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RESPONSE OF A STRAND TO AXIAL AND TORSIONAL DISPLACEMENTS

by

S. Machida and A. J. Durelli

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Department of Civil and Mechanical Engineering

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ABSTRACT

Taking advantage of geometric considerations, and following previous studies by Hruska, the authors give explicit expressions for the determination of axial force, bending and twisting moments in helical wires, and for axial force and twisting moment in the core of a 7-wire strand subjected to axial and torsional displacements. The equations given are linear, could be similarly developed for strands of any number of wires or for strands subjected to large displacements, and should be found practical by designers. Measurements on oversize epoxy models of the strand show good correlation with the theory and support the observation that axial load has no effect on the effective torsional rigidity of the strand. Results of experimental strain analyses on steel strands and on models of strands, as well as other theoretical studies on helices and strands, will be published in other papers.

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INTRODUCTION

The importance of cables in almost all fields of technology from electric power transmission to ship towing and dentistry, cannot be overemphasized. Two extensive surveys of publications have been prepared, one by Laura and Casarella[1] and the other by Parsons and Casarella[2]. Surprisingly, few scientific investigations have been conducted so far to determine the load transfer and stress and strain distribution in strands and cables. Simultaneously with the preparation of this paper, Chi[3],[4] prepared two reports in which he extends the previous work of Hruska [5],[6],[7] and compares elongations and strains obtained theoretically, with some of those obtained experimentally by the present authors in a steel strand. The subject also deserved the attention of Leissa[8] and of Starkey and Cress[9]. They, following Drucker and Tachau[10], emphasized the importance of the contact loads between wires.

The present paper is part of a series in which the authors will attempt to study the problem of load transfer and stress and strain distribution in strands, both theoretically and experimentally from several points of view and with the main purpose of increasing the knowledge that may, in the future, help designers to manufacture better cables. In one of the papers consideration will be given to the stress and strain distribution in a single helix, the elementary component of the strand. In other papers experimental stress analysis results on steel strands will be reported. It is believed that these experimental analyses are the first ones to be conducted on helices and strands. In another paper an attempt will be made at the study of vibrations in submerged cables.

In the experimental work measurements are taken not only on steel strands (as received from a manufacturer) but also on plastic oversized models of strands. This permits a better understanding of the differences between the results of theoretical analyses, and the results of experiments in strands and cables. An attempt is made in the theoretical as well as in the experimental papers to present the problem with graphical representations for clearer understanding of its complexity, and to emphasize the frequently erratic behavior of actual cables.

The theoretical developments to be presented in this paper introduce simplifications that make the equations linear and therefore easier to handle by designers. These equations are applied to the case when the deformations of the strand are small, but the reasoning behind them will also permit the application to the case of large deformations. These equations could also be applied to the analysis of wire-ropes made of strands, considering each strand in the wire-rope analogously to the situation of a helical wire, in the strand.

GEOMETRY AND LOADING OF THE STRAND

Consider a simple strand geometry as the one shown in Fig. 1 where a straight core wire is wrapped around by a layer of helical wires. The number of helical wires is six for the most typical and simple strand, which is the one considered in this paper, but the equations to be given in the later part of the paper can be extended to the case where a strand has an arbitrary number of helical wires. Each helical wire is assumed to have a circular cross section in a plane normal to its axis (a helix) (Fig. 1), the diameter of which is small in comparison with the pitch of the helix. It is also assumed that each helical wire is in contact with

the two adjacent wires, with the core, or with both core and adjacent wires.

A strand can be loaded by three principal static types of loadings: 1) pulling (or axial loading of the strand), 2) torsion, and 3) bending. Besides these three principal types of loadings, a strand could be subjected to thermal loadings, if there are temperature gradients in its environment, impact and vibrations. In the analysis, torsion and pulling are conveniently associated and will be treated together. Bending, (as produced when the strand is wrapped around a drum) may develop when the strand is subjected to different levels of prestressing (by pulling). It may also develop when the strand is used as a component of a wire-rope, and the wire-rope is subjected to axial loading or torsional loading.

The loadings considered in this paper are axial tension, torsion and combined torsion with tension. These loadings cause elongation and rotation in the strand. Because of geometric restrictions the state of stress and strain associated with these loadings is constant all the way along the axial line of each helical wire. Therefore, stresses and loads in the strand can be described by the stresses and loads on a single transverse cross section of a helical wire and core.

In case of bending of the strand, the stresses and loads on the transverse cross section change along its axial line. This makes the analysis more complicated. This problem is out of the scope of this paper and would be dealt with somewhere else.

In the present analysis the radial dimensions of the cross section in the unloaded strand and its position with respect to the core are

assumed to remain constant under load; the interwire contact deformation and Poisson's effect due to axial strain are neglected. In other words, the axial lines of helical wire after deformation in the strand are assumed to remain on the cylindrical surface (Fig. 1) on which they were before deformation (This cylinder will be called "reference cylinder" and its radius will be called R.). This assumption is reasonable because each helical wire is restrained by the core and by its neighbors. This restriction makes the main difference in the deformation characteristic between a free helical wire and a helical wire in a strand.

The deformation of a helical wire in a strand will be better understood with the following statement: Consider material line segments, say $\overline{AB}$ and $\overline{CD}$ in a helical wire which are the principal normal and binormal to the axial line (helix) at point 0, respectively (Fig. 2). After deformation these material line segments are assumed to displace in such a way that they still are the principal normal and binormal to the deformed axial line ($\overline{A'B'}$ and $\overline{C'D'}$ in Fig. 2).

The following is the nomenclature used in the analysis:

A: cross sectional area of wire (used with c or h subscript)

A to D: coefficients functions of geometry and material properties

c: subscript used to identify the core

d: wire diameter (used with c or h subscript)

E: Young's modulus

G: shear modulus

h: subscript used to identify the helical wire

I: moment of inertia of the cross section of helical wire

I_p: polar moment of inertia of the cross section of wire (used with c or h superscript)

M: bending moment acting on the helical wire in a plane containing the axis of helical wire and the principal normal (positive M is shown in Fig. 7)

N: axial tensile load applied to the strand

N_c: axial force acting in the core

N_h: axial force acting in the helical wire

P: resultant contact force acting on a helical wire (per unit length)

P_c: contact force acting between core and a helical wire (per unit length)

P_h: contact force acting between two adjacent helical wires (per unit length)

p: pitch of helical wire

R: radius of strand measured from the center of strand to the center of helical wire (radius of reference cylinder shown in Fig. 1)

r: distance on the transverse cross section of a wire from the centroid (Fig. 7)

s: arc length of helical wire

T: torsional load applied to the strand

T_h: twisting moment acting on the helical wire (positive T_h is shown in Fig. 7)

T_c: twisting moment acting on the core (positive T_c is shown in Fig. 7)

β: helical wire lay angle (Fig. 1)

γ: normalized rotation per original one pitch length of the strand and defined by $\gamma = \Delta/2\pi$

$\gamma_{z\theta}^{tc}$: shear strain on the transverse cross section of core associated with twisting moment T_c acting on the core

$\gamma_{s\theta}^{th}$: shear strain on the transverse cross section of helical wire associated with twisting moment T_h acting on the helical wire

Δ: rotation per original pitch length applied to the strand (positive in winding rotation) (in radians)

δ: axial displacement per original pitch length applied to the strand (positive in elongation) (in inches)

ε: axial displacement of the strand per unit length and defined by $\varepsilon = \delta/p$

ε_z^{ac}: axial strain in the core associated with the axial force N_c acting in the core

ε_s^{ah}: axial strain in the helical wire associated with the axial force N_h acting in the helical wire

ε_s^{bh}: axial strain in the helical wire associated with bending moment M acting on the helical wire

ζ: radius of twist of the axis of helical wire

η: distance on the transverse cross section of helical wire from its neutral axis for bending (Fig. 7)

ν: Poisson's ratio

ρ: radius of curvature of the axis of helical wire

σ_z^{ac}: normal stress corresponding to ε_z^{ac}

σ_s^{ah}: normal stress corresponding to ε_s^{ah}

$\tau_{z\theta}^{tc}$: shear stress corresponding to $\gamma_{z\theta}^{tc}$

$\tau_{s\theta}^{th}$: shear stress corresponding to $\gamma_{s\theta}^{th}$

Quantities after deformation are identified by prime symbols (e.g. s', p', β', etc....)

FORCES IN A HELICAL WIRE ASSOCIATED WITH AXIAL AND TORSIONAL DISPLACEMENTS APPLIED TO THE STRAND

Consider a segment of strand (length l, which can be taken arbitrary) as shown in Fig. 3. It is assumed that, because of geometric constraints, whether the strand is pulled axially, or is twisted about its axis, the core can only displace axially, and the wire can displace axially as well as rotate in contact with the core. The deformed state is shown by heavier lines in the figure. The axial line of helical wire EF (undeformed helix) moves to another helix EF' (deformed helix). This deformation consists of two components: one is a displacement in the axial direction of strand. Call δ this displacement per pitch (positive in elongation). The other component is a rotation around the axis of the strand. Call Δ in radians this rotation per pitch (positive in winding direction). The forces in the helical wire associated with the above mentioned deformation can be categorized into four types: (1) axial force, (2) bending in the plane containing the axial line of the helical wire and the principal normal of the helix, (3) twisting around axis of helical wire and (4) contact force, the resultant force of which lies on the plane containing the axial line of helical wire.

Strains and stresses acting on the cross section of the helical wire and associated with each of the above four types of forces will be expressed as functions of δ and Δ. The strains and stresses will be described in terms of their components in the plane normal to the axial line of the helix. Cylindrical coordinates (r, θ, s) will be used (Fig. 1).

Axial Force in the Wire

The displacement of the axial line segment from $\overline{EF}$ to $\overline{EF'}$ (Fig. 3) requires a change in the arc length. This is associated with an axial strain and with an axial stress and axial force (uniaxial state of stress in the wire is assumed).

The axial strain can be expressed in terms of δ and Δ as follows: Call l the length of undeformed strand. The initial length s of the axial line of a helical wire is given by

$$s = \frac{l}{p} \sqrt{p^2 + (2\pi R)^2} \tag{1}$$

The final length s' is given by

$$s' = \frac{l}{p} \sqrt{(p+\delta)^2 + [(2\pi+\Delta)R]^2} \tag{2}$$

If it is assumed that the helical wire is subjected to uniform axial stress distribution, the axial strain of helical wire is

$$\begin{aligned} \varepsilon_s^{ah} &= \frac{s' - s}{s} \\ &= \frac{p}{\sqrt{p^2 + (2\pi R)^2}} \sqrt{(1+\frac{\delta}{p})^2 + (\frac{2\pi R}{p})^2 (1+\frac{\Delta}{2\pi})^2} - 1 \\ &= \cos\beta \sqrt{(1+\varepsilon)^2 + (1+\gamma)^2 \tan^2\beta} - 1 \end{aligned} \tag{3}$$

where $\varepsilon = \delta/p$, $\gamma = \Delta/2\pi$

Assuming small deformation, i.e. ε & $\gamma \ll 1$, and neglecting higher order small quantities:

$$\varepsilon_s^{ah} \simeq \varepsilon \cos^2\beta + \gamma \sin^2\beta \tag{4}$$

Thus the quantities related to axial force acting in a helical wire become:

$$\varepsilon_s^{ah} = \varepsilon\cos^2\beta + \gamma\sin^2\beta$$

$$\sigma_s^{ah} = E\,(\varepsilon\cos^2\beta + \gamma\sin^2\beta) \tag{5}$$

$$N_h = A_h E\,(\varepsilon\cos^2\beta + \gamma\sin^2\beta)$$

Bending Moment in the Wire

The deformation of the axial line of the helical wire (Fig. 3) causes a change in the radius of curvature and this is associated with a bending moment in the plane containing the helix and the principal normal.

Using the Cartesian coordinates (X, Y, Z), the parametric expression of the helix EF is given by

$$X = R\cos\varphi$$

$$Y = R\sin\varphi \tag{6}$$

$$Z = \frac{P}{2\pi}\varphi$$

The radius of curvature ρ can be computed from

$$\frac{1}{\rho^2} = \left[\left(\frac{d^2X}{d\varphi^2}\right)^2 + \left(\frac{d^2Y}{d\varphi^2}\right)^2 + \left(\frac{d^2Z}{d\varphi^2}\right)^2 - \left(\frac{d^2s}{d\varphi^2}\right)^2\right] \Big/ \left(\frac{ds}{d\varphi}\right)^4 \tag{7}$$

where ds is a line increment of the helix defined as

$$ds^2 = (dX)^2 + (dY)^2 + (dZ)^2$$

Using Eq. (7) the radius of curvature of the undeformed helix EF is

$$\rho = R + \left(\frac{P}{2\pi}\right)^2\frac{1}{R} \tag{8}$$

The radius of curvature after deformation is given by

$$\rho' = R + \left(\frac{P'}{2}\right)^2\frac{1}{R} \tag{9}$$

The deformed pitch p' can be expressed in terms of the original pitch p, the axial and rotational deformations of the strand

$$p' = (p + \delta)\left(\frac{2\pi}{2\pi+\Delta}\right) = p\,\frac{1+\varepsilon}{1+\gamma} \tag{10}$$

In the case of the strand analyzed experimentally (7-wire), the radius of curvature is large compared with the diameter of the helical wire. The angle $\beta \simeq 8.5°$ and $\rho \simeq 50\ d_h$. Therefore, the shift of the neutral axis from the centroid of the cross section is negligible and the elementary theory of bending can be used.

Using Eqs. (8), (9) and (10), the following quantities related to bending of the helical wire can be determined for a given deformation of the strand (δ and Δ, or ε and γ).

$$\begin{aligned} \varepsilon_s^{bh} &= \left(\frac{\rho' - \rho}{\rho\rho'}\right)\eta \\ \sigma_s^{bh} &= E\left(\frac{\rho' - \rho}{\rho\rho'}\right)\eta \\ M &= EI\left(\frac{\rho' - \rho}{\rho\rho'}\right) \end{aligned} \tag{11}$$

When strains are small ε, $\gamma \ll 1$, then:

$$p' = p\,(1 + \varepsilon - \gamma) \tag{12}$$

and Eq. (11) becomes:

$$\begin{aligned} \varepsilon_s^{bh} &= 2(\varepsilon - \gamma)\,\frac{\eta}{R}\cos^2\beta\sin^2\beta \\ \sigma_s^{bh} &= 2E\,(\varepsilon - \gamma)\,\frac{\eta}{R}\cos^2\beta\sin^2\beta \\ M &= 2EI\,(\varepsilon - \gamma)\cos^2\beta\sin^2\beta/R \end{aligned} \tag{13}$$

The positive direction of the bending moment is shown in Figs. 7 and 8.

Twisting Moment in the Wire

The line segments $\overline{AB}$ and $\overline{CD}$ embedded in the helical wire (Fig. 2) can be considered as lines of geometric reference. When moving along the axial line of the helical wire, these lines rotate around the axial line. Call $\Delta\psi$ the angle of rotation resulting from travelling through distance Δs along the axial line. Then the quantity

$$\frac{1}{\zeta} = \lim_{\Delta \to o} \frac{\Delta\psi}{\Delta s} \tag{14}$$

is the angle of twist, per unit length of the plane of geometric reference containing a line segment such as $\overline{AB}$ or $\overline{CD}$ (Fig. 4). It should be noted that this has nothing to do with twisting strain. ζ in Eq. (14) is called the radius of twist (or the second radius of curvature) of a spatial line.

Denoting the original and final values of the radius of twist as ζ and ζ' respectively, a segment of helical wire s undergoes twisting deformation around its axis through an angle which is given by

$$\Delta\psi' - \Delta\psi = \frac{\Delta s}{\zeta'} - \frac{\Delta s}{\zeta} \tag{15}$$

Thus the angle of twist per unit length due to the deformation is given by

$$\lim_{\Delta s \to o} \frac{\Delta\psi' - \Delta\psi}{\Delta s} = \frac{\zeta - \zeta'}{\zeta\zeta'} \tag{16}$$

Further computations can be made by assuming that the elementary theory of twist of circular bar can be applied to this problem. The radius of twist ζ of any spatial line is given by

$$\frac{1}{\zeta^2} = \left(\frac{d\lambda}{ds}\right)^2 + \left(\frac{d\mu}{ds}\right)^2 + \left(\frac{d\nu}{ds}\right)^2 \tag{17}$$

where λ, μ and λ are the direction cosines of the binormal at the point considered. In case of helix ζ is constant and is expressed as follows

$$\text{Before deformation:} \quad \zeta = \left(\frac{p}{2\pi}\right) + R^2 / \left(\frac{p}{2\pi}\right)$$

$$\text{After deformation :} \quad \zeta' = \left(\frac{p'}{2\pi}\right) + R^2 / \left(\frac{p'}{2\pi}\right) \tag{18}$$

Using Eqs. (10), (16), (18), the following quantities related to twisting of the helical wire can be determined for a given deformation of the strand.

$$\gamma_{s\theta}^{th} = \left(\frac{\zeta - \zeta'}{\zeta\zeta'}\right) r$$

$$\tau_{s\theta}^{th} = G \left(\frac{\zeta - \zeta'}{\zeta\zeta'}\right) r \tag{19}$$

$$T_h = GI_p^h \left(\frac{\zeta - \zeta'}{\zeta\zeta'}\right)$$

Assuming again small strain (ε, $\gamma \ll 1$), Eq. (19) becomes:

$$\gamma_{s\theta}^{th} = \frac{(\gamma - \varepsilon) r}{4R} \sin 4\beta$$

$$\tau_{s\theta}^{th} = \frac{G(\gamma - \varepsilon) r}{4R} \sin 4\beta \tag{20}$$

$$T_h = \frac{GI_p^h (\gamma - \varepsilon)}{4R} \sin 4\beta$$

The positive direction of the twisting moment is shown in Figs. 7 and 8.

Contact Forces between Wires

Contact problem is by its nature non linear. The stress distribution is complex at the points of contact. But the effect is considered local and the details of the stress distribution could be determined theoretically using the Hertz solution.

If the strand is well lubricated, the frictional force can be neglected and the resultant force due to contact is a force directed outward normally to the axial line of the helical wire as shown in Fig. 5.

The resultant contact force per unit length, P, is

$$P = 2P_h \cos 60^\circ + P_c = P_h + P_c \tag{21}$$

From the consideration of equilibrium for a short segment of helical wire (Fig. 6), on which all of the forces discussed in the preceding sections are acting, the following relation can be obtained with regard to contact force

$$P = \frac{N_h}{\rho'} \simeq \frac{N_h}{\rho} \tag{22}$$

FORCES IN A CORE ASSOCIATED WITH AXIAL OR TORSIONAL DISPLACEMENT OF THE STRAND

If the core of a strand is connected with the surrounding helical wires at both ends of the strand the quantities representing the deformation applied to the strand, δ and Δ, also represent the deformation of the core. Since a core is a straight circular rod, estimation of strains, stresses and forces can be made easily.

When there is contact between core and helical wires the core will act as a simple bar in tension and torsion with the addition of some loading due to 6 lines of spiral contact from helical wires. This suggests that the deformed cross section of the core may not remain plane, but may be warpt. This effect could be neglected for the first approximation since it is considered that the localized radial forces

due to contact would have small contribution to overall deformation of the core.

The axial deformation δ of the strand produces an average axial strain $\frac{\delta}{p}$ ($=\varepsilon$) in the core. The rotational deformation Δ of the strand results in a twisting deformation per unit length $\frac{\Delta}{p}$ ($=\frac{2\pi\gamma}{p}$) of the core. Therefore the core is subjected to axial force and twisting moment.

Strains, stresses and forces expressed in terms of ε and γ are:

$$\begin{aligned}
\varepsilon_z^{ac} &= \varepsilon \\
\sigma_z^{ac} &= E\varepsilon \\
N_c &= A_c E\varepsilon \\
\gamma_{z\theta}^{tc} &= \frac{2\pi\gamma r}{p} \\
\tau_z^{tc} &= \frac{2\pi G\pi r}{p} \\
T_c &= GI_p^c \frac{2\pi\gamma}{p}
\end{aligned} \tag{23}$$

FORCES ACTING ON THE STRAND CORRESPONDING TO THE APPLIED DISPLACEMENTS

From the preceding considerations, it follows that each helical wire is subjected to 1) axial load, 2) bending moment in a plane containing the axial line and principal normal, 3) twisting moment around the axial line. The core is subjected to 1) axial load and 2) twisting moment. The stress and strain distribution associated with these types of loading can be obtained approximately by the theory of strength of materials. Main results are summarized in Fig. 7.

Resultant forces on the strand cross section can easily be expressed in terms of forces acting on the cross sections of each of the seven wires. This is done in Fig. 8 and the following relations between internal forces and external forces are obtained from consideration on equilibtium:

N	External axial force of the strand	$N = N_c + 6N_h \cos\beta'$	(24)
T	External torque of the strand	$T = T_c + 6\,(\Gamma_h \cos\beta' - M \sin\beta' + N_h R \sin\beta')$	(25)

where β' is lay angle of helical wire after deformation and is expressed in terms of ε and γ as follows:

$$\beta' = \tan^{-1}\left(\frac{1+\gamma}{1+\varepsilon}\tan\beta\right) \tag{26}$$

From the assumption of small deformation, we can put

$$\beta' \simeq \beta \tag{27}$$

The approximation of Eq. (27) reduces Eqs. (24) and (25) to much more simple forms which are linear functions of ε and γ.

Using Eqs. (5), (13), (20), (23) and Eqs. (24) and (25) can be expressed in terms of ε and γ as fo

$$N = A\,\varepsilon + B\,\gamma \tag{28}$$

$$T = C\,\varepsilon + D\,\gamma \tag{29}$$

where A, B, C and D are constants determined by the geometry of the strand and the elastic constants of the strand material, and are given by

$$A = A_c E + 6A_h E \cos^3\beta$$

$$B = 6A_h E \sin^2\beta \cos 4\beta$$

$$C = 6A_h RE \sin\beta \cos^2\beta - \frac{3GI_p^h \sin 4\beta \cos\beta}{2R} - \frac{12EI \cos^2\beta \sin^3\beta}{R} \tag{30}$$

$$D = 6A_h RE \sin^3\beta + \frac{3GI_p^h \sin 4\beta \cos\beta}{2R} + \frac{12EI \cos^2\beta \sin^3\beta}{R} + \frac{2\pi GI_p^c}{p}$$

From Eq. (28) and (29), we have

$$\left.\begin{aligned} \varepsilon &= \frac{D}{AD-CB} N - \frac{B}{AD-CB} T \\ \gamma &= -\frac{C}{AD-CB} N + \frac{A}{AD-CB} T \end{aligned}\right\} \tag{31}$$

Substituting Eq. (31) into (5), (13), (20), (23) and (24), strains, stresses, forces and moments acting in the helical wires and the core can be expressed in terms of the external axial force N and torque T which can be treated as applied force and applied torque for the strand.

APPLICATIONS

Using the equations developed in the preceding sections, the response of the strand to an applied load will be given in what follows, as a function of that load. Since the equations in the previous chapter are all expressed in linear and explicit form, the procedure of computations is easy.

Axial Loading with Unrestricted Ends (Simple Tension of Strand)

In this case since no torque is applied to the strand,

$$T = 0 \tag{32}$$

Then, recalling Eq. (29)

$$\gamma = -\frac{C}{D}\varepsilon \tag{33}$$

Since approximately it can be stated that for the 7-wire strand:

$$d_h \simeq d_c \text{ and } R \simeq \frac{1}{2} d_h + \frac{1}{2} d_c \tag{34}$$

Then C/D from Eq. (30) is given by

$$\frac{C}{D} = \frac{48\ (1+\nu)\ \cos^2\beta\ \sin\beta\ (8-\sin^2\beta)\ -\ 3\ \sin 4\beta\ \cos\beta}{4\ \tan\beta + 3\ \sin^4\beta\ \cos\beta\ +\ 48(1+\nu)\ \sin^3\beta(\cos^2\beta + 8)} \tag{35}$$

Equation (33) determines γ or unwinding motion of strand for a given value of ε or elongation of strand.

Forces and moments can be expressed in terms of the applied force N as follows:

Axial force in the core

$$N_c = A_c E\ \frac{D}{AD-CB}\ N \tag{36}$$

Twisting moment in the core

$$T_c = -\frac{2\pi G I_p^c}{p}\ \frac{C}{AD-CB}\ N \tag{37}$$

Axial force in the helical wire

$$N_h = A_h E\ \frac{D\cos^2\beta\ -\ C\sin^2\beta}{AD-CB}\ N \tag{38}$$

Bending moment in the helical wire

$$M = \frac{2EI\ \cos^2\beta\sin^2\beta}{R}\ \frac{C+D}{AD-CB}\ N \tag{39}$$

Twisting moment in the helical wire

$$T_h = \frac{G I_p^h\ \sin 4\beta}{4R}\ \frac{C+D}{AB-CB}\ N \tag{40}$$

The effective tensile rigidity of the strand defined by N/ε is given by

$$\frac{N}{\varepsilon} = \frac{AD-BC}{D} \tag{41}$$

From Eq. (33), the unwinding motion of the strand due to the applied N is given by

$$\Delta = -\frac{2\pi C}{AD-CB} N \tag{42}$$

Torsional Loading

In this case, since no axial force is applied:

$$N = 0 \tag{43}$$

From Eq. (28)

$$\varepsilon = -\frac{B}{A}\gamma \tag{44}$$

If the same approximations of Eq. (34) are made, B/A is given by

$$\frac{B}{A} = \frac{6 \sin^2\beta\cos\beta}{1 + 6 \cos^3\beta} \tag{45}$$

Equation (44) means that the strand should contract when winding torque is applied.

The torsional loading considered here refers to "winding" of the strand only. Unwinding causes separation between wires and thus change in radial dimensions of the cross section of the strand.

Forces and moments in the core and the helical wires associated with winding torque T applied to the strand are:

Axial force in the core

$$N_c = -A_c E \frac{B}{AD-CB} T \tag{46}$$

Twisting moment in the core

$$T_c = \frac{2\pi G I_p^c}{p} \frac{A}{AD-CB} T \tag{47}$$

Axial force in the helical wire

$$N_h = A_h E \frac{A \sin^2\beta - \beta\cos^2\beta}{AD-CB} T \tag{48}$$

Bending moment in the helical wire

$$M = -\frac{2EI \cos^2\beta\sin^2\beta}{R} \frac{A + B}{AD-CB} T \tag{49}$$

Twisting moment in the helical wire

$$T_h = \frac{GI_p^h \sin 4\beta}{4R} \frac{A + B}{AD-CB} T \tag{50}$$

The effective torsional rigidity of the strand defined by $T/\frac{\Delta}{p}$, is given by

$$T/\frac{\Delta}{p} = \frac{AD-BC}{A} \frac{p}{2\pi} \tag{51}$$

Axial Loading with Restricted Ends (Tension combined with Restricting Torque)

In this case since unwinding motion is restricted.

$$\Delta = o \quad \text{or} \quad \gamma = o \tag{52}$$

For a given elongation of strand ε, the axial tensile load N and torque T required from the support to restrict unwinding of strand are from Eqs. (28) and (29)

$$N = A\,\varepsilon \tag{53}$$

$$T = C\,\varepsilon = \frac{C}{A} N \tag{54}$$

In this case the applied torque is linearly related to the applied tension.

Forces and moments in the core and the helical wires expressed in terms of applied axial load N are

Axial force in the core

$$N_c = A_c E \frac{1}{A} N \tag{55}$$

Twisting moment in the core

$$T_c = o \tag{56}$$

Axial force in the helical wire

$$N_h = A_h E \frac{\cos^2\beta}{A} N \tag{57}$$

Bending moment in the helical wire

$$M = \frac{2EI \cos^2\beta \sin^2\beta}{R} \frac{1}{A} N \tag{58}$$

Twisting moment in the helical wire

$$T_c = \frac{GI_p^h \sin 4\beta}{4R} \frac{1}{A} N \tag{59}$$

The effective tensile rigidity of the strand with the ends restrained is given by

$$N/\varepsilon = A \tag{60}$$

Combined torsion with tension

The point of interest in this case is whether the axial tensile load has an effect on the effective torsional rigidity. This can be discussed on the basis of foregoing considerations.

For a given axial load $\overline{N}$, we have from Eq. (28)

$$\overline{N} = A\varepsilon + B\gamma \tag{61}$$

Then (29) can be written as

$$\begin{aligned} T &= \frac{C.\overline{N}}{A} + \left(\frac{AD-BC}{A}\right) \gamma \\ &= \frac{C.\overline{N}}{A} + \left(\frac{AD-BC}{A}\right) \frac{\Delta}{2\pi} \end{aligned} \tag{62}$$

The angle Δ is the twist per pitch to be measured with no applied tensile loads. The strand will unwind when $\bar{N}$ is applied. The unwinding deformation due to $\bar{N}$ is obtained from Eqs. (28) and (29) by putting $N = \bar{N}$ and $T = o$. The computed unwinding angle of twist per pitch Δ_u is

$$\Delta_u = \frac{C.\bar{N}}{AD-CB} 2\pi \tag{63}$$

Let the angle of twist per pitch caused by applied torque T be measured with the strand under load $\bar{N}$ as the original state and let it be denoted as $\bar{\Delta}$, then

$$\bar{\Delta} = \Delta + \Delta_u \tag{64}$$

Substituting (63) and (64) into (62) we have

$$T = \left(\frac{AD-BC}{A}\right) \frac{\bar{\Delta}}{2\pi} \tag{65}$$

Thus the effective torsional rigidity is given by

$$\frac{T}{\bar{\Delta}/p} = \left(\frac{AD-BC}{A}\right) \frac{p}{2\pi} \tag{66}$$

which is the same as the case of torsion without axial load Eq. (51)

In other words, the axial load has no effect on the effective torsional rigidity of strand when the deformation of strand remains small.

EXPERIMENTAL ANALYSIS

An oversized epoxy model of a 7-wire strand was prepared and measurements of displacements were taken in the four cases of loading described in the previous section.

The dimensions of the model were: diameter of the core: $d_c = 0.53$ in., diameter of the helical wire: $d_h = 0.53$ in. ($= d_c$), pitch: $p = 22$ in., lay angle: $\beta \approx 8° 36'$, length of the strand (between sockets): 24.5 in., overall length of the model (including sockets): 33.5 in.

The material properties of the model were: $E = 5.1 \times 10^5$ psi

$\nu = 0.35$

The set-up for the axial tensile loading is shown in Fig. 9. Elongations and rotations between sockets were measured with dial gages and protractors. The figure shows also a Huggenberger extensometer used to determine strains. Results obtained from the extensometer, as well as those obtained from brittle coatings, electrical strain gages and three-dimensional photoelasticity will be reported in another paper.

Measured elongation and rotations associated with axial loading are shown in Figs. 10 and 11. They compared well with those computed from Eqs. (41), (42) and (60). The maximum deviation is about 6%.

Figure 11 shows N vs. T for the case of restricted ends. The torque was estimated from the measured unwinding angle in the case of simple tension with free ends and the measured torque vs. angle of twist in the case of torsional loading. The results compare well with those of Eq. (54).

Figure 12 shows the measured angle of twist per unit length of strand subjected to axial load when torque is applied to the strand. All the measurements fall with small scatter on the theoretical prediction of Eq. (51) and (66) proving the validity of the conclusions obtained theoretically.

ACKNOWLEDGEMENTS

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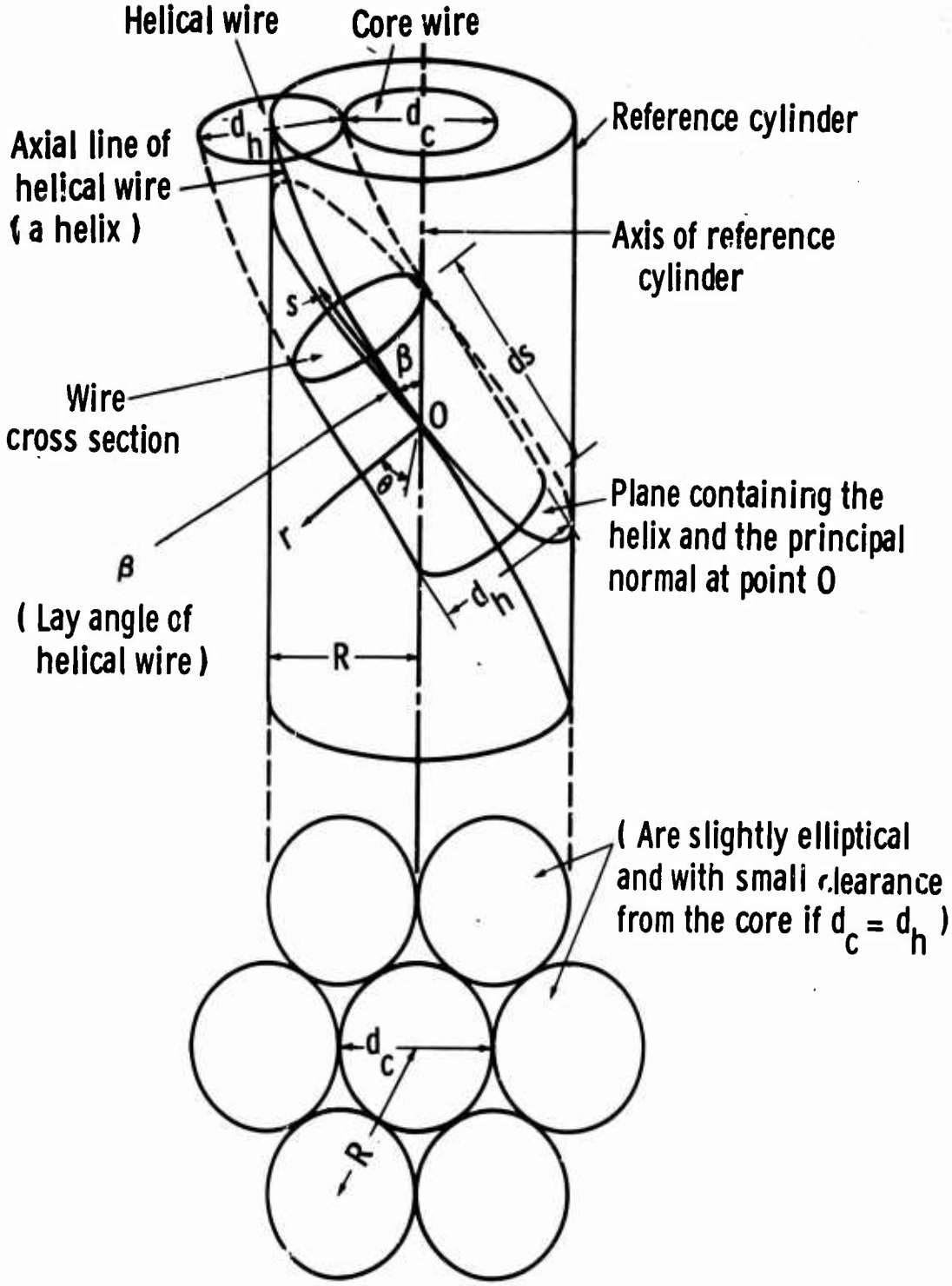
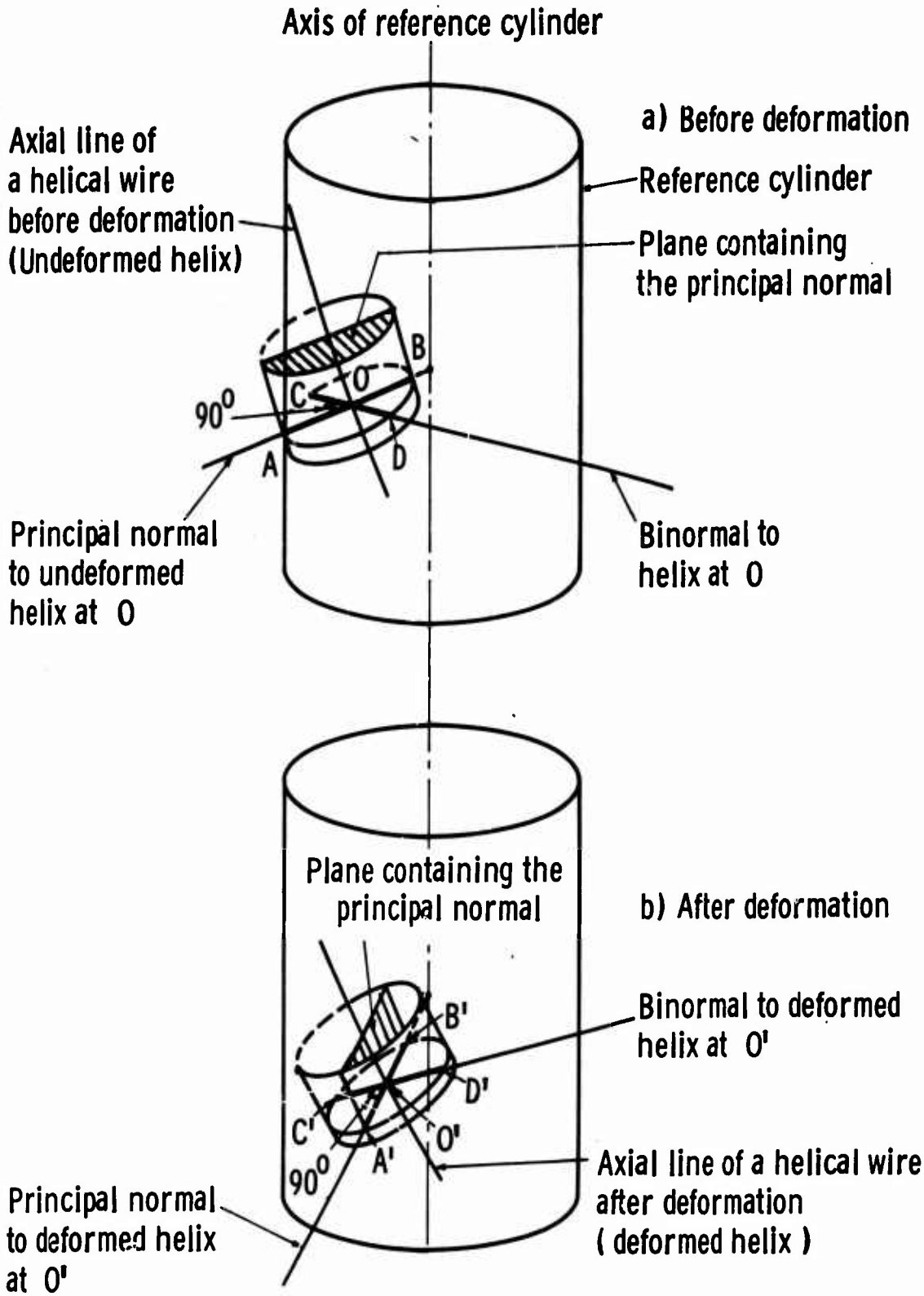


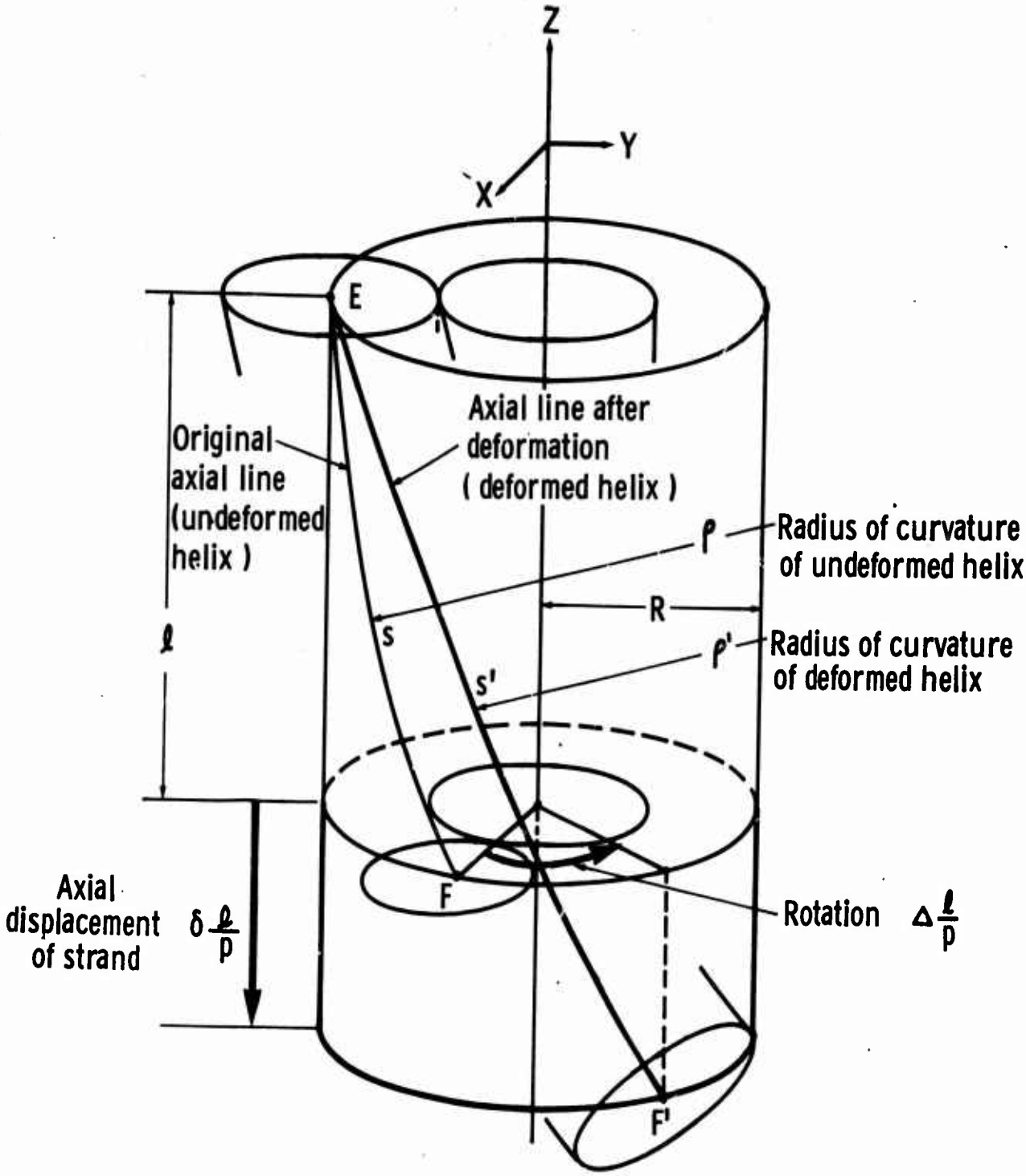
3001

FIG. 1 GEOMETRY OF A HELICAL WIRE WRAPPED AROUND A CORE



3002

FIG. 2 DEFORMATION OF A CROSS-SECTION OF A HELICAL WIRE

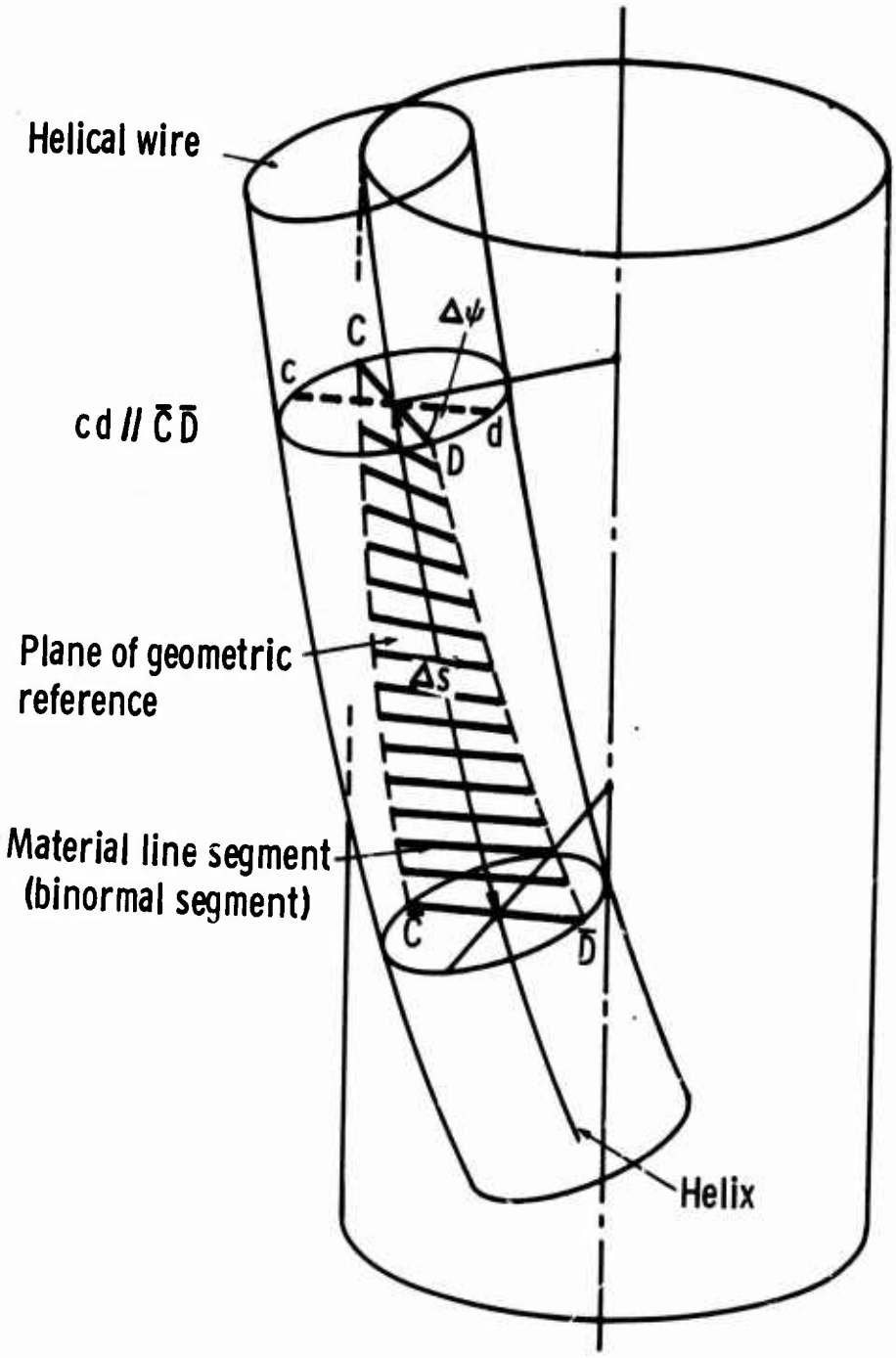


Displacement of strand

{ Axial displacement of strand — δ per original pitch length
{ Winding rotation — Δ rad. per original pitch length

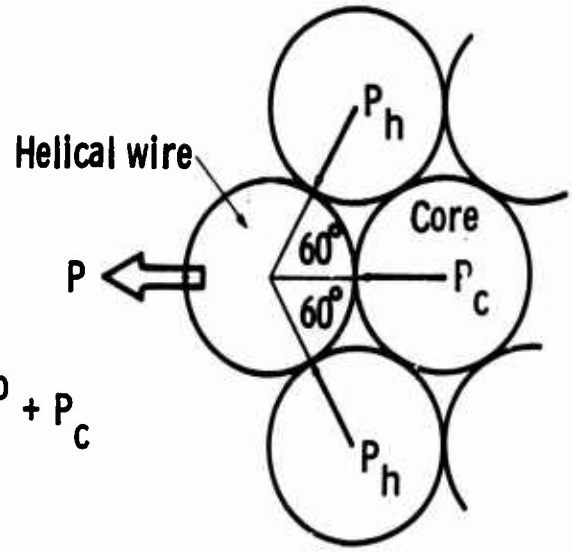
3003

FIG. 3 ELONGATION AND ROTATION IN A HELICAL WIRE



3004

FIG. 4 PLANE OF GEOMETRIC REFERENCE IN HELICAL WIRE FOR CONSIDERATION OF TWISTING DEFORMATION



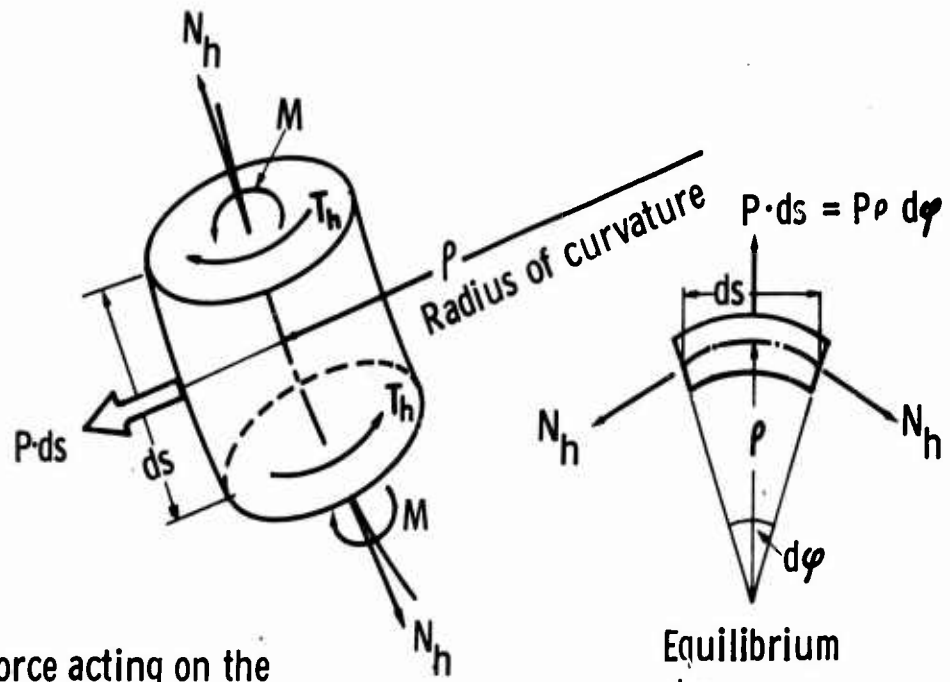
$$P = 2P_h \cos 60^0 + P_c$$
$$= P_h + P_c$$

P_h: Contact force(per unit length) due to contact between two adjacent helical wires

P_c: Contact force(per unit length) due to contact between helical wire and core

3005

FIG. 5 RESULTANT CONTACT FORCE IN THE TRANSVERSE CROSS-SECTION OF HELICAL WIRE



N_h: Axial force acting on the cross section of helical wire

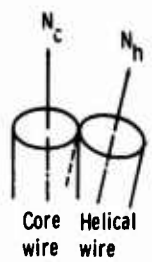
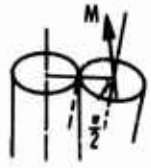
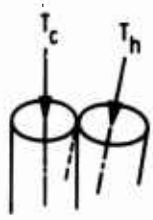
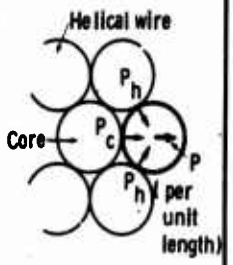
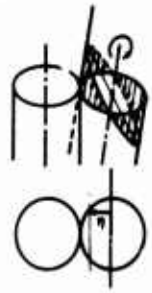
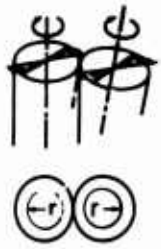
Equilibrium

$$2N_h \frac{d\varphi}{2} = P \cdot ds = P\rho d\varphi$$

$$\therefore P\rho = N_h$$

3006

FIG. 6 EQUILIBRIUM OF FORCE IN AN ELEMENT OF A HELICAL WIRE

		Axial load	Bending moment (plane containing helix and normal)	Twisting Moment	Contact force
Force and Moment Vector					
Stress distribution					
Formulae for strains and stresses	Core	$\varepsilon_z^{ac} = \varepsilon$ $\sigma_z^{ac} = E\varepsilon$ Eq. (23)	—	$\gamma_{z\theta}^{tc} = \frac{2\pi\gamma r}{p}$ $\tau_{z\theta}^{tc} = \frac{2\pi G\gamma r}{p}$ Eq. (23)	
	Helical wire	$\varepsilon_s^{ah} = \varepsilon\cos^2\beta + \gamma\sin^2\beta$ $\sigma_s^{ah} = E(\varepsilon\cos^2\beta + \gamma\sin^2\beta)$ Eq. (5)	$\varepsilon_s^{bh} = \frac{2(\varepsilon-\gamma)\eta}{R}\cos^2\beta\sin^2\beta$ $\sigma_s^{bh} = \frac{2E(\varepsilon-\gamma)\eta}{R}\cos^2\beta\sin^2\beta$ Eq. (13)	$\tau_{s\theta}^{th} = \frac{(\gamma-\varepsilon)r}{4R}\sin 4\beta$ $\gamma_{s\theta}^{th} = \frac{G(\gamma-\varepsilon)r}{4R}\sin 4\beta$ Eq. (20)	
Formulae for forces and moments	Core	$N_c = A_c E\varepsilon$ Eq. (23)	—	$T_c = -\frac{2\pi G I_p^c \gamma}{p}$ Eq. (23)	
	Helical wire	$N_h = A_h E(\varepsilon\cos^2\beta + \gamma\sin^2\beta)$ Eq. (5)	$M = 2EI(\varepsilon-\gamma)\cos^2\beta\sin^2\beta/R$ Eq. (13)	$T_h = \frac{G I_p^h(\gamma-\varepsilon)}{4R}\sin 4\beta$ Eq. (20)	$P = N_h/\rho$ Eq. (22)

$\frac{\Delta}{2\pi} = \gamma$, $\frac{\delta}{p} = \varepsilon$ } Assumed as small $\gamma, \varepsilon \ll 1$

3007

FIG. 7 FORCES, MOMENTS, STRAINS, AND STRESSES DEVELOPED IN THE HELICAL WIRES AND THE CORE ASSOCIATED WITH EXTENSIONAL AND ROTATIONAL DEFORMATIONS APPLIED TO A STRAND

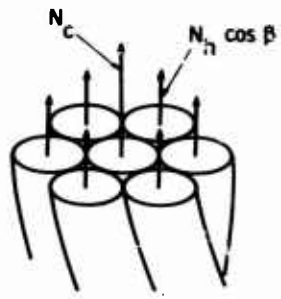
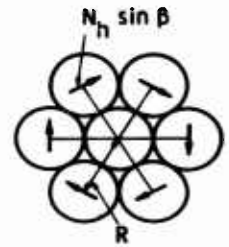
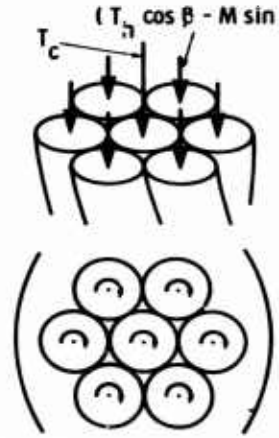
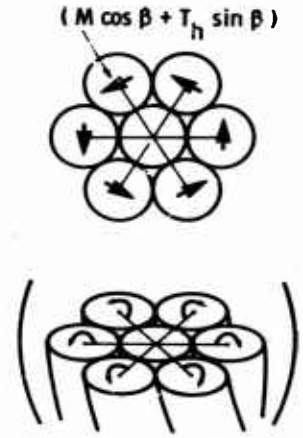
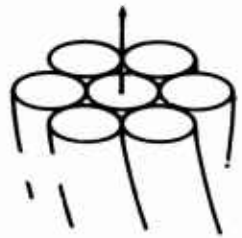
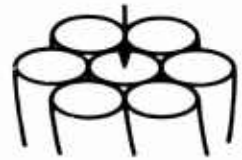
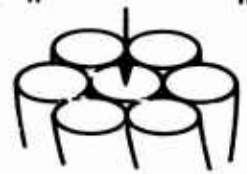
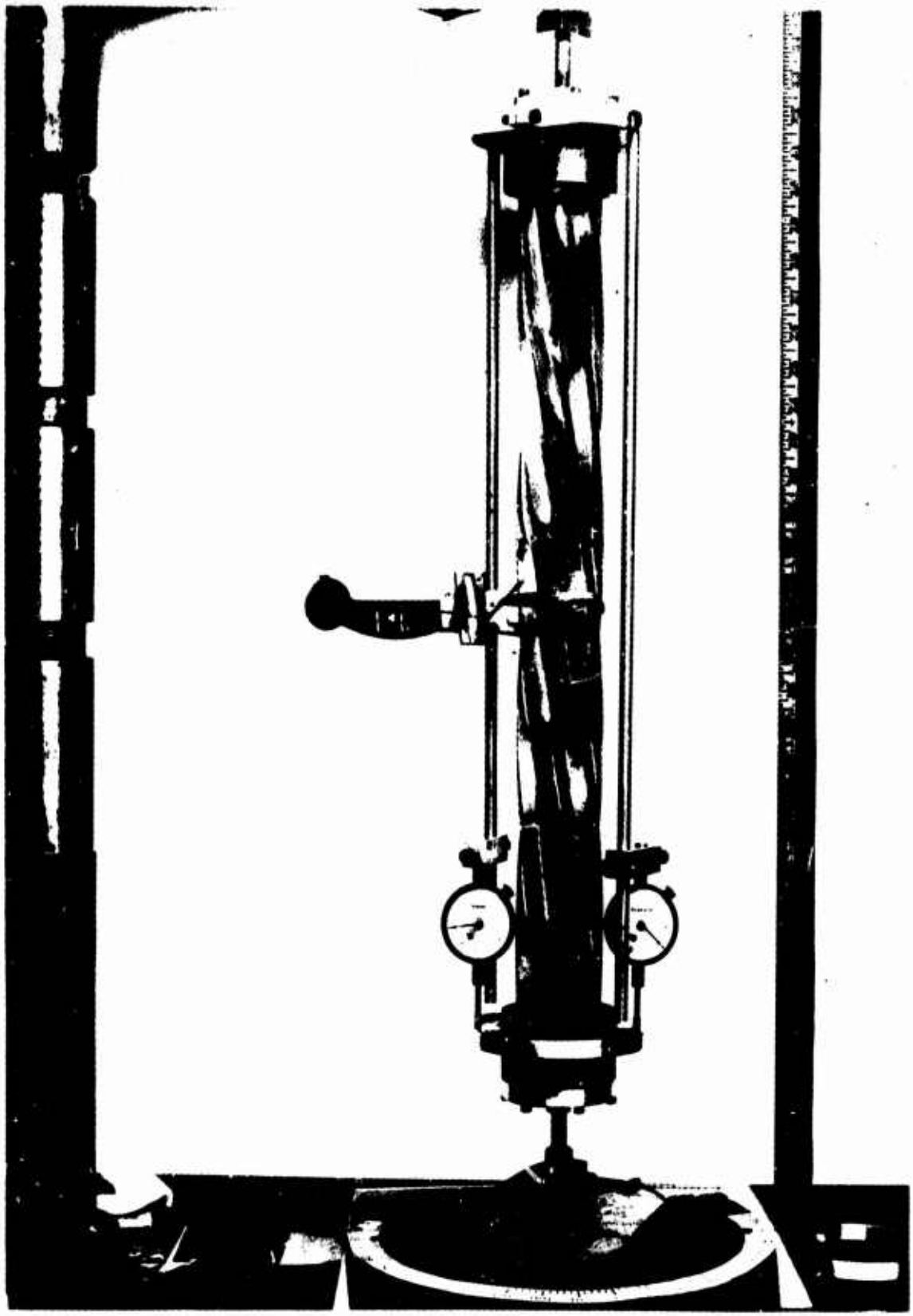
	FORCE VECTORS PERPENDICULAR TO THE PLANE	FORCE VECTORS IN THE PLANE	MOMENT VECTORS PERPENDICULAR TO THE PLANE	MOMENT VECTORS IN THE PLANE
FORCES AND MOMENTS ACTING IN EACH WIRE ON THE PLANE NORMAL TO THE AXIS OF STRAND				
RESULTANT FORCES AND MOMENTS ACTING ON THE STRAND CROSS SECTION	$(N_c + 6N_h \cos\beta)$ 	$6N_h R \sin\beta$ 	$T_c + 6(T_h \cos\beta - M \sin\beta)$	NO RESULTANT MOMENT VECTOR ACTING ON THE STRAND CROSS SECTION
		$T_c + 6(T_h \cos\beta - M \sin\beta + N_h R \sin\beta)$ 		
EQUILI-BRIUM	$(N_c + 6N_h \cos\beta) =$ N (External axial force applied to strand)	$T_c + 6(T_h \cos\beta - M \sin\beta + N_h R \sin\beta) =$ T (External torque applied to strand)		

FIG. 8 FORCES AND MOMENTS ON A TRANSVERSE CROSS-SECTION OF A STRAND



3009

FIG. 9 EPOXY MODEL OF A 7 WIRE STRAND SET UP IN A TESTING MACHINE TO MEASURE ELONGATIONS, ROTATIONS AND DETERMINE STRAINS

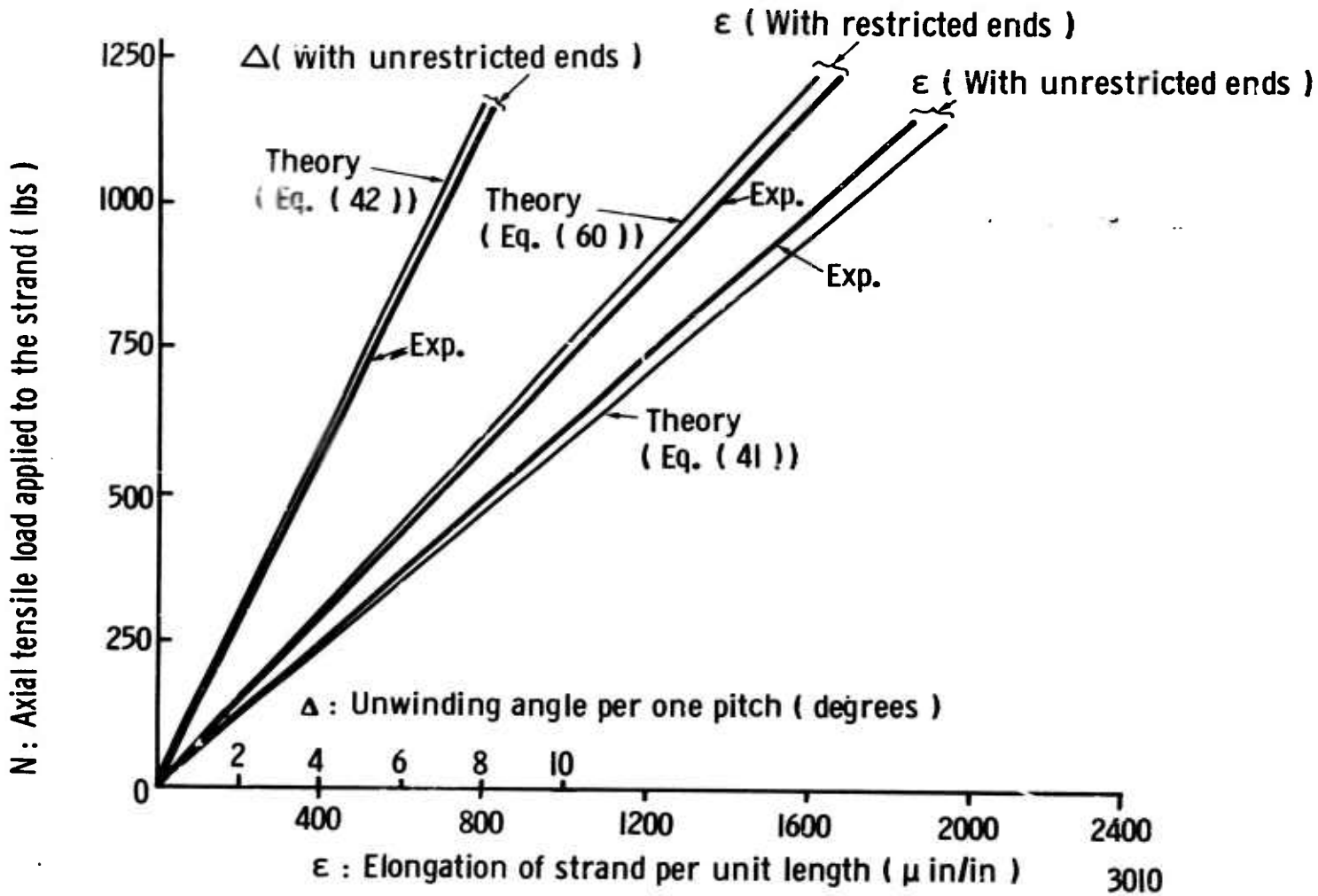
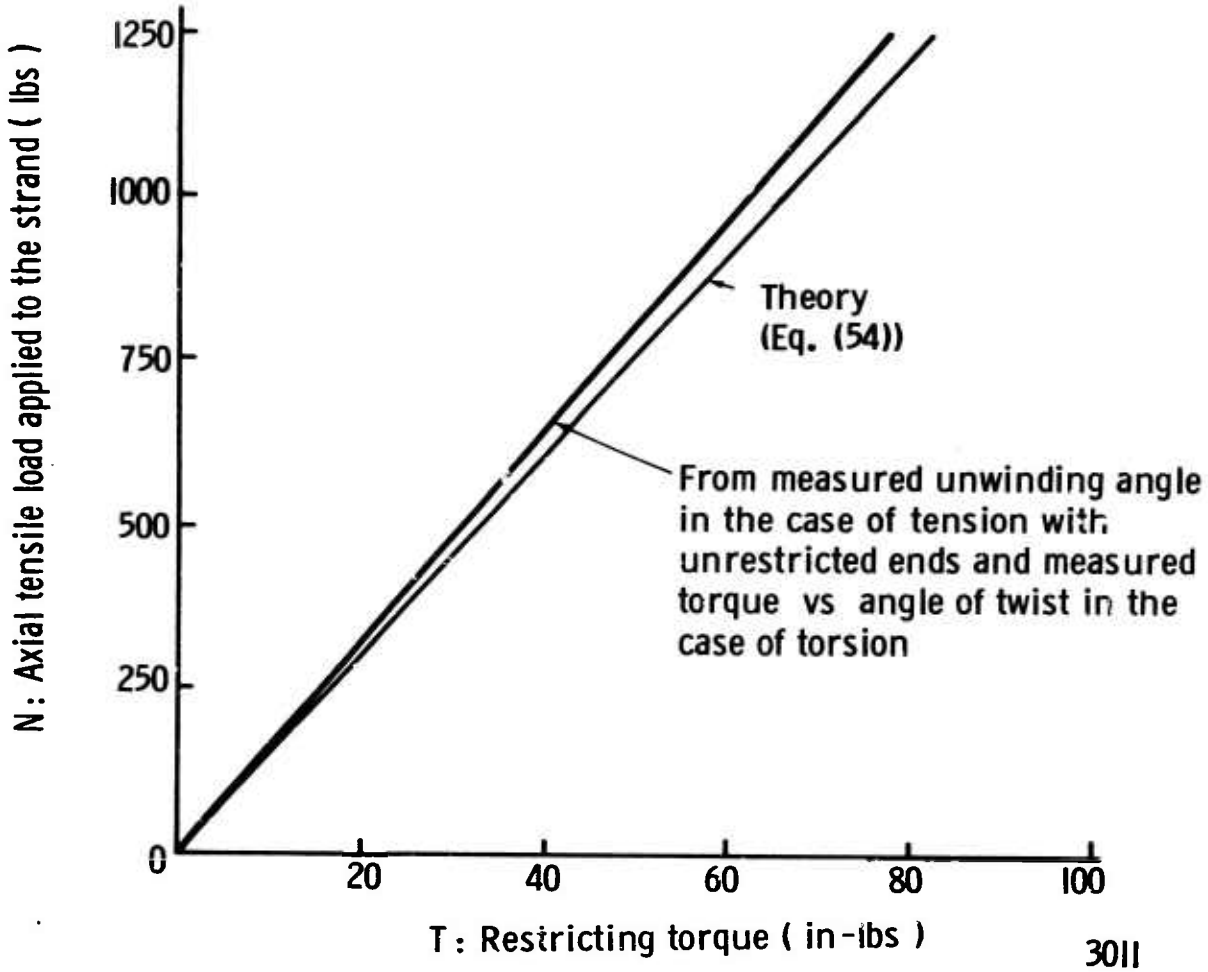
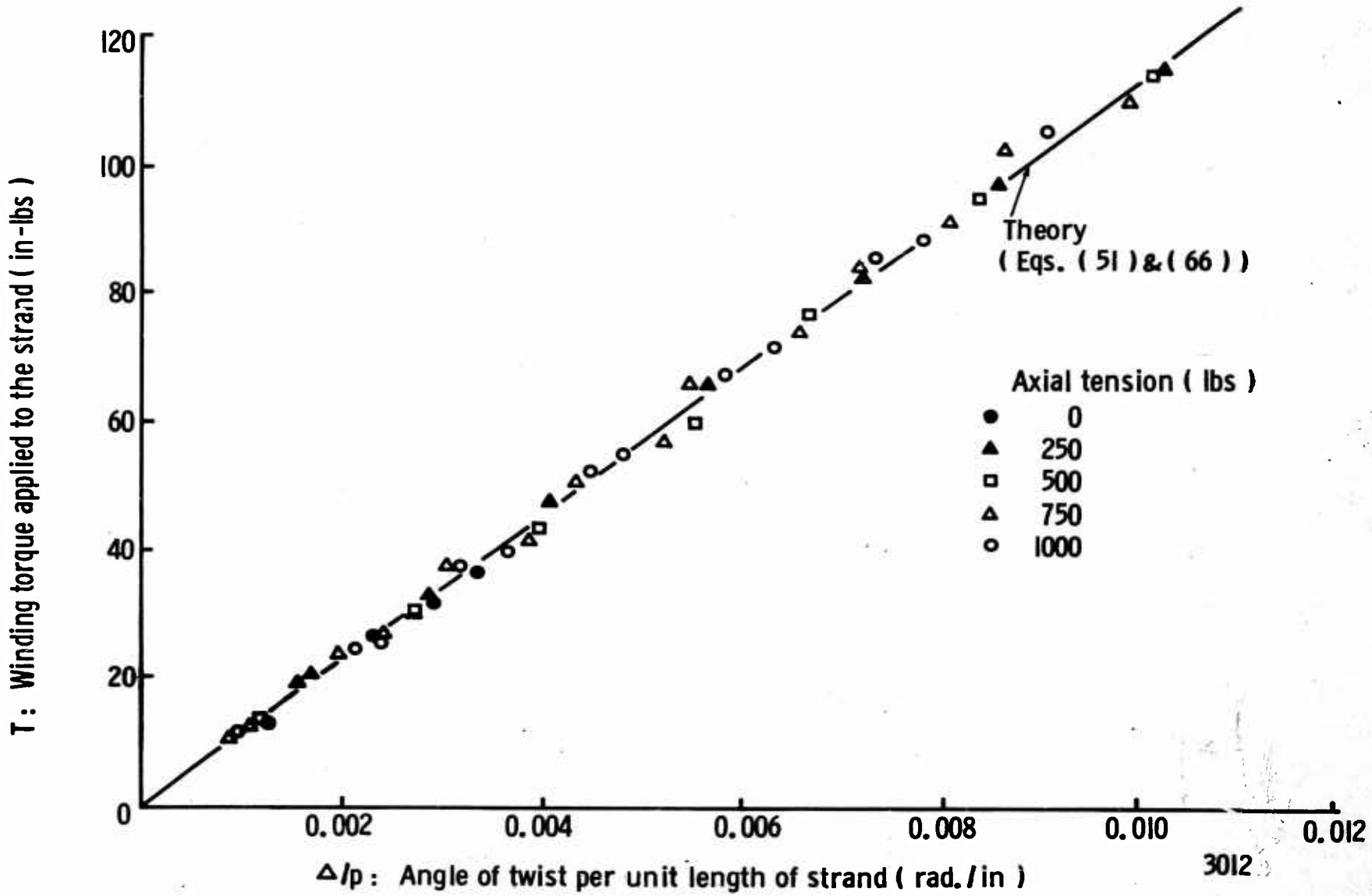


FIG. 10 DEFORMATIONS ASSOCIATED WITH AXIAL LOADING OF A STRAND
a) FREE TO ROTATE, b) WITH RESTRICTED ENDS, THEORETICALLY AND EXPERIMENTALLY OBTAINED



3011

FIG. 11 TORQUE DEVELOPED IN A STRAND WITH RESTRICTED ENDS, WHEN SUBJECTED TO AXIAL LOADING



3012

FIG. 12 ANGLE OF TWIST ASSOCIATED WITH TORSIONAL LOADING OF A STRAND WITH AND WITHOUT AXIAL TENSION, THEORETICALLY AND EXPERIMENTALLY OBTAINED