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SPECIAL OPERATIONAL DATA COLLECTION PLANS

RESEARCH REPORT

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By

Lawrence Weglarz

Approved by

Dr. Roger J. McNichols
Chairman

Dr. J. W. Foster

Dr. D. L. Shreve

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ABSTRACT

This paper developes techniques to determine sample size and sampling duration when measuring mean time between failures and mean time to repair. The exponential, normal and log normal distributions are considered.

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Life cycle analyses and the determination of system parameters, such as mean time to repair and mean time between failures, are dependent on obtaining sufficiently accurate and timely data from the organizational or user unit. The Army had been collecting this data under TAERS (The Army Engineering Report System) and is presently collecting it under TAMMS (The Army Materiel Management System). The hope of these systems was to collect all information on failure times, repair times, and corrective actions on nearly every item of equipment in the Army inventory. The result was a huge quantity of mostly useless information that required much paper work on the part of Army maintenance men and data collectors. Computer storage of all the data developed into a monumental task. The most unfortunate aspect of this system, however, is the inability to get specific management information. In most instances, even if the data elements such as localization time were recorded, the data is subject to inaccuracy and omitions since in many cases, personnel untrained in data collection are providing the information.

To conserve time and resources, the Army has now decided to discontinue TAMMS on all but a few major items and has

implemented the concept of Special Operational Data Collection. According to AMC Regulation No, 750-43, Paragraph 3B, Special Operational Data Collection is defined as:

> A data collection, processing, and analysis system controlled by the equipment proponent and specifically designed to gather accurate, timely, and complete data necessary to evaluate the performance effectiveness of Army materiel in the field.

It is the hope of this paper to present an organized and general approach to providing reliability and maintainability parameters in such a system. Because of the great size of the area of investigation, only a general framework and a few specific examples will be presented in this paper.

The problem of developing a general sampling plan for this system is complicated by the amount of non-statistical external constraints. The basic approach will be to detail the decisions necessary to arrive at a sampling technique and to fit this technique into the constraints of the specific problem. These decisions are detailed in chapter three.

As will be shown in chapter two, most literature on sampling techniques pertains to acceptance sampling where a forehand knowledge of acceptable and unacceptable characteristics allows for the development of soundly founded "optimal" acceptance sampling plans. These plans are developed in terms of α and β risks which define the probability of rejection of items with acceptable characteristics (α risk) or acceptance of items with undesireable characteristics (β risk).

The basic difference in designing sampling plans for maintenance information retrieval is that there are no acceptable or unacceptable characteristics on which to design a test. This does not mean the statistical techniques used in determining sample size are different from acceptance sampling techniques. It just means that one must have a different terminology and a different approach. One must now determine confidence intervals for the parameters of interest. It is unfortunate that these confidence intervals depend on both sample size and variance. It will be shown that even when the mean, variance and distribution are assumed known, the task of providing the most cost-effective sampling plan is prodigious. The information which is required by the Army when requesting a special data collection plan is as follows:[1]

1. Security guide.
2. Project description.
3. Basic characteristics to be measured.
4. A materiel description.
5. Proposed start and completion dates.
6. Type of units expected to participate.
7. Geographical locations.
8. Resources estimates.

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1 AMC REGULATION NO. 750-43, 22 April 70.

9. Mission objectives.

10. Description of information desired.

11. A preliminary design.

The information that lies within the scope of this report is numbers 3, 5, and 8.

CHAPTER II

LITERATURE SEARCH

The goal of the literature search will be to determine the basic Reliability and Maintainability parameters to be measured, determine and define the data elements to be collected, and provide methods to determine sampling plans. Also, the problems of external factors such as geographical location, sampling costs, and other external constraints will be considered in the Maintainability and Reliability testing program.

Reliability is defined in ARINC (1)[1] as:

> Reliability is the probability that a system will perform satisfactorily for at least a given period of time when used under stated conditions.

In order to apply this definition of Reliability, one must define what a failure is and the conditions under which the item is to be used (mission, geographical location, types of personnel).

The reliability function is the same probability expressed as a function of time. It is one of the parameters which may be used to define reliability in a quantitative fashion. It is usually measured by a survival cure.

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1 The number in the parenthesis is the location of the text in the reference list.

However, because of the increasing importance of the system characteristic known as availability, a more popular measure of reliability is Mean Time Between Failures (MTBF). It has been pointed out in ARINC (1), however, that when one does specify this parameter one also must specify the distribution of time between failures to go with it. MTBF by itself is not a measure of reliability. It is this parameter MTBF, along with the maintainability parameters, for which tests will be primarily designed.

Maintainability is defined in MIL-STD 778 as follows:

> Maintainability is a characteristic of design and installation which is expressed as the probability that an item will conform to specified conditions within a given period of time when maintenance action is performed in accordance with prescribed procedures and resources.

The period of time required to restore an item to a specified condition is called down time. Down time has been broken into many parts. The two major parts being active down time and delay down time. In active down time, one usually makes a distinction between preventive (scheduled) down time and corrective down time. Corrective down time may also be broken down into localization, isolation, repair-replace, and check out time. It is therefore very important to determine which, if not all, of these times are important to the problem at hand and design the data collection system to fit these needs.

The maintainability parameters usually measured are Mean Time to Repair (MTTR), Mean Preventive Action Time ($\bar{M}_{pt}$), Mean Active Corrective and Preventive Action Time ($\bar{M}$), and of course Mean Down Time (MDT). It must also be kept in mind that these parameters do not have a full meaning until the distribution of times is specified.

MTTR, $\bar{M}_{pt}$, and $\bar{M}$ are usually observed to be log normally distributed. Therefore by taking the logarithm of each repair time, the resulting distribution is usually normal. Care must be taken when using this technique since the antilog of the mean of the normal is not the mean of the log normal, but the median of the log normal. Delay down time does not possess this [illegible] behavior, and it is therefore a good practice to separate at least the delay times from active repair times. Delay time is usually further subdivided into logistics time and administrative time. The comparison of Mean Delay Time to MTTR is also very valuable to the maintainability engineer in determining whether system (item) design is inadaquate or whether repair facility lay out or sparing levels are inadaquate.

The majority of the statistics necessary for the determination of a sampling plan and the analysis of the data is presented in Quality Control and Industrial Statistics by Duncan (3). Most sampling plans presented in this text are acceptance sampling plans or test of hypothesis. These plans are concerned with whether a sample statistic falls within

an interval or outside an interval. Of interest to this paper are the methods of determining this interval, hopefully prior to the analysis of data.

The following chapter presents the logic necessary to arrive at a sampling technique.

CHAPTER III

DECISIONS NEEDED TO ARRIVE AT A SAMPLING PLAN

The following sections were prepared to itemize the decisions and considerations necessary to formulate a sampling plan. Only two such plans have been considered in this paper. Other plans, such as those to measure maximum time to repair or combinations of the various parameters, have been excluded since the purpose of this paper is to present only a general framework and a few specific examples. The two parameters MTBF and MTTR were chosen because of their importance in determining the availability of a piece of equipment.

The following discussion should be applicable whether one is looking at an entire system, a subsystem, or just one element of a system. It should be noted that the particulars of the plan, such as the data collection forms and the type of data collectors, will be dependent on the item in question, but the method of determining sample size and sampling time, which are presented in the following chapters, should not be dependent on these particulars.

The following section will discuss the mean time between failures (MTBF) parameter.

Mean Time Between Failures

In most cases, where a representative and sufficiently large sample exists, the distribution of the sampled mean time between failures should be approximately a normal distribution about the true mean time between failures. It is for this reason that MTBF and MTTR are particularly nice parameters to measure. If one knows the variance of time between failures, one may always determine a sample size N required to measure MTBF to the desired accuracy, as will be shown in subsequent chapter. It is the form of the distribution, however, which will determine the amount of time necessary to accomplish the N failures and will determine how the data is analysed. The exponential time between failures will be discussed in Chapter IV and the normal time between failures will be discussed in Chapter V.

Figure 1 presents the decisions necessary to arrive at a desired sampling plan. The first consideration in developing a sampling plan concerns the suspension of scheduled maintenance. If the item under consideration is replaced or parts of the item are replaced or repaired in the process of scheduled maintenance, and these actions will interfere with the acquisition of the required data, then the suspension of scheduled maintenance may be required.

If the item to be sampled is a new system, the time between failures may not be characteristic of that item in

later operation. It is for this reason that subsequent samples may be deemed necessary.

If no good estimate of the mean is available, the duration of the test is indeterminable. The occurrence of this situation is not usually very common since the procurement and design phases of life cycle usually provide this information. If this case does arrise, however, one may choose to test a small, handy sample for the mean and use this as an estimate to determine this parameter with a higher confidence in a subsequent test. If a plan with an indefinite termination date is acceptable, the need for a special mean test is not necessary.

The confidence with which the mean is measured will depend on the sample variance and the size of the sample. If a target confidence on MTBF exists, the sample size will be determined from consideration of the variance. If no estimate of the variance is available, then one may test for the variance. If the sample size chosen for this test was such that a "t" test provided sufficient confidence on the mean, one need not perform a second test. However, this is unlikely.

Finally, one must consider the form of the distribution. If the form can be assumed exponential or normal, then the methods of Chapters IV and V can be applied. If the distribution is known to be of some other form, a plan may be formed using the considerations of Chapters IV and V. If the

form of the distribution is unknown, a non-parametric test[12] should be used.

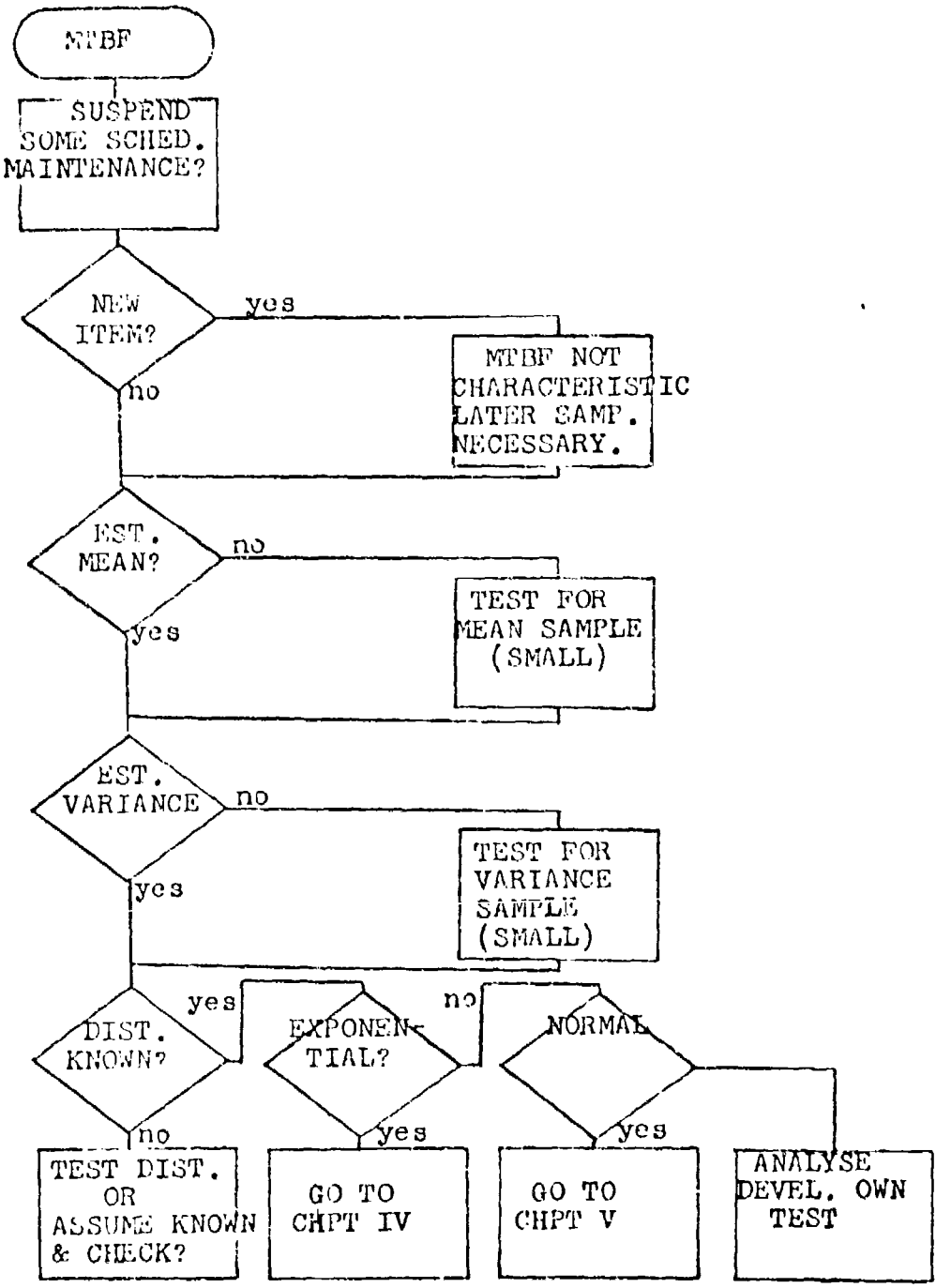

Figure 1. DECISIONS TO DEVELOPE A MTBF SAMPLING PLAN

Mean Time To Repair

The distribution of the sampled mean time to repair will also approach a normal distribution for a sufficiently representative and large sample. The sample size N needed to obtain the desired accuracy is easily calculated once the variance of time to repair is known. If one wishes to induce failures, no problem in obtaining this required number is expected. If one is using naturally occurring failures, however, one needs an estimate of the failure rate to determine the time necessary to observe the required number of failures.

Figure 2 presents the decisions necessary to arrive at a desired sampling plan. The first decision necessary concerns whether the item is newly fielded or not. Special consideration must be given to a newly fielded item since the repair times may not be representative of those times at a later point in the life of the item. One must consider performing a subsequent test to determine the changes in time to repair.

Next, we must consider the form of the distribution of time to repair. The case of log normal time to repair is covered in Chapter VI. Other distributions would have to be considered on their own merit.

A prior estimate of the mean time to repair is not usually necessary in determining the test duration if the collection of the data uses naturally occurring failures.

In this case, the time constraint is usually more dependent on the amount of time that is required to obtain the necessary failures than it is on the time required to perform the repairs. If the repair time is a significant factor in determining the test duration, it may be necessary to sample a small convenient population to get an estimate.

The sample size will be determined by the desired accuracy and the variance of the time to repair. If no estimate of this variance is available, a convenient sample should be tested for the variance. If the mean is determined to the desired accuracy by means of a "t" test, further sampling is unnecessary.

If the MTTR is to be determined within as short a time as possible, artificially induced failures should be considered. There are several problems with this method. The first is determining a representative set of failures to be induced, since this will effect the MTTR. One must also consider the facilities necessary to handle the work load.

If artificially induced failures are not used, an estimate of the failure rate or time between failures is necessary to determine the duration of a test. A convenient sample may be tested to determine this parameter if it is not known. It may even be necessary to know the variance of time between failures if the sample size is not sufficiently large.

The next chapter presents the considerations necessary to determine a sampling plan for mean time between failures when

the distribution of time between failures is normal.

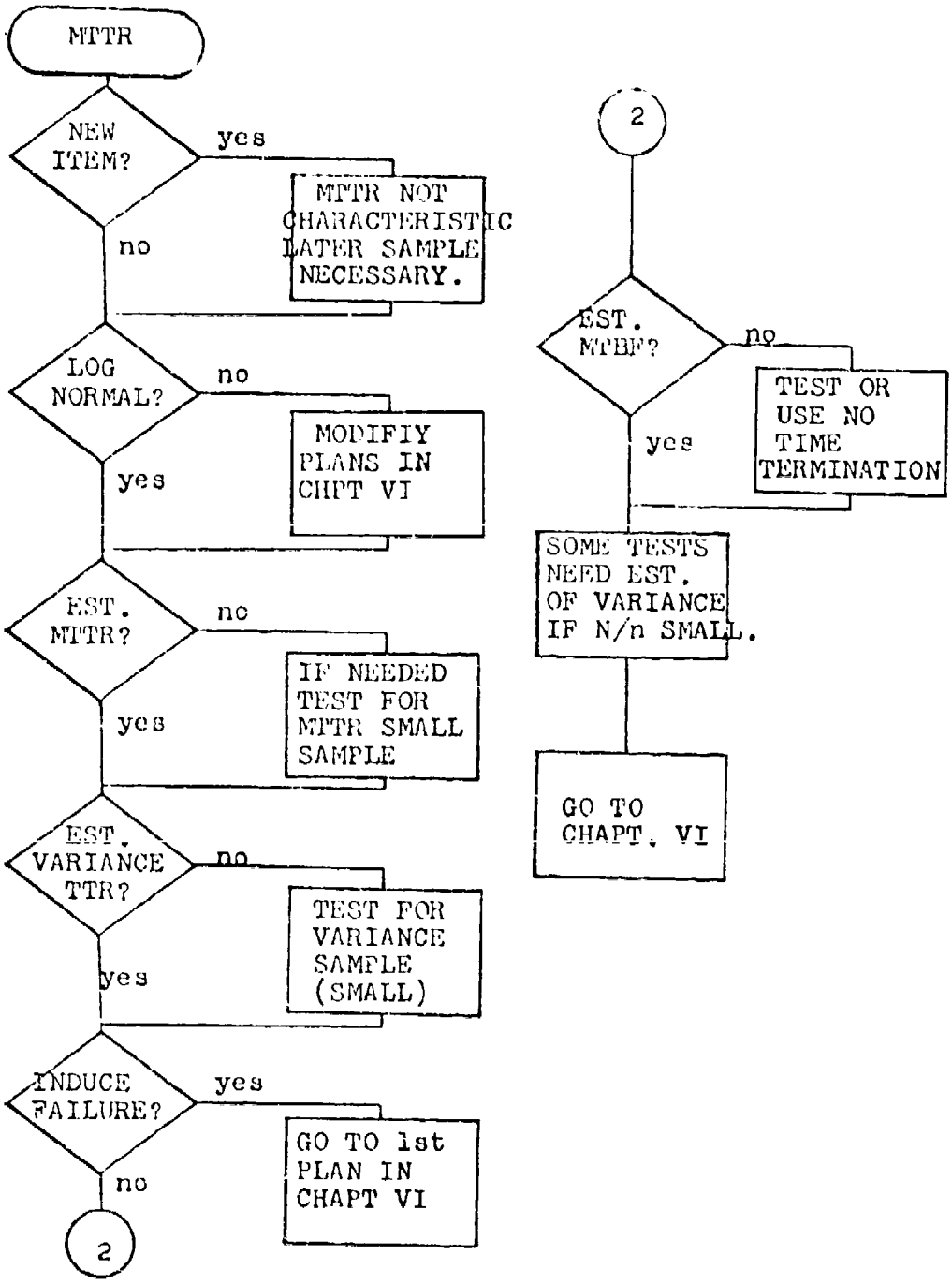

Figure 2. DECISIONS TO DETERMINE A MTTR SAMPLING PLAN

CHAPTER IV

EXPONENTIALLY DISTRIBUTED TIME BETWEEN FAILURES

The following method is useful in determining sample size and sampling time necessary to attain desired accuracy, when the distribution is assumed to be exponential and one has an estimate of mean time between failures.

One can assume that the distribution of the mean will be normal for a sufficiently large sample ($N>15$) and that the variance of the mean will be equal to the variance of the times between failure divided by the sample size.

The form of the exponential distribution is

$$f(t) = (1/\theta)\, e^{-t/\theta}$$

$$\text{with mean} = \theta$$

$$\text{and variance} = \theta^2 .$$

Therefore the variance of the normally distributed MTBF will be θ^2/N and the standard deviation will be $\sigma = \theta/\sqrt{N}$.

If one wishes a 95% confidence interval on MTBF the interval will be (from normal tables $Z_{.975} = 1.96$)

$$\theta \pm 1.96\,\sigma$$

or

$$\theta\,(1 \pm 1.96/\sqrt{N})$$

It is because the exponential distribution is a one parameter distribution that one gets the dependence of variance on the mean.

This dependence allows us to determine sample size independently of the mean. It is accomplished by dividing the confidence interval by the mean giving a percent confidence interval. For example, if one wishes the true MTBF to lie within 10% of the estimate, with 95% confidence, one calculates the sample size by the following method:

$$Z_{.975}\sigma/\theta = .10$$

$$1.96\ (\theta/\sqrt{N}\ )/\theta = .10$$

$$1.96\ /\sqrt{N} = .1$$

$$\sqrt{N} = 19.6$$

$$N = 384$$

It was by this method that Table 1 was calculated by USATECOM [5]. This number N represents the number of failures that must be observed to get the desired accuracy. The problem now is to determine a sample size and a sampling time.

The expected number of failures from a sample of size n with mean θ in a period of time T is

$$N(T) = n(1 - e^{-T/\theta})$$

This formula assumes sampling without replacement. This formula will be acceptable if the probability of more than one failure is small (large n, small T). If more than one failure per item is allowed, a different approach is necessary. This approach will be covered later. The value N(T) has already been determined from Table 1 and a curve of T/θ

TABLE 1

Failure Observation Requirements to Limit Errors in MTBF Estimates

PERCENTAGE ERROR IN MTBF	REQUIRED FAILURE OBSERVATIONS			
	PROBABILITY THAT ERROR IS NOT EXCEDED			
	.85	.90	.95	.99
5	830	1082	1537	2655
10	207	271	384	664
15	92	120	171	295
25	33	43	61	106
35	17	22	31	54

Note. Assuming an exponential distribution of times between failures.

Reference: Guidebook for Systems Analysis/Cost Effectiveness, USATECOM, March 1969.[1]

[1] Inquiries regarding this document may be forwarded to the Commanding General, USATECOM, ATTN: AMXRD-AMO.

verses N/n may be traced. The functional relationship is

$$T/\theta = -\ln(1 - N/n)$$

If the cost associated with time is $C_1(T)$ and the cost associated with sample size is $C_2(n)$, the total cost will be $C = C_1(T) + C_2(n)$. To determine a particular value of T and n one must minimize the total cost, subject to the constraint $T + \theta \ln(1 - \frac{N}{n}) = C$. To accomplish this, replace T by $-(1/\theta)\ln(1-N/n)$ in $C_1(T)$ therefore producing a cost which is dependent on only one variable n.

$$C(n) = C_1(-(1/\theta)\ln(1-N/n)) + C_2(n)$$

To determine the maxima or minima values, set the derivative of this function equal to zero. A negative second derivative implies a local minimum. If there is more than one local minimum, choose the one which yields the lowest total cost. Care must be taken, however, since n must be an integer.

The above calculations were made using the expected number of failures. It is therefore reasonable to assume that about one half the time this method is used, one is not going to receive the number of observations necessary to obtain the accuracy for which one has planned. This occurs because the distribution of the number of events in a time interval is a Poisson distribution and for most cases can be approximated by a normal. If having the specified accuracy is very important, one would like to design the test in a way that one would have a high confidence of meeting

the accuracy.

The distribution of events in an interval, when time between events is exponential, is a Poisson distribution. The mean and variance of this distribution is N. If one approximates the Poisson by a normal, which is a good approximation for large N, one can compute an effective N_1

$$N_1 = N + Z_\gamma N$$

Where Z_γ is the N(0,1) coordinate which gives the desired assurance γ. One may now use N_1 in the place of N in the previous calculations. The cost has obviously increased to get assurance of accuracy.

The question is now, "How good was the initial estimate of MTBF?". Obviously, the confidence interval had to be greater than what was desired or ther would be no need for further sampling. If the accuracy desired is needed with a high probability then knowing the variance of the estimate one may use an effective mean θ_o such that

$$\theta_o = \theta \ (1 + Z\gamma / \sqrt{N_o} \ ).$$

Z_γ is the normal coordinate for the assurance desired and N_o is the sample size used to make the first estimate of the mean θ. This modification will also cost money since it will increase the sampling time.

If ones allows the items in the sample to have more than one failure, a decrease in sample size or a decrease in the sampling time necessary to obtain the same accuracy can be expected. This occurs because the number of failures to

be observed remains constant while the number of failures per item increases. The estimate for MTBF in this case is

$$\hat{\theta} = (nT_f - T_D) / N$$

Where T_f is the duration of the test, T_D is the total down time of all failed items, n is the number of items on test, and N is the number of failures. The quantity $2N\hat{\theta}/\theta$ is a Chi Square variable with 2N degrees of freedom χ^2_{2N}. For values of $2N > 30$ one can assume that the quantity

$$\sqrt{4N\hat{\theta} / \theta} - \sqrt{4N - 1}$$

has a normal distribution with a mean θ and a variance 1.

If one wishes $\hat{\theta}$ to be within 10% of θ with a 95% confidence, the above quantity must lie between -1.96 and + 1.96. If $\hat{\theta} = 0.9\theta$ then the normal coordinate is -1.96 and

$$\sqrt{4\ N\ (.9\ \theta) / \theta} - \sqrt{4\ N - 1} = -1.96$$

If one neglects the -1 compared to large N, one calculates N = 384. One will note that this is the same solution as before, and therefore one is allowed to use Table 1 to determine N. The difference lies in the fact that we now have a different relationship between test duration and sample size.

$$T = (N\theta + T_D) / n$$

A sampling plan can be determined in much the same manner as for the one failure per item approach, the only difference being the duration-sample size relationship. One drawback, however, is the need for an estimate of total down time of all failed items. An estimate of this param-

eter is

$$T_D = N*MDT$$

where MDT is the mean down time.

The final approach to be considered is to terminate the sampling plan after a specified number of failures. This plan has the great advantage of simplicity. One need only look in Table 1 for the cut-off number for the desired accuracy and choose a sample size (usually about the size of the cut-off number). The big disadvantage, however, is the unpredictability of the plan's duration. One cannot schedule work loads and assignments with much confidence.

If one has no prior estimates of θ, however, it is probably the easiest method to obtain this estimate. Using a rather broad confidence interval and, therefore, a comparitively small N and sample size, one can sample items conveniently at hand and in a hopefully short time have the estimate. If desired, a specified termination date override may be used to prevent extremely good items from consuming too much time. The full sampling problem can then be approached in the more complete manner.

The next chapter will present methods to determine mean time between failures when the distribution of time between failures is normal.

CHAPTER V

NORMALLY DISTRIBUTED TIME BETWEEN FAILURES

If one has an estimate of the variance of the normal distribution, the number of failures necessary for any desired confidence interval may be determined from

$$N = (2Z_{(1-\alpha/2)} \sigma / R)^2 \qquad (1)$$

where σ is the estimate of variance, $Z_{(1-\alpha/2)}$ is the normal coordinate for the desired confidence $(1-\alpha)$, and R is the difference between the upper and lower limits of the confidence interval. The confidence interval commonly used is 95% confidence interval, where $Z_{.975} = 1.96$.

If one has an estimate of MTBF and one determines the duration of a sampling plan equal to this estimate, the expected number of failures would equal one half of the number of items on test.[1] In practice this would not provide a very effective test since all the times to failure which would have exceded the mean are not observed, therefore complicating the analysis of data and making it difficult to determine if the underlying distribution was actually normal. The most desirable condition occurs when all the items on test have failed. The probability an item will fail in a time T is γ if $T = MTBF + Z_\gamma \sigma$. The probability

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1 One must assume here that all items on test start at time zero in new or repaired condition.

that all items in a sample of size n will fail in T is γ^n. It is obvious that as n becomes large, one should not expect all items on test to fail. However, with a large number of data points covering a sufficient range of the distribution, one may perform a least squares fit to determine the parameters of the distribution. This test requires that the data be grouped. Obviously, the last group would consist of those items that have not yet failed. The expected size of this group is $(1-\gamma)$ n, is the sample size.

For small samples, n 30, or samples with little confidence in the original estimate of the variance, one would have to use a "t" test to determine the confidence interval.

It must be noted that only one failure per item is recorded. The sample size is then calculated to be $n = N/\gamma$.

If a sample of size n is not available or is inconvenient to test, one may specify that each item on test fail a specific number of times p. This requirement must be imposed since, if some items fail many times and others fail fewer times, one will bias the sample mean to be shorter than the actual mean of the underlying distribution. The sample size is now n/p, but the duration of the test must now be increased. The mean time to the pth failure is p times the MTBF and the variance will be p times the estimate of time between failure variance. One may use this mean variance and go through the same procedure as for one failure per item to determine the duration of the test.

If one has an estimate of MTBF but no estimate of the variance, one has no good method by which to determine a sample size. A best guess of variance may be used and a plan may be designed to these specifications. The confidence interval on the mean will be determined by a "t" test. One may have to take a second sample if the desired accuracy is not obtained. One could also have tested a small sample to get an estimate of the variance and then designed the test using this estimate.

A procedure similar to the one above may be used when one has neither estimate. In this case, the testing of a small sample is probably more advantageous than guessing a mean variance. In the latter case, the probability of having to take a second large sample is greatly increased.

The cost trade-offs to be made in the above plans do not lend themselves to analytical methods of solution. If the cost of sampling $C_2(n)$ and the time costs $C_1(T)$ are known, a brute force method of minimizing cost may be used. A computer program could be designed to do this.

The trade-off between accuracy and total cost may be performed more easily. It is a reasonable assumption, in this case, that the total cost is a function of N. This is true since the sampling duration T will not vary much with N once an optimal n and T have been determined. The difficulty here lies in determining the worth of accuracy W(R) and the cost relationship C(N).

The quantity W(R)-C(N) represents the value of the information after the cost of obtaining this information has been considered. To optimize the sampling plan, one must maximize this quantity. The relationship between R and N is given by Equation (1) and the solution may be obtained by simple substitution of a variable and solving for a maximum.

Finally, one must consider the age of the items being sampled. If the parameter to be measured is MTBF, the sample should include all ages with equal probability. If this condition is not met, the mean measured is biased by the age of the equipment. It is therefore important to take more than one sample when one is dealing with newly fielded items. Subsequent samples can be scheduled annually or as needed. This process reveals more information about the instantaneous failure rate of the system.

The next chapter will present methods to determine sampling plans for mean time to repair when the distribution of time to repair is log normal.

CHAPTER VI

LOG NORMAL TIME TO REPAIR

The most commonly used distribution for time to repair is the log normal distribution. The MTTR will approach a normal distribution with a variance equal to the variance of the time to repair distribution divided by N the sample size. N can therefore be calculated by the method of chapter V if the variance is known. In this case, however, one need not be concerned that the data is incomplete since there will be a repair time for every failure.

If failures are to be induced, the test duration will depend only on the amount of time necessary to induce the failure, the mean and maximum time to repair, and the number of repairs that can be handled at one time. One can assume the time to induce a failure is negligible. If the number of repairs that can be handled at one time is less than N, one is faced with a Queueing Theory problem, otherwise one can assume the duration of a test equal to the observed maximum time to repair. The observed maximum time to repair is a random variable.

If one wishes to terminate the test after a preassigned duration, one would like to have all repairs accomplished by this time. The confidence of this happening is γ^N, where γ is the probability of repair in the time allowed.

In most cases a very high confidence of completing all repairs may not require an exceptionally long period of time.

In most cases, naturally occurring failures are preferable to induced failures. Induced failures would usually be used when one wishes to study a particular repair operation. When using naturally occurring failures, one must be careful if dealing with a particular age group. It is very possible that a large percentage of the repairs are characteristic of that age group and therefore bias the measure of repair time. It is important to get as uniformly distributed an age group as possible when measuring MTTR. In cases where this is impossible, such as with a newly fielded item, one or more subsequent samples should be taken to determine the effect of age.

The duration of the test will now depend on the instantaneous failure rate and sample size. The instantaneous failure rate is the inverse of the mean time between failure.[1] The sample size for this problem may be several times larger than the number of failures we wish to observe. Even if the distribution of time to failure is not exponential, the expected number of failures with n items on test for a time T_o is $N = nT_o/\theta$. The instantaneous failure rate may be

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1 A thorough blend of ages and states of repair must be assumed.

used as an approximation even if the failure rate is not constant as long as the failure rate dose not change significantly during the test interval.

The test duration would then equal $T_o + T_{Dmax}$, where T_{Dmax} is the maximum down time. A cost trade-off could be performed in the same manner as in Chapter IV. The above method requires n/N be large. If one cannot satisfy this condition, the distribution of time between failures must be considered, and a sampling duration must be determined in a manner similiar to that in Chapter V. This duration is, of course, modified by adding T_{Dmax}.

If the needed estimates are not available, one must guestimate the parameters and design a plan accordingly, knowing that a further sample may be necessary.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Chapters III thru VI have been concerned with determining sample sizes and test durations for any type item, system, and subsystem in general. The specific item considerations of a sampling plan such as data collection forms, geographic locations of the samples, and training skill of the data collectors have not been emphisized in this paper. The cost considerations mentioned in the text would obviously fall in this category. These considerations are very important to a sampling plan and further work should be continued in this area.

The work of this paper may be further extended to include plans measuring many other parameters and combinations thereof and may include more general considerations of distribution. Hopefully, in the future, the Army would be able to combine all this information into a handbook or set of regulations which will guide the equipment proponent in the formulation of a complete data collection plan. Until then, individual work should continue in accumilating this information.

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