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on

The Accumulation of Fatigue Damage in Aircraft Materials and Structures

by

J.Schijve

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THE ACCUMULATION OF FATIGUE DAMAGE IN AIRCRAFT MATERIALS AND STRUCTURES

by

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This AGARDograph was prepared at the request of the Structures and Materials Panel of AGARD.

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FOREWORD

The Structures and Materials Panel of the NATO Advisory Group for Aerospace Research and Development (AGARD), is composed of engineers, scientists and technical administrators from industry, governmental establishments and universities in the NATO nations. Our part in assisting the Mission of AGARD, recorded on a previous page, is reflected in the nature of the Panel activities which cover more than 20 different technical subjects in progress at the present time.

It is many years since the Panel observed, in relation to the assessment of cumulative fatigue damage, that the Palmgren-Miner rule was frequently used without an appreciation of the assumptions upon which the rule is based. Many who used the rule in important design calculations, even when aware of its limitations, had no more certain rule to guide them and this situation may not change markedly until a rational law for the assessment of fatigue damage is available. The Panel agreed that an investigation of the state of knowledge surrounding fatigue damage accumulation should be undertaken, in the terms recorded in Professor Schijve's introduction to this report.

We hoped to provide a sound background to those concerned with fatigue problems in design and to identify significant lines of further investigation for research workers.

The task has taken many years and the Panel was fortunate in having Professor Schijve to undertake this work and is indebted to him for the skill and tenacity he has applied in bringing it to the present state of completion.

Anthony J.Barrett
Chairman,
AGARD, Structures and Materials Panel

SUMMARY

The available literature is surveyed and analysed. Physical aspects of fatigue damage accumulation are discussed, including interaction and sequence effects. Empirical trends observed in variable-amplitude tests are summarized including the effects of a high preload, periodical high loads, ground-to-air cycles and the variables pertaining to program loading, random loading and flight-simulation loading. This also includes results from full-scale fatigue test series. Various theories on fatigue damage accumulation are recapitulated. The significance of these theories for explaining empirical trends as well as for estimating fatigue properties as a design problem is evaluated. For the latter purpose reference is made to the merits of employing experience from previous designs. Fatigue testing procedures are discussed in relation to various testing purposes. Emphasis is on flight-simulation tests. Finally several recommendations for further work are made.

CONTENTS

LIST OF SYMBOLS

D	damage
ΔD	damage increment
kc	kilocycles (1000 cycles)
K	stress intensity factor
K_t	theoretical stress concentration factor
ℓ	crack length for edge crack
	semi crack length for central crack
$\Delta\ell$	increment of ℓ
$d\ell/dn$	crack propagation rate
n	number of cycles
n'	number of cycles of a particular load spectrum
N	fatigue life
N'	fatigue life in program test or random test
N_ℓ	fatigue life to obtain a crack of length ℓ
P	load
r	radius
R	stress ratio (S_{min}/S_{max})
S	stress
S_a	stress-amplitude
$S_{a,max}$	maximum S_a in a variable-amplitude test
$S_{a,min}$	minimum S_a in a variable-amplitude test
S_m	mean stress of stress cycle
S_{max}	maximum stress of stress cycle
S_{min}	minimum stress of stress cycle
S_{rms}	root-mean square value of stress in random load test
S_u	ultimate tensile stress
$S_{0.1}$, $S_{0.2}$	yield stresses for 0.1 and 0.2 percent permanent yield
S_{1g}	mean stress in flight
ΔS	stress range ($S_{max} - S_{min}$)
t	sheet th[illegible]
ε	stra[illegible]
[illegible]	exponent [illegible] relation (SN[illegible] = constant)
μ	micron = 0.001 mm

THE ACCUMULATION OF FATIGUE DAMAGE IN AIRCRAFT MATERIALS AND STRUCTURES

J. Schijve

1. INTRODUCTION

In a classic fatigue test the load is varying sinusoidally with a constant mean load and a constant load amplitude. The fatigue load on a structure under service conditions, however, generally has a more or less arbitrary or random character. Nevertheless it may well be assumed that the accumulation of fatigue damage under such an arbitrary fatigue load is a process which occurs in the material in a similar way as in the classic fatigue test. It is just one step further to state that the fatigue life for an arbitrary load-time history can be predicted from fatigue life data obtained in classic fatigue tests. The well-known Palmgren-Miner rule ($\Sigma n/N = 1$) was based on such assumptions, which also applies to more complex laws proposed by others.

It has to be admitted, however, that a rational law for the calculation of fatigue damage accumulation is not yet available. There is an abundant literature on fatigue which has revealed several characteristic features of the fatigue process in metallic materials. Fatigue tests with a varying load-amplitude were also carried out by many investigators. This has indicated many empirical trends for which physical explanations were sometimes given. Moreover, calculation rules for the accumulation of fatigue damage were published from time to time. Nevertheless the present situation is far from satisfactory, even from an engineering point of view.

The purpose of this report is to survey the various aspects of fatigue damage accumulation and to analyse the problems associated with this phenomenon. The implications of the present knowledge for making life estimates in the design phase of an aircraft and for planning fatigue tests will be considered also.

In summary the aims of the report are:

<u>a</u> To review the present state of knowledge about fatigue damage accumulation (Chapters 2 and 3).
<u>b</u> To summarize the empirical trends obtained in tests with variable-amplitude loading and to see whether they can be explained (Chapter 4).
<u>c</u> To survey the various life calculation theories (Chapter 5).
<u>d</u> To analyse the design problem of estimating fatigue lives and crack propagation rates (Chapter 6).
<u>e</u> To assess the merits and the limitations of various fatigue testing procedures adopted for fatigue life evaluations (Chapter 7).

The report is completed (Chapter 8) by sections giving a summary of the present study and recommendations for future work.
It should be pointed out that aspects associated with elevated temperature due to aerodynamic heating have been excluded from the survey.

It is hoped that this report will provide a background to those dealing with fatigue life problems in the aircraft industry. On the other hand, it is also hoped that it will give a better picture of the real problem to scientists in universities and laboratories when approaching fatigue damage accumulation from a more theoretical point of view.

2. THE FATIGUE PHENOMENON IN METALLIC MATERIALS

Our present knowledge about fatigue in metals has to a large extent been obtained by means of the microscope. In 1903 Ewing and Humfrey observed that fatigue cracks were nucleated in slip bands. Around 1930 classical studies were conducted by Gough and his co-workers, who further emphasised the significance of slip systems and resolved shear stresses. After 1945 the number of microscopical investigations has considerably increased and the information becoming available has broadened for a variety of reasons. It turned out that the observations could be dependent on the type of material, the type of loading and

and the level of magnification. The electron microscope has added a number of details unknown before. It will be tried in this chapter to recapitulate briefly the main points of the numerous phenomenological investigations. More detailed surveys are given in references 1-6.

Three phases in the fatigue life

An important observation is that cracks may nucleate relatively early in the fatigue life. As an illustration figure 2.1 shows results of optical microscopy during fatigue tests on aluminum alloy specimens (Ref.7). Cracks of 0.1 millimeter (100 μ) were present after 40 percent of the fatigue life had elapsed. The electron microscope has revealed cracks at earlier stages, almost from the beginning of a fatigue test. Nevertheless the lower part of figure 2.1 suggests that nucleation is relatively more difficult at stress levels near the fatigue limit.

It appears useful to divide the fatigue life into three phases, namely: crack nucleation, crack propagation and final failure, see figure 2.2. A difficulty thus introduced is that of the definition of the transition from the nucleation phase to the propagation phase.

Slip

It is well established that fatigue requires cyclic slip. The present state of our knowledge about dislocations and metal physics leave no doubt about the essential contribution of slip to fatigue.

Fatigue on the atomic level, decohesion

If there were no decohesion there would be no fatigue. In principle decohesion may occur by sliding-off, by cleavage or by vacancy diffusion. Disruption of atomic bonds is involved in any case.

Although it is difficult to rule out cleavage type conceptions, it is thought that sliding-off is the more plausible mechanism for relatively ductile materials. Sliding-off implies that dislocations are cutting through the free surface which may also be the tip of a fatigue crack. A second possibility is that dislocations are generated at the tip of a crack. In the latter case the tip of a crack acts as a dislocation source rather than a dislocation sink. In general terms fatigue may be visualised as the conversion from cyclic slip into crack nucleation and crack growth.

Chemical attack may facilitate the decohesion process but the environmental effect on the atomic level is not well understood.

Fatigue on the microscopic level, striations

Cross sections of fatigue cracks, as viewed through the optical microscope, usually show the crack to be transgranular. The path of the crack appears to have a fairly irregular orientation at this level of magnification.

Replicas from the fatigue fracture surface studied in the electron microscope have revealed the so-called striations, see for an example figure 2.3. Such striations clearly prove that crack extension occurred in every load cycle. This type of evidence was mainly obtained for macro-cracks, while for micro-cracks striations cannot be observed for several reasons. However, if crack propagation occurs as a cyclic sliding-off mechanism it appears reasonable to assume that crack growth of a microcrack also occurs in every load cycle.

For aluminium alloys evidence is available that strongly suggest crack extension and striation formation to occur as a co-operative sliding-off on two differently oriented $\{111\}$ slip planes (Refs.9,10).

Type of loading (tension vs. torsion)

Brief reference may be made here to the work of Wood et al. (Ref.11) concerning torsion fatigue tests on copper specimens. It turned out that crack nucleation occurred by the forming of pores and this was a process of a relatively long duration. It appears that the process may be essentially different from fatigue under cyclic tension because in pure tension the planes with a maximum shear stress have a zero tensile stress. It is thought that this will allow a much slower crack nucleation and even a different dislocation mechanism may be applicable. Since fatigue in aircraft structures is associated with cyclic tension, torsion will not be considered in the present report.

Nucleation sites

In fatigue tests on unnotched specimens the probability to observe more than one fatigue crack in the same specimen is increasing at higher stress amplitudes. At low stress levels near the fatigue limit quite frequently only one crack nucleus is observed. This observation may inspire statisticians to develop a weakest-link theory to explain size effects. Anothe r consequence, not generally recognized, is that special fatigue sensitive conditions apparently exist at the site of crack nucleation. Grosskreutz and Shaw (Ref.12) in this respect have studied crack nucleation at intermetallic particles in an Al alloy (see also Ref.13). Nucloation at inclusions in high strength steels were also reported. Other special conditions can easily be thought of, such as cladding layer, surface scratches, local inhomogeneity of the material.

Plane strain vs. plane stress conditions

Macroscopically a slowly propagating fatigue crack is growing in a plane perpendicular to the maximum tensile stress (main principal stress). However, if the crack rate is accelerating the growth will continue on a plane at 45° to the maximum tensile stress. This transition occurs gradually, see figure 2.4, starting at the free surface of the material with the development of shear lips. It is generally accepted that this is to be related to the transition from plane-strain conditions to plane-stress conditions at the tip of the crack. After the transition has occurred, it is more difficult to observe the striations but there are still indications that crack extension occurs in every load cycle.

The transition from the tensile mode (plane strain) to the shear mode (plane stress) and Forsyth's Stage I/Stage II (Ref.2) proposition should not be confounded. Stage I was associated with the initial and very slow growth along a slip plane and Stage II with later growth perpendicular to the tensile stress. Stage II should correspond to the tensile mode. Stage I, however, is thought to occur only at the free surface of the material at both low and fast propagation rates (Ref.6). It is promoted by the lower restraint on slip at the free surface.

Cyclic strain-hardening (and softening)

Since fatigue and crack growth are a consequence of cyclic slip, cyclic strain-hardening (or softening) will occur. That means that the structure of the material will be changed. The spatial configurations of the dislocations will change. Dislocation multiplication may occur as well as dislocation reactions and pinning. According to Grosskreutz, a cel structure will be formed (Ref.14).

If the material was already work-hardened, re-arrangement of the dislocation distribution may lead to cyclic strain softening. Anyhow, crack growth will occur in a material that will not have the same dislocation structure as the virgin material or the materia l in the "as received" condition.

A major problem is to define quantitatively the structure of the cyclically strain-hardened material in terms of dislocations. Secondly the significance for crack growth is not fully clear.

Rate effects

Fatigue being a consequence of cyclic slip may well be a loading rate sensitive phenomenon, because slip itself may be a function of the loading rate (creep). Fatigue as it is considered in this report, is outside the creep domain. However, there are more reasons why rate effects may occur. Chemical attack from the environment may be significant at the surface of the material (nucleation) but also in the crack at its very apex (propagation). Secondly, diffusion in the material may affect the mobility of the dislocations and the fracture mechanism.

It is very difficult to get beyond speculative arguments. However, a few empirical trends seem to be well established. Decreasing the loading frequency of a sinusoidal loading may decrease fatigue lives and increase crack rates. These effects will depend on the type of material and on the environment. Especially crack propagation in aluminium alloys got much attention. It was clearly observed that the crack rate was reduced if the humidity of the environment was lower, while this effect was dependent on the loading frequency (Refs.15,16).

Type of material

It can hardly be a surprise that fatigue does not manife st itself as exactly the same phenomenon in all materials. Striations have been noticed on many materials, but differences were found, such as ductile

striations in the 2024 alloy and brittle striations in the 7075 alloy (Ref.2). There are also materials (some types of steel) where striations are hard to observe.
Since fatigue is a consequence of cyclic slip, it will be clear that fatigue is dependent on the possibilities for slip (available slip systems, ease of cross slip), the hardening mechanisms present in the matrix, the break-down of such mechanisms, cyclic strain-hardening (or softening), etc. This implies that the picture can be different for different materials. Crack nucleation may also depend on the material due to the presence of second phase particles or inclusions, that means on the cleanness of the material.

Concluding remarks

It is trivial to state that various details of the fatigue mechanism will be different for different conditions. At this stage, it is more relevant to see whether fatigue in technical alloys under various conditions has still enough features in common to postulate a simple fatigue model, that could be useful for a discussion of fatigue damage accumulation under variable-amplitude loading. It is thought that a model with the following characteristics could satisfy this need, while still being in agreement with the observations discussed before.

1. Since we are concerned with "finite life" problems, this implies that crack nucleation starts early in the life. Hence the nucleation period may be neglected.
2. Crack growth occurs by sliding-off at the tip of the crack, either by dislocations moving into the crack or by dislocations emitted by the crack, that means it occurs by slip, which is local plastic deformation.
3. As a consequence, the growth rate is dependent on the amount of cyclic slip and on the effectivity of converting cyclic slip into crack extension. Obviously, the amount of slip is a function of the local condition of the material and the local stresses. The condi tion of the material is dependent on the preceding strain history, while the local stress is a function of the applied stress and the geometry of the specimen, including the length of the crack.
4. The conversion of cyclic slip into crack extension will also depend on the local tensile stress (fracture mechanism, disruption of bonds, strain energy release). This stress should include residual stress induced by the preceding fatigue loading.

More comments on fatigue damage accumulation will be given in the following chapter.

3. FATIGUE DAMAGE ACCUMULATION

In the previous chapter the fatigue phenomenon has been discussed in qualitative terms, tacitly assuming that the fatigue loading did not vary during the test (constant mean, constant amplitude). If the fatigue load does vary, how will this picture be affected? This will be discussed in the present chapter.
Pertinent questions are:

a Is fatigue under a variable fatigue loading still the same process as fatigue under constant amplitude loading?
b How does fatigue damage accumulation occur under a variable fatigue loading?
c To answer the previous question, the following question has also to be answered:
How do we describe fatigue damage?

With respect to the first question it has to be expected that the qualitative description of fatigue given in chapter 2 is still valid. It does occur in the same material, again as a consequence of cyclic slip. This does not imply that the fatigue process will also be the same in a quantitative way. It need not even be the same in constant-amplitude tests at high and low amplitudes (high-level fatigue and low-level fatigue). The discussion of quantitative aspects first requires a definition of fatigue damage.

3.1 Fatigue damage

Fatigue damage is most generally defined as being the changes of the material caused by fatigue loading. The amount of cracking, apparently, is the most prominent aspect of these changes. However, there are other changes in the material than crackling alone, for instance cyclic strain-hardening and the development of residual stresses.

Geometry of the crack

It should be recognized that the crack is not completely defined by giving a crack length or a cracked area. Considering a crack as a separation in the material its size, as a first approximation, can be defined by the position and the orientation of the crack.front. The crack front need not be a single straight line or a circular arc. On a microscopic level it certainly will not be a straight line through the various grains. At a macroscopic level the orientation of the crack front will be different for a plane-strain crack (tensile mode) and a plane-stress crack (shear mode) (Fig.2.4).

The geometry of the crack tip is another aspect to be considered. On the atomic level a detailed picture is a matter of imagination, but even on a microscopic level this is a difficult problem. It has to be expected that the tip will be blunted after application of a high tensile load, while reversing the load will induce resharpening of the crack tip. Blunting and resharpening both will depend on the local ductility of the material and on the magnitude of the load applied.

If a cracked sheet is loaded in compression the crack will be closed. Hence it will be no longer a severe stress raiser since it can transmit compressive loads. This argument was suggested by Illg and McEvily (Ref.17) who confirmed it by comparing crack propagation data obtained in tests with the same S_{max}, but with $S_{min} = 0$ in one case ($R = 0$) and $S_{min} = - S_{max}$ in the other case ($S_m = 0$, $R = -1$). Approximately the same crack rates were found. This result was more applicable to 7075-T6 sheet material than to 2024-T3 sheet material. The latter was explained by the higher ductility of the 2024 alloy, implying more crack opening due to plastic deformation in the crack tip area. Hence a larger compressive stress was required before crack closure occurs.

Recently, Elber (Refs.18,19) observed that crack closure may occur while the sheet is still loaded in tension. According to Elber plastic elongation will occur in the plastic zone of the growing crack. This plastic deformation will remain present in the wake of the crack and it will cause crack closure before complete unloading of the specimen. This phenomenon was confirmed in an exploratory investigation at NLR. The data in figure 3.1 illustrate the conception. As a consequence of crack closure the crack opening as a function of applied stress shows a non-linear behaviour. For increasing stress, the crack is gradually opened until at $S = S_o$ it is fully open. During the fatigue tests 1 and 2, see lower graph of figure 3.1, the crack was partly closed during a considerable part of the stress cycle. For tests 3 and 4, the S_o - level could only be determined after unloading the specimen below S_{min}.
The above aspects of the crack geometry have been listed in figure 3.2.

Strain-hardening effects

As said in chapter 2, cyclic slip will affect the structure of the material. In view of the stress concentrating effect of the crack, changes of the structure will have a localized character with large gradients. Since it is already difficult to describe the changes in a qualitative way, it will be clear that a quantitative description is a termendous problem.

Residual stress

Plastic deformation at the tip of the crack will occur in the ascending part of a load cycle. If this deformation is not fully reversed in the descending part it will leave residual stresses in the crack tip region. In the fatigue model outlined in the previous chapter, the efficiency of converting slip into crack extension is dependent on the tensile stress in the crack tip region. Residual stresses have to be added to the stresses induced by the applied loads. As a consequence, residual stresses will affect the fatigue damage accumulation and for this reason they are an essential part of fatigue damage.
A calculation of the distribution and the magnitude of the residual stresses will be extremely difficult in view of the cyclic plastic behaviour of the material, the large strain gradients and the crystallographic nature of the material.
The picture is further complicated by crack closure as described above. It will turn out later that several empirical trends, attributed to residual stresses in the crack tip region, may also be explained by crack closure.

3.2 Fatigue damage accumulation. Interaction

Aspects of the previous discussion are summarized in figure 3.2 which will be discussed further in this section. In general terms fatigue damage may also be formulated as follows:

$$\text{Fatigue damage} = \left(\begin{array}{c}\text{changes of the material}\\ \text{due to cyclic loading}\end{array}\right) = \left|\begin{array}{l}\text{crack geometry}\\ \text{cyclic strain hardening}\\ \text{residual stresses}\end{array}\right| \qquad (3.1)$$

Fatigue damage accumulation means an accumulation of damage increments in every load cycle. A damage increment according to equation (3.1) involves incremental changes of the crack geometry, the cyclic strain-hardening and the residual stress. If these three aspects were uniquely correlated, fatigue would be the same process irrespective the magnitude of the fatigue loading. The damage could then be fully described by one single damage parameter, for instance the crack length. Unfortunately such a unique correlation does not exist. Compare as an example high-level fatigue and low-level fatigue. Crack propagation occurs in both cases, but the amount of cyclic strain-hardening and the residual stress at a certain length of the crack will be different. Even the crack will not be the same. It may be a shear mode crack for high-level fatigue and a tensile mode crack for low-level fatigue. This implies that quantitatively, fatigue is not the same process, irrespective of the magnitude of the fatigue loading. Consequently, it is impossible to describe the damage by a single damage parameter.

For a variable fatigue loading, the problem is still more complex than for constant-amplitude loading. A crack propagation test with a constant-amplitude loading and a few intermittent high loads is a relatively simple case, while at the same time it is a very illustrative example.
As shown in figure 3.3, three upward peak loads had a large delaying effect on the crack propagation, compare C and A. If the upward peak load was immediately followed by a downward one (sequence B), the delaying effect is much smaller, but nevertheless the increase of life is noticeable. Some comments on these results may now be made.
During the peak load crack extension does occur. Although, being small from a macroscopic point of view the extension could be observed. The question is whether this increment of the crack length would have been the same if the crack had been grown up to the same length by peak loads only (compare also A and C in Fig.3.2). There are various reasons to believe that this is not true.
1. The orientation of the crack front would be different because peak load cycles would produce a shear mode fracture, whereas the low-amplitude cycles produced a tensile mode fracture. In other words the peak loads in figure 3.3 are faced with an orientation of the crack front that is not compatible with their own magnitude. This incompatibility or mismatch between load amplitude and crack front orientation is illustrated by figure 3.5 for some simple load sequences.
2. The low-amplitude cycles will produce a sharper crack tip than cycles of the peak load magnitude would have done. This may also affect the crack extension of a single peak load.
3. The cyclic strain history is obviously different for low-amplitude cycles and high-amplitude cycles.
It is extremely difficult to quantify these three aspects.

Let us now consider the crack growth during the low-amplitude cycles after a peak load was applied, that means during the delayed growth period. The delay can also be explained by various mechanisms.
1. The high peak load in test C induced compressive residual stresses in the crack tip region. This will not necessarily restrain cyclic slip but according to the model outlined in the previous chapter, it will suppress the conversion into crack extension. In test B, the subsequent downward peak load reversed the sign of the residual stress but this occurs in a smaller plastic zone because the crack is closing under compression. Hence the crack tip is surrounded by a small zone with tensile residual stresses and a larger zone with compressive residual stresses, see figure 3.4 In agreement with this picture the crack growth started faster. then slowed down and finally resumed normal speed.
2. The observations in figures 3.3 and 3.4 can also be explained by Elber's crack closure argument. This was recently studied by Von Euw (Ref.21). The argument is that the delaying effect of the positive peak load caused by crack closure should occur after the crack has penetrated the plastic zone with the residual compressive stresses. Consequently the crack rate should reach a minimum after some further growth.

Von Euw could substantiate this view by fractographic observations (see also Ref.26).
3. Crack blunting might qualitatively explain the delay in test C. However, the delay would be very large for a crack that is blunted on a microscopic scale only, whereas it is still a sharp crack on a macroscopic scale. Further, it is difficult to see that crack blunting can explain the delay in test B since the downward peak load should resharpen the crack tip again.
4. Strain-hardening in the crack tip region is also a mechanism to explain the observations, although in this case it also is difficult to reconcile the large differences between the delays in tests B and C. It would require a more detailed picture about strain-hardening under cyclic load.

Another example is given in figure 3.5b. The crack extension due to the batch of low-amplitude cycles was smaller than in a constant-amplitude test with the same low amplitude. Arguments mentioned before, such as residual stress, crack closure, incompatible crack front orientattion, cyclic strain hardening and crack blunting may all be relevant in this case. It is indeed difficult to design a test and means for observation such that just one mechanism can be studied separately.

Interaction effects

As illustrated by the above tests, crack extension in a load cycle is depending on the fatigue damage being present. This damage is again dependent on the load history that produced the damage. In other words, a damage increment in a certain load cycle will be a function of the damage done by the preceding load cycles. A recapitulation of the various aspects is given in figure 3.2.
It may also be said that the damage produced in a certain load cycle will affect the damage produced in the subsequent load cycles. These effects were labelled in the past as interaction effects, as it was supposed to be an interaction between the damaging effects of load cycles of different magnitude. We will still use the word "interaction effect" in order to refer ~~to~~ damage accumulation under variable fatigue loading as being different from damage accumulation under constant-amplitude loading.

3.3 Fatigue damage at final failure

The end of the fatigue life could be defined as the presence of a specified amount of fatigue cracking. In most theories, however, the end of the fatigue life is associated with complete failure. Obviously, the length (or the area) of the fatigue crack will then be a function of the highest load occurring in the test, as indicated by Valluri (Ref.23). This applies to both constant-amplitude tests as well as variable-amplitude tests. For the former type of testing it is illustrated by figure 3.6, which has been drawn for this illustrative purpose only. Unfortunately this aspect is ignored by most cumulative damage theories to be discussed in chapter 5.

In a variable-amplitude test the occurrence of the final failure will be dependent on the maxima of the load history and the size of the growing crack. One may ask whether the condition of the material at the tip of the crack could also affect the occurrence of the final failure. Broek's work (Ref.24) suggests that this will hardly be true. The final failure (unstable crack growth) will be preceded by a small amount of stable crack growth. Moreover, he found that saw cuts and fatigue cracks gave similar residual strength values. It thus appears to be justified to apply the fracture toughness conception for the prediction of the final failure, i.e. the end of the fatigue life.

3.4 Micro and macro aspects

The various possibilities for interaction effects during the accumulation of fatigue damage are summarized in figure 3.2. It is good to realise how we arrived at the knowledge of the recognition of the existence of such interaction mechanisms. It then has to be admitted that macroscopic concepts (stress and strain e.g.) were quite frequently employed. Microscopic observations (striations) were usually obtained for macrocracks. Crack growth delays were also observed for macrocracks. For microcracks the growth rate is so low that detailed observations are extremely difficult. Nevertheless, it is thought that the damage accumulation picture outlined before will qualitatively apply in the micro range also. However, since the picture for macrocracks is also largely qualitative it will be clear that there is a good deal of intuitive speculation involved in our conceptions. It is expected that our knowledge for a long time will still have a qualitative character.

4. EMPIRICAL TRENDS OBSERVED IN VARIABLE-AMPLITUDE TESTS

As explained in the previous chapter, fatigue damage accumulation is a fairly complex phenomenon characterized by various mechanisms for interaction effects. In this chapter it will be analysed whether variable-amplitude tests have revealed systematic trends with respect to interactions. For this purpose we will first consider the methods for measuring interaction effects. Secondly various types of variable-amplitude loading will be listed. The major part of the chapter is covered by summarising empirical trends observed in various test series (Secs.4.3-4.18). It is not the intention to give a complete compilation of all available data. Representative data will be shown, however, to illustrate the various trends.

4.1 How to measure interaction effects?

In chapter 3 the interaction effect was defined as the effect on the damage increment in a certain load cycle as caused by the preceding load-time history. It can be similarly defined as the effect of the damage being present on subsequent damage accumulation.

Fractography

In view of the significance of cracking for fatigue damage, the best method for measuring interaction effects would be by fractographic means. With the electron microscope striations can be observed, that means crack length increments of individual load cycles. It is beyond any doubt that fractography is the most direct method to measure interaction effects. However, there are limitations because striations cannot always be observed, especially in the microcrack range. Moreover, interpretation problems may also arise. Reference may be made here to the work of McMillan, Pelloux, Hersberg (Refs.25,26) and Jacoby (Ref.4). More investigations of this nature are thought to be very worthwhile.

Visual crack growth observations

The examples of interaction discussed in the previous chapter (Sec.3.2), were studied by visual observation of the crack growth. The effects could still directly be observed because there were considerable crack growth delays. A similar observation was made (Refs.27,28) after changing the stress amplitude from a high to a low value (two-step test), as illustrated by figure 4.1a. When changing the amplitude from a low to a high value, the crack apparently resumed immediately the propagation rate pertaining to the high stress amplitude, see figure 4.1b. In other words, macroscopically an interaction effect could not be observed in the second case. Nevertheless, a significant interaction effect during a small number of cycles could easily escape such visual crack growth observations. Electron fractography is then required and there are indeed some indications (Ref.21) that the crack rate immediately after a low-high step was higher during a few cycles.

Fatigue life

In the majority of variable-amplitude test series reported in the literature, observations on crack growth were not made. Since favourable interaction effects increase the life, whereas unfavourable effects will shorten it, interaction effects can also be derived in an indirect way from fatigue life data.

Damage values $\Sigma n/N$

Since the value of $\Sigma n/N$ at the moment of failure may be considered as a relative fatigue life, this value may also be adopted for studying interaction effects. We may expect $\Sigma n/N > 1$ to be the result of a favourable interaction effect, whereas $\Sigma n/N < 1$ would indicate an unfavourable interaction effect. Other reasons for deviations from $\Sigma n/N = 1$ are defined in section 5.3.2.

The value of $\Sigma n/N$ can give an indication of interaction only if the fatigue load is varied no more than once in a test, see figure 4.2, A1 and A2. If the fatigue load is changed more than once, see for a simple example figure 4.2B, a value $\Sigma n/N > 1$ may again be interpreted as an indication of favourable interaction effects. However, it is impossible to say whether it was a favourable interaction of the high-amplitude cycles on subsequent damage accumulation during low-amplitude cycles or the reverse. It is even possible that there were unfavourable and favourable interaction effects both, with the latter ones predominating. Hence, in general, the $\Sigma n/N$ value will only indicate some average of all possible interaction effects.

4.2 Various types of variable-amplitude loading

There is obviously a multitude of load-time histories deviating from the fatigue load with constant mean and constant amplitude. A survey of several types applied in test series reported in the literature and the nomenclature to be used, are given in figures 4.2 and 4.3. The more simple ones are presented in figure 4.2. The number of variables is small and the variables can easily be defined. For the more complex load-time histories shown in figure 4.3, a statistical description of the loads has to be given. This may be the distribution function of the load amplit udes. The function may be a stepped one, as for instance for the programloading F and the randomized block loading G in figure 4.3. An example of such a stepped function is given in figure 4.22.

Program loading was proposed in 1939 by Gassner (Ref.29), while the randomized block loading was advised by NASA (Refs.30,31) as a variant of program loading. In a program test, the blocks with load cycles of the same magnitude are applied in a systematic sequence, whereas this sequence is a random one for the randomized block loading.

If random loading is a stationary Gaussion process, it is fully described by its power spectral density function (PSD-function). Other statistical parameters characterizing the random load are the root mean square value of the load (S_{rms}) and the ratio between the number of peaks and the number of mean-load crossings. For a narrow-band random loading, the latter ratio is approaching one, while the distribution function of the amplitudes is a Rayleigh distribution. Aspects of describing random loads are discussed in the literature (for instance Refs.32-34).
The sequence of peak loads of a quasi-random or pseudo-random loading is derived from random numbers, in such a way that there is no correlation at all between the magnitude of successive load cycles.

In a realistic flight simulation test (M in Fig.4.3), flight loads are applied in sequence which are different from flight to flight, see also figure 7.3. The load-time history may be a calculated one, whereas actual load records obtained in flight can be adopted if available (Branger, Ref.35). In the past, many full-scale structures have been tested with simplified flight-simulation loadings such as shown in figure 4.3, all flights being identical.

In figures 4.2 and 4.3, only the major types of fatigue loadings are given. The list is not complete since many variants on the examples shown can be thought of. For instance in a program test, the mean load need not be constant but may vary from block to block. As another example in a random load test, the S_{rms} need not be constant but can be varied from time to time as proposed by Swanson (Ref.33). Nevertheless, the list is complete enough for the discussion in the following section on systematic trends in the results of variable-amplitude loading. The merits of several testing methods are discussed in more detail in chapter 7.

4.3 Trends observed in tests on unnotched specimens

If an unnotched specimen is axially loaded, the stress distribution will be homogeneous. Exceeding the yield limit will not induce residual stress on a macro scale. This is an important difference as compared to notched specimens. Consequently, a significant mechanism for interaction effects will not occur in axially loaded unnotched specimens.
If unnotched specimens are loaded in rotating bending, the mean stress is equal to zero and the sign of the stress will change in each cycle. This is again an important difference with notched specimens

loaded at a positive mean stress.
As a consequence, we have to expect that the cumulative damage behaviour of unnotched specimens especially if loaded at $S_m = 0$, may be significantly different from the behaviour of notched specimens loaded at a positive mean stress. For instance it may be said that $\Sigma\, n/N < 1$ is a fairly common observation for unnotched specimens loaded in rotating bending, whereas $\Sigma\, n/N > 1$ is a relatively common observation for notched specimens loaded at a positive mean stress. An example of different sequence effects in unnotched and notched specimens is given in figure 4.4. It is thought that the explanation for the sequence effect of the unnotched specimens is mainly a matter of crack nucleation. Nucleation will more readily occur with the high stress amplitude at the beginning of the tests. Subsequently, cycles with a lower amplitude may then carry the crack to failure. For the notched specimens, residual stresses are responsible for the reversed sequence effect, see the following section.
Unnotched specimen data were reviewed in references 38,39. In view of their limited practical significance the data will not be further considered in this report.

4.4 The effect of a high preload

Various investigators have studied the effect of a single high preload on the subsequent fatigue life of notched elements. A survey is given in table 4.1 which shows that the effect of preloading was studied for a variety of materials and specimens including built-up structures, while the fatigue loading encompasses constant-amplitude loading, program loading and random loading.
Without any exception an increased fatigue life due to the preload was found in all the investigations. This was generally attributed to residual stresses at the root of the notch. Already Heyer in 1943 (Ref.41) attributed the increased life to compressive residual stresses. It is shown in figure 4.5 how these stresses are introduced by a high load. The compressive residual stress at the root of the notch implies that the local mean stress in subsequent fatigue testing will be reduced with an amount equal to the residual stress. Two examples of the effect of a preload on the S-N curve are shown in figure 4.6, one for constant-amplitude loading and one for random loading.
As a general trend, the investigations mentioned in table 4.1 also indicate that the preload effect is larger for higher preloads. This is illustrated by Heywood's results in figure 4.7.
In some investigations the effect of a negative preload (compressive load) was also studied (Refs.41-43, 47,49) and reductions of the life were found indeed, see figure 4.7. These losses are to be attributed to tensile residual stresses.

4.5 Residual stresses

Compressive residual stresses will increase the life for reasons discussed in chapter 3. Unfortunately residual stresses may be released by subsequent cyclic loading. Crews and Hardrath introduced a new technique for measuring the residual stresses at the root of a notch by means of very minute strain gauges (Refs.52,53). With the strain gauges the local strain history is measured. The corresponding stress history is then deduced from tests on unnotched specimens to which the same strain history is applied. Some results from Haibach, Schütz and Svenson (Refs.54,55) for a simple flight simulation loading, are given in figure 4.3. After the peak load P the local mean stress is lower than before the peak load and this will reduce the damage rate. However, the downward load A (ground-to-air cycle) has a reversed effect and hence it is unfavourable for a long fatigue life. Similar measurements were reported by Edwards (Ref.56).
The residual stresses at a notch will remain present only if the local stress range does not cause local yielding. This is obviously depending on the fatigue load applied, the geometry of the specimen (including cracks) and the cyclic stress-strain behaviour of the material. When cyclic plastic deformation occurs, either at the root of the notch or in the crack tip region, relaxation of residual stresses will occur. Obviously the residual stress can be restored by a new high load. Consequently periodic repetition of high loads will have a much larger effect on the fatigue life than a single preload of the same magnitude. Examples will be discussed later on.

Imig (Ref.49) performed fatigue tests on edge-notched Ti-alloy specimens and he found a life increase from 2[illegible] to 1[illegible] cycles due to a preload of [illegible] kg/mm^2 (cyclic stress range 0-35 kg/mm^2). He could largely eliminate the residual stress induced by preloading, by applying a new heat cycle (288°C) to the specimens. This reduced the life from 145 000 cycles to 55000 cycles.

It might be expected that residual stresses can fade away if given enough time. This could apply to strain-ageing materials, such as mild steel. However, it has not been observed in aluminium alloys. Smith (Ref.45) found the same fatigue life for preloaded 7075-T6 specimens when tested immediately after preloading or tested half a year later. Preloading had more than doubled the life. Program tests of Gassner (Ref.57) on a tube with 3 holes may also be mentioned here. Frequent interruptions of these tests for two days rest periods did not systematically affect the life.

4.6 Periodic high loads and residual stresses

Investigations on this topic have been listed in table 4.2. Fatigue tests are interrupted from time to time for the application of a high load (Fig.4.2D). The general trend is that these periodic high loads are considerably more effective in increasing the life than a single preload. An illustration of this observation is presented by figure 4.7. The delaying effect on crack propagation was already discussed in chapter 3, see figure 3.3. The effect will be larger for higher periodic loads (Refs.26,60,62).

The relaxation and restoration of residual stresses is illustrated by the results of reference 39. Riveted lap joints were tested under program loading, see figure 4.9. The periodic high loads considerably increased the life. If the application of the high loads was stopped after the 50th period (series 6a), the residual stresses could be relaxed by the subsequent fatigue loading and failure occurred after 8 additional periods. Similarly, applying the high loads after each 2 periods (series 6b) also allowed more relaxation of residual stress and gave a three times shorter life. It is also noteworthy that the application of the program loading in the Hi-Lo sequence (series 17) instead of the Lo-Hi sequence, gave a much shorter life. Apparently, applying the maximum amplitude immediately after the periodic high load reduced the residual stresses and the subsequent lower amplitude cycles could be more damaging than in test series 6.

In some investigations, listed in table 4.2, it was studied whether a high negative load would reduce the life increasing effect of a high positive load. This was true in all cases. An illustration concerning crack propagation was already discussed in chapter 3, see figure 3.3 Another example for the fatigue life of riveted joints is shown in figure 4.10. If a single load cycle with a very high amplitude is applied, it is apparently very important whether this cycle starts either with the positive peak or the negative peak. The last peak load applied has a predominant effect on the damage accumulation, see discussion in section 3.2.

Hudson and Raju (Ref.62) also performed constant-amplitude tests with intermittent batches of 5, 10, 20 or 28 high load cycles. The effect of crack propagation in aluminium alloy sheet material was studied and it turned out that the crack growth delays were larger than for single high loads. It may be assumed that more high load cycles will further increase the compressive residual stresses in the plastic zone. It may also be assumed that the size of the plastic zone will still become larger. Another explanation is to attribute the increased growth delay to a more intensive strain hardening in the crack tip zone. It is difficult to indicate the significance of the various contributions. It is noteworthy that Heywood (Ref.42) found a few test results indicating that 10 high preloads on a notched element induced a larger increase of the fatigue life than a single high preload.

4.7 The damaging effect of periodic negative loads on GTAC

For wing structures, ground-to-air cycles (GTAC), also called ground-air-ground transitions (GAG), are frequently recurring load cycles. A survey of investigations on the effect of GTAC on fatigue life is presented in table 4.3. The GTAC has the reputation to be very damaging. It is true indeed that GTAC are reducing the life considerably, that means to a much greater extent than the Palmgren-Miner rule predicts (see for summaries Refs.76 and 79). In flight-simulation tests, life reduction factors in the range 2-5 are common.

A GTAC may be damaging for two reasons. First, it generally is a severe load cycle which certainly will contribute to crack growth. Second, it will partly eliminate compressive residual stresses as explained in section 4.5. see also figure 4.8. These two arguments explain the results of Barrois (Ref.70) in figure 4.11, which illustrates that the life in cycles is shorter if there are more GTAC.
From the above arguments it has to be expected also, that the damaging effect will be larger if the minimum load in the GTAC is going farther down into compression. This is illustrated by results of Naumann (Ref.67) and Imig and Illg (Ref.80), see figure 4.12.

The effect of GTAC was also studied for macrocrack propagation. The effect in simplified flight-simulation tests was observed to be small (see Ref.72). It was more significant with realistic flight simulation loading (Refs.77,78) as shown by the results in figure 4.13. The reduction factors for the crack propagation life are nevertheless noticeably smaller than the usual values for notched specimens and structures (range 2-5). Hence the damaging effect of the GTAC appears to be smaller for crack propagation. Reversing the load on a notched element implies that the stress at the root of the notch is also reversed and may thus reverse the sign of the residual stress if plastic deformation occurs. The reversion would also occur if small microcracks are present. However, for a macro crack, reversing the load from tension to compression implies that the crack will be closed thus being able to transmit compressive loads as discussed in section 3.2. The crack then is no longer a stress raiser.
As a consequence of the above reasoning, it appears that GTAC are more damaging for crack nucleation (including micro-crack growth) than for macrocrack propagation.

4.8 Sequence effects in two-step tests

In the previous sections the effects of high loads were discussed and it turned out that residual stresses could well explain the trends observed. As a consequence, a high peak load cycle could extend the life if it started with the negative half cycle and ended with the positive half cycle. Reversing the sequence of the two high loads had a detrimental effect on the fatigue life.
Another example of a sequence effect is given in figure 4.4b. In this figure the first block of high-amplitude cycles apparently exerted a favourable interaction effect ($\Sigma n/N = 5.35$) on the remaining life under the second block of low-amplitude cycles. This effect may again be due to residual stresses, although cyclic strain hardening and other interaction effects may also have been active.
The following illustrative example has been drawn from Wallgren (Ref.81). In figure 4.14 results are shown from two series of two-step tests that are almost identical, since the same S_m and S_a values apply to the first and the second block. The only difference is in the transition from the first block to the second one, which had a significant effect on the life. The life is relatively short if the first block ends up with S_{min} and relatively long if the block ends with S_{max}, which is just a matter of one additional half cycle. This observation is strongly in favour of residual stress as the major mechanism for interaction. The observation is also in good agreement with Edwards' measurements of residual stresses at the root of a notch (Ref.56), showing that the sign of the residual stress may change in each cycle if the applied stress range is large enough.

With respect to macrocrack propagation in sheet specimens, crack growth delays after a high-low amplitude step have been mentioned before (Section 4.1). It was also emphasized that an interaction effect after a low-high step, being significant during a few cycles only, could easily escape macro observations, but it can be detected by electron fractography. Crack growth acceleration after such a low-high step was successfully explained by Elber (Ref.19), using the crack-closure argument. During the low-amplitude cycling little plastic deformation is left in the wake of the crack. Consequently, after changing over to the high amplitude there is less crack closure and more crack opening as compared to crack growth at the high amplitude only. After some further crack extension the crack closure is again representative for the high amplitude (Ref.21).

4.9 Sequence effects in program tests

In a two-step test the stress amplitude is changed only once. In a program test it is changed many times, both by increasing and decreasing its value. As a consequence, results of two-step tests will not necessarily allow a direct interpretation of sequence effects in program tests.
There is another reason why explaining sequence effects in program tests may be problematic. In most program tests a change of S_a is supposed to occur stepwise. If this were true, it is important whether the change is made either after the minimum or after the maximum of the last cycle of a step, see the previous section and figure 4.14. Unfortunately this information is rarely given in the literature for those cases where the change is really step-wise (manual operation, slow-drive machine, closed-loop machine with load control on individual cycles). Many program tests were carried out on resonance fatigue machines, which implies that changing the amplitude from one level to another level did occur gradually, that means in a rather large number of cycles. Apparently, there is a poor definition of details of the load sequence in program tests although these details could be important for interaction effects and hence for the fatigue life.

Gassner proposed the program test in 1939 (Ref.29) and shortly afterwards he studied already the effect of period size (number of cycles in one period, see figure 4.3F) and the effect of the sequence of amplitudes in a period (Ref.82). A survey of investigations on the methods of program testing is given in table 4.4.

Size of period

The investigations listed in table 4.4 indicate that the fatigue life may depend on the size of the period but unfortunately a clearly systematic trend was not found in all cases. Reducing the size of the period in several but not in all cases, reduced the life.
Reducing the size of the period to relatively small numbers of cycles while maintaining the same load spectrum, implies that the highest amplitudes occur less than once in a period. The amplitudes then have to be applied in a limited number of periods. Adopting this procedure, Lipp and Gassner (Refs.94,95) and Breyan (Ref.98) reported a systematic effect on the program fatigue life. The results, as shown in figure 4.15, indicate that the effect was far from megligible. In an NLR study (Refs.96,97) on crack propagation, a similarly large effect of the period size was found, see figure 4.16, while the load spectrum of amplitudes was exactly the same for the short and the long period.

Sequence of amplitudes

Sequences frequently applied are:
a increasing amplitudes (Lo-Hi)
b increasing-decreasing amplitudes (Lo-Hi-Lo)
c decreasing amplitudes (Hi-Lo)
d randomized sequence of blocks with the same amplitude.

Various comparative studies are reported in the literature. The effect of the sequence is illustrated by the NASA results in figure 4.17, and for crack propagation by the NLR results in figure 4.16. The results are generally systematic in a way that the life for the Lo-Hi-Lo sequence is always in between that of the Lo-Hi and the Hi-Lo sequence. Unfortunately, the results are not systematic with respect to the comparison between the Lo-Hi and the Hi-Lo sequence. In both figures 4.16 and 4.17, the fatigue life was longer for the Hi-Lo sequence, a trend also confirmed by tests on wings reported by Parish (Ref.91). However, results of Gassner (Ref.81) and NLR tests on riveted joints (Refs.39,58) showed the opposite trend, that means longer fatigue lives for the Lo-Hi sequence. As said before, the way of changing from one amplitude to another one may be important for having either favourable or unfavourable interaction effects.

Both the effect of the size of the period and the effect of the sequence of amplitudes indicate that the damage accumulation rate is a function of the frequency of changing the amplitude (period size) and the pattern of changing the amplitude (sequence). From a fatigue point of view it cannot be surprising that these variables will affect the damage accumulation and hence the fatigue life. However, a detailed picture about how interactions could explain the data, would ask a good deal of speculation.

4.10 High-amplitude cycles in program tests

In a program test the statistical distribution function of the amplitudes is usually based on an assumed load spectrum. Assessing the maximum value of the stress amplitude to be applied in a program test, is making a more or less arbitrary choice. Sometimes the choice is dictated by the possibilities of the available fatigue machine. In view of the large effect that periodic high loads could have on the fatigue life (see Sec.4.6), it has to be expected that the assessment of $S_{a,max}$ in a program test may be a critical issue. A survey of relevant investigations has been given in table 4.4.

High-amplitude cycles may either extend or reduce the fatigue life for the following reasons:

a These cycles will be damaging since they will substantially contribute to crack nucleation and propagation. They may contribute to crack growth even more than in a constant-amplitude test carried out at $S_{a,max}$, because of unfavourable interactions caused by cycles with lower S_a values.

b High-amplitude cycles will also reduce the life because final failure will occur at a shorter crack length.

c On the other hand, high-amplitude cycles may extend the life if they introduce compressive residual stresses which is not unlikely. The crack closure argument also appears to be applicable.

In view of these arguments it will be clear that fully systematic results cannot be expected. The trend could be dependent on the question whether there are relatively many high amplitude cycles (manoeuvre spectrum) or just a few (gust spectrum). Secondly, the question whether compressive or tensile residual stresses are introduced will be dependent on $S_{a,max}$, the detailed load sequence, the geometry of the notch and the material. Consequently, it should not be surprising that data from the literature indicate both life extension and reductions if higher amplitudes are applied in a program test. Illustrations of both are given in figure 4.18. The results of the tailplanes reported by Rosenfeld (Ref.48), were obtained with S_{min} = + 13.3 % P_L (P_L = limit load) and hence the increased life obtained by adding higher load cycles may well be due to introducing compressive residual stress, which apparently outweighed the damaging effect of these cycles per se. In Naumann's tests (Ref.87) on the edge notched specimens, the addition of higher load cycles was coupled to negative minimum loads which may have eliminated compressive residual stress and thus the cycles were damaging only. Effects as found in other investigations were generally smaller then those in figure 4.18.

Kirkby and Edwards carried out narrow-band random load tests on lug type specimens (Ref.99). They also performed test series with three S_{rms} values in a programmed sequence, see figure 5.2. Omission of the highest S_{rms} reduced the life 2.5 times. Apparently, the higher-amplitude cycles had a beneficial effect in the first tests. Comments on high-amplitude cycles in flight-simulation tests are given in section 4.13.

4.11 Low-amplitude cycles in program tests

In aircraft structures fatigue load cycles with a low amplitude usually occur in relatively large numbers. Consequently, if such cycles could be omitted from a test a large proportion of the testing time would be saved. This topic was studied in several investigations employing program loading, see for a survey table 4.4.

Low-amplitude-cycles may be damaging for more than one reason:

a Due to the large numbers, they may induce fretting corrosion damage and thus enhance crack nucleation.

b Low-amplitude cycles may contribute to crack growth as soon as a crack has been created by higher-amplitude cycles. This implies that cycles with an amplitude below the fatigue limit can be damaging.

c Low-amplitude cycles may enhance the crack growth at subsequent cycles with a higher amplitude, see the discussion in section 4.8.

It is well-known that fretting corrosion can have a most detrimental effect on the fatigue limit and on the lower part of the S-N curve. However, the effect is relatively small at high S_a-values because crack nucleation does occur quite early and is less dependent on the assistance of frettingcorrosion. Similarly, we may expect fretting corrosion to be less important in program tests. Nevertheless, Gassner (Ref.100) still found a 50 percent life reduction if fretting was applied at the root of a notched 2024-T4 specimen. Jeomans (Ref.89) also in program tests, found a life reduction of about 65 percent when comparing dry and greased bolted joints of the 2014 alloy.

Program tests from which low-amplitude cycles were omitted, always indicated either a negligible effect on the fatigue life (in periods) or an increase of the life. In other words the available data confirm that cycles with amplitudes below the fatigue limit may be damaging. An example is given in figure 4.19 with results reported by Wallgren (Ref.83). The last column of the table illustrates the reduction of testing time obtained when omitting low-amplitude cycles.

4.12 Comparison between the results of program tests and random tests

In comparison to a random load test, the variation of the stress amplitude in a program test occurs in a simple and systematic way. For random loading the amplitude (as well as S_{max} and S_{min}) may be significantly different from cycle to cycle. In a program test, however, the amplitude may remain unchanged during large numbers of cycles. The number of amplitude changes is relatively small. In view of the present knowledge about interaction effects, it has to be expected that the fatigue damage accumulation rate may be different for the two types of loading. Any similarity between the results of random tests and program tests cannot be claimed on physical arguments but has to be shown by tests.

A second aspect of the comparison between random and program loading is concerned with the concept "random". A random signal may be stationary or non-stationary, it may be Gaussion or non-Gaussion (Refs.32-34). If it is a stationary Gaussion process, the sequence is still dependent on the power-spectral density function (PSD-function). An illustration is given in figure 4.20 by two record samples of Hillberry (Ref.101). The effect of the shape of the PSD-function on the random load fatigue life was studied in some investigations, see table 4.5. As a gene ral trend, it was found that the effect was either small or negligible. It is thought, however, that these data are still too limited to justify a generalization.
The importance of the "randomness" for fatigue life has also been studied under different pseudo random loading conditions. In figure 4.21, results of Naumann (Ref.67) illustrate that the fatigue life is apparently depending on the question whether we consider full cycles (starting and ending at S_m) or half cycles (also starting and ending at S_m). It should be pointed out that the statistical distribution functions of the maxima and the minima were exactly the same for all test series in figure 4.21. It should also be pointed out that the statistical distribution functions of stress ranges (differences between successive values of S_{max} and S_{min}) are not the same for these test series, which will be evident after a closer look at the sequence samples in figure 4.21.
A second example of sequence effects in random load tests is shown in figure 4.16, giving data from NLR tests on crack propagation. In two test series exactly the same random sequence of complete load cycles were applied. In the first series the cycle started with the positive half cycle, whereas in the second series it started with the negative one. Also here it is true that the statistical distribution functions were the same for the peak values but different for the stress ranges, which apparently has some effect on the crack propagation life, although the effect was small.

A comparison between the results of random tests and program tests was recently published by Jacoby (Refs.107,108). Some new results became available since then. A survey of comparative investigations is given in table 4.6. As Jacoby pointed out, there is no unique relation between the fatigue lives for random loading and for program loading. He mentioned (Ref.109) various aspects that could affect the comparison. Some important ones are the type of random loading, the type of program loading, the maximum stress in the test, the mean stress and the shape of the load spectrum.
It is difficult to draw general trends from the investigations listed in table 4.6. In general, the life in the "equivalent" program test is larger than in the random test. In several investigations the difference is not very large. However, Jacoby (Ref.107) arrived at program fatigue lives that were about six times longer than in random load tests. In figure 4.16, NLR results on crack propagation indicate about

three times longer lives if the comparison is made with program loading with 40000 cycles in a period. Fractographic observations also indicated different cracking modes. For the short period (average 40 cycles), the difference between random and program loading was small. This is in agreement with the observation that the sequence effect in the program tests (Lo-Hi, Lo-Hi-Lo and Hi-Lo) was small for the short period (although still systematic). Gassner's and Lipp's results (Refs.94,95, see fig.4.15) also point to a small difference between random and program test results if the period of the program is short. There are some indications that the differences may also be smaller for a more severe load spectrum. Unfortunately the large differences mentioned above still give some uneasy feelings about the equivalence of random tests and program tests.

4.13 Trends observed in flight-simulation tests

A survey of investigations on flight-simulation testing is presented in table 4.7. In these investigations several trends were observed that are qualitatively more or less similar to those discussed before.

Sequence effects

Naumann's tests on the effect of the random sequence of either complete cycles or half cycles also included flight-simulation tests. As the results in figure 4.21 show, a similar sequence effect was found in the random tests and in the flight-simulation tests. However, the effect was much smaller in the flight-simulation tests. A similar observation was made by Jacoby (Ref.107).

The sequence effect in flight-simulation tests was one of the topics studied in a recent NLR investigation (Refs.77,78) on crack propagation in 2024 and 7075 sheet material. The gust load spectrum applied is shown in figure 4.12, while different sequences are presented in figure 4.23. In these tests 10 different types of weather conditions were simulated in different flights. Apart from test series H the gust sequence in each flight was random, while the sequence of the various flights was also random. In figure 4.23 a comparison is made between a random sequence of complete cycles, the same sequence of "reversed" complete cycles and a Lo-Hi-Lo programmed sequence. As the data in the figure show, the sequence effect was practically negligible. This result is in good agreement with the small sequence effect found in figure 4.16 when comparing random loading and program loading with a short period.

A small sequence effect was also found by Gassner and Jacoby (Refs.66,73), and by Imig and Illg (Ref.80) with one exception. Gassner and Jacoby, testing 2024-T3 specimens ($K_t = 3.1$), found fatigue lives of 2500, 2800 and 5800 flights for a random gust sequence, a Hi-Lo-Hi gust sequence and a Lo-Hi-Lo gust sequence respectively. The latter result is considerably higher than the former two results. They applied 400 gust cycles in each flight which is a relatively high number.

Low-amplitude cycles

As said in section 4.11, low-amplitude cycles may be significantly damaging in a program test. In such a test these cycles are applied in blocks of large numbers of cycles. In random loading the low-amplitude cycles are randomly dispersed between cycles with higher amplitudes. This implies that the information from program tests is not necessarily valid for random loading.

Some investigations on flight-simulation testing have also explored this aspect, see table 4.7. Average results are collected in figure 4.24. Naumann (Ref.67) found a very small increase of the fatigue life when omitting low-amplitude gust cycles, while Branger (Ref.110) found a small reduction of the life. Gassner and Jacoby (Ref.73), however, found a significant increase. They omitted 370 low-S_a cycles from 408 cycles in each flight. Both numbers are large, which may have contributed to the result. The NLR results on crack propagation are recapitulated in figure 4.25. Here also it is evident that omitting low-S_a cycles increases the life.

With respect to omitting taxiing loads from the ground-to-air cycles, the trend appears to be that this has a minor effect on life. It is thought that the taxiing cycles were hardly damaging because they occurred in compression. Consequently the low damaging effect of the taxiing loads will not be applicable if the mean stress of the GTAC is a tensile stress (upper skin of wing structure).

High-amplitude cycles

In section 4.6 it turned out that periodically applied high loads could considerably increase the fatigue life. It then may be expected that high-amplitude cycles in a flight-simulation test may also have a similarly large effect, if applied now and then in a few flights. This aspect was not intensively studied so far, see table 4.7. Gassner and Jacoby (Ref.73) reported 25 percent longer life if increasing the maximum stress amplitude from 0.55 S_m to 1.1 S_m (S_m is mean stress in flight). In these tests the gust loads in each flight were applied in a programmed sequence. Branger (Ref.111), employing a manoeuvre spectrum, found 10 to 40 percent longer lives when raising the maximum peak loads with 15 percent.

At the NLR we performed one test series on a sheet specimen with a central hole and several series on crack propagation in sheet specimens (Refs.77,78). The results of the hole notched specimens are shown in figure 4.26. Load sequences were similar to those shown in figure 4.25 (sequence B), while the load spectrum given in figure 4.23 was applicable. In three comparative test series the spectrum was truncated at $S_{a,max}$ = 4.4 , 6.6 and 8.8 kg/mm^2 respectively. Truncation implies that cycles, which should have higher amplitudes according to the load spectrum, were applied with an amplitude equal to $S_{a,max}$ (<u>truncation level</u>). Figure 4.26 clearly shows a systematic effect of the truncation level on both the nucleation period and the crack propagation life. Both periods are longer for higher $S_{a,max}$ values. More data from the crack propagation tests are collected in figure 4.27, which clearly confirms the longer fatigue life if higher amplitudes are included in the flight-simulation test. In one test on a 7075-T6 specimen the gust spectrum from figure 4.22 was applied without truncation, that means $S_{a,max}$ = 12.1 kg/mm^2. The crack rate was extremely low and decreased as the crack grew longer. The test had to be stopped in view of excessive testing time.

It is thought that the predominant effect of high gust load cycles, as illustrated by figures 4.26 and 4.27), has to be explained by the effect of compressive residual stresses on crack growth and by crack closure. Practical aspects of the effect of the truncation level are discussed in chapter 7.

Some remarks on the effects of loading frequency and environment as observed in flight-simulation tests are made in section 4.17.

4.14 <u>The effects of the design stress level and the type of load spectrum</u>

The design stress level will obviously affect the fatigue life of an aircraft. Empirical studies for a long time could only be made by program tests. Gassner started the work about 30 years ago (Refs.29,57, 82); another early publication is from Wallgren (Ref.83), see also table 4.4 The major part of this type of work was carried out in Gassner's laboratory at Darmstadt. Much of this work was recently summarized by W. Schütz (Ref.115).

From a large amount of program data obtained with standardized load spectra (Ref.116), Gassner found a linear relation between log stress level and log program fatigue life. This trend is illustrated by figure 4.28. The relation can bewritten as:

$$N' \, S_{a,max}^{\kappa} = \text{constant} \tag{4.1}$$

Many tests indicated the trend for κ to be in the order of 5-7. In equation (4.1), N' is the program life and $S_{a,max}$ is the maximum amplitude of the standardized load spectrum which is truncated at a level occurring once in 500 000 cycles.

Since $S_{a,max}$ as well as S_m are linearly related to the ultimate design stress level, the merit of the above relation is that it immediately indicates the change of life associated with a certain percentage change of design stress level. The question is, however, whether equation (4.1) would also be valid for realistic service load-time histories. This could not be checked empirically until the electrohydraulic fatigue machine with closed loop load control became available. As pointed out in section 4.12 it remains to be explored whether trends valid for program tests are also applicable to random loading.

In the more recent literature some test results are presented regarding the effect of design stress level on fatigue life in flight-simulation tests, see table 4.7. These, by now, are apparently the most realistic data available to judge this effect. The data are summarized in figure 4.29. It should be pointed out that for each test series in this figure changing the stress level did not affect the shape of the load time history. Hence the shape of the load spectrum also remained the same. Changing the stress level only implied that all load levels were multiplied by the same factor. Evidently, the data in figure 4.29 are too limited for deriving a general trend. The relation of equation (4.1) is not applicable to the NLR crack propagation data. In the other graphs the slope factor κ is outside the range 5-7 usually found for program tests. Three graphs indicate a higher value (average 8), while the data of Branger and Ronay indicate a very low κ-value. Further comments on this topic are made in section 5.3.7.

In variable-amplitude tests based on a service load spectrum, a gust spectrum or a manoeuvre spectrum was usually adopted. The investigations, in general, do not allow a direct comparison between the two spectra, since there were more variables than the spectrum shape alone (for instance stress ratios, truncation level).
Nevertheless, manoeuvre spectra are generally considered to be more severe than gust spectra, because the proportion of higher-amplitude cycles is larger.
The effect of the spectrum shape was systematically studied in one investigation only, namely by Ostermann (Ref.117). He performed program tests on notched 2024-T3 specimens and kept all variables constant except the spectrum shape. The number of cycles in one period was also constant. The test results indeed confirmed that the life became shorter if the proportion of high-amplitude cycles increased (and the proportion of low-amplitude cycles decreased). Some further comments on this work are made in section 5.3.5.

4.15 Observations from full-scale fatigue test series

This section is partly similar to Appendix J of reference 76, entitled "The influence of the loading history on the indication of fatigue-critical components".
As explained before, high loads will introduce local stress redistributions around notches and the effect on the fatigue life may be different for different notches, depending on K_t, stress gradient and nominal stress level. The consequence is that the indications of the most fatigue-critical component in a structure may depend on the selected load spectrum and the truncation level. Fatigue tests on large structures reported in the literature give some information on this question. They are summarized below.

Tests on Mustang wings

Results of an extensive test program on Mustang wings were reported in references 44,74 and 118. The following types of tests were carried out:
(1) Constant-amplitude tests, various P_a and P_m values
(2) Program tests with 3 amplitudes, P_{max} = 33 % P_u
(3) Random load tests, gust spectrum, P_{max} = 63 % P_u
(4) Random load tests with GTAC, same gust spectrum, P_{min} for GTAC = - 24 % P_u
(5) Random load tests, manoeuvre spectrum, including negative manoeuvre loads, P_{max} = 75 % P_u.
The 1-g load level for test series 2-5 was 20 % P_u. The stresses at P_u were in the order of 28 kg/mm^2.
Cracks were mainly found in two areas, indicated as the tank bay area and the gun bay area. For the two areas intersecting S-N curves were found in test series no.1, both for initial cracking and final failure. This shows that a certain component, which under constant-amplitude loading is more fatigue-critical than another component, can be less critical at another load level.
In test series 2-5 the initial failure was always first observed in the gun bay area. However, cracking in the tank bay area could be more serious. The final failure occurred in both areas in test series 2 (lower P_{max} value) and in the tank bay area only in test series 3, 4 and 5. In the random gust tests without GTAC the gun bay area was then in an advanced stage of cracking, whereas this failure was almost completely suppressed in the random gust tests with GTAC. The latter was partly true also for test series 5.

Tests on Commando wings

Results of tests on Commando wings were reported by Huston in reference 119. Three types of tests were conducted, viz.:

(1) Constant amplitude tests (P_{max} values $\leq 59\ \%\ P_u$)

(2) Program tests with a gust spectrum ($P_{max} \sim 75\ \%\ P_u$)

(3) Program tests with a manoeuvre spectrum ($P_{max} \sim 78\ \%\ P_u$) .

A limited amount of service experience was available. The program tests were randomized step tests. The stress at P_u was low, viz. about 19 kg/mm^2.

Constant-amplitude tests revealed only 1 or 2 fatigue-critical locations, which were different for high and low amplitudes. In test series 2 and 3, cracks were found at 7 different locations. With respect to the first crack that appeared, the crack at location F (code of Ref.119) was the most frequent one in test series 2 and 3, whereas this location was not very important in the constant-amplitude tests. The most critical crack with respect to final failure was found at location B in the constant-amplitude tests and at location III in the program tests.

A comparison between the cracks found in service (4 aircraft) and in the program tests (gust spectrum), yielded a reasonable agreement regarding the locations at which cracks were found.

Tests on Dakota wings

In reference 71 Winkworth reported the results of testing 4 Dakota wings and a comparison with service experience.

The following four tests were carried out:

(1) Gust cycles only, constant amplitude, P_a corresponding to 12 ft/sec gust.

(2) Simplified flight simulation, 15 gust cycles (as applied in test 1) per flight.

(3) Same as test 2, except 5 instead of 15 gust cycles per flight.

(4) GTAC only, P_{max} at 1-g level, $P_{min} < 0$.

Cracks occurred at three different locations, A, B and C. The most critical crack in tests 1 and 2 occurred at location A and in tests 3 and 4 at location C. Cracks at location B were found in all tests. In service cracks were predominantly found at location B and cracks at location A did not occur. Cracks were also found in service at a location at which no cracks were found in the tests. It cannot be said that a fair agreement between service experience and testing was obtained. This may be partly due to the simplified flight-simulation load sequence adopted for the tests.

Tests on a swept back wing

Results of constant-amplitude tests and program tests on a wing of a fighter were reported by Rosenfeld (Ref.48). In the program tests two different manoeuvre spectra were used. In one test series GTAC were inserted (in batches), which in this case were upward loads rather than downward loads. Values for P_{max} from 55 to 100 % P_L (P_L is limit load) were used in the constant-amplitude tests and from 35 to 125 % P_L in the program tests. P_{min} was 13.3 % P_L in all tests.

In each wing, failure always occurred at a bolt hole. In the program tests (4 different programs) failures occurred at locations A (6 times), C (once), E (twice) and F (once). In the constant-amplitude tests, failures occurred at locations A (7 times), especially at the higher load levels), B (twice), C (once) and D (twice, at the lowest load level only).

Tests on the pre-mod F-27 center section wings

Random and program tests were carried out, both with and without ground-to-air cycles (Ref.76). Constant-amplitude tests were carried out representing GTAC loading and gust load cycles. In the random and the program tests a very severe gust spectrum was adopted, the maximum load being $P_{max} = 65\ \%\ P_u$ where P_u is the ultimate design load. In the constant-amplitude tests, P_{max} values covered a range from 35 to 47 % P_u.

Although the same type of crack was the most critical one in all tests, considerable differences were found between the random and the program tests at the one hand and the constant-amplitude tests on the other hand. Contrary to Huston's findings, the number of locations at which cracks were found was larger in the constant-amplitude tests. Secondly, some types of cracks occurred predominantly if not exclusively in the random and the program tests, whereas other types of cracks were found in the constant-amplitude tests only.

Tests on Venom wings

Branger (Refs.120,121) has reported interesting data on the indication of fatigue critical locations in the structure. Information was available from:

(1) One constant-amplitude test ($P_{max} \hat{=} -1g$)

(2) Two program tests ($P_{max} \hat{=} 7.25\ g$, $P_{min} \hat{=} -0.87\ g$)

(3) Six half wings tested with a most realistic flight-simulation loading ($P_{max} \hat{=} 6.5\ g$, P_{min} -30% P_{max})

(4) Service experience.

In the flight-simulation tests, two explosive failures occurred. One of these failures had not been detected in the constant-amplitude test and the program tests. Initial cracking corresponding to this failure was observed in service. On the other hand, five main failures occurring in the constant-amplitude test and the program tests did not occur in the flight-simulation tests and in service.

One general trend emerging from the available evidence is that the picture of fatigue-critical elements in an aircraft structure is significantly depending on the load-time history applied. This emphasizes the need for realistic load-time histories for application to full-scale testing, see chapter 7.

4.16 Fatigue by two superimposed sinusoidal loads with different frequencies

The superposition of two cyclic loads with different amplitudes and frequencies may occur in certain components under service conditions. This especially applies if a component is subjected to high-frequency vibrations, while at the same time a low-frequency fatigue loading occurs. Even gust loads and taxiing loads may be considered as high-frequency loads superimposed on the ground-to-air cycle.

Apart from the technical significance, the superposition of two cyclic loads is an intrigueing variable-amplitude load sequence to check certain assumptions about fatigue damage accumulation. Two examples are shown in figure 4.30, which can be written as

$$S = S_m + S_{a1} \sin \omega_1 t + S_{a2}\ \omega_2 t \tag{4.2}$$

with $\omega_1 \ll \omega_2$, ω being the angular frequency.

Investigations on superimposed cyclic loads have been listed in table 4.8 which shows that there is a good deal of variety between the various studies. Nevertheless some general findings may be reported. If S_{a2} is small enough to be below the fatigue limit, the Palmgren-Miner rule would suggest the cyclic load $S_{a2} \sin \omega_2 t$ to be non-damaging. Consequently the life should be N_1 if N_1 is the fatigue life associated with S_{a1}. However, it turns out that the life is shorter. This has to be expected since the cyclic load $S_{a2} \sin \omega_2 t$ will anyhow increase the stress range of the low-frequency component from $2\ S_{a1}$ to $2\ (S_{a1} + S_{a2})$, see figgure 4.30a. In other words, a life N_1' associated with an amplitude $S_{a1} + S_{a2}$ should be expected at most. Usually a shorter life is found depending on the ratios S_{a1}/S_{a2} and ω_2/ω_1. Apparently, apart from increasing the stress range, the high-frequency cyclic load itself is also contributing some damage.

The example shown in figure 4.30b has more the character of a cyclic load ($S_{a2} \sin \omega_2 t$) with a slowly varying mean. Available results indicate this varying mean to be damaging, implying that the life will be shorter than N_2, if N_2 is the life associated with S_{a2}. Here also the stress range is increased to $2\ (S_{a1} + S_{a2})$ and even in case that $S_{a1} \ll S_{a2}$, the material will remember this to some extent depending on ω_2/ω_1.

In some investigations the low-frequency component was cyclically changed step-wise or following a triangular wave form. With respect to the high-frequency component, a randomly varying S_{a2} value has been applied (Ref.129). This is further complicating the picture but it is more similar to practical conditions. Such complex load histories raise the problem of how to define a load cycle. This already applies to the examples in figure 4.30. A cycle with a range $2\ (S_{a1} + S_{a2})$ does in fact not occur in these examples, although the range has still some meaning for the fatigue life. The problem how to define cycles for moren complex load-time histories is given more attention in chapter 6. It may be noted here that fatigue under superimposed cyclic loads is also being studied by following the strain histories (Refs.128,129).

4.17 Effects of environment and loading frequency

In chapter 2 brief reference was made to the possible effects of environment and loading frequency. Effects were observed in constant-amplitude tests and although the humidity of the environment appears to be important, a full understanding of these effects has not yet been obtained. Only a few investigations have been made under variable-amplitude loading.

Environment

In a comparative investigation at NLR (Ref.106) on 2024 and 7075 sheet material, crack propagation was simultaneously studied in an indoor and an outdoor environment. Program loading and random flight-simulation loading were used. The results indicated a negligible effect for the 2024 material, but for the 7075 alloys the crack growth outdoors was 1.5 to 2 times faster than indoors. Figge and Hudson (Ref.130) in a recent study found a similar trend. Branger (Ref.111), testing two-hole specimens under flight-simulation loading, found a doubling of the life when testing in pure nitrogen instead of air. The specimens were produced from 7075-T6 bar material.

Loading frequency

In a recent NLR investigation crack propagation tests have been carried out on 2024 and 7075 sheet specimens under flight-simulation loading. The variables being studied are of the design stress level and the loading frequency. Three frequencies have been adopted, namely 10 cps, 1 cps and 0.1 cps. The investigation is not yet complete, but abailable data (see Ref.64) indicate a rather small and not fully systematic influence. Although such a small effect is a very convenient result, it is not yet justified to generalize this empirical observation.
Branger (Ref.111), in flight-simulation tests on light alloy specimens notched by two holes, found a slightly lower life at 96 cpm (cycles per minute) as compared to 173 cpm (1.6 cps and 2.9 cps respectively). Surprisingly enough he found a reversed frequency effect in another test series with frequencies of 210, 40 and 5.4 cpm (3.5, 0.7 and 0.09 cps respectively). The longer life was obtained at the lower frequency.

4.18 Aspects related to the type of material

The majority of variable-amplitude tests was performed on aluminium alloy specimens and structures. There is some work available on titanium alloys and low-alloy steels. (Tabls.4.1-4.8). The question now is whether these materials show empirical trends similar to those of the aluminium alloys. Indications of a significantly different behaviour have not been obtained so far.
There are some reasons why certain materials may show a similar cumulative fatigue damage behaviour. The accumulation of fatigue damage has been described in chapter 3. The interaction mechanisms, see figure 3.2, were related to cracking, residual stress at the tip of the crack due to local plastic deformation, crack closure, crack blunting, cyclic strain-hardening, etc. All these mechanisms are related to the ductility of the material. Consequently it is thought that materials with a similar plastic behaviour could show a cumulative damage behaviour that is qualitatively similar. With respect to preloading notched elements this was clearly confirmed (Refs.41,46,49).
A qualitative similarity, however, does not yet imply a quantitative similarity. This can be illustrated by comparing data for the two well-known aluminium alloys 2024 and 7075. Both alloys are neither extremely ductile nor brittle, but the ductility of the 7075 alloy is certainly smaller than that of the 2024 alloy. Favourable interaction effects have been noted for both alloys. Nevertheless, Hardrath, Naumann and Guthrie (Refs.30,31,88) found systematically higher $\Sigma n/N$ values for the 7075 alloy. Similarly figure 4.27 shows that the 7075 alloy is indeed more sensitive to the effect of high-amplitude cycles. Larger favourable interaction effects are also confirmed by the NLR crack propagation data in figure 4.29, the more so since constant-amplitude data suggested a much longer life for the 2024 alloy as compared to the 7075 alloy. An increasing ductility will imply that the residual stresses will be smaller and that relaxation of residual stresses due to cyclic straining will be easier.

As a general conclusion, similar qualitative trends may be expected within certain limits. The similarity should no longer be expected if the material has a significantly different ductility, for instance a very high ductility (low strength alloys) or responds to unstable yielding (mild steel). Brittle materials for which a very small crack may be disastrous and for which the life is mainly occupied by crack nucleation, may also behave differently.

5. THEORIES ON FATIGUE DAMAGE ACCUMULATION

5.1 Introduction

In the literature a variety of cumulative damage theories have been presented. The question now is whether these theories can account in a realistic way for the trends described in the previous chapter. In this chapter an attempt will be made to give a systematic survey of the various aspects characterizing the theories. In view of this goal, some salient features of fatigue damage will be summarised first (section 5.2). Secondly, damage theories will be discussed in three groups, each group being characterized by a certain similarity of the damage accumulation model adopted (section 5.3). Finally the physical and practical limitations of the theories are discussed in section 5.4. The significance of the limitations for practical applications is a topic also covered by chapter 6.

5.2 Fatigue damage

Fatigue and damage accumulation in metallic materials have been discussed in chapters 2 and 3. The fatigue life was divided in some periods, for instance (see also figure 2.2):
- crack nucleation
- crack propagation
- final failure.

These periods are recognized by some theories but certainly not by all. An obvious difficulty is the definition of the termination of the first period and the start of the second period. This problem does not occur in those theories that assume crack growth to start in the very beginning of the fatigue life. Since the most essential part of fatigue damage was described in chapter 3 as decohesion of the material, a physical theory should incorporate crack growth as a minimum requirement.
However, it was explained in chapter 3 that the amount of cracking alone could not give a complete description of the state of fatigue damage. Several additional damage aspects were mentioned, see figure 3.2. From these aspects only residual stress has been incorporated in a few theories. The other aspects have been mentioned in the literature to explain certain trends observed in tests, but these aspects are not an explicit part of a quantitative theory.
The occurrence of the final failure should be a function of the crack length and the applied maximum stress. A few theories try to account for this aspect by employing fracture toughness criteria.
Several theories predict fatigue life only, without any reference to the physical damage occurring between the beginning and the end of the fatigue life. Moreover, the end of the life in most theories means "complete failure" without any further specification.

5.3 Theories

5.3.1 General survey

The number of cumulative fatigue damage theories is large. This is certainly true if we keep in mind that the theories try to solve the same problem, which is to predict the fatigue life (or crack propagation life) under variable-amplitude loading from available data. For a good appreciation of the various theories, three different approaches may be recognised, as listed in table 5.1.

In the incremental damage theories it is assumed that each cycle or each batch of cycles causes a certain damage increment. This increment is quantitatively equal to the percentage of fatigue life consumed by those cycles. The complete life expires and failure will occur at the moment that the sum of all damage increments becomes equal to one.

The similarity approach refers to those theories, which presume that similar loading conditions at the fatigue-critical locations in two different specimens should produce similar fatigue lives. The stress history or the strain history could be adopted for characterizing the loading condition. For crack propagation the similarity approach implies that similar stress intensity factors should produce the same crack rates.

The interpolation methods appear to be the most direct approach, since interpolation is made between available fatigue data. Nevertheless, the interpolation procedure may be a critical issue. Interpolation can be made for a large variety of variables, such as S_m, K_t, load spectrum shape, etc.
More details of the theories are given in the following sections. An evaluation of the theories is given in section 5.4.

5.3.2 The Palmgren-Miner rule

This rule is the most well-known representative of the incremental damage theories. Palmgren (Ref.131), as early as 1923, assumed that n_i load cycles with the same mean load and load amplitude will consume a portion of the fatigue life equal to n_i/N_i where N_i is the life to failure in a constant-amplitude test with the same mean and amplitude. Secondly, Palmgren assumed that failure will occur if the sum of the consumed life portions equals 100 percent. This implies that the condition for failure is:

$$\sum n_i/N_i = 1 \tag{5.1}$$

Without any detailed knowledge about fatigue in metals, Palmgren's assumptions are the most obvious ones to be made. One might well ask how many times the assumptions were made independently afterwards. Well-known is the publication of Miner in 1945 (Ref.132) and curiously enough less well-known is the earlier publication by Langer (Ref.133) in the same journal. The assumptions were also independently made in a Dutch publication in 1940 by Biezeno and Koch (Ref.134).

Langer should be especially quoted, since he already made the refinement to divide the life into a crack nucleation period and a crack propagation period, Langer suggested

$$\sum n_i'/N_i' = 1 \quad \text{and} \quad \sum n_i''/N_i'' = 1 \tag{5.2}$$

where n_i' and n_i'' are numbers of cycles spent in the crack nucleation period and the crack propagation period, while N_i' and N_i'' are the corresponding crack nucleation life and crack propagation life. Obviously the problem is how to define and to determine the moment that the first period terminates and the second one starts. Langer's assumptions were also repeated, namely by Grover in 1960 (Ref.135) and by Manson et al in 1966 (Ref.136).

In the literature, the Palmgren-Miner rule is also referred to as the linear cumulative damage rule. Miner indeed assumed that the damage in a constant-amplitude test is a linear function of the number of cycles. However, Bland and Putnam (Ref.137) in the discussion on Miner's paper, indicated that the linearity was not required in order to obtain $\Sigma\, n/N = 1$. It was sufficient to assume that the damage rate was a function of n/N which is independent of the magnitude of the cyclic stress. Moreover, these authors emphasized that the material should be insensitive to load cycle sequences. A similar assumption was made by Nishihara and Yamada (Ref.138) by stating that the degree of fatigue damage D was a function of the cycle ratio n/N, independent of the stress amplitude (affine damage curves after Shanley, Ref.139):

$$\eta = f\,(n/N) \tag{5.3}$$

In reference 140 the present author argued that $\Sigma\, n/N = 1$ requires that:

1 The fatigue damage is fully characterised by a single fatigue damage parameter D,

2 The damage D is indeed a single valued and monotonously increasing function of the cycle ratio n/N, which is the same in any constant-amplitude test (stress independent after Kawchele, Ref.141). Hence

failure will always occur at the same amount of damage.
A consequence of the first statement is that interaction effects will not exist. The second one implies $\Sigma\, n/N = 1$ at failure.

5.3.3 Incremental damage theories

Theories based on constant-amplitude data, ignoring sequence effects

Several objections have been raised against the Palmgren-Miner rule, associated with interaction effects, sequence effects, damage due to cycles below the fatigue limit, favourable effect of positive peak loads, etc., which all lead to $\Sigma\, n/N \neq 1$. These effects have been illustrated in chapter 4. Moreover, from a physical point of view it appears incorrect to state that a single damage parameter can uniquely indicate the state of fatigue damage. This has been discussed in chapter 3.
The short-comings of the Palmgren-Miner rule have stimulated several new theories which still preserve the idea of progressive damage accumulation and also the concept of summing damage increments cycle by cycle. A survey of the theories is given in table 5.2.

It is not the purpose of this report to give a complete list and full details of all theories. Several surveys have been given in the literature, see for instance references 47, 142, 143. A recent survey has been given by O'Neill (Ref.144). It will be tried here to indicate essential features of the main groups of theories. There are two important questions in this respect, see table 5.2. The first question is whether the theory employs constant-amplitude fatigue data or variable-amplitude fatigue data. The majority still employs the first type of data. A second question is whether fatigue damage accumulation is sensitive to variations of the load sequence.

The first group of theories to be mentioned is characterized by some kind of adjusted S-N curves. Freudenthal and Heller (Refs.145,146) started from the idea that damage increments in a random load test will be affected by stress interaction effects. They finally arrive at formulas which they call a "quasi-linear rule of cumulative damage". Their failure criterion can be written as

$$\Sigma \frac{n_i}{N_i/\omega_i} = 1 \qquad (5.4)$$

where the "interaction factor" ω_i is either constant or a simple function of S_{ai}, to be determined from fatigue tests. It may also depend on the type of load spectrum. Equation (5.4) indeed implies the application of the Palmgren-Miner rule to adjusted S-N curves. Since these authors assume $\omega_i > 1$, the curves are reduced life curves.

Marsh (Ref.147) suggests to adopt a hypothetical S-N curve with a different slope and a lower fatigue limit (80 %) as compared to the original curve. He recognises the problem of arriving at such an adjusted curve in order to match the empirical data with $\Sigma\, n/N_a = 1$, where N_a is derived from the adjusted curve.

Haibach (Ref.148) also allows for load cycles below the fatigue limit by stating that the fatigue limit is continuously decreasing as a result of increasing fatigue damage. For random and program loading his analytical evaluation is equivalent to applying $\Sigma\, n/N = 1$ to a S-N curve, which is adjusted below the fatigue limit only, see figure 5.1.

Henry (Ref.149) assumes that fatigue damage may be described as a notch in the material which will proportionally lower the S-N curve over the entire stress range. A damage increment is an incremental shift of the S-N curve.

Smith (Ref.45) suggested that $\Sigma\, n_i/N_i = 1$ could not be valid for a program test because residual stresses introduced at the higher amplitudes affected the damage accumulation at the lower stress amplitudes. He therefore proposed that N_i in the Palmgren-Miner rule should be replaced by the fatigue life of the specimen preloaded to the maximum stress occurring in the program test. The preloading should induce the same residual stress being present in the program test. This presumes that a relaxation of the residual

stress will not occur.
In later publications (Refs.150,151) Smith proposed two other theories. In the "linear strain theory" it is assumed that the strain at the root of the notch will be $K_t.\varepsilon_{nominal}$, also after local plastic deformation. With the aid of a stress-strain curve the residual stress at the root of the notch may then be determined. The stress at the root of the notch is then $S = K_t.S_{nominal} + S_{residual}$. Employing this stress-value, corresponding N-values are obtained from unnotched S-N curves for different R-values. These N-values are used for the Palmgren-Miner rule.
The second theory ("Smith method") starts from the idea that the maximum stress at the root of a notch in a program test will be of the order of the yield stress, provided local plastic deformation occurs. The residual stress is now determined indirectly from a constant-amplitude test on the component, tested at the maximum load cycle to be applied in the variable-amplitude test. The fatigue life obtained in this test and the assumption about S_{max} at the root of the notch in conjunction with the unnotched S-N data, will then indicate the applicable R-value and hence S_{min} at the root of the notch. This is sufficient for determining the complete stress history at the root of the notch for the variable-amplitude test. Knowledge of the K_t-value is not required. Again the Palmgren-Miner rule and the unnotched fatigue data are used for the life calculation.
In both proposals Smith has assumed that the material at the root of the notch behaves elastically after the residual stress has been introduced by plastic deformation induced by the maximum load cycle. Secondly, a relaxation of the residual stress should not occur. Since he adopts the Palmgren-Miner rule sequence effects are ignored.

Several authors were reasoning that damage accumulation is progressive crack growth. Shanley (Ref.139) assumes an exponential crack growth law for each constant-amplitude test:

$$\ell = \alpha\, e^{C S_a^{\beta} n} \qquad (5.5)$$

where α, β and C are constants and n is the number of cycles. Failure should occur at a constant crack length ℓ_c independent of the cyclic stress

$$\ell_c = \alpha\, e^{C S_a^{\beta} N} \qquad (5.5a)$$

Damage accumulation was assumed to be the summing of crack length increments without interaction effects. Equation (5.5) and (5.5a) can be written as

$$\ell/\ell_c = (\ell_c/\alpha)^{(\frac{n}{N}-1)} \qquad (5.6)$$

which is of the type of equation (5.3). Consequently, Shanley's formulas imply the validity of the Palmgren-Miner rule. More comments on Shanley's formulas are given in reference 38.

Valluri (Ref.23) also adopted the idea that damage accumulation was a cumulative process of crack growth increments without interaction effects. However, he stated that the crack length at failure was depending on the highest stress amplitude applied, see section 3.3. Since the N-values in $\Sigma\, n/N = 1$ are in fact related to different amounts of cracking depending on the stress cycle, Valluri does not arrive at the Palmgren-Miner rule.

Corten and Dolan (Ref.152) included interaction effects in their crack propagation concept. They postulated that in a program test the maximum load cycle will be decisive for the initial damage, since it will determine the number of loci at which crack growth will start. After this number has been established crack growth is again assumed to be a cumulative process without any interaction. For a program test they arrive at the formula

$$N_g = \frac{N_1}{\sum \alpha_i \left(\frac{S_{ai}}{S_{a1}}\right)^d} \qquad (5.7)$$

where N_g is the program fatigue life, S_{a1} is the maximum stress amplitude with the corresponding constant-amplitude fatigue life N_1, α_i is the percentage of cycles applied at amplitude S_{ai} and d is a constant, that should follow from tests. Since $\alpha_i N_g = n_i$ equation (5.7) can be rewritten as:

$$\sum \frac{n_i}{N_i} \left| \frac{N_i}{N_1} \left(\frac{S_{a1}}{S_{ai}} \right)^d \right| = 1 \qquad (5.8)$$

Note the similarity with equation (5.4). Further, if the S-N relation could be written as $N_i S_{ai}^{-d}$ = constant, equation (5.8) reduces to the Palmgren-Miner rule.

In the last 10 years high-level low-cycle fatigue got much attention. Many constant-strain amplitude tests were carried out involving large amounts of plastic strain. Ohji, Miller and Marin (Ref.153) have suggested that the Palmgren-Miner rule should apply to variable-strain-amplitude tests. In this case n_i is the number of cycles with strain amplitude ε_{ai}, while N_i is the constant-strain amplitude fatigue life associated with ε_{ai}. The same concept has recently been adopted by Dowling (Ref.129), but he first splits up the life in a nucleation period and a propagation period, similar to Langer's treatment, see section 5.3.2.

Incremental damage theories based on constant-amplitude data, including sequence effects

For several years sequence effects were almost exclusively attributed to residual stresses only, these stresses being caused by local plastic yielding at the root of a notch or the tip of a crack. Consequently a theory predicting sequence effects should include the evaluation of the residual stresses during a variable-amplitude test. Presently available theories are deriving residual stresses from the strain-history at the root of a notch. This work was started by Crews and Hardrath (Refs.52,53) as discussed in section 4.5.

A few theories for life calculations employing the above concept have now been published. The basic line of reasoning includes the following steps:

1 The starting data are: a The load-time history, b the material and c the geometry of the specimen.

2 The second step consists of calculating the strain history and the stress history for the fatigue critical location of the specimen. For a notched specimen this is the root of the notch. The strain history will include plastic strains and the stress history will include the local residual stress.

3 The strain history or the stress history calculated in the previous step is split up into individual cycles. Each cycle is assumed to cause a damage increment ΔD equal to $1/N$, where N is the corresponding constant-amplitude life. The failure criterion is again $\Sigma \Delta D = 1$.

The second step is a difficult issue. The strain history may be measured and the stress history may then be derived from the strain history by additional testing. This was discussed in section 4.5, see also figure 4.8. However, a life calculation theory requires that these data be obtained by means of calculation rather than experiment. Morrow and co-workers (Refs.51,154,155) at the University of Illinois are working on this topic. They adopt the Neuber equation, relating the stress and the strain at the root of the notch by:

$$K_\sigma \cdot K_\varepsilon = K_t^2 \qquad (5.9)$$

where K_σ is the stress concentration factor including plasticity, K_ε is the strain concentration factor, also including plastic strain, and K_t is the well-known stress concentration factor for elastic behaviour. Equation (5.9) seems to be satisfactorily substantiated. Morrow et al. then adopt the cyclic stress-strain behaviour from unnotched material. For aluminium alloys this appears to be justified by the observation that the stress-strain hysteresis loop rapidly stabilises, also after a change of amplitude. The strain history and stress history can then be calculated. Morrow et al. also carried out test series to check the theory and the data reported look promising. The evaluation of this concept is still not yet complete and further work is going on.

Impellizzerri (Ref.156) has been reasoning along similar lines. However, since results as shown in figure 4.9, indicate a relaxation of residual stress, he introduces the relaxation into his calculation.

The rate of change of residual stress d $S_{res.}/dn$ in each cycle is assumed to be equal to:

$$\frac{d\,S_{res.}}{dn} = a\,S_{res.}\,\varepsilon_R\,\frac{S_R}{S_{yield}}$$

where $S_{res.}$ is the residual stress, ε_R and S_R are the applied strain and stress range at the root of the notch, and "a" is an empirically determined constant of proportionality. With a computer program Impellizzerri has treated program fatigue test data from NLR, NASA and his own data. He found a very good agreement. In a recent publication (Ref.157) Martin, Topper and Sinclair also introduced stress relaxation. Moreover, their stress-strain model also allows cyclic strain hardening or softening to be accounted for. Agreement with low-cycle variable-amplitude data was good.

With respect to the third step, both Morrow et al., Martin et al. and Impellizzerri adopt $\Sigma\, n/N = 1$. Morrow and Martin prefer constant-strain amplitude data whereas Impellizzerri employs constant-stress amplitude data. In fact, the former are mainly working in the low-cycle fatigue range whereas Impellizzerri applies his calculations to high-cycle fatigue data.
One difficulty in calculating $\Sigma\, n/N$ may still be mentioned here. For a variable-amplitude loading, a load-time history in general will not consist of a sequence of complete load cycles. This also applies to the stress and strain history at the root of a notch if residual stresses are included. The definition of n in $\Sigma\, n/N$ then becomes a problem. This issue is discussed in section 6.2.

Incremental damage theories based on variable-amplitude data, ignoring sequence effects

In the Palmgren-Miner rule $\Sigma\, n/N = 1$, the values of N are derived from constant-amplitude fatigue data. However, it is also possible to adopt variable-amplitude fatigue data for this purpose. Gassner (Ref.158) proposed to use program fatigue test data while Kirkby and Edwards (Ref.99) suggested to adopt narrow-band random load fatigue data. The basic idea comprises the following steps:

1 A load spectrum for a variable-amplitude test should be standardized.

2 Variable-amplitude tests with this load spectrum should be carried out for different intensities of the load spectrum. The results can be plotted as S'- N' curves, where S' is a stress value characterizing the intensity of the load spectrum and N' is the fatigue life in cycles obtained in the variable-amplitude tests.

3 An arbitrary load spectrum can now be decomposed into the sum of a number of the standardized load spectra. If n_i' is the number of cycles of the spectrum characterized by S_i' the failure criterion is

$$\Sigma\,\frac{n_i'}{N_i'} = 1 \tag{5.10}$$

The procedure is illustrated by figure 5.2 for narrow-band random load fatigue data, although the principle is essentially the same for program fatigue test data (Ref.159). In a narrow-band random load test, the load spectrum of the stress peaks is in accordance with a Rayleigh distribution. This distribution is the standardized load spectrum mentioned in step 1 above. The characteristic stress value S' is most conveniently taken as the root-mean-square value S_{rms} of the variable stress. Kirkby and Edwards carried out tests on lug specimens and the results have been plotted in figure 5.2a (step 2). In subsequent tests, the S_{rms} was varied periodically, see figure 5.2b. This was done in such a way that the sum of the three spectra A, B and C was approximately similar to a gust spectrum (step 3). In other words, the gust spectrum can be decomposed into the three spectra A, B and C all obeying a Rayleigh distribution. For the tests in figure 5.2b, both $\Sigma\, n'/N'$ and the classical $\Sigma\, n/N$ values were calculated, see figure 5.2d. From these results the authors drew the following conclusions:

a Damage calculations according to the proposed method ($\Sigma\, n'/N'$) gave results superior to the classical Palmgren-Miner $\Sigma\, n/N$ values, i.e. the values were deviating less from 1.

b The $\Sigma\, n'/N'$ values still deviate considerably from 1.

As shown in figure 5.2b, the S_{rms} value was programmed in a Lo-Hi sequence. However, some tests carried out in a Hi-Lo sequence gave similar fatigue lives. It should be noted that the calculation of $\Sigma\, n'/N'$ still ignores the sequence.

Kirkby and Edwards also performed some tests from which the most severe step C was omitted. As mentioned in chapter 4, this reduced the life 2.5 times. Unfortunately also the Σ n'/N' value was considerably affected, see fig.5.2d. This implies that the new concept in this case does not well account for the change in load spectrum.

5.3.4 The similarity approach

The similarity approach based on stress

In section 5.3.1 the similarity approach was defined as the conception that similar loading conditions at the fatigue critical locations in two different specimens of the same material should produce similar fatigue results. This concept is easily illustrated by referring to the notch effect under constant-amplitude loading.
Compare a notched and an unnotched specimen. If the cyclic stress at the root of the notch is the same as in the unnotched specimen, the same fatigue life may be expected. This had led to the well known $K_f = K_t$ relation, where K_f is the fatigue strength reduction factor and K_t is the theoretical stress concentration factor assuming elastic behaviour. The shortcomings of this relation are also well recognized. Part of them are due to plasticity effects and another part may be attributed to differences between the volumes of highly stressed material (stress gradient effect). Actually, these arguments imply that the conditions at the fatigue critical locations were not really the same.
With respect to plasticity effects an improvement was the work of Crews and Hardrath, and Morrow et al. mentioned before. By accounting for the plastic deformation at the root of the notch, the cyclic stress at that location could be described more accurately. This cyclic stress was again compared with the same stress in an unnotched element.

For joints and other complicated components, K_t values are usually unknown. For such cases effective stress concentration factors, K_{eff} have been adopted in the literature. A survey was given by Schütz (Ref.115). The background is in fact a similarity approach. It is assumed that a component to which a certain K_{eff}-value applies, will show the same fatigue life as a simple notched specimen for which the K_t-value is equal to K_{eff}. This should be valid for any cyclic stress. The empirical determination of K_{eff} for a certain component should therefore be made by comparative testing of the component and of simple notched specimens with a sufficient range of K_t-values.
Obviously the conception again ignores the effects of size and plasticity. Moreover, the fact that different components may exhibit intersecting S-N curves, see figure 7.2, is not easily reconciled with this K_{eff} concept. It requires that for a component having a lower K_{eff} than another component, the S-N curve should be superior at all stress amplitudes.

The K_{eff} conception need not be restricted to constant-amplitude loading conditions. Gassner and Schütz (Ref.159) have proposed the application to the results of program fatigue tests. They performed numerous tests of this type on 2024-T3 specimens with a range of K_t-values. They suggest to use the data for estimating the program fatigue life of a component by assuming some K_{eff}-value for the component. The life is then obtained by interpolation between the specimen data for adjacent K_t-values.

The similarity approach based on strain

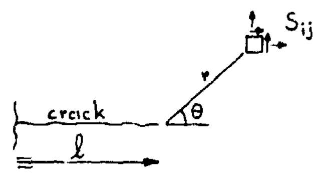
The similarity approach based on strain was adopted in the measurements described in section 4.5, see also figure 4.8. The assumption made was that similar strain history, i.e. at the root of a notch and in an unnotched specimen, should produce similar stress histories. Another assumption is that similar strain histories should also produce similar fatigue lives.
For practical application the similarity approach based on strain is not yet easily used. The strain history has to be either measured or calculated. Subsequently, life data for a similar history if not available, should be determined empirically.

The similarity approach for crack propagation

The application of the stress intensity factor K to the correlation of fatigue crack propagation data is a most outstanding example of the similarity approach.

The stress distribution around the tip of crack can be written in the following form (Ref.160)

$$S_{ij} = \frac{C\,S\sqrt{\ell}}{\sqrt{2\pi r}}\, f_{ij}(\theta) \qquad (5.11)$$



S is the nominal stress on the specimen, ℓ is the crack length and r and θ are polar co-ordinates. The geometry of the specimen is accounted for by the non-dimensional constant C. It should be pointed out that equation (5.11) is essentially a solution for linearized elasticity. Moreover it is assymptotically valid only for small values of r/ℓ, that means for the crack tip region. Further, different values of $f_{ij}(\theta)$ apply to the three crack opening modes. It is sufficient here to consider only the tension opening mode (tensile mode, 90° mode in fig.2.4).

Equation (5.11) can be written as

$$S_{ij} = \frac{K}{\sqrt{2\pi r}}\, f_{ij}(\theta) \qquad (5.12)$$

with the stress intensity factor $K = C\,S\sqrt{\ell}$ (5.13)

Equation (5.12) implies that the stress distribution at the crack tip is fully determined by K.

The similarity approach can now be defined. Compare two different specimens, with different geometry, crack length and loading stress, but the same stress intensity factor. Then the stress distribution at the crack tip according to equation (5.12) will be the same. For a cyclic stress, Paris et al. (Refs.161, 162) proposed that equal K-values should imply equal crack propagation rates, and consequently the crack rate should be a unique function of the stress intensity factor

$$\frac{d\ell}{dn} = f(K) \qquad (5.14)$$

A cyclic stress is determined by two of the quantities S_{max}, S_{min}, S_a, S_m. The ratios between these quantities are constant for fatigue tests with a constant stress ratio R ($= S_{min}/S_{max}$). Hence it should be expected (Ref.163) that f (K) in equation (5.14) is dependent on R:

$$\frac{d\ell}{dn} = f_R(K) \qquad (5.15)$$

There is now abundant evidence from constant-amplitude tests confirming the applicability of equation (5.15). The similarity approach here implies that crack growth in a certain type of specimen can be predicted from relation (5.15) determined empirically on another type of specimen. The latter may be a simple sheet specimen.

Available data mainly comes from macrocrack growth observations. It was stimulating to see, however, that the concept was still applicable to corner cracks in different 2024-T3 specimens with sizes as small as $\ell = 0.2$ mm, see figure 5.3. Nevertheless, there should be a lower limit to the validity of the K concept, which implies that it cannot be used in the crack nucleation period (Ref.163).

Limitations should also be expected from other arguments. The K-value is essentially an elastic conception, whereas fatigue crack propagation is due to cyclic plastic strain. If the plastic zone is small, as compared to the region where equation (5.12) is still approximately valid, the stress intensity factor may also be a characteristic value for the amount of plasticity in the small plastic zone. However, for large plastic zones the validity of equation (5.15) should get lost. Nevertheless, it is surprising to see that crack rates at fairly high values of stress and crack length could still be correlated by K, despite the fact that the transition from a 90°-mode crack to a 45°-mode crack had occurred already.

Another stimulus was offered by comparing results of two differently loaded specimens, see figure 5.4 (Ref.164). One specimen was stressed at the ends, whereas the other specimen was loaded at the opposite edges of the crack. For crack growth under constant-amplitude loading this implies that K will increase in the former specimen, whereas K will decrease in the latter one, see figure 5.4b. This is clearly confirmed by the two crack propagation curves shown in figure 5.4a. Nevertheless, the two types of specimen loading produced the same $d\ell/dn$-K relation, as illustrated by figure 5.4c.

The crack rate in each cycle of a variable-amplitude test could be drawn from equation (5.15), that means from data obtained in constant-amplitude tests. However, this is only allowed if there are no interaction effects. Since there are such effects this procedure cannot be valid. Crack growth delays, as discussed in chapter 3, amply illustrate this point.
The similarity approach could still be applied to variable-amplitude tests by comparing crack rate data obtained in different variable-amplitude tests. Paris (Ref.162) suggested that K should be applicable to random load tests if S_{rms} is substituted in equation (5.13). Limited evidence available (Refs.165-167) seems to confirm this viewpoint. A recent NLR test program is concerned with crack propagation under flight-simulation loading (Ref.64) with the design stress level as a variable. It was tried to correlate the data for different design stresses with the stress intensity factor. Unfortunately the attempt was unsuccessful. In fact the similarity approach is not satisfied by similar K-values alone in order to predict similar crack rates. A second requirement should be that the crack tip regions should have gone through similar K-histories. In general the two requirements are incompatible. For constant-amplitude tests, the second requirement apparently is not important. It may be more important for variable-amplitude tests due to interaction effects. Further investigations are required to solve this problem. It would indeed be of practical significance if the K-concept could be applied to random loading and flight-simulation loading.

5.3.5 Interpolation methods

Interpolation, extrapolation and generalisation of empirical trends may be illustrated by some proposal made by Gassner and Schütz in reference 159. The type of information employed has already been discussed in section 4.14. Figure 5.5a is illustrating equation (4.1) once again. The relation can be written as:

$$\log N' + \kappa \log S_{a,max} = C \tag{5.16}$$

where N' is the program fatigue life, $S_{a,max}$ is the maximum stress amplitude of the standardized load spectrum applied in the test and C is a constant. This linear relation was confirmed by many tests. From a graph as given in figure 5.5a, the program fatigue life can be read for any value of $S_{a,max}$, either by interpolation or extrapolation. Usually the slope factor κ had a value in the order of 6.
In figure 5.5b the slope of figure 5.5a has been adopted to draw the line through a single available data point. Obviously, this is a generalisation of an empirical trend.
The effect of the shape of the load spectrum has also been mentioned in section 4.14. Results from Gassner's group for four different shapes shown in figure 5.5c, (spectrum D is a constant-amplitude loading) produced a curve as plotted in figure 5.5d. Curves shown in figure 5.5a can now be adjusted to another load spectrum by generalizing the applicability of figure 5.5d (Ref.159). If figure 5.5a is valid for spectrum A, it can be adjusted to spectrum B by multiplying the N' values with a life reduction factor N'_B/N'_A drawn from figure 5.5d. This implies a horizontal shift of the curve over a distance $\log N'_B - \log N'_A$, while the slope remains the same. This is another case of generalising an empirical trend.

Employing empirical trends is similar to applying past experience. It is better justified if more experience is available. It is fully justified only if the trend is obeying a recognised physical law. Unfortunately such laws are not yet established for fatigue. The quality of the results obtained will therefore depend on the amount of available information and the personal ability of interpreting and judging the validity of the data.

The comparison between the results of random load tests and program tests, as discussed in section 4.12, is an example where generalizing trends is apparently unjustified. Trends indicated by one type of tests are not necessarily valid for the other type of testing. For flight-simulation tests the trends may again be different, at least quantitatively. In fact figure 4.29 is already illustrating this point.

Obviously, the safe way of handling available information is to interpolate between data rather than to extrapolate data by generalizing empirical trends. For the purpose of making life estimates, interpolation will produce only relevant information if we start from realistic test data. It is now believed that the most realistic data should come from flight-simulation tests. As a consequence of this position, the present author in references 64 and 96 has proposed the following concept, which might be labelled as the "flight-simulation interpolation method". In order to avoid extrapolation extensive data obtained in flight-simulation tests should be compiled. The data should cover the main variables of this type of testing. Life estimates can then be made by interpolation. The following test program was proposed:
Random flight-simulation tests for a certain structural material should be carried out, including the following variables:

a Specimens. Representative riveted and bolted joints should be used.

b Shape of load spectrum. Some typical shapes should be used, for instance representing gust spectra and manoeuvre spectra.

c Design stress level. Some values should be adopted in order to study the effect of the stress level in a similar way as Gassner has done for program tests.

d Ground-to-air cycle. The number and the magnitude may be varied.

Taking for example four cases for each item a-d this would imply $4^4 = 256$ test conditions if all possible combinations are made.
Evidently it is a large test program, but it would serve more than one purpose. Firstly, the data could indeed be used in the design stage for making life estimates by interpolation between the data. Secondly, the results would reveal the effects of several variables under flight-simulation conditions, which are not well known up to now. Thirdly, without actually having to design a standardized test one could use the data as a standard for comparison when checking the fatigue quality of a new component. A handbook with this type of data could be extended from time to time.

Of course the above flight-simulation interpolation method will not exclude all extrapolations, mainly because the geometry of an actual component will deviate from those of the specimens tested. However, some additional flight-simulation tests on the actual component may indicate the applicability of the available data. Moreover, this will add to the compilation of flight-simulation test data.

5.4 Evaluation of the theories

In the introduction of this chapter the question was asked whether one of the proposed theories could account in a realistic way for the trends described in chapter 4. Actually, if one of the theories could do so the present report would not have been prepared. Nevertheless, some theories can account for one or a few trends because these theories were based on those trends. The principal question now is:
What do we expect from a theory, which requirements should it satisfy, which questions should it answer, accepting the fact that it will be unable to answer all questions.
There are two different approaches to this question, the physical one and the practical one. With respect to the first approach it is important that the theory has a physical base which appears to be sound rather than speculative. The theory as a physical model should be in agreement with the phenomenological observations on crack nucleation and crack growth. The agreement may be qualitative, but anyhow the model should make sense.
With respect to the practical approach the question is whether a theory can give reasonable answers to life prediction problems as they occur in aircraft design or in service.
The two approaches are specified in some more detail by a number of questions listed in the table 5.3. More questions could easily be formulated, but those in the table are pertinent ones for a discussion on the significance of the theories presented in the previous sections.

5.4.1 Physical significance of the theories

Most incremental damage theories in table 5.2 ignore the existence of crack nucleation and crack growth and for that reason these theories are to be labelled as non-physical. In fact most theories ignore all aspects a-d from table 5.3.
Fatigue damage in some theories (Shanley, Valluri and Corten and Dolan), is associated with a fatigue crack, while the accumulation of damage is then similar to crack growth. However, these theories do not explain such simple sequence effects as illustrated in figures 3.3, 4.4 and 4.14. The effect of single overloads may be qualitatively explained by Elber's argument, which is the plastic deformation left in the wake of the crack. Recently Von Euw showed this argument to be applicable (see section 3.2). However, a quantitative treatment of the delay has not yet been achieved. This would require the solution of extremely difficult problems related to cyclic stress-strain distributions around cracks.

A residual stress concept was introduced by Smith in a simplistic way. A more advanced approach is the prediction of stress strain histories at the root of a notch (Crews and Hardrath, Jo Dean Morrow et al., Impellizzerri). This is indeed a refinement of the description of the load history at the fatigue critical location which then appears to be sensitive to load sequences. The effect of high preloads on the fatigue life of notched elements are well predicted by this approach. However, for more complex sequences the translation of the stress-strain history into fatigue lives is just another problem.

According to the incremental damage theories, the damage increment per cycle ΔD equals $1/N$, if N is the constant-amplitude life associated with the magnitude of the individual strain cycles or stress cycles. The N-value is derived from adjusted S-N curves in some theories. From a physical point of view, $\Delta D = 1/N$ is at most "plausible" but in fact it is pure speculation. It also implies that interaction effects on the damage accumulation are accounted for only by adopting the real stress-strain history at the root of the notch, which includes residual stress. Other interaction effects and damage parameters as outlined in chapter 3 are ignored. Moreover, the nucleation and the growth of cracks do not form a part of these theories.

In conclusion it has to be admitted that available theories are physically speaking rather incomplete and hence fairly primitive. The complexity of fatigue damage accumulation as a physical phenomenon is much better recognized than in the early days. At the same time this offers tremendous problems for postulating a quantitative physical theory.

5.4.2 Practical significance of the theories

If a life prediction is made for a practical problem, the reliability of the result may be limited for several reasons. These reasons are only partly associated with the validity of the cumulative damage rule, adopted, as will be discussed in chapter 6. Here the discussion is restricted to the cumulative damage theory itself and the fatigue data required for its application.
The majority of the incremental damage theories are based on constant amplitude data, see table 5.2. Frequently, it is tacitly assumed that these data are available or can easily be estimated. Since practical questions are associated with components and joints, this is incorrect. Moreover, an empirical determination of S-N data of joints is usually costly and time consuming. Apparently, the S-N data are a weak link in the application of the incremental damage theories. More comments on this aspect are given in chapter 6.

Prediction of fatigue life until visible cracks (topic e1, table 5.3)
Most theories do not pay much attention to the definition of the end of the fatigue life. In general it should be understood to be the life until a visible crack is present, or the life until complete failure of a small component. It is not realistic to consider the life until complete failure of the entire aircraft structure, unless it is a safe-life structure. In a fail-safe structure the propagation of the visible crack should be treated separately.
Assuming that S-N data are available, the Palmgren-Miner rule is the most simple rule to be used. Many

investigations were carried out to check the validity of the rule and considerable deviations were found. Although the deviations do not show a clear pattern, it may be said that most values of $\Sigma\, n/N < 1$ were found for zero mean stress and for unnotched material (Ref.168). For positive mean stress and notched elements, quite a lot of $\Sigma\, n/N$ values are in the range 0.5 - 2.0 (Refs.168,169). It was concluded elsewhere (Ref.78) that the Palmgren-Miner rule in many practical applications will be good enough for a rough life estimation, provided realistic S-N data are available.
If unconservative estimates have to be feared, for instance in case of zero mean stress or many cycles below the fatigue limit, the application of the Palmgren-Miner rule to an adjusted S-N curve, such as proposed by Haibach (see figure 5.1), may be recommended. For the combination of flight loads with GTAC some conservatism could be added by simply starting from $\Sigma\, n/N = 0.5$.

A comparison between a number of incremental damage theories was made by several authors (Refs.47,142,170). In general the conclusion was that it is hard to prefer one of the rules to the Palmgren-Miner rule. Better predictions were sometimes found by the Corten-Dolan theory if the constant "d" in equation (5.7) could be adapted to the test results. Actually, a more accurate validity for practical loading conditions until now, cannot be claimed by any of the incremental damage theories based on constant-amplitude fatigue data.

It may be expected that theories based on stress-strain histories at the root of a notch have the potentiality to give more accurate life predictions. This is thought to be true because they may account for residual stress effects, which probably play an important part in fatigue damage accumulation. Even relaxation of residual stress could be incorporated. Results published by Impellizzerri look promising but a wider exploration is required before definite conclusions can be drawn. One improvement of the situation may be mentioned here. Present computers allow damage calculations to be made from cycle to cycle, even if the total life is a high number of cycles.

Another improvement, as compared to the Palmgren-Miner rule, appears to be obtained by the procedures proposed by Gassner and by Kirkby and Edwards ($\Sigma\, n'/N' = 1$). Also here a general validity will probably not apply. Similar to the application ofthe Palmgren-Miner rule, the availability of relevant fatigue curves (S'- N' data) may be a problem. Moreover, the advantage of these procedures may be weakened by the problem of how to account for deterministic loads such as the ground-to-air cycles. The most logical consequence then appears to be to start from data obtained in flight-simulation tests. This is the proposal discussed in section 5.3.5. The purpose is indeed to minimize extrapolation as far as possible. More comments are given in chapter 6.

<u>Prediction of macro-crack propagation</u> (topic e2, table 5.3)
The most logical approach appears to be an estimation by employing the stress-intensity factor. A crack growth curve may be calculated by integrating $\Delta\ell$ for each cycle, where $\Delta\ell$ is derived from $d\ell/dn = f_R\,(K)$ (Eq.5.15) as obtained in constant amplitude tests. Interaction effects are ignored by this procedure and since such effects are predominantly favourable to macro-crack growth, a conservative result will be obtained. Moreover, it will be necessary to calculate K-values for the structure as a function of crack length. The relevance of this procedure was recently proven (Refs.171,172) for cracks in stiffened panels. A good agreement between prediction and test results (constant-amplitude loading) was found. More comments on predicting crack growth are made in section 6.4.

<u>Complicated sequence effects</u> (topic <u>f</u>, table 5.3)
In section 4.13 it was indicated that a change of the load sequence in a flight-simulation test probably had a small effect on life only. This is a convenient trend because almost all incremental damage theories do not account for different sequences. The exceptions are theories based on stress strain histories which, however, are not yet sufficiently checked for practical load histories.

<u>Effect of a change of the load spectrum</u> (topic <u>g</u>, table 5.3)
If an aircraft is used for two different missions, two different load spectra will apply. It is a practical question to ask how the fatigue lives associated with these missions will compare. A similar problem

occurs if the service load spectrum turns out to be different from the load spectrum adopted in design calculations and full-scale fatigue tests.

If the lives for load spectra A and B are L_A and L_B respectively, the ratio L_B/L_A may be calculated by means of the Palmgren-Miner rule. It is sometimes suggested that the invalidity of the rule, which could lead to misleading values of both L_A and L_B, would have a smaller effect on the accuracy of the calculated ratio L_B/L_A. This suggestion is unjustified.

Although the rule may give rough life estimates it is in fact unable to give indications of the damage contributions of the various load amplitudes S_{a_i}. Some may be larger than n_i/N_i, others may be smaller. If the modification of the load spectrum is in a range of S_a-values, where n_i/N_i gives a false indication of the damage contribution, the ratio L_B/L_A will also be a false indication.

The so-called "level of maximum damage", was defined in reference 173 (see also Ref.168) as the load amplitude of a load spectrum giving the largest contribution to $\Sigma\, n_i/N_i$. However, it is far from sure that it will indeed give the largest contribution. All the misleading suggestions tacitly assumed the absence of interaction effects, which do exist, however.

Some simple examples of misleading indications were discussed in chapter 4. If a gust spectrum is changed by having some more high-amplitude cycles the Palmgren-Miner rule predicts a slightly shorter life. In reality the life may be much longer. A second simple example is offered by modifying the spectrum in the low-amplitude range. According to the Palmgren-Miner rule this will have no effect at all, contrary to test results.

Unfortunately, the fact that the Palmgren-Miner rule is unreliable to account for modifications of the load spectrum, also applies to the other incremental damage theories. The only way out is testing, that means comparative flight-simulation tests with different load spectra. Once again we arrive at the proposal for systematic flight-simulation tests made in section 5.3.5. Curves similar to the curve in figure 5.5d, determined by Gassner and co-workers for program fatigue tests, would be required.

Effect of a few high-amplitude cycles (topic h, table 5.3)

According to most theories the effect of a few high-amplitude cycles will be negligible, whereas in reality it may be very large. Theories including a residual stress concept try to account for the effect of a few high-amplitude cycles. This applies to the Smith theory and the theories based on the strain-history at the fatigue critical location. As said before, a satisfactory solution has not yet been obtained but a further exploration is certainly worthwhile.

Effect of many low-amplitude cycles (topic j, table 5.3)

If low-amplitude cycles are below a fatigue limit, most incremental damage theories will predict these cycles to be non-damaging. The exceptions are the theories that include a reduced fatigue limit. Unfortunately these theories do not account for the effect of a few high-amplitude cycles.

In summary

With respect to making life estimates, a theory that is distinctly superior to the Palmgren-Miner rule, is not available. Secondly, a life estimate obtained with the Palmgren-Miner rule is a rough life estimate only. Its accuracy is not only dependent on deviations from the rule but also on the reliability of the S-N curves used. Thirdly, for the purpose of estimating the effect of modifications of the load spectrum the Palmgren-Miner rule is unreliable. The same applies to the other rules.

There is some prospect for the theories based on stress-strain histories at the fatigue critical locations, but a further evaluation and empirical checking is necessary.

Until now most theories had the character of incremental damage theories. The similarity approach, in principle, is a fully justified approach, but unfortunately there are still considerable limitations. The $K_{eff.}$-method being the most prominent exponent for life estimates is not satisfactory. For crack propagation, the stress-intensity factor is successful for constant-amplitude tests. There are indications that the stress-intensity factor will not work for service load-time history, but a further exploration is desirable.

The interpolation approach will probably give the most accurate life predictions, provided that it can be based on realistic flight-simulation tests data. Such data are still largely to be collected.

6. ESTIMATING FATIGUE PROPERTIES AS A DESIGN PROBLEM

6.1 Survey of aircraft fatigue problems

A large variety of empirical trends has been reported in chapter 4. Cumulative damage theories have been discussed in chapter 5 and it has to be admitted that the picture of theories, in view of explaining the empirical trends, is not a bright one. There is some qualitative understanding of the trends, but the possibilities for quantitative predictions is still rather limited as yet.
It should now be analysed how this affects the determination of fatigue properties of aircraft structures, both in the design stage and later on. It cannot be the purpose of this report to discuss all aspects of fatigue in aircraft structures. It is sufficient here to list the most prominent aspects and this has been done in table 6.1 drawn from reference 78. The list is not necessarily complete, but sufficient for the present analysis. It is important to note that there are three phases in the history of an aircraft, namely, the design phase, the construction of the first aircraft and the performance of test flights, and finally, the utilisation of the aircraft in service.
In the first phase the designer has to start with the actual design work, including general lay-out of the structure, joints, detail design specification of the materials, etc. He then has to estimate the fatigue performance of the structure, which broadly outlined involves the steps indicated in table 6.2.

The description of the fatigue environment is a complex problem, not only because it involves a good deal of guesswork but also in view of the large variety of aspects. This is illustrated by table 6.3. Several topics in the table will be briefly touched upon when discussing the merits of life calculations and fatigue tests.

The second step of table 6.2 includes the dynamic response of the structure to arrive at the fatigue loads in the structure. A modern trend in this area is the application of power spectral density (PSD) methods. The calculation of the fatigue loads in the structure is beyond the scope of this paper, but it has to be said that the aircraft response may be a significant source of uncertainties. It can be partly circumvented by direct load measurements in flight.

The last step in table 6.2 is concerned with the estimation of fatigue lives and crack propagation. In the present chapter comments will be made on the statistical description of service load-time histories (Sec.6.2), while the problem of estimating fatigue lives and crack propagation data is dealt with in the remainder of the chapter. Aspects of testing procedures, in order to get relevant information, are discussed in chapter 7.

6.2 The description of the service load-time history

In general service load-time histories are described by statistical means. There are two different approaches, namely:
1. Counting methods
2. The power spectral density method .
Before making comments on these methods some thought should be given to the definition of a load cycle.

Definition of a load cycle

In constant-amplitude tests the fatigue life N is given as a number of cycles until failure. In the Palmgren-Miner rule, $\Sigma\, n_i/N_i = 1$, the meaning of n_i also is a number of cycles applied at a certain cyclic load. Mathematically it may seem attractive to define the size of the load cycle by specifying its mean and amplitude, for instance S_m and S_a (Fig.6.1a). However, considering fatigue as a damaging phenomenon in the material, the moments of reversing the loading direction are the more important milestones of the load-time history. In other words, the load peaks, i.e. minima and maxima, are the characteristic occurrences of the cyclic load. Consequently from a physical point of view it would be better to define a cycle by its minimum and maximum, see figure 6.1b. In order to complete the definition of a load cycle it should be said that it consists of a rising and a falling part, that means it

consists of two half cycles. For the time being the loading rate will not be considered.
A few variable-amplitude load sequences are given in figure 6.1 c-f. Figure 6.1c shows the case where the amplitude is reduced, while maintaining the same minimum. The definition of load cycles 1-4 gives no problem.
If the amplitude is reduced while the mean remains the same, this can be done in two ways, see figures 6.1d and e. Clearly enough cycle 3 in figure 6.1d and cycle 2 in figure 6.1c cannot be defined by a single value for S_{max} and S_{min}. As shown by the empirical trends in figure 4.14, the difference between the two cases is not irrelevant for fatigue damage accumulation.
Another problem is illustrated by figure 6.1f. The small intermediate load fall BC implies an additional maximum and minimum. Hence one might say that there are two cycles in this figure, although the material from a damaging point of view will respond to it as a single cycle ADE. Considering the two gust load records in figure 6.2 it will be clear that a definition of load cycles in these practical cases is far from simple.

Counting methods

The aim of counting methods is to give a statistical distribution of characteristic magnitudes of the load-time histories. Whether useful data can be produced in this way will be discussed later. In the following a survey of some counting methods is given to illustrate the type of data obtained.
Characteristic occurrences of a load-time history adopted for counting may be either peaks, level crossings or ranges. This has led to a variety of counting methods (Refs.174-176). Examles are illustrated by figure 6.3, while a more complete list is given by Van Dijk (Ref.176).
In methods a and b peaks are counted. In the second one only the most extreme peak between two mean-crossings is counted. The purpose of this is to ignore smaller load variations which are thought to be irrelevant to fatigue. The VGH records were evaluated with this peak-between-mean crossings count method.
In method c level crossings are counted. One might assume that the number of maxima above a certain level is equal to the number of crossings of that level (with positive slope). This, however, is incorrect although it is approximately valid under certain conditions.
Method d is a variant of method c involving a second condition to be met before a level-crossing count is made. A level-crossing in the upward direction is counted only if the load has gone downwards to a lower level. This eliminates level crossings from smaller load variations. The well-known Fatiguemeter is operating according to this method.
In the simple range count method (Fig.6.3e), positive and negative ranges between successive peak values are counted. The basic idea is that ranges are more important for fatigue than the absolute peak values. The range count method has one serious disadvantage. The counting result is extremely sensitive to the smallest load variations still to be considered. In figure 6.1f, the range AD will not be counted, but instead of one large range three smaller ranges AB,BC and CD have to be counted.
With the range-pair exceedance count method, range exceedances are counted in pairs of equal magnitude and opposite sign. The disadvantage of the range count method is thus eliminated.
Recently, De Jonge (Ref.177) proposed the NLR counting method (referred to as the range-pair-range count method by Van Dijk (Ref.176). This method is a further development and extension of the range-pair-exceedance counting method. It also gives information on the mean of the range counted. This implies that peak load levels can also be derived from the counting result. Moreover, counts for a relatively small time interval can be processed separately. As a consequence, the "memory" for pairing positive and negative ranges has now been set to certain limits. Hence the method is thought to give more relevant information from a fatigue damaging point of view as compared to previous methods. It should be noted that the rain-flow counting method (Ref.129) can produce similar information as the NLR counting method.

The power spectral density concept

The application of power spectral analysis to randomly varying loads, especially to gust loads, is being explored to an ever-increasing extent. The mathematical frame work for application to random loads cannot be discussed here. It may be found, for instance, in references 32-34,178. It is assumed that the external load, for instance turbulent air, may be considered to be a stationary Gaussian process during a certain period. Such a process is fully described by its power spectral density function $\phi_e(\omega)$, where ω is an angular frequency. If the response of the structure to the external load is linear, the power spectral density function of the internal load $\phi_i(\omega)$ can be calculated from $\phi_e(\omega)$ and the transfer function of the structure. This is an elegant way to account for the dynamic behaviour of the structure. It is possible to calculate from $\phi_i(\omega)$ the statistical distribution functions for level crossings and peak loads. The weakness is that all crossings and all peaks are obtained, also if they are caused by very small load fluctuations. Results obtained by counting methods based on ranges, cannot be calculated. Nevertheless, $\phi(\omega)$ is fully characteristic for a random sequence and any information that cannot be calculated mathematically can be determined by measurements from a signal with the same spectral density function. The effect of $\phi(\omega)$ on the irregularity of the random load was already illustrated by figure 4.20. It may be emphasized here that the power spectral density method requires the external load to be a Gaussian phenomenon.

<u>The usefulness of the counting methods</u>

The question about the usefulness has to be related to the purpose of the data being collected. Three main objectives may be mentioned here.

1. Collecting data for load spectra to be used for future aircraft design.
2. Establishing data for the application to a full-scale fatigue test.
3. Collecting data for estimating the consumed life of individual aircraft.

The first topic is of interest to the designer, the second one to the test engineer, while the third one is important for the aircraft operator.
Starting with the last objective there is a fairly extensive literature on collecting in-flight data for this purpose. A discussion would be beyond the scope of this report. Reference may be made here to a recent publication by De Jonge (Ref.177). He made a proposal for a fatigue load monitoring system. Two essential features are: (1) Load statistics should be derived from strain records instead of acceleration measurements. (2) The strain record should be analysed by the NLR counting method. The first recommendation was made because the relation between accelerations and loads in the structure is not unequivocal in many cases. The second recommendation is made because it is thought that the NLR counting method gives more appropriate indications about the fatigue load environment.
The second objective mentioned above is discussed in chapter 7. Some comments will be given there. The first objective is important for estimating fatigue lives which is the subject of the present chapter.

Various counting methods were compared for the application to gust load records (Ref.174) and manoeuvre load records (Ref.176). It turned out indeed that the range method was fully inadequate in view of its sensitivity to small load fluctuations as mentioned before. Also the simple level-crossing count method and the simple peak count method are believed to count toomany irrelevant occurrences.
The restricted-level-crossing count method (Fatiguemeter) and the peak-between-mean crossings count method (VGH records) give more relevant information. Moreover, the two methods show relatively small differences between their counting results. The range-pair exceedance count method appears attractive but the disadvantage is that no information is obtained about peak values of the load-time history. This disadvantage is eliminated in the NLR counting method. Unfortunately, almost no data obtained with this method are available as yet.

The question whether a counting method gives useful information for calculating fatigue lives does not yet allow a definite answer. If statistical data obtained with some counting method could give a realistic estimation of the actual load-time history, we are still left with another problem. This is how to calculate the fatigue life from this load-time history. In the previous chapter it had to be admitted that this problem is not yet solved. As a consequence, a comparison between service life and calculated life can neither prove nor disprove the quality of the counting method since the uncertainties about the life calculation method are involved also. At best we may ask whether statistical counting results allow a relevant estimation of the service load-time history. The estimation is considered to be relevant if tests with the real load-time history and the estimated one would produce similar fatigue lives. Keeping in mind this criterion some more comments will be made later.

<u>Sequence of loads</u>

It should be pointed out that all counting methods do not give any information about the sequence of the loads counted. (Some information about possible sequences will be retained by the NLR counting method).

The results of statistically counted loads are usually presented as exceedance curves, such as shown in figure 6.4 for gusts and for manoeuvres. For gusts it is assumed that the load spectra for positive and negative gusts are symmetric which allows a presentation by a single curve. For manoeuvre loads the same procedure cannot be adopted since positive manoeuvre load increments are usually much more severe than negative increments. A presentation by two curves is necessary, see figure 6.4b.
Since curves as shown in figure 6.4 do not contain information about the sequence of the loads, the usual procedure for estimating fatigue lives is to combine upward peak loads and downward peak loads, that have the same frequency of occurrence, in order to form complete load cycles. This is illustrated for a single cycle in both figures 6.4a and b. Actually, load records such as shown in figure 6.2 do not justify this procedure. However, it is generally thought to be conservative since it has the character of maximizing load cycle amplitudes. The counting results produced by Van Dijk (Ref.176) suggested that it most probably will be conservative for manoeuvre loadings.

Ground-to-air cycles

Contrary to gusts and manoeuvre loads, the ground-to-air cycle has a more or less deterministic character. Several loads occur once per flight such as the transition from the static load on the ground to the static load in flight, fuselage pressurisation, flap loads and empennage loads during take-off and landing, etc. The flight-load profile thus consists of a mixture of these deterministic loads and other loads that are mainly statistical in nature with respect to occurrence and magnitude.

Flight-load profiles

The assessment of flight-load profiles for a certain type of aircraft requires an analysis of the various missions to be performed by the aircraft. Both deterministic and stochastic loads can then be estimated. As a result of combining these loads, synthetical flight-load profiles will be obtained. Two simplified examples applying to wing bending, are shown in figure 6.5. A number of comments on this figure should be made:

1. Those portions of the flight during which the load is not varying have been omitted.
2. Gusts manoeuvres and taxiing loads were assumed to occur as complete cycles. As said before this is probably a conservative procedure.
3. The sequence of gusts, manoeuvres and taxiing loads was assumed to be random without any sequence correlation. For gust loads the PSD-method might allow a more realistic determination of the sequence.
4. Flight-load profiles may be different from flight to flight. For gusts this has to be expected since flights both in good weather and in poor weather will occur.
5. The two examples in figure 6.5 show that there may be one important additional cycle per flight. Basically it is the static ground-air-ground transition, but it is enlarged by additional loads both at the ground and in flight. The minimum and the maximum of this additional cycle are indicated in figure 6.5. The magnitude of this cycle may vary from flight to flight. In view of the discussion in the previous chapters this cycle may well be expected to give a significant damage contribution. Buxbaum (Ref.179) measured the maximum load cycle of each flight of a transport aircraft and he made a statistical evaluation of its magnitude.

In summary: The assessment of flight-load profiles implies the prediction of the sequence of various fatigue loads from flight to flight, i.e. the service load-time history. It is flying the aircraft by imagination at a moment that it still has to go into service. This is necessary for planning a realistic full-scale test, see chapter 7. However, it is also necessary in view of defining the cycles associated with the ground-air-ground transition. Such cycles are certainly important enough to be considered in making life estimates.

6.3 Methods for estimating fatigue lives

The fatigue life in this section should be understood to be the life until a visible crack is present, or the life until complete failure of a small component. If life estimates as accurate as possible are required, realistic flight-simulation tests are essential. This will be discussed in chapter 7. However, if provisional estimates have to be made several procedures can be adopted. The basic elements involved in such estimates have been listed in table 6.4. The first topic of the table is the estimated service load-

time history. This aspect was discussed in the previous section. The second element is the structure or component for which a life estimate is requested. It will be assumed that the dimensions and the material have been chosen already. Insufficient fatigue life could of course modify these data.
The third element of table 6.4 includes a variety of possibilities concerning available fatigue data. A survey is given in table 6.5. As an example of similar materials one may adopt fatigue data from 2024-T3 material for applications where 2024-T8 or 7075-T6 material was selected. Obvously, this may affect the quality of the life estimate to be based on the data.
The type of specimen for which data are available may vary from the unnotched specimen to the component itself. This implies that it may vary from a highly unrealistic representation to the most realistic representation of the component for which the life estimate has to be made. Also for the type of loading the qualification of available data may vary from a low similarity (constant-amplitude loading) to a high similarity (experience in service).
The fourth topic in table 6.4 is the fatigue life calculation theory. This subject has been analysed in chapter 5. It was made clear that there were no reasons to be optimistic with respect to the accuracy of available theories.
For completeness, additional fatigue tests have been mentioned as the last aspect in table 6.4. It will be clear that the quality of a life estimate can be improved by additional tests.

Several procedures for making life estimates can now be specified. A survey is given in table 6.6 which will be discussed below. For all cases it is assumed that estimated service load statistics were collected already.

Methods based on available data

As illustrated by table 6.6, the calculated life will depend on the type of available data, on corrections made to these data by accounting for deviating aspects and on the life calculation theory. Apparently there may be several weak links.
The most simple type of fatigue data would be S-N curves for unnotched specimens. The S-N curves for the component under consideration have to be derived from these data and this will introduce unknown inaccuracies. The reliability of the required S-N data would be improved by starting from S-N data for notched specimens, while data for components could be a still better starting point. Methods for obtaining optimal S-N data, improved by accounting for material, S_m, K_t, size, etc., are beyond the scope of this report (see for instance Refs.180,181). It may be said here that the relevance and the quality of S-N data to be obtained are a matter of judgement and ability to evaluate available information.
As said in chapter 5, a better cumulative damage rule than the Palmgren-Miner rule does not appear to be available yet. Starting from fatigue data obtained under a more complex fatigue loading, such as program loading, random loading or flight-simulation loading, has as its aim to reduce or eliminate uncertainties about the life calculation theory (methods 1c,1d,1e in table 6.6). This approach is still hampered by insufficiently available fatigue data. Other aspects have been discussed in chapter 5.

Methods based on service experience from previous designs

A new design may have a high similarity with a previous design. It even may be a further development of the previous one. In this situation it will be clear that most valuable information should come from the service record of the older design (see for instance Ref.182). This information can be utilized in a rather general way by adopting the stress level, that for a certain material allowed a satisfactory service behaviour in the past (method 2a in table 6.6). If cracks did not occur in the previous design, the life drawn from its service experience is a lower limit. The life should be longer. Obviously it is necessary to prove that the new design is at least as good as the previous one. The proof could be given analytically, for instance by considering K_t-values for improved detail design. For joints the stress severity factor proposed by Jarfall (Refs.183,184) may turn out to be a useful criterion for judging the fatigue quality (Ref.185). The alternative to the analytical comparison is comparative testing, see section 7.4.

A further evaluation of past experience, method 2b in table 6.6, includes a consideration of all relevant conditions instead of considering the stress level only. This implies that service experience is now considered to be a fatigue test on specific components. The conditions for the old design have

now to be translated to the new design by accounting for deviating aspects. This offers similar problems as correcting S-N data, and in addition one new problem, which is accounting for a different load spectra. As said before, the Palmgren-Miner rule is unreliable for the latter purpose. Comparative testing, see section 7.4, is the best solution. If this is not feasible conservative assumptions may be made.
The attractive feature of employing past experience is that we start from data obtained under most realistic conditions. Secondly, the information may come from a large number of aircraft, which increases the statistical confidence. Limitations already mentioned are associated with deviations between the new and the old design. Another limitation is that the load-time history for the old design, in many cases, will not be accurately known, which requires estimates to be made.

Testing of a new component or a complete structure

This method (method 3 in table 6.6) will be advisable in many cases. Nevertheless it appears to be fully justified only if a realistic load-time history will be applied in the test. That means that a flight-simulation test should be carried out. This topic is discussed further in chapter 7.

Comparison of the methods

Speaking in general terms there are three alternatives apart from new methods still under development:

1. Estimates based on S-N data employing the Palmgren-Miner rule.
2. Estimates based on the evaluation of the experiences obtained with previous designs.
3. Estimates based on flight-simulation tests.

The first method will give rough life indications only. It would not be fair to say that the Palmgren-Miner rule is the only weak link in this method. The estimated S-N curves may also be a source of inaccurancies. Obviously additional constant-amplitude component testing could improve the situation.
The second method has a good appeal for reasons mentioned above. There are also limitations. However, if a careful analysis is incorporated into this method it should be preferred to the first method. Comparative flight-simulation tests may significantly add to the value of the second method.
The third method, still to be discussed in chapter 7, is to be recommended only if a carefully planned flight-simulation test will be carried out. The method is certainly preferable to the first method. In comparison to the second method, testing the component or structure itself implies a more realistic simulation in this respect. This may also apply to the load-time history adopted in the test. The second method, however, may be more realistic with respect to the environment (corrosive effects, rate effects), while the statistical confidence may also be superior. It is difficult to say which of the two methods will give the best answers. An evaluation of past experience should be recommended in any case. However, a realistic flight-simulation test on a component need not be a relatively costly effort if a modern fatigue machine is available. Hence in many cases such tests are recommendable as well. A full-scale fatigue test with a realistic flight-simulation should anyhow be recommended, since such a test serves more purposes than obtaining life indications only (see chapter 7).

6.4 Methods for estimating crack propagation rates

Problems of estimating crack propagation rates are to a large extent similar to those involved in estimating lives. Some specific features will be discussed.
Information about the propagation of macro-cracks is desirable in view of judging the safety of an aircraft. For assessing the quality of a fail-safe design this information is even indispensable. It may be tried to give a similar survey as given in table 6 for estimating fatigue lives. This has been done in table 6.7.

The amount of available data from constant-amplitude tests is steadily increasing and as discussed in section 5.3.4, such data allow a presentation as

$$d\ell/dn = f_R(K) \qquad (6.1)$$

It should be pointed out that almost all data in the literature were obtained by testing sheet material under axial loading. However, in thick sections with predominantly plane-strain conditions the crack

rate may be higher. The crack rate in a pressurized fuselage will also be higher in view of bulging of the crack due to the pressure on the edges of the crack.
The most simple case would be to predict the crack propagation rate in an axially loaded sheet metal structure for which K-values can be calculated. Tests on stiffened panels have shown that equation (6.1) is indeed capable of correlating the crack propagation rates obtained under constant-amplitude loading (Refs.171,172). For a service load-time history the crack rate could be estimated by the formula:

$$\frac{d\ell}{dn}(\ell = \ell_j) = \frac{\sum_i n_i \frac{d\ell_j}{dn}(K_{ji})}{\sum_i n_i} \qquad (6.2)$$

where the subscript i is referring to the various stress levels involved.
Equation (6.2) gives a weighted average crack rate, but the same formula is obtained if the Palmgren-Miner rule is applied to the fatigue lives required for an incremental crack length extension. This implies that equation (6.2) is ignoring any interaction effect between successive stress cycles with different magnitudes. Since interaction effects are predominantly favourable for macro-crack growth, equation (6.2) will produce a conservative result and it even may be very conservative depending on the type of load spectrum (see for instance Ref.186).

More realistic estimates will be obtained if data from flight-simulation tests can be adopted. Such tests were recently carried out at NLR (see table 4.7), but so far this appears to be the only source.

Additional testing is to be recommended because crack propagation may be sensitive to the type of alloy. Different crack rates may even be found for the same alloy produced by different manufacturers while also batch to batch variations have been noted (Ref.163). The recommendation for additional testing is easily made since simple and inexpensive specimens can be used for this purpose.

Service experience from previous designs with respect to crack propagation will in general not be available. It is common practice to repair a crack in service immediately after it was found. However, data from full-scale tests on previous designs may give useful indications about crack rates to be expected in a new design. Testing the new design itself obviously should give the most direct information. This is discussed in chapter 7.

6.5 The significance of life estimates

The significance of a life estimate and the accuracy required or desirable will depend on the consequences that the estimated life values may have. This part of the problem has several aspects. Some aspects are briefly mentioned below in order to further complete the picture of the practical problem.

The consequences of the life estimate will obviously depend on the result obtained. The estimated life may be highly insufficient, it may be of the correct order of magnitude, and it also may be much larger than required. Obviously these three cases will ask for different actions to be taken. It will be clear that the follow-up of the estimate should also depend on the quality and reliability of the estimate. Moreover, scatter of fatigue properties has to be considered.
Decisions to be made will also depend on design aspects. If the design is an entirely new type for which no experience is available, a realistic life estimate seems desirable. Another aspect is the question whether the structure has a fail-safe or a safe-life character. In the latter case accurate life estimates are again requested. For coming to decisions it may also be important whether a redesign is easily possible. Test facilities available are another aspect of the problem in view of complementary testing.

The question whether the result of a life estimate should be considered as being satisfactory, in many cases, will not allow an easy answer due to the many aspects involved. The problem will not be discussed any further here. It may be said, however, that the philosophy of the aircraft firm with respect to designing for safety and economics, airworthiness requirements and requests of the aircraft operator may also affect the answer.

7. FATIGUE TESTING PROCEDURES

7.1 Survey of testing procedures and testing purposes

With respect to testing purposes, the main aspects characterizing a fatigue test are the type of specimen and the type of fatigue load applied. From a testing point of view the fatigue testing machine is important. In this connection the question is whether available fatigue equipment is capable of handling the specimen and of applying the fatigue load sequences required. Some general comments on the above aspects will be made in this section, while the usefulness of various testing methods for specific testing purposes will be discussed in subsequebt sections.

Test purposes

Being confronted with the vast amount of literature on fatigue investigations, it is useful to recognize some categories of purposes. In general terms three groups may be mentioned.
1. Basic fatigue studies.
2. Empirical investigations to explore the effect of various factors on fatigue life.
3. Test series to provide specific data for design purposes.

The purpose of the first group is to increase our physical understanding about fatigue, to describe the fatigue phenomenon qualitatively, and if possible also quantitatively. Investigations to improve our knowledge about fatigue damage accumulation are in this group.
The second group comprises the investigations on the effects of notches, metallurgical conditions, surface treatment, fretting corrosion, production aspects, etc. Experience obtained in such investigations provides useful qualitative information for the designer with respect to the selection of materials, dimensions, production techniques, etc.
The main purpose of test series in the third group is to provide test data for estimating fatigue properties of structures and its components. This type of data was referred to in tables 6.6 and 6.7. Some more specific testing purposes of the third category are indicated in figure 7.1.
Since basic fatigue studies are important here with respect to damage accumulation, some comments on testing procedures for this purpose will be made in section 7.2. The second category is largely outside the scope of the present report. Planning fatigue tests for the third category requires knowledge about trends in damage accumulation under variable-amplitude loading. Tests will be discussed in sections 7.3 - 7.7.

Type of specimen

A survey of different types of specimens is given in table 7.1. Specimens with simple notches may reveal the notch sensitivity of a material. Since most cracks in a structure are starting in joints, a simple notched specimen is not yet a relevant representation of a fatigue critical location in a structure. In a joint fretting corrosion is generally significant. This can be simulated in simple joint specimens. The advantage of a component over a simple joint specimen is that all dimensions and the production technique are fully realistic. Testing a full-scale structure is obviously still more realistic. This may eliminate questions about the correct loads for the various components of the structure. Moreover, unknown eccentricities of the loads on the various components are automatically simulated.

Load sequences in fatigue tests

A survey of possible load sequences has previously been given in figures 4.2 and 4.3, while examples of variants are given in several other figures. With respect to design purposes the discussion will be restricted to constant-amplitude tests, program tests, random tests and flight-simulation tests, see figure 7.1. For basic studies on damage accumulation simple variable-amplitude load sequences may be attractive.

Fatigue testing machines

For a long time fatigue machines were primarily designed for carrying out constant-amplitude tests. If the machine was designed as a resonance system, high loads and high loading frequencies could relatively easily be obtained. Such machines were not well suited for variable-amplitude tests but a slow variation of the amplitude was possible. Hence program tests could be carried out. A major difficulty was to apply small numbers of high-amplitude cycles. This had to be done either manually or by non-resonant slow-drive loading mechanisms.

In some laboratories resonance fatigue machines have been successfully adapted for carrying our narrow-band random load fatigue tests (Refs.99,187).

A break-through in this situation was the development of the electro-hydraulic fatigue machine with closed-loop load control. (Refs.33,188). Each load-time sequence that could be generated as an electrical signal could be applied. By now several fatigue machines of this type are commercially available. In many full-scale tests hydraulic jacks operating according to the same principles have been employed. It is true, however, that a test in such a machine will be more expensive than a constant-amplitude test in an old machine. The problem may then be whether the more relevant information from a complex load sequence is worth the price.

7.2 Tests for basic fatigue studies

In chapter 3 fatigue has beendescribed as a cumulative process. Although cracking was the most prominent feature of fatigue damage, the accumulation of damage turned out to be a complex phenomenon. Under variable-amplitude loading a number of different interaction mechanisms could be operating. Although there is some qualitative understanding it is not free from speculation. In fact there is still ample room for studying the damage accumulation phenomenon. Partly this should occur by refining our knowledge about local stress-strain history. For another part microscopical observations on damage accumulation should be very worthwhile.

Load sequences for basic fatigue studies should be simple sequences in order to bring out the observations to be made as explicitly as possible. Two-step tests, interval tests and tests with periodic high loads may be most appropriate. With a more complex sequence the risk of mixing up a variety of favourable and unfavourable interaction effects is present. It may be impossible then to distinguish the various effects. It will be clear that for detailed observations the electron microscope and fractography are indispensable tools. It should also be said here that basic studies will not immediately solve the life estimating problems of the designer. However, a basic understanding is a prerequisite for arriving ultimately at qualitative improvements of the present situation.

7.3 Determination of fatigue data for making life estimates

As illustrated by fig.7.1, different types of tests could be adopted for the determination of basic data for life estimates. The merits and limitations of constant-amplitude tests, program tests and random tests have been discussed in section 6.3. Further it was indicated that data from flight-simulation tests could provide the most relevant information for this purpose, see also section 5.3.5.

From the discussion in chapter 4 it follows that the test results of program tests, random tests and flight-simulation tests will depend on some variables associated with the loads applied. The main variables are listed in table 7.1. The need for standardising is apparent and in fact Gassner has made proposals for the program test. Standardising is justified only if we know the effect of the variables

to be standardised on the test result. In this respect the program test, the random test and the flight-simulation test are all sensitive to the maximum load amplitude allowed in the test. With respect to the sequences, the effect is probably small for random loading and flight-simulation loading, whereas it may be significant for the program test. This aspect and the uncertain ratio between the results of program tests and random load tests have led to a preference for random loading instead of program loading (see also section 4.12).
For a narrow-band random load test, the distribution function of the load peaks is a Rayleigh distribution, while for broad-band random load the same distribution is approximately valid accept for the lower amplitudes.
A flight-simulation test can be carried out only on a fatigue machine with closed loop load control. This implies that any load spectrum can still be adopted. Standardizing a flight-simulation test at this stage appears to be somewhat premature, since the influence of several variables has still to be explored in greater detail. For this purpose the test program in section 5.3.5 was proposed. Nevertheless, some recommendations can be made already now, for instance with respect to sequence and truncation. This is discussed later in this chapter.

Finally, some unbalanced approaches with respect to determining fatigue data for life estimates may be mentioned here. It is not realistic to carry out flight-simulation tests on unnotched specimen. This is combining an advanced testing method with a primitive and unrepresentative specimen. Similarly, it is an unbalanced approach to apply a constant-amplitude test on a full-scale structure, which is the most simplified test on the most realistic simulation of the structure.

7.4 Comparative fatigue tests

Many people still feel that constant-amplitude tests are a good means for comparing alternative designs, production techniques, etc. However, the possibility of intersecting or of non-parallel S-N curves is making this very dubious. In figure 7.2 comparative tests at stress level S_{a1} would indicate design A to be superior to design B. At stress level S_{a3} the reverse would apply, whereas at S_{a2} both designs would be approximately equivalent. Some comments on this issue were made in section 5.3.4 when discussing the $K_{eff.}$ concept. Fretting corrosion is one aspect where constant-amplitude tests may give a misleading of its effect in service.

The numerous test series with program loading carried out by Gassner and his co-workers suggest that the risk of a misjudgement would be smaller if program loading were adopted for comparative testing. This will apply also to random loading. Nevertheless, if flight-simulation loading can be adopted it appears that it is the most preferable solution. Real problems should be tackled with realistic testing methods if possible. Recently, Ronay (Ref.189) adopted random flight-simulation loading for exploring the fatigue behaviour of a high-strength steel. Imig and Illg (Ref.80) adopted this test method for studying the effect of temperature on the endurance of notched titanium alloy specimens. Schütz and Lowak (Ref.190) studied the effect of plastic hole expansion on the fatigue life of an open hole 2024 alloy specimen by employing flight-simulation loading. At the NLR, as part of an ad-hoc problem, we compared two alternative types of joints with random flight-simulation loading. Some aircraft firms have already started comparative testing for design purposes employing a kind of flight-simulation loading.
As an illustration of different answers to the same question, a recent investigation (Ref.64) indicated that the crack propagation in 7075-T6 was four times faster than in 2024-T3 according to constant-amplitude loading. However, under flight-simulation loading the ratios were only 1 to 2 (see Fig.4.29).

7.5 Direct determination of fatigue life and crack propagation data by flight-simulation testing

In the previous chapter it was concluded that life estimates based on available data may have a low accuracy. If a better accuracy is required, a realistic test is necessary. This implies that both the specimen and the load sequence should be representative for service conditions. For the specimen this means that the test should be carried out on the actual component or a complete part of the structure.

With respect to the fatigue load, a flight-simulation test representative for service loading is required. An exact simulation of the load-time history in service would be the preferable solution. Usually a service record will not be available, but in case that it can be measured before the fatigue test it is the best starting point as advocated by Branger (Ref.35). For reasons of time and economy, periods during which the load does not vary could be left out.
In general, a load-time history will have to be designed on the basis of mission analysis and load statistics obtained with other aircraft. It is thought that it is possible to compose a representative load-time history (see the discussion in section 6.2). A good knowledge of the empirical trends is essential for this purpose. As an illustration, figure 7.3 shows a sample of a load record from the test on the F-28 wing. Different types of weather conditions were simulated in accordance with statistical information. The sequence of the gust loads in each flight was a random sequence without any sequence correlation. This may be a deviation from the random sequence in service, but fortunately the deviation will probably have a minor effect as discussed in section 4.13.

A major problem is the assessment of the highest load level to be applied in the flight-simulation test. As discussed before, this level may have a predominant effect on the life and the crack propagation. If the load level that will be reached (or exceeded) once in the target life of the aircraft is applied in a test, we know that it may have a favourable effect on the fatigue life. It then should be realized that this load level is subject to statistical variations, that means some aircraft will meet this load more than once in the target life, whereas other aircraft will never be subjected to it. In view of this aspect and the flattering effect of high loads, it was proposed elsewhere (Refs.58,76) that the load spectrum should be truncated at the load level exceeded ten times in the target life (see Fig.7.4 for illustration).
In reference 76 a similar recommendation was made for crack growth studies, but then, instead of the aircraft life, one has to consider the inspection period. The predominant influence of high loads on crack growth was illustrated by the results presented in sections 3.2 and 4.6. If a full-scale structure with cracks is tested to study the crack rate, high loads will considerably delay the crack growth. The application of high loads may again considerably flatter the test result. Therefore a truncation is necessary to avoid unsafe predictions for those aircraft of the fleet that will not meet the high loads.

Sometimes fail-safe loads are applied at regular intervals during a full-scale fatigue test to demonstrate that the aircraft is still capable of carrying the fail-safe load. If this load exceeds the highest load of the fatigue test, the result may be that a number of cracks that escaped detection so far, will never be found because of crack growth delay. In other words, this precedure could eliminate the possibility of obtaining the information for which the fatigue test is actually carried out. The crack growth delay in a full-scale structure was recently confirmed (Refs.63,64) in additional tests on the F-28 wing. The certification test was completed after simulating 150 000 flights. Then fail-safe loads (limit load) were applied. In a subsequent research program it turned out that several cracks did not grow any further as shown in figure 7.5. New artificial cracks, however, showed a normal growth.

The significance of low-amplitude cycles has been discussed in section 4.13. Leaving out these cycles from a flight-simulation test will considerably reduce the testing time. For the F-28 wing, omitting the gust cycles with the lowest amplitude reduced the testing time per flight from 115 seconds to 46 seconds. However, since such cycles may contribute to crack nucleation (fretting) and crack growth, the cycles were not omitted during the certification tests.
Taxiing load cycles can be omitted under certain conditions. In fact it appears admissable only if the cycles occur in compression for the components being tested (see section 4.13). Care should be taken that the ground-to-air cycle reaches the most extreme minimum load occurring on the ground, including dynamic loads (see Fig.6.5).

The development of hydraulic loading systems with closed-loop load control has considerably affected the present state of the art. By now it seems inadmissable to simplify the loading program in a flight-simulation test for experimental reasons to a sequence of the type shown in figure 4.31.

The result of a flight-simulation test will be a fatigue life in numbers of flights or a crack propagation rate in millimeters per flight. There are limitations to the meaning of these data, which are discussed in the section 7.7.

7.6 Full-scale fatigue tests

A full-scale test on a new aircraft design is an expensive test. Hence there should be good reasons to carry out such a test (Refs.191-194). In most general terms the test is carried out to avoid fatigue trouble in service. Reference may be made to table 7.2 giving a survey of several types of fatigue problems and possible consequences. In view of these consequences and the costs of the test there is every reason to require that the test gives realistic and relevant information. As said before, a full-scale fatigue test should be carried out with a carefully planned realistic representation of the service load-time history.

Comments on the application of high-amplitude and low-amplitude cycles were made in the previous section (see also Ref.76). It may be emphasised once again that the application of a high pre-load for static testing purposes (strain measurements for instance) or high fail-safe loads during the test, should be prohibited. Such loads may have a large flattering effect on fatigue lives and crack rates, and quantitative indications from the test may become worthless. Some more information from test series in full-scale structures bearing on this aspect were recapitulated in section 4.15.

Several aspects can be mentioned that make full-scale testing of a new aircraft structure desirable. It is thought that the most important ones are listed below (Ref.76).

(1) Indication of fatigue critical elements and design deficiencies.
(2) Determination of fatigue lives until visible cracking occurs.
(3) Study of crack propagation, inspection and repair methods.
(4) Measurements on residual strength.
(5) Economic aspects.

Items 4 and 5 are beyond the scope of the present discussion.

With respect to the first purpose mentioned above, Harpur and Troughton (Ref.191) observed that in several cases fatigue cracks occurring in service were not found in the full-scale test, because the structure tested was not sufficiently representative. This was not only due to manufacturing differences and modifications, but also to simplifying the test article. Due consideration should therefore be given to the structural completeness of the specimen.
Fatigue critical elements will only be indicated in the correct order if the fatigue lives obtained in the full-scale tests are correct indications of the service life. The test should not indicate component A to be more critical than component B if service experience indicate the reverse order. This risk can be avoided only by a realistic and representative test. In section 4.15, several examples of misleading information obtained in full-scale tests due to unrealistic fatigue loadings have been mentioned.
It will be clear that a representative fatigue loading also implies, that due consideration has to be given to simulate all types of fatigue loads that may be significant for certain parts of the structure.

The full-scale test is also a training experiment with respect to inspection techniques. This problem will not be discussed in this report. If a structure is a good design, inspecting for cracks during a full-scale test is a tough job because cracks will hardly occur.
In order to obtain information about crack propagation rates it is common practice to apply artificial cracks to the structure for initiating fatigue crack growth. Usually this is done by making saw-cuts. The information about crack growth is needed in order to establish safe inspection periods. As suggested in the previous section, the truncation level of the load spectrum should be lowered after application of the artificial cracks. This was in fact done during the certification tests on the F-28 wing.

The limitations of the information obtained in a full-scale test are discussed in the following section.

7.7 Limitations of flight-simulation tests

Assuming that the load-time history to be applied in a flight-simulation test was carefully planned, there are still some limitations to the information obtained in the test. Aspects to be briefly mentioned here are associated with loading rate, corrosive influences, scatter and deviating load spectra in service.

Loading rate

A full-scale test on a structure is an accelerated flight simulation that may last for 6 to 12 months, while representing 10 or more years of service experience. A flight-simulation test on a component in a modern fatigue testing machine may take no more than a week.
Considering loading rate effects, one should not simply compare testing time with flying time, but rather the times that the structure is exposed to the high loads. Orders of magnitude are given in figure 7.6. This argument is speculating on the fact that any effect of the loading rate is a matter of some time-dependent dislocation mechanisms occurring at high stresses. It might imply that the effect is relatively small for a full-scale test but it could have some effect in a component test in a fatigue machine running at a relatively high load frequency (see also the discussion in section 4.17).

Corrosive influences

Differences between testing time and service life also imply different times of exposure to corrosive attack. Therefore, if corrosion is important for crack nucleation (corrosion fatigue) one certainly should consider this aspect. In practice cracks frequently originate from bolt holes and rivet holes where the accessibility of the environment is usually poor and the corrosion influence probably not very significant. However, as soon as macro-cracks are present the environment will penetrate into the crack and the effect on crack growth should be considered. Safety factor should be applied to inspection periods depending on the material and the environment (see also the discussion in section 4.17).

Scatter

Testing a symmetric structure generally implies that at least two similar parts are being tested. However, fatigue properties may vary from aircraft to aircraft because the quality of production techniques and materials will not remain exactly constant from year to year. It will not be tried here to speculate on the magnitude of the scatter, although some interesting data are available in the literature. It is recognized, however, that the shortest fatigue lives in a large fleet of aircraft, in general, will be shorter than the result of the full-scale test as a consequence of scatter.

Deviating load spectrum

Load measurements in service may indicate that the service load history is significantly deviating from the load history applied in the test. Suggestions were heard in the past that the test result could be corrected for such deviations by calculation, employing the Palmgren-Miner rule and some S-N curves. However, as explained in section 5.4.2 this rule is highly inaccurate for this purpose.
If the structure has good fail-safe properties, the question of deviating load spectra in service is probably less important. This is certainly true if the impression is that the service load spectrum is less severe than the test spectrum. However, if one feels that the service loading could be more severe than the test loading, it appears that additional testing is indispensable for a safe-life component.

Comparison between test and service experience

After having summarised several limitations of a full-scale flight-simulation test, the proof actually is the comparison between service experience and test results. A few papers on this issue have been presented in the literature (Refs.120,191,196,197) and some comments will be made.
As far as data are available, the service life is usually shorter than the test life, although there are some cases where the agreement is reasonable. However, if the service life is from 2 to 4 times shorter than the test life, further clarification is obviously needed.
There are a number of reasons why discrepancies between test results and service experience may occur. Several of them have been listed above, for instance scatter and environmental effects. Secondly, a fair comparison requires that the test is a realistic simulation of the service load history and this is a

severe restriction on the comparisons that could be made in the past. Thirdly, if a test reveals a serious fatigue failure it is likely that the aircraft firm will modify the structure, thus eliminating the possibility of a comparison.

In summary: The accuracy of the quantitative results from a full-scale test is limited by the above aspects. It is difficult to quantify these aspects. Depending on the possible consequences associated with the fatigue indications obtained in the test, scatter factors or safety factors may be applied. The selection of these factors is again a matter of philosophy, as briefly commented on in section 6.5.

8. SURVEY OF PRESENT STUDY AND RECOMMENDATIONS

8.1 Survey of the present study

Several features of the fatigue phenomenon have been recapitulated in chapter 2. The fatigue process was described as a sequence of crack nucleation, micro-crack growth, macro-crack growth and final failure. Subsequently, it was tried in chapter 3 to describe fatigue damage and damage accumulation. In most general terms fatigue damage is a change of the material as caused by cyclic loading. Fatigue cracking, i.e. decohesion, is the most prominent feature of fatigue damage. However, the amount of cracking alone is insufficient to describe the damage. A definition of the fatigue damaged material should include a description of all aspects of the fatigue crack geometry and the condition of the material around the fatigue crack, including cyclic strain-hardening and residual stress distributions. A survey is given in fig.3.2. Damage accumulation in a certain load cycle therefore implies incremental changes of all these aspects. The incremental changes will depend on the intensity of the load cycle, but at the same time they will be a function of the damage already present. This has led to the definition of interaction effects, which in general terms means: the damage increment due to a certain load cycle will depend on the damage caused by the preceding load cycles. This also implies that the damage caused by a certain load cycle will affect the damage increments of subsequent load cycles. Interaction effects may be either favourable or unfavourable, which means that they may either decelerate or accelerate the damage accumulation. As a consequence, it is important for the damage accumulation in which sequence load cycles of various magnitudes will be applied. Such sequence effects have been observed in many test series.

Various examples of interaction effects and sequence effects are presented in chapter 4, which gives a survey of empirical trends observed in tests with a variable fatigue load. This includes the effects of high preloads, periodically applied high load cycles, ground-to-air cycles and the effects of several variables of program loading, random loading and flight-simulation loading. The investigations are summarised in tables 4.1 - 4.8, while various illustrative test results are shown in figures 4.1 - 4.30. The empirical trends can sometimes be explained qualitatively, fatigue cracking and residual stresses being the main arguments. Nevertheless, the trends clearly confirm that fatigue damage accumulation is a complex phenomenon which will not easily allow a satisfactorily quantitative treatment.

Some thought was given to the comparison between fatigue under program loading and random loading. In a random load test the load amplitude is varied from cycle to cycle. The load amplitude in a classic program test, however, is varied infrequently and in a systematic way, i.e. in a programmed sequence. Consequently sequence effects on the damage accumulation may be different, which implies that a strict correlation between the fatigue lives in the two types of tests may not be expected. Empirical evidence has substantiated this view. In fact, a realistic simulation of fatigue damage accumulation as it occurs in service, requires a test that preserves the essential features of the service load-time history. With the present knowledge it can be concluded that a flight-simulation test may satisfy this requirement, whereas a more simple test will not do so.

A survey of theories for life calculations has been given in chapter 5. An important question is whether the theories are capable of predicting the empirical trends as summarised in chapter 4. The theories were grouped in three categories, which are (1) the incremental damage theories, (2) theories based on a similarity approach and (3) interpolation procedures. Most theories are in the first group, see table 5.2. However, they poorly satisfy the picture about fatigue damage accumulation and it is not surprising that the prediction of empirical trends is also poor. Also the similarity approach, although being less dependent on knowledge about fatigue damage, does not yield accurate life prediction. The quality of predictions with the interpolation methods is apparently dependent on the quality of the data from which the interpolation is made. In view of the knowledge of interaction effects and sequence effects, interpolations should preferably be based on results of flight-simulation tests. Since such data are hardly available, a proposal is made for a systematic test program that could fill this gap.

In chapter 6 the consequences of the present state of the art for estimating fatigue lives and crack propagation rates in the aircraft design phase are analysed. First a survey is given of the various aspects of designing an aircraft from the point of view of fatigue. This indicates that the fatigue theory is not the only weak link in predicting fatigue properties. One aspect briefly discussed is the definition of load cycles if the load is varying in some random way as occurs in service. Attention is then paid to estimation methods based on available fatigue data and, as an alternative method employing service experience from previously designed aircraft. The advantages and limitations of both approaches are emphasized. Reliable and accurate information of fatigue properties in many cases can be obtained only by carrying out relevant tests.

Several testing methods are discussed in chapter 7. Different testing purposes are listed first, which are: (1) basic fatigue damage studies, (2) test series exploring the effects of various factors on fatigue life and (3) estimation problems with respect to fatigue life and crack propagation as a design effort. Some comments are made on the first topic, but major emphasis is on the last one. Four types of tests are considered, which are constant-amplitude loading, program loading, random loading and flight-simulation loading. Specific goals are: (1) compiling basic data for life estimates, (2) comparative design studies and (3) determination of direct estimates of life and crack propagation. The conclusion is that the flight-simulation tests should be preferred to the other types of tests. It is emphasized that comparative tests on different designs may give unreliable information if constant-amplitude tests are used. Actually, if realistic answers are required realistic testing procedures have to be adopted. This appears to be a trivial conclusion. Nevertheless, it is well substantiated by present day knowledge of fatigue damage accumulation and by empirical evidence from a vast amount of variable-amplitude test series.

For a full-scale fatigue test, a realistic flight-simulation loading is a necessity. If a simplified fatigue loading is adopted, the test may give incorrect indications of fatigue critical components and misleading information about fatigue lives and crack propagation rates. Relevant evidence from test series on full-scale structures was summarized in chapter 4 (section 4.15). It is also emphasized that the application of fail-safe loads during a full-scale test may fully obliterate the relevance of the test results. By introducing residual stresses, such a high load may considerably increase the fatigue life and it may completely stop the growth of fatigue cracks. Comments are also made on the significance of truncating the load spectrum, omitting low-amplitude cycles and other aspects of flight-simulation testing.

In 1965, Herbert Hardrath (Ref.198) presented a review on cumulative fatigue damage. He then came to the conclusion that new break-throughs of our knowledge should not be expected in the near future. In fact this has been true for the past six years. Hardrath's review is still relevant to day but some aspects have become more clear since then. These aspects are listed below in view of making recommendations for future research.

1 Electro-hydraulic cylinders and fatigue machines with closed-loop load control are now being used in many laboratories. Actually this is some sort of a break-through with respect to the possibilities of performing fatigue tests with any required load-time history. Advantages already exploited are related to an increasing knowledge about fatigue damage accumulation and to more realistic testing methods for practical problems.

2 Our phenomenological knowledge about fatigue damage accumulations is steadily increasing. The phenomenon appears to be more complex than thought before, but this trend is not uncommon in science.

3 Sequence effects and interaction effects are better recognized than before.

4 Progress has been made with respect to predicting the strain and stress history at the root of a notch. Calculations to be made from cycle to cycle are no longer objectionable in view of computer capabilities.

8.2 Recommendations for future work

Basic research and empirical research are still required both. Basic research cannot be neglected because the evaluation of trends, as observed in empirical studies, requires a physical understanding of the phenomenon occurring in the material. Empirical investigations on the other hand are necessary because we simply cannot wait until our physical understanding is good enough to answer a number of practical questions. Apparently we are learning slowly and extensive efforts have to be made to improve our knowledge. Therefore, it has to be emphasized that all laboratories should be fully aware of the various aspects of the practical problem. We should avoid to look for solutions of problems that do not exist by outlining what the real problems are. It is hoped that the present report will prove to be helpful in this respect. Some more specific recommendations will now be made.

Basic research

1. Microscopic studies

The phenomenological picture of fatigue damage accumulations is still highly qualitative in nature. Microscopic studies providing quantitative data about the various aspects of the damaging process should therefore be welcome. Studies with both the optical and the electron microscope can be useful. There is still ample room for fractographic observations of crack growth under variable-amplitude loading. At the same time thin-foil studies of fatigue damaged material appear to be worthwhile. It then should be kept in mind that fatigue is a highly localized occurrence, that means that the local state of the material may differ from that of the bulk material.

2. Stress-strain histories at the notch

In this report reference was made to investigations on the prediction of stress-strain histories at the root of a notch under variable-amplitude loading. Such studies included speculative assumptions about incremental damage accumulation. Nevertheless the stress-strain histories at a notch root may be considered in its own right. As such it is a more detailed description of the fatigue environment for the material at the critical location. Without this information it is difficult to see how a rational cumulative fatigue damage theory for notched elements could be formulated. Already for this reason alone this type of investigations should be recommended.

3. Crack closure

Crack closure as described by Elber, has recently entered our picture of fatigue crack growth. More quantitative information about this phenomenon under various conditions should be welcomed.

4. Plastic stress-strain distributions

Theoretical calculations of stress-strain distributions around notches and cracks, including plasticity, is a difficult problem. It is even more difficult for cyclic loading since the cyclic stress-strain behaviour of the material can usually not be described in a simple way. It should be recommended to exploit the potential usefulness of finite element methods to this problem.

5. Environmental effects

Laboratory results to be extrapolated to service conditions are still afflicted by the possibility of unknown environmental effects. Investigations to improve our physical knowledge about the mechanisms of environmental effects are to be recommended.

Empirical investigations

6. Flight-simulation testing

For many practical problems, flight-simulation testing was recommended in this report. Our knowledge of the effects of several variables pertaining to flight-simulation testing, is still insufficient. For that reason investigations on notched elements exploring these effects should be advised. Such investigations may be somewhat similar to test series carried out by NLR on fatigue crack propagation (section 4.13).

7. Compilation of flight-simulation test data

In this report the compilation of flight-simulation test data was advocated. A test program for this purpose was described in section 5.3.5. A handbook with this type of data may be useful for estimating fatigue properties. Moreover, it could be a reference for comparative testing in order to judge the fatigue quality of a new design.

8. Service experience

In chapter 6 it was emphasized that experience on fatigue in service gives most useful information. Such data are obtained under highly realistic conditions with respect to load-time histories and environment. The statistical reliability may be high if many aircraft are involved. It would be extremely useful if such data could be collected and analysed, and be made generally available.

9. Statistical analysis of service load-time histories

The prediction of fatigue properties in the design phase, the performance of realistic flight-simulation tests, and the monitoring of fatigue life in service all require information about service load-time histories. Investigations on the question how relevant information can be obtained should be recommended. Problems involved are partly associated with measurement techniques, while another aspect is the statistical analysis in relation to fatigue damage accumulation.

10. Fatigue machines

A flight-simulation test requires a fatigue machine with closed-loop load control. Electro-hydraulic machines of this type are now available. It would certainly stimulate more realistic testing procedures if these machines could be produced at a lower price. Secondly, the generation of electrical signals for controlling the load in such a machine is also a topic where a development of new and cheaper apparatus is desirable.

A final recommendation is related to the dissemination of information. Several times it was noticed that good solutions were not reached because of insufficient knowledge about the real problems, although the information was available elsewhere. Equally regretful is the situation where prejudice prevents improved solutions. In this respect, flight-simulation testing is sometimes labelled as a "sophisticated" type of testing. Actually a flight-simulation test is a rather trivial solution because it is aiming at a simulation of service loading. A constant-amplitude test on the other hand, being a convenient type of test from an experimental point of view, is in fact a highly artificial simulation of service loading. We should be careful that progress is not hampered by historical traditions starting at the time of August Wöhler. The problem is partly a matter of education and dissemination of information. It is hoped that the present report may be helpful also in this respect by outlining the various aspects of the aircraft fatigue problem and by the analysis of and cross references to the literature.

9 REFERENCES

1.	Barrois, W.G.	Manual on fatigue of structures. Fundamental and physical aspects. AGARD-MAN.8-70, 1970.
2.	Forsyth, P.J.E.	The physical basis of metal fatigue. Blackie and Son, London 1969.
3.	Grosskreutz, J.C.	Fatigue mechanisms in the sub-creep range. Am.Soc.Testing Mats, STP (to be published).
4.	Jacoby, G.H.	Application of microfractography to the study of crack propagation under fatigue stresses. AGARD Report 541, 1966.
5.	Plumbridge, W.J. Ryder, D.A.	The metallography of fatigue. Metallurgical Reviews, Vol.14, Review 136, 1969, pp 119-142.
6.	Schijve	Analysis of the fatigue phenomenon in aluminium alloys. NLR TR M.2122, Amsterdam 1964.
7.	Schijve, J. Jacobs, F.A.	Fatigue crack propagation in unnotched and notched aluminium alloy specimens. NLR TR M.2128, Amsterdam 1964.
8.	Broek, D. Van der Vet, W.J.	Systematic electron fractography of fatigue in aluminium alloys. NLR TR 68002, Amsterdam, 1968.
9.	Pelloux, R.M.N.	Mechanisms of formation of ductile striations. ASM Trans.Quarterly, Vol.62, p.281, 1969.
10.	Bowles, C.Q. Broek, D.	On the formation of fatigue striations. To be published in Int.J.of Fract.Mech. Also NLR MP 69014, Amsterdam 1969.
11.	Wood, W.A. Cousland, S. Sargant, K.R.	Systematic microstructural changes peculiar to fatigue deformation. Acta Met., Vol.11, 1963, p.643.
12.	Grosskreutz, J.C. Shaw, G.G.	Critical mechanisms in the development of fatigue cracks in 2024-T4 aluminum, Fracture 1969, Ed.P.L.Pratt, p.620, Chapman and Hall, London 1969.
13.	Bowles, C.Q. Schijve, J.	The role of inclusions in fatigue crack initiation in an aluminum alloy. NLR MP 70019, Amsterdam 1970.
14.	Grosskreutz, J.C. Shaw, G.G.	Microstructures at the tips of growing fatigue cracks in aluminum alloys. Fatigue crack propagation, Am.Soc.Test.Mats, STP 415, p.226, 1967.
15.	Hartman, A.	On the effect of oxygen and water vapor on the propagation of fatigue cracks in 2024-T3 Alclad sheet. Int.J.of Fract.Mech., Vol.1, 1965, p.157.
16.	Bradshaw, F.J. Wheeler, C.	The influence of gaseous environment and fatigue frequency on the growth of fatigue cracks in some aluminum alloys. Int.J.of Fract.Mech., Vol.5, 1969, p.255.
17.	Illg, W. McEvily, A.J.	The rate of fatigue crack propagation for two aluminum alloys under completely reversed loading. NASA TN D-52, 1959.
18.	Elber, W.	Fatigue crack closure under cyclic tension. Fracture Mechanics, Vol.2, 1970, p.37.
19.	Elber, W.	The significance of fatigue crack closure. Paper 1970 Annual Meeting ASTM, Toronto, June 1970.
20.	Schijve, J. Groep, F.F.	Some tests on Elber's crack closure. NLR report, Amsterdam, to be issued shortly.
21.	Von Euw, E.F.J.	Effect of overload cycles on subsequent fatigue crack propagation in 2024-T3 aluminum alloy. Doctor Thesis, Lehigh University, March 1971.
22.	Schijve, J.	Fatigue damage interaction due to incompatibility between crack front orientation and stress amplitude. NLR report, Amsterdam, to be issued shortly.
23.	Valluri, S.R.	A unified engineering theory of high stress level fatigue. Aerospace Eng., Vol.20, 1961, p.18.
24.	Broek, D.	The residual strength of high alloy sheets containing fatigue cracks. Aerospace Proceedings 1966, p.811. McMillan 1967.
25.	McMillan, J.C. Pelloux, R.M.N.	Fatigue crack propagation under random and program loads. Fatigue Crack Propagation. Am.Soc.Test.Mats, STP 415, 1967, p.505.
26.	McMillan, J.C. Hertzberg, R.W.	The application of electron fractography to fatigue studies. Electron Fractography, Am.Soc.Test.Mats, STP 436, 1968, p.89.
27.	Schijve, J.	Fatigue crack propagation in light alloy sheet material and structures. Advances in Aeronautical Sciences, Vol.3, Pergamon Press, 1961, p.387. Also NLR MP 195, Amsterdam 1960.
28.	Hudson, C.M. Hardrath, H.F.	Effects of changing stress amplitude on the rate of fatigue crack propagation in two aluminum alloys. NASA TN D-960, 1961.
29.	Gassner, E.	Festigkeitsversuche mit wiederholter Beanspruchung im Flugzeugbau. Luftwissen, Vol.6, p.61, 1939. Translation: NACA TM 1087.
30.	Hardrath, H.F. Naumann, E.C.	Variable-amplitude fatigue tests of aluminum alloy specimens. Am.Soc.Test.Mats, STP 274, 1960, p.125.
31.	Naumann, E.C. Hardrath, H.F. Guthrie, E.C.	Axial-load fatigue tests of 2024-T3 and 7075-T6 aluminum alloy sheet specimens under constant- and variable-amplitude loads. NASA TN D-212, 1959.
32.	Bendat, J.S.	Principles and applications of random noise theory, John Wiley and Sons, New York, N.Y. 1958.
33.	Swanson, S.R.	Random load fatigue testing: A state of the art survey. Materials Research and Standards, Vol.8, No.4, p.11, April 1968.
34.	Kowalewski, J.	Beschreibung regelloser Vorgänge. Fortschritt Berichte, VDI-Z, Reihe 5, Nr.7, 1969.
35.	Branger, J.	Second seminar on fatigue design. Columbia Un., Institute for the study of Fatigue and Reliability, TR No.5, 1964.
36.	Macro, S.M. Starkey, W.L.	A concept of fatigue damage. Trans.ASME, Vol.76, 1954, p.627.
37.	Schijve, J. Jacobs, F.A.	Fatigue tests on notched and unnotched Clad 24 S-T sheet specimens to verify the cumulative damage hypothesis. NLR Report M.1982, Amsterdam 1955.
38.	Schijve, J. Jacobs, F.A.	Research on cumulative damage in fatigue of riveted aluminum alloy joints. Nat.Aeronautical Research Inst., Amsterdam, Report M.1999, January 1956.

39. Schijve, J. Jacobs, F.A. — Program fatigue tests on notched light alloy specimens of 2024 and 7075 material. Nat.Aeronautical Research Inst., Amsterdam, Technical Report M.2070, 1960.

40. Forrest, G. — Some experiments on the effects of residual stresses on the fatigue of aluminium alloys. J.Inst.Metals,Vol.72, 1946, pp 1-17.

41. Heyer, K. — Mehrstufenversuche an Konstruktionselementen. Lilienthal-Gesellschaft für Luftfahrtforschung, Bericht 152, 1943.

42. Heywood, R.B. — The effect of high loads on fatigue. Colloquium on Fatigue. Ed. by W. Weibull and F.K.G. Odqvist. Springer Verlag 1956, p.92.

43. Heywood, R.B. — The influence of preloading on the fatigue life of aircraft components and structures. RAE Report Structures 182, June 1955.

44. Payne, A.O. — Determination of the fatigue resistance of aircraft wings by full-scale testing. Fatigue Testing of Aircraft Structures, ICAF Symp., Amsterdam 1959, ed. by F.J.Plantema and J.Schijve. Pergamon 1961, p.76.

45. Smith, C.R. — Fatigue-service life prediction based on tests at constant stress levels. Proc. SESA, Vol.16, No.1, 1958, p.9.

46. Boissonat, J. — Experimental research on the effects of a static preloading on the fatigue life of a structural components. Fatigue of Aircraft Structures, ICAF Symp., Paris 1961, ed. by W.Barrois and E.L.Ripley, Pergamon, 1963, p.97.

47. Mordfin, L. Halsey, N. — Programmed maneuver-spectrum fatigue tests of aircraft beam specimens. Symp. Fatigue Tests of Aircraft Structures, Am.Soc.Test.Mats, STP 338, 1963, p.251.

48. Rosenfeld, M.S. — Aircraft structural fatigue research in the Navy. Symp. Fatigue Tests of Aircraft Structures, Am.Soc.Test.Mats, STP 338, 1963, p.216.

49. Imig, L.A. — Effect of initial loads and of moderately elevated temperature on the room-temperature fatigue life of Ti-8Al-1Mo-1V titanium alloy sheet. NASA TN D-4061, 1967.

50. Kirkby, W.T. Edwards, P.R. — Variable-amplitude loading approach to material evaluation and component testing and its application to the design procedure. Proc.ICAF Symp., Stockholm 1969, ed. G.Wallgren and S.Eggwertz, FFA. Also RAE TR 69182.

51. Morrow, J.D. Wetzel, R.M. Topper, T.H. — Laboratory simulation of structural fatigue behavior. Symp.Effects of Environment and Complex Load History on Fatigue Life. Am.Soc.Test.Mats, 1970, STP 462, pp 74-91.

52. Crews, J.H. Hardrath, H.F. — A study of cyclic plastic stresses at a notch root. J.Soc.Ex.Stress Analysis, Vol.6, 1966, p-313.

53. Crews, J.H. — Elastoplastic stress-strain behavior at notch roots in sheet specimens under constant-amplitude loading. NASA TN D-5253, 1969.

54. Schütz, D. — Die Betrachtung der elastischen-plastischen Beanspruchung in Kerbgrund als Ausweg aus einer phänomenologisch betriebenen Schwingfestigkeitsforschung. LBF-Colloquium May 1968, LBF Bericht TB-80, Darmstadt 1968.

55. Heibach, E. Schütz, D. Svenson, O. — Zur Frage des Festigkeitsverhaltens regellos im Zug-Schwell-Bereich beanspruchter gekerbter Leichtmetallstäbe bei periodisch eingestreuten Druckbelastungen. LBF Bericht FB-78, Darmstadt 1969.

56. Edwards, P.R. — An experimental study of the stress histories at stress concentrations in aluminium alloy specimens under variable-amplitude loading sequences. RAE TR 70 004, 1970.

57. Gassner, E. — Ueber bisherige Ergebnisse aus Festigkeitsversuchen im Sinne der Betriebsstatistik. Lilienthal Gesellschaft für Luftfahrtforschung, Bericht 106, I.Teil, 1939. Translation NACA TM 1266, 1950.

58. Schijve, J. — The endurance under program-fatigue testing. Full-scale Fatigue Testing of Aircraft Structures, 1st ICAF Symposium, Amsterdam 1959, ed. by F.J.Plantema and J.Schijve, Pergamon 1961, p.41. Also NLR MP 178, Amsterdam 1959.

59. Schijve, J. Broek, D. De Rijk, P. — Fatigue-crack propagation under variable-amplitude loading. NLR Report M.2094, Amsterdam 1961.

60. Hudson, C.M. Hardrath, H.F. — Investigation of the effects of variable-amplitude loadings on fatigue crack propagation patterns. NASA TN D-1803, August 1963.

61. Smith, S.H. — Random-loading fatigue crack growth behavior of some aluminum and titanium alloys. Symp.Structural Fatigue in Aircraft, Am.Soc.Test.Mats, STP 404, 1966, p.76.

62. Hudson, C.M. Raju, K.N. — Investigation of fatigue-crack growth under single variable-amplitude loading, NASA TN D-5702, 1970.

63. Schijve, J. De Rijk, P. — Crack propagation in a full-scale wing structure under random flight-simulation loading. NLR TR 71043, Amsterdam 1971.

64. Schijve, J. — Fatigue tests with random flight-simulation loading. Paper 6th ICAF Symp., Miami, May 1971. Also NLR MP 71008, Amsterdam 1971.

65. Gassner, E. Horstmann, K.F. — Einfluss der Start-Lande Lastwechsels auf die Lebensdauer der böenbeanspruchten Flügel von Verkehrsflugzeugen. Advances in Aero.Sc., Vol.4, Pergamon 1961, p.763.

66. Gassner, E. Jacoby, G. — Betriebsfestigkeitsversuche zur Ermittlung zulässiger Entwurfsspannungen für die Flügelunterseite eines Transportflugzeuges. Luftfahrttechnik-Raumfahrttechnik, Vol.9, Jan.1964, p.6.

67. Naumann, E.C. — Evaluation of the influence of load randomization and of ground-to-air cycles on fatigue life. NASA TN D-1584, 1964.

68. Melcon, M.A. McCulloch, A.J. — Simulation of random aircraft service loadings in fatigue tests. ICAF Symp. 1963: Current Aeronautical Fatigue Problems, ed. by J.Schijve, J.R.Heath-Smith and E.R.Welbourne, Pergamon 1965, p.347.

69. McCulloch, A.J. Melcon, M.A. Crichlow, W.J. Foster, H.W. and Rebman, J. — Investigation of the representation of aircraft service loadings in fatigue tests. ASD TR 61-435, Sep.1961.

70. Barrois, W. — Physical interpretation of metal fatigue. Minutes 5th ICAF Conference, Brussels 1957.

71. Winkworth, W.J. — Fatigue behavior under service and ground test condition. RAE TN Structures 306, 1961.

72. Schijve, J., De Rijk, P. — The effect of ground-to-air cycles on the fatigue crack propagation in 2024-T3 Alclad sheet material. NLR TR M.2173, Amsterdam 1966.

73. Gassner, E., Jacoby, G. — Experimentelle und Rechnerische Lebensdauerbeurteilung von Bauteilen mit Start-Lande-Lastwechsel. Luftfahrttechnik-Raumfahrttechnik, Vol.11, 1965, p.138.

74. Mann, J.Y., Patching, C.A. — Fatigue tests on Mustang wings and notched aluminium alloy specimens under random gust loading with and without ground-to-air cycles of loading. Note ARL/SM.268, Melbourne 1961.

75. Finney, J.M., Mann, J.Y. — Fatigue behavior of notched aluminium alloy specimens under simulated random gust loading with and without ground-to-air cycles of loading. ICAF Symp.1961: Fatigue of Aircraft Structures, ed. by W.Barrois and E.Ripley, Pergamon 1963, p.151.

76. Schijve, J., Broek, D., De Rijk, P., Nederveen, A., Sevenhuijsen, P.J. — Fatigue tests with random and programmed load sequences with and without ground-to-air cycles. A comparative study on full-scale wing center sections. NLR Report S.613, Amsterdam 1965. Also AFFDL-TR-66-143, Oct. 1966.

77. Schijve, J., Jacobs, F.A., Tromp, P.J. — Crack propagation in aluminium alloy sheet materials under flight-simulation loading. NLR TR 68117, Amsterdam 1968.

78. Schijve, J. — Cumulative damage problems in aircraft structures and materials. The 2nd F.J.Plantema Memorial Lecture, ICAF Conference, Stockholm 1969. The Aeronautical Journal, Vol.74, 1970, p.517. Also NLR MP 69005, Amsterdam 1969.

79. Jacoby, G. — Comparison for fatigue life estimation processes for irregularly varying loads. Proc. 3rd Conf. on Dimensioning and Strength Calculations, Hungarean Ac.of Sc., Budapest 1968, p.81.

80. Imig, L.A., Illg, W. — Fatigue of notched Ti-8Al-1Mo-1V titanium alloy at room temperature and 550°F (560°K) with flight-by-flight loading representative of a supersonic transport. NASA TN D-5294, 1969.

81. Wallgren, G. — Review of some Swedish investigations on fatigue during the period June 1959 to April 1961. FFA-TN-HE-879, Stockholm 1961.

82. Gassner, E. — Auswirkung betriebsähnlicher Belastungsfolgen auf die Festigkeit von Flugzeugbauteilen. Doctor Thesis, Darmstadt 1941. Also Bericht der DVL, Cf 407/5, 1941.

83. Wallgren, G. — Fatigue tests with stress cycles of varying amplitude. FFA Report No.28, Stockholm 1949.

84. Wallgren, G., Petrelius, S. — Program utmattningsprov med bultförband. FFA Report HU-429:2, Stockholm 1954.

85. Fisher, W.A.P. — Some fatigue tests on notched specimens with programme loading for a "ground-attack" aircraft. RAE TN Structures 235, 1958.

86. Fisher, W.A.P. — Programme fatigue tests on notched bars to a gust load spe um. RAE TN Structures 236, 1958.

87. Naumann, E.C., Schott, R.L. — Axial-load fatigue tests using loading schedules based on maneuver-load statistics. NASA TN D-1253, 1962.

88. Naumann, E.C. — Variable-amplitude fatigue tests with particular attention to the effects of high and low loads. NASA TN D-1522, 1962.

89. Yeomans, H. — Programme loading fatigue tests on a bolted joint. RAE TN Structures 327, 1963.

90. Corbin, P.L., Naumann, E.C. — Influence of programming techniques and varying limit load factors on maneuver load fatigue test results. NASA TN D-3149, 1966.

91. Parish, H.E. — Fatigue test results and analysis of 11 piston Provost wings to determine the effect of programmed load. Min.of Tech., U.K., S and T Memo 5/67, 1967.

92. Parish, H.E. — Fatigue test results and analysis of four piston Provost wings tested in an ascending-descending order of loading. Min. of Aviation, S and M Memo 1/68, 1958.

93. Dunsby, J.A. — Fatigue tests on notched specimens of 2024-T351 aluminum alloy under a low altitude load spectrum. Nat.Res. Council Canada, NAE LR-504, 1968.

94. Lipp, W. — Unterschiede in der Lebensdauerangabe nach Betriebsfestigkeitsversuchen gegenüber den Ergebnissen aus Fahrversuchen. LBF Bericht Nr. TB-80, Darmstadt 1968, p.67.

95. Gassner, E. — Betriebsfestigkeit gekerbter Stahl- und Aluminiumstäbe unter betriebsähnlichen und betriebsgleichen Belastungsfolgen. Materialprüfung, Vol.11, 1969, p.373. Also ICAF Symp., Stockholm 1969.

96. Schijve, J. — Load sequences for fatigue testing of components and full-scale aircraft structures. ICAS Congress Rome 1970, Paper No. 70-32. Also NLR MP 70012, Amsterdam 1970.

97. Schijve, J. — The effect of load sequence on fatigue crack propagation under random loading and program loading. NLR TR 71014, Amsterdam 1971.

98. Breyan, W. — Effects of block size, stress level, and loading sequence on fatigue characteristics of aluminum-alloy box beams. Effects of Environment and Complex Load History on Fatigue Life, ASTM STP 462, 1970, p.127.

99. Kirkby, W.T. — Constant-amplitude or variable-amplitude tests as a basis for design studies. ICAF Symp., Munich 1965. Fatigue Design Procedures, ed. by E.Gassner and W.Schütz, Pergamon 1969, pp 253-290.

100. Gassner, E. — On the influence of fretting corrosion on the fatigue life of notched specimens of an AlCuMg2 alloy. Fatigue of Aircraft Structures, ICAF Symp., Paris 1961, ed. by W.Barrois and E.L. Ripley, Pergamon 1963, p.87.

101. Hillberry, B.M. — Fatigue life of 2024-T3 aluminum alloy under narrow- and broad-band random loading. Effects of Environment and Complex Load History on Fatigue Life. ASTM STP 462, 1970, p.167.

102. Kowalewski, J. — On the relation between fatigue lives under random loading and under corresponding program loading. Full-Scale Fatigue Testing of Aircraft Structures, 1st ICAF Symp., Amsterdam 1959, ed.by F.J.Plantema and J.Schijve, Pergamon 1961, p.60.

103. Fuller, J.R. Research on techniques of establishing random type fatigue curves for broad band sonic loading. SAE Paper 671 C, April 1963 (see also ASD-TDR-62-501, Oct. 1962).
104. Naumann, E.C. Fatigue under random and programmed loads. NASA TN D-2629, Febr. 1965.
105. Clevenson, S.A., Steiner, R. Fatigue life under random loading for several power spectral shapes. NASA TR R-266, 1967.
106. Schijve, J., De Rijk, P. The crack propagation in two aluminum alloys in an indoor and an outdoor environment under random and programmed load sequences. NLR TR M.2156, Amsterdam 1965.
107. Jacoby, G. Comparison of fatigue lives under conventional program loading and digital random loading. Effects of environment and complex load history on fatigue life. ASTM STP 462, 1970, p.184.
108. Jacoby, G. Vergleich der Lebensdauer aus Betriebsfestigkeits-, Einzelflug- und digital programmierten Random-Versuchen sowie nach der linearen Schadenakkumulationshypothese. Fortschritt Bericht VDI-Z, Reihe 5, Nr. 7, 1969, p.63.
109. Jacoby, G. Beitrag zum Vergleich der Aussagefähigkeit von Programm- und Random-Versuchen. 1970 Zeitschrift für Flugwissenschaften, Vol.18, 1970, p.253.
110. Branger, J. A review of Swiss investigations on aeronautical fatigue during the period June 1965 to April 1967. Eidgenössisches Flugzeugwerk, Emmen, F+W, 1967, S.196.
111. Branger, J. The influence of modifications of a fatigue history loading program. Paper 6th ICAF Symposium, Miami, May 1971.
112. Branger, J., Ronay, M. Investigation of high strength steels under history program fatigue. Columbia Un., Institute for the Study of Fatigue and Reliability, TR No.56, 1968.
113. Schijve, J., Jacobs, F.A., Tromp, P.J. Crack propagation in 2024-T3 Alclad under flight-simulation loading. Effect of truncating high gust loads. NLR TR 69050, 1969.
114. Schütz, D. Einzelflugversuche und Einstufenversuche an Augenstäben aus AlZnMgCuHg (3.4354.7) zur Ueberprüfung der linearen Schadenakkumulation. LBF TM 53/70, Darmstadt 1970.
115. Schütz, W. Ueber eine Beziehung zwischen der Lebensdauer bei konstanter und bei veränderlicher Beanspruchungsamplitude und ihre Anwendbarkeit auf die Bemessung von Flugzeugbauteilen. Doctor Thesis Technische Hochschule München, October 1965. A shorter version is published in Z.für Flugwissenschaften, Vol.15, 1967, p.407.
116. Gassner, E. Effect of variable load and cumulative damage on fatigue in vehicle and airplane structures. Int.Conf.Fatigue of Metals, The Inst.of Mech.Eng., London 1956, p.304.
117. Ostermann, H. Einfluss der Kollektivform auf die Schwingfestigkeit der Aluminiumlegierung 3.1354.5 (AlCuMg2). LBF TM 36/68, Darmstadt 1968.
118. Jost, G.S. The fatigue of 24 S-T aluminium alloy wings under asymmetric spectrum loading. Report ARL/SM.295, Melbourne, Febr. 1964.
119. Huston, W.B. Comparison of constant-level and randomized step tests of full-scale structures as indicators of fatigue-critical components. Symp. Full-scale Fatigue Testing of Aircraft Structures, Amsterdam 1959. Pergamon 1961, p.193.
120. Branger, J. Life estimation and prediction of fighter aircraft. Int.Conf.Structural Safety and Reliability, April 1969, Smithsonian Institution, Washington D.C. Also Eidgenössisches Flugzeugwerk, Emmen, F+W, S.207.
121. Branger, J. Private communication.
122. Locati, L. Essais de fatigue par flexion avec fréquences superposées. Colloquium on Fatigue, Stockholm 1955, ed. by W.Weibull and F.K.G.Odquist, Springer Verlag, 1956. p.160.
123. Starkey, W.L., Macro, S.M. Effects of complex stress-time cycles on the fatigue properties of metals. Trans.ASME, Vol.79, 1957, p.1329.
124. Nishihara, T., Yamada, T. Fatigue life of metals under varying repeated stresses. Proc.6th Japan Nat.Congres for Appl. Mech., 1956, p.61.
125. Gassner, E., Svenson, O. Einfluss von Störschwingungen auf die Ermüdungsfestigkeit. Stahl und Eisen, Vol.82, 1962, p.276.
126. Jacoby, G. Observation of crack propagation on the fracture surface. ICAF Symp. Current Aero. Fatigue Problems, Rome 1963, ed. by J.Schijve, J.R.Heath Smith and E.R.Welbourne, Pergamon 1965, p.155.
127. Yamada, T., Kitagawa, S. Fatigue strength of aluminium alloy under actual service loads. Proc.12th Japan Congress on Mats Res., 1969, p.56.
128. Nowack, H. Biharmonische Beanspruchungsabläufe zur Beurteilung der Schädigung unregelmässiger Beanspruchungen, VDI-Z, Fortschritt Berichte, Reihe 5, No.7, 1969, p.91.
129. Dowling, N.E. Fatigue failure predictions for complicated stress-strain histories. Un.of Illinois, T. and A.M.Report 337, 1971.
130. Palmgren, A. Die Lebensdauer von Kugellager. Z.V.D.I., Vol.68, 1924, pp 339-341.
132. Miner, M.A. Cumulative damage in fatigue. J.Appl.Mech., Vol.12, 1945, pp A159-A164.
133. Langer, B.F. Fatigue failure from stress cycles of varying amplitude. J.Appl.Mech., Vol.4, 1937, pp A160-A162.
134. Koch, J.J., Biezeno, C.B. Over een verkorte methode ter bepaling van den kerffactor β bij wisselende buigen wringbelasting. De Ingenieur, 1940, No.46.
135. Grover, H.J. An observation concerning the cycle ratio in cumulative damage. Symp. on Fatigue of Aircraft Structures, Am.Soc.Test.Mats, STP 274, 1960, pp 120-124.
136. Manson, S.A., Freche, J.C., Ensign, C.R. Application of a double linear damage rule to cumulative fatigue. Fatigue Crack Propagation, ASTM STP 415, Am.Soc.Testi.g Mats, 1967, p.384.
137. Bland, R.B., Putnam, A.A. Cumulative damage in fatigue. Discussion on Ref.132. J.Appl.Mech., Vol.13, 1946, pp A169-A171.
138. Nishihara, T., Yamada, T. The fatigue strength of metallic materials under alternating stresses of varying amplitude. The Japan ScienceReview, Vol.1, No.3, 1950, pp 1-6.
139. Shanley, F.R. A proposed mechanism of fatigue failure. Colloquium on Fatigue, ed.by W.Weibull and F.K.G.Odquist. Springer Verlag, 1956, pp 251-259.
140. Schijve, J. Critical analysis of the fatigue damage concept and some consequencies for fatigue testing of aircraft structures. Minutes 4th Conf. of the Int.Comm.on Aero.Fatigue, Zürich 1956, Appendix 2.

141. Kaechele, L. Review and analysis of cumulative-fatigue-damage theories. The Rand Corp. Memo RM-3650-PR, 1963.

142. Crichlow, W.J., McCulloch, A.J., Young, L., Melcon, M.A. An engineering evaluation of methods for the prediction of fatigue life in air-frame structures. ASD-TR-61-434, Wright Patterson AFB, 1962.

143. Edwards, P.R. Cumulative damage in fatigue with particular reference to the effects of residual stresses. RAE TR 69 237, 1969.

144. O'Neill, M.J. A review of some cumulative damage theories. Aero Res.Labs, Melbourne, Report ARL/SM 326, 1970.

145. Freudenthal, A.M., Heller, R.A. Accumulation of fatigue damage. Proc.Int.Conference on Fatigue on Aircraft Structures, Academic Press, 1956, p.146.

146. Freudenthal, A.M., Heller, R.A. On stress interaction in fatigue and a cumulative damage rule. J.Aerospace Sc., Vol.26, 1959, pp 431-442.

147. Marsh, K.J. Direct-stress cumulative fatigue damage tests on mild-steel and aluminium alloy specimens. Nat.Engineering Lab. Glasgow, NEL Report No.204, 1965.

148. Haibach, E. Modifizierte lineare Schadensakkumulations-Hypothese zur Berücksichtigung des Dauerfestigkeitsabfalls mit fortschreitender Schädigung. Laboratorium für Betriebsfestigkeit, TM Nr.50/70, July 1970.

149. Henry, D.L. A theory of fatigue damage accumulation in steel. Trans.ASME, Vol.77, 1955, p.913.

150. Smith. C.R. Small specimen data for predicting life of full-scale structures. Symp.Fatigue Tests of Aircraft Structures: Low-Cycle, Full-Scale and Helicopters, Am.Soc. Testing Mats, STP 338, 1963, pp 241-250.

151. Smith, C.R. Linear strain theory and the Smith method for predicting fatigue life of structures for spectrum-type loading. Aerospace Res.Lab., WPAFB, Ohio, Report ARL 64-55, 1964.

152. Corten, H.T., Dolan, T.J. Cumulative fatigue damage. Proc.Int.Conf.on Fatigue of Metals, London 1956, pp

153. Ohji, K., Miller, W.R., Marin, J. Cumulative damage and effect of mean strain in low-cycle fatigue of a 2024-T351 aluminum alloy. J.Basic Eng., Trans.ASME, Vol.88 (D), 1966, pp 801-810.

154. Topper, T.H., Sandor, B.Z. Effects of mean stress and prestrain on fatigue-damage summation. Symp. Effects of Environment and Complex Load History on Fatigue Life. Am.Soc.Test.Mats., STP 462, pp 93-104.

155. Topper, T.H., Sandor, B.I., Morrow, J.D. Cumulative fatigue damage under cyclic strain control. Aero.Struct.Lab., Report No.NAEC-ASL-1115, 1967.

156. Impellizzerri, L.F. Cumulative damage analysis in structural fatigue. Symp.Effects of Environmental and Complex Load History and Fatigue Life. Am.Soc.Test.Mats, STP 462, 1970, pp 40-68.

157. Martin, J.F., Topper, T.H., Sinclair, G.M. Computer based simulation of cyclic stress-strain behavior with applications to fatigue. Materials Res. and Standards, Vol.11, No.2, 1971, pp 23-28,50.

158. Gassner, E. Performance fatigue testing with respect to aircraft design. Fatigue in Aircraft Structures, Academic Press, New York 1956, pp 178-206.

159. Gassner, E., Schütz, W. Assessment of the allowable design stresses and the corresponding fatigue life. ICAF Symp., Munich 1965, Fatigue Design Procedures, ed. by E.Gassner and W.Schütz, Pergamon 1969, pp 291-307.

160. Paris, P.C., Sih, G.C. Stress analysis of cracks. Symp. Fracture Toughness Testing and its Applications. Am.Soc.Test.Mats, STP 381, 1965, pp 30-83.

161. Paris, P.C., Gomez, M.P., Anderson, W.E. A rational analytic theory of fatigue. The Trend in Engineering, Vol.13, 1961,p.9.

162. Paris, P.C. The growth of cracks due to variations in load. Doctoc Thesis, Lehigh University 1962.

163. Schijve, J. Significance of fatigue cracks in micro-range and macro-range. Symp.Fatigue Crack Propagation, Am.Soc.Test.Mats, STP 415, 1967, pp 415-457.

164. Figge, I.G., Newman, J.C. Fatigue crack propagation in structures with simulated rivet forces. Symp.Fatigue Crack Propagation, Am.Soc.Test.Mats, STP 415, 1967, pp 71-93.

165. Paris, P.C. The fracture mechanics approach to fatigue. Fatigue, an Interdisiplinery Approach, ed.J.J.Burke, N.L.Reed and V.Weiss, Syracuse Un.Press, 1964, p.107.

166. Smith, S.H. Fatigue crack growth under axial narrow and broad band random loading. Acoustical Fatigue in Aerospace Structures, ed. W.J.Trapp and D.M.Forney, Syracuse Un.Press, 1965, p.331.

167. Swanson, S.R., Cicci, F., Hoppe, W. Crack propagation in Clad 7079-T6 aluminum alloy sheet under constant and random amplitude fatigue loading. Symp.Fatigue Crack Propagation, Am.Soc.Test.Mats, STP 415, 1967, pp 312-350.

168. Schijve, J. Estimation of fatigue performance of aircraft structures. Symp.Fatigue of Aircraft Structures, STP 338, Am.Soc.Test .Mats, 1962, p.193. Also NLR MP 212.

169. Jacoby, G. Comparison for fatigue life estimation processes for irregularly varying loads. Proc.3rd Conference on Dimensioning and Strength Calculation, Hungarian Academy of Sciences, Budapest 1968, p.81.

170. Deneff, G.V. Fatigue prediction study. Flight Dynamics Lab, WADD TR 61-153, 1962.

171. Poe, C.C. Fatigue crack propagation in stiffened panel. Paper ASTM Symp.Crack Propagation and Damage Tolerance in Aircraft Structures, Toronto, June 1970.

172. Lazzarino, L., Salvetti, A. A theoretical and experimental research on the fatigue behavior of [illegible] ced sheets. ICAS Paper No.70-35, ICAS-meeting, Rome, Sep.1970.

173. Chilver, A.H. The estimation of fatigue damage in aircraft wing structures. J.Roy.Aero.Soc., Vol.58, 1954, p.396.

174. Schijve, J. The analysis of random-load-time histories with relation to fatigue tests and life calculations. Fatigue of Aircraft Structures. ICAF Symp., Paris 1961, ed.by W.Barrois and E.L.Ripley, Pergamon 1963, p.115. Also NLR MP 201, Amsterdam 1961.

175. Buxbaum, O. Statistische Zählverfahren als Bindeglied zwischen Be..spruchungsmessungen und Betriebsfestigkeitsversuch. LBF TR No.TB-64, Darmstadt 1966.

176. Van Dijk, G.M. Statistical load data processing. Paper 6th ICAF Symp., Miami, May 1971. Also NLR MP 71004, Amsterdam 1971.

177. De Jonge, J.B. The monitoring of fatigue loads. ICAS Congres Rome 1970, Paper No. 70-31. Also NLR MP 70010, Amsterdam 1970.

178. Swanson, S.R. A review of the current status of random load testing in America. Proc.ICAF-meeting, Stockholm 1969, ed.by G.Wallgren and S.Eggwertz, p.2.3/1.

179. Buxbaum, O. Extreme value analysis and its application to c.g. vertical accelerations measured on transport airplanes of type C-130. AGARD Report No. 579, 1971.

180. Heywood, R.B. Designing against fatigue. Chapman and Hall, London 1962.

181. Data sheets. Fatigue. Roy.Aero.Soc., London.

182. Stone, M. Structural reliability through detail design and development testing. Proc. Air Force Conf. on Aircraft Structures and Materials. Miami 1969, AFFDL TR 70-144, 1970, p.579. Also Douglas Paper 5683.

183. Jarfall, L. Optimum design of joints. The stress severity factor concept. Paper 5th ICAF Symp. Melbourne, May 1967.

184. Jarfall, L. Fatigue cycling of riveted or bolted joints. FFA Report HF-1239, Stockholm 1969.

185. Spencer, M. The 747 fatigue integrity program. Paper 6th ICAF Symp. on Design of Current Aircraft Systems, Miami, May 1971.

186. Schijve, J., Broek, D. Crack propagation under variable-amplitude loading. Aircraft Engineering, Vo.34, 1962, p.314. Also NLR Report MP 208, Amsterdam 1961.

187. White, D.J., Lewszuk, J. Narrow band random fatigue testing with Amsler Vibraphore machines. J.Mech.Engrg. Science, Vol.11, 1969, p.598.

188. Laudert, H., Jacoby, G., Nowack, H., Weber, H.D. Einige Möglichkeiten der Ermüdungsprüfung bei Random-Beanspruchungen. Fortschritt Bericht, VDI-Z, Reihe 5, No.7, April 1969, p.29.

190. Schütz, D., Lowak, H. Einzelflugversuche an gekerbten Flachstäben aus AlCuMg2 (3.1354.5) mit plastischer Aufweitung des Kerbgrundes. LBF TM 54/70, Darmstadt 1971.

191. Harpur, N.F., Troughton, A.J. The value of full-scale fatigue testing. Fatigue Design Procedures, 4th ICAF Symp. Munich 1965, ed. by E.Gassner and W.Schütz, Pergamon 1969, p.343.

192. Morgan, R.C. Some thoughts on the economics of fatigue. Paper presented at the 5th ICAF Symp. Melbourne, May 1967.

193. Axisa, R., Graff, D. Economic aspects of fatigue in commercial airlines. Paper presented at the 5th ICAF Symposium, Melbourne, May 1967.

194. Rhomberg, H. Economic and operational aspects of fatigue figures of a Swiss ground attack-fighter. Paper presented at the 5th ICAF Symposium, Melbourne, May 1967.

195. Schijve, J. Fatigue of aircraft structures. 4th Von Karman Lecture, Israel J.of Technology, Vol.8, 1970, p.1.

196. Raithby, K.D. A comparison of predicted and achieved fatigue lives of aircraft structures. Fatigue of Aircraft Structures, 2nd ICAF Symposium, Paris 1961, ed.by W.Barrois and E.L.Ripley, Pergamon 1963, p.249.

197. Lowndes, H.B., Miller, W.B. The U.S. Air Force Weapon Systems fatigue certification program. Fatigue Design Procedures, 4th ICAF Symposium, Munich 1965, ed.by E.Gassner and W.Schütz, Pergamon 1969, p.139.

198. Hardrath, H.F. A review of cumulative damage. Paper presented to the AGARD Structures and Materials Panel, Munich, June 1965.

Table 4.1 Investigations on the effect of a preload on fatigue life

Investigation	Material	Specimen	Type of fatigue loading [a]
Forrest (1946, Ref.40)	Al-alloy (2014)	V-notch	Rotating bending, C.A. (preload in tension)
Heyer (1943, Ref.41)	Al Zn Mg alloy Al Cu Mg alloy Cr Mo steel	Lug V-notch	Axial, C.A. with positive mean
Heywood (1955, Refs.42,43)	Al-alloys	Lug Hole notched Channel specimen Wing joints Meteor tail plane	Axial, C.A. with positive mean Bending, C.A. with positive mean
Payne (1955, Ref.44)	2024-alloy	Mustang wing	Bending, C.A. with positive mean
Smith (1958, Ref.45)	7075-alloy	Hole-notched	Axial, C.A. with positive mean
Boissonat (1961, Ref.46)	Al-alloys Ti-alloy Low alloy steel	Edge-notched Strap joint Edge-notched Wing fitting Tensile bolt	Axial, C.A. R = 0.1
Mordfin and Halsey (1962, Ref.47)	7075-alloy	Riveted box beam	Bending, C.A. with constant pos.S_{min}
Rosenfeld (1962, Ref.48)	7075-alloy	Wing structure	Bending, program loading wing manoeuvre spectrum
Imig (1967, Ref.49)	Ti-alloy (8-1-1)	Edge-notched	Axial, C.A. R = 0
Kirkby and Edwards (1969, Ref.50)	Al-alloy (2014)	Pinned lug and clamped lug	Axial, random loading with positive mean
Jo Dean Morrow, Wetsel and Topper (1970, Ref.51)	2024-alloy	Hole notched	Axial, C.A. with positive mean

(a) C.A. = constant-amplitude tests

Table 4.2 Investigations on the effect of periodic-high loads on fatigue life and crack propagation

Investigation	Material	Specimen	Type of high load (a)	Type of fatigue load (b)
Heywood (1955, Ref.42)	Al-alloys	Lug Channel specimen Meteor tailplane		C.A.
	7075-alloy	Channel specimen		
Schijve and Jacobs (1959,1960, Refs.39,58)	2024-alloy 7075-alloy	Riveted lap joint		program loading
Schijve, Broek, De Rijk (1960,1961, Refs.27,59)	2024-alloy	Sheet specimen (c)		C.A.
Boissonat (1961, Ref.46)	Al-alloys	Edge-notched specimen		C.A.
Nordfin and Halsey (1962, Ref.47)	7075-alloy	Riveted box beam		C.A.
Hudson and Hardrath (1963, Ref.60)	2024-alloy	Bar specimen (c)		C.A.
Smith (1966, Ref.61)	Ti-alloy (8-1-1)	Sheet specimen (c)		C.A.
McMillan and Hertzberg (1968, Ref.26)	2024-alloy	Sheet specimen (c)		C.A.
Hudson and Raju (1970, Ref.62)	7075-alloy	Sheet specimen (c)		C.A.
Schijve and De Rijk (1971, Refs.63,64)	2024 and 7075 alloy	Wing structure (c)		flight-simulation loading

(a) periodic high positive load
" " negative "
" " load cycle starting with positive part
" " " " " " negative "

(b) C.A. = constant-amplitude loading

(c) crack propagation was studied in these investigations.

Table 4.3 Investigations on the effect of ground-to-air cycles on fatigue life (comparative testing with and without GTAC)

Investigation	Material	Specimen	Type of fatigue loading
Gassner and Horstmann (1961, Ref.65)	2024 alloy	Central notch	Program loading
Gassner and Jacoby (1964, Ref.66)	2024 alloy	Central notch	
Naumann (1964, Ref.67)	7075 alloy	Edge notched specimen	
Melcon and McCulloch (1961,1965, Refs.68,69)	7075 alloy	Elliptical hole specimens	
Barrois (1957, Ref.70)	2024 alloy	Riveted lap joint	Simplified flight simulation
Winkworth (1961, Ref.71)	2024 alloy	Dakota wings	
Schijve and De Rijk (1966, Ref.72)	2024 alloy	Sheet specimen, crack propagation	
Gassner and Jacoby (1964,1965, Refs.66,73)	2024 alloy	Central notch	Programmed flight simulation[a]
Mann and Patching (1961, Ref.74)	2024 alloy	Mustang wings	Random flight simulation
Finney and Mann (1963, Ref.75)	4-Al alloys	Round specimens with V-notch	
Melcon and McCulloch (1961,1965, Refs.68,69)	7075 alloy	Elliptical hole specimens	
Naumann (1964, Ref.67)	2024 and 7075 alloy	Edge notched specimen	
Schijve et al. (1965, Ref.76)	7075 alloy	Wing center section	
Schijve, Jacobs, Tromp (1968,1970, Refs.77,78)	2024 and 7075 alloy	Sheet specimen, crack propagation	

(a) Randomised block loading

Table 4.4 Investigations on program testing

Investigation	Material	Specimen	Variables studied: Sequence	Size of period	Low S_a cycles	High S_a cycles	Design stress level	Load spectrum
Gassner (1941, Ref.82)	2 Al-alloys	Hole notched specimens	x	x	x		x	gust, mixed
Wallgren (1949, Ref.83)	2024 and 7075 alloys	Hole notched specimen and riveted joints			x		x	gust, manoeuvre
Wallgren and Petrelius (1954, Ref.84)	2024 and 7075 alloys	Lug					x	manoeuvre
Fisher (1958, Ref.85)	AlZnCuMg	Edge notched specimen				x	x	manoeuvre
Fisher (1958, Ref.86)	AlZnCuMg	Edge notched specimen		x	x			gust
Hardrath et al. (1959, Refs.30,31)	2024 and 7075 alloys	Edge notched specimen	x	x			x	gust
Schijve and Jacobs (1959, Refs.39,58)	2024 and 7075 alloys	Riveted lap joint	x	x	x	x		gust
Naumann and Schott (1962, Ref.87)	7075 alloy	Edge notched specimen		x	x	x		manoeuvre (a)
Naumann (1962, Ref.88)	2024 and 7075 alloys	Edge notched specimen			x	x	x	gust, manoeuvre
Rosenfeld (1963, Ref.48)	7075 alloy	Wing, tailplane			x	x		manoeuvre
Nordfin and Halsey (1963, Ref.47)	7075 alloy	Boxbeam			x			manoeuvre
Jeomans (1963, Ref.89)		Bolted joint, greased and dry	x					gust
Corbin and Naumann (1966, Ref.90)	7075 alloy	Edge notched specimen			x	x	x	manoeuvre (a)
Parish (1967/1968, Refs.91,92)	Al alloy	Wing	x					manoeuvre
Dunsby (1968, Ref.93)	2024 alloy	Edge notched specimen			x		x	gust
Lipp and Gassner (1968, Refs.94,95)	Cr steel	Hole notched specimen loaded in bending, S_m=0		x			x	gust
Schijve (1970, Refs.96, 97)	2024 alloy	Sheet specimen, crack propagation	x	x				gust
Breyan (1970, Ref.98)	7075 alloy	Box beam		x			x	manoeuvre
Impellizzerri (1970, Ref.156)	7075 alloy	Hole notched specimen, including crack propagation				x		manoeuvre

(a) In these investigations randomized block loading was applied.

Table 4.5 Investigations on the shape of the spectral density function for random load fatigue life

Investigation	Material	Specimen	Type of loading
Kowalewski (1959, Ref.102)	2024-T3	Notched, K_t = 1.8	Bending, S_m = 0
Fuller (1963, Ref.103)	2024-T3	Unnotched	Bending, S_m = 0
Naumann (1965, Ref.104)	2024-T3	Edge-notched, K_t = 4	Axial, S_m = 12.2 kg/mm^2
Smith (1966, Ref. 61)	2024-T3, 7075-T6 Ti-8-1-1, Ti-6-4	Sheet, crack propagation	Axial, S_m = 8.4 - 13.3 kg/mm^2
Clevenson and Steiner (1967, Ref.105)	2024-T4	Notched, K_t = 2.2	Axial, S_m = 0
Hillberry (1970, Ref.101)	2024-T3	Mildly notched	Bending, S_m = 0

Table 4.6 Investigations on the comparison between the results from program tests and random tests

Investigation	Material	Specimen	Load spectrum	Remarks
Kowalewski (1959, Ref.102)	2024 alloy	Notched, K_t = 1.8	Rayleigh distribution	S_m = 0, bending
Melcon and McCulloch (1961/63, Refs.68,69)	7075 alloy	Elliptical hole, K_t = 4 and 7	Gust and manoeuvre	Tests with and without GTAC
Rosenfeld (1962, Ref.48)	7075 alloy	Wing	Manoeuvre	
Naumann (1964, Ref.67)	7075 alloy	Edge notched specimen, K_t = 4	Severe gust spectrum	Tests with and without GTAC
Naumann (1965, Ref.104)	2024 alloy	Edge notched specimen, K_t = 4	Severe gust spectrum	Different types of random and program tests
Corbin and Naumann (1966, Ref.90)	7075 alloy	Edge notched specimen, K_t = 4	Manoeuvre	3 load spectra
Schijve et al. (1965, Ref.76)	7075 alloy	Wing center section	Severe gust spectrum	Tests with and without GTAC
Schijve and De Rijk (1965, Ref.106)	2024 and 7075 alloys	Sheet specimen (c)	Severe gust spectrum	Tests with and without GTAC, tests indoors and outdoors
Lipp and Gassner (1968, Refs.94,95)	Cr steel	Hole notched specimen	Gust spectrum, S_m=0	S_m = 0, bending
Jacoby (1970, Refs.107,108)	2024-T3	Elliptical hole specimen, K_t = 3.1	Gust spectrum	2 design stress levels
Jacoby (1970, Ref.109)	CoCrNiW alloy Ti6Al 4V alloy 2024-T3	Circumferential notch K_t = 3.1 Elliptical hole, K_t=3.1	Rayleigh distribution	3 S_m-values
Breyan (1970, Ref.98)	7075 alloy	Riveted box beam	Manoeuvre	2 design stress levels 4 load spectra
Schijve et al. (1970, Refs.96,97)	2024-T3	Sheet specimen (c)	Gust	Different types of random and program loading

(c) crack propagation specimen

Table 4.7 Investigations on flight-simulation testing

Investigation	Material	Specimen	Load spectrum	Variables studied					
				Load sequence	Flight loads		GTAC		Design stress level
					Low-S_a cycles	High-S_a cycles	S_{min}	Taxiing loads	
Naumann (1964, Ref.67)	7075-T6 2024-T3	Edge notched specimen, K_t=4	Severe gust	x	x		x x		
Gassner and Jacoby (1964/65, Refs.66,73)	2024-T4	Elliptical hole specimen K_t = 3.1	gust	x	x	x		x	
Jacoby (1970, Refs.107,108)	2024-T4	same	gust	x					
Branger (1967,1971, Refs.100,111)	7075 bar 2014 plate	Hole notched specimen K_t = 3.6	manoeuvre		x	x		x	x
Branger and Ronay (1968, Ref.112)	CrNi steel	Hole notched specimen K_t = 2.3	manoeuvre						x
Imig and Illg (1969, Ref.80)	Ti-8Al 1Mo1V	Elliptical hole specimen K_t = 4	typical for supersonic aircraft	x			x	x	x
Schijve, Jacobs, Tromp (1968, 1969, Refs.77,78)	2024-T3 7075-T6			x	x	x	x	x	
Schijve, Jacobs, Tromp (1969, Ref.113)	2024-T3	Sheet specimen, crack propagation	gust			x			
Schijve (1971, Ref.64)	2024-T3 7075-T6								x
D. Schütz (1970, Ref.114)	7075-T6	Lug-type specimen	gust						x
Schijve, De Rijk (1971, Refs.63,64)	2024-T3 7075-T6	Wing structure, crack propagation	gust		x	x			

Investigations on the effect of omitting GTAC are mentioned in table 4.3

Table 4.8 Investigations on the superposition of two cyclic loads

Investigation	Material	Specimen	Type of loading	ω_2/ω_1	Variables studied
Locati (1956, Ref.122)	2024-T3 Steel	Unnotched	Bending	14 7 and 19	S_{a2}/S_{a1}
Nishihara and Jamada (1956, Ref.123)	Carbon steels	Notched	Bending	115-740	S_{a2}
Starkey and Macro (1957, Ref.124)	AlZn alloy SAE 4340 steel	Unnotched	Axial	2	S_{a2}/S_{a1}, phase angle
Gassner and Svenson (1962, Ref.125)	Mild steel	Notched	Bending	30	S_{a2}/S_{a1} , S_{a2} also programmed
Jacoby (1963, Ref.126)	AlMg alloy	Crack propagation	Axial	500	Fractographic observations
Jamada and Kitawaga ([illegible], Ref.127)	7075 alloy 17 ST4	Unnotched	Bending		S_{a2}/S_{a1}
Nowack (1969, Ref.128)	2024-T3	Notched	Axial	2	S_{a2}/S_{a2} , phase angle
Dowling (1971, Ref.129)	2024-T4	Unnotched	Axial	2-600	S_{a2}/S_{a1} , S_{a2} also random

Table 5.1 Three different approaches for calculating fatigue life

• Incremental damage theories, see table 5.2	
• Similarity approach based on	stress strain stress intensity factor
• Interpolation methods	

Table 5.3 Some test cases for the significance of cumulative damage theories

- **Aspects of the physical relevance of a theory**
  - a crack nucleation and crack growth
  - b residual stress
  - c other damage parameters
  - d simple sequence effects
- **Aspects of the practical usefulness of a theory**
  - e 1. prediction of life until visible cracks
    2. prediction of macro crack propagation
  - f complicated sequence effects
  - g effect of a change of the load spectrum
  - h effect of a few high-amplitude cycles
  - i effect of many low-amplitude cycles

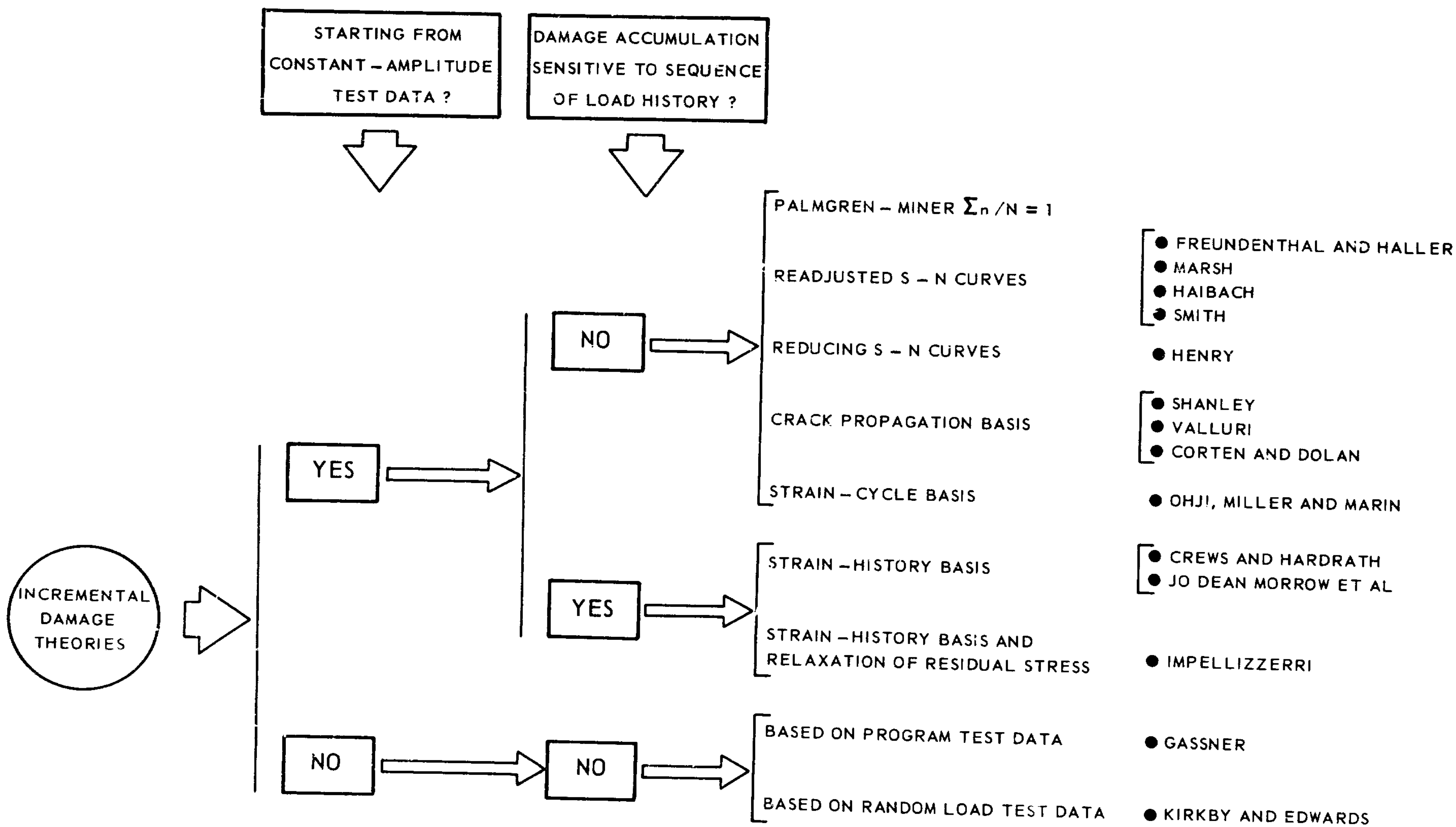


TABLE 5.2 SURVEY OF INCREMENTAL DAMAGE THEORIES

Table 6.1 Survey of aircraft fatigue problems (Ref.78)

DESIGN PHASE	Design efforts	. Type of structure, fail-safe characteristics . Joints . Detail design . Materials selection . Surface treatments . Production techniques	
		. Airworthiness requirements	
	Estimations Calculations Testing	. Prediction of fatigue environment mission analysis load statistics required target life	
		. Dynamic response of the structure	
		. Estimation of fatigue properties fatigue lives crack propagation fail-safe strength . Exploratory fatigue tests for design studies support of life estimates	 o o o o
CONSTRUCTION OF AIRCRAFT PROTOTYPES TEST FLIGHTS		. Load measurements in flight . Proof of satisfactory fatigue properties by testing components or full-structure . Allowances for service environment . Structural modifications . Inspection procedures for use in service	 o o
AIRCRAFT IN SERVICE		. Load measurements in service . Corrections on predicted fatigue properties . Cracks in service, relation to prediction . Structural modifications	o o o

Problems involving aspects of fatigue damage accumulation are indicated by o

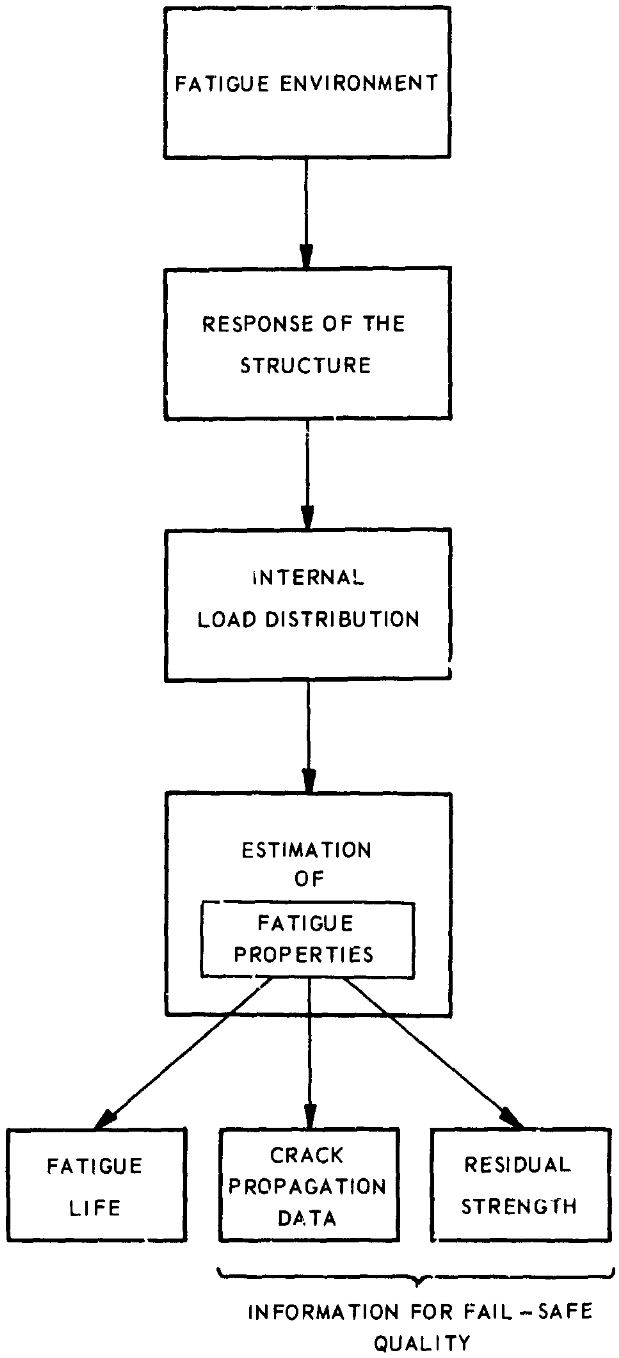


TABLE 6.2 SEVERAL PHASES OF ESTIMATING FATIGUE PROPERTIES

Table 6.3 Various aspects of the aircraft fatigue environment (Ref.78)

Load-time history	. Mission analysis, flight profiles . Fatigue loads gusts manoeuvres GTAC ground loads acoustic loading etc. . Statistical description of fatigue loads Counting of peaks, ranges, etc. PSD approach Unstationary character of environment Scatter of environmental conditions . Sequence of fatigue loads . Loading rate Time-history Wave form Rest periods
Temperature-time history	. Fatigue at low and high temperature . Thermal stresses . Interaction creep-fatigue
Chemical environment	. Corrosion, influence on crack initiation crack propagation . Interaction stress corrosion-fatigue

Table 6.4 Basic elements of fatigue life estimating procedures in the design stage

a	Estimated service load-time history
b	Structure, component: dimensions and material
c	Available fatigue data
d	Fatigue life calculation theory
e	Additional fatigue tests

Table 6.5 Aspects of available fatigue data for making life estimates

Aspect	Specification of available data
Material	• Similar material • Same material
Type of specimen	• Unnotched • Simply notched specimens • Similar structural element • Same component
Type of loading	• Constant-amplitude test data • Data from more complex fatigue load sequences • Loading in service

Table 6.6 Various procedures for making life estimates in the design stage

Starting point	Type of data	Improvement of data by accounting for :	Life calculation
1. Available fatigue life data	S-N data for: 1a. simple specimens 1b. components	Material S_m K_t and size	$\sum \frac{n}{N} = 1$
	Program test data or random load test data for: 1c. simple specimens 1d. components		$\sum \frac{n'}{N'} = 1$ (Sect.5.3.3)
	1e. Flight-simulation test data		Interpolation between available data (Sect.5.3.5)
2. Service experience from previous design	2a. Stress level giving sufficient crack free life		New design is superior to old design. Result: At least same life
	2b. Crack free life for specific components	Material, S_m, K_t, size and load spectrum	Similar life as for old design
3. Testing of new component or structure			

Table 6.7 Procedures for estimating crack propagation rates in the design stage

Starting point	Type of data
Available crack propagation data	Data from constant-amplitude tests
	Data from flight-simulation tests
Experience from previous design	Data from panel tests of full-scale fatigue tests
Testing of new component or full-scale structure	

Table 7.1 Survey of fatigue specimens

Type of specimen	Remarks
Unnotched specimen	$K_t \sim 1$
Simple notched specimen	Examles: Edge notched specimens, specimens with central notch. Specimens mainly characterized by K_t-value and notch root radius r.
Simple joint specimen	Lap joint, strap joint, either riveted or bolted. Lug type specimens.
Component	Part of a structure, full size. Examples: joint, skin panel with fatigue critical details, brackets, etc.
Full-scale structure	Large part of an aircraft structure. Examples: wing, fuselage, empennage, or large parts of these items, for instance nose section of fuselage, tailplane, etc.

Table 7.2 Survey of different types of fatigue trouble in service (Ref.195)

Type of fatigue trouble		Possible consequence
Insufficient fatigue strength	Safe-life component	Catastrophe
	Fail-safe component	Expensive repair
Deficiencies of the detail design		Extra maintenance by repair, modification or replacement
Fatigue cracks in components that were assumed to carry no cyclic load		
Fatigue cracks in secondary structure		
Fatigue cracks due to incidental service damage		Depends on type of component

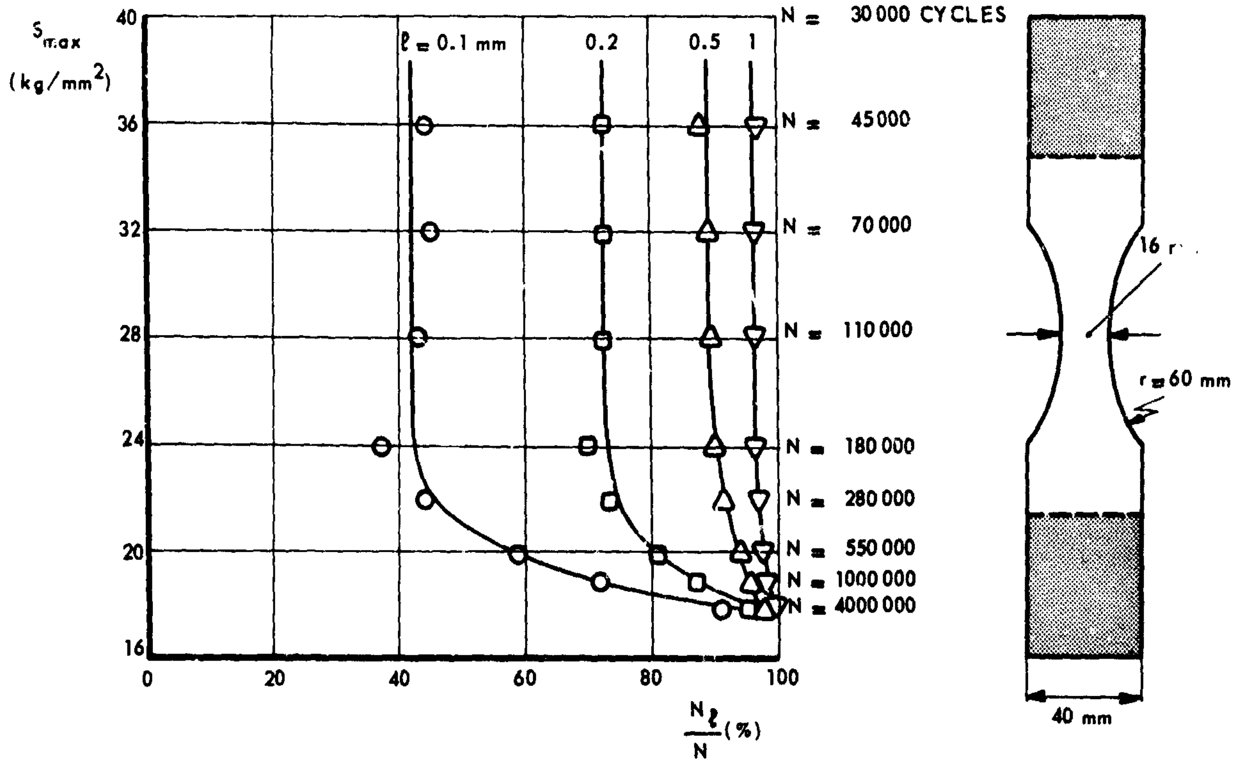


FIG. 2.1 PERCENTAGE OF FATIGUE LIFE COVERED BY CRACK PROPAGATION IN 2024-T3 SPECIMENS UNTIL A CRACK LENGTH ℓ HAS BEEN REACHED (REF. 7). TESTS AT R = 0

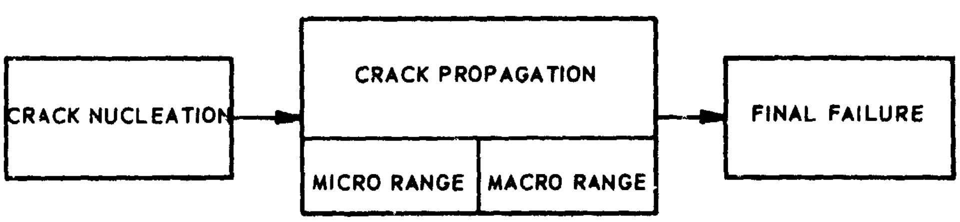
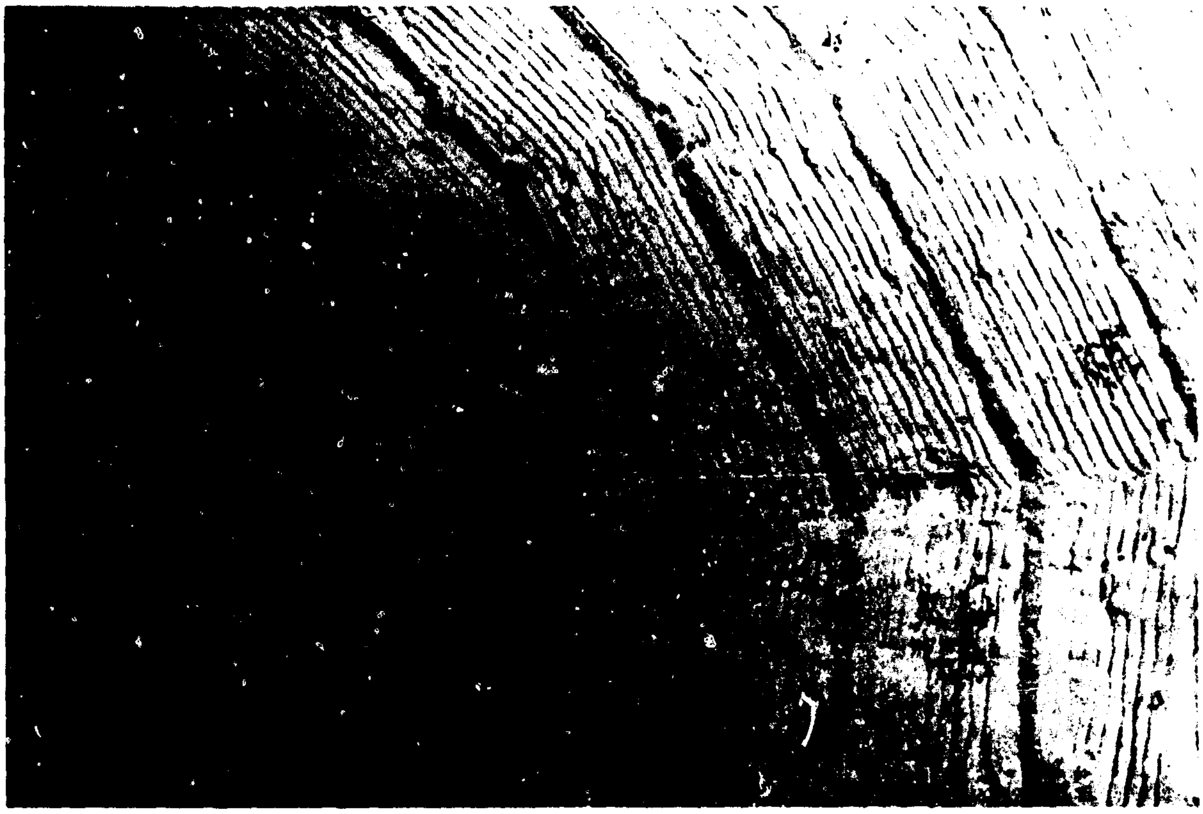


FIG. 2.2 THREE PHASES IN THE FATIGUE LIFE.



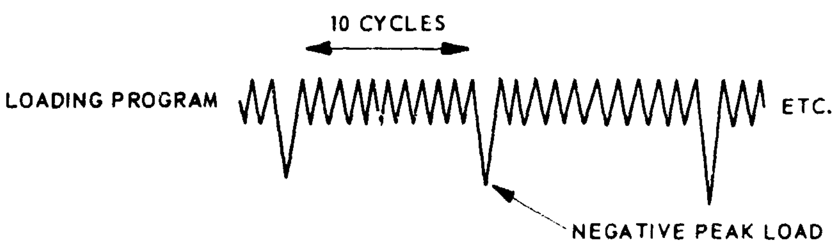


FIG. 2.3 ELECTRON MICROGRAPH FROM A FRACTURE SURFACE OF A FATIGUE CRACK.(REF.8)
PROPAGATION FROM LEFT TO RIGHT, MATERIAL 2024 – T3 ALCLAD SHEET.
EACH STRIATION CORRESPONDS TO A SINGLE LOAD CYCLE.

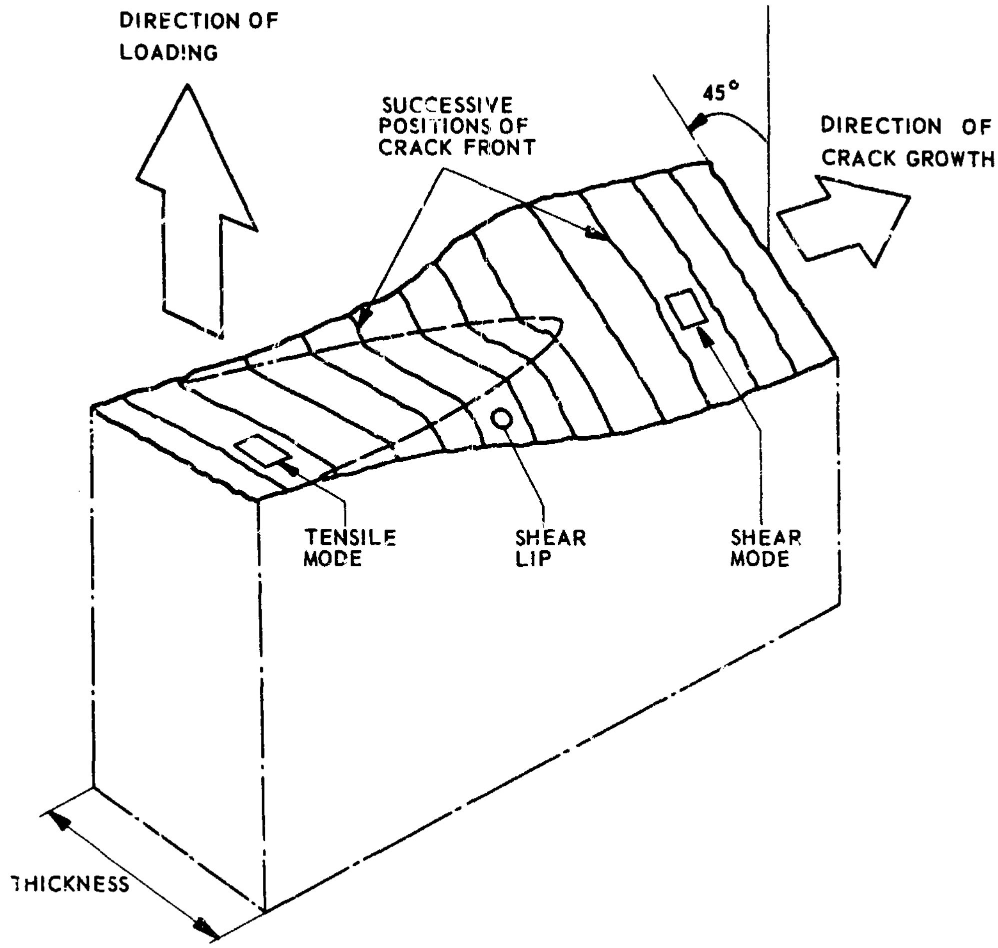


FIG. 2.4 THE SURFACE OF A FATIGUE FRACTURE IN SHEET MATERIAL DURING THE TRANSITION FROM THE TENSILE MODE (90° MODE) TO THE SHEAR MODE (45° MODE), SEE ALSO FIGURE 3.5.

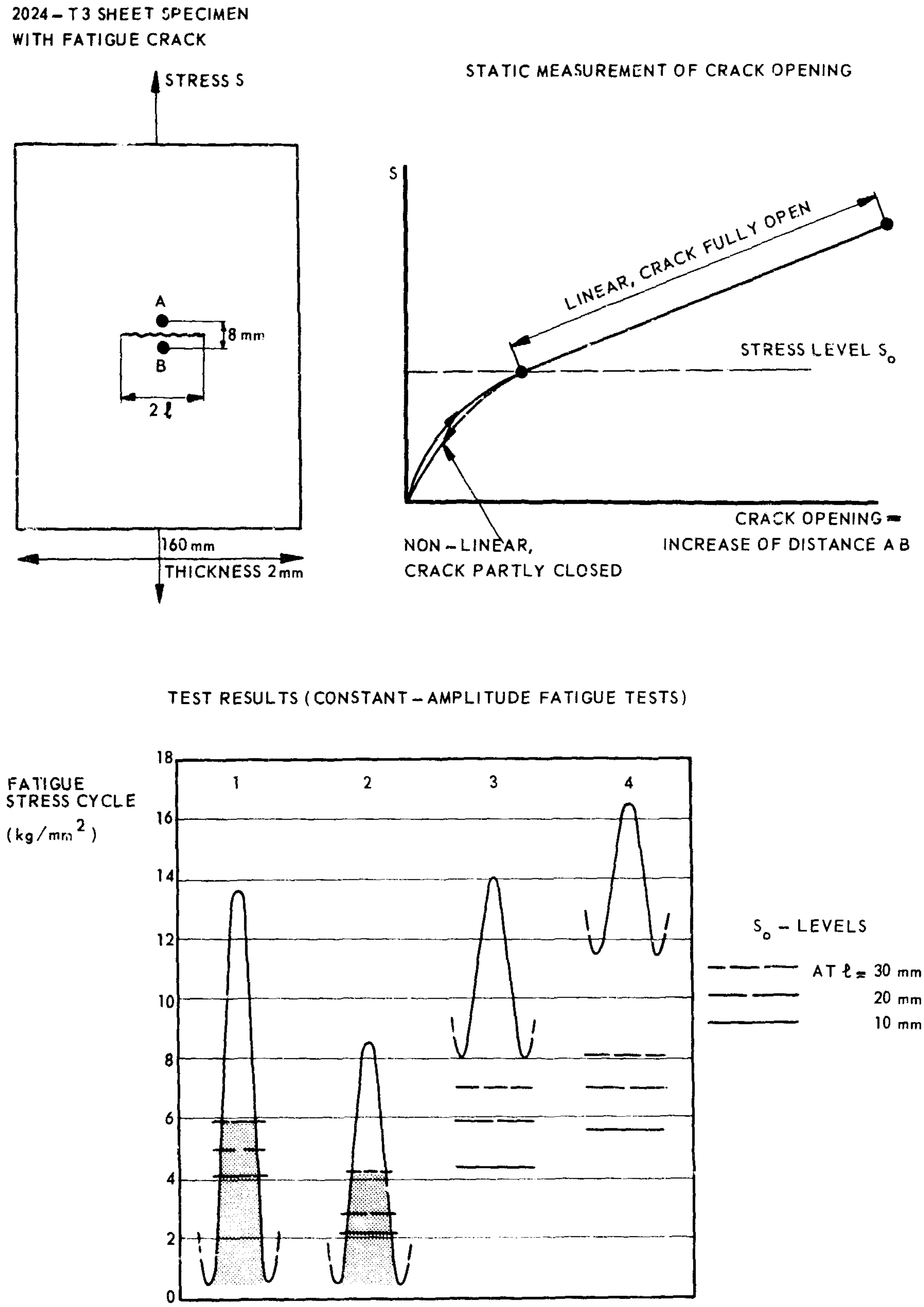
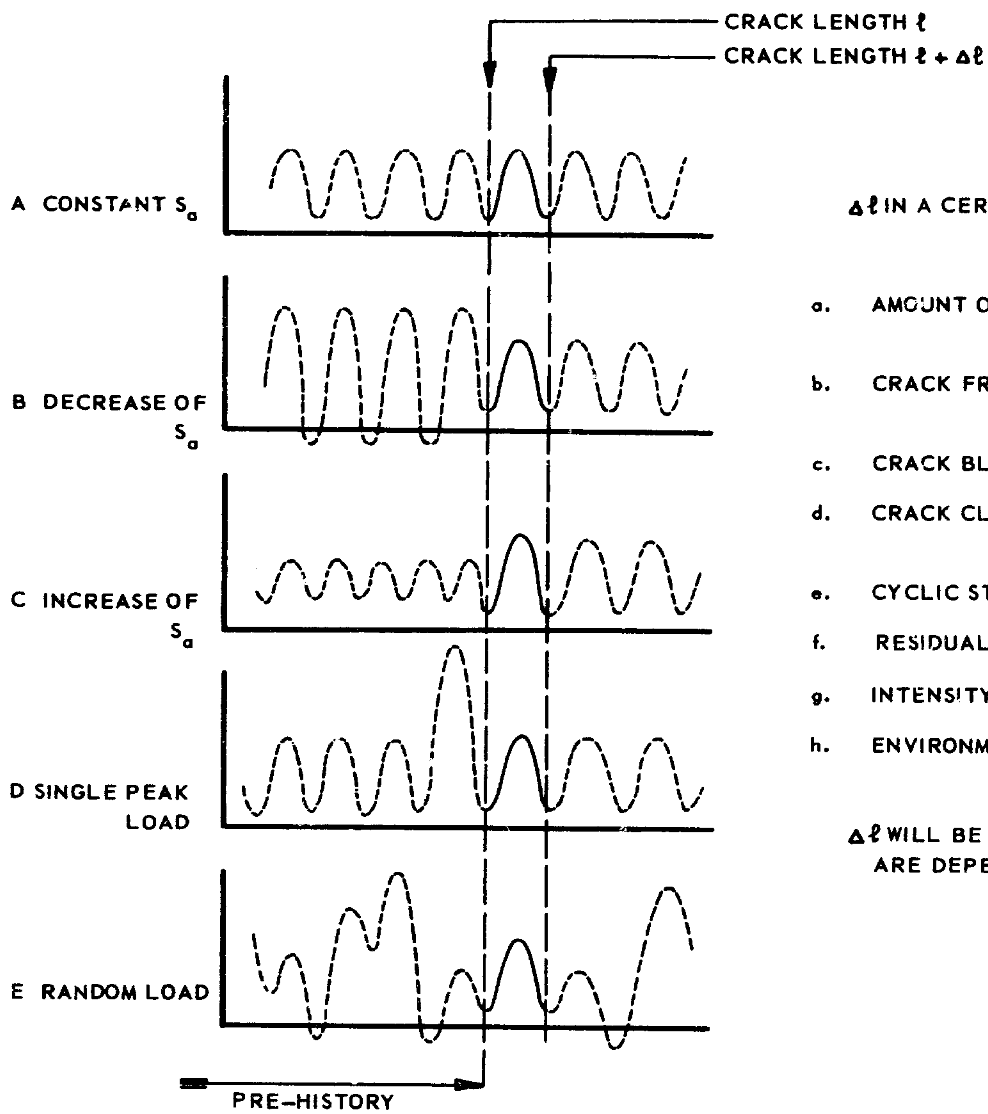


FIG. 3.1 CRACK CLOSURE ACCORDING TO ELBER (REF. 18). RESULTS FROM NLR TESTS (REF. 20).



$\Delta\ell$ IN A CERTAIN LOAD CYCLE WILL DEPEND ON :

- a. AMOUNT OF CRACKING (CRACK LENGTH ℓ, CRACKED AREA)
- b. CRACK FRONT ORIENTATION (TENSILE MODE OR SHEAR MODE)
- c. CRACK BLUNTING (SHAPE OF CRACK TIP)
- d. CRACK CLOSURE (PLASTIC DEFORMATION IN THE WAKE OF THE CRACK)

$\Big\}$ CRACK GEOMETRY

- e. CYCLIC STRAIN–HARDENING
- f. RESIDUAL STRESSES

$\Big\}$ CONDITION OF THE MATERIAL AT THE TIP OF THE CRACK

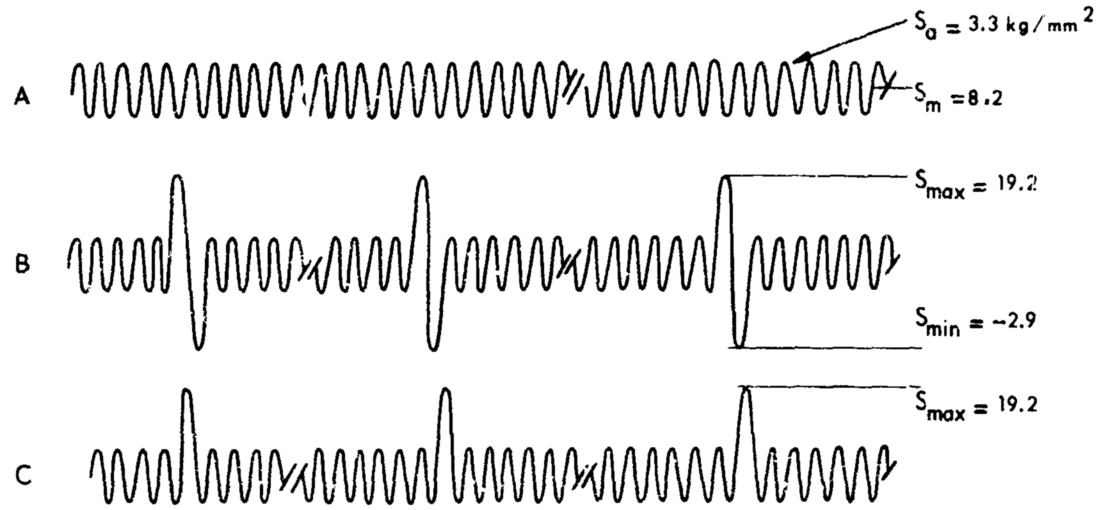
(a–f) $\Big\}$ FATIGUE DAMAGE BEING PRESENT

- g. INTENSITY OF LOAD CYCLE (ΔS, S_{max})
- h. ENVIRONMENT

$\Big\}$ EXTERNAL CONDITIONS

$\Delta\ell$ WILL BE DIFFERENT FOR A, B, C, D AND E BECAUSE b, c, d, e and f ARE DEPENDENT ON THE PRE–HISTORY

FIG. 3.2 CRACK EXTENSION AS EFFECTED BY THE LOAD HISTORY. INTERACTION MECHANISMS.



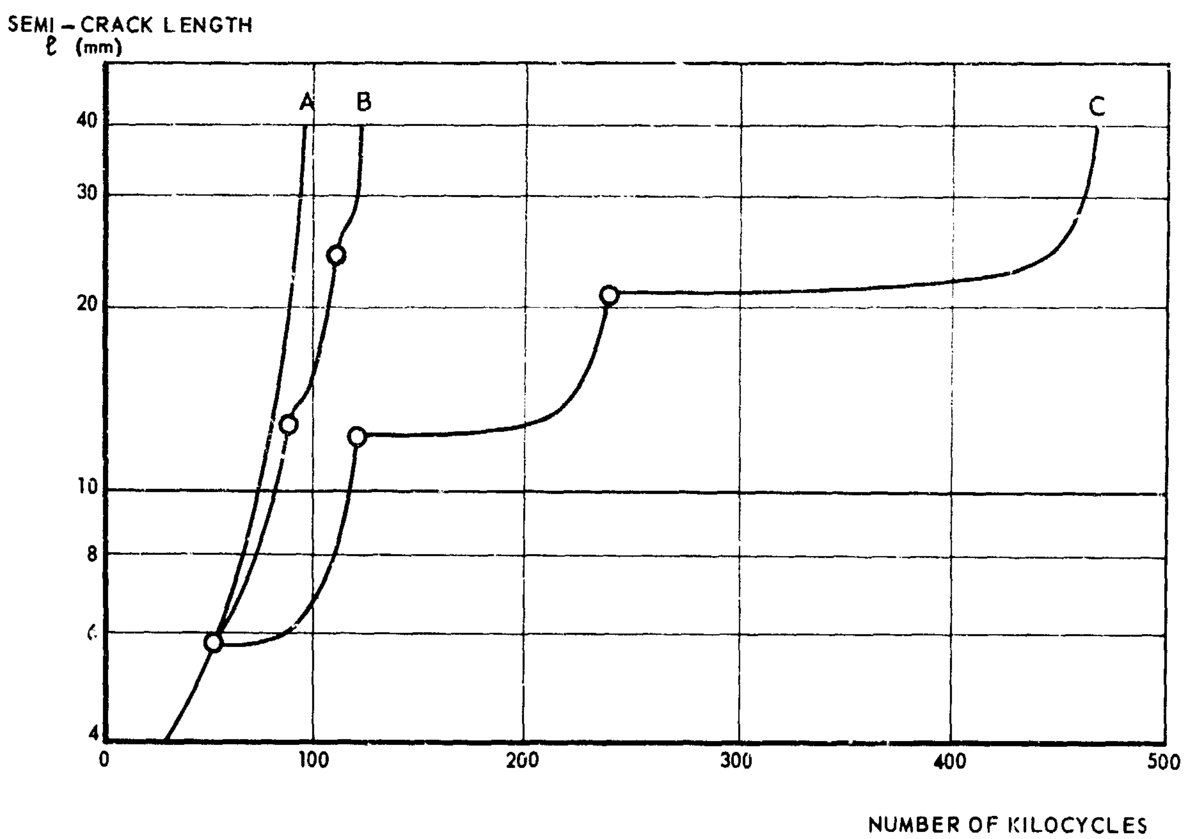


FIG. 3.3 THE DELAYING EFFECT OF PEAK LOADS ON CRACK PROPAGATION IN 2024-T3 ALCLAD SHEET SPECIMENS, WIDTH 160 mm, THICKNESS 2 mm (REF. 27).

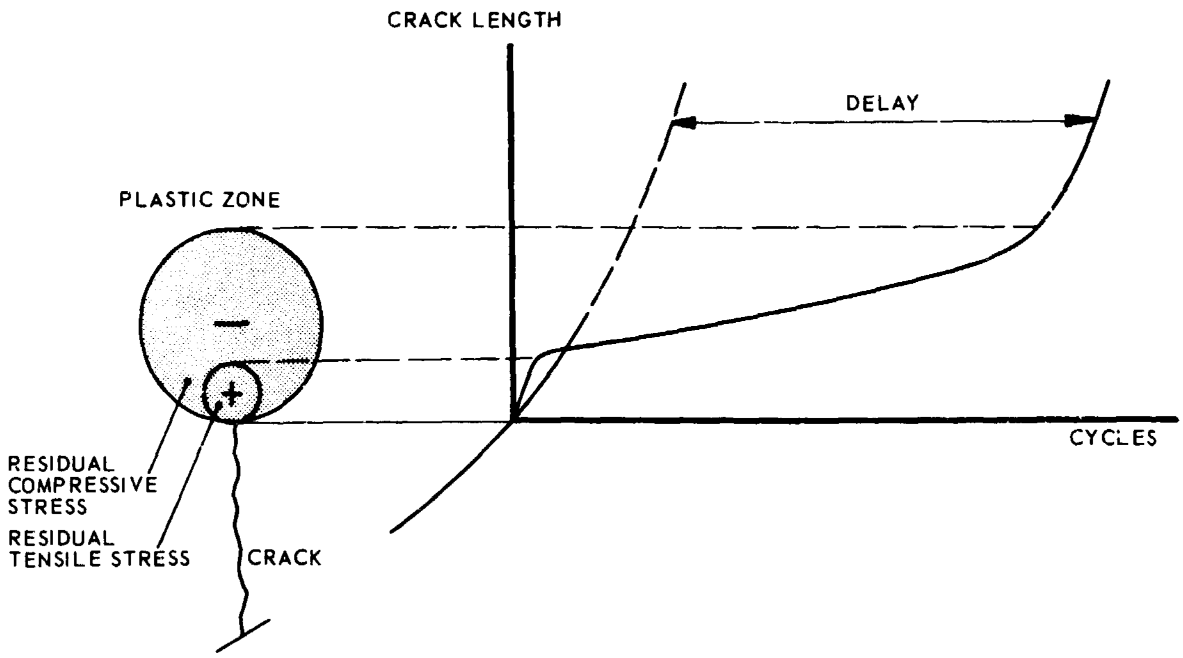
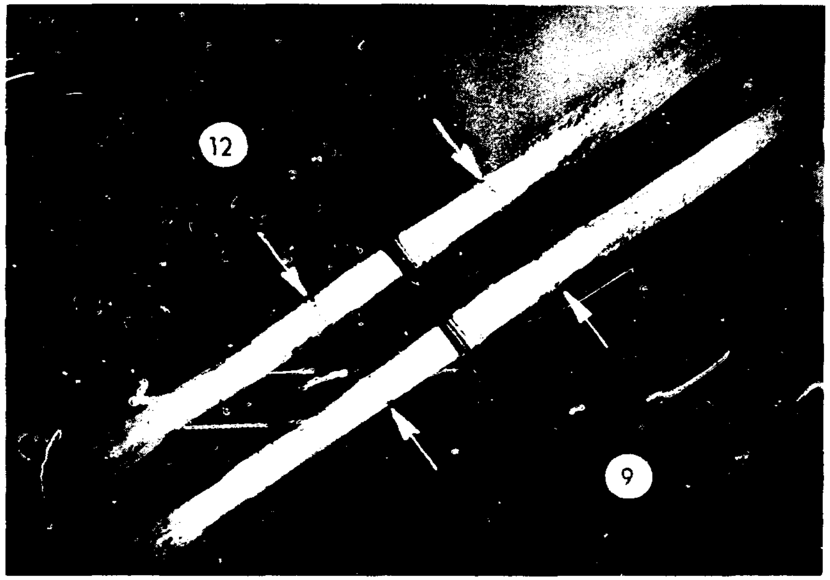
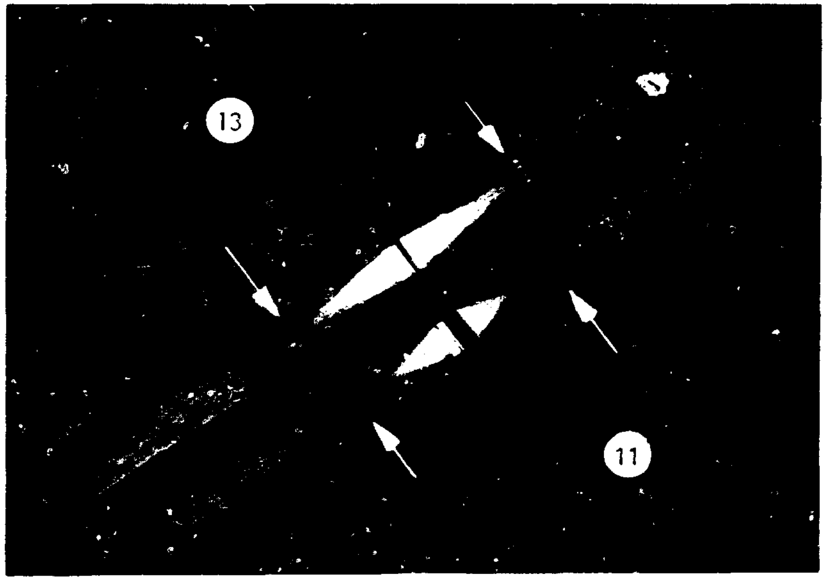
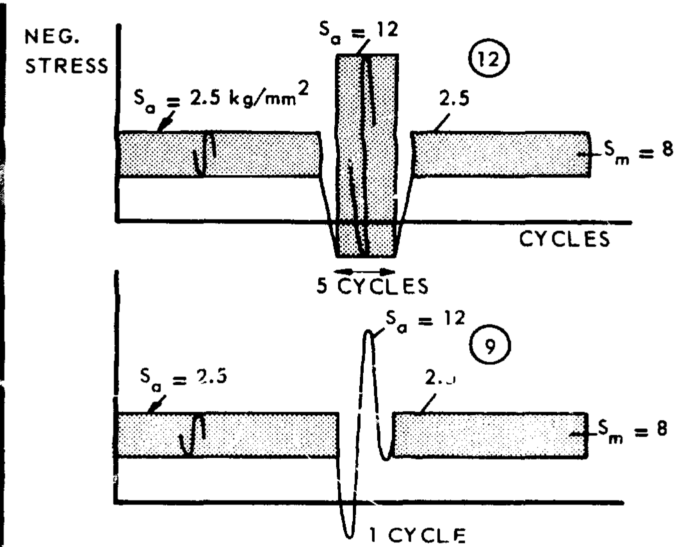


FIG. 3.4 CRACK GROWTH AFTER A PEAK LOAD CYCLE, SEQUENCE B IN FIG. 7.



ARROWS INDICATE CRACK EXTENSION BY INTERMITTENT CYCLES

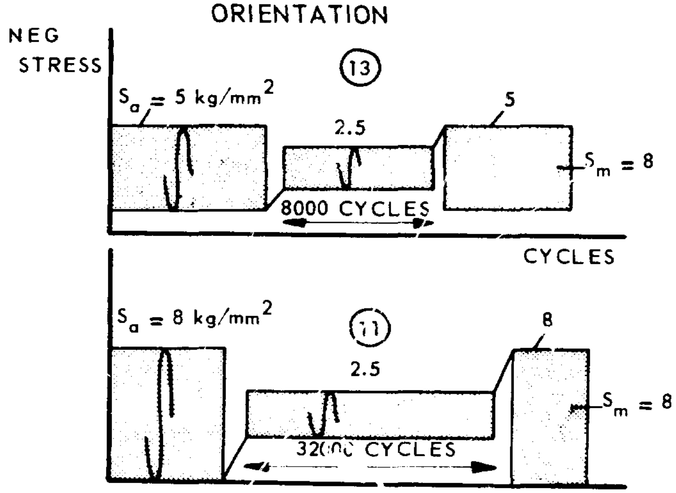




SPECIMENS 2024 – T3 ALCLAD
WIDTH 80 mm, THICKNESS 4 mm

INTERMITTENT
HIGH – AMPLITUDE CYCLES

FIG. 3.5a HIGH – AMPLITUDE CYCLES OCCURRING AT 90° CRACK FRONT ORIENTATION



INTERMITTENT BLOCK
OF LOW AMPLITUDE CYCLES

FIG. 3.5b LOW – AMPLITUDE CYCLES OCCURING AT 45° CRACK FRONT ORIENTATION.

FIG. 3.5 INCOMPATIBILITY BETWEEN CRACK FRONT ORIENTATION AND STRESS AMPLITUDE (REF. 22).

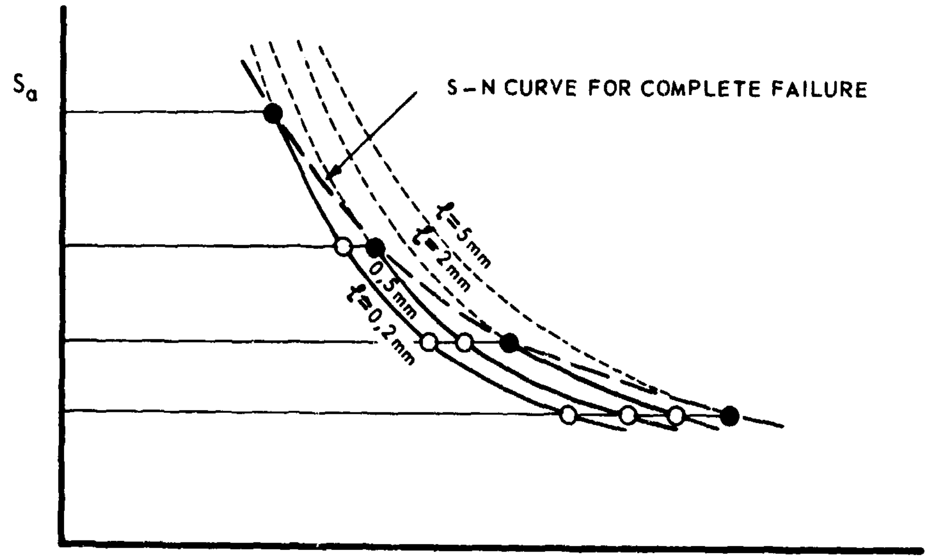
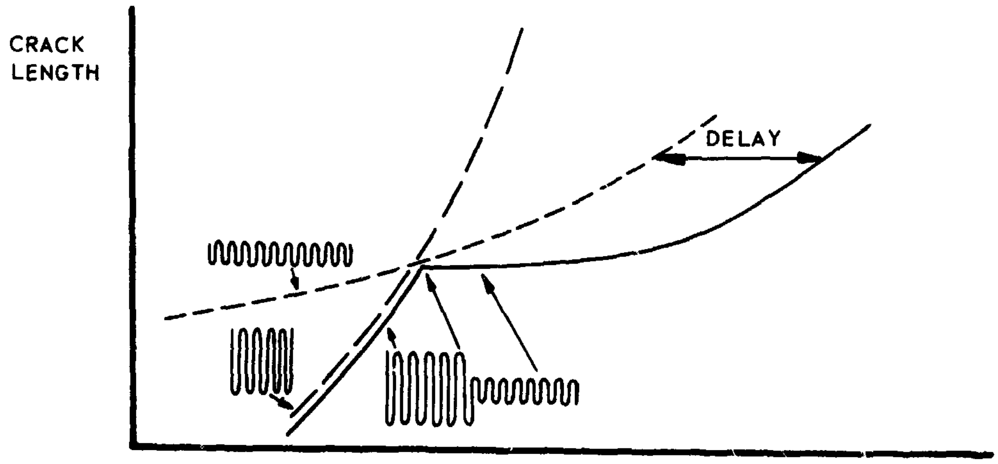
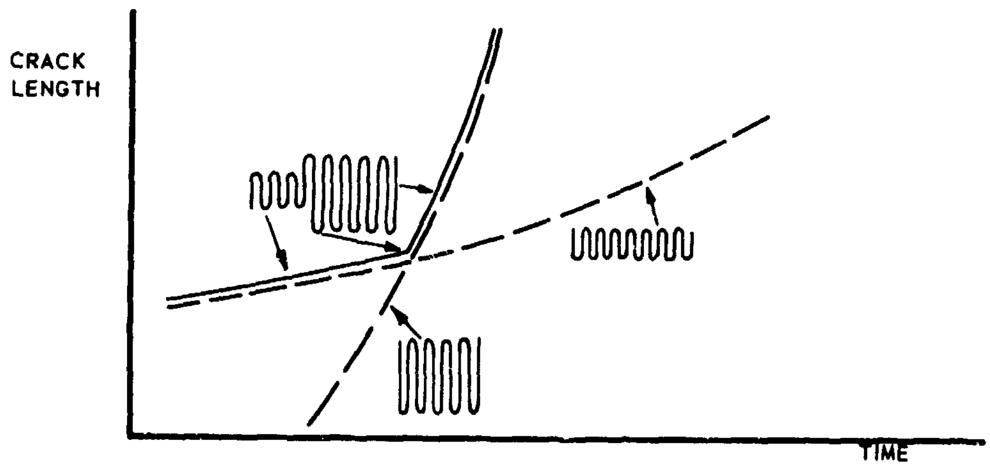


FIG. 3.6 S – N CURVES FOR CONSTANT AMOUNTS OF CRACKING (ℓ IN MILLIMETERS) AND S – N CURVE FOR COMPLETE FAILURE.



A CRACK GROWTH DELAY IN TWO–STEP TEST.



B APPARENT ABSENCE OF INTERACTION EFFECT IN TWO–STEP TEST

FIG.4.1 MACROCRACK PROPAGATION, INTERACTION EFFECTS IN TWO–STEP TESTS.

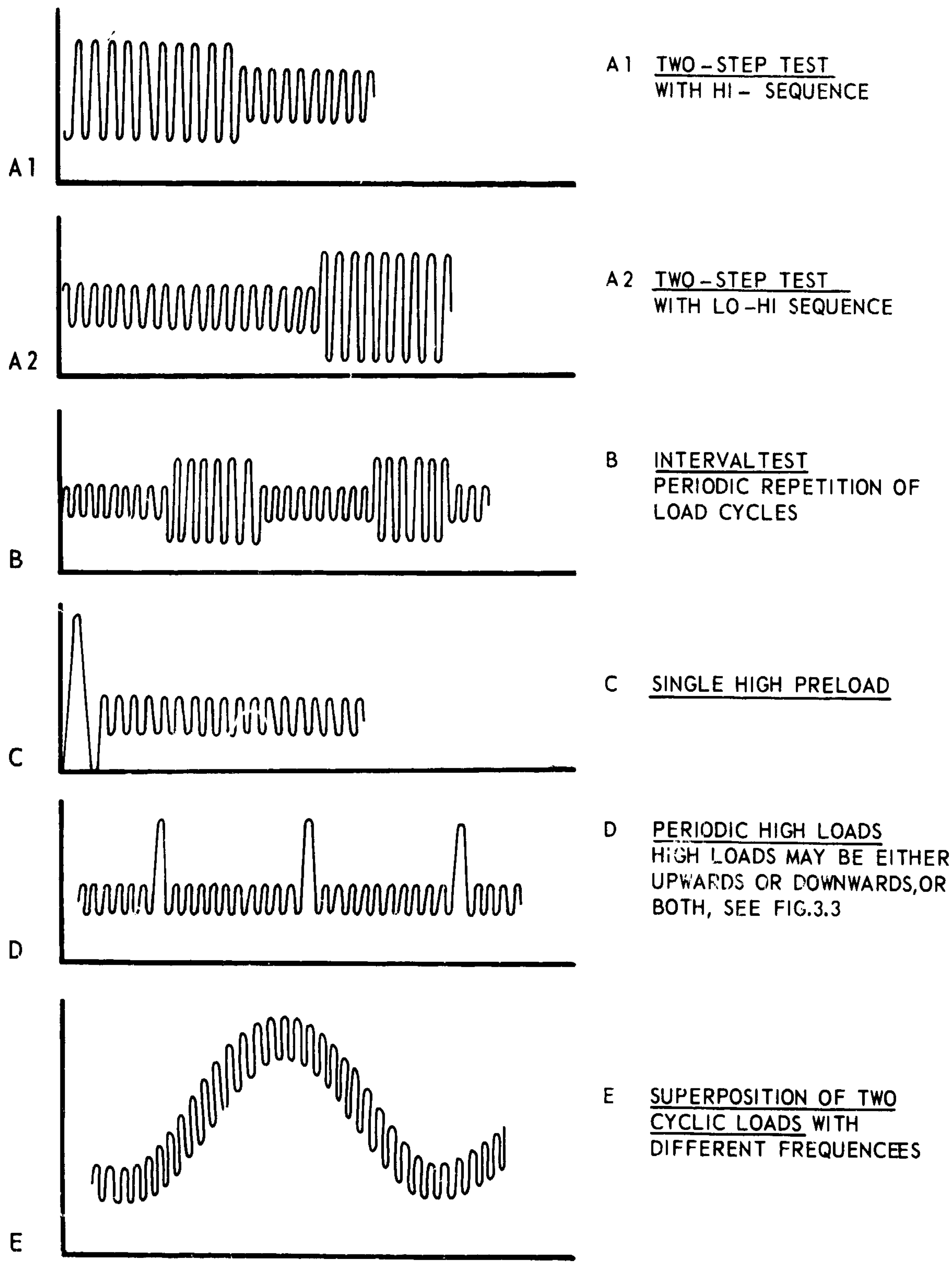
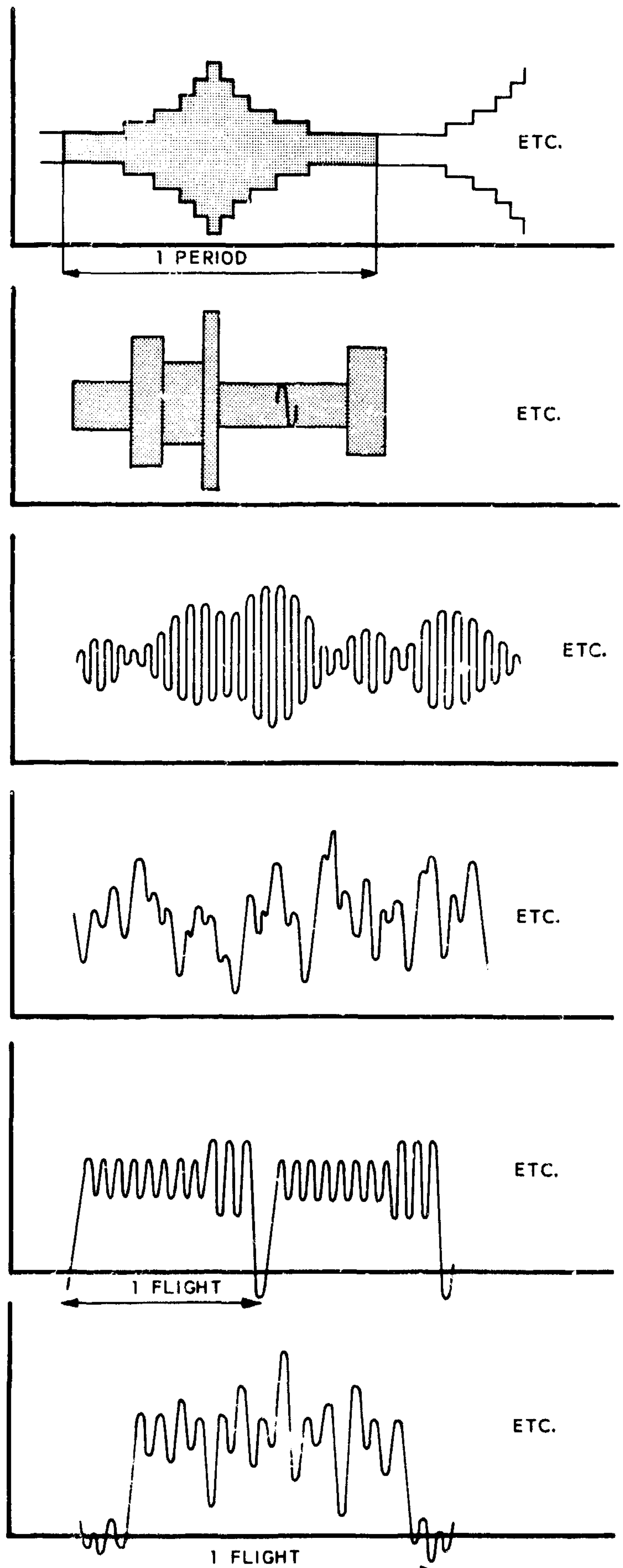


FIG. 4.2 SEVERAL SIMPLE TYPES OF VAVIABLE-AMPLITUDE LOADING.



F PROGRAM LOADING WITH LO-HI-LO SEQUENCE. LO-HI AND HI-LO SEQUENCES WERE ALSO ADOPTED, SEE FIGURE 4.17

G RANDOMIZED BLOCK LOADING SIMILAR TO PROGRAM LOADING, HOWEVER, BLOCKS OF CYCLES IN RANDOMIZED SEQUENCE

H NARROW-BAND RANDOM LOADING (SEE FIG 4.20)

J BROAD-BAND RANDOM LOADING (SEE FIG 4.20)

K QUASI-RANDOM LOADING PSEUDO-RANDOM LOADING

L SIMPLE FLIGHT-SIMULATION LOADING ALL FLIGHTS SIMILAR

M COMPLEX FLIGHT-SIMULATION LOADING DIFFERENTS FLIGHTS, SEE FIG. 7.3

FIG.4.3 SEVERAL TYPES OF COMPLEX FATIGUE LOAD HISTORIES.

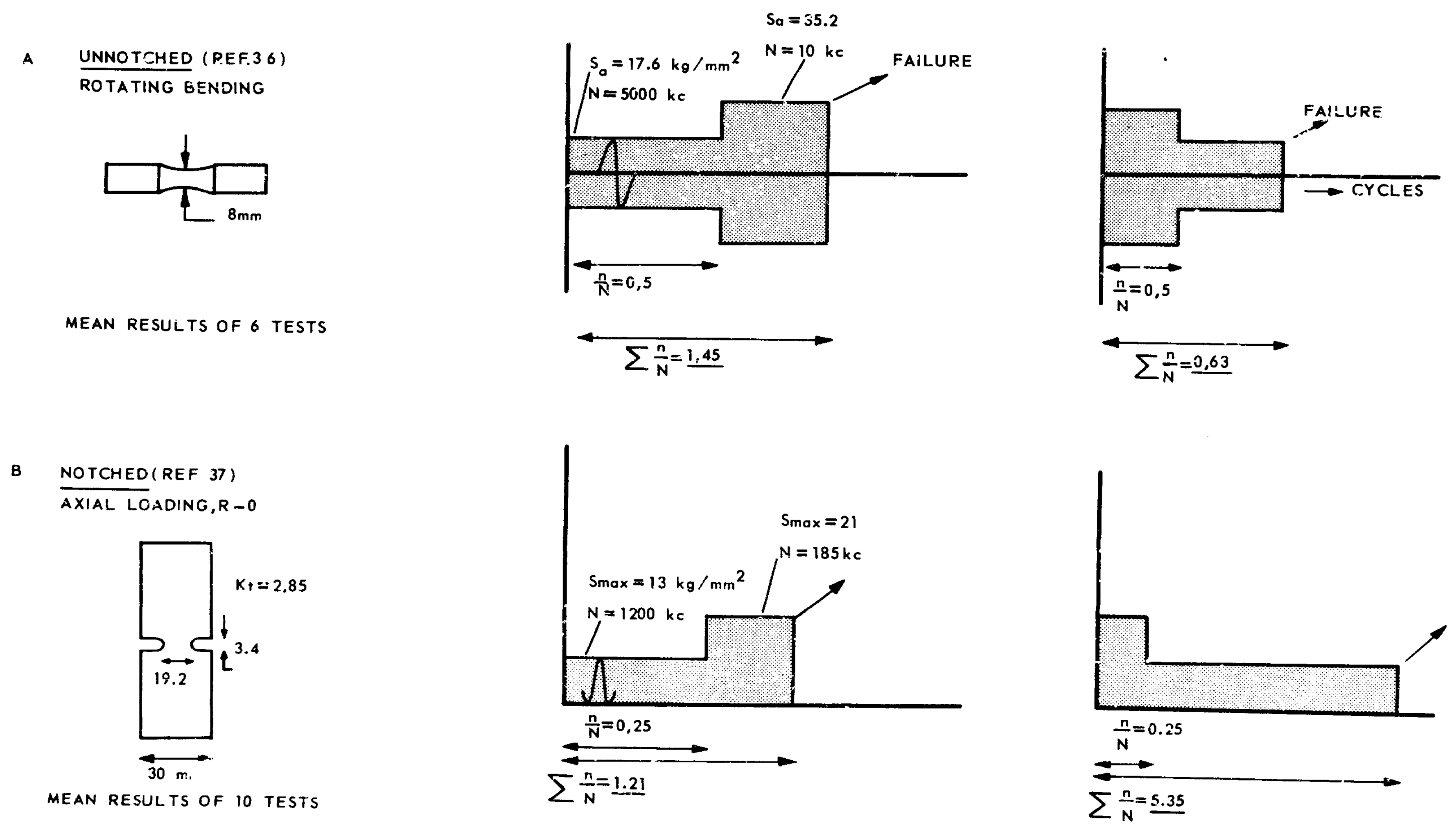


FIG. 4.4 OPPOSITE INTERACTION EFFECTS IN TWO – STEP TESTS ON UNNOTCHED AND NOTCHED AL – ALLOY SPECIMENS.

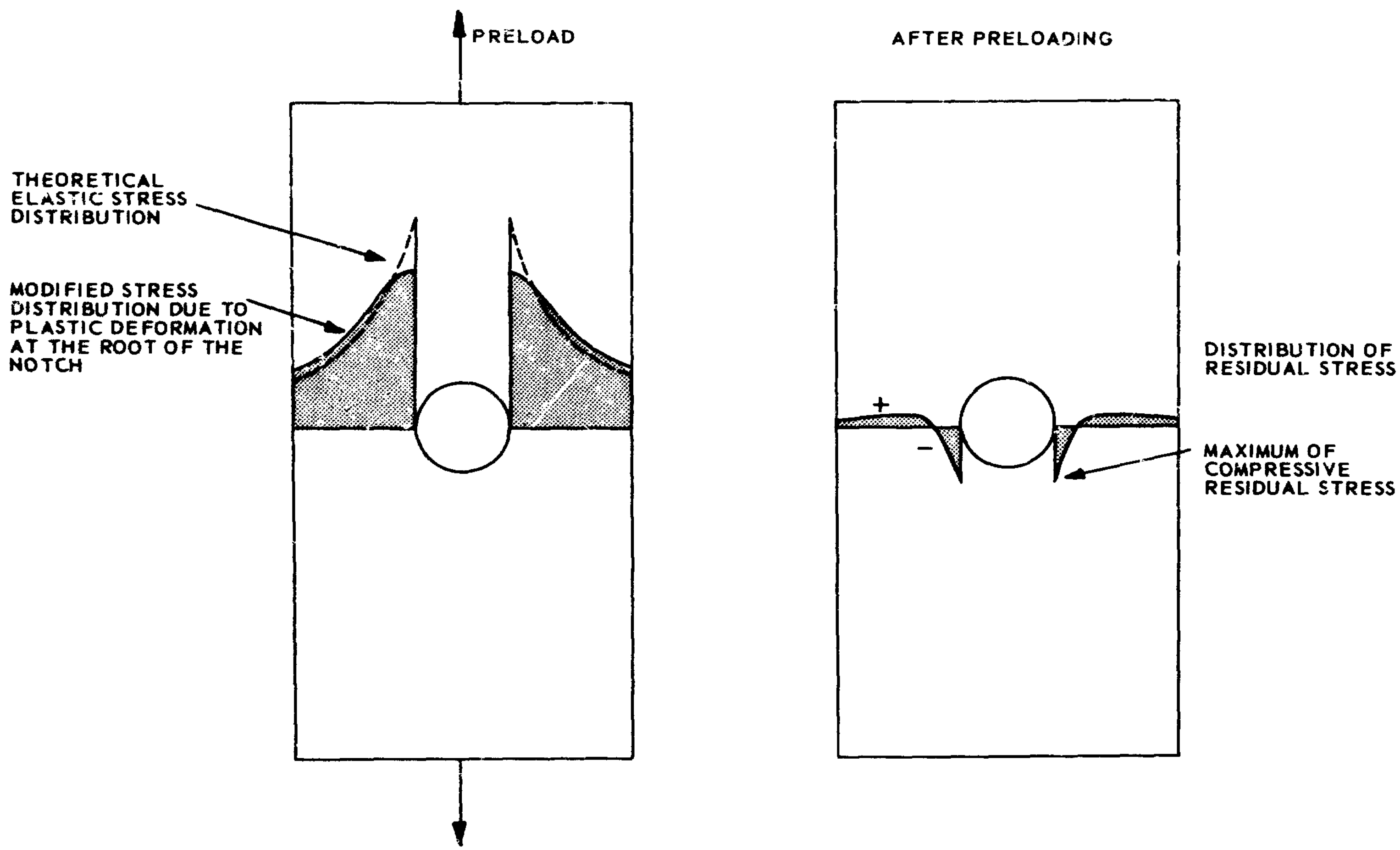
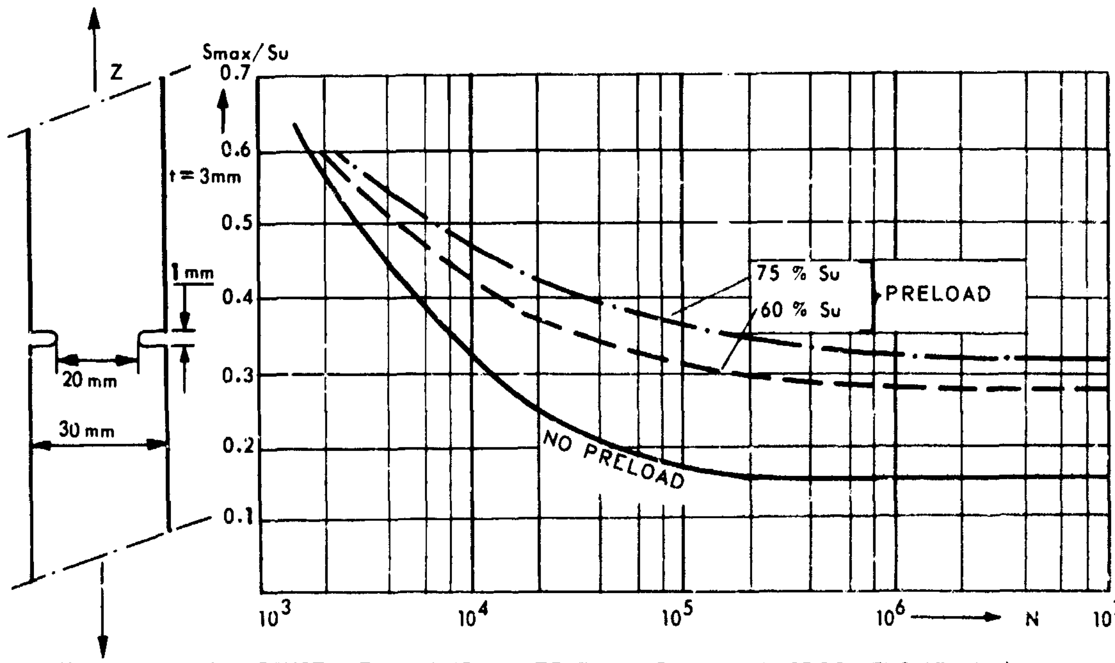
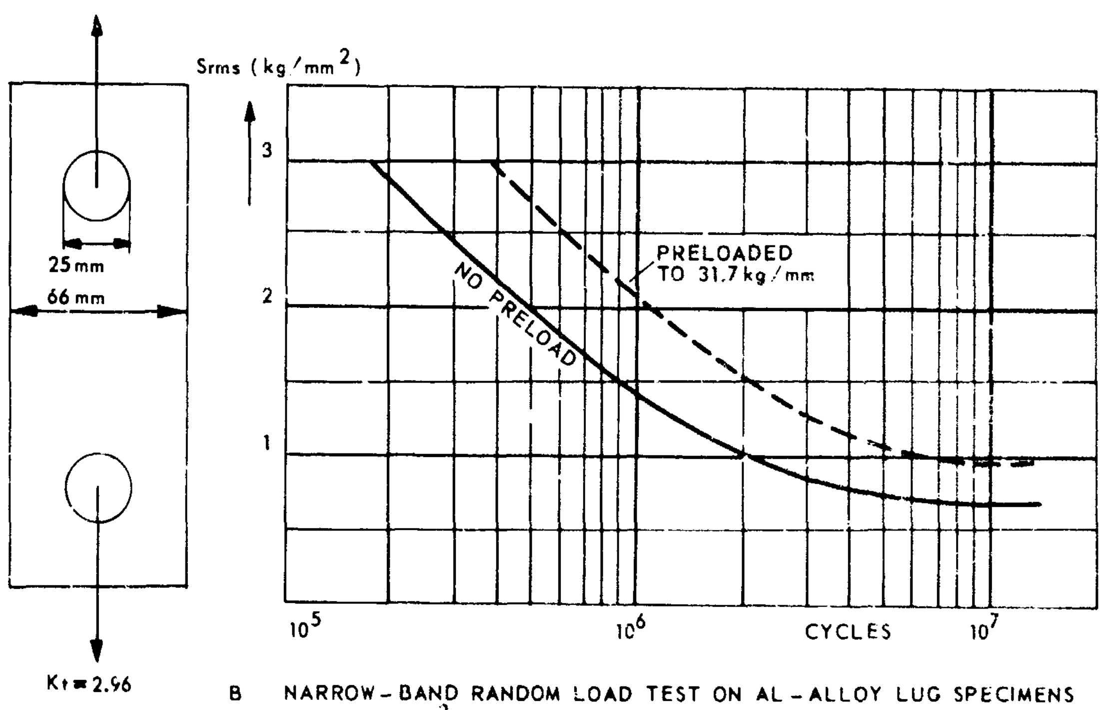


FIG. 4.5 RESIDUAL STRESS INTRODUCED BY PRELOADING IN TENSION. SPECIMEN NOTCHED BY A CENTRAL HOLE

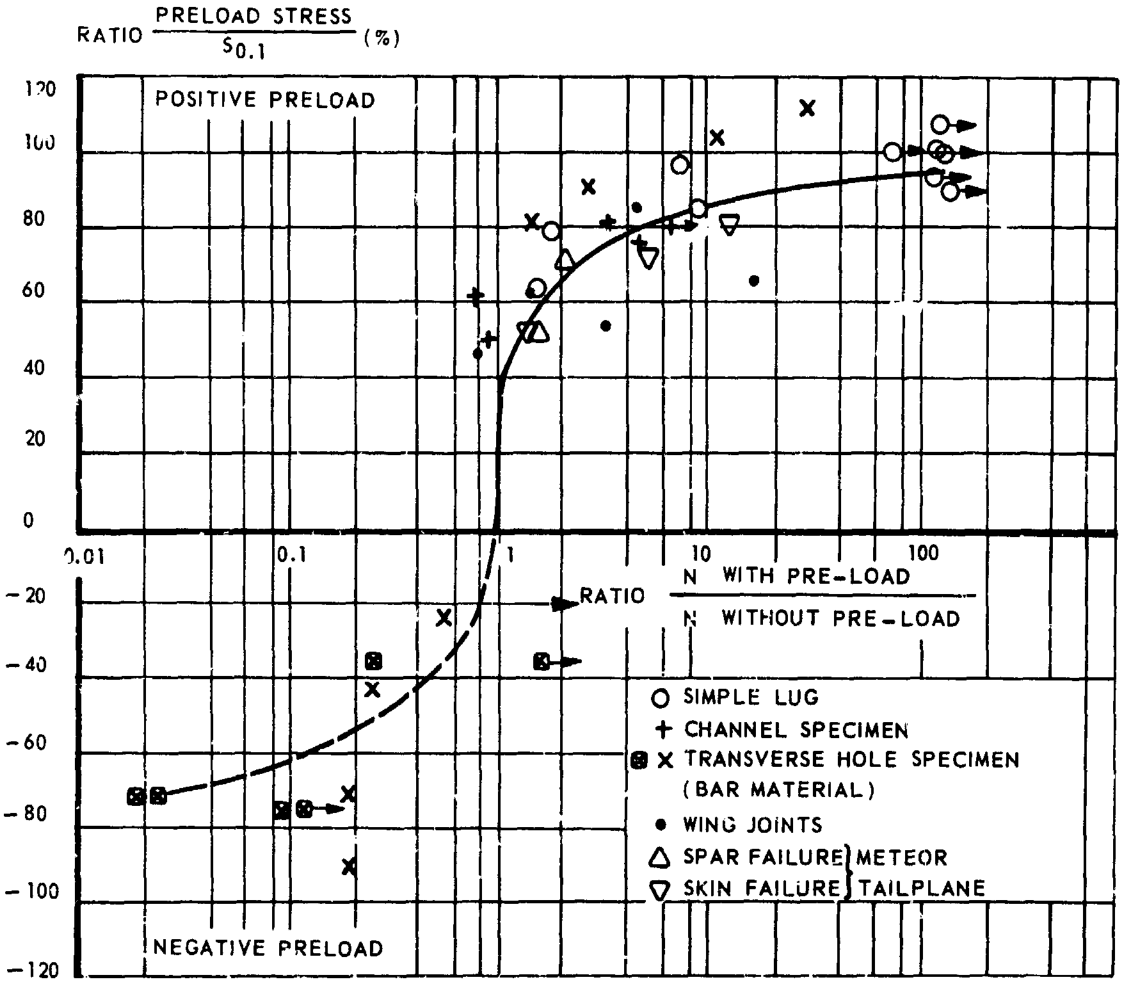


A CONSTANT-AMPLITUDE TESTS ON TI-ALLOY SPECIMENS (R = 0.1) RESULTS FROM BOISSONAT (REF.46)



B NARROW-BAND RANDOM LOAD TEST ON AL-ALLOY LUG SPECIMENS (S_m = 10 kg/mm^2) RESULTS FROM KIRKBY AND EDWARDS (REF.50)

FIG.4.6 THE EFFECT OF A PRELOAD ON FATIGUE CURVES FOR CONSTANT-AMPLITUDE LOADING AND NARROW-BAND RANDOM LOADING.



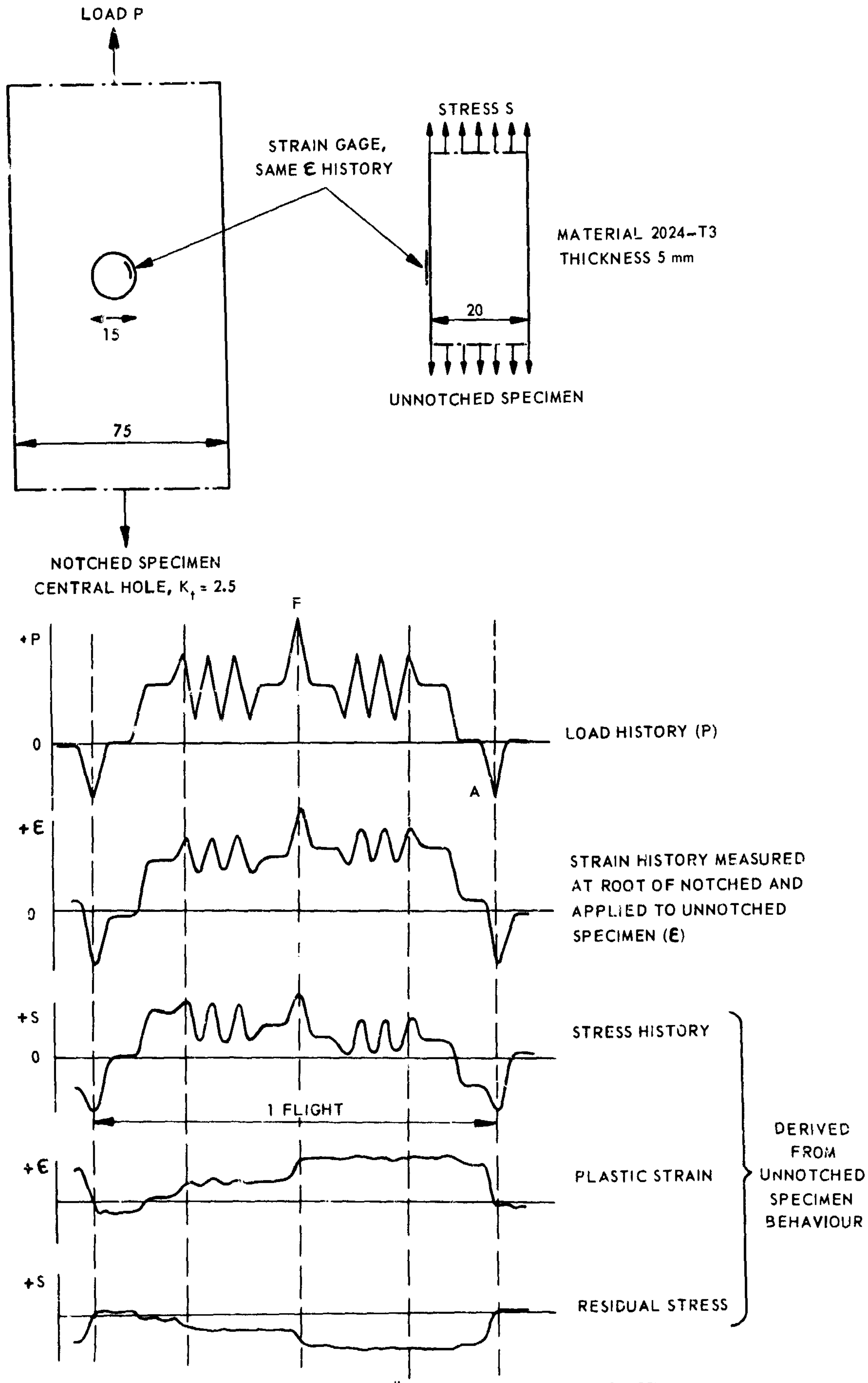
MEAN STRESSES 3.4 – 14.2 kg/mm^2

STRESS AMPLITUDES 2.7 – 5.4 kg/mm^2

N WITHOUT PRELOAD 80 kc TO 850 kc EXCEPT FOR ⊠ (7700 kc)

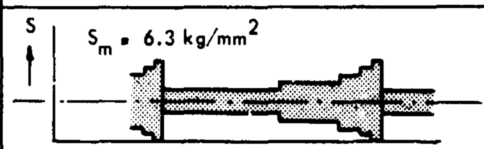
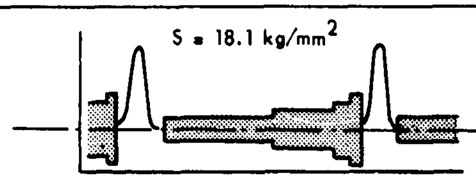
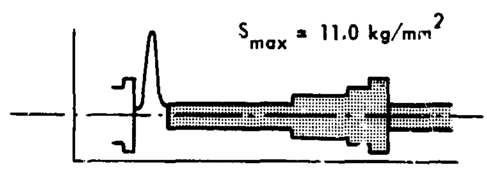
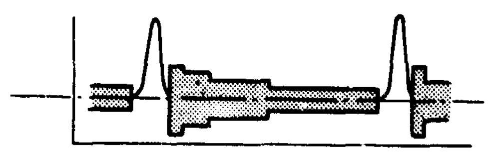
RESULTS REPORTED BY HEYWOOD (REF. 42)

FIG. 4.7 EFFECT OF THE MAGNITUDE OF A PRELOAD ON FATIGUE LIFE UNDER CONSTANT-AMPLITUDE LOADING.



MEASUREMENTS FROM HAIBACH, D. SCHÜTZ AND SVENSON (REF. 54, 55)

FIG. 4.8 DETERMINATION OF RESIDUAL STRESS AT THE ROOT OF A NOTCH DURING A SIMPLE FLIGHT–SIMULATION LOADING

TEST SERIES	LOADING SEQUENCE	REMARKS	FATIGUE LIFE (PERIODS)
10		PROGRAMMED GUST CYCLES	11
6		1 HIGH LOAD PER PERIOD	126
6a		HIGH LOADS OMITTED AFTER 50th PERIOD	58
6b		1 HIGH LOAD PER 2 PERIODS	40
17		SIMILAR TO SERIES 6 BUT GUST CYCLES IN REVERSED ORDER	22

RESULTS FROM REF. 39
FATIGUE LIFE IN PROGRAM PERIODS, 1 PERIOD = 81500 CYCLES
EACH RESULT IS THE MEDIAN OF 7 TESTS.

FIG. 4.9 THE EFFECT OF PERIODIC HIGH LOADS ON THE PROGRAM – FATIGUE LIFE OF 7075 – T6 RIVETED LAP JOINTS.

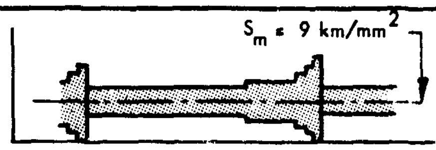
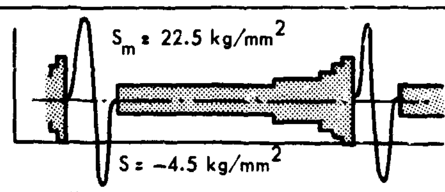
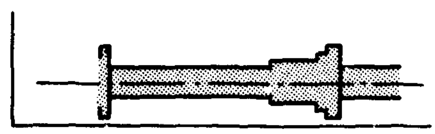
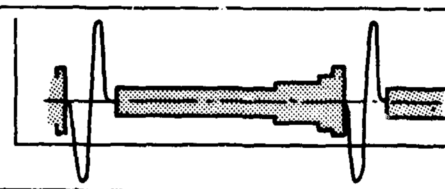
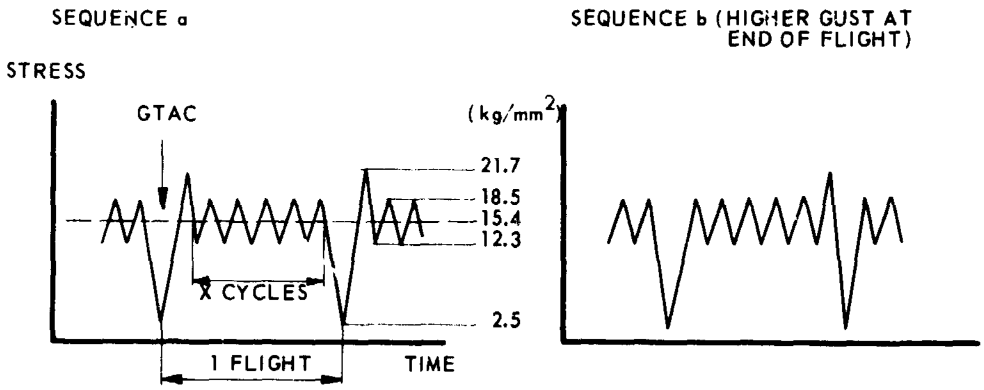
TEST SERIES	LOADING SEQUENCE	REMARKS	FATIGUE LIFE (PERIODS)
21		PROGRAMMED GUST CYCLES (5 ampl.)	31
27		HIGH LOAD CYCLE IN POS.-NEG. SEQUENCE	11
24		PROGRAMMED GUST CYCLES (4 AMPL.)	14
28		HIGH LOAD CYCLE IN NEG.-POS. SEQUENCE	85

FIG. 4.10 THE EFFECT OF THE SEQUENCE OF A PERIODIC HIGH LOAD CYCLE ON THE PROGRAM-FATIGUE LIFE OF 2024-T3 RIVETED LAP JOINTS.
RESULTS FROM REF. 39
FATIGUE LIFE IN PROGRAM PERIODS, 1 PERIOD = 432300 CYCLES
EACH RESULT IS THE MEDIAN OF 7 TESTS



SPECIMEN : RIVETED LAP JOINT, 2 ROWS OF 5 RIVETS, 2024 – T3 MATERIAL

CYCLES PER FLIGHT (X)			5	10	49	99	999
LIFE	CYCLES	a	19740	30380	58400	59850	125800
		b	22400	28350	51500	55431	84600
	FLIGHTS	a	3290	2762	1168	598	126
		b	3733	2577	1010	554	85
$\sum n/N$		a	0.53	0.51	0.44	0.37	0.63
		b	0.50	0.48	0.38	0.34	0.42

ALL DATA ARE THE MEAN OF THREE TESTS

FOR GUSTS ONLY N = 205300 CYCLES. FOR GTAC ONLY (S_{max} = 21.7 AND S_{min} = 2.5 kg/mm^2)

N = 7360 CYCLES

RESULTS REPORTED BY BARROIS (REF. 70)

FIG. 4.11 THE EFFECT OF GROUND–TO–AIR CYCLES ON THE FATIGUE LIFE IN A SIMPLIFIED FLIGHT SIMULATION TEST.

MATERIAL	LOAD SPECTRUM	S_{min} IN GTAC (kg/mm^2)	LIFE (FLIGHTS)
7075 – T6	SEVERE GUST SPECTRUM	0	2699
		–7.0	1334
Ti–8Al–1Mo–1V	FAIRLY SEVERE SPECTRUM REPRESENTATIVE OF A SUPERSONIC TRANSPORT. TESTS AT ROOM TEMPERATURE	0	>52000
		–10.5	16600
		–21.1	8500

7075 – T6 SHEET SPECIMENS, TWO EDGE NOTCHES, K_t = 4

Ti–8Al–1Mo–1V, Ti ALLOY SHEET SPECIMENS, QUASI ELLIPTICAL HOLE, K_t = 4

ALL DATA ARE THE MEAN OF 5 – 7 TESTS

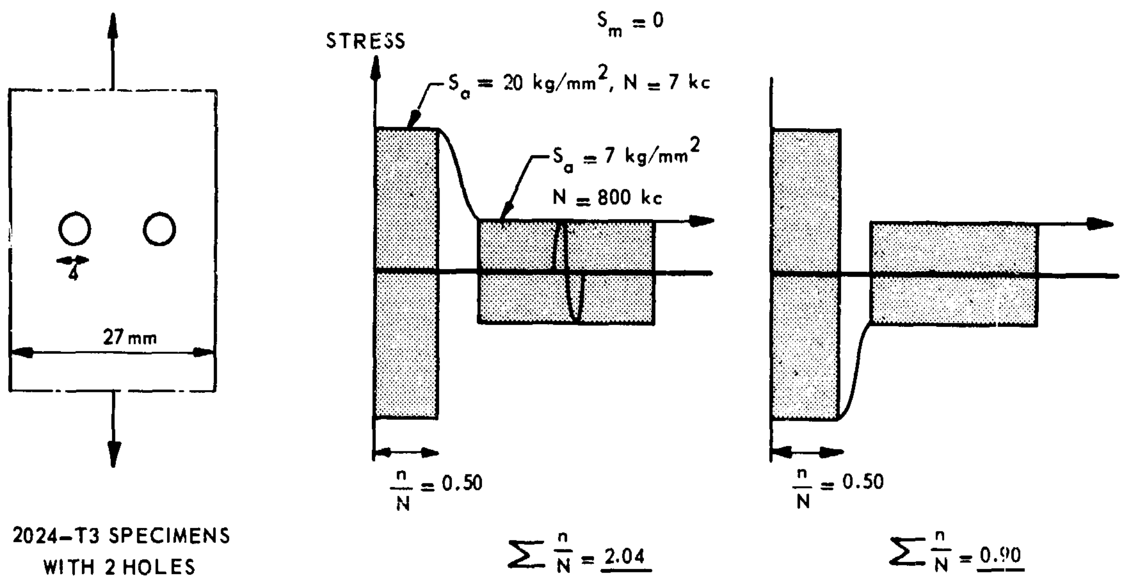
RESULTS REPORTED BY NAUMANN (REF. 67) AND BY IMIG AND ILLG (REF. 80).

FIG. 4.12 THE EFFECT OF THE MINIMUM STRESS OF THE GROUND–TO–AIR CYCLE ON THE FATIGUE LIFE IN RANDOM FLIGHT SIMULATION TESTS.

MATERIAL	CRACK PROPAGATION LIFE (FLIGHTS)		RATIO
	WITHOUT GTAC	WITH GTAC	
7075–T6	7518	5062	1.5
2024–T3	20869	11781	1.8

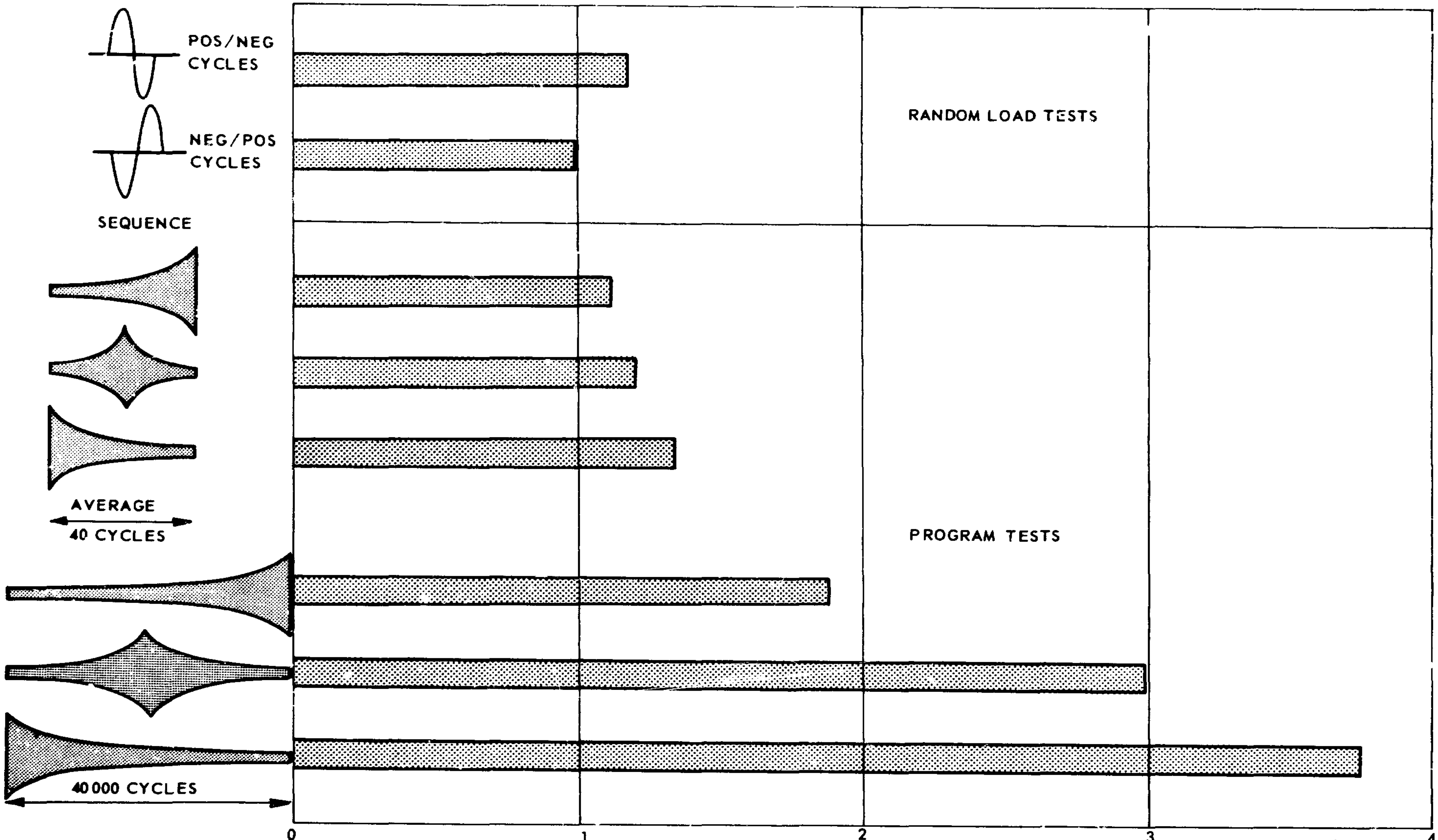
SHEET SPECIMENS WITH A CENTRAL CRACK, SPECIMEN WIDTH 160 mm.
CRACK LIFE COVERS PROPAGATION FROM $2\ell = 20$ mm TO COMPLETE FAILURE.
FLIGHT SIMULATION LOADING WITH GUST SPECTRUM, $S_m = 7.0$ kg/mm^2, S_{min} IN GTAC $= -3.4$ kg/mm^2
ALL DATA ARE MEAN RESULTS OF 4 TESTS.
RESULTS REPORTED BY SCHIJVE, JACOBS AND TROMP (REFS. 77, 78)

FIG. 4.13 EFFECT OF THE GTAC ON CRACK PROPAGATION LIFE UNDER FLIGHT SIMULATION LOADING.



RESULTS REPORTED BY WÅLLGREN (REF. 81), MEAN VALUES OF 9 TESTS

FIG. 4.14 TWO–STEP TESTS WITH DIFFERENT WAYS FOR CHANGING THE AMPLITUDE



CRACK PROPAGATION TESTS ON 2024 – T 3 ALCLAD SHEET SPECIMENS (WIDTH 160 mm, $t = 2$ mm)
RESULTS FROM REFS. 96, 97 MEAN VALUES OF 2 – 4 TESTS.

FIG. 4.16 EFFECTS OF LOAD SEQUENCE AND SIZE OF PERIOD ON PROGRAM FATIGUE LIFE. COMPARISON WITH RANDOM LOAD TEST RESULTS.

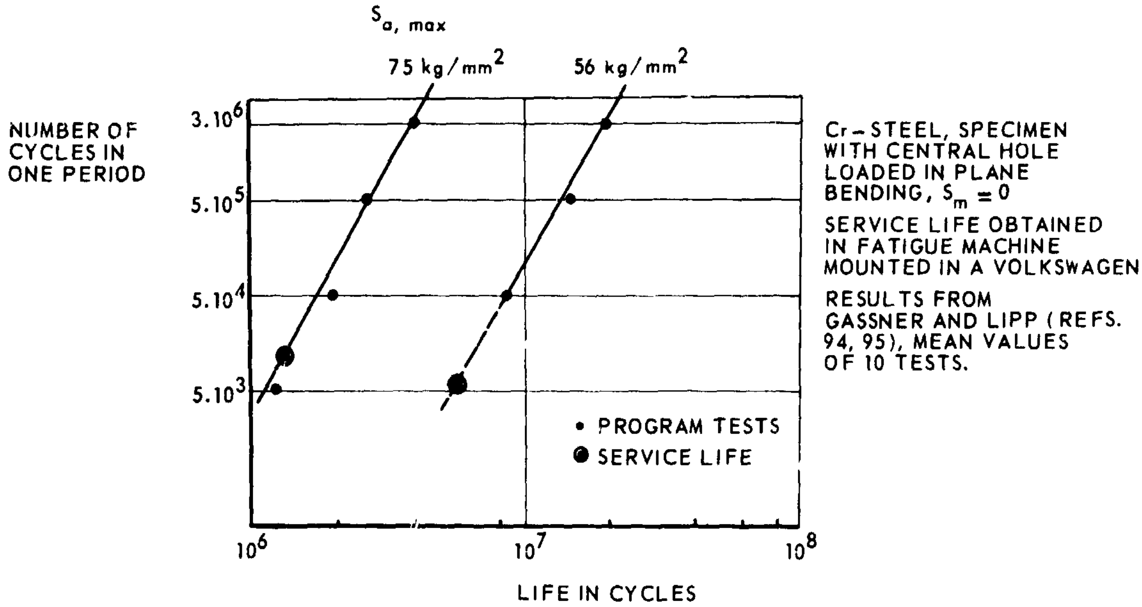
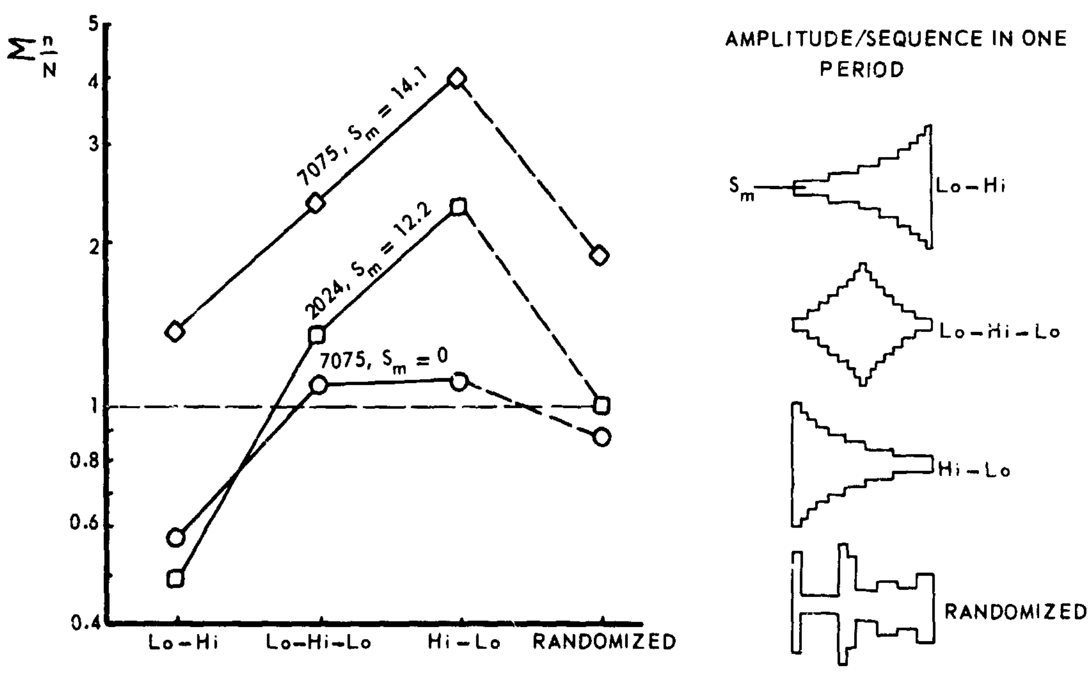
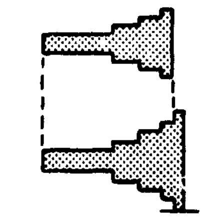
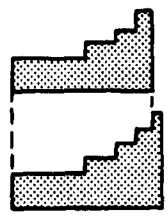
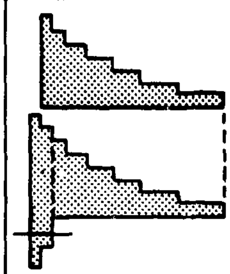


FIG. 4.15 EFFECT OF SIZE OF PERIOD ON PROGRAM FATIGUE LIFE AND COMPARISON WITH RANDOM LOAD FATIGUE LIFE



TESTS ON EDGE NOTCHED SPECIMENS, $K_t = 4$, S_m IN kg/mm² RESULTS REPORTED BY HARDRATH ET AL (REFS. 30,31), MEAN VALUES OF 3 – 4 TESTS.

FIG. 4.17 THE EFFECT OF THE AMPLITUDE SEQUENCE ON THE FATIGUE LIFE IN PROGRAM TESTS.

SPECIMEN SPECTRUM	S_a- SEQUENCE IN ONE PERIOD	LIFE		LIFE RATIO		EFFECT OF ADDING
		CYCLES	$\Sigma n/N$	CYCLES/CYCLES	Σ/Σ	HIGH S_a- CYCLES
RIVETED JOINT 2024–T3 GUST SPECTRUM (REF. 39)		6 130 000 13 060 000	1.31 2.90	2.14	2.21	INCREASED LIFE
TAIL PLANE 7075–T6 MANEUVER SPECTRUM (REF. 48)		44 000 91 800	3.0 6.5	2.08	2.17	INCREASED LIFE
EDGE NOTCHED SPECIMEN 7075–T6 MANEUVER SPECTRUM (REF. 87)		41 520 17 730	2.16 1.31	0.43	0.61	REDUCED LIFE

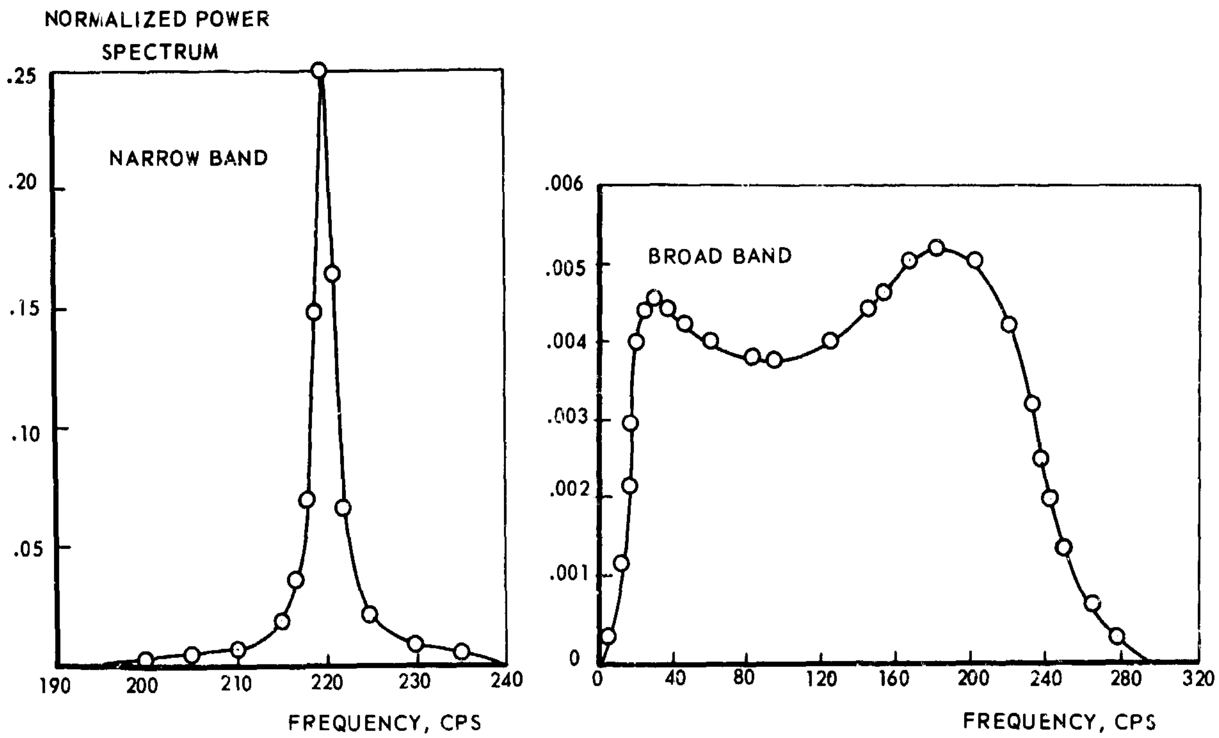
DATA ARE MEAN VALUES OF 7,7, 3, 1, 7 AND 6 TESTS RESPECTIVELY.

FIG. 4.18 DIFFERENT EFFECTS OF HIGH–AMPLITUDE CYCLES IN PROGRAM TESTS.

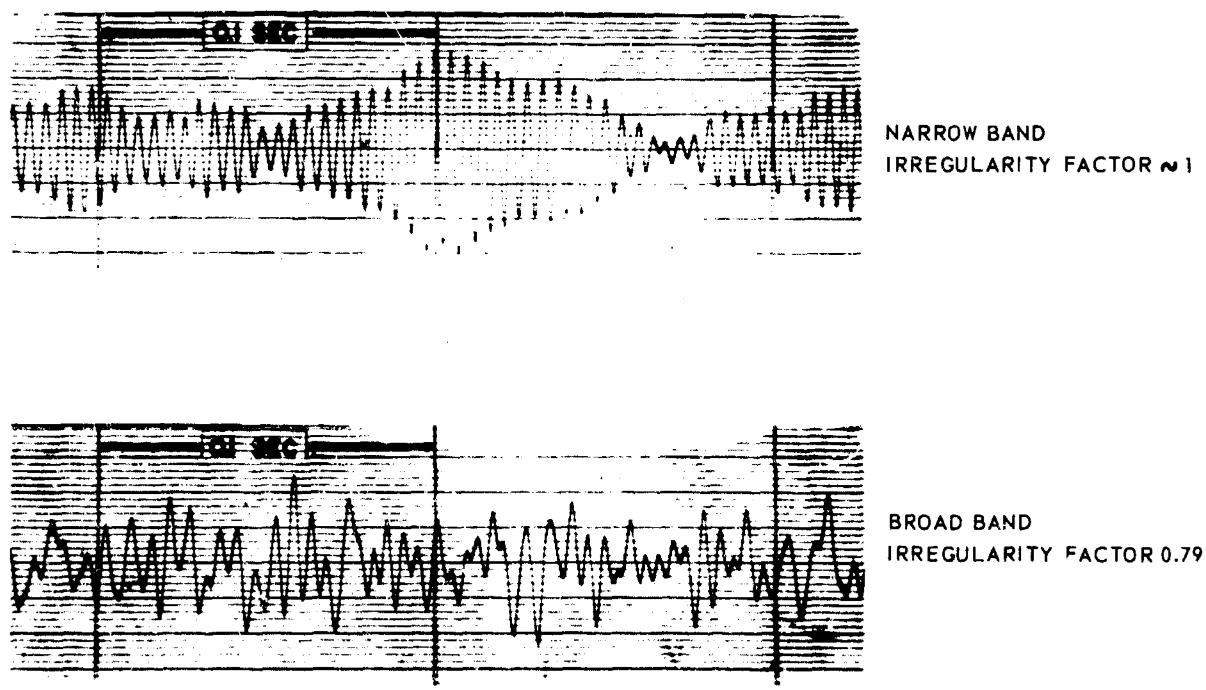
RIVETED JOINT	S_m (kg/mm^2)	S_a–VALUES BELOW FATIGUE LIMIT	LIFE		RATIO	
			PERIODS	CYCLES(KC)	PERIOD/PERIOD	CYCLE/CYCLE
2024–T3 DOUBLE LAP JOINT	3.5	3	15	166 000	1.1	160
		OMITTED	16	1 040		
7075–T6 SINGLE STRAP JOINT	9.3	1	98	3 080	1.3	3.3
		OMITTED	129	900		
	4.7	2	20	107 000	1.7	16
		OMITTED	34	6 500		

RESULTS REPORTED BY WALLGREN (REF. 83). ALL DATA ARE MEAN VALUES OF 2–4 TESTS.

FIG. 4.19 THE EFFECT OF OMITTING LOW–AMPLITUDE CYCLES FROM A PROGRAM TEST ON FATIGUE LIFE AND TESTING TIME.

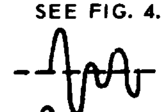
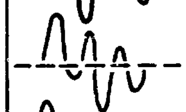
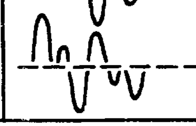


a. POWER SPECTRA NORMALIZED TO GIVE UNIT AREA UNDER THE CURVE.



b. STRESS-TIME HISTORY FOR NARROW-BAND AND BROAD-BAND LOADING.

FIG. 4.20 ILLUSTRATION OF THE EFFECT OF THE POWER SPECTRAL DENSITY FUNCTION ON THE LOAD-TIME HISTORY (REF. 101)

SEQUENCE	EXAMPLE OF SEQUENCE	RANDOM LOAD TESTS FATIGUE LIFE (CYCLES)	RANDOM LOAD TESTS RATIO	FLIGHT – SIMULATION TESTS FATIGUE LIFE (FLIGHTS)	FLIGHT – SIMULATION TESTS RATIO
RANDOMIZED BLOCK (PROGRAM TEST)	SEE FIG. 4.17	617 000	1.56		
RANDOM SEQUENCE OF COMPLETE CYCLES		395 500	1	1 334	1
RANDOM SEQUENCE OF HALF CYCLES, ALTERNATELY POS. AND NEG.		504 700	1.28	1 515	1.14
RANDOM SEQUENCE OF HALF CYCLES NO RESTRICTION ON SEQUENCE OF POS. AND NEG.		598 300	1.51	1 588	1.19

TESTS ON EDGE NOTCHED 7075 – T6 SPECIMENS ($K_t = 4$). GUST SPECTRUM
RESULTS REPORTED BY NAUMANN (REF. 67), MEAN VALUES OF 6 TESTS

FIG. 4.21 THE EFFECT OF THE CYCLE SEQUENCE IN RANDOM LOADING AND FLIGHT–SIMULATION LOADING.

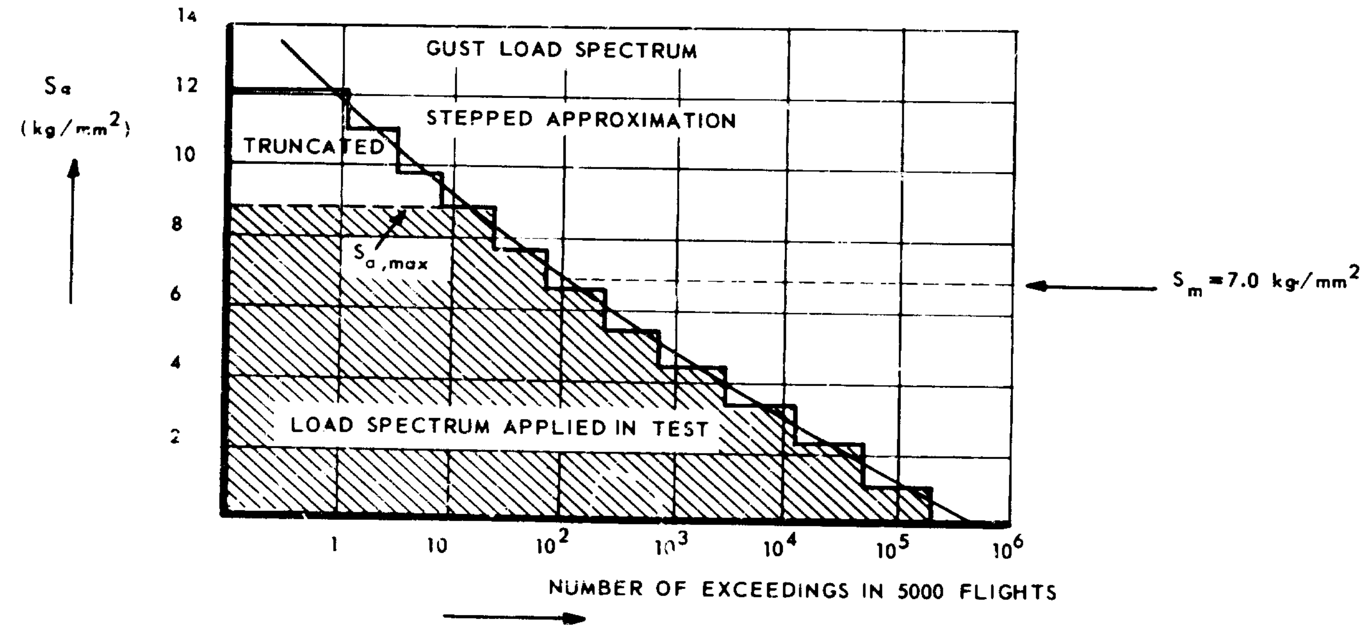
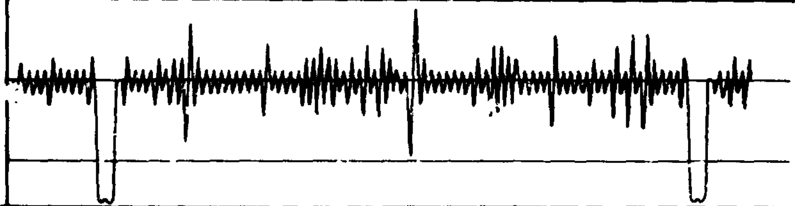
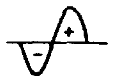


FIG.4.22 LOAD SPECTRUM APPLIED IN NLR– TESTS ON CRACK PROPAGATION UNDER RANDOM FLIGHT – SIMULATION LOADING (REFS 77,78)

	LOAD SEQUENCE (FLIGHT No. 19, TYPE F)	REMARKS	CRACK PROPAGATION LIFE (a)			
			FLIGHTS		RATIO	
			2024 – T 3	7075 – T 6	2024 – T3	7075 – T6
B			11 781	5062	1	1
C		GUST CYCLES IN REVERSED SEQUENCE 	11 184	4851	0.95	0.96
H		PROGRAMMED SEQUENCE OF GUST CYCLES	11 365	5061	0.96	i.00

(a) THE CRACK LIFE COVERS PROPAGATION FROM $2\ell = 20$ mm TO COMPLETE FAILURE OF THE SHEET SPECIMEN, WIDTH 160 mm. ALL DATA ARE MEAN VALUES OF 4 OR 6 TESTS.

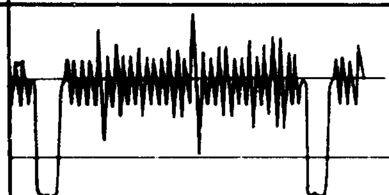
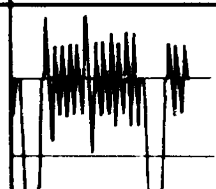
FIG. 4.23 THE EFFECT OF THE CYCLE SEQUENCE IN FLIGHT-SIMULATION TESTS. CRACK PROPAGATION TESTS ON SHEET MATERIAL. (REFS. 77, 78)

INVESTIGATION (a)	PERCENTAGE INCREASE OF LIFE IN FLIGHTS CAUSED BY	
	OMITTING LOW-AMPLITUDE CYCLES	OMITTING TAXIING LOADS (b)
NAUMANN (1964, REF. 67)	6 % AND 17 %	
GRASSNER AND JACOBY (1964, 1965, REF. 66, 73)	150 %	0
BRANGER (1967, REF. 110)	− 14 %	− 23 %
IMIG AND ILLG (1969, REF. 80)		2 % TO 12 %
NLR (SEE FIG. 4.25)	18 % TO 93 %	− 8 % AND + 16 %
SCHIJVE AND DE RIJK (1971, REF. 63)	0 − 50 %	

(a) FOR MORE INFORMATION SEE TABLE 4.7

(b) S_{min} IN THE GTAC HAD THE SAME VALUE WITH AND WITHOUT TAXIING LOADS

FIG. 4,24 THE EFFECT OF OMITTING LOW-AMPLITUDE CYCLES FROM FLIGHT-SIMULATION TESTS.

	LOAD SEQUENCE (FLIGHT NO. 19, TYPE F)	REMARKS	CRACK PROPAGATION LIFE (a)			
			FLIGHTS		RATIO	
			2024–T3	7075–T6	2024–T3	7075–T6
A		RANDOM FLIGHT SIMULATION	10 876	5 889	0.92	1.16
B		TAXIING LOADS OMITTED	11 781	5 062	1	1
D		SMALL GUST CYCLES OMITTED (S_a = 1.1 kg/mm^2)	13 924	7 006	1.18	1.38
E		MORE SMALL GUST CYCLES OMITTED S_a = 1.1 AND 2.2 kg/mm^2	20 759	9 779	1.76	1.93
F		ONE GUST LOAD PER FLIGHT ONLY (THE LARGEST ONE)	36 583	14 556	3.11	2.88

(a) THE CRACK LIFE COVERS PROPAGATION FROM 2ℓ = 2 mm TO COMPLETE FAILURE OF THE SHEET SPECIMEN, WIDTH 160 mm
ALL DATA ARE MEAN VALUES OF 4 OR 6 TESTS.

FIG. 4.25 THE EFFECT OF OMITTING LOW–AMPLITUDE GUST CYCLES AND TAXIING LOADS IN FLIGHT–SIMULATION TESTS. CRACK PROPAGATION TESTS ON SHEET MATERIAL (REFS. 77, 78)

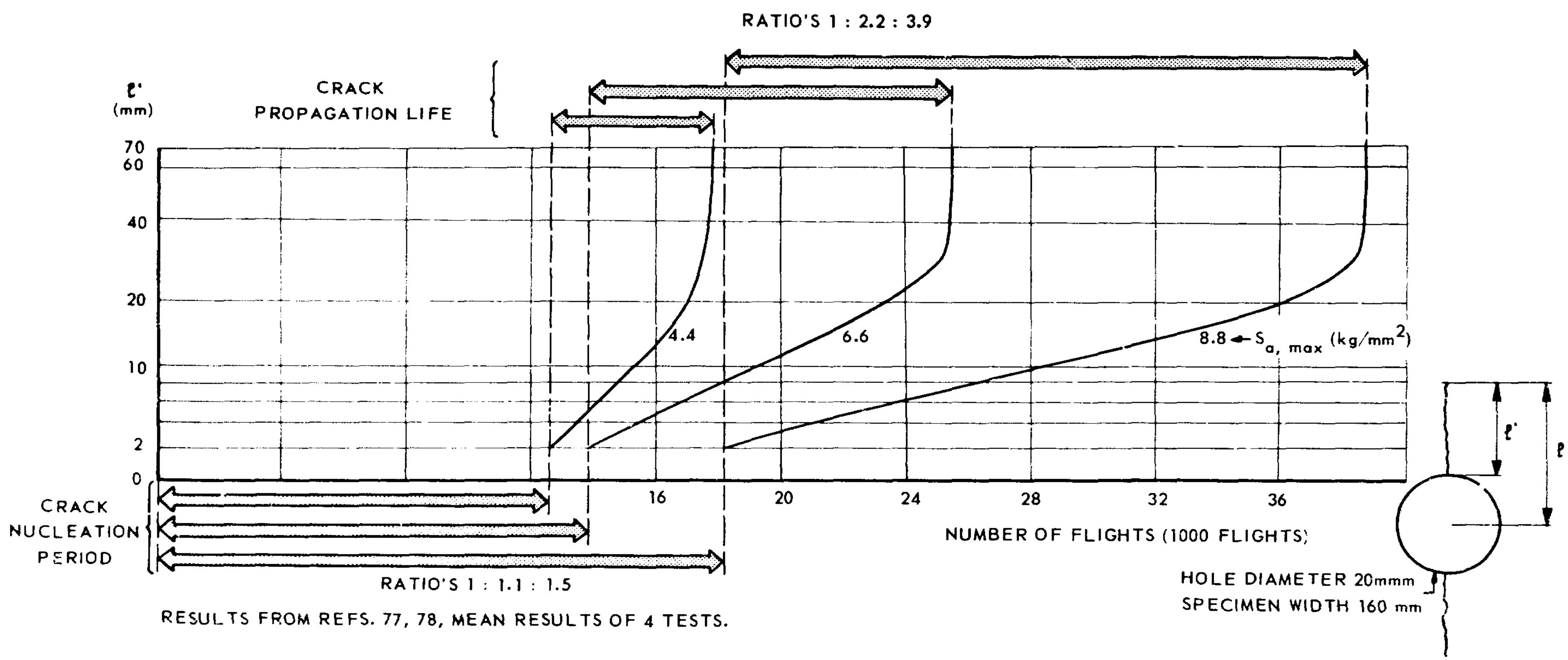
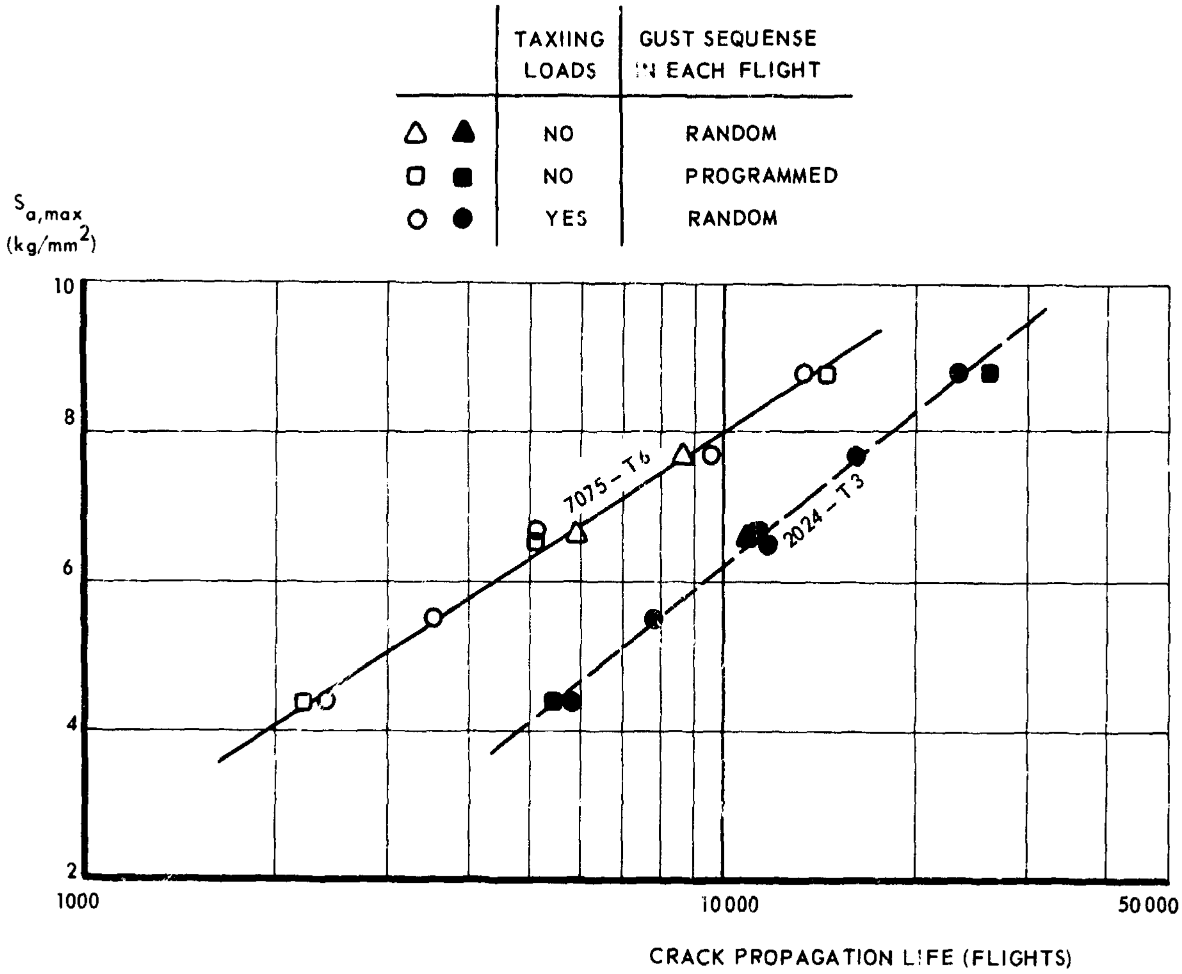
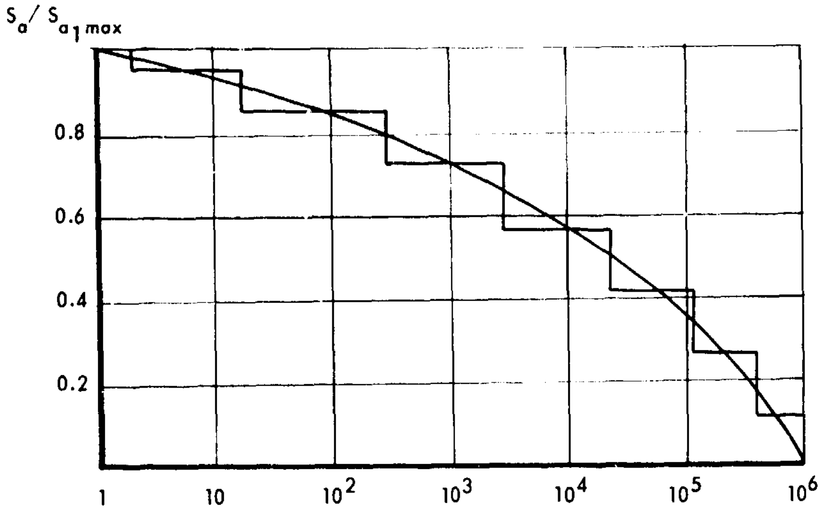


FIG. 4.26 EFFECT OF TRUNCATION LEVEL ($S_{a,\ max}$) ON THE CRACK NUCLEATION PERIOD (TO $\ell' = 2$ mm) AND THE CRACK PROPAGATION LIFE. RANDOM FLIGHT-SIMULATION TESTS ON 2024-T3 ALCLAD SHEET SPECIMENS WITH A CENTRAL HOLE.



ALL DATA POINTS ARE MEAN VALUES OF 4–6 TESTS. THE CRACK LIFE COVERS PROPAGATION FROM 2ℓ = 20 mm TO COMPLETE FAILURE. SPECIMEN WIDTH 160 mm.

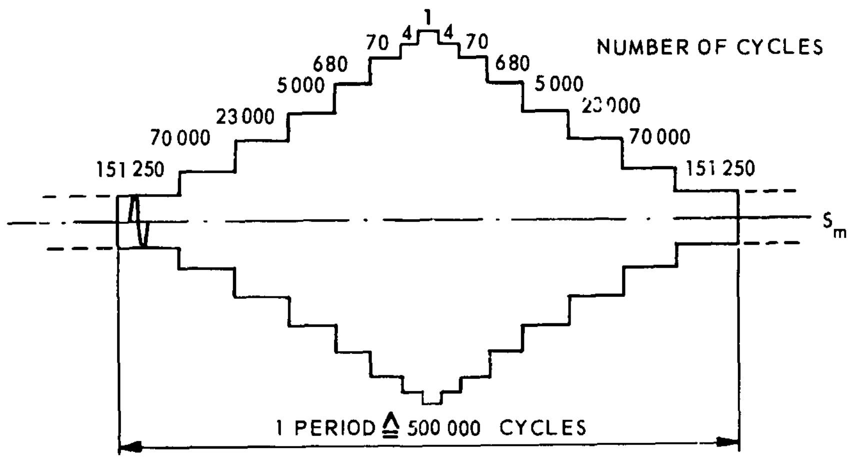
FIG. 4.27 THE EFFECT OF TRUNCATING THE GUST LOAD SPECTRUM ON THE CRACK PROPAGATION LIFE IN FLIGHT-SIMULATION TESTS (REFS. 77, 78)



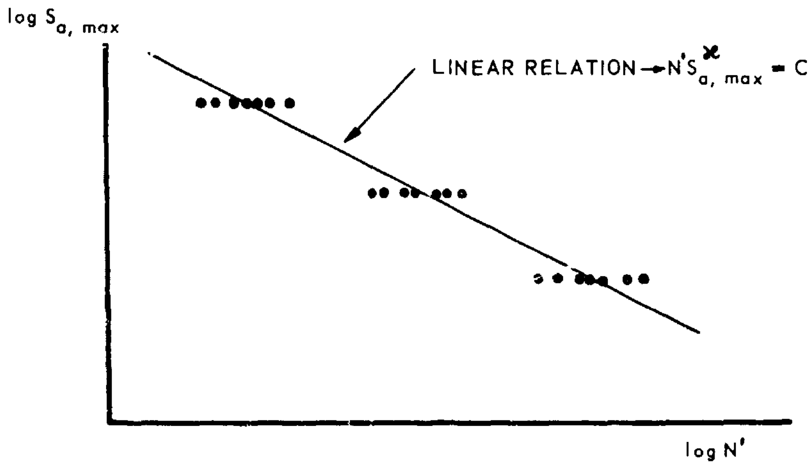
LOG BINOMIAL DISTRIBUTION

STEPPED APPROXIMATION	
$S_a / S_{a,\ max}$	NUMBER OF EXCEEDINGS
1	2
0.95	18
0.85	298
0.725	3018
0.575	23000
0.425	115000
0.275	395000
0.125	1000000

a. STANDARDIZED LOAD SPECTRUM



b. LOAD SEQUENCE IN PROGRAM TEST AND NUMBERS OF CYCLES IN ONE PERIOD



c. ENDURANCE CURVE, RESULTS FROM TESTS WITH DIFFERENT $S_{a,\ max}$ VALUES, BUT SAME $S_{a,\ max}/S_m$ RATIO.

FIG. 4.28 ENDURANCE CURVE OBTAINED IN STANDARDIZED PROGRAM TEST ACCORDING TO GASSNER

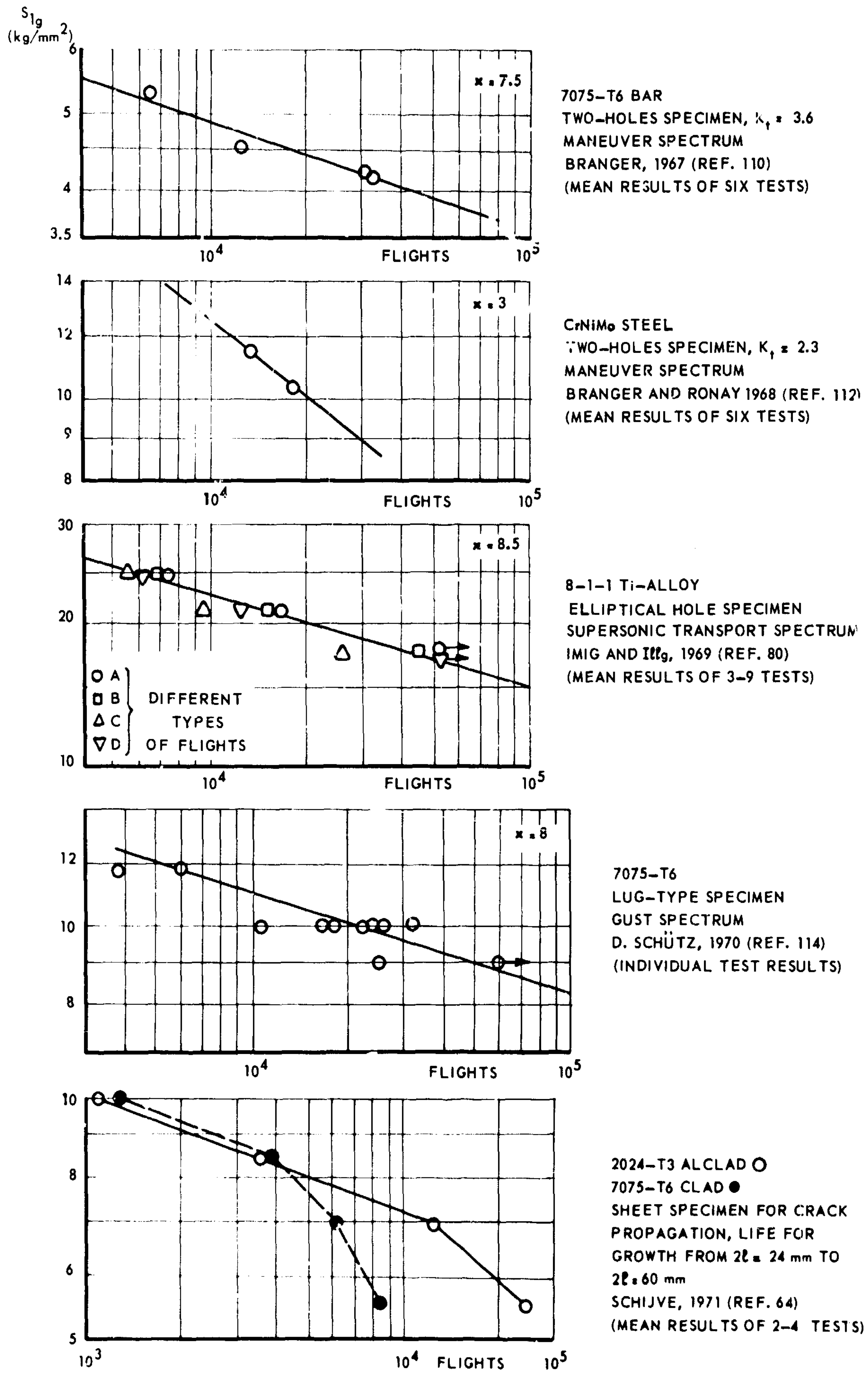


FIG. 4.29 THE EFFECT OF THE DESIGN STRESS LEVEL ON FATIGUE LIFE UNDER FLIGHT-SIMULATION LOADING. S_{1g} = CHARACTERISTIC 1g–STRESS LEVEL IN FLIGHT.

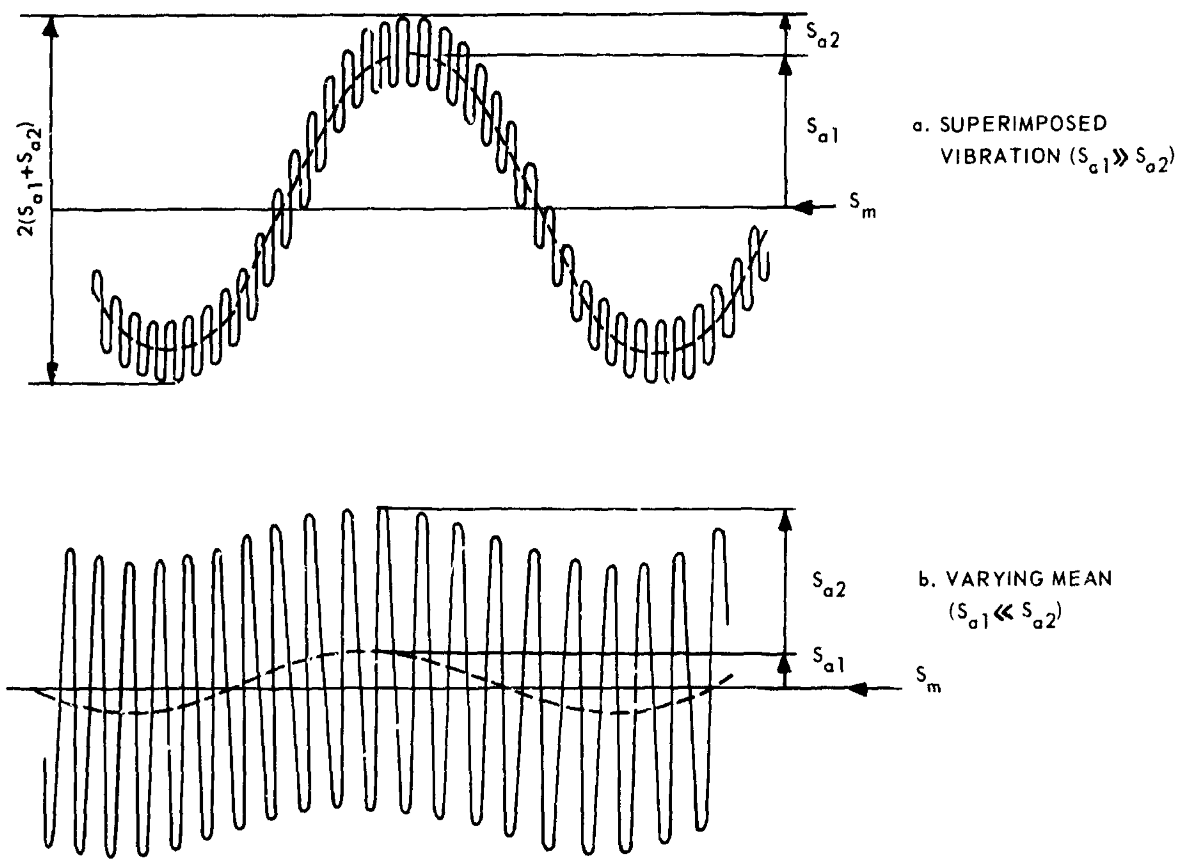


FIG. 4.30 TWO TYPES OF SUPERIMPOSED CYCLIC LOADS.

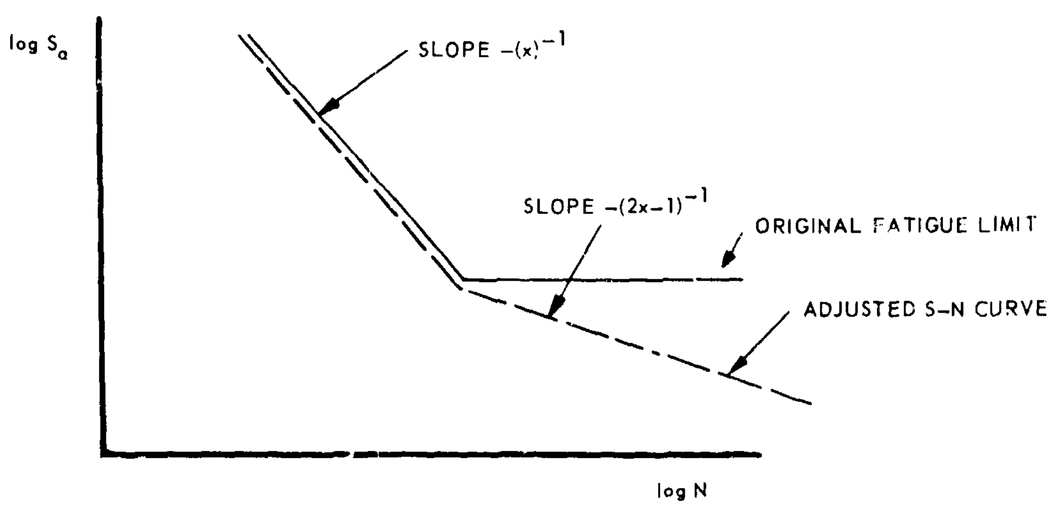


FIG. 5.1 ADJUSTED S–N CURVE FOR LIFE CALCULATIONS ACCORDING TO HAIBACH (REF.148)

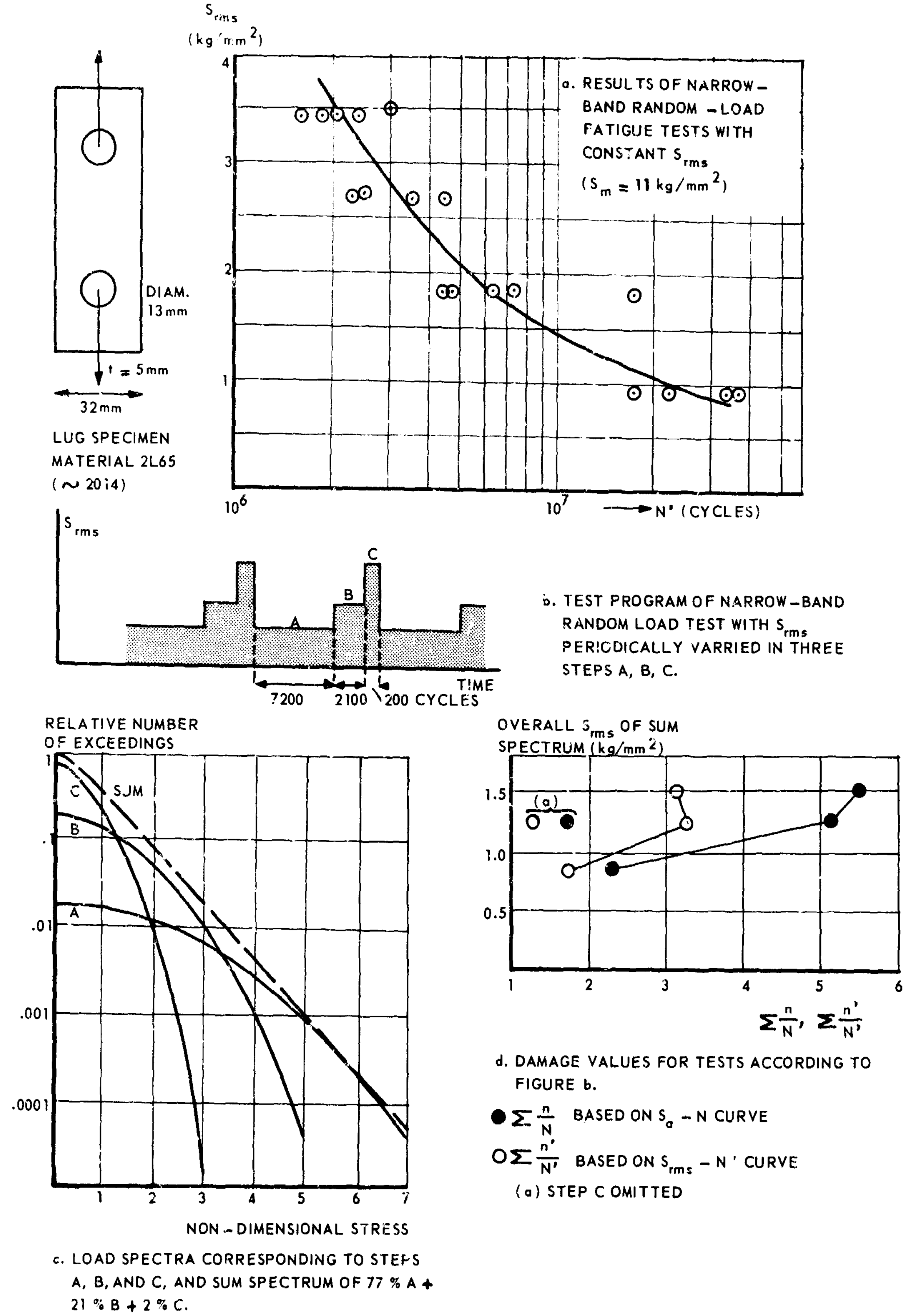


FIG. 5.2 NARROW-BAND RANDOM LOAD TESTS WITH CONSTANT S_{rms} AND PROGRAMMED S_{rms}. RESULTS FROM KIRKBY AND EDWARDS (REF. 99).

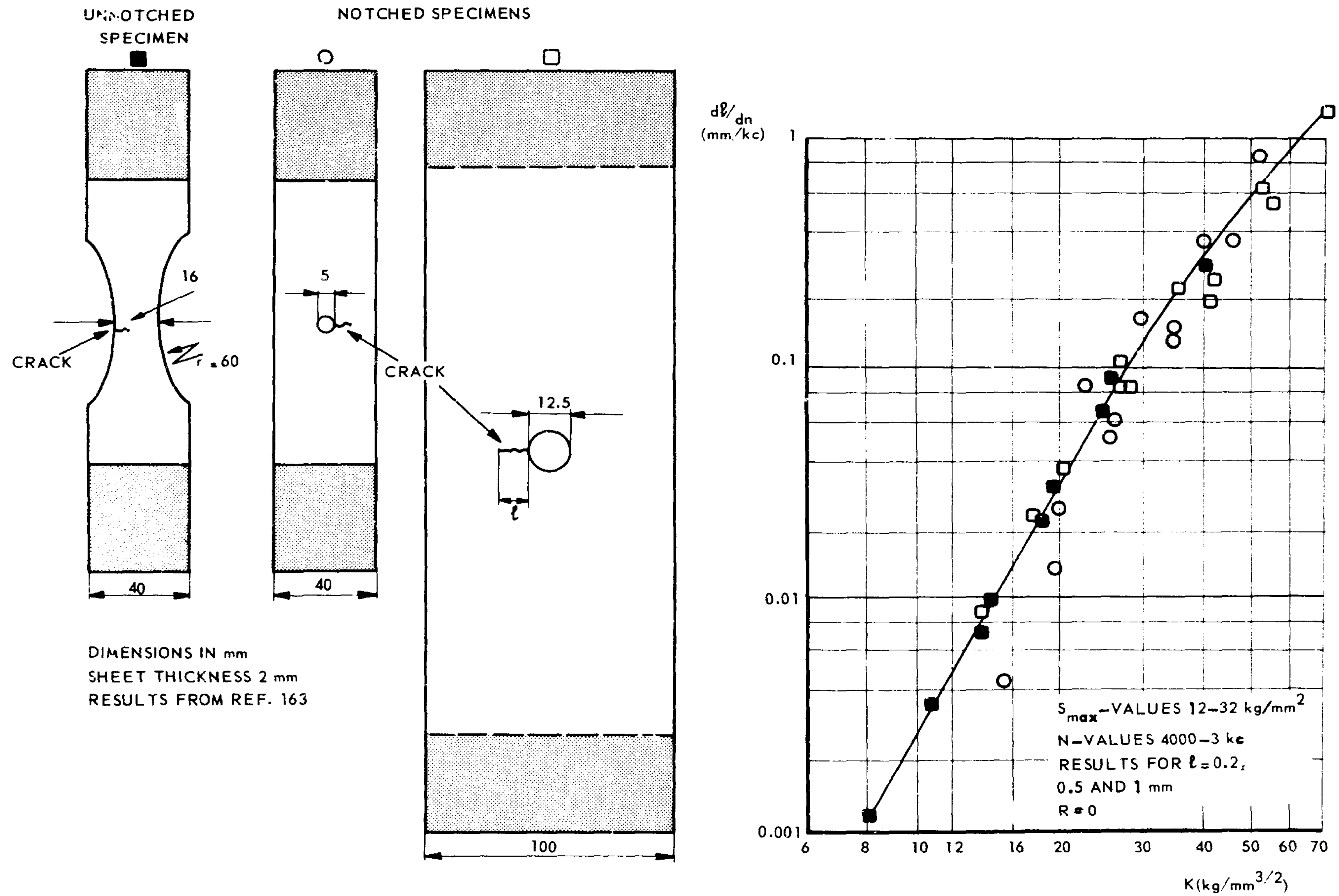
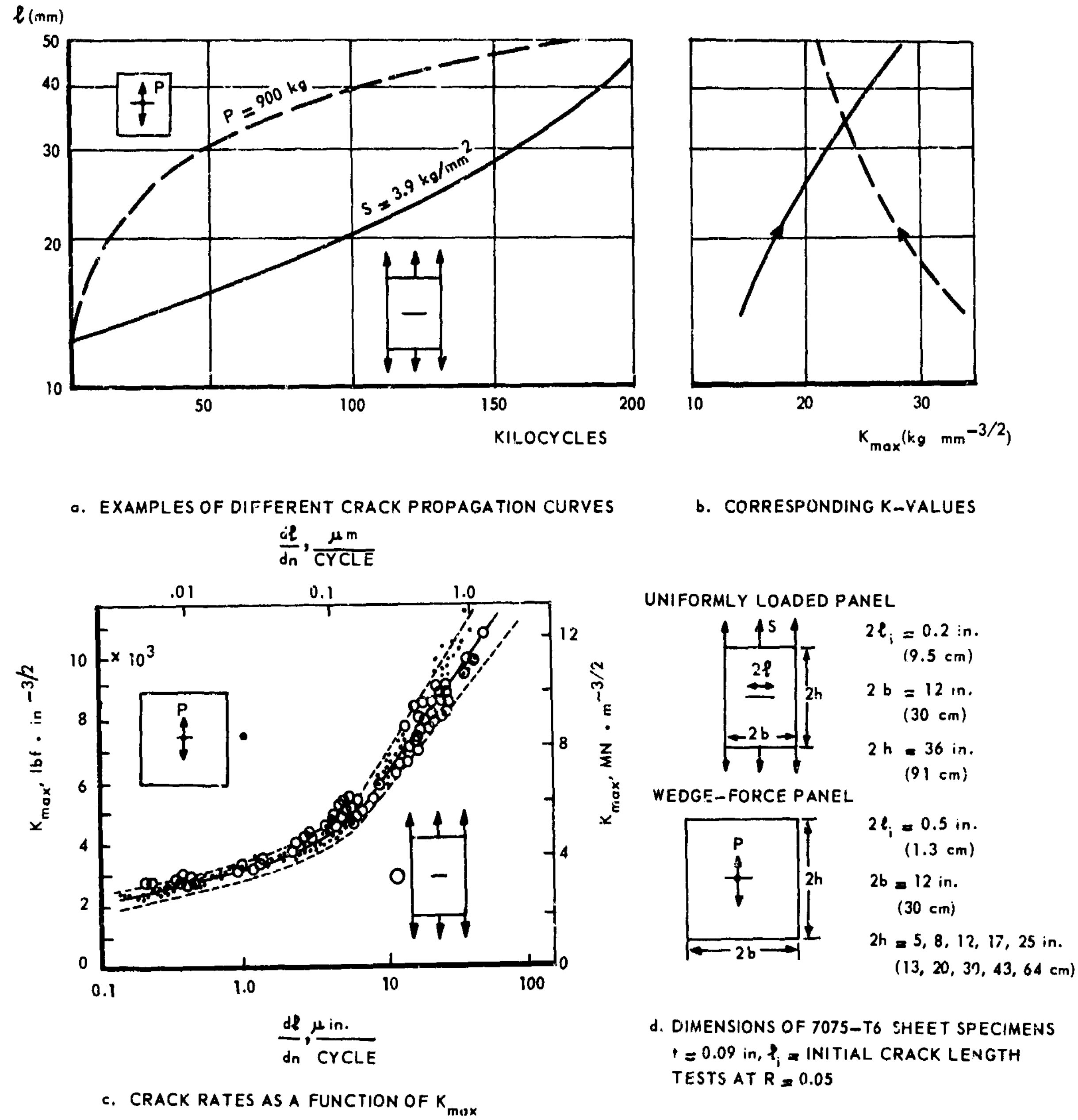


FIG. 5.3 THE CRACK RATE AS A FUNCTION OF THE STRESS INTENSITY FACTOR FOR SMALL CRACKS (0.2 – 1.0 mm)



a. EXAMPLES OF DIFFERENT CRACK PROPAGATION CURVES

b. CORRESPONDING K-VALUES

c. CRACK RATES AS A FUNCTION OF K_{max}

d. DIMENSIONS OF 7075-T6 SHEET SPECIMENS
t = 0.09 in, ℓ_i = INITIAL CRACK LENGTH
TESTS AT R = 0.05

FIG. 5.4 A COMPARISON BETWEEN THE CRACK RATES IN UNIFORMLY LOADED AND WEDGE - FORCE LOADED SPECIMENS
RESULTS FROM FIGGE AND NEWMAN (REF. 164).

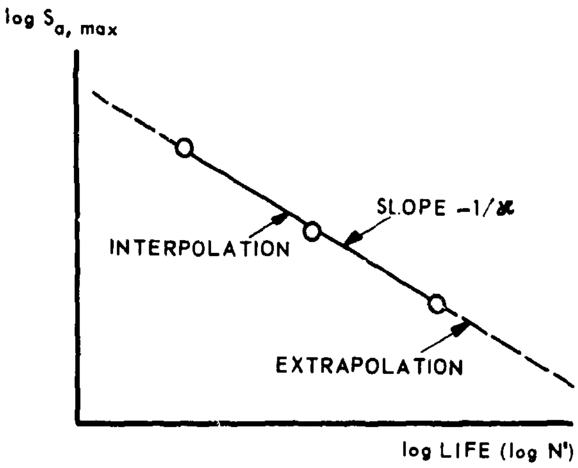


FIG. 5.5 a

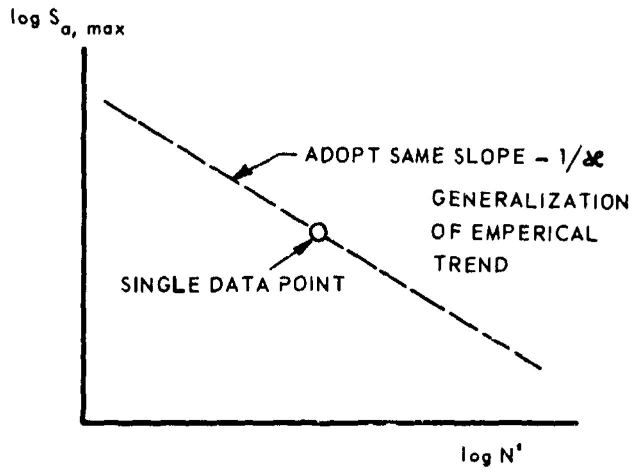


FIG. 5.5. b

EFFECT OF DESIGN STRESS LEVEL IN PROGRAM FATIGUE TESTS

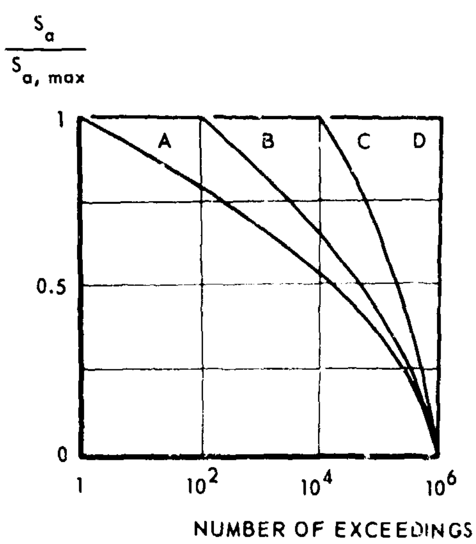


FIG. 5.5 c

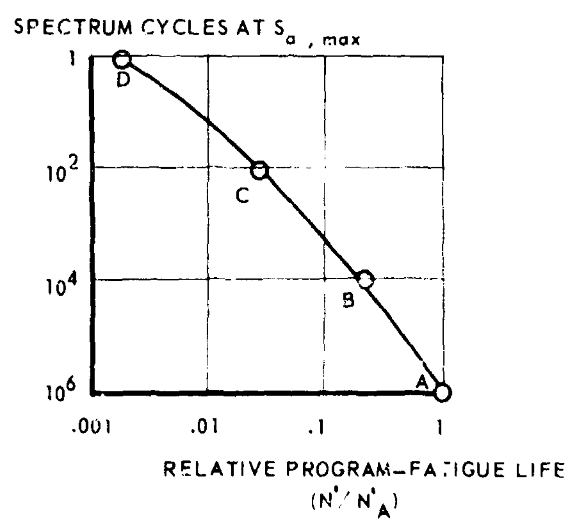


FIG. 5.5 d

EFFECT OF LOAD SPECTRUM SHAPE (A–D) ON PROGRAM FATIGUE LIFE

FIG. 5.5 SOME CONCEPTS OF GASSNER AND CO-WORKERS FOR EMPLOYING PROGRAM FATIGUE TEST DATA.

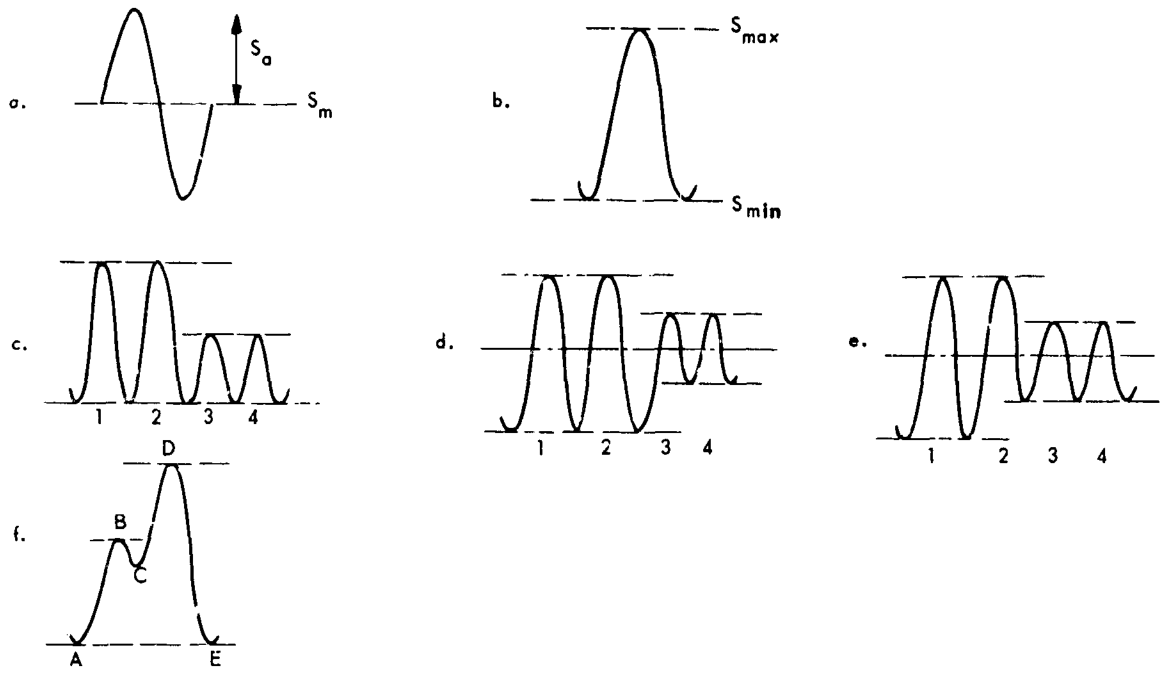


FIG. 6.1 EXAMPLES OF SIMPLE LOAD VARIATIONS REGARDING THE DEFINITION OF LOAD CYCLES

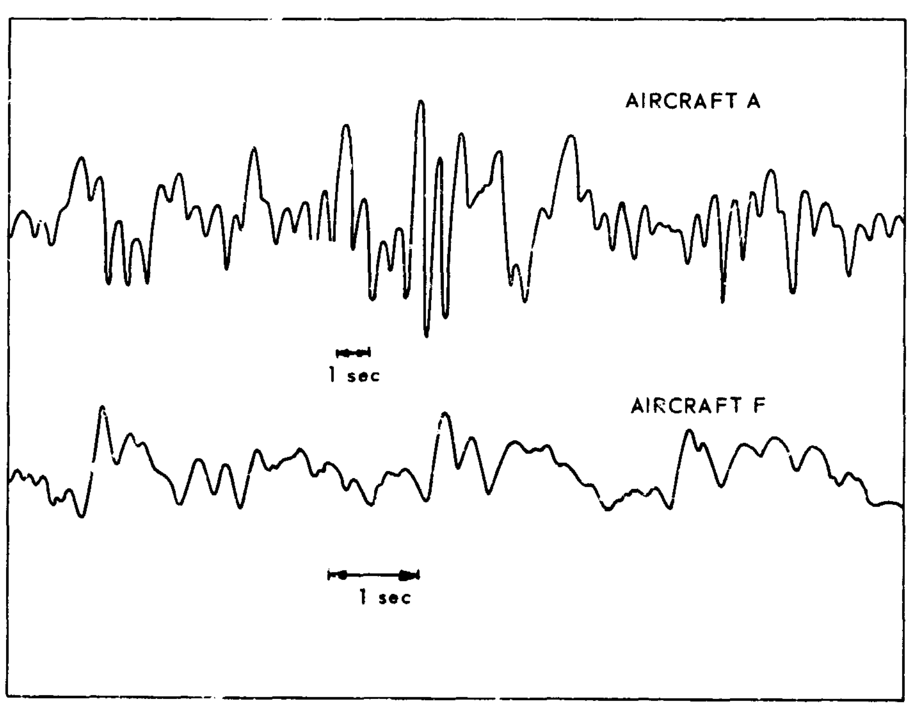


FIG. 6.2 STRAIN GAGE RECORDS OF THE WING BENDING MOMENT OF 2 AIRCRAFT FLYING IN TURBULENT AIR (REF. 174)

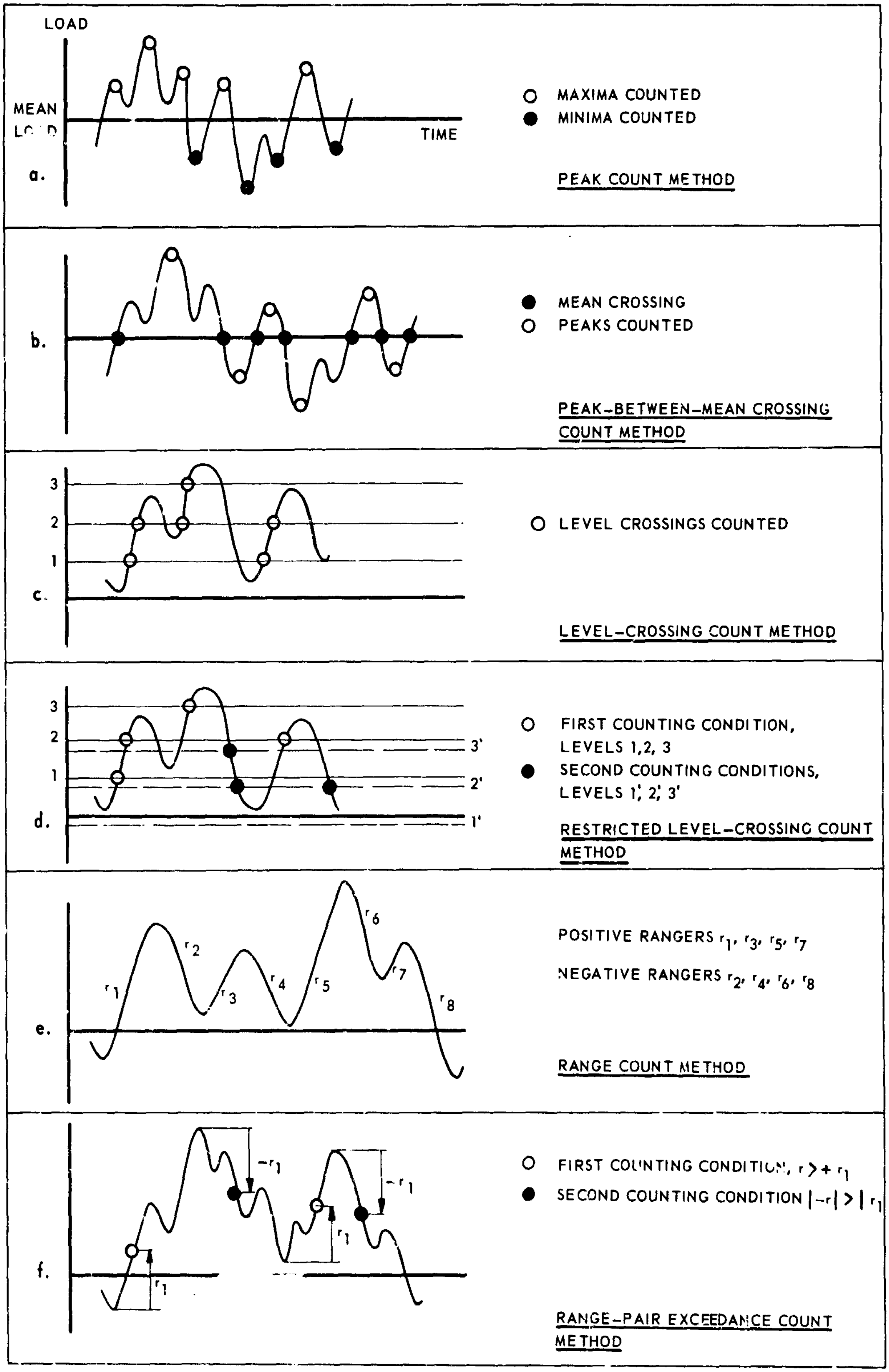


FIG. 6.3 SEVERAL COUNTING METHODS FOR A STATISTICAL DESCRIPTION OF A LOAD-TIME HISTORY. NAMES OF METHODS AFTER VAN DIJK (REF.176)

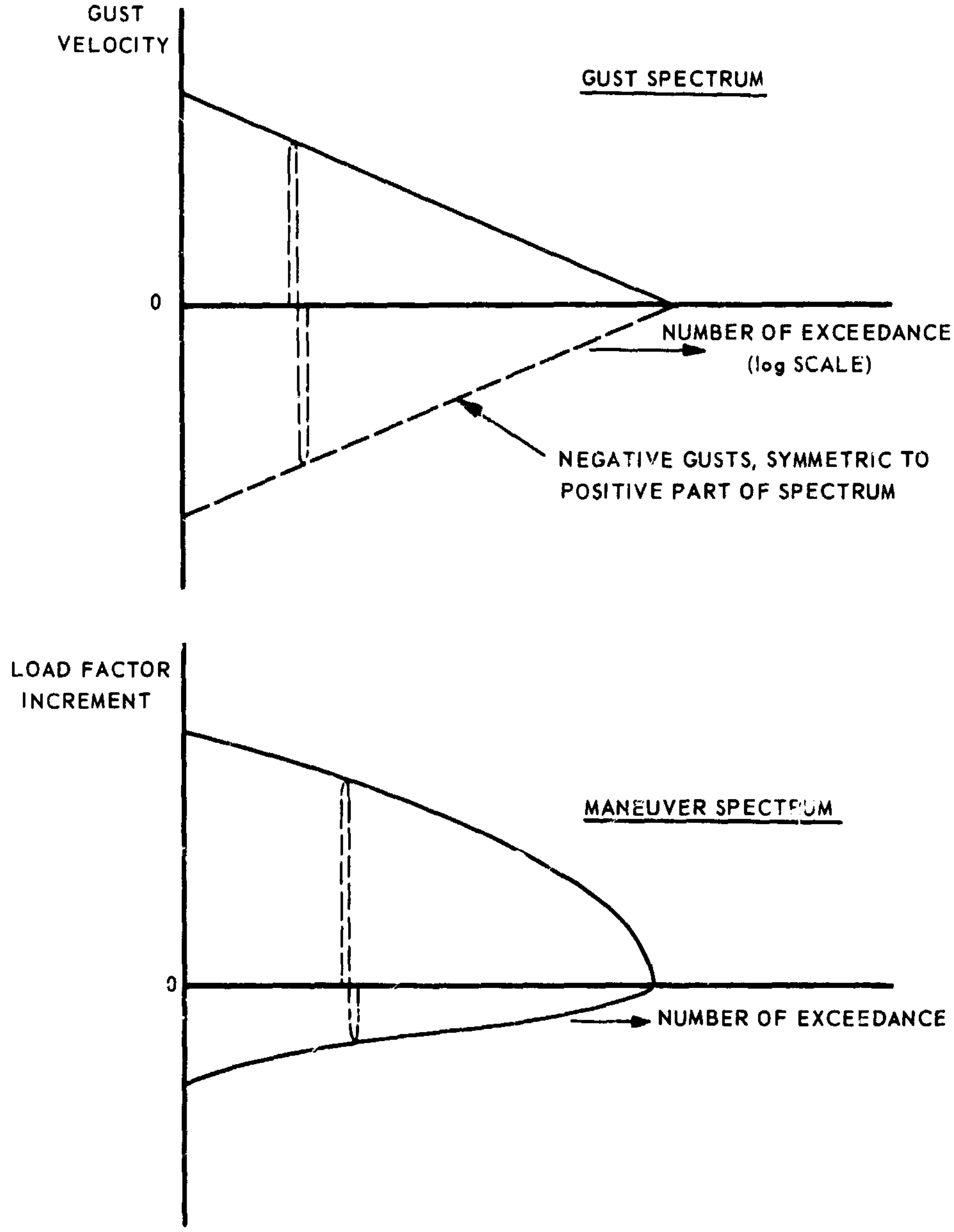


FIG. 6.4 DIFFERENCES BETWEEN GUST AND MANEUVER SPECTRA

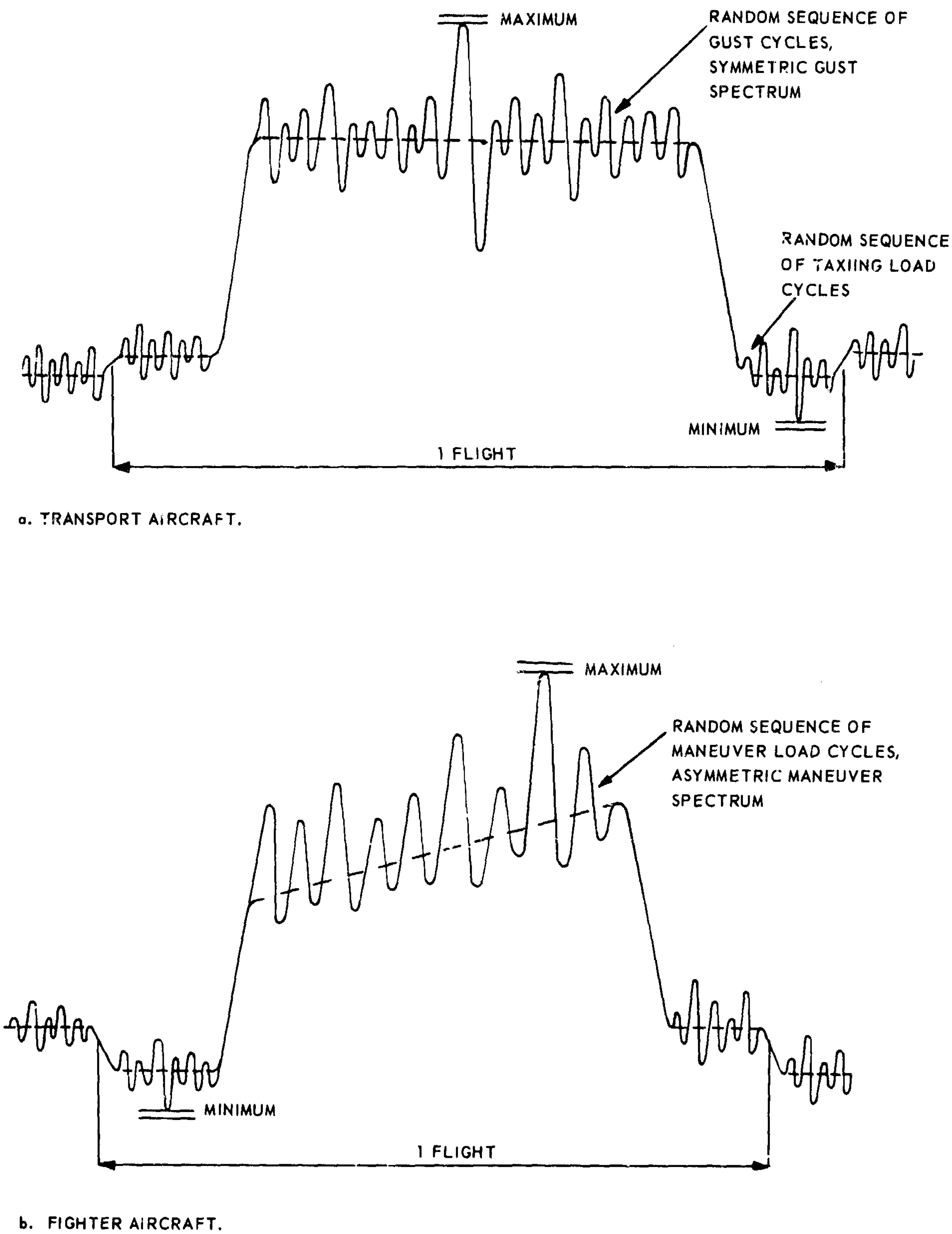


FIG. 6.5 TWO SIMPLIFIED EXAMPLES OF ESTIMATED FLIGHT-LOAD PROFILES FOR THE AIRCRAFT WING ROOT STRUCTURE.

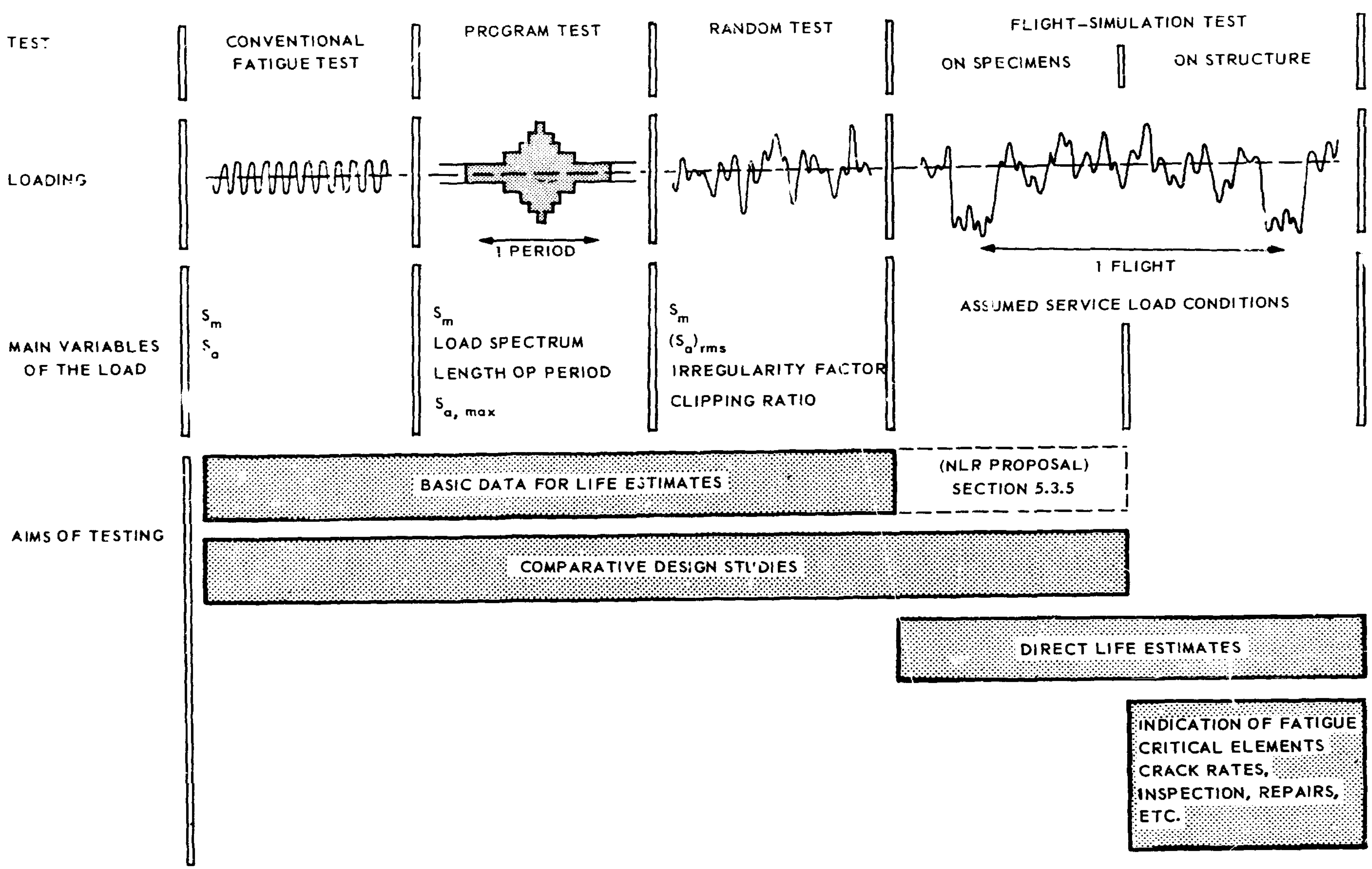


FIG. 7.1 SOME FATIGUE TEST LOAD SEQUENCES, MAIN VARIABLES AND TESTING PURPOSES.

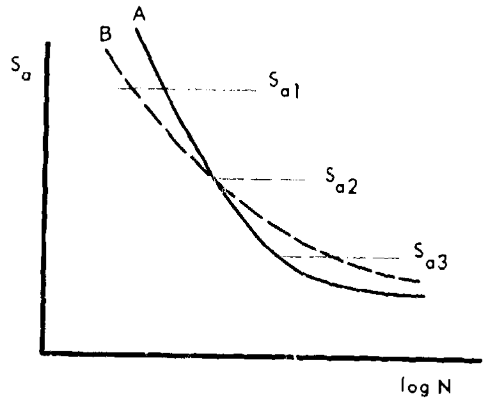


FIG. 7.2 TWO INTERSECTING S–N CURVES

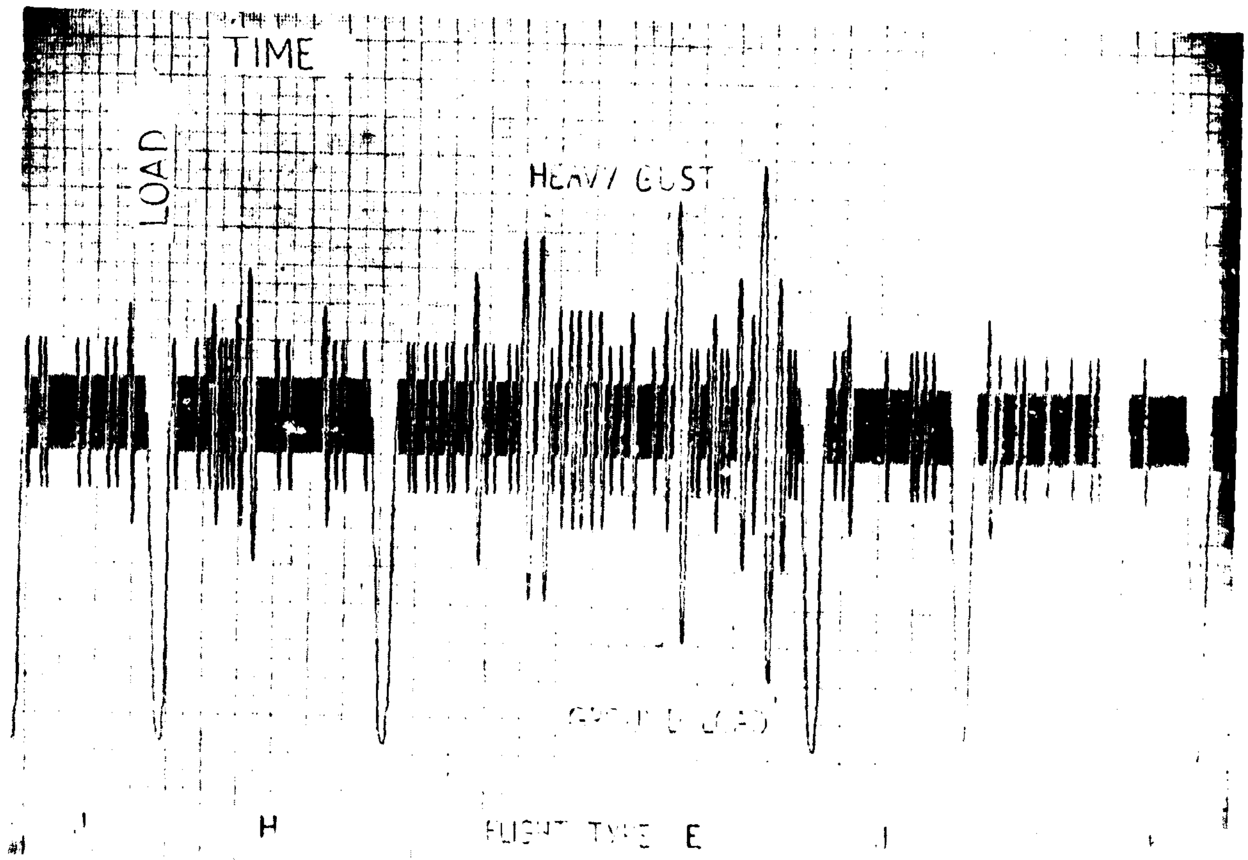


FIG. 7.3 SAMPLE OF A LOAD RECORD, ILLUSTRATING THE LOAD SEQUENCE APPLIED IN THE F-28 WING FATIGUE TEST. TEN DIFFERENT TYPES OF WEATHER CONDITION ARE SIMULATED, FLIGHT TYPE E CORRESPONDS TO FAIRLY SEVERE STORM WHILE FLIGHT TYPE K IS FLOWN IN GOOD WEATHER (REF. 96)

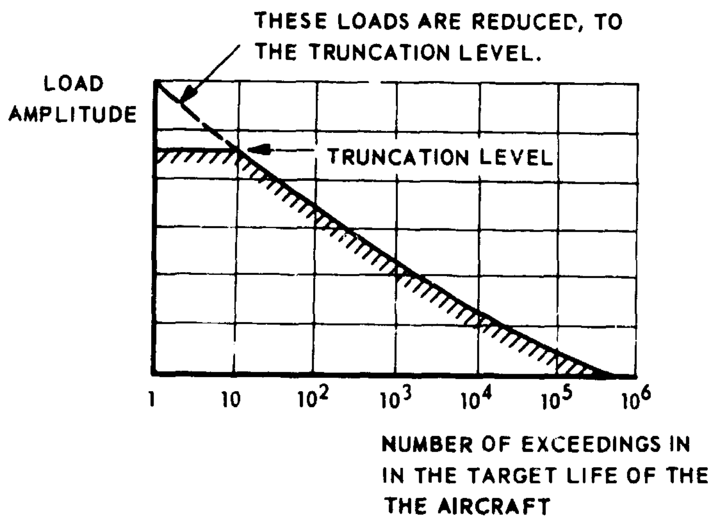
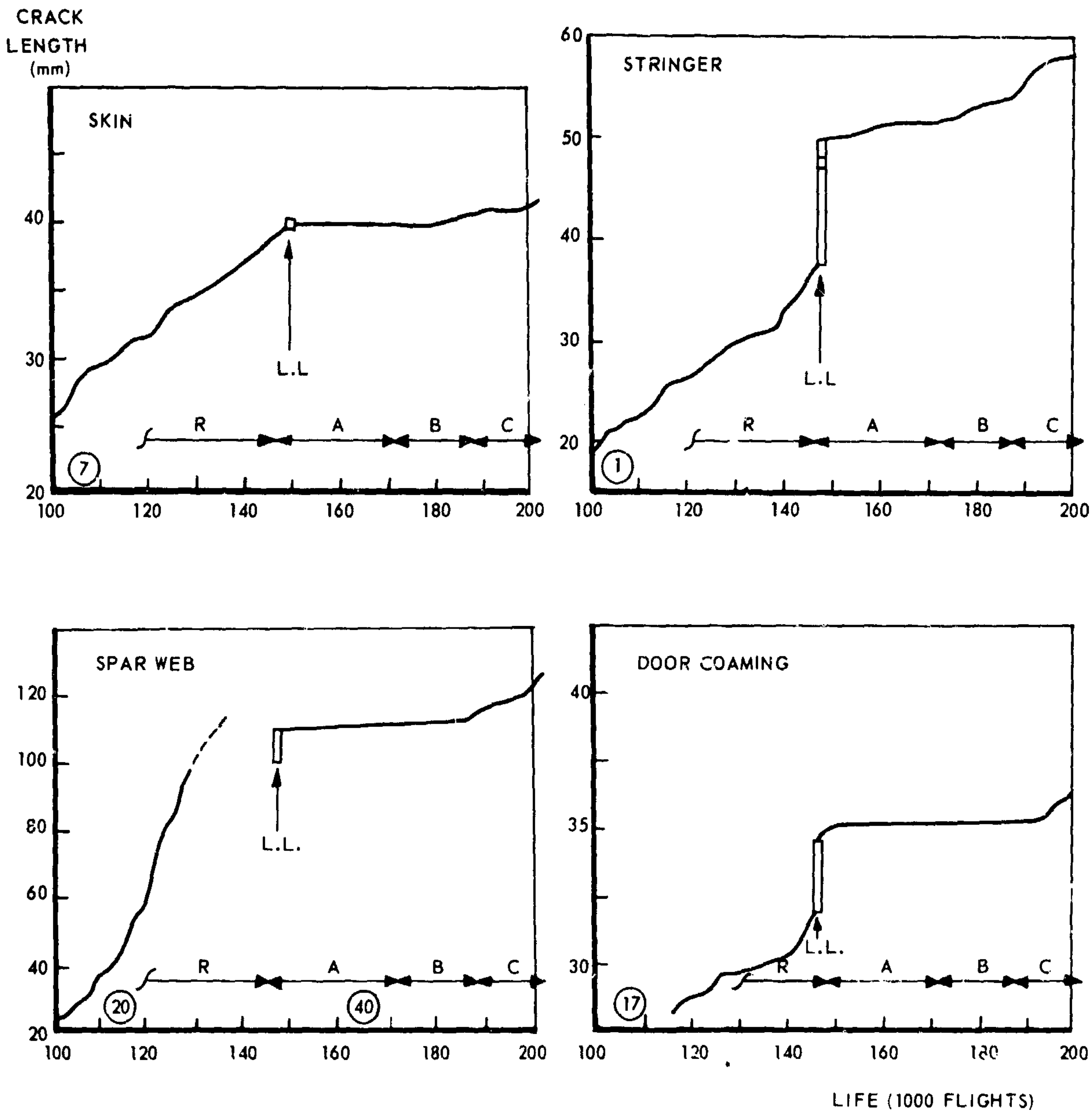


FIG. 7.4 EXAMPLE OF TRUNCATING THE INFREQUENTLY OCCURRING HIGH AMPLITUDES OF A LOAD SPECTRUM.



R: : CERTIFICATION TESTS
LL : : CRACK EXTENSION DUE TO LIMIT LOAD APPLICATION AT END OF CERTIFICATION TESTS
A, B, C : PERIODS OF SUBSEQUENT RESEARCH PROGRAM
A : LOW-AMPLITUDE GUST CYCLES OMITTED
B : LOWER TRUNCATION LEVEL
C : LOAD LEVELS INCREASED 25 PERCENT

FIG. 7.5 THE EFFECT OF LIMIT LOAD APPLICATION ON CRACK PROPAGATION.

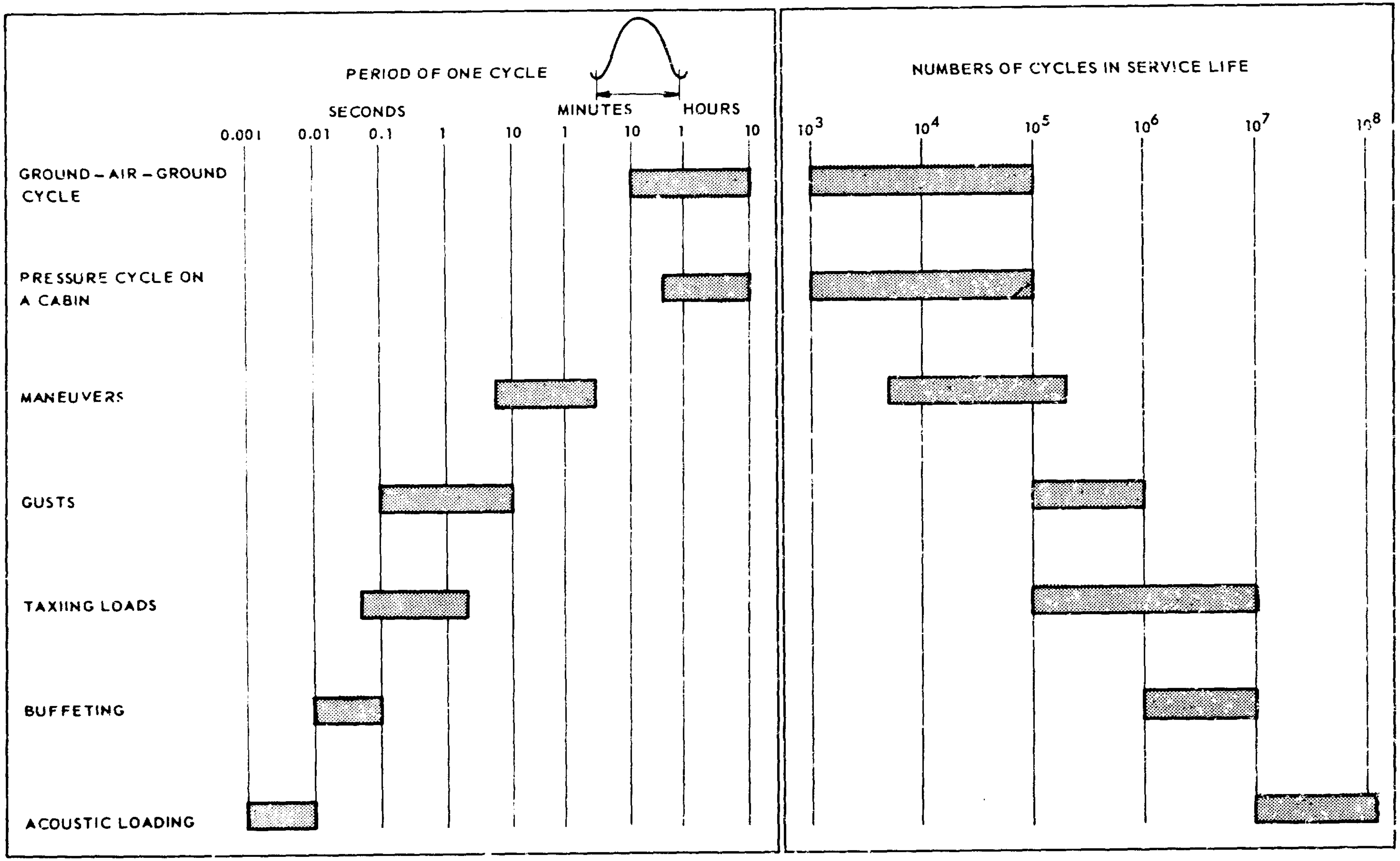


FIG. 7.6 PERIODS AND NUMBERS OF SEVERAL TYPES OF AIRCRAFT FATIGUE LOADS (REF. 195). ORDERS OF MAGNITUDE.