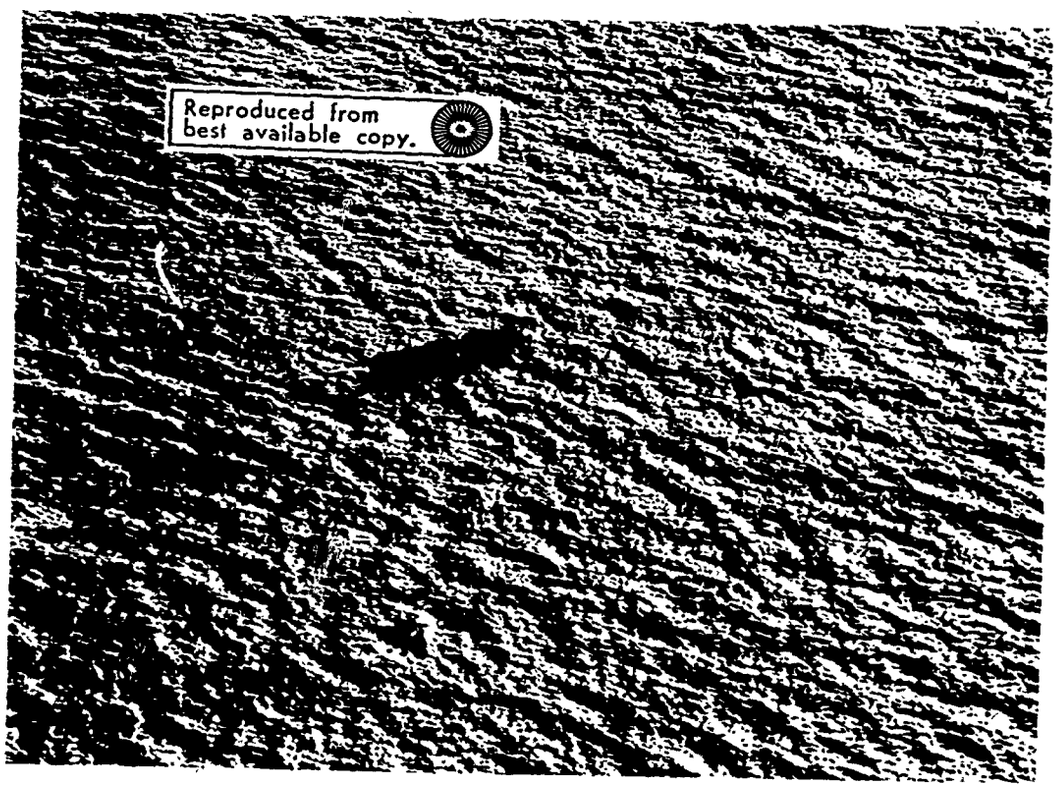
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FIGURE 1.1 Aerial photo of sea waves.

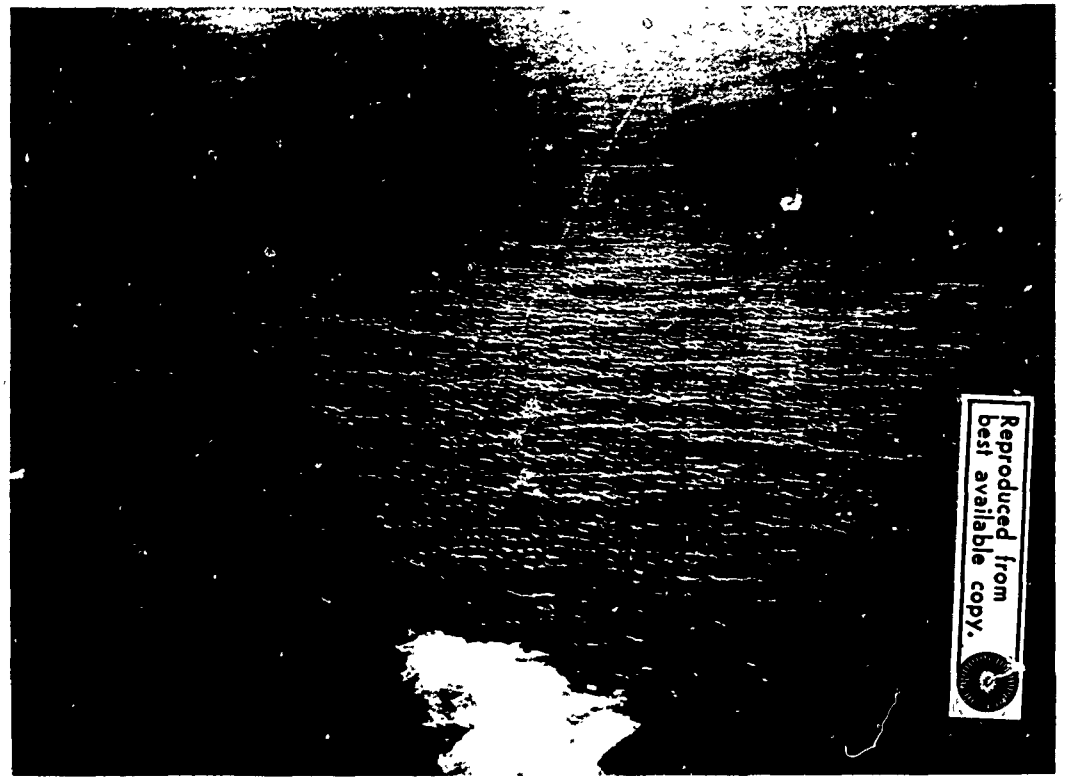

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FIGURE 1.2 Aerial photo of swell waves.

Properties of Swell

The important properties of swell are shown in figure 1.2. Some low chop is also present, but the underlying swell can be readily seen. The swell waves are low with rounded tops. The wave crests can be followed by eye for a considerable distance, at least six or seven times the distance between crests. The waves following each other are nearly of the same height. It is possible to follow a series of crests all the way across the picture.

Waves Recorded at a Point

A wave-recording instrument exists which can be put overboard and which will record the heights of waves as they pass the instrument. The instrument is a long spar which is weighted on the bottom and has a large flat disk around its bottom to keep it from bobbing up and down. As the waves go by, it records the height of the water against the spar and makes a graph on a piece of paper. For sea waves, the records look like those shown in figure 1.3. For swell waves, the records look like those shown in figure 1.4.

Sea Wave Records

Figure 1.3 shows some sea wave records. The waves are quite high at some places but quite low at other places in the same record. Look at the uppermost record very carefully. First there is 1 medium wave followed by a flat-topped one. Then another medium one comes along. The fourth wave has 2 small waves superimposed on it. The fifth is low. The sixth is medium. The seventh is something like the second. Then there are 3 in a row, each a little lower than the

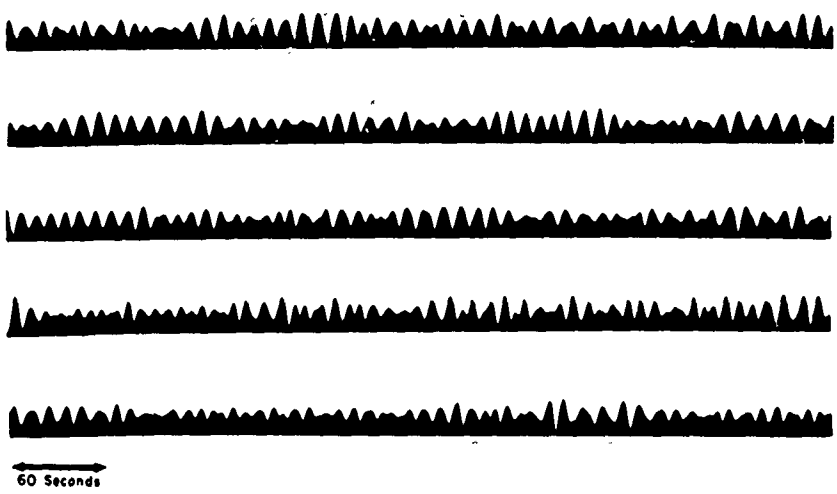

FIGURE 1.3 Sea wave records.

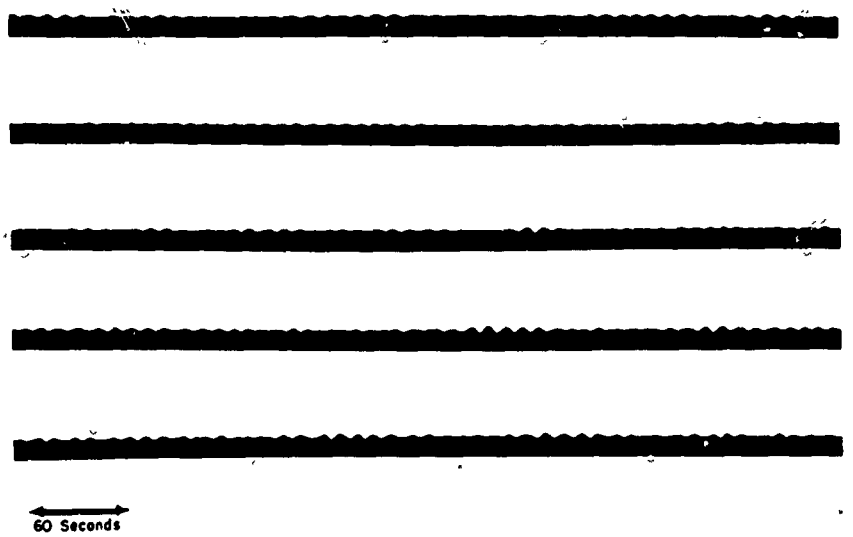

FIGURE 1.4 Swell wave records.

one before. They are followed by 2 very small waves. There are then 3 in a row each higher than the one before, followed by 4 medium high waves. Then come 3 big ones all of nearly the same height. The record illustrates the point to be made: sea waves are very, very irregular.

High waves follow low waves in a completely random and mixed-up way. In a sea it is very difficult to say whether a wave passing a point will be higher or lower than the one which has just passed. The third wave in sequence is completely unpredictable. Suppose that on the average it takes 15 seconds for a wave to pass. Then, in a sea, it is not possible to tell how high the wave will be which will pass in 45 seconds.

Under certain conditions experienced navigators sometimes can estimate approximately the behavior of the storm sea for a little time ahead, and they use such experience in carrying out ship maneuvers. On a ship under way, a navigator is not limited to the wave which is right *at* the ship. Just knowing what goes on at one point does not give much information, as is shown by figure 1.3. A navigator on the bridge can look out ahead of the ship and see a high wave approaching from a considerable distance; hence, he knows both what is happening at the ship and all around the ship. Thus, in the old days, a ship's captain well knew when to take the best chance in coming about with a sailing vessel in a heavy "sea." The captain would look out toward approaching waves, and if no high waves could be seen in the distance the turn would be made. If high waves could be seen, he would hold his course and make the turn at a later, more favorable time.

Swell Wave Records

Figure 1.4 shows some swell records. Many actual wave conditions are intermediate between figures 1.3 and 1.4, but these records are representative of swell conditions. Look at the top record of figure 1.4. The waves are lower than those appearing in figure 1.3 (sea conditions). First, there is a group of 5 waves followed by a practically flat trace over a time interval of 20 seconds. Again there are 5 waves in a row, with the highest wave in the middle, followed by another flat trace. Then there are 8 waves in a row, all about the same height. Other parts of the various records show groups of 5, 6, and 7 waves in a row, all smoothly varying in height. The swell record is therefore much more regular. If it were predicted that the next wave would be nearly the same height as the one which had just passed, the prediction would be very nearly correct every time. If the waves are increasing in height, a forecast that the next wave will be a little higher than the one just before will usually be right. If the waves are decreasing in height, a forecast that the next wave will be a little lower than the one before will usually be right.

In another sense, swell is also unpredictable. It is never possible to tell whether there will be 4, 5, 6, or more groups of high waves in a group of waves. There is no way to forecast, for example, the exact height of the wave which will occur five minutes in the future.

A Basic Difference Between Sea and Swell

A basic difference between sea and swell is the difference in the short-range predictability of the waves. Sea is chaotic, irregular, and unpredictable. It is very difficult to guess what will happen next. Swell is predictable in a short-range sense. What will happen in the very near future depends to a certain extent on what has just happened.

Wave Amplitudes and Heights

Forecast Depends on One Number

There is one number that permits a forecast of some of the information needed about wave amplitudes and heights. It can be found by taking any wave record such as those in figures 1.3 and 1.4 and making a simple computation. The number will be called E, and it has the dimensions of ft.^{2} To compute E, the value of the wave record is read off at a hundred or so equally spaced points with any arbitrary zero reference level. An average value is computed from the points read off, and this average value is subtracted from each of the original readings. Each resultant number is squared, and an average of the sum of the squares is found. The result, multiplied by two, is the number E. ONE DEFINITION OF E, THEN, IS THAT IT

EQUALS TWICE THE VARIANCE OF A LARGE NUMBER OF VALUES FROM POINTS EQUALLY SPACED IN TIME AS CHOSEN FROM A WAVE RECORD. Methods will be given which will make it possible to forecast this number, E, for sea and swell conditions.

Illustrative Analogy

When a pair of dice is thrown, there is 1 chance in 36 of getting 2 ones. There is 1 chance in 36 of getting 2 sixes. There are 2 chances in 36 of getting a total of 3, and 2 chances in 36 of getting a total of 11. There are odds for getting any total from 2 to 12. Suppose that a pair of dice is thrown 360 times. Then, about 10 of the throws should be 2 ones; 20 throws should total 3; 60 throws on the average should total 7, and so on.

Wave Amplitude Data

The same thing, in a way, is true of ocean waves as they pass a fixed point of observation, or as they encounter a ship under way. For any given E value, there are certain odds that a wave of a certain amplitude (as measured from a crest to mean sea level or from mean sea level to a trough) will be observed; and for a large number of waves a certain number on the average will have a certain amplitude.

Once the value of E is known, tables 1.1, 1.2, and 1.3 give the properties of the waves which can be forecast. If one takes wave records such as those in figures 1.3 and 1.4 and measures the amplitude (crest to sea level or trough to sea level) and then writes down the amplitudes of *all* of the waves in the record, and not just the highest, it will be found that ten percent of the waves have amplitudes equal to or less than $0.32\sqrt{E}$. Ten percent will have amplitudes greater than $1.52\sqrt{E}$. Ten percent will have amplitudes between $0.83\sqrt{E}$ and $0.96\sqrt{E}$. The average amplitude of all the waves will be $0.886\sqrt{E}$.

Exceptionally High Amplitudes

Table 1.3 gives information on the exceptionally high amplitudes that can occur. This table is not as useful as table 1.6 to be described later, because it is difficult to measure wave amplitudes. However, it is useful in a study of the motion of ships, and a way to use table 1.3 in predicting ship motions is described in Chapter VII (page 228).

Heights and Amplitudes

The wave records shown in figures 1.3 and 1.4 are not simple sine waves. Consequences of this fact will be pointed out later. If waves were simple sine waves, then twice the amplitudes would equal the height from crest to trough of all of the waves. The values given for wave amplitudes in tables 1.1, 1.2, and 1.3 are very nearly exact for the amplitudes in any wave condition, and it might seem that

Table 1.1—Wave Amplitude Data

TEN PERCENT RANGES (TEN PERCENT OF THE WAVES WILL HAVE AMPLITUDES BETWEEN THE GIVEN RANGE OF VALUES):

10%	between	0	and	$0.32\sqrt{E}$ ft.
10%	"	$0.32\sqrt{E}$	"	$0.47\sqrt{E}$ ft.
10%	"	$0.47\sqrt{E}$	"	$0.60\sqrt{E}$ ft.
10%	"	$0.60\sqrt{E}$	"	$0.71\sqrt{E}$ ft.
10%	"	$0.71\sqrt{E}$	"	$0.83\sqrt{E}$ ft.
10%	"	$0.83\sqrt{E}$	"	$0.96\sqrt{E}$ ft.
10%	"	$0.96\sqrt{E}$	"	$1.10\sqrt{E}$ ft.
10%	"	$1.10\sqrt{E}$	"	$1.27\sqrt{E}$ ft.
10%	"	$1.27\sqrt{E}$	"	$1.52\sqrt{E}$ ft.
10%	greater than	$1.52\sqrt{E}$		

CUMULATIVE ASCENDING 10% VALUES:

10%	less than	$0.32\sqrt{E}$	ft.
20%	" "	$0.47\sqrt{E}$	ft.
30%	" "	$0.60\sqrt{E}$	ft.
40%	" "	$0.71\sqrt{E}$	ft.
50%	" "	$0.83\sqrt{E}$	ft.
60%	" "	$0.96\sqrt{E}$	ft.
70%	" "	$1.10\sqrt{E}$	ft.
80%	" "	$1.27\sqrt{E}$	ft.
90%	" "	$1.52\sqrt{E}$	ft.

CUMULATIVE DESCENDING 10% VALUES:

10%	greater than	$1.52\sqrt{E}$	ft.
20%	" "	$1.27\sqrt{E}$	ft.
30%	" "	$1.10\sqrt{E}$	ft.
40%	" "	$0.96\sqrt{E}$	ft.
50%	" "	$0.83\sqrt{E}$	ft.
60%	" "	$0.71\sqrt{E}$	ft.
70%	" "	$0.60\sqrt{E}$	ft.
80%	" "	$0.47\sqrt{E}$	ft.
90%	" "	$0.32\sqrt{E}$	ft.
100%	" "	$0.00\sqrt{E}$	ft.

(after S. O. Rice and M. S. Longuet-Higgins)

just doubling the values would give the distribution of the crest-to-trough heights. This is not strictly true, however. For swell waves such an approximation is nearly correct, but for sea waves it is not exactly right. If the wave records shown on page 4 are studied, it can be seen that the vertical distance from mean sea level to a given trough is not equal to the vertical distance from mean sea level to either

Table 1.2—Average Wave Amplitude Data

The Most Frequent Wave Will Have an Amplitude of

$$0.707\sqrt{E}\text{ ft.}$$

The Average Amplitude of all the Waves is

$$0.886\sqrt{E}\text{ ft.}$$

The Average Amplitude of the ⅓ Highest Waves is

$$1.416\sqrt{E}\text{ ft.}$$

The Average Amplitude of the $\frac{1}{10}$ Highest Waves is

$$1.800\sqrt{E}\text{ ft.}$$

(after Longuet-Higgins)

Table 1.3—Greatest Wave Amplitude Data

(see explanation, table 1.6)

N	*5% lower*	*most frequent*	*average*	*5% higher*
20	$1.40\sqrt{E}$	$1.73\sqrt{E}$	$1.87\sqrt{E}$	$2.44\sqrt{E}$
50	$1.69\sqrt{E}$	$1.98\sqrt{E}$	$2.12\sqrt{E}$	$2.62\sqrt{E}$
100	$1.88\sqrt{E}$	$2.15\sqrt{E}$	$2.28\sqrt{E}$	$2.75\sqrt{E}$
200	$2.05\sqrt{E}$	$2.30\sqrt{E}$	$2.43\sqrt{E}$	$2.87\sqrt{E}$
500	$2.26\sqrt{E}$	$2.50\sqrt{E}$	$2.60\sqrt{E}$	$3.03\sqrt{E}$
1000	$2.41\sqrt{E}$	$2.61\sqrt{E}$	$2.73\sqrt{E}$	$3.14\sqrt{E}$

(after Longuet-Higgins)

adjacent crest. When the distances are nearly equal, then doubling the values in tables 1.1, 1.2, and 1.3 yields nearly correct values for the heights which can occur. When the waves are very irregular, this procedure is not exactly correct.

Unfortunately, doubling the values given in tables 1.1, 1.2, and 1.3 is the best theoretical approach that can be made at the present time. The problem of the exact height distribution is a very difficult one. Tables 1.4, 1.5, and 1.6 give the approximate crest-to-trough height ranges that can be expected to occur in a long series of height observations.

THE HEIGHT OBSERVATIONS AND THE VERIFICATION OF HEIGHT FORECASTS MUST BE MADE ON THE BASIS OF PROCEDURE B AS DESCRIBED IN CHAPTER IV. The crest-to-trough heights of the waves must be estimated at a fixed point on the sea surface or with respect to a point moving along a straight line on the sea surface. Low waves must be counted as well as high waves.

Exceptionally High Waves

In any wave system, after a long enough time, an exceptionally high wave will occur. These monstrous outsized waves are improb-

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Table 1.4—Approximate Wave Height Data

TEN PERCENT RANGES (TEN PERCENT OF THE WAVES WILL HAVE CREST-TO-TROUGH HEIGHTS BETWEEN THE GIVEN RANGE OF VALUES):

10%	between	0.00	and	$0.64\sqrt{E}$ ft.
10%	"	$0.64\sqrt{E}$	"	$0.94\sqrt{E}$ ft.
10%	"	$0.94\sqrt{E}$	"	$1.20\sqrt{E}$ ft.
10%	"	$1.20\sqrt{E}$	"	$1.42\sqrt{E}$ ft.
10%	"	$1.42\sqrt{E}$	"	$1.66\sqrt{E}$ ft.
10%	"	$1.66\sqrt{E}$	"	$1.92\sqrt{E}$ ft.
10%	"	$1.92\sqrt{E}$	"	$2.20\sqrt{E}$ ft.
10%	"	$2.20\sqrt{E}$	"	$2.54\sqrt{E}$ ft.
10%	"	$2.54\sqrt{E}$	"	$3.04\sqrt{E}$ ft.
10%	greater than	$3.04\sqrt{E}$		

CUMULATIVE ASCENDING 10% VALUES:

10%	less than		$0.64\sqrt{E}$ ft.
20%	"	"	$0.94\sqrt{E}$ ft.
30%	"	"	$1.20\sqrt{E}$ ft.
40%	"	"	$1.42\sqrt{E}$ ft.
50%	"	"	$1.66\sqrt{E}$ ft.
60%	"	"	$1.92\sqrt{E}$ ft.
70%	"	"	$2.20\sqrt{E}$ ft.
80%	"	"	$2.54\sqrt{E}$ ft.
90%	"	"	$3.04\sqrt{E}$ ft.

CUMULATIVE DESCENDING 10% VALUES:

10%	greater than		$3.04\sqrt{E}$ ft.
20%	"	"	$2.54\sqrt{E}$ ft.
30%	"	"	$2.20\sqrt{E}$ ft.
40%	"	"	$1.92\sqrt{E}$ ft.
50%	"	"	$1.66\sqrt{E}$ ft.
60%	"	"	$1.42\sqrt{E}$ ft.
70%	"	"	$1.20\sqrt{E}$ ft.
80%	"	"	$0.94\sqrt{E}$ ft.
90%	"	"	$0.64\sqrt{E}$ ft.
100%	"	"	$0.00\sqrt{E}$ ft.

(after S. O. Rice and M. S. Longuet-Higgins)

Table 1.5—Average Wave Height Data

THE MOST FREQUENT WAVE HEIGHT WILL BE	$1.41\sqrt{E}$
AVERAGE HEIGHT	$1.77\sqrt{E}$
"SIGNIFICANT" HEIGHT: AVERAGE OF THE HEIGHTS OF THE ⅓ HIGHEST WAVES	$2.83\sqrt{E}$
AVERAGE OF THE HEIGHTS OF THE 1/10 HIGHEST WAVES	$3.60\sqrt{E}$

(after Longuet-Higgins)

able but still possible; hence they do happen. They can happen at any time, and the exact time of occurrence of such an outsized wave can never be predicted.

Something can be said about the probability that the highest wave out of a total of N waves will have a height within a certain range. The probable height of the highest wave out of N waves that passes a given point depends on the size of N and the value of E.

To see how this might be determined, consider the dice-throwing problem again. It is possible, but not very probable, to throw five dice and have five ones come up. The chance of doing it is one in 7,776. That is, if five dice are thrown 7,776 times, there ought to be one time when five aces come up. It could happen twice, more than twice, or not at all. But on the average it will happen once.

Consider a very long wave record with N waves in it, say, 100, 200, or 1,000 waves. Let the value of E be constant for the entire record. Pick out the highest wave height of all the N wave heights. Repeat the experiment for a large number of wave records containing N waves each and then average the values of the highest wave height out of each record of N waves. The result is the average value of the highest wave out of N waves from the total of, say, M individual records with N waves in each record.

Also it would be possible to consider the M different values which result from such an analysis. They would not all be equal for a given set of values from M records containing N waves each. In one wave record the highest wave might have a height of 4 feet in another the wave might have a height of 5 feet, and in still a third the highest wave might be only 2 feet, depending on which particular record was studied. Thus, the height of the highest wave out of N waves is a random number just as the observed heights of individual waves are random numbers.

However, the properties of the distribution of this highest wave height out of N waves has been derived, and certain things can be said about it. Table 1.6 summarizes the data as follows:

Table 1.6—Exceptionally High Waves

During conditions where E is constant, suppose that a total of N wave heights is observed, and that the height of the highest wave of N waves is recorded. Suppose that the observations are repeated M times, so that M different values of the highest wave of N waves are tabulated. Then

1) 5 percent of the M values will be less than the value given in the first column of the table,
2) the most frequent value of the M values will be near the value given in the second column of the table,
3) the average value of the M values will be given by the third column of the table,
4) 5 percent of the M values will be greater than the value given in the last column of the table,
5) 9 times out of 10 a particular value of the highest wave out of N waves will be between the values given in the first and last columns of the table.

Height in Terms of $\sqrt{E}$

N	5% lower	most frequent	average	5% greater
20	$2.81\sqrt{E}$	$3.46\sqrt{E}$	$3.74\sqrt{E}$	$4.89\sqrt{E}$
50	$3.37\sqrt{E}$	$3.96\sqrt{E}$	$4.24\sqrt{E}$	$5.25\sqrt{E}$
100	$3.75\sqrt{E}$	$4.29\sqrt{E}$	$4.56\sqrt{E}$	$5.50\sqrt{E}$
200	$4.11\sqrt{E}$	$4.60\sqrt{E}$	$4.86\sqrt{E}$	$5.75\sqrt{E}$
500	$4.53\sqrt{E}$	$4.99\sqrt{E}$	$5.20\sqrt{E}$	$6.06\sqrt{E}$
1000	$4.82\sqrt{E}$	$5.26\sqrt{E}$	$5.46\sqrt{E}$	$6.28\sqrt{E}$

Exceptionally High Waves in Terms of Significant Heights

(See explanation above, $\overline{H}_{1/3}$ is the significant height)

N	5% lower	most frequent	average	5% greater
20	$.99\overline{H}_{1/3}$	$1.22\overline{H}_{1/3}$	$1.32\overline{H}_{1/3}$	$1.73\overline{H}_{1/3}$
50	$1.19\overline{H}_{1/3}$	$1.40\overline{H}_{1/3}$	$1.50\overline{H}_{1/3}$	$1.86\overline{H}_{1/3}$
100	$1.33\overline{H}_{1/3}$	$1.52\overline{H}_{1/3}$	$1.61\overline{H}_{1/3}$	$1.94\overline{H}_{1/3}$
200	$1.45\overline{H}_{1/3}$	$1.63\overline{H}_{1/3}$	$1.72\overline{H}_{1/3}$	$2.03\overline{H}_{1/3}$
500	$1.60\overline{H}_{1/3}$	$1.76\overline{H}_{1/3}$	$1.84\overline{H}_{1/3}$	$2.14\overline{H}_{1/3}$
1000	$1.70\overline{H}_{1/3}$	$1.86\overline{H}_{1/3}$	$1.93\overline{H}_{1/3}$	$2.22\overline{H}_{1/3}$

(after Longuet-Higgins)

Table 1.6 summarized the predictable characteristics of the highest wave of N waves. For example, if 20 waves are observed to pass a

given point of observation, then 9 out of 10 times the highest wave will have a height between $2.81\sqrt{E}$ and $4.89\sqrt{E}$. One time in 20 it will be lower than the first value, and 1 time in 20 it will be higher than the second value. Most of the time a height nearly equal to $3.46\sqrt{E}$ will be observed; if the observation of the highest wave in 20 waves is repeated 50 or 60 times during conditions such that E is constant, the average value of the heights of the highest wave in 20 waves will be $3.46\sqrt{E}$.

It is not possible to tell exactly what the height will be. It is not even possible to tell when this high wave will occur, but it is important to know that a wave with a height within the range indicated will occur 9 out of 10 times during the next 20 waves.

In terms of significant height as given in the second part of the table, the entries for an N of 20 show that one time in 20, for example, the highest wave of 20 waves will be lower than the significant height and that one time in 20 the highest wave of 20 waves will be 1.73 times the significant height. This shows that a few observations are extremely unreliable in determining the true significant height of the waves. This important fact will be shown in another way in Chapter IV.

As a numerical example, suppose that 1,000 waves are expected to pass during conditions where E is a constant, equal to 100 units. Then from table 1.6, there is one chance in 20 that the highest waves of these 1,000 waves will be less than 48 feet in height. If less than 48 feet in height, incidentally, these waves would probably not be much less than 48 feet. There are 9 chances in 10 that the highest wave of these 1,000 waves will be between 48 and 63 feet in height. There is one chance in 20 that this highest wave will be greater than 63 feet in height. Most probably the highest wave height will be near 53 feet. Finally, if many observations of 1,000 waves were made for the same value of E, the average of these highest waves would be 55 feet. The difference between the values of 48, 53, and 55 feet is insignificant in practice, because wave heights cannot be determined visually on the open ocean with sufficient accuracy to distinguish between these values. However, the fact that 1 time in 20 the highest wave in 1,000 can exceed 63 feet is important because there is always the possibility that the highest wave can be still higher than the value suggested by the table.

The explanation of the table is, of course, quite logical. If a sequence of 1,000 wave-height values is considered, then the last row of the table must be used, for somewhere in these 1,000 observations there could be a wave $5.26\sqrt{E}$ units high. When these 1,000 values are separated into sets of 20 waves each, one of these sets of 20 height observations must contain this high wave. When the values in the

table for $N=20$ are applied, it is seen that this high wave in the series of 1,000 is one of the examples for the series of 20 heights where the height of the wave exceeds the range in which the highest wave can be expected to be found 9 times out of 10.

Limitations on Table 1.6

The exceptionally high waves recorded as rare occurrences in table 1.6 are actually observed; they are often reported by seafaring men. There has been much speculation as to how and why they form. These high waves form because they are a basic property of the randomness of the waves.

These waves occur both in sea and swell conditions. In a swell with a significant wave 2.83 feet high, little notice is paid to a wave 5.3 feet high. (E equals 1; N equals 1,000.) In a sea with a significant wave 28.3 feet high, everyone notices a wave 53 feet high! (E equals 100; N equals 1,000.) Thus, usually these outsized waves are reported only in heavy seas.

In heavy seas, or for fully developed seas, these outsized waves are very unstable. They may break at the crests and produce a wall of plunging white water out in the middle of the open ocean. These outsized waves, then, are destroyed by their very height. Thus, in a heavy sea they may be rarer than predicted by the above tables. The experience of the forecaster can be used to modify these tables for conditions in heavy seas.

Significant Wave Height

The significant wave height is defined as the average value of the heights of the one-third highest waves in a given observation. From wave records such as figures 1.3 or 1.4, tabulate the heights of all the waves in the particular record. Then count the number of waves and divide by three. Pick out the highest wave, the next highest, and so on until one-third of the total number has been selected. Then average these heights, and the result is the significant wave height.

Table 1.7 illustrates the procedure employed in computing the significant height. The data were taken from a wave record made at Long Branch, New Jersey, by the Beach Erosion Board. The average wave height and the average height of the one-tenth highest waves are also given in table 1.7.

Since the average height is 3.0 feet and since $3.0=1.77\sqrt{E}$, the value of $\sqrt{E}$ is 1.69. From table 1.5, the significant height is 2.83 times 1.7, or 4.8 feet; and the average of the one-tenth highest waves is 3.60 times 1.7, or 6.1 feet.

The value of the significant height found from table 1.7 is 4.5 feet. In terms of the average height and table 1.5, it is 4.8 feet. The dif-

Table 1.7—Sample Computation of Wave-Height Characteristics

Wave height in feet	*Number of waves*	*Cumulative number*	*Average height*	*Significant height*	*Average 1/10 highest*
1	7	102	7		
2	33	95	66		
3	30	62	90	6(3x2)	
4	20	32	80	80	
5	7	12	35	35	25(5x5)
6	3	5	18	18	18
7	1	2	7	7	7
8	1	1	8	8	8
Total			311	154	58

Average height=311/102=3.0 feet
Significant height=154/34 =4.5 feet
Average 1/10 highest= 58/10 =5.8 feet

ference is 0.3 feet. For the one-tenth highest, the difference is also 0.3 feet. The agreement between the two ways of computing the values is therefore close.

An interesting experiment can be carried out with table 1.7. Let the number of waves 1 foot high be reduced to 5, and increase the number 8 feet high to 3. The average height is then 3.2 feet, the significant height 5.0 feet, and one-tenth highest height 7.4 feet. A slight change in the data produces a slight change in the average height and much bigger changes in the other two values. Thus, the average height is a more reliable number, and it is not so dependent on the randomness of the waves when only a few heights are observed.

From table 1.4, 41 out of 102 waves should be less than 2.40 feet high. Table 1.7 shows that there are 40 waves 2 feet high or lower. There should be 20 waves higher than 4.3 feet. There are actually 12 waves 5 feet high or higher. The discrepancies can be explained by the fact that the wave heights are reported only to the nearest foot. Of the 20 waves reported to be 4 feet high, eight might easily be between 4.2 and 4.5 feet high, and they would be tabulated as if they were 4 feet high by rounding off the values. From table 1.6, 9 times out of 10 for 100 waves the highest wave will have a height between 6.3 and 9.3 feet. There is one wave 8 feet high.

From this example, it can be seen that there is nothing particularly significant about the significant height. It is a value which is convenient to work with at times, but the particular height which results is no more important than any other statistical value from the tables.

Significant Wave Heights as Reported by Ships

Frequently ships report that waves being observed from a ship have a certain height. The procedure is often for some observer to look out

over the sea surface and make a quick guess as to the wave height. The value reported is *assumed* to be the significant height. The significant height is *supposed* to be the predominant (basic or characteristic) height of the waves. The observer certainly does not record the heights of even 100 waves and does not pick out the 33 highest to average, so it is interesting to speculate on how he is supposed to know that the value he gives is actually the significant height.

Such a procedure is highly subjective. One observer on a small ship may think that waves 10 feet high in a given sea are important. He reports some guess as to the wave height. Another observer on a larger ship may not be impressed by any wave less than 15 feet high. His guess would be quite different from the guess of the first observer in the same wave conditions.

Accurate wave forecasts cannot be verified or made on the basis of what someone guesses the significant wave height to be. In Chapter IV, techniques for observing wave heights correctly will be given. If forecasts are made by the methods of the manual and verified by an inadequate observation at the point at which the forecast is made, the forecaster can expect discrepancies of the order of 30 percent, and yet his forecast may still be correct.

A Height Forecast: Example 1.1

E is forecast to be 25 ft.[2] Then some of the various forecasts that can be made are given in table 1.8.

Table 1.8—A Height Forecast Example

	Amplitude crest (or trough) to sea level feet	*Height crest to trough feet*
Average	4.43	8.85.
Significant	7.08	14.15
Average 1/10 highest	9.00	18.00.
10% between	0 and 1.60	0 and 3.20.
	1.60 and 2.35 (etc,)	3.20 and 4.70 (etc.)
	6.35 and 7.6	12.7 and 15.2.
10% higher than	7.6	15.2.
Half of the waves will exceed	4.15	8.30.
One wave out of 1,000 may be between	12.0 and 15.7	24.1 and 31.4 (may be breaking)

Comments on Height Forecasting

The amount of detail to be incorporated in a forecast of wave heights depends very much on the operational use of the forecast. This fact should be kept in mind at all times, and the user of the forecast should know of the possibilities that can influence his action. Some hypothetical examples of the use to which the forecast can be put may be given.

Example 1.2.—A seaplane is dispatched to land near a ship on the North Atlantic, pick up a passenger, and rush him back to a hospital. The forecast is for a moderate sea with a significant wave height from crest to trough of 6 feet. The forecast is correct. The average of the one-third highest waves is 6 feet. The pilot lands his plane in an area where the waves were relatively low, but before the plane can slow down, a steep wave 10 feet high rises up suddenly in front of it and staves in the hull. The plane sinks; and the pilot and crew are rescued by the ship they were trying to aid.

This hypothetical example illustrates a forecast which was correct before the extreme-variability of wave height was thought to be important. No one would have thought to forecast that 1 wave out of 20 could be *ten* feet high for the same conditions or that in a sea a wave of extra great height could form before one's eyes in a few seconds.

In a swell for the same significant height the landing would be much safer, because, had the pilot found an area of low waves, the waves would have remained low for the time required for the landing. Further comments on landing seaplanes will be given in Chapter VII. The present example emphasizes the importance of height variability.

Example 1.3.—Two ships are transferring fuel in the Pacific. One is a fleet auxiliary vessel, the other a destroyer escort with a very low freeboard. The significant height is forecast to be 15 feet, and the operation is proceeding satisfactorily. Two crewmen are working with the fuel hoses near the ship's rail, when 2 waves 24 feet high approach one right after the other. Green water comes over the deck suddenly, and one man is thrown against the rail. His arm is broken and he is almost lost overboard. The two men should have been told that although *most* of the time conditions would be safe, the decks would be awash from about one wave out of 100 during the next hour.

This is another example of a correct forecast which does not forecast wave variability. It is possible to think of many other situations where such wave variability considerations would be of extreme importance.

Wave Periods and Wave Lengths

"Period" vs. *period*

The figures which have been shown of wave records taken as a function of time at a fixed point show that the waves are very irregular. Compare these figures with figure 1.5 which shows the graph of a simple sine wave. This simple sine wave can be produced in a laboratory by moving a large paddle back and forth with a sinusoidal motion at the end of a long tank of water. The record was taken by

FIGURE 1.5 A simple sine wave.

placing a recorder in the water and allowing the passing waves to draw a graph on a sheet of paper. Figure 1.5 is quite different from figures 1.3 and 1.4. In earlier studies of ocean waves an assumption was made which replaced the irregularity of figures 1.3 and 1.4 by the regularity of figure 1.5. This resulted in a very serious mistake because the formulas which describe waves like those in figure 1.5 are completely different from the formulas which describe waves like those in figure 1.3 or figure 1.4. The sea surface is actually like figures 1.3 and 1.4, and therefore it cannot be represented by the formulas which apply to figure 1.5.

The reason this was done previously was partly because of the word "period." This word has two different meanings, and when it is used there is often a great deal of confusion as to which meaning applies. Mathematicians use the word to mean one very precise thing, whereas most people use the word in a much less precise sense. It is necessary, consequently, to study what this word actually means and to be very precise in its use in this manual.

Figure 1.5 is a portion of a wave record which could be periodic in the exact definition of the word. It has a *period*. The definition of the word *period* is given below.

DEFINITION: A *PERIOD* IS A TIME INTERVAL, T, IN WHICH A WAVE RECORD REPEATS ITSELF EXACTLY; THAT IS, IF $F(t+T)=F(t)$ FOR ALL t, AND T IS THE SMALLEST NUMBER GREATER THAN ZERO FOR WHICH SUCH A RELATION HOLDS, THEN T IS THE *PERIOD* OF THE WAVE RECORD.

As an example, consider equation (1.1). It states that the cosine of $t+T$ is exactly the same at all times as the cosine of t, if T is the *period* of the function. The cosine repeats itself exactly every T seconds.

$$\begin{aligned} &\cos[2\pi(t+T)/T] \\ &=\cos 2\pi t/T \cos 2\pi - \sin 2\pi t/T \sin 2\pi \\ &=\cos 2\pi t/T \end{aligned} \tag{1.1}$$

Another way to see what this exact definition of the word *period* means is to notice that if someone had begun to graph figure 1.5 at a time T seconds later than it was actually graphed, it would have been impossible to tell the difference between the resulting graph and the graph shown in figure 1.5. A truly *periodic* function repeats itself

exactly. Every crest has exactly the same height as all the others. This statement, as we have seen, is not true of actual ocean waves.

Usually when the word, *period* is used, the exact definition as given above is not meant. "Period" has a popular definition as given below.

DEFINITION: A "PERIOD" IS "THE TIME OF ONE COMPLETE CYCLE OF A VIBRATION, CURRENT, ETC."

Then, if the word "cycle" needs to be defined, it is defined as the "PERIOD OF TIME OF A COMPLETE PROCESS OF GROWTH OR ACTION THAT REPEATS ITSELF IN THE SAME ORDER." These popular definitions are from a well-known dictionary, and they show that it is difficult to define the word "period" in the popular sense, since "period" is defined in terms of "cycle" and "cycle" is defined in terms of the word "period."

Measurement of "Periods" [1]

The definitions given above for the words "period" and "cycle" can be made more precise by considering figure 1.6. It shows a portion of a wave record which could easily have come from one of the wave records shown before. This portion is enlarged in order to show the details. It is a graph as a function of time, and the time scale is shown below the graph. Now, according to the popular definition of the word "period," it is necessary to find the time of one complete cycle of the record. The cycle begins, usually, at an easily recognizable part of the record such as, possibly, the time when the record goes through a maximum. The first maximum of this record occurs a few seconds after the beginning of the record, and a little mark is shown at the maximum. Then the graph goes down through zero, through a trough, and goes back up again to another maximum. This then, is a "cycle," and the "period" is the time interval between the first maximum and the second.

This record, in this sense of the word, has many different "periods." The next time interval from the second maximum of the record to the

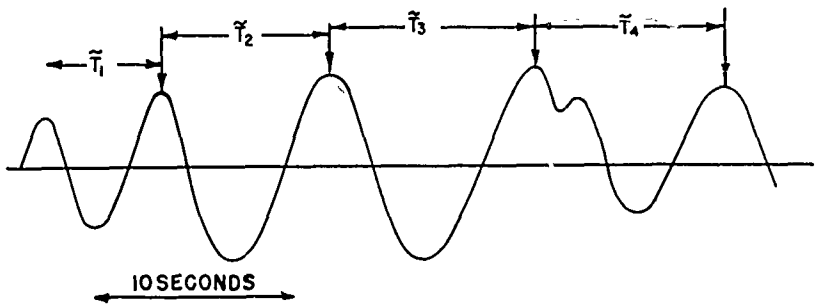

FIGURE 1.6 The "periods" in a wave record.

[1] For actual techniques, see Chapter IV.

third maximum of the record is not equal to the first one, and the third time interval is not necessarily equal to either of the other two. It would be possible to write down a complete list of all of the "periods" in this segment of the record. A little difficulty occurs at the last part of the record where there are two maximums in the record without a corresponding minimum that goes beneath the zero line of the record. By definition for the purposes of this manual, the time intervals measured with respect to the higher of the two maximums will be defined to be the "periods" of the waves shown.

The result of the analysis of an actual ocean wave record is then a whole list of different "periods." The time intervals between successive crests can vary considerably from wave to wave. They can range from 5 seconds to 10 or 11 seconds in a given state of the sea, and certain values occur more often than others. There certainly is not one number, T, which describes the wave record completely.

If every "period" is observed in a wave record, or as the waves pass a fixed point, then it is possible to define a new term; namely, the average "period." THE AVERAGE "PERIOD" IS THE AVERAGE OF ALL THE VALUES OF THE INDIVIDUAL "PERIODS" IN A GIVEN OBSERVATION. For example, the various values indicated in figure 1.6 would be measured, added up, and divided by the total number observed. The result would be the average "period." This is a useful number to know in order to discuss the average time it takes the wave to pass a fixed point of observation. This average "period" cannot be used in the exact sense of the word *period*. For example, if the average "period" is 10 seconds and if a wave crest is passing at a certain time (say, 21 minutes and 20 seconds after 0500Z) then it would not be possible to conclude on the basis of the average "period" that in exactly 60 seconds the crest of the sixth wave would follow. Since the waves are not truly *periodic*, this conclusion cannot be made.

Definition of Terms

Two meanings of the word "period" have been used in the above discussion. At times in this manual, it will be necessary to use the word in the exact sense. When the first definition is meant and when the exact sense is meant, the word, *period*, will be italicized as it has been above. On the other hand when the popular definition is meant the word will have quotation marks around it. The true *period* of the waves cannot be measured, but the average "period" can be measured and it has a meaning. The true *periods* are a property of the spectrum of the waves, and when this spectrum is discussed the word *period* will be used in order to describe the waves.

Wave Length vs "Wave Length" [2]

The wave length of a simple sine wave is also *periodic*, in that as a function of distance in a direction perpendicular to the crests the graph of the waves is another sine wave. For such sine waves, then, the *wave length* is similar in meaning to the *period*. Also, in the actual irregular sea surface, as figures 1.1 and 1.2 show, the "wave length" along a line perpendicular to the crests varies in a way similar to the "period." Therefore, in the same sense that the word means the time between successive maximums of a wave record, the word "wave length" will mean the distance between two successive crests on the sea surface.

Symbols To Be Used

In order to point out the difference between the various terms mentioned above, whenever the word *period* is meant in the exact sense a T will be used. Whenever the *wave length* is meant in the exact sense an L will be used. The symbol $\widetilde{T}$ will be used whenever the word "period" is meant. The symbol $\widetilde{\widetilde{T}}$ will be used to represent the average "period." The symbol $\widetilde{L}$ will be used whenever the word "wave length" is meant. The symbol $\widetilde{\widetilde{L}}$ will be used to designate the average "wave length."

Definition of Frequency

THE FREQUENCY (f) OF A SINUSOIDAL WAVE IS THE RECIPROCAL OF THE PERIOD ($f=1/T$). For example, a wave with a *period* of 10 seconds has a frequency of one-tenth cycle per second. Stated another way, one-tenth of this complete sine wave passes a given point in one second. In defining the wave spectrum and forecasting swell, the wave frequency is more useful than the wave *period*.

Frequencies are used in connection with electromagnetic waves more than with ocean surface waves, but frequencies and *periods* are interchangeable terms. For example, a 60-cycle alternating current has a *period* of $1/60$ second.

Some New Formulas

Now that the above definitions have been given it is possible to make a very important point. For a simple sine wave which is truly *periodic*, it is true that equation (1.2) holds. In equation (1.2), L is measured in feet, and T is measured in seconds. Equation (1.2) comes from classical theory, and if the wave is a simple harmonic progressive wave the formula holds. The formula can be verified and

[2] For techniques of measuring "wave lengths," see Chapter IV.

measured exactly in wave tanks and in places where waves can be generated artificially.

$$L=5.12T^2 \tag{1.2}$$

However, in terms of the popular definition of the word it is not true that equation (1.3) holds. In terms of the popular definitions, equation (1.3), if it actually was true, would have to mean that the average "wave length" as shown in pictures such as figure 1.1 would have to be equal to 5.12 times the square of the average "period" in a wave record such as figure 1.3 (with the assumption that both are observed at the same time and place). There is no reason why this particular formula should hold, and, in fact, it does not. The average "wave length" is not equal to 5.12 times the average "period" squared when the "period" is measured in seconds and the "wave length" in feet. The classical formula simply does not hold for the irregular sea surface. It does not hold because the assumptions under which the classical formula was derived are not true; consequently, there is no reason to expect the formula to hold. It will be possible to give correct formulas similar to equation (1.3) later in terms of what is actually observed on the sea surface when the quantities described above in terms of averages are measured.

$$\widetilde{L}=5.12\widetilde{T}^2 \qquad \text{(Equation not true)} \tag{1.3}$$

Why a Sea Is Different From a Swell

There may seem to be something strange in what has just been said about wave heights and wave "periods." The same height tables forecast wave heights for both sea and swell, and yet sea and swell have been shown to be very different. The reason for the difference between sea and swell has to do with the spectrum of the waves and not with the height distribution, which follows the same rules for all wave conditions with sufficient accuracy for our purposes.

Look at an incandescent lamp; you see white light. Hold some red cellophane between your eyes and the lamp; you now see red light. White light contains in reality all colors, and the red cellophane stops all the colors except the red from coming through to your eye. Blue cellophane would similarly let only the blue light through.

Physicists would say that the white light has a continuous spectrum of colors, and that the colored cellophane acts as a filter by letting only one color come through. Other examples of wave filters could be given from electronics.

It has been found that ocean waves also have a spectrum which is a function of frequency and direction. THE SPECTRUM OF THE OCEAN WAVES ASSIGNS THE CORRECT VALUE OF THE

SQUARE OF THE WAVE HEIGHT TO EACH FREQUENCY AND DIRECTION BY MEANS OF A FUNCTION OF WAVE FREQUENCY AND DIRECTION. Whether the waves are sea waves or swell waves is determined by the range of frequencies covered by the spectrum and the shape of the spectrum. The spectrum also assigns angular directions to the frequencies of the waves.

If the spectrum of the waves covers a wide range of frequencies and directions the result is a sea. The "periods" are distributed over a wide range of values. If the spectrum covers a narrow range of frequencies and directions, the result is swell. The "periods" are distributed over a narrow range of values. If the spectrum of the waves can be forecast, then the properties of the waves can be forecast, but first the properties of the spectrum must be given.

The Structure of the Waves

Now that it has been shown that the classical formulas for wave motion are wrong when applied to sea conditions, it is necessary to say that the classical formulas are still the only ones completely understood for wave motion. The winds over the ocean, by not producing simple sine waves, have complicated the problem of understanding actual ocean waves and of forecasting them. The classical formulas must be applied to ocean waves in a different way than before in order to get valid pictures of the waves and forecast them correctly.

To see how this can be done, consider figure 1.7. It shows a great number of sine waves piled up on top of each other. Think, for example, of a sheet of corrugated iron which would represent a simple sine wave on the surface of the ocean frozen at an instant of time. Below this in figure 1.7, there is another simple sine wave traveling in a slightly different direction from the one on top. Below this second one is a third simple sine wave with a third direction. Below the third is a fourth traveling in another direction and with a different wave length. There are shown a great many more, one below the other. Each is a classical simple sine wave. Each has the same wave height everywhere along every crest. (The heights should be thought of as being very low.) Each wave system has either a different direction or different *period* from all the others shown. Each wave system obeys the classical formulas which give the speed of the crests and the *wave length*.

Now to get an approximation of the actual sea surface, add all these simple sine waves together, one on top of the other. At some places, the crests of a number of different sine waves will add together to produce a tall mound on the actual sea surface. The mound will not be very long because the sine waves are traveling in different directions and the crests do not stay in line. At other places, the

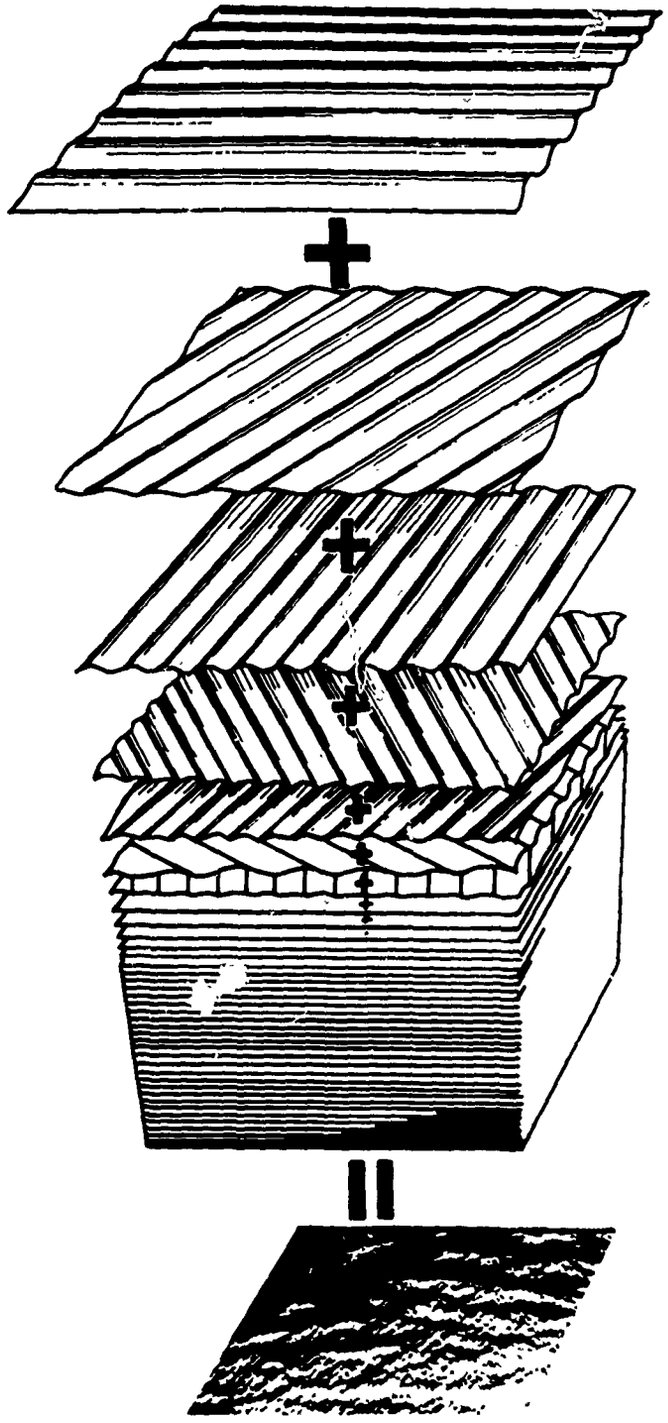

FIGURE 1.7 A sum of many simple sine waves makes a sea.

crests of some of the sine waves will cancel out with some of the troughs of the other sine wave systems; the result is a low mound, if any, on the sea surface.

It can be shown that as the number of different sine waves in the sum is made larger and larger and the heights are made smaller and smaller, and as the *periods* and directions are picked closer and closer together (but never the same and always over a considerable range of values), the result is a sea surface just like those actually observed with a range of heights which is very nearly like the range described in tables 1.4, 1.5, and 1.6. These features have been verified by observers in England, New Zealand, and the United States, and no naturally generated wave system has ever been found which did not very nearly have these properties.

Sea contains sine waves with a large range of wave *periods* and directions. These waves reinforce to produce a high wave at one point; then, since the *wave lengths* and *periods* are so very different, they get out of phase rapidly and produce a low wave next to the high wave. The sea is thus extremely irregular because the spectrum covers a wide range of frequencies (*periods*) and directions. The classical formulas are correct for each individual sine wave, but they are not correct when applied to the sum of all the different sine waves when the sine waves are added together.

Swell contains sine waves with a much smaller range of wave *periods* and directions. When they reinforce to make a big wave, the next wave must also be a big one because they cannot get out of phase in the short time required for the next wave to pass. Swell is thus much more regular than sea because the spectrum covers a narrower range of frequencies and directions. The classical formulas are then almost, but never exactly, correct for swell conditions.

THE NUMBER *E* CAN THEN ALSO BE DEFINED AS THE SUM OF THE SQUARES OF THE INDIVIDUAL AMPLITUDES OF THE INDIVIDUAL SINE WAVES WHICH GO TO MAKE UP THE COMPLICATED IRREGULAR PATTERN OF THE ACTUAL WAVE MOTION. Then by making the assumption that any part of each sine wave could be passing a fixed point, instead of, say, all the crests at once, it is possible to derive the numbers given in the tables theoretically and to prove that the ranges of heights shown can be expected to occur.

A Summary of the Properties of Sea

There have been many recent new and important advances in the study of ocean waves. Better observation methods and analysis methods have been developed although they are not used as extensively as they should be. A summary of these results can give a very

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accurate description of waves. Given below is a summary of the known and verified properties of actual ocean waves.

In sea, the waves are irregular, chaotic, short-crested, mountainous, and unpredictable. High waves follow low waves in a completely mixed-up way. The crests are only 2 or 3 times as long as the distance between crests. The wave that is passing tells us little about the height of the wave after that. Individual crests can appear to be traveling in different directions, varying by as much as 20° or 30° from the dominant direction. There are waves on top of waves and crests with depressions in the top. There is great variability in the "periods." The average "wave length" is not equal to 5.12 times the square of the average "period."

The pattern of sea never repeats or duplicates itself. Patterns change rapidly with time. No two aerial photographs or wave records of a sea will ever be exactly alike. The high waves die down as they travel along and soon disappear. New waves, which were once very low, form and build up to take their places. Individual wave crests can disappear completely as they travel a distance as short as 500 feet. In a flat place on the ocean surface a high wave can form right before your eyes in 30 or 40 seconds.

A Summary of the Properties of Swell

In swell the waves are more regular, longer crested, and more predictable than in sea. High waves (compared with the average) follow high waves and low waves follow low waves. When the waves are high, 5 or 6 waves of nearly the same height will pass in a row. When the waves are low, they can remain low for maybe as much as a minute and a half. If the waves are increasing in height, the next wave will probably be higher. If the waves are decreasing in height, the next wave will probably be lower. A guess about the heights of the next 2 or 3 waves to pass, on the basis of whether the wave which has just passed is high or low and on the basis of whether the heights are increasing or decreasing, will be right very often. A swell is predictable in a short-range sense.

The crests are much longer than in sea, as much as 6 or 7 times the distance between the crests. The average "wave length" is almost equal to 5.12 times the square of the average "period," and frequently the value is correct to within 10 or 15 percent. The different values of the successive "periods" are more nearly the same.

The pattern is still as unpredicatable over longer time intervals as is that of a sea. However, high waves can be present for many minutes before they die down and disappear. Low waves take a long time to become high waves.

The In-Between State

Sea is a clear-cut state because its waves are being generated by winds right in the storm. A swell, however, requires time to acquire the properties given above. A few hundred miles outside the generating area, waves cannot be precisely described by either of the above descriptions. They are in an in-between state represented by the records shown in figure 1.8, which are not quite as regular as a well-defined swell and also not quite as irregular as a fully-developed sea. A fresh swell which has only slightly *dispersed* (this term will be defined later) is half-way between the two states described above. As skill is developed in the use of the new methods given in this manual, it will be possible to tell when the above descriptions can be safely applied to the forecast situation.

Forecasting

The way to forecast waves is to forecast the spectrum of the waves. From the spectrum, many of the other properties of the waves can be calculated, such as the heights of the waves, the average "period," and the average "wave length." In the next chapter, ways to forecast the wave spectrum, the value of E, and the properties of "sea" will be given. Then in Chapter III, ways to forecast the swell spectrum and the properties of swell waves from the sea spectrum will be given.

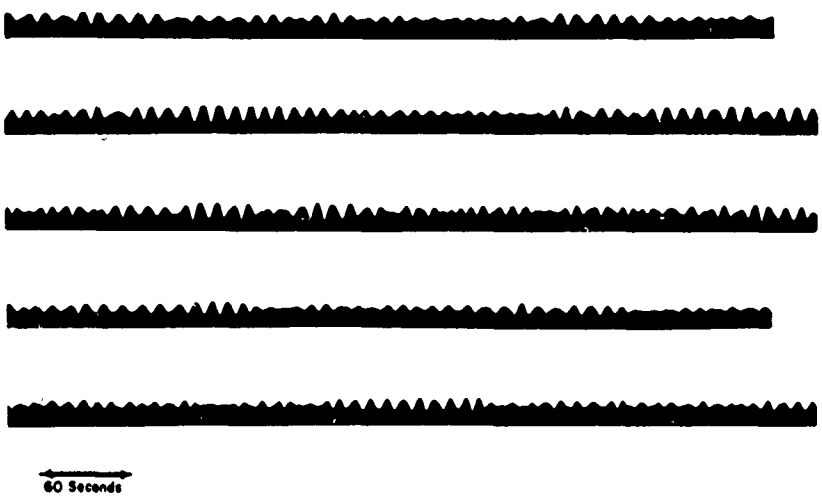

FIGURE 1.8 The records of in-between waves.

Chapter II

THE GENERATION AND FORECASTING OF SEA WAVES

Introduction

This chapter deals with the generation of composite wave motion in deep water under the influence of wind and with forecasting the characteristics of the waves in the generating area. "Deep water," in this sense, means that the depth of the water is large compared with the wave length of the longest wave generated by the wind.

Deep-Water Waves

In general, waves may be considered deep-water waves when the depth of the water layer is greater than one-half wave length. In this case the velocity of propagation of a single sinusoidal wave, that is, C, the speed with which a wave crest of such a wave component travels along the water surface, is (almost) independent of the water depth and proportional to the square root of the wave length, L. *Periods*, T, or frequencies $f=1/T$, of single wave components in the complex sea are interrelated to the speed, C, and *wave length*, L, by simple formulas.

$$C=1.34\sqrt{L}=3.03\ T \tag{2.1}$$

$$L=0.557\ C^2=5.12\ T^2 \tag{2.2}$$

$$T=0.442\sqrt{L}=0.33\ C \tag{2.3}$$

where C is given in knots, L in feet, and T in seconds.

If the depth of the water is small compared with the wave length, not only does the velocity of wave propagation change, but also the growth and the decay of the waves, which are influenced by bottom effects including increased dissipation of wave energy by friction.

The term "deep water" therefore has a relative meaning because it depends on the *ratio* between depth and wave length. Thus, the techniques given in this manual can also be applied in coastal regions of shallow water, in bays, lakes, or channels if the water can be considered as "deep" with respect to the longer waves generated in the

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composite wave spectrum. Such cases may apply to slight or moderate wave motion at lower wind velocities in shallow coastal waters, whereas heavy storm seas or long swell may "feel the bottom" strongly and therefore be affected by the limited depth while propagating through regions of shallow water.

Wave Conditions Limited by the Fetch or by the Duration

Relationship between the growth of complex deep-water waves, the wind velocity at the sea surface, the area of water over which the wind blows (the "fetch"),[1] and the length of time that the wind has blown (the "duration"), have been established for two principal cases. One of these deals with the growth of the sea over an unlimited fetch, x, where the waves grow at all localities at the same rate with increasing time. The wave characteristics depend then on the duration of the wind and the wind velocity. The other case considers the duration of wind action, t, as long enough to produce a steady state at all different localities, but the fetch, x, is limited, and the stage of wave development is given as a function of x. The wave characteristics depend then only on the length of the fetch and on the wind velocity.

Intermediate Conditions

Between these two principal cases, many other possibilities of wave generation under wind action have to be considered in the process of wave forecasting. Very often it happens that the wind encounters an "old sea" which has been generated by a wind just previously. For instance, the wind over a large area of the ocean may change in direction and speed in a few hours by a change in the weather situation. If the wind opposes the old waves, the newly arisen wind has to destroy the "old sea," or at least to destroy it partly, and to generate a new wave motion. In such cases allowances must be made for waves that are present when the wind under consideration starts blowing. In another case, if the wind increases with time, the sea grows continuously with increasing speed.

It is not possible to give simple graphs or tables in this manual which cover individually all possible cases. In practice, innumerable possibilities of different conditions of fetch and duration will be encountered by the wave forecaster. He must use his experience and judgment in the interpretation of consecutive synoptic weather maps so that the given graphs and tables will be used in the most effective way to make good wave forecasts. Therefore, it is necessary to understand clearly the physical significance and limitations of the

[1] See the definition of "Fetch" in Chapter V (page 165). In this manual, the word "fetch" will be used in two different ways. Its meaning in a particular sentence should be determined from the context. At times, the word "fetch" will mean the area in which the waves are generated; at other times, it will mean the length of the area in which the waves are generated measured upwind from the point of interest.

methods presented in this manual. Tests of the methods by trained personnel have been made with considerable success, and their experience will be discussed in Chapters V and VIII.

The Growth of Waves

Initial Wavelets

When a breeze comes up, the sea surface instantaneously becomes covered with tiny ripples which form more or less regular arcs of long radius. They increase rapidly in height until they attain a maximum steepness where the pointed crests take on a "glassy" appearance, indicating small breaking processes. The further development of these wavelets into the state of more irregular composite wave motion, called sea, is a very complicated mechanism because of the limitations on the growth of individual component wave trains caused by breaking. The total wave energy begins to spread out over a range of *periods*, and with increasing wave development this range extends more and more to long *period* waves. A spectrum of ocean waves is being formed. At a given wind velocity and a given state of wave development, each spectrum has a maximum band where most of the total energy is concentrated.

Short Choppy Waves

The growth of sea beyond the state of initial wavelets is largely determined by the breaking of large and small waves, by the energy dissipation connected with this turbulent motion, and by energy exchanges from one part of the spectrum to another. The waves can grow only if they receive more energy from the wind than they lose by turbulence in the complex breaking sea.

Energy Balance

Energy is transferred to the waves by the pushing and dragging forces of the wind. These forces depend upon many different factors, such as the difference between wind and wave velocity, the wave form, and the roughness conditions of the sea surface. Thus, the energy transfer from wind to waves depends upon the present state of wave development itself. If there are component wave trains in the complex sea that outrun the wind, they will meet an air resistance by opposing wind relative to the wave form. These waves actually do a net amount of work on the wind field and lose energy. This effect and that of the internal eddy viscosity tend to slow down the undulatory wave motion.

The Fully Developed State

The fully developed state over a long fetch with a sufficiently long duration finally will be attained when the wave-generating tractions

have increased the total wave energy to the point where dissipation balances the work done by pushing and dragging surface forces. The *total amount of wave energy* accumulated in the composite wave motion is then *distributed* over a wide range of *wave lengths* or *periods* (frequencies). THE FUNCTION THAT DESCRIBES MATHEMATICALLY THE DISTRIBUTION OF THE SQUARE OF THE WAVE HEIGHT (WAVE ENERGY) WITH FREQUENCY IS CALLED THE SPECTRUM OF THE WAVE MOTION. The square of the wave height is related to the potential energy of the sea surface so that the spectrum will at times also be called the energy spectrum. The first step in wave forecasting is to forecast the spectrum of the waves in the generating area.

The Spectrum of Wind-Generated Ocean Waves

Imagine an infinite number of individual wave components in the composite pattern of a fully arisen sea at a given wind velocity. As described in Chapter I (fig. 1.7), the sum of these single waves makes up the wave pattern at the sea surface as it appears to an observer. For an approximation, all the waves grouped around an average frequency, f_1, and distributed over the small range $\frac{1}{2}\Delta f$ on each side can be picked out from the continuous sequence of individual wave components. If this accumulation of wave components is continued at intervals of Δf, a finite number of simple sine waves with average frequencies $f_1, f_2, f_3 \ldots f_n$, starting with lower frequencies and ending with the highest frequencies present in the composite wave pattern, is obtained. By this procedure, the continuous distribution of an *infinite* number of wave components is approximated by a *finite* number of individual sine waves with slightly different average frequencies, f.

Each of these sine waves can be considered as "filtered out" of the total sum, like the individual waves in figure 1.7. Then, the characteristics of these waves are given with a fair degree of approximation by the average frequency, f; *period*, T; *wave length*, L; and average wave amplitude.

If the rectangles whose areas are equal to the squares of the amplitudes, $A^2(f)$, of each component wave train are plotted against the average frequency, f, (fig. 2.1) a "stairway" approximation to the spectrum is obtained. The graph covers the entire range of periods. The wave energy per unit sea surface area contained in each wave train of the average frequency, f, is proportional to the square of the amplitude of each individual wave train.

If the difference, Δf, is made smaller and smaller, the number of individual wave trains increases correspondingly, and approaches infinity. At the same time the amplitude $A(f)$ of each individual

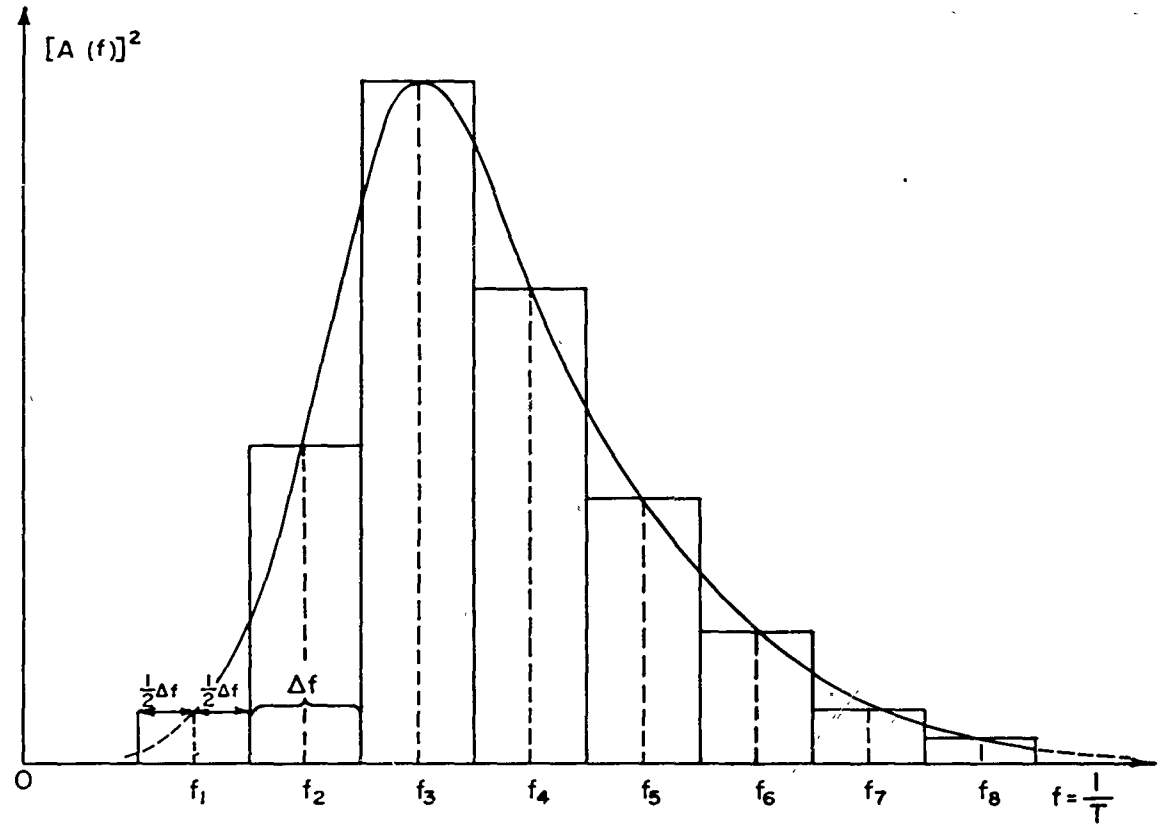

FIGURE 2.1 "Stairway" approximation to the wave spectrum. (The area of a given rectangle is proportional to the square of the height of a sine-wave with the associated frequency.)

wave component is also made smaller and smaller, but the total amount of energy accumulated in the composite wave motion remains the same. Only the distribution of wave energy as a function of $f=1/T$ changes from the "stairway" graph in figure 2.1 into the continuous curve. This continuous curve between $f=0$ and $f=\infty$ represents the spectrum of the waves.

By empirical evidence and theoretical considerations, the wave spectrum has been determined in a general way for wind-generated wave motion. THIS SPECTRUM PROVIDES THE BASIC THEORY FOR THE METHODS DEVELOPED IN THIS MANUAL.

The Properties of the Spectrum

Without going into the mathematical details of the spectrum, its properties can best be described by means of some examples. Figure 2.2 represents the spectra of the waves by curves for a fully developed sea at wind velocities of 20, 30, and 40 knots. The ordinate $[A(f)]^2$

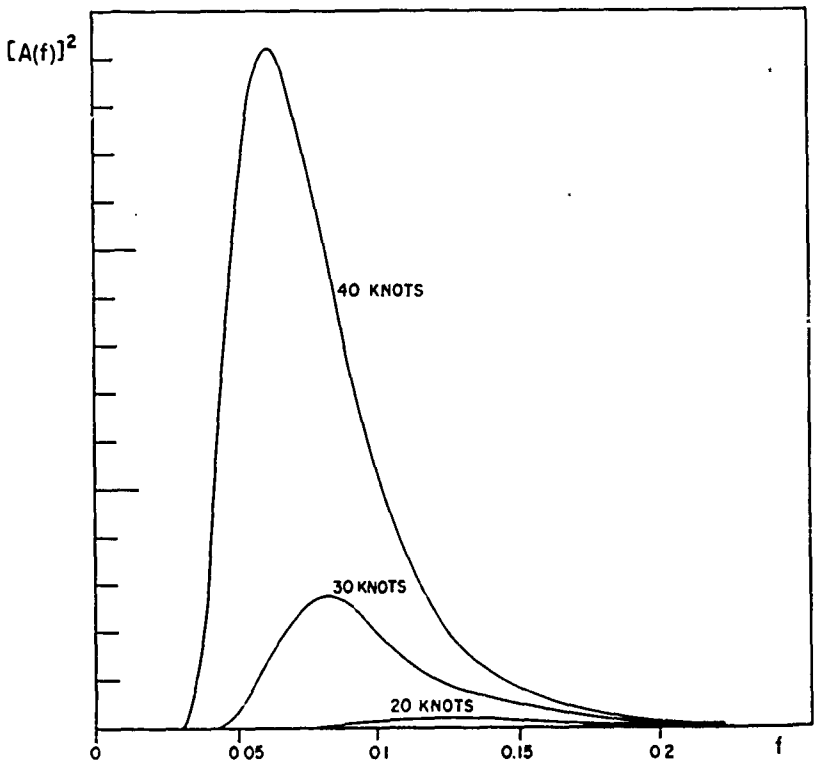

FIGURE 2.2 Continuous wave spectrum for fully arisen sea at a wind speed of 20, 30, and 40 knots, respectively. Note the displacement of the optimum band (maximum of spectral energy) from higher to lower frequencies with increasing wind speed.

has the dimensions of ft.^{2}-sec., so that the area under the spectrum, upon integration over a frequency range, has the dimensions of ft^2. The abscissa shows the scale for the frequencies, f. It extends theoretically from zero to infinity, but only that part which is of interest has been shown in figure 2.2.

Depending upon the wind velocity, the range of waves with a significant or noticeable amount of energy covers a more or less broad band on the f scale. The relatively small spectral wave heights at 20 knots cover significant frequencies between $f=0.083$ and $f=0.3$. In terms of *periods* the range is from 12 seconds to 3 seconds ($f=0.3$ is not included in the range of the f scale). The maximum of spectral wave energy is concentrated in a band around $f=0.124$ or $T=8.1$ seconds.

With increasing wind velocity the range of significant frequencies extends more and more toward smaller f values (or higher *periods*); at 30 knots the range is from about $f=0.048$ to $f=0.24$, that is, for *periods* from $T=17$ to $T=5$ seconds. The maximum band is displaced to lower frequencies; at 30 knots, this band is to be found around $f=0.0826$ or $T=12.1$ seconds.

The curve of the spectrum at 40 knots shows a great increase of wave energy compared with those of 20 and 30 knots. It also shows how the significant wave components in the composite wave pattern extend to lower frequencies with the characteristic shift of the optimum band. The frequency, f_{max}, of the maximum band at different wind velocities is given by equation 2.4 where v is in knots.

$$f_{max}=\frac{2.476}{v} \tag{2.4}$$

Table 2.1 gives the f_{max} values for different wind velocities between $v=10$ knots and $v=56$ knots. This band and its frequency are important factors in wave forecasting and will be used in various parts of the following chapters.

The Important Spectral Components for a Given Wind Velocity

Another important fact has to be mentioned at this point. As the spectral wave energy in a broad band around the frequency of the maximum f_{max} increases with increasing wind velocity, the contributions of wave energy at the lower end of the curves (fig. 2.2) near the abscissa become less and less significant compared with the amount of wave energy concentrated around the maximum. This means that wave components with very high or very low frequencies are negligible and do not noticeably affect the dominating wave pattern of the sea. Thus, at a wind velocity of 20 knots the wave components concentrated around $f=0.33$ ($T=3.0$ seconds) may still be noticeable

Table 2.1—Frequency (f_{max}) and Period (T_{max}) of the Maximum Band in the Spectrum Where Most of the Spectral Wave Energy is Concentrated

Wind Vel. (knots)	10	12	14	16	18	20	22	24	26
f_{max}	0. 248	0. 206	0. 177	0. 155	0. 138	0. 124	0. 113	0. 103	0. 095
T_{max}	4. 0	4. 8	5. 7	6. 5	7. 3	8. 1	8. 9	9. 7	10. 5

Wind Vel. (knots)	28	30	32	34	36	38	40	42
f_{max}	0. 0884	0. 0825	0. 0774	0. 0728	0. 0688	0. 0652	0. 0619	0. 0590
T_{max}	11. 4	12. 1	12. 9	13. 7	14. 5	15. 3	16. 2	17. 0

Wind Vel. (knots)	44	46	48	50	52	54	56
f_{max}	0. 0563	0. 0538	0. 0516	0. 0495	0. 0476	0. 0459	0. 0442
T_{max}	17. 7	18. 6	19. 4	20. 2	21. 0	21. 8	22. 6

and significant. But at 40 knots in a heavier sea with a much longer and higher average wave motion, these waves with periods of about 3 seconds (wave length 45 feet) are less significant in their contribution to the dominating wave pattern, and the observer will probably pay little attention to them in a high sea. Naturally, these waves are always present in the complex sea as long as the sea is under the maintaining influence of the wind. Even smaller waves, which make up the continuation of the spectrum curve with increasing f values, are present, including ripples. However, they are less significant for wave forecasting. (In some instances even the short-period (or high-frequency) portion of the spectrum may be important, e. g., in the study of optical or acoustical properties of the sea surface.)

At the other end, with low frequencies (or high *periods*), the curves drop rapidly to small energy values or amplitudes. These extreme long waves in the wind-generated wave pattern are relatively low and therefore extremely flat. They are almost completely masked by the dominating bands in the wave spectra around the energy maxima; therefore, they are also without significance in the complex wave motion of the wind-generated sea. However, these long-period waves may be much more important in the decay area as flat swell or as surf running against the shores in shallow water.

For wave forecasting in the generating area, it is sufficient to consider only the *significant part of the wave spectrum*. For this purpose, the spectrum can be cut off at high and low frequencies for values below a certain minimum amount of spectral wave energy. This procedure will be explained in another section. The basic curves

used for wave forecasting in this manual will not be given as graphs of the spectrum like those in figure 2.2 but will be presented in a slightly different form. This has been done for practical reasons which will be explained when the co-cumulative spectrum is defined.

Co-Cumulative Spectra (C. C. S.)

Derivation

The number E which has been defined in Chapter I as the quantity used to forecast the needed information about wave heights, can be derived from the spectrum by the procedure of integration. The number E represents the sum of the squares of the amplitudes of the individual component wave trains which make up the actual wave motion as it is observed.

At the top of figure 2.3 the spectrum for a fully arisen sea at a given wind velocity is plotted against frequency, f, like the curves in figure

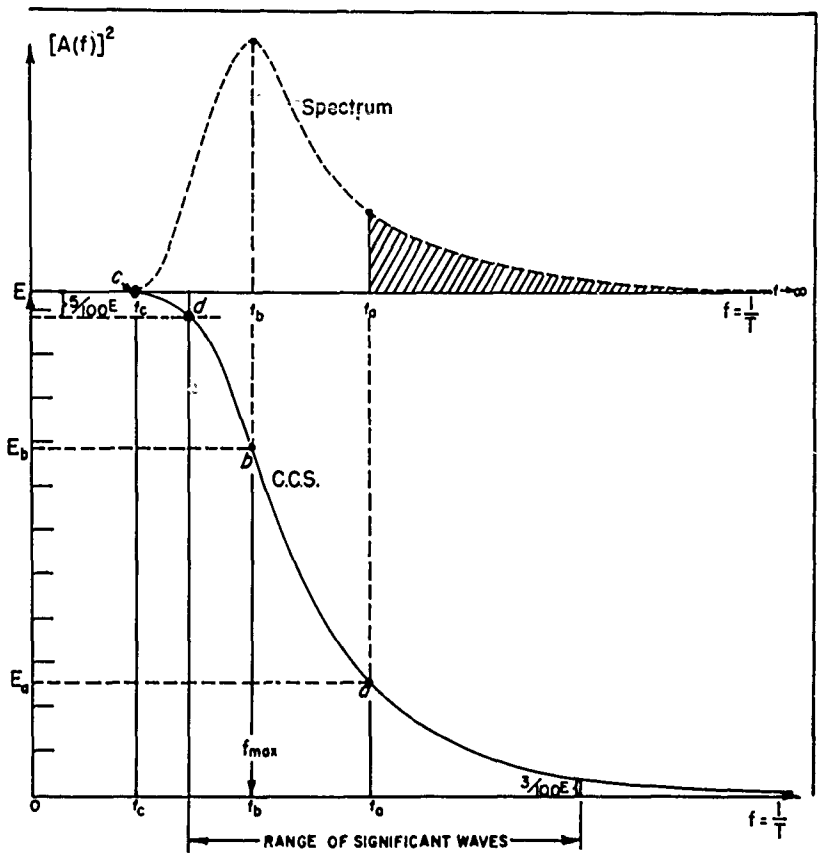

FIGURE 2.3 Wave spectrum and co-cumulative spectrum (C. C. S.)

2.2. The summation of the squares of the amplitudes of individual wave components $[A(f)]^2$ is begun at very low periods and the process is continued up to a certain frequency, f_a. Then the sum equals the area under the spectrum limited by the ordinate at the frequency, f_a, and the abscissa. This area is indicated in figure 2.3 by hatching. The numerical value, E_a, of this sum is plotted against frequency, f_a, in the lower graph of figure 2.3 at point *a*.

Continue further by summing to the frequency, f_b, which in figure 2.3 coincides with the frequency of the maximum band. The value, E_b, of the sum is given by the area under the spectrum between the ordinate at f_b, the abscissa, and $f=\infty$. It has been plotted as point *b* on the lower graph. When the summation is extended to the frequency, f_c, nearly all the area under the spectrum between $f=0$ and $f=\infty$ has been taken into account. The value E_c, therefore, practically will not differ from the total sum of the squares of the amplitudes, $[A(f)]^2$, obtained by the integration between the limits $f=0$ and $f=\infty$.

THE CURVE WHICH CONNECTS THE POINTS, a, b, and c IN THE LOWER GRAPH IS CALLED THE CO-CUMULATIVE SPECTRUM (ABBREVIATED C. C. S.). These C. C. S. have been computed for wind velocities between 10 knots and 56 knots, and they are presented in figures 2.4*a* through 2.4*f* for use in this manual. *E* values are proportional to the total wave energy to the right of the frequency shown.

The Properties of the Co-Cumulative Power Spectra

Evidently, the curves of the C. C. S. are closely related to the spectrum. For instance, the frequency, f_{max}, of the band where most of the spectral energy is concentrated is the frequency at which the maximum slope of the C. C. S. curve (point *b* in figure 2.3) is found.

The ordinate values, *E* of the C. C. S. curves have the dimensions ft.^{2} and permit one to determine the height characteristics of the composite wave motion in the sea and, as will be shown later, the height characteristics of swell in the decay area. Rules for computing the wave height data from a given *E* value are presented in tables 1.4, 1.5, and 1.6.

The procedure of "cutting out" the significant part of the wave spectrum of the sea in the generating area has already been mentioned (p. 36). This procedure can be applied much more easily to the C. C. S. curves. Toward the low frequency end of the C. C. S. the slopes of the curves level off rapidly, and the curves continue toward the left side almost exactly as a horizontal line at a constant value of *E*. This is the case in figure 2.3 beyond the point *c* for frequencies smaller than f_c. In this stage (point *c*) the sea is almost fully arisen, and the wave components with higher periods or lower frequencies

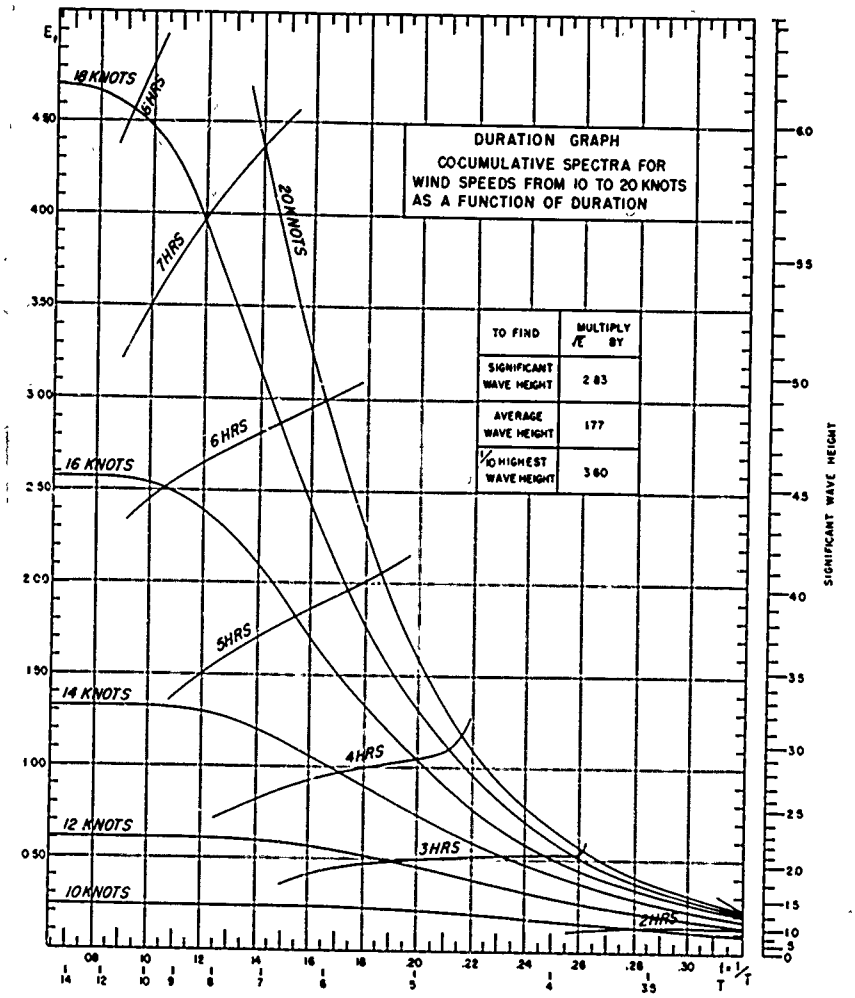

FIGURE 2.4a.

than f_c (approximately) contribute such a small amount of energy to the total that they can be neglected.

In order to forecast and to describe the properties of fully-arisen wind-generated sea, the horizontal or nearly horizontal parts of the C. C. S. curves can be thought to be cut off. Physically, this means that certain high- and low-period portions of the wave spectrum can be filtered out without any significant change of the dominating wave pattern. As a rule, about 5% of the total E value in the fully-developed state of wind-generated sea can be cut off at the upper part of the C. C. S. curves, as shown in figure 2.3 by the horizontal dashed line through point d on the C. C. S. curve. By this procedure, the periods of dominating waves are practically limited at a certain upper

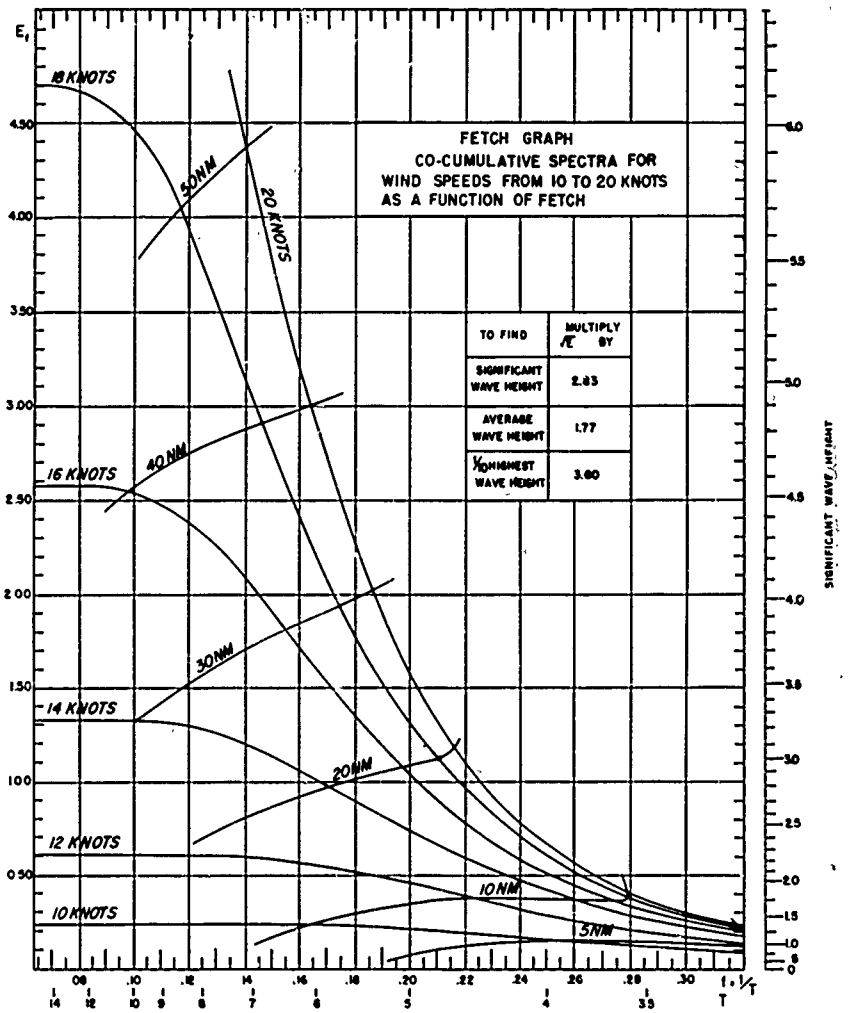

FIGURE 2.4b.

end of the spectra. However, this practice can be applied only in the wind area where the long, flat waves with periods higher than this limit are more or less masked by the sum of the other components. In the decay area even this "cut off" part can be significant if the wave components with higher periods or lower frequencies are all that are present.

Reason for the Co-Cumulative Spectra

The C. C. S. curves are used instead of the spectrum curves for forecasting the waves in order to accomplish once and for all the task of integrating the spectra. The area under a given spectrum from a

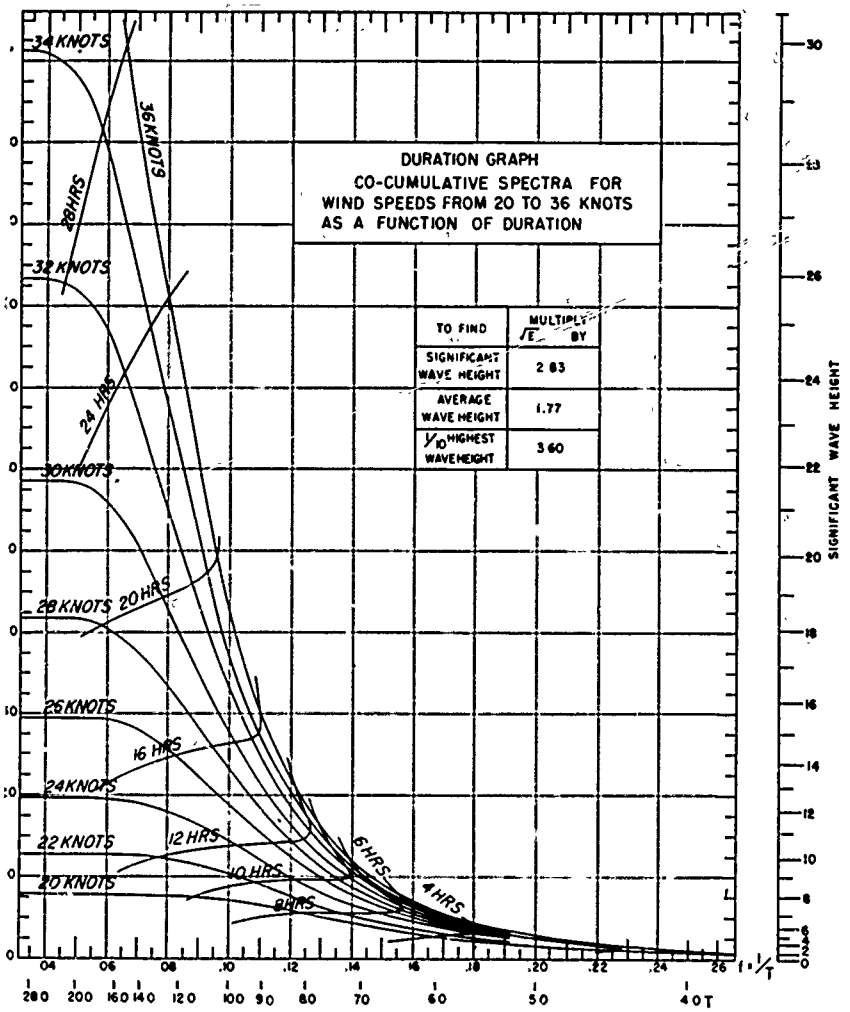

FIGURE 2.4c.

particular frequency value to infinity is given by the value of the C. C. S. curve at that frequency. It will also be possible, as will be shown in Chapter III, to find quite easily the area under any portion of the spectrum. This additional property is important in forecasting swell.

Description of the Wind-Generated Sea in the Fully Developed State

Definition of the Fully Developed State

THE SEA IS FULLY DEVELOPED AT A GIVEN WIND SPEED, WHEN ALL POSSIBLE WAVE COMPONENTS IN

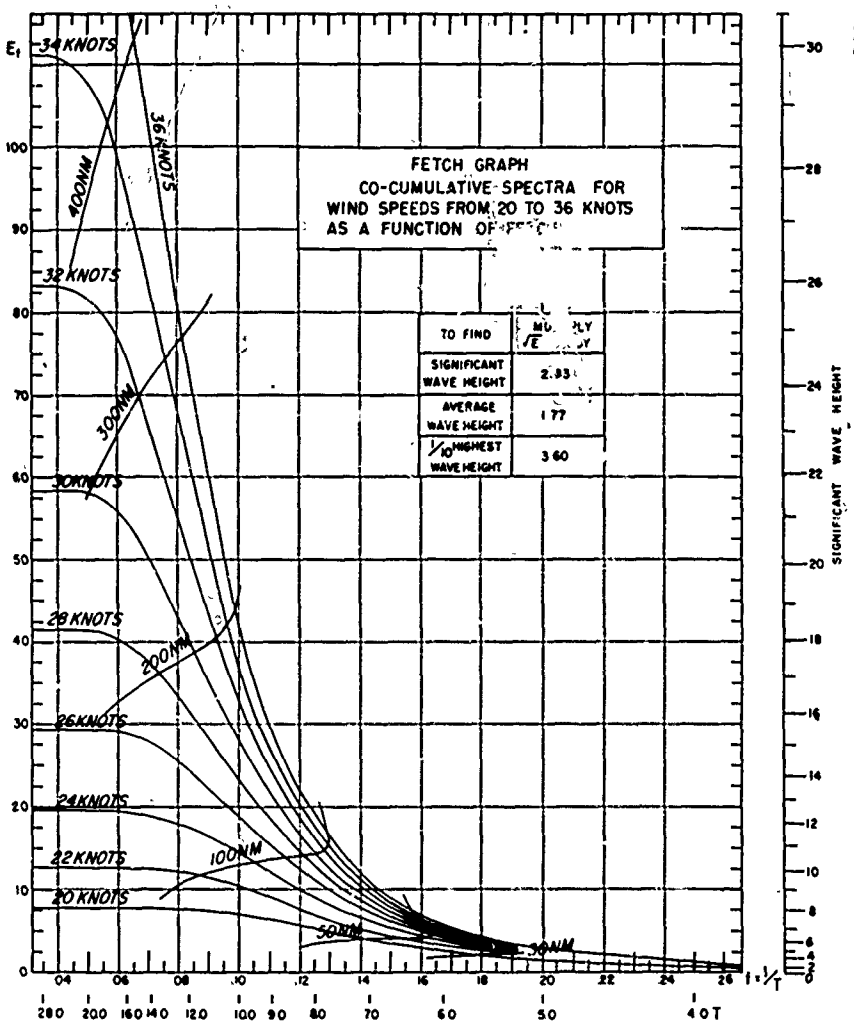

FIGURE 2.4d.

THE SPECTRUM BETWEEN $f=0$ and $f=\infty$ ARE PRESENT WITH THEIR MAXIMUM AMOUNT OF SPECTRAL ENERGY. The spectrum in this case looks like the upper curve in figure 2.3, and the C. C. S. looks like the lower curve in the same figure. Theoretically, this stage will be reached exactly at an infinitely long duration of wind action over an infinitely long fetch. But practically, and with a good mathematical approximation, the fully developed state is attained at a certain *finite* duration, and over a certain *finite* fetch, which can be called the "*minimum duration*," t_m, and the "*minimum fetch*," F_m, needed for the generation of the "fully arisen sea." The reason this "fully arisen sea" is defined is that the amount of

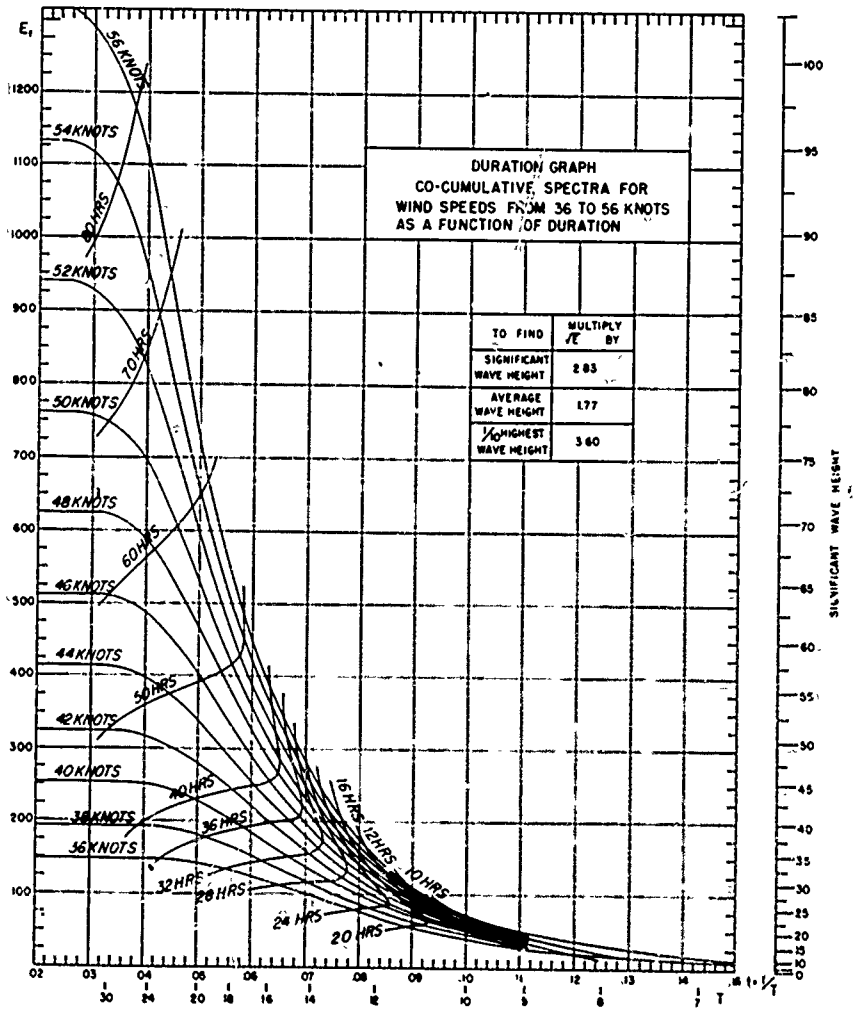

FIGURE 2.4c.

energy added to the spectrum beyond a certain limit in order to "build up" the mathematically fully developed spectrum is so small that it can be neglected. The minimum duration and the minimum fetch needed for reaching the stage of the "fully arisen sea" depend upon the wind velocity. Their values are shown in table 2.2. It is seen that duration and minimum fetch increase rapidly with increasing wind velocity.

Storm seas at wind velocities of 50 knots and more reach the fully-arisen state only in rare cases. Table 2.2 shows that even in cases where the fetch is long enough the needed duration of continuous wind action is far too long in most practical examples. However,

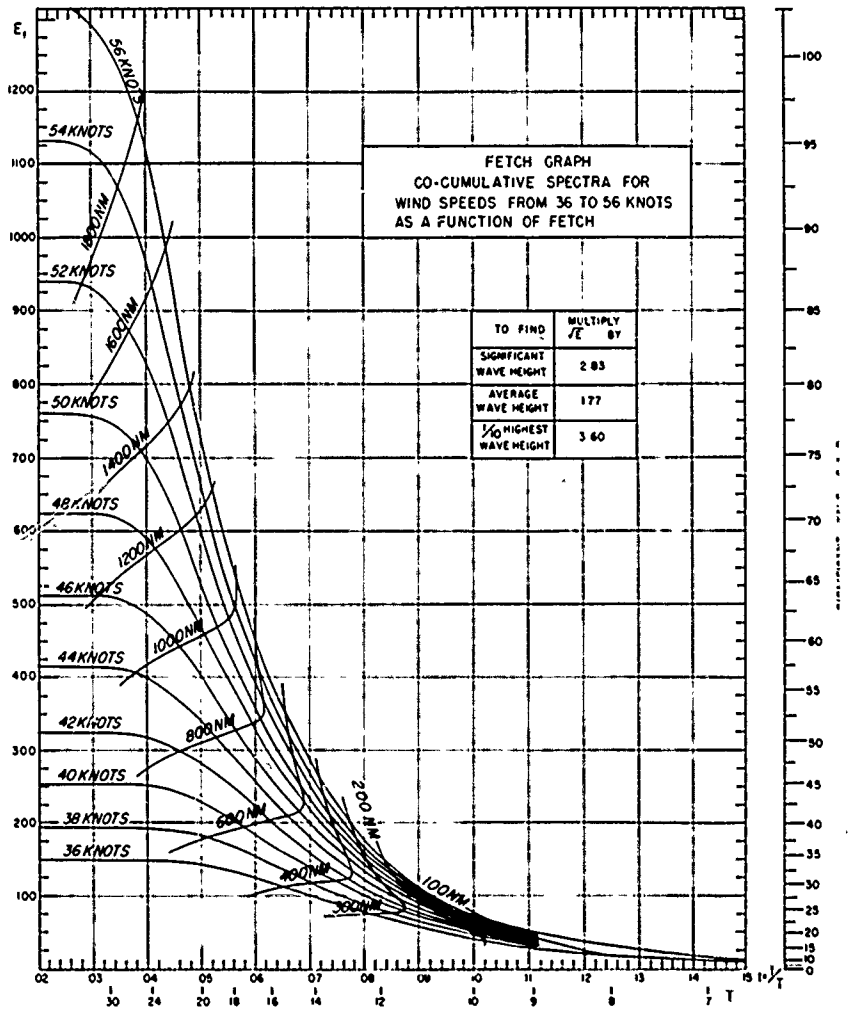

FIGURE 2.4f.

at low and moderate wind velocities in the open ocean, the fetch and duration in many practical cases will be long enough to provide conditions for the generation of a fully developed sea.

In some instances, however, it may happen that even with extreme wind velocities of 50 knots and more the fully arisen state will be approached. This may occur over long fetches, in the Antarctic, for example, when one heavy storm follows another, and the waves do not die down too much during the less stormy intervals. With each new storm the swell remaining from the previous storm is built up again. By this process very long and high waves may be generated which almost reach the fully arisen state at the given maximum wind

Table 2.2—Minimum Fetch and Minimum Duration Needed to Generate a Fully Developed Sea for Various Wind Velocities

V knots	10	12	14	16	18	20	22	24	26	28
F_m NM	10	18	28	40	55	75	100	130	180	230
t_m hr	2.4	3.8	5.2	6.6	8.3	10	12	14	17	20

V knots	30	32	34	36	38	40	42	44	46
F_m NM	280	340	420	500	600	710	830	960	1100
t_m hr	23	27	30	34	38	42	47	52	57

V knots	48	50	52	54	56
F_m NM	1250	1420	1610	1800	2100
t_m hr	63	69	75	81	88

speed. Such a region, known for its extreme storm sea conditions in winter is Cape Horn in the Southern Hemisphere.

E Values and Potential Energy for a Fully Arisen Sea

The number, E, used in this manual to forecast wave height characteristics of the sea, is a value that has to be determined from the C. C. S. curves in figures 2.4*a* through 2.4*f* for given wind velocities. In the fully arisen state of the sea it is the largest value of the C. C. S. curves at the $E(f)$ scale, that is, where the C. C. S. curves for a given wind speed approach the ordinate horizontally. As explained in the derivation of the C. C. S., this E value is proportional to the total amount of energy accumulated in the composite wave motion.

If for some reason it is necessary to determine the total wave energy accumulated in fully arisen sea at a given wind speed, the E values with the dimensions $(\text{ft.})^2$ have to be converted into the "potential energy of the wave motion" per unit sea surface area. The total wave energy (potential+kinetic) is approximately twice this value. Table 2.3 shows the potential wave energy of fully arisen sea at different wind velocities. To get the total wave energy, double the values in table 2.3. This table shows that the total wave energy per unit sea surface area (U) increases with the fifth power of the wind velocity ($U \propto v^5$).

Wave Heights in a Fully-Arisen Sea

The wave height characteristics of a fully arisen sea as derived from figures 2.4*a* to 2.4*f* by means of the number, E, and the values given in table 1.5 (p. 11) are shown in table 2.4. The value of E for the fully developed sea is given by equation (2.5) where v is in knots.

$$E=0.242(v/10)^5 \qquad (2.5)$$

Table 2.3—Average Potential Wave Energy, U, in Ergs/cm² of the Fully Arisen Sea at Different Wind Speeds (v in Knots)

v (knots)	10	12	14	16	18	20	22	24	
U (ergs/cm.²)	0.55	1.4	3.0	6.0	10.4	17.6	28.3	43.7	x 10^5
v (knots)	26	28	30	32	34	36	38	40	
U (ergs/cm.²)	6.5	9.5	13.4	18.4	25.0	33.2	43.5	56.3	x 10^6
v (knots)	42	44	46	48	50	52	54	56	
U (ergs/cm.²)	7.2	9.1	11.3	14.0	17.2	20.9	25.2	30.3	x 10^7

Table 2.4—Wave Height Characteristics of the Fully Arisen Sea at Different Wind Speeds

Wind Vel. (knots)	10	12	14	16	18	20	22	24
E (ft.²)	0.24	0.60	1.30	2.54	4.6	7.7	12.5	19.3
Av. Ht. (ft.)	.87	1.37	2.02	2.82	3.8	4.9	6.3	7.8
Sig. Ht. (ft.)	1.39	2.20	3.23	4.51	6.1	7.9	10.0	12.4
1/10 highest (ft.)	1.77	2.79	4.11	5.73	7.7	10.0	12.7	15.8
Wind Vel. (knots)	26	28	30	32	34	36	38	40
E (ft.²)	28.8	41.6	58.8	81.2	110.0	146.3	191.7	247.8
Av. Ht. (ft.)	9.5	11.4	13.6	15.9	18.6	21.4	24.5	27.9
Sig. Ht. (ft.)	15.2	18.3	21.7	25.5	29.7	34.2	39.2	44.5
1/10 highest (ft.)	19.3	23.2	27.6	32.4	37.7	43.5	49.8	56.7
Wind Vel. (knots)	42	44	46	48	50	52	54	56
E (ft.²)	316.3	399.1	498.4	616.6	756.3	920.1	1111.2	1332.8
Av. Ht. (ft.)	31.5	35.4	39.5	44.0	48.7	53.7	59.0	64.6
Sig. Ht. (ft.)	50.3	56.5	63.2	70.3	77.8	85.8	94.3	103.3
1/10 highest (ft.)	64.0	71.9	80.4	89.4	99.0	109.2	120.0	131.4

The Range of "Periods" in a Fully Arisen Sea

The range of "periods" in a fully arisen sea has to be determined by the range of significant waves, as indicated in figure 2.3. Within this range of frequencies or *periods*, component waves of the spectrum are to be found with a *significant* amount of spectral energy. That is, at lower frequencies the waves with an E value less than about 5 percent of the total E value and at larger frequencies the waves with an E value less than about 3 percent of the total E value can be thought to be filtered out. Of course, these very long and very short waves are

always present in the fully arisen sea, but they do not contribute significantly to the observed dominating wave pattern. (See page 35 on the important spectral components for a given wind velocity in the discussion of the spectrum of a fully arisen sea.)

Table 2.5 shows the range of significant "periods" by tabulating the *period* such that for the indicated wind the fraction of E contributed by *periods* greater than T_U is 5 percent and the fraction of E contributed by *periods* less than T_L is 3 percent. For a fully arisen sea, "periods" outside of this range are rarely observed, if ever. Note that 1.515 times T_U gives a group velocity in knots that is less than the wind velocity. This important point will be discussed in Chapter III.

Table 2.5—Significant Range of *Periods* in the Fully Arisen Sea at Different Wind Speeds (v). (T_L = Lower Limit, T_U = Upper Limit of Significant Periods)

v(knots)	10	12	14	16	18	20	22	24	26	28
T_L(sec.)	1.0	1.0	1.5	2.0	2.5	3.0	3.4	3.7	4.0	4.5
T_U(sec.)	6.0	7.0	7.8	8.8	10.0	11.1	12.2	13.5	14.5	15.5

v(knots)	30	32	34	36	38	40	42	44	46
T_L(sec.)	4.7	5.0	5.5	5.8	6.2	6.5	7.0	7.0	7.0
T_U(sec.)	16.7	17.5	18.5	19.7	20.8	21.7	23.0	24.0	25.0

v(knots)	48	50	52	54	56
T_L(sec.)	7.5	7.5	8.0	8.0	8.5
T_U(sec.)	26.0	27.0	28.5	29.5	31.0

"Periods"

The "periods" observed at a fixed position at the sea surface, viz.; the time intervals between succeeding crests, are very important wave characteristics for practical purposes. The range of these "periods" is practically the same as the range of significant *periods*. The *average* of the time intervals between succeeding crests, $\tilde{\tilde{T}}$, can be determined mathematically from the spectrum. For a given wind velocity, v, this average "period" can be forecast by the use of equation (2.6).

$$\tilde{\tilde{T}} = 0.285\, v \tag{2.6}$$

where v is given in knots. This formula applies *only* for a fully arisen sea.

Although the *average* "period" at a fixed position certainly is of value in practical wave forecasting, it might be of more interest to know in what manner the time intervals between succeeding wave crests scatter around this average value. Some information can be given by carefully made visual observations of the "periods" which are given below.

Empirical Distribution Function of the "Periods"

A set of curves is shown in figures 2.5a, 2.5b, and 2.5c. The abscissa of each curve gives the "periods" (time intervals) as they will be observed at a fixed point of the sea surface between dominating wave crests. The ordinate gives the relative chance of occurrence of different values of the *significant* "periods." Thus, the *most frequent "period" or time interval between succeeding wave crests* is given by the *maximum* point of the curves, and by comparison with the ordinate of this point, an estimate can be made of how frequently the other "periods" will occur at this place.

These curves are empirical results, obtained by smoothing the histograms of many "period" observations. They apply to the fully

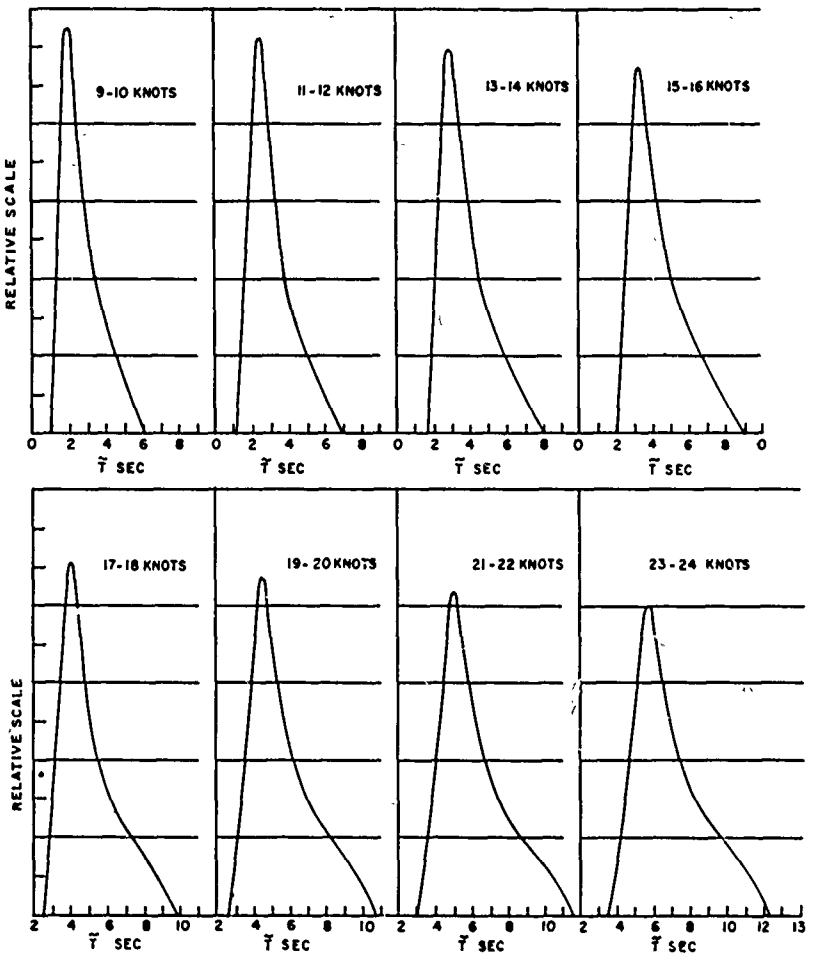

FIGURE 2.5a Empirical distribution function of the "periods" in a fully arisen sea for wind velocities from 9 to 24 knots.

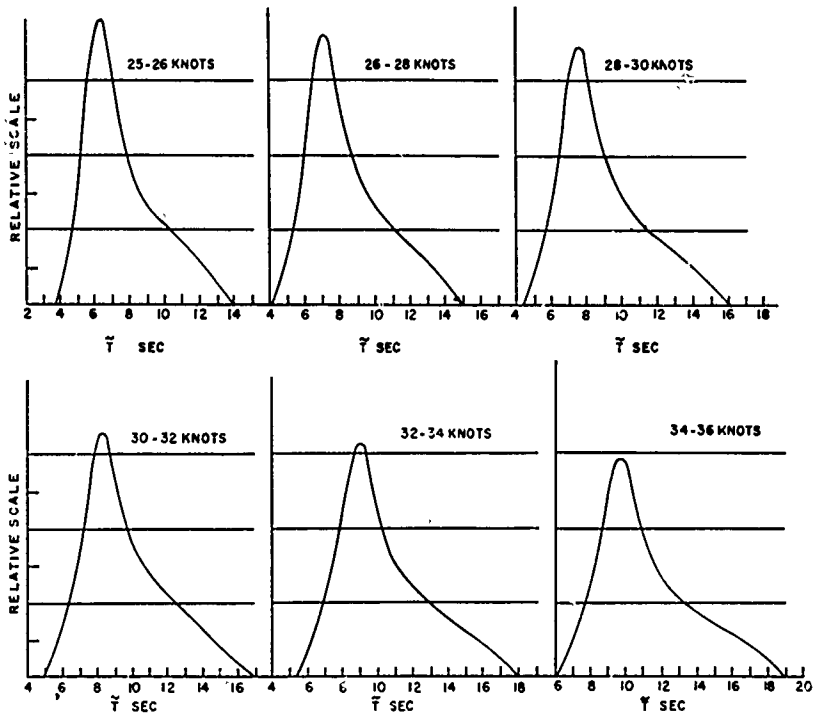

FIGURE 2.5b Empirical distribution function of the "periods" in a fully arisen sea for wind velocities from 25 to 36 knots.

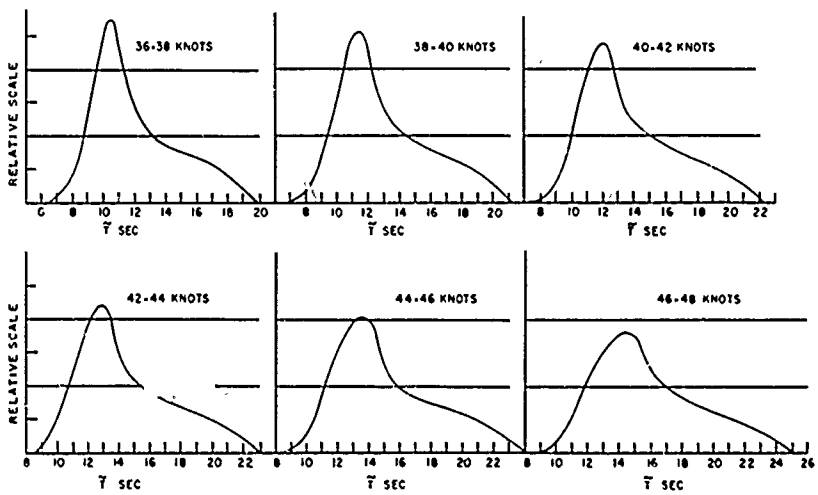

FIGURE 2.5c Empirical distribution function of the "periods" in a fully arisen sea for wind velocities from 36 to 48 knots.

developed sea at a given wind speed. The comparison of these curves with theory shows that the range of observed "periods" along the abscissa of the curves in figures 2.5a, 2.5b, and 2.5c coincides to a high degree of accuracy with the range of *significant* periods in the C. C. S. curves for the given wind velocity in the fully arisen sea (table 2.5). The most frequent time interval between succeeding crests of dominating "waves" at a fixed point is approximately the same as the average period $\tilde{\tilde{T}}$ according to equation (2.6).

Average "Period" and Average "Wave Length" in a Fully Arisen Sea

The average "period" as given in equation (2.6) can be used to determine the average "wave length" in a fully arisen sea. For a fully arisen sea, the average "wave length" is given by equation (2.7).

$$\tilde{\tilde{L}}=3.41\ \tilde{\tilde{T}}^2=\frac{2}{3}\,(5.12)\tilde{\tilde{T}}^2 \qquad (2.7)$$

The distribution of "wave lengths" is not known. It appears that "wave lengths" are much more variable than "periods" in a fully arisen sea. There are a great many short waves which decrease the value of the average "wave length" to a value considerably less than would be expected from the use of the average "period." Table 2.6 gives the values for the average "period" and the average "wave length" for fully developed seas at various wind velocities. Methods for observing "periods" and "wave lengths" will be given in Chapter IV.

Table 2.6—The Average "Period" and Average "Wave Length" of Fully Arisen Seas at Different Wind Speeds

v(knots)	10	12	14	16	18	20	22	24
$\tilde{\tilde{T}}$ (sec.)	2.85	3.42	3.99	4.56	5.13	5.70	6.27	6.84
$\tilde{\tilde{L}}$ (ft.)	27.7	39.9	54.3	70.9	89.7	111.	134.	160.
v(knots)	26	28	30	32	34	36	38	40
$\tilde{\tilde{T}}$ (sec.)	7.41	7.98	8.55	9.12	9.7	10.3	10.8	11.4
$\tilde{\tilde{L}}$ (ft.)	187.	217.	249.	284.	321.	362.	398.	443.
v(knots)	42	44	46	48	50	52	54	56
$\tilde{\tilde{T}}$ (sec.)	12.0	12.5	13.1	13.7	14.3	14.8	15.4	16.0
$\tilde{\tilde{L}}$ (ft.)	491.	533.	585.	640.	697.	747.	809.	873.

Forecast Example for a Fully Arisen Sea at a Given Wind Speed

The application of the methods explained in the preceding sections of this chapter in the case of fully arisen sea is shown in example 2.1.

EXAMPLE 2.1.—WHAT ARE THE CHARACTERISTICS OF

THE FULLY-DEVELOPED SEA FOR A WIND SPEED OF 30 KNOTS?

Method of solution:

1) Table 2.2 shows that at $v=30$ knots the sea approaches the fully arisen state if the fetch is larger than 280 NM and if the duration is longer than 23 hours.

2) Figure 2.4c or 2.4d shows at the ordinate for the C. C. S. curve of 30 knots that the maximum value is given by $E=58.5$ (feet)^2; so $\sqrt{E}=7.65$ (feet). With the aid of table 1.5 the following *wave height data are computed:*

   Average height=13.5 feet

   Significant height=21.6 feet

   Average height of 1/10 highest waves=27.6 feet.

   The same values can be found in table 2.4 for a wind speed of 30 knots where they are tabulated for a fully arisen sea at various wind velocities.

3) *The optimum band* of the wave spectrum where most of the wave energy is concentrated can be found by equation (2.4). It is around the frequency

   $f_{max}=0.0826$ which corresponds to a

   period, $T_{max}=12.1$ seconds.

4) Take 5 percent of E and subtract it from E, that is

$$58.5-2.9=55.6$$

$$E\ (f)=55.6\ (\text{feet})^2.$$

   With this value go from the ordinate horizontally to the C. C. S. curve for 30 knots. Read off the frequency, $f=1/T$, or the period T, of the intersection at the abscissa:

$$f=0.06,\ T=16.7 \text{ seconds.}$$

   Take 3 percent of E, that is $E\ (f)=1.76$.

   With this value go from the ordinate horizontally to the 30-knot C. C. S. curve. Read off the frequency, or period of the intersection at the abscissa: $f=0.225$, $T=4.5$ seconds. (If all the curve is not shown on the diagram, extrapolate the value. This will be sufficiently accurate for practical purposes.) The range of significant "periods" is from 4.5 seconds to 16.7 seconds.

5) *The average "period"* is determined by equation (2.6). $\bar{\bar{T}}=8.55$ seconds.

6) *The most frequent "period"* can be found by means of figure 2.5b. Interpolation between the curves for 29 and 31 knots shows that the maximum point of the curve is at a "period" half way between 7.8 seconds and 8.3 seconds, so that the most frequent "period" is about 8 seconds.

7) *The range of "periods"* as the waves pass a fixed position is, according to the graphs in figure 2.5b, between 4 and 17 seconds. This is the same as the range of significant *periods* given by the C. C. S. curves as tabulated in table 2.5.
8) From the same graphs in figure 2.5b it is seen that waves with time intervals of 10 seconds between succeeding crests will pass this position only about half as often as waves with the most frequent "period" of about 8 seconds. Similarly, wave crests with a "period" of about 6.5 seconds will pass half as often as waves with an 8-second "period." "Periods" of 14 seconds are relatively few. On the average they occur about one-fifth as often as the most frequent "periods."
9) The average "wave length" from table 2.6 is 250 feet.

Summary of Example 2.1 (Extracting Most Important Part of the Data Given Above)

Fully Arisen Sea With a Wind Velocity of 30 Knots:

Average Wave Height	13.6 feet.
Significant Wave Height	21.6 feet.
Average Height of the 1/10 Highest Waves	27.6 feet.
Range of Dominating "Periods"	4 to 17 seconds.
Average "Period" and Approximately Most Frequent "Period."	8 seconds.
Period of Energy Maximum in Wave Spectrum.	11.8 seconds.
Average "Wave Length"	250 feet.

The Nonfully Developed State of the Sea

In cases where the fetch is not long enough, or the duration of wind action is too short to raise the composite wave motion to the point of a fully developed sea, allowances must be made for such limitations in describing the "sea" in the generating area.

If the rate of energy transfer from wind to waves and the rate of energy dissipation in every phase of wave development are known, it is possible to establish differential equations from which relationships among waves, wind speed, and fetch and duration are obtained as solutions. As mentioned in the introduction to this chapter, two principal cases have been considered.

Case A: Wave Characteristics Limited by the Wind Duration

A constant wind with mean velocity v (knots) blows over an unlimited stretch of water (fetch: nautical miles, N. M.). The energy added to the wave motion is the same everywhere so that the waves grow at all localities at the same rate with time, t (duration, hours). The stage of development of the sea depends only upon the duration of

wind action and is given by the DURATION GRAPHS in figures 2.4a, 2.4c, and 2.4e.

Case B: Wave Characteristics Limited by the Length of the Fetch

If the duration, t, of the wind action is found to be long enough to produce a steady state, but the fetch, x, is limited, the state of the sea depends only upon the stretch of water over which the wind has blown. This case is given by the FETCH GRAPHS in figures 2.4b, 2.4d, and 2.4f.

The Reason for the Fetch and Duration Lines

Both the duration and fetch graphs show the C. C. S. curves for different wind velocities between 10 and 56 knots. The duration graphs show a set of lines of equal duration in hours. The fetch graphs show lines of equal fetch length in nautical miles (N. M.). The intersection points of the C. C. S. curves with the duration or fetch lines show the limit of the development of the composite wave motion at the given duration or fetch. Physically, this means that the state of development is limited by a certain maximum amount of total energy which the wave motion can absorb from the wind under the given considerations. As an approximation, only the frequencies to the right of the point determined from the curve are present. The *E value* of the ordinate of this intersection point is a practical measure of the total energy accumulated in the wave motion of this nonfully developed state, as it was in the case for the fully arisen sea where the *maximum E* value of a given C. C. S. curve describes the waves.

Average "Period" and Average "Wave Length" for the Nonfully Arisen Sea

The spectrum of the nonfully arisen sea can be obtained from the spectrum of the fully arisen sea by discarding that part of the curve below a certain frequency. For example, in figure 2.3 the shaded portion stands for that part of the spectrum which would be present after a certain limited duration over a very long fetch (or at the end of a limited fetch after a long time). For a nonfully developed sea, the C. C. S. levels off with a constant value at a certain frequency. Thus the fetch and duration graphs also show the C. C. S. curves for *all* possible states of the sea. Actually the spectrum is cut off sharply but not vertically as deduced from the graphs. The approximation is sufficiently valid, but a better approximation is given below, where an additional small correction for the lower limit of significant frequencies is introduced.

The intersection of a fetch or duration line with the C. C. S. curve for a given wind velocity determines a frequency, f_1, such that, as an approximation, no frequencies less than f_1 are present in the waves.

For example, the intersection of the 38-knot C. C. S., with the 400-NM fetch line shows that no frequencies less than 0.07 are present.

The average "period" for a nonfully developed sea depends on the wind velocity and f_1. First, compute the ratio of the crest speed of the lowest frequency present to the wind speed, that is $3.03/(f_1 v)$, and read off the value of the term $F(3.03/(f_1 v))$ as given in table 2.7.

The average "period" is then given by equation (2.8)

$$\tilde{\tilde{T}}=0.285v\cdot F(3.03/(f_1 v)). \tag{2.8}$$

More quickly, if $3.03/(f_1 v)$ is less than 0.5, the average "period" is given approximately by equation (2.9):

$$\tilde{\tilde{T}}=0.77T_1. \tag{2.9}$$

Table 2.7—Values of $F(3.03/(f_1 v))$

$3.03/(f_1 v)$	$F(3.03/(f_1 v))$	$3.03/(f_1 v)$	$F(3.03/(f_1 v))$
0.1	0.089	0.8	0.670
0.2	0.179	0.9	0.720
0.3	0.268	1.0	0.780
0.4	0.350	1.2	0.870
0.5	0.430	1.4	0.930
0.6	0.510	1.6	0.970
0.7	0.588	2.0	1.000

For a nonfully developed sea, the average "wave length" is given approximately by equation (2.10) if the value of $3.03/(f_1 v)$ is less than 0.5. If the value of $3.03/(f_1 v)$ is greater than 0.5 and less than 1.4, then $\tilde{\tilde{L}}$ cannot be found easily.

$$\tilde{\tilde{L}}=2.56\ T_1^2 \tag{2.10}$$

If $3.03/(f_1 v)$ is greater than 1.4, then $\tilde{\tilde{L}}$ is given approximately by the value of $\tilde{\tilde{L}}$ for the fully developed sea.

A Correction To Be Used Inside the Generating Area

The wave spectrum does not stop sharply at a particular frequency for the nonfully developed sea. Some energy is present at frequencies less than f_1, although it is quite small. The lower limit of important frequencies can be found by subtracting a small correction, Df, as given by equation (2.11), from the value of f_1 as determined from the fetch and duration graphs. However, this correction should be applied *only in the case* where the value f_1 of the intersection is larger than or equal to the frequency f_{max} in the spectrum of the

fully developed sea at the given wind speed. The value of f_{max} for different wind speeds is tabulated in table 2.1 or can easily be computed by equation (2.4).[2]

The new frequency, f_u, is then found from equation (2.12).

$$Df=0.15\,f_1 \tag{2.11}$$

$$f_u=f_1-Df=0.85\,f_1 \tag{2.12}$$

Note that f_u is less than f_1. Therefore, T_u is greater than T_1. "Periods" corresponding to $1/f_u$ will be observed occasionally in the nonfully developed sea.

More Complicated Cases in Which Both Fetch and Duration are Limited

Under actual conditions, often both fetch and duration are limited, and the E value for any given situation in most cases will be different for the fetch and the duration. In such cases the smaller of the two E values must be taken. The reason for this will be explained in example 2.5.

When developing the theory of the growth of complex wave motion, the wind in the generating area has been assumed to be constant in time and space. In practice this will be true only approximately in some cases. In most cases the generating winds will be more complicated, and the determination of "fetch" and "duration" in the generating area is probably the most subjective factor in the process of wave forecasting. It is difficult to set up firm rules for determining these important parameters of wave forecasting, but in Chapters V and VIII some practical rules are given from first-hand experience. A wave forecaster can become proficient only by observing the sea and making tests of his forecasts at sea as often as possible.

Some Examples on the Use of the Fetch and Duration Graphs

In order to explain the use of the fetch and duration graphs in the case of a nonfully developed sea at given wind velocities, it is best to start with some simple examples.

Example 2.2.—FOLLOWING A PERIOD OF VERY WEAK SHIFTING WINDS OR NO WIND AT THE TIME, $t=0$, THE WIND FRESHENED TO 20 KNOTS WITHIN A RELATIVELY SHORT TIME. IT BLEW OVER A FETCH OF AT LEAST 200 NM WITH AN AVERAGE WIND VELOCITY OF 20 KNOTS. FORECAST AND DESCRIBE THE WAVES AT THE END OF

[2] Actually, a small correction should be applied in cases where f_1 is smaller than f_{max}, but this correction becomes smaller and smaller as the wave pattern approaches the fully arisen sea, and therefore it can be neglected for most practical purposes. This is the reason why, in this manual, a suggestion is made to apply the correction Df only in cases where f_1 is larger than f_{max}.

Table 2.8—Wave Characteristics for Limited Durations at a Wind Speed of 20 Knots

Duration	4 hours	6 hours	8 hours	12 hours
E value	1.18	2.99	5.55	7.7.
Frequency (period) of intersection $f_i(T_i)$	0.216 (4.6)	0.165 (6.1)	0.124 (8.1)	fully arisen sea $f_{max}=0.124$ $T_{max}=8.1$ sec.
3% of E	0.035	0.09	0.166	0.240.
f_L and T_L	>0.4; <2.5	~ 0.4; ~ 2.5	0.35; 2.9	0.333; 3.0.
f_u and T_u	0.183; 5.5	0.140; 7.1	0.105; 9.5	0.09; 11.1
$\tilde{T}$ sec	3.41	4.3	5.1	5.7.
$\tilde{L}$ ft	$\sim$	$\sim$	$\sim$	111.
$\sqrt{E}$	1.09	1.73	2.36	2.83.
Average ht Sig. ht Av. ht. 1/10 highest	1.9 ft 3.1 ft 3.9 ft	3.1 ft 4.9 ft 6.2 ft	4.2 ft 6.7 ft 8.5 ft	4.9 ft. 7.9 ft. 10.0 ft.

THIS FETCH AFTER 4, 6, 8, and 12 HOURS OF WIND ACTION. (This example describes conditions in a region of fresh trade winds.)

Method of Solution

It is seen from table 2.2 that a fetch of 200 NM is more than sufficient for the development of fully arisen sea at $v=20$ knots. Thus, only the duration graph, figure 2.4c, comes into consideration (Case A). Furthermore, it is seen from table 2.2 that after a continuous duration of wind action of 10 hours the sea is fully arisen.

The forecast is made according to the scheme in table 2.8. The single steps are explained in the following text.

The first step is to read off the E values and frequencies of the intersection points between the 20-knot C. C. S. curve and the given duration lines from the graph in figure 2.4c. These values are listed in table 2.8 as "E value" and "Frequency (period) of intersection." The period can be computed more accurately from the frequency, f_i, in using the formula, $T_i=1/f_i$, than it can be read off the T scale. The periods are given in parentheses.

The next step is to take 3 percent of the E value at given durations as shown in the table. Use these values to determine the *lower limit* of significant periods, T_L, from f_L. This is done by taking 3 percent of E as the ordinate and reading off the frequency, f_L, of the intersections with the 20-knot C. C. S. curve. At 4, 6, and 8 hours these intersections are beyond the range of frequencies on the abscissa at rather small periods corresponding to an f-value of more than 0.35. In this range the period scale is more and more exaggerated with increasing frequencies, and the small values of the periods can be estimated accurately enough for the purposes of this manual. At 8 hours the lower limit, T_L, is found with f_L approximately 0.35, or $T_L=2.85$ seconds. At 12 hours $f_L=0.319$, or $T_L=3.1$ seconds.

The *lower limit* of important frequencies, f_u, is determined by the "Frequencies of intersection," f_i, as given in the second row of the table, minus the additional small correction, Df for f_i larger than or equal to f_{max}, as given in equations (2.11) and (2.12). Thus, with a duration of 4 hours it follows that

$$f_u=0.85\ (0.216)=0.183$$

for the frequency of the upper limit of the significant "period" range as given by equations (2.11) and (2.12). The period $T_u=1/0.183=5.5$ seconds. Similarly with a duration of 6 hours it follows that

$$f_u=0.85\ (0.165)=0.140$$

or $T_u=7.1$ seconds, and so on. The f_L and f_u values, or T_L and T_u

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values, respectively, are listed in the table. The average "period" is computed from equation (2.8). It is 4.3 seconds.

For the fully arisen sea, the upper and lower limits of the range of significant frequencies, or periods, are shown in table 2.5 for different wind velocities. This table has been given for practical use in order to avoid repeated computations for cases of a fully arisen sea. For wind velocities between the given values, interpolation will be sufficient.

For a wind velocity of 20 knots, table 2.5 gives the range of significant frequencies and periods in a fully arisen sea. The range is from

$$T_L = 3.0 \text{ seconds}, f_L = 0.333$$

$$\text{to} \quad T_u = 11.1 \text{ seconds}, f_u = 0.09$$

These values are shown in table 2.8 of example 2.2 in the corresponding row at 12 hours of duration. The average "period" is 5.7 seconds. The average "wave length" is 111 feet.

The *height characteristics* of the sea at the given durations are computed according to the rules given in Chapter I. In the table for this example the square roots of the E values have been entered first. Below this value the computed height characteristics are given as "average height," "significant height," and "average height of the 1/10 highest waves."

From Table 2.8 the Summarized Forecast, for Example, at a Duration of 6 Hours Would be as Follows:

Answer to Example 2.2.—THE SEA HAS AN AVERAGE HEIGHT OF 3 FEET AND A SIGNIFICANT HEIGHT OF 5 FEET. (THE COMPUTED VALUES, 3.1 FEET AND 4.9 FEET, RESPECTIVELY, SHOULD BE ROUNDED OFF IN A REASONABLE WAY.) THE AVERAGE HEIGHT OF THE 1/10 HIGHEST WAVES WILL BE ABOUT 6 FEET, AND WAVES WITH HEIGHTS OF 6 TO 7 FEET MAY APPEAR OCCASIONALLY. THE RANGE OF "PERIODS" WILL BE FROM ABOUT 2 TO 7 SECONDS. IT MAY HAPPEN THAT A TIME INTERVAL BETWEEN SUCCEEDING CRESTS IS A LITTLE LARGER THAN 7 SECONDS, BUT THE *SIGNIFICANT* WAVE PATTERN WILL BE DESCRIBED BY THE 2- TO 7-SECOND INTERVALS.

With 12 hours of duration a similar description of the sea can be given. The most important wave characteristics are:

The significant height (1/3 highest): 8 feet,

Range of "Periods" (time intervals between succeeding crests): 3 seconds to 11 seconds.

For forecasting and describing the fully arisen sea, the forecaster is referred to the description of the fully arisen sea given at the start of this chapter in the appropriate tables, figures, and equations. Additional information for the case of a fully developed sea can be given by use of the graphs of the frequency distribution of time intervals between succeeding crests at a fixed position as explained on page 48. In this example refer to figure 2.5a for a wind speed of 20 knots. In all these examples, the computations have been carried out even for the fully developed state. If the forecaster is sure that the sea is fully developed, he can use the tables and figures and omit the computations.

Waves Limited by the Fetch

Example 2.3.—THERE ARE NAVAL OPERATIONS OFFSHORE AT A DISTANCE OF 100 NM TO 300 NM. THE WIND HAS BLOWN FROM THE LAND TO THE SEA WITH A VELOCITY OF 30 KNOTS FOR ABOUT 24 HOURS. FORECAST AND DESCRIBE THE SEA PATTERN AT A DISTANCE OF 100 NM, 200 NM, AND 300 NM OFF THE SHORE.

Method of Solution

Table 2.2 shows that with a duration of 24 hours a wind of 30 knots will generate a fully arisen sea, if the fetch is larger than 280 NM. Thus, for the forecast, the fetch graph (fig. 2.4d) has to be used (Case B). The forecast is made up as in table 2.9.

The single steps in forecasting the sea (table 2.9) are the same as in example 2.2, with the only difference that the fetch graph, figure 2.4d, is used. The values of E are read from the intersection points of the C. C. S. with the 100 and 200 NM lines. A fetch of 300 NM gives the fully arisen sea (minimum fetch 280 NM, according to table 2.2).

In order to determine f_L and f_u, or T_L and T_u, respectively, for the nonfully developed state, again use 3 percent of E, and equations (2.11) and (2.12) as in the preceding example. For the forecast at 300 NM (which is larger than the "minimum fetch") apply table 2.5.

The height characteristics have been computed from the E values, by multiplying $\sqrt{E}$ by the factors given in table 1.5. Again, the height characteristics for a fully arisen sea can be taken from table 2.4.

The average "period" for different fetches, $\widetilde{T}$, at each fixed position is found from equation (2.8). Information about the frequency of

Table 2.9—Wave Characteristics for Limited Fetches at a Wind Speed of 30 Knots

Fetch (NM)	100 NM	200 NM	300 NM
E value	14.5	38.8	58.5.
Frequency (period) of intersection, f_i (T_i).	0.125 (8.0)	0.085 (11.8)	fully arisen sea $f_{max}=0.0826$ $T_{max}=12.1$ seconds
3% of E	0.435	1.16	1.76.
f_L and T_L	0.28; 3.6	0.24; 4.2	0.21; 4.7.
f_u and T_u	0.106; 9.4	0.072; 13.9	0.06; 16.7.
$\widetilde{\widetilde{T}}$ sec	5.8	7.4	8.6.
$\widetilde{\widetilde{L}}$ ft	~	~	253.
$\sqrt{E}$	3.81	6.23	7.65.
Average ht	6.7 feet	11.0 feet	13.5 feet.
Sig. ht	10.8 feet	17.6 feet	21.6 feet.
Av. ht. 1/10 highest	13.7 feet	22.4 feet	27.5 feet.

time intervals at a fixed position in the fully developed sea at a wind speed of 30 knots can be given by the use of figure 2.5b.

ANSWER TO EXAMPLE 2.3.—ONE HUNDRED NM FROM THE COAST, THE WAVES WILL HAVE A SIGNIFICANT HEIGHT OF 11 FEET AND A "PERIOD" RANGE FROM 3.6 TO 9.4 SECONDS. THE AVERAGE "PERIOD" WILL BE 5.6 SECONDS.

TWO HUNDRED NM FROM THE COAST, THE WAVES WILL HAVE A SIGNIFICANT HEIGHT OF 18 FEET AND A "PERIOD" RANGE FROM 4.2 TO 13.9 SECONDS. THE AVERAGE "PERIOD" WILL BE 7.4 SECONDS.

THREE HUNDRED NM FROM THE COAST, THE WAVES WILL BE FULLY DEVELOPED AS DESCRIBED IN PREVIOUS SECTIONS OF THIS CHAPTER. (See the appropriate tables and graphs for a fully arisen sea.)

EXAMPLE 2.4.—A PLANE DITCHED 200 NM FROM SHORE. THE WIND VELOCITY WAS 40 KNOTS AND THE WIND DIRECTION WAS AT AN ANGLE OF 45° FROM LAND TO SEA. THE VELOCITY OF THE WIND HAD BEEN 40 KNOTS FOR 24 HOURS, AND ON THE DAY BEFORE THE WIND HAD INCREASED STEADILY FROM 30 KNOTS TO 40 KNOTS. FIGURE 2.6 ILLUSTRATES THE SITUATION. THE AEROLOGIST ABOARD A CARRIER 500 NM FROM THE COAST

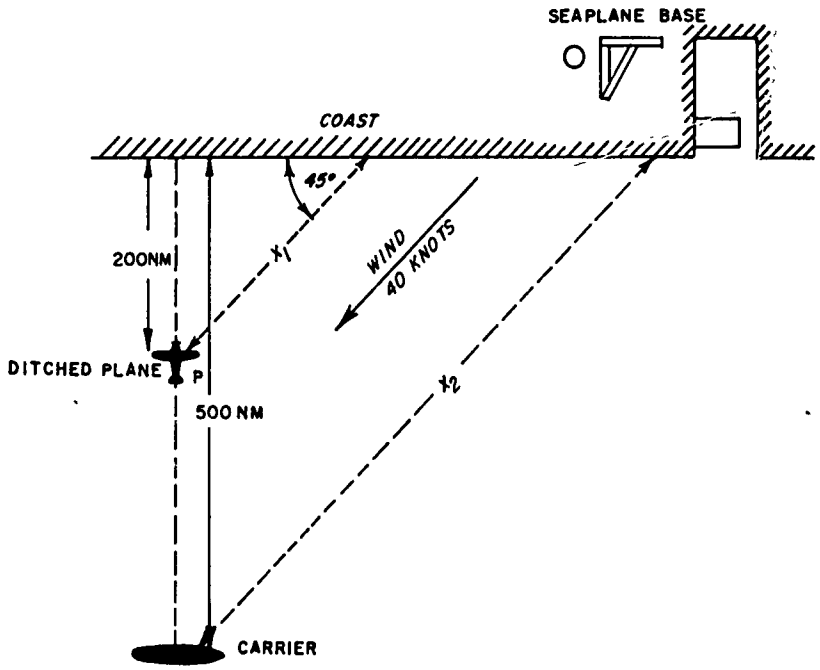

FIGURE 2.6 Forecast conditions for an offshore wind.

HAS TO FORECAST THE SEA CONDITIONS AT THE POINT WHERE THE PLANE DITCHED AS SHOWN IN FIGURE 2.6 IN ORDER TO DECIDE WHETHER A SEAPLANE FROM THE LAND BASE SHOULD RESCUE THE PILOT OR WHETHER A SLOWER SHIP SHOULD BE SENT.

Method of Forecast.—The *effective fetch* for the landing place of the plane is

$$x_1=\frac{200}{\sin 45^\circ}=\frac{200}{0.707}\text{ NM}$$

$$x_1=280\text{ NM approximately.}$$

Here is an example of more complicated duration conditions. The wind of 30 knots, two days ago, raised a fully developed sea with an E value (for $v=30$ knots) of 58.5 ft.^{2} To find approximately the increase of wave development while the wind freshened steadily to 40 knots during the 24-hour interval, take $E=58.5$ ft.^{2} on the C. C. S. curve of 35 knots (35 knots is considered an average wind velocity during the period of increasing wind). The values for 35 knots should be interpolated between 34 and 36 knots.

The value $E=58.5$ on the interpolated 35-knot curve in the duration graph, figure 2.4c, indicates an "equivalent" duration of about

22 hours. Add 22 hours and 24 hours and follow the interpolated 35-knot C. C. S. curve with increasing duration up to 46 hours. It is found that this time is more than sufficient to raise a fully developed sea with 35 knots (duration graph, figure 2.4c). Thus, during this increase of wind, the sea continued to grow. As the wind increased in velocity, the sea attained the fully developed state (almost) at each time during this 24-hour interval. Further, it is seen that with the value of $E=58.5$, even on the 40-knot C. C. S. curve given in figure 2.4e, corresponding to an "equivalent duration" of 18 hours, the sum 18+24 hours=42 hours on the 40-knot curve gives nearly the fully arisen sea. Thus, since the 40-knot wind blew after the period of increase for another 24-hour interval with the same speed, it is certain that this duration is enough to produce steady conditions, and that only the fetch graph for forecasting the sea at the end of the fetch of 280 NM has to be used.

For 40 knots and 280 NM, the graph in figure 2.4f gives the following values.

$$E=75 \text{ (ft.)}^2$$

$$f_t=0.089$$

ANSWER TO EXAMPLE 2.4.

Forecast of Wave Conditions at Point Where Plane Ditched

Average height	15.3 feet
Significant height	24.5 feet
Average ht. of 1/10 highest	31.1 feet
"Period" range	5 seconds to 13 seconds

Obviously the Seaplane Can Never Land at the Proposed Landing Place. The Ship Will Have To Be Sent.

The wave forecaster on the carrier should make a forecast at the same time for his own position. Not only does he gain a lot of experience by this practice, but also will he be able, in case of discrepancies, to correct his forecast for the landing place of the plane. In case of discrepancies, the theory is not necessarily at fault. With complicated fetch and duration conditions it may be that the parameters employed, fetch and/or duration, are improperly determined. Such sources of error can be corrected by observing the sea and making tests as often as possible.

In the case of example 2.4, the effective fetch for the carrier is

$$x_2=\frac{500}{\sin 45^\circ}=710 \text{ NM}.$$

With a wind velocity of 40 knots, a distance of 710 NM is just the minimum fetch necessary to raise a fully developed sea according to table 2.2. The sea at the position of the carrier will be practically fully arisen. The forecaster simply has to use table 2.4 for the *height characteristics:*

Average height	28 feet
Significant height	45 feet
Average ht. of 1/10 highest	58 feet

(Round off the values in table 2.4 in a reasonable way.)

The state of the sea at the carrier can then be found from the results of this chapter which apply to a fully developed sea.

Thus the plane 200 NM off the shore will not find conditions which make a landing possible, and the sea conditions 500 NM offshore at the position of the carrier are very rough indeed. It would be necessary to send a ship to rescue the pilot.

Example 2.5.—FIND THE WAVE CONDITIONS AT ALL POINTS OVER A 1000-NM FETCH AFTER A WIND WITH A VELOCITY OF 34 KNOTS HAS BLOWN FOR 6 HOURS. FIND THE CONDITIONS AFTER 10 HOURS, AFTER 16 HOURS, AFTER 24 HOURS, AND AFTER 30 HOURS. (THIS IS AN EXAMPLE OF A GENERAL CASE OF LIMITED FETCH AND DURATION.)

Method of Solution

On page 55 it was said that under conditions where both fetch and duration are limited the smaller of the two E values, as found by the fetch or duration graphs, respectively, has to be taken. This statement will be explained in the following example.

The growth of the wave pattern at a wind velocity of 34 knots as determined by the graphs and tables in this chapter is illustrated in figure 2.7. It shows the height and period characteristics of the composite wave pattern as functions of the distance from the coast (fetch in NM) for various wind durations (hours). When the wind has blown for 6 hours after a period of calm, one finds that with increasing distance from the coast the waves increase rapidly out to a distance of 50 NM. The significant height at this point is 7.5 feet, the average wave height is 4.5 feet, and the height of the one-tenth highest waves is 9.2 feet. The average "period," $\overline{\overline{T}}$, is 4.6 seconds, and T_u, is 7.4 seconds in the upper limit of significant periods.

Beyond 50 NM the wave pattern is the same at all distances, as indicated in figure 2.7 by the horizontal lines for the given duration of 6 hours. If the wind continues to blow, the waves will grow only at distances larger than 50 NM off the coast, and the wave pattern inside 50 NM will remain the same at any given point. That is, a *steady*

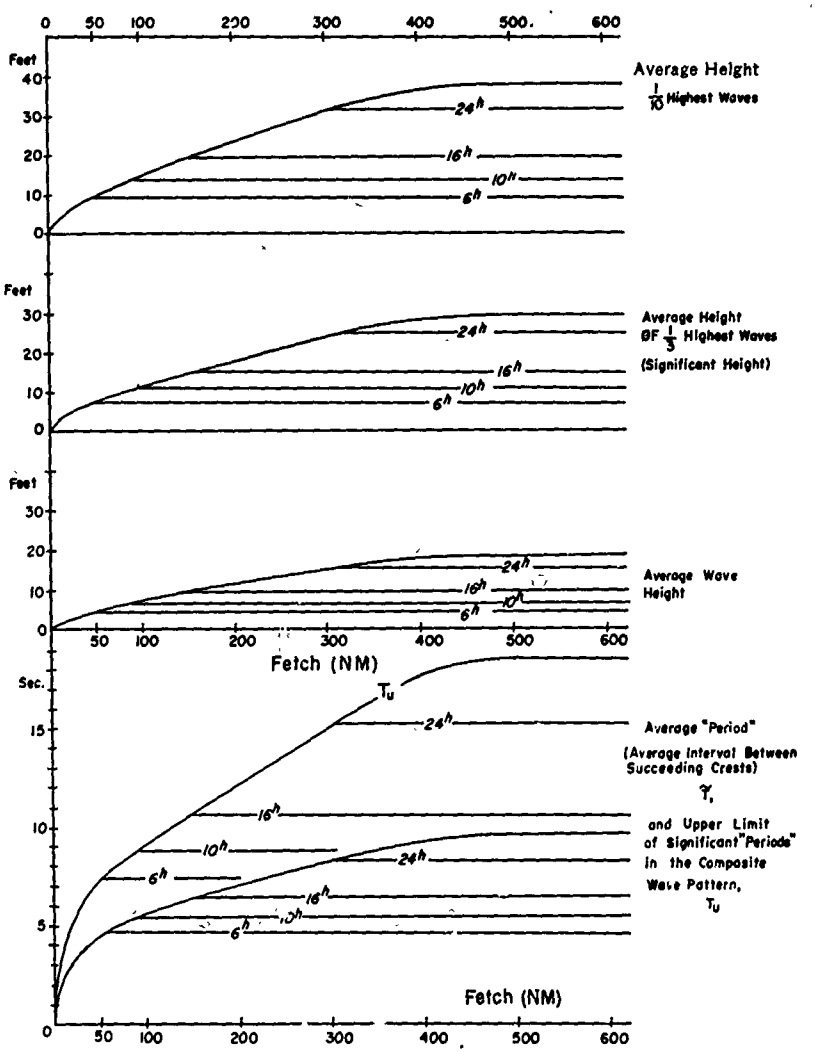

FIGURE 2.7 Fetch-limited waves. Characteristics of the composite wave pattern at a wind velocity of 34 knots, shown as a function of the distance from the coastline (fetch) at different durations (h=hours) ~~after wind~~ started.

state has been reached at fetches shorter than 50 NM, and the state of the sea is determined only by the distance (fetch) off the coast.

After 16 hours a steady state has been established to a distance of 150 NM. The characteristics of the wave pattern between the coast and a point 150 NM off the coast are now given by the curves in figure 2.7 from the zero point up to the point where the 16-hour line branches off horizontally. Beyond the 150-NM fetch, the waves are everywhere the same and their characteristics are given by the horizontal lines labeled 16 hours.

After 24 hours, the steady state has reached a distance of about 300 NM. At this point the wave characteristics according to the curves in figure 2.7 are as follows:

Average ht. of 1/10 highest	31.0 feet
Significant height	24.3 feet
Average height	15.2 feet
Average "period" $\tilde{\tilde{T}}$	8.3 seconds
Upper limit of significant "periods" T_u	15.3 seconds

The fully arisen state of the sea at a wind velocity of 34 knots is practically attained when the fetch exceeds 420 NM, and the duration is 30 hours from the time when the wind started to blow with a constant speed over an undisturbed water surface. The wave characteristics of fully arisen sea for a wind speed of 34 knots are also shown in tables 2.4, 2.5, and 2.6.

In cases where fetch and duration are limited the fetch graph has to be used when the E value from the fetch graph is smaller than the E value from the duration graph, and vice versa. This becomes evident when considering conditions in example 2.5, figure 2.7. Assume that a wind has blown with a constant speed for 16 hours, and that a forecast has to be made for a fetch of 100 NM. The steady state has developed from the coast to a distance of 150 NM offshore, as shown by the curves in figure 2.7. Thus, the limiting factor is the fetch of 100 NM, which, of course, gives a lower E value than the 16-hour duration line. (Compare figures 2.4c and 2.4d.)

Answer to Example 2.5.—THE WAVE CHARACTERISTICS FOR ANY FETCH AT ANY DURATION ARE AS IN FIGURE 2.7.

Distorted C. C. S. Curves

Advantages and Disadvantages of the C. C. S. Curves

The C. C. S. curves in figures 2.4a, b, c, d, e, and f are plotted on a linear E-scale and a linear f-scale. In order to cover ranges of E from 0 to 1200 on both fetch and duration graphs, it was necessary to plot six different sets of curves for different ranges; and yet for short fetches or durations at high winds, the E values cannot be read. Thus supplementary enlarged portions of the C. C. S. curves would also have to be drawn up in order to present the complete range of forecast situations.

Just as much space on the frequency scale is needed to cover *periods* from, say, 30 seconds to 12 seconds as is needed to cover *periods* from 12 seconds to 7 seconds. In addition, if E is increased by a factor of four, the significant height is increased only by a factor of two; hence, the significant height scale on the right is very much condensed for low heights. Also, only about one-third of the area of the diagram

contains curves which can be actually used in forecasting, and thus a lot of space is wasted.

The above considerations show a number of disadvantages in graphing the C. C. S. curves on linear E and frequency scales. There are a number of advantages, however, and these advantages are described below.

As will be described in Chapter III, swell forecasts require the computation of the difference between two E values on a C. C. S. curve as read off at two frequency values. Equal differences in E values are of equal importance in forecasting swell. Also, frequencies enter more easily in the theory of swell forecasting. Thus, the scales in these figures are the natural ones for swell forecasts.

The Distorted C. C. S. Curves

The disadvantages of the C. C. S. curves can be eliminated by plotting distorted C. C. S. curves. These distorted curves are obtained from the C. C. S. curves by plotting the E value on a nonlinear scale and the frequencies over part of the range on a reciprocal scale. New disadvantages are also introduced, but the advantages outweigh the disadvantages.

Figure 2.8 compares the C. C. S. curve for 32 knots on the left with the distorted C. C. S. curve on the right, which results from the use of different scales. On the distorted curve, the significant height is linear, and the E values for equal increments bunch at the top of the scale. Over 3 times as much of the scale is used to go from zero to ten in the distorted curve as is used in the regular curve, but only

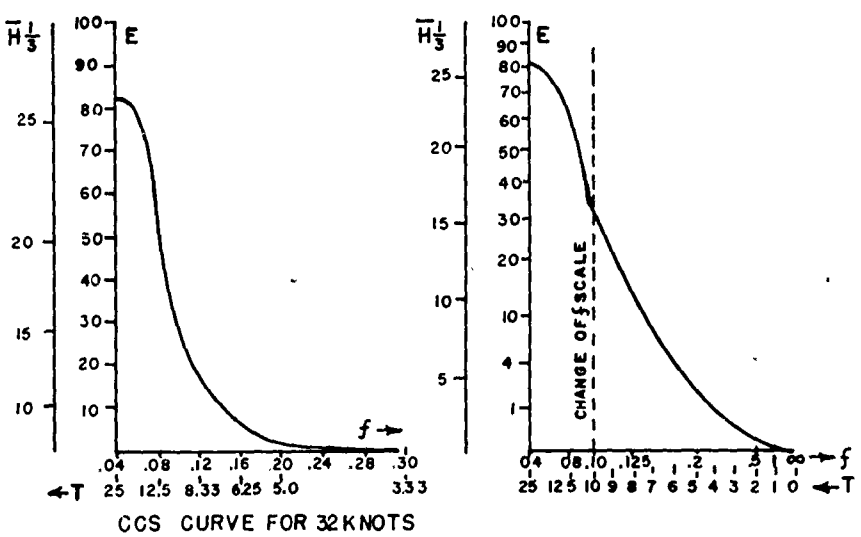

FIGURE 2.8 Comparison of C. C. S. curves plotted in different coordinate systems.

half as much distance is used to go from ninety to one hundred as is used on the undistorted scale.

On the distorted scale in figure 2.8, frequencies from 0.04 to 0.10 are plotted linearly, but then *periods* from ten seconds to zero seconds are plotted linearly in descending order. The result is that frequencies from 0.10 to infinity all occur on the frequency scale, but the frequency scale is not linear to the right of the dashed line at 0.10.

The distorted C. C. S. curve is found simply by reading the E values for given frequencies on the regular curve and plotting these values at the correct points in the distorted scale. The low E values are thus magnified, and the curve is moved out more into the center of the area of the graph paper.

When a family of distorted C. C. S. curves is plotted, the result is that the full range can be covered in just four diagrams and that the values for short fetches and low durations can be read off easily on the curves.

The distorted C. C. S. curves are given in figures 2.9 a, b, c, and d. They are also enclosed in the envelope of loose charts and tables which accompanies this manual for use in day-to-day forecasting. Of course the original C. C. S. curves can also be used in some cases just as easily.

On these distorted C. C. S. curves, dashed lines which show the values of T_u and T_{max}. are shown. Note that the distorted C. C. S. curves do not have a maximum slope at the value, T_{max}. However, the E value increases most rapidly as a function of f_1 at this point.

Disadvantages of the Distorted C. C. S. Curves

The advantages of the distorted C. C. S. curves are many, as can be seen by simply inspecting them. The disadvantages occur only in swell forecasting. These disadvantages will be discussed in Chapter III, and ways to avoid errors in their use will be given.

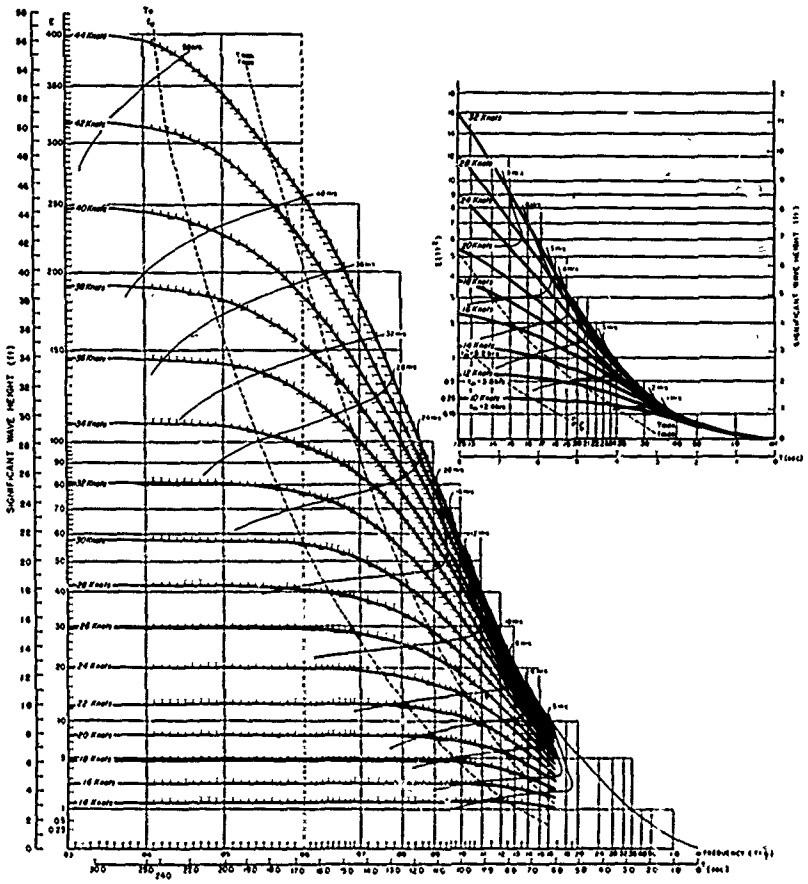

FIGURE 2.9a Duration graph. Distorted co-cumulative spectra for wind speeds from 10 to 44 knots as a function of the duration.

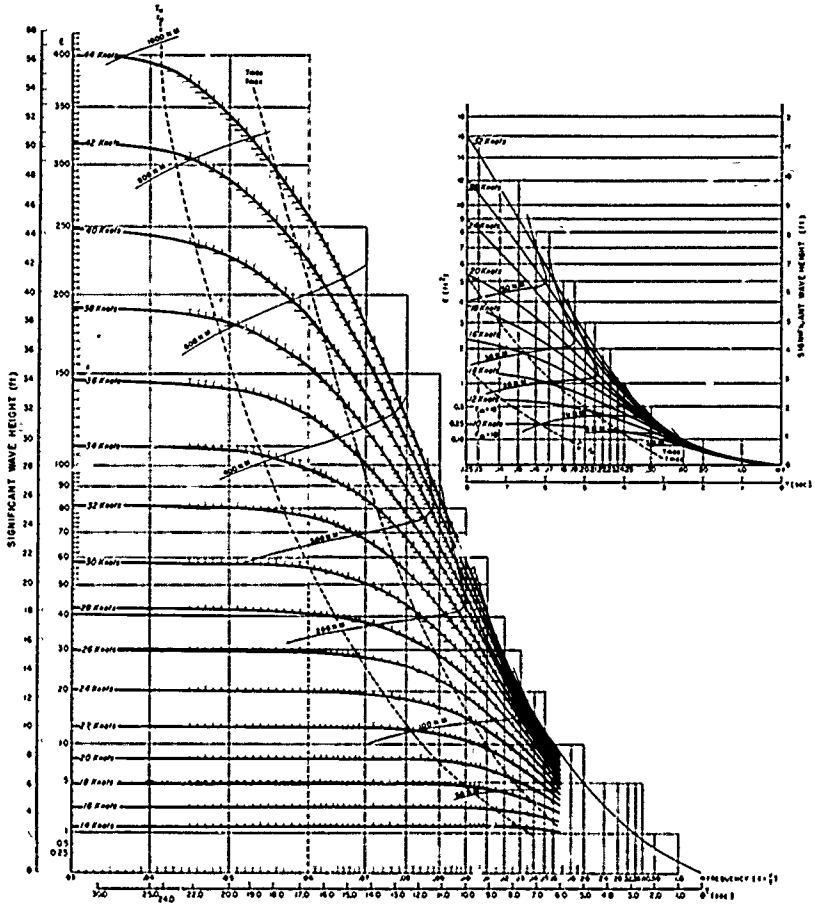

FIGURE 2.9b Fetch graph. Distorted co-cumulative spectra for wind speeds from 10 to 44 knots as a function of the fetch.

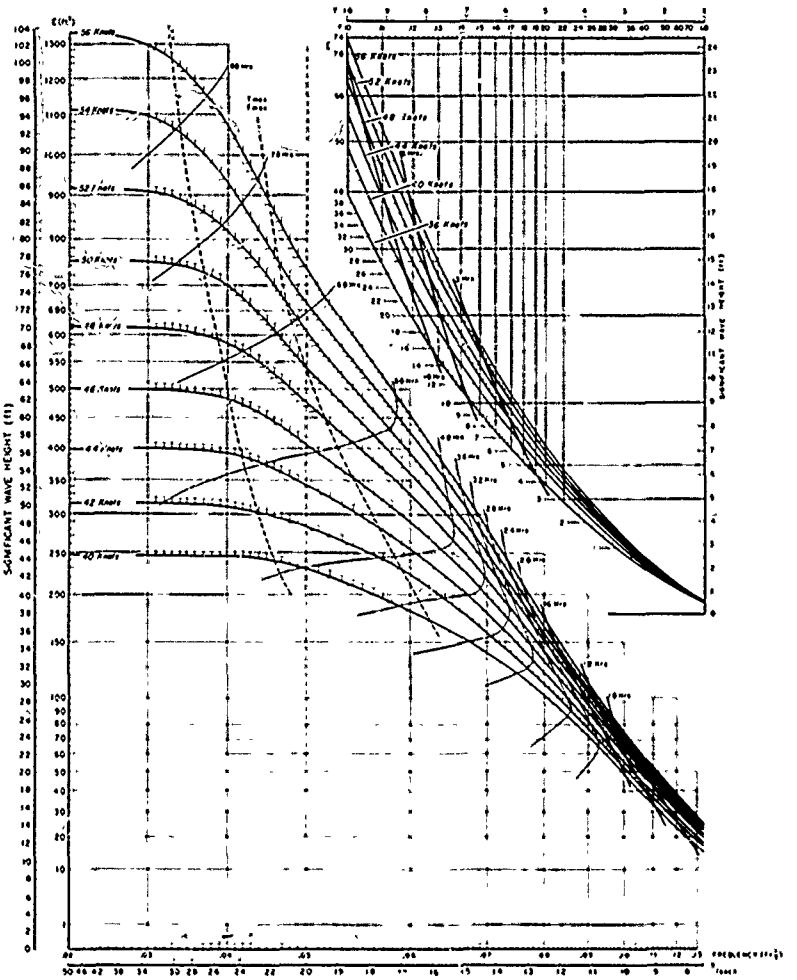

FIGURE 2.9c Duration graph. Distorted co-cumulative spectra for wind speeds from 36 to 56 knots as a function of the duration.

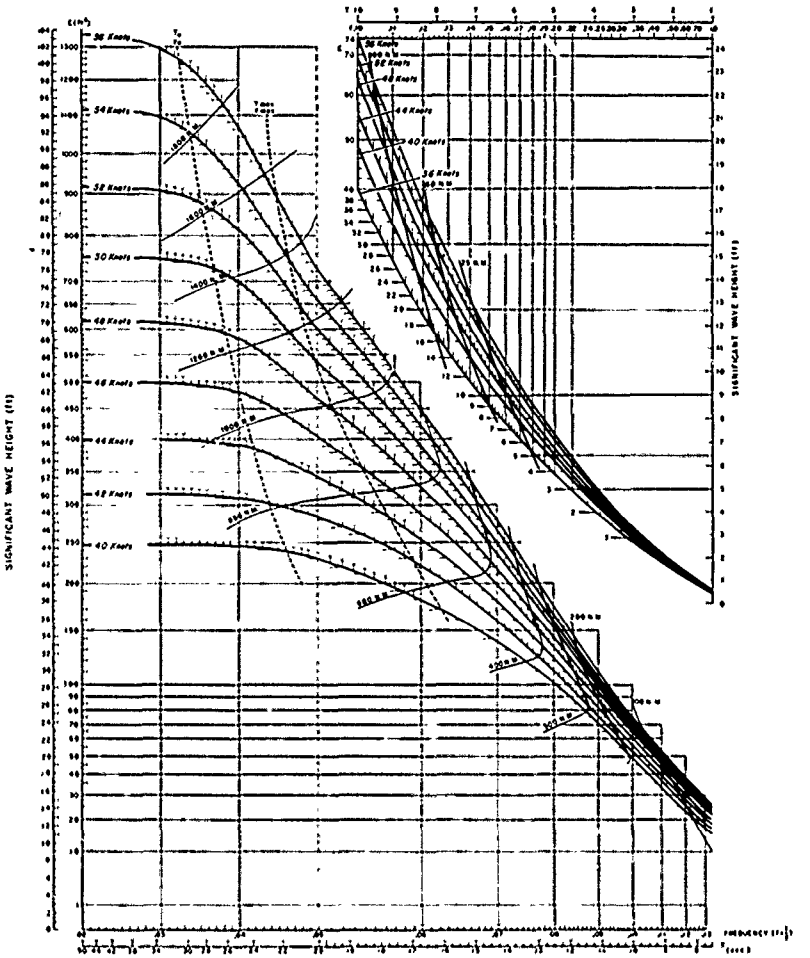

FIGURE 2.9d Fetch graph. Distorted co-cumulative spectra for wind speeds from 36 to 56 knots as a function of the fetch.

Chapter III

WAVE PROPAGATION AND FORECASTING SWELL WAVES—SIMPLE MODELS

Introduction

There would be no problem of the propagation of ocean waves if the waves were to last forever. A simple harmonic progressive wave never starts and never stops. Methods must be developed to take care of the fact that waves build up under the action of wind with time, propagate out of the generating area, and eventually, in some particular generating area, die down and vanish. One of the inadequacies of past theories was that it was not possible to forecast when the waves would *cease*. It will be found that the time when the wind ceases in a generating area is just as important as the time when it begins when one attemps to forecast how long the waves from a given generating area will last. The dimensions of the storm are also of extreme importance, as pointed out by DeLeonibus (1955).

A Very Simple Forecast

The wind generates a sum of many sine waves in order to make a "sea." Or, more precisely, the wind generates pieces of sine waves to produce the sea surface in the generating area. What happens to pieces of simple harmonic progressive waves will be worked out first, and then the results will be generalized to real wave conditions.

Consider a very special and idealized storm over the North Atlantic. It begins and intensifies over the northwest corner of the North Atlantic with the storm center about 600 NM off the North American continent. At a certain time, $t=0$, real sinusoidal waves start to leave the leeward edge of the storm along a line 600 NM long. They last for 40 hours and then stop just as suddenly. The storm and the coordinates considered are shown in figure 3.1.

Now these waves are *not* what would be recorded as a function of time if a simple harmonic wave were passing the line $x=0$. A simple harmonic progressive wave never starts and never stops. The disturbance caused by this special idealized storm does start and stop. It is also of finite width, being only 600 NM wide. Suppose that the

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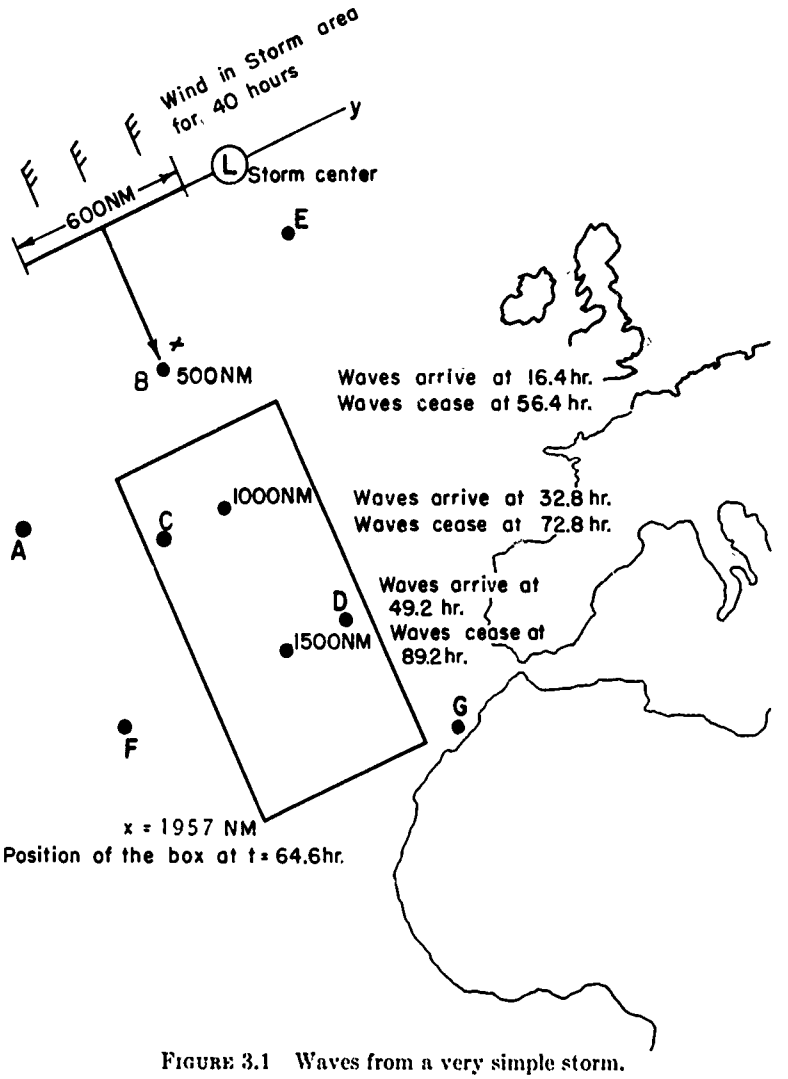


FIGURE 3.1 Waves from a very simple storm.

wave amplitude (crest to sea level) is 10 feet, and that the time interval between successive waves is exactly 20 *seconds*. There is certainly a disturbance present at $x=0$. What time will it arrive at other values of x, and what will it look like then?

A solution to the problem posed above can be given. Let $W_s/2$ be half the width of the disturbance, and let D_w be the 40 hours that the waves last at the source. Also, note that there is an x, y coordinate system defined in figure 3.1 with $x=0$, $y=0$ at the center of the leeward edge of the storm. The symbol, T^*, stands for 20 seconds in this particular example. A very simple visual picture can be given

of the nature of the solution. Imagine a huge box 10 feet high and 600 NM wide. Let the leeward edge of the box be at $x=0$ and $t=0$. Now start moving the leeward edge down wind in the positive x direction and at a speed given by 1.515 T^* knots (30.3 knots). Forty hours later, for this example, the windward end of the box will pass the line $x=0$. The box will be about 1,215 NM long.

The result is an imaginary invisible box scooting along at a speed of 30.3 knots. It is 600 NM wide and 1,215 NM long. The location of the box 64.6 hours after the waves started is shown in the figure. Underneath the box let there be a simple sine wave 10 feet high with a form given by

$$10 \sin \left[\frac{4\pi^2 x}{gT^{*2}} - \frac{2\pi t}{T^*}\right].$$

Anywhere outside of the box let the sea be as calm as a pond on a windless day. When the box arrives at a given point, there suddenly appears a great number of 10-foot high waves which will last 40 hours. A wave record taken while the waves were present would look exactly like figure 1.5. There will therefore be about 7,200 waves since there is one wave every 20 seconds.

The wave crests travel with a speed of 60.6 knots ($3.03T^*$). They travel twice as fast as the box. Therefore they appear to form at the rear or windward edge of the box, race forward through the box, and disappear at the front or leeward edge of the box.

Group Velocity

These results depend upon the assumption that the waves are like a simple sine wave as shown by figure 1.5. When such a simple sine wave is started and stopped along a given line source such that it has a certain width and lasts a certain time, it can be shown that the energy of the system travels forward with the *group velocity* of sine waves in very deep water, which equals one-half of the velocity of the crests of the waves. That is the reason for advancing the front and rear edges of the box, which are boundaries of the *envelope* of the disturbance, at a speed given by 1.515 T^* knots. That the side edges do not spread very far also has just recently been shown by theoretical considerations.

A formal solution would smooth out the effects of the sharp sides of the box. Over the whole interior of the box to within a few nautical miles of the sides of the box, the above description is adequate. To be more precise, it would be necessary to permit a little of the disturbance to leak out into the surrounding still water. A few very low waves run out ahead of the forward edge of the box. The walls on the side smooth out a little so that the once sharp edges become rounded.

The discussion given in H. O. Pub. No. 604 explains the properties of the group velocity by a physical example and some computations involving the heights of the waves near the forward edge of the envelope. The same arguments would also apply in reverse to the rear edge of the envelope. They show that the approximation made here is sufficiently accurate for all practical applications when applied to simple sine waves.

Interpretation of Results

These results are extremely interesting in many ways. After all, inside the wave system, for as far as the eye can see and for 40 long hours, it will not be possible to tell the difference between these waves and pure sine waves which last forever. While the waves are present, they can be treated as though they were pure sine waves lasting forever, but the formulas in a sense tell when the waves are turned on or off.

Another important point is that the waves never arrive at points to the side of the storm. No waves ever arrive at the point, say, $y=(W_s/2)+(W_s/10)$. A large part of the ocean area is never affected by these waves.

Finally, the results show that the waves are still as high when they reach the distant points as they were at the start. At this point, some theorists will object and say that viscosity and friction are the major causes that account for the observed fact that actual waves decrease in amplitude as they propagate out of the generating area. If actual ocean waves were like this model, then the effects of viscosity and friction would have to be taken into account in order to make the waves of the model die down. This is *not* the case. Actual ocean waves are *not* like the model just described. In many cases, the decrease of wave height can be explained by two processes: dispersion and angular spreading. (The terms dispersion and angular spreading will be defined later.)

The Results of the Simple Forecast

As a review, the forecasts which would be made from this simple model can be summarized. Seven points, A, B, C, D, E, F, and G are shown on figure 3.1. Point A is not in the path of the waves. Therefore at point A the forecast is always no waves. Point B is 500 NM from $x=0$. In 16.4 hours the waves will arrive at this point. There will be one wave exactly 10 feet high every 20 seconds for the next 40 hours. Then the waves will cease at point B at exactly 56.4 hours after they started from $x=0$. Point C is 1,000 nautical miles from $x=0$. The waves arrive at 32.8 hours and cease at 72.8 hours. Similarly for point D, 1,500 nautical miles away, the waves arrive 49.2 hours after they leave the source and cease 89.2 hours after they

leave the source. Other points E, F, and G, are shown. No waves are ever observed at these points. Note in figure 3.1 how much a change in the direction of the waves by 5° at the source would affect the forecast for distant points.

A Forecast for a Sum of Sine Waves

Consider the same line source for the waves as was assumed for the simple forecast above. Instead of a simple sinusoidal wave emanating from the storm, consider a sum of sine waves. Table 3.1 tabulates wave amplitude in feet, wave frequencies ($1/T$), and the direction toward which each wave is traveling. Each wave is a pure sine wave for which the classical formulas hold.

Table 3.1—Directions, Amplitudes, and Frequencies for the Terms of a Sum of Sine Waves (Amplitude in Feet)

Frequencies	−22.5°	−15°	−7.5°	0°	7.5°	15°	22.5°
1/20	------	1/8	1/4	1/2	1/4	1/8	------
1/15	1/8	1/4	1/2	1	1/2	1/4	1/8
1/12	1/8	1/8	1/4	1/2	1/4	1/8	1/8
1/10	1/8	1/8	1/4	1/4	1/4	1/8	1/8
1/6	1/8	1/8	1/8	1/8	1/8	1/8	1/8

At the leeward edge of the storm, let all these wave trains start out at the same time. Let the front of the storm be 600 NM wide as before. For simplicity, for the trains traveling toward 7.5°, let the center of the leeward edge of the storm be the forward edge of a box similar to the one described above, but let the forward edge of the box be inclined at an angle of 7.5° with the y axis. Let the width along this line be given by 600 cos 7.5°. Similarly, let the forward edges of the other various components be at corresponding angles to the y axis and let the width be appropriately modified.

Suppose that all the above 33 sine waves start out from $x=0$, $y=0$, $t=0$, and let each component at the source last for 10 hours. What does the disturbance look like in the vicinity of the source? There are 33 terms in the sum. If the complete phase reinforcement were ever added at some time and place, there would be momentarily a wave 7.625 feet from mean sea level to the crest. Yet the sum of the squares of the amplitudes, which is equal to E, is equal only to 2.86 ft.^{2} The waves at places are thus almost as high as the simple sine waves of the previous model, and yet the value of E is very much less, since in the previous model, E was equal to 100 ft.^{2}

How does the disturbance propagate? Each term in the disturbance travels down wind with its own group velocity; namely, 20×1.515, 15×1.515, 12×1.515, 10×1.515, and 6×1.515, respectively. Each

disturbance travels off in its own direction which ranges from $-22.5°$ to $+22.5°$. The position of the box which surrounds each term 64 hours after the start of the storm is shown in figure 3.2. The boxes for the waves with a 20-second period are 1,965 NM from the point $x=0$, $y=0$. They are about 300 NM long, and *if* they arrive at a given point, they pass that point in 10 hours. The boxes for the waves with a 15-second period are about 1,470 NM from the source, and they are 225 NM long. The values for the other terms in the sum of sine waves are given on the figure.

These results show that the various sine waves *disperse* and *spread angularly* as they leave the source and travel into the area of calm. First, 20-second waves reach a given point. Then 15-second waves arrive, followed by 12-second waves and so on. Waves arrive at points to the side of the storm that they could not possibly reach if the storm were of finite width and the sine waves were all traveling in the same direction. Waves are short-crested in the generating area, and they must travel out of the storm at angles to the dominant direction of the wind.

Forecasts can be made for various points in the diagram just as for the simpler cases first described. It is not necessary to be as precise as in the first example, and the general features are evident from the figure. Since the individual terms of the sum of sine waves travel off in different directions, the waves will be lower at points a greater distance from the source owing to this effect. Since the waves travel at different group velocities, they arrive at different times at the distant points; thus, the combined effect is lower.

At very great distances from the source, something occurs which seems strange in nature but is not strange in this particular model. Each rectangular area spreads out so far from the others that calm water appears in between them. At a great distance along the x axis, an observer would first observe a portion of a sine curve with a 20-second period one-half foot high. It would last for 10 hours and then cease. The ocean would be perfectly calm. It would be necessary to wait some number of hours, and then waves one foot high with a *period* of 15 seconds would arrive and last 10 hours. Later on, the 12-second waves would show up, followed by the 10-second waves; and finally, after the appropriate interval, the 6-second waves would come along. A diagram such as figure 3.2 could be constructed for a storm lasting 40 hours, but the various terms would overlap so much that the figure would be confusing.

This example is quite unrealistic because of the calm water between the different areas. The reason it is unrealistic is that the model is unrealistic. It is important to consider an infinite number of infini-

Frequency	Period	Distance traveled in nautical miles	Length of disturbance in nautical miles
.050	20	1960	300
.067	15	1470	225
.083	12	1180	180
.100	10	980	150
.167	6	590	90

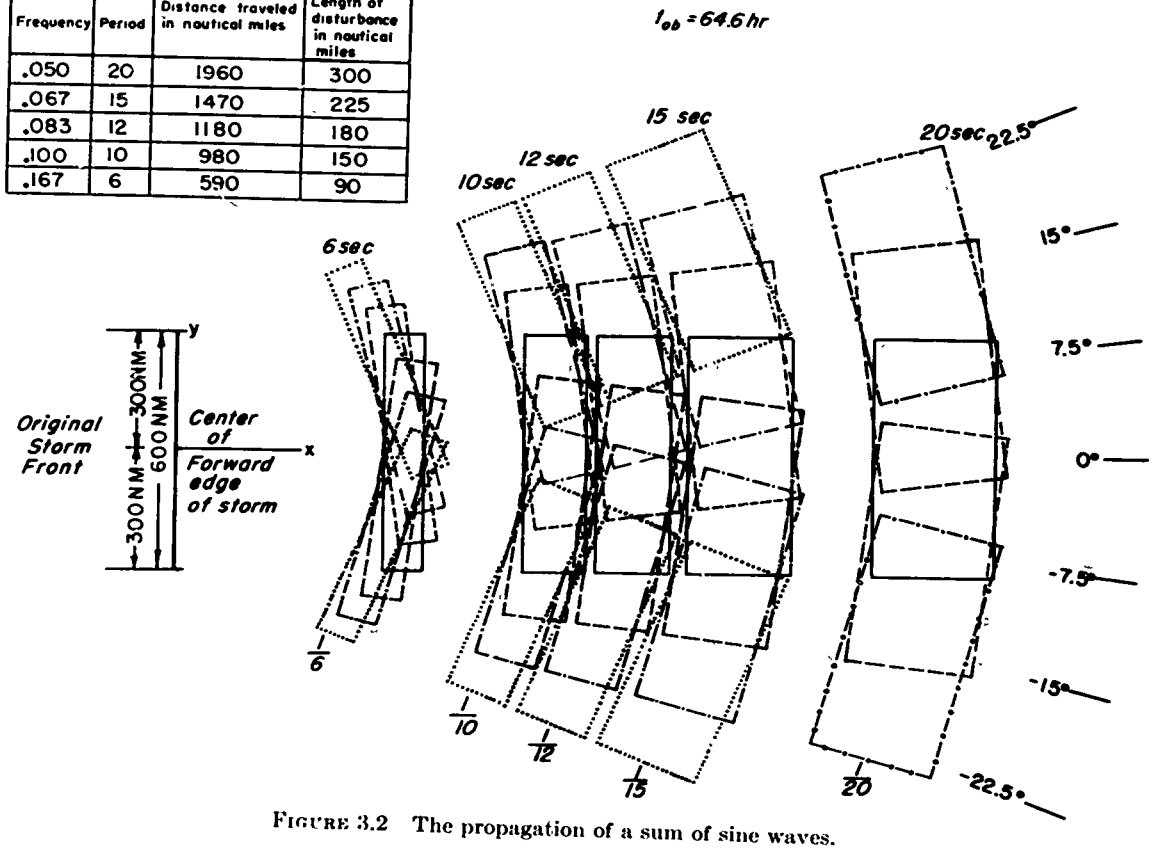


FIGURE 3.2 The propagation of a sum of sine waves.

tesimally high sine waves which are represented by a continuous spectrum as in Chapter II.

A Forecast for a Very Large Sum of Very Low Sine Waves

As the next step, forecast procedures for actual ocean waves will be described. It would be difficult to construct the infinite number of boxes required for the actual continuous spectrum, although it can easily be done for a finite sum of a few terms. The easier way is to consider what can happen to any one particular term and then generalize to a large number of terms.

Dispersion

The verb "to disperse" means "to spread out." In ocean wave theory deep water is a dispersive medium. This means that an originally limited irregular disturbance spreads out as it travels. The dispersiveness of waves in deep water is explained by the fact that the energies of different spectral frequencies travel with different group velocities. DISPERSION, IN OCEAN WAVE THEORY, IS DEFINED TO BE THE SPREADING-OUT EFFECT CAUSED BY THE DIFFERENT GROUP VELOCITIES OF THE SPECTRAL FREQUENCIES IN THE ORIGINAL DISTURBANCE AT THE SOURCE.

In figure 3.2, the positions of 33 different rectangular areas are shown in the x,y plane at a certain time. Consider any fixed point in the x,y plane at this instant of time. Either a sine wave system is present at the point or it is not present. If there were several thousand rectangular areas present in the x,y plane there could be quite a few present at the point and time of observation.

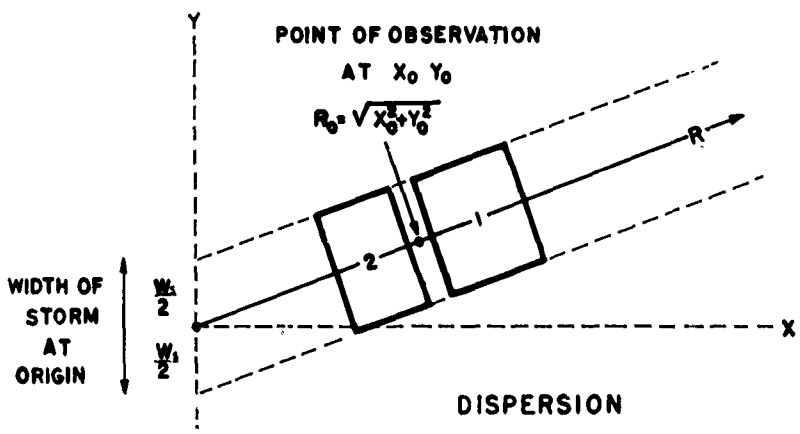


FIGURE 3.3 The effect of variability in frequency.

Figure 3.3 shows an x,y coordinate system with a fixed point, x_p,y_p, which can be thought of as the point at which the forecast at time, t_{ob}, is to be made. Some of the rectangular areas are traveling toward the point x_o,y_o, so that they must, at some time, reach and pass the point of observation after they start out at $t=0$. At the time, t_{ob}, for waves which last only D_w hours at the source ($x=0$, $y=0$), some rectangular areas must already have passed and some have not yet arrived.

The first step is to find the *period* of that particular sine wave which could have reached the point, x_o,y_o, at the time, t_{ob}. The particular sine wave, represented by rectangular area No. 1 in figure 3.3 a few minutes after t_{ob}, had a *windward* edge which started from the source at D_w hours ($t=0$ is the time the leeward edge started). The *windward* edge traveled R_o nautical miles (given by $\sqrt{x_0^2+y_0^2}$) as measured from the center of the leeward edge of the storm to the point of observation. It traveled at a speed given by 1.515 T_1 for a total time of $t_{ob}-D_w$ hours (since the windward edge started out D_w hours after the waves in the storm). Since velocity times time equals distance, it follows that

$$(t_{ob}-D_w)1.515\ T_1=R_o. \tag{3.1}$$

For any value of T greater than T_1, the windward edge of the rectangular area associated with that particular *period* will have already passed. There can be a great many such rectangular areas which have already passed, once the waves from the storm have arrived at the point of observation under study.

The next step is to find the *period* of that particular sine wave which could just have arrived at the point, x_o,y_o, at the time, t_{ob}. The sine wave, represented by rectangular area No. 2, a few minutes before, has a leeward edge which started from the source at zero hours. By the same argument as before, it follows that

$$t_{ob}1.515\ T_2=R_o. \tag{3.2}$$

This rectangular area is traveling more slowly than the ones that have already passed. Any rectangular area associated with a *period* less than T_2 will not have arrived at the point of the forecast.

The value of T_1 is greater than the value of T_2, and therefore the value of $1/T_1$ is less than $1/T_2$. These reciprocals represent frequencies; for example, one can say that a wave with a period of 10 seconds goes through one-tenth of a cycle per second, or that the wave frequency is one-tenth cycle/sec.

The above two equations can be solved for the frequencies involved, and the result is the following two equations:

$$f_2=\frac{1}{T_2}=\frac{t_{ob}\,1.515}{R_o} \tag{3.3}$$

$$f_1=\frac{1}{T_1}=\frac{(t_{ob}-D_w)\,1.515}{R_o}. \tag{3.4}$$

The value of f_1 can be subtracted from the value of f_2 to find the difference between the two frequencies. The result is that

$$\Delta f=f_2-f_1=\frac{1.515\,t_{ob}}{R_o}-\frac{1.515\,(t_{ob}-D_w)}{R_o}$$

$$=\frac{1.515\,D_w}{R_o}. \tag{3.5}$$

This means that for the type of storm under study, which is treated like a line source for the waves, the difference in frequencies is always a constant for a particular point of observation. This depends on the duration of the storm and the distance the waves travel to reach the point of the forecast.

Some values of the *period* are not greater than T_1 and not less than T_2. They are the *periods* of the rectangular areas that started out *toward* the point of observation and *that are there now* at the time, t_{ob}. They are the only *periods* that can possibly be present at the time of observation.

EXAMPLE 3.1.—A STORM BEGAN 60 HOURS AGO AND LASTED 20 HOURS. THE POINT AT WHICH THE FORECAST IS TO BE MADE IS 500 NAUTICAL MILES DOWN WIND FROM THE CENTER OF THE LEEWARD EDGE OF THE STORM. WHAT *PERIODS* (FREQUENCIES) COULD BE PRESENT AT THE POINT OF THE FORECAST? (THE WORDS *COULD BE* ARE USED BECAUSE SOME FREQUENCIES ARE NEVER GENERATED (AS SHOWN IN CHAPTER II), AND THEREFORE THEY NEVER ARRIVE.)

ANSWER

$$T_2=\frac{R_o}{1.515\,t_{ob}}=\frac{500}{(60)\,1.515}=\frac{500}{91}=5.5 \text{ seconds.}$$

$$T_1=\frac{R_o}{1.515\,(t_{ob}-D_w)}=\frac{500}{(40)\,1.515}=\frac{500}{60.5}=8.27 \text{ seconds.}$$

$$f_2=\frac{1}{T_2}=.182 \text{ cycle/sec.}$$

$$f_1=\frac{1}{T_1}=.121 \text{ cycle/sec.}$$

$$\Delta f=.061 \text{ cycle/sec.}$$

ALL *PERIODS* BETWEEN 8.27 AND 5.5 SECONDS ARE PRESENT.

EXAMPLE 3.2.—SAME AS ABOVE EXCEPT THAT THE POINT IS 1,000 NAUTICAL MILES AWAY.

ANSWER

$$T_2=\frac{R_o}{1.515\,t_{ob}}=\frac{1{,}000}{60\,(1.515)}=\frac{1{,}000}{91}=11 \text{ seconds.}$$

$$T_1=\frac{R_o}{1.515\,t_{ob}}=\frac{1{,}000}{40\,(1.515)}=\frac{1{,}000}{60.5}=16.5 \text{ seconds.}$$

$f_2=.091$ cycle/sec.

$f_1=.0605$ cycle/sec.

$\Delta f=.0305$ cycle/sec.

NOTE THAT IF R IS DOUBLED Δf IS HALVED

Usually a forecast of the waves at a point of interest for just one particular time is not wanted. A continuous forecast of the waves is better. The high wave *periods* arrive first, followed by the medium *periods*, and finally the low *periods* come along. This is very easy to forecast.

EXAMPLE 3.3.—A STORM LASTED FOR 18 HOURS AT THE SOURCE OF THE WAVES. THE FORECAST POINT IS 1,000 NAUTICAL MILES AWAY. THE HIGHEST *PERIOD* GENERATED IN THE STORM WAS 15 SECONDS. WHEN DO THE FIRST WAVES ARRIVE AND WHAT *PERIODS* ARE PRESENT AT 6-HOUR INTERVALS THEREAFTER FOR ONE AND ONE-HALF DAYS AND AT 12-HOUR INTERVALS FOR THE NEXT DAY?

ANSWER

WHEN THE LEEWARD EDGE OF THE 15-SECOND RECTANGULAR AREA ARRIVES, THE FIRST WAVES WILL BEGIN TO SHOW UP. SOLVE FOR t_{ob} IN THE FORMULA FOR T_2.

$$t_{ob}=\frac{R_o}{1.515\,T_2}=\frac{1{,}000}{(1.515)\,(15)}=\frac{1{,}000}{22.7}=44 \text{ hours.}$$

Start with $t_{ob}=44$ hours, and compute T_1 and T_2 for 50 hours, 56 hours, etc.

t_{ob}	hrs.	$\frac{R_o}{(t_{ob}-D_W)(1.515)}$	$\frac{R_o}{t_{ob}(1.515)}$	T_1	T_2	f_1	f_2	Δf
1 day 20 hrs.	44		$\frac{1,000}{44(1.515)}$		15		.0667	~
2 days 2 hrs.	50	$\frac{1,000}{32(1.515)}$	$\frac{1,000}{50(1.515)}$	20.6 15	13.2	.0455 .0667	.0756	~
2 days 8 hrs.	56	$\frac{1,000}{38(1.515)}$	$\frac{1,000}{56(1.515)}$	17.85 15	11.8	.056 .0667	.085	~
2 days 14 hrs.	62	$\frac{1,000}{44(1.515)}$	$\frac{1,000}{62(1.515)}$	15	10.6	.0667	.0940	.027
2 days 20 hrs.	68	$\frac{1,000}{50(1.515)}$	$\frac{1,000}{68(1.515)}$	13.5	9.7	.0756	.103	.027
3 days 2 hrs.	74	$\frac{1,000}{56(1.515)}$	$\frac{1,000}{74(1.515)}$	11.8	8.92	.085	.112	.027
3 days 8 hrs.	80	$\frac{1,000}{62(1.515)}$	$\frac{1,000}{80(1.515)}$	10.6	8.27	.0940	.121	.027
3 days 20 hrs.	92	$\frac{1,000}{74(1.515)}$	$\frac{1,000}{92(1.515)}$	8.92	7.19	.112	.139	.027
4 days 8 hrs.	104	$\frac{1,000}{86(1.515)}$	$\frac{1,000}{104(1.515)}$	7.67	6.35	.130	.1575	.027

Note that Δf is a constant after the 15-second sine wave has passed.

Note that the successive differences between the f_1's are constant and equal to 0.009. Also note that the successive differences between the f_2's are constant and equal to 0.009. The differences between the *periods* are not constant.

Angular Spreading

Waves in an area of generation are short crested. This means that there are components traveling in many different directions as shown in figure 1.7. ANGULAR SPREADING IS A PROCESS WHICH TAKES INTO ACCOUNT THE FACT THAT WAVES IN THE GENERATING AREA ARE TRAVELING IN MANY DIFFERENT DIRECTIONS.

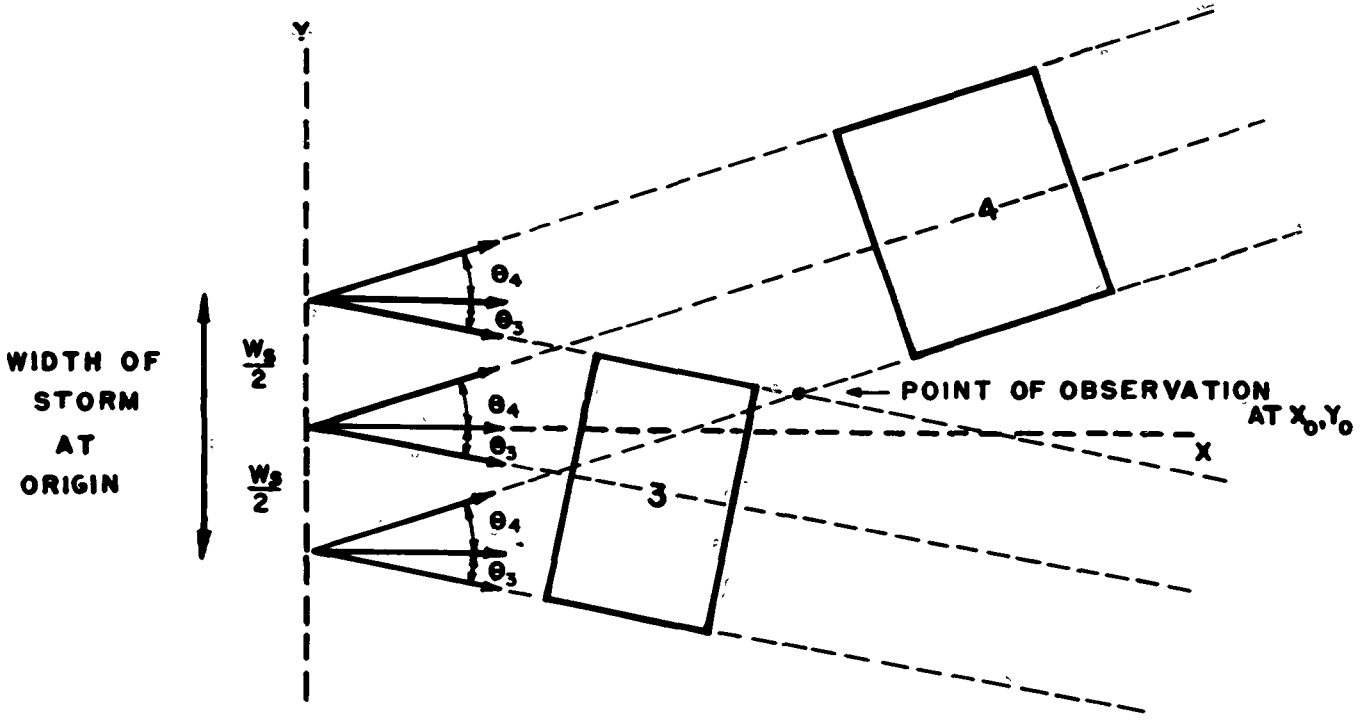
In figure 3.2 not only did the rectangular areas spread out because they traveled with different speeds, but they also spread out sideways over a very large area because they started out in different directions. Some of these rectangular areas will never arrive at the point of forecast because they pass to its side.

Two such rectangular areas are shown in figure 3.4. Each has a width given approximately by W_s (the width of the storm) and more accurately by $W_s \cos\theta_4$ and $W_s \cos\theta_3$. Area number four started out at the angle, θ_4; it has just passed *above* the point of the forecast, x_0, y_0. Area number 3 started out at an angle, θ_3; it will soon pass *below* the point of the forecast, x_0, y_0. Neither area can possibly affect the point of the forecast. Any other rectangular area which starts out in a direction less than θ_4 or greater than θ_3 can also never affect the point of the forecast.

By definition, the angles should be measured with reference to the edges of the fetch as shown in figure 3.5. The edges of the fetch should be drawn as closely as possible parallel to the predominant wind and sea direction in the fetch.

Angles measured clockwise from the above lines to the lines connecting the leeward corners of the fetch and the forecast point are positive; angles measured similarly but in a counterclockwise sense are negative. When looking to leeward, θ_3 is the angle on the left, θ_4 the angle on the right.

To minimize the time spent by the forecaster in preparing a wave forecast, the above angles should be measured by protractor, although this is not the most accurate method. The angles can be determined more accurately by measuring the lengths of the triangles formed and applying standard trigonometric formulas. In practice, however, two points favor the former method. One is the ease and simplicity of using a protractor. The other is the difficulty in selecting the boundaries of a fetch. No two forecasters will select exactly the same fetch width for a given storm, and a small change in width can mean an appreciable angle change. Thus the accuracy of computing the angle is much higher than the accuracy of the triangle dimensions, and the accuracy inherent in trigonometric calculation is not valid since the storm width is not sharply defined.



ANGULAR SPREADING

FIGURE 3.4 Angular spreading, the effect of variability in direction.

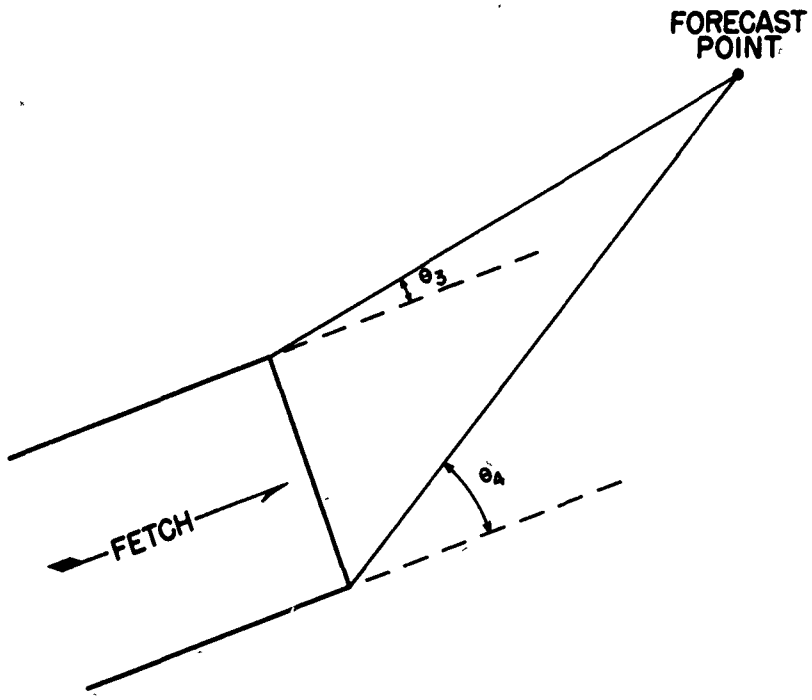


FIGURE 3.5 Definition of θ_3 and θ_4.

If an objective method can be devised for determining storm width, then the more accurate method of computing θ_3 and θ_4 should be used. Until that time, the protractor is sufficiently accurate.

Only those rectangular areas which start out in a direction between θ_4 and θ_3 can ever arrive at the point of the forecast. This range of angles can be very small indeed if the point of forecast is far from the storm compared to the width of the storm. The width of a storm is very important in making a correct forecast of the wave properties in the area into which the waves travel when they leave a particular storm.

A Wave-Forecasting Filter (Filter I)

It has just been shown that for a certain type of storm model only certain wave *periods* (or frequencies) and certain wave directions can be present at a fixed point in the area outside a given storm at any given time after the waves have left the storm. Only *periods* between T_1 and T_2 and directions between θ_4 and θ_3 can possibly be present at a given point at a given time. The *periods* present are given by

$$T_1 < T < T_2 \text{ (or } f_2 < f < f_1) \tag{3.6}$$

and the directions are given by

$$\theta_4 < \theta_D < \theta_3. \tag{3.7}$$

In a storm when sea is present, there are sine waves traveling in many different directions and with many different *periods*. At the point of forecast, only a much smaller range of *periods* and directions is present. Those that are present add up to make the waves at the point of the forecast, but since only a fraction of what was present in the storm is now present at the forecast point, they must add up to much lower waves. If the range of *periods* and the range of directions can be forecast, then E can be forecast, the distribution of the irregular wave heights can be computed, and the times required for successive crests to pass a given point can be computed. The irregularity of the waves can be described.

In effect, a very simple process has been carried out on the spectrum of the waves in the storm. A certain range of frequencies and a certain range of directions of those present at the source have been kept, and the rest of the values have been discarded. The procedure is similar to what happens when a piece of red cellophane is held before a lamp. All colors in the light are present up to one side of the cellophane, but only the color red comes through to reach your eye. The cellophane *filters* the light by letting only one color out of many come through to the eye.

To forecast the waves outside of the storm, all that is needed is to forecast a filter to be applied to the spectrum at the source in order to forecast the spectrum at the point and time of the forecast. A filter is a device that rejects unwanted quantities and retains wanted quantities.

A WAVE-FORECASTING FILTER IS A SET OF FORMULAS THAT GIVE THE FREQUENCIES AND DIRECTIONS TO BE KEPT AT THE POINT OF FORECAST. The actual ocean surface itself is the physical filter (like the red cellophane) and the formulas tell how the filter acts. (The formula for the light frequencies actually passed by the cellophane is probably very complicated.) The ocean surface acts as a filter because the waves generated on the ocean surface contain many frequencies and directions which disperse and spread out angularly. The deep ocean is a *dispersive* medium as can be seen from the figures which have been shown. Sea waves that cover a very small area near or in a storm can spread out and cover an area many times greater as they travel out of the storm to distant points. A disturbance that lasts only 20 hours in a storm could require 100 or more hours to pass a point a great distance from the storm after it had finally reached that point.

The particular filter just described is satisfactory for certain types of storms, but for other types of storms other types of filters are needed. This filter has probably a limited application since it is a limiting case of a more general filter.

Other Filters

A Storm With a Fetch (Filter II)

Consider a stationary *area* over which the winds blow in a uniform direction and with a uniform velocity for D_s (duration of storm) hours after the waves in the storm have been fully developed. Let the storm be W_s NM wide and F NM long. The leeward edge of each rectangular area propagates just as before. The windward edge of each rectangular area, however, starts out not from $x=0$ as before but from $x=-F$. Therefore, it has to travel F nautical miles to get to the leeward edge of the storm. The coordinates and the dimensions of the storm and of a typical rectangular area are shown in figure 3.6

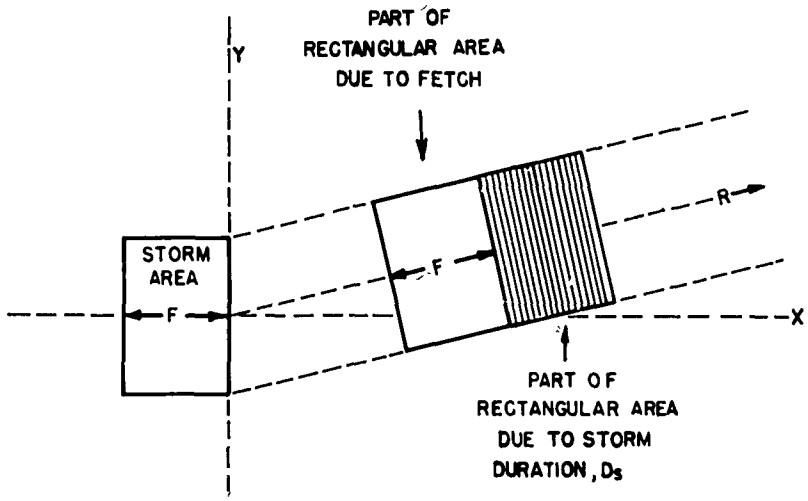


FIGURE 3.6 A storm with a fetch.

Each rectangular area which accompanies a particular frequency and direction has two pieces. The piece at the leeward edge behaves just like the ones treated before. Somewhere inside there is a line which would be the end of that particular frequency if the fetch were not there. However, the part from the fetch follows right behind without a break, and an area the length of the fetch in the storm must still pass.

The angles θ_3 and θ_4 are still the same as before. The leeward edge of each rectangular area is still determined by T_2. The only difference

is that T_1 can no longer be used. Instead, the *period* of that particular sine wave which started from F nautical miles to windward of the leeward edge of the storm, left $t_{ob}-D_s$ hours ago, and has just passed the point of observation, must be found. This period is given by T_5, and it can be found by the use of equation (3.8).

$$(t_{ob}-D_s)1.515\ T_5=R_0+F \tag{3.8}$$

The *period*, T_5, is greater than T_1 since R_0+F is greater than R_0. The frequency range for the sine waves present at the point and time of the forecast is thus greater for a storm with a fetch than for the previous model.

A Storm Moving to Leeward (Filter III)

The rectangular areas associated with the different wave frequencies travel with the group velocity associated with that frequency. In table 2.5 of Chapter II it was shown that the highest important *period*, or lowest frequency, for a given wind velocity was such that the group velocity associated with that *period* was less than the wind velocity which generated the waves. Consider a high-wind area with winds in it of a certain velocity, moving to leeward. The high-wind area will frequently move to leeward with the speed of the winds, as illustrated in figure 3.7.

At the point, P, no waves will be observed from the high-wind area until the high-wind area arrives at the point of the forecast. The wave energy cannot outrace the winds. The waves arrive with the storm winds and appear to reach full development as soon as the winds

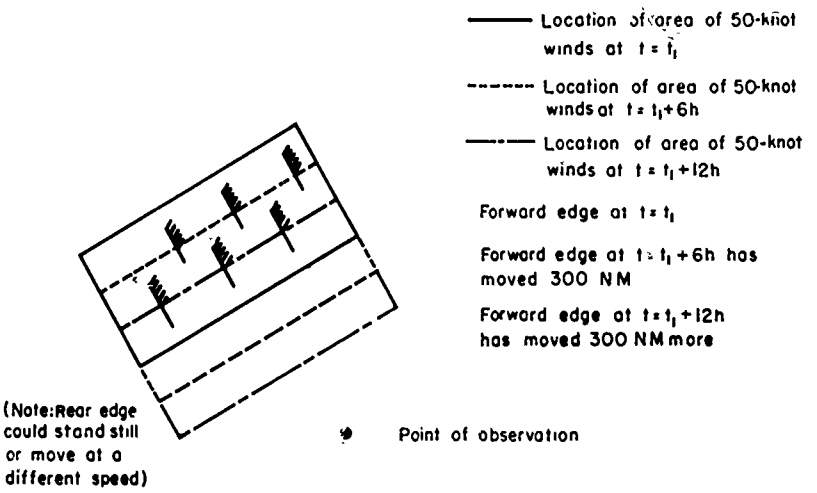


FIGURE 3.7 A traveling area of high winds

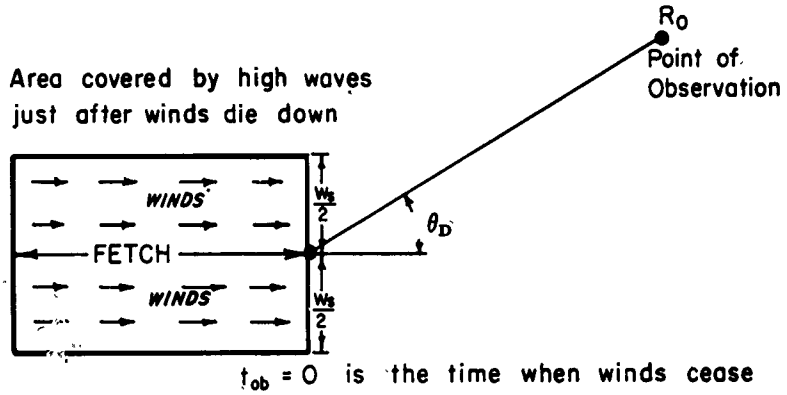


FIGURE 3.8 Forecast conditions for the waves propagating from a storm which has just dissipated over an area at a distance from the point of observation.

build up at the point of the forecast. A rapidly moving cold front can produce just such a condition.

Another condition can also exist. The area of high winds can develop, travel across a wide swath of the ocean, and dissipate. The waves will be nearly fully developed in the area of high winds, but outside the area of high winds the waves will be quite low.

The area of high winds can dissipate at some time during its travels, and when it does the waves generated propagate out of the area to distant points. Figure 3.8 shows the conditions which exist just after the winds over a large storm area have died down.

In this case θ_3 and θ_4 are still the same, although it might be more accurate to measure them by using angles from the center of the storm. There is no value corresponding to D_s or D_w. The value of T_2 is still the same, but the value of T_1 is no longer appropriate. Instead T_6 is used as given by equation (3.9).

$$t_{ob}\ 1.515\ T_6 = R_0 + F \tag{3.9}$$

The value of T_6 is greater than the value of T_1 since R_0+F is greater than R_0 so that a range of frequencies is still present. This filter, θ_3, θ_4, T_2, and T_6, is a strange one indeed compared to others. It seems to agree best with the methods given in forecasting procedures from other references.

The Decrease of Waves in the Fetch (Filter IV)

When the winds cease in a generating area, the waves in the area decrease in height, often quite rapidly. Each frequency disappears at the point of forecast as soon as its windward edge can travel to that point. Let F_0 be the distance in the fetch to the windward edge of the fetch. Then the windward edge of each rectangular area must

travel this distance before the height contribution due to this particular *period* will pass by. The *period*, T_7, can be associated with this passage of the windward edge of the rectangular area, according to the following equation:

$$t_{ob}\ 1.515\ T_7 = F_0 \tag{3.10}$$

The *period* T_2 is already present since the point of forecast is in the generating area, and θ_3 and θ_4 are not needed. The time, t_{ob}, is measured from the time that the winds die down.

When some examples are given of how rapidly the waves can die down in the generating area after the winds cease, the results will be very surprising. There is nothing particularly complicated about the loss of the lower frequency components. This loss is not due to viscosity or friction effects. Only dispersion is needed (in this case) to explain the major part of the decrease of wave height. Of course, the short chop (*periods* less than about three seconds) dies out rapidly owing to turbulent friction.

The Use of the Filters to Forecast the Waves at the Point of Observation

Statement of the Properties of a Filter

Four different filters have been described. These filters make it possible to forecast the waves at points outside the fetch area or at points inside the fetch area after the winds have ceased to blow. As stated above, A WAVE-FORECASTING FILTER IS A SET OF FORMULAS WHICH GIVE THE FREQUENCIES AND DIRECTIONS TO BE KEPT AT THE POINT OF FORECAST. Each filter is defined by two frequencies and two directions. Only those frequencies in the original wave spectrum that lie between the two frequencies defined by the filter can be present at the point and time at which the forecast is being made. Only those waves that started out in directions between the two angles given by the filter can be present at the point of observation because of the finite width of the storm. The original spectrum of the waves at the source contains energy at many other wave frequencies, traveling in many other directions. Since this wave energy is not present at the point at which the forecast is being made, the waves are lower. The forecasting problem, then, for the point of observation is to add up the amount of wave energy present at the time of the forecast and to compute the value of E which represents this wave energy. Then the wave height can be forecast from this value of E. The frequency band of the filters gives the "periods" present in the wave record, and the direction band of the filter gives the directions toward which the wave will appear to be traveling at the point of observation.

Effect of Frequencies

Consider, as an example, the filter which employs f_5 and f_2. Suppose that the two values are known. Then, as in figure 2.3, they can be marked off on the spectrum of the waves in the generating area. Those frequencies less than f_5 are frequencies in the original spectrum which have already passed the point of the forecast. Those frequencies greater than f_2 are frequencies which have not yet arrived at the forecast point. Therefore, the energy which can be present at the point is at most equal to the area under the spectrum between these two frequencies. As figure 3.9 shows, this area is a small fraction of the total area under the spectrum. In terms of the spectrum it would be necessary to compute this area, but this has already been done in the co-cumulative spectra which are used as the forecasting diagrams. As figure 3.9 shows, the area under the spectrum to the right of frequency f_2 is equal to $E(f_2)$. Also, the area under the spectrum to the

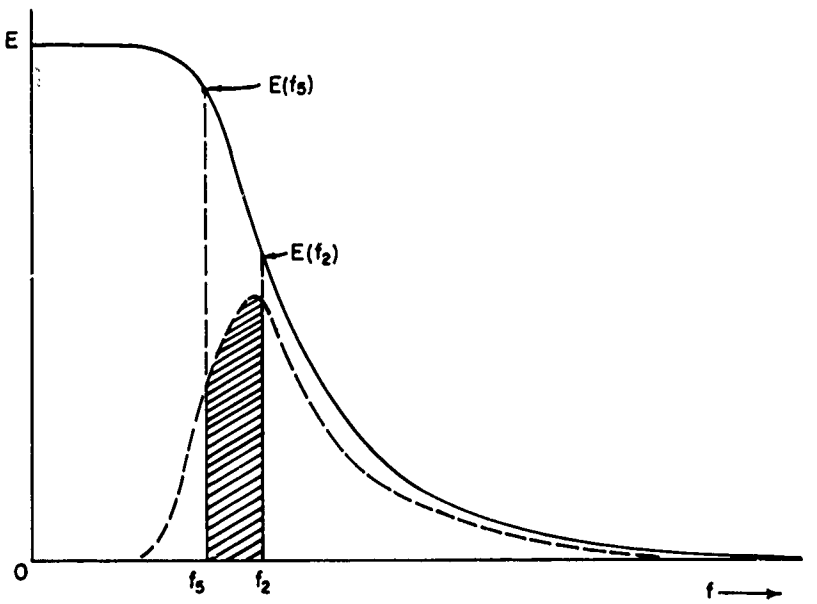


FIGURE 3.9 The effect of the frequencies in the filter.

right of f_5 is equal to $E(f_5)$. The area to the right of f_5 minus the area to the right of f_2 is equal to the area under the spectrum between f_5 and f_2. This in turn is simply equal to $E(f_5)$ minus $E(f_2)$. Therefore, in order to find the value of E which can be present at the point of forecast, simply compute the two frequencies given by the filter and subtract the corresponding values of E on the co-cumulative spectra.

The Effect of Direction

The spectra shown in Chapter II have not treated the effect of variability in wave direction and the fact that the waves are short-crested. The waves vary not only in frequency but also in direction inside the area in which they have been generated. Figure 3.10 shows the way in which the waves are spread out. The lower curve shows the relative amount of wave energy present at each direction within the storm. Some energy, although not very much, is theoretically present

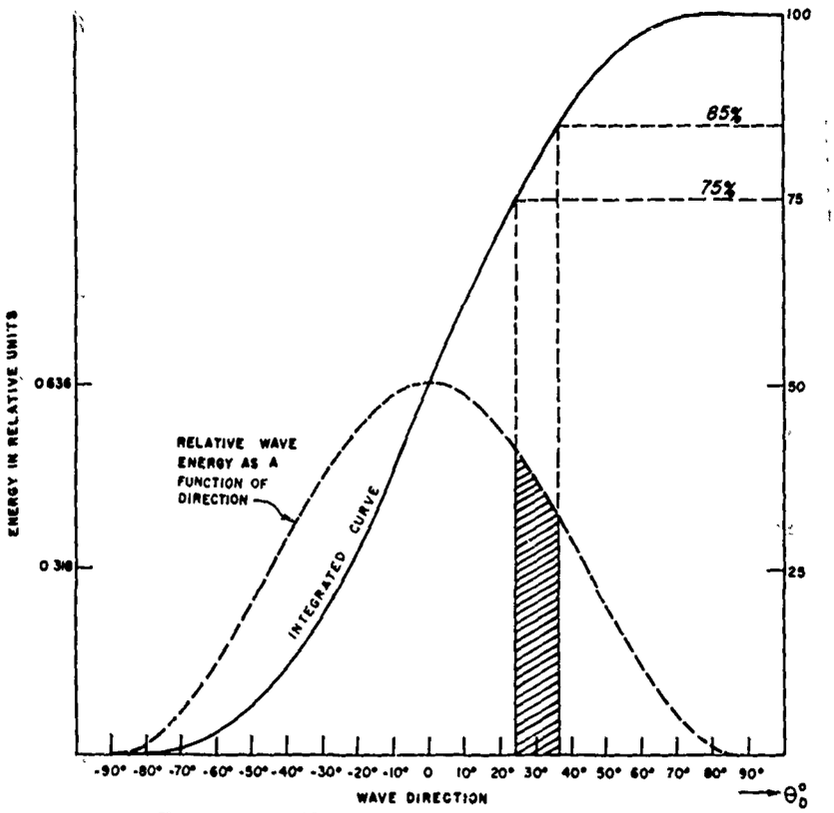


FIGURE 3.10 The effect of directions in the filter.

at all directions between $-90°$ and $+90°$ to the direction of the wind in the storm. Most of it is concentrated within an angular range of $\pm 30°$. Some is still present at $\pm 45°$, and a very small amount is present beyond $\pm 60°$. The area under this curve should be equal to unity, so that when the total variability in direction is summed the result would be the variation in frequency as described in the frequency spectra given in Chapter II. This curve can also be integrated from $-90°$ to any value of the angle θ_a greater than $-90°$ and finally

up to $+90°$. Since the area under this curve is equal to unity, the total range of the integral will be one. Expressed in percentages on the right hand side, the total range will be 100 percent.

At the point at which the forecast is being made, only waves that started out within a direction between θ_4 and θ_3 can possibly be present. The area under the angular spreading curve to the left of θ_3 is given by the integrated curve up to θ_3, and the area to the left underneath the curve up to θ_4 is given by the integral of the curve up to θ_4. Therefore, the percentage of wave energy that can be present at the point of forecast is given by the area under the lower curve between the angle θ_3 and θ_4; this in turn is equal to the difference in the percentage values as read on the integrated curve at θ_4 and θ_3. The difference between the two values read off on the integrated curve will always be some number between 0 and 100 percent.

Combined Effect

To combine the effect of dispersion and the effect of angular spreading, the spectrum given as a function of frequency in figure 3.9 must be multiplied by the percentage obtained in figure 3.10 for each point. However, since interest lies only in the energy between f_5 and f_2, it is sufficient to multiply the spectrum between f_5 and f_2 by the percentage obtained in figure 3.10. Figure 3.11 illustrates the total effect of the filter. The same result is obtained if $E(f_5)-E(f_2)$ is

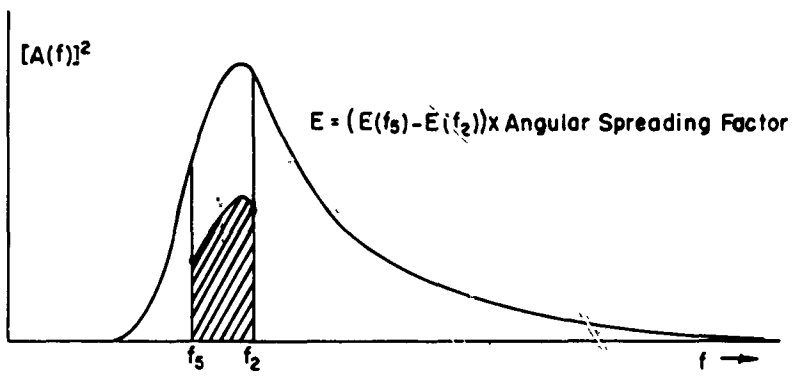


FIGURE 3.11 The total effect of the filter.

multiplied by the percentage obtained in figure 3.10. The result is then the value of E associated with the waves at the point of observation. The filter consequently cuts down on the energy that can be present at the point of observation in two different ways. The first is an effect of dispersion; the second is an effect of angular spreading. The combined effect makes the waves quite low at the point of observation.

The other features of the waves at the point of observation can also be determined from the filter. The "periods" can be found from the range of frequencies present, and the direction toward which the waves are traveling can be found from the angle θ_n. An indication of the short-crestedness of the waves can be found from the angular range between θ_3 and θ_4. The wider the angular range, the shorter are the crests of the waves; and the narrower the angular range, the longer are the crests of the waves. Note that at angles to the storm and at great distances from the storm the waves no longer appear to be traveling in the direction of the wind in the storm. They can be traveling at angles as great as 45° or 50° to the original direction in which the waves in the storm appear to be traveling. This is a very important fact: when a swell is observed a at distant point from the storm, it is not sufficient simply to go back in the direction from which the swell has come and look for winds blowing in that direction to find the generating cause of the swell.

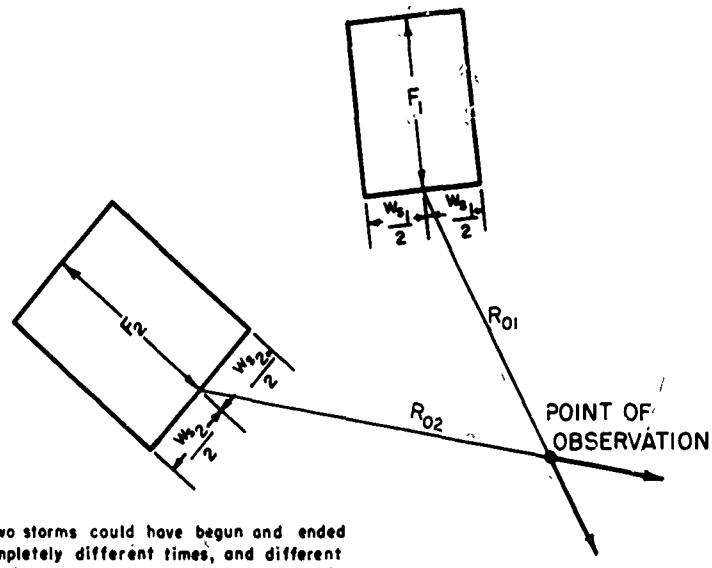
This combined effect of angular spreading and dispersion as summarized in a wave-forecasting filter seems to account for the entire decrease of wave height with distance traveled outside of the storm. There are those who believe that viscosity also decreases the height of the waves as they travel outside the storm. Further comments on this point will be made later in this chapter. However, the important point to be made now is that the waves spread out as they travel outside the storm and cover a very large area compared to the size of the initial disturbance. If they cover a very large area, even if no energy is lost, the energy per unit area must be much lower than it is in the storm. This therefore explains a very large part of the decrease in wave height as the waves travel out of the storm. Computations will be given later that will show the magnitude of this effect in preparing wave forecasts. The effect of superimposing viscosity on dispersion and angular spreading appears to cut down the height of the waves too much at the point where the forecast is being made.

Waves from Several Storms

Just as an infinite number of sine waves is added together to obtain the sea surface in a particular storm, no difficulty occurs when the rectangular areas from two or more different generating areas arrive at a point of observation. The numbers, E, which are forecast for each generating area are added together to give the E of the total disturbance. The wave frequencies (or *periods*) give an interference pattern which is quite recognizable especially if two narrow frequency

bands are present and do not overlap. The central frequency of each band produces waves with time intervals between crests near that frequency at various times. At other times, waves with time intervals between crests near the value given by the mean of the center of the two frequency bands are present.

Figure 3.12 illustrates such a situation where waves from two different storms have arrived at a point of observation. The waves from the two different storms produce a *cross sea*. Sometimes the waves appear to be traveling toward the direction of one arrow. At other times, they appear to be moving in the direction of the other



The two storms could have begun and ended at completely different times, and different parts of each spectrum could be present for the time of observation at the point of observation.

FIGURE 3.12 Waves from two different storms at a point of observation.

arrow. Then they appear to be moving in directions somewhere between. The high waves are the points of reinforcement of the cross sea. Since there is no such thing as a wave *period* in the actual waves, the forecast must be worded to show the range of "periods."

If the frequency bands of a cross sea are sufficiently separated, it is possible to identify time intervals between crests associated with each band. The records look as if two different swell spectra have simply been added together. Some parts of the record show one "period." Other parts show another. Figure 3.13 shows several such records. A study of these records shows the features discussed quite clearly.

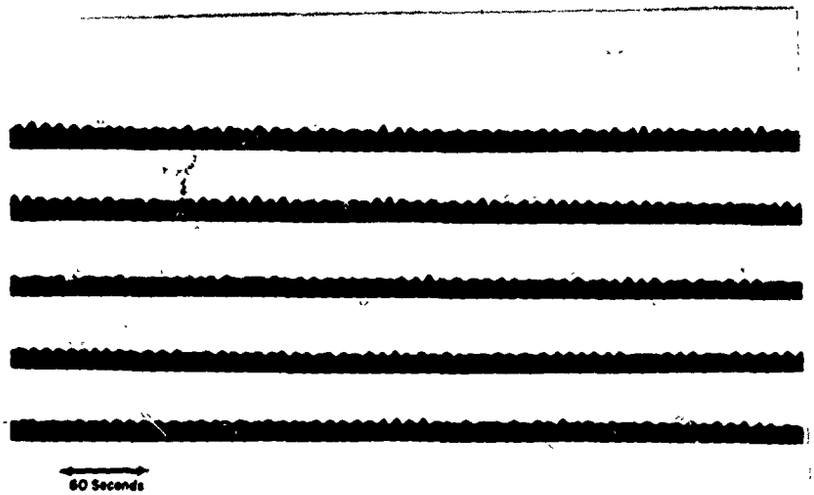


FIGURE 3.13 Wave records with two distinct frequency bands present.

EXAMPLE 3.4.—AT THE POINT OF OBSERVATION SHOWN IN FIGURE 3.12 FOR A CERTAIN TIME OF FORECAST, TWO WAVE SYSTEMS ARE FORECAST TO BE PRESENT AT t_{ob}. THEIR PROPERTIES ARE TABULATED BELOW.

	Frequency band		*E*	*Dominant direction (toward which waves are traveling)*
Storm I	0.12	0.08	0.64 ft.^{2}	100°
Storm II	0.06	0.04	0.36 ft.^{2}	140°

PROBLEM: WHAT WILL THE WAVES LOOK LIKE AT THE POINT OF OBSERVATION?

ANSWER: THE WAVES FROM STORM I HAVE A FREQUENCY BAND CENTERED AT 0.10 CYCLE/SEC. THUS, THE "PERIODS" OF THE WAVES TRAVELING TOWARD 100° WILL BE NEAR 10 SECONDS WITH A SPREAD FROM 8.3 SECONDS TO 12.5 SECONDS. THE WAVES FROM STORM II HAVE A CENTRAL BAND NEAR 0.05. THUS, THE WAVES TRAVELING TOWARD 140° HAVE "PERIODS" NEAR 20 SECONDS WITH A SPREAD BETWEEN 16.7 SECONDS AND 25 SECONDS. THESE WAVES WILL BE LOWER BECAUSE THEIR VALUE OF E IS LOW.

OTHER WAVES TRAVELING TOWARD 120° (OR MORE LIKELY 115° BECAUSE E_1 IS GREATER THAN E_2) WILL APPEAR WITH A FREQUENCY GIVEN BY APPROXIMATELY $(0.10+0.05)\frac{1}{2}=.1500/2=.0750$. THUS, ONCE IN A WHILE, "PERIODS" OF ABOUT 13.3 SECONDS WILL BE OBSERVED.

THE WAVE HEIGHTS ARE FORECAST BY TAKING THE SUM OF THE TWO FORECAST VALUES OF E GIVEN BY

$E_I + E_{II} = 0.64 + 0.36 = 1 = E_{total}$. THE SIGNIFICANT HEIGHT (CREST-TO-TROUGH) IS GIVEN BY 2.83 FEET. ONE WAVE OUT OF 100 WILL BE ABOUT 4.56 FEET CREST-TO-TROUGH.

Additional Forecasting Considerations

The rules and typical forecast situations which have been given all depend on one concept. *If the various frequencies and directions of the sine waves in the generating area can be kept track of, the waves can be forecast at other points or in the generating area by accounting for their dispersion and angular spreading.* As the forecaster becomes more adept at the use of this one basic concept, he will be able to extend it to more general cases. Generating areas have been treated mainly as stationary, and as if they were rectangular in shape. At times, it is possible to break up an irregularly shaped generating area into a number of rectangular areas, and, by the method just given above, superpose the forecasts for the different areas to get the total effect. Moving storms can sometimes be treated as if they move by jumps and last a certain number of hours at each point. These techniques need to be developed, extended, and made more precise. For actual forecasting procedures, the methods described above can be summarized into four basic charts to make the forecasts easy, quick, and practical.

Forecasting Diagrams for Dispersion and Angular Spreading

Summary of Filters

Now that the various filters used in forecasting the waves outside of the generating area (or after the waves have ceased in the generating area) have been described, the next step is to summarize these four filters and then to give the diagrams that can be used to calculate the needed values given by the filters easily.

So far, four types of storms and the filters associated with them have been described. Suppose that frequencies are considered instead of *periods*. Then f_1 corresponds to $1/T_1$, f_2 corresponds to $1/T_2$, and so on. Also, it would be advisable at this time to divide both sides of the dispersion equations by the number 1.515. The reciprocal of 1.515 is 0.66.

Table 3.2 presents a summary of the various filters that have been described. Filter I applies to a line-source storm. The parameters needed are (1) t_{ob}, the time of observation; (2) D_W, the duration of the waves at the leeward edge of the storm; (3) R_o, which is the distance from the storm to the point of observation and (4) θ_3 and θ_4, the angles made by the wind direction in the storm and lines to the point outside the storm at which the forecast is to be made as defined in figure 3.5.

Table 3.2—Summary of Filters

Storm type	Observed quantities	Formulas for filters	Calculated quantities
Line-source FILTER I	t_{ob} (time measured in hours after waves start from storm) D_W (duration of the waves at the source) R_o (distance from storm to point of observation) θ_3 (angle to forecast point) θ_4 (angle to forecast point)	$t_{ob}-D_W=(0.66R_o)f_1$ $t_{ob}=(0.66R_o)f_2$	f_1 f_2 θ_4 θ_3
Stationary storm with a fetch FILTER II	t_{ob} (as above) D_s (duration of storm) R_o (as above) F (length of storm) θ_3 (as above) θ_4 (as above)	$t_{ob}-D_s=0.66(R_o+F)f_5$ $t_{ob}=0.66R_o f_2$	f_5 f_2 θ_4 θ_3
A storm moving to leeward FILTER III (velocity of fetch equals velocity of winds in the fetch.)	t_{ob} (time measured in hours after winds cease in storm, waves start then) R_o, F, θ_3, θ_4 (as above)	$t_{ob}=0.66(R_o+F)f_6$ $t_{ob}=0.66\ R_o f_2$	f_6 f_2 θ_4 θ_3
Waves in the fetch FILTER IV	t_{ob} (time measured in hours after winds cease over the fetch) F_o (distance from point of observation to windward edge of fetch)	$t_{ob}=0.66\ F_o f_7$	f_7 $(f_2=\infty)$ $(\theta_4=+90°)$ $(\theta_3=-90°)$

The other filters are defined in a similar way. With them, a fetch is needed also. The value, R_o, is the distance from the center of the leeward edge of the storm to the point of observation at which the forecast is to be made, and F is the distance from the leeward edge to the windward edge of the storm. As a first approximation, the distance from the windward edge of the storm to the point of observation is equal to the distance from the windward edge of the storm to the leeward edge of the storm plus the distance from the leeward edge of the storm to the point of observation.

In each filter four calculated quantities are needed, two frequencies and two directions. When these four quantities have been computed, it is then possible to forecast the waves at a point outside the storm (or inside the storm after the winds cease). In the last filter, Filter IV, three of the values are already known, f_2 is infinite, and the two angles are $+90°$ and $-90°$, since the point of observation is inside the storm area.

Forecasting Diagrams

Two additional basic diagrams are needed to forecast the waves at any point outside or inside the storm after the winds have ceased to blow in the storm. In addition, the co-cumulative spectra, which were discussed in Chapter II, are needed. However, only two new diagrams are needed for the completion of the forecasting theory. The two diagrams are the basic diagrams for forecasts after the winds have ceased to blow. They must cover a large range of values. Therefore, they have been duplicated so that in reality there are four. Three are dispersion diagrams and one is an angular spreading factor diagram. In any one forecast situation usually only one of each of the different kinds of diagrams needs to be used, and the forecaster will quickly learn which one fits the purpose at hand. The ranges shown are chosen to make it possible to cover all values conveniently and quickly.

Dispersion Diagrams: Figures 3.14, 3.15, and 3.16

Consider all the formulas involving frequencies in table 3.2. The desired frequencies, which are to be computed from the parameters of a particular storm, all depend on a time and a distance. The time is to be measured in hours, and the distance is to be measured in nautical miles. Therefore, all these equations simplify to a formula of the form, $t=0.66\ Rf$. If t is known and R is known, f can be found. Sometimes t stands for the quantity $t_{ob}-D_s$. At other times it stands for t_{ob} alone, but in any case it is a time measured in hours. R sometimes stands for R_o, the distance from the leeward edge of the storm to the point of observation. At other times it might stand for F_o, the distance from the point of observation inside the storm to the

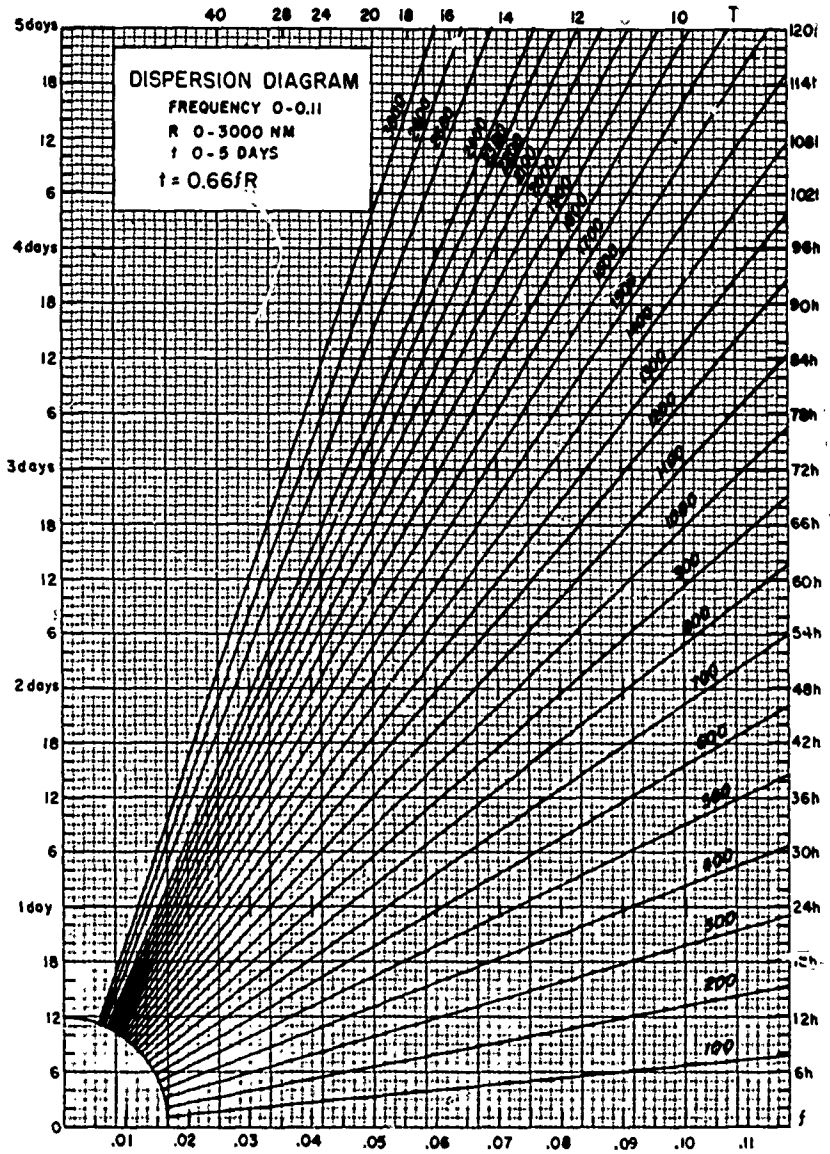


FIGURE 3.14.

windward edge of the storm. But in any case it is a distance measured in nautical miles.

This simple equation, $t=0.66\ Rf$, is an equation which needs to be solved every time a forecast is to be made. The dispersion diagrams do this in the simplest possible way. For each value of R, this equation is an equation simply of a straight line through the zero-frequency

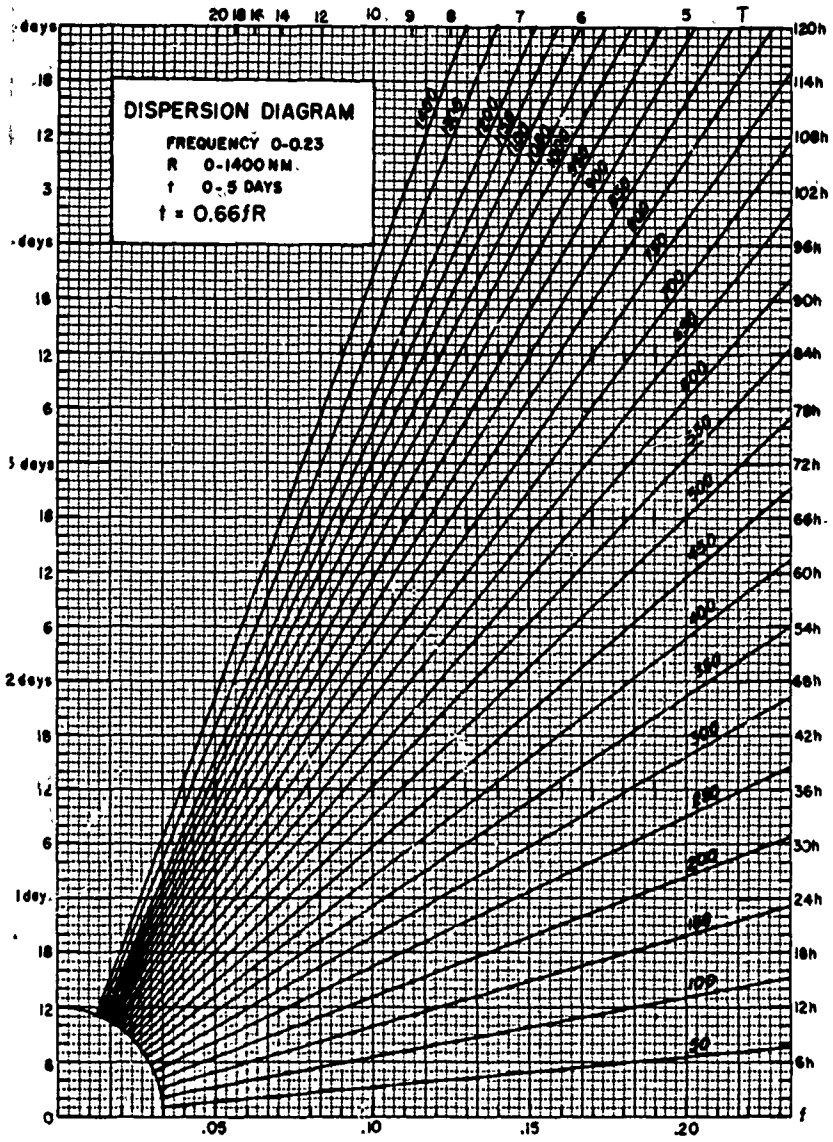


FIGURE 3.15.

value and the zero-time value with a slope which becomes steep for large values of R. The dispersion diagrams then cover time intervals from 0 to 5 days and from 0 to 2½ days and frequencies from 0 to 0.11 and from 0 to 0.23, all for various values of R. These forecasting diagrams are all essentially the same except for the way that the scales have been changed. By remembering the formula, $t=0.66\ Rf$, a

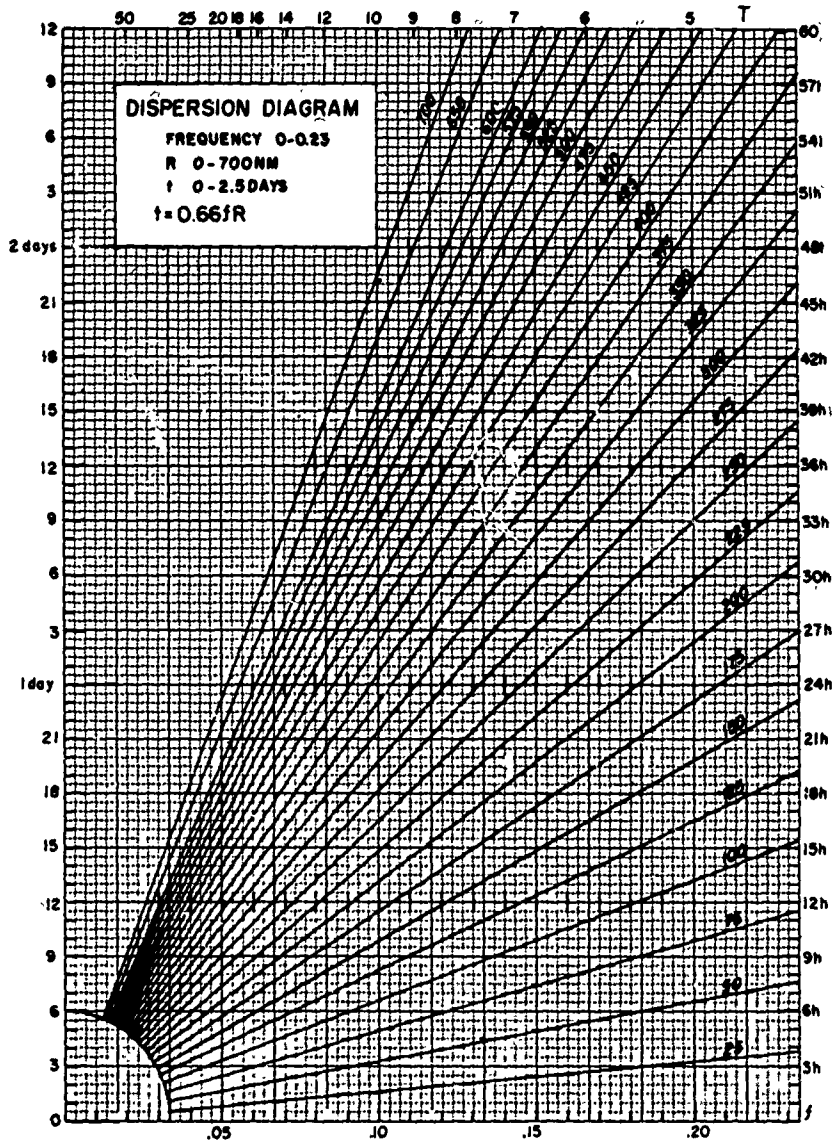


FIGURE 3.16.

forecaster can duplicate these diagrams in just a few minutes on any sheet of ordinary graph paper. Graph paper with heavy lines every six units in the vertical direction makes it more convenient to mark off 6-hourly time increments and time intervals of the order of days which occur quite frequently in forecasting.

The forecasting diagrams for dispersion are used very simply. Some examples of how they can be used are given below.

EXAMPLE 3.5. USE OF DISPERSION DIAGRAMS.

CASE 1. FILTER I. Given values $t_{ob}=24$ hrs., $R_o=1{,}000$ NM

Answer: $f_2=0.0367$, $T_2=27.2$ sec.

CASE 2. FILTER III, $t_{ob}=48$ hrs., $R_o+F=500$ NM

Answer: $f_5=0.145$, $T_6=6.9$ sec.

CASE 3. FILTER IV, $t_{ob}=12$ hrs., $R_o=100$ NM

Answer: $f_7=0.1816$, $T_7=5.55$ sec.

To solve any one of these, say, the first one, enter the dispersion diagram which contains the values for $R=1{,}000$ and $t=24$ hrs, find the intersection of the $R=1{,}000$ line with the value $t=24$ hrs, and read off the correct frequency on the bottom. The various frequencies needed for the filters described above can then be evaluated very quickly from the diagrams.

Sometimes, though rarely, the diagrams will not cover the needed range of frequencies, times, and distances. A schematic table, table 3.3, helps provide off-scale values in the dispersion diagram. It states quite simply that if the value of R is doubled, the value of t should be doubled if f is kept the same. If the value of R is doubled and the value of t is just the same, the value of f should be divided by two. All possible combinations are given in this table, and it extends the use of the diagrams to any possible values.

On figure 3.14, there are six scale divisions for a frequency of 0.01. It would be convenient for the forecaster if he would enter on the margin of this diagram the values 0.0017, 0.0033, 0.0050, 0.0067, 0.0083, and 0.0100. Then for these intermediate values the correct interpolated value is known immediately. On the other diagrams the frequency scale divisions are simply 0.0033 and 0.0067.

Table 3.3—Off-Scale Values in the Dispersion Diagrams

$2R$	$2t$	f	$2t$	$2R$	f	$2f$	$2t$	R
$2R$	t	$f/2$	$2t$	R	$2f$	$2f$	t	$2R$
$R/2$	$t/2$	f	$t/2$	$R/2$	f	$f/2$	$t/2$	R
$R/2$	t	$2f$	$t/2$	R	$f/2$	$f/2$	t	$R/2$

Angular Spreading Factor: Figure 3.17

The values of θ_4 and θ_3 are very important. A storm 400 NM wide which sends waves out to a point 1,000 miles away from the storm produces a completely different effect from a storm 800 NM wide which sends out some waves to a point 1,000 miles away from the

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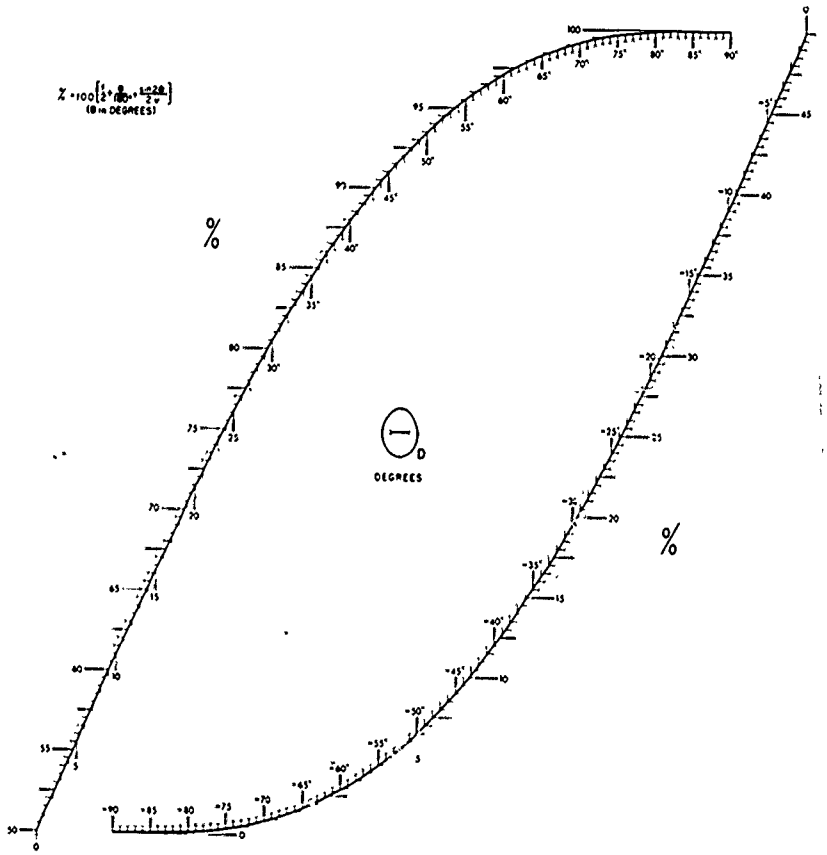

FIGURE 3.17 Angular spreading factor.

storm. If the waves inside the storm have the same spectrum and if all other things are equal, the waves outside the storm at the point of observation will be $\sqrt{2}$ times higher in the second case than they are in the first case, simply because of the effect of the width of the storm. The size of the storm determines in a very critical way the height of the waves at any point outside of the storm. These θ_n angles are values that take into effect the width of the storm, and they must be measured very carefully.

The co-cumulative spectra shown so far are functions of frequency only. Waves are really short-crested at all times. Thus, for any frequency band shown in the co-cumulative spectrum, the energy there is spread out over a large range of angles. Consequently, outside the storm, waves will be observed to arrive at points which they could not possibly reach if they traveled only in the dominant direction of the

wind in the storm. As an example, if waves did not travel out of a storm at an angle, the point shown in figure 3.8 and labeled R_0 could not possibly be reached by any of the waves in the indicated storm. At a point far enough away even a sine wave traveling at an angle of 10° to the dominant direction of the wind in the storm reaches points well outside the width of the storm. (This width is W_s in figure 3.8.) In fact, for an angle of 10° the waves get outside the width of the storm at a distance of only 2.8 times the width of the storm.

The height forecast for waves at points outside the storm must therefore be corrected for this effect of angular spreading. Wave forecasting theories formerly stated that any point within an angle of ±30° to the dominant direction of the wind in the storm should be treated in exactly the same way as points directly in front of the storm. This theory extends these past results and gives a better correction for waves traveling at angles greater than ±30°.

The angular spreading factor (fig. 3.17) makes it possible to compute the value which accounts for this effect. It is a percentage number and will always lie between 0 and 100 percent. To use the diagram, enter the values of θ_4 and θ_3 as determined from the map, read off the percentages on the other side of the curve, subtract the smaller value from the larger, and obtain a number which is the desired angular spreading factor. The graph for the angular spreading factor is the curve in figure 3.10 cut in half and doubled over to make the scale greater. The angles are on the inside. The percentage values are on the outside.

Some examples are given below

EXAMPLE 3.6. ANGULAR SPREADING FACTOR.

CASE 1. Given values: $\theta_4=35.7$, $\theta_3=24.8$. (See fig. 3.10)

FIND: Percent at 35.7°=85%, Percent at 24.8°=75%.

ANSWER: Angular spreading factor: 85%−75%=10%.

CASE 2. Given values $\theta_4=4.1°$, $\theta_3=-4.1°$.

FIND: Percent at 4.1°=54%, Percent at −4.1°=46%.

ANSWER: Angular spreading factor: 54%−46%=8%.

It should be pointed out at this point that only a particular type of chart can be used for accurate angular spreading computations. This is due to the earth's spheroidal shape and the difficulty in transferring sections of it to a plane surface without distortion. Angles and distances measured or computed along straight lines of a map are not always true, the error depending upon the projection used.

Ocean waves follow the shortest distance between fetch and forecast point. On a sphere, this is a great circle, defined as a circle on the surface of the earth, the plane of which passes through the earth's

center. Thus, it is obvious that a chart that presents great circles as straight lines would be the most suitable for determining the angles between wave travel and fetch orientation.

There are several such charts available, the two most popular being the gnomonic and the Lambert conformal conic projections. The latter chart is the more widely used; most weather maps used by the U. S. Weather Bureau, the military services, and commercial weather firms are of this type. On a Lambert conformal chart all meridians appear as straight lines converging to the pole. All parallels of latitude appear as arcs of concentric circles. A straight line on a Lambert chart will represent approximately a segment of a great circle, and the intersection of any two lines will correctly indicate their angular relation. A distance scale is usually given from which distance in nautical miles can be determined, or the mid-latitude, using 60 NM to the degree, can be employed.

All the examples in this manual are from standard Weather Bureau or Coast Guard maps, using Lambert conformal projections.

In dealing with short distances, on the order of a few hundred miles, the errors assumed by using maps not on the above projections are small, but for larger distances a Lambert conformal is advisable. Special care should be taken to avoid use of a Mercator projection in angular spreading computations. This is a widely used projection for navigational charts, but does not accurately represent great circle relations and distances.

If there is no choice but to use a Mercator chart, a correction must be applied to the computed angles. A table of correction values is given in H. O. Pub. No. 9, American Practical Navigator, as Table 1. This table actually gives the correction for converting radio (great circle) bearing to Mercator bearing, but can be used in reverse to convert Mercator bearing to great circle bearing.

Corrections are given as a function of the middle latitude between the forecast point and the fetch point and the difference of longitude between the same two points. Once the two angles necessary for angular spreading correction have been computed, the correction for a Mercator map projection is as follows:

(1) Determine latitude and longitude of forecast point and points in fetch.
(2) Compute middle latitude and difference in longitude for both sets of points.
(3) Enter table 1 in H. O. Pub. No. 9 to read off correction values.
(4) In north latitude, when fetch point is $\frac{\text{eastward}}{\text{westward}}$ of forecast point the correction is $\pm$. For south latitude, the sign is reversed.

After the Wind Ceases

Each of the examples given above describes the state of the sea over an area both inside and outside the fetch area occupied by winds of velocity V at a time determined by the length of time that the winds have been blowing. In examples 3.9 and 3.10 the winds are assumed to cease at the time that the computation was made, but the same procedure could be continued were the winds to continue to blow.

When the wind ceases, the generation process ceases, and dispersion and angular spreading immediately begin to modify the waves. The best way to forecast the waves after the winds cease, in each case, is to break up the spectrum for the given wind velocity into frequency bands and direction bands, just as was done in the chapter on wave refraction. Then the E value for a given small frequency band can be associated with an area over the ocean occupied by these spectral components with a given average (central) frequency and a given direction. These areas will not be the same size and, in general, the area covered by a high-frequency component will be larger and completely surround an area covered by a low-frequency component. These increments of energy are then propagated in the appropriate direction with the appropriate group velocity for a length of time determined by the time at which the forecast is to be made. The areas which arrive at the point of forecast are reassembled, and the height and period range can then be computed.

Range of E

The various forecasting diagrams that have been presented furnish a forecast of the value of E when properly employed. Then from the results of Chapter I any desired height quantity can be computed. The value of E can vary over a considerable range of numbers; yet the height will not change very much because it is related to $\sqrt{E}$. For example, E can vary from 28.1 to 36.1 and the significant height will change only from 13 feet to 15 feet. For most cases, extremely precise calculations of $\sqrt{E}$ and the various derived heights are not needed. Table 3.4 eliminates this last step by tabulating typical values of the significant height from 0.5 to 100 feet. Corresponding to these significant height values, there is a range of E values such that if the forecast value of E falls within this range of values, the significant height given in the table is sufficiently accurate for most purposes. Of course, the calculation of the correct value is possible any time, but this table will simplify the problem when quick forecasts are needed.

Some Sample Forecasts

Diagrams Needed

The needed forecasting tools are now assembled, and they have all been discussed in this chapter. In addition, a slide rule will be very

Table 3.4—Range of E for Typical Height Values

Range of E	$\sqrt{E}$	E	Av. Ht.	Sig. Ht.	Av. 1/10 Ht.
.008–.06	.18	.03	.32	.5	.65
.06–.19	.35	.12	.62	1.0	1.26
.19–.38	.53	.28	.94	1.5	1.91
.38–.64	.71	.50	1.26	2.0	2.56
.64–.94	.88	.77	1.56	2.5	3.17
.94–1.54	1.06	1.12	1.88	3	3.82
1.54–2.53	1.41	1.99	2.50	4	5.08
2.53–3.76	1.77	3.13	3.13	5	6.37
3.76–5.29	2.12	4.49	3.75	6	7.63
5.29–7.02	2.47	6.10	4.37	7	8.89
7.02–9.00	2.83	8.01	5.01	8	10.2
9.00–11.3	3.18	10.1	5.63	9	11.4
11.3–15.1	3.53	12.5	6.25	10	12.7
15.1–21.1	4.24	18.0	7.50	12	15.3
21.1–28.1	4.95	24.5	8.76	14	17.8
28.1–36.1	5.65	31.9	10.1	16	20.3
36.1–45.0	6.36	40.4	11.3	18	22.9
45.0–60.4	7.08	50.1	12.5	20	25.5
60.4–84.5	8.48	71.9	15.0	24	30.5
84.5–112	9.89	97.8	17.8	28	35.6
112–144	11.3	128	20.0	32	40.7
144–180	12.7	161	22.5	36	45.7
180–225	14.1	199	25.0	40	50.8
225–282	15.9	253	28.1	45	57.2
282–346	17.7	313	31.3	50	63.7
346–412	19.4	376	34.3	55	69.8
412–488	21.2	449	38.8	60	76.3
488–571	23.0	529	40.7	65	82.8
571–702	24.7	610	43.7	70	88.9
702–900	28.3	801	50.1	80	102
900–1,129	31.8	1,010	56.3	90	114
1,129–1,376	35.3	1,250	62.5	100	127

useful for some of the calculations. However, for many practical purposes, only table 3.4 is required. The needed diagrams are listed below along with the other information necessary for making a complete wave forecast.

Equipment Needed to Make a Wave Forecast

1. Weather map and weather data.
2. Co-cumulative spectra. (Figure 2.4a through 2.4f)
3. Dispersion diagrams. (Figures 3.14, 3.15, or 3.16)
4. Angular spreading factor diagram. (Figure 3.17)
5. Height table (table 3.4), or slide rule.

Steps in a Wave Forecast

For many wave forecasts, the steps listed below must be carried out. In some forecasts, as in the generation area while the waves

are being generated, some of the intermediate steps are omitted; but for a forecast involving wave propagation, each of the steps listed below must be carried out.

Steps in Preparing a Wave Forecast

1. Analyze the weather maps.
2. Forecast the co-cumulative spectrum in the storm.
3. Determine type of storm and filter required and evaluate observed quantities needed for the filter.
4. Evaluate θ_4 and θ_3.
5. Evaluate angular spreading factor.
6. For various times of observation find frequency band present. (Compute range of "periods" present, also.)
7. Find value of E present at point of observation owing to the effect of dispersion by subtracting the value of E at the upper frequency present from the value of E at the lower frequency present.
8. Multiply by angular spreading factor to find the forecast value of E.
9. From E compute needed height data.

EXAMPLE 3.7. WAVE FORECASTS FOR A TYPICAL WEATHER SITUATION.

FIGURE 3.18 SHOWS A TYPICAL WEATHER SITUATION IN JANUARY. AT 301800Z A COLD FRONT IS SHOWN PUSHING OFF THE COAST OF SOME LAND MASS. AT THE CENTER OF THE FRONT, THERE IS AN AREA OF RATHER HIGH WINDS, AND THE WINDS FAN OUT TO EACH SIDE OF THIS AREA AND BECOME QUITE LOW. THIS COLD FRONT PUSHES OUT DURING THE NEXT 30 HOURS UNTIL 010000Z, WHERE IT IS SHOWN IN ITS LAST POSITION. ON THE NEXT 6-HR. MAP, (NOT SHOWN) THE PUSH BEHIND THE COLD FRONT HAS STOPPED. THE 32-KNOT WINDS, WHICH OCCUPY AN AREA 400 NM WIDE AND 600 NM LONG ON THE 0000Z CHART, DIE OUT IN A FEW HOURS.

IN THESE MAPS EACH FULL BARB REPRESENTS 10 KNOTS, THE AVERAGE WIND OVER THE HIGH WIND AREA WAS 32 KNOTS, AND IT FELL OFF RATHER SHARPLY TO 15 KNOTS AND LESS OUTSIDE THIS HIGH WIND AREA. THREE POINTS ARE SHOWN ON FIGURE 3.18. THEY ARE THE POINTS A, B, AND C. IN THIS EXAMPLE THE WAVES WILL BE FORECAST FOR EACH OF THESE POINTS BEGINNING WITH POINT A.

Analysis of Forecast for Point A (Example 3.7)

The steps in preparing a forecast for point A are given below.

1. An analysis of figure 3.18 in terms of the scale shown in the figure and in terms of the wind field shows that at the time when the winds cease in the storm (010000Z of February) point A is 1,000 NM from the center of the leeward edge of the high wind area. The fetch, that is, the area covered by the winds of 32 knots, is 600 NM long. The width of the leeward edge of the storm is 400 NM. The time, $t_{ob}=0$, will be 010000Z of February, 1953.

2. The 32-knot winds existed to windward of the storm area from the time that the front first passed out over the ocean. There was a gap of several hundred miles between the high wind area and the coast on 311200Z of January; the winds moved forward with the storm area thereafter. Under these conditions, it is probably safe to assume that the spectrum in the area designated by the rectangular block is fully-developed and that it can be represented by the curve labeled 32 knots in either of the appropriate co-cumulative spectra charts of figure 2.4. The 32-knot winds have acted for over 24 hours upon the high spectral components which travel with the group velocity, and the lower spectral components, of course, are raised quite rapidly. Therefore the steady-state sea condition over this area ought to be represented most easily by this particular spectral chart. This spectrum can extend all the way back to the windward edge of the high wind area since the high wind area has been to windward of the line shown at the latest time and there was at least a 300-NM fetch up to the windward edge of the fetch a few hours before. More refined forecasts would have to consider the much lighter winds elsewhere as producing some low "period" waves of very low height. In this particular forecast all wind outside this high wind area will be neglected, and only the effect of this one generating area will be considered.

3. This is a forward-moving storm which has died out in intensity Therefore Filter III is needed. At $t_{ob}=0$, which is 010000Z of February, it is assumed that waves given by the 32-knot co-cumulative spectrum are present over this entire area. The wave-forecasting problem is then to find out what spectral components can arrive at point A, when they arrive, and how long they last.

4. Measure θ_3 and θ_4. Since they are both to the right, their values are positive; $\theta_3=32.6°$ and $\theta_4=49.4°$.

5. On figure 3.17 the reading at 49.4° is 93 percent, and at 32.6° is 82 percent. The difference yields an angular spreading factor of 11 percent.

6. From figure 2.9a, the lowest important frequency present is 0.0570. From figure 3.15, it takes 38 hours for the first spectral

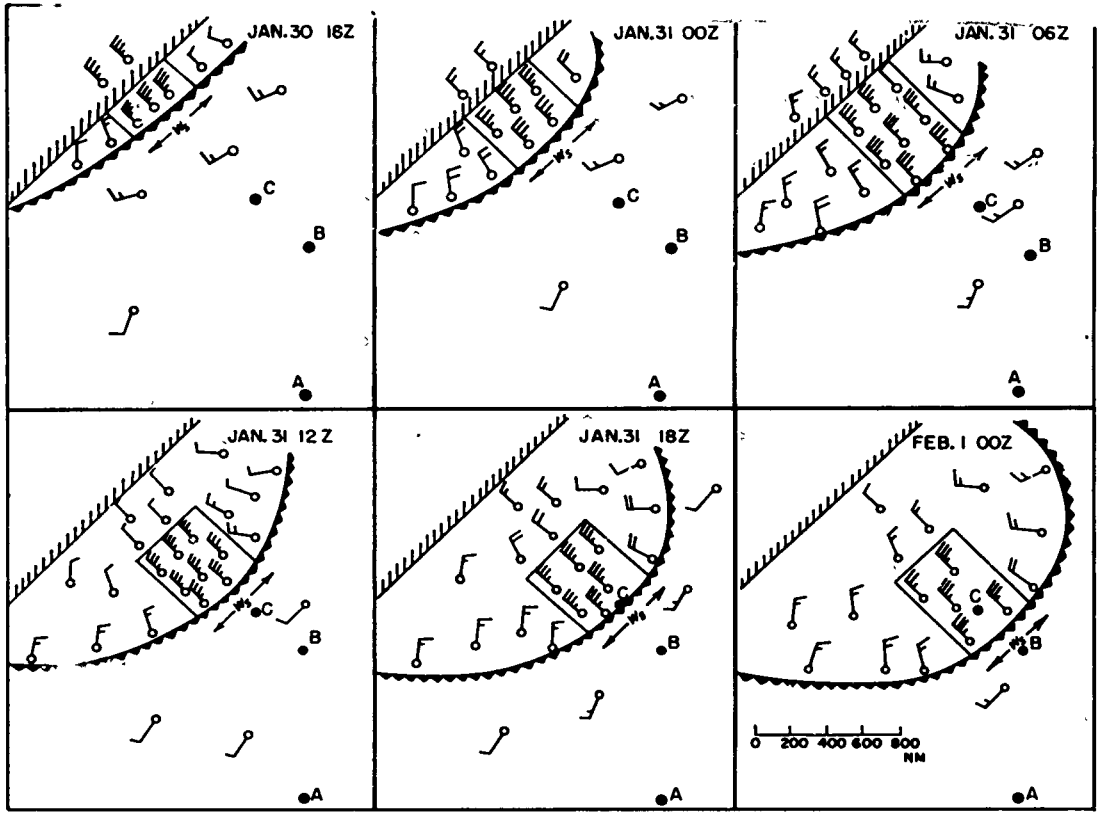

FIGURE 3.18 A typical weather situation. Forecasts to be made at points A, B, and C.

frequencies to travel the 1,000 NM to point A. For convenience, the forecast verifying times will be for 36 hours, 48 hours, and 60 hours after 010000Z, and at 12-hour intervals thereafter.

7. Table 3.5 has been prepared in order to carry out the forecast for the times suggested in step 6. Use the dispersion diagram, and find the values of f_2 for the time indicated in table 3.5. Use a value of R_0 of 1,000 NM. Also, find the values of T_2 from the diagram (or compute $1/f_2$). These values are entered in row 1 of table 3.5.

8. Use the dispersion diagram again, and find the values of f_6 and T_6 for $R_0+F=1{,}600$ NM. These values are entered in row 2.

9. Use the co-cumulative spectrum for 32 knots. Find the values of E for the different values of f_2. These values are entered in row 3 as $E(f_2)$.

10. Use co-cumulative spectrum for 32 knots. Find the values of E for the different values of f_6. These values are entered in row 4 as $E(f_6)$.

11. Find the value of $E(f_6)-E(f_2)$. That is, subtract the smaller number obtained in step 9 from the larger number obtained in step 10 and enter the result in row 5 as the difference. This is the value of E which would be present at the point of observation if there were no angular spreading in the storm.

12. Multiply the value obtained in row 5 by 11 percent as given in step 5 above.

13. Take the square root of the result of step 12. Row 7 tabulates $\sqrt{E}$.

14. Compute the needed height data. For example, the significant heights are given in row 8.

Final Forecast for Point A (Example 3.7)

AT POINT A, IF THIS IS THE ONLY STORM PRESENT THAT CAN AFFECT THE FORECAST, THE RESULTS OF TABLE 3.5 SHOW THAT A LONG LOW SWELL TRAVELING TOWARD THE SOUTH WITH THE SIGNIFICANT HEIGHT OF 1.38 FEET WILL APPEAR IN 36 HOURS (021200Z OF FEBRUARY). IT WILL HAVE AN AVERAGE "PERIOD" OF ABOUT 19 SECONDS. THE VALUE OF 29 SECONDS IS FICTITIOUS BECAUSE NO ENERGY IS PRESENT IN THE SPECTRUM AS HAS BEEN SHOWN IN CHAPTER II. TABLE 3.6 LISTS THE FORECASTS WHICH CAN BE MADE AT LATER TIMES FOR A NUMBER OF DAYS AFTERWARD AS THE WAVES CONTINUE TO ARRIVE AT POINT A FROM THE ORIGINAL HIGH WIND AREA.

Table 3.5—Forecast for Point A (Example 3.7)

Time Days, hrs. Date-time, Feb.	36 hrs. 1 day 12 hrs. 021200Z	48 hrs. 2 days 030000Z	60 hrs. 2 days 12 hrs. 031200Z	72 hrs. 3 days 040000Z	84 hrs. 3 days 12 hrs. 041200Z	96 hrs. 4 days 050000Z
1. f_2—(T_2)	.055 (18.2)	.073 (13.7)	.091 (11)	.11 (9.5)	.127 (7.9)	.145 (6.9)
2. f_6—(T_6)	.034 (29)	.046 (22)	.057 (17.5)	.068 (14.7)	.080 (12.5)	.091 (11)
3. E (f_2)	79.5	64.0	41.9	25.0	15.7	10
4. E (f_6)	80.0	82.0	79.8	70.0	54.1	41.8
5. Difference	0.5	18.0	37.9	45.0	38.4	31.8
6. 11%	0.06	1.98	4.16	4.95	4.12	3.5
7. $\sqrt{E}$	0.25	0.45	0.64	0.705	0.64	0.59
8. Sig. Ht	0.71	1.27	1.81	2.00	1.81	1.67

Time Days, hrs. Date-time, Feb.	108 hrs. 4 days 12 hrs. 051200Z	120 hrs. 5 days 060000Z	132 hrs. 5 days 12 hrs. 061200Z	144 hrs. 6 days 070000Z
1. f_2—(T_2)	.16 (6.25)	.178 (5.6)	.206 (4.9)	.224 (4.5)
2. f_6—(T_6)	.102 (9.8)	.114 (8.8)	.126 (7.9)	.138 (7.3)
3. E (f_2)	7	4.4	2.3	2
4. E (f_6)	31	23	15.8	12
5. Difference	24	18.6	13.5	10
6. 11%	2.64	2.06	1.48	1.1
7. $\sqrt{E}$	0.51	0.45	0.39	0.33
8. Sig. Ht	1.44	1.27	1.10	0.93

Table 3.6—Final Forecast Data for Point A

Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.	Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.
020600Z	0	---------	050000Z	5	7–11
021200Z	1.5	18–21	051200Z	5	6–10
030000Z	4	15–21	060000Z	4	6–9
031200Z	5	11–17	061200Z	4	5–8
040000Z	6	10–15	070000Z	3	4–7
041200Z	6	8–13			

Comments

This example of a forecast for point A looks as though it might be a lot of work. But consider that a forecast has been prepared for the point of observation for 7 days, as far as this particular storm is concerned. The E values at this point can be added to any other E values that may arrive at the same time for other storms to give the total effect of the waves present. Note also that the forecast tells when the waves will cease, and it gives a great amount of detail about how the waves change from day to day. The waves at point A will actually last about 5 days, and 36 hours will be required for the first waves to arrive. Thus the forecast extends in time for 7 days, and the effect of this storm takes 7 days to become negligible at point A.

Forecast for Point B (Example 3.7)

THE 32-KNOT WIND AREA NEVER REACHES POINT B. ON 010000Z OF FEB., POINT B IS 200 NM AWAY FROM THE CENTER OF THE LEEWARD EDGE OF THE STORM. AT THIS PARTICULAR TIME THE WINDS DIE DOWN VERY RAPIDLY IN THE STORM. THE PROBLEM THEN IS TO FORECAST THE WAVES AT POINT B FROM 010000Z ON INTO FUTURE TIME.

Analysis of Forecast for Point B (Example 3.7)

The steps in the preparation of a forecast for point B are given below.

1. The weather map data were given previously in figure 3.18. From the data given R_0 is 200 NM, the fetch F=600 NM, the width of the storm W_s is still 400 NM, and t_{ob}=0 is still 010000Z of February 1953.
2. The same spectrum is present in the storm as in example 3.7 for the forecast of the waves at point A.

3. The same filter is needed, but R_o is only 200 NM; R_o+F is only 800 NM. Filter III is needed.
4. Measure θ_3 as $-45°$ and θ_4 as $+45°$.
5. Enter diagram for angular spreading factor. The reading at 45° is 91 percent; the reading at $-45°$ is 9 percent; therefore the angular spreading factor is 82 percent.
6. The lowest important frequency present is 0.05. It takes 6½ hours for the waves of this frequency to arrive. It takes a small amount of time for the waves to build up after they arrive, so for point B a forecast for the waves after 9 hours, 12 hours, 18 hours, 24 hours, and so on will be prepared until the waves cease at point B.
7. Repeat step 7 as in previous case for point A Use R_o=200 NM.
8. Repeat as in previous example for point A. Use R_o+F=800 NM.
9. Repeat as in previous example for point A.
10. Repeat as in previous example for point A.
11. Repeat as in previous example for point A.
12. Multiply the value found tabulated in row 5 of table 3.7 by 82 percent as given in step 5 above.
13. Repeat as in forecast for point A.
14. Compute height data.

Table 3.7 gives the values which are obtained for the forecast for point B. The waves are forecast at 6-hour intervals for the first day, and then at 12-hour intervals when the height of the waves becomes less important.

Final Forecast for Point B (Example 3.7)

AT POINT B, IF THIS IS THE ONLY STORM THAT CAN AFFECT THE FORECAST, THE WAVES WILL BEGIN TO ARRIVE IN 6 HOURS, AFTER 010000Z OF FEBRUARY. THEY WILL BE TRAVELING TOWARD THE SOUTHEAST, AND THEY WILL BE QUITE SHORT-CRESTED BECAUSE OF THE WIDE RANGE OF ANGLES GIVEN IN THE ANGULAR FILTER. THE WAVES WILL INCREASE RAPIDLY IN HEIGHT DURING THE FIRST 18 HOURS, REMAIN FAIRLY CONSTANT FOR THE NEXT DAY, AND DIE DOWN COMPLETELY DURING THE NEXT TWO DAYS. TABLE 3.8 SUMMARIZES THE FORECAST DATA.

Table 3.7—Forecast for Point B (Example 3.7)

Time Days, hrs. Date-time, Feb.	9 hrs. 9 hrs. 010900Z	12 hrs. 12 hrs. 011200Z	18 hrs. 18 hrs. 011800Z	24 hrs. 1 day 020000Z	30 hrs. 1 day 6 hrs. 020600Z	36 hrs. 1 day 12 hrs. 021200Z	42 hrs. 1 day 18 hrs. 021800Z
1. f_2—($\tilde{T}$)	.07(14.3)	.092(11)	.137(7.4)	.183(5.5)	.226(4)	.274(3.7)	.3(≅0)
2. f_6—($\tilde{T}$)	.0167(∼)	.023(∼)	.034(∼)	.045(22)	.057(18)	.068(15)	.08(12.5)
3. $E(f_2)$	67.0	40	12.2	3.9	1.4	.4	.27
4. $E(f_6)$	81.1	81.1	81.0	81.0	78.0	69.4	55
5. Difference	14.1	41.1	68.8	77.1	76.6	69.0	54.73
6. 82%	11.5	33.8	56.5	63.2	62.9	56.6	44.9
7. $\sqrt{E}$	3.40	5.71	7.55	7.94	7.92	7.55	6.70
8. Sig. Ht	9.62	16.16	21.37	22.47	22.41	21.37	18.96

Time Days, hrs. Date-time, Feb.	48 hrs. 2 days 030000Z	54 hrs. 2 days 6 hrs. 030600Z	60 hrs. 2 days 12 hrs. 031200Z	72 hrs. 3 days 040000Z	84 hrs. 3 days 12 hrs. 041200Z	96 hrs. 4 days 050000Z
1. f_2—($\tilde{T}$)	.3(≅0)	.3(≅0)	.3(≅0)	.3(0≅)	.3(≅0)	.3(≅0)
2. f_6—($\tilde{T}$)	.091(11.5)	.103(9.8)	.113(9.1)	.137(7.5)	.16(6.2)	.182(5.5)
3. $E(f_2)$	.27	.27	.27	.27	.27	.27
4. $E(f_6)$	42	30	25.2	12.5	7	4.0
5. Difference	41.73	29.73	24.93	12.23	6.73	3.73
6. 82%	34.2	24.4	20.4	10.0	5.5	3.06
7. $\sqrt{E}$	5.86	4.94	4.52	3.16	2.34	1.75
8. Sig. Ht	16.58	13.98	12.79	8.94	6.62	4.95

Table 3.8—Final Forecast Data for Point B

Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.	Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.
010600Z	0	~	030000Z	17	0–12
010900Z	10	15–21	030600Z	15	0–10
011200Z	16	11–21	031200Z	12	0–9
011800Z	21	7–21	040000Z	9	0–7
020000Z	23	6–21	041200Z	7	0–6
020600Z	22	4–18	050000Z	4	0–5
021200Z	21	0–15	050600Z	~4	0–5
021800Z	19	0–13			

Comments

The results of the forecast for point B show that the waves arrive more quickly, build up higher, and die down sooner than they do at point A. The forecast extends over a forecast time of 4 days instead of 7 days as in the example for point A. Only 9 hours are required for the first waves to arrive as contrasted with 36 hours at point A, and the waves last only 3½ days instead of 5 days as they did at point A. The closer a point is to a storm, the sooner the waves pass that point.

Forecast for Point C (Example 3.7)

FIGURE 3.18 SHOWS THAT THE COLD FRONT PASSES POINT C AT 311800Z OF JANUARY. NO SPECTRAL COMPONENT IN THE WAVES GENERATED BY THE 32-KNOT WINDS BEHIND THE FRONT WILL TRAVEL AHEAD OF THE FRONT BECAUSE THEIR GROUP VELOCITIES ARE TOO LOW. CONSEQUENTLY, THE WAVES PRESENT BEFORE THE FRONTAL PASSAGE WILL BE LOW SOUTHERLY WAVES CAUSED BY THE WINDS SHOWN IN ADVANCE OF THE COLD FRONT. AFTER THE FRONT PASSES, THE WAVE HEIGHT WILL JUMP TO A VALUE CORRESPONDING TO THE VALUE OF *E* AT 27 HOURS ON THE DURATION GRAPH OF THE CO-CUMULATIVE SPECTRUM FOR 32 KNOTS. THIS IS TRUE BECAUSE THE WINDS HAVE ACTED ON THE WAVES JUST BEHIND THE FRONT EVER SINCE IT CAME OFF THE LAND, AND THIS DURATION IS AT LEAST 27 HOURS. THE HIGHER PERIOD COMPONENTS IN THE SPECTRUM THEREFORE TRAVEL RIGHT ALONG WITH THE WINDS AT ALMOST THE VELOCITY OF THE WINDS, AND THE WINDS HAVE ACTED CONTINUOUSLY TO GENERATE THEM. THE AREA COVERED BY THE 32-KNOT WIND IS OVER 400 NM LONG, AND TO WINDWARD OF THE AREA SHOWN ON THE LAST MAP

WHERE THE WINDS ARE 32 KNOTS, THE WINDS ONCE WERE 32 KNOTS IN STRENGTH. THEREFORE IT IS PRACTICAL (AS STATED AT THE BEGINNING OF THIS EXAMPLE) TO ASSUME THAT THE AREA IS COVERED BY A SPECTRUM OF FULLY GENERATED WAVES CORRESPONDING TO A CO-CUMULATIVE SPECTRUM FOR 32 KNOTS. CONSEQUENTLY, AT THE TIME OF THE FRONTAL PASSAGE (311800Z), THE WAVES WILL JUMP TO A SIGNIFICANT HEIGHT OF 25.8 FEET AND HAVE A "PERIOD" RANGE FROM 5 SECONDS TO 17.5 SECONDS. THIS JUMP TO THIS HEIGHT WILL OCCUR IN JUST A FEW HOURS AFTER THE PASSAGE OF THE COLD FRONT, POSSIBLY IN AS SHORT A TIME AS 45 MINUTES AFTER THE PASSAGE OF THE FRONT.

THE WAVES WILL MAINTAIN THIS HEIGHT AND "PERIOD" RANGE FOR THE NEXT 6 HOURS WHILE THE WINDS CONTINUE TO BLOW AT 32 KNOTS. THEN THE PUSH OF THE COLD FRONT WILL BE WEAKENED, AND THE WINDS IN THIS AREA WILL DIE DOWN IN JUST A FEW HOURS, AS ASSUMED BEFORE. THEN THE PROBLEM OF THE WAVE FORECAST AT POINT C IS TO FORECAST THE WAVES AT 6-HOUR INTERVALS UNTIL THEY DIE DOWN AFTER THE WINDS HAVE CEASED BLOWING OVER THIS 600 NM LONG, 400 NM WIDE AREA IN WHICH THE WAVES EXIST.

Analysis of Forecast for Point C (Example 3.7)

The steps in preparing a forecast for point C are given below.

1. The point at which the forecast is to be made is 400 NM from the windward edge of the area in which the waves exist. It is also within the fetch area.
2. The spectrum is fully developed, and it is still given by the curve for 32 knots in the co-cumulative spectrum.
3. Filter IV is needed since the decrease of the waves in the fetch is needed.
4. No computation of θ_4 and θ_3 is needed.
5. The angular spreading factor is 100%.
6. A full spectrum is present at 010000Z. The problem is to forecast waves for 6-hour intervals thereafter.
7. $f_2 = \infty$ for each forecast because all spectral components are present at the start of the forecast time.
8. Use the dispersion diagram. Find the value of f_7 and T_7 for $F_0 = 400$ NM. See row 2 of the table.
9. Repeat all steps 9 through 14 as in previous examples.

The results of the computations for forecasting the waves at point C are given in table 3.9, as referred to in the above steps of the previous examples.

Final Forecast for Point C (Example 3.7)

AT 311800Z THE WAVES WILL JUMP TO A SIGNIFICANT HEIGHT OF 26 FEET WITH A "PERIOD" RANGE FROM 5 TO 18 SECONDS WHEN THE COLD FRONT PASSES WITH 32-KNOT NORTHWEST WINDS BEHIND IT. THE WAVES WILL APPEAR TO BE TRAVELING TOWARD THE SOUTHEAST AND WILL BE VERY SHORT-CRESTED AND QUITE CHOPPY. THIS SIGNIFICANT WAVE HEIGHT AND "PERIOD" RANGE WILL BE MAINTAINED FOR 18 HOURS AFTER THE WAVES FIRST BUILD UP TO THE HEIGHT OF 26 FEET. IN THE NEXT 6 HOURS THE WAVES WILL DIE DOWN ONLY 3 FEET IN HEIGHT. THEREAFTER, THE WAVES WILL DECREASE IN HEIGHT, AT FIRST GRADUALLY AND THEN RAPIDLY UNTIL ON 030600Z THEY WILL BE ONLY 3 FEET HIGH AND VERY SHORT IN "PERIOD."

THE FINAL FORECAST DATA FOR POINT C ARE TABULATED IN TABLE 3.10. NOTE THAT THE WAVES DIE DOWN TO PRACTICALLY NOTHING TWO DAYS AFTER THE WINDS CEASE OVER THE AREA IN WHICH THE WAVES HAVE BEEN GENERATED.

A Summary of the Forecast Results

In this rather detailed example which has just been carried out, three points have been chosen at which a forecast is to be made, given the current and past weather maps. A forecaster would have to know that at 010000Z the winds died down markedly in intensity behind the front. This forecast example then shows the major effects of dispersion and angular spreading at the three points of interest. The farther away the point of interest is from the fetch area, the lower the waves will be and the longer it will take for them to pass. If the point of interest is at a moderate distance, the waves are quite high and take a shorter time to pass as compared with the distant point. If the point of interest is within the generating area, the important thing to note is the distance from the point of interest to the windward edge of the fetch area. Then with the use of Filter IV, the time it takes the waves to die down after the wind has ceased can be forecast.

This forecast example shows quite markedly many features which are of great importance to the practical forecaster. He not only wants to know the height of the highest waves which will pass a given point, but he also wants to know how long the waves will last and how they

Table 3.9—Forecast for Point C (Example 3.7)

Time Days, hrs. Date-time, Feb.	0 hrs. 0 hrs. 010000Z	6 hrs. 6 hrs. 010600Z	12 hrs. 12 hrs. 011200Z	18 hrs. 18 hrs. 011800Z	24 hrs. 1 day 020000Z
1. $f_2-(\tilde{T})$	∞ (0)	∞ (0)	∞ (0)	∞ (0)	∞ (0)
2. $f_7-(\tilde{T})$	0 (∞)	. 022	. 045 (22)	. 068 (15)	. 091 (11)
3. $E(f_2)$	0	0	0	0	0
4. $E(f_7)$	81. 1	81. 1	80. 5	69. 8	41. 8
5. Difference	81. 1	81. 1	80. 5	69. 8	41. 8
6. 100%	81. 1	81. 1	80. 5	69. 8	41. 8
7. $\sqrt{E}$	9. 1	9. 1	8. 96	8. 35	6. 47
8. Sig. Ht.	26	26	25. 3	23. 6	18. 3

Time Days, hrs. Date-time, Feb.	30 hrs. 1 day 6 hrs. 020600Z	36 hrs. 1 day 12 hrs. 021200Z	42 hrs. 1 day 18 hrs. 021800Z	48 hrs. 2 days 030000Z	54 hrs. 2 days 6 hrs. 030600Z
1. $f_2-(\tilde{T})$	∞ (0)	∞ (0)	∞ (0)	∞ (0)	∞ (0)
2. $f_7-(\tilde{T})$	. 114 (8. 9)	. 137 (7. 5)	. 160 (6. 3)	. 182 (5. 5)	. 205 (5)
3. $E(f_2)$	0	0	0	0	0
4. $E(f_7)$	24	12. 2	7	4	2. 3
5. Difference	24	12. 2	7	4	2. 3
6. 100%	24	12. 2	7	4	2. 3
7. $\sqrt{E}$	4. 9	3. 5	2. 64	2. 0	1. 52
8. Sig. Ht.	13. 9	9. 9	7. 5	5. 7	4. 3

Table 3.10—Final Forecast Data for Point C

Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.	Date-time Feb.	Significant ht. (rounded off) feet	"Period" range (rounded off) sec.
311800Z.....	26	(0)5–18	020600Z.....	13	(0)5– 8
010000Z.....	26	(0)5–18	021200Z.....	10	(0)5– 8
010600Z.....	26	(0)5–18	021800Z.....	8	(0)5– 6.6
011200Z.....	25	(0)5–18	030000Z.....	6	(0)5– 5.5
011800Z.....	24	(0)5–15	030600Z.....	3	(0)(5–5)
020000Z.....	18	(0)5–11			

will change from day to day. This forecasting procedure gives these data and makes it possible to compute in a logical way the entire sequence of waves that pass a given point and to say something about when the waves will stop and what they will look like over a long period of time.

This forecast example is also very important because it graphically illustrates the major feature of this wave-forecasting theory. This major feature is very simple: if the various spectral components in the total sea in the fetch area can be kept track of and propagated in the proper direction with the proper group velocity, then they can be reassembled by computing their E values at any point of observation and the height and "period" range at the point of observation can be found. Thus the waves can be forecast at any point after the winds which generated them have ceased.

There is one last very important fact about this forecast example. It is that the forecast example oversimplifies nature. It neglects the effects of the winds of lower velocity surrounding the generating area. It also oversimplifies by assuming very sharp boundaries for the fetch area. For more critical forecasts where such simplifications cannot be employed, it is possible to obtain bounds on the forecast heights by assuming the fetch to be at a minimum of 500 NM and at a maximum of about 700 NM. Then by carrying out a double computation, using each value for the length of the fetch, a range of heights can be obtained, and the true value will be known to lie between these two computed values. Similarly, if any parameter of the filter cannot be determined precisely, then it is advisable to use the best estimate, a greatest possible value, and a least possible value for this parameter. Forecasts for each of the three conditions can be carried out and the best height forecast can then be bracketed by a least height and a greatest height forecast. The forecaster can then assign a certain amount of reliability to the final results in terms of these bounds.

A Slightly More Complicated Case (Example 3.8)

SOMETIMES IT IS NOT SUFFICIENTLY ACCURATE TO ASSUME THAT THE SAME SPECTRUM COVERS THE ENTIRE AREA IN WHICH THE WAVES HAVE BEEN GENERATED. FOR EXAMPLE, CONSIDER A POINT AT WHICH A FORECAST IS TO BE MADE WHICH IS LOCATED 400 NM OFFSHORE FROM A LARGE LAND MASS. SUPPOSE THAT WIND OF 36 KNOTS HAS BEEN BLOWING FROM THE LAND TO THE SEA FOR OVER 3 DAYS. THEN BY THE RESULTS OF CHAPTER II IT WOULD BE POSSIBLE TO FORECAST WHAT WAVES WERE PRESENT WHILE THE WIND WAS BLOWING AT 36 KNOTS. NOW SUPPOSE THAT THE 36-KNOT WIND CEASED TO BLOW AT 030600Z, OF MARCH 1953, AND THAT THE WIND DIED DOWN TO, SAY, 10 OR 15 KNOTS AND REMAINED LOW FOR SEVERAL DAYS. THE FORECAST PROBLEM IS TO FORECAST HOW RAPIDLY THE WAVES WILL DIE DOWN AFTER THE WIND HAS CEASED TO BLOW.

Analysis of Forecast for Example 3.8.

At the time that the wind ceases, the waves present are characterized by the intersection of the co-cumulative spectral curve for 36 knots and the 400-NM fetch line on figure 2.4d. For these conditions $E=113$ ft.^{2}. The lowest frequency present is approximately 0.065, which corresponds to a highest "period" present of 15 seconds. Thus while the wind was blowing, the significant height was 31 feet and the "period" range was from 15 seconds to about 5.6 seconds, as shown in Chapter II.

The important point of this example is that E is not 113 ft.^{2} over the entire fetch; and if Filter IV is used, the windward edge of the fetch, namely F_o, is different for the different spectral frequencies present at the points of observation.

Figure 3.19 illustrates this. At the point 400 NM from the shore the wave conditions just described are present; but if a point 300 NM from the shore is picked, E is only 76 ft.^{2}, the significant height is only 24.7 ft., and the lowest frequency present is 0.082. Points closer to the shore and farther away from the point of observation have lower and lower values of E and consequently lower and lower values of significant heights and higher values of minimum frequency. All these values are shown in figure 3.19b; they can be read off in a matter of seconds from figure 2.4d, the co-cumulative spectrum.

Over the first 100 NM to windward of the point at which the forecast is to be made, E is greater than 76 ft.^{2} because E increases continuously the farther away from the coast the point of observation

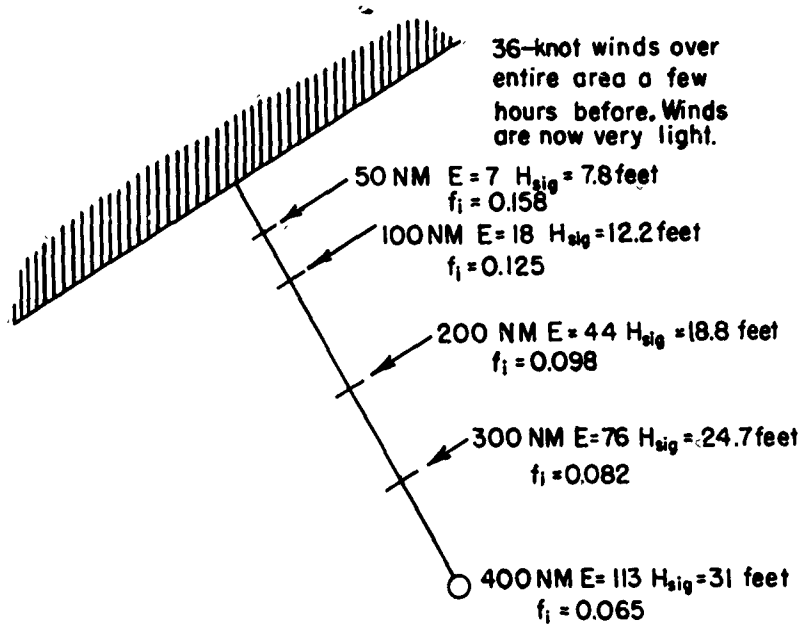

FIGURE 3.19 Fetch-limited waves.

is located. Therefore, all the energy in the waves at the point 400 NM from the coast which is associated with frequencies between 0.065 and 0.082 is present only over this short distance of 100 NM. When the spectral frequency associated with the point labeled 300 NM which is 100 NM to windward of the point of forecast moves past the point of forecast after the wind ceases, the value of E at the point at which the forecast is to be made will drop from 113 ft.^{2} to 76 ft.^{2} The problem then is to determine how long it takes a spectral frequency of 0.082 to travel 100 NM. From the dispersion diagram (fig. 3.16), it can be determined that the time required is 5.5 hours. Therefore, 5.5 hours after the wind stops, the value of E will decrease to 76 ft.^{2}, the significant height of the waves will drop to 24.7 ft., and the "period" range will decrease to values between 12.2 and 5.6 seconds.

A point 200 NM to windward of the point of forecast has an E value of 44 ft.^{2} and a significant height of 18.8 feet. The frequency associated with the significant height is 0.098, and it takes this spectral frequency 13 hours to travel the 200 NM. Therefore, 13 hours after the wind stops, the significant height at the point of observation will be 18.8 feet, and the "period" range will be from 10.2 to 5.6 seconds.

These results can be summarized in table 3.11 for the remaining points of interest. Table 3.11 shows, for example, that after 37 hours E will drop to 7 ft.^{2} and the significant height will drop to 7.8

Table 3.11—The Decrease of Waves in an Area of Wave Generation Limited by a Fetch

F_e (Distance) NM	Frequency per second	Time hours	E ft.²	Sig. ht. feet	"Period" range seconds
0	0.065	0	113	31	15 –5.6
100	0.082	5.5	76	24.7	12.2–5.6
200	0.098	13	44	18.8	10.2–5.6
300	0.125	26	18	12.2	8 –5.6
350	0.158	37	7	7.8	6 –5.6

feet. The important point is that only a fraction of the total fetch has low frequency spectral components in it, and they, therefore, will pass the point of observation in a much shorter time than would be required if they had to travel the entire 400 NM from the coast to the point of observation.

Final Forecast for Example 3.8

AFTER THESE SIMPLE COMPUTATIONS HAVE BEEN MADE ON THE BASIS OF THE VARIATION OF WAVE HEIGHT WITH DISTANCE FROM THE COAST, THE RESULTS OF THE COMPUTATIONS CAN BE TURNED AROUND. THE RESULT IS TABLE 3.12, WHICH IS THE FINAL FORECAST FOR EXAMPLE 3.8.

Table 3.12—Forecast for Example 3.8

Date-time, March	Significant height, feet	"Period" range seconds
030600Z	31	15 –5.6
031130Z	25	12.2–5.6
031900Z	19	10.2–5.6
040800Z	12	8 –5.6
041900Z	8	~6 –5.6

Comments

If the wind stops at 030600Z, and the waves are 31 feet high at that time, they will drop to 25 feet in 5.5 hours, to 19 feet in 13 hours, and to 12 feet in 26 hours, simply because of the effect of dispersion. The waves die down only because the rectangular areas associated with the frequencies involved have passed the point at which the forecast is being made. The waves are not dissipated by eddy friction, but merely have moved out of the particular forecast area. The frequencies present at the rear edge of every storm and the times associated with them, which they require to travel given distances, are just as important as the frequencies at the forward edge of every storm and the times they require to travel a given distance. These frequencies define the area covered by the envelope of the sine wave

associated with that particular frequency. The application of this principle in all forecasts permits a forecast of how long the waves will last at a given point.

The Use of the Distorted C. C. S. Curves

The distorted C. C. S. curves that were given in Chapter II can be used to forecast wave propagation as easily as they can be used to forecast wave generation. However, the E values at the various frequencies must be read off very carefully in the preparation of the forecast. For low frequencies, very small increments on the E scale are as significant as large increments for high frequencies. The E values should be estimated to at least the nearest square foot in computing all differences, and the frequencies should be measured from the curve as carefully as possible.

Moving Fetches

Limitations on Filters

Filter II is a filter which can be applied if the generating area is stationary. Such conditions can occur if, for example, there is a strong gradient between a stationary high and a stagnating low. Filter III is a filter which can be applied if the generating area is moving in the direction of the wind with the velocity of the wind in the generating area. Such conditions, as described before, can occur behind a well-developed cold front.

Isobaric configurations and the winds at the surface are in part determined by changes at upper levels; hence, it is possible for the winds over a generating area to blow with a velocity, V, and for the generating area to move in any direction with another velocity, v_r. Such conditions are not completely treated by the filters that have been given, although in many cases one or the other will be a fairly good approximation.

Forecasts for Moving Fetches

It would appear that the sea in such generating areas can build up by successive finite increments. The waves build up over the generating area for, say, 3 hours. Then the generating area as well as the spectral components are found where they would have traveled in three hours. Then if the spectrum up to a frequency, f_i, is present, one moment of duration equal to the next three hours is added on to the spectrum. The process is repeated at 3-hour intervals until the wind stops or until the time at which the forecast is to be made, whichever is earlier.

After the wind ceases, the various spectral components cover areas of different sizes on the sea surface. However, each area travels in

some direction with a known group velocity, and by computing which frequencies can be present at a given point of forecast at a given time, a forecast of the waves can be made at times after the wind ceases.

The problem of formalizing the procedures to be employed in forecasting waves for such cases has led to the introduction of some new filters, which take into account the motion of the fetch.

Filter M-V

The first filter considered is designated as M–V. It pertains to the following situation: A fetch of length, F, forms at $t=0$. The wind blows with a speed V. The fetch moves with velocity, v_F, in the same direction as the wind. The problem is to determine the energy in the wave spectrum at any desired point either inside or outside the fetch area and at any time after $t=0$.

As the fetch moves with velocity v_F, the length of time that the wind blows at each point in the path of the fetch can be found. If the windward edge of the fetch takes a certain number of hours to move past a certain point, then the duration at that point is given by this number of hours, provided that the point was within the initial fetch area. The "duration" at a point in advance of the initial fetch area is modified by the flux of energy (at group velocity) into the area in addition to the energy received by the sea from the wind stresses. This latter effect causes a reduced apparent duration time for points in advance of the initial position of the fetch in using the co-cumulative spectrum graphs. Or, stated another way, a given E value in the partially developed sea is attained sooner than would be expected from a computation based on the observed duration of the wind at a point to leeward of the initial position of the fetch. It results in an increased gradient in the energy distribution at the leeward edge of the fetch. In order to determine the energy distribution at the leeward edge of the fetch, the following relation is used for a region called the *duration control zone*, the zone between the leeward edge of the fetch and the region within the fetch where a steady state has been developed.

$$D=\int_0^{\tau} (v_F-1.515/f)\,d\tau$$

$$=\sum_{\tau=1}^{N} (v_F-1.515/f_i)\,\Delta\tau \qquad (3.11)$$

$$\text{where } \sum_{1}^{N} \Delta\tau=\tau$$

In equation (3.11), D is the distance from the leeward edge of the fetch to a zone to windward where the component has duration time equal to τ. This duration time, τ, holds only if the fetch has been in

existence at least that long; otherwise the actual time of existence of the fetch should be used to determine E from the duration graphs. Furthermore, it should be mentioned that the choice of $\Delta\tau$ in the above formula depends on how rapidly $1.515/f$ (group velocity) varies with the duration, τ. Smaller values of $\Delta\tau$ should be used in conjunction with the larger variations of $1.515/f$ with τ.

To find the distribution of the energy at the windward edge of the fetch, consider the wave components that are left behind at the windward edge in the *dispersion zone*, where the fetch moves to leeward faster than the group velocity of the higher frequency components. In order to do this, determine the duration of the wind at a point located at the windward edge of the fetch area. This locates the rear of the spectral component with the same duration time. It is then necessary to follow this component at its group velocity. This can be done for any windward fetch position after the fetch first forms.

Example 3.9 (Filter M-V)

A FETCH 700 NM LONG STARTS TO MOVE DOWN WIND WITH A SPEED OF 30 KNOTS. THE WIND OVER THE FETCH HAS A VELOCITY OF 30 KNOTS IN THE SAME DIRECTION AS THE MOTION OF THE FETCH. THE WIND LASTS 24 HOURS. WHAT IS THE STATE OF THE SEA AT VARIOUS DISTANCES FROM THE WINDWARD EDGE OF THE FETCH AT $t=24$ HOURS?

In this example $\Delta\tau$ will be chosen as one hour since this is as fine a reading as can be made from the duration graphs. The distribution of energy at the leeward edge of the fetch is determined in table 3.13.

Table 3.13—Energy Front of the Moving Fetch in the Duration Zone. (1-Hour Interval)

$D_i=(vF-1.515/f_i)\Delta\tau$ (NM)	$\Sigma\Delta D_i=D$ (NM)	E_i at 1 hr. Duration for 30-knot wind	$D_i=(vF-1.515/f_i)\Delta\tau$ (NM)	$\Sigma\Delta D_i=D$ (NM)	E_i at 1 hr. Duration for 30-knot wind
25.9	25.9	.13	17.0	272.2	18.
25.4	51.3	.21	16.4	288.6	21.
23.8	75.1	1.0	15.6	304.2	23.7
22.1	97.2	2.9	15.0	319.2	26.4
21.7	118.9	3.6	14.1	333.3	31.0
21.2	140.1	4.8	13.0	346.3	35.
20.6	160.7	6.1	11.8	358.1	40.
20.0	180.7	7.0	10.3	368.4	44.8
19.6	200.3	9.0	8.0	376.4	51.5
18.9	219.2	11.3	5.4	381.8	56.0
18.3	237.5	13.1	2.3	384.1	57.0
17.7	255.2	15.2	0	384.1	58.7

The distribution of energy in the dispersion zone to the windward of the fetch can be determined as follows:

The travel time of the windward edge of the fetch from its initial position to any point X gives the maximum duration time of the wind for that point. The windward edge of the zone having a spectral component corresponding to this duration is thus located. The difference between the time of observation (t=24 hours in the example) and this duration time gives the travel time for the windward edge of the zone having this particular component. This method provides a complete check on the component (and therefore the energy) in the dispersion zone to windward of the fetch area.

Following is the solution to example 3.9 which shows the way the data obtained from the above procedures can be collected.

It can be observed from the tables that the fully developed sea (E=59 ft.^{2}) occurs at a distance somewhat less than 400 NM to windward of the leeward edge of the fetch. If one were to neglect the wave energy flux, the fully developed sea would be computed to occur 720 NM to windward of the fetch. This would mean that a fully developed sea would be forecast to occur nearly 11 hours later than is indicated above.

In table 3.14 there are 300 NM of fully developed sea indicated, whereas neglect of wave flux will yield no area of fully developed sea. The effect of energy flux is therefore of the utmost importance in determining the distribution of energy near the leeward edge of the advancing fetch area.

Filter M-VI

If the wind blows in a direction opposite to the direction of propagation of the fetch, then the only difference that occurs in the above considerations is in the relation:

$$D=\int_0^{\tau} (v_F+1.515/f)d\tau$$

$$\cong\sum_1^N (v_F+1.515/f_i)\Delta\tau_i \qquad (3.12)$$

where the plus sign now appears in place of the minus sign.

The cause of this is the flux of energy (at group velocity) out of the fetch area (since the energy now moves opposite to the direction of propagation of the fetch). This effect requires an increased apparent duration time for a given energy to be produced near the leeward edge of the fetch area.

Table 3.14—Distribution of E in Area Passed Over by Moving Fetch of Example 3.9

Distance from windward edge of fetch at $t=0$, (NM), measured to leeward	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400
Duration of wind at above point (hrs.)	0	3.3	6.7	10.0	13.3	16.7	20.0	23.3	20.7	17.3	14.0	10.7	7.3	4.0	0.7
$_{max}T$ reached at new positions listed below	0	4.5	6.2	7.2	8.7	10.3	13.0	19.2	20.5	20.6	20.6	9.9	7.3	5.4	2.5
$_{min}f=\frac{1}{_{max}T}$	∞	.22	.16	.14	.11	.10	.08	.05	.05	.05	.05	.10	.14	.18	.40
Travel time after wind ceases (hr.)	24.0	20.7	17.3	14.0	10.7	7.3	4.0	0.7	0	0	0	0	0	0	0
Distance traveled at frequency $_{min}f$ (NM)	0	141	162	153	141	114	79	20	0	0	0	0	0	0	0
New positions of spectral components at $t=24$ hrs. (NM from initial windward edge	0	241	362	453	541	614	679	720	800	900	1000	1100	1200	1300	1400
E (ft.^{2})	0	2	6	10	22	27	44	59	59	59	59	26	12	4	0
Sig. Ht. (ft.)	0	4.0	6.9	9.0	13.3	14.7	18.8	21.7	21.7	21.7	21.7	14.4	9.8	5.7	0
$\bar{T}$ (sec.)	0	3.4	4.6	5.1	6.2	6.8	7.7	8.6	8.8	8.8	8.8	6.7	5.1	4.2	1.9

Example 3.10

AS AN EXAMPLE OF THIS CASE, CONSIDER A FETCH AREA 880 NM LONG, MOVING WITH A SPEED OF 30 KNOTS. THE WIND STARTS TO BLOW AT 30 KNOTS OPPOSITE TO THE DIRECTION OF MOTION OF THE FETCH. THE WIND AGAIN LASTS FOR 24 HOURS. THE PROBLEM IS TO DETERMINE THE DISTRIBUTION OF ENERGY AT VARIOUS DISTANCES FROM THE LEEWARD EDGE OF THE INITIAL POSITION OF THE FETCH. NOTE THAT THE LEEWARD EDGE IN THIS CASE IS *NOT* THE ADVANCING EDGE.

Table 3.15 shows that there is no region of fully developed sea (E–59 ft.^{2}) anywhere. The effect of wave energy flux is to cause the total wave energy to be distributed throughout the fetch with a very small gradient from windward to leeward. The difference in energy distribution between example 3.9 and example 3.10 is due to the energy being distributed over a large ocean area when the fetch moves against the wind (example 3.10 but over a much smaller area when the fetch moves with the wind (example 3.9).

Filter M-VII

Finally, consider a very simple filter for a fetch moving along a coast at a constant velocity with the wind blowing toward the coast at an angle to the direction of propagation of the fetch. For simplicity, consider an east wind in a fetch moving north parallel to a coast orientated north-south. It is assumed that the wind has been blowing a sufficient time for all significant components to have reached the coast.

The purpose of this filter is to project these significant components upon the coast and thereby determine the distribution of E along the coast. It should be mentioned that the coast need not be parallel to the direction of motion. In fact, even if the coast is quite irregular the projection method (to be described below) holds.

Example 3.11

AN EAST WIND IS BLOWING WITH A VELOCITY V WITHIN A FETCH THAT IS MOVING NORTHWARD WITH A VELOCITY V_F. WHAT IS THE DISTRIBUTION OF ENERGY AND THE STATE OF THE SEA FOR ANY POINT P AND ANY TIME t?

In order to determine the distribution of E along the coast as shown in figure 3.20, the following procedure is used:

Determine from the duration or fetch graphs the limiting frequencies of components having significant energy. Corresponding to these frequencies which will be denoted by f_1, and $f_2(f_2>f_1)$ there exist the group velocities $1.515/f_1$ and $1.515/f_2$ $(1.515/f_1>1.515/f_2)$ and duration

Table 3.15—Distribution of E in Area Passed Over by Moving Fetch of Example 3.10

(Waves are traveling toward x=0, fetch is traveling toward x=1600)

Distance from leeward edge of fetch at $t=0$, (NM)	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
Duration of wind at above point (hrs.)	0	3.3	6.7	10.0	13.3	16.7	20.0	23.3	24.0	23.3	20.0	16.7	13.3	10.0	6.7	3.3	0
$_{max}T$ reached at new positions listed below	0	4.5	6.2	7.2	8.7	10.3	13.0	14.0	12.5	10.7	9.6	8.6	7.6	6.6	5.6	3.8	0
$_{min}f=\frac{1}{_{max}T}$	∞	.22	.16	.14	.11	.10	.08	.07	.08	.09	.11	.12	.13	.15	.18	.27	∞
Travel time after wind ceases (hr)	24.0	20.7	17.3	14.0	10.7	7.3	4.0	0.7	0	0	0	0	0	0	0	0	0
Distance traveled at frequency $_{min}f$ (NM)	0	−141	−162	−153	−141	−114	−79	−20	0	0	0	0	0	0	0	0	0
New positions of spectral components at $t=-24$ hrs. (NM from initial leeward edge)	0	−41	38	147	259	386	521	680	800	900	1000	1100	1200	1300	1400	1500	1600
E (ft.^{2})	0	2	6	10	22	27	44	51	44	36	23	17	15	8	4	1	0
Sig. Ht. (ft.)	0	4.0	6.9	9.0	13.3	14.7	18.8	20.2	18.8	17.0	13.5	11.6	10.9	8.0	5.7	2.8	0
$\tilde{T}$ (sec.)	0	3.4	4.6	5.1	6.2	6.8	7.7	8.0	7.7	6.7	6.5	6.0	5.4	4.8	4.1	2.8	0

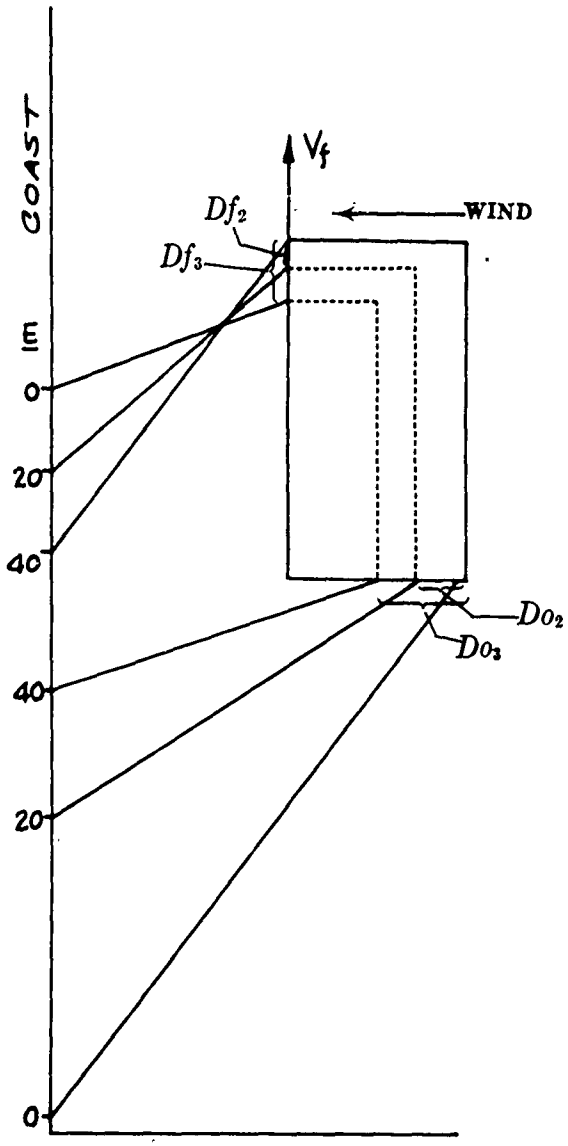

FIGURE 3.20 Fetch moving parallel to a straight coast at velocity V with a wind blowing at right angles to the direction of fetch motion.

times τ_1 and τ_2 ($\tau_1 > \tau_2$), respectively. These components will be generated at a distance $Df_1 = v_F\tau_1$ and $Df_2 = v_F\tau_2$ from the north edge of the fetch and at a distance $Do_1 = \int_0^{\tau_1} (1.515/f)dt$ and $Do_2 = \int_0^{\tau_2} (1.515/f)dt$ from the windward edge of the fetch. In a unit time the fetch will move north the distance v_F, the fastest component will move

west the distance $1.515/f_1$, and the slowest component will move west the distance $1.515/f_2$. Adding these displacements vectorially as indicated in the diagram, the projection is obtained, which determines the boundaries of the significant components produced in the fetch. In a similar manner the boundaries of all the components with frequencies greater than f_1, and less than f_2 can be obtained. In this way the entire spectrum can (in a sense) be projected onto the coast as is indicated in figure 3.20 and the energy $E(f)$ determined along the coast or anywhere over the ocean between the fetch and the coast. In this example, it is seen that this projected system will move with the speed of the fetch along the coast. As mentioned before the projection can still be made for irregularly shaped coast. For a wind blowing at some angle to the direction of propagation of the fetch, the projection is easily carried out where Df_1 now equals $\int_0^{\tau_1} [v_F - (1.515/f) \cos\theta]\, dt$, $Df_2 = \int_0^{\tau_2} [v_F - (1.515/f) \cos\theta]\, dt$, $Do_1 = \int_0^{\tau_1} (1.515/f) \sin\theta\, dt$, and $Do_2 = \int_0^{\tau_2} (1.515/f) \sin\theta\, dt$ where θ is the angle between the wind and fetch velocity vectors. One has to subtract the fetch velocity vector from the component velocity vector which is always aligned with the wind.

How Sea Changes Into Swell

The results of this chapter also explain how sea changes into swell. Figure 3.9 shows how the sea spectrum is filtered so that only a narrow band of frequencies is present at a point outside the generating area. As was explained in Chapter I, a swell has a narrow spectrum. The more distant a point of observation is from a given storm, the narrower the band of frequencies present at any given time and the more regular the swell. Compare, in example 3.7, the frequency band present at point A at different times (table 3.6) with the frequency band present at point B at different times (table 3.8). At point A, for comparable parts of the spectrum, the band is much narrower than at point B.

The "Period" Increase of Ocean Swell

The highest swell to pass a given point outside the generating area is associated with the frequency, f_{max}, as given in table 2.1. The *period* associated with f_{max} is greater than the average "period," $\tilde{\tilde{T}}$, observed inside the generating area. For a fully developed sea, it is equal to $\sqrt{2}\ \tilde{\tilde{T}}$. Therefore, the "period" of the highest swell will become equal to $\sqrt{2}\ \tilde{\tilde{T}}$ at great distances, but it will never exceed

this value. If the average "period" of the waves in the generating area is, say, 10 seconds, the average "period" of the highest swell will not be much greater than 14 seconds.

Effects of Viscosity

Added Possible Effect of Viscosity

In all these forecast examples the waves have decreased in height with time at a certain point because of the fact that the high *periods* (low frequencies) have traveled away from the point at which the forecast is being made. In each case, toward the end of the forecast period, the frequencies present are high and the "periods" associated with them are low. These low "periods" have seldom been observed and reported on the open ocean. This is partly because there is nearly always some light local wind present, and thus they may be associated with the local winds instead of with the distant storm. Also, these low "period" waves take so long to arrive that no one looks for them. The storm that caused them is "ancient" history. It also may be due to the fact that in addition to the effects of dispersion and angular spreading these low "period" components are dissipated by the effect of eddy viscosity. If this is the case, the height forecast in all these examples toward the end of the forecast time will be too high because the eddy viscosity effect will decrease the height of the waves even further.

Question Not Settled

At the present time not enough correctly obtained wave observations of actual conditions have been made to permit a correct computation of the effect of viscosity. The wave forecaster, as he develops experience, may be able to take this effect into account quantitatively. If these forecast low "period" waves do not show up in clear cases where they should, then this effect of viscosity is important; but, until careful observations are made in well-defined situations where these low "period" waves should show up, this question cannot for the moment be settled one way or the other.

Cross and Head Winds

The above statements have to do with very simple cases in which the waves propagate through areas of relative calm. In situations where waves from one storm travel at an angle to waves generated by local wind, with frequencies the same as for the traveling waves, the traveling waves will be dissipated. Little white caps and breakers will form every time a wave builds up beyond a certain height. Each time breaking occurs, a certain amount of energy will be dis-

sipated. Part of the dissipated energy will be restored to the local waves as the local wind continues to blow. However, the energy dissipated from the waves traveling from the distant storm cannot be replenished. It will be lost and result in a corresponding decrease of those waves. Consequently, in any practical case where the traveling waves encounter obliquely a local sea having the same spectral components, they should not be forecast to arrive at the distant point of forecast.

As already stated, not enough is yet known about the effect of viscosity for quantitative results to be given at the present time. It is suggested that the forecaster keep records of such situations and verify his forecasts very carefully in order to form an estimate in his own mind of these effects, so that his forecasts can be modified by experience when it is suspected that the effects of viscosity will be important.

Forecasts Finally Merge Into Local "Sea"

Of course, in all these examples the point at which the forecast is to be made will have some light local wind after the stronger wind, which produces the really important waves, has died down. If the forecast table shows that the waves will finally decrease to heights which are comparable to those produced by the light local winds, then the forecast should be stopped at this point. Thus in example 3.10, if at 041900Z of March the local winds instead of being exactly zero were 14 knots, then shortly after 041900Z, the forecast values for the point of observation would merge with a significant height of 3.26 feet, and it would not be worthwhile to continue carrying out the calculations on the basis of the waves caused by the 36-knot wind at 030600Z.

Chapter IV

WAVE OBSERVATION TECHNIQUES

Introduction

Purpose of Chapter

The purpose of this chapter is to describe how to observe waves correctly. A poor wave observation might lead to the conclusion that the forecast was wrong, whereas verification and not the forecaster would be at fault. This is most easily shown by figure 4.1 which gives wave records taken at a very reduced speed. As a consequence, the waves appear all run together. The records are several hours long, and the time scale shows that there are intervals on the record where the waves are quite low for as much as 5 or 10 minutes. There are other intervals on the record where the waves are quite high. If an observation is made for a time interval of the order of 5 or 6 minutes during which the waves are low, the average wave height will be considerably lower than the forecast value. Conversely, if the observations are made for a period 4 or 5 minutes long during which the waves are quite high, the observed average wave height will be considerably higher than the forecast value.

If an observation is made for at least 15 minutes and preferably longer, and if enough individual values are recorded, then the average wave height will be truly representative of the forecast value. The reasons for this effect were given in Chapter I, where it was shown that certain height ranges, depending upon the value of E, would occur. A series of waves, all of a nonrepresentative height range, can persist for only a fraction of this time. An average based upon a shorter time of record will not be adequate.

Theory of Correct Observation Methods

Establishment of correct wave observation procedures involves considerations of sampling theory. The problem is, basically, how long must the waves be observed in order to get a reliable average height, or a reliable average "period." Also, how many individual values should be recorded before the total group of values will begin to approximate the distribution given in the tables of Chapter I.

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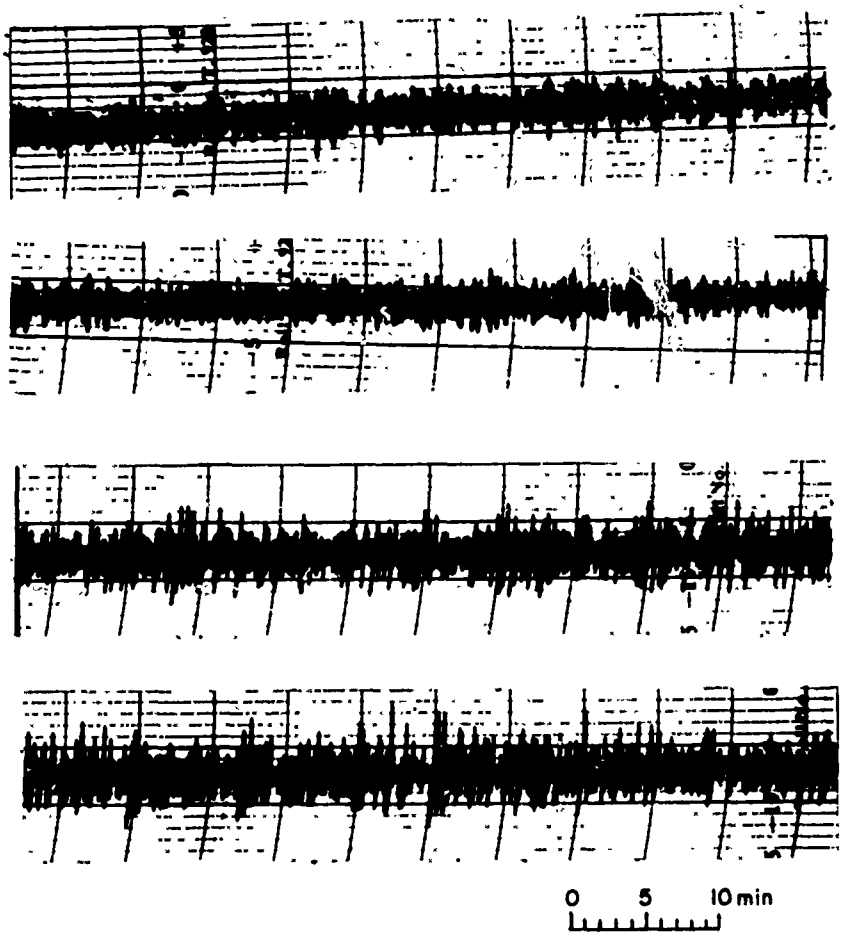

FIGURE 4.1 Some very condensed wave records.

If only 5 or 10 values of the wave height are recorded, they can differ markedly from those distributions. It will be very difficult to establish the correct value of E for the particular observation. It can be shown that if about 100 wave heights are observed, a distribution will be obtained that will suffice for the vast majority of cases. The observation will then be reliable. When there is not enough time to get as many observations as 100, possibly 50 will do; but any small number of wave heights observed at random is likely to give a very unreliable representation of the wave height distribution and of the value of E.

As an example, consider a box of colored marbles, with 40 percent red marbles and 60 percent black ones. If a person were to reach into the box of marbles and pull out a handful, and from that handful try

to decide without knowing beforehand, the percentage of one color as compared with the other, he would get different results depending upon the size of the handful he took. If he took only two marbles, he could get either two red, two black, or one red and one black. No ratio of the sample values would then correspond exactly to the 40 percent red and 60 percent black. Consequently, he could not know what the true value was on the basis of just two marbles. If the person were to pick 20 marbles, then of a large number of the different possible handfuls of 20, a great many would approximate the values of 40 percent and 60 percent. Some handfuls though, would still be off one way or another and show a wrong percentage. There is always the probability, no matter how remote, that a given series of observations will give a poor value of E, but if the observation consists of a large number of values such as 50 or 100 individual heights, the probability becomes extremely small.

Similar statements also hold true for the observation of "periods" and "wave lengths." The observation of heights, "periods," and "wave lengths" will be discussed in this chapter.

Two Purposes for Wave Observations

The observation techniques described in this chapter are given for two purposes. The first purpose is to show the forecaster how to verify a forecast. A forecast verified by a poor observation will not be verified correctly, and erroneous conclusions will therefore be drawn. The second purpose of this chapter is to show how to collect adequate wave data for scientific purposes. Most wave data today are taken and organized very poorly and do not show the basic features of the sea surface, in which scientists have become interested. These better techniques are needed in order to improve, extend, and unify the methods given in this forecasting manual.

General Observations

Before each wave observation is made for which a record is to be taken and forwarded to some central collection agency, it is necessary to record some general data which are very useful in working up the observation. These data are shown below in table 4.1. Many kinds of information are requested concerning the nature of the weather and the type of ship on which the observer is located.

It is also necessary to include a copy of the logged weather report for the past 24 hours or for the time since the last reported wave observation. This will permit people who are studying the records to obtain reliable estimates of the average wind speed, of the variability of the wind, and of the other important factors in order to prepare wave forecasts for themselves and to check the observations with the theory.

Table 4.1—General Data Needed for all Observations

Date .....
Time of observation from ..... Z to ..... Z.
Ship location ..... Lat. ..... Long.
Length of ship .... feet. Beam (width) of ship .... feet.
Speed of ship .... knots ..... course.
Wind speed (true)[1] .... knots. Wind direction (true)[1] .....
Character of wind. .... Very gusty. ..... Gusty.
..... Strong and steady. ..... Weak and steady.
..... Weak and variable. ..... Calm.
Sea temperature ..... Air temperature .....
Type and class of ship .....
Height of metacenter .....
Natural rolling period .....

[1] If the air mass is unstable and/or wind speeds are high, variations in wind direction and gustiness are common. Record maximum gust ..... knots and variations in observed wind direction over a period reading of as much as ±40°( ), ±30°( ), ±20°( ), ±10°( ), ±0°( ). Assuming the total time for all observations to be an hour or more, take five readings during this period of the wind and use the average for the wind speed and direction recorded above. Note also minimum reading of wind speed ..... knots and maximum reading .... knots of the five readings.

The above tabulation of general data includes information on the type of ship, on the dimensions of the ship, and on the various atmospheric conditions which exist at the time of observation. The data on the type and physical characteristics of the ship are needed because it may be possible to derive methods for actually forecasting how the ship will respond to a given wave system. If this can be done, one of the ultimate goals of the forecast will be realized, because in many applications to practical cases, people do not really care what the waves are doing. They are really interested in what the ship does because of the waves. If this can be forecast, the information will be most valuable. Some applications of this information on the characteristics of the ship will be given in Chapter VII. The information requested about the wind is a little more detailed than usual, but this is because it is believed that a very gusty wind, even with the same average velocity, will cause a type of wave structure different from that produced by a steady wind of the same average velocity. The turbulence in the wind for the gusty case might conceivably produce a broader, higher spectrum.

The Observation of the "Periods"

There is no such thing as a *period* in an ocean wave record or in actual ocean waves because the wave pattern never repeats itself. However, an important visual observation consists of the tabulation of the time intervals between successive crests, that is, the "periods," as defined in Chapter I. Consider the wave records shown in Chapter I. It should be possible, while watching the waves pass a fixed point,

to start a stopwatch at the time that a given crest passes that point and stop it when the next crest goes by. This procedure gives a certain time interval. If enough of these values are recorded and tabulated, the result is a frequency distribution of the "periods." (See figures 1.6, 2.5a, 2.5b, and 2.5c.)

Difference Between the Frequency Distribution and the Spectrum

This frequency distribution has something to do with the spectrum of the waves, but it is not the same thing as their spectrum. Consider, for example, a very low wave with a time interval between crests of 5 seconds and a considerably higher wave with a time interval between crests of, say, 10 seconds. Now suppose low waves with a time interval between crests of 5 seconds occur 20 times, and high waves with a time interval between crests of 10 seconds occur only 10 times. Then in the distribution of these time intervals the maximum will be near the 5-second time interval, and the lower value will be associated with the 10-second interval. Yet, looking at the distribution, one would think that the 5-second waves were more important. Obviously, that is not the case.

Difference Between a Sea and a Swell

In a sea the time intervals between crests can vary over a considerable range, as shown in Chapter II. As an example, for a wind speed of 32 knots, it was shown that the "periods" could vary from 5 to 17.5 seconds, and any intermediate value could occur. It was also shown in Chapter II that the average "period" was related to the shape of the spectrum, and that the averages of all these values were considerably lower in "period" than the *period* associated with the maximum energy in the spectrum of the waves. In a swell the "periods" are clustered more nearly about the period corresponding to the central frequency of the range of frequencies in the band present at the time of the observation.

Method of Observation

The best place to observe the time intervals between successive crests of the wave system is from a high point on the ship such as the mast or the deck above the bridge. Look one or two ship lengths ahead to windward. Pick out a mark, e. g., a large foam patch, a clump of seaweed, or any other drifting object which is easily seen and at a relatively fixed point on the sea surface. A piece of an empty wooden food crate makes a good marker. It can be thrown overboard and followed for a long time if the ship is not moving too rapidly. By keeping the mark in sight as long as possible and recording the time interval between the crests as they pass this mark, a representative series of observations can be obtained. When the

mark passes out of sight, a new one can be used and the process repeated until a sufficient number of values has been obtained.

The mark will bob up and down as the waves pass. Start a stopwatch when the mark is at the top of the wave and stop it when it is at the top of the next one. Write down the time interval that results. Repeat such measurements for 50 to 100 observations and report each individual value of the readings.

If two observers are available, team work helps in getting the observations made more quickly. In fact, continuous measurements of the "periods" can be taken.

The two observers take a high place of observation together. Each observer should have a stopwatch. Observer A observes the mark while observer B is ready to note the individual values. A starts his watch when the mark is at the crest of a wave. When he stops his stopwatch he loudly says, "stop," and at the same moment B starts his watch. While keeping the sign in sight, A shows his stopwatch to B who records the time interval on A's watch. This can be done quickly, and B says, "OK" to indicate that A clears his watch to zero again. Meanwhile the mark has gone through a cycle and approaches the next crest. A says, "stop" at the moment when the mark is at the crest of the next wave. This means that B has to stop his watch while A starts his watch again. B reads off his watch, notes the time interval of his observation, and clears his watch to zero again until A says, "stop." A says, "stop" at the moment when the mark is at the top of the third crest. The process can then be repeated. A observes the first, the third, the fifth, and so on of the "periods" recorded, while B observes the second, the fourth, and the sixth. Thus, a continuous series of observations can be made in half the time it takes one man to do it alone.

These individual values are very important, and averages based on them are also very important. About 50 individual values of the "periods" must be tabulated in order to obtain a reliable value of the average "period." It is a rather difficult problem theoretically to go from the individual values and the average of these individual values to the properties of the spectrum, but enough is known at the present time to make it possible to deduce some very important conclusions about the nature of the waves on the basis of these data.

The Observation of Wave Heights

There are three different procedures which can be used to observe wave heights. In order to see how they differ, consider again the aerial photograph of the sea shown in figure 1.1 of Chapter I. These waves are very short-crested, very mountainous, and very irregular.

Procedure A

One procedure, designated as procedure A, is to look out over the sea surface and pick out the highest part of each of the visible crests. Then an estimate or a measurement of the vertical distance from the crest of the wave to the trough of the wave right next to it can be made. This series of observations gives a set of numbers which represent the highest part of each of the short-crested waves within the field of vision of the observer.

However, if the waves are observed in this way they will not have the distribution of heights given in Chapter I. Only high values will be recorded, and the result will be the distribution of the highest parts of the short-crested waves within the field of vision of the observer. The low waves will be neglected or ignored by this procedure almost entirely. Such a procedure of observation is useful only if the procedure to be described next cannot be used due to a lack of time of the observer.

Procedure B

Again consider figure 1.1. The ship shown is moving through the waves. She will not encounter the highest part of each wave that she runs into. In fact, most of the time she will be on the side of one of the high waves and will encounter a considerably lower height than one would expect as indicated by the first method of observation. It is this range of heights that really affects the ship, and it is this particular distribution of heights that was given in Chapter I.

Similarly, a wave-recorder of some type placed out in the ocean will record the heights of the waves that pass a fixed point of observation. The highest part of each wave to pass within the vicinity of the wave-recorder will not necessarily pass over the recorder, and thus a wave-recorder will record values similar to the values encountered by a ship under way. The fact that an observation of waves passing a fixed point is different from an observation of the highest part of each short-crested wave is a very important practical consequence of wave theory. It shows that what is really needed is an observation similar to the type of observation that a wave-recorder actually gives.

In order to observe waves by what will be designated as procedure B, therefore, it is necessary to try to estimate the height of that particular part of each wave a ship encounters and not the height of the highest part of each wave. It is also satisfactory to estimate the heights of those waves that pass a fixed point. It is also possible to think of all the waves encountered at a point moving with the ship's speed at a fixed azimuth and distance from the moving ship. The observer should look out from the bow of the ship at an angle of, say, 45° to the direction of motion of the ship and keep his eyes fixed

on a convenient point a number of yards out in this particular direction. Certain waves will be passing that point; and, as the point moves along at the same speed as the ship, it will encounter the same distribution of heights that the ship does. The heights will occur in a different order, but the distribution will be the same. This is important because waves near the ship are distorted, and it is better to observe at a point some distance off.

The procedure, then, is to estimate the crest-to-trough heights of the waves as they pass this point of observation. This is not too difficult to do, but the important thing to remember is that the height of *each* wave must be noted. There is a very strong tendency to neglect the lower waves and emphasize the higher waves. There will be times when it appears that no waves are present or that what is happening is unimportant compared to what will happen when higher waves pass or what has happened when higher waves were passing. This is not the case because those times when the sea is low are just as important in characterizing the overall pattern as the times when the sea is high. *Every* height should be recorded as the waves pass this point of observation.

Procedure C

There is also a third procedure for the observation of wave heights. Procedure B may at times be very difficult to apply. Then procedure C can be used. If the low waves are omitted in procedure B, the computed average wave height is too great. The value of $\sqrt{E}$ is incorrect, and thus an incorrect forecast of the complete distribution of wave heights results.

If the wave observer finds that it is very difficult to observe and actually count the lower waves that pass a fixed point with reference to a moving ship, he should then attempt to observe the height of *all* waves in excess of a certain fixed minimum height. It is not sufficient to attempt to estimate the heights of the one-third highest waves and compute a number which is supposed to be the significant height from such an observation. How is it possible to know what the one-third highest waves are if all the waves are not counted and put in order as was done in Chapter I? Some fixed lower limit is needed as a reference point from which these averages can be computed.

Suppose that in a given state of the sea an observer records the heights of all waves in excess of some fixed height, say, for example, 4 feet. Then a certain percentage of waves is omitted from the observations, all with a height less than 4 feet. The average of the heights of those waves greater than 4 feet will, consequently, be higher than the average of the heights of all the waves.

As an example, midshipmen at Annapolis are selected so that they are taller than a certain minimum height. The average height of the

midshipmen at Annapolis is therefore greater than the average height of the total male population in this country. Similarly, the average height of all the waves observed in excess of a certain fixed minimum height will be greater than the average height of all the waves that actually occur.

Table 4.2 shows the average values obtained when all heights in a given state of the sea are recorded for waves in excess of 4, 10, and 20 feet. If the average of the heights of all waves in a given state of the sea is computed *for all waves greater than 4 feet* and if this average, for example, results in the value of 4.86 feet, then the *average height of all the waves,* as shown in table 4.2, is 2.50 feet. The significant height is 4 feet. This shows that the average is based on some fraction of the waves which is even less than one-third of the total number of the waves. In fact, as the last column shows, 86.6 percent of the total number of waves has been omitted, and only 13.4 percent of the waves has been recorded. As another example, if the average height of all waves *in excess of 4 feet* is computed and if this average turns out to be 12.3 feet, then the *average height of all the waves* is 11.3 feet and the significant height is 18 feet. Under these conditions, a considerably larger percentage of the total number of waves which pass is observed, and only 10 percent of the total number of waves is omitted. If the average height of the waves whose heights are greater than either 10 feet or 20 feet is computed, the results are similar, as table 4.2 shows.

The average height as computed by the methods of procedure C, always is greater than the average height of all the waves, and it is connected to the significant height and the E values by the values given in table 4.2.

The reason for this method of observation is that it gives a fixed lower bound to the observed heights. It is not too difficult to count all waves greater than a certain height, and once this fixed lower bound is marked, the computations can be carried out easily. The theoretical results presented in procedure C are based on a truncated probability distribution function, and they offer a sound method for obtaining wave height observations. The important point is, though, that a guess is not sufficient. A precise technique for recording the heights of a representative and complete sample of the waves that pass a fixed point on the sea surface, or a point moving along on the sea surface in a fixed direction and with a fixed speed, is absolutely necessary.

Examples of the Use of Procedure C

If only the heights of the waves in excess of a certain height are recorded and tabulated, it is not sufficient to go back and use the results of Chapter I on the observed height distribution. Parts of the

Table 4.2—Data for the Application of Procedure C, the Average Value of the Heights of all Waves in Excess of 4, 10, or 20 feet

Average height of all waves greater than 4 feet	4.86	5.30	5.70	6.16	6.64	7.22	7.68	8.80	9.82	11.04	12.3
Average height of all waves	2.50	3.13	3.75	4.37	5.01	5.63	6.25	7.50	8.76	10.0	11.3
Significant height	4	5	6	7	8	9	10	12	14	16	18
Percent of waves omitted	86.6	72.2	59.0	48.2	39.4	32.9	27.6	20.0	14.5	12.4	10.0
Average height of all waves greater than 10 feet		12.06	12.88	13.50	14.7	15.7	16.6	18.52	21.08		
Average height of all waves		6.25	7.50	8.76	10.00	11.3	12.5	15.0	17.8		
Significant height		10	12	14	16	18	20	24	28		
Percent of waves omitted		86.5	75.1	63.9	54.2	46.3	39.4	29.5	23.2		
Average height of all waves greater than 10 feet		22.7	25.4	27.5	30.4	33.4	36.8	39	43		
Average height of all waves		20.0	22.5	25.0	28.1	31.3	34.3	38.8	40.7		
Significant height		32	36	40	45	50	55	60	65		
Percent of waves omitted		18.0	14.5	12.4	10	7.5	6.5	5.6	4.6		
Average height of all waves greater than 20 feet		33.16	35.3	38.3	42	42.8	47.2	48.4	54.8	63	66
Average height of all waves		25	28.1	31.3	34.3	38.8	40.7	43.7	50.1	56.3	62.5
Significant height		40	45	50	55	60	65	70	80	90	100
Percent of waves omitted		39.4	32.9	27.6	23.7	20	17.5	14.5	11.5	10	7.5

data are actually missing and must be recovered through the use of table 4.2 as given below. These examples demonstrate the way in which the height ranges and the other wave data given in Chapter I can be computed in terms of the data obtained in procedure C.

EXAMPLE 4.1.

ALL WAVES IN EXCESS OF A HEIGHT OF 4 FEET WERE RECORDED AND THE AVERAGE HEIGHT OF THESE WAVES WAS 6.64 FEET. 50 WAVES WERE OBSERVED. FIND THE SIGNIFICANT WAVE HEIGHT AND THE OTHER WAVE DATA WHICH CAN BE COMPUTED FROM CHAPTER I.

Method of Solution

From table 4.2, if the average height of the waves is 6.64 feet for those waves which exceed 4 feet in height, then the average height of all the waves is 5.01 feet, and the significant height is 8 feet. Since the significant height is 8 feet, it can be found, from table 3.4 of Chapter III, that $\sqrt{E}=2.83$. The last column of table 4.2 shows that 39.4 percent of the waves have been omitted by the procedure of recording only those over 4 feet in height. Since 50 waves were observed and since approximately 40 percent of the waves which passed the fixed point of observation were omitted, this means that 60 percent of a total of some N waves (if all had been counted) was observed. Since 60 percent of N is 50, N is 83. That is, if all waves had been observed as they passed this fixed point of observation, there would have been 83 waves in the total number observed.

If it is desired to use table 1.4 in order to compute the 10 percent ranges of the waves and compare the observations with the theory, it is necessary to omit the first four 10 percent ranges because approximately 40 percent of the waves have been omitted. The heights of the first 8 waves (i. e., 10 percent of 83), which are very low and lie within the first height range, were not observed, the second 8 waves were not observed, nor the third and fourth. However, of the 83 waves which should have passed this point during the time of observation, 8 waves should have heights between 4.0 and 4.7 feet. Thus, the 10 percent ranges and the cumulative 10 percent ranges can be computed for that part of the complete range of wave heights which is contained in the data. The only information that cannot be gotten from the observations is that pertaining to the 40 percent of the 83 waves which were not observed because they were lower than 4 feet in height. Of course the observer can then say that of the 33 waves omitted, 8 had a height from zero to 1.8 feet, 8 had a height from 1.8 to 2.7 feet, 8 more had a height from 2.7 to 3.4 feet, and the last 8 (getting close to the total of 33) had a height from 3.4 to 4.0 feet. These waves were actually not observed because only waves in excess of 4 feet in height were recorded.

ANSWER TO EXAMPLE 4.1.

BY USING TABLE 4.2 THE NEEDED INFORMATION ABOUT THE DISTRIBUTION OF HEIGHTS CAN BE COMPUTED. BY ENTERING TABLE 4.2 WITH THE AVERAGE HEIGHT OF ALL THE WAVES GREATER THAN 4 FEET AND FINDING THE VALUE 6.64, IT IS POSSIBLE TO GET THE SIGNIFICANT HEIGHT AND THE AVERAGE HEIGHT. FROM THE SIGNIFICANT HEIGHT AND TABLE 3.4, OR BY SIMPLE CALCULATIONS, ANY OTHER QUESTION ABOUT THE DISTRIBUTION OF WAVE HEIGHTS CAN BE ANSWERED.

EXAMPLE 4.2.

IN A MODERATELY HEAVY SEA AN OBSERVER RECORDED ALL WAVES IN EXCESS OF A 10-FOOT HEIGHT. THE AVERAGE HEIGHT OF THE 50 WAVES OBSERVED WAS 22.7 FEET. THE PROBLEM IS TO DESCRIBE THE AVERAGE OF ALL WAVES, THE SIGNIFICANT HEIGHT, THE VALUE OF E, AND THE OTHER NEEDED INFORMATION ABOUT THE DISTRIBUTION OF WAVE HEIGHTS.

Method of Solution

From table 4.2, if the average height of all waves in excess of 10 feet is 22.7 feet, the average height of all waves is 20 feet, the significant height is 32 feet, and 18 percent of the total number of the waves was omitted. If the 50 waves recorded were 82 percent, then a grand total of 61 waves must have passed the fixed point during the time of observation. There were 11 waves left out of the observations or approximately 20 percent. The first 6 waves (10 percent of 61) omitted would have had a height between 0 and 7.2 feet. The next 6 would have had a height between 7.2 and 10.6 feet. The use of table 1.4 then gives values for the 10 percent ranges of all the heights, which are actually tabulated in the data since $\sqrt{E}$ is 11.3 feet.

ANSWER TO EXAMPLE 4.2. SINCE FOR THIS EXAMPLE THE AVERAGE HEIGHT IS 22.7 FEET FOR ALL WAVES GREATER THAN 10 FEET, IT FOLLOWS FROM TABLE 4.2 THAT THE SIGNIFICANT HEIGHT IS 32 FEET AND FROM TABLE 3.4 THAT $\sqrt{E}$ IS 11.3 FEET. FROM THESE TWO VALUES, ANY OTHER DESIRED PROPERTY OF THE DISTRIBUTION OF THE WAVE HEIGHTS CAN BE OBTAINED.

Additional Facts About Procedure C

A study of table 4.2 shows that as the average height increases (for a given minimum observed height), the error resulting from the use of the average height of all waves greater than this minimum

height instead of the true average height becomes less and less. For example, in the first row, where the average height of all waves greater than 4 feet is found, if the average height of the waves is 12.3 feet, the true average height is 11.3 feet; the error involved is not too serious because only 10 percent of the waves are omitted. In this case, with some waves 18 feet and higher, a wave 4 feet high becomes fairly unimportant, and it is difficult for the observer to record a wave so low in a sea where the dominating waves are much higher. This is why it is very important to try to measure the heights of all waves or at least to try to fix a lower bound which is the height of the waves which will be omitted. If a set of wave heights is recorded in which the observer does not take care to do this, the average value will then have very little meaning in a statistical sense, and it will not give a reliable value of $\sqrt{E}$.

Relative Importance of Procedures A, B, and C

Three procedures can be used in reporting wave heights, and it should be noted which procedure is used so that the proper theoretical conclusions can be drawn from the reported results. In all cases it is preferable to use procedure B because at the present time much more is known about the statistical properties of such a series of observations. If this is not possible, procedure C is the next best method. It yields consistent results if the values obtained are interpreted correctly. On the other hand, if all that can be followed is procedure A, then this type of observation is better than no observation at all.

Reliability of Wave Height Observations

After a number of wave heights have been recorded according to procedure B, the average height of the observed waves can be computed. With this average value and the use of table 1.5, $\sqrt{E}$ can be found by dividing the average wave height by 1.77. However, if there are not enough heights recorded, the average of the *observed* heights may not be equal to the average of the heights of all the waves. Just as a small handful of marbles in the example given previously can give an incrorrect value for the percentage of red and black marbles, so also the observation of too few wave heights can give an incorrect value for the average wave height and for the value of $\sqrt{E}$.

If $\sqrt{E_m}$ is the value calculated on the basis of N wave heights, table 4.3 shows how far off the correct value of $\sqrt{E}$ may actually be. Suppose, for example, that 9 wave heights are recorded, and that $\sqrt{E_m}$ is 10 feet. Then all that can be said theoretically is that 90 percent of the time the true value of $\sqrt{E}$ will be between 7.8 feet and 14 feet. There is one chance in ten that the true value will either be

less than 22 percent lower or greater than 40 percent higher than the calculated value. The range of values that the true value can have on the basis of just these 9 observations is 6.2 feet, and the calculated value, 10 feet. Thus, very little precision can be assigned to the value of $\sqrt{E_m}$ computed on the basis of only 9 wave heights.

On the other hand, if 100 wave heights are observed, and if, for example, $\sqrt{E_m}$ is 10 feet, the true value will theoretically lie between 9.2 and 10.9 feet 90 percent of the time. The range in which the correct value can lie 90 percent of the time covers only 1.7 feet. It is thus possible to place a high degree of confidence on an average height or a value of E computed from 100 wave heights, but a value computed from only 9 heights cannot be trusted. However, one case in ten will give a value outside the range indicated.

Table 4.3—Confidence Values of $\sqrt{E_m}$

If $\sqrt{E_m}$ is calculated on the basis of N measured wave heights, then 90 percent of the time the true value of $\sqrt{E}$ as calculated on the basis of many more wave heights will be found between the value tabulated below. Both the theoretical value and a value with an added safety factor are shown.

N	Lower value (Safety factor)	Lower value (Theoretical)	Upper value (Theoretical)	Upper value (Safety factor)
9	0.71 $\sqrt{E_m}$	0.78 $\sqrt{E_m}$	1.40 $\sqrt{E_m}$	1.68 $\sqrt{E_m}$
16	0.76 $\sqrt{E_m}$	0.82 $\sqrt{E_m}$	1.23 $\sqrt{E_m}$	1.44 $\sqrt{E_m}$
25	0.80 $\sqrt{E_m}$	0.85 $\sqrt{E_m}$	1.21 $\sqrt{E_m}$	1.33 $\sqrt{E_m}$
50	0.85 $\sqrt{E_m}$	0.89 $\sqrt{E_m}$	1.12 $\sqrt{E_m}$	1.21 $\sqrt{E_m}$
100	0.89 $\sqrt{E_m}$	0.92 $\sqrt{E_m}$	1.09 $\sqrt{E_m}$	1.12 $\sqrt{E_m}$
200	0.92 $\sqrt{E_m}$	0.94 $\sqrt{E_m}$	1.06 $\sqrt{E_m}$	1.09 $\sqrt{E_m}$

Table 4.3 emphasizes the importance of making adequate wave observations. A wave forecaster spends many hours each day analyzing weather maps and preparing a wave forecast. If the forecast is then verified against an observation based on the average of 10 or 20 wave heights, the observed values can differ considerably from the forecast value, and yet the forecast value can be correct. AT LEAST FIFTY WAVE HEIGHTS ARE NEEDED BEFORE ANY CONFIDENCE CAN BE PLACED IN THE COMPUTED AVERAGE WAVE HEIGHT.

The theoretical values in table 4.3 were computed on the basis of the assumption that the waves were completely random. The effect of a run of waves of nearly the same height as illustrated in figure 4.1 was not considered. Thus the theoretical values given in table 4.3 are the best that can be hoped for, and an actual observation may be even more in error than the table shows. On the basis of an assumption that very crudely estimates the added effect of a run of

waves of the same height, the value indicated with a safety factor may be more nearly correct for actual ocean waves.

Table 4.3 can also be applied to wave height data computed from procedure C. All the modifications made in going from procedure B to procedure C tend to increase the reliability of the values obtained in procedure C. Thus if 50 actual heights are observed in procedure C, the results of table 4.3 can be applied to determine the reliability of the values obtained.

Methods of Height Observation (Heavy Seas)

A rather good method can be given which permits the use of procedure A in heavy seas. The method is to find some place aboard ship from which the crests of the waves appear to be level with the horizon when the ship is in the trough of a wave and on an even keel. The heights of the waves are then equal to the height of the observer's eye above the water line. Stated another way, a ship is rolling and pitching because of the heavy seas. Every once in a while the ship is in a trough of the wave train. If the observer is then advantageously located with respect to the maximum depression of the ship in the trough, he will see, if he looks out on the horizon, that the crests appear in many cases to reach up to his eye level. There will be some crests higher and others lower than his eye level with respect to the horizon, but the average crest will appear to be at eye level. The best position for such an observation is amidships, and the observer should move up and down in order to pick that point where these conditions hold. The height of the observer's eyes above the water level of the ship in this situation then will represent a number related to the average height of all the waves. The value of course depends upon the trough depression of the particular wave in which the ship happens to be located. But enough observations made at these times will give a reliable estimate of the average height. Then, from this advantageous position, the observer can look out and tabulate a number of heights both above and below this average value and give the variation of the crest-to-trough heights of the visible waves with relation to the maximum part of each of the visible crests. It would be quite difficult under these conditions to employ procedure B because of the fact that such heavy seas are present.

Methods of Height Observation (Reference Line)

If the waves are low, the winds light, and a low local sea is present, or if a low or a moderate swell is running, and it is convenient to stop the ship, a good method of observation consists of heading into the waves and dropping a weighted line with graduated marks (like a leadline) over the bow. The line must go quite a distance below the water. Then both the lowest and the highest levels that the water

reaches during the passage of a given wave are recorded. The difference is the crest-to-trough height of the wave. A series of such observations then gives the data needed for a forecast verification. Of course, this application will be useful only if the ship is not pitching too much with the wave motion, which means that the "wave length" must be short compared to the length of the ship. Otherwise, the line itself will move up and down with the waves, and only a very small variation in height will be recorded. For low local short "period" waves, such a method will be useful.

Such a technique can also be very useful at the end of a dock or where any fixed platform can be used. At times it is necessary to verify forecasts on beaches, and this could be a method of recording the heights of the waves passing some fixed marker in the water some distance out from a beach. If the waves are reflected from a vertical cliff and hence do not break at the beach, this method should not be used.

Methods of Height Observation (Visual)

In the application of either procedure B or procedure C there are a number of possible techniques. The first is a simple visual estimate of the heights that pass a fixed point with reference to the moving ship. This is adequate for many applications, and one who is practiced in such a procedure should get a reliable range of values, especially if the observer is careful not to omit the low values of the waves which pass. Other techniques that might be useful employ theodolites, range finders, and any other such device aboard ship which can magnify a part of the sea surface and give some sort of a scale once the distance from the ship to the point of observation is known. There are undoubtedly many techniques and many improvements that a person really interested in the practical aspects of such a problem can suggest and carry out on his own initiative. Such data are really needed, and the more precisely they can be made the more useful they will be.

Procedures for Measuring the "Wave Lengths"

General Comments

Just as there is no such thing as A *wave period* there is no such thing as A *wave length*. Also, there is no such thing as A *wave speed*. Various wave crests will travel with various speeds. Some waves will outrace others and appear to travel through them. The distance between the crests in a direction perpendicular to the dominant direction of the crests is not a constant. "Wave lengths" and "speeds" will vary in the same way that "periods" do.

The measurement of "wave lengths" is more difficult than the

measurement of "periods" because the shorter waves superimposed on the longer waves tend to mask the distances which represent the important "wave lengths." A wave should have a trough about as much below sea level as its crest is above sea level; if the shorter waves on top of the longer waves are reported, this will not be the case. The following procedure makes it possible to define "wave length" for the purposes of forecasting and observation.

Definition of "Wave Length"

One method for measuring "wave lengths" is to use what is called a chip log. A "wave length" can most easily be defined in terms of what an observer will see during an observation carried out by this method. If a chip log is not available, any floating object easily visible, such as a life buoy, can be tied securely to a line. The ship can be under way during the entire course of these observations. The line will be more useful if it is marked at 50-foot intervals, either by staining the line dark for 50 feet and leaving it plain for another 50 feet or by marking it with painted floats or bits of colored cloth at 50-foot intervals. The line may need to be 800 or more feet long. It is payed out from the taffrail and should trail out directly in the wake of the ship, partly floating and partly submerged. Consider a graph of the height of the sea surface as a function of distance along the line at some instant of time. The situation will look something like that shown in figure 4.2 except that the height scale is exaggerated.

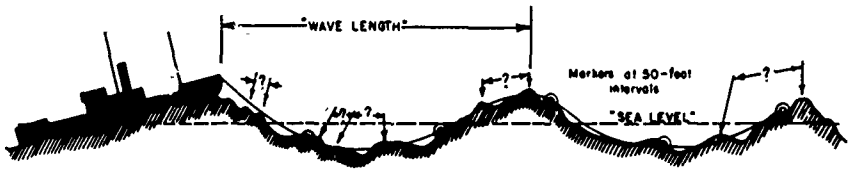

FIGURE 4.2 The measurement of a "wave length."

For a simple sine wave, there is no trouble in defining a *wave length*, but it is more difficult to define a "wave length" for actual waves. In fact, two possible definitions can be given. The first definition is that a "wave length" is the horizontal distance between two successive maximums where a maximum is a part of the record where the water height is lower both before and after it. There are a great many such "wave lengths" shown in this figure. The second definition is that a "wave length" is the horizontal distance between the two highest parts of the two masses of water above sea level separated by a distance where the sea surface is below sea level. As shown in the figure, an imaginary line representing sea level separates the graph of the waves into mounds of water above sea level and trenches in the water

below sea level. The "wave length" is then defined to be the horizontal distance from the highest part of one mound of water (which happens to be under the stern of the ship in figure 4.2) to the highest part of the next following mound of water.

The first definition leads to great difficulty in application. Each little ripple then has a "wave length." A developing "sea" has ripples on top of wavelets, wavelets on top of small waves, small waves on top of longer waves, and so on. As figure 4.2 shows, practically any distance can be called a wave length according to this definition. If the ripples are ignored, should the wavelets be ignored? If the wavelets are ignored, should the small waves be ignored? Figure 4.3 shows how very difficult it would be to apply such a definition if every ripple were counted as a "wave length." An observer trying to record the "wave lengths" according to this definition, will soon be ready to give the whole business up as a bad job.

The first definition also leads to great theoretical difficulty. The average "wave length" according to this definition can be shown to be only one or two inches; but, although these short irregularities on top of the more dominant waves may be important in radar problems, they certainly do not characterize the "wave length" of the dominant big waves. For these reasons, then, the first definition of "wave length" will not be used.

The second definition of "wave length" will, consequently, be accepted. A "WAVE LENGTH" IS THE HORIZONTAL DISTANCE BETWEEN THE HIGHEST PARTS OF TWO SUCCESSIVE CRESTS ABOVE SEA LEVEL WHICH ARE SEPARATED BY A TROUGH WHICH IS BELOW SEA LEVEL. The average "wave length" is the average of these distances, and the values given in Chapter II for "sea" conditions are the average "wave lengths" according to this definition. For swell, the average "wave length" is a little less than the theoretical value of 5.12 $\widetilde{\widetilde{T}}^2$.

The Chip Log Method

The observational procedure is to wait until a crest is just under the stern, as is shown in figure 4.2. There will be another crest visible somewhere out along the line. From the spacing of the markers, the "wave length" of this particular wave can be recorded. The observations can then be repeated until about 50 values have been recorded. The distribution of the various "wave lengths" and the average "wave length" computed from these 50 values will then form a reliable set of data for wave research and for various theoretical studies. The values will vary through a considerable range of distances. The length of line payed out should be long enough to measure all of the different "wave lengths" which occur.

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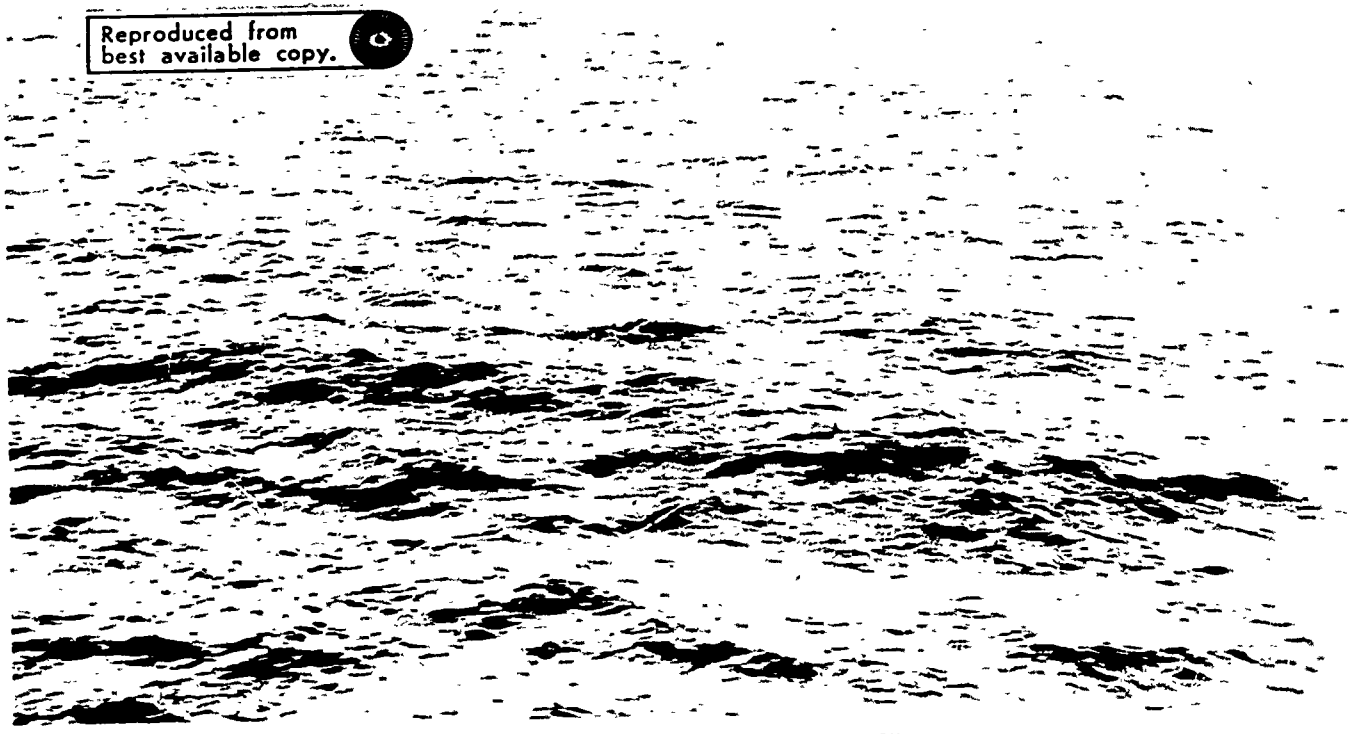
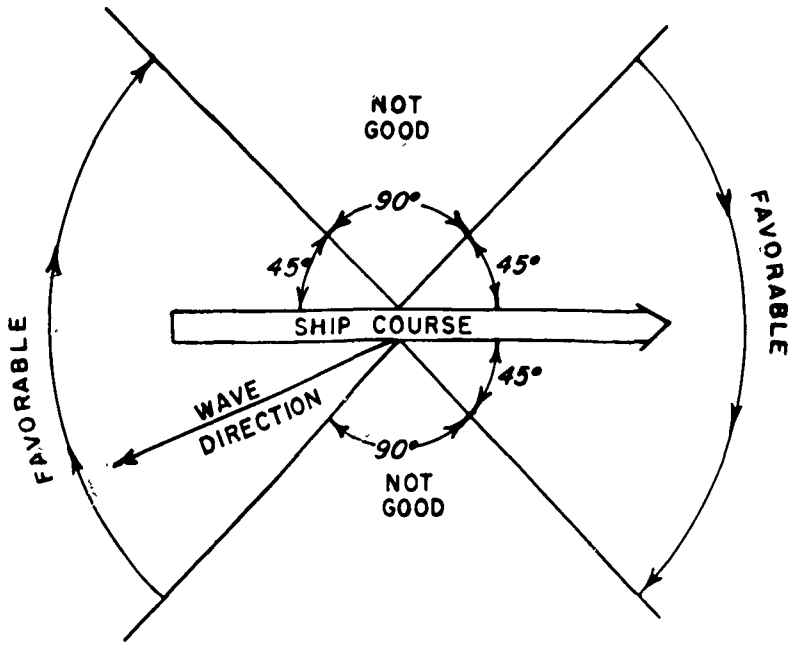


FIGURE 4.3 What is the "wave length?"



ANY WAVE DIRECTION INSIDE THE LIMITS SHOWN IS FAVORABLE. ARROW INDICATES DIRECTION TOWARD WHICH WAVES ARE TRAVELING.

FIGURE 4.4 Favorable conditions for a chip log observation.

The chip log method cannot be used if the ship is headed at an angle too oblique to the waves. If the waves are advancing within 45° of either beam, this procedure is impractical (figure 4.4). Likewise, method is unsafe for use in very high waves, and the ship's wake destroys the wave pattern when the waves are too low and short-crested. Usually waves from 5 to 20 feet high are best for this type of observation.

Also, if there are two or more dominant wave systems present traveling toward directions which differ by more than 45° and if they are of equal importance in height, do not make this type of observation. On the other hand, if one wave system dominates the others, then it might be possible to try to measure the "wave lengths" connected with just this one system.

The Use of the Ship as a Scale Factor

If the observer is in a fairly long ship and if telephone communication is possible from the wing of the bridge to the stern, it is possible to observe "wave lengths" with the use of the ship as a scale factor. With telephone communication one observer stations himself on the

wing of the bridge and the other as near the stern as possible. They both look out toward the same side of the ship and should have an unobstructed view of the waves along the entire length of the side of the ship. The ship should preferably be traveling in the same direction as the waves. If these conditions are satisfied, a wave crest will appear at intervals directly abeam of the forward observer. When this occurs the forward observer should say, "mark" into the telephone. The after observer, stationed a known number of frames aft, then looks out and sights the next crest in order as defined above. This crest may not be exactly opposite him; but if he is opportunely stationed, it will not be difficult for him to estimate accurately the relatively short distance between his position and the point where the wave crest is abeam. The two observers know their distance apart. Suppose, for example, this is 200 feet and that the after observer at the instant "mark" sees that the wave crest nearly opposite him is abeam of a point 20 feet farther aft. Then he knows that the "wave length" of that particular wave is 220 feet, and he can tabulate this value as an observed value of the "wave length." The next crest to be observed might be 30 feet nearer ahead of his point of observation, and the "wave length" of that wave would be 170 feet. The after observer under these conditions is thus in a position where he can estimate a small correction factor in tens of feet to be added to the distance between him and the first observer forward. A sufficient number of "wave lengths" observed under these conditions then makes it possible to tabulate the distribution of "wave lengths" and to compute the average "wave length" which was defined in Chapter I.

This method of course will not work whenever the average wave length is much greater than the distance two observers can separate themselves on shipboard and still remain in communication.

It can be carried out most easily if the waves are running with the ship and not moving against it. Under these conditions, the ship's speed may be comparable to that of the wave crests, and then the wave crests opposite the observers will appear relatively stationary. It is then possible for the estimate to be made with care and precision. As an example, in a sea where the dominant spectral components are near 5 or 6 seconds, wave crests will appear to be almost stationary with respect to a ship moving in the same direction at 15 knots. Waves with "periods" of 8 or 9 seconds will overtake the ship at the slow relative speed of about 10 knots.

Conversely, if the ship is heading into the waves at a speed of 15 knots (for example), the "wave lengths" become very difficult to measure by this method. Waves with "periods" of 5 or 6 seconds go past the ship at a relative speed of 30 knots, and waves with

"periods" of 8 or 9 seconds shoot past with a relative speed of 40 knots.

A very interesting phenomenon can be observed under conditions where the waves are moving by rather slowly. It is possible for the wave observer to keep an eye on a particular crest for a considerable length of time. The particular crest under observation will not maintain its height. It will grow at first (possibly), and then gradually shrink in height until there is practically no wave at all. Then due to the slight relative motion which is always present, a new crest will slowly move into position along the side of the ship, and that crest will be seen to change its height very slowly as it travels along. In terms of the definition of the sea surface which was given in Chapter I, this means that the various simple sine waves as illustrated in figure 1.7 are gradually getting out of phase at this point, so that they add up to a lower and lower height as time goes on. An observer watching the actual wave which is the reinforcement of a great many simple sine waves then sees this one particular wave first gradually grow in height and then die down until it tends to disappear.

Visual Estimation of the "Wave Length"

Quite frequently, the detailed procedure necessary to carry out the techniques just described cannot be employed because of lack of time, equipment, or personnel. From a high point on the ship, the length of the waves in a direction perpendicular to the crests, can be estimated visually by the aid of other ships, other scale factors, such as the length of the ship, or by the judgment of the observer. Such estimates are useful if they are made with care and if a sufficient number are taken so that the averages given are representative. One or two estimated lengths are not sufficient.

The Measurement of Wave "Speeds"

The wave crest "speeds" relative to the ship's speed can also be observed by the chip log method. With the same operational setup as in the chip log method, it can be noted whether the crests are overtaking the ship or being overtaken. Then, as a given wave crest passes one reference point, a stop watch can be started. When that same crest passes the other reference point, the watch can be stopped. The result is a number which is the length of time it took the wave crest to travel the distance from the first point of observation to the second. The speed of the ship and the angle between the wave direction and the ship are needed to compute the "speed" correctly. The result is a measurement of the speed of that particular crest. Fifty observations then give a reliable observation of the average "speed."

In an irregular state of the waves there will be some crests which

will not travel the entire length of line payed out. They will disappear as they travel from one reference point to the other. It would be interesting if the number of such waves were recorded, since theoretically it is not possible to tell how such a number can be predicted.

No methods for forecasting wave crest "speeds" or the average wave "speed" have been formulated. It is known that $\widetilde{\widetilde{L}}$ does not equal $5.12\widetilde{\widetilde{T}}^2$ for sea waves. It is not known whether or not $\widetilde{\widetilde{C}}$ equals $3.03\ \widetilde{\widetilde{T}}$, or whether $\widetilde{\widetilde{L}}$ equals $\widetilde{\widetilde{C}}\,\widetilde{\widetilde{T}}$ for sea waves. Consequently, the observation of wave crest "speeds" is of great interest to those who are working with the theory of actual ocean waves, because they do not know what the observed results will be. For swell, the classical formulas are approximately correct when applied to $\widetilde{\widetilde{L}}$, $\widetilde{\widetilde{C}}$, and $\widetilde{\widetilde{T}}$.

Check List of Visual Wave Properties

There are many properties of waves that are difficult to describe other than by a careful study of an aerial photograph. However, a qualitative estimate of these properties can be made in the form of a check list. Given below is such a check list, which attempts to describe some of the statistical properties of waves and to point out the differences between sea and swell. The farther away the waves are from the generating area the lower they will be, the longer their crests will appear to be, and the more regular the crests will be, i. e., the crests in a group of crests will be uniformly varying in height instead of rapidly and erratically varying in height. At times, when two wave systems arrive at the same point of observation from different generating areas, the sea will be a cross sea. The check list attemps to define these properties in terms of just a few numbers and to summarize some of the variability of the waves in a convenient form.

Table 4.4—Check List of Visual Wave Properties

Check the phrases which best describe the waves being observed.

(1) ( ) Irregular mountainous crests. (Check if yes.)
  ( ) Crest length is about 1, 2, 3, 4, or 5 times the distance between successive crests. (Record one value.)
  ( ) On the average there are 2, 3, 4, or 5 high waves in a group of waves. (Record one value.)

(2) ( ) Regular crests. (Check if yes.)
  ( ) Crest length is about 4, 5, 6, 7, or 8 times the distance between successive crests. (Record one value.)
  ( ) On the average there are 3, 4, 5, 6, or 7 high waves in a group of waves. (Record one value.)

(3) ( ) Smooth very regular crests. (Check if yes.)
( ) Crest length is about 7, 8, 9, 10, 11, or 12 times the distance between successive crests. (Record one value.)
( ) On the average there are 5, 6, 7, 8, 9, or 10 high waves in a group of waves. (Record one value.)

(4) ( ) YES
There appears to be only one dominant wave system.
( ) NO

(5) If the answer to (4) is no, report the dominant direction of each wave system to the nearest 10° and estimate the average height and average "period" of each system.

DIRECTION				
AV. HEIGHT				
AV. "PERIOD"				

(6) If only one wave system is present, the individual crests can still appear to be traveling toward directions at an angle to the dominant direction.
( ) Some of the waves appear to be traveling toward directions at an angle as much as ±40°, ±30°, ±20°, ±10° to the dominant direction (Record one value.)

CONCLUSION

Visual and instrumental methods have been given for quantitatively determining the statistical properties of the waves. These methods are time-consuming, but in order to verify a forecast correctly and in order to make a reliable wave observation, the detail and care described are needed. It may be possible to develop wave-recording instruments which will make it possible to eliminate the great amount of time necessary to make such detailed observations. However, these techniques will always be needed where such instruments are not available, and they should become an important part of the wave forecaster's knowledge so that he will understand completely the irregularity of the waves. This irregularity is a basic and important feature of the waves and must be understood and measured carefully in order to obtain reliable, useful, and practical results.

Chapter V

SYNOPTIC FORECASTING METHODS AND WEATHER MAP ANALYSIS

Introduction

The preceding chapters have described the properties and the theory of ocean waves. Most of the concepts are new; some are time-proven; and all are important. Before attempting a forecast, the forecaster should read those sections carefully to grasp the physical reasoning behind the theory. The forecaster who knows how the waves are generated, dispersed, and spread out angularly, and who understands the method of multivalued (in place of single-valued) forecasts, will make better forecasts than the man who uses the graphs without understanding the theory on which they are based.

The results presented so far in the manual can be summarized as follows. Chapter I described wave properties, with particular emphasis on the difference between sea and swell. It introduced E, the variable used in forecasting wave heights. The second chapter discussed the generation of waves as a function of fetch and duration. The co-cumulative spectrum was introduced, and some model forecasts were given. Wave propagation, as the next logical step, is covered in Chapter III, which also discussed the effect on wave propagation of dispersion and angular spreading. Knowledge of wave directions and frequencies in the generating area was shown to determine the propagation of waves. Chapter IV gave methods for observing the statistical properties of waves.

From those chapters the reader learned what waves are, how they are generated and propagated, and how to observe them. Before making a forecast, however, he must also learn how to interpret data from the weather maps and determine the values needed in the various graphs. The purpose of this chapter is to discuss the analysis of consecutive synoptic weather maps in order to prepare wave forecasts.

Wind Field, Actual and Required

What weather data are required for wave forecasting? Obviously, wind velocity and duration and the dimensions of the fetch area are necessary. These quantities could be described collectively as the

wind field; that is, the magnitude and direction of the wind over an area of the ocean for a certain interval of time. The theory so far covers only conditions in which velocities are constant over the duration time. An idealized wind field is a wind of 20 knots blowing for twelve hours over a fetch exactly 300 NM long and 500 NM wide. This is the required wind field, but the actual wind field usually consists of winds of 20±5 knots blowing for 12±3 hours over a fetch 300±50 NM long and 500±50 NM wide.

The reasons for the range of values in the actual weather data are that the wind is never constant, the fetch area usually is moving, and the value for the duration time is indeterminate between 6-hourly maps. These factors result in a continuous variation in time and space of the four quantities that are required in the theory as single values.

Limitations of the Weather Data

With all the detailed theory and forecasting graphs, a wave forecast can be no better than the weather data used to make it. Thus, it is evident that the weather map must be as accurate and as complete as possible. Because of the lack of observed values, it is often necessary to compute the geostrophic wind. This operation in itself necessitates a careful isobaric analysis consistent both with the current observations and with past analyses.

Even with a carefully analyzed weather map containing a dense network of observations, the choice of values is subjective. This is due partly to the range of values possible in both velocity and duration, and partly to the variation in ship reports. On many weather maps, the velocity is plotted in the Beaufort scale. The difference between the highest and the lowest velocities can range from 2 to 9 knots, depending upon the Beaufort value. Thus a Beaufort 5 wind can be interpreted as having a velocity anywhere from 17 to 21 knots. It is evident from the graphs that a wind 4 knots too high can cause a significant error in the wave-height forecast.

Wind duration is also difficult to determine more closely than within 2 or 3 hours. Data sufficient for maps are seldom furnished oftener than 6-hourly.

When a wind field changes between maps, there is a 4-hour range in which to locate the exact time of shift. As an illustration of the effect of these ranges, consider a Beaufort 5 wind reported on two maps out of four (starting between 0600Z and 1200Z and ending after 1800Z). The minimum and maximum values of velocity and duration are 17 knots for 8 hours and 21 knots for 16 hours, respectively, and the resulting forecast of significant height is either 5.0 feet or 9.0 feet, depending upon the values used. The wind certainly blew for a little over 6 hours. It could not have blown for exactly 18 hours.

Therefore, to the nearest whole hour, the values just given are the minimum and maximum durations.

It is evident that some improvement is necessary in selecting the wind values if the forecast is to be useful. Two methods for obtaining more precise values will be mentioned here and then described in more detail in later sections.

For wind velocity, the best procedure is first to average the wind reports over the fetch and then check the result with a computed geostrophic wind. Since the wind observations are coded as a single value in knots, the best procedure is to plot the actual wind value and direction rather than the speed in the conventional Beaufort force. There are any number of ways to do this, but an arrow with the actual velocity value entered at the tail is about the simplest. For reading the maps quickly, a full barb for every 10 knots and a piece of a barb proportional to the last digit is useful. The last digit of the velocity can be written in over the inside barb.

The duration can be determined more accurately if an estimate of wind changes between maps is made. For example, suppose a cold front is moving at 30 knots across the eastern United States and at 0600 is located 90 NM inland. On the 1200Z map the front will be over the water. If it is assumed that the front moved at 30 knots for the entire 6-hour period, it then can be said that the wind started at 0900 and has blown for 3 hours at points near the coast. Unless this extrapolation of the movement of the front was considered, the duration could be any value from 0 to 6 hours.

For the open ocean, 6-hourly maps are the best that can be hoped for. The forecaster can, however, keep track of the wind changes at his position at hourly intervals. Also for coastal regions and forecasts at a land station, the hourly teletype reports are useful for finer computations of the duration and of the changes in the wind field.

Simplifying the Problem of Determining the Wind Field

The term, wind field, is all-inclusive. It implies the determination of all the information needed for forecasting. In order to treat this problem in adequate detail, it must be broken up into smaller subjects. Thus, in the following sections, ways to locate the fetch, ways to determine wind velocity accurately, and ways to determine duration will be discussed in that order.

Ways to Locate the Fetch

Some Typical Fetches

In all these cases the first step toward a wave forecast is locating a fetch. The term *fetch* has been used extensively in this manual, but its definition is important enough to bear repeating. A FETCH IS

AN AREA OF THE SEA SURFACE OVER WHICH A WIND WITH A CONSTANT DIRECTION AND VELOCITY IS BLOWING. Some typical fetch areas are shown in figure 5.1. The ideal fetch over the open ocean is rectangular, with the winds constant in both direction and velocity. The ideal rectangular fetch is described by a length, F, and a width, W_s. If the angle between the wind direction in the fetch and the line connecting the center of the leeward

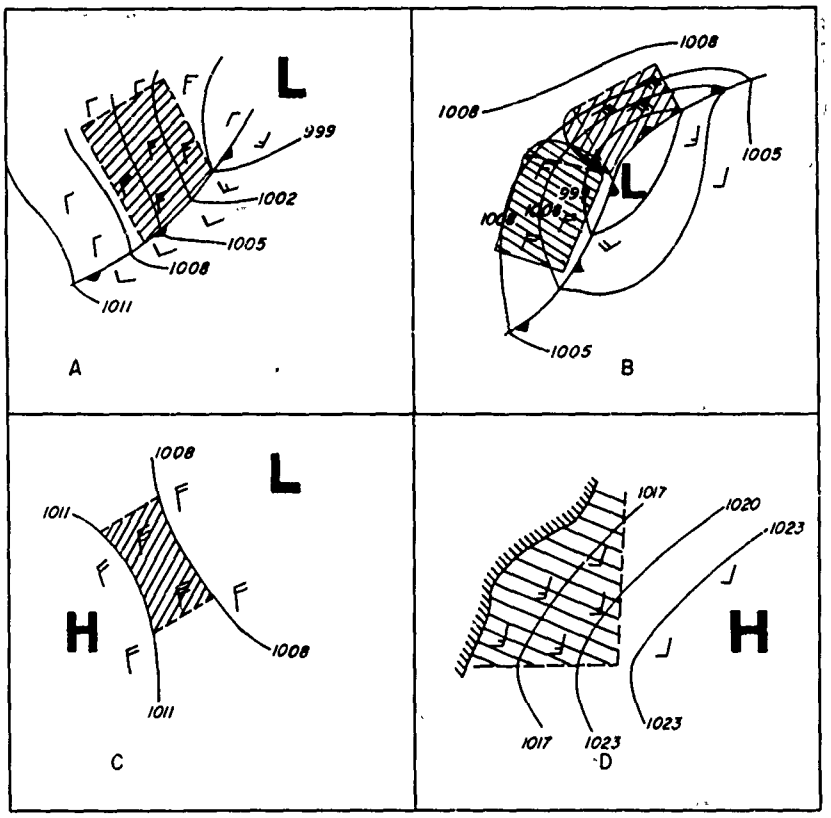


FIGURE 5.1 Typical fetch locations.

edge of the fetch to the point at which a forecast is to be made is 60° or less, the waves from that particular fetch may affect the forecast.

As shown in figure 5.1, most fetch areas are bounded by coastlines, frontal zones, or a change in the isobars. In cases where the curvature of the isobars is large, it is a good practice to use several fetch areas instead of one, as shown in figure 5.1B. When two fetch areas generate waves that affect the forecast area, the spectra must be superimposed as discussed in Chapter III (page 96).

Moving Fetches

So far in this chapter the fetch areas have been described as if they were stationary, but this is not always true. Although semipermanent pressure systems have stationary fetch areas, and some storms may move in such a manner that the fetch is practically stationary, there are also many moving fetch areas.

There are, as a simplification, three possibilities for moving fetch areas. Fetch areas can move to windward, to leeward, or perpendicular to the wind system. The problem is to determine what part of a moving fetch area to consider as fetch for a duration time, say, of 6 hours. Figure 5.2A shows three cases where a fetch, AB, has moved to the position, CD, 6 hours later.

CASE 1. The fetch moves normal to the direction of the winds in the wind field, as shown in figure 5.2A. The best approximation is a fetch with a 6-hour duration, CB. Therefore, in a forecast involving this type of fetch, use only the fetch area that appears on two consecutive maps. The remaining fetch does contain waves, but they are lower than those in the overlap area.

CASE 2. The fetch moves to leeward: a typical situation as shown by the cold front in figure 5.2B. Since the waves are moving forward through the fetch area, the area to be used here as fetch is CD. As the front passes, if the duration of the wind behind the front is long enough, the wave height jumps to the full amplitude maintained by the wind field.

CASE 3. The fetch moves to windward. Figure 5.2C shows the movement described here. The area, CB, occurs on both maps and naturally is a good choice as the fetch. Since the waves move toward A, the region, AC, will have higher waves than the area BD. Experience has shown that in this case, AB is the most accurate choice for a fetch.

Much more theoretical work needs to be done for the case of moving fetches, especially for cases 1 and 3. Filter III can be used for case 2 if the fetch area moves with the speed of the wind over the fetch.

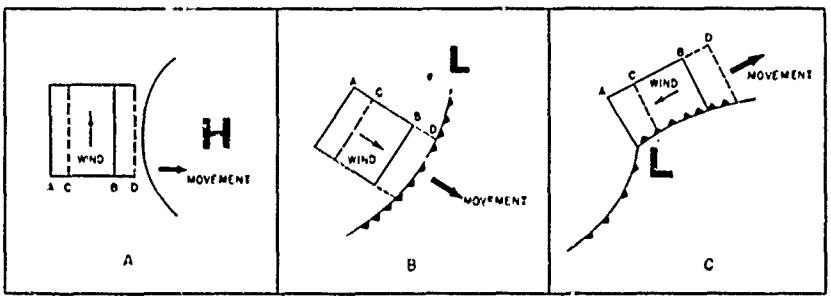


FIGURE 5.2 Fetch movement.

Accurate Determination of the Wind Velocity

Three Ways to Determine the Velocity

There are three ways to determine the wind velocity. The first way is to average the reported wind velocities over the fetch of interest, both for a given map and for successive maps if the winds over the fetch have nearly the same velocities for successive maps. The second way is to compute the geostrophic wind from the isobar spacing and employ corrections for curvature and air mass stability to get the surface wind velocity. The third way is to do both and compare the observed and computed velocities.

Reported Winds

The first way is relatively simple to apply in theory, but it is difficult and sometimes inaccurate in practice. As mentioned before, the procedure here is to compare the observed wind values over the fetch to determine the velocity that best represents the wind field. If the winds have been plotted as observed, and not in Beaufort values, an average usually gives an accurate velocity value. This way of determining the wind velocity has both advantages and disadvantages. The wind is the actual reported wind at anemometer level; therefore, it does not need to be corrected. Difficulties such as supergradient winds and an anomalous angle of wind direction to isobars when isallobaric effects affect the wind are no problem because the winds are the actual winds. Some disadvantages are that the values which are reported may not be representative, the report may be in error, and needed data may be missing.

Corrected Geostrophic Winds

The second way to determine the wind velocity is, at times, the only possible way to find the needed values. Wind reports may be missing over a fetch of importance, and yet there may be enough data around the fetch to make it possible to determine the isobar analysis over the fetch from consistency and continuity principles. If it is also possible to determine the sea-air temperature difference, then an estimate of the surface wind can be obtained by correcting the geostrophic wind for curvature and air mass stability (see also P. W. Johnson, 1955).

The reason for the geostrophic wind corrections is that the geostrophic wind actually applies to a level above the friction layer (2,000 to 3,000 feet). It is correct for straight isobars only. When the isobars curve, other forces enter into the computations, and the wind increases or decreases depending upon whether the system is anticyclonic or cyclonic in nature. The stability correction is a measure of the turbulence in the layer above the water. Cold air

over warm water is unstable and highly turbulent, making the surface wind more nearly equal to the geostrophic wind. Conversely, warm air over cold water produces a stable air mass and results in the surface wind being much smaller than the geostrophic wind.

Correction for Curvature

A practical method for correcting the geostrophic wind for curvature is given below.

1. For moderately curved to straight isobars—no correction applied.
2. For great anticyclonic curvature—add 10 percent of the velocity. (Radius of curvature less than 16° of latitude.)
3. For great cyclonic curvature—subtract 10 percent of the velocity. (Radius of curvature less than 16° of latitude.)

In the majority of cases the curvature correction can be neglected since isobars are usually relatively straight. The gradient wind can always be computed if more refined computations are desired.

Correction for Air Mass Stability

In order to correct for air mass stability another observed quantity is needed. This is the sea-air temperature difference which can be estimated from ship reports in or around the fetch aided by climatic charts of average monthly sea temperatures. An average sea-air temperature difference for the whole fetch is needed.

The stability correction is applied to the geostrophic wind after it has been corrected for curvature. The correction to be applied is given in table 5.1. The symbol T_s stands for the temperature of the sea surface, and T_a stands for the air temperature.

Table 5.1—Correction to Geostrophic Wind for the Sea-Air Temperature Difference

T_s-T_a	*Percent of geostrophic winds used*
0 or negative	60
0 to 10	65
10 to 20	75
20 or above	90

Advantages and Disadvantages

The greatest advantage of the use of the corrected geostrophic wind is that it can be used in cases where observed winds are not available. There are many disadvantages since errors in isobar analysis and errors in computations can combine to give values which are seriously in error. Careful weather map analysis and carefully made pressure gradient measurements are essential in the use of this method.

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Combined Reported Winds and Corrected Geostrophic Winds

At times just a few winds are reported over the fetch; however, an adequate isobar analysis is possible based on reported pressures in and around the fetch. The third way to obtain an accurate determination of the wind velocity is to combine reported winds with corrected geostrophic winds.

The geostrophic wind at a representative point in the fetch can be computed. The isobars should be in agreement with the observed winds. The geostrophic wind, corrected for curvature and stability, should be compared with the observed velocities to determine the velocity actually used. This requires judgment and experience as sometimes there is considerable difference between the computed and observed values. Always give more weight to the observed winds unless there is reason to believe that they were coded improperly or incorrectly obtained.

Advantages and Disadvantages

This third way has many advantages. It provides a check on the consistency of the values since they are obtained in two independent ways. If the results agree, then all is well. If they disagree, then the analysis, the computations, and the reported values should be checked again to find the source of error. Another advantage is that practice is obtained in using corrected geostrophic winds, which can be checked against observations. Then, when only the geostrophic wind is available, it can be applied with much more confidence because of the added experience. The disadvantages are that it takes longer to obtain the winds and that the values obtained can still be incorrect, for the reasons given above.

The Accurate Determination of Duration

Aids to Interpolation

Once a fetch has been determined and the velocities have been found, the next step is to determine the duration of the wind over the fetch. It is very unlikely that the wind will begin or end exactly at one of the 6-hourly maps, hence an accurate value must be interpolated. In most forecasts a simple interpolation of the successive maps will be sufficient to locate the bounds of the wind field, but in some cases the following rules might help.

A. A cyclonic center moves with approximately the same speed as the warm front and somewhat more slowly than the cold front.

B. Warm fronts move with a speed 60 percent to 80 percent of the geostrophic wind component normal to the front.

C. Cold fronts move with a speed 90 percent to 100 percent of the geostrophic wind component normal to the front.

D. Occluded fronts move with a speed 80 percent of the geostrophic wind component normal to the front.

Waves Already Present

Another problem arises when there are waves, traveling to leeward, already present in the generating area. These waves are compensated for by using a longer duration value. In effect, it is assumed that the wind started earlier and produced the waves itself. The exact length of time to add to the duration is found from the generation graph and can be explained best by an example.

Assume there are 4-foot significant waves over an area. Then assume that a 20-knot wind blows for 2 hours. The waves build up from 4 feet to a new height in 2 hours owing to the new wind. What will the new height be? The first part of the problem is to determine how long the 20-knot wind will have to blow to generate 4-foot waves. To find the answer, just reverse the normal procedure for forecasting. That is, instead of going from wind velocity and duration to E, taking the square root of E, and multiplying by 2.83 to get the significant wave height, work the problem backwards. Divide 4 feet by 2.83, square the result, and find that $E=1.99$ ft^2. Then enter the graph at $E=1.99$ ft.^{2}, move horizontally to the 20-knot line, and read off the duration value. This is the time necessary for a 20-knot wind to generate 4-foot waves. In this case it is 5 hours, so the effective duration is 5 plus 2 or 7 hours. The new E value after 2 hours is thus 4.30 ft.^{2}, and the significant height 5.9 feet.

A similar procedure for a slightly different situation is worked out in example 2.3, Chapter II, for a case where the wind increased over a steady sea state.

This method should be applied only if the spectrum present covers the same frequency range as the spectrum being generated. The old waves should be traveling in the same direction as the new wind. A low frequency swell from a distance would be independent of the newly generated sea. The E values should be added in this case, and the waves described as sea plus swell.

If a full spectrum is present and the wind starts to blow against the waves, then a new wave system is generated which travels in the new wind direction against the old sea. The old sea must then be destroyed (in part) before the new sea can build up. Just how long this takes and how it should be forecast is not yet known. The forecaster should study such situations carefully and determine from observation how such situations should be treated.

Variable Wind Velocities

The forecasting method is simplified when the wind has a constant velocity for the entire duration. If this does not occur, an effective

duration must be computed. For example, suppose the wind has been blowing for 24 hours with velocities of 10 knots for 6 hours, 15 knots for 12 hours, and 20 knots for 6 hours. The duration is 24 hours but the velocity value to be used is in question since only one velocity is allowed and there are three distinct values. There are two possibilities. One possibility is to find an equivalent velocity for the 24-hour duration. The other is to use three durations with the observed velocities and build up the spectrum according to definite rules. Both methods work, but the latter is more consistent with the wind field as it actually occurred. As the duration changes, care must be taken that the limitations imposed by short fetches do not complicate the problem. The effect of a limited fetch can be handled by an appropriate modification where needed.

Slowly Varying Winds

For slowly varying winds the procedure is to assign a single velocity to the duration. Suppose the wind blows for 12 hours and during that time increases in velocity from 10 to 20 knots. What velocity would be representative of the duration? The last value is too high since the wind blew not for 12 hours at 20 knots but for 7.2 minutes at 10 knots, 7.2 minutes at 10.1 knots, 7.2 minutes at 10.2 knots and so on, up to the final velocity. The average velocity for the period, 15 knots, is a close approximation to a single value of the velocity for the duration of 12 hours, but experience shows that this value is too low. When the change in wind velocity is small, say, only 5 knots in 12 hours, then the average wind is accurate enough.

Studies and experience have shown that cases of variable winds can be assigned a single value for the velocity and duration if the velocity change is relatively small. The following two rules can be applied under these conditions.

1. Average the wind velocities when the change is gradual, or increasing and then decreasing. Apply the average to the entire duration time.
2. Use the last velocity when the velocity changes in the first few hours and then remains constant. Apply that velocity to the entire duration time.

A Rapid Increase in Wind Velocity

If the wind blows steadily for a period of time at a constant velocity and then increases rapidly to a new value for another period of time without significant change in direction, then the method explained above for waves already present in the area can be used. The E value produced by the first low winds can be found from the appropriate C. C. S. curve. When the velocity increases to a new and higher value, the fictitious duration which is required on the C. C. S.

curve for the higher wind velocity to give the old E value can be found. This fictitious duration (which is always less than the duration of the weaker wind) is then added to the duration of the stronger wind to obtain the effective duration of the stronger wind.

If the wind increases rapidly and steadily from low values to high values of the velocity, the generation of the waves can be predicted by successively applying the procedure described above. An example of the way these techniques can be used is given below.

EXAMPLE 5.1.

AN EAST WIND BLOWS OVER AN UNLIMITED FETCH. FOR 4 HOURS THE VELOCITY IS 16 KNOTS. THEN FOR 5 HOURS IT IS 20 KNOTS. THEREAFTER, IT IS 24 KNOTS FOR 6 HOURS, 30 KNOTS FOR 7 HOURS, 36 KNOTS FOR 10 HOURS, AND 50 KNOTS FOR 10 HOURS. WHAT IS THE E VALUE FOR THE HEIGHTS AS THE WAVES GROW?

Method of solution.—A wind of 16 knots for 4 hours generates waves with an E value of 1.05 ft^2. A 20-knot wind for 3.8 hours produces the same E values. Therefore, at the end of the first 4 hours, the results are the same as if a 20-knot wind had blown for 3.8 hours. The fictitious duration of 3.8 hours plus an additional actual duration of 5 hours is the effective duration of the 20-knot wind. The effective duration is, consequently, 8.8 hours. A 20-knot wind for 8.8 hours generates waves with an E value of 7.0 ft^2. A 24-knot wind blowing for 8.2 hours also generates waves with an E value of 7.0 ft^2. Therefore, at the end of the first 9 hours, the results are the same as if a 24-knot wind had blown for 8.2 hours. The fictitious duration of 8.2 hours plus the actual duration of 6 hours gives an effective duration of 14.2 hours at 24 knots. A duration of 14.2 hours at a velocity of 24 knots generates waves with an E value of 19.2 ft^2. This procedure can be repeated for as long as the wind velocity continues to increase.

ANSWER TO EXAMPLE 5.1.

THE COMPLETE DETAILS OF THE ANSWER TO EXAMPLE 5.1 ARE GIVEN IN TABLE 5.2. NOTE THAT THE EFFECTIVE NUMBER OF HOURS IS LESS THAN THE ACTUAL NUMBER OF HOURS. THE EFFECTIVE NUMBER OF HOURS NEED NOT INCREASE CONTINUOUSLY. IN FACT, IN THE LAST ROW IT DROPS FROM 29.3 TO 27.7 HOURS.

Decreasing Winds

Suppose that a wind of high velocity has blown long enough to generate waves with a spectrum associated with that velocity, and that the wind velocity then decreases to some lower value. A wave

Table 5.2—The Growth of Waves with Increasing Wind Velocities

Actual Velocity and Duration	Effective Velocity and Duration	Hours Elapsed	Effective Hours at ( ) knots	E
16 knots 4 hours	16 knots, 4 hours or 20 knots, 3.8 hours	4	4(16)	1.05
20 knots 5 hours	20 knots, 8.8 hours or 24 knots, 8.2 hours	9	8.8(20)	7.0
24 knots 6 hours	24 knots, 14.2 hours or 30 knots, 12.9 hours	15	14.2(24)	19.2
30 knots 7 hours	30 knots, 19.9 hours or 36 knots, 19.3 hours	22	19.9(30)	45.0
36 knots 10 hours	36 knots, 29.3 hours or 50 knots, 17.7 hours	32	29.3(36)	122.0
50 knots 10 hours	50 knots, 27.7 hours	42	27.7(50)	196.0

spectrum will eventually be established which corresponds to the low wind velocity. Thus the waves over the fetch decrease in height. They die down because the energy in the fetch travels out of the fetch in the manner given by Filter IV mentioned in Chapter III (page 91). They also die down from the effect of viscosity at the high-frequency end. As stated in Chapter III, not enough is yet known about the effect of viscosity to permit any computation of its effects.

The effect of dispersion, however, can be taken into account quite easily. Compute the time it takes the various spectral frequency components to propagate past the point at which the forecast is being made. After these times the energy associated with these frequencies for the higher wind spectrum has passed the forecast point. However, if there is some energy present for these same frequencies, from the spectrum for the lower winds, it must be included in the determination of the total E value of the waves still present.

Viscosity Effects

The forecaster can best determine the effects of viscosity by a series of careful observations of the waves in actual forecasting situations.

Past observations have so completely intermixed the effect of dispersion with the effect of viscosity that little is known about each effect separately. If the fetch is very long so that a steady state due to duration alone is present over a very long distance, and if the time required for the waves to die down owing to dispersion is very long, and if, then, the waves die down before this required time has elapsed, the decrease in wave height is due to viscosity. Careful observation of the waves during such situations will make it possible for the forecaster to determine the effect of viscosity.

As can be seen, there is a multitude of situations that arise in wave forecasting. Only by actually forecasting and acquiring experience and judgment can a forecaster become proficient in selecting the proper variables in any individual case.

A Summary Chart Helps Determine the Wind Field

The Total Effect of the Fetch, Velocity, and Duration

Taken all together, the fetch, velocity, and effective duration determine the wind field. After the three component parts of the total field are determined, they must be studied as a unit before the forecast is prepared. A summary chart is useful in keeping track of the total wind field. The computed and observed winds can be plotted for successive synoptic map times on the same base map by plotting the winds at one time in one color, the winds 6 hours later in a second color, and so on. Often one base map can be used to keep track of the winds for several days. Frontal positions and high and low centers can also be added in the appropriate colors. The preparation of this one chart is simple. It saves the forecaster a great amount of time because he does not have to hunt back and forth through the past maps in order to determine the changes in the wind field.

"Sea" and "Swell" Forecasts

For some wave-forecasting situations, the current and past weather are all that is necessary. Forecasting the propagation of waves, or swell, is in this category, because a forecast up to a certain time in the future can be made on the basis of past or current weather. When a forecast is desired for a given area at some future time, not only the present and past weather are needed but also the future weather. This necessitates making a prognosis of at least the important wind fields and perhaps the entire map. Three forecasting cases which are typical of all wave problems that might arise are:

Case 1. Two ships will rendezvous at a designated point within a few hours. There is no wind in the rendezvous area, but there is some swell. Will the swell increase or decrease? For this case, the

forecaster uses the current and past maps to locate the fetch producing the swell and also the past maps to compute the spectrum present in the fetch and the needed filter characteristics. Then, a forecast is made following the principles given in Chapter III to determine whether the waves will increase or decrease. Only past and current weather data are needed in this case.

Case 2. A seaplane will land at a certain point. What are the wave conditions there now? The forecaster uses the present and past maps again to compute the wave conditions. Again, only the past and current weather data are needed.

Case 3. A ship is damaged and is just making headway in high waves and severe winds. Are the waves going to decrease, and if so, when? In this case the forecaster has to forecast the weather and wind changes for the next 24 hours and then forecast the related changes in the waves. (Chapter II and possibly Chapter III.)

Synoptic Wave Charts

Forecasting Waves by Synoptic Methods

A method developed by the U. S. Navy Hydrographic Office for forecasting waves over a large area of the ocean offers great promise. It is a technique that combines some of the principles of wave forecasting with the methods used in weather forecasting. This procedure has been in use at the Hydrographic Office. It results in construction of a very practical chart which can be used in the preparation of forecasts for a great many points on the ocean within a few minutes.

Construction and Analysis of the Chart

In addition to the ordinary meteorological charts prepared in the forecasting office, a chart is plotted which shows just the synoptic wave reports for a given time as they come in from the various ships over the ocean. An arrow is plotted showing the direction of the waves, and a fraction, having wave "period" above and significant height below is put beside the arrow. The forecasting techniques given in this manual are used to fill in the empty spaces of the chart and to give the dominant wave direction, the average "period," and the significant wave height associated with the waves not actually reported. From the meteorological analysis, the appropriate fronts are entered on the chart. The centers of the highs and lows are also shown. Then the chart is broken up by dashed red lines into areas where the waves have the same dominant direction. This can be done by assigning the same direction to all waves in a warm-front sector, for example, and to all the waves in a particularly strong area of polar outbreak behind the cold front. Large red arrows are entered in each

of these areas to show this dominant wave direction at a glance. The next step is to draw isolines of constant significant heights. These isolines look like isobars on an ordinary weather map. They form a definite pattern on the map so that it is possible to tell at a glance how high the waves are at any point over the area of the ocean which has been analyzed. The result is a synoptic wave map.

Various techniques are employed in the analysis of the charts in order to make them consistent from one chart to the next. One technique is the method of continuity. The high wave areas move according to the laws developed in this manual. They persist from day to day and, for example, would be found under an area of high wind traveling across the ocean in a cyclonic storm. It is desirable to try to keep the high wave areas roughly rectangular in shape. The use of the fetch and duration graphs helps determine the height contours and their exact locations. For example, with a strong wind which has been blowing off a continent for a number of hours in a uniform direction, a height contour will be the same distance (roughly speaking) from the shore everywhere along a considerable length of the contour. The location of that height contour, where the waves reach full amplitude for a fully developed spectrum at a given velocity, can be determined easily from the charts given in Chapter II. With the principles of continuity and the methods given in this manual it is not difficult to prepare a consistent analysis.

Forecasts from the Synoptic Wave Charts

From the synoptic charts, it is possible to carry out a forecast using principles similar to those used in weather forecasting. The resultant chart is called a synoptic wave prognostic chart. The principles of continuity, the idea of using the weather prognostic chart in order to aid in forecasting the wave conditions, and the prediction of wave heights and directions from the methods in this manual then make it possible to draw prognostic charts for 12, 24, and 48 hours into the future.

This method has been tried by the Hydrographic Office. The results it gives on prognostic charts are comparable in accuracy, at least, to the prognostic charts of the weather on which they are based.

Some Additional Suggestions

Additional refinements are possible. The analysis of the chart can be made in greater detail by keeping track of the wave systems from each storm on the successive prognostic charts and outlining the height contours that go with a given wave system in a given color. Storm 1 can have its height contours drawn in red, storm 2 in blue, and so on. It is also possible to keep track of swell as it travels out of a given storm area and spreads across the ocean, becoming lower and

lower. Areas where several wave trains interfere with each other can also be detected. Quite frequently, in addition to the waves that are present behind the cold front in a storm, there is also swell from some other storm there, and the E value for that swell needs to be added to the E value for the locally-generated waves in order to get the total E value.

Figure 5.3 is an illustration of such a synoptic wave chart. It shows the various analysis features just discussed. Such a chart is not difficult to prepare. Its use in wave forecasting centers will be a valuable aid.

Revising the Forecast

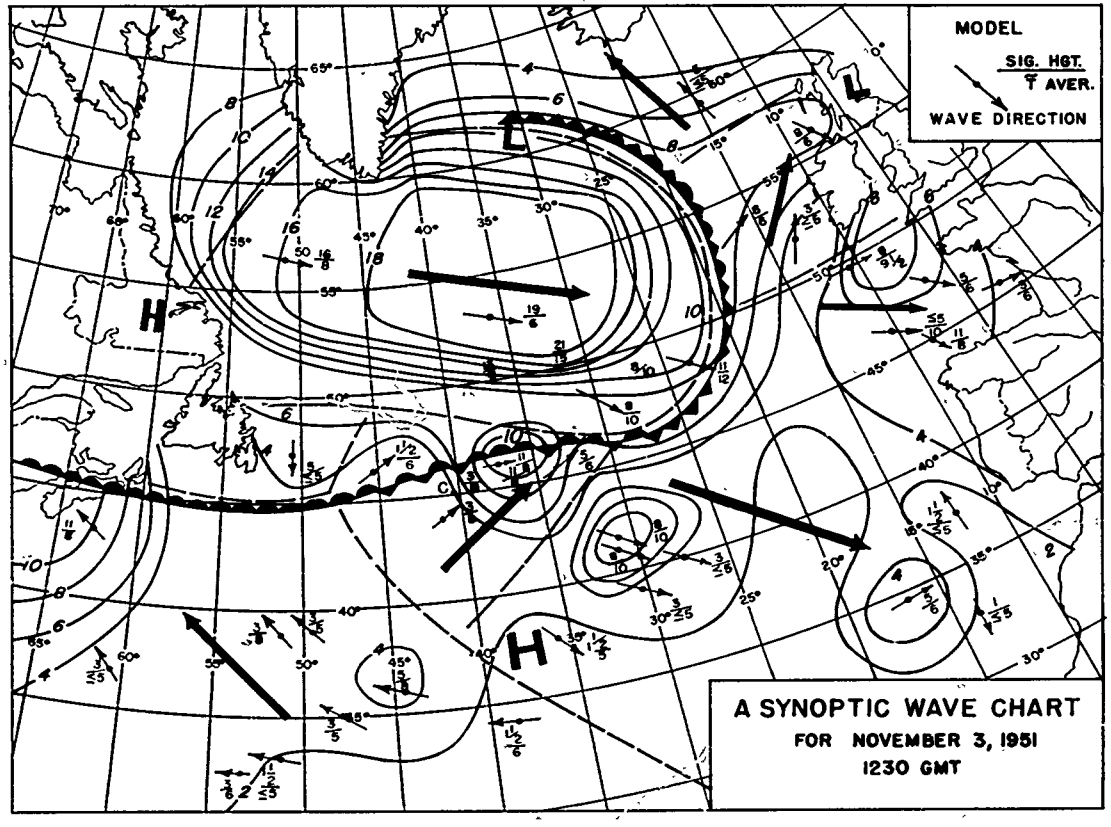
Even after the forecast has been completed, the forecaster must keep a careful check on the weather, with an eye to revising the wave forecast on the basis of new weather data.

Forecasting the onset of wave-height decreases is very important and since a change in the wind field is indicative of wave changes, it pays to watch the wind changes carefully. A sudden shift of winds or a weakening of the pressure gradient can be just enough to allow bad sea conditions to change to operational sea conditions within a few hours.

It is also a good idea to give, along with the forecast, some indication of the validity of the data. That is, if the wind field is likely to change owing to complex frontal patterns, give the forecast but add that it is subject to change. Naturally, any forecast is subject to change, but some forecasts are quite stable, such as one a forecaster might make with a semipermanent high present. One procedure is to grade each forecast as good, fair, or poor, depending upon the stability of weather conditions and the reliability of the data on which the forecast is based.

In forecasting there is often a chance for choosing between two values. This type of subjective procedure is liable to a constant error of prejudice. That is, the forecaster, given a choice between force four and force five winds may always lean toward the higher value. Since there are many such choices before every forecast is completed, many forecasters find they consistently forecast on the same side of the true value.

The only way to correct this error of prejudice, besides being very careful and consistent in preparation of the forecast, is to verify as many forecasts as possible and graph the observed and forecast wave heights. If the forecasts run consistently on the same side, either too high or too low, try to allow for this tendency in future forecasts.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WAVE DIRECTION
A SYNOPTIC WAVE CHART
FOR NOVEMBER 3, 1951
1230 GMT

FIGURE 5.3.

Wave Forecasting Check List

A check list is given below for the general procedure up to the use of the generation graphs and filters. In some cases the forecast will not require every step, so that the procedure is simpler.

1. **Fetch**
   - A. Locate all areas that affect the forecasting point.
   - B. Determine areas valid for a given duration.
   - C. Adjust fetch dimensions for moving fetches.
2. **Velocity**
   - A. Find average velocity over each fetch from actual observations.
   - B. Compute geostrophic wind over each fetch.
     1. Correct for stability.
     2. Correct for curvature.
   - C. Compare observed winds with corrected geostrophic winds.
3. **Duration**
   - A. Interpolate between maps.
   - B. Correct for waves already present.
   - C. Determine correct durations for increasing winds.
     1. By assigning one representative velocity to the duration.
     2. By computing effective durations for increasing winds.
   - D. For decreasing winds, apply filter theory to determine how fast the waves will die down.

Chapter VI

WAVE REFRACTION

Introduction

The results presented in the previous chapters apply only to those parts of the ocean, the Great Lakes, or other bodies of water where the depth of water is greater than one-half the wave length associated with the highest spectral *period* present. In shallow water other effects become important. Refraction bends the waves so that their crests become more and more parallel to the shore. The wave crests, which in a particular case may be at an angle of 45° to the beach out in deep water, may turn until the crests of the waves near the shore are at an angle of only 10° to the coastline just before the waves break. Refraction affects waves in several ways as they travel from deep water over the shoal waters off the coast and finally break upon the shore.

If the effect of refraction can be computed for a simple sine wave, then the effect of refraction can be computed for actual ocean waves. The theory is simply to consider each of the sine waves in the sum that makes up the complete sea surface and refract each sine wave separately and independently of all the others. Then, add up the E values of each of the waves at a point in shallow water in order to find the total E value at that point. With this E value it is possible to forecast the height distribution of the waves at this point.

The theory presented in this chapter will work well for all depths between very deep water out at sea and the point where the waves begin to break just before they rush up on the beach. No techniques will be given for forecasting breaker heights or their behavior during the last 50 yards before they break. The breaker heights will be associated with the heights of the waves just before breaking, and this will be sufficient in many cases to describe the breakers qualitatively. If the waves offshore in 30 feet of water are known to be 10 feet high, on the average, for one portion of a coast line and only 5 feet high for another portion under the same wave conditions in deep water, then the region where the waves are only 5 feet high will be the logical place for an operation in which low surf is necessary.

The Refraction of One Sine Wave

Description of Refraction

Just as the sum of a great many sine waves can be considered and used theoretically to forecast the properties of the actual sea surface, so is it possible to forecast the effects of wave refraction by first studying the refraction of one sine wave and then adding the effects of many simple sine waves to obtain the total sea surface disturbance. A simple sine wave is characterized by a height, a *period*, and a direction. Unlike the actual ocean waves discussed in previous chapters, it has an infinitely long crest. A simple sine wave in deep water can approach a coast from any direction depending upon where it was generated in the storm which caused it. Figure 6.1 illustrates a portion of a simple sine wave as it approaches a region where the

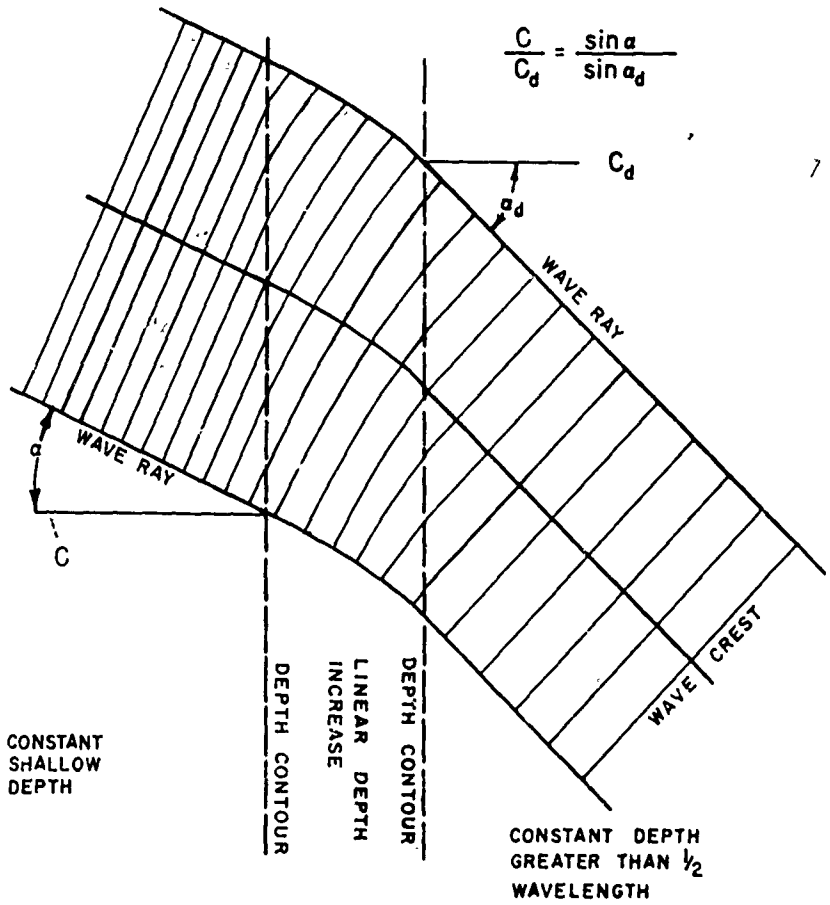

FIGURE 6.1 The refraction of a simple sine wave.

depth of water becomes shallow over a rather narrow transition zone. To the right in figure 6.1 the water is deep in the sense that the depth is greater than one-half the indicated *wave length*. To the left, in the region designated as shallow, the water has a depth less than one-half of the deep-water *wave length*. The wave crests bend as they approach the shallow water zone because the part that enters the shallow water zone first slows down. The part still in the deeper water continues to speed ahead, and the crest turns. As a result, after the wave has advanced through and into the shallow water zone, the crests are traveling in a different direction. In a simple case such as this the crests are still straight lines, although they are aligned in a different direction.

Snell's Law

The deep-water wave *speed* is C_d, and their shallow-water *speed* is C. The bending of the wave crests obeys a law which is called Snell's law and is given in equation (6.1).

$$C_d/C = \sin \alpha_d / \sin \alpha \tag{6.1}$$

Since C is smaller than C_d, $\sin \alpha$ is smaller than $\sin \alpha_d$, and the crests bend so that they are more nearly parallel to the depth contours, as indicated in figure 6.1.

Wave Rays

Shown also on figure 6.1 are the wave rays. A WAVE RAY IS A *LINE* DRAWN EVERYWHERE PERPENDICULAR TO THE WAVE CRESTS ON A REFRACTION DIAGRAM. The best way to study the effects of refraction in a practical case is to construct the wave rays for as many simple sinusoidal waves as time permits, approaching the coast under consideration from a great many possible directions and with a great many different frequencies. It is then possible to make a computation which shows how the waves change, and this permits forecasting the properties of the waves in the shallow water zone. The methods for constructing the wave rays according to Snell's law will be given later in this chapter.

The Changes Undergone by a Refracted Sine Wave

Four things happen to a simple sine wave as it travels from deep water into the shallow water bordering the coast line. Observation of the wave at a fixed point in the shallow water region will yield a wave height different from the height in deep water. In the shallow water zone the wave crest near this point will appear to be traveling in a direction different from the direction that it had in deep water. The horizontal distance between the crests of the waves will be less than in deep water, and the *speed* of the waves will be lower. Finally,

the crests, which in the ideal case are infinitely long and straight in deep water, become curved in shallow water. The methods given in this chapter permit a forecast of the height and direction changes for actual ocean waves; but nothing will be said about the *length* of the waves, the *speed* of the waves, or the curvature of the crests at a point of forecast in a shallow water zone. The height can be shown to vary from point to point. Forecasts for many points are possible, but no actual technique will be given which will make it possible to demonstrate the curvature of the crests.

Effect of Complex Contours

Complex bottom contours can produce great distortion of a simple sine wave. The Hudson Canyon, which extends out to 60 miles from the coast of New York and New Jersey, is an example of a very complex bottom contour system. A simple sine wave approaching this system from a deep-water direction of 112.5° and a *period* of 12 seconds is bent and distorted by the effect of the bottom contours as shown in figure 6.2. The scale of this figure is such that only every 45th wave crest is shown. It will be observed that the original straight line of the deep-water wave crest is broken up into three segments, each of which proceeds independently. They cross and form a very complicated pattern. Each new segment can be treated separately, and the combined effects can be found by the same techniques as used in Chapter III for combining waves from two different storms at a point of forecast.

The Refraction of Two Sine Waves

Apparent Lengthening of Wave Crests

There is an interesting effect concerned with the refraction of actual ocean waves which can most easily be studied by showing the effect of the refraction of two simple sine waves of the same period but approaching a coast from two slightly different directions. This effect is the apparent lengthening of the wave crests. Suppose that the crests of the two sine waves are approaching a coastline and that the angle between the crests of the sine waves is, say, 15°. The interference effect in the simple case produces rows of alternate elevations above and depressions below the sea surface. The individual short-crested waves are shaped like long ellipses, and a given contour of equal height made on the interference pattern would be roughly elliptical in shape. The crests cross at some angle out in deep water; if each of the two infinitely long crests that make up the visible interference pattern is refracted separately, the effect is *always* to make the crests more nearly parallel in shallow water if the *period* is the same. If the individual crests are more nearly parallel in shallow water for the sum of

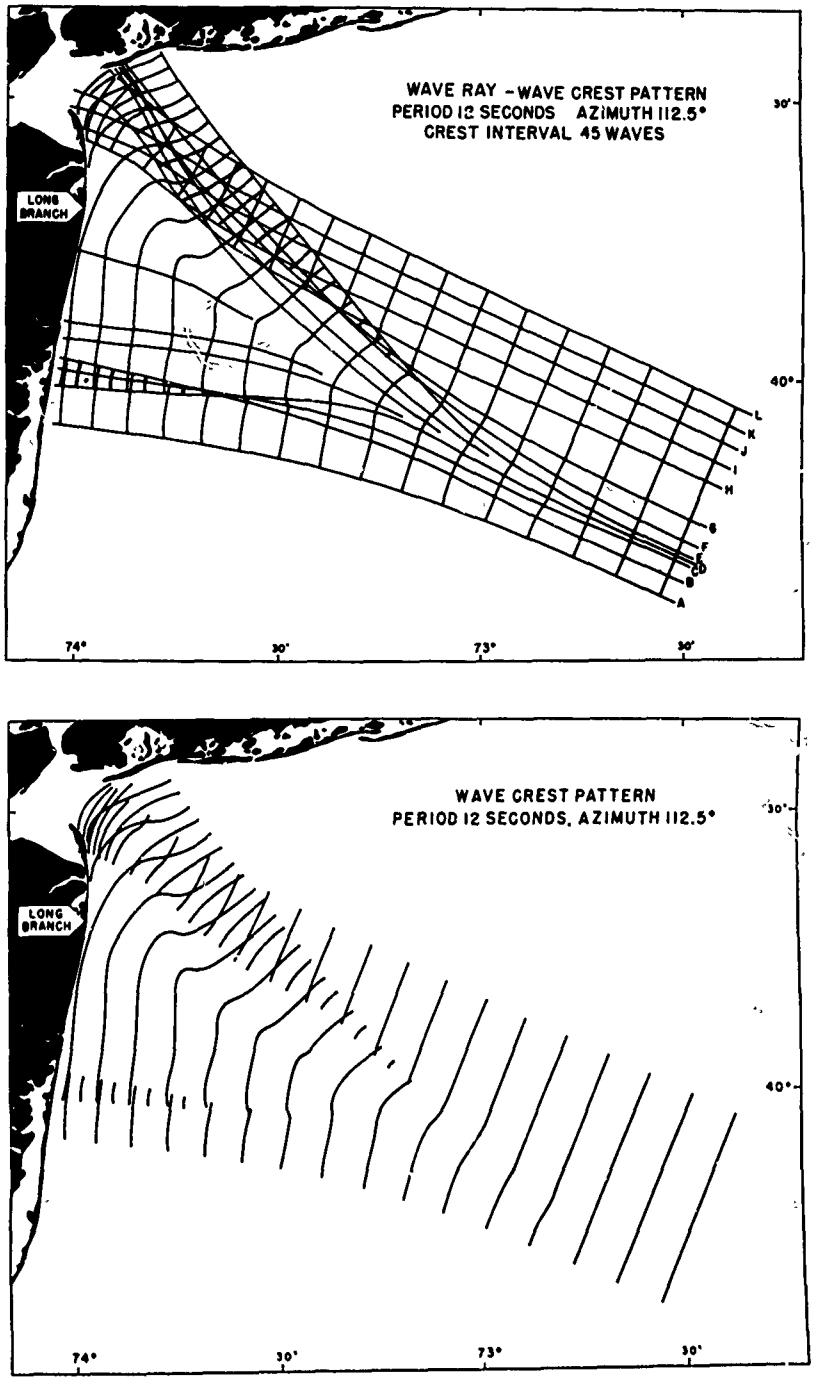

FIGURE 6.2 The effect of the Hudson Canyon on a simple sine wave.

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these two simple sine waves, then the interference pattern stretches along the crests. Height contour ellipses that may have been only 100 or 200 yards long out in deep water stretch to three or four times that length in shallower water near the beach. This means that the visible wave crests appear to stretch out to great lengths along the crests as they approach the beach. This effect is illustrated in figure 6.3a. It is a very pronounced one which can be recognized in many aerial photographs. The refraction of a sum of several sine waves produces the same effect in many cases.

Echelon Waves

A second interesting effect can be obtained from a consideration of the refraction of two simple sine waves by studying the effect of refraction upon two sine waves which are traveling in deep water in exactly the same direction but with slightly different *periods*. The wave lengths are also different, and from the discussion given in Chapter I such a pattern will appear to be individual groups of waves; that is, there will be high waves for a certain distance followed by low waves and then high waves again. As this pattern is refracted, the two different *periods* are bent by different amounts in entering shallower water. This effect is shown in figure 6.3b. The once infinitely long crests in deep water become short crests in shallow water.

When this effect is applied to a long, narrow spectrum, i. e., to a wave spectrum where frequencies vary over a wide range but where directions vary over only a narrow range, the result is an echelon pattern. Consider waves arriving from a storm which has lasted a long time but is so far away that the range from θ_4 to θ_3 is very small, say, of the order of only 5° or 10°. A given wave will have a long crest, but as it is followed in its progression toward shore it will become less pronounced in height and tend to disappear. The next wave, instead of lining up directly behind the first, appears to one side and it dies down in its turn. The third wave likewise forms to the side of the second rather than directly behind. The resulting pattern somewhat resembles the bow waves at one side of a ship advancing through calm water. This echelon pattern can be seen in many aerial photographs of ocean waves.

The Refraction of Actual Ocean Waves

Some Misleading Effects

When actual ocean waves are refracted, the illustrations just given no longer apply because actual ocean waves cannot be treated as if they were just one or two simple sine waves. As an approximation, actual ocean waves can be treated as the sum of 50 or 60 simple sine waves and very reliable results will be obtained, but to use only the

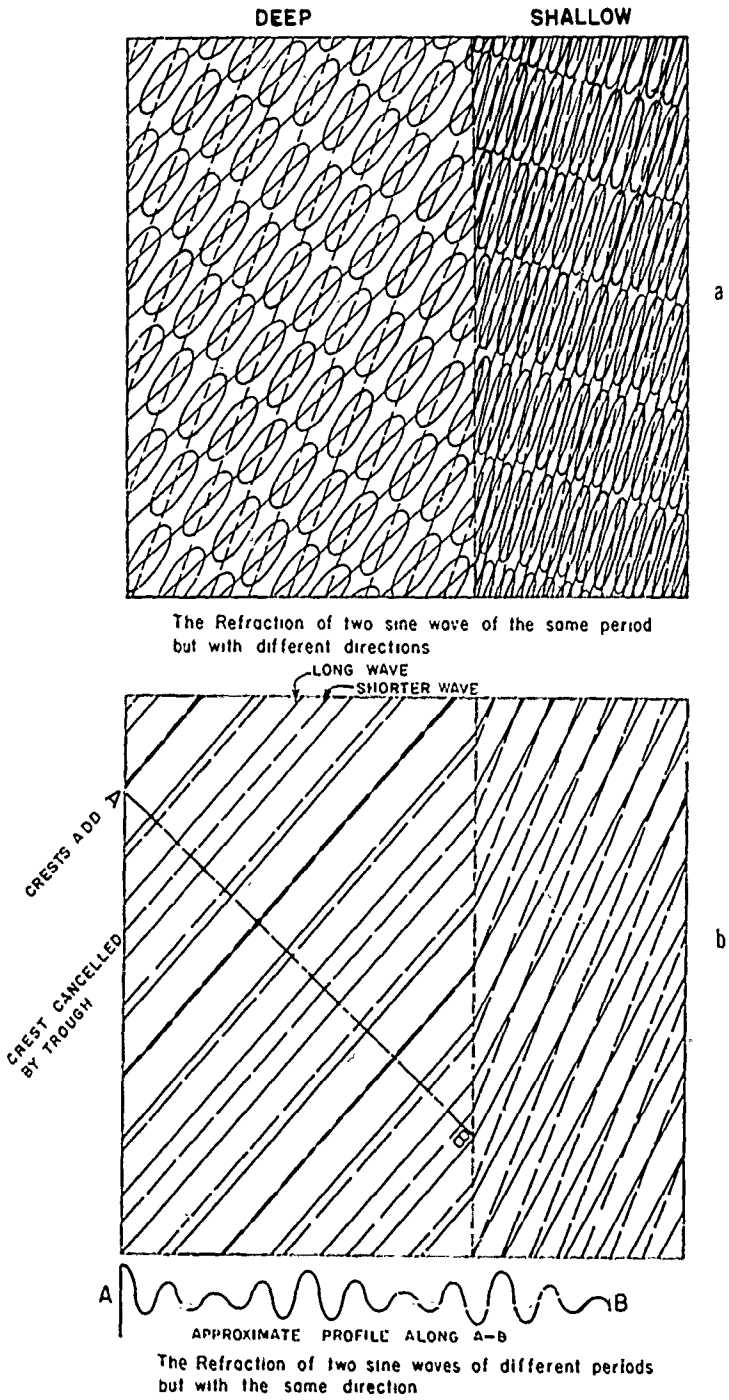

FIGURE 6.3 The refraction of two simple sine waves.

significant height and the average "period" of the waves in deep water and to refract the waves with these two numbers will lead to totally unrealistic results. The use of an average "period" in deep water or any single "period" will fail completely because the average "period" of the waves in deep water will be completely different from the average "period" of the waves in the shallow water zone for which the forecast is being made.

As individual waves in the sum of many simple sine waves are refracted, the effect of refraction is often to focus a great amount of wave energy associated with a certain *period* band in the actual spectrum at one point and to deflect it away from another point along the coast. When this occurs, one part of the *period* band in the complete spectrum will show up at one point on the coast and produce a wave with one average "period" and at another point along the coast there, will be a completely different average "period" owing to the effect of focusing for that particular spectral *period*.

As an example, the northern New Jersey shore (from Sandy Hook to a point near Asbury Park, a distance of about 9 or 10 miles) has been studied in great detail in connection with the effect of refraction. It was found that if waves were approaching this coastline from a southerly direction in deep water and if the *periods* of these waves were high enough, very little of the wave energy would ever reach Sandy Hook. A sine wave system 10 feet high in deep water with a *period* of 14 seconds will be only one foot high near Sandy Hook. The same sine wave system will be 14 feet high at a point near Asbury Park. Lower-*period* waves will be the same in height at both places. The total effect on a continuous spectrum, such as those discussed in Chapter II, is that at Sandy Hook the waves are quite low, and the energy in the waves is associated only with the higher frequency components in the original spectrum. Thus, the average "period" of the waves near Sandy Hook for a typical situation will be only 6 or 7 seconds, and the significant wave height only 4 or 5 feet. At the same time and for the same wave spectrum, the focusing of the lower frequencies on Asbury Park produces waves with an average "period" of 14 seconds. The significant height of the waves at Asbury Park will be about 15 feet. Thus at two points along the coast, separated by only a few miles, completely different significant heights and average "periods" will be observed for exactly the same wave characteristics out in deep water 60 miles beyond the influence of the Hudson Canyon. The conclusion, then, is that waves are focused, distorted, bent and piled up at different points separated by only a short distance along the coastline. Visual observations at only one of these points will be completely misleading, since they will tell nothing about the nature of the waves in deep water. The "periods" observed along a coast can be quite

different from those observed out beyond the effect of refraction, especially under conditions similar to those just described.

Aerial Photographs

Aerial photographs of waves approaching a coastline are perhaps the best and simplest way to obtain information about the effect of refraction. They reveal many facts about the nature of the offshore bottom contour configurations and give information on those regions where the waves can be expected to be highest for given types of storm situations. If a portion of a coastline is under study, the methods given in previous chapters can be used to forecast the type of spectrum present in deep water and then aerial photographs can be used to determine the effect of refraction along the beaches without going into the difficult computations which will be given later. If a qualitative estimate is sufficiently accurate, then a detailed and careful study of aerial photographs can give some important information.

The next five figures show aerial photographs taken over points along the coast of the United States. Enlargements of portions have been made to illustrate some of the very pronounced features of wave refraction.

Aerial Photograph of Ocracoke (Figure 6.4)

Figure 6.4 is an aerial photograph taken by the U. S. Coast and Geodetic Survey of a wave pattern off the coast of North Carolina at Ocracoke. The white area on the left is sand dunes, as the shallows indicate. The coast in the upper part of the figure slopes slowly to the ocean. The depth contours are roughly parallel to the coastline with the depth increasing gradually. At the bottom of the figure there is an underwater ridge which extends a considerable distance into the ocean in such a way that the water is very shallow over this ridge for a great distance from the coast. The waves in deep water are approaching from the southeast. As the many different spectral components approach the coastline, the waves are held back by the ridge, because they must travel more slowly in the shallow water, and they bend to cross over the ridge from both sides. The waves form a very confused interference pattern over the ridge. Notice the areas where the water is so shoal that the waves break offshore and form whitecaps out over the ridge in three rather distinct rows. To the south of the ridge the waves are bent so that they travel almost north. The waves to the north of the ridge are bent so that they travel east. The crossover pattern is very pronounced over the ridge. The waves from south of the ridge go right on north through the shoals over the ridge and can be seen to interfere with the waves coming in from deep water to the north, which are unaffected by the ridge. There is a very definite pattern of interference north of the ridge in a region where the water

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FIGURE 6.4 Aerial photograph of Ocracoke.

is deeper than it is over the ridge. At the top of the picture, the echelon effect discussed before is quite pronounced.

Aerial Photograph of Swash Inlet (Figure 6.5)

A second photograph taken in the same way as the first shows a wave pattern at Swash Inlet, N. C. Here the coastline is relatively straight and undisturbed, and the depth contours are parallel to the coast out to a great distance from the coast. The strong crossover effect seen at Ocracoke is lacking. There are two wave trains. One

FIGURE 6.5 Aerial photograph of Swash Inlet.

is a long "wave length," high "period" swell approaching the coastline in deep water from south-southeast. Superimposed on the long swell a low "period" local chop moving almost directly north is clearly visible. The angle between the local chop and the swell is about 30° in the deepest water at the far right of the photograph. The swell could have been formed by a storm many hundreds of miles to the southeast. The chop, has been generated locally probably by wind with a velocity of 15 or 20 knots.

Following the local chop in the photograph from right to left, one

can see that the direction toward which it is traveling is very nearly the same at the center of the photograph as on the far right. This is because the water is still so deep at the center of the photograph that the very short "wave length" waves have not refracted. They are not bent even half way toward the coast from the center of the picture, where the depths must be less. It is necessary to look at a point about one-quarter of the way from the coast on the left side of the photograph to see where the crests of the local chop become curved, and then they curve rather sharply and travel toward the shore.

The swell, which is much higher in its average "period" and which has a much longer "wave length," is refracted in much deeper water than the low local chop. The angle between the crests and the coast is much smaller near the center of the photograph than on the far right. It is evident that the crests continually become more nearly parallel to the shore as they travel from deep to shallow water. This means that the effect of refraction on these longer "wave lengths" is such that it is operating continuously over the entire area of the photograph.

Another interesting feature is the length of the crests of the swell. On the far right they are quite short; near the center they grow to considerable length; and then near the coast the crests can be followed a considerable distance before they become irregular and tend to disappear. This is a very important feature. The waves frequently appear more short crested in deep water and long crested near the coast owing to the effect of refraction.

Enlargement Over Ocracoke (Figure 6.6)

Figure 6.6 is an enlargement of a portion of figure 6.4. This enlargement was selected from the top part of figure 6.4 to illustrate the short crestedness of actual ocean waves. It also shows a very pronounced example of the echelon effect, some effect of low local chop on top of the longer swell, and the behavior of the waves as they approach the coast and begin to form breakers.

Enlargement Over Ocracoke (Figure 6.7)

Figure 6.7 is another enlargement of the area over the ridge in figure 6.4. It shows the extreme complexity of the waves over such an area and the resultant crisscross pattern. The water over this ridge is so shoal that the individual wave peaks are short and steep, and the intervening troughs are long and shallow. The interference pattern over the ridge is different from that at the upper left of the same figure. The crosshatch effect is not apparent in the upper left, and yet the waves there are formed by crossing of waves from east and south just as they are over the ridge. Such a ridge is a very dangerous place for small boats or landing craft because the waves

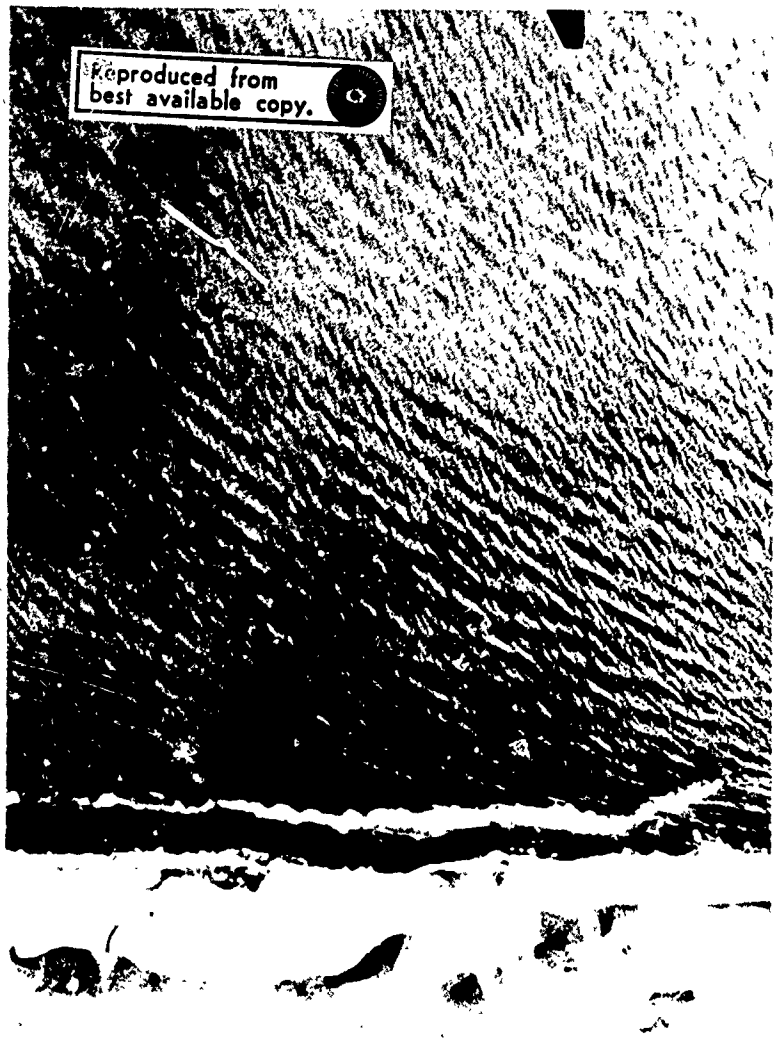

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FIGURE 6.6 Enlargement of Ocracoke.

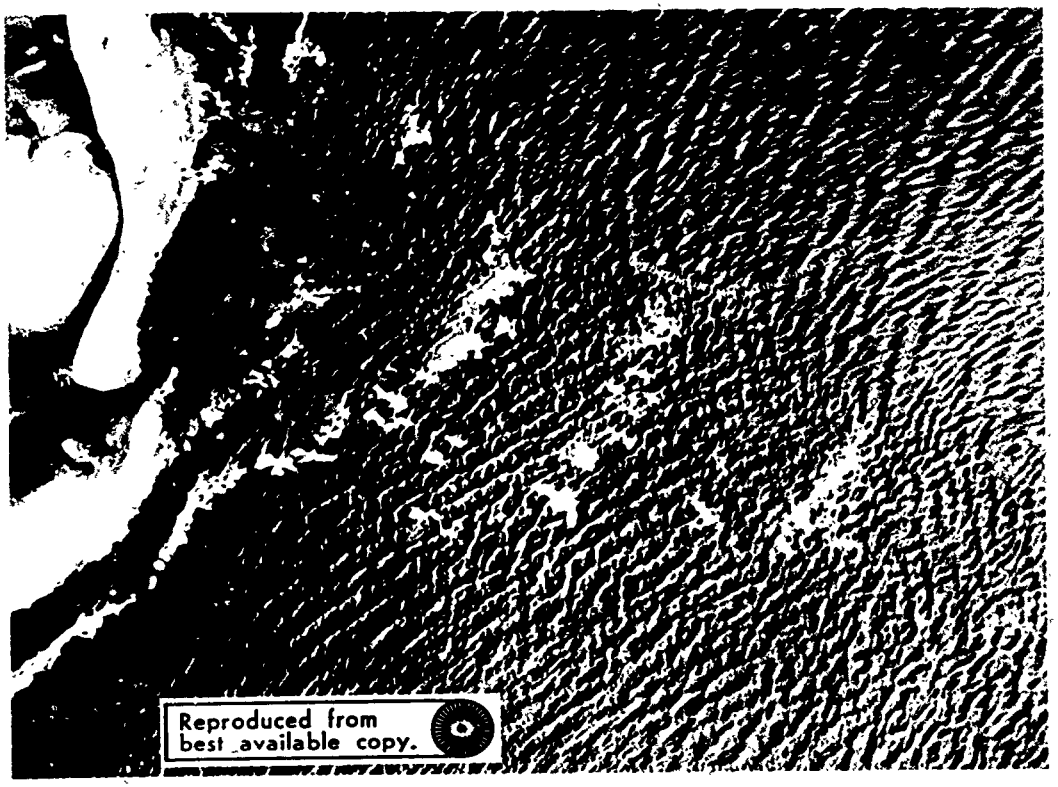

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FIGURE 6.7 Enlargement of Ocracoke.

attack the vessel from two different directions. First, a wave higher than average will advance from starboard and roll the boat to port and then another wave approaching from the port side, from a direction almost 90° (in this case) from the first, will swamp the craft while she still rolls from the first wave. The crisscross effect also tends to focus the energy along these ridges, and very steep high waves form as a result.

Inland Continuation of Swash Inlet (Figure 6.8)

Figure 6.8 shows part of figure 6.5 and the coast farther west at a larger scale. At the right edge the swell crests are at an angle of about 30° to the coastline. The crests of the low local chop are at an angle of about 80° to the coastline. As the swell approaches the shore, the crests become very high and peaked and the troughs become long and shallow. The swell crests take on a distinctly different appearance, which can best be visualized by looking at the photograph very carefully and imagining how the height of the waves would vary along a line perpendicular to the crests. Nearer the shore, the swell is peaking up and becoming ready to break. The waves are very unstable in this region, and the crests are very long and quite well defined along a sharp ridge. At the same time the low "period" local chop is still traveling just as if the swell were not there at all. It is moving in superimposed on the swell in a pattern that is evidently unaffected by the presence of the swell.

The fact that the low local chop, corresponding to a spectrum containing high frequencies, behaves independently from the swell and travels just as if the swell were not present, is a very important feature of the method that will be given later for forecasting wave heights in the refraction zone. This effect becomes visible in the photograph, but theoretical considerations indicate that, even for spectral frequencies separated by only a small amount, the individual sine waves represented by these spectral frequencies behave independently of each other as far as refraction is concerned. It then follows that it is possible to break up any spectrum such as those forecast in Chapter II or Chapter III into a sum of a great many sine waves, refract each sine wave independently, and arrive at a conclusion as to the height distribution and the characteristics of the waves after they have been refracted.

Computation of Refraction Effects

Introductory Remarks

The computation of refraction effects is difficult. So far in this manual, it has been possible to avoid carrying out an integration to find a required value of E. This cannot be done when waves are

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FIGURE 6.8 Inland continuation of Swash Inlet.

being refracted. So it is necessary to work with the actual spectrum of the waves instead of with the co-cumulative spectrum. The computations are very time-consuming. It is not possible to predict the effect of refraction by means of a computation which can be accomplished in 15 or 20 minutes, as can be done for a wave forecast prepared by the techniques given. Diagrams similar to figure 6.2 must be constructed for a complete range of deep-water frequencies and directions. If interest is concentrated at a certain point along the

coast and if the bottom is not too complex, only a few wave rays need to be drawn for each refraction diagram in order to straddle the points of major interest. This simplifies to a certain extent the amount of work that must be done. If a considerable length of a particular coastline is to be studied, then the work becomes even more complicated because a large number of refraction diagrams have to be constructed and the wave rays on each diagram must be drawn for an entire length of coastline, as in figure 6.2.

The computation of refraction effects should not be undertaken unless it is of extreme importance to know about them in a particular locality. To accomplish this job correctly often will require as much as 2 or 3 man-months in constructing refraction diagrams. Each forecast may require about 40 minutes of computation after the refraction diagrams are available. The importance of the tasks to be performed and the importance of the accuracy of the forecast of the height of the waves in the shallow water at the point of interest should be weighed very carefully against the amount of work necessary in order to do the task properly.

Steps To Be Followed

An overall outline of the steps that must be followed in preparing a forecast for wave heights and wave characteristics at a point in shallow water near a coast is given below. This outline will be gone over in great detail, and the steps will be described in the later sections of this chapter.

(1) Construct refraction diagrams for many different values of f and θ.

(2) Plot the refraction effect in polar coordinates in order to show the variations of two functions which will be called $[K(f,\theta)]^2$ (the refraction function) and $\Theta(f,\theta)$ (the direction function). These two functions will be described in what follows.

(3) Forecast the spectrum of the waves to be refracted. Locate the bounds of the spectrum, the upper and lower cutoff frequencies, and the values of θ_4 and θ_3. Divide this spectrum into an approximating sum of simple sine waves. Find those fractions of energy from the total energy of the whole spectrum that apply to small pieces of the spectrum and label them ΔE. Associated with each ΔE will be a central frequency, f, and a direction θ, which will describe the simple sine wave whose contribution to the total sum is characterized by these three numbers.

(4) Evaluate the following equations to obtain the directions and energies of each simple sine wave fraction of the spectrum:

$$\Delta E_h = [K(f,\theta)]^2 \Delta E \tag{6.2}$$

$$\theta_h = \Theta(f,\theta) \tag{6.3}$$

(5) Add up the ΔE_h values for all the individual sine waves and forecast the height characteristics of the waves at the point of interest in the shallow water. Interpret the range of θ_h as the crest length of the waves at the point in shallow water.

Each of the five steps will be described in detail in the text that follows. The first two steps need to be done only once for a given place and the resulting graphs can be used to prepare wave forecasts at that place for any wave spectrum present in deep water.

The Construction of Refraction Diagrams for a Simple Sine Wave

Introductory Remarks

The methods that will be given for the construction of refraction diagrams are taken from a paper by R. S. Arthur, W. H. Munk, and J. D. Isaacs (1952). This new method for the construction of refraction diagrams makes all earlier techniques obsolete. One of the earliest methods consisted in the location of the successive positions of the wave crests as the waves travel in toward the shore. This method leads to serious cumulative errors in the computation of effect of refraction. A later method constructed the wave rays (often called orthogonals), but this method also had errors in it if the depth changed rapidly. The new technique that will be described here eliminates these difficulties and gives solutions corresponding very closely to those obtained by exact mathematical methods.

Depth Chart

The first step is to obtain a bathymetric chart of the region near the point at which the forecast is to be made. It should show depth contours out to soundings greater than one-half wave length of the highest spectral *period* of interest in the construction of the refraction diagrams. The contours should smooth out because the minor irregularities are not very important in the effect of refraction. For the higher *periods*, the smoothing of the contours should be more extensive than for the shorter *periods*. It is advisable to take the original depth chart and smooth it considerably (over distances comparable to one *wave length*) before constructing the wave rays for the higher *periods* and to smooth it somewhat less before constructing the wave

rays for the shorter *periods*. The shorter waves are of course affected by a much narrower offshore zone of contours near the point of interest, and so the wave rays need not be constructed to such great depths in this case.

After the contours have been smoothed, it is advisable to use a semitransparent overlay such as rice paper on which the wave rays can be constructed. Fix the overlay in position on the chart by ticking the intersection of latitude and longitude lines, and make a set of these overlays for each spectral frequency and direction of interest.

The depth contours also represent lines of constant wave crest *speed* for a given frequency for the waves as they come in from deep water and approach the coast.

The ratio of the wave speed which the simple sine waves being refracted should have on each of these contours to the wave speed in deep water can be obtained readily from figure 6.9. Given the depth of the water in fathoms or in feet and the wave period, draw a line connecting these two values on the outer edges of this alignment chart. The intersection of this line with the center line on the alignment chart then yields the ratio of the wave speed at the depth indicated to the wave speed in deep water. In each zone bounded by two depth contours, the desired quantity is actually the ratio of the wave speed in the deeper water to the wave speed in the shallower water. Thus, for example, if the ratio of the wave speeds (C_1/C_d) at one contour in the deeper water is 0.60 and the ratio of the wave speeds (C_2/C_d) at the next contour in the shallower water is 0.50, then the ratio of the two wave speeds (C_1/C_2) is given by 0.60/0.50, or 1.20. It is necessary to compute these individual values of the ratios and enter them on the overlay for each successive pair of contours from deep water into the shallow water. These ratios are all that is needed to construct the wave rays, since the technique is essentially an application of Snell's law, with finite differences.

The Construction of a Wave Ray

As a wave ray corresponding to a certain discrete spectral frequency approaches the shore from deep water it curves until, theoretically, it is exactly perpendicular to the beach in zero depth of water at the beach. (Of course, the waves actually break in water of finite depth before this occurs.) For any change in depth, Snell's law determines the curvature of the wave ray. It must intersect a contour at an angle determined by Snell's law for the successive changes in depth. The tangent to the wave ray must make the angle, α, with a line perpendicular to the contour, C_1, at the point where the ray intersects the contour. Also, the wave ray must curve properly as it undergoes a change in depth. That is, Snell's law may be satisfied at a dis-

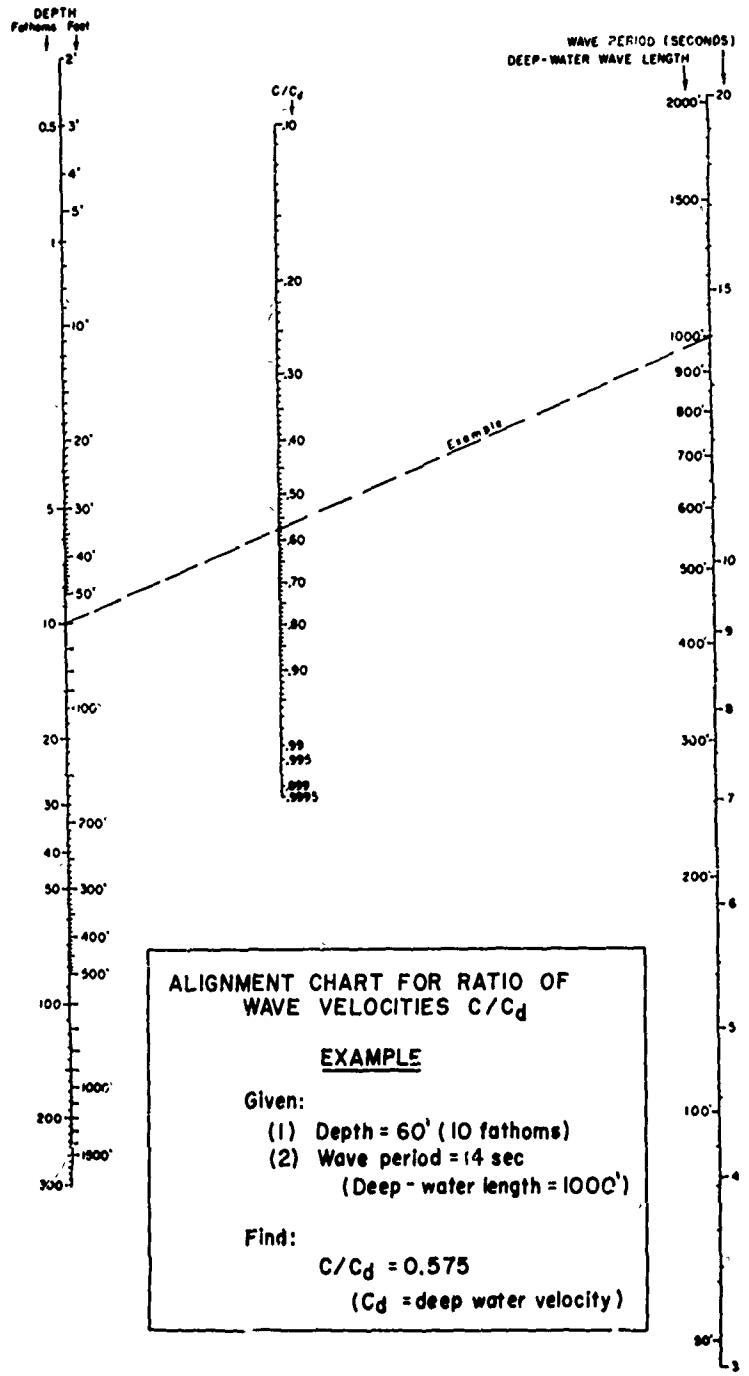

FIGURE 6.9 Alignment chart for wave velocity ratios (after Arthur, Munk, and Isaacs).

crete set of points given by the intersection of the ray with a certain set of contours; yet the ray can get out of alignment so that for a finer division of contours it will not be correct. The method given by Arthur, Munk, and Isaacs (1952) eliminates this difficulty by developing a way to keep the curvature of the ray correct as it goes from one depth contour to the next.

As the wave ray crosses the contour corresponding to the wave speed C_1, as shown in figure 6.10, the wave crest makes the angle α_1, with a line drawn parallel to the smoothed contour. Since the wave crest is continuously changing direction, it must make a new angle,

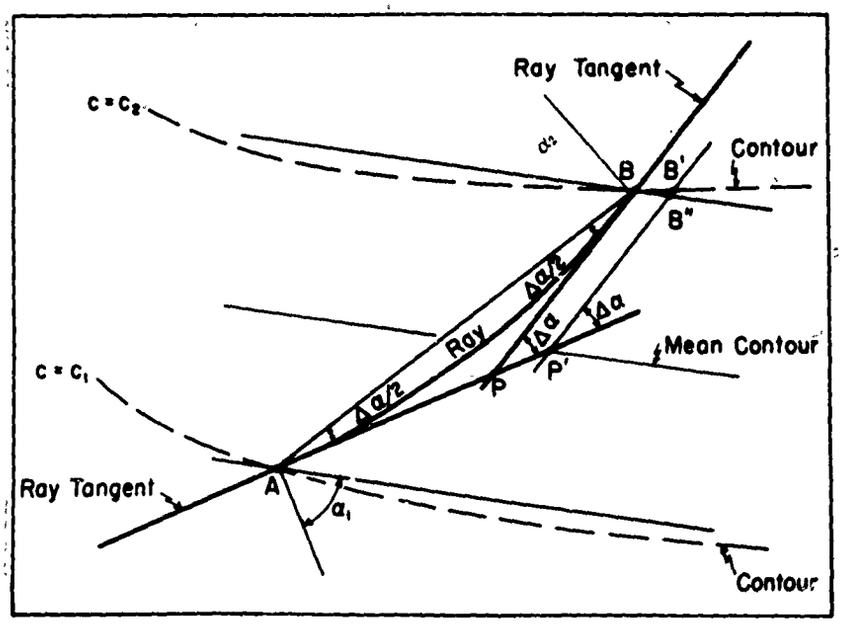

FIGURE 6.10 The ray tangents as the depth contours are crossed (after Arthur, Munk, and Isaacs).

α_2, with the smoothed contour corresponding to the wave speed, C_2, when it reaches that contour. The change in angle is $\Delta\alpha$. Then at the two contours corresponding to wave speeds, C_1 and C_2, Snell's law holds as defined in equation (6.1), since the wave crests intersect the contours at the correct angles. The important point of this construction method is that the two ray tangents shown are connected by an arc of a circle which determines the exact path of the wave ray from point A to point B. It is evident that if the ray tangent at point B be moved a little to the left or right, the equation given by Snell's law would still be satisfied at that contour. However, the curve connecting points A and B would no

longer be a true wave ray. This construction thus gives the correct shape of the wave ray as it connects two points located on two given depth contours.

Constructing Wave Rays by Graphical Methods

The technique for the actual construction of a wave ray is summarized in figure 6.11. The initial direction of the wave ray in deep water is determined by the deep-water direction of the waves, and when the wave ray crosses the first contour at which the depth is less than half the deep-water wave length, the construction begins

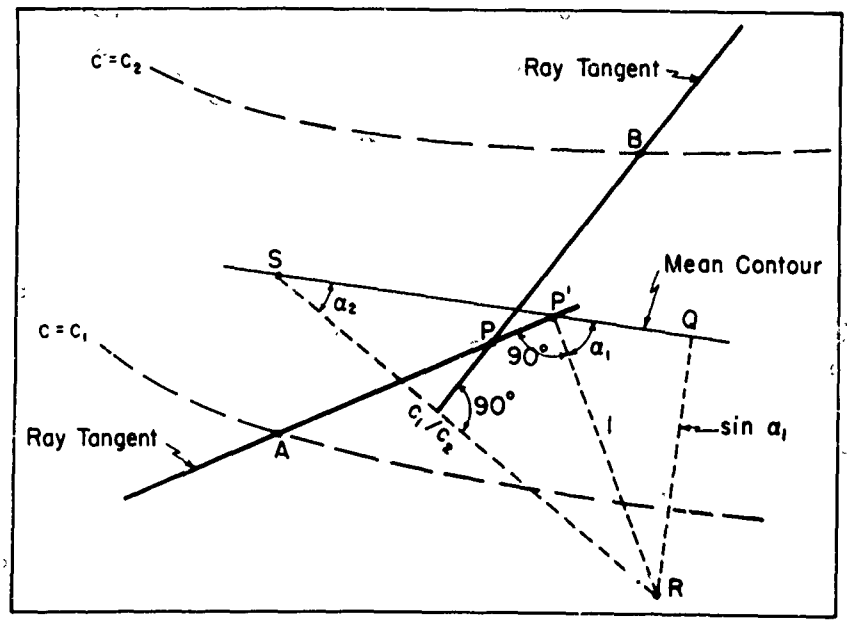

FIGURE 6.11 Method for construction of wave rays (after Arthur, Munk, and Isaacs).

according to the following rules. These yield ray tangents which cross each contour at the proper point and which make the wave ray a continuous arc of a circle between the two points.

(1) Establish point A, where the deep-water ray crosses the first contour. C_1 is the wave speed corresponding to this contour, and C_2 the speed corresponding to the second contour.

(2) Find C_1/C_d and C_2/C_d from figure 6.9. From them determine C_1/C_2 (i. e., C_1/C_d divided by C_2/C_d). C_1/C_2 is greater than one except where the ray is passing toward deeper water (e. g., after crossing a ridge).

(3) Draw by eye a mean contour for the interval.

(4) Extend the deep-water ray beyond point A until it intersects the mean contour at P′. To the seaward of point A, the line AP′ is the deep-water ray; shoreward it is the tangent to the ray at point A.

(5) At the point P′ construct a line perpendicular to the line AP′ and mark on it a point R, such that P′R has ana rbitrary length of one unit.

(6) With R as the center of a circle draw an arc with a radius of C_1/C_2 units of the same arbitrary scale. This arc intersects the mean contour at point S.

(7) Draw the line RS and on it erect a perpendicular such that (as judged by eye), it intersects AP′ at a point P equidistant from the two contours (i. e., AP=PB). The intersection of this perpendicular with the second contour at B is the point at which the wave ray crosses the next contour corresponding to wave speed C_2. Even if judged by eye the value will be sufficiently accurate. PB is now the tangent to the wave ray at point B, just as AP was the tangent at point A.

(8) To continue the ray, the next step involves the contour through the point B and the next contour toward the shore. The same procedure is repeated as described above except that now C_2 becomes C_1 for this new interval and C_2 corresponds to the new contour.

This procedure is continued until the wave ray crosses the depth contour of interest for the point of forecast. In this manner one wave ray can be constructed. It is necessary to construct a number of wave rays. Finally by good luck, an intelligent guess, or hard work, two wave rays will be constructed which straddle the point in shallow water where the wave forecast is to be made. They should not be too far apart when they straddle this point and it is sometimes necessary to construct additional wave rays between the first few in order to refine the calculations.

A Useful Ray Plotter

When a great many wave rays have to be constructed, it is advisable to make a plotter out of plastic as shown in figure 6.12, and then the various steps given above for the construction of a wave ray can be carried out quite easily. This ray plotter provides an arbitrary unit scale and gives an indication of the value of $\Delta\alpha$.

The Construction of a Family of Wave Rays

The wave rays constructed by this method give the pattern that the wave crests must follow as they proceed into shallow water. In fact, the wave crests are perpendicular to these rays everywhere. Thus, as a wave crosses a point at which a forecast is to be made

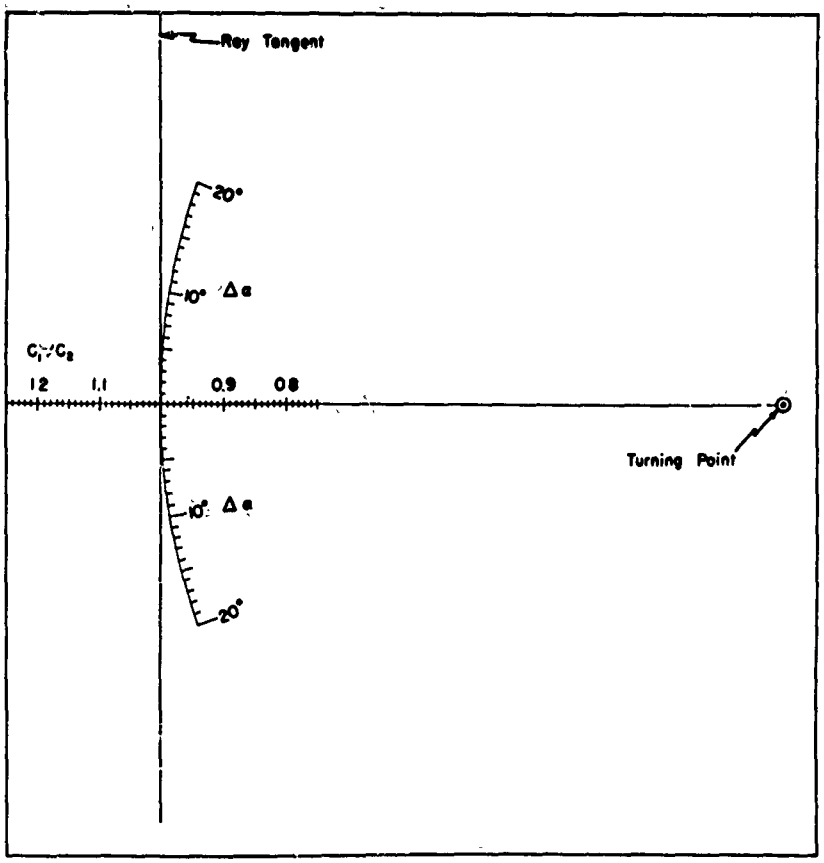

FIGURE 6.12 A useful ray plotter (after Arthur, Munk, and Isaacs).

in shallow water, it tells the direction toward which that particular spectral component will be traveling. The separation of the rays is an indication of the height of the waves. Where the rays converge the waves are higher; where the rays spread out the waves are lower.

Occasionally, it happens that the wave rays which start in deep water and finally arrive at the point of forecast can undergo some involved changes. In fact, it is possible for wave rays from two different points in deep water to cross the same point in shallow water. For example, figure 6.4 shows crossed wave crests, and since the wave rays are everywhere perpendicular to the crests, they must also cross each other over the ridge. When this occurs, it means that the original infinitely long wave crest (compared with the dimensions of the problem) in deep water has broken into a number of pieces and that these individual pieces are crossing the points at which the forecast is to be made. When wave rays cross each other, very com-

plicated things occur and the exact effects are not completely known; but if the laws of geometrical optics can be applied to this problem, certain rules can be given which will make it possible to estimate the effect of crossed wave rays. Care must be taken that a sufficiently dense network of wave rays is constructed so that it is possible to be sure that energy from other points in deep water cannot reach the point for which the forecast is being made.

The Determination of $[K(f,\theta)]^2$ and $\Theta(f,\theta)$

The Effect of Refraction on a Simple Sine Wave

After a given refraction diagram has been constructed, it is possible to determine the height that a simple sine wave will have at a given point in shallow water if the height of that sine wave is known in deep water and if the refraction diagram is available. Two effects combine to change the height of the wave at the point in shallow water: shoaling, which changes the rate at which energy is propagated toward the shore; and convergence of the wave rays. Under the assumptions of wave refraction theory, it is assumed that all wave energy is propagated along the wave rays and that no energy crosses rays. This assumption is valid if the scale of the contours is large compared to the *wave length*. Consequently, if the wave rays are closer together at a point in shallow water than in deep water, the waves must be higher per unit length of crest since the amount of energy between the two rays must remain the same.

Both these effects are combined in one formula for the function, $[K(f,\theta)]^2$, given by equation (6.4). It will later be used in equation (6.2) to forecast the changes in the spectral wave heights.

$$[K(f,\theta)]^2=\frac{(b_d/b)}{\left(\frac{C}{C_d}\right)+\frac{4\pi d}{L}\left[\frac{1}{\sinh\frac{4\pi d}{L}\frac{C_d}{C}}\right]} \tag{6.4}$$

If equation (6.4) is evaluated for a particular simple sine wave, the height at the point in shallow water can be forecast. Suppose, for example, the value of this function is 2.0 for a 10-second sine wave in deep water approaching the coast from some known angle. Then the wave height at the point near the coast for which this function has been evaluated will be equal to $\sqrt{2}$ times the wave height in deep water. If $[K(f,\theta)]^2$ is 0.01, the wave height in shallow water is one-tenth the deep-water wave height.

The refraction function given by equation (6.4) depends upon the ratio, C/C_d, the depth of the water, d, and the deep-water wave *length*,

L. It also depends on the ratio of two numbers, b_d and b. The denominator of equation (6.4) can be evaluated for any point in shallow water in terms of the ratio, C/C_d, and d and L. The ratio, C/C_d, can be obtained from figure 6.9. (Note that C_d/C equals $1/(C/C_d)$.) The value, $4\ d/L$, is given by equation (6.5), in terms of frequency, f.

$$\frac{4\pi d}{L}=\frac{4\pi d f^2}{5.12}=2.46df^2 \tag{6.5}$$

The numerator of equation (6.4) is b_d/b. The first quantity, b_d, is the perpendicular distance between the orthogonals in deep water, and b is the distance between the same two orthogonals as they straddle the point of interest in shallow water as shown in figure 6.13. The distance b should be measured in the shortest possible sense along a curved line perpendicular to both orthogonals at the point of interest in shallow water. If both b_d and b are measured with the same scale, any units can be used. The depth contours are not shown in figure 6.13. Whatever their configuration is, they are definitely not parallel.

A value greater than one for the ratio b_d/b means convergence of the wave rays and greater wave heights; a value less than one means divergence and lower wave heights in shallow water. Since the denominator of the function in equation (6.4) does not vary with direction, the numerator, b_d/b, determines the variation of the refraction function with direction at a given frequency. For the same frequency with different directions, b_d/b can be smaller than 0.01 for one direction and as high as 3.0 for another direction when the wave rays converge very strongly at a given point.

Thus, for each refraction diagram, after two wave rays have been constructed, that are fairly close together and straddle the point of interest in shallow water, the value of $[K(f,\theta)]^2$ can be determined for that particular frequency and direction of a simple sine wave in deep water. Now, this refraction function is a function of the deep-water frequency and direction of the waves; and if a large number of different deep-water frequencies and directions are taken, the values of the refraction function can be determined for a large number of points.

It then finally becomes possible to plot the values of this function on a polar coordinate diagram frequency, f as the radius and θ as the direction. When enough values are plotted, the diagram can be isoplethed, as in figure 6.15, and the result is the function used in equation (6.2) to forecast the changes of wave height. This will be demonstrated later. A considerable number of points is needed to make this function precise. This might entail using as many as eight or nine deep-water directions and five or six, or even ten, different frequencies. Thus as many as eighty wave refraction diagrams must be constructed

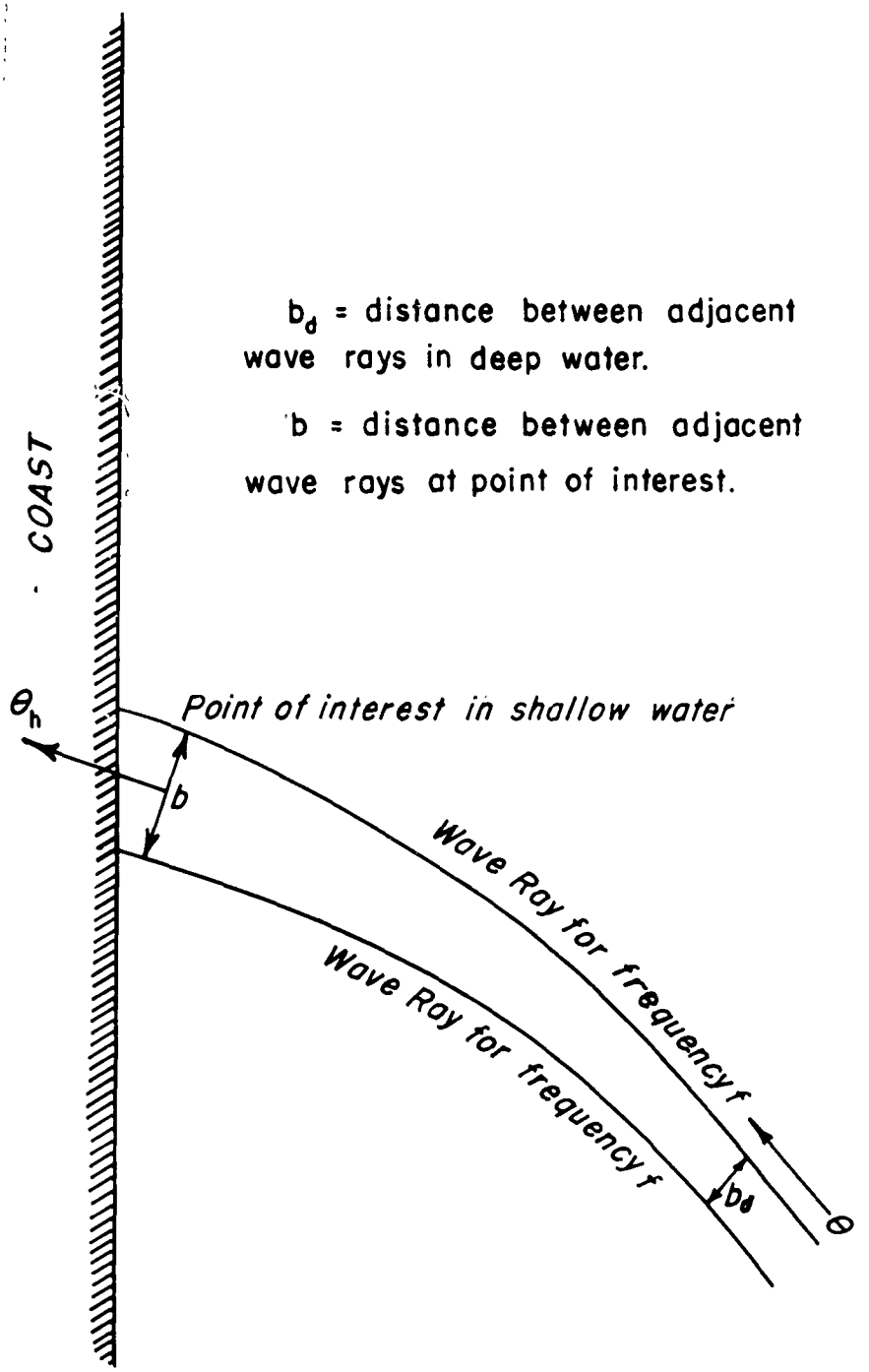

FIGURE 6.13 The determination of (b/b_d) and $\Theta(f, \theta)$.

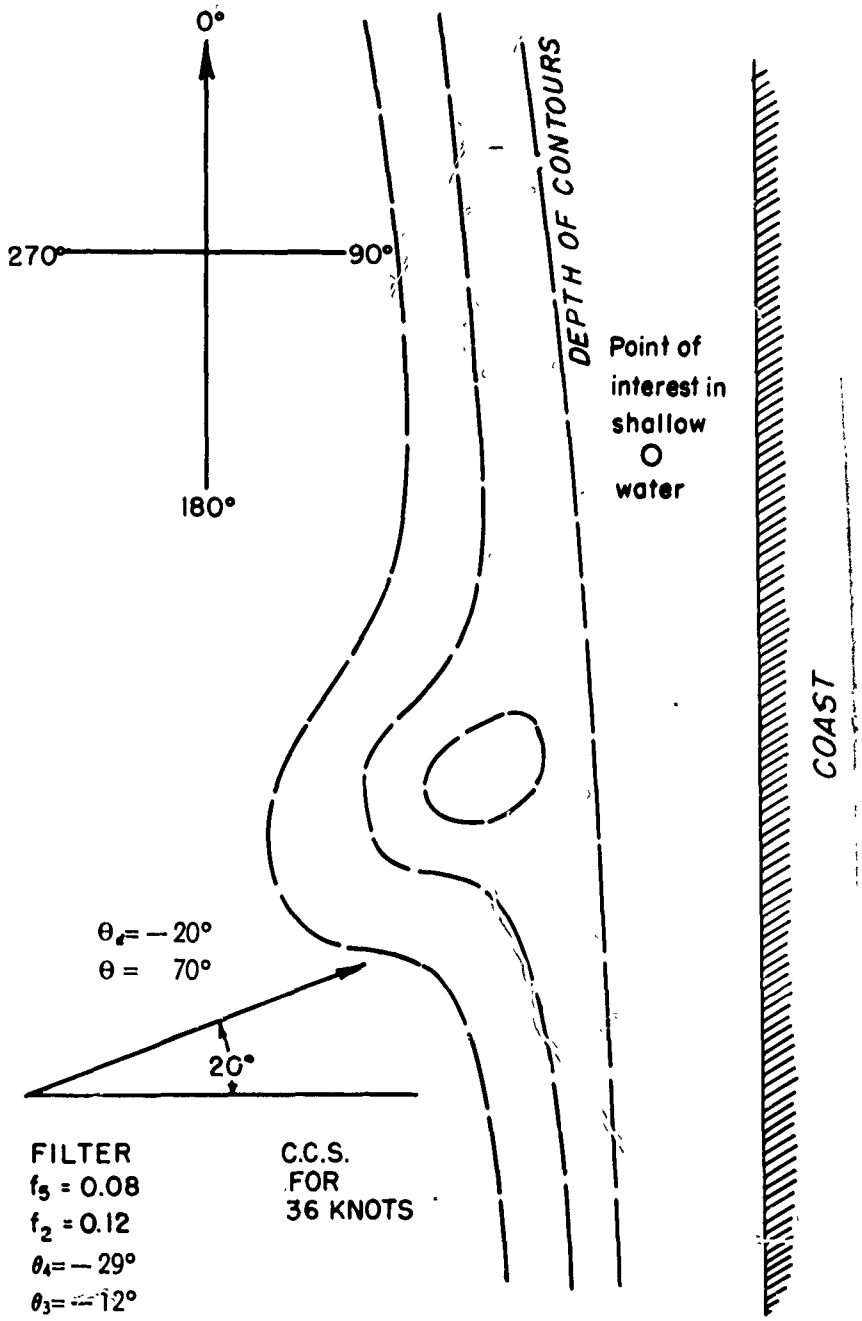

FIGURE 6.14 Conditions for a refraction problem.

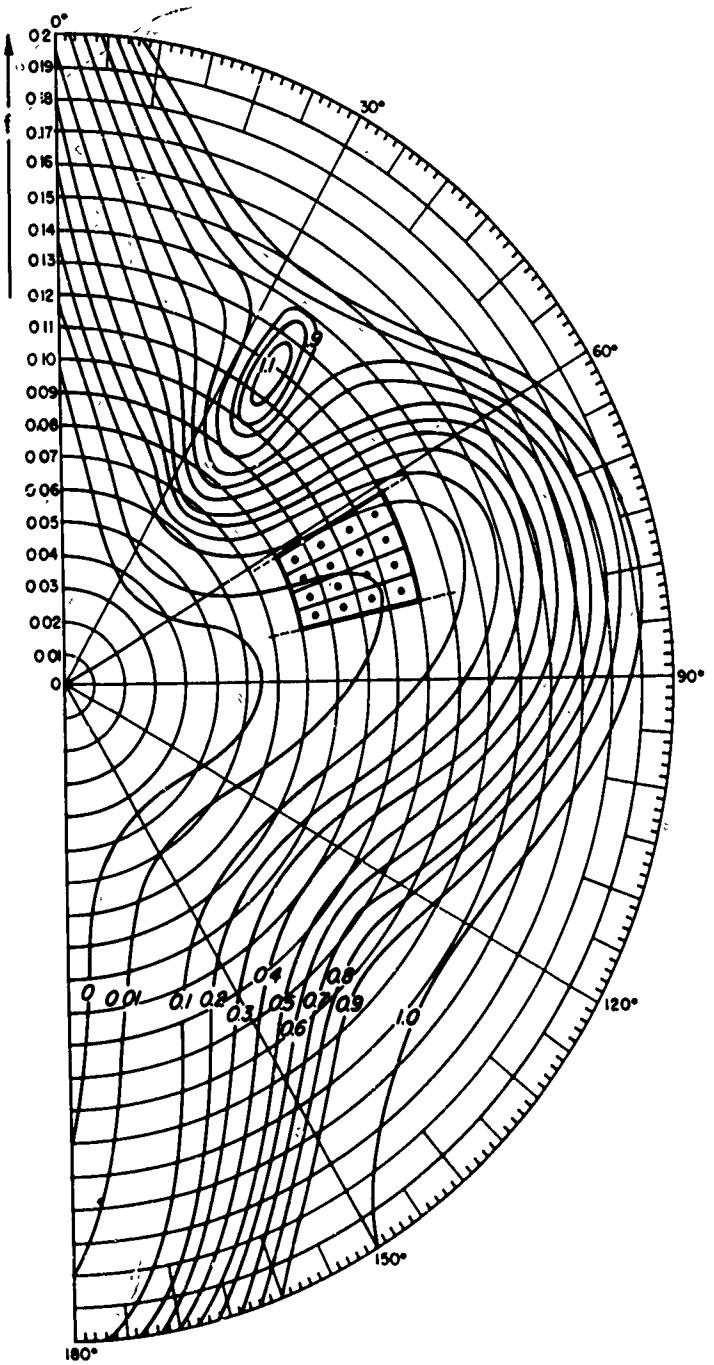

FIGURE 6.15 Hypothetical refraction function $[K(f,\theta)]^2$.

in order to compute and correctly determine the refraction function given in equations (6.2) and (6.4). Note that in equation (6.4), the refraction function is treated as if a particular combination of frequency and direction is being evaluated; in equation (6.2) the function is treated as if it were a continuously varying function of frequency and direction.

The Determination of the Direction Function

The waves in deep water have a direction given by the angle θ. In shallow water they have a new direction given by θ_h, as shown in figure 6.13. The direction of the wave rays is the same as the direction toward which the waves are traveling at each of the points, and thus the direction function can be determined from the refraction diagram at the same time that the refraction function is determined. The angle, θ_h, is a function of wave frequency and deep-water direction. Its angular values can also be plotted on another polar diagram in which the frequency is the radial variable and θ is the angular variable. The net result, then, is that the functions needed in equation (6.2) and (6.3) are known.

Crossed Wave Rays

If the wave rays cross during the construction of a refraction diagram, the rays involved should be continued shoreward as in figure 6.2a. Construct a number of individual rays sufficient to obtain a well-defined pattern.

If two rays cross that were not adjacent in deep water, it simply means that the original wave crest in deep water is broken into pieces, and that two pieces of the crest cross over each other (as in figure 6.7) where the rays cross.

If two adjacent wave rays cross, equation 6.4 states that the wave will be infinitely high at the point of crossing since b is zero and b_d/b is therefore infinite. However, the construction of additional wave rays between the two that crossed usually results in a new location of the points of crossing of adjacent rays. Compare, for example, rays E, F, and G in figure 6.2a. No finite amount of wave energy is ever really focused at a point. What usually results is an imperfect focusing which in optical theory is called a caustic curve. By constructing a great many rays, it is possible to see that the points of crossing of adjacent rays form a set of points on the refraction diagram which can be connected by a curved line to determine the caustic curve.

As a wave crest approaches a caustic curve, it breaks into three pieces. Two pieces simply overlap owing to the crossing over of nonadjacent rays. A short third piece is produced between the crossing adjacent rays. The wave crests do not appear to become very high as

they approach the quasi-focus of the caustic, and no energy appears to be lost by the wave due to breaking.

The net result is that at points near the shore (of interest after the crests have passed through a caustic in deeper water), three values of $[K(f,\theta)]^2$ need to be determined instead of just one as in the simpler case.

This effect is illustrated in figure 6.2a, where rays B, C, and D cross each other. Three separate pieces of the original unbroken crest in deep water are approaching a portion of the coast. Suppose that the point marked by the dot in figure 6.2a is the one at which refraction is being studied. Then for the southernmost piece of a crest b_d/b would be determined from the distance from ray A to ray B over the point marked and from the distance from ray A to ray B in deep water. For the northernmost crest, the corresponding distances from ray D to ray E would be used. (Note that these rays diverge greatly so that the northernmost crest will be exceptionally low.) The ratio, b_d/b, for the short piece of crest would be determined from the appropriate distances between rays C and D.

The polar coordinate plot then becomes more complex since for certain frequencies and directions, three values of $[K(f,\theta)]^2$ and three values of $\Theta(f,\theta)$ occur at one point on the diagram. The best procedure appears to be to make an overlapping analysis of the $[K(f,\theta)]^2$ values associated with the two parts of the original deep-water crest which did not go through a quasi-focus, i. e., the crest segment associated with rays D and E in figure 6.2a. The two pieces of analysis then continue out into the single-valued portion of the refraction function quite easily. The third set of values associated with the crossed rays can be analyzed on a separate chart. It would appear that $[K(f,\theta)]^2$ should not be permitted to be greater than 2.0 near a caustic.

General Discussion of the Refraction and Direction Functions

The functions just described can vary markedly from point to point along the coast. They can also act in a highly selective manner by suppressing some frequencies in the original deep-water wave spectrum and amplifying others. For example, the refraction diagrams for the northern New Jersey coast actually show that only the high frequencies can reach Sandy Hook. The low frequencies, as they come into Asbury Park are amplified by a factor of two in amplitude, whereas the low frequencies which attempt to come into Sandy Hook are decreased by a factor of ten. Thus, a simple sine wave in deep water, with a certain critical direction, if it had a high period of 14 seconds or so, and a deep-water height of 5 feet, would have a height of 0.5 at Sandy Hook and 10 feet at Asbury Park.

Once these two functions have been determined, they can be used

every time in forecasting the waves at the point of interest in shallow water. If the phase of the work described is accomplished once for a particular location, it need not be repeated. It is, therefore, advisable to take considerable care in the preparation of the refraction diagrams and in the determination of these two functions so that reliance can be placed on any forecast that will be made with the use of these functions at a later time.

The Refraction of a Particular Spectrum

By the methods given in Chapters II and III, it is possible to forecast the cumulative spectrum of a sea at a point in deep water offshore from the point in shallow water at which the forecast is to be made. It is also possible to forecast the frequency and direction bands present in a filtered spectrum if a swell is present offshore from the point at which the forecast is to be made. In either case, since the co-cumulative spectrum contains all the information about the original wave spectrum that the spectrum will contain (either with or without the application of a filter), it is then possible to operate on the forecast spectrum in deep water in order to forecast the spectrum in shallow water.

The refraction is accomplished by an approximation similar to that which was used in Chapter I, figure 1.7, to describe the waves, or to the one used in Chapter II to show how the wave spectrum was derived. The spectrum that really represents an infinite sum of infinitesimally small sine waves is broken up in such a way that it represents a finite sum of small sine waves of approximately equal importance. The actual number to be used depends on the shapes of the refraction and direction functions. Suppose, however, that a forecast for a given sea condition is to be made. Then by the use of the co-cumulative spectrum for that particular sea condition and the angular spreading factor as given in Chapter III, figure 3.17, it is possible to approximate the continuous spectrum by a sum of sine waves.

This approximation is found by marking off the frequency range into a number of pieces. At the center of each of these frequency bands, the center frequency can be used to represent the frequency of a simple sine wave, and the difference between the E value at the first frequency and the E value at the second frequency is the E value associated with that sine wave. This procedure gives the correct values for the frequencies, but it is also necessary to have the correct values for the directions. Thus, each of these selected frequencies should be broken up into directions, and by the use of the angular spreading factor, a certain percentage energy for each frequency can

be assigned to each different direction. The result, after this operation is carried out, is that the spectrum of the waves will be represented by a number of simple sine waves, possibly with as many as ten different frequencies and six different directions or sixty individual waves. Each sine wave will have a small fraction of the total value of E associated with the complete spectrum assigned to it. These are the values designated by ΔE in equation (6.2). A two-way table, with, say, frequency in vertical columns under each direction, can be made up and the ΔE values for each frequency and direction can be assigned in this table. Note that the sum of all these values of ΔE should be equal to the total E for the forecast spectrum in the deep water.

The next step is to go to the refraction function, read off the value of $[K(f, \theta)]^2$, and tabulate the values for the appropriate frequencies and directions in the first step. A third table then tabulates the values of ΔE_h for each of the values of ΔE in the first table by multiplying ΔE by the appropriate value from the second table. When these values are added, the result is a new value of E equal to the value of E_h of the waves at the point of interest in shallow water.

If the refraction function varies very rapidly from one frequency to another, then the values of ΔE should be made quite small and a large number should be taken. If the refraction function is relatively flat over a large range of directions and frequencies, larger values of ΔE can be taken. The net effect is to approximate the infinite sum of sine waves in the spectrum by an appropriately chosen finite sum of sine waves and to refract each sine wave as if it were a true simple harmonic progressive wave according to the theory which has been described. Once the value of E_h is forecast, the height-distribution of the waves in shallow water can then be forecast, since the significant height, the average height, the average of the one-tenth highest waves, and so on are all obtained from this one number.

If the spectrum is greatly distorted by the effect of refraction, some frequencies will be suppressed and other frequencies will be amplified. Then the average "period", $\widetilde{T}$, will be quite different in shallow water from the value it had in deep water. No convenient formulas can be given to forecast this value of $\widetilde{T}$ in a simple way but by inspection, a suitable estimate of the average "period" can be made by noting that it will be slightly lower than the "period" of the center of mass of the new spectrum as a function of frequency. If the spectral components near 10 seconds in a sea with an average "period" of 5 seconds are amplified by a very great amount, then at the point of interest in shallow water the average "period" might be 8 seconds or so owing to this effect. The average "period" does

not remain constant, and it is not sufficient simply to forecast the average "period" in deep water and then to forecast it to be the same at a point in shallow water. The narrowing of the spectrum is a measure of the lengthening of the wave crests; the narrower the angular variability in the shallow water region and the closer the values of θ_h for each of the simple sine waves in the partial sum that represents the spectrum, the longer crested the waves will appear to be at this point. At the present time, it is not possible to give precise formulas for this effect, but it can be estimated qualitatively from the function defined in equation (6.3).

Alignment Problems

In the forecasting of waves discussed in earlier chapters, the angles θ_3 and θ_4 were defined in relation to the wind direction in the fetch. Waves in refraction theory, however, must be referred to a fixed system of angles with respect to the compass; that is, north should be 0°, east should be 90° and so on. This means that, in forecasting the spectrum at the point offshore from the point of interest in shallow water, it is necessary to be sure that the angles in the refraction theory are defined properly.

To facilitate this, two new angles will be defined, Θ_d and Θ. The angle between the wind direction in the fetch and a line drawn to the forecast point from the center of the windward edge of the fetch is Θ_d. It lies between the values for θ_3 and θ_4. The angle Θ is Θ_d measured with respect to north; thus it gives the geographical direction of the waves.

With these new definitions it will be easy to determine the absolute direction in which each given spectral component will be traveling, and therefore not difficult to line up the various simple sine waves properly in deep water so that they can be refracted.

An Example of the Refraction of a Wave Spectrum

A Hypothetical Example

Figure 6.14 illustrates a situation in which wave refraction should be an important factor in forecasting wave heights. This is a hypothetical problem and does not apply to any particular location. Figure 6.14 shows the coastline and the point of interest in shallow water at which a forecast is to be made. It also shows a number of depth contours as they might appear on a hydrographic chart. The depth contours are fairly regular except for an area slightly to the south of the point of interest. There they bulge out in an underwater ridge with a little knoll on top. This underwater ridge affects the refraction values at the point of interest in shallow water.

Also shown on the chart is the convention for wave direction used in refraction theory, with north at 0°. An angle of 0° tells that the spectral component is traveling toward due north; an angle of 90° due east, and so on.

It will be assumed in this example that the wave spectrum was originally generated by a 36-knot wind so that the co-cumulative spectrum for 36 knots will be used. The angular filter values for this particular time of forecast are given by $\theta_3 = -12°$ and $\theta_4 = -29°$. Thus Θ_d falls between 12° and 29° and is −20° in this case. The wind direction in the storm is assumed to be due east, so that Θ is 70°.

The Hypothetical Refraction Function

A refraction function has been assumed for the point of interest in shallow water. It is given by figure 6.15. It is a realistic refraction function in that waves approaching the coastline traveling in an absolute direction toward 70° or 80° will spread out and become lower as they pass just to the north of the underwater ridge. Waves traveling more toward directions of 30° and 35° will be focused by this underwater ridge and the little knoll so that when they arrive at the point of interest they will be amplified. Waves approaching this point from the west or any direction north of west will be affected only by parallel depth contours. This refraction diagram is drawn up according to these ideas. The isopleth value of zero near the origin means that theoretically, at least, no low-frequency energy can possibly reach the shore. For the higher frequencies, the value of the refraction function becomes 1.0. This means that the water is so deep at the point of interest compared with the frequencies involved that the waves are essentially unrefracted. The refraction function equals one at a frequency of about 0.2, which corresponds to a *period* of 5 seconds. A *period* of 5 seconds corresponds to a *wave length* of 125 feet, so that the point of interest is at a depth of about 65 feet.

The Spectrum of the Waves in Deep Water

The spectrum of the waves in deep water is determined from the forecasting filter for a point in deep water offshore from the point of interest in shallow water. The spectrum is given by either the values measured in angles with respect to the storm or in angles with respect to the coast. Also the frequency band is needed. Suppose that the frequency band present ranges from 0.08 to 0.12 for the particular swell present and that the θ_d band, as measured with respect to the storm, ranges from −29° to −12°.[1] In the absolute coordinate system of the refraction diagram given in figure 6.15 these angles range from 61° to 78°.

[1] θ_d = the angle between the direction of the various wave angular components and the axis of the fetch. It varies between θ_3 and θ_4. $\Theta_{abs} = \Theta_d$ measured with respect to north.

Since the frequency band is from 0.08 to 0.12 figure 2.4c, the C. C. S. diagram for 36 knots, yields 81.5 ft.^{2} for E at a frequency of 0.08 and 22 ft.^{2} for E at 0.12. The difference in the two values of E is 59.5 ft^2. Figure 3.17 gives 20.4 percent and 37 percent for the angular spreading factors for $-29°$ and $-12°$. Multiplying 59.5 ft.^{2} by 16.6 percent or 0.166 yields the E value for the waves in deep water just offshore from this point of interest as 9.88 ft^2. This gives a significant height in deep water of 8.9 feet and a "period" range from roughly 12 to 8 seconds, with a center value near 10 seconds.

The Decomposition of the Spectrum

Figure 6.15 shows a network of lines and points from the frequency, 0.08, to the frequency, 0.12, and from 61° to 78° superimposed upon the refraction function. This area is broken up into 16 subareas of approximately equal size, except that the ones at the top have a five-degree spread and all the others have a four-degree spread. The procedure is to break up this spectrum with a total E of 9.88 ft.^{2} into a set of small E values each of which applies to a narrow direction range and a narrow frequency range so that the refraction function for the narrow frequency range can be considered constant.

Decomposition into Directions

The first step is to breakup the spectrum into directions as in table 6.1. Table 6.1 tabulates this breakup of the spectrum into directions. The first row shows the angular coordinates measured with respect to the storm, and the second row shows they are converted to the absolute coordinate system on figure 6.14. The range of angles from $-29°$ to $-12°$ is broken up by nearly equal increments into $-29°$, $-24°$, $-20°$, $-16°$, and $-12°$. Midway between these values, the means of $-26.5°$, $-22°$, $-18°$, and $-14°$ are entered. Then the angular correction factors given in the third row are entered from figure 3.17. The fourth row is then the successive differences in these angular correction factors. The first value, 5.1 percent, is the angular correction factor which would apply, for example, to a θ_4 and a θ_3 of $-29°$ and $-24°$, respectively, because it is the difference between 20.47 percent and 25.57 percent. These percentages apply to the angles above them. For example, the angular spreading factor of 5.1 percent applies to the angle, $-26.5°$, as measured with respect to the storm or 63.5° as measured with respect to the coastline and the refraction function.[1] In table 6.1 the sum of the percentages, 16.6 percent, is a check on the computations of the individual values.

Decomposition Into Frequencies

The second step is to break up the spectrum into frequencies, table 6.2. The frequency range of 0.08 to 0.12 is broken up in the

[1] Note that the θ_d angles increase as the θ (absolute) angles decrease. In mathematical theory angles usually increase positively in a counterclockwise direction, but on the modern mariner's compass they increase positively in a clockwise direction.

Table 6.1—The Decomposition of the Spectrum Into Directions

θ_d	−29°	−26.5°	−24°	−22°	−20°	−18°	−16°	−14°	−12°
$\theta_{(abs.)}$	61°	63.5°	66°	68°	70°	72°	74°	76°	78°
%	20.4%		25.5%		28.5%		32.5%		37.0%
Difference		5.1%		3.0%		4.0%		4.5%	

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Table 6.2—The Decomposition of the Spectrum Into Frequencies

f	.08	.085	.09	.095	.10	.105	.11	.115	.12		
$E(f)$	81.5		60		43		30		22[81.5−22=59.5]		
Difference		21.5		17		13		8	[Sum=59.5]	Check	
					59.5	16.6%=9.88					

top row of this table. The E values for 36 knots are then read off the appropriate co-cumulative spectrum, and the successive differences are entered in the last row. For example, the difference between the E values associated with the frequencies, 0.09 and 0.10, is 17 ft^2. Note that the total of the bottom row is equal to 59.5, a check of the accuracy of this computation. The difference values apply to the frequencies midway between the two values for which $E(f)$ has been read off. For example, in table 6.2 the frequency 0.105 has a difference value assigned to it of 13 ft^2.

The Total Decomposition

The E values given in table 6.2 have now been spread out over different frequencies. They also need to be spread out over different directions. This is accomplished by taking the E value assigned to frequencies between 0.08 and 0.09 and multiplying it by each of the percentages in table 6.1. The result is then entered into a two-way table (6.3), which shows the total decomposition into the E values. The number 1.110, for example, is the product of 21.5 ft.^{2} and 5.11 percent. The value, 0.360, in the lower right-hand corner of the table is the product of 8 ft.^{2} (associated with the frequency 0.115) and the value 4.5 percent which comes from table 6.1 under the angle, 76°. Thus, in table 6.3 the total energy of 9.83 ft.^{2}, which is associated with the whole swell wave system, is broken up into 16 smaller values of the energy, ranging from 1.110 down to 0.360. A check of the computations can be made at this point. The sum of all entries in table 6.3 should equal the value computed in table 6.2, namely, 9.88.

Table 6.3—Total Decomposition into ΔE Values

$\theta_{(abs)}$		63.5°	68°	72°	76°
θ_d		−26.5°	−22°	−18°	−14°
f	.085	1.110	.645	.860	.970
f	.095	.870	.510	.680	.770
f	.105	.660	.390	.520	.585
f	.115	.410	.240	.320	.360
			Sum=9.83		

It adds up to 9.83 ft.^{2} and thus shows that the computations are correct to two significant figures.

The Refraction of the Sine Waves in the Sum

The amount of energy assigned to each frequency and direction in table 6.3 now covers only a relatively small range of frequencies and directions. This energy can be assumed to be assigned to a simple harmonic progressive wave traveling toward the coast with that direction and frequency. Under this assumption, then, each value for E can be refracted individually according to the laws of wave refraction just as if it applied to the square of the height of a simple sine wave. The next step is to evaluate the refraction function, $[K(f,\theta)]^2$, for each point given by these energy sections. The necessary values of the refraction function are shown on figure 6.15 by the dots at the center of each piece of the superimposed grid. Table 6.4 gives the values of $[K(f,\theta)]^2$ at the appropriate points.

Table 6.4—Values of $[K(f,\theta)]^2$

	$\theta_{(abs.)}$	63.5°	68°	72°	76°
	θ_d	−26.5°	−22°	−18°	−14°
f	.085	.06	.01	.008	.005
f	.095	.08	.03	.008	.007
f	.105	.08	.04	.01	.009
f	.115	.09	.06	.03	.03

The Computation of ΔE_h

The computation of the various values of ΔE_h is the last step that needs to be carried out in wave refraction. Each value in table 6.4 is multiplied by the corresponding entry in table 6.3, and the result is the effect of refraction on each elemental sine wave in the sum of the 16 sine waves. When this is done, the numbers appearing in table 6.5 are the result. They are the values of ΔE_h that apply to the point of interest in the shallow-water zone. These 16 numbers, in this particular example, are then added up to find the total E_h at this point of interest, and the sum is 0.331 ft^2. The square root of this value of E_h is 0.575; therefore the significant height of the waves at the point of interest in shallow water is 1.61 feet.

Comments on the Example

In addition to calculating the significant height at the point of interest, one may obtain the direction toward which the waves are traveling by evaluating the function $\Theta\ (f,\theta)$. The waves will be seen to be heading more nearly into the beach at this point than they were

Table 6.5—Values of ΔE_h

$\theta_{(abs.)}$		63.5°	68°	72°	76°
θ_d		−26.5°	−22°	−18°	−14°
f	.085	.0666	.0065	.0069	.0049
f	.095	.0696	.0153	.0054	.0054
f	.105	.0528	.0156	.0052	.0053
f	.115	.0369	.0144	.0096	.0108

Sum $= E_h = .331$
$\sqrt{E_h} = .575$
Significant height = 1.61 feet.

in deep water, and the angular band width of the spectrum will be narrower than it was in deep water.

It is a very grave mistake to refract ocean waves as if they had one "period" and just one significant height. In this particular example, suppose that the waves were assigned a "period" of 10 seconds and a direction of 70°. Then the value of the refraction function will be approximately 0.01, and the forecast height at the point of interest in shallow water will be 0.87 feet instead of 1.61 feet as obtained by the correct method. The wider the angular spread of the spectrum and the wider the range of frequencies present, the more in error will be the refraction of the waves by one height and one "period." It is left to the forecaster to carry out several other examples of refraction. It is very interesting in this particular example to see what happens when the spectrum used is turned through about 20° toward a more northerly direction so that the center of the spectrum lies on a line given by $\theta = 35°$ in the absolute coordinate system.

Bottom Friction and Percolation

Not only are the waves refracted and thus changed in height as they come in toward the shore, but two other effects appear to be of great importance in forecasting waves in the shallow water zone. They result from bottom friction and percolation. As the waves travel in over shallow water they lose considerable energy because of the stress of the moving water against the bottom of the ocean and this causes the waves to die down in height. If the waves are moving over a sandy bed, they tend to set up currents inside the sand which are very rapidly attenuated by the effects of friction. This effect is called percolation. Percolation can therefore also decrease the height of the waves. In both cases the effect is to make the waves lower than

the refraction function forecasts them to be at a point in shallow water. However, it is not possible to go into the theory of the effects of friction and percolation on actual ocean waves at the present time because of the great complexity of the theory of these effects on even simple sine waves.

Final Comments on Refraction Theory

From the results of the above example and from the description of the work necessary to carry out the refraction of an actual storm spectrum correctly, it is evident that there is a great amount of work involved. In many cases the experienced forecaster should be able to forecast the wave conditions at a point of interest in shallow water by taking these effects into account qualitatively. Photographs and the descriptions given at the start of this chapter may be sufficient aids in a great many cases. The one thing *not* to do, however, is to attempt to refract the waves by using just one "period". This can introduce tremendous errors and the forecast heights, because of this one error alone, can be wrong by 50 percent or 100 percent. Attempts have been made to correlate deep-water wave forecasts with observations made in shallow water, and this has been one point where big errors have been introduced. It has made it very difficult in many cases to verify wave forecasts because of the fact that the verifying observations have been made in quite shallow water.

Chapter VII

OPERATIONAL APPLICATIONS OF FORECASTING METHODS

Introduction

The success or failure of major naval operations and even of the daily work of the Navy and Coast Guard can depend on the accuracy of forecasts of sea and swell conditions and on the interpretation of the forecasts in terms of the tasks to be performed. The unceasing battle between ships and the sea affects the history of every nation that sends men to sea in ships. Not only are single ships lost in the fury of the sea, but there have been times in history when whole fleets have been lost in a storm.

During World War II, the 3rd Fleet encountered a typhoon in the Pacific. Three destroyers were capsized and lost with heavy personnel casualties. One hundred and forty-six planes were blown overboard or damaged on carrier decks. Thirteen ships required major repairs and nine others minor repairs. As much damage resulted from the heavy seas produced by the typhoon as from any major battle with the enemy. Throughout the war the cost and time lost in the repair of ships damaged by heavy seas were as great as for battle damage.

The effects of sea conditions upon naval operations are just as important as any other factors in planning and execution. Advances in ship design, weather forecasting and wave forecasting make it possible to extend the range of tasks that can be accomplished, but the weather and wave effects still often set the upper limit on capabilities. The present day naval task force is just as vulnerable to the sea under modern demands of performance as the Spanish Armada of earlier times was in the tasks it carried out.

The fast carrier task force of the modern Navy is a potent weapon because it can travel quickly over great distances to deliver its punch. A task force has many types of ships. When a heavy sea develops some ships must slow down; consequently, the whole task force is slowed down. Thus, the ability of the task force to deliver its punch is weakened. The ability to forecast the speed that a ship can

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make good in a seaway is thus a critical factor in determining whether the schedule of an entire naval operation can be met.

Amphibious landings also involve correct wave forecasts. The surf conditions are of critical importance both for the initial landings and for supplying the beachhead. The operations in Normandy in 1944 were seriously affected a few days after the initial landings by the fact that a heavy storm damaged some structures that had been towed to the coast and scuttled to form breakwaters and piers. The transportation of supplies from ships to the beach by various types of cargo carriers was so hindered by the storm that the supply of munitions and equipment for the men on the beachheads was critically endangered.

Antisubmarine warfare is likewise vitally affected by the state of the sea. The background noise of breaking waves decreases the ability of sonar to detect a submarine. The scatter of radar waves on a rough sea and the consequent reflection pattern on a radarscope masks the presence of submarine snorkels and periscopes which might be easily detected if the sea were calm.

In Coast Guard search and rescue operations, a major problem is to forecast the state of the sea for the landing of a seaplane on the open ocean. This is often the limiting factor because waves in excess of a rather low height make such landings impossible to carry out.

The routine operational value of the procedures given in the past chapters depends on the ability of the forecaster not only to prepare accurate forecasts but also to state his forecast results so that they can be of value to those who are to use them. This last step, putting the forecast in a practical form, is a very important step. The wave forecaster should be able to carry out at least part of this problem so that his knowledge can be used.

The purpose of this chapter is to suggest ways in which the results of these wave forecasting methods can be stated in terms of the answers to practical operational problems. The following problems will be discussed:

1) Ship motions
2) Factors affecting the speed of ships
3) General operational problems
    a) Refueling
    b) Aircraft carrier operation in sea and swell
    c) Hurricanes
    d) Storm damage
4) Seaplane landings and takeoffs

The applications of wave refraction theory were discussed in Chapter VI so they will not be discussed again.

The wave forecaster who works daily on practical problems will

discover many other ways in which this manual can be used. Much still needs to be done to develop these practical aspects. Other factors, which cannot readily be solved theoretically enter into the problem. The demands of a situation often outweigh the results of the forecast so that in one situation for the same wave conditions a task would still be undertaken, whereas in another situation for exactly the same wave conditions the task might be postponed for another day or another time.

Ship Motions

In many cases, a forecaster, or the person who is to use the forecast, is not really interested in the properties of the waves themselves. He is actually interested in the application of this knowledge to a particular operational or practical problem. For example, if the response of a ship to a given wave system could be stated, after the wave system itself has been forecast, the captain would find the forecast of ship motion much more useful than the wave forecast alone. However, waves act on a ship in such a complicated way that ship motion forecasts are not yet completely feasible.

Kinds of Ship Motions

In perfectly calm water a carefully steered ship moves along a straight course. A point which corresponds to the center of gravity of the ship also moves along this straight line at a constant velocity. In a wave system, however, this point deviates from this simple, straight-line motion in six different ways:

1) HEAVE, the up-and-down motion of this point as it travels along.
2) SURGE, the fore-and-aft motion of this point as the ship speeds up and slows down when she encounters waves.
3) SWAY, the athwartships motion as the point departs from a straight-line path.
4) ROLL, the athwartships angular rotation about this point which occurs as ship heels first to one side and then to the other.
5) PITCH, the fore-and-aft angular rotation about this point which occurs as the bow and stern alternately rise and fall.
6) YAW, the horizontal angular rotation about this point which occurs as the direction of ship's keel is deflected from the direction of her course.

The first three motions are motions such that the center of gravity (approximately) actually departs from its straight-line motion. Heave is the most apparent of these three motions. The last three motions are motions such that the center of gravity does not depart from its position. Other points on the ship do move, however. For example, a point on the deck twenty feet to port of the center of gravity will move

down 3½ feet if the ship rolls 10° to port. Roll and pitch are the most apparent of these three motions.

A Ship in a Simple Sine Wave

One of the difficulties involved in forecasting the motion of a ship arises because the *period* of the motion of a ship is not equal to the *period* of the waves through which the ship is moving. To illustrate the complications which are involved, consider for example, a ship steaming along in a simple sine wave such as illustrated in figure 1.5, page 18. The simple sine wave has infinitely long crests, and every wave is the same in height; but it can travel in different directions with respect to the ship. Suppose, for example, that the simple sine wave has a *period* of 10 seconds. Then the crests of the waves are traveling at a speed of approximately 30 knots. Now suppose that the ship heads directly into the waves at 15 knots. Then the ship encounters the waves more frequently than once every 10 seconds. In fact, it will encounter a wave every 6.67 seconds so that the *period* of encounter is much lower than 10 seconds. Consider, also, the opposite condition where the waves are overtaking the ship from directly abaft. Then the speed of the waves relative to the ship is only 15 knots, and fewer waves encounter the ship in a given time interval. Under these conditions the *period* of encounter would be 20 seconds, which is twice the wave *period* of 10 seconds.

As the direction toward which the waves are traveling changes with respect to the direction toward which the ship is traveling and as the speed of the ship changes, all sorts of possibilities can result. Consider, for example, 5-second waves approaching a 15-knot ship from astern. Then, approximately, the crests of the waves are traveling at the same speed as the ship, and the ship will remain in a given trough or on a given crest, theoretically, forever. The frequency with which the ship encounters the waves under these conditions is, consequently, zero and the *period* of encounter is infinite.

A ship has natural *periods* for roll, pitch, and heave. These *periods* are different for each of the different types of motions. In a simple sine wave system, the response of the ship to certain *periods* of the waves is very great, because the wave *periods* coincide with her natural *periods* of motion. At wave *periods* much different from the natural ship *periods*, the ship motion response is much less. Thus as the direction of the waves with respect to the ship changes, different responses of the various motions occur with respect to the same sine wave.

The "Periods" of a Ship's Motion

Some large ships have a natural rolling *period* of approximately 20 seconds. Suppose that the ship is steaming on a given course at 10

knots and that waves with, say, a 9-second average "period" (speed 17 knots) are overtaking the ship from the quarter. Then it is quite possible for the average "period" of encounter of these waves to be near 20 seconds. The ship will be rolling back and forth with an average time interval between successive rolls to port of 20 seconds, and yet the waves will have an average "period" of only 9 seconds.

In a sea spectrum many frequencies and many directions are present. Since the responses of the various motions of the ship are at different *periods*, the ship can select the energy from the spectrum near its own *period* for each type of motion. The net result is that the ship can have one particular average rolling "period," a completely different average pitching "period," and still a third average heaving "period" in the same wave spectrum.

The Amplitudes of Ship Motions

The motions of a ship obey the same statistical law that was given in Chapter I for the properties of the waves. Ship motion records have the same appearance as the wave records in Chapter I. For example, a very interesting series of observations was made on the U.S.C.G.C. *Halfmoon* by Atlantic Weather Patrol personnel. The observations are of the amplitude of the rolling motion of the ship as measured by the inclinometer on the bridge, in degrees. One hundred values were tabulated, and the results are shown in table 7.1. For example, there were three cases in which the roll of the ship either from center to starboard or center to port was 2°. There were seven cases in which the roll was 3°. There were twelve cases in which the roll was 4°. The average amplitude of the rolling motion was 8°; if

Table 7.1—Roll Angle for 100 Observations

Inclinometer Roll Angle (degrees from zero)	Number of Cases	Cumulative Number	Inclinometer Roll Angle (degrees from zero)	Number of Cases	Cumulative Number
1	0	0	11	6	83
2	3	3	12	1	84
3	7	10	13	5	89
4	12	22	14	4	93
5	10	32	15	2	95
6	9	41	16	2	97
7	12	53	17	0	97
8	9	62	18	2	99
9	7	69	19	0	99
10	8	77	20	1	100

Sum = 795 $(2\times3)+(3\times7)+(4\times12)$ etc.

Average Roll Angle = 8°

Therefore $0.866\sqrt{E_s} = 8°$

$\sqrt{E_s} = 8°/0.866 = 9.24°$

this value of 8° is used to compute E, or something similar to it, say, E_s, for the ship's rolling motion, it is found that the square root of E_s is given by 8/(0.866) or 9.24°.

With the aid of the tables in Chapter I, the range of values such that 10 percent of the total number of observations will lie within given ranges can be predicted. For example, there should be ten observations between 0° and 2.96° according to the first entry in table 1.1. The value, 2.96°, is so close to 3° that the value 3° should be included in that first range, since the degrees are estimated only to the nearest whole number. Table 7.1 actually contains ten values equal to 3° or less for the rolling angle as tabulated in this series of observations. Table 7.2 shows the predicted number of rolls which should have the amplitude in degrees indicated on the left as compared to the observed number. The predicted values are all 10, since there were 100 observations. The observed values fluctuate slightly above and below the predicted value. For example, there should have been only 10 rolls of 4° and there were 12; hence there is an error of 2. The errors range from 0 to 4, and most of them are rather small. Of the 10 errors, 7 are 2 or less in a predicted value of 10.

The expected value of the greatest roll out of 100 on the basis of $\sqrt{E_s}=9.24°$ should be 21°. From table 1.3, 9 times out of 10 the highest roll out of 100 would be between 17° and 25°. One 20° roll was observed.

This is about as close as these numbers will come in the usual run of affairs, and the difference between the predicted and observed values for this particular example is due to the fact that only 100 observations were taken, and to the fact that the values can be read only to the nearest whole degree. A reading of 10°, for example, could actually be a value anywhere between 9.5° and 10.5°. This means that a little freedom has to be taken in column two of table 7.2 in assigning

Table 7.2—Predicted and Observed Values for the Rolling Angle of a Ship

Computed 10% Ranges	Nearest or most representative whole value of the angles	Predicted Number	Observed Number	Error
0°–2.96°	0° to 3°	10	10	0
2.96°–4.35°	4°	10	12	+2
4.35°–5.54°	5°	10	10	0
5.54°–6.55°	6°	10	9	−1
6.55°–7.65°	7°	10	12	+2
7.65°–8.89°	8°	10	9	−1
8.89°–10.2°	9°	10	7	−3
10.2°–11.8°	10° to 11°	10	14	+4
11.8°–14.0°	12° to 14°	10	10	0
14.0°–∞	15° or more	10	7	−3

the most representative whole value of the angles. A great many more accurate observations, which would take a much longer time, would give closer percentage agreement, and fewer observations would make it very difficult to check the theory because of the scatter in the values.

The rolling motion of a ship can become extreme in certain types of seaways. The behavior of the U. S. S. *Langley* (CVL-27) during an extreme roll is shown in figure 7.1. The ship was operating in a task force in the South China Sea when this photograph was taken.

Similar remarks can be made about any other motion of a ship although the problem, as stated above, is a very complicated one. Current theory indicates that the rolling motion, the heaving motion, and the pitching motion of a ship should all have the same theoretical type of distribution as given in Chapter I, and the motions should all be governed by the same mathematical laws of probability and chance. The amplitudes of the motion of a point on the deck of a ship can, therefore, be estimated, and they should have the properties described above.

Factors Affecting the Speed of Ships

A recent paper by E. V. Lewis of the Experimental Towing Tank at Stevens Institute of Technology has described the problem of maintaining the speed of a ship in storm seas. The paper, "The Influence of Sea Conditions on the Speed of Ships," is a very thorough summary of this problem. At the risk of repeating parts of the previous discussion, a section of this paper is quoted in the following pages. Where deletions have been made in the original text, dots (. . .) will be found in this text. Insertions by the authors of this manual are enclosed in brackets ([ ]).

"Let us consider first of all the direct effects of waves and winds on increasing the resistance. This added resistance results from the following:

(1) Wind resistance—the magnitude of which has been well established for different ship types,
(2) Wave reflection effects—the increased resistance caused by the distortion by the moving hull of the encountered waves,
(3) Effect of rolling—a factor which is comparatively small,
(4) Effect of heaving and pitching—an oscillating fore and aft force resulting from the motion being out of phase with the wave, this force having a net component aft. . . . This effect has been studied in model tests and theory for regular waves only; extension to complex seas is needed. But we do not know that the irregularity *reduces* the effect. . . .

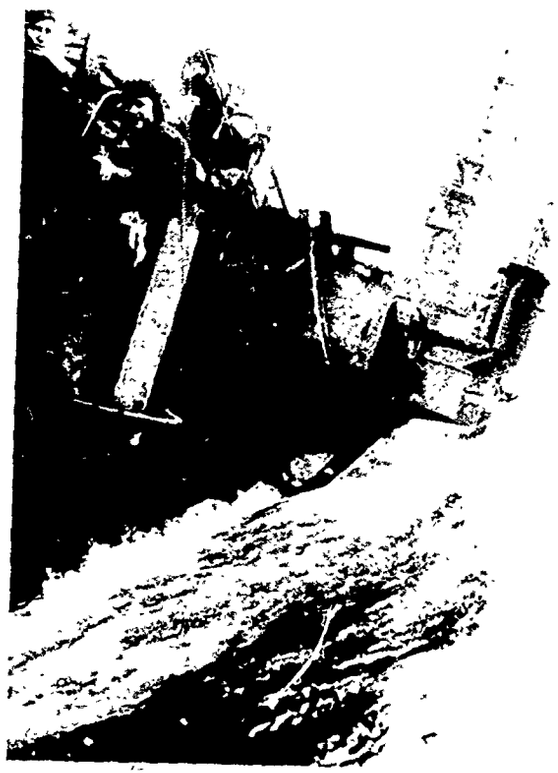

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FIGURE 7.1 Extreme roll of the U. S. S. *Langley* (CVL–27).

(5) Finally, the indirect effect of the added resistance on propulsion—this is not serious in itself for a turbine-driven ship, if the propeller remains well immersed.

"In good weather trade routes and for a considerable portion of the time on even bad weather runs, the above added resistance considerations are basic. However, the other group of factors becomes increasingly important as sea conditions grow worse—these are the indirect effects of motions on speed. For every ship, there are limiting sea conditions beyond which the power must be reduced in order to ease the motions of the vessel. Even the new *United States* must reduce power in heavy weather, and it is reported that the most comfortable condition for the *Queens* in really rough going is to turn and run with the sea, which they sometimes do if necessary for passengers' comfort.

"To illustrate the relative importance of the two types of factors affecting speed, let us refer to figure 7.2. We have recently analyzed some log data for typical cargo ships in winter North Atlantic service—north of the British Isles to Scandinavia—undoubtedly the most severe of the important trade routes in the world. The graph shows first the expected trend of ship speed with increasing average wave height, if

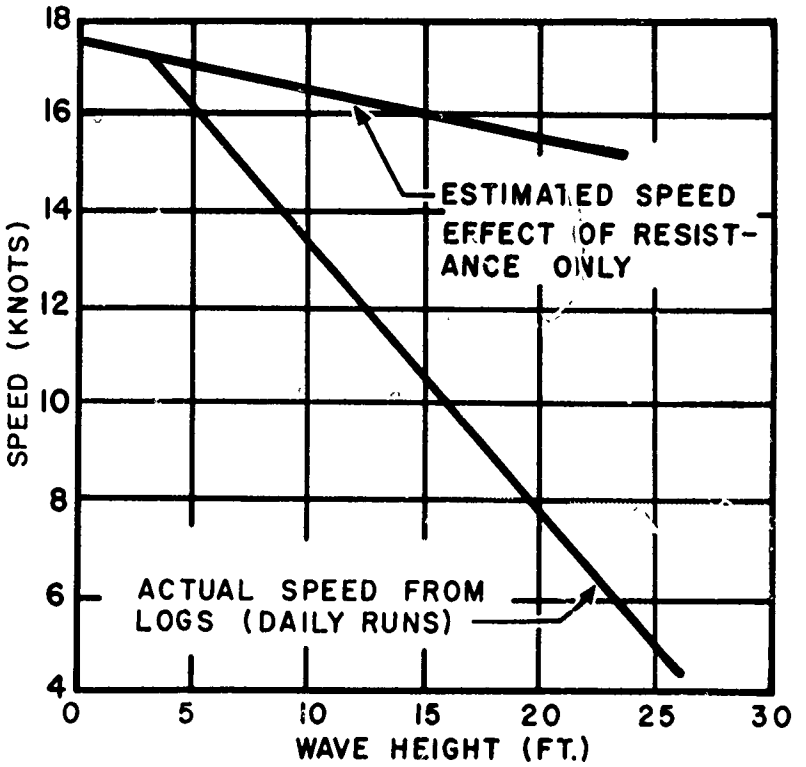

FIGURE 7.2 Effect of sea on ship speed (Victory ship, North Atlantic).

full power were maintained and only the added resistance effects were taken into account. The other steeper line shows the actual trend of speed found by plotting average daily speeds from log records. The difference between the two lines is due almost entirely to the voluntary reduction of power made necessary by the violent motions of the ship.

"Thus, it is clear that simply increasing the power of ships is not alone the answer to our problem of speed. The effect of motions on speed involves many factors—gleaned from log records, discussions with officers, and from the reports of a comparatively small number of voyages in which actual data have been taken. . . . Although different people stress different things, the effects of motions may be classified as follows:

(1) Effects of amplitudes of motions:
Shipping water—with danger to fittings and deck cargo,
Pounding and slamming—with danger to structure,
Racing of propeller—with reduced efficiency of propulsion,
Difficulty in steering—with danger of being unable to maintain the desired course.

(2) Effects of accelerations:
Shifting cargo,
Dislodging objects on board, furniture, etc.,
Discomfort and danger to passengers and crew.

"This is one field in which more qualitative data are needed, for strange to say, very little is known exactly as to what are the limits of amplitudes and accelerations which are acceptable on different types of ships.

Effect of Ship Motions on Speed

"Assuming that criteria on allowable amplitudes and accelerations will be forthcoming in the near future, the most urgent problem of ship speeds narrows down to the problem of motions. How can we determine them in advance and hence modify them so that the limits of speed can be raised for vessels of all types?

"This question is being studied on the basis of both theory and experiment. For convenience, the motions are classified in accordance with a ship's six "degrees of freedom:" angular—roll, pitch, yaw; translation—heave, surge, and sway. Of course, they can be dealt with separately only to a limited extent, because each motion usually has an effect on the other. The most important motions are roll, pitch, and heave.

Rolling

". . . Rolling has received the greatest share of attention; and from the point of view of the present discussion of speed, it may be considered to be understood. Rolling in a regular swell is a forced simple harmonic oscillation—excited by harmonic wave forces and resisted

by small damping forces (wavemaking and viscous). The ship has a natural frequency of roll depending on its geometry and weight distribution, and when the exciting forces are tuned to this frequency, large resonant rolling can result. In regular waves a small change of course or speed usually suffices to avoid the synchronous roll.

"When the waves are *not* regular, however, the rolling is *not* in general regular or harmonic. But observations at sea show that as the rolling becomes heavy in rough seas, the period does begin to approximate the natural period of roll. . . . The explanation for this behavior lies in the fact . . . that a rough sea contains a wide range of frequencies, modified of course by the ship's speed and direction. The exciting wave forces act upon an oscillating system—the ship—which has a sharply tuned response curve—with the peak at its natural period of roll. *Consequently, ships at sea single out the resonant frequencies and roll predominantly in their natural period. Under such sea conditions, it may be difficult or impossible to avoid heavy rolling by a change of course or speed.*

"But from the point of view of ship speed, it may be said that rolling is not a serious problem. First, rolling does not itself have a very great effect on resistance. And second, the use of stabilizing fins seems to have made it possible to reduce rolling very drastically. The idea of fins appears to have been worked out independently in 1889 by Thornycroft in England and Motora in Japan. In the last few years they have been successfully installed in a large number of ships in the form of the Denny-Brown stabilizer. . . .

Pitch and Heave

"We come now to the most important aspect of ship motions in respect to speed: the motions of heave and pitch. . . .

"The question arises: why should speed be governed by the violence of pitching and heaving motions? In particular, why does a reduction of speed usually make a ship more comfortable? The absolute value of the added resistance due to waves is found both theoretically and experimentally to be affected very little by the ship's speed. Why shouldn't the motions show the same small effect? If we knew the answer to these questions, perhaps we can find other ways to ease the motions than by reducing speed.

"*It so happens that observations show the most important wave lengths in storms at sea to be of the same order of magnitude as the lengths of ships, so that large exciting forces are always present in rough weather, regardless of speed.* Considering as an example the case of a ship heading into a regular swell of its own length, it is clear that the forces causing pitching will be very high—because at the same time that a crest is pushing the bow up, a hollow is letting the stern drop.

"The wave length does not have to be exactly the same length as the ship. Waves of ¾ the ship's length begin to cause appreciable exciting forces, and the forces continue large until the swell is much longer—say more than twice the ship's length. The important thing is that the range of significant wave lengths usually present in a storm wave pattern covers the normal range of ship lengths—including that of even the *Queen Elizabeth* in a bad storm. And *these length relationships are unchanged by the speed of the ship, for no matter what its speed, when the crest of a wave is at the bow, the hollow will be at the same position at the stern.* On the other hand, if the ship meets the waves at some direction other than head on or directly astern, the effective lengths of the waves will be increased, and, therefore, the shorter waves will be more important. Although some particular heading may result in reduced exciting forces, change of speed does not have this effect.

"The reason for the influence of speed on pitching and heaving then is in the period relationships—as in the case of rolling. It is the combination of large exciting forces *plus* resonance which seems to give the really violent motion. We shall discuss this important matter first for a ship in uniform waves—closely approximating a swell condition at sea. This simple case has been studied both experimentally and analytically. Pitching will be considered, but the picture with respect to heaving is analogous.

"The significant thing about speed is that it affects the period of the exciting forces—i.e., the period of encounter with the waves. If the ship speed is 0, the period of encounter in a regular seaway will be that corresponding to the wave velocity past the ship. As the ship is given forward speeds against the waves, the period of encounter becomes less. (In a following sea, it becomes greater.) Although the pitching will always be heavy in waves near the ship length, it will reach its greatest value in the vicinity of the speed giving synchronism between the period of encounter and the ship's natural pitching period. *It so happens that for most ships in waves of their own length this condition occurs somewhere near their normal speeds; consequently, a reduction in speed ameliorates the situation.*

"*There is another equally important consideration*, and that is the phase relationship between the regular waves and the motion. It is well known in the theory of oscillations that at synchronism a lag of 90° in phase is characteristic between the applied force and the response—for moderate damping. For the normal ship with almost vertical sides this applies closely and results in a situation shown clearly in model tests, in which the ship at synchronism pitches down into the crest, *usually shipping green water even in waves of moderate height.* See the model photographs in figure 7.3.

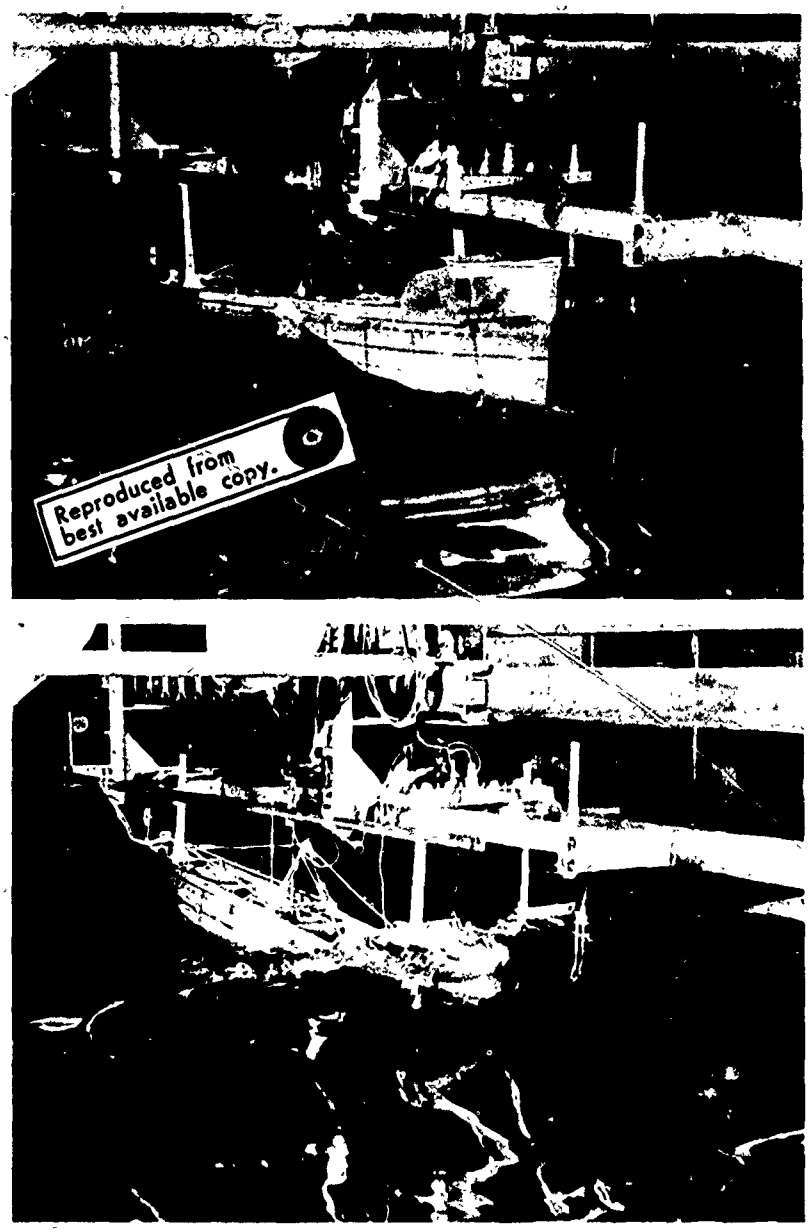
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FIGURE 7.3 A ship model pitching bow down in a wave crest.

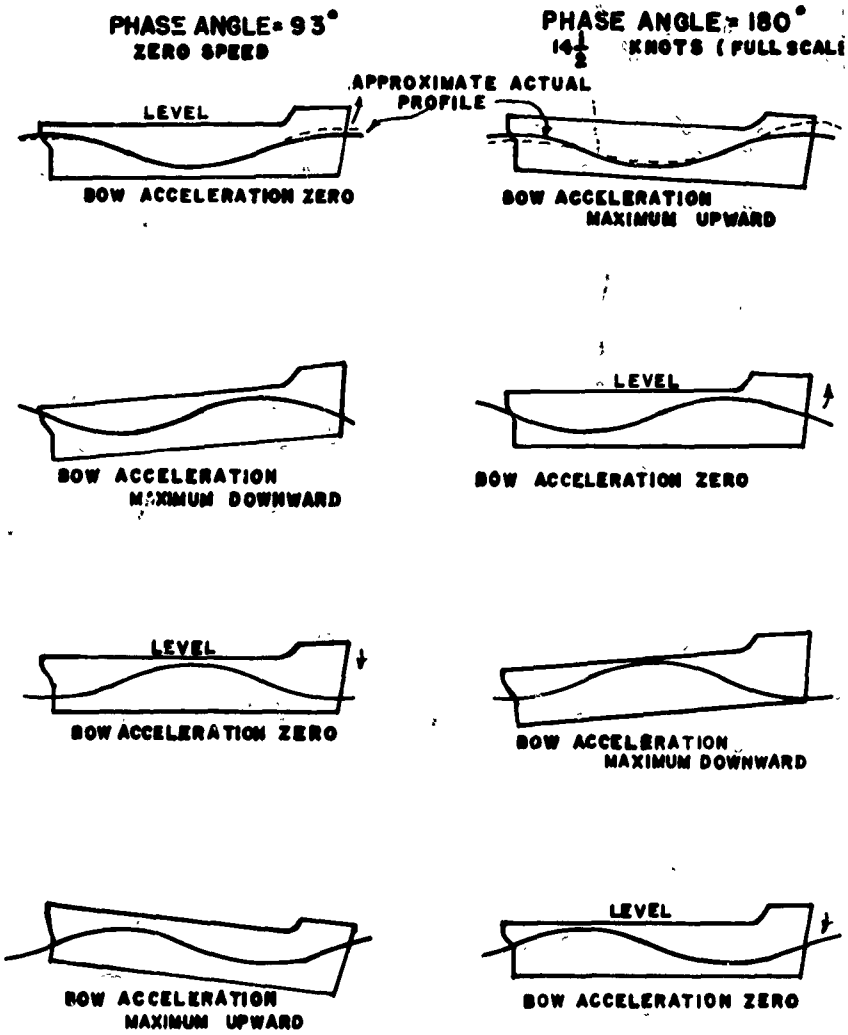

FIGURE 7.4 Model test of merchant ship.

"A reduction of speed serves not only to reduce the amplitude of motions and accelerations, but it changes the phase relationships so as to keep the decks drier. The ship follows the wave slope instead of going against it. Figure 7.4 shows these phase relationships for a typical merchant ship in regular waves of length equal to the ship length.

"The possibility remains of increasing the speed to avoid serious resonance. For a high-powered vessel in a regular swell this may well be possible, so that the pitching becomes less severe as the speed is either increased or decreased.

Pitch and Heave in an Irregular Sea

"We come now to the situation of a ship pitching in a complex storm sea—a problem which is just beginning to be studied in detail . . . Neither the *wave lengths* nor *periods* of the component waves . . . are directly observable in the seaway. Only the combined effect of the superimposed components can be [illegible]. But it is possible to observe and record the apparent "period[illegible]" and "wave lengths", which can be defined as the time and the length, respectively, between successive crests observed at a fixed point.

"It is safe to say then that if there are component *wave lengths* near the ship length present in a seaway—which . . . is always the case when the sea is rough enough—there will be appreciable irregular pitching. If further, the ship's speed and heading give *periods* of encounter with these components which are near the ship's natural *period* the pitching will at times be more violent. Sea observations show this to be the case in storms, with motions tending to follow a *period* of oscillation close to the natural *period*, but with great irregularity in amplitudes.

"The significance of all this for ship speed is that (1) *it is impossible to eliminate pitching and heaving when heading into a bad storm, because wave lengths near the ship's length cannot be avoided (a change of course may be helpful in some cases). It is possible to avoid synchronism with the most serious component frequencies by reducing speed. If the ship turns and runs with the sea at slow speed, the frequencies of encounter with important components will be removed still more from synchronism.* This is no doubt one of the reasons that books on navigation suggest that steamships heave to with the stern to wind and sea.

"On the other hand, if speed is increased in an irregular storm sea, resonance will occur with lower frequency components present, which may be equally important. Hence, pitching can be reduced by *increasing* speed only in moderate seas or in a short swell.

"Thus in really bad weather a ship heaves to, with speed near zero, heading into the sea or nearly so. Or the ship may turn and run with the wind and sea. Pitching and heaving are then much less severe, since synchronism with the waves having the largest exciting forces are avoided. An additional favorable effect is the tendency of the ship to follow the seas rather than to pitch into them, so that less water is shipped forward."

General Operational Problems

Refueling at Sea

Refueling at sea can be a difficult and dangerous task, as figure 7.5 illustrates. This photograph shows a group of men on the

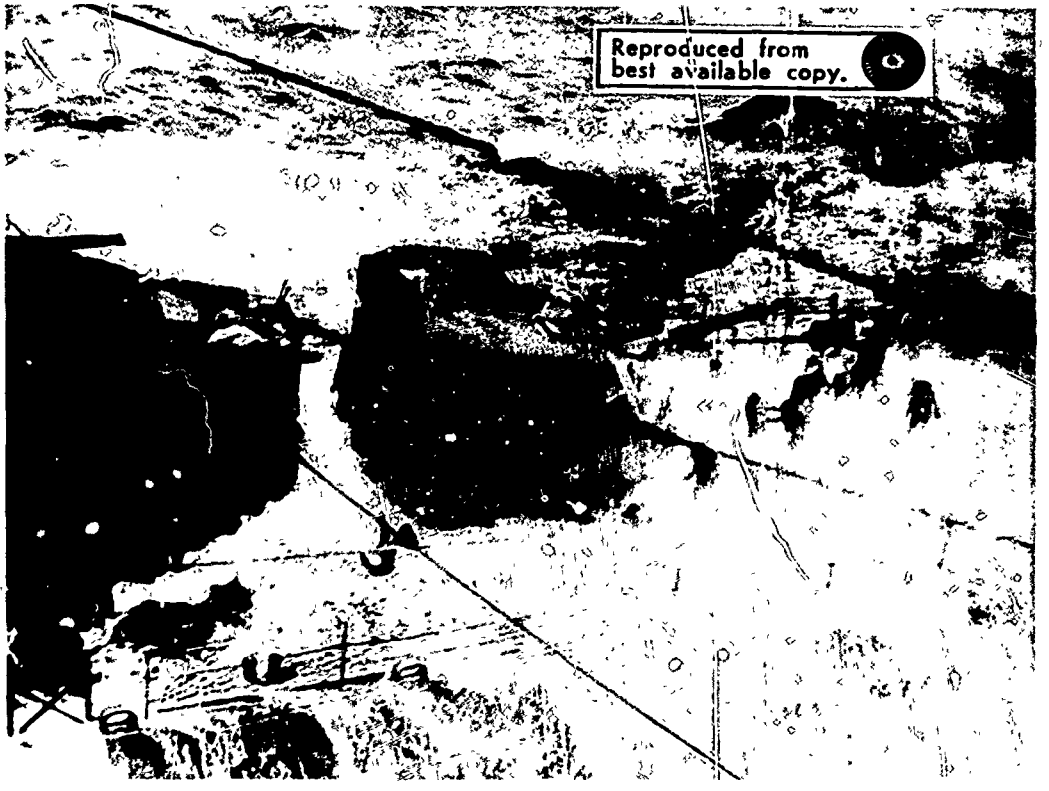

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FIGURE 7.5 Refueling a DD during maneuvers.

fantail of a destroyer attempting to refuel her during maneuvers in October 1947. The deck is awash, and the operation is extremely hazardous to the personnel involved. Relatively small ships, such as destroyer escorts and destroyers, respond to every wave. In heavy seas, or even in moderate seas, it is difficult to keep them on course. They roll, pitch, yaw, and sway much more than a larger vessel. Thus, the fuel lines between the destroyer and the oiler are frequently torn apart.

In heavy seas, therefore, the refueling problem becomes very critical, and it may involve extensive delays. It takes a long time to refuel if the ships are responding to the waves erratically. In very heavy seas, refueling becomes impossible. The demands for refueling are not always flexible; and the timing problem, that is, the choice of when a ship can be refueled, is often determined by many contingent circumstances. Thus, one very useful wave forecast is one which gives advance warning of difficult refueling conditions. Given a reliable forecast, the decision may be made to refuel although the ship has only partially exhausted her fuel supply. Another important forecast is one which tells when it would become possible for a ship to refuel, if conditions at the time of the forecast were such that it was very difficult to refuel.

The problem of refueling depends essentially on how the ship responds to a given seaway. Each class and type of vessel responds differently. Therefore, in order to meet this problem best, it is advisable to keep records of the types of sea present and of the difficulties encountered in such sea conditions. Then when these sea conditions are forecast, it will be possible to predict the degree of difficulty that will be encountered in an attempt to refuel.

Aircraft Carrier Behavior in Sea and Swell

In a heavy sea or a high swell, aircraft carrier operation becomes impossible, in that aircraft can neither take off from nor land on a carrier whose decks are moving up and down due to the passage of the waves. The state of the sea when operations must cease depends on the class of the carrier. It varies from ship to ship just as the difficulties of refueling vary with the type of ship involved. Again, records of the wave spectrum present and of the behavior of the ship in such a seaway are important in determining the conditions and in making it possible to prepare better forecasts at future times.

One particular difficulty encountered in aircraft carrier operation occurs when a light wind is present and the carrier is in a long, heavy swell. Because of the light wind the carrier must speed up in order to get a sufficient wind velocity over the flight deck to permit the aircraft to take off. This produces excessive pitching and causes

excessive vertical motions of the flight deck to such an extent that it becomes an extremely dangerous operation to launch and recover aircraft.

At the same time the winds aloft and the general weather situation can be excellent for aircraft operations. Nevertheless the whole maneuver must be held up until such time as the swell dies down. To be able to forecast just when the swell will die down thus becomes very important in this situation. The problem, of course, can be solved by the techniques of this manual, and such a forecast is an ideal problem for the application of the methods given herein.

Hurricanes

A hurricane is the most dangerous condition that a ship can encounter. The severity of the attack on a ship is such that many have been lost and many more have been damaged by hurricanes. The methods given in this manual unfortunately do not make it possible to forecast the state of the sea in a hurricane. The area in which the waves are generated does not have the property of a uniform direction and velocity over a large area. Hurricanes have a circular vortex wind field with winds decreasing in velocity from the center, and under these wind conditions the theories derived in the past chapters simply do not apply. The problem is being studied, but it is an extremely difficult one to solve.

The real problem, as far as a hurricane is concerned, is making a good weather forecast so that the hurricane can be avoided. The problem of preparing a good weather forecast and of having enough advance warning of the presence of a hurricane (or a typhoon) is beyond the scope of this manual.

Ship Damage

Damage to ships in heavy seas can be severe. As mentioned in the introduction to this chapter, it can limit the operational effectiveness of a task force and can cause severe losses in time and money by delaying equipment transport across the seas. Very little is known about the conditions which cause ship damage or about how they can be avoided. In this section two photographs illustrate the effect of the fury of heavy seas.

The first photograph is shown in figure 7.6. The U. S. S. *Hornet* had her flight deck ripped up 30 feet back from the bow at the height of a typhoon which had wind speeds over 100 miles per hour. The photograph shows a high wave breaking over the bow and portions of the flight deck being torn away by the wave action. Note the protective barricade, shown in the foreground, which was put up in order to protect the aircraft. The *significant* waves must have been well over 40 feet high in order to produce the damage that resulted.

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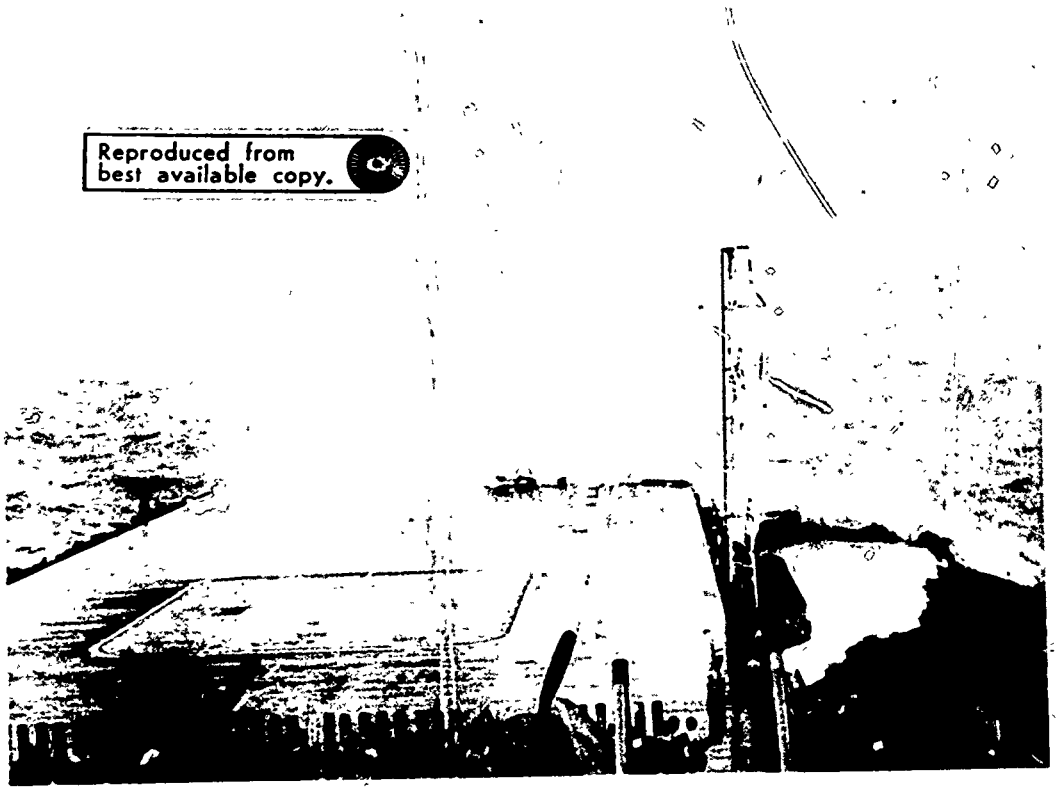

FIGURE 7.6 Damage to the U. S. S. *Hornet* (CV–12) during a typhoon.

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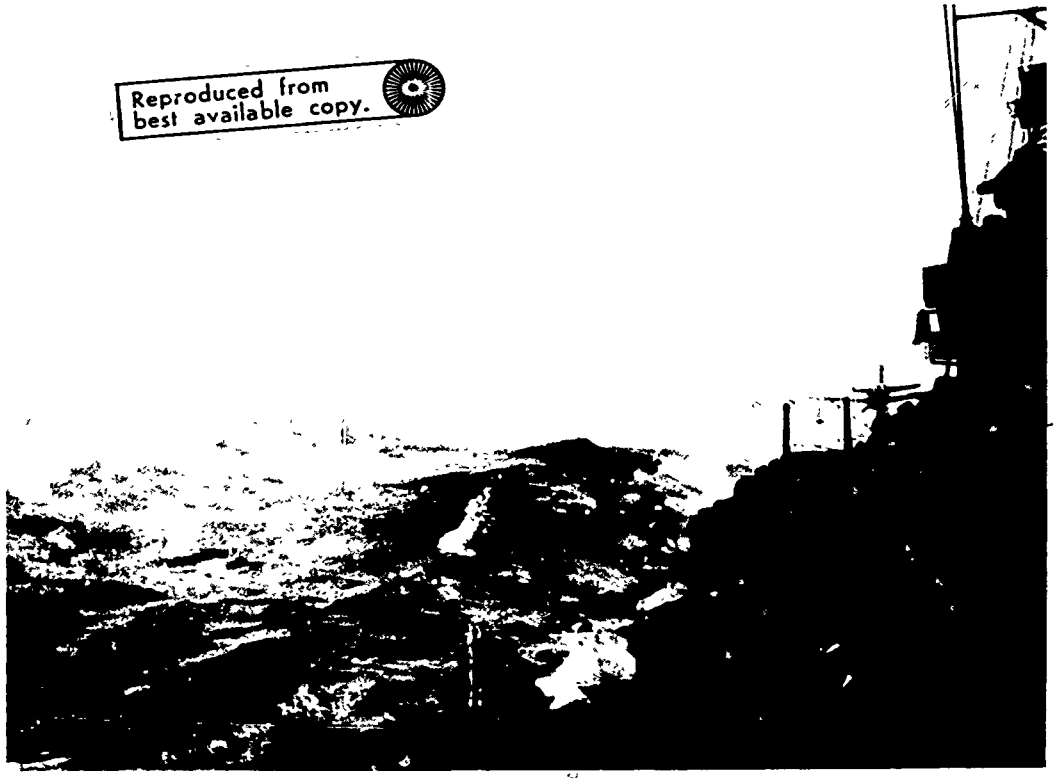

FIGURE 7.7 U. S. S. *Pittsburgh* loses bow during typhoon.

The second photograph is shown in figure 7.7. During a typhoon in the Pacific on 5 June 1945, the U. S. S. *Pittsburgh* lost her bow. The cruiser steamed 900 miles to Guam under her own power, making 9 knots despite the damage. This photograph was taken on the starboard side of the cruiser looking aft during the typhoon. Notice the mountainous irregular wave in the foreground and the tremendous trench which looks something like the Grand Canyon before the crests of the next two waves which are dimly visible in the background. It is extremely difficult to estimate the height of this particular wave from the photograph, but it certainly must have been tremendous.

Forecasting Ship Motions, Ship Speed, and Operational Factors

The problem of forecasting ship motions, the effect of sea conditions on the speed of a ship, and the effects of waves on operations problems, requires a knowledge of two factors.

The first factor is an accurate description of the spectrum of the waves present. This can be forecast by the methods of this manual.

The second factor is that of deciding the effect of this forecast spectrum on a particular ship operating at a particular heading in the waves associated with the forecast spectrum. The considerations which enter here cannot be treated fully in this manual because of the many different types of ships and because of the many subjective considerations involved in each operational problem. Some information has been given in this chapter on what is known about such problems, but much more is needed.

One suggestion can be given: keep adequate records of the way a given ship behaves under given sea conditions. Careful study of such records is the best way for a forecaster to build up a backlog of experience as to what a ship will do under given conditions. Such a backlog of experience is a part of every forecaster's training, but by keeping adequate records and by correlating what happens in a particular operation with a given state of the sea, this backlog of experience can increase more rapidly and become more reliable.

Landing and Takeoff Conditions for Seaplanes

General Information

Landing and taking off seaplanes on the open ocean are difficult and dangerous operations. The decision as to whether an attempt should be made is always left to the pilot. The information given in this manual will aid the pilot and the wave forecaster in planning a possible search and rescue operation.

Of the many models of seaplanes in use by naval activities, four may be mentioned specifically. One is the PBY which is being replaced

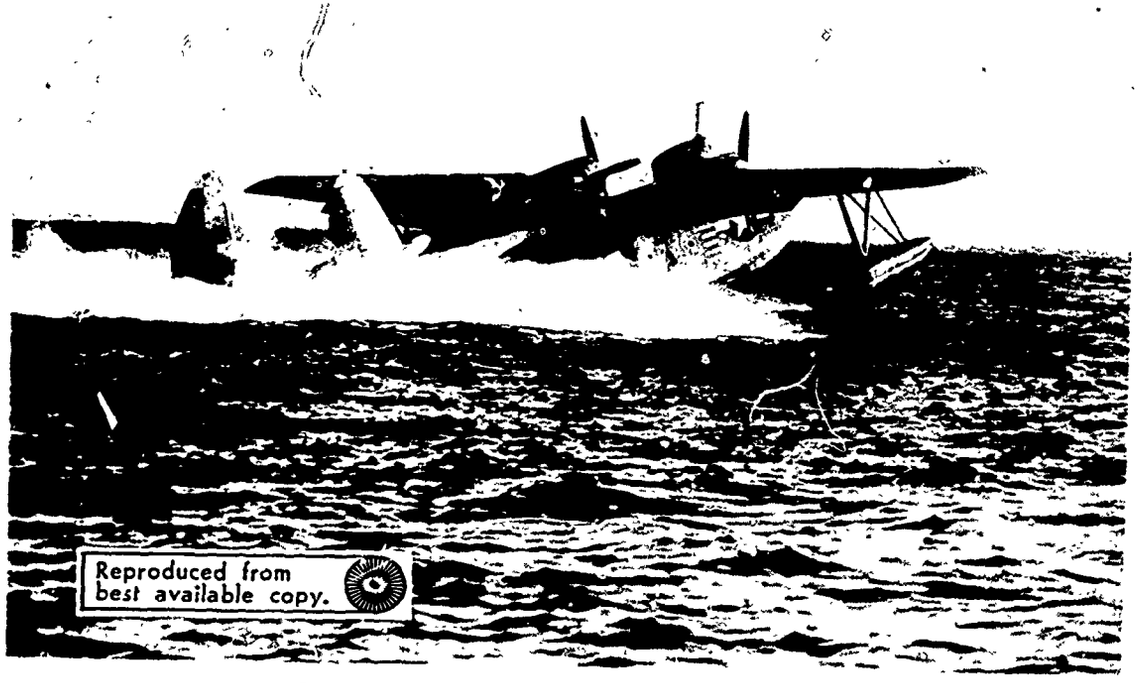

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FIGURE 7.8 Landing a PBM parallel to the swell.

by the SA-16 (the Grumman twin-engine albatross), and another is the PBM. The other two are the XP5M and the UF. It is recommended that landing of the UF be confined to well-defined swells and that landings be made parallel to the crests. The PBY is, relatively speaking, a fragile aircraft compared to the PBM. The worst seas that the PBY can land on are limited by the manufacturer to a height of 3 feet. The manufacturer probably means a significant height of 3 feet. The PBM is a larger, heavier aircraft, as shown in figure 7.8 and can land in waves with a significant height from 6 to 8 feet. These values probably represent the maximum possible heights for safe landings.

A Study of the PBM

In 1944 and 1945, a study of the problem of landing the PBM was made at the U. S. Coast Guard Air Station at San Diego, Calif. The results were published in a report entitled "Report of Open Sea Landing Tests and Study." The comments and the remarks which will be made here on this problem will largely be quoted from this report. In addition, Capt. D. B. MacDiarmid, USCG, who wrote the report, has supplied additional information based on further tests. The following text also includes quotations from a letter from him on the subject.

The results presented in the previous chapters of this manual when combined with these tests make it possible to give some very valuable advice on the problem of handling the PBM on the open ocean. The conclusions apply to any other aircraft of comparable size and performance. "Generally speaking one may say—'The bigger the seaplane the smaller the sea', but aircraft which must touch down at high speeds or which have idiosyncracies in water handling may be most difficult to land in a sea."

In the introduction to the report, the following statement was made, "THE MECHANICS OF A COMPLEX SEA, IN PARTICULAR, ARE VERY OBSCURE: AND PROBABLY A CLEAR UNDERSTANDING OF THESE WOULD HELP TO THROW A GREAT WHITE LIGHT ON SOME OF THE BAFFLING FORCES THAT SOMETIMES SUDDENLY DISCONCERT SEAPLANE PILOTS IN THE MIDDLE OF A LANDING OR TAKEOFF RUN AT SEA."

At the time that the report was written, it was not known that every seaway, from the simplest swell produced by one and only one distant storm to the mountainous waves in a storm area for a fully developed spectrum, could be represented by the superposition of the sum of a great many simple sine waves. This one fact makes it possible to interpret many of the observations in this report and to clarify

and explain the reasons given for the best landing and takeoff techniques. This report, in fact, is probably the earliest one ever written which recognized the features of the waves described in Chapter I and reemphasized here in order to show the best techniques for landing and taking off seaplanes.

In the report, 54 landings and 54 takeoffs were made in a variety of sea and swell conditions. "Since the completion of the report quoted above, more than 300 other landings have been made in seas from the coast of northern California to the coast of western Mexico, 1,000 miles south of the border; in the Gulf of Mexico; off the Florida Keys in the Atlantic; off the Virginia Capes; off New York; and off the coast of New Hampshire. About 200 landings were made in the PBM5; 106 landings and takeoffs in the PBM5A (amphibious version of PBM5); 25 landings and takeoffs in the XP5M and 10 landings and takeoffs in the UF.

Continuous and carefully taken records, motion pictures, accelerometer records, stress and strain records, and other data were all gathered to make it possible to decide what actually happened during the takeoffs and landings. The conclusions arrived at are therefore based upon sound and careful study, and they will be quoted extensively in this section to explain the procedures which should be used.

These landings and takeoffs were made under all possible conditions. They were made heading into the swell, traveling with the swell, and parallel to the swell. They were made under different wind conditions, and they were made with local seas present on top of the swell. The analysis of the results shows that the complexity of the landing increased as the complexity of the sea increased and as the wind became stronger. The simplest cases will be discussed first, and then the more complex cases will be described.

Landing a Seaplane on a Simple Sinusoidal Wave

No pilot of a conventional landplane would be very happy if he were forced to land his aircraft on a runway which had a sinusoidal profile with a wave length of 500 feet and with elevations and depressions with a total swing of 5 feet. Landing a seaplane is just as difficult, although one might think that the water would, in some way, be different from a concrete runway. This is true to a certain extent owing to the fact that the seaplane has a hull shaped like that of a ship, but the impact forces and the accelerations which the plane undergoes as it moves up and down on the humps are comparable in force to those which a landplane would encounter in landing on a bumpy runway.

Consider the problem of a plane landing on a sea surface actually composed of one simple harmonic progressive wave. Suppose this

wave to be sinusoidal with a *period* of 10 seconds. Suppose that first the seaplane is heading into the waves as it lands. The stalling speed of the seaplane is approximately 65 knots so that as it comes in to be set down on the sea surface it is moving with this speed. The waves are moving toward the plane at 30 knots. Consequently, the aircraft encounters the waves as if they were standing still and the plane was traveling at a speed of 95 knots against the waves. A simple computation then shows that the plane passes from one crest to the next in 3.2 seconds. The pilot then has to react to very rapid and erratic motions of the plane as it travels up and down over the waves.

Conversely, suppose that the plane is traveling with the 30-knot waves as it lands with a speed of 65 knots. The plane is moving relative to the crests at 35 knots and encounters one every 11.7 seconds. It still overtakes the individual crests so that the crests appear to the pilot to be coming toward him as he lands. However, in this condition, the time between the successive crests is much greater, and the pilot has more chance to react to the responses of the plane to the waves.

Figure 7.9 illustrates the conditions that can occur as a plane lands on a simple sine wave. The top part of the figure shows the successive positions of the plane with respect to the wave profile as the plane lands, with the pilot holding the nose of the plane high. Under these conditions, the aircraft lands on the first wave crest, takes off from the second wave crest, and hits the third crest with a bad shock. If the pilot jockeys the nose of the aircraft up and down, as in the center part of the figure, the altitude achieved by the plane as it flies off the second crest is considerably less, and the shock upon hitting the third crest is greatly lessened. If the pilot misses the first crest of the wave system and lands in a trough, as is illustrated by the bottom part of the figure, the plane then achieves considerable altitude as it flies off the second crest, and the worst shock is encountered when it hits the third crest.

In summary, the pilot should attempt to land on a crest of a wave or just slightly beyond the crest of the wave if he must land in a direction such that the crests of the waves are perpendicular to the incoming aircraft. He should try to keep the plane on the water by jockeying the nose up and down. If the crest is missed and the plane falls down into the trough, the worst conditions are encountered.

Landing a seaplane perpendicular to the swell was for many years considered to be the only possibility. Captain Mac Diarmid's report also describes attempts to land the seaplane parallel to the swell. This was found to be a quite satisfactory procedure. In an idealized simple sinusoidal wave system, suppose that the pilot attempts to set the plane down on a crest of a wave as he flies parallel to the waves.

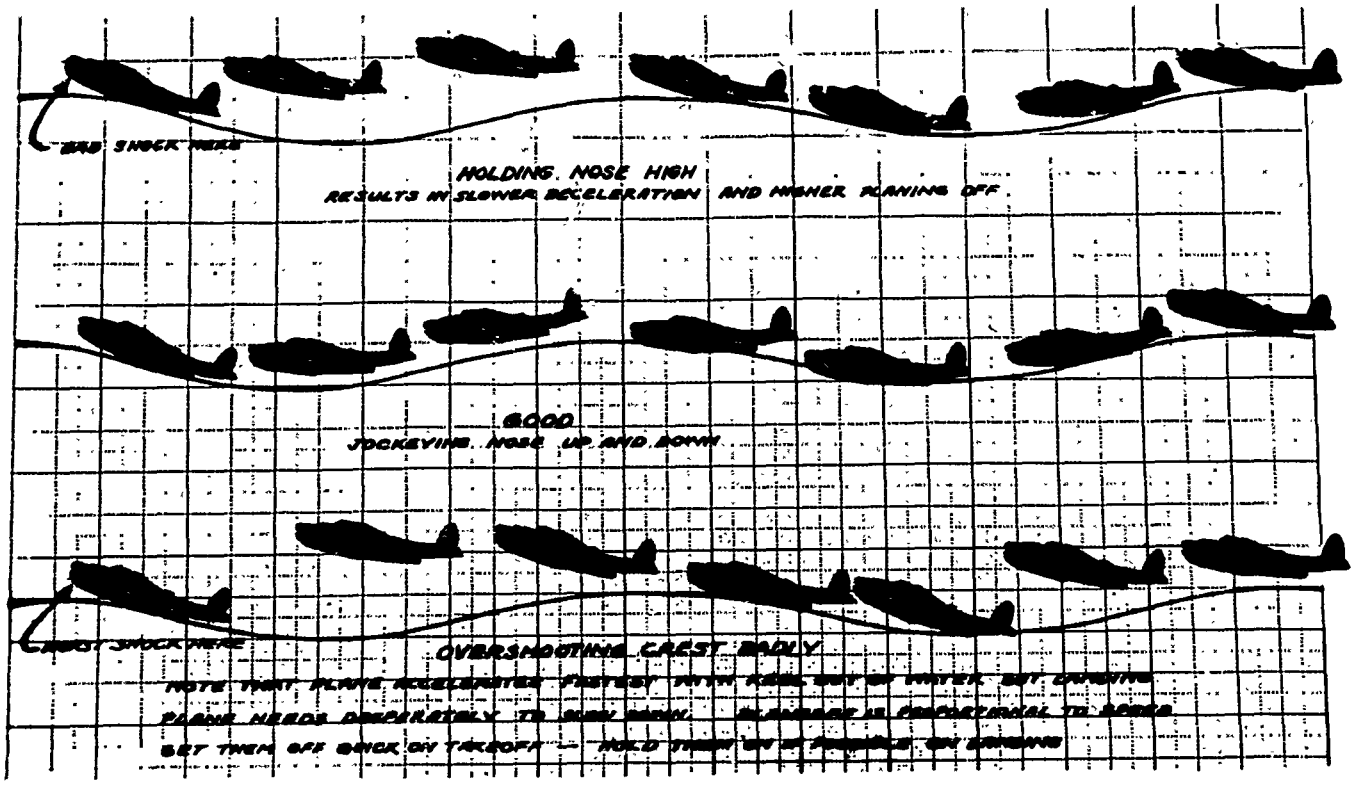

FIGURE 7.9 Schematic drawing of a PBM landing on a simple sine wave.

The plane rises and falls with a ten-second *period* due to the passage of the waves. However, and this is the important point, the plane does not fly up and down hill because of going down into one trough and up on the crest of the next since the hull of the plane is always on a horizontal line. The planing-off and the shock that results upon smashing down into the next crest is therefore avoided to a large extent if the plane lands parallel to the swell. Stated another way, the nose of the plane does not move up and down if the plane is landed parallel to the swell. If the wave length is great enough, there is no danger of damaging a wing tip float under these conditions. Also, if the wave length is great enough, it is also possible to land on the side of the wave or in the trough of the wave without difficulty and without any serious modification of the above statements.

In summary, then, upon considering the effects that a plane encounters when landing on a simple sinusoidal wave, one can see that to land into the waves so that the waves rush toward the plane with their velocity as the plane moves into the waves with its velocity is the most dangerous because the pilot does not have sufficient time to react to the responses of the plane. Landing with the waves is the second best choice because this gives the pilot more time to respond to the motions of the airplane. However, there is still the danger of planing off the top of one wave and crashing down violently into the crest of the next. The third way, that of landing parallel to the swell, is the safest, apparently, because the plane is not tilted as it goes over the sides of the waves and the whole airplane can then be kept on an even keel.

Landing in a Swell

As shown in figures 1.2 and 1.4 even the simplest swell is not like a simple sine wave. Even in a swell with a narrow frequency band width and a narrow angular band width which is arriving from one single distant source there still are areas over the sea surface where the swell is low and areas where the swell is high. The wave heights are distributed according to the laws given in Chapter I, even for the most regular swell. The important thing, though, in this case, is that low waves come in groups and high waves come in groups, so that there are large areas of the sea surface where the waves are quite low. Therefore, the procedure for landing in such a regular swell will be discussed first. Minor low "period" local chop can also be present as in figure 1.2 or figure 6.5. If this local chop is not too high, it can be ignored in the discussion of the landing procedure.

The best landing procedure is to land parallel to the swell. "With winds of less than 20 knots the direction of the wind is a consideration to the pilot definitely second to the necessity of planning a landing or

takeoff run with regard to the swell alone." With the winds less than 20 knots, therefore, the important thing is to land parallel to the swell. The best landing approach in a regularly formed sea with light winds is parallel to the swell, touching down on the crest of the swell."

The landing procedure follows. "The direction of the wind, unless it be 20 knots or more, should always be considered a hazard secondary to the sea conditions in planning a landing." The final approach for the landing is made parallel to the swell. "The final approach for the landing should be dragged in low a mile or so short of the proposed landing spot with the propellers in low pitch, the flaps down full, and the tabs set to require a slight down pressure on the yoke. This approach should be made at not more than 15 knots above the stalling speed of the aircraft as loaded. A careful watch should be kept on the sea well ahead during this run; and if the sea ahead suddenly appears relatively smooth, the aircraft should be stalled as quickly and as short as possible. Running the plane's speed out on the landing can be done with the plane's nose very high or dragging the nose slightly, but the pilot should in either case be alert to play the nose of the plane up and down to ease the shock of its passage through the waves and to avoid planing off the top of any wave higher than is absolutely necessary as long as he has control with his elevators."

"Touching down in the trough while running parallel to the swell does not embarrass the pilot very much. The popular fear that a wing tip float will probably be dragged in on such a landing is not supported by experience. If the landing is cross wind, the pilot may either crab into the wind as necessary, slip slightly, or approach with wings level, ignoring drift. The sidewise motion does not appear to affect the landing characteristics of the plane when stalled in the open sea nearly as much as it does in a smooth harbor, probably because there are so many other factors affecting the landing that this one becomes relatively unimportant."

"The touchdown landing parallel to a long swell may be made very safely along the crest, the trough, or anywhere on the slope of a swell. Most pilots will instinctively dress their wings to the surface under them. A touch down along the crest is not a critical necessity."

In the additional tests described above, "landings were made downwind successfully in winds of over 18 knots and in crosswinds up to 23 knots. It was found good practice for a pilot landing in a strong crosswind to kick some downwind rudder while still running fast on the sea and attempting to make a slight downwind turn. This helps to cancel out the weather-cocking tendency of the aircraft."

If the local wind is higher, the local chop will also be higher; and after a certain state for the local sea, the local sea may actually obscure a swell which is running underneath. If the winds are much higher than

20 or 23 knots in velocity, then the wind direction becomes an important factor in deciding the heading the aircraft should take in making a landing. A down-swell landing is the next best procedure if a landing parallel to the swell cannot be made. Again, the plane should be dragged in low as described, and the pilot should look for an area in the distance where the waves appear to be relatively low over a rather large area. "The second best landing heading in a well-formed and regular swell is down swell, touching down on the crest of a swell or within a few feet beyond it and making every effort to prevent the airplane from being thrown high in the air as she planes up over successive swells. To accomplish this the nose should be deliberately pushed down as the plane races up the back of the swell and approaches the crest. As the plane starts to fall back on again, the nose should again be pulled up. This technique requires very fast pilot reaction, but it is believed safer than to hold the nose high at all times because the principal damage suffered in these tests was from the shock of falling back hard after a high bounce."

In this condition the frequency with which the plane encounters the waves is quite low and the "period" of encounter is therefore correspondingly high. This gives the pilot more time to react to the various responses which the airplane makes as she goes over the waves.

Landing into a swell is the most hazardous of all procedures. "A landing should be made into a fast swell only when the wind is blowing from that direction with a force so great that a landing on any other heading will be very hazardous owing to the wind." In this series of tests landings were made for cross winds up to 23 knots and in downwind conditions of 18 knots without damage to the aircraft. Thus, when the wind gets to be much higher than 20 or 23 knots, it may then be necessary to land into the swell. "The hills and valleys caused in the sea surface by the swell affect an aircraft landing or takeoff almost the same as skips or bumps in a runway affect landplanes." A landing into the swell is more difficult than a landing down swell only because the aircraft is hitting the swells more often and hitting more of them.

An Analysis of the Results of Landings at Different Headings

In the 54 landings of the original report and in the 180 landings of the PBM5 "an attempt was made in every instance to make landings and takeoffs into the sea, down swell, and in two directions parallel to the swell, and the only operations aborted were when the aircraft had been badly damaged or, in a few cases, where operations into the swell were so severe the pilot nearly lost control of the aircraft. So as far as it was possible, conditions were the same for landings on all headings except that landings were made with success parallel to the swell and down swell when they could not be accomplished into the swell in the same sea."

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The original report tabulates the total number of landing bounces which the plane made for each of the landings, and the data can be summarized very briefly. Of 22 landings parallel to the swell, the plane bounced 4 times in 5 of the landings, 3 times in 4 of the landings, twice in 2 of the landings, once in 4 of the landings, and not at all in 5 of the landings. The average number of bounces per landing, for landings parallel to the swell, was therefore 2.3. For landings down swell, there was one case in which the plane bounced 6 times, 1 case in which the plane bounced 5 times, 3 cases in which the plane bounced 3 times, 2 cases in which the plane bounced twice, and 3 cases in which the plane did not bounce at all, for a total of 10 down-swell landings. The average number of bounces per landing for down-swell landings was therefore 2.4. For up-swell landings, there was one case in which the plane bounced 6 times, 3 cases in which the plane bounced 4 times, 1 case in which the plane bounced 3 times, 1 case in which the plane bounced twice, and 1 case in which it did not bounce at all, for a total of 7 up-swell landings. The average number of bounces per up-swell landing was therefore 3.3.

Thus for 22 different types of sea and wind conditions, 22 landings parallel to the swell could be made. Of the 22 different sea and wind conditions, at least 12 were such that it was not even advisable to attempt a down-swell landing. (There may have been more than 12.)

In about 10 cases of the original 22 sea and swell conditions, it was possible to make a down-swell landing. These sea and swell conditions in which down-swell landings were made must have been the most favorable of the original 22, and yet the average number of bounces per landing still exceeded the value for landings parallel to the swell.

Similarly, since up-swell landings are even more risky, at least 15 of the original 22 sea and wind conditions had to be eliminated. In only about 7 of the original sea and wind conditions was it advisable to attempt an up-swell landing. Yet for the safest 7 conditions the average number of bounces per up-swell landing exceeded the values in the other two cases.

Takeoff in a Swell

The problem of taking off in a swell is similar to the problem of landing in a swell since the plane's behavior as the wave crests are encountered for different headings is essentially the same as in a landing after the aircraft has gotten up to the speed at which it can leave the surface of the water. "The greatest difficulty—though not the greatest danger—in any takeoff other than into the wind is in getting the aircraft up to steerable speed on the heading desired. It is practically impossible to start a cross-wind takeoff run in the

open sea with the PBM-3 airplane starting the takeoff headings from rest. The solution to this is to head either down wind or up wind and accelerate to steerable speed and then ease around parallel to the swell for the takeoff run. Accelerating speed downwind is believed better than upwind as the spray and pounding are less, and the plane can be brought from downwind to cross wind much more handily than from upwind to cross wind." The best takeoff is parallel to the swell. "A takeoff run parallel to the swell may be accomplished either on a constant heading or by easing the nose around downwind to stay on the crest of a selected swell. The latter election is almost instinctive with the pilot and does not require a turn of more than a few degrees." As the plane is accelerated to takeoff speed the pilot should keep an eye out for an area of the ocean where the waves are relatively low. When such an area is entered, the aircraft should be dragged off at the earliest possible moment regardless of whether it slaps successive waves or not.

"The second best takeoff heading in a well-formed and regular and fast moving swell is down swell, disregarding the wind unless it be 20 knots or more. The best technique for the down-swell takeoff is to accelerate the aircraft until it is running as fast as it can without planing off the tops of the swell, the nose being jockeyed up and down freely to hold it on. This run should be continued patiently until a condition with a large swell with smaller swells ahead is overtaken, or overtakes the aircraft; then the plane should be nosed down on the top of the swell, the throttles opened smartly to takeoff power and every effort made to reach takeoff speed and stay in the air when the aircraft planes off on the top of the next swell overtaken. This operation sounds difficult in the telling, but it has been found surprisingly easy when smartly executed."

The worst takeoff heading is up swell. "A takeoff into the swell in a calm or light wind is to be avoided like death. Such takeoffs have been accomplished in these tests in a high slow moving swell; but in a long low Pacific swell moving at 25 knots the pilot found that when he headed into the swell and accelerated, the swells were coming at him so fast they gave the impression of a very rough high sea. Under the same conditions, a down swell cross wind takeoff was made smoothly and easily. If the pilot decides that he must make a takeoff into the swell, the technique is the same as for a down-swell takeoff, that is, to get as much way as possible on the aircraft without planing off and then suddenly race down the back of a long swell and attempt to stay in the air when she planes off on the next swell. Pilots should be warned that in a takeoff into a fast swell the aircraft will strike successive swells so fast that the effect is pounding rather than planing." With an aircraft accelerating to

takeoff speed at approximately 65 knots and running into a swell with a 10-second *period*, the plane would encounter the waves every 3.2 seconds. The jarring and pounding of the aircraft under these conditions would be extremely severe.

Landing in a Sea

Landing an aircraft in a swell with a significant height of 5 or 6 feet is quite different from landing an aircraft in a sea with exactly the same significant height. The swell is composed of a narrow band of frequencies, and the "period" of the swell is quite high. This means, first of all, that the slopes of the sea surface are not great. Secondly, it means that if an area of the sea surface is found over which the waves are relatively low, then the properties of the swell make it possible to guarantee (almost) that the swell will remain low during the time it takes to set the plane down. A locally generated sea of the same significant height will be produced by the local winds and it will cover a much wider frequency band. The average "period" and the average "wave length" will be much lower. Since the waves have the same significant height, this means that the slopes of the sea surface will be much steeper. In addition, owing to the irregularity of the sea waves as shown in figures 1.1 and 1.3, there is no guarantee that if a relatively flat area of the sea surface is found by the pilot it will remain low during the time it takes to make the landing. In fact, the hypothetical example in Chapter I for the landing of a seaplane shows exactly what can happen under these conditions. Consequently, to land a seaplane in a sea is far more dangerous even for the same significant height than to land the same seaplane in a swell. It probably should be recommended that landings in a sea spectrum be restricted to significant heights which are much lower than those the airplane can safely land on in an equivalent swell spectrum. The rules and procedures just given for landing in a swell follow with modifications for landing in a sea. The pilot should expect much bumpier and much more irregular conditions and the whole procedure will be more hazardous.

Takeoff in a Sea

Exactly the same statements can be made for takeoffs in a sea. Looking out ahead and picking out an area where the sea is relatively flat will not guarantee in many instances that the sea in front of the aircraft will remain relatively flat during the time it takes to get off of the water. Most of the tests cited in this report were made for swell conditions, and much less is known about what can be encountered by an aircraft when it attempts to take off in a sea. There are also intermediate conditions where the waves are not quite so well defined as swell, and yet where they are not as irregular as a

locally generated sea. The same suggestion which was given above when discussing ship motions and ship operations can be made here. Records of the actual spectrum present during such landings and takeoffs and of what happened will be the only way to obtain reliable data so that in future times the feasibility of such operations can be determined with more accuracy, providing greater safety for all concerned.

More Complex Conditions

A frequent condition which is encountered consists of swell with a 4- or 5-foot significant height occurring simultaneously with a local sea with a 3- to 4-foot significant height. The sea is less dangerous than the underlying swell in these conditions, and yet the sea may completely mask the swell unless great care is taken to detect it. The conditions are not the same as in figure 6.5, because the sea is considerably higher and the swell is considerably lower so that the swell underneath the sea cannot be easily seen.

"The commonly encountered sea for offshore rescue operations nearly always includes a wind-driven chop or sea partially concealing one or more swell systems." Hence, a typical condition is one in which there are cross swells which consist of two different systems arriving from different storms at a distance, with a local sea on top of them. This produces very hazardous landing and takeoff conditions because of the simultaneous presence of three different wave systems. In all of the cases mentioned, sea plus one swell, or sea plus several swells, the same law would hold: there should be areas of the sea surface where the waves are relatively low, and these are the areas over which an attempt should be made to land or take off the aircraft.

"The major hazard to a seaplane operating in difficult wind-driven seas is from a wind-driven sea 'peaking' on top of a swell. On several occasions these seriously damaged the aircraft—in one case carrying away a flap and in another case leading to the loss of a float. Where extended flaps have low freeboard it is expedient to partially retract the flaps immediately after touch and down when operating in a confused wind-driven sea."

In such complex conditions the wave forecaster can help very much in aiding the pilot. With the techniques given in this manual the wave forecaster should be able to say that certain conditions exist from a careful study of his maps and a careful preparation of a forecast for the point at which the operation is to be carried out. Therefore briefing the pilot on the state of the sea to expect and the number of independent wave systems present is a job which the forecaster should be able to do.

If this cannot be done, suggested techniques for the pilot to carry out on his own have been given in the reference which has been cited. These techniques are summarized now.

Suggested Pilot Techniques for Determining Sea and Swell Conditions

"A pilot contemplating a landing in the open sea should make an orderly study of that sea checking the following points particularly:

(a) From two thousand feet or higher, is a definite ground swell perceptible? If so, turn and parallel it and set the directional gyro on zero.

(b) Drop a smoke float on the water and circle it keeping it sharply in sight. As it rises on the crest of a swell, start timing it with a stop watch or sweep second hand. Count its passage over three or five or so successive swells, and clock it again on the crest of a swell. Divide the total time in seconds by the number of swells passed to get the period of the swell in seconds. The velocity of the swell in knots is roughly equal to 3 times the period of the swell in seconds. The distance between successive swells in feet is equal to approximately five times the square of the period of the swell expressed in seconds. Now the pilot knows a little about the swell.

(c) Go down to several hundred feet and study the swell again. Fly parallel to it. If your directional gyro is not very close to zero or 180, the wave system you are now looking at is entirely different from the one you measured at 2,000 feet. If there are prominent groups of swells larger than their fellows but all swells appear from the same direction, assume that two wave systems of different periods are rolling in the same direction. Measure the period and compute the length and velocity of the swell noted at low altitude just as you did for the big ground swell, which you now cannot see.

(d) Estimate the force and direction of the surface wind by seaman's eye from the surface sea condition and the smoke float observed at a very low altitude.

(e) Make a wide circle at 40 to 50 feet steadying momentarily on cardinal points to observe on which heading the sea appears less boisterous. The pilot's best heading for a landing is probably the one on which the sea appears smoothest when observed from low altitude if such a heading will not throw him into the face of either observed wave system.

(f) Study the sea, looking for groups of waves which appear markedly higher than those about them. If such waves are strikingly apparent, the landing is apt to be either very easy if the landing run avoids the seas in phase or very dangerous if it strikes them.

(g) Having weighed what he has learned about the swell, the surface conditions, and the wind, the pilot should decide on a heading for his landing which will give him a minimum hazard of running into the face of either swell and which will provide the best promise of a relatively smooth and short runout of speed after landing."

Comments on Suggested Pilot Techniques

The above quotation is an excellent description of the complexity of the sea surface. Just one comment on sections (c) and (f) needs to be made. The presence of very high waves and irregular sea as observed under these conditions may simply mean that a broad frequency spectrum is present from just one storm. A broad frequency spectrum, by its very nature, means that the waves are irregular. Then the conclusion which is given in section (f) follows quite logically. The wave forecaster, when he has correctly determined the frequency band width of the spectrum on which the seaplane is to land, can therefore say something about the irregularity of the waves.

Reverse Propellers

"In the sea landing tests with the PBM5A, XP5M, and UF, reverse propellers were available and these were found most effective in shortening the landing run and reducing the punishment the hull had to take from the sea. In more than 100 landings the propellers were reversed with full throttle immediately upon touchdown and this was found safe even though on a few occasions reverse failed to work on one side and power had to be taken off the backing engine very quickly. After approximately sixteen different pilots had made these landings, they all agreed that they would employ reverse thrust on a sea landing if they had it available. The water flaps now available on the P5M were not installed when the XP5M was tested in the sea and these should also be advantageous for shortening the landing run and that uphappy moment when the aircraft is neither fish nor fowl."

Jet Assisted Takeoff (Jato)

"The use of jet or rocket power to assist in making a quick takeoff is one of the most important contributions ever made to safe seaplane operations in rough water. The use of jato gear during these tests improved the control of the aircraft. If the jets are fired after the airplane has got good steerage way, the failure of one jet to fire does not substantially affect the controllability of the airplane, and will not embarrass any competent pilot except in that he receives only half the jet thrust expected." Jato is recommended for takeoffs in rough water. The best procedure "is to fire maximum jet power immediately after takeoff power taken from both engines."

Summary

The results just outlined in connection with seaplane operations give the wave forecaster some practical suggestions by which he can help the pilot in such a situation. It is unfortunate that space does not permit a more detailed summary of the results of the quoted report, which contains a wealth of information. In this manual an attempt has been made to summarize those parts of the report which actually apply to the wave conditions involved.

New Seaplane Designs

As reported by Francis and Katherine Drake in an article entitled "Biggest News Yet in Jet Flying" (Reader's Digest, July 1953), new and radical seaplane designs have been developed by Ernest C. Stout of Convair. One new model has retractable hydro-skis as landing gear. When the jet plane settles in the water upon slowing down, it floats since its fuselage is water-tight. These new planes are designed "to land and take off in seas far rougher than anything an oldtime flying boat could handle."

In a few years, consequently, the wave forecaster may have many requests for forecasts of open-sea landing conditions for these new planes. It appears that the same basic concepts discussed above will still be valid for such aircraft. The waves will still be like a bumpy concrete runway, but the frequency with which the plane will encounter the bumps will be much higher than in the case of a flying boat landing at a slower speed because of the high speed with which jet planes land.

Conclusion

The problems treated in this chapter are of necessity not covered fully. There are not many references in the literature which are as practical and useful as the ones on ship speed and seaplane operations which were quoted in this chapter. More data and more papers on such problems are definitely needed. The wave forecaster can help to eliminate this lack of information by applying the concepts of this manual to the many problems which arise in the operation of planes and ships on the surface of the sea.

Chapter VIII

VERIFICATION BY FORECASTS AND OBSERVATIONS

Introduction

The purpose of this chapter is: (1) to carry out some forecasts that illustrate the procedures used; (2) to verify these forecasts by observations; (3) to test other forecasts against independent observations; and (4) to give data that verify the values of the "period" range and the average "period" given in Chapter II. The results will show how well the procedures actually work.

Some of the forecast procedures are short and simple. Others are more involved. Each forecaster must learn the most efficient and convenient procedure for his own capabilities. This is accomplished only by experience and by absorbing the theory behind the forecasting procedure. In many cases, especially in those involving the propagation of waves, a fundamental knowledge of such terms as spectrum, frequency band, and dispersion will help the forecaster devise time-saving short cuts or simplify complex weather situations.

Four wave forecasts are discussed. The first is a simple case of wave generation under the influence of the wind. The next two show the procedure followed when the wind stops over a fetch and the waves decay. The last case is the more difficult Filter III storm. It is difficult because a more complicated filter has to be used.

In all cases, wave observations made at the forecast time are given for comparison with the forecast values. Verification is important in forecasting for both the improvement of technique and for the possible revision of long-range wave forecasts.

The results of some observations made by Weather Bureau personnel on the Atlantic Weather Patrol are used to make an independent verification of the forecast procedures. The results show that the forecasting procedures and the height observation methods given in Chapter IV work quite well.

Data from the observation of fully developed seas in the North Atlantic are shown which verify the theory of the "period" range and

the average "period." The verification is good for average "periods" up to 9 seconds, but there may be a tendency for the observer to neglect the shorter "periods" in heavy seas, and thus obtain a slightly higher average "period" than is predicted by these methods.

Forecasting Waves in a Generating Area

Example 8.1

On 4 May 1948, a low pressure area developed off the east coast of New Jersey and subsequently intensified into a mature storm. Figure 8.1 shows the storm at 1330, when the wind was still moderate and the waves were just starting to increase, and at 1930, when the wind and waves reached their maximum values.

Suppose that a forecast is required for the northern New Jersey coast over the period of time between 1330 and 1930. The procedure used in making such a forecast follows.

Step 1. Locate Fetch

The fetch area is the area of ENE winds bounded by the shore on two sides, the occluded front on a third, and the curvature of the isobars on the last. Although the dimensions change slightly from the first map to the second, there is an area influenced by ENE winds in both maps that is approximately 375 NM long.

Step 2. Determine Wind Velocity

Normally the average of the observed winds is compared with the corrected geostrophic wind to get the actual wind velocity. In this case, however, the curvature of the isobars is enough to make geostrophic wind computations questionable. Since there are enough ship and coastal reports to only give a representative wind velocity, the observed winds will be used.

When the wind reports over the fetch are averaged, it is found that a high Beaufort 5 occurred at 1330, and that the wind increased to a Beaufort 6 by 1930. Coastal reports, in addition, show that the velocities should be 20 knots and 24 knots, respectively.

Step 3. Determine Wind Duration

From the earlier maps (not shown here), it was found that the wind had a velocity of 20 knots for 10 hours previous to the 1330 map. The wind velocity increased slowly from 20 knots at 1330 to 24 knots at 1930 and the durations which should be used are 10 hours for 20 knots, and 6 hours for 24 knots.

The latter duration must be corrected however, for the waves present at 1330. Follow the procedures described in Chapter V for this correction. Enter the generation graph with the lower velocity (20 knots) and 10 hours to find that a 20-knot wind raises an E value

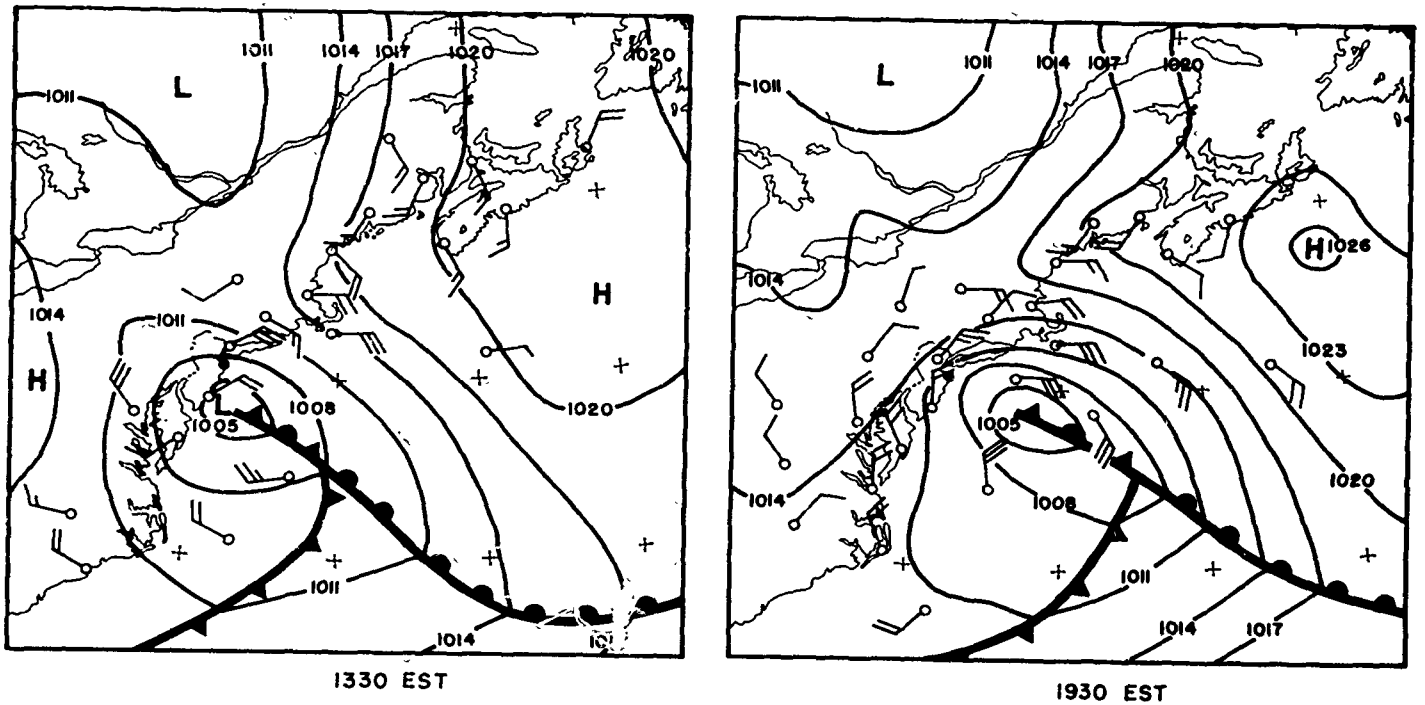


FIGURE 8.1 Weather conditions during an East Coast storm, 5 May 1948.

of 3 ft². in 10 hours. Move horizontally to the 24-knot line and read off 9 hours as the time required for a 24-knot wind to raise the same E value. Then 9 plus 6, or 15 hours, is the effective duration time for 24 knots.

Step 4. Use C. C. S. Graphs

From steps 1 through 3 the following values have been determined.

Time	*Wind Velocity*	*Wind Duration*	*Direction*	*Fetch*
051330R	20 knots	10 hrs.	ENE	375 NM
051930R	24 knots	15 hrs. (effective)	ENE	375 NM

In a choice between fetch or duration graphs the parameter giving the lowest E value is always used. This is explained in Chapter II, but in this case the sea is fully developed so table 2.4 can be used instead of the C. C. S. curves.

With these values the forecast would be as given in table 8.1.

Table 8.1—Forecast and Observed Wave Conditions for Example 8.1

	Forecast	*Observed*
	051330R	051200R
E, ft.²	7.7	---------
$\sqrt{E}$, ft.	2.78	---------
Av. Ht. ft.	4.9	4.8
Sig. Ht., ft.	7.9	7.6
Av. 1/10 Ht. ft.	10.0	8.9
"Period" Band, sec.	3–11	3–11
$\widetilde{\widetilde{T}}$, sec.	5.7	6.0
	051930R	052000R
E, ft.²	22.0	---------
$\sqrt{E}$, ft.	3.5	---------
Av. Ht. ft.	6.3	5.9
Sig. Ht., ft.	10.0	10.0
Av. 1/10 Ht. ft.	12.7	14.0
"Period" Band, sec.	4–14	3–12
$\widetilde{\widetilde{T}}$, sec.	6.3	6.6

The period band is found by taking all the periods from 3 percent to 95 percent of E as explained in Chapter II. Since this is a fully developed sea, the average period is found from formula (2.5) where

$$\widetilde{\widetilde{T}}=0.285\,v$$

and v is the wind velocity in knots. The values can also be found from the tables in Chapter II.

Step 5. Verify Forecast

The fifth and last step is to check the forecasts against observations of the waves. During the day a wave gage mounted on a pier at

Long Branch, New Jersey, recorded the waves at 1200 and 2000. The observed values, given in table 8.1, were computed from these records.

These observed values check very well with the computed heights and periods. There is a slight tendency for the observed heights to be low, but since the pier is in relatively shallow water, this error might be due to shallow water effects. Waves from this direction are not affected as strongly by refraction as waves from the southeast.

Forecasting Waves in a Generating Area After the Winds Stop

One of the most frequent wave forecasting problems is to forecast the decrease of the waves in a fetch once the winds have stopped. The length of time required for the waves to disappear depends on the fetch length and the wave spectrum present. Thus, for a short fetch with high period waves, the waves will decrease in a very short time. The theory is explained in Chapter III, so only some examples of this type of forecast will be given here.

Example 8.2

Figure 8.2 shows the isobars and wind observations at 101830Z of a storm which lasted two days and produced a fully developed sea. Twelve hours later, at 110630Z, the ridge moved over the ship (point A), and the winds decreased to almost calm. The forecast required in this example is to determine when after 110630Z the waves will be low enough to permit marine operations. Filter IV is used and the procedure below.

Step 1. Determine fetch

The fetch is measured from the forecast point to the shore, a distance of 250 NM. In this example, the fetch width is unimportant, since the forecast point is within the fetch.

Step 2. Determine wind velocity and duration

In a problem like this, the forecaster usually has been following the weather, map by map, and he is waiting for signs of a wind shift or a decrease in wind velocity. Thus, assume that the forecaster knows that a 24-knot wind has produced a steady state and that the 1830Z map is the latest map received.

To find the characteristic wind velocity at 1830Z, he will first compute the geostrophic wind and then compare it with the observed winds to get the final wind velocity. The isobars are nearly straight, and they are approximately 1.6° of latitude apart. At 35°N, this gives a geostrophic wind of 43 knots before correction.

Since the isobars are straight there is no curvature correction. For the temperature correction, it is found that the airmass is just a

few degrees colder than the sea temperature, so 65 percent of 43 or 28 knots is used as the surface wind.

The average of the observed wind reports results in a Beaufort 6 wind. This is a velocity of approximately 24 knots. The average

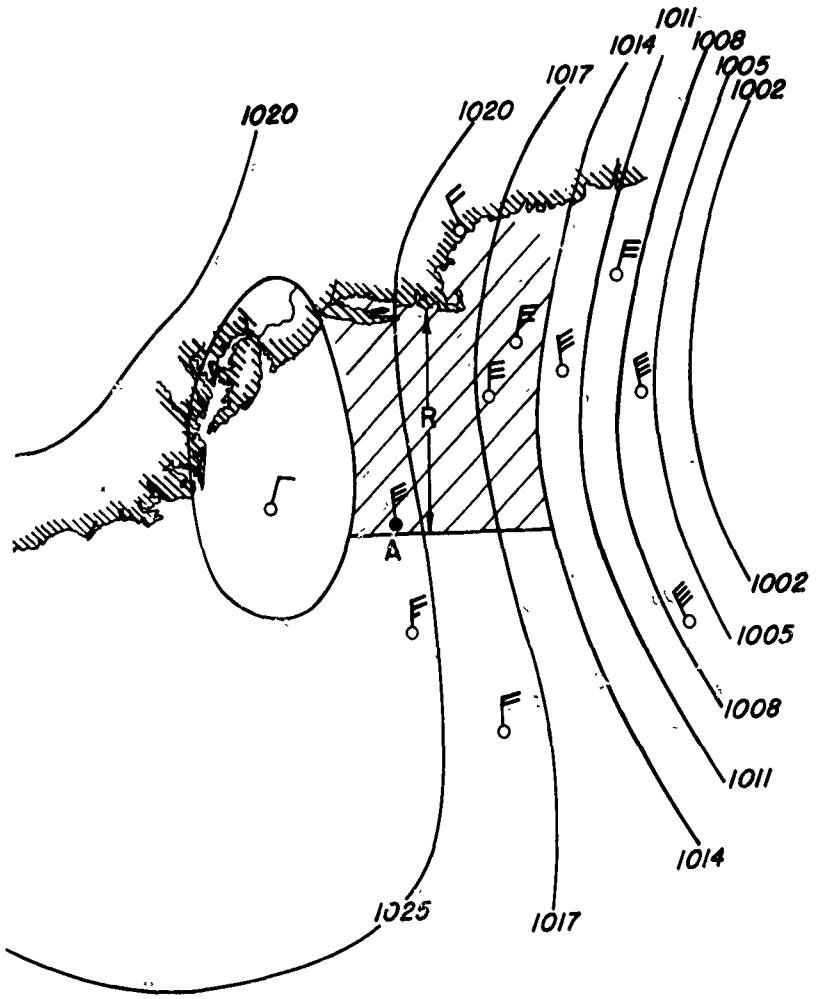


FIGURE 8.2 Weather map for 1830Z 10 Feb. 1953.

of the computed and observed wind velocities is 26 knots, and this value of the velocity is assigned to the 1830Z map.

The duration of this 26-knot wind is 6 hours, but since the 24-knot wind had an 18-hour duration (from previous maps), the duration at 1830Z will be 24 hours.

Step 3. Compute variation of E over the fetch

Since the fetch is limited by land, the waves will naturally be highest at the seaward end and almost calm at the shoreward end of the fetch. The E values will thus increase as a function of increasing distance from shore (since the wind velocity and duration are constant).

To find the E values over the fetch, enter the C. C. S. graph with 26 knots and fetch lengths of 25, 50, 100, 150, 250 NM, respectively. From the intersection of these fetch values with the 26-knot line read off the E values appropriate to the different locations in the fetch. Compute the significant height for each E value and record these data as in table 8.2.

Step 4. Compute Frequency Band

The frequency band can be read from the bottom scale in the process of determining the E values of step 3. As the distance from shore increases, the frequency band increases.

Step 5. Compute Travel Time of Each Frequency

Use the dispersion graph for this step. Enter the graph with the lowest f value found at each distance, and the distance from the ship. This gives the time necessary for each frequency to travel from its location in the fetch to the forecast point. The frequency selected at each point is the lowest frequency present. This means that this particular frequency will move past the ship, and only higher frequencies will be observed thereafter.

Step 6. Verify Forecast

Wave observations were made at ship A at 111230Z and 112130Z. The heights recorded at those times were 13 feet and 9 feet, respectively. Compare these values with the forecast height of 13.6 ft. at 111230Z, 10.6 ft. at 111930Z, and 6.3 ft. at 120330Z. There is very good agreement on the rate of wave height decrease.

The final forecast might be worded:

110630Z WAVE HEIGHT NOW 15 FEET. IT WILL DECREASE TO 10 FEET. BY 111700Z. SEA CONDITIONS WILL BE GOOD BY EARLY MORNING OF 12 FEBRUARY.

Table 8.2—Computations for Example 8.2

Distance from ship R, NM	Distance from shore F, NM	E ft^2	Sig. Ht. ft	Frequency band	Travel time hrs.	Travel time date	time	Month
0	250	29.8	15.5	.06–∞	0	11	0630	Feb.
100	150	24.0	13.8	.085–∞	6	11	1230	Feb.
150	100	15.0	10.9	.110–∞	11	11	1730	Feb.
200	50	4.8	6.2	.160–∞	21	12	0330	Feb.
225	25	1.8	3.8	.200–∞	30	12	1230	Feb.

Example 8.3. Short Fetch With Offshore Winds

Another example of a Filter IV storm is discussed now, but this time the fetch is very short even though the wind velocity is the same. In a case like this the waves decrease rapidly as the longer periods move past the forecast point within a short time.

On 20 and 21 November the height observations given in table 8.3 were made on a ship 85 miles off the coast.

Table 8.3—Forecast and Observed Significant Wave Heights, Feet

Date	*Time*	*Month*	*Observations*	*Forecast*
20	2130	Nov.	--------	9.7
21	0030	Nov.	8	7.5
21	0330	Nov.	--------	4.8
21	0630	Nov.	5	---------
21	0930	Nov.	--------	4.0
21	1230	Nov.	3	---------
21	1830	Nov.	2	---------
21	2130	Nov.	--------	1.0
22	0030	Nov.	1.0	---------
22	0630	Nov.	calm	calm

To explain this rapid decrease in wave height, consider the weather and fetch at 20 1830. The forecast situation is shown in figure 8.3 where a 26-knot wind has raised a fully developed sea over an 85-NM fetch. In the next 3 hours the wind decreased to 5 knots with variable directions. If a Filter IV is used, the decrease can be understood.

Assume that the forecaster already knows that a 26-knot wind has raised waves over the fetch and that at 20 2130 (3 hours after map), the wind stopped and the wave decay started. The first step is to compute the E variation and the significant height over the fetch area. Table 8.4 shows these calculations and the other necessary data. The procedure is exactly similar to example 8.2, although the waves decrease much more rapidly.

Table 8.4—Computations for Example 8.3

Distance from ship R, NM	Distance from shore F, NM	E ft^2	Sig. Ht. ft	Frequency band	Travel time hrs.	Travel time date	time	Month
0	85	12	9.8	.12–.∞	0		202130	Nov.
25	60	7	7.5	.12–.∞	3		210030	Nov.
50	35	3	4.9	.15–.∞	6		210330	Nov.
75	10	2.6	4.2	.24–.∞	12		210930	Nov.
80	5	1.1	1.8	.30–.∞	24		212130	Nov.

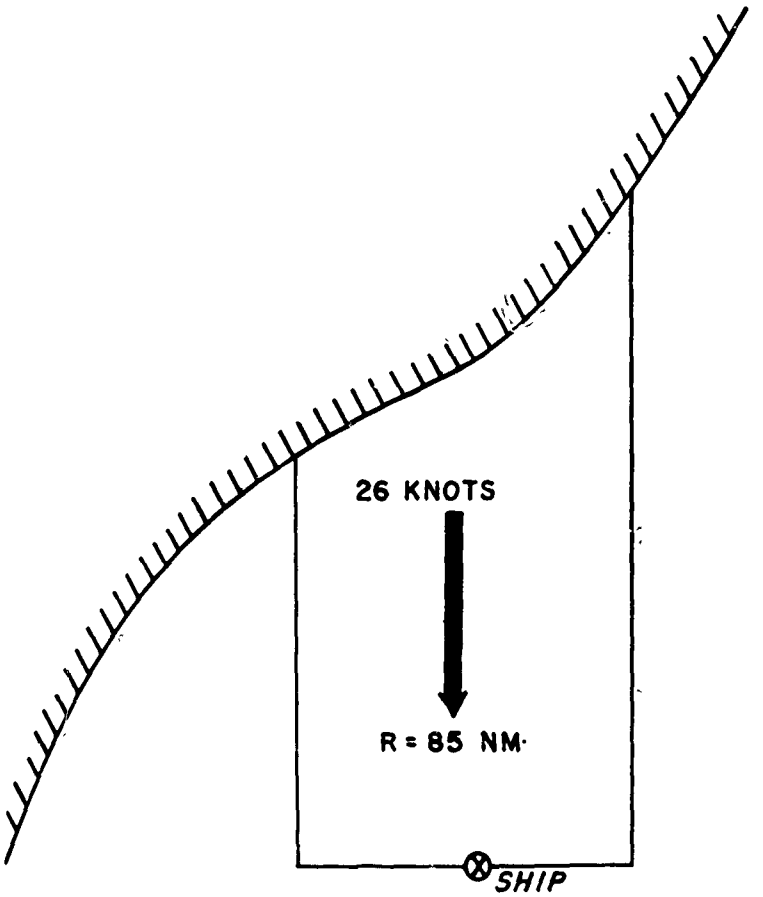


FIGURE 8.3 Fetch and ship position at 1830Z 20 Nov. 1951.

The Propagation of Waves Outside of the Fetch

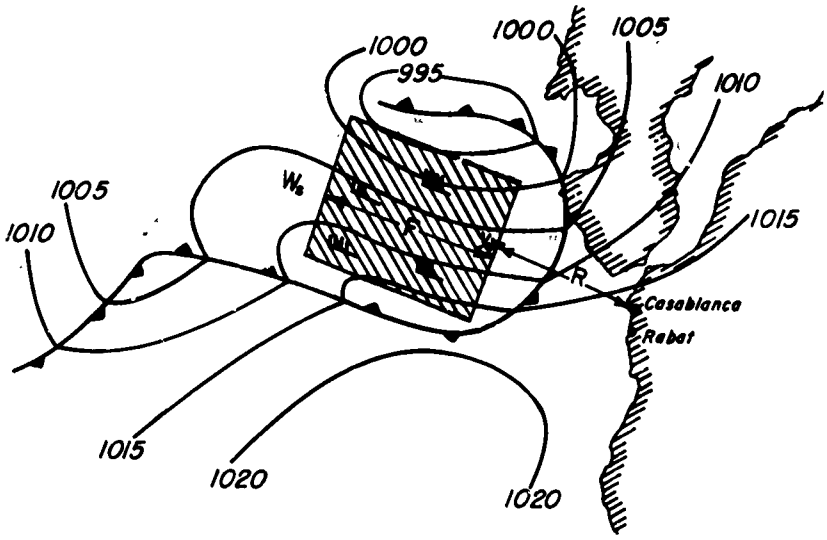
Example 8.4

The storm which produced the waves to be forecast is shown in figure 8.4 where the important distances are indicated.

A summary of previous weather maps shows that for 24 hours previous to this map a cold front had been moving eastward. Its speed of movement was high enough to prevent any waves from moving out ahead of the fetch. At 1300, however, the front slowed down and a secondary low to the east started to intensify. As the secondary low intensified, the wind field over the fetch changed to a cross wind from the south. The front meanwhile moved slowly eastward but without the strong winds evident at 1300.

Since there was a well-developed steady state sea at 1300 over the

fetch, and the fetch itself did not move (as the wind died out or changed direction), the waves in that fetch can be treated as stationary at 1300. The results might be questionable since a light following wind occurred over most of the period of decay, but if its effect is considered negligible, then the Filter III method is valid.



$W_s = 600$

$\theta_d = 6°$

$x_o = R = 600$

$F = 800$

FIGURE 8.4 Weather situation over the North Atlantic at 1300Z, 7 Nov. 1931.

Step 1. Measure Fetch and Decay Dimensions

The forecast point for this example is Casablanca, as indicated on the map. WNW winds extended from just behind the front to where the isobars curve. This is the fetch (indicated by hatching), and it is 800 NM long and 600 NM wide. The distance from the leeward edge of the fetch to the forecast point is 600 NM.

Step 2. Determine Wind Velocity and Duration

From the average of the observed velocities and the computed geostrophic velocity, a representative velocity of 28 knots was selected for this map. Investigation of the previous maps shows that the wind was approximately 28 knots for the last 24 hours. Thus the velocity and duration to be used at 071300Z are 28 knots and 24 hours.

Step 3. Compute θ_3, θ_4, and the Angular Spreading Factor

From figure 8.4, values of θ_4 and θ_3 are $-20°$ and $+35°$. The angular correction factor therefore is 70 percent−16 percent or 54 percent. The value, 54 percent, represents the percent of the energy in a given frequency band which leaves the storm and arrives at the forecast point.

Step 4. Determine Travel Time of Smallest Frequency

The smallest frequency is found from the generation graph where for a 28-knot wind at 24 hours the frequency band is from .064 to infinity.

Enter the dispersion graph with f=.064 and R=600 NM. This gives t_{ob}, the travel time, as 25 hours. That is, no waves of appreciable height will arrive at Casablanca from this storm before 081400Z.

Step 5. Compute f_2 and f_6

The forward edge of the envelope of each frequency at the front of the storm travels 600 NM. The frequencies as they arrive are indicated by f_2. The rear edge of the envelope of each frequency travels 600+800 or 1400 NM. The frequencies which have passed are given by f_6. Use the dispersion graph and consecutive values of t_{ob}=24+6, 12 . . . . hours for both R and $R+F$. This gives the band of frequencies from f_6 to f_2 that is present at Casablanca at each of the times.

Table 8.5 shows the results of these calculations. The time continues until the waves start to decrease, and further computations of the same type would extend the forecast until the waves decreased to zero. Chances are, however, the weather locally would change enough to make any such long-range forecast invalid.

Table 8.5—Computations for Example 8.4

Elapsed Time, hr. Date-Time	24 081300	30 081900	36 090100	42 090700	48 091300	54 091900	60 100100
f_2	.062	.076	.092	.106	.122	.136	.152
T, sec.	(16)	(13)	(11)	(9.4)	(8)	(7.3)	(6.6)
f_6	.050	.050	.050	.050	.052	.058	.064
T, sec.	(20)	(20)	(20)	(20)	(19)	(17)	(15.5)
E_2, ft.^{2}	40.5	35.5	28	20.5	13.5	9.5	7.0
E_6, ft.^{2}	41.5	41.5	41.5	41.5	40.0	40.5	39.5
E, ft.^{2}	1.5	6.0	13.5	21	27.5	31.0	32.5
54%	.8	3.2	7.3	11.0	15.0	16.7	17.6
$\sqrt{E}$, ft.	.9	1.80	2.7	3.3	3.9	4.1	4.2
Sig. Ht. ft.	2.5	5.1	7.6	9.4	11.0	11.6	11.9
T Band 16-20	16-20	13-20	11-20	9.4-20	8-19	7.3-17	6.6-15.5
$\tilde{T}$ (est) sec.	18	16.5	15.5	15.2	13.5	12.1	11.0

Step 6. Compute *E* Values

The procedure is similar to the two previous examples except that in this case both frequencies have E values. The E values decrease much more slowly in this case than in the Filter IV cases as both the length of the fetch and the distance of decay tend to make changes slow.

Multiply E by 2.83 for the significant height and average the period band (for each time) to obtain the final forecast values.

Step 7. Verify

Observations at Casablanca and Rabat for the morning of 9 November indicate close agreement with these forecast heights and periods. Both observations are presented in table 8.6 for comparison.

Table 8.6—Observed and Forecast Values for Example 8.4

	Observed at Rabat	Observed at Casablanca	Forecast for morning
Sig. Ht. Ft.	9	13	9.5 to 11
$\bar{T}$ sec.	15	15	13 to 15

Filter II Forecasts

No example of Filter II forecasts has been given, because no clear-cut documented weather situation could be found in literature or in reported weather situations. Cases in which the weather pattern called for Filter II occurred frequently, but when they occurred, there was no wave observation to verify the forecasts.

A Filter II forecast differs from a Filter III forecast in that the fetch moves more slowly than the wind or remains stationary. A strong gradient between a stationary high and a stagnating low frequently provides such a situation. A full wave spectrum is generated over the fetch area. But the wind continues, and the various frequencies propagate out from the leeward edge of the storm for a number of hours until the winds cease. In a sense, the conditions before the wind ceases can be thought of as a Filter I forecast with a Filter III forecast begun just as the wind ceases. The combination is a Filter II forecast.

A Test of the Forecasting Methods

Procedure Followed

During the period from 8 February through 21 February 1953, Weather Bureau personnel on Coast Guard ocean station vessels made wave observations according to the methods and procedures described in Chapter IV. These data were studied and analyzed. The data

showed that low waves were rarely reported because the observers tended to see only the larger waves. When the wave height data were analyzed according to procedure C as described in Chapter IV, the reported heights followed the theories described in Chapter I quite closely.

After the data were analyzed, a series of twelve observations was selected which contained reports of the highest significant waves and which appeared to be the most complete in the sense that weather data from the ships were available and the data were internally consistent.

The dates, times, and locations of these twelve observations were given to a forecaster who did not know what the wave observations had been or the results of the analysis of the data. He prepared wave forecasts for each of the twelve observations. No checks were permitted, and no wave observations for past times were given him. The only thing he knew was what the weather had been.

Such a verification procedure is a very severe test. A practical forecaster usually knows how well his past forecasts have worked out, and often he knows the wave conditions for past weather conditions. His forecasts can, therefore, be more accurate because he has more knowledge to apply to the forecasts. The test applied to these theories is, therefore, quite severe. Unfortunately in any type of forecasting theory, if the forecaster knows what the answer is supposed to be, as was the case in the preceding examples, the answer can usually be obtained by adjusting a few numbers here and there. This could not be done in this test because the answer was not known.

Results of Forecasts

The results of this test are tabulated in table 8.7. The observed values as reported by the U.S.C.G.C. *Mendota*, *Barataria*, *Duane*, and *Unimak* are given along with the forecast values as prepared by the forecaster.

The observed significant heights, computed according to procedure C and the results of Chapter I, range from 28 feet to 9 feet. The average of all the significant heights was 20 feet.

Since there were 50 height observations for each forecast, the 90 percent confidence limits of the reported significant heights can be computed according to the results of table 4.3. For example, a reported significant height of 28 feet as in the first entry, when based on 50 height observations, means that 90 percent of the time the true value of the significant height will lie between the values of 25 feet and 32 feet if many more observations are made.

The column marked "Error" is the difference between the forecast height and the observed height. Thus the first forecast was 8 feet too low. Of the twelve forecasts, seven were correct to within ± 5 feet.

Table 8.7—Observed and Forecast Wave Data During February 1953

USCGC	Date time	Position	Observed Sig. Ht. Ft.	90% Confidence Limit Ft.	Forecast Sig. Ht. Ft.	Error, Ft.	Band Error, Ft.	Observed ($\tilde{T}$) Range, Sec.	Observed Average ($\tilde{T}$) Period, Sec.	Forecast ($\tilde{T}$) Range, Sec.	Forecast Average ($\tilde{T}$) Period, Sec.	Average ($\tilde{T}$) Period Error, Sec.
Mendota	2/091200Z	51°43′N	28	25–32	20	−8	−5	7–11	8.8	4–14	8	−0.8
	091800Z	39°24′W	21	19–24	11	−10	−8	8–12	10.1	3–12	6	−4.1
Barataria	101200Z	36°40′N	17	15–19	22	+5	+3	5–13	8.1	4–14	9	+0.9
	101800Z	69°35′W	14	12–16	24	+10	+8	3–15	7.2	4–13	8.5	+1.3
	111200Z	69°35′W	13	11–15	11	−2	0	6–15	10	3–12	8	−2
	112100Z	69°35′W	9	8–10	6	−3	−2	4–15	9.3	3–10	6	−3.3
	161200Z	69°35′W	27	24–31	24	−3	0	4–11	7.2	3–11	5.7	−1.5
	162100Z	69°35′W	26	23–30	14	−12	−9	8–15	11.4	4–12	6.5	−4.9
Duane	201800Z	35°00′N	18	16–20	13	−5	−3	4–9	6.2	4–13	7	+0.8
	211200Z	48°00′W	18	16–20	18	0	0	6–13	8	4–15	8	0
Unimak	141300Z	44°00′N	27	24–31	18	−9	−6	4–16	11.2	3–10	5.1	−5.1
	141800Z	41°00′W	21	19–24	18	−3	−1	4–14	7.6	3–11	5.7	−1.9
		SUM	239		199	70	45		105.1		83.5	26.6
			19.9 ft.		16.8 ft.	5.8 ft.	3.8 ft.		8.76 sec.		6.96 sec.	2.2 sec.
		BIAS	(−3.3)				BIAS	(−2.0)				

The average error, without regard to sign, was 5.8 feet. There was a tendency to forecast heights which were too low as shown by the mean of all the errors with regard to sign of −3.3 feet.

The column marked "Band Error" is the difference between the forecast value and the nearest value of the 90 percent confidence limits. This procedure checks the forecasts against the observations in a way that does not penalize the forecast for chance inaccuracies of the observations. Even the most carefully made observations must still have this statistical error. Three of the forecasts were within the 90 percent confidence limits. Five more were within ± 5 feet of the 90 percent confidence limits. The average error was 3.8 feet.

As a summary of the accuracy of the height forecasts, there were three exact hits (within 90 percent confidence limits), four near misses (within ± 3 feet of 90 percent confidence limits), one miss (within ± 5 feet of 90 percent confidence limits), and four busts (more than ± 5 feet of 90 percent confidence limits).

The observed and forecast "period" range and the observed and forecast average "period" are also given in table 8.7. A miss in these values corresponds usually with a miss in the forecast significant height. Four forecasts of the average "period" are within one second of the observed values, and four more are within two seconds of the observed values. The average forecast average "period" error was 2.2 seconds.

The Reason for the Misses

After it was all over, the forecaster was asked if he could explain the bad misses which were made. The first two (*Mendota*, 091200Z and 091800Z) were explained because the wind data were not extensive enough, and a good wind velocity could not be found.

The miss for the *Barataria* observation at 162100Z was also explained. The wind had died down shortly after 1200Z, and the forecaster felt that the waves ought to have died down also. The fetch, however, was a long fetch, and a forecast according to the principles of Filter IV would have resulted in a correct forecast.

Summary

If the conditions under which the test was carried out are kept in mind, it can be seen that the above test is a conclusive verification of the forecasting methods. As more experience is gained in applying these methods, the accuracy should be even better.

"Period" Range and Average "Period" Verification

Verification in Above Forecasts

The forecast "period" band in example 8.1 was 3 to 11 seconds for 1330R and 4 to 14 seconds for 1930R. The observed values were 3 to 10 seconds and 3 to 11 seconds, which shows fairly good verification. The forecast average "periods" were 5.7 seconds and 6.3 seconds, and the observed values were 6.0 seconds and 6.6 seconds. This shows good verification.

The other cases were examples of the use of the filters. Example 8.4 verified well as to the "period" of the swell.

The observed and forecasted values of the average "period" and the "period" range in table 8.7 check fairly well. The differences are due to three effects. The first is the forecast error. The second is the observation error. The third could be due to error in the theory. Thus, verification of this part of the theory cannot be undertaken with these data because the three effects cannot easily be separated.

Theoretical Versus Observed Values

Observations of the distribution of the "periods" in conditions where the sea was fully developed are available. The average "periods" and the range of "periods" can be computed directly from these observations, and the empirical distribution of the "periods" can be illustrated.

These calculations are an independent verification of the theory given in Chapter II. The theoretical values of the average "period" ($\tilde{\tilde{T}}$) (equation 2.6) and of the "periods," T_L and T_u, were obtained from the theoretical energy spectrum of the waves. The theoretical energy spectrum of the waves cannot be observed directly, but the quantities which are derivable from the theoretical spectrum can be verified by observation.

The statement that T_{max} equals $\sqrt{2}\,\tilde{\tilde{T}}$ can be derived from the theoretical spectrum. There is not much direct observational evidence at present to prove that the highest swell has a period equal to T_{max} at a great distance from the generating area. The indirect evidence is quite substantial, however.

If an observer were to measure the "periods" of *all* waves (even the smallest) to pass a given point and average these values, he should theoretically obtain the value, $\tilde{\tilde{T}}$, for a fully developed sea. The conditions for counting a "period" in the observation are that the crest at which the measurement is begun must be above sea level, the trough must be below sea level, and the crest that follows must come back up above sea level. Thus, fluctuations in time similar to

those shown in space in figure 4.2 and indicated by question marks should not be counted.

In practice, small ripples which occur on the sides of big waves may satisfy these conditions, and yet they may not be observed and included in the data. Thus, these low values of the individual observed "periods" may not be observed; this may have an effect on the value of $\tilde{\tilde{T}}$ as computed from these observations when it is compared with the theoretical value.

The values of T_L and T_u, however, do give information on the part of the spectrum of practical importance to naval architects and others. The much shorter ripples are not important in describing the significant wave pattern.

Theoretically, the average "period," $\tilde{\tilde{T}}$ will not be seriously affected if it is computed from a theoretical spectrum in which the high frequency ripples have been eliminated. Thus, the neglect of very low "period" waves in the observations should not affect the value of $\tilde{\tilde{T}}$ when it is computed from visual observations. Some results will now be presented which show how well the theoretical and actual values check in a fully developed sea.

Data on Fully-Developed Seas

The statistical treatment of carefully taken visual observations indicates that in a long series of measurements, with at least 50 or 100 individual "period" observations, a reliable value of the average "period" can be obtained even in the case where the smaller, insignificant apparent waves are neglected. This is in agreement with the theoretical deductions just given. Of course, the individual observer may ask what the smallest "periods" are that must be recorded. This smallest "period" depends on the total appearance of the wave pattern and, thus, on the wind velocity. In addition, this smallest "period" depends on the skill of the observer in making the observations. A good observer will soon learn the value of the smallest "period" that can be measured in any given sea state condition. The lower limit of the significant, or dominating, wave pattern is really quite well defined for a given state of the sea.

With regard to the "periods," percentage histograms of the observed "periods" (the time intervals between succeeding crests, as defined in Chapter I) are of special interest. Given such a histogram, the average "period" is also easily computed.

A percentage histogram of the observed "periods" is prepared in the following way. First, 50 to 100 or more individual "period" observations are made. Then the number of times that the observed "periods" fall within different class intervals, say 3.00 to 3.49 seconds,

3.50 to 3.99 seconds, and so on, is counted. The ratio of these values to the total number of observations as a percentage is then plotted as a horiztontal bar over the appropriate range. The result is a percentage histogram of the observed values.

A number of facts can be obtained from these histograms. Thus if a percentage value is 25 percent, there are 25 chances in 100 that a "period" within the indicated range will occur. The average "period" can be computed by multiplying the center of the "period" range by the appropriate percentage value, summing over all terms, and dividing by 100. The "period" range is obtained by inspection of the histogram.

Incidentally, it has not been possible to derive these curves theoretically with the use of the theoretical spectrum and the principles of mathematical statistics. The problem seems to be a very difficult one. The curves shown in figures 2.5a, 2.5b, and 2.5c are smoothed results obtained from a large number of such histograms.

Histograms are available for some detailed observations made at wind velocities between 10 and 39 knots. They are based on many individual stopwatch observations of "periods" according to the procedure given in Chapter IV. Many more such histograms were employed in the preparation of figures 2.5a, 2.5b, and 2.5c. See also the section entitled, Empirical Distribution Functions of the "Periods." These explain the actual observations.

Figure 8.5 shows six histograms lettered a, b, c, d, e, and f for wind velocities of 10.6–12.4 knots, 14.5–16.3 knots, 20.4–22.1 knots, 22.3–24.1 knots, 28.1–29.9 knots, and 31 knots, respectively.

As an example, consider the histogram for winds between 28.1 and 29.9 knots (average wind velocity 29 knots) in figure 8.5(e). According to table 2.5 the significant range of "periods" in a fully arisen sea of 29 knots is between 4.6 and 16.1 seconds. The average "period" equals 8.3 seconds. The observations taken in the fully arisen sea with wind speeds between 28.1 and 29.9 knots show that the observed significant "periods" scatter between 4 and 15 seconds. This range agrees well with the theoretical range in table 2.5. The computed average of all individual "period" observations will be denoted by the symbol $\bar{\bar{T}}_b$. From the values given in the histogram, $\bar{\bar{T}}_b$ equals 8.76 seconds. Thus $\bar{\bar{T}}_b$ is slightly larger than the theoretical value of 8.30 seconds. The slight displacement of the observed average "period," $\bar{\bar{T}}_b$, toward higher values, when compared with the theoretical average "period," $\bar{\bar{T}}$, seems to be typical for visual observations at *higher* wind speeds. This may be explained by the fact that with higher seas the observer tends to neglect more of the smaller apparent waves in the composite pattern.

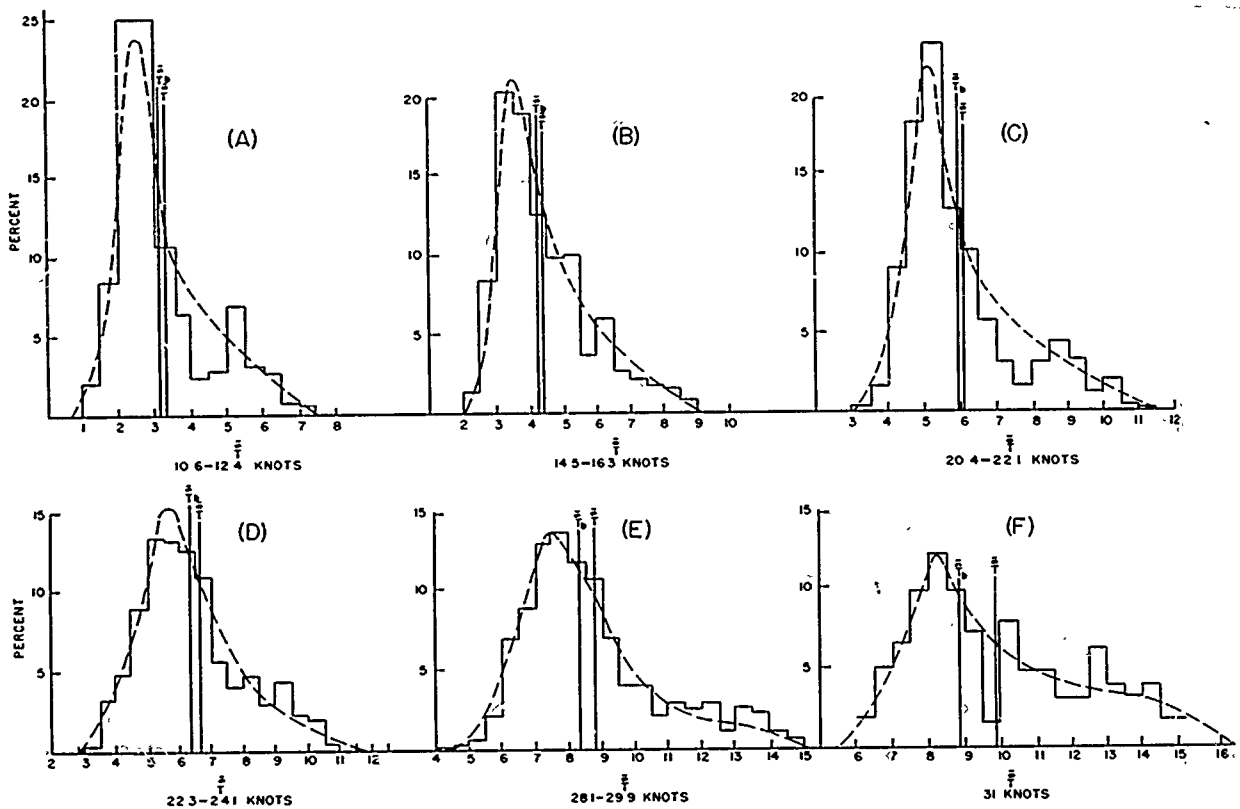


FIGURE 8.5 Wave "period" histograms.

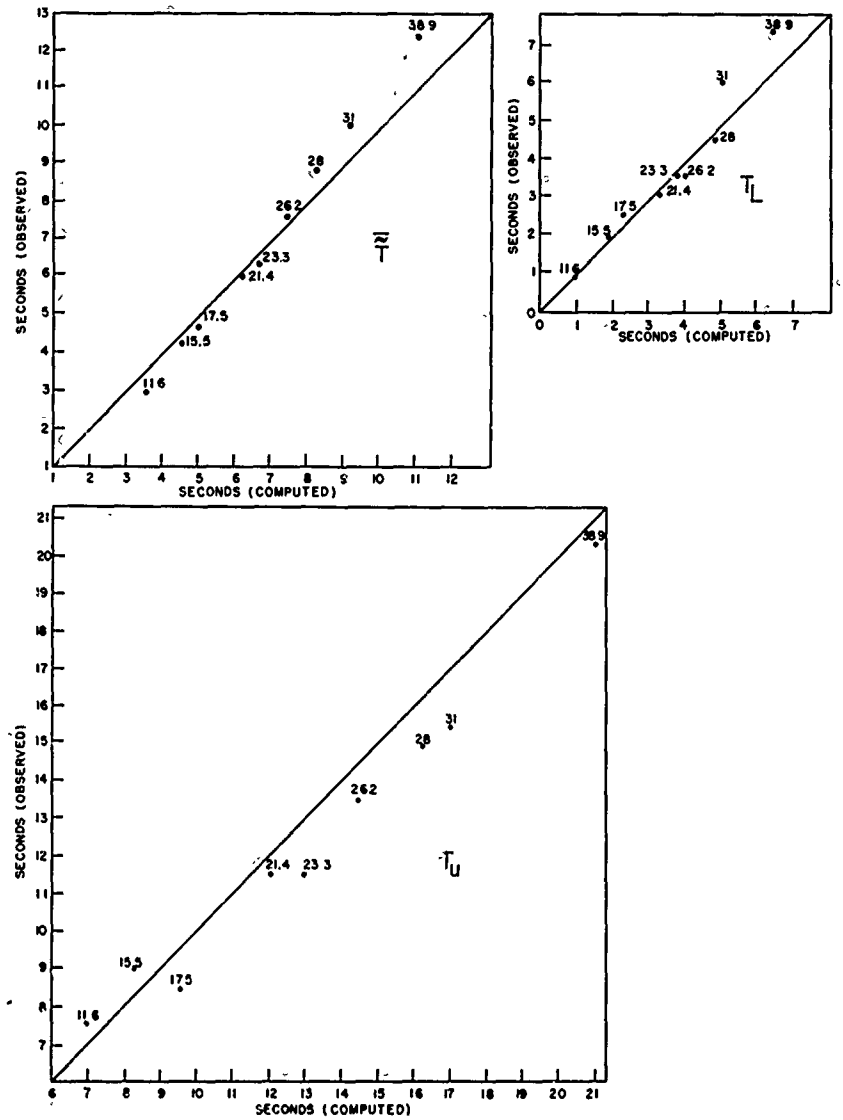


FIGURE 8.6 Verification of $\tilde{T}$, T_L, and T_u.

With light and moderate winds the agreement between the observed and the theoretical average "period" is remarkably good. The same holds for the upper and lower limits of the significant range of "periods." In figures 8.5(a)–(f), the "periods" $\tilde{T}$ and $\tilde{T}_b$ are indicated by vertical lines. The difference between the observed ($\tilde{T}_b$) and theoretical ($\tilde{T}$) values is quite small for the histograms based on many observations (a through e), and it is not too big even for case f.

Parts a, b, and c of figure 8.6 are correlation graphs of the observed and the theoretical values for the average "period" $\bar{\bar{T}}$, and the lower and the upper limits of the significant range of periods, T_L and T_u in the case of nine different series of observations. The ordinate shows the observed value and the abscissa the computed value. The figure beside each point in the graphs gives the mean wind speed in knots for the observations. Figure 8.6(a) indicates clearly the slight displacement of the observed average "period" to higher values when compared with the computed average "period" at wind velocities of more than 26 knots. This displacement seems to occur simultaneously with a corresponding displacement of the "period" of the lower limit of the significant range. If the two series with 31-knot and 38.9-knot winds permit a conclusion, the shorter apparent waves in the composite wave motion are probably more and more neglected with increasing wind speed. This necessarily must lead to a displacement of the value of the observed average "period," $\bar{\bar{T}}_b$, to higher periods.

As a whole, the differences between theory and observations appear relatively small. Even with the "period" of the upper limit of the significant range (T_u), where the deviations from the theoretical value are largest, the differences do not exceed 1.5 seconds, and in most cases they are smaller than 1.0 seconds.

Conclusions

The methods for forecasting waves which have been described in this manual are not difficult to understand and to apply. After the *ideas* behind the methods are understood, so that a forecast is no longer just a mechanical application of a set of rules, the preparation of a forecast becomes extremely simple and straightforward. After a forecaster has understood the *ideas* involved, he will see how to forecast for situations not covered explicitly in the text, such as for points in the shadow of a land mass and for wave generation areas that cannot be approximated by a rectangular shape.

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Figure 4.3.—What is the "Wave length?" Photograph courtesy Woods Hole Oceanographic Institution.

Figures 6.4, 6.5, 6.6, 6.7, and 6.8.—Aerial photographs of wave refraction taken by the U. S. Coast and Geodetic Survey and furnished to the authors by Mr. Dean F. Bumpus of Woods Hole Oceanographic Institution.

Figures 6.9, 6.10, 6.11, and 6.12.—Figures from *The direct construction of wave rays*, by R. S. Arthur, W. H. Munk, and Isaacs. Trans. A. G. U., v. 33, no. 6, December 1952. Scripps Institution of Oceanography.

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Figure 8.4.—Weather situation. Map and data from *Wind Waves and Swell*, Principles in Forecasting. Hydrographic Office, H. O. Misc. 11,275.

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