MAINTENANCE OF
SUPPLIES AND EQUIPMENT

TECHNIQUES FOR DETERMINING
OPTIMAL OPERATIONAL
READINESS FLOAT

HEADQUARTERS, U. S. ARMY MATERIEL COMMAND

JUNE 1971
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The techniques presently in use for calculating operational readiness float levels do not take into consideration the random nature of failures or the desirability to allocate stock levels to minimize the associated costs. A more sophisticated mathematical model taking these important points into account is presented herein. This procedure determines stock levels required to meet a desired float-availability goal at a minimum cost by using past demand data or appropriate estimates. To assist the reader in understanding the techniques presented in this pamphlet, it is suggested that he read appendix E. Definition of Terms and Phrases, before reading the rest of the pamphlet.
INTRODUCTION

1-1. Purpose. a. This pamphlet presents techniques for determining the optimum allocation for operational readiness float. The float is optimized on the basis of achieving a desired float-availability at a minimum cost. The procedure may be divided into two separate parts. One part is used for calculating float levels based on estimated parameters. The other part is used for calculating float levels based on historical float demands. The procedure provides the user with a dual capability: (i) he is able to determine initial float allocation levels for systems with little or no usage data, and (ii) when historical data become available, he is able to modify initial float levels to reflect past occurrences. To assist the user in obtaining the optimal float allocation levels, a computer program for this procedure is also presented.

b. This model is formulated to determine the operational readiness float allocations for a set of identical end items. However, the formulation is still valid if applied to a group of different end item types. The float-availability resulting from such application will be applicable to the group of end item types. The respective float-availabilities of each end item type comprising the group will be greater than the float-availability of the group.

1-2. Scope. This pamphlet applies to Headquarters, U.S. Army Materiel Command (AMC); AMC major subordinate commands; project/product managers; and separate installations and activities reporting directly to Headquarters, AMC.

1-3. General. a. The analytical derivation of the procedure is detailed in chapters 2 and 3. Chapter 2 presents the general procedure including the optimizing technique, and chapter 3 discusses the procedure's formulation with and without historical demand information. Chapter 4 presents the coding formats for each part of the procedure.

b. The appendixes contain the computer program and sample input and output data for the two parts of the procedure. Possible model variations are also briefly discussed in the appendixes.

c. The source deck for the operational readiness float allocation procedures (appendix C) is available upon request from the AMC Maintenance Support Center, Applied Science Division, Letterkenny Army Depot, Chambersburg, Pennsylvania 17201 (AUTOVON 242-7739).
GENERAL PROCEDURE

2-1. General. The model presented in this pamphlet is used to determine component-end-item-float levels for operational readiness float. The objective of this model is to determine float levels which enable operational readiness float to meet a prespecified float availability goal at a minimum cost.\(^1\)

2-2. Assumptions. a. Assumptions which are inherent in the general formulation of this model are listed below.

(1) The end item can be subdivided into mission essential component end items.

(2) Component end item failures are independent.

(3) The failure rate of a component end item in float is negligible compared to its operational failure rate.

(4) The float demands for each component end item follow a Poisson process.

(5) The float cost of any component end item is directly proportional to the quantity of that end item in float.

b. Assumptions in a(1) and (2) above are inherent in AP 750-19. These assumptions require that a maintenance analysis be performed to segment the end item into component end items which are mission essential and that fail independently. The determination of mission essentiality is made in reference to an overall mission objective of the end items to be supported, not just one particular field mission.

c. The Poisson demand assumption specifies that the occurrence of a failure of a component end item of type I, from a population of \(N\) component end items of type I, is described by a Poisson process. This is not stipulating that any one component end item fails exponentially, i.e., by a Poisson process.

2-3. Overview of Computation Procedures. a. The first step in obtaining the optimum float levels is to determine for each component end item the probability that the quantity of that component end item in the repair/resupply channel is less than or equal to the quantity of that end item in float. The quantity of component end items in float may vary from zero to a predetermined float stock level limit. Calculation of the probability can be performed by two methods. The choice of methods is

\(^1\)The cost is a minimum for the calculated float-availability which may be equal to or greater than the float-availability goal.
dependent upon the existence of historical demand information. If demand data on component end items are unavailable, as in the case of a new end item, estimated component end item reliability and maintainability parameters will be used in conjunction with the Poisson distribution to arrive at the float-availability associated with the various float levels of each component end item. If sufficient demand data are available, they are utilized through application of Bayesian inference and the Poisson distribution to arrive at the float-availability. These two methods are individually discussed in Chapter 3.

b. The float-availability for a particular set of float levels is represented by the product of the component-end-item float-availabilities of the appropriate component end items. The component-end-item-float-availability increases as the float level of that component end item is increased, thus increasing float-availability.

c. There are numerous float level combinations that enable the operational readiness float to meet a prespecified float-availability. The problem is to find the least expensive combination based upon the unit cost of the component end items.

d. The minimum cost float allocation is determined by an incremental process. Beginning with no component end items in float, the float-availability is calculated. If this value is below the float-availability goal, an additional component end item for float is selected on the basis of utility, i.e., having the greatest increase in the float-availability per dollar expended. For each component end item, the utility differs with the quantity of that component end item in float. This process is repeated until the desired float-availability is achieved. The optimum float allocation is that allocation first encountered in the process which achieves the desired float-availability.

2-4. Incremental Process Example. a. To help illustrate the incremental process, an example is included below. The end item considered for this example consists of two component end items, unit A and unit B. The component-end-item-float-availability calculated for each component end item relative to its respective float level and unit costs is listed in Table 2-1. (See Chapter 3, equations (1) and (3) for calculation procedures.)

<table>
<thead>
<tr>
<th>Component End Item</th>
<th>Component End Items In Float</th>
<th>Unit Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>0.368</td>
<td>0.736</td>
</tr>
<tr>
<td>B</td>
<td>0.607</td>
<td>0.910</td>
</tr>
</tbody>
</table>

2-2
b. The steps and example results of the incremental process are as follows.

(1) Compute the natural logarithms for each component end item's set of float availabilities, corresponding to the various float levels (Table 2-2).

Table 2-2. Natural Logarithm of Component-End-Item-Float-Availabilities

<table>
<thead>
<tr>
<th>Component End Item</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.999</td>
<td>-0.307</td>
<td>-0.083</td>
<td>-0.019</td>
<td>-0.004</td>
<td>-0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>-0.499</td>
<td>-0.094</td>
<td>-0.014</td>
<td>-0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(2) For each component end item, successively compute the change in the natural logarithm of its component-end-item-float-availability resulting from the addition of one more unit of that component end item to float. This is the marginal increase. (See Table 2-3 for results.)

Table 2-3. Marginal Increase in Natural Logarithm of Component-End-Item-Float-Availability

<table>
<thead>
<tr>
<th>Component End Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.692</td>
<td>0.224</td>
<td>0.064</td>
<td>0.015</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>B</td>
<td>0.405</td>
<td>0.080</td>
<td>0.012</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(3) Divide the marginal increase by the unit cost to obtain the marginal utility resulting from the addition of each unit to float. (See Table 2-4 for results.)

Table 2-4. Marginal Utility

<table>
<thead>
<tr>
<th>Component End Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.93x10^-4</td>
<td>2.23x10^-4</td>
<td>6.42x10^-5</td>
<td>1.52x10^-5</td>
<td>3.01x10^-6</td>
<td>1.0x10^-6</td>
</tr>
<tr>
<td>B</td>
<td>8.10x10^-5</td>
<td>1.60x10^-5</td>
<td>2.42x10^-6</td>
<td>4.00x10^-7</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(4) For each component end item order the marginal utilities from the largest to the smallest.
(5) Compute the natural logarithm of the float-availability goal. (No goal is presented in the example.)

(6) Allocate component end item float stockage in an optimal manner by adding a float, a unit of the component end item with the greatest marginal utility, until the float-availability goal is achieved or exceeded (table 2-5).

(7) Optimal component end item float stockage levels are those that result in the calculated float-availability being equal to or greater than the float-availability goal.

c. The two methods used to calculate the probability of adequate float are discussed in chapter 3.

Table 2-5. Allocations

<table>
<thead>
<tr>
<th>Unit A</th>
<th>Unit B</th>
<th>Float-Availability</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>.224</td>
<td>$ 0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>.446</td>
<td>1,000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>.560</td>
<td>2,000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>.839</td>
<td>7,000</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>.892</td>
<td>8,000</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>.968</td>
<td>13,000</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>.981</td>
<td>14,000</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>.985</td>
<td>15,000</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>.997</td>
<td>20,000</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>.998</td>
<td>21,000</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>=1.000</td>
<td>26,000</td>
</tr>
</tbody>
</table>
CHAPTER 3
DEVELOPMENT OF FLOAT MODELS

3-1. General. This chapter presents the underlying principles and the development of the two float level calculation methods presented and discussed in this pamphlet. The understanding of the derivations is not essential for the successful use of either model. The derivations are short and briefly discussed. They are presented to aid in understanding the approach taken for calculating operational readiness float levels.

3-2. Method 1 - Operational Readiness Float Without Prior Data. a. Introduction and formulation of problem. In the absence of any past demand data it is necessary to derive a model which will allow the use of engineering judgment and past experience with like end items in the formulation of demand estimates. This situation can occur with new end items or with older end items that lack appropriate or meaningful data. This procedure is an application of Palm's Theorem and follows closely work done by G. J. Fenney and C. C. Sherbrooke.

b. Additional assumptions.

(1) Assumptions used in this model, in addition to those already presented are:

(a) The mean-time-between-failures, the mean-time-to-repair, the mean-transportation-time, and the mean-time-awaiting-repair are constant, at least over the interval of time for which float levels are being computed.

(b) A prior maintenance engineering analysis of failure rates and repair/resupply data has been conducted; thus all data required for this analysis are available.

(2) The first assumption is not restrictive, providing that the time between recalculations is not large, and that it describes a representative portion of the useful life of the end item(s). AR 750-19 stipulates such float factor calculations be made once a year.

(3) The second assumption is implicit in AR 750-19.

c. Development of model.

(1) The development of this model is based on a queuing theorem developed by Palm, which states that if input of a process is Poisson, then the quantity of units in the service cycle in the steady state is also Poisson for any distribution of service. The Poisson-state probabilities depend on the mean of the resupply distribution, but not on the distributional form.

(2) Stated simply, using the Poisson demand assumption, it is possible to calculate the probability of having X component end items of type I in the repair/resupply cycle at any random point in time.
Thus, the probability of having at least one unit of component end item type I available from float (probability of having X or less component end items of type I in the repair/resupply cycle) can be related to the respective float level X of component end item type I.

Define:

\( N \) = Quantity of end items assigned to the using unit.

\( M \) = Total number of types of component end items which are candidates for float stock.

\( X(I) \) = Quantity of component end items, of type I in repair/resupply at any random point in time.

\( MTBF_f(I) \) = Mean-time-between-failures requiring float for component end item type I. (In calendar time.)

\( MTTR_f(I) \) = Mean-time-to-repair/resupply for component end item type I. (In calendar time)

\( T(I) \) = Mean-transportation-time for component end item type I.

\( B(I) \) = Mean-time-awaiting-repair/resupply for component end item type I.

\( W(I) \) = Mean-repair/resupply-time for component end item type I. \( (MTTR_f(I) + T(I) + B(I)). \)

\( F(I) \) = Float stock level for component end item type I.

\( C(I) \) = Unit cost to purchase, stock, and maintain in float, component end item type I.

\( R[F(I)|MTBF_f(I), W(I), N] \) = Component-end-item-float-availability for component end item type I when float stock equals \( F(I) \), given parameters \( MTBF(I), W(I) \), and \( N \).

\( RG \) = Float-availability goal.

\( F[X(I)|MTBF_f(I), W(I), N] \) = Probability of having X units of component end item type I in the repair/resupply channel.

\( SL \) = Maximum quantity of any component end item type which can be stocked in float.
(3) The probability that, at a random point in time, none of the end items are inoperable for the lack of float for component end item type I, is simply the component-end-item-float-availability of component end item type I.

This is given as:

\[ R(F(I) | \text{MTBF}_f(I), W(I), N) = \sum_{X(I)=0}^F(I) F(X(I) | \text{MTBF}_f(I), W(I), N) \]

For \( I=1, 2, 3, \ldots, M \).

(4) The float-availability of the complement of end items is the probability that, at a random point in time, no end item is inoperable for lack of float for any component end item. This is given by:

\[ \text{Float availability} = \prod_{I=1}^M R(F(I) | \text{MTBF}_f(I), W(I), N) \]

(5) Under the Poisson demand assumption and Palm's theorem,

\[ F(X(I) | \text{MTBF}_f(I), W(I), N) = \frac{N \cdot W(I)^X(I)}{\text{MTBF}_f(I)^{X(I)}} \cdot X(I)! \cdot e^{-N \cdot W(I)} \]

For \( I=1, 2, 3, \ldots, M \), and \( X(I)=0, 1, 2, \ldots, SL \); otherwise the total expression = 0.

(6) For convenience, let:

\[ A(I) = \frac{N \cdot W(I)}{\text{MTBF}_f(I)} \]

expected number of component end item type I in the repair/resupply cycle from the N end items assigned to the user.
Substituting equations (1) and (3) into equation (2) yields:

\[
\text{Float availability} = \prod_{I=1}^{M} \left[ \sum_{I} \frac{X(I) - A(I)}{A(I) e^{X(I) - A(I) X(I) = 0} X(I)!} \right]
\]

(4)

Therefore, the problem is to minimize:

\[
\sum_{I=1}^{M} C(I) \cdot F(I)
\]

(5)

Satisfying the constraint:

\[
\prod_{I=1}^{M} \mathbb{P}(F(I) | A(I)) \geq RG
\]

For \( F(I) \leq SL, \) and

\( I = 1, 2, 3, \ldots, M. \)

(6)

Satisfying the constraint:

\[
M
\sum_{I=1}^{M} C(I) \cdot F(I)
\]

(5)

Satisfying the constraint:

\[
\prod_{I=1}^{M} \mathbb{P}(F(I) | A(I)) \geq RG
\]

For \( F(I) \leq SL, \) and

\( I = 1, 2, 3, \ldots, M. \)

(6)

The procedure used to solve the above objective function, subject to the one constraint, is described in section II.


a. Introduction and formulation of problem. This technique uses the demand over a fixed past interval of time to estimate the float-availability resulting from operational readiness float, by allocating float levels to minimize float stock cost. The problem can be formulated as follows:

It is desired to choose float stock levels for component end items, \( F(I) \), for each of \( M \) different types of component end items which have experienced specific demands \( D(I) \) from a group of \( N \) end items over some fixed past interval of time so that the float-availability shall be greater than or equal to a prespecified float-availability requirement. The solution will also yield a minimum float stock cost.

b. Additional assumptions.

(1) Assumptions used in this model in addition to those already presented are:

(a) The mean-demand for any component end item during a fixed interval of time is a gamma\(^1\) distributed random variable \( \theta \) given by \( f(\theta, \alpha, \beta) \); where \( \alpha \) and \( \beta \) are parameters of the gamma distribution.

\(^1\)This gamma distribution of mean demands is referred to as the prior gamma distribution in the latter discussion.
(b) Historical demand data over a fixed interval of time are available for each candidate component end item for which float stock is to be established.

(2) The first assumption is supported by work done by Sherbrooke. The failure of any particular component end item in an end item composed of many such component end items is a random variable. The mean-demand of any one component end item is an unknown constant, but the mean-demand of all component end items in an end item will form a distribution of mean-demands. This is the reasoning behind the mean-demand distribution assumption. The use of the log-normal distribution as well as the gamma distribution has been suggested for such applications. Although both methods are available for computer application, the gamma distribution is used in the discussion.

(3) Historical demand data necessary to satisfy the second assumption above are available by proper application of the documentation of float usage procedures specified in AR 750-19. The demand data must represent the total demands, from all end items being supported by float, for a float component end item type.

(4) The necessity of having past demand data available eliminates the use of this procedure for new end items or for end items with no failure data. This procedure will yield a result for float requirements even if all demands are zero for the inputs, providing the prior distribution parameters are estimated. However, unless it is known that zero demands really resulted from zero failures requiring float and not because of a lack of proper data collection procedures, or because the component end item is relatively new, this procedure should not be used with such inputs.

c. Development of model.

(1) The derivation of this model uses the principles of Bayesian statistics in estimating future demands for component end items. Bayesian statistics is primarily concerned with predicting a future state of nature based on assumptions about past states, or knowledge gained on past states of nature through some experiment, or by means of a set of data tied to past experience. By assuming a distributional form for the past mean-demand for float of all component end items in an end item, it is possible to use the assumption to calculate the probabilities of X demands in the future. Instead of trying to give a point estimate of true demand for float for the component end item, this approach estimates the probability that the mean-demand for float for the component end item has various values.
Define:

- **M** = Total number of types of component end items which are candidates for float stock.

- **D(I)** = Total quantity of past demands from all end items for component end item type I over some fixed interval of time.

- **C(I)** = Unit cost of component end item type I.

- **α** = Shape parameter of the prior gamma distribution of mean-demands for component end item type I.

- **β** = Scale parameters of the prior gamma distribution of mean-demands for component end item type I.

- **P** = The number of cells into which the prior gamma distribution of mean-demands for component end item type I is divided.

- **Γ(α)** = Gamma function with argument α, \( Γ(α) = (α-1)! \)

- **θ_k** = Mean of the kth cell, where k = 1, 2, 3, ..., P.

- **Pr[D(I)|θ_k]** = Conditional probability of D(I) demands given a mean-demand of \( θ_k \).

- **T** = The fixed past interval of time over which component end item demands were observed.

- **λ** = \((AT/T)\) the ratio of the weighted average repair/resupply time (AT) for all component end item types, to the fixed past demand observation time (T).

- **Pr[X(I)|λθ_k]** = Conditional probability of X demands during a mean-repair/resupply time given a mean-demand of \( λθ_k \).

- **F(I)** = Float stock level for component end item type I.

- **R_f[F(I)|λθ_k]** = Conditional component-end-item-float-availability for component end item type I for float level F(I), given a mean-demand \( λθ_k \) during a mean-repair/resupply time.

2 The division of the distribution of mean-demands into P cells is mathematically necessary for numerical integration involved in the calculation of the Bayesian inverse probabilities. The actual number of cells is determined through a trade-off. As the number increases, the accuracy, computation time, and core requirements increase. For component end items with mean-demands of less than ten, it was found that twenty cells operate effectively and efficiently.
$R(F(I)|D(I)) = \text{Conditional component-end-item-float-availability for component end item type I for float level F(I), given that the component end item experienced D(I) demands during some fixed past interval of time.}$

$z = \text{Factor to reflect anticipated changes in usage or end item density.}$

$SL = \text{Maximum quantity of any component end item type which can be stocked in float.}$

$RG = \text{Float-availability goal.}$

(2) By the method of moments, the parameters of the prior gamma distribution can be determined as such:

\[
\text{Expected value of } \theta = \alpha \beta = \frac{M}{\sum_{I=1}^{M} D(I)/M} \\
\text{Var}(\theta) = \alpha \beta^2 = \frac{\left[ \frac{M}{\sum_{I=1}^{M} D^2(I)} - \frac{1}{M} \left[ \frac{M}{\sum_{I=1}^{M} D(I)} \right] \right]^2}{M-1}
\]

Therefore,

\[
\alpha = \left[ \frac{M}{\sum_{I=1}^{M} \frac{D(I)}{M}} \right]^{2/ \left( M-1 \right)} \left[ \frac{M}{\sum_{I=1}^{M} D^2(I)} - \frac{1}{M} \left[ \frac{M}{\sum_{I=1}^{M} D(I)} \right] \right]^2 \\
\beta = \frac{\left[ \frac{M}{\sum_{I=1}^{M} D^2(I)} - \frac{1}{M} \left[ \frac{M}{\sum_{I=1}^{M} D(I)} \right] \right]^2}{(M-1) \left[ \frac{M}{\sum_{I=1}^{M} D(I)} \right]}
\]

(3) Once $\alpha$ and $\beta$ are obtained from equations (7) and (8), the prior gamma distribution defining the past mean-demands for all component end items is described. However, the data available concern past demands...
for each component end item and not mean-demands. The objective is to use these past demands to calculate the float-availability for each type of component end item in light of the past demand data.

(4) It is convenient to subdivide the prior gamma distribution into \( P \) cells. This essentially says that, for computational purposes, the prior gamma distribution is composed of \( P \) cells which generate a mean-demand of \( \theta_k \) (mean for the \( k \)th cell). This makes it possible to calculate the probability of the occurrence of a particular mean-demand \( \theta_k \) given that a past demand \( D \) has occurred.

(5) The mean of the first and last cell (\( \alpha_1 \) and \( \alpha_p \), respectively) must be calculated at some percentile point, since it is obviously impossible to subdivide an infinite scale into \( P \) parts. The .01 and .99 percentile levels are used here, but any other level could be used. The result of narrowing the limits (e.g., .05 and .95 levels) would be to weight the extremes of the distribution by moving the extreme estimates of the mean-demands corresponding to the .05 and .95 percentile levels toward the center of the distribution. This narrowing would have little effect on the stock levels calculated. If carried too far, it would tend to decrease the calculated float levels, because cells representing high demands would not be weighted proportionally. Therefore, with the range from the .01 to the .99 percentile included within the distribution, the results should be of sufficient accuracy for most applications.

(6) The mean-demands, \( \theta_1 \) and \( \theta_p \), at the lower and upper limits, for the .01 and .99 percentiles respectively, are calculated from the following equations.

\[
\theta_{.01} = \int_0^{\theta_1} \frac{y^{(\alpha-1)} e^{-(y/\beta)}}{\Gamma(\alpha) \beta^\alpha} \, dy
\]

\[
\theta_{.99} = \int_0^{\theta_p} \frac{y^{(\alpha-1)} e^{-(y/\beta)}}{\Gamma(\alpha) \beta^\alpha} \, dy
\]

(7) The computer program presented herein evaluates equations (9) and (10) by transforming the gamma distribution into a chi-square distribution\(^3\) since chi-square tables are more readily available and easier to use.

\(^3\) The transformation is \( y = 2X/\beta \) where \( X \) is a chi-square distributed random variable.
(8) With $\theta_1$ and $\theta_2$ calculated, the other means are calculated by equation (11).

$$\theta_k = \theta_1 + (k-1) \frac{\theta_p - \theta_1}{p-1}$$

(11)

For $k = 1, 2, 3, \ldots, P$.

(9) The upper and lower limits $L_{(k-1)}$ and $L_k$ respectively, of each of the $P$ cells are obtained from the following equation:

$$L_k = \theta_k + \frac{\theta_p - \theta_1}{2(p-1)}$$

(12)

For $k = 1, 2, 3, \ldots, (P-1)$;

with $L_0 = 0$ and $L_P = \infty$.

(10) With these values, the probability of occurrence of a mean-demand of $\theta_k$ is:

$$P(\theta_k) = \int_{L_{(k-1)}}^{L_k} \left[ \frac{\Gamma(\alpha) \beta^{\alpha}}{\Gamma(\alpha+1)} \right] y^{\alpha-1} e^{-(y/\beta)} dy$$

(13)

For $k = 1, 2, 3, \ldots, P$.

(11) Using the Poisson demand generation assumption, the probability of observing a particular demand $D(I)$ given the occurrence of a mean-demand of $\theta_k$ is:

$$P(D(I) | \theta_k) = \frac{e^{D(I) \theta_k} \theta_k^{-D(I)} (\theta_k D(I) !)^{-1}}{D(I)!}$$

(14)

This probability is calculated for each type of component end item for all $\theta_k$'s.
(12) Using Bayesian inference, the probability of any \( \theta_k \) occurring, given an occurrence of \( D(I) \) is:

\[
Pr[\theta_k | D(I)] = \left[ \frac{Pr[D(I)|\theta_k] \cdot Pr(\theta_k)}{\sum_{k=1}^{P} Pr[D(I)|\theta_k] \cdot Pr(\theta_k)} \right]
\]

(13) The demand data used in this method is that obtained over a fixed post interval of time \( T \). The float problem is concerned with supplying enough float of component end items to protect against supply shortages over the repair/resupply time interval. Therefore, the historical values need adjustment to convert to the projected values \( \theta'_k \), over the required time scale. This yields:

\[
\theta'_k = Z_k \theta_k
\]

For \( k=1, 2, 3, \ldots, P \).

(14) The \( Z \) value is a factor which allows the analyst to adjust the data to fit future operational needs which may not be compatible with the usage that generated the demand data. Possible applications of the \( Z \) factor may reflect changes in environmental and/or usage conditions. If conditions have changed so that twice as many demands for float are expected, the value of \( Z \) would be equated to 2.0. With the new mean demands, the probability of a demand of size \( X \) arising for each component end item type \( I \), given the mean demand of \( \theta'_k \) is:

\[
Pr[X(I)|\theta'_k] = \left[ \frac{\theta'_k \cdot X(I)}{\theta'_k \cdot e} \right]^X(I) \cdot e^{-X(I)}
\]

For \( X(I)=0, 1, 2, \ldots, F(I) \).

(15) Using equation (17), the probability of having demands for \( F(I) \) or less component end items of type \( I \) is:

\[
R[F(I)|\theta'_k] = \sum_{X(I)=0}^{F(I)} Pr[X(I)|\theta'_k]
\]

For \( F(I)=0, 1, 2, \ldots, SL \).

3-10
(16) With equations (15) and (18), the weighted average utility of each additional float of component end item type I, given that the component end item had a demand of D(I) in the past, is:

\[ R(F(I) | D(I)) = \sum_{k=1}^{P} R(F(I) | \theta_k') Pr(\theta_k'|D(I)) \]  

(19)

(17) Equation (19) gives the float-availability of component end item I given a particular float stock level of F(I). The float-availability of a complement of end items is:

\[ \text{Float-availability} = \prod_{I=1}^{M} R(F(I) | D(I)) \]  

(20)

(18) The problem is then reduced to finding the set of stock levels F(I) which minimizes the cost of float stock and yields the desired float-availability goal. Therefore, the problem is to minimize

\[ \sum_{I=1}^{M} C(I)F(I), \text{ satisfying:} \]

\[ \prod_{I=1}^{M} R(F(I) | A(I)) \geq RG \]  

(21)

(19) The number of end items being supported with component end items stocked in float does not enter into the calculations of this method. Demand data are generated by all end items; using such data in this method accounts for all end items, regardless of the actual number. If there is an anticipated change in the number of end items in use, the value of the \( z \) must reflect the percentage of change.

3-4. Conclusion. a. These methods provide the user the capability to:

(1) Allocate operational readiness float to achieve a float-availability goal.

(2) Perform the allocation in a manner which minimizes the cost to achieve a float-availability goal.

(3) Perform allocations which consider the random nature of end item failure and component end item failures.

b. These important considerations are not available to the user in present methods of allocating operational readiness float.
CHAPTER 4

USER'S GUIDE

4-1. General. A computer program was developed to perform the calculation inherent in the previously discussed float allocation procedure. The coding procedures required for the utilization of this computer program are presented in the following chapter. The presentation consists of two parts which coincide with the two procedures for float allocation: (i) without and (ii) with historical demand data.

4-2. Float Allocation Without Historical Data. a. Data inputs. The following set of data is necessary to utilize this method:

(i) The code number -- the value is 1 for this method.

(ii) The number of float-availability goals and their values, for which float allocation is desired. (Maximum number is 10.)

(iii) Float stock level limit -- the maximum number of any type of component end item to be stocked in float.

(iv) The quantity of end items assigned to the user.

(v) For each floatable component end item:

(a) Description or user-distinguishable code.

(b) Unit cost to float each component end item.

(c) $MTBF(I)$ -- the mean-time-between-failures for each component end item of type I which requires float.

(d) $MTF(I)$ -- the mean-time-between-float-replenishment for each component end item of type I. This value is the mean interval between the time that a failed end item generates a float demand and the time that the floated component end item is replaced or returned to float status. This time period may be subdivided into: the mean-time-to-repair/resupply, the mean-transportation-time, and the mean-time-waiting-repair/resupply.

b. Coding procedure. The data inputs are discussed in four parts, each of which contains an example coded data card. The data positions on the card and the inclusion or exclusion of decimal points must be strictly followed. The specific formats are illustrated on the sample data cards.

(1) Initialization card 1:

(a) The code number value of 1 is entered in column A.
(b) The number of float-availability goals to be used in the float calculation is right justified in columns 5-8. (Maximum number is 10.)

(c) The float stock level limit is right justified in columns 9-12.

(d) The number of different floatable component end items is right justified in columns 13-16.

(e) An example coded initialization card 1 is shown below.

(2) Initialization card 2:

(a) Enter the values of the desired float-availability goals in descending order (highest value first) from left to right on the card in consecutive ten column fields. Each value must be less than 1.0 and must contain a decimal point. For the computer program in this pamphlet, a maximum of four decimal places can be used.

(b) An example coded initialization card 2 is shown below.

(3) Initialization card 3:

(a) Quantity of end items assigned to the user. This number must be right justified in columns 1-4.

(b) An example coded initialization card is shown below.

(4) Item characteristics card:

(a) For each floatable component end item, enter the identification
(b) The unit cost to float each component end item is entered in columns 41-50, in dollars and cents. (Maximum is 9999999.99.)

(c) The \( W(I) \) is entered in columns 51-60.

(d) The \( \text{MTBF}_f(I) \) is entered in columns 61-70.

The unit cost, \( \text{MTBF}_f(I) \), and the \( W(I) \) must each contain a decimal point; the dimensional units for \( \text{MTBF}_f(I) \) and \( W(I) \) must be identical for all component end items, i.e., Cost = $1000.00, \( W(I) = 5.00 \) hours, \( \text{MTBF}_f = 245.00 \) hours.

(e) An example item characteristics card is shown below.

\[
\text{PSN XXXXX, XX} 1000.0 5.0 245.0
\]

c. Data deck sequence. The set of data cards must be ordered as shown below:
4-3. Float Allocation With Historical Demand Data. a. Data inputs.
The following set of data is required as input for the second method.

(1) The code number -- the value is 2 for this method.

(2) The number of float-availability goals and their values, for
which float allocation is desired. (Maximum number is 10.)

(3) Float stock level limit -- the maximum number of any type of
component end item to be stocked in float.

(4) The quantity of component end items assigned to the user.

(5) For each floatable component end item:
   (a) Description or user-distinguishable code.
   (b) Unit cost to float each component end item.
   (c) The number of past demands for float of each component end
       item, accumulated over some interval of time.

(6) A projected usage/density factor. This factor is used to bias
the input data whenever it is known that float is being computed for end
items that will have different usage rates and/or different quantities
than the end items that generated the original data.

(7) The mean-repair/resupply time for component end items being
floated.

(8) The interval of time over which the demand data was obtained.

b. Coding procedure. The data inputs are discussed in four parts,
each of which contains an example coded data card. The data positions on
the card and the inclusion or exclusion of decimal points must be strictly
followed. The specific formats are illustrated on the sample data cards.

(1) Initialization card 1:
   (a) The code number value of 2 is entered in column 4.
   (b) The number of float-availability goals to be used in the float
       calculation is right justified in columns 5-8. (Maximum number is 10.)
   (c) The float stock level limit is right justified in columns
       9-12.
   (d) The number of different floatable component end items is right
       justified in columns 13-16.
(e) An example coded initialization card 1 is shown below.

(2) Initialization card 2:

(a) Enter the values of the desired float-availability goals in descending order (highest value first) from left to right on the card - in consecutive ten column fields. Each value must be less than 1.0 and must contain a decimal point. For the computer program in this pamphlet, a maximum of four decimal places can be used.

(b) An example coded initialization card 2 is shown below.

(3) Item characteristics card:

(a) For each floatable component end item, enter the identification code (the FSN or other identification) in columns 1-40. A short description can also be included.

(b) The unit cost to float each component end item is entered in columns 41-50, in dollars and cents. (Maximum is 9999999.99.)

(c) The past demand is entered in columns 51-55. The demand must be read in as a real number, for computational purposes only to one decimal place.

(d) An example coded item characteristics card is shown below.

(4) Data characteristics card:

(a) The usage/density factor is entered in columns 1-10 with a maximum of three decimal places.
(b) The mean-repair/resupply time is entered in columns 11-20 with a maximum of three decimal places.

(c) The time interval over which the data were collected is entered in columns 21-30 with a maximum of three decimal places. The dimensional units must be the same as for (b) above (e.g., both hours, or days, etc.).

(d) An example coded data characteristics card is shown below.

```
1.25 32.0 365.0
```

c. Data deck sequence. The set of data cards must be ordered as shown below.
Appendix A

MODEL VARIATIONS

This appendix is used to discuss situations which deviate from those idealized in the formulation of this model.

A-1. **Condition** - An item is a component end item, and it is impossible or undesirable to allocate that item as float.

**Action** - Such an item may be disregarded when applying this model. However, the float-availability indicated by the float allocation must be modified by the user to reflect the float-availability of the nonfloatable component end item.

Two modifications are possible that permit the model to obtain a float-availability which includes the effect of the nonfloatable component end item.

a. The unit cost of stocking the component end item in float may be assigned a large value, i.e., $9999999.99.

b. Alternatively and preferably, the float-availability goal can be divided by the float-availability of the nonfloatable component end item (the probability of zero demands on the nonfloatable component end item). This new value can then be input into the model as the float-availability goal.

A-2. **Condition** - The float limit varies with component end item.

**Action** - The model assumes the maximum float limit is the same for each floatable component end item. It is possible that variable float limits will have no effect on the presently formulated model. The effect can be determined by setting the maximum float limit to a high value (less than 100). This value should allow the float-availability goals to be achieved without reaching this limit. If, upon reviewing the model allocation, it is found that the actual float limits of the respective component end items are not exceeded, the solution has been reached. However, if float limits are exceeded, computer program modification will be necessary.

A-3. **Condition** - The incremental cost of float for one or more component end items is not constant.

**Action** - The average incremental cost can be used, if the associated error is acceptable to the user.
Model redefinition is necessary to arrive at an optimal solution if the inclusion of variable incremental costs of float is desired.

A-4. Condition - It is necessary to maximize float-availability constrained to a fixed amount of funds.

Action - This problem is a slight variation of the problem addressed in this pamphlet (obtain a float-availability goal at a minimum cost). Only minor modification of the computer program is required.
This appendix contains two examples to illustrate the dual usage of the model as well as the output format of the computer adaption. The first example gives a float allocation for an end item without demand data. The second example gives an allocation for an end item with demand data.

B-1: System Without Prior Data. a. The end item under consideration is to meet a float-availability goal of 0.95 at a minimum cost. The failure and repair related data and the cost for the four component end items are as follows:

<table>
<thead>
<tr>
<th>Component End Items</th>
<th>W(hours)</th>
<th>'TBF(hours)</th>
<th>Cost/Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN (XXXX.XX1)</td>
<td>5.0</td>
<td>245.0</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX2)</td>
<td>6.0</td>
<td>194.0</td>
<td>$2,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX3)</td>
<td>5.0</td>
<td>445.0</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX4)</td>
<td>5.0</td>
<td>120.0</td>
<td>$500.00</td>
</tr>
</tbody>
</table>

The number of end items authorized float support is 50.

b. The computer inputs and outputs for this example are shown on table B-1. An output has been included which shows the float stock levels required to meet a float-availability goal of 0.99 for comparison purposes. The total cost for float stock to meet the 0.95 float availability goal is seen to be $25,000. The float-availability goal of 0.99 can be obtained for $33,500. Therefore, the float-availability can be increased by another 4% with the expenditure of $8,500. This illustrates the importance of computing the float levels for a number of float-availability goals instead of merely one. The achieved float-availabilities for the float levels calculated are 0.96 and 0.99, which are the closest values obtained that meet or exceed the requirements of 0.95 and 0.99. This is not saying that a system of float levels cannot be computed that yields a lower float-availability closer to the desired value even at a slightly smaller cost. It simply means that on a dollar/float-availability basis, a float-availability of 0.96 is the best rate that exceeds specifications.

B-2: System With Prior Data. a. The operational readiness float for the end items under consideration is to meet a float-availability goal of 0.95 at a minimum cost. Each end item is composed of four component end items. The historical demand data for a fixed interval of time and the cost per unit are listed below.
<table>
<thead>
<tr>
<th>Component End Items</th>
<th>Demands/Period</th>
<th>Cost/Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN (XXXX.XX1)</td>
<td>1.0</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX2)</td>
<td>2.0</td>
<td>$2,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX3)</td>
<td>0.0</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>FSN (XXXX.XX4)</td>
<td>2.0</td>
<td>$500.00</td>
</tr>
</tbody>
</table>

It was assumed the condition of usage, number of end items in the group, and period for float allocation are equal to the condition of the past demand period.

b. The computer inputs and outputs for this example are shown on table B-2. The output for the 0.99 float-availability goal is included for comparison purposes.
Table B-1. Float Allocations Without Prior Data

**INPUT LISTING TABLE**

NUMBER OF END ITEMS FIELDED = 50

<table>
<thead>
<tr>
<th>ITEM</th>
<th>COST</th>
<th>UNIT</th>
<th>MTTR</th>
<th>MTBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>1000.00</td>
<td>5.00</td>
<td>245.00</td>
<td></td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>2000.00</td>
<td>6.00</td>
<td>194.00</td>
<td></td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>5000.00</td>
<td>5.00</td>
<td>445.00</td>
<td></td>
</tr>
<tr>
<td>FSN XXXXX.XX4</td>
<td>500.00</td>
<td>5.00</td>
<td>120.00</td>
<td></td>
</tr>
</tbody>
</table>

REQU FLOTAVALABILITY -- 0.9500

FLOAT AND ASSOCIATED COST TO OBTAIN THE ACTUAL FLOTAVALABILITY

<table>
<thead>
<tr>
<th>ITEM ID. NO. AND DESCRIPTION</th>
<th>REQUIRED ITEM FLOAT</th>
<th>TOTAL ITEM COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>4</td>
<td>4000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>4</td>
<td>8000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>2</td>
<td>10000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX4</td>
<td>6</td>
<td>3000.00</td>
</tr>
</tbody>
</table>

TOTAL COST OF FLOAT STOCKAGE 25000.00

REQU FLOTAVALABILITY -- 0.9900

FLOAT AND ASSOCIATED COST TO OBTAIN THE ACTUAL FLOTAVALABILITY

<table>
<thead>
<tr>
<th>ITEM ID. NO. AND DESCRIPTION</th>
<th>REQUIRED ITEM FLOAT</th>
<th>TOTAL ITEM COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>5</td>
<td>5000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>5</td>
<td>10000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>3</td>
<td>15000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX4</td>
<td>7</td>
<td>3500.00</td>
</tr>
</tbody>
</table>

TOTAL COST OF FLOAT STOCKAGE 35500.00
Table B-2. Float Allocations With Prior Data

INPUT LISTING

<table>
<thead>
<tr>
<th>ITEM</th>
<th>COST/UNIT</th>
<th>DEMAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>1000.00</td>
<td>1.0</td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>2000.00</td>
<td>2.0</td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>5000.00</td>
<td>0.1</td>
</tr>
<tr>
<td>FSN XXXXX.XX4</td>
<td>500.00</td>
<td>2.0</td>
</tr>
</tbody>
</table>

USAGE FACTOR= 1.00
AVERAGE REPAIR RESUPPLY TIME= 1.00
DATA COLLECTION PERIOD= 1.00

REQUIRED FLOAT-AVAILABILITY= 0.9500
ACTUAL FLOAT-AVAILABILITY= 0.9597

FLOAT AND ASSOCIATED COST TO OBTAIN THE ACTUAL FLOAT-AVAILABILITY

<table>
<thead>
<tr>
<th>ITEM ID.NO. AND DESCRIPTION</th>
<th>REQUIRED ITEM FLOAT</th>
<th>TOTAL ITEM COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>5</td>
<td>5000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>6</td>
<td>12000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>3</td>
<td>15000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX4</td>
<td>7</td>
<td>3500.00</td>
</tr>
</tbody>
</table>

TOTAL COST OF FLOAT STOCKAGE 35500.00

REQUIRED FLOAT-AVAILABILITY= 0.9900
ACTUAL FLOAT-AVAILABILITY= 0.9931

FLOAT AND ASSOCIATED COST TO OBTAIN THE ACTUAL FLOAT-AVAILABILITY

<table>
<thead>
<tr>
<th>ITEM ID.NO. AND DESCRIPTION</th>
<th>REQUIRED ITEM FLOAT</th>
<th>TOTAL ITEM COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSN XXXXX.XX1</td>
<td>7</td>
<td>7000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX2</td>
<td>7</td>
<td>14000.00</td>
</tr>
<tr>
<td>FSN XXXXX.XX3</td>
<td>5</td>
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TOTAL COST OF FLOAT STOCKAGE 50500.00
Appendix C

FORTRAN PROGRAM FOR FLOAT ALLOCATION PROCEDURE

This appendix contains a FORTRAN IV computer program which can be used to determine operational readiness float based on the two methods previously discussed. Coding procedures for this program are discussed in chapter 4. This computer program was structured to handle low demand items. High demand data will cause the logarithm and exponential functions to overflow and underflow. If such application is desired, modification of the computer program is necessary.
C - FORTRAN IV G Level 18

DATE = 76/96 20/14/01 PAGE 22A1

D001 DIMENSION W(6),MTBF(100),C(100),A(I),B(I)
LND(100,3),MAG+H(100,3),FLAT(100),L(100,4),AL+G(100,3),
V(100),CH(100,3),PHU(100,3),PHQ(100,3),
3ICHT(2,20),SOUND(3),PRQUT(3,3),M(250)

D003 DIMENSION COST(3)

D003 DATA CH(150),H(400),6,63.,.,.2231,9,21.,.,.2231,9,21.,.,.2231,9,21.,.,.2231,9,21.
.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.,.
C-3
C-4
C-5
Appendix D

FORTRAN SEGMENT FOR PRIOR DATA USING LOG-NORMAL DISTRIBUTION

D-1. If the demand data indicate that the distribution of means follows a log-normal distribution, another program segment must be substituted into the source deck to allow for a log-normal representation. The log-normal segment shown on table D-1 must be substituted for the segment between markers A + and B + in the source program which is contained in appendix C.

D-2. To use the log-normal segment, it is necessary to include, following the data elements required by chapter 4, data for the standardized normal distribution. This data set is shown in table D-2 and must be entered in ascending order.
Table D-1. Log-normal Segment

A 130 CONTINUE
095 TIN = IN
096 ANI = NI
097 AP = SUMDI/ANI
099 B = SQDI-ANI*AP**2.0 / (ANI-1.0)
100 VAR = ALOG (1.0+B/AP**2.0)
101 AMEAN = ALOG (AP)-VAR/2.0
102 SDEV = SQRT (VAR)
103 Z01 = -2.33
104 X01 = EXP (Z01*SDEV+AMEAN)
105 X99 = EXP (-Z01*SDEV+AMEAN)
106 SUMP = .01
107 TEMP 2 = (X99-X01)/TIN
108 NT = IN-1
109 DO 230 K = 1, IN
200 Q(K) = X01+(K-1)*TEMP 2
201 LIM (K) = Q (K) + TEMP 2/2.0
202 SUMPT = 0.0
203 READ (5,1012) (ZN (I), I = 1, 234)
204 DO 1K=1, NT
205 JZ = ((ALOG (LIM (K)) - AMEAN)*100.0/SDEV)
206 IF (JZ-0) 4, 7, 7
207 4 JZ = -1*JZ+1
208 ZN (JZ) = 1.0-ZN (JZ)
209 IF (K-1) 8, 8, 9
2010 PR (K) = ZN (JZ)
2011 URP = ZN (JZ)
2012 GO TO 1
2013 7 JZ = JZ + 1
2014 9 PR (K) = ZN (JZ)-URP
2015 URP = ZN (JZ)
2016 IF (K-NT) 1, 3, 1
2017 3 PR (IN) = 1.0-ZN (JZ)
2018 1 CONTINUE
2019 B 130 CONTINUE
2020 DO 430 I = 1, NI
Table D-2. Normal Table Data for Log-normal Program Segment

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D-3
DEFINITION OF TERMS AND PHRASES

Mean-time-awaiting-repair -- Mean time a failed component end item spends in a queue awaiting repair.

Mean-transportation time -- Mean time spent in transporting a component end item to the repair facility from the location of failure, plus transportation time from the repair facility to the float stock location.

Component end item -- A group of assemblies, subassemblies, and parts, which, although part of a larger end item, are connected together in such a manner as to be capable of operating independently of the larger end item, e.g., transmitter, receiver, power supply unit, etc. These are also called end items of equipment.

Component-end-item-float-availability (as applied to operational readiness float) -- The probability that, at a random point in time, none of the end items supported by operational readiness float are inoperable for lack of operational readiness float for that component end item.

Component end item float level -- The quantity of a specific component end item which is stocked in operational readiness float.

Demand data -- Historical information describing the quantity of each component end item requested from operational readiness float during a specific interval of time.

End item (JCS) -- A final combination of end products, component parts, and/or materials which is ready for its intended use, e.g., ship, tank, mobile machine shop, aircraft, etc.

Float-availability (as applied to operational readiness float) -- The probability that, at a random point in time, none of the end items supported by operational readiness float are inoperable for lack of any item which is authorized operational readiness float.

Float-availability goal -- The float-availability desired for the end items supported by operational readiness float.

Float item -- A term used collectively to denote a component end item which is to be stocked for use as operational readiness float.

Float stock cost -- The total cost of buying, stocking, and maintaining a component end item allocated to operational readiness float.
Operational readiness float -- Per AR 750-19, "End items of mission essential, maintenance significant equipment authorized for stockage by maintenance support units or activities to replace unserviceable repairable equipment to meet operational commitments."

Optimal float allocation -- An allocation, such that no other allocation can meet or exceed the float-availability achieved by that allocation, at less cost.

Utility -- A measure of the increase in float-availability per dollar expended which corresponds to increasing by one unit the quantity of a component end item kept in float.
FOR THE COMMANDER:

OFFICIAL:  CHARLES T. HORNER, JR.
           Major General, USA
           Chief of Staff

W. J. PHILLIPS
Colonel, GS
Chief, HQ Admin Mgt Ofc

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