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Lovick Edward Cannon, III

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13. ABSTRACT

Pseudo random number generating programs coded in machine language for the IBM 360/65 computer have been prepared for producing random values from the normal, exponential, and gamma distributions as well as from any discrete probability distribution. The schemes are based largely on combinations of sophisticated techniques first suggested by Marsaglia. The distributions simulated are exact within the word size of the computer and average production time per number is very short, e.g. 10 to 40 microseconds depending upon the particular distribution.

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PSEUDO RANDOM NUMBER GENERATORS FOR STATISTICAL APPLICATIONS

LOVICK EDWARD CANNON, III

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ROLF BARGMANN
PRINCIPAL INVESTIGATOR

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THE UNIVERSITY OF GEORGIA
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CHAPTER 1
INTRODUCTION

With the increasing use of electronic computers, random sampling methods have become a very useful tool for providing solutions to problems involving probability as well as for problems of a deterministic nature. The availability of sequences of numbers which appear to be drawn at random from particular probability distributions is a vital ingredient in the random sampling process. Such numbers are referred to as pseudo-random numbers or, for convenience, simply as random numbers. This paper is concerned with the rapid and accurate generation of such numbers in a stored computer program.

The use of random sampling to estimate distribution functions originated with Student [17] in 1908. At that time and for some time to follow, necessary random numbers were obtained by drawing cards from a deck, counters from an urn, or by rolling dice. Such processes are very slow and make it quite difficult to insure randomness.

To facilitate the use of random numbers large tables of random digits were compiled. The first such table, published by Tippett [19] in 1927, consisted of 41,600 digits taken at random from census reports and combined into 10,400 four digit numbers. The requirement for a larger set of numbers led to the publication of 100,000 random digits in 1939 by Kendall and Babington-Smith [7]. These digits were generated by means of

a mechanical process and were the first to be so produced. Kendall and Babington-Smith are also responsible for developing many of the tests frequently applied to sequences of random numbers (5,6). Other such tables have since been constructed, primarily by the use of physical devices, culminating in the most extensive, "A Million Random Digits with 100,000 Normal Deviates" published by the RAND Corporation [15] in 1955.

The use of a physical device for the generation of random numbers on line with a computer is both expensive and difficult to maintain. The storing of a table of random numbers on magnetic tape or on cards is also an unsatisfactory method of generating numbers for use in a computer. This would necessitate the use of an input device; also the time required to read in the numbers would be excessive.

The development of arithmetic procedures for the generation of random numbers began in the 1940's with the introduction of computers. The first such procedure was suggested by von Neumann and Metropolis in 1946 and is described in [4]. It was also during this time that calculations involving random numbers received the picturesque name "Monte Carlo." The "middle-square" method proposed by von Neumann and Metropolis is simple, fast, and requires only an initial starting value. In this procedure each new number is produced by taking the middle n digits of the square of the previous n-digit number. As pointed out in [4] and in [16] the sequence of numbers produced using this method sooner or later degenerates to a cycle which often is very small, and at worst consists of only a single number. In addition some of the statistical tests performed on samples generated by the middle-square procedure have resulted in failures. Improvements on the von Neumann method are plagued by similar difficulties

[2] and [20].

Of the various arithmetic procedures sequences of random numbers with the best statistical properties and longest periods are generated by means of the congruence relation

$$x_{n+1} = a \cdot x_n + c \cdot (\text{mod } m). \tag{1.1}$$

The representation (1.1) is termed the mixed congruential method. The multiplicative congruential method is defined by taking c = 0 in (1.1) Lehmer [8] is credited with the invention of the multiplicative method. In their very informative survey Hull and Dobell [3] prescribe conditions for a, m, c, and x_0 which insure maximum period. Other procedures for the genration of random numbers based upon reduced Fibonacci series [21] and upon transcendental numbers [3] are inferior to those based on the congruence relation (1.1) (See [4]). In [9] MacLaren and Marsaglia present a table look-up scheme which seems to offer some promise.

Random numbers generated by arithmetic procedures are not "truly random" in that the entire sequence is determined in advance and can be reproduced simply by using the same starting value, x_0. The statistical behavior of sequences generated by both the mixed and multiplicative congruential methods is quite good, and, in fact, numbers genrated in this fashion do appear to be drawn at random from the uniform (0,1) distribution

$$f(u) = 1, \quad 0<u<1. \tag{1.2}$$

The reproducibility of sequences is an advantage in debugging and in certain calculations it may be desirable to reproduce a given sequence.

Bargmann [1] has provided a procedure for the generation of independent uniform (0,1) random numbers on a binary computer. This procedure, which is a form of the multiplicative congruential method, is defined by

$$u_{n+1} = a \cdot u_n (\text{mod } 2^{32}) \tag{1.3}$$

where a is chosen such that a + 1 and a - 1 end in as few zero bits as possible. This is insured by requiring that neither a - 1 nor a + 1 be divisible by 2^k for k = 3,4,.... Choosing a as an odd power of 5 determines that the low order 3 digits will be 125, and neither 124 nor 126 is divisible by 2^k for k = 3,4,.... Hence, $a = 5^{13}$ is a reasonable choice. This procedure requires that u_0, the starting value, be an odd positive integer.

The use of this procedure results in a very fast computer program requiring only 3 operations: load, multiply, and store. A description of the usage of this program along with time and storage requirements is given in Chapter 6 of this paper. Results of statistical tests performed on this procedure are presented in Chapter 5, and a listing of the program is given in Appendix A.

The preceding discussion has dealt only with the generation of random numbers which appear to be uniformly distributed. In principle it should be very easy to obtain any other distribution from the uniform distribution, requiring only a solution to the equation

$$u = F(y) \tag{1.4}$$

for y, where u is uniformly distributed on the interval (0,1) and F is the

required cumulative distribution function. When the inverse of F is known, as for the exponential distribution, this is a simple matter:

$$y = -\ln(u). \tag{1.5}$$

However, the evaluation of the ln function may be somewhat time consuming. An alternative approach might be to store a value of y for each possible value of u based on the relation

$$y = F^{-1}(u). \tag{1.6}$$

To generate y, simply generate a uniform (0,1) random number u which will determine the location of a stored value of y. Let u be given to 8 digits and let β be the base of the number system in which the digits are represented, then for this procedure a total of β^8 storage locations are required.

Various approximations have been proposed for the normal distribution. Most of these involve taking the sum of a fixed number of uniform (0,1) deviates or the use of Chebyshev approximations [14] or a combination of the two [18]. Such approximations frequently lack accuracy and are either slow or space consuming.

The procedures suggested by Marsaglia et al. in [11],[12], and [13] for transforming uniform (0,1) random numbers to random numbers having other distributions are superior to any encountered. They are fast, require minimal space, and are simple to program. In addition these procedures are completely accurate; the precision of the result is dependent only on the word size of the computer. These procedures along with programs and results of statistical tests are presented in detail in this paper.

CHAPTER 2

GENERATION OF DISCRETE RANDOM VARIABLES IN A COMPUTER

Let Y be a discrete random variable with point probabilities $p_i = Pr(Y = y_i)$ for $i = 1,2,\ldots$. The direct way to generate Y in a computer is to generate a uniform (0,1) random number u and put $Y = y_i$ if $p_1 + p_2 + \ldots + p_{i-1} < u \leq p_1 + p_2 + \ldots + p_i$. However, techniques based on this method lead to complicated programs that are excessively time consuming.

An alternative method proposed by Marsaglia [11] is simple to program and requires minimal time and storage. Let p_i for $i = 1,2,\ldots,$ n be expressed by k digits as $p_i = .\delta_{1i}\delta_{2i} \ldots \delta_{ki}$. If the domain of the random variable is infinite, the probability distribution must be truncated at some p_n. The fastest method for generating Y is as follows.

Let β be the base of the number system in which the δ_{ji}'s are represented. In memory locations 0 to (β^k-1) store $\delta_{11}\delta_{21}\delta_{31} \ldots \delta_{k1}$ y_1's, $\delta_{12}\delta_{22} \ldots \delta_{k2}$ y_2's, ..., $\delta_{1n}\delta_{2n} \ldots \delta_{kn}$ y_n's. If u is a uniform (0,1) random number, $u = .d_1d_2 \ldots d_k$, look up the number in location $d_1d_2\ldots d_k$ and let that be Y.

Though this may be the fastest method, it clearly requires an exessive amount of storage space. Even if the p_i's are truncated to four digits, β^4 memory locations will be required.

Marsaglia [11] suggests a technique that offers a considerable reduction in storage space with very little sacrifice in execution time. For convenience assume that the p_i's are truncated to four digits. We have the following situation:

Value of Y	Probability
y_1	$\cdot\ \delta_{11}\delta_{21}\delta_{31}\delta_{41}$
y_2	$\cdot\ \delta_{12}\delta_{22}\delta_{32}\delta_{42}$
y_n	$\cdot\ \delta_{1n}\delta_{2n}\delta_{3n}\delta_{4n}$

Define

$$P_0 = 0, \qquad P_r = \beta^{-r} \sum_{i=1}^{n} \delta_{ri} \tag{2.0.1}$$

for r = 1,2,3,4, and

$$\Pi_s = \sum_{j=1}^{s} \sum_{i=1}^{n} \delta_{ji} \tag{2.0.2}$$

for s = 1,2,3,4 and $\Pi_0 = 0$. Segment memory locations 0 to (Π_4-1) into four mutually exclusive sets such that set 1 consists of locations 0 to (Π_1-1), set 2 comprises locations Π_1 to (Π_2-1), and in general the sth set occupies memory locations Π_{s-1} to (Π_s-1) for s = 1,2,3,4. In each set s for s = 1,2,3,4 each y_i is stored in δ_{si} locations for i = 1,2, ..., n, a total of $\Pi_s - \Pi_{s-1}$ locations. The total memory requirement is then $\Pi_4 = \sum_{j=1}^{4} \sum_{i=1}^{n} \delta_{ji}$ locations. Now choose set 1 with probability P_1, set 2 with probability P_2, set 3 with probability P_3, or

set 4 with probability P_4. Having chosen a set, select at random a location within that set; the number occupying that location is the desired random variable.

This procedure gives the required discrete distribution, as can be seen by defining:

$$q_{ri} = \delta_{ri} / \sum_{i=1}^{n} \delta_{ri}$$

for $i = 1,2, \ldots, n$ and $r = 1,2,3,4$. Thus q_{ri} is simply the probability of choosing y_i from set r. Then the probability of generating $Y = y_i$ is

$$\sum_{r=1}^{4} P_r \cdot q_{ri} = \sum_{r=1}^{4} \beta^{-r} \delta_{ri} = .\delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i} \quad ,$$

which is the probability $P(Y = y_i)$.

In order to select the proper set and location within that set, generate a uniform (0,1) random variable $u = .d_1 d_2 \ldots$, and let $A\{j\}$ denote the contents of memory location j. Then if

$$\sum_{r=0}^{s-1} P_r \leq u \leq \sum_{r=0}^{s} P_r$$

put

$$Y = A\ \{d_1 d_2 \ldots d_s + \Pi_{s-1} - \beta^s \sum_{r=0}^{s-1} P_r\ \}.$$

The following example illustrates this procedure. The hexadecimal (β=16) number system is used as it is the representation of the IBM 360/65,

i	y_i	p_i
1	0	0.A000
2	1	0.3800
3	2	0.14A2
4	3	0.0F50
5	4	0.0202
6	5	0.020C

Defining P_r and Π_s as before gives

$$P_0 = 0$$

$$P_1 = 16^{-1} \sum_{i=1}^{6} \delta_{1i} = 0.E$$

$$P_2 = 16^{-2} \sum_{i=1}^{6} \delta_{2i} = 0.1F$$

$$P_3 = 16^{-3} \sum_{i=1}^{6} \delta_{3i} = 0.00F$$

$$P_4 = 16^{-4} \sum_{i=1}^{6} \delta_{4i} = 0.0010$$

and

$$\Pi_0 = 0$$

$$\Pi_1 = \sum_{j=1}^{1} \sum_{i=1}^{6} \delta_{ji} = E$$

$$\Pi_2 = \sum_{j=1}^{2} \sum_{i=1}^{6} \delta_{ji} = 2D$$

$$\Pi_3 = \sum_{j=1}^{3} \sum_{i=1}^{6} \delta_{ji} = 3C$$

$$\Pi_4 = \sum_{j=1}^{4} \sum_{i=1}^{6} \delta_{ji} = 4C.$$

The four sets are stored as follows:

A - TABLE

SET 1		SET 2				SET 3		SET 4	
Loc.	Con.	Loc.	Con.	Loc.	Con.	Loc.	Con.	Loc.	Con.
0	0	E	1	1E	3	2D	2	3C	2
1	0	F	1	1F	3	2E	2	3D	2
2	0	10	1	20	3	2F	2	3E	4
3	0	11	1	21	3	30	2	3F	4
4	0	12	1	22	3	31	2	40	5
5	0	13	1	23	3	32	2	41	5
6	0	14	1	24	3	33	2	42	5
7	0	15	1	25	3	34	2	43	5
8	0	16	2	26	3	35	2	44	5
9	0	17	2	27	3	36	2	45	5
A	1	18	2	28	3	37	3	46	5
B	1	19	2	29	4	38	3	47	5
C	1	1A	3	2A	4	39	3	48	5
D	2	1B	3	2B	5	3A	3	49	5
		1C	3	2C	5	3B	3	4A	5
		1D	3					4B	5

TABLE 2.1

To generate the desired random variable, let $u = .d_1d_2 \ldots$ be a uniform (0,1) random number and let A{j} denote the contents of memory location j and proceed as follows:

1) If $0 \leq u < 0.E$ put $Y = A\{d_1\}$

2) If $0.E0 \leq u < 0.FF$ put $Y = A\{d_1d_2 - D2\}$

3) If $0.FF0 \leq u < 0.FFF$ put $Y = A\{d_1d_2d_3 - FC3\}$

4) If $0.FFF0 \leq u$ put $Y = A\{d_1d_2d_3d_4 - FFB4\}$.

Examples:

$u = 0.2170 \ldots Y = A\{2\} = 0$

$u = 0.EF10 \ldots Y = A\{EF-D2\} = A\{1D\} = 3$

$u = 0.FFFE \ldots Y = A\{FFFE - FFB4\} = A\{4A\} = 5.$

In this example $4C_{(16)}$ memory locations are required as compared to 16^4 memory locations for the fastest method. Suppose the times for certain operations in the computer are:

Operation	Time
Compare two integers	P
Subtract two integers	S
Look up an addressed location	L

Then the fastest method for generating Y requires a total time of P + L and 16^4 memory locations. In the example presented only $4C_{(16)}$ memory locations are required and an average generation time of

$$0.E \cdot (P + L) + 0.F \cdot (2P + S + L) + 0.00F \cdot (3P + S + L) + 0.001 \cdot (3P + S + L) = 1.21_{(16)} \cdot P + 0.20_{(16)} \cdot S + L$$

A schemacic for storing the four sets of discrete variables and the subsequent generation of the desired random variables is presented in figure 2.1.

FIGURE 2.1

Flowchart for the Generation of Random Numbers from a Discrete Distribution

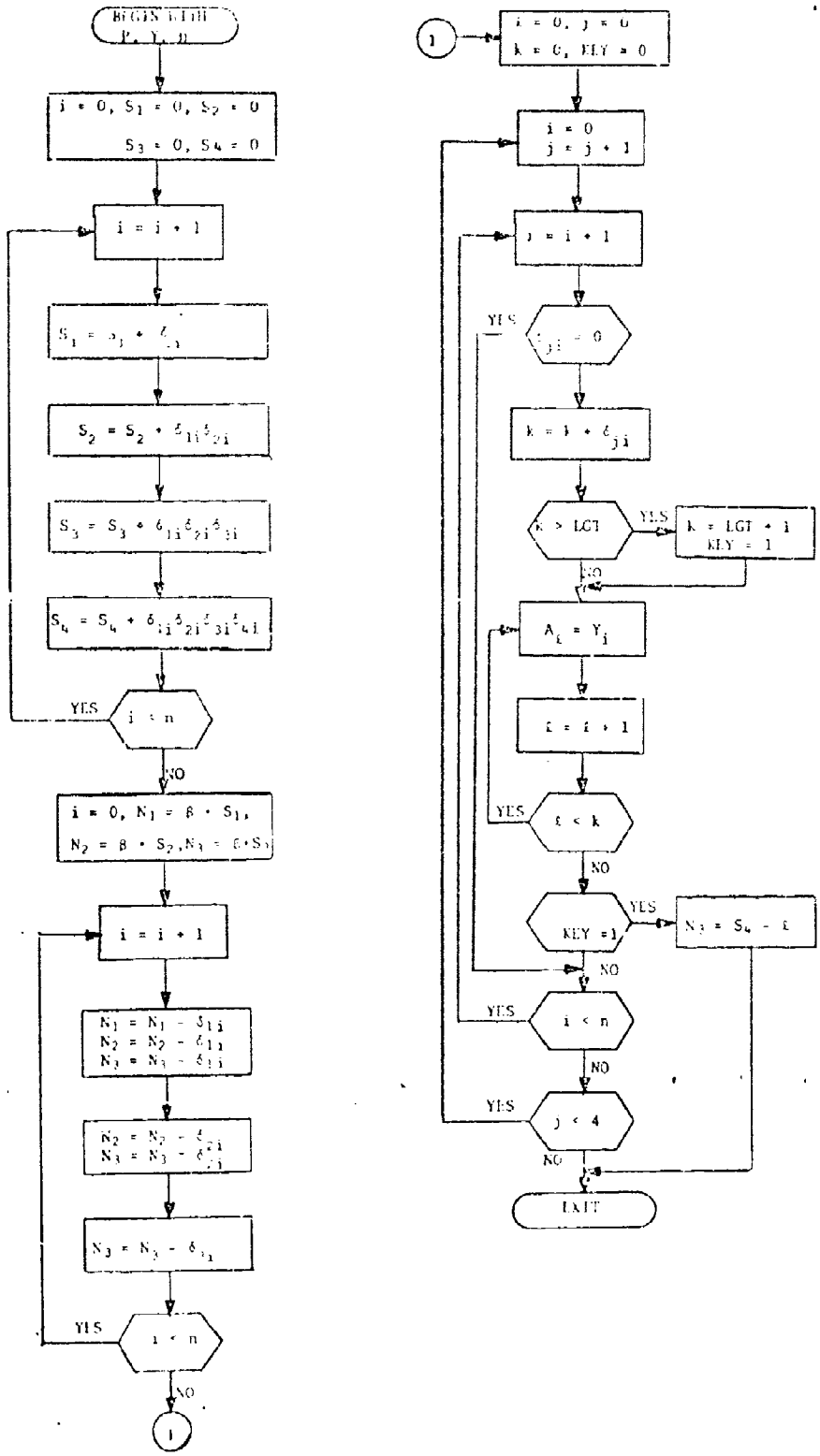
P is a vector of probabilities ($p_i = .\delta_{1i}\delta_{2i}\delta_{3i}\ldots$)

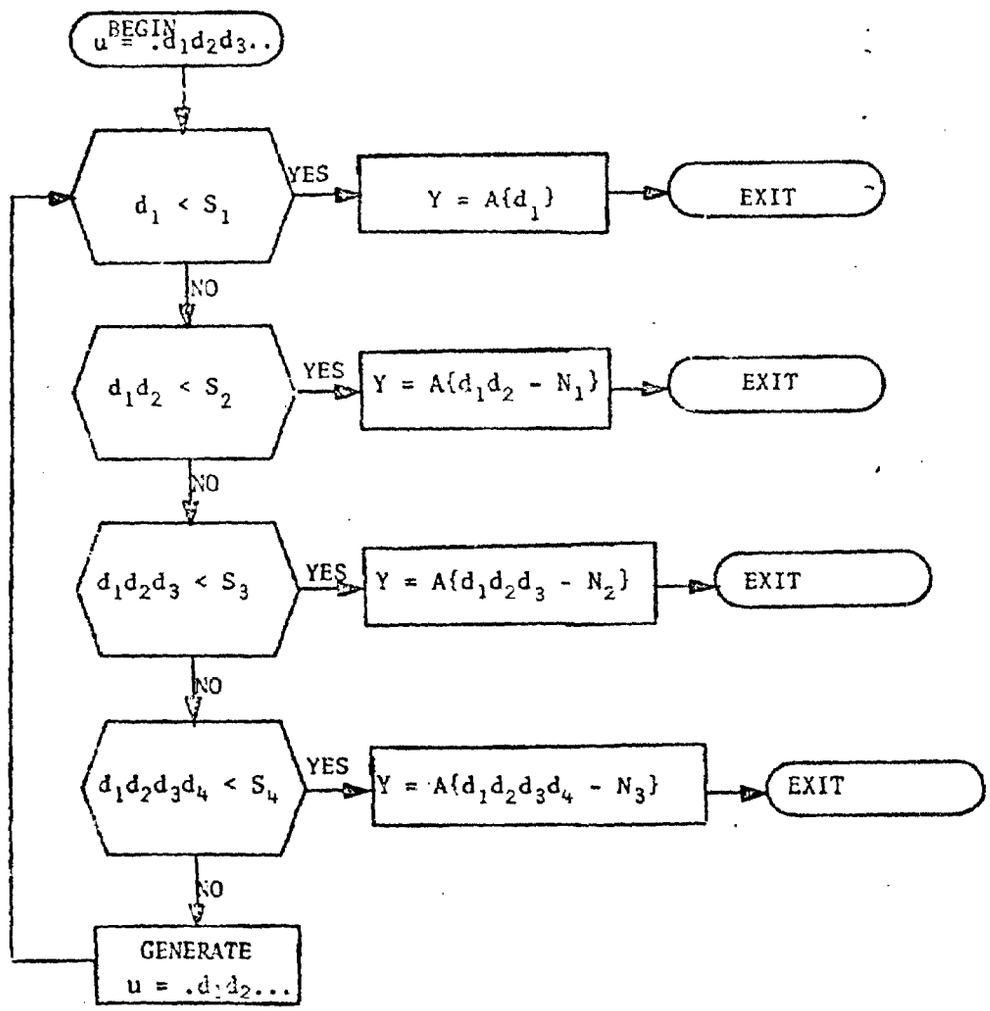
Y is a vector of the corresponding discrete variables (integer mode)

n is the number of elements in P and in Y.

β is the base of the number system in which the δ_{ij}'s are represented (for the 360/65 $\beta = 16$, for a decimal machine $\beta = 10$).

LGT is the maximum length of the A-Table. If LGT is exceeded, the table loading process ceases and N_3 is adjusted (since N_3 was calculated on the basis that loading would be complete).


BEGIN WITH
P, Y, n
i = 0, S1 = 0, S2 = 0
S3 = 0, S4 = 0
i = i + 1
S1 = S1 + δ1i
S2 = S2 + δ1iδ2i
S3 = S3 + δ1iδ2iδ3i
S4 = S4 + δ1iδ2iδ3iδ4i
YES
i < n
NO
i = 0, N1 = β + S1,
N2 = β + S2, N3 = β + S3
i = i + 1
N1 = N1 - δ1i
N2 = N2 - δ1i
N3 = N3 - δ1i
N2 = N2 - δ2i
N3 = N3 - δ2i
N3 = N3 - δ3i
YES
i < n
NO
1
1
i = 0, j = 0
k = 0, KEY = 0
i = 0
j = j + 1
j = i + 1
YES
δji = 0
k = k + δji
k > LGT
YES
k = LGT + 1
KEY = 1
NO
Ai = Yi
i = i + 1
YES
i < k
NO
KEY = 1
YES
N3 = S4 - i
NO
YES
i < n
NO
YES
j < 4
NO
EXIT


BEGIN
u = .d1d2d3..
d1 < S1
YES
Y = A{d1}
EXIT
NO
d1d2 < S2
YES
Y = A{d1d2 - N1}
EXIT
NO
d1d2d3 < S3
YES
Y = A{d1d2d3 - N2}
EXIT
NO
d1d2d3d4 < S4
YES
Y = A{d1d2d3d4 - N3}
EXIT
NO
GENERATE
u = .d1d2...

CHAPTER 3

GENERAL TECHNIQUES FOR THE GENERATION OF PSEUDO-RANDOM NUMBERS HAVING A CONTINUOUS DISTRIBUTION FUNCTION

Certain techniques of a rather general nature are used in some of the routines described in this paper. They are especially well suited for use in very fast computer programs. The principle of each technique is described below and reference will be made to these techniques as they are used in the specific generator programs.

3.1 The Composition Technique

The fundamental procedure used for generating pseudo-random numbers from a continuous probability distribution, as suggested by Marsaglia [10], is based upon a decomposition of the density function f into a mixture of 3 densities:

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t) \qquad (3.1.1)$$

where $p_1 > p_2 > p_3$, $p_1 + p_2 + p_3 = 1$ and p_1 is very close to 1.

A random number from f(t) is obtained as either a number from $g_1(t)$, from $g_2(t)$ or from $g_3(t)$ with respective probabilities p_1, p_2, p_3. Consider a density f(t) defined from all $t \geq a$ (figure 3.1).

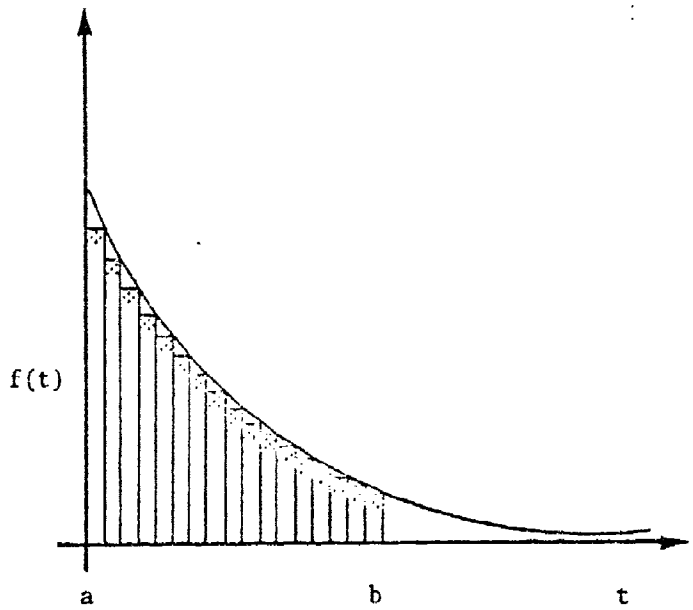


FIGURE 3.1

The density $g_1(t)$ is represented by the rectangles, appropriately standardized, including the shaded upper portions, and is defined for $a \leq t < b$. The width of the rectangles is Δ, a quantity whose value is dictated by the number system used within the computer. For a binary machine using hexadecimal arithmetic, $\Delta = 0.1_{(16)} = 0.0625_{(10)}$. The total area represented by the rectangles is p_1 which clearly is very close to 1. The density $g_2(t)$ is represented by the "triangular" regions, appropriately standardized, lying below $f(t)$ and above $g_1(t)$, and is also defined for $a \leq t < b$. The total area occupied by the "triangles" is p_2. The density $g_3(t)$ is defined for $t \geq b$ and represents

the tail of $f(t)$. Its area is p_3.

Random variables having density $g_1(t)$ may be rapidly generated as follows:

Define

$$N = (b-a)/\Delta \tag{3.1.2}$$

and

$$t_i = a + (i-1) \cdot \Delta \quad \text{for } i = 1,2,3, \ldots, N. \tag{3.1.3}$$

Now assign each discrete t_i a probability

$$P_i = \Delta \cdot f(t_{i+1}) = .\delta_{1i}\delta_{2i}\ldots. \tag{3.1.4}$$

The P_i's are simply the areas of the individual rectangles comprising $g_1(t)$. Based on (say) the high order 4 digits of the P_i's store the discrete t_i's in a table according to the technique described in Chapter 2 of this paper. Thus to generate a random variable Y from this portion of g_1 (unshaded in figure 3.1), generate a uniform (0,1) random number $u = .d_1d_2 \ldots$. If

$$u < \sum_{i=1}^{N} .\delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i}$$

then allow the 4 high order digits of u, $d_1d_2d_3d_4$, to locate a particular t_i from the discrete table, as described in Chapter 2, and set $Y = t_i + \Delta \cdot (.d_5d_6\ldots)$.

Random variables from the residual of g_1, corresponding to the remaining digits of the P_i's $(.0000\delta_{5i}\delta_{6i}\ldots)$ and represented by the shaded region in figure 3.1, are generated according to the following scheme: Let N, t_i, and P_i be defined as before (3.1.2), (3.1.3), and (3.1.4); and define:

$$H_i = \delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i} \tag{3.1.5}$$

(area of the unshaded portion of the ith rectangle),

$$R_i = P_i - H_i \tag{3.1.6}$$

(area of the shaded portion of the ith rectangle),

$$T_i = F(t_{i+1}) - F(t_i) - P_i \tag{3.1.7}$$

where

$$F(t_i) = \int_a^{t_i} f(t_i)dt.$$

(T_i is the area of the "triangular" region above the ith rectangle), and

$$P_i' = \sum_{i=1}^{N} H_i \tag{3.1.8}$$

(total area occupied by the unshaded rectangles). It is obvious from the above definitions that

$$p_2 = \sum_{i=1}^{N} T_i \quad , \tag{3.1.9}$$

$$p_1 = \sum_{i=1}^{N} P_i \quad , \text{ and} \tag{3.1.10}$$

$$p_1 - p_1' = \sum_{i=1}^{N} R_i \tag{3.1.11}$$

(See(3.1.1) for the significance of p_1 and p_2). Now store the t_i's in N consecutive locations. The order in which the t_i's are stored is not important*. However, for convenience let D(1) denote the t-value occupying

---

*They may be overlapped with the previously stored t-values in order to save storage space.

the first location, D(2) denote the t-value occupying the second location, etc. For each t_i define a pair of values C(k) and B(k) as follows:

$$C(1) = p_1' + R[D(1)], \tag{3.1.12}$$

$$B(k) = C(k) + T[D(k)], \text{ and} \tag{3.1.13}$$

$$C(k+1) = C(k) + R[D(k+1)] \tag{3.1.14}$$

for k = 1,2, ..., N. R[D(k)] and T[D(k)] denote respectively the area of the rectangle residue (shaded in figure 3.1) and the area of the "triangular" region corresponding to the t-value occupying location D(k). It is apparent that

$$B(N) = p_1 + p_2. \tag{3.1.15}$$

If u is a uniform (0,1) random number such that $p_1' \leq u < p_1 + p_2$, a random variable Y is generated from either the rectangular residues of g_1 or the "triangular" regions of g_2 as follows:

If $B(k-1) \leq u < C(k)$ then generate a new uniform (0,1) random number v and set $Y = D(k) + \Delta \cdot v$. A random variable generated in this fashion will have the density represented by the shaded rectangles in figure 3.1. If $C(k) \leq u < B(k)$, then the random variable Y must be generated from the "triangular" density of g_2 corresponding to the t-value occupying location D(k). The method used depends upon the particular function f(t). The same applies for random variables from g_3 for $t > b$.

The techniques used for the exponential and normal densities will be described subsequently.

Since p_1 is close to 1, most of the time a random variable from g_1 is generated. Though g_2 and g_3 may be complicated and require longer running time, the average time per generation is small since they must be handled so rarely.

3.2 The Acceptance-Rejection Method

Consider the density $h(t)$ defined on the interval (ξ_0,ξ_1). See figure 3.2. Now if u,v is a pair

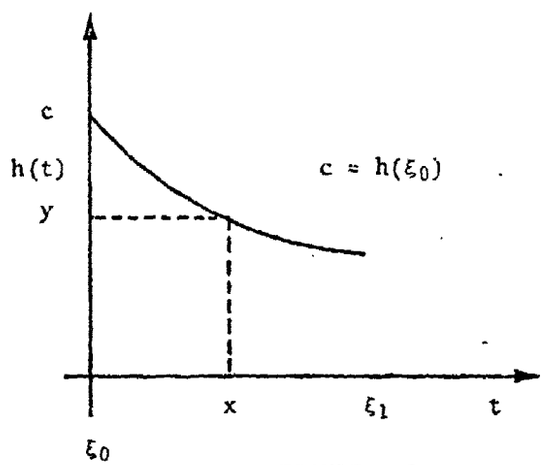


FIGURE 3.2

of independent uniform (0,1) random numbers, $x = \xi_0 + u \cdot (\xi_1-\xi_0)$ and $y = v \cdot c$ will have uniform densities on (ξ_0,ξ_1) and $(0,c)$ respectively. Suppose that $y \leq h(x)$. Then the conditional distribution function of x is given by

$$P[X \leq x \mid Y \leq h(x)] \propto \int_{\xi_0}^{x} \int_{0}^{h(t)} dydt = \int_{\xi_0}^{x} h(t)dt. \qquad (3.2.1)$$

Consequently the conditional density of x given $y \leq h(x)$ is $h(x)$.

When a pair of independent uniform (0,1) numbers, u and v, are generated and the resulting x and y satisfy the inequality $y \leq h(x)$, x is then a required random variate from $h(x)$. Otherwise the pair is rejected and a new pair is generated and checked. It is clear that the inequality will be satisfied with a high probability only if the value $c \cdot (\xi_1 - \xi_2)$ is close to one. If not, the procedure will produce a large proportion of inadmissable (u,v) pairs and reduce the efficiency of the scheme. Even when this proportion is small, the time required to check $y \leq h(x)$ is generally rather long compared with that of the table look-up procedures. Consequently, the acceptance-rejection method is best used for generating those infrequent values from the "triangular" and tail regions described in section 3.1.

3.3 Generation of Numbers with a Triangular Density

Let u and v be independent uniform (0,1) random numbers. Then $T = \min(u,v)$ has the distribution function

$$G(t) = 1 - P[T>t] = 1 - P[u>t,\ v>t] = 1 - (1-t)^2,\ 0<t<1, \qquad (3.3.1)$$

and the density function

$$g(t) = 2(1-t), \quad 0<t<1. \qquad (3.3.2)$$

See figure 3.3a. A linear transformation $T' = a + bT$ produces a random variable with density

$$g(t') = \frac{2}{b} - \frac{2}{b^2}(t' - a), \quad a<t<a+b \qquad (3.3.3)$$

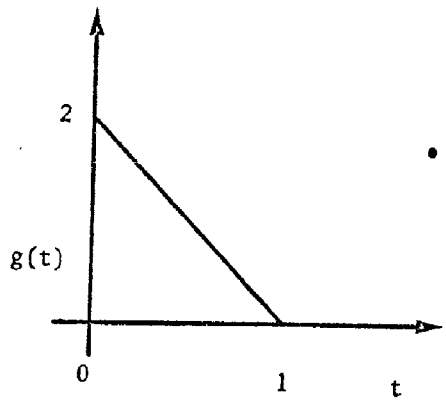


FIGURE 3.3(a)

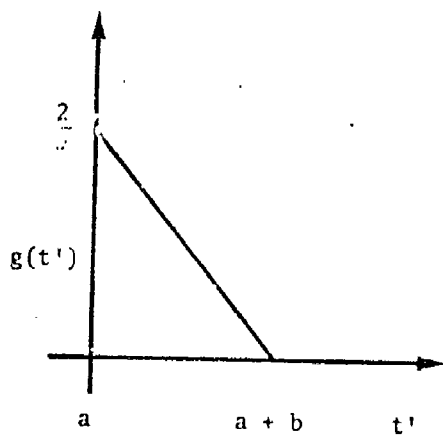


FIGURE 3.3(b)

3.4 The Distribution of the Minimum of a Random Number of Uniform (0,1) Random Variates: Some Special Results for the Exponential Distribution

Let $x = c \cdot \min(u_1, u_2, \ldots, u_n)$ where the u_i are random independent uniform (0,1) variates, c is a positive constant, and N is a discrete valued random variable with probability function $P(N=n) = q(n)$ for $n = 1,2, \ldots$. Then the distribution function of x can be expressed as follows:

$$F(x) = \sum_{n=1}^{\infty} P[X \leq x \mid N = n] \cdot P[N = n]$$

$$= \sum_{n=1}^{\infty} [1 - (1 - x/c)^n] \cdot q(n)$$

$$F(x) = 1 - \sum_{n=1}^{\infty} (1 - x/c)^n \cdot q(n) \qquad 0 < x < c \qquad (3.4.1)$$

Now consider the special case

$$q(n) = c^n/[n!(e^c-1)] \qquad n = 1,2, \ldots.$$

The distribution of x becomes

$$F(x) = 1 - \sum_{n=1}^{\infty} (c-x)^n/n!(e^c-1) = 1 - (e^{c-x}-1)/(e^c-1)$$

$$F(x) = (1-e^{-x})/(1-e^{-c}) \qquad 0 < x < c \qquad (3.4.3)$$

Thus x has the exponential distribution truncated on the right at the value c.

If, instead of the probability function given in (3.4.2), N has the distribution specified by

$$g(n) = \Delta^n/n!(e^{\Delta}-1-\Delta) \quad n = 2,3, \ldots, \tag{3.4.4}$$

where Δ is a constant as defined in section 3.1, then the distribution function of x becomes

$$F(x) = 1 - (e^{\Delta-x}-1-\Delta+x)/(e^{\Delta}-1-\Delta), \quad 0<x<\Delta, \tag{3.4.5}$$

with density

$$f(x) = (e^{-x}-e^{-\Delta})/[1 - e^{-\Delta}(1+\Delta)] \ , \quad 0<x<\Delta. \tag{3.4.6}$$

This is the distribution function of a random variable from the "tooth" of the exponential distribution between 0 and Δ. See figures 3.4(a) and 3.4(b).

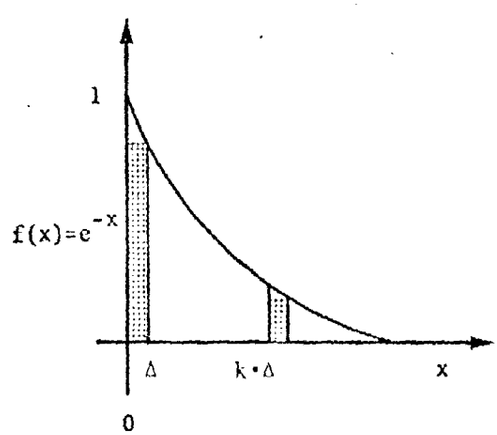


FIGURE 3.4(a)

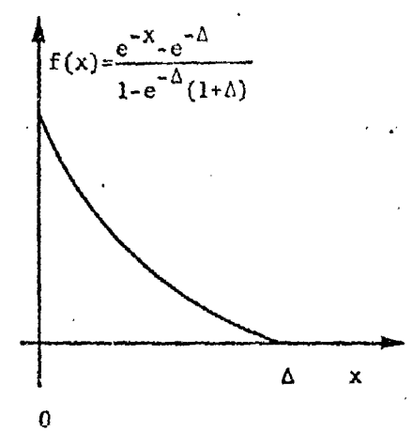


FIGURE 3.4(b)

Figure 3.4(c) shows the density of a random variable from any other such "tooth" defined by $k \cdot \Delta < z < (k+1) \cdot \Delta$.

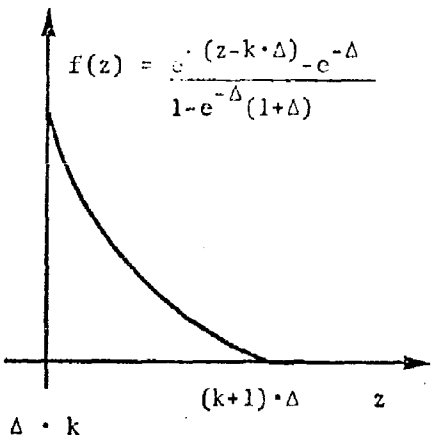


FIGURE 3.4(c)

It is obvious that all such densities are identical, a unique characteristic of the exponential distribution. Consequently, a random variable from the $(k+1)^{st}$ "tooth" is always produced by taking $z = k \cdot \Delta + x$ where the value $k \cdot \Delta$ is chosen by the technique described in section 3.1 and x is generated by taking Δ times the minimum of N uniform (0,1) random numbers where N has the distribution (3.4.4). Its expected value when $\Delta = 0.1_{(16)}$ is 2.02 or in general

$$E[N] = \Delta \cdot (e^{\Delta}-1)/(e^{\Delta}-1-\Delta) \qquad (3.4.7)$$

The same unique "no memory" characteristic of the exponential distribution used above is also used for obtaining random variates from g_3, the tail of the exponential density where $t > c$.

Let $Y = W + X$ where W is discrete valued with probability function

$$P[W = w = c \cdot k] = Ae^{-w} = Ae^{-c \cdot k}, \quad k = 1,2,3, \ldots, \qquad (3.4.8)$$

$A = (1 - e^{-c})/e^{-c}$ and x is independent of w and has the distribution function (3.4.3). The distribution function of Y can be written as

$$G(y) = P[Y \leq y = w + x] = P[Y \leq w + c] + P[W = w, X \leq x] \qquad (3.4.9)$$

Since the distribution function of W is

$$P[W \leq w = c \cdot k] = A \sum_{j=1}^{k} e^{-j \cdot c} = 1 - e^{-c \cdot k}$$

the expression for $G(y)$ becomes

$$G(y) = 1 - e^{-(w-c)} + Ae^{-w}(1-e^{-x})/(1-e^{-c}) = 1 - e^{c-y}, \qquad (3.4.11)$$

$$y > c.$$

This is the required distribution function for variates from the density e^{-y} truncated on the left at $y = c$. Thus values of t from g_3 in the exponential case are generated as the sum of a discrete variate taking values c, $2c$, $3c$, ... and a continuous variate from the truncated exponential density on $(0,c)$.

3.5 Generating Numbers from a 'Nearly Triangular' Density: Application to the Half Normal Density

The results of this section are applied to the half normal density

$$f(t) = \sqrt{2/\pi}\, e^{-\frac{1}{2}t^2} \qquad 0 < t < \infty \qquad (3.5.1)$$

for the generation of random normal deviates to be described in chapter 4. Consider the concave and convex triangular densities (members of g_2) depicted in figures 3.5(a) and 3.5(b).

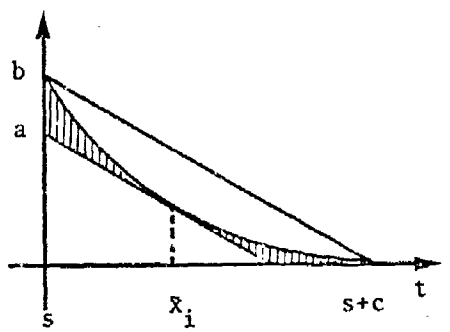


FIGURE 3.5(a)

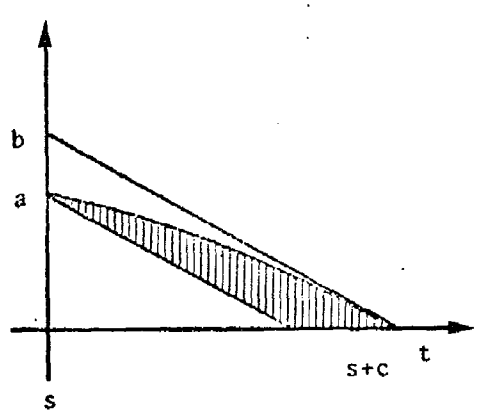


FIGURE 3.5(b)

The parallel chords and tangents enclosing f(t) are determined by construction as indicated in the figures. In both cases, the inner right triangle represents most of the area under the density F(t). This will be true in general as long as the ratio a/b is close to one. A random variate from the density represented by this triangle is generated as described in section 3.3. Infrequently, a random variate from the shaded area must be generated by the acceptance-rejection technique described in section 3.2. Marsaglia et al. [12] have combined these two procedures as follows.

1) Generate independent uniform (0,1) random variables u and v.

2) If $\max(u,v) \leq a/b$, put $t = s + c \cdot \min(u,v)$

3) If not, test $b|u-v| \leq f(s + c \cdot \min(u,v))$.

   If yes, put $t = s + c \cdot \min(u,v)$.

   If no, go to step 1 and try again.

In order to show that this procedure produces a variate t with the required density, we conveniently take $s = 0$ without loss of generality and proceed as follows.

Define

$$m = \min(u,v)$$

$$M = \max(u,v)$$

$$x = c \cdot m$$

$$y = b(M-m) = b(u-v) \qquad (3.5.2)$$

Then the pair (x,y) is uniformly distributed over the triangle in figure 3.5(c)

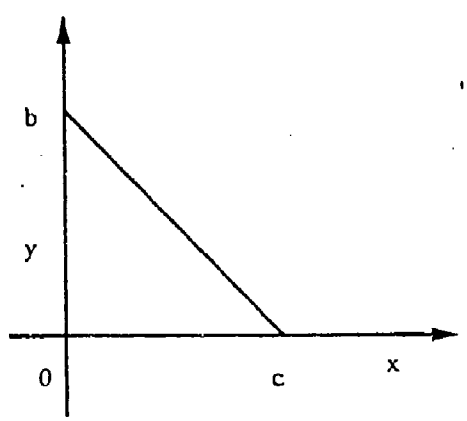


FIGURE 3.5(c)

To verify this, consider the joint distribution function of X and Y:

$$F(x,y) = P[X \leq x,\ Y \leq y] = P[m \leq x/c,\ M - m \leq y/b]$$

This is just the area of the cross hatched region in figure 3.5(d) which is seen to be

$$F(x,y) = 2 \cdot (y/b) \cdot (x/c) = 2xy/bc\ ,\quad 0<x<c\ ,\quad 0<y<b\ . \tag{3.5.3}$$

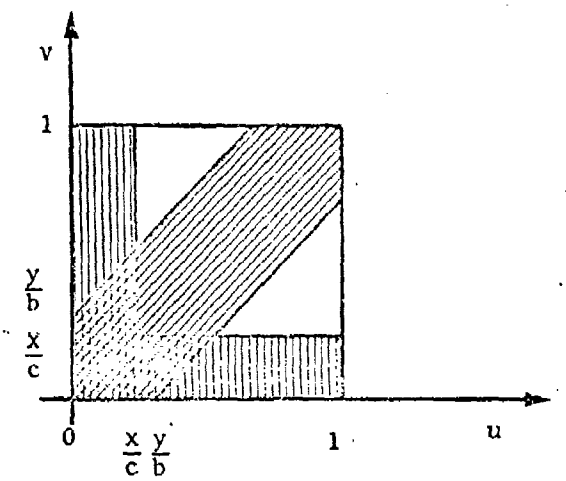


FIGURE 3.5(d)

Thus, (x,y) is a point randomly chosen from the larger triangle in figures 3.5(a) and 3.5(b). By the acceptance-rejection technique, if this point falls within the smaller triangle of figures 3.5(a) and 3.5(b), the value $x = t$ has the triangular density $-bx/c + a$. Now the condition $\max(u,v) \leq a/b$ is equivalent to:

$$bM \leq a$$

$$b(M-m) \leq a - bm = -bx/c + a$$

or

$$y \leq -bx/c + a \qquad (3.5.4)$$

which means that (x,y) falls in the small triangle. If this condition is not satisfied, a second check is made to determine whether the point (x,y) falls within the shaded area between f(t) and y_2 in figures 3.5(a) and 3.5(b). If so, then clearly

$$y = b(M - m) = b|u - v| \leq f(x) = f(c \cdot \min(u,v)) \qquad (3.5.5)$$

as required by the second test. The acceptance-rejection principle insures that every x value passing at least one of these two tests will have the density f(t). The first test is rapidly made; the second test involves evaluation of f(x) and is time consuming. The first test will be passed with a probability

$$P[M \leq a/b] = (a/b)^2$$

as given by the distribution function of M = max(u,v). It is therefore desirable to have $(a/b)^2$ close to 1 to achieve a short average execution time. This will be the case if f(t) is "nearly triangular."

For the half normal density the a_i/b_i ratios are obtained as follows.

Case (a) "Concave Triangles" (See figure 3.5(a)).

Let $t_i = i \cdot c = s$

$$t_{i+1} = (i+1)c = s + c, \quad i = 0,1,2, \ldots \qquad (3.5.6)$$

The equation of the chord and the tangent are respectively:

$$y_1 = - b_i(\quad s)/c + b_i$$

$$y_2 = - b_i(x-s)/c + a_i \tag{3.5.7}$$

The value of b_i is clearly

$$b_i = f(t_i) - f(t_{i+1}) = f(s) - f(s+c) \tag{3.5.8}$$

The slope of both lines, $- b_i/c$, must equal $f'(\bar{x}_i)$. Thus

$$-b_i/c = - \sqrt{2/\pi}\, x_i e^{-\frac{1}{2}x_i^2} = - \bar{x}_i f(\bar{x}_i) \tag{3.5.9}$$

This equation must be solved iteratively for $\bar{x}_i$, the abscissa of the point of tangency. Finally, this value is substituted into the tangent's equation to give

$$f(\bar{x}_i) = f(t_{i+1}) = (- b_i/c)(\bar{x}_i-s) + a_i. \tag{3.5.10}$$

Substituting the expression for b_i/c and solving for a_i gives

$$a_i = f(\bar{x}_i)[1 + \bar{x}_i(\bar{x}_i-s)] - f(s+c). \tag{3.5.11}$$

Consequently for "concave triangles"

$$a_i/b_i = [f(\bar{x}_i)\{1 + \bar{x}_i(\bar{x}_i-s)\} - f(s+c)]/[f(s) - f(s+c)] \tag{3.5.12}$$

Case (b) "Convex Triangles" (See figure 3.5(b))

In this case the point of tangency occurs at the end of the interval where $x_i = t_{i+1} = s + c$. The slope of both lines is given by

$$- b_i/c = f'(s+c)$$

or
$$b = -cf'(s+c). \tag{3.5.13}$$

The value of a_i is simply

$$f(t_i) - f(t_{i+1}) = f(s) - f(s+c).$$

Consequently, for "convex triangles"

$$a_i/b_i = [f(s) - f(s+c)]/[-cf'(s+c)] \tag{3.5.14}$$

3.6 Generation of Random Variates from the Tail of the Half Normal Distribution

The procedure suggested by Marsaglia et al. [12] for the generation of random variates from the tail of the half normal distribution is based on the rejection principle described in section 3.2. The procedure is to generate a pair of independent half normal variates x_1 and x_2 such that the point (x_1, x_2) lies outside the quarter circle (see figure 3.6(a)).

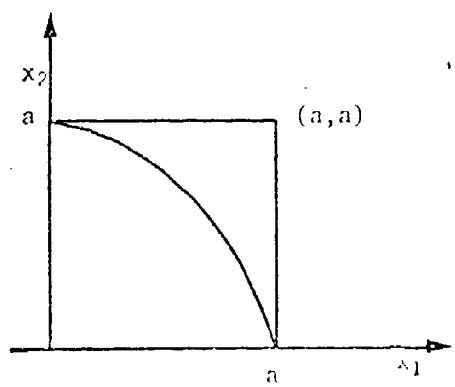


FIGURE 3.6(a)

This is done by generating pairs of uniform (0,1) random numbers u_1 and u_2 until $0 < u_1^2 + u_2^2 \leq 1$ and setting

$$x_1 = \sqrt{a^2 - 2\ln(u_1^2 + u_2^2)} \cdot \sqrt{u_1^2/(u_1^2 + u_2^2)} \tag{3.6.1}$$

$$x_2 = \sqrt{a^2 - 2\ln(u_1^2 + u_2^2)} \cdot \sqrt{u_2^2/(u_1^2 + u_2^2)} \tag{3.6.2}$$

To show that the point (x_1,x_2) lies outside the quarter-circle, let $w = -\ln(u_1^2 + u_2^2)$ where $0 < u_1^2 + u_2^2 \leq 1$. Then take the sum $x_1^2 + x_2^2 = a^2 + 2w$ where $0 \leq w < \infty$ so that $x_1^2 + x_2^2 \geq a^2$. If the point (x_1,x_2), generated in this fashion, lies within the square (figure 3.6(a)), it is rejected and a new pair of uniform (0,1) random numbers is generated and tested. Otherwise the variable, x_1 or x_2, whose value exceeds a is taken as the required random number.

In order to show that this procedure produces the desired result, let u and u be a pair of uniform (0,1) random numbers conditioned by $0 < u_1^2 + u_2^2 \leq 1$. Then the joint density of u_1 and u_2 is

$$f(u_1,u_2) = 4/\pi, \quad 0 < u_1 < 1, \quad 0 < u_2 < 1, \quad 0 < u_1^2 + u_2^2 \leq 1. \tag{3.6.3}$$

Now define:

$$X = u_1^2 + u_2^2, \quad 0 < X < 1, \text{ and} \tag{3.6.4}$$

$$Y = u_2/u_1, \quad 0 < Y < \infty. \tag{3.6.5}$$

Then the probability distribution of X is

$$F(x) = P(X \leq x) = (\pi \cdot x/4)/(\pi \cdot 1^2/4) = x \tag{3.6.6}$$

and the density of x is

$$f(x) = 1, \quad 0 < x < 1. \tag{3.6.7}$$

Consequently X is uniform on (01). The joint density of X and Y is:

$$g(x,y) = f(u,v) \cdot |J| = (4/\pi)[1/2(1+y^2)] = 2/\pi(1+y^2) = g_1(x) \cdot g_2(y). \tag{3.6.8}$$

Hence X and Y are stochastically independent. Substitution of (3.6.4) and (3.6.5) into (3.6.1) and (3.6.2) yields:

$$x_1 = \sqrt{a^2 - 2\ln X} \cdot \sqrt{1/(1+Y^2)} \tag{3.6.9}$$

$$x_2 = \sqrt{a^2 - 2\ln X} \cdot \sqrt{Y^2/(1+Y^2)}. \tag{3.6.10}$$

The inverse transformation is:

$$X = e^{-\frac{1}{2}(X_1^2+X_2^2-a^2)} \tag{3.6.11}$$

$$Y = X_2/X_1 \tag{3.6.12}$$

The joint density of X_1 and X_2 is given by:

$$\begin{aligned} f(x_1,x_2) &= g(x,y) \cdot |J| \\ &= [2/\pi(1+y^2)](1+y^2) \cdot e^{-\frac{1}{2}(x_1^2+x_2^2-a^2)} \\ &= (\sqrt{2/\pi}\, e^{-\frac{1}{2}x_1^2})(\sqrt{2/\pi}\, e^{-\frac{1}{2}x_2^2})\, e^{a^2/2} \end{aligned} \tag{3.6.13}$$

$$a^2 \leq x_1^2 + x_2^2 < \infty .$$

Clearly x_1 and x_2 are independent random variables from the tail of the half normal distribution ($x_1, x_2 > a$).

CHAPTER 4

GENERATION OF PSEUDO-RANDOM NUMBERS FROM THE EXPONENTIAL, GAMMA, AND NORMAL DISTRIBUTIONS

Before presenting the procedures for the generation of random numbers from the exponential, gamma, and normal distributions, a simple example is given demonstrating the use of the techniques discussed in Chapter 3 for obtaining random variates from any continuous distribution defined on a finite interval. In the interest of clarity all numbers are expressed to the base 10. Consider the distribution defined by

$$F(y) = (y/2)(3-y^2) \quad 0 < y < 1 \tag{4.0.1}$$

with density

$$f(y) = (3/2)(1-y^2) \quad 0 < y < 1. \tag{4.0.2}$$

Using the composition technique presented in 3.1, $f(y)$ is represented as a mixture of 2 densities (since there is no tail):

$$f(y) = p_1 g_1(y) + p_2 g_2(y). \tag{4.0.3}$$

As before, g_1 is a series of rectangles and g_2 represents the nearly triangular regions between g_1 and f. See figures 4.0(a), (b), and (c).

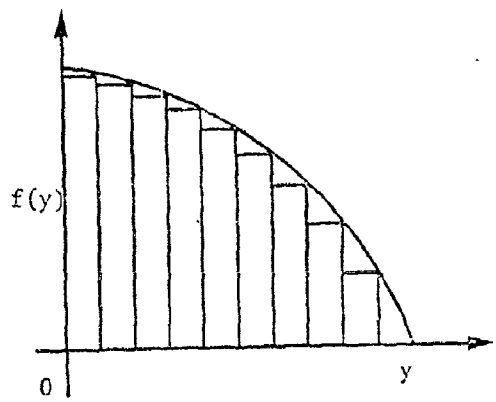


FIGURE 4.0(a)

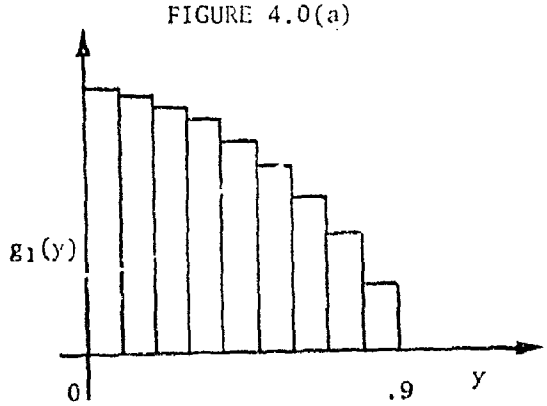


FIGURE 4.0(b)

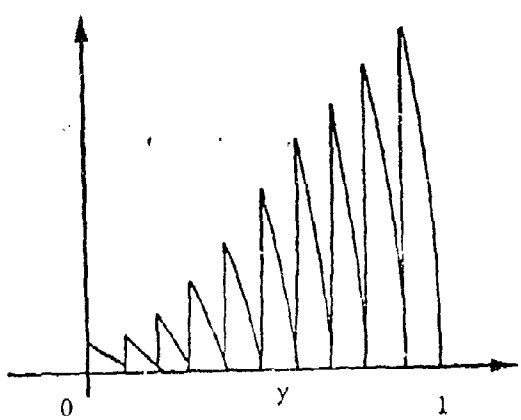


FIGURE 4.0(c)

As described in 3.1, define discrete variables

$x_i = (i-1) \cdot \Delta$ for $i=1,2,\ldots,10$ and in this example $\Delta=0.1_{(10)}$. (4.0.4)

Now assign each x_i a probability P_i as follows:

$$P_i = \Delta \cdot f(x_{i+1}). \tag{4.0.5}$$

Now define a value T_i as given in (3.1.7), the area of the triangular region above the ith rectangle:

$$T_i = F(x_{i+1}) - F(x_i) - P_i. \tag{4.0.6}$$

These values are recorded in table 4.0.1. At this point it is obvious that the values of p_1 and p_2, defined in (4.0.3), are $p_1 = 0.9225$ and $p_2 = 0.0775$. Next, the four sets of the discrete values are stored, based on the high order of digits of the P_i's.

x_i	$P_i = 0.1 \cdot f(x_{i+1})$	$F(x_{i+1}) - F(x_i)$	$T_i = F(x_{i+1}) - F(x_i) - P_i$
0.0	0.1485	0.1495	0.0010
0.1	0.1440	0.1465	0.0025
0.2	0.1365	0.1405	0.0040
0.3	0.1260	0.1315	0.0055
0.4	0.1125	0.1195	0.0070
0.5	0.0960	0.1045	0.0085
0.6	0.0765	0.0865	0.0100
0.7	0.0540	0.0655	0.0115
0.8	0.0285	0.0415	0.0130
0.9	0.0000	0.0145	0.0145
Total	0.9225	1.0	0.0775

TABLE 4.0.1

Since the P_i's have only 4 digits, application of this technique will account for all of g_1 and there is no rectangular residue in this simple example. Since the composition of these 4 sets is constant (the distribution is parameter free), the sets may be overlapped and considerable storage space saved. Looking at table 4.0.2, set 1 occupies locations 2 through 6, set 2 occupies locations 4 through 40, set 3 occupies locations 18 through 67, and set 4 occupies locations 46 through 70. Locations 0 through 9 are also used for the generation of variates from g_2, as described subsequently.

TABLE 4.0.2

A-TABLE

Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con
0	0.9	10	0.6	20	0.7	30	0.5	40	0	50	0.3	60	0.2
1	0.8	11	0.6	21	0.7	31	0.3	41	0.8	51	0.6	61	0.2
2	0.3	12	0.6	22	0.7	32	0.3	42	0.3	52	0.6	62	0.2
3	0.1	13	0.6	23	0.7	33	0.2	43	0.3	53	0.6	63	0
4	0.4	14	0.6	24	0.6	34	0.1	44	0.3	54	0.6	64	0
5	0.2	15	0.5	25	0.5	35	0.1	45	0.3	55	0.6	65	0
6	0	16	0.5	26	0.5	36	0.1	46	0.8	56	0.4	66	0
7	0.7	17	0.2	27	0.5	37	0.1	47	0.8	57	0.4	67	0
8	0.6	18	0.8	28	0.5	38	0	48	0.8	58	0.2	68	0.4
9	0.5	19	0.8	29	0.5	39	0	49	0.8	59	0.2	69	0.4
												70	0.4

As presented in Chapter 2, define:

$$Q_0 = 0, \quad Q_r = 10^{-r} \cdot \sum_{i=1}^{10} \delta_{ri} \tag{4.0.7}$$

and

$$\Pi_0 = 0, \quad \Pi_r = \sum_{j=1}^{r} \sum_{i=1}^{10} \delta_{ji} \quad \text{for } r=1,2,3,4. \tag{4.0.8}$$

Using the above definitions define:

$$S_0 = 0, \quad S_k = 10^k \sum_{r=1}^{k} Q_r \tag{4.0.9}$$

and

$$N_k = \Pi_{k-1} - 10 \cdot S_{k-1} \quad \text{for } k=1,2,3,4. \tag{4.0.10}$$

The N_k's are identical to the quantity

$$\Pi_{s-1} + \beta^s \cdot \sum_{r=0}^{s} P_r$$

in (2.0.3). Because of overlapping between the four sets, the N_k's must be adjusted. Their values and the values of the S_k's are recorded in table 4.0.3.

S-TABLE		N-TABLE	
Loc	Con	Loc	Con
0	5	0	2
1	87	1	-46
2	920	2	-852
3	9225	3	-9154

Table 4.0.3

The procedure for generating random numbers from g_1 is:

1) Generate a uniform (0,1) random number $u = 0.d_1d_2\ldots$.

2) If $d_1 < S(1)$, set $Y = A\{d_1 + N(1)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (3).

3) If $d_1d_2 < S(2)$, set $Y = A\{d_1d_2 + N(2)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (4).

4) If $d_1d_2d_3 < S(3)$, set $Y = A\{d_1d_2d_3 + N(3)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (5).

5) If $d_1d_2d_3d_4 < S(4)$, set $Y = A\{d_1d_2d_3d_4 + N(4)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise generate a random variate from g_2.

In order to generate random variates from g_2, define:

$$C(1) = S_4 + T[A(1)] \text{ and} \tag{4.0.11}$$

$$C(k+1) = C(k) + T[A(k)] \quad \text{for } k=1,2,\ldots 9. \tag{4.0.12}$$

This is identical to the definition given in (3.1.13), except that in this example there is no rectangular residue. The notation $T[A(k)]$ denotes the area of the triangular region (the T_i's in table 4.0.1) corresponding to the discrete value stored in the kth location of the A-table (Table 4.0.2). The values of the $C(k)$'s are given in table 4.0.4. If u is a uniform (0,1) random number such that $u \geq S_4$, the proper triangle of g_2 may be chosen simply by testing:

$u < C(k)$ until the condition is satisfied (note that $C_{(10)} = 1.0$)

The discrete value occupying location k of the A-table denotes the correct triangle.

Having selected the proper triangle of g_2, a random variate having the density of that triangle is to be generated. This is accomplished

through the use of the acceptance-rejection principle discussed in 3.2 and the technique given in 3.5. Consider a particular triangle or tooth from g_2 (figure 4.0(d)). The tangent and the chord are constructed as indicated in 3.5

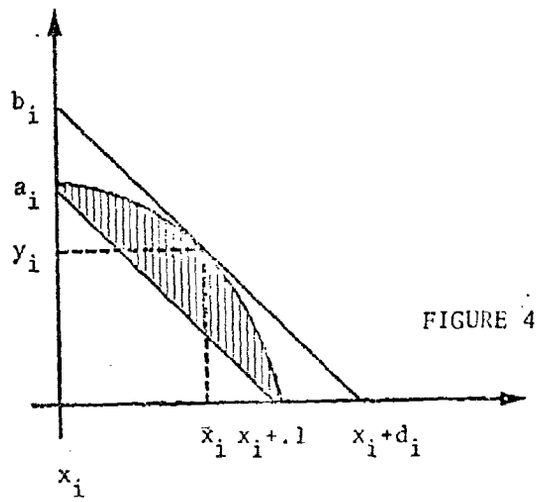


FIGURE 4.0(d)

The inner right triangle encloses most of the area under $f(x)$. A random variate from the density of this triangle is generated as described in 3.3. Occasionally, a random variate from the shaded region must be generated by the acceptance-rejection technique. The values of a_i, b_i, and d_i are obtained as follows:

$$-(a_i/0.1) = f'(\bar{x}_i) = 3\bar{x}_i \tag{4.0.13}$$

where x_i is the abscissa of the point of tangency and

$$a_i = f(x_i) - f(x_{i+1}). \tag{4.0.14}$$

Substituting (4.0.14) into (4.0.13) and solving for $\tilde{x}_i$ yields:

$$\tilde{x}_i = [f(x_i) - f(x_{i+1})]/0.3 \tag{4.0.15}$$

The ordinate of the point of tangency is:

$$y_i = f(\tilde{x}_i) - f(x_{i+1}). \tag{4.0.16}$$

At $x = \tilde{x}_i$, $y_1 = y_i$ and

$$b_i = y_i + (a_i/0.1)(\tilde{x}_i - x_i) \tag{4.0.17}$$

The value of d_i may be obtained by noting that:

$$(a_i/0.1) = (b_i/d_i) \text{ and hence } d_i = (0.1 \cdot b_i/a_i). \tag{4.0.18}$$

The values of a_i/b_i, d_i, and b_i are recorded in table 4.0.4 in the R-table, D-table, and B-table respectively.

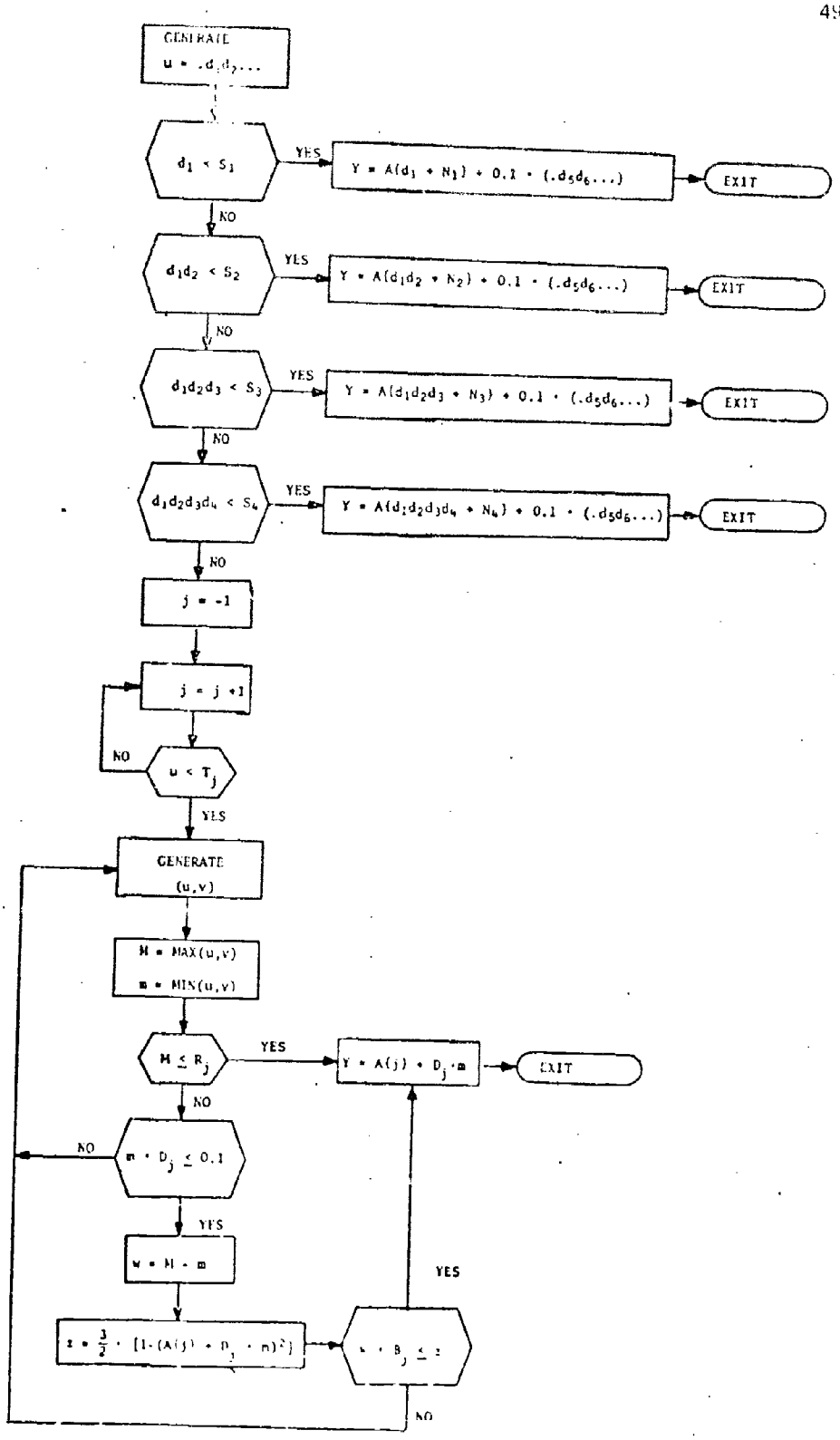
C-Table		R-Table		D-Table		B-Table	
Loc	Con	Loc	Con	Loc	Con	Loc	Con
0	.9370	0	.9870	0	.1013	0	.28875
1	.9500	1	.9855	1	.1015	1	.25875
2	.9555	2	.9655	2	.1036	2	.10875
3	.9580	3	.9231	3	.1083	3	.04875
4	.9650	4	.9730	4	.1028	4	.13875
5	.9690	5	.9524	5	.1050	5	.07875
6	.9700	6	.8000	6	.1250	6	.01875
7	.9815	7	.9836	7	.1017	7	.22875
8	.9915	8	.9811	8	.1019	8	.19875
9	1.0000	9	.9778	9	.1023	9	.16875

Table 4.0.4

FIGURE 4.0(e)

Flowchart for the Generation of Random Variates from

$f(y) = (3/2)(1 - y^2), \quad 0 < y < 1$


GENERATE
u = .d1d2...
d1 < S1
YES
Y = A(d1 + N1) + 0.1 · (.d5d6...)
EXIT
NO
d1d2 < S2
YES
Y = A(d1d2 + N2) + 0.1 · (.d5d6...)
EXIT
NO
d1d2d3 < S3
YES
Y = A(d1d2d3 + N3) + 0.1 · (.d5d6...)
EXIT
NO
d1d2d3d4 < S4
YES
Y = A(d1d2d3d4 + N4) + 0.1 · (.d5d6...)
EXIT
NO
j = -1
j = j + 1
NO
u < Tj
YES
GENERATE
(u,v)
M = MAX(u,v)
m = MIN(u,v)
M ≤ Rj
YES
Y = A(j) + Dj · m
EXIT
NO
NO
m · Dj ≤ 0.1
YES
w = M - m
YES
z = 3/2 · [1-(A(j) + Dj · m)²]
w · Bj ≤ z
NO

The procedure for obtaining random variates from g_2 is:

1) Let u be a uniform (0,1) random number such that $u \geq S_4$.
2) Test $u < C(k)$ for $k = 0, 1, \ldots, 9$ until the inequality is satisfied.
3) Generate independent uniform (0,1) random numbers u_1 and u_2.
4) If $\max(u_1,u_2) \leq R(k)$, then set $Y = A(k) + D(k) \cdot \min(u_1,u_2)$; otherwise go to (5).
5) If $B(k) \cdot |u_1 - u_2| \leq f[A(k) + D(k) \cdot \min(u_1,u_2)]$, set $Y = A(k) + D(k) \cdot \min(u_1,u_2)$; otherwise go to step (3) and try again.

A schematic of the entire procedure for generating random variates from the density f(y), given in (4.0.2), is presented in figure 4.0(e).

4.1 Generation of Exponentially Distributed Random Numbers

Applying the composition principle presented in 3.1, the exponential density function,

$$f(t) = e^{-t}, \; 0 \leq t \leq \infty, \tag{4.1.1}$$

is represented as a mixture of three densities

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t). \tag{4.1.2}$$

As before, g_1 is a series of rectangles defined for $t \leq 4$, g_2 represents the toothlike regions between g_1 and f defined for $t \leq 4$, and g_3 is the tail of f for $t \geq 4$. The probabilities p_1, p_2, and p_3 are respec-

tively the areas represented by g_1, g_2, and g_3.

As described in 3.1 a random variable Y having density g_1 is generated as follows:

Define

$M = 4/\Delta$, where $\Delta = 10^{-1}$ for a decimal machine or $\Delta = 16^{-1}$ for a binary machine; and discrete variates (4.1.3)

$t_i = (i-1) \cdot \Delta$ for $i = 1,2, \ldots, M$. (4.1.4)

Each t_i is assigned a probability

$$P_i = \Delta \cdot f(t_{i+1}) = \Delta \cdot e^{-t_{i+1}} = \delta_{1i}\delta_{2i}\delta_{3i}\ldots \tag{4.1.5}$$

Now, using the high order 4 octal digits (binary machine) or the high order 3 decimal digits (decimal machine), store the t_i's according to the technique described in Chapter 2. As in the case of the example given in the beginning of this chapter, the sets of discrete variates are stored in the manner that permits maximum overlap. See the D-table in the program listing of GEN1 in the appendix; set 1 occupies locations 49A thru 4BE, set 2 occupies locations 40C through 4AC, and set 3 occupies locations 37D through 453. Locations 374 through 3B5 are used for the generation of variates from g_2 and the residual of g_1 as described in 3.1.

At this point define:

$$Q_0 = 0, \quad Q_r = \beta^{-r} \cdot \sum_{i=1}^{M} \delta_{ri} \tag{4.1.6}$$

$$\Pi_0 = 0, \quad \Pi = \sum_{j=1}^{r} \sum_{i=1}^{M} \delta_{ji} \quad \text{for } r = 1,2,3,4 \tag{4.1.7}$$

for binary machines where the δ's are octal digits and $\beta = 8$ or $r = 1,2,3$

for decimal machines where the δ's are decimal digits and $\beta = 10$. Also define:

$$S_0 = 0, \; S_k = \beta^k \cdot \sum_{r=1}^{k} Q_r \tag{4.1.8}$$

and

$$N_k = \Pi_{k-1} - \beta \cdot S_{k-1} \text{ for } k = 1,2,3,4 \tag{4.1.9}$$

in the binary case ($\beta = 8$) or $k = 1,2,3$ in the decimal case ($\beta = 10$). The S-values and the N-values are recorded in the program listing of GEN1 in the appendix. Note that the N-values have been adjusted to allow for overlap between the sets of the D-table.

The procedure for the generation of random variates from this truncated portion of g_1 is: (binary case)

1) Generate a uniform (0,1) random number $u = .d_1d_2\ldots$.
2) If $d_1d_2 < S_2$, set $Y = D\{d_1d_2 + N_2\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (3).
3) If $d_1d_2d_3 < S_3$, set $Y = D\{d_1d_2d_3 + N_3\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (4).
4) If $d_1d_2d_3d_4 < S_4$, set $Y = D\{d_1d_2d_3d_4 + N_4\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise generate a random variate from the residual of g_1, g_2, or g_3.

Note that in the above procedure the testing begins with S_2. This is because $\delta_{1i} = 0$ for all i since $0 < f(x) < 1$ for $0 < x < 4$.

Random variates from the residual of g_1 and from g_2 are generated by first selecting one of the rectangles of g_1 with appropriate probability and then testing to determine whether the required variate should

come from the tooth or from the residual corresponding to that rectangle. This is accomplished as follows:

In M consecutive locations store each of the t_i's once (locations 374 through 3B3 of the D-table in the listing of GEN1 in the appendix). For each t_i define a pair of values as given in (3.1.12), (3.1.13), and (3.1.14)

$$C(1) = S_4 + R[D(1)], \tag{4.1.10}$$

$$B(k) = C(k) + T[D(k)] \text{ and} \tag{4.1.11}$$

$$C(k+1) = C(k) + R[D(k+1)]. \tag{4.1.12}$$

In the exponential case

$$T[D(k)] = e^{-D(k)} - (1 + \Delta) \cdot e^{-(D(k)+\Delta)} \tag{4.1.13}$$

and

$$R[D(k)] = P[D(k)] - .\delta[1,D(k)]\delta[2,D(k)]\delta[3,D(k)]\delta[4,D(k)], \tag{4.1.14}$$

where $P[D(k)]$ is as defined in (4.1.5). The values $T[D(k)]$ and $R[D(k)]$ denote respectively the areas of the triangular region and rectangular residue lost upon truncation of $P[D(k)]$ corresponding to the t-value occupying location $D(k)$. The values of the $B(k)$'s and the $C(k)$'s are also recorded in the listing of GEN1 in the appendix. A random variate with a rectangular distribution is generated as described in 3.1, and in the exponential case a variate having the density of one of the teeth of g_2 is generated by the technique given in 3.4. The procedure for generating random variates from either the residual of g_1 or from g_2 is summarized as follows:

1) Let u be a uniform (0,1) random number such that $S_4 \leq u < p_1 + p_2 = 1 - e^{-4}$.

2) Test $u < B(J)$ for $J = 1,2, \ldots, M$, until the inequality is satisfied (note $B(M) = 1 - e^{-4}$) for same $J = J'$.

3) If $u < C(J')$, generate a new uniform (0,1) random number v and set $Y = D(J') + \Delta \cdot v$ (this produces a number from g_1); otherwise go to (4).

4) Generate a new uniform (0,1) random number v and test $v < Q(n) = \sum_{j=2}^{n} \Delta^j/[j!(e^{\Delta} - 1 - \Delta)]$ for $n = 2,3, \ldots$ until the inequality is satisfied for $n = n'$.

5) Generate n' independent uniform (0,1) random numbers and set $Y = D(J') + \Delta \cdot \min(u_1,u_2, \ldots, u_{n'})$. This produces a number from g_2.

A random variate Y having density g_3 is generated as a sum

$$Y = W + X \tag{4.1.15}$$

where X is exponential on (0,4) and W has the distribution

$$P[W = w = 4 \cdot k] = e^{-4k}(e^4 - 1), \quad k = 1,2, \ldots. \tag{4.1.16}$$

The procedure for generating a random variate from g_3 is summarized as follows:

1) Let u be a uniform (0,1) random number such that $u \geq p_1 + p_2 = 1 - e^{-4}$.

2) Test $u < 1 - e^{-4(k+1)}$ for $k = 1,2, \ldots$ until the inequality is satisfied for some $k = k'$.

3) Set $W = 4 \cdot k'$ and generate a new uniform (0,1) random number v.

4) Set $u = (p_1 + p_2) \cdot v = (1 - e^{-4}) \cdot v$ and enter the procedure for generating exponentially distributed random variates on the interval (0,4) with $u = (1 - e^{-4}) \cdot v$ as the initial uniform number and generate X.

5) Set $Y = W + X$.

In [13] Marsaglia presents a schematic and the necessary constants for the generation of random variates from the exponential distribution. Application of this method results in a very fast generator program. As is evident from the discussion herein the method is also exact; the accuracy of the result is dependent only on the word size of the computer.

4.2 Generation of Gamma Distributed Random Variates

In order to produce random variates having the gamma distribution, the following result is used.

If x_i is a random variable with density

$$f(x_i) = e^{-x_i}, \; 0 < x_i < \infty \quad \text{for } i = 1,2, \ldots, n \qquad (4.2.1)$$

then the random variable

$$Y = \sum_{i=1}^{n} x_i \qquad (4.2.2)$$

has the gamma density

$$g(y) = (1/\Gamma(n))y^{n-1} e^{-y}, \; 0 < y < \infty \qquad (4.2.3)$$

when the x_i's are stochastically independent. Consequently, the generation of random variates from the gamma distribution involves only a simple

modification of the exponential method.

1) Using the method given in 4.1, generate n exponentially distributed random numbers $x_1, x_2, \ldots, x_n$.

2) Set $Y = \sum_{i=1}^{n} x_i$ (if a scale parameter $B \neq 1$ is desired, set $Y = B \cdot \sum_{i=1}^{n} x_i$).

4.3 Generation of Random Variates having the Standard Normal Distribution

Using the composition principle of 3.1, the absolute normal density

$$f(t) = \sqrt{2/\pi}\ e^{-\frac{1}{2}t^2},\ t \geq 0 \tag{4.3.1}$$

is represented as a mixture of three densities

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t). \tag{4.3.2}$$

Again, g_1 is a series of rectangles defined for $t \leq 3$, g_2 represents the toothlike regions above g_1 and below f defined for $t \leq 3$, and g_3 is the tail of the density (4.3.1) defined for $t \geq 3$.

Random variates from g_1 are generated by means of the technique given in 3.1. Define

$$M = 3/\Delta \quad \text{where } \Delta = 10^{-1} \text{(decimal machine)}$$
$$\text{or } \Delta = 16^{-1} \text{(binary machine), and} \tag{4.3.3}$$

discrete variables

$$t_i = (i - 1) \cdot \Delta \quad \text{for } i = 1, 2, \ldots, M. \tag{4.3.4}$$

Assign each t_i a probability

$$P_i = \Delta \cdot f(t_{i+1}) = \Delta \cdot \sqrt{2/\pi}\; e^{-\frac{1}{2}t_{i+1}^2} = .\delta_{1i}\delta_{2i}\ldots \qquad (4.3.5)$$

Applying the storage technique described in Chapter 2, store the t_i's based on the high order 4 octal digits (binary machine) or the high order 3 decimal digits. Note that as in the exponential case $\delta_{1i} = 0$ for all i, and that the sets of discrete values are overlapped (see the A-table in the listing of GEN3 in the appendix). Now define values

$$Q_0 = 0, \quad Q_r = \beta^{-r} \sum_{i=1}^{M} \delta_{ri} \qquad (4.3.6)$$

and

$$\Pi_0 = 0, \quad \Pi_r = \sum_{j=1}^{r} \sum_{i=1}^{M} \delta_{ji}; \qquad (4.3.7)$$

also define

$$S_0 = 0, \quad S_k = \beta^k \cdot \sum_{r=1}^{k} Q_r \qquad (4.3.8)$$

and

$$N_i = \Pi_{k-1} - \beta \cdot S_{k-1}, \qquad (4.3.9)$$

for a binary machine $r,k = 1,2,3,4$ and $\beta = 8$ for a decimal machine, $r,k = 1,2,3$ and $\beta = 10$. The S-values and N-values are also recorded in the program listing of GEN3 (note that the N-values have been adjusted to allow for overlap between sets).

The procedure for the generation of random variates from this truncated portion of g_1 is: (binary case)

1) Generate a uniform (0,1) random number $u = .d_1d_2\ldots$
2) If $d_1d_2 < S_2$, set $Y = A(d_1d_2 + N_2) + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (3).

3) If $d_1d_2d_3 < S_3$, set $Y = A\{d_1d_2d_3 + N_3\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (4).

4) If $d_1d_2d_3d_4 < S_4$, set $Y = A\{d_1d_2d_3d_4 + N_4\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise generate a random variable from g_2, g_3, or the residual of g_1.

Random variates from the residual of g_1 and from g_2 are produced by first choosing one of the rectangles of g_1 with appropriate probability and then testing to determine whether the required variate should come from the rectangular residue of g_1 or from the toothlike region of g_2. As presented in 3.1, this is accomplished as follows:

In M consecutive locations store each t_i once, and for each t_i define a pair of values as in (3.1.12), (3.1.13), and (3.1.14):

$$C(1) = S_4 + R[A(1)], \tag{4.3.10}$$

$$B(k) = C(k) + T[A(k)], \tag{4.3.11}$$

$$C(k+1) = C(k) + R[A(k+1)]. \tag{4.3.12}$$

Note that in the listing of GEN3 this portion of the A-table occupies locations 690 thru 6BF. However, for convenience it is assumed that location 690 correspond to A(1). R[A(k)] and T[A(k)] denote respectively the area of the rectangular remnant and the area of the toothlike region corresponding to the t-value occupying location A(k). For the normal case:

$$R[A(k)] = P[A(k)] - .\delta[1,A(k)]\delta[2,A(k)]\delta[3,A(k)]\delta[4,A(k)] \tag{4.3.13}$$

where $P[A(k)]$ is defined in (4.3.5) as

$$P[A(k)] = \Delta \cdot f(A(k) + \Delta) = \Delta \cdot \sqrt{2/\pi} \cdot e^{-\frac{1}{2}(A(k)+\Delta)^2}$$

$$= .\delta[1,A(k)]\delta[2,A(k)]\ldots, \qquad (4.3.14)$$

and

$$T[A(k)] = F(A(k) + \Delta) - F(A(k)) - P[A(k)] \qquad (4.3.15)$$

where

$$F(x) = \sqrt{2/\pi} \int_0^x e^{-\frac{1}{2}t^2} dt \qquad (4.3.16)$$

Once having determined whether a variate with a rectangular density (g_1) or a variate with a toothlike density (g_2) is required, the schemes given in 3.1 and in 3.5 are used to produce the variate as required. This procedure is summarized as follows:

1) Let u be a uniform (0,1) random number such that $S_4 \leq u < p_1 + p_2 = F(3)$.
2) Test $u < B(k)$ for $k = 1,2, \ldots, M$ until the inequality is satisfied for some $k = k'$.
3) If $u < C(k')$, then generate a new uniform (0,1) random number v and put $Y = A(k') + \Delta \cdot v$ (this produces a variate from g_1); otherwise go to (4).
4) Generate independent uniform (0,1) random numbers u_1 and u_2.
5) If $\max(u_1,u_2) \leq D(k')^*$, set $Y = A(k') + \Delta \cdot \min(u_1,u_2)$. This produces a variate from g_2 as described in 3.5; otherwise go to (6).
6) Set $W = -\frac{1}{2}(\Delta \cdot \min(u_1,u_2) - \Delta) \cdot [2 \cdot A(k') + \Delta \cdot \min(u_1,u_2) + \Delta]$

---

* The D(k)'s are tabled values of the a_i/b_i ratios given in (3.5.12) and in (3.5.14) corresponding to the t_i stored in A(k').

and test $|u_1 - u_2| < E(k') \cdot (e^W - 1)$**. If yes, set $Y = A(k') + \Delta \cdot \min(u_1, u_2)$. If no go to step (4) and try again.

A random variate from the density g_3 is produced by the technique presented in 3.6. An absolute normal variable $|Y|$, conditioned by $|Y| > 3$, is required. The procedure is summarized as follows:

1) Generate pairs of uniform (0,1) random numbers u_1 and u_2 until the condition $u_1^2 + u_2^2 \leq 1$ is satisfied.

2) Form pairs x_1 and x_2 as described in (3.6.1) and (3.6.2)

$$x_1 = u_1 \cdot [\{9 - 2\ln(u_1^2 + u_2^2)\}/\{u_1^2 + u_2^2\}]^{\frac{1}{2}}$$

$$x_2 = u_2 \cdot [\{9 - 2\ln(u_1^2 + u_2^2)\}/\{u_1^2 + u_2^2\}]^{\frac{1}{2}}$$

3) Test $x_1 > 3$. If yes, set $Y = x_1$. If no, test $x_2 > 3$. If yes, put $Y = x_2$. If no, go to step (1) and repeat the procedure.

In [12] Marsaglia presents a schematic and the required constants for the generation of random variates from the absolute normal distribution. This method results in random variates having the density (4.3.1). In order to obtain random variates from the standard normal distribution.

$$f(t) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}t^2},\ -\infty < t < \infty \quad , \qquad (4.3.17)$$

a random + or - sign must be attached at some point in the procedure. In GEN3 this is done by generating a uniform (0,1) random number u and testing $u < \frac{1}{2}$. If yes, a - sign is affixed to the variate; otherwise a positive variate is returned.

The procedures presented in this chapter derive their speed from the

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** This expression is simply an alternative way of testing $b_i|u-v| \leq f(A(k') + \Delta \cdot \min(u_1, u_2))$.

fact that the vast majority of variates generated come from the truncated portion of g_1. It requires only the generation of a uniform (0,1) random number, a series of compare instructions, and a table lookup. While the generation of variates from g_2 and g_3 may be somewhat time consuming, such variates are required only rarely. Consequently, that average time per generation is quite fast. As is evident these methods are exact. Precision is limited only by the word size of the computer.

CHAPTER 5
RESULTS OF TESTS PERFORMED ON SEQUENCES OF PSEUDO-RANDOM NUMBERS

5.1 Random Numbers From The Uniform (0,1) Distribution

The tests performed on sequences of independent uniform (0,1) random numbers were suggested by MacLaren and Marsaglia in [9]. The stringency of these tests is justified in that the procedures for the generation of random variates from other distributions depend heavily upon the method used to obtain uniform (0,1) random numbers.

The tests made were chi-square (χ^2) goodness of fit tests on the distribution of the random numbers, pairs of random numbers, triples of random numbers, and the maximum and minimum of n random numbers. In general a sequence of ℓ variables $Y_1, Y_2, \ldots, Y_\ell$ was calculated from the sequence of random uniform variates. The range of the Y_i was divided into m cells of equal probability, p, and the number of occurrences, O_i, in each cell counted. The χ^2 statistic

$$\chi^2 = \sum_{i=1}^{m} (O_i - \ell \cdot p)^2/\ell \cdot p \qquad (5.1.1)$$

with m - 1 degrees of freedom was calculated and transformed to a standard normal deviate

$$T = (2 \cdot \chi^2)^{\frac{1}{2}} - (2 \cdot (m - 1) - 1)^{\frac{1}{2}} \qquad (5.1.2)$$

for each test. This form is valid for $m \geq 31$. The significance level of this normal deviate was then computed as

$$S = \int_0^T (1/\sqrt{2\pi})\, e^{-\frac{1}{2}t^2} dt \,. \qquad (5.1.3)$$

In each test a total of 100,000 uniform numbers was generated. For convenience the numbers were generated in 10,000 sets of 10 uniform numbers each. Five separate runs were made and the tests performed as follows:

1) Uniformity. The unit interval was divided into 1000 segments. Each segment has an expected value of 100 occurrences. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{1000} (O_i - 100)^2/100 \qquad (5.1.4)$$

where O_i is the number of observed occurrences in the ith segment.

2) Pairs. Successive pairs of uniform numbers were taken as the coordinates of a point in the unit square. The unit square was partitioned into 100 cells, each with an expected value of 500 occurrences. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{10} (O_{ij} - 500)^2/500 \qquad (5.1.5)$$

where O_{ij} is the number of observed occurrences in the ij^{th} cell.

3) Triples. Successive triples of uniform numbers were taken as the coordinates of a point in the unit cube (every tenth number was skipped). The unit cube was partitioned into 1000 cells with an expected

value of 30 occurrences in each cell. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{10} \sum_{k=1}^{10} (O_{ijk} - 30)^2/30 \qquad (5.1.6)$$

where O_{ijk} is the number of observed occurrences in the ijk^{th} cell.

4) Maximum of n random values. If $u_1, u_2, \ldots, u_n$ are independent uniform (0,1) deviates, then

$$W = \max(u_1, u_2, \ldots, u_n) \qquad (5.1.7)$$

should have the distribution

$$P(W \leq a) = F(a) = a^n \quad \text{for } 0 < a < 1, \text{ and} \qquad (5.1.8)$$

$$F(W) = [\max(u_1, u_2, \ldots, u_n)]^n \qquad (5.1.9)$$

should be uniformly distributed over the interval (0,1). A total of 10,000 W's were generated for each n, and F(W) was tested for uniformity using the χ^2 goodness of fit test for 100 equal subintervals. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{100} (O_i - 100)^2/100 \qquad (5.1.10)$$

where O_i is the number of observed occurrences in the ith subinterval. In this test n takes the values 2 and 5.

5) Minimum of n random values. This test is the same as that for the maximum of n except

$$W = \min(u_1, u_2, \ldots, u_n) \text{ and} \qquad (5.1.11)$$

and

$$F(W) = 1 - (1 - W)^n \tag{5.1.12}$$

which should be uniformly distributed over the unit interval. The 10,000 W's were segmented into 100 equal subintervals and the χ^2 test computed as in (5.1.10). In this test n takes the values 3 and 10.

The results of these tests are reported in Table 5.1.1. The results compare favorably with those presented in [9] and justify the use of this particular uniform (0,1) random number generator.

TABLE 5.1.1

RUN #1

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	975.400	999	0.30186688
PAIRS	89.6720	99	0.25987495
TRIPLES	1034.7333	999	0.78918171
MAX OF 2	97.1800	99	0.46241028
MAX OF 5	80.7800	99	0.09257838
MIN OF 3	109.5200	99	0.77766504
MIN OF 10	75.0400	99	0.03713434

RUN #2

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1047.1600	999	0.85902187
PAIRS	92.8440	99	0.34129536
TRIPLES	1022.4667	999	0.70302930
MAX OF 2	104.4800	99	0.66267689
MAX OF 5	96.9800	99	0.45671365
MIN OF 3	102.1000	99	0.60032472
MIN OF 10	97.7400	99	0.47836695

TABLE 5.1.1 (continued)

RUN #3

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1006.3400	999	0.56949803
PAIRS	93.8720	99	0.36930366
TRIPLES	1052.0000	999	0.88131258
MAX OF 2	93.5400	99	0.36019046
MAX OF 5	104.5000	99	0.66318213
MIN OF 3	102.7400	99	0.61749364
MIN OF 10	107.0600	99	0.72480337

RUN #4

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	970.2600	999	0.26223433
PAIRS	103.9040	99	0.64798213
TRIPLES	1084.4667	999	0.97021768
MAX OF 2	83.2400	99	0.12861638
MAX OF 5	96.4200	99	0.44078290
MIN OF 3	76.7000	99	0.04945168
MIN OF 10	116.5000	99	0.89040197

TABLE 5.1.1 (continued)

RUN #5

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1052.4000	999	0.88303445
PAIRS	82.5080	99	0.11705985
TRIPLES	1013.5333	999	0.63124447
MAX OF 2	92.6200	99	0.33528034
MAX OF 5	98.8400	99	0.50965471
MIN OF 3	140.6600	99	0.99687357
MIN OF 10	136.3000	99	0.99333696

5.2 Random Numbers Having A Discrete Distribution

Sequences of random numbers from the binomial density

$$f(x) = \binom{N}{x} \cdot p^x \cdot (1 - p)^{N-x}, \quad 0 < p < 1, \quad x=0,1,\ldots N \qquad (5.2.1)$$

were generated for a variety of parameter combinations. The range of the variable was divided into k intervals of probability q_i, such that the expected value of each interval, $m \cdot q_i$, was at least 10. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{k} (O_i - m \cdot q_i)^2/m \cdot q_i \qquad (5.2.2)$$

where O_i is the number of observed occurrences in the ith interval, and m is the size of the sample generated. The significance level was computed as

$$S = (1/\Gamma(\nu/2)) \cdot \int_0^{\chi^2/2} e^{-t} \cdot t^{(\nu/2 - 1)} dt \qquad (5.2.3)$$

where ν = degrees of freedom, in this case $\nu = k-1$. The results of these tests are recorded in table 5.2.1. A sample size of 1000 random numbers were generated for each test.

Random numbers from the Poisson density

$$f(x) = \lambda^x e^{-\lambda}/x! \ , \ 0 < \lambda < \infty \ , \ x = 0,1,\ldots,\infty \qquad (5.2.4)$$

and from the negative binomial density

$$f(x) = \binom{r+x-1}{x} \cdot p^r \cdot (1-p)^x, \ r \geq 0, \quad 0 < p < 1, \qquad (5.2.5)$$

$$x = 0,1,2,\ldots,\infty;$$

were tested in the same manner as random numbers from the binomial density (5.2.1). The results of these tests are recorded in table 5.2.2 and 5.2.3 respectively.

TABLE 5.2.1

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS
ON RANDOM NUMBERS FROM THE BINOMIAL DENSITY

P	N	D.F.	CHI-SQUARE	SIGNIFICANCE
0.100	10	4	6.03494	0.8034458581
0.100	20	6	6.44102	0.6243600635
0.100	30	7	2.52631	0.0748969943
0.100	40	9	6.33123	0.2936355148
0.100	50	9	12.92191	0.8338284965
0.200	10	5	7.56431	0.8180614798
0.200	20	8	16.34847	0.9623445588
0.200	30	10	15.13733	0.8728654417
0.200	40	11	9.00770	0.3788180126
0.200	50	13	12.00444	0.4727203102
0.300	10	7	7.97911	0.6655606411
0.300	20	9	10.98540	0.7232888983
0.300	30	11	17.78713	0.9133467551
0.300	40	13	12.08372	0.4792085395
0.300	50	15	21.66071	0.8829858823
0.400	10	7	7.29406	0.6010818180
0.400	20	10	12.18194	0.7269356404
0.400	30	12	20.64248	0.9441325898
0.400	40	14	10.05124	0.2415723570
0.400	50	16	12.35961	0.2811236434
0.500	10	8	6.86156	0.4483592431
0.500	20	10	12.57912	0.7518355135
0.500	30	12	10.41700	0.4205727006
0.500	40	14	23.28293	0.9441810635
0.500	50	16	9.42171	0.1049637735
0.600	10	7	6.92477	0.5632454466
0.600	20	10	18.27415	0.9494885520
0.600	30	12	10.17474	0.3993657394
0.600	40	14	11.25580	0.3341658613
0.600	50	16	10.03645	0.1352818684

TABLE 5.2.1 (continued)

P	N	D.F.	CHI-SQUARE	SIGNIFICANCE
0.700	10	7	3.47126	0.1617396643
0.700	20	9	10.70221	0.7033259190
0.700	30	11	12.02199	0.6380075901
0.700	40	13	13.88283	0.6178529373
0.700	50	15	18.60988	0.7680404078
0.800	10	5	4.53985	0.5254310881
0.800	20	8	16.74590	0.9671335640
0.800	30	10	10.97150	0.6402573630
0.800	40	11	9.02074	0.3800224541
0.800	50	13	15.05620	0.6961423824
0.900	10	4	3.31137	0.4928666476
0.900	20	6	4.40204	0.3775593406
0.900	30	7	6.78871	0.5487935778
0.900	40	9	6.74965	0.3368342656
0.900	50	9	5.04256	0.1694175512

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON
RANDOM NUMBERS FROM THE POISSON DENSITY.

LAM	D.F.	CHI-SQUARE	SIGNIFICANCE
5.00	10	13.57652	0.8068012578
10.00	15	8.51890	0.0986988385
15.00	17	16.33853	0.5000245564
20.00	20	18.35403	0.4359007928
25.00	21	28.70494	0.8787325544
30.00	23	24.94425	0.6468336304
35.00	25	26.91360	0.6397852204
40.00	27	43.19749	0.9750174127
45.00	28	22.55633	0.2450911949
50.00	30	23.26187	0.1956567510
55.00	30	42.01434	0.9286283430
60.00	31	34.81330	0.7087406320
65.00	33	29.49011	0.3573849816
70.00	34	29.73472	0.3231975827
75.00	35	26.18810	0.1410044103
80.00	35	26.80832	0.1619395204
85.00	36	21.61020	0.0278060381
90.00	37	28.29852	0.1527661009
95.00	38	30.51639	0.1992224404
100.00	39	37.10850	0.4435994584

TABLE 5.2.2

TABLE 5.2.3

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE NEGATIVE BINOMIAL DENSITY.

P	R	D.F.	CHI-SQUARE	SIGNIFICANCE
0.100	2	41	36.08627	0.3115372475
0.100	4	57	51.64328	0.3244292796
0.100	6	67	77.44254	0.8201664431
0.100	8	72	54.21390	0.0583581269
0.100	10	76	58.52535	0.0683503608
0.200	2	23	20.00081	0.3581367902
0.200	4	33	27.73007	0.2730820886
0.200	6	40	43.00986	0.6563763897
0.200	8	45	42.83166	0.4357673160
0.200	10	49	38.69528	0.1455417220
0.300	2	15	11.10495	0.2548804445
0.300	4	22	25.94653	0.7460151638
0.300	6	28	17.46899	0.0612580512
0.300	8	31	25.10221	0.2368731801
0.300	10	34	18.65952	0.0151991805
0.400	2	11	12.32864	0.6605459138
0.400	4	16	7.20666	0.0309309696
0.400	6	20	20.18198	0.5534007984
0.400	8	23	23.12594	0.5466209213
0.400	10	25	22.04679	0.3669595654
0.500	2	8	4.96920	0.2391366628
0.500	4	12	13.04976	0.6345655810
0.500	6	15	16.03409	0.6202105433
0.500	8	17	25.16863	0.9090016643
0.500	10	19	24.55579	0.8243107298
0.600	2	6	13.11971	0.9588261108
0.600	4	9	14.42509	0.8920130256
0.600	6	11	13.29194	0.7253265963
0.600	8	13	17.84469	0.8364937782

TABLE 5.2.3 (continued)

P	R	D.F.	CHI-SQUARE	SIGNIFICANCE
0.600	10	14	13.95647	0.5470410000
0.700	2	5	6.42472	0.7329438096
0.700	4	7	11.10162	0.8657526004
0.700	6	8	9.98931	0.7342226732
0.700	8	10	4.82568	0.0974859792
0.700	10	11	14.96953	0.8161040822
0.800	2	3	1.03482	0.2071720727
0.800	4	5	2.88184	0.2818034287
0.800	6	6	1.48994	0.0398398009
0.800	8	7	4.59512	0.2907654804
0.800	10	8	2.13612	0.0234479888
0.900	2	2	0.67942	0.2880246110
0.900	4	3	0.23420	0.0281120703
0.900	6	3	4.99992	0.8281970919
0.900	8	4	2.59324	0.3719796012
0.900	10	4	2.34658	0.3276973945

5.3 Random Numbers From Continuous Distribution Functions

Sequences of random numbers from the exponential

$$f(t) = e^{-t} \ , \ 0 < t < \infty \tag{5.3.10}$$

and normal

$$f(x) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}x} \ , \ -\infty < x < \infty \tag{5.3.2}$$

densities were generated, and segmented into 20 intervals of approximately equal probability, $p_i = 0.05$. Samples of $m = 1000$ random numbers were generated and the χ^2 statistic computed as

$$\chi^2 = \sum_{i=1}^{20} (O_i - m \cdot p_i)^2/m \cdot p_i \tag{5.3.3}$$

where O_i is the number of observed occurrences in the ith interval. The significance level was calculated as given in (5.2.3) with $\nu = 19$. The results of these tests are given in tables 5.3.1 and 5.3.2.

Sequences of random numbers from the gamma density

$$f(t) = (1/\Gamma(n))\ t^{n-1}\ e^{-t} \ , \ \ 0 < t < \infty, \tag{5.3.4}$$

were tested in a somewhat different manner. If t is a variable with density (5.3.4) then

$$y = 2 \cdot (t)^{\frac{1}{2}} - (4 \cdot n - 1)^{\frac{1}{2}} \tag{5.3.5}$$

should have an approximate standard normal density when n is large. Sequences of m gamma distributed random numbers were generated and using the transformation (5.3.5), transformed to m approximately standard normal variables. As before, the normal distribution was partitioned into 20

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE EXPONENTIAL DENSITY, F(X)=EXP(-X).

D.F.	CHI-SQUARE	SIGNIFICANCE
19	20.76000	0.6497876
19	11.24000	0.0844989
19	17.64000	0.4534074
19	22.68000	0.7482694
19	13.68000	0.1979842
19	18.52000	0.5120074
19	11.48000	0.0933650
19	17.36000	0.4345072
19	24.24000	0.8128932
19	37.24000	0.9925960

TABLE 5.3.1

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE NORMAL DENSITY

D.F.	CHI-SQUARE	SIGNIFICANCE
19	19.59699	0.5808149783
19	25.42150	0.8528693863
19	18.29836	0.4974018037
19	6.71247	0.0044039802
19	14.75770	0.2621316817
19	19.91207	0.6001134403
19	20.76446	0.6500395713
19	23.50146	0.7840244000
19	11.96570	0.1129139927
19	18.02468	0.4792096579

TABLE 5.3.2

intervals of approximately equal probability $p_i = 0.05$, and the χ^2 statistic computed as before. The results of these tests are recorded in table 5.3.3.

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON TRANSFORMED GAMMA RANDOM NUMBERS

SAMPLE SIZE	N	D.F.	CHI-SQUARE	SIGNIFICANCE
100	50	19	12.30156	0.1276659399
100	100	19	14.27597	0.2326276646
100	200	19	13.79290	0.2043727108
200	50	19	13.99520	0.2160288470
200	100	19	20.25710	0.6207407158
200	200	19	23.61244	0.7885588803
500	50	19	23.13291	0.7684615439
500	100	19	21.91446	0.7114659110
500	200	19	32.02104	0.9689153752

TABLE 5.3.3

5.4 Discussion of Results.

It should be emphasized that no test or series of tests can assure the suitability of a given sequence of random numbers for a particular problem. When possible the sequence should be tested on a similar problem with known solution. However, the results presented here establish the fundamental reliability of the generation procedures presented in this paper. The significance levels obtained should follow the uniform distribution on (0,1) if the corresponding null hypothesis tested are valid. The above results are in agreement with this condition.

CHAPTER 6
DESCRIPTION AND USE OF GENERATOR PROGRAMS

6.1 GENO

GENO supplies the user with a very fast procedure for the generation of independent uniform (0,1) random numbers with density

$$f(u) = 1, \quad 0 < u < 1. \tag{6.1.1}$$

The method used was suggested by Dr. Rolf Bargmann [1] and is a form of the multiplicative congruential procedure,

$$u_{n+1} = a \cdot u_n \bmod 2^{32} \tag{6.1.2}$$

where $a = 5^{13}$. This procedure requires that u_0, the starting value, be an odd positive integer.

GENO is written as a subroutine with entry point URAN. It is called as follows:

$$U = \text{URAN(IOD)}, \tag{6.1.3}$$

where "IOD" is the starting value and must be an odd positive integer. The desired random number U in (6.1.3), is returned to the calling program in single precision real mode.

GENO requires 20 words (80 bytes) of storage space and an average time per generation of 25 usec.

6.2 GEN1

GEN1 provides the user with a fast procedure for the generation of exponentially distributed random numbers

$$f(x) = e^{-x}, \quad 0 < x < \infty. \tag{6.2.1}$$

The method used is that described in 4.1, and presented by Marsaglia in [13].

GEN1 is written as a subroutine with entry point RANEXP. It is called from a FORTRAN main program as follows:

$$X = \text{RANEXP(IOD)}. \tag{6.2.2}$$

The parameter "IOD" primes the scheme for the generation of uniform (0,1) random numbers, "IOD" must be an odd positive integer to insure the proper generation of u. The required random number x in (6.2.2) is returned to the calling program in single precision real mode.

The average time per generation is approximately 70 usec. Memory requirement for GEN1 is 307 words (1228 bytes).

6.3 GEN2

GEN2 supplies the user with a fast procedure for the generation of gamma ($\alpha = n$, $\beta = 1$)

$$g(y) = 1/\Gamma(n) \cdot y^{n-1} \cdot e^{-y}, \quad 0 < y < \infty \tag{6.3.1}$$

distributed random numbers. The technique used is described in 4.2.

GEN2 is written as a subroutine with entry point RANGAM, and is called from a FORTRAN main program as follows:

$$Y = \text{RANGAM(IOD,N)}. \qquad (6.3.2)$$

As before, "IOD" must be an odd positive integer and n = "N" in (6.3.1). The required variate is returned in single precision real mode.

GEN2 requires 313 words of memory space and approximately 65 · N usec per generation for N > 1.

6.4 GEN3

GEN3 supplies the user with a fast procedure for the generation of random numbers from the standard normal density

$$f(x) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}x^2}\ ,\ -\infty < x < \infty. \qquad (6.4.1)$$

The procedure is described in 4.3 and by Marsaglia in [12].

GEN3 is written as a subroutine and is called from a FORTRAN main program as follows:

$$X = \text{RANORM(IOD)}. \qquad (6.4.2)$$

The parameter "IOD" must be an odd positive integer, and the required variate X in (6.4.2) is returned to the calling program in single precision real mode.

Memory requirement for GEN3 is 442 words. The average time per generation is approximately 65 usec.

6.5 GEN4

GEN4 provides a fast procedure for the generation of random numbers from the binomial distribution

$$f(x) = \binom{n}{x}\ p^x(1 - p)^{n-x},\ 0 < p < 1,\ x=0,1,\ldots,n. \qquad (6.5.1)$$

GEN4 is written as a subroutine with two entry points, BSETUP and RANBI. The user calls BSETUP with the parameters "P" and "N" e.g.

$$\text{CALL BSETUP(P,N)}. \tag{6.5.2}$$

BSETUP calculates point probabilities using the recursion relation

$$f(0) = (1 - P)^N \text{ .and} \tag{6.4.3}$$

$$f(x+1) = f(x) \cdot P \cdot (N-x)/((x+1) \cdot (1-P)).$$

The four sets of discrete values are then stored using the procedure described in Chapter 2. BSETUP need be called only once for a given "P" and "N", but must be called at least once prior to calling RANBI.

RANBI addresses the very fast scheme that generates the required variates. It is called as follows:

$$X = \text{RANBI(IOD)} \tag{6.4.4}$$

where "IOD" must be an odd positive integer, and X is returned in single precision real mode.

In its present form GEN4 requires the "P" be in the real mode, $0 < P < 1$, and that "N" be in the integer mode and not exceed 255. The program requires 948 words of memory and an average time per generation of approximately 35-40 usec. Figure 2.1 describes the flowchart of GEN4 except for the section that computes point probabilities.

If the user desires random numbers returned in the integer mode, he need only delete

```
STMT
199    ST    0, RES
200    MVI   RES, X'46'
201    AE    0, RES
```

and change the designation RANBI to IANBI in his calling statement and in statements

```
170              ENTRY RANBI
171              USING RANBI, 15
173              DC    CL7' ..RANBI'
174RANBI         STM   1,3, 24(13)
```

Storage requirements may also be altered simply by changing

```
STMT
146              C       7, = F'2000'
222              DS      2000XL1.
```

6.5 GEN5

GEN5 supplies a fast routine for the generation of random numbers from the poisson density

$$f(x) = \lambda^x e^{-\lambda}/x! \ , \ 0 < \lambda < \infty \ , \ x = 0,1,\ldots. \qquad (6.5.1)$$

Since the domain of x is infinite, it is neccessary to truncate the distribution. This is done at both low and high point values whenever a point probability is less than 16^{-4}.

GEN5 is written as a subroutine with two entry points, PSETUP and RANPOI. The user calls PSETUP with the parameter λ, e.g.

CALL PSETUP(ALAM). (6.5.2)

PSETUP calculates point probabilities using the recursion relation

$$f(0) = e^{-\lambda}$$

$$f(x+1) = f(x) \cdot \lambda/(x+1), \qquad (6.5.3)$$

and stores the four sets of discrete values by means of the technique described in Chapter 2. PSETUP need be called only once for a given λ, but must be called at least once before the initial call to RANPOI.

The entry point RANPOI addresses the very fast scheme that performs the actual generation of the required variate. It is called as follows:

$$X = \text{RANPOI(IOD)}, \tag{6.5.4}$$

where "IOD" must be an odd positive integer and the desired random number X is returned in single precision real mode.

GEN5 requires 972 words of memory space and an average time per generation of approximately 35-40 usec. The flow of GEN5 follows that presented in figure 2.1 except for the section that computes the point probabilities.

If the user desires his result returned in the integer mode, he need only delete

```
STMT
 190                    ST          0,RES
 191                    MVI         RES, X'46'
 192                    AE          0,RES
```

and change the designation RANPOI to IANPOI in his calling statement and in statements

```
STMT
 161                                ENTRY  RANPOI
 162                                USING  RANPOI
 164                                DC     CL7'   RANPOI'
 165 RANPOI                         STM    1,3,24(13)
```

Storage requirement may also be altered by changing

```
STMT
 137                    C     7, = F'2000'
 211  A                 DS    2000XL1      .
```

6.6 GEN6

GEN6 provides a fast routine for generating random numbers from the negative binomial density

$$f(x) = \binom{x+r-1}{x} p^r (1-p)^x \quad , 0 < p < 1, x = 0,1, \ldots, \qquad (6.6.1)$$

$$\text{and } r \geq 0.$$

The domain of x is infinite, consequently, the distribution is truncated at both low and high point values whenever a point probability is less than 16^{-4}.

GEN6 is written as a subroutine with two entry points, NBSETU and RANEBI. NBSETU receives the parameters, p and r, and calculates point probabilities using the recursion relation

$$f(0) = p^r \qquad (6.6.2)$$

$$f(x+1) = f(x) \cdot (1-p) \cdot (x+r)/(x+1).$$

Having computed the point probabilities, NBSETU then stores the four sets of discrete values using the procedure described in Chapter 2. NBSETU is called as follows:

$$\text{CALL NBSETU(P,IR)}, \qquad (6.6.3)$$

where "IR" must be in the integer mode and 0 < "P" < 1. NBSETU need be

called only once for a given "P" and "IR", but must be called at least once prior to the initial call to RANEBI.

RANEBI addresses the very fast scheme for the generation of the required variates, and is called as follows:

$$X = \text{RANEBI}(\text{IOD}), \tag{6.6.4}$$

where "IOD", the primer, must be an odd positive integer, and X is returned in single precision real mode.

GEN6 requires 944 words of storage space and an average time per generation in the 35-40 usec range. The flow-chart of GEN6 follows that of figure 2.1 except for the section of GEN6 that calculates the point probabilities.

6.7 GEN7

GEN7 provides a fast procedure for generating discrete random variables with any specified probability distribution denoted by a vector $P = \{p_1, p_2, \ldots\}$. It is written as a subroutine with two entry points, DSETUP and RANDIS. DSETUP is called with 3 parameters, "P", "IX", "N"; where "P" is a vector of probabilities (real mode), "IX" is a vector of corresponding discrete values (integer mode), and "N" is the number of elements in vectors, e.g.

$$\text{CALL DSETUP}(P, IX, N). \tag{6.7.1}$$

The discrete values of the "IX" vector are stored in four sets based on the high order 4 digits of the elements of the "P" vector. This procedure is given in Chapter 2 and illustrated in figure 2.1. DSETUP need be called

only once for a particular set of parameters, however, it must be called at least once prior to the initial call to RANDIS.

RANDIS addresses the very fast scheme that performs the actual generation of the required variates. It is called as follows:

$$X = \text{RANDIS(IOD)}, \tag{6.7.2}$$

where "IOD" must be an odd positive integer and "X", the generated variate is returned in single precision real mode.

In its present form GEN7 requires that the elements of the "IX" vector be in the integer mode and that no value exceed 255. Also, the parameter "N", the number of elements in the vectors, may not exceed 256. If the user wishes his discrete variables, members of the "IX" vector, to be in the real mode or to be in the integer mode but with values greater than 255, he need only make the changes indicated by table 6.1. If he desires to have his result returned in the integer mode he need only delete the following

```
STMT
 149          ST          0,RES
 150          MVI         RES, X'46'
 151          AE          0,RES,
```

and change the designation RANDIS to IANDIS in his calling statement and in the following

```
STMT
 120          ENTRY       RANDIS
 121          USING       RANDIS, 15
 123          DC          CL7'   RANDIS'
 124 RANDIS   STM         1,3,24(13).
```

Storage requirement may also be altered simply by changing

```
STMT
 97                C          7, = F'2000'
 168 A             DS         2000XL1
```

In its present form GEN7 requires 904 words of memory space and 35-40 usec per generation.

STMT			INTEGER > 255		REAL MODE	
87	L	6, = F'1'	LA	6,4	LA	6,4
97	C	7, = F'2000'	C	7, = F'SPAC'	C	7, = F'SPAC'
99	L	2, 0(3,11)	L	2, 0(3,11)	LE	2, 0(3,11)
101	STC	2, A(12)	ST	2, A(12)	STE	2, A(12)
106	CR	0,4	C	0, = F'16'	C	0, = F'16'
148 EX	IC	0, A(2)	EX SLL	2, 2	EX SLL	2, 2
168 A	DS	2000 XL1	A DS	SPAC XL4	A DS	SPAC XL4
			ALSO INSERT BETWEEN		ALSO INSERT BETWEEN	
			STMT 95 AND STMT 96		STMT 95 AND STMT 96	
			SLL	8, 2	SLL	8, 2
					AND DELETE STMT 149	
					THROUGH STMT 151.	

TABLE 6.1

REFERENCES

[1] Bargmann, R. E. private communication, Department of Statistics, University of Georgia, (November, 1969).

[2] Forsythe, G. E. "Generation and Testing of Random Digits at the National Bureau of Standards, Los Angeles, Monte Carlo Method.", Nat. Bur. Stan., Appl. Math. Series 12, pp. 34-35, (1951).

[3] Hull, T. E., and A. R. Dobell, "Random Number Generators." SIAM Rev., Vol. 4, pp. 230-254, (1962).

[4] Jansson, B. Random Number Generators, Stockholm, Almquist and Wiksell (distr.), (1966)

[5] Kendall, M. G., and B. Babington-Smith, "Randomness and Random Sampling Numbers." J. Roy. Stat. Soc. 101, pp. 147-166, (1938)

[6] Kendall, M. G., and B. Babington-Smith, "Second Paper on Random Sampling Numbers." J. Roy. Stat. Soc. Supplement 6, pp. 51-61, (1939).

[7] Kendall, M. G., and B. Babington-Smith, "Tables of Random Sampling Numbers." Tracts for Computers, No. 24, Cambridge, (1939).

[8] Lehmer, D. H. "Mathematical Methods in Large Scale Computing Units." Annals Comp. Laboratory Harvard Univ. 26, pp. 141-146, (1941).

[9] MacLaren, M. D., and G. Marsaglia, "Uniform Random Number Generators." J. ACM 12, No. 1, pp. 83-89, (January, 1965).

[10] Marsaglia, G. "Expressing a Random Variable in Terms of Uniform Random Variables." Ann. Math. Stat., Vol. 32, pp. 894-898, (1961)

[11] Marsaglia, G. "Generating Discrete Random Variables in a Computer." Comm. ACM 6, No. 1, pp. 37-38, (January, 1963).

[12] Marsaglia, G, and M. D. MacLaren, and T. A. Bray, "A Fast Procedure for Generating Normal Random Variables." Comm. ACM 7, No. 1, pp. 4-10, (January, 1964).

[13] Marsaglia, G, and M. D. MacLaren, and T. A. Bray, "A Fast Procedure for Generating Exponential Random Variables." Comm. ACM 7, No. 5, pp. 298-300, (May, 1964).

[14] Muller, M. E. "An Inverse Method for the Generation of Random Normal Deviates on Large Scale Computers." Math. Tables Other Aids 12, pp. 167-174, (1958).

[15] Rand Corporation, A Million Random Digits with 100,000 Normal Deviates, Free Press, Glencoe, (1955).

[16] Strawderman, W. E. "Generation and Testing of Pseudo-Random Numbers.", Technical Report No. 171, N00014-67-A-0112-0053 (NR-042-267), Stanford Univ., (February, 1971)

[17] Student, "The Probable Error of a Mean." Biometrika, Vol. 6, pp. 1-25, (1908).

[18] Teichroew, D. "Distribution Sampling with High Speed Computers.", Diss., Univ. of North Carolina, (1953).

[19] Tippett, L. H. C. "Random Sampling Numbers." Tracts for Computers, No. 15, Cambridge, (1927).

[20] van Hoerner, S. "Herstellung von Zafallszahlen auf Rechenautomaten." Angew. Math. Physik, Vol. 8, pp. 26-52, (1957)

[21] Wall, D. D. "Fibonacci Series Module n." The Am. Math. Monthly, Vol. 67, pp. 525-532, (1960).

APPENDIX A

Assembler Listing of GEN0

NOT REPRODUCIBLE

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F08APR70   6/30/71

000000                                  GENO     START 0
                                        ***********************************************************************
                                        * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., APR 1970                  *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING UN-*
                                        *          IFORM(0,1) RANDOM NUMBERS.                                 *
                                        * USAGE----X=URAN(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER. THE RE-*
                                        *          QUIRED VARIATE IS RETURNED IN SINGLE PRECISION REAL MODE.  *
                                        * EXAMPLE:X=URAN(56865)  'X' WILL BE UNIFORM ON (0,1).                *
                                        * METHOD---THE METHOD IS THE MULTIPLICATIVE CONGRUENTIAL PROCEDURE    *
                                        *                          X(I+1)=X(I)*A                             *
                                        *          WHERE A=5**13, AS SUGGESTED BY DR. R. F. BARGMANN.         *
                                        ***********************************************************************
                                                 ENTRY URAN
000008                                           USING URAN,15
000000 07                                        DC    XL1'07'
000001 404040E4D9C1D5                            DC    CL7'   URAN'
000008 9013 D018            00018        URAN    STM   1,3,24(13)
00000C 3800                                      SER   0,0           CLEAR FPRO FOR RESULT.
00000E 47F0 F028            00030        BRANCH  B     TEMP          BRANCH TO TEMP.
000012 5020 F044            0004C                M     2,BPAT        GENERATE U IN R3.
000016 5030 F048            00050                ST    3,LA          STORE U FOR NEXT CALL.
00001A 8830 0008            00008                SRL   3,8           CONVERT TO FLOATING POINT.
00001E 5030 F040            00048                ST    3,RES
000022 9240 F040      00048                      MVI   RES,X'40'
000026 7A00 F040            00048                AE    0,RES         NORMALIZED RESULT IN FPRO.
00002A 9813 0018            00018                LM    1,3,24(13)
00002E 07FE                                      BCR   15,14         RETURN TO CALLING PROGRAM.
000030 5830 F048            00050        TEMP    L     3,LA
000034 D203 F006 F028 0000E 00030                MVC   BRANCH,TEMP   REPLACE INSTR. AT BRANCH BY INSTR. AT TEMP.
00003A 5830 1000            00000                L     3,0(,1)       LOAD ADDRESS OF PRIMER.
00003E 5830 3000            00000                L     3,0(,3)       LOAD PRIMER.
000042 47F0 F00A            00012                B     BRANCH+4      BRANCH TO INSTR. FOLLOWING BRANCH.
000048                                   RES     DS    1F
00004C 48C27395                          BPAT    DC    XL4'48C27395'
000050                                   LA      DS    1F
                                                 END
```

APPENDIX B

Assembler Listing of GEN1

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000000                                  GEN1     START 0
                                        ***********************************************************************
                                        * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., APR 1970                 *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING EX-*
                                        *          PONETIAL DISTRIBUTED RANDOM NUMBERS.                      *
                                        * USAGE----X=RANEXP(I). 'I' MUST BE AN ODD INTEGER, IT PRIMES THE GEN-*
                                        *          ERATING SCHEME.                                           *
                                        * EXAMPLE--X=RANEXP(55695).                                          *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN  'COMMUNI- *
                                        *          CATIONS OF ACM',VOL.7,NO.5,MAY 1964. THE PROGRAM DOCUMEN-  *
                                        *          TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.         *
                                        ***********************************************************************
000008                                          ENTRY RANEXP       PROLOGUE, STANDARD FOR 360/65.
000000 07                                       USING RANEXP,15
000001 4009C1D5C5E7D7                           DC    XL1'07'
000008 9019 0018          00018         RANEXP  STM   1,9,24(13)
00000C 4780 F0AE          00086         BRANCH  B     TEMP         EXECUTED ONLY ON THE FIRST CALL,
000010 3800                                     SER   0,0          RESULT SUMMED IN FPR0.
000012 5040 F130          00138                 M     4,BPAT       OBTAIN NEW U.
000016 5050 F129          00130                 ST    5,LA         STORE FOR NEXT GENERATION.
00001A 1866                                     SR    6,6
00001C 1844                                     SR    4,4          CLEAR R4.
00001E 8040 0006          00006                 SLDL  4,6          TWO HIGH ORDER OCTAL DIGITS OF U IN R4(D1D2)
000022 5940 F488          004C0                 C     4,=F'37'     D1D2 COMPARED WITH 37(BASE 10)=45(BASE 8).
000026 4780 F02E          00036                 BC    11,NA        D1D2>=37 BRANCH NA.
00002A 4364 F452          0049A                 IC    6,D+294(4)   OBTAIN DISCRETE RANDOM VARIABLE FROM D.
00002E 8950 0006          00006                 SLL   5,6          R5=D5D6D7...
000032 4780 F05A          00062                 BC    15,EX        BRANCH EX.
000036 8040 0003          00003         NA      SLDL  4,3          R4=D1D2D3.
00003A 5940 F48C          004C4                 C     4,=F'456'    456(BASE 10)=710(BASE 8).
00003E 4780 F046          0004E                 BC    11,NB        D1D2D3>=455 BRANCH NB.
000042 4364 F20C          002F4                 IC    6,D-144(4)   OBTAIN DISCRETE RANDOM VARIABLE FROM D.
000046 8950 0003          00003                 SLL   5,3          R5=D5D6D7...
00004A 4780 F05A          00062                 BC    15,EX        BRANCH EX.
00004E 8040 0003          00003         NB      SLDL  4,3          R4=D1D2D3D4.
000052 5940 F4C0          004C8                 C     4,=F'3863'   3863(BASE 10)=7427(BASE 8).
000056 4780 F072          0007A                 BC    11,NC        D1D2D3D4>=3863 BRANCH NC.
00005A 5840 F4C4          004CC                 S     4,=F'3639'   OBTAIN PROPER ADDRESS FOR D TABLE.
00005E 4364 F3AC          00374                 IC    6,D(4)       OBTAIN DISCRETE VARIABLE FROM D.
000062 1846                                     LR    4,6          R4= DISCRETE VARIABLE.
000064 8C40 0010          00010         FL      SRDL  4,16         R5= DISCRETE VARIABLE + HI ORDER 16 BITS OF
000068 5050 F124          0012C                 ST    5,RES        R5(NOW LO ORDER 16 BITS) WITH HI ORDER 8
00006C 9241 F124    0012C                       MVI   RES,X'41'    BITS ZEROED OUT FOR CONVERSION TO FLOAT-
000070 7400 F124          0012C                 AE    0,RES        ING POINT. FPR0 CONTAINS RESULT + 'W'.
000074 9817 0018          00018                 LM    1,9,24(13)     RESTORE'REGISTERS.
000078 07FE                                     BCR   15,14      BRANCH TO MAIN LINE.
00007A 5850 F128          00130         NC      L     5,LA         RESTORE U TO R5.
00007E 4120 0004          00004                 LA    2,4
000082 1833                                     SR    3,3
000084 5553 F354          0035C                 CL    5,P(3)       U>=P(0) BRANCH NO.
000088 4780 F0FE          00106                 BC    11,NO
00008C 1832                                     SR    3,2
00008E 1432                                     AR    3,2          R3=IB(IN WORDS, 4 BYTES=1 WORD).
000090 5553 F154          0015C                 CL    5,B(3)       U>=B(IB) BRANCH NE(CONTINUE LOOP).
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000094 4780 F086          0008E   56          BC    11,NF
000098 5553 F234          0025C   57          CL    5,C(3)       C(IB)=<U<B(IB) BRANCH NF.
00009C 4780 F0C8          00000   58          BC    11,NF
0000A0 5C40 F130          00138   59          M     4,BPAT       OBTAIN NEW U.
0000A4 5050 F128          00130   60          ST    5,LA         STORE FOR NEXT GENERATIO.
0000A8 1B44                       61 GO       SR    4,4          CLEAR R4.
0000AA 8830 0002          00002   62          SRL   3,2          CONVERT IB TO BYTES.
0000AE 4343 F36C          00374   63          IC    4,D(3)       OBTAIN DISCRETE VARIABLE.
0000B2 47F0 F05C          00064   64          BC    15,FL        BRANCH TO FL.
0000B6 5850 F128          00130   65 TEMP     L     5,LA         REPLACES INSTR. AT BRANCH.
0000BA 0203 F004 F0AE 0000C 00086   66          MVC   BRANCH,TEMP  REPLACE BRANCH BY TEMP.
0000C0 5850 1000          00000   67          L     5,0(,1)      ADDRESS OF PRIMER IN R5.
0000C4 5850 5000          00000   68          L     5,0(,5)      PRIMER IN R5.
0000C8 5050 F128          00130   69          ST    5,LA         STORE PRIMER FOR USE IN URAN.
0000CC 47F0 F008          00010   70          B     BRANCH+4,    BRANCH TO INSTRUCTION FOLLOWING 'BRANCH'.
0000D0 5C40 F130          00138   71 NF       M     4,BPAT       OBTAIN NEW U.
0000D4 1882                       72          LR    8,2          USED FOR INDEXING.
0000D6 1482                       73          AR    8,2          R8=IQ.
0000D8 5558 F134          0013C   74          CL    5,0(8)       U>=Q(IQ) CONTINUE LOOP.
0000DC 4780 F0CF          000D6   75          BC    11,*-6
0000E0 5C40 F130          00138   76          M     4,BPAT       OBTAIN NEW U.
0000E4 1882                       77          SR    8,2
0000E6 1865                       78          LR    6,5
0000E8 1192                       79          LNR   9,2          R9=-4.
0000EA 1482                       80          SR    8,2
0000EC 5C40 F130          00138   81 NH       M     4,BPAT       OBTAIN NEW U.
0000F0 1556                       82          CLR   5,6          R5>=R6 BRANCH TO CONTINUE.
0000F2 4780 F0F0          000F8   83          BC    11,*+6
0000F6 1865                       84          LR    6,5          LOAD SMALLEST U IN R6.
0000F8 8689 F0F4          000FC   85          BXH   8,9,NH       CONTINUE(R9 HAS BOTH INCREMENT & DELIMITER).
0000FC 5050 F128          00130   86          ST    5,LA
000100 1856                       87          LR    5,6          R5=MIN(U1,U2,...,U(IQ)).
000102 47F0 F0A0          000A8   88          BC    15,GO
000106 1432                       89 ND       AR    3,2          R3=IP.
000108 5553 F354          0035C   90          CL    5,P(3)       U>=P(IP) CONTINUE LOOP.
00010C 4780 F0FE          00106   91          BC    11,ND
000110 5030 F12C          00134   92          ST    3,J          STORE IP INTO J.
000114 9246 F12C     00134        93          MVI   J,X'46'      CONVERT IP TO FLOATING POINT.
000118 7400 F12C          00134   94          AE    0,J          NORMALIZED AND LOADED INTO FPRO 'W'.
00011C 5C40 F130          00138   95          M     4,BPAT       OBTAIN NEW U.
000120 5050 F128          00130   96          ST    5,LA         STORE FOR NEXT GENERATION.
000124 5C46 F354          0035C   97          M     4,P(6)       U=U*P(0).
000128 47F0 F014          0001C   98          BC    15,N1        BRANCH N1.
00012C                            99 RES      DS    1F
000130                           100 LA       DS    1F
000134                           101 J        DS    1F
000138 48C27395                  102 BPAT     DC    XL4'48C27395'
00013C 00000000                  103 Q        DC    X'00000000'    BASE 10 0.0
000140 00000000                  104          DC    X'00000000'    BASE 10 0.0
000144 FA81CEA7                  105          DC    X'FA81CEA7'    BASE 10= 0.97927562333
000148 FFFAD84B                  106          DC    X'FFFAD84B'    BASE 10= 0.99957719882
00014C FFFFBC72                  107          DC    X'FFFFBC72'    BASE 10= 0.99999597344
000150 FFFFFF4C                  108          DC    X'FFFFFF4C'    BASE 10= 0.99999995812
000154 FFFFFFFE                  109          DC    X'FFFFFFFE'    BASE 10= 0.99999999963
000158 FFFFFFFF                  110          DC    X'FFFFFFFF'    BASE 10= 1.00000000000
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
000158	F1F2A021			111 B	DC	X'F1F2A021'	BASE	10	0.94510842156
000160	F23FF28D			112	DC	X'F23FF28D'	BASE	10	0.94628827211
000164	F28C810C			113	DC	X'F28C810C'	BASE	10	0.94745642234
000168	F2C13730			114	DC	X'F2C13730'	BASE	10	0.94826073593
000160	F2FDE51B			115	DC	X'F2FDE51B'	BASE	10	0.94894248986
000170	F307428B			116	DC	X'F307428B'	BASE	10	0.94932953916
000174	F317F405			117	DC	X'F317F405'	BASE	10	0.94958329319
000178	F326185C			118	DC	X'F326185C'	BASE	10	0.94980003601
00017C	F32DF665			119	DC	X'F32DF665'	BASE	10	0.94992008199
000180	F380051C			120	DC	X'F380051C'	BASE	10	0.95190460160
000184	F42015AE			121	DC	X'F42015AE'	BASE	10	0.95361457350
000188	F489094E			122	DC	X'F489094E'	BASE	10	0.95525892423
00018C	F4F4020F			123	DC	X'F4F4020F'	BASE	10	0.95686066501
000190	F5592594			124	DC	X'F5592594'	BASE	10	0.95839152226
000194	F582107A			125	DC	X'F582107A'	BASE	10	0.95974907150
000198	F60A63D7			126	DC	X'F60A63D7'	BASE	10	0.96109651596
00019C	F65F0C8F			127	DC	X'F65F0C8F'	BASE	10	0.96237258570
0001A0	F6A471A9			128	DC	X'F6A471A9'	BASE	10	0.96344671629
0001A4	F6E808E1			129	DC	X'F6E808E1'	BASE	10	0.96447806832
0001A8	F729F034			130	DC	X'F729F034'	BASE	10	0.96548445247
0001AC	F761312A			131	DC	X'F761312A'	BASE	10	0.96632678298
0001B0	F7941C80			132	DC	X'F7941C80'	BASE	10	0.96710374973
0001B4	F7C586EF			133	DC	X'F7C586EF'	BASE	10	0.96786063536
0001B8	F7F1E012			134	DC	X'F7F1E012'	BASE	10	0.96853524471
0001BC	F81D047F			135	DC	X'F81D047F'	BASE	10	0.96920558701
0001C0	F849D468			136	DC	X'F849D468'	BASE	10	0.96986129414
0001C4	F871E734			137	DC	X'F871E734'	BASE	10	0.97048802404
0001C8	F897DE94			138	DC	X'F897DE94'	BASE	10	0.97106734391
0001CC	F8B9DC57			139	DC	X'F8B9DC57'	BASE	10	0.97158600928
0001D0	F8DAC69F			140	DC	X'F8DAC69F'	BASE	10	0.97208825453
0001D4	F8F8106D			141	DC	X'F8F8106D'	BASE	10	0.97254093522
0001D8	F914289F			142	DC	X'F914289F'	BASE	10	0.97305557854
0001DC	F938A989			143	DC	X'F938A989'	BASE	10	0.97352084757
0001E0	F9568658			144	DC	X'F9568658'	BASE	10	0.97397651342
0001E4	F9740441			145	DC	X'F9740441'	BASE	10	0.97442652313
0001E8	F9900A1A			146	DC	X'F9900A1A'	BASE	10	0.97485411775
0001EC	F9AA4357			147	DC	X'F9AA4357'	BASE	10	0.97525425709
0001F0	F9C36880			148	DC	X'F9C36880'	BASE	10	0.97563794277
0001F4	F9DC3773			149	DC	X'F9DC3773'	BASE	10	0.97601648869
0001F8	F9F46745			150	DC	X'F9F46745'	BASE	10	0.97638554992
0001FC	FA0A550A			151	DC	X'FA0A550A'	BASE	10	0.97672015570
000200	FA20049B			152	DC	X'FA20049B'	BASE	10	0.97705105581
000204	FA33E7C0			153	DC	X'FA33E7C0'	BASE	10	0.97735451174
000208	FA4667F8			154	DC	X'FA4667F8'	BASE	10	0.97763681231
00020C	FA580771			155	DC	X'FA580771'	BASE	10	0.97791811485
000210	FA6B3FD0			156	DC	X'FA6B3FD0'	BASE	10	0.97819899411
000214	FA706E58			157	DC	X'FA706E58'	BASE	10	0.97847642580
000218	FA8EC18A			158	DC	X'FA8EC18A'	BASE	10	0.97874079506
00021C	FA9F650C			159	DC	X'FA9F650C'	BASE	10	0.97899467042
000220	FAAF65EE			160	DC	X'FAAF65EE'	BASE	10	0.97923892340
000224	FABF09A2			161	DC	X'FABF09A2'	BASE	10	0.97947750293
000228	FACEA835			162	DC	X'FACEA835'	BASE	10	0.97971601531
00022C	FADD203A			163	DC	X'FADD203A'	BASE	10	0.97993661330
000230	FAEB949A			164	DC	X'FAEB949A'	BASE	10	0.98015717294
000234	FAF96A01			165	DC	X'FAF96A01'	BASE	10	0.98036825688

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT					F08APR70 6/30/71
000238	F8060436			166		DC	X'F8060436'	BASE	10	0.98357295160
00023C	F8143588			167		DC	X'F8143588'	BASE	10	0.98077711667
000240	F81F2894			168		DC	X'F81F2894'	BASE	10	0.98094419111
000244	F8291DCA			169		DC	X'F8291DCA'	BASE	10	0.98109613598
000248	F832FE11			170		DC	X'F832FE11'	BASE	10	0.98126587942
00024C	F83CA379			171		DC	X'F83CA379'	BASE	10	0.98139402130
000250	F8448484			172		DC	X'F8448484'	BASE	10	0.98151460399
000254	F84C437C			173		DC	X'F84C437C'	BASE	10	0.98163244057
000258	F84FAA91			174		DC	X'F84FAA91'	BASE	10	0.98169436111
00025C	F177D5FD			175	C	DC	X'F177D5FD'	BASE	10	0.94323490080
000260	F1FF3941			176		DC	X'F1FF3941'	BASE	10	0.94529539518
000264	F24FC2FB			177		DC	X'F24FC2FB'	BASE	10	0.94651430666
000268	F2943837			178		DC	X'F2943837'	BASE	10	0.94757146938
00026C	F2CCD989			179		DC	X'F2CCD989'	BASE	10	0.94843821067
000270	F2F6A495			180		DC	X'F2F6A495'	BASE	10	0.94907597217
000274	F3136908			181		DC	X'F3136908'	BASE	10	0.94951504656
000278	F3108030			182		DC	X'F3108030'	BASE	10	0.94947253651
00027C	F3320C27			183		DC	X'F3320C27'	BASE	10	0.94985597021
000280	F33CA978			184		DC	X'F33CA978'	BASE	10	0.95014449778
000284	F3838926			185		DC	X'F3838926'	BASE	10	0.95196110898
000288	F4260070			186		DC	X'F4260070'	BASE	10	0.95370563837
00028C	F49AFC59			187		DC	X'F49AFC59'	BASE	10	0.95549089478
000290	F4F9846A			188		DC	X'F4F9846A'	BASE	10	0.95693234495
000294	F5509921			189		DC	X'F5509921'	BASE	10	0.95864135204
000298	F58FF212			190		DC	X'F58FF212'	BASE	10	0.95996017747
00029C	F6055556			191		DC	X'F6055556'	BASE	10	0.96116289522
0002A0	F65F7807			192		DC	X'F65F7807'	BASE	10	0.96237915936
0002A4	F6A5C471			193		DC	X'F6A5C471'	BASE	10	0.96359303254
0002A8	F6F39363			194		DC	X'F6F39363'	BASE	10	0.96465303933
0002AC	F72F0177			195		DC	X'F72F0177'	BASE	10	0.96554574164
0002B0	F7640587			196		DC	X'F7640587'	BASE	10	0.96637302930
0002B4	F7238BE7			197		DC	X'F7238BE7'	BASE	10	0.96732393493
0002B8	F7C77089			198		DC	X'F7C77089'	BASE	10	0.96788774270
0002BC	F7FFCF83			199		DC	X'F7FFCF83'	BASE	10	0.96873186240
0002C0	F823515C			200		DC	X'F823515C'	BASE	10	0.96928987209
0002C4	F8480408			201		DC	X'F8480408'	BASE	10	0.96987974044
0002C8	F87C79AC			202		DC	X'F87C79AC'	BASE	10	0.97064928261
0002CC	F8A495E7			203		DC	X'F8A495E7'	BASE	10	0.97126042281
0002D0	F8C7E209			204		DC	X'F8C7E209'	BASE	10	0.97180092548
0002D4	F8E15379			205		DC	X'F8E15379'	BASE	10	0.97219820298
0002D8	F8F00158			206		DC	X'F8F00158'	BASE	10	0.97261055461
0002DC	F921F2CE			207		DC	X'F921F2CE'	BASE	10	0.97317426251
0002E0	F93E5398			208		DC	X'F93E5398'	BASE	10	0.97360757562
0002E4	F9663071			209		DC	X'F9663071'	BASE	10	0.97421630871
0002E8	F97E5995			210		DC	X'F97E5995'	BASE	10	0.97458419708
0002EC	F9939911			211		DC	X'F9939911'	BASE	10	0.97503048590
0002F0	F9374014			212		DC	X'F9374014'	BASE	10	0.97545242919
0002F4	F9CC990C			213		DC	X'F9CC990C'	BASE	10	0.97577828454
0002F8	F9E059C9			214		DC	X'F9E059C9'	BASE	10	0.97607968974
0002FC	FA004CC4			215		DC	X'FA004CC4'	BASE	10	0.97656636054
000300	FA195797			216		DC	X'FA195797'	BASE	10	0.97695777372
000304	FA2E064F			217			[illegible]E064F'	BASE	10	0.97727717817
000308	FA337605			218		DC	X'FA337605'	BASE	10	0.97743933414
00030C	FA52557E			219		DC	X'FA52557E'	BASE	10	0.97781981658
000310	FA668368			220		DC	X'FA668368'	BASE	10	0.97816010835

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LOC	OBJECT CODE	ADDR1 ADDR2	STMT	SOURCE STATEMENT			
000314	FA73F667		221	DC	X'FA73F667'	BASE 10	0.97833194768
000318	FA8355E5		222	DC	X'FA8355E5'	BASE 10	0.97856652118
00031C	FA94E954		223	DC	X'FA94E954'	BASE 10	0.97894151090
000320	FAA8798A		224	DC	X'FAA8798A'	BASE 10	0.97913322095
000324	FAB8172F		225	DC	X'FAB8172F'	BASE 10	0.97941727549
000328	FAC5C620		226	DC	X'FAC5C620'	BASE 10	0.97959029068
00032C	FAD78818		227	DC	X'FAD78818'	BASE 10	0.97985429215
000330	FAE434D6		228	DC	X'FAE434D6'	BASE 10	0.98004465[illegible]
000334	FAF1907C		229	DC	X'FAF1907C'	BASE 10	0.98024840[illegible]
000338	FB046F58		230	DC	X'FB046F58'	BASE 10	0.98053642181
00033C	FB115246		231	DC	X'FB115246'	BASE 10	0.98073305333
000340	FB146DE4		232	DC	X'FB146DE4'	BASE 10	0.98078047595
000344	FB245AF4		233	DC	X'FB245AF4'	BASE 10	0.98102348782
000348	FB2FA860		234	DC	X'FB2FA860'	BASE 10	0.98119594912
00034C	FB36E548		235	DC	X'FB36E548'	BASE 10	0.98130639089
000350	FB40D549		236	DC	X'FB40D549'	BASE 10	0.98145802546
000354	FB49B004		237	DC	X'FB49B004'	BASE 10	0.98159104689
000358	FB4C97A1		238	DC	X'FB4C97A1'	BASE 10	0.98163745593
00035C	FB4FAA91		239 P	DC	X'FB4FAA91'	BASE 10	0.98168436111
000360	FFFA03DE		240	DC	X'FFFA03DE'	BASE 10	0.99966453737
000364	FFFF98EA		241	DC	X'FFFF98EA'	BASE 10	0.99999385579
000368	FFFFFE1C		242	DC	X'FFFFFE1C'	BASE 10	0.99999988746
00036C	FFFFFFF7		243	DC	X'FFFFFFF7'	BASE 10	0.99999999794
000370	FFFFFFFF		244	DC	X'FFFFFFFF'	BASE 10	0.99999999996
000374	00		245 D	DC	X'00'	BASE 10	0.0
000375	0A		246	DC	X'0A'	BASE 10	0.6250
000376	08		247	DC	X'08'	BASE 10	0.6875
000377	10		248	DC	X'10'	BASE 10	1.0000
000378	15		249	DC	X'15'	BASE 10	1.3125
000379	20		250	DC	X'20'	BASE 10	2.0000
00037A	35		251	DC	X'35'	BASE 10	3.3125
00037B	28		252	DC	X'28'	BASE 10	2.6875
00037C	36		253	DC	X'36'	BASE 10	3.3750
00037D	01		254	DC	X'01'	BASE 10	0.0625
00037E	02		255	DC	X'02'	BASE 10	0.1250
00037F	03		256	DC	X'03'	BASE 10	0.1875
000380	05		257	DC	X'05'	BASE 10	0.3125
000381	04		258	DC	X'04'	BASE 10	0.2500
000382	06		259	DC	X'06'	BASE 10	0.3750
000383	08		260	DC	X'08'	BASE 10	0.5000
000384	07		261	DC	X'07'	BASE 10	0.4375
000385	09		262	DC	X'09'	BASE 10	0.5625
000386	0C		263	DC	X'0C'	BASE 10	0.7500
000387	0D		264	DC	X'0D'	BASE 10	0.8125
000388	0F		265	DC	X'0F'	BASE 10	0.8750
000389	0F		266	DC	X'0F'	BASE 10	0.9375
00038A	14		267	DC	X'14'	BASE 10	1.2500
00038B	11		268	DC	X'11'	BASE 10	1.0625
00038C	16		269	DC	X'16'	BASE 10	1.3750
00038D	13		270	DC	X'13'	BASE 10	1.1875
00038E	12		271	DC	X'12'	BASE 10	1.1250
00038F	18		272	DC	X'18'	BASE 10	1.5000
000390	1C		273	DC	X'1C'	BASE 10	1.7500
000391	1E		274	DC	X'1E'	BASE 10	1.8750
000392	19		275	DC	X'19'	BASE 10	1.5625

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT			
000393	17			276	DC	X'17'	BASE 10	1.4375
000394	18			277	DC	X'18'	BASE 10	1.6875
000395	1A			278	DC	X'1A'	BASE 10	1.6250
000396	23			279	DC	X'23'	BASE 10	2.1875
000397	1F			280	DC	X'1F'	BASE 10	1.9375
000398	22			281	DC	X'22'	BASE 10	2.1250
000399	25			282	DC	X'25'	BASE 10	2.3125
00039A	21			283	DC	X'21'	BASE 10	2.0625
00039B	1D			284	DC	X'1D'	BASE 10	1.8125
00039C	28			285	DC	X'28'	BASE 10	2.5000
00039D	30			286	DC	X'30'	BASE 10	3.0000
00039E	33			287	DC	X'33'	BASE 10	3.1875
00039F	24			288	DC	X'24'	BASE 10	2.2500
0003A0	2F			289	DC	X'2F'	BASE 10	2.9375
0003A1	3E			290	DC	X'3E'	BASE 10	3.8750
0003A2	29			291	DC	X'29'	BASE 10	2.5625
0003A3	26			292	DC	X'26'	BASE 10	2.3750
0003A4	39			293	DC	X'39'	BASE 10	3.5625
0003A5	2E			294	DC	X'2E'	BASE 10	2.8750
0003A6	37			295	DC	X'37'	BASE 10	3.4375
0003A7	2A			296	DC	X'2A'	BASE 10	2.6250
0003A8	32			297	DC	X'32'	BASE 10	3.1250
0003A9	2C			298	DC	X'2C'	BASE 10	2.8125
0003AA	2C			299	DC	X'2C'	BASE 10	2.7500
0003AB	3F			300	DC	X'3F'	BASE 10	3.9375
0003AC	3C			301	DC	X'3C'	BASE 10	3.7500
0003AD	27			302	DC	X'27'	BASE 10	2.4375
0003AE	34			303	DC	X'34'	BASE 10	3.2500
0003AF	3A			304	DC	X'3A'	BASE 10	3.6250
0003B0	31			305	DC	X'31'	BASE 10	3.0625
0003B1	38			306	DC	X'38'	BASE 10	3.5000
0003B2	3D			307	DC	X'3D'	BASE 10	3.8125
0003B3	38			308	DC	X'38'	BASE 10	3.6875
0003B4	02			309	DC	X'02'	BASE 10	0.1250
0003B5	C3			310	DC	X'03'	BASE 10	0.1875
0003B6	03			311	DC	X'03'	BASE 10	0.1875
0003B7	C3			312	DC	X'03'	BASE 10	0.1875
0003B8	03			313	DC	X'03'	BASE 10	0.1875
0003B9	03			314	DC	X'03'	BASE 10	0.1875
0003BA	03			315	DC	X'03'	BASE 10	0.1875
0003BB	05			316	DC	X'05'	BASE 10	0.3125
0003BC	0F			317	DC	X'0F'	BASE 10	0.9375
0003BD	0F			318	DC	X'0F'	BASE 10	0.9375
0003BE	14			319	DC	X'14'	BASE 10	1.2500
0003BF	14			320	DC	X'14'	BASE 10	1.2500
0003C0	14			321	DC	X'14'	BASE 10	1.2500
0003C1	12			322	DC	X'12'	BASE 10	1.1250
0003C2	12			323	DC	X'12'	BASE 10	1.1250
0003C3	12			324	DC	X'12'	BASE 10	1.1250
0003C4	12			325	DC	X'12'	BASE 10	1.1250
0003C5	1A			326	DC	X'1A'	BASE 10	1.6250
0003C6	25			327	DC	X'25'	BASE 10	2.3125
0003C7	25			328	DC	X'25'	BASE 10	2.3125
0003C8	25			329	DC	X'25'	BASE 10	2.3125
0003C9	25			330	DC	X'25'	BASE 10	2.3125

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
0003CA 21                         331          DC     X'21'        BASE 10  2.0625
0003CB 21                         332          DC     X'21'        BASE 10  2.0625
0003CC 1D                         333          DC     X'1D'        BASE 10  1.8125
0003CD 1D                         334          DC     X'1D'        BASE 10  1.8125
0003CE 3C                         335          DC     X'3D'        BASE 10  3.0000
0003CF 2F                         336          DC     X'2F'        BASE 10  2.9375
0003D0 2F                         337          DC     X'2F'        BASE 10  2.9375
0003D1 3E                         338          DC     X'3E'        BASE 10  3.8750
0003D2 3E                         339          DC     X'3E'        BASE 10  3.8750
0003D3 3F                         340          DC     X'3F'        BASE 10  3.8750
0003D4 26                         341          DC     X'26'        BASE 10  2.3750
0003D5 26                         342          DC     X'26'        BASE 10  2.3750
0003D6 26                         343          DC     X'26'        BASE 10  2.3750
0003D7 39                         344          DC     X'39'        BASE 10  3.5625
0003D8 39                         345          DC     X'39'        BASE 10  3.5625
0003D9 39                         346          DC     X'39'        BASE 10  3.5625
0003DA 39                         347          DC     X'39'        BASE 10  3.5625
0003DB 39                         348          DC     X'39'        BASE 10  3.5625
0003DC 2E                         349          DC     X'2E'        BASE 10  2.8750
0003DD 2E                         350          DC     X'2E'        BASE 10  2.8750
0003DE 2E                         351          DC     X'2E'        BASE 10  2.8750
0003DF 37                         352          DC     X'37'        BASE 10  3.4375
0003E0 37                         353          DC     X'37'        BASE 10  3.4375
0003E1 37                         354          DC     X'37'        BASE 10  3.4375
0003E2 37                         355          DC     X'37'        BASE 10  3.4375
0003E3 37                         356          DC     X'37'        BASE 10  3.4375
0003E4 37                         357          DC     X'37'        BASE 10  3.4375
0003E5 2D                         358          DC     X'2D'        BASE 10  2.8125
0003E6 2D                         359          DC     X'2D'        BASE 10  2.8125
0003E7 2D                         360          DC     X'2D'        BASE 10  2.8125
0003E8 2D                         361          DC     X'2D'        BASE 10  2.8125
0003E9 2C                         362          DC     X'2C'        BASE 10  2.7500
0003EA 2C                         363          DC     X'2C'        BASE 10  2.7500
0003EB 2C                         364          DC     X'2C'        BASE 10  2.7500
0003EC 2C                         365          DC     X'2C'        BASE 10  2.7500
0003ED 2C                         366          DC     X'2C'        BASE 10  2.7500
0003EE 3F                         367          DC     X'3F'        BASE 10  3.9375
0003EF 3F                         368          DC     X'3F'        BASE 10  3.9375
0003F0 3F                         369          DC     X'3F'        BASE 10  3.9375
0003F1 3C                         370          DC     X'3C'        BASE 10  3.7500
0003F2 3C                         371          DC     X'3C'        BASE 10  3.7500
0003F3 3C                         372          DC     X'3C'        BASE 10  3.7500
0003F4 3C                         373          DC     X'3C'        BASE 10  3.7500
0003F5 27                         374          DC     X'27'        BASE 10  2.4375
0003F6 27                         375          DC     X'27'        BASE 10  2.4375
0003F7 3A                         376          DC     X'3A'        BASE 10  3.6250
0003F8 3A                         377          DC     X'3A'        BASE 10  3.6250
0003F9 3A                         378          DC     X'3A'        BASE 10  3.6250
0003FA 3A                         379          DC     X'3A'        BASE 10  3.6250
0003FB 3A                         380          DC     X'3A'        BASE 10  3.6250
0003FC 31                         381          DC     X'31'        BASE 10  3.0625
0003FD 38                         382          DC     X'38'        BASE 10  3.5000
0003FE 38                         383          DC     X'38'        BASE 10  3.5000
0003FF 38                         384          DC     X'38'        BASE 10  3.5000
000400 38                         385          DC     X'38'        BASE 10  3.5000
```

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
000431	38			386	DC	X'38'	BASE 10	3.5000	
000432	38			387	DC	X'38'	BASE 10	3.5000	
000433	3D			388	DC	X'3D'	BASE 10	3.8125	
000434	3D			389	DC	X'3D'	BASE 10	3.8125	
000435	3D			390	DC	X'3D'	BASE 10	3.8125	
000436	3D			391	DC	X'3D'	BASE 10	3.8125	
000437	38			392	DC	X'38'	BASE 10	3.6875	
000438	38			393	DC	X'38'	BASE 10	3.6875	
000439	38			394	DC	X'38'	BASE 10	3.6875	
00043A	38			395	DC	X'38'	BASE 10	3.6875	
00043B	38			396	DC	X'38'	BASE 10	3.6875	
00043C	02			397	DC	X'02'	BASE 10	0.1250	
00043D	02			398	DC	X'02'	BASE 10	0.1250	
00043E	05			399	DC	X'05'	BASE 10	0.3125	
00043F	05			400	DC	X'05'	BASE 10	0.3125	
000440	05			401	DC	X'05'	BASE 10	0.3125	
000441	05			402	DC	X'05'	BASE 10	0.3125	
000442	05			403	DC	X'05'	BASE 10	0.3125	
000443	04			404	DC	X'04'	BASE 10	0.2500	
000444	04			405	DC	X'04'	BASE 10	0.2500	
000445	06			406	DC	X'06'	BASE 10	0.3750	
000446	06			407	DC	X'06'	BASE 10	0.3750	
000447	06			408	DC	X'06'	BASE 10	0.3750	
000448	06			409	DC	X'06'	BASE 10	0.3750	
000449	07			410	DC	X'07'	BASE 10	0.4375	
00044A	07			411	DC	X'07'	BASE 10	0.4375	
00044B	0D			412	DC	X'0D'	BASE 10	0.8125	
00044C	0E			413	DC	X'0E'	BASE 10	0.8750	
00044D	0E			414	DC	X'0E'	BASE 10	0.8750	
00044E	0E			415	DC	X'0E'	BASE 10	0.8750	
00044F	0E			416	DC	X'0E'	BASE 10	0.8750	
000450	0F			417	DC	X'0F'	BASE 10	0.9375	
000451	0F			418	DC	X'0F'	BASE 10	0.9375	
000452	11			419	DC	X'11'	BASE 10	1.0625	
000453	11			420	DC	X'11'	BASE 10	1.0625	
000454	16			421	DC	X'16'	BASE 10	1.3750	
000455	16			422	DC	X'16'	BASE 10	1.3750	
000456	16			423	DC	X'16'	BASE 10	1.3750	
000457	12			424	DC	X'12'	BASE 10	1.1250	
000458	18			425	DC	X'18'	BASE 10	1.5000	
000459	18			426	DC	X'18'	BASE 10	1.5000	
00045A	18			427	DC	X'18'	BASE 10	1.5000	
00045B	18			428	DC	X'18'	BASE 10	1.5000	
00045C	1E			429	DC	X'1E'	BASE 10	1.8750	
00045D	1E			430	DC	X'1E'	BASE 10	1.8750	
00045E	1E			431	DC	X'1E'	BASE 10	1.8750	
00045F	19			432	DC	X'19'	BASE 10	1.5625	
000460	18			433	DC	X'18'	BASE 10	1.6875	
000461	18			434	DC	X'18'	BASE 10	1.6875	
000462	18			435	DC	X'18'	BASE 10	1.6875	
000463	1A			436	DC	X'1A'	BASE 10	1.6250	
000464	1A			437	DC	X'1A'	BASE 10	1.6250	
000465	1A			438	DC	X'1A'	BASE 10	1.6250	
000466	1A			439	DC	X'1A'	BASE 10	1.6250	
000467	1A			440	DC	X'1A'	BASE 10	1.6250	

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT			
000438	23			441	DC	X'23'	BASE 10	2.1875
000439	1F			442	DC	X'1F'	BASE 10	1.9375
00043A	22			443	DC	X'22'	BASE 10	2.1250
00043B	22			444	DC	X'22'	BASE 10	2.1250
00043C	22			445	DC	X'22'	BASE 10	2.1250
00043D	25			446	DC	X'25'	BASE 10	2.3125
00043E	25			447	DC	X'25'	BASE 10	2.3125
00043F	21			448	DC	X'21'	BASE 10	2.0625
000440	21			449	DC	X'21'	BASE 10	2.0625
000441	21			450	DC	X'21'	BASE 10	2.0625
000442	1D			451	DC	X'1D'	BASE 10	1.8125
000443	1D			452	DC	X'1D'	BASE 10	1.8125
000444	1D			453	DC	X'1D'	BASE 10	1.8125
000445	1D			454	DC	X'1D'	BASE 10	1.8125
000446	28			455	DC	X'28'	BASE 10	2.5000
000447	28			456	DC	X'28'	BASE 10	2.5000
000448	30			457	DC	X'30'	BASE 10	3.0000
000449	2F			458	DC	X'2F'	BASE 10	2.9375
00044A	29			459	DC	X'29'	BASE 10	2.5625
00044B	26			460	DC	X'26'	BASE 10	2.3750
00044C	26			461	DC	X'26'	BASE 10	2.3750
00044D	2E			462	DC	X'2E'	BASE 10	2.8750
00044E	32			463	DC	X'32'	BASE 10	3.1250
00044F	2D			464	DC	X'2D'	BASE 10	2.8125
000450	2C			465	DC	X'2C'	BASE 10	2.7500
000451	27			466	DC	X'27'	BASE 10	2.4375
000452	27			467	DC	X'27'	BASE 10	2.4375
000453	31			468	DC	X'31'	BASE 10	3.0625
000454	00			469	DC	X'00'	BASE 10	0.0000
000455	00			470	DC	X'00'	BASE 10	0.0000
000456	00			471	DC	X'00'	BASE 10	0.0000
000457	01			472	DC	X'01'	BASE 10	0.0625
000458	04			473	DC	X'04'	BASE 10	0.2500
000459	04			474	DC	X'04'	BASE 10	0.2500
00045A	04			475	DC	X'04'	BASE 10	0.2500
00045B	0B			476	DC	X'0B'	BASE 10	0.6875
00045C	0B			477	DC	X'0B'	BASE 10	0.6875
00045D	0B			478	DC	X'0B'	BASE 10	0.6875
00045E	0B			479	DC	X'0B'	BASE 10	0.6875
00045F	0B			480	DC	X'0B'	BASE 10	0.6875
000460	0B			481	DC	X'0B'	BASE 10	0.6875
000461	0C			482	DC	X'0C'	BASE 10	0.7500
000462	0C			483	DC	X'0C'	BASE 10	0.7500
000463	0C			484	DC	X'0C'	BASE 10	0.7500
000464	0C			485	DC	X'0C'	BASE 10	0.7500
000465	0C			486	DC	X'0C'	BASE 10	0.7500
000466	0D			487	DC	X'0D'	BASE 10	0.8125
000467	0D			488	DC	X'0D'	BASE 10	0.8125
000468	0D			489	DC	X'0D'	BASE 10	0.8125
000469	10			490	DC	X'10'	BASE 10	1.0000
00046A	10			491	DC	X'10'	BASE 10	1.0000
00046B	16			492	DC	X'16'	BASE 10	1.3750
00046C	16			493	DC	X'16'	BASE 10	1.3750
00046D	16			494	DC	X'16'	BASE 10	1.3750
00046E	16			495	DC	X'16'	BASE 10	1.3750

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
00046F	18			496	DC	X'18'	BASE	10	1.5000
000470	18			497	DC	X'18'	BASE	10	1.5000
000471	1C			498	DC	X'1C'	BASE	10	1.7500
000472	1C			499	DC	X'1C'	BASE	10	1.7500
000473	1C			500	DC	X'1C'	BASE	10	1.7500
000474	1C			501	DC	X'1C'	BASE	10	1.7500
000475	1C			502	DC	X'1C'	BASE	10	1.7500
000476	1E			503	DC	X'1E'	BASE	10	1.8750
000477	19			504	DC	X'19'	BASE	10	1.5625
000478	19			505	DC	X'19'	BASE	10	1.5625
000479	19			506	DC	X'19'	BASE	10	1.5625
00047A	19			507	DC	X'19'	BASE	10	1.5625
00047B	19			508	DC	X'19'	BASE	10	1.5625
00047C	17			509	DC	X'17'	BASE	10	1.4375
00047D	17			510	DC	X'17'	BASE	10	1.4375
00047E	17			511	DC	X'17'	BASE	10	1.4375
00047F	17			512	DC	X'17'	BASE	10	1.4375
000480	17			513	DC	X'17'	BASE	10	1.4375
000481	17			514	DC	X'17'	BASE	10	1.4375
000482	17			515	DC	X'17'	BASE	10	1.4375
000483	18			516	DC	X'18'	BASE	10	1.6875
000484	18			517	DC	X'18'	BASE	10	1.6875
000485	23			518	DC	X'23'	BASE	10	2.1875
000486	23			519	DC	X'23'	BASE	10	2.1875
000487	1F			520	DC	X'1F'	BASE	10	1.9375
000488	1F			521	DC	X'1F'	BASE	10	1.9375
000489	1F			522	DC	X'1F'	BASE	10	1.9375
00048A	20			523	DC	X'20'	BASE	10	2.0000
00048B	20			524	DC	X'20'	BASE	10	2.0000
00048C	20			525	DC	X'20'	BASE	10	2.0000
00048D	20			526	DC	X'20'	BASE	10	2.0000
00048E	33			527	DC	X'33'	BASE	10	3.1875
00048F	24			528	DC	X'24'	BASE	10	2.2500
000490	24			529	DC	X'24'	BASE	10	2.2500
000491	24			530	DC	X'24'	BASE	10	2.2500
000492	29			531	DC	X'29'	BASE	10	2.5625
000493	35			532	DC	X'35'	BASE	10	3.3125
000494	2A			533	DC	X'2A'	BASE	10	2.6250
000495	2A			534	DC	X'2A'	BASE	10	2.6250
000496	2B			535	DC	X'2B'	BASE	10	2.6875
000497	2B			536	DC	X'2B'	BASE	10	2.6875
000498	34			537	DC	X'34'	BASE	10	3.2500
000499	36			538	DC	X'36'	BASE	10	3.3750
00049A	C0			539	DC	X'00'	BASE	10	0.0000
00049B	00			540	DC	X'00'	BASE	10	0.0000
00049C	C0			541	DC	X'00'	BASE	10	0.0000
00049D	C1			542	DC	X'01'	BASE	10	0.0625
00049E	01			543	DC	X'01'	BASE	10	0.0625
00049F	01			544	DC	X'01'	BASE	10	0.0625
0004A0	04			545	DC	X'04'	BASE	10	0.2500
0004A1	C4			546	DC	X'04'	BASE	10	0.2500
0004A2	C8			547	DC	X'08'	BASE	10	0.5000
0004A3	C8			548	DC	X'08'	BASE	10	0.5000
0004A4	C7			549	DC	X'07'	BASE	10	0.4375
0004A5	08			550	DC	X'08'	BASE	10	0.6875

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LCC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    FOBAPR70   6/30/71
0004A6 09                          551          DC     X'09'          BASE 10  0.5625
0004A7 CC                          552          DC     X'0C'          BASE 10  0.7500
0004A8 0D                          553          DC     X'0D'          BASE 10  0.8125
0004A9 CE                          554          DC     X'0E'          BASE 10  0.8750
0004AA 10                          555          DC     X'10'          BASE 10  1.0000
0004AB 13                          556          DC     X'13'          BASE 10  1.1875
0004AC 02                          557          DC     X'02'          BASE 10  0.1250
0004AD 02                          558          DC     X'02'          BASE 10  0.1250
0004AE 02                          559          DC     X'02'          BASE 10  0.1250
0004AF 03                          560          DC     X'03'          BASE 10  0.1875
0004B0 03                          561          DC     X'03'          BASE 10  0.1875
0004B1 03                          562          DC     X'03'          BASE 10  0.1875
0004B2 05                          563          DC     X'05'          BASE 10  0.3125
0004B3 05                          564          DC     X'05'          BASE 10  0.3125
0004B4 C6                          565          DC     X'06'          BASE 10  0.3750
0004B5 C6                          566          DC     X'06'          BASE 10  0.3750
0004B6 C7                          567          DC     X'07'          BASE 10  0.4375
0004B7 0A                          568          DC     X'0A'          BASE 10  0.6250
0004B8 0A                          569          DC     X'0A'          BASE 10  0.6250
0004B9 09                          570          DC     X'09'          BASE 10  0.5625
0004BA 0F                          571          DC     X'0F'          BASE 10  0.9375
0004BB 14                          572          DC     X'14'          BASE 10  1.2500
0004BC 15                          573          DC     X'15'          BASE 10  1.3125
0004BD 11                          574          DC     X'11'          BASE 10  1.0625
0004BE 12                          575          DC     X'12'          BASE 10  1.1250
                                   576          END
0004C0 00000025                    577                 =F'37'
0004C4 000001C8                    578                 =F'456'
0004C8 00000F17                    579                 =F'3863'
0004CC 00000E37                    580                 =F'3639'
```

NOT REPRODUCIBLE

APPENDIX C

Assembler Listing of GEN2

```
                                                                                                                  PAGE    1
LOC     ECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  FORAPR70   6/30/71
00000                                  GEN2      START 0
                                       ***********************************************************************
                                       * SOURCE---L.F.CANNON, UGA, DEPT. OF STAT., APR 1970                  *
                                       * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING EX-*
                                       *          PONETIAL AND GAMMA DISTRIBUTED RANDOM NUMBERS.             *
                                       * USAGE----X=RANGAM(I,N). 'I' MUST BE AN ODD INTEGER. IT PRIMES THE   *
                                       *          GENERATING SCHEME. 'N' IS THE NUMBER OF EXPONETIAL RANDOM  *
                                       *          NUMBERS SUMMED IN GENERATING A RANDOM NUMBER THAT IS GAMMA *
                                       *          (A=N,B=1).                                                 *
                                       * EXAMPLE:X=RANGAM(55595,50)  'X' WILL BE GAMMA(A=50,B=1).            *
                                       * NOTE-----IF N=1 AN EXPONETIALLY DISTRIBUTED RANDOM NUMBER WILL BE   *
                                       *          GENERATED.                                                 *
                                       * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN  'COMMUNI- *
                                       *          CATIONS OF ACM',VOL.7,NO.5,MAY 1964. THE PROGRAM DOCUMEN-  *
                                       *          TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.         *
                                       ***********************************************************************
000008                                         ENTRY RANGAM       PROLOGUE, STANDARD FOR 360/65.
000000 07                                      USING RANGAM,15
000001 40D9C105C7C104                          DC    XL1'07'
000008 90EC D00C                 0000C         DC    CL7' RANGAM'
00000C 5880 1004                 00004 RANGAM  STM   14,12,12(13)
000010 5880 800D                 00000         L     11,4(,1)     LOAD ADRESS OF N INTO R11.
000014 41C0 0001                 00001         L     11,0(,11)    LOAD N INTO R11
000018 4140 0001                 00001         LA    12,1         R12=1, USED AS AN INDEX.
00001C 3800                                    LA    10,1         R10=1, USED AS AN INCREMENT.
00001E 47F0 F08A                 00092         SER   0,0          RESULT SUMMED IN FPR0.
000022 5C40 F148                 00150 BRANCH  B     TEMP         EXECUTED ONLY ON THE FIRST CALL.
000026 5050 F140                 00148         M     4,BPAT       OBTAIN NEW U.
00002A 3822                                    ST    5,LA         STORE U FOR NEXT GENERATION.
00002C 1866                                    SER   2,2          SET FPR2=0, 'W'=0.
00002E 1844                                    SR    6,6
000030 8040 0006                 00006 N1      SP    4,4          CLEAR R4.
000034 5940 F400                 00408         SLDL  4,4          TWO HIGH ORDER OCTAL DIGITS OF U IN R4(D1D2)
000038 4780 F040                 00048         C     4,=F'37'     D1D2 COMPARED WITH 37(BASE 10)=45(BASE 8).
00003C 4364 F44A                 00482         BC    11,NA        D1D2>=37 BRANCH NA.
000040 8950 0006                 00006         IC    6,D+294(4)   OBTAIN DISCRETE RANDOM VARIBLE FROM D.
000044 47F0 F06C                 00074         SLL   5,6          R5=D5D6D7...
000048 8040 0003                 00003         BC    15,EX        BRANCH EX.
00004C 5940 F404                 004DC NA      SLDL  4,3          R4=D1D2D3.
000050 4780 F058                 00060         C     4,=F'456'    456(BASE 10)=710(BASE 8).
000054 4364 F2F4                 002FC         BC    11,NB        D1D2D3>=456 BRANCH NB.
000058 8950 0003                 00003         IC    6,D-144(4)   OBTAIN DISCRETE RANDOM VARIABLE FROM D.
00005C 47F0 F06C                 00074         SLL   5,3          R5=D5D6D7...
000060 8040 0003                 00003         BC    15,EX        BRANCH EX.
000064 5940 F4D8                 004E0 NB      SLDL  4,3          R4=D1D2D3D4..
000068 4780 F0A4                 000AC         C     4,=F'3863'   3863(BASE 10)=7427(BASE 8).
00006C 5840 F4DC                 004E4         BC    11,NC        D1D2D3D4>=3863 BRANCH NC.
000070 4364 F384                 0038C         S     4,=F'3639'   OBTAIN PROPER ADDRESS FOR D TABLE.
000074 1846                                    IC    6,D(4)       OBTAIN DISCRETE VARIABLE FROM D.
000076 8C60 0010                 00010 EX      LR    4,6          R4= DISCRETE VARIABLE.
00007A 5050 F13C                 00144 FL      SPOL  4,16         R5= DISCRETE VARIABLE + HI ORDER 16 BITS OF
00007E 9241 F13C          C0144                ST    5,RES        R5(NOW LO ORDER 16 BITS) WITH HI ORDER 8
000082 7A20 F13C                 00144         MVI   RES,X'41'    BITS ZEROED OUT FOR CONVERSION TO FLOAT-
000086 3A02                                    AE    2,RES        ING POINT. FPR2 CONTAINS RESULT + 'W'.
                                               AER   0,2          FINAL RESULT SUMMED IN FPR0.
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    FORAPH.70   6/30/71
000088 87CA F016            0001E   56          BXLE  12,10,BRANCH   PERFORM BRANCH LOOP N TIMES.
000080 98EC 000C            00000   57          LM    14,12,12(13)  RESTORE REGISTERS.
000090 07FE                         58          BCR   15,14       BRANCH TO MAIN LINE.
000092 5850 F140            00149   59 TEMP     L     5,LA         REPLACES INSTR. AT BRANCH.
000094 0203 F016 F08A 0001E 00092   60          MVC   BRANCH,TEMP  REPLACE BRANCH BY TEMP.
000098 5850 1000            00000   61          L     5,0(,1)      ADDRESS OF PRIMER IN R5.
000040 5850 5000            00000   62          L     5,0(,5)      PRIMER IN R5.
000044 5050 F140            00148   63          ST    5,LA         STORE PRIMER FOR USE IN BRAN.
000048 47F0 F01A            00022   64          B     BRANCH+4     BRANCH TO INSTRUCTION FOLLOWING 'BRANCH'.
00004C 5850 F140            00148   65 NC       L     5,LA         RESTORE U TO R5.
000050 4120 0004            00004   66          LA    2,4
000054 1833                         67          SR    3,3
000056 5553 F36C            00374   68          CL    5,P(3)       U>=P(0) BRANCH ND.
000058 4780 F116            0011E   69          BC    11,ND
00005A 1833                         70          SR    3,3
00005C 1832                         71 NE       AR    3,2          R3=IB(IN WORDS, 4 BYTES=1 WORD).
00005E 5553 F16C            00174   72          CL    5,B(3)       U>=B(IB) BRANCH NE(CONTINUE LOOP).
000060 4730 F0B8            00000   73          BC    11,NE
000064 5553 F26C            00274   74          CL    5,C(3)       C(IB)=<U<B(IB) BRANCH NE.
000068 4780 F0F0            00008   75          BC    11,NE
00006C 5040 F148            00150   76          M     4,RPAT       OBTAIN NEW U.
000070 5050 F140            00148   77          ST    5,LA         STORE FOR NEXT GENERATION.
000074 1844                         78 GO       SR    4,4          CLEAR R4.
000076 9833 0002            00002   79          SPL   3,2
00007A 4343 F3B4            00380   80          IC    4,D(3)       OBTAIN DISCRETE VARIABLE.
00007E 47F0 F08E            00076   81          BC    15,FL
000082 5040 F148            00150   82 NF       M     4,RPAT       OBTAIN NEW U.
000086 1882                         83          LR    8,2          USED FOR INDEXING.
000088 1882                         84          AR    8,2          R8=IQ.
00008A 5559 F14C            00154   85          CL    5,D(8)       U>=D(IQ) CONTINUE LOOP.
00008E 4780 F0F6            00008   86          BC    11,*-4
000092 5040 F148            00150   87          M     4,RPAT       OBTAIN NEW U.
000096 1882                         88          SR    8,2
000098 1865                         89          LR    6,5
00009A 1192                         90          LNR   9,2          R9=-4.
00009C 1982                         91          SR    8,2
00009E 5040 F148            00150   92 NM       M     4,RPAT       OBTAIN NEW U.
0000A2 1555                         93          CLR   5,6          R5>=R6 BRANCH TO CONTINUE.
0000A4 4780 F108            00110   94          BC    11,*+6
0000A8 1865                         95          LR    6,5          LOAD SMALLEST U IN R6.
0000AA 8A90 F0FC            00104   96          BXH   8,9,NM       CONTINUE(R9 HAS BOTH INCREMENT & DELIMITER).
0000AE 5050 F140            00148   97          ST    5,LA         STORE FOR NEXT GENERATION.
0000B2 1856                         98          LR    5,6          R5=MIN(U1,U2,...,U(IQ)).
0000B4 47F0 F0D2            00DDA   99          BC    15,GO
0000B8 1832                         100 ND      AR    3,2          R3=IP.
0000BA 5553 F36C            00374   101         CL    5,P(3)       U>=P(IP) CONTINUE LOOP.
0000BE 4780 F116            0011E   102         BC    11,ND
0000C2 5030 F144            0014C   103         ST    3,J          STORE IP INTO J.
0000C6 9245 F144            0014C   104         MVI   J,X'46'      CONVERT IP TO FLOATING POINT.
0000CA 7820 F144            0014C   105         AE    2,J          NORMALIZED AND LOADED INTO FPR2 'W'.
0000CE 5040 F148            00150   106         M     4,RPAT       OBTAIN NEW U.
0000D2 5050 F140            00148   107         ST    5,LA
0000D6 5046 F36C            00374   108         M     4,P(6)       U=U*P(0).
0000DA 47F0 F026            0002E   109         BC    15,N1        BRANCH N1.
0000DE                              110 RES     DS    1F
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    F08APP70   6/30/71
000148                          111 LA      DS    1F
00014C                          112 J       DS    1F
000150 48C27395                 113 BPAT    DC    XL4'48C27395'
000154 00000000                 114 Q       DC    X'00000000'    BASE 10 0.0
000158 00000000                 115         DC    X'00000000'    BASE 10 0.0
00015C FA81CEA7                 116         DC    X'FA81CEA7'    BASE 10= 0.97927562333
000160 FFFA0848                 117         DC    X'FFFA0848'    BASE 10= 0.99967719382
000164 FFFF8C72                 118         DC    X'FFFF8C72'    BASE 10= 0.99999597344
000168 FFFFFF4C                 119         DC    X'FFFFFF4C'    BASE 10= 0.99999995912
00016C FFFFFFFF                 120         DC    X'FFFFFFFF'    BASE 10= 0.99999999963
000170 FFFFFFFF                 121         DC    X'FFFFFFFF'    BASE 10= 1.0000000000
000174 F1F24021                 122 R       DC    X'F1F24021'    BASE 10 0.94510942156
000178 F23FF28D                 123         DC    X'F23FF28D'    BASE 10 0.94629827211
00017C F28C810C                 124         DC    X'F28C810C'    BASE 10 0.94745642234
000180 F2C13730                 125         DC    X'F2C13730'    BASE 10 0.94826073593
000184 F2F0F518                 126         DC    X'F2F0F518'    BASE 10 0.94894248886
000188 F3074288                 127         DC    X'F3074288'    BASE 10 0.94932953916
00018C F317F405                 128         DC    X'F317F405'    BASE 10 0.94958329319
000190 F326185C                 129         DC    X'F326185C'    BASE 10 0.94980003601
000194 F32[illegible]665                 130         DC    X'F32[illegible]665'    BASE 10 0.94992008199
000198 F38[illegible]51C                 131         DC    X'F38[illegible]51C'    BASE 10 0.95190460160
00019C F42015AE                 132         DC    X'F42015AE'    BASE 10 0.95361457350
0001A0 F4880D4F                 133         DC    X'F4880D4F'    BASE 10 0.95525892423
0001A4 F4F4020F                 134         DC    X'F4F4020F'    BASE 10 0.95606066501
0001A8 F5592594                 135         DC    X'F5592594'    BASE 10 0.95839152226
0001AC F5[illegible]2107A                 136         DC    X'F5[illegible]2107A'    BASE 10 0.95974907159
0001B0 F60A68D7                 137         DC    X'F60A68D7'    BASE 10 0.96109651586
0001B4 F65F0CBE                 138         DC    X'F65F0CBE'    BASE 10 0.96237258579
0001B8 F6A471A9                 139         DC    X'F6A471A9'    BASE 10 0.96344671629
0001BC F6F[illegible]8F1                 140         DC    X'F6F[illegible]8F1'    BASE 10 0.96447806832
0001C0 F729F034                 141         DC    X'F729F034'    BASE 10 0.96548445247
0001C4 F761312A                 142         DC    X'F761312A'    BASE 10 0.96637678298
0001C8 F7941C80                 143         DC    X'F7941C80'    BASE 10 0.96710374973
0001CC F7C586EF                 144         DC    X'F7C586EF'    BASE 10 0.96796063536
0001D0 F7F1E012                 145         DC    X'F7F1E012'    BASE 10 0.96853524[illegible]1
0001D4 F81D087F                 146         DC    X'F81D087F'    BASE 10 0.96920558791
0001D8 F843046B                 147         DC    X'F843046B'    BASE 10 0.96986129414
0001DC F871F734                 148         DC    X'F871F734'    BASE 10 0.97048802494
0001E0 F8970F94                 149         DC    X'F8970F94'    BASE 10 0.97106734391
0001E4 F8B90C57                 150         DC    X'F8B90C57'    BASE 10 0.97158600928
0001E8 F8DAC69E                 151         DC    X'F8DAC69E'    BASE 10 0.97209825453
0001EC F8F81060                 152         DC    X'F8F81060'    BASE 10 0.97258093522
0001F0 F91A289F                 153         DC    X'F91A289F'    BASE 10 0.97305557854
0001F4 F93A8989                 154         DC    X'F93A8989'    BASE 10 0.97352084759
0001F8 F9568658                 155         DC    X'F9568658'    BASE 10 0.97397651342
0001FC F9740441                 156         DC    X'F9740441'    BASE 10 0.97442652313
000200 F9900A1A                 157         DC    X'F9900A1A'    BASE 10 0.97485411775
000204 F9A34357                 158         DC    X'F9A34357'    BASE 10 0.97525425799
000208 F9C36880                 159         DC    X'F9C36880'    BASE 10 0.97563794277
00020C F9DC3773                 160         DC    X'F9DC3773'    BASE 10 0.97601648369
000210 F9F46745                 161         DC    X'F9F46745'    BASE 10 0.97638554992
000214 FAC4550A                 162         DC    X'FAC4550A'    BASE 10 0.97672[illegible]
000218 FA20C498                 163         DC    X'FA20C498'    BASE 10 0.97705[illegible]
00021C FA33E7C0                 164         DC    X'FA33E7C0'    BASE 10 0.977[illegible]
000220 FA4667F8                 165         DC    X'FA4667F8'    BASE 10 0.97763681231
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LOC	OBJECT CODE	ADDR1 ADDR2	STMT	SOURCE STATEMENT				
000224	FA58D771		166	DC	X'FA58D771'	BASE	10	0.97791911485
000228	FA683FD0		167	DC	X'FA683FD0'	BASE	10	0.97819899411
00022C	FA7D6F58		168	DC	X'FA7D6F58'	BASE	10	0.97847642590
000230	FA8FC18A		169	DC	X'FA8FC18A'	BASE	10	0.97874079506
000234	FA9F650C		170	DC	X'FA9F650C'	BASE	10	0.97899467042
000238	FAAF66FF		171	DC	X'FAAF66FF'	BASE	10	0.97923892349
00023C	FABFC9A2		172	DC	X'FABFC9A2'	BASE	10	0.97947750293
000240	FACFA835		173	DC	X'FACFA835'	BASE	10	0.97971601531
000244	FADD203A		174	DC	X'FADD203A'	BASE	10	0.97993661330
000248	FAF3949A		175	DC	X'FAF3949A'	BASE	10	0.98015717294
00024C	FAF96A91		176	DC	X'FAF96A91'	BASE	10	0.98036925689
000250	FB060436		177	DC	X'FB060436'	BASE	10	0.98057295161
000254	FB143589		178	DC	X'FB143589'	BASE	10	0.98077711667
000258	FB1F2894		179	DC	X'FB1F2894'	BASE	10	0.98094419111
00025C	FB201DCA		180	DC	X'FB201DCA'	BASE	10	0.98109613594
000260	FB32FE11		181	DC	X'FB32FE11'	BASE	10	0.98124587942
000264	FB3CA379		182	DC	X'FB3CA379'	BASE	10	0.98139402139
000268	FB448A84		183	DC	X'FB448A84'	BASE	10	0.98151460390
00026C	FB4C437C		184	DC	X'FB4C437C'	BASE	10	0.98163244057
000270	FB4FAA91		185	DC	X'FB4FAA91'	BASE	10	0.98168436111
000274	F177D5FD		186 C	DC	X'F177D5FD'	BASE	10	0.94323490089
000278	F1FE3941		187	DC	X'F1FE3941'	BASE	10	0.94528539518
00027C	F24FC2F8		188	DC	X'F24FC2F8'	BASE	10	0.94651430666
000280	F294C837		189	DC	X'F294C837'	BASE	10	0.94757146939
000284	F2CCD889		190	DC	X'F2CCD889'	BASE	10	0.94843921067
000288	F2F65495		191	DC	X'F2F65495'	BASE	10	0.94907597217
00028C	F3136908		192	DC	X'F3136908'	BASE	10	0.94951504656
000290	F31C9030		193	DC	X'F31C9030'	BASE	10	0.94967253451
000294	F329C2C7		194	DC	X'F329C2C7'	BASE	10	0.94965597071
000298	F33CA878		195	DC	X'F33CA878'	BASE	10	0.95014449779
00029C	F383B926		196	DC	X'F383B926'	BASE	10	0.95196110838
0002A0	F4260D7D		197	DC	X'F4260D7D'	BASE	10	0.95370563497
0002A4	F49AFC59		198	DC	X'F49AFC59'	BASE	10	0.95549289478
0002A8	F4F9A446		199	DC	X'F4F9A446'	BASE	10	0.95693234495
0002AC	F5504921		200	DC	X'F5504921'	BASE	10	0.95845135294
0002B0	F58FF212		201	DC	X'F58FF212'	BASE	10	0.95996010742
0002B4	F60FC556		202	DC	X'F60FC556'	BASE	10	0.96115288522
0002B8	F65E7807		203	DC	X'F65E7807'	BASE	10	0.96237915935
0002BC	F6AF0971		204	DC	X'F6AF0971'	BASE	10	0.96359302254
0002C0	F6F39063		205	DC	X'F6F39063'	BASE	10	0.96455303830
0002C4	F72F0177		206	DC	X'F72F0177'	BASE	10	0.96554574164
0002C8	F7640687		207	DC	X'F7640687'	BASE	10	0.96637002930
0002CC	F7A299E7		208	DC	X'F7A299E7'	BASE	10	0.96732383403
0002D0	F7C77089		209	DC	X'F7C77089'	BASE	10	0.96788774270
0002D4	F7FFCF83		210	DC	X'F7FFCF83'	BASE	10	0.96873186240
0002D8	F823618C		211	DC	X'F823618C'	BASE	10	0.96928987209
0002DC	F84A0A08		212	DC	X'F84A0A08'	BASE	10	0.96987974844
0002E0	F87C78AC		213	DC	X'F87C78AC'	BASE	10	0.97064928261
0002E4	F8A485E7		214	DC	X'F8A485E7'	BASE	10	0.97125042281
0002E8	F8C7F209		215	DC	X'F8C7F209'	BASE	10	0.97180092549
0002EC	F8F15379		216	DC	X'F8F15379'	BASE	10	0.97218820298
0002F0	F8FD0158		217	DC	X'F8FD0158'	BASE	10	0.97261055461
0002F4	F921F2CE		218	DC	X'F921F2CE'	BASE	10	0.97317426251
0002F8	F93E5898		219	DC	X'F93E5898'	BASE	10	0.97360757562
0002FC	F9663071		220	DC	X'F9663071'	BASE	10	0.97421630871

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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT			
000300	F9755995			221	DC	X'F9755995'	BASE 10	0.9745861970B
000304	F9989911			222	DC	X'F9989911'	BASE 10	0.9750304B590
000308	F9874018			223	DC	X'F9874018'	BASE 10	0.97545242919
00030C	F9CC980C			224	DC	X'F9CC980C'	BASE 10	0.97577828654
000310	F9E059C9			225	DC	X'F9E059C9'	BASE 10	0.97607968974
000314	FA33460C4			226	DC	X'FA33460C4'	BASE 10	0.97656636056
000318	FA19E797			227	DC	X'FA19E797'	BASE 10	0.97695777372
00031C	FA2ED54F			228	DC	X'FA2ED54F'	BASE 10	0.97727717817
000320	FA397605			229	DC	X'FA397605'	BASE 10	0.97763933614
000324	FA52557F			230	DC	X'FA52557F'	BASE 10	0.97781881658
000328	FA6B9368			231	DC	X'FA6B9368'	BASE 10	0.97816016935
00032C	FA73F667			232	DC	X'FA73F667'	BASE 10	0.97633194768
000330	FA8355E5			233	DC	X'FA8355E5'	BASE 10	0.97855652118
000334	FA99E954			234	DC	X'FA99E954'	BASE 10	0.97890151993
000338	FAA7998A			235	DC	X'FAA7998A'	BASE 10	0.97913322195
00033C	FAB9172F			236	DC	X'FAB9172F'	BASE 10	0.97941727549
000340	FAC5C620			237	DC	X'FAC5C620'	BASE 10	0.97958029048
000344	FAD7B018			238	DC	X'FAD7B018'	BASE 10	0.97985429215
000348	FAE43406			239	DC	X'FAE43406'	BASE 10	0.98004465327
00034C	FAF1937C			240	DC	X'FAF1937C'	BASE 10	0.98026848031
000350	FB146E58			241	DC	X'FB146E58'	BASE 10	0.98053542181
000354	FB115246			242	DC	X'FB115246'	BASE 10	0.98073305333
000358	FB145BE4			243	DC	X'FB145BE4'	BASE 10	0.98078947595
00035C	FB245BE4			244	DC	X'FB245BE4'	BASE 10	0.98102349782
000360	FB2FA960			245	DC	X'FB2FA960'	BASE 10	0.98119594912
000364	FB36E548			246	DC	X'FB36E548'	BASE 10	0.98130639099
000368	FB409549			247	DC	X'FB409549'	BASE 10	0.98145902546
00036C	FB498004			248	DC	X'FB498004'	BASE 10	0.98159104699
000370	FB4C97A1			249	DC	X'FB4C97A1'	BASE 10	0.98163745593
000374	FB4FA391			250 P	DC	X'FB4FA391'	BASE 10	0.98168436111
000378	FFEA030E			251	DC	X'FFEA030E'	BASE 10	0.99966453737
00037C	FFFF9AFA			252	DC	X'FFFF9AFA'	BASE 10	0.99999385579
000380	FFFFFE1C			253	DC	X'FFFFFE1C'	BASE 10	0.99999988746
000384	FFFFFFF7			254	DC	X'FFFFFFF7'	BASE 10	0.99999999794
000388	FFFFFFFF			255	DC	X'FFFFFFFF'	BASE 10	0.99999999996
00038C	00			256 D	DC	X'00'	BASE 10	0.0
000380	0A			257	DC	X'0A'	BASE 10	0.6250
00038E	0B			258	DC	X'0B'	BASE 10	0.6875
000390	10			259	DC	X'10'	BASE 10	1.0000
000390	15			260	DC	X'15'	BASE 10	1.3125
000391	20			261	DC	X'20'	BASE 10	2.0000
000392	35			262	DC	X'35'	BASE 10	3.3125
000393	2B			263	DC	X'2B'	BASE 10	2.6875
000394	36			264	DC	X'36'	BASE 10	3.3750
000395	C1			265	DC	X'01'	BASE 10	0.0625
000396	02			266	DC	X'02'	BASE 10	0.1250
000397	03			267	DC	X'03'	BASE 10	0.1875
000398	05			268	DC	X'05'	BASE 10	0.3125
000399	04			269	DC	X'04'	BASE 10	0.2500
00039A	C6			270	DC	X'06'	BASE 10	0.3750
000399	08			271	DC	X'08'	BASE 10	0.5000
00039C	07			272	DC	X'07'	BASE 10	0.4375
000390	09			273	DC	X'09'	BASE 10	0.5625
00039E	0C			274	DC	X'0C'	BASE 10	0.7500
00039F	0D			275	DC	X'0D'	BASE 10	0.8125

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
0003A0 0E                        276        DC    X'0E'        BASE 10  0.8750
0003A1 0F                        277        DC    X'0F'        BASE 10  0.9375
0003A2 14                        278        DC    X'14'        BASE 10  1.2500
0003A3 11                        279        DC    X'11'        BASE 10  1.0625
0003A4 16                        280        DC    X'16'        BASE 10  1.3750
0003A5 13                        281        DC    X'13'        BASE 10  1.1875
0003A6 12                        282        DC    X'12'        BASE 10  1.1250
0003A7 18                        283        DC    X'18'        BASE 10  1.5000
0003A8 1C                        284        DC    X'1C'        BASE 10  1.7500
0003A9 1E                        285        DC    X'1E'        BASE 10  1.8750
0003AA 19                        286        DC    X'19'        BASE 10  1.5625
0003AB 17                        287        DC    X'17'        BASE 10  1.4375
0003AC 1B                        288        DC    X'1B'        BASE 10  1.6875
0003AD 1A                        289        DC    X'1A'        BASE 10  1.6250
0003AE 23                        290        DC    X'23'        BASE 10  2.1875
0003AF 1F                        291        DC    X'1F'        BASE 10  1.9375
0003B0 22                        292        DC    X'22'        BASE 10  2.1250
0003B1 25                        293        DC    X'25'        BASE 10  2.3125
0003B2 21                        294        DC    X'21'        BASE 10  2.0625
0003B3 1D                        295        DC    X'1D'        BASE 10  1.8125
0003B4 29                        296        DC    X'28'        BASE 10  2.5000
0003B5 30                        297        DC    X'30'        BASE 10  3.0000
0003B6 33                        298        DC    X'33'        BASE 10  3.1875
0003B7 24                        299        DC    X'24'        BASE 10  2.2500
0003B8 2F                        300        DC    X'2F'        BASE 10  2.9375
0003B9 3F                        301        DC    X'3F'        BASE 10  3.8750
0003BA 29                        302        DC    X'29'        BASE 10  2.5625
0003BB 26                        303        DC    X'26'        BASE 10  2.3750
0003BC 39                        304        DC    X'39'        BASE 10  3.5625
0003BD 2E                        305        DC    X'2E'        BASE 10  2.8750
0003BE 37                        306        DC    X'37'        BASE 10  3.4375
0003BF 2A                        307        DC    X'2A'        BASE 10  2.6250
0003C0 32                        308        DC    X'32'        BASE 10  3.1250
0003C1 2D                        309        DC    X'2D'        BASE 10  2.8125
0003C2 2C                        310        DC    X'2C'        BASE 10  2.7500
0003C3 3F                        311        DC    X'3F'        BASE 10  3.9375
0003C4 3C                        312        DC    X'3C'        BASE 10  3.7500
0003C5 27                        313        DC    X'27'        BASE 10  2.4375
0003C6 34                        314        DC    X'34'        BASE 10  3.2500
0003C7 3A                        315        DC    X'3A'        BASE 10  3.6250
0003C8 31                        316        DC    X'31'        BASE 10  3.0625
0003C9 38                        317        DC    X'38'        BASE 10  3.5000
0003CA 3D                        318        DC    X'3D'        BASE 10  3.8125
0003CB 3B                        319        DC    X'3B'        BASE 10  3.6875
0003CC 02                        320        DC    X'02'        BASE 10  0.1250
0003CD 03                        321        DC    X'03'        BASE 10  0.1875
0003CE 03                        322        DC    X'03'        BASE 10  0.1875
0003CF 03                        323        DC    X'03'        BASE 10  0.1875
0003D0 03                        324        DC    X'03'        BASE 10  0.1875
0003D1 03                        325        DC    X'03'        BASE 10  0.1875
0003D2 03                        326        DC    X'03'        BASE 10  0.1875
0003D3 05                        327        DC    X'05'        BASE 10  0.3125
0003D4 0F                        328        DC    X'0F'        BASE 10  0.9375
0003D5 0F                        329        DC    X'0F'        BASE 10  0.9375
0003D6 14                        330        DC    X'14'        BASE 10  1.2500
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
0003D7	14			331	DC	X'14'	BASE	10	1.2500
0003D8	14			332	DC	X'14'	BASE	10	1.2500
0003D9	12			333	DC	X'12'	BASE	10	1.1250
0003DA	12			334	DC	X'12'	BASE	10	1.1250
0003DB	12			335	DC	X'12'	BASE	10	1.1250
0003DC	12			336	DC	X'12'	BASE	10	1.1250
0003DD	1A			337	DC	X'1A'	BASE	10	1.6250
0003DE	25			338	DC	X'25'	BASE	10	2.3125
0003DF	25			339	DC	X'25'	BASE	10	2.3125
0003E0	25			340	DC	X'25'	BASE	10	2.3125
0003E1	25			341	DC	X'25'	BASE	10	2.3125
0003E2	21			342	DC	X'21'	BASE	10	2.0625
0003E3	21			343	DC	X'21'	BASE	10	2.0625
0003E4	1D			344	DC	X'1D'	BASE	10	1.8125
0003E5	1D			345	DC	X'1D'	BASE	10	1.8125
0003E6	3C			346	DC	X'3C'	BASE	10	3.0000
0003E7	2F			347	DC	X'2F'	BASE	10	2.9375
0003E8	2F			348	DC	X'2F'	BASE	10	2.9375
0003E9	3E			349	DC	X'3E'	BASE	10	3.8750
0003EA	3C			350	DC	X'3E'	BASE	10	3.8750
0003EB	3C			351	DC	X'3E'	BASE	10	3.8750
0003EC	26			352	DC	X'26'	BASE	10	2.3750
0003ED	26			353	DC	X'26'	BASE	10	2.3750
0003EE	26			354	DC	X'26'	BASE	10	2.3750
0003EF	39			355	DC	X'39'	BASE	10	3.5625
0003F0	39			356	DC	X'39'	BASE	10	3.5625
0003F1	39			357	DC	X'39'	BASE	10	3.5625
0003F2	39			358	DC	X'39'	BASE	10	3.5625
0003F3	39			359	DC	X'39'	BASE	10	3.5625
0003F4	2E			360	DC	X'2E'	BASE	10	2.8750
0003F5	2E			361	DC	X'2E'	BASE	10	2.8750
0003F6	2E			362	DC	X'2E'	BASE	10	2.8750
0003F7	37			363	DC	X'37'	BASE	10	3.4375
0003F8	37			364	DC	X'37'	BASE	10	3.4375
0003F9	37			365	DC	X'37'	BASE	10	3.4375
0003FA	37			366	DC	X'37'	BASE	10	3.4375
0003FB	37			367	DC	X'37'	BASE	10	3.4375
0003FC	37			368	DC	X'37'	BASE	10	3.4375
0003FD	2D			369	DC	X'2D'	BASE	10	2.8125
0003FE	2D			370	DC	X'2D'	BASE	10	2.8125
0003FF	2D			371	DC	X'2D'	BASE	10	2.8125
000400	2D			372	DC	X'2D'	BASE	10	2.8125
000401	2C			373	DC	X'2C'	BASE	10	2.7500
000402	2C			374	DC	X'2C'	BASE	10	2.7500
000403	2C			375	DC	X'2C'	BASE	10	2.7500
000404	2C			376	DC	X'2C'	BASE	10	2.7500
000405	2C			377	DC	X'2C'	BASE	10	2.7500
000406	3F			378	DC	X'3F'	BASE	10	3.9375
000407	3F			379	DC	X'3F'	BASE	10	3.9375
000408	3F			380	DC	X'3F'	BASE	10	3.9375
000409	3C			381	DC	X'3C'	BASE	10	3.7500
00040A	3C			382	DC	X'3C'	BASE	10	3.7500
00040B	3C			383	DC	X'3C'	BASE	10	3.7500
00040C	3C			384	DC	X'3C'	BASE	10	3.7500
00040D	27			385	DC	X'27'	BASE	10	2.4375

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

00040E 27                        386          DC    X'27'        BASE 10  2.4375
00040F 3A                        387          DC    X'3A'        BASE 10  3.6250
000410 3A                        388          DC    X'3A'        BASE 10  3.6250
000411 3A                        389          DC    X'3A'        BASE 10  3.6250
000412 3A                        390          DC    X'3A'        BASE 10  3.6250
000413 3A                        391          DC    X'3A'        BASE 10  3.6250
000414 31                        392          DC    X'31'        BASE 10  3.0625
000415 38                        393          DC    X'38'        BASE 10  3.5000
000416 38                        394          DC    X'38'        BASE 10  3.5000
000417 38                        395          DC    X'38'        BASE 10  3.5000
000418 38                        396          DC    X'38'        BASE 10  3.5000
000419 38                        397          DC    X'38'        BASE 10  3.5000
00041A 38                        398          DC    X'38'        BASE 10  3.5000
00041B 30                        399          DC    X'30'        BASE 10  3.8125
00041C 30                        400          DC    X'30'        BASE 10  3.8125
00041D 30                        401          DC    X'30'        BASE 10  3.8125
00041E 30                        402          DC    X'30'        BASE 10  3.8125
00041F 38                        403          DC    X'38'        BASE 10  3.6875
000420 38                        404          DC    X'38'        BASE 10  3.6875
000421 38                        405          DC    X'38'        BASE 10  3.6875
000422 38                        406          DC    X'38'        BASE 10  3.6875
000423 38                        407          DC    X'38'        BASE 10  3.6875
000424 02                        408          DC    X'02'        BASE 10  0.1250
000425 02                        409          DC    X'02'        BASE 10  0.1250
000426 05                        410          DC    X'05'        BASE 10  0.3125
000427 05                        411          DC    X'05'        BASE 10  0.3125
000428 05                        412          DC    X'05'        BASE 10  0.3125
000429 05                        413          DC    X'05'        BASE 10  0.3125
00042A 05                        414          DC    X'05'        BASE 10  0.3125
00042B 04                        415          DC    X'04'        BASE 10  0.2500
00042C 04                        416          DC    X'04'        BASE 10  0.2500
00042D 06                        417          DC    X'06'        BASE 10  0.3750
00042E 06                        418          DC    X'06'        BASE 10  0.3750
00042F 06                        419          DC    X'06'        BASE 10  0.3750
000430 06                        420          DC    X'06'        BASE 10  0.3750
000431 07                        421          DC    X'07'        BASE 10  0.4375
000432 07                        422          DC    X'07'        BASE 10  0.4375
000433 00                        423          DC    X'00'        BASE 10  0.8125
000434 0E                        424          DC    X'0E'        BASE 10  0.8750
000435 0F                        425          DC    X'0F'        BASE 10  0.8750
000436 0F                        426          DC    X'0F'        BASE 10  0.8750
000437 0F                        427          DC    X'0F'        BASE 10  0.9375
000438 0F                        428          DC    X'0F'        BASE 10  0.9375
000439 0F                        429          DC    X'0F'        BASE 10  0.9375
00043A 11                        430          DC    X'11'        BASE 10  1.0625
00043B 11                        431          DC    X'11'        BASE 10  1.0625
00043C 16                        432          DC    X'16'        BASE 10  1.3750
00043D 16                        433          DC    X'16'        BASE 10  1.3750
00043E 16                        434          DC    X'16'        BASE 10  1.3750
00043F 12                        435          DC    X'12'        BASE 10  1.1250
000440 18                        436          DC    X'18'        BASE 10  1.5000
000441 18                        437          DC    X'18'        BASE 10  1.5000
000442 18                        438          DC    X'18'        BASE 10  1.5000
000443 18                        439          DC    X'18'        BASE 10  1.5000
000444 1E                        440          DC    X'1E'        BASE 10  1.8750
```

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
000445	1E			441	DC	X'1E'	BASE 10	1.8750	
000446	1F			442	DC	X'1F'	BASE 10	1.8750	
000447	19			443	DC	X'19'	BASE 10	1.5625	
000448	1B			444	DC	X'1B'	BASE 10	1.6875	
000449	1B			445	DC	X'1B'	BASE 10	1.6875	
00044A	1B			446	DC	X'1B'	BASE 10	1.6875	
00044B	1A			447	DC	X'1A'	BASE 10	1.6250	
00044C	1A			448	DC	X'1A'	BASE 10	1.6250	
00044D	1A			449	DC	X'1A'	BASE 10	1.6250	
00044E	1A			450	DC	X'1A'	BASE 10	1.6250	
00044F	1A			451	DC	X'1A'	BASE 10	1.6250	
000450	23			452	DC	X'23'	BASE 10	2.1875	
000451	1F			453	DC	X'1F'	BASE 10	1.9375	
000452	22			454	DC	X'22'	BASE 10	2.1250	
000453	22			455	DC	X'22'	BASE 10	2.1250	
000454	22			456	DC	X'22'	BASE 10	2.1250	
000455	25			457	DC	X'25'	BASE 10	2.3125	
000456	25			458	DC	X'25'	BASE 10	2.3125	
000457	21			459	DC	X'21'	BASE 10	2.0625	
000458	21			460	DC	X'21'	BASE 10	2.0625	
000459	21			461	DC	X'21'	BASE 10	2.0625	
00045A	1D			462	DC	X'1D'	BASE 10	1.8125	
00045B	1D			463	DC	X'1D'	BASE 10	1.8125	
00045C	1D			464	DC	X'1D'	BASE 10	1.8125	
00045D	1D			465	DC	X'1D'	BASE 10	1.8125	
00045E	28			466	DC	X'28'	BASE 10	2.5000	
00045F	28			467	DC	X'28'	BASE 10	2.5000	
000460	30			468	DC	X'30'	BASE 10	3.0000	
000461	2F			469	DC	X'2F'	BASE 10	2.9375	
000462	29			470	DC	X'29'	BASE 10	2.5625	
000463	26			471	DC	X'26'	BASE 10	2.3750	
000464	26			472	DC	X'26'	BASE 10	2.3750	
000465	2E			473	DC	X'2E'	BASE 10	2.8750	
000466	32			474	DC	X'32'	BASE 10	3.1250	
000467	2D			475	DC	X'2D'	BASE 10	2.8125	
000468	2C			476	DC	X'2C'	BASE 10	2.7500	
000469	27			477	DC	X'27'	BASE 10	2.4375	
00046A	27			478	DC	X'27'	BASE 10	2.4375	
00046B	31			479	DC	X'31'	BASE 10	3.0625	
00046C	00			480	DC	X'00'	BASE 10	0.0000	
000460	00			481	DC	X'00'	BASE 10	0.0000	
00046E	00			482	DC	X'00'	BASE 10	0.0000	
00046F	01			483	DC	X'01'	BASE 10	0.0625	
000470	04			484	DC	X'04'	BASE 10	0.2500	
000471	04			485	DC	X'04'	BASE 10	0.2500	
000472	04			486	DC	X'04'	BASE 10	0.2500	
000473	08			487	DC	X'08'	BASE 10	0.6875	
000474	08			488	DC	X'08'	BASE 10	0.6875	
000475	08			489	DC	X'08'	BASE 10	0.6875	
000476	08			490	DC	X'08'	BASE 10	0.6875	
000477	08			491	DC	X'08'	BASE 10	0.6875	
000478	08			492	DC	X'08'	BASE 10	0.6875	
000479	0C			493	DC	X'0C'	BASE 10	0.7500	
00047A	0C			494	DC	X'0C'	BASE 10	0.7500	
00047B	0C			495	DC	X'0C'	BASE 10	0.7500	

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT				
000470	0C			496	DC	X'0C'	BASE	10	0.7500
000470	0C			497	DC	X'0C'	BASE	10	0.7500
00047E	0D			498	DC	X'0D'	BASE	10	0.8125
00047F	0D			499	DC	X'0D'	BASE	10	0.8125
000480	0D			500	DC	X'0D'	BASE	10	0.8125
000481	10			501	DC	X'10'	BASE	10	1.0000
000482	10			502	DC	X'10'	BASE	10	1.0000
000483	16			503	DC	X'16'	BASE	10	1.3750
000484	16			504	DC	X'16'	BASE	10	1.3750
000485	16			505	DC	X'16'	BASE	10	1.3750
000486	16			506	DC	X'16'	BASE	10	1.3750
000487	18			507	DC	X'18'	BASE	10	1.5000
000488	18			508	DC	X'18'	BASE	10	1.5000
000489	1C			509	DC	X'1C'	BASE	10	1.7500
00048A	1C			510	DC	X'1C'	BASE	10	1.7500
00048B	1C			511	DC	X'1C'	BASE	10	1.7500
00048C	1C			512	DC	X'1C'	BASE	10	1.7500
00048D	1C			513	DC	X'1C'	BASE	10	1.7500
00048E	1E			514	DC	X'1E'	BASE	10	1.8750
00048F	19			515	DC	X'19'	BASE	10	1.5625
000490	19			516	DC	X'19'	BASE	10	1.5625
000491	19			517	DC	X'19'	BASE	10	1.5625
000492	19			518	DC	X'19'	BASE	10	1.5625
000493	19			519	DC	X'19'	BASE	10	1.5625
000494	17			520	DC	X'17'	BASE	10	1.4375
000495	17			521	DC	X'17'	BASE	10	1.4375
000496	17			522	DC	X'17'	BASE	10	1.4375
000497	17			523	DC	X'17'	BASE	10	1.4375
000498	17			524	DC	X'17'	BASE	10	1.4375
000499	17			525	DC	X'17'	BASE	10	1.4375
00049A	17			526	DC	X'17'	BASE	10	1.4375
00049B	1B			527	DC	X'1B'	BASE	10	1.6875
00049C	1B			528	DC	X'1B'	BASE	10	1.6875
00049D	23			529	DC	X'23'	BASE	10	2.1875
00049E	23			530	DC	X'23'	BASE	10	2.1875
00049F	1F			531	DC	X'1F'	BASE	10	1.9375
0004A0	1F			532	DC	X'1F'	BASE	10	1.9375
0004A1	1F			533	DC	X'1F'	BASE	10	1.9375
0004A2	20			534	DC	X'20'	BASE	10	2.0000
0004A3	20			535	DC	X'20'	BASE	10	2.0000
0004A4	20			536	DC	X'20'	BASE	10	2.0000
0004A5	20			537	DC	X'20'	BASE	10	2.0000
0004A6	33			538	DC	X'33'	BASE	10	3.1875
0004A7	24			539	DC	X'24'	BASE	10	2.2500
0004A8	24			540	DC	X'24'	BASE	10	2.2500
0004A9	24			541	DC	X'24'	BASE	10	2.2500
0004AA	29			542	DC	X'29'	BASE	10	2.5625
0004AB	35			543	DC	X'35'	BASE	10	3.3125
0004AC	2A			544	DC	X'2A'	BASE	10	2.6250
0004AD	2A			545	DC	X'2A'	BASE	10	2.6250
0004AE	2B			546	DC	X'2B'	BASE	10	2.6875
0004AF	2B			547	DC	X'2B'	BASE	10	2.6875
0004B0	34			548	DC	X'34'	BASE	10	3.2500
0004B1	36			549	DC	X'36'	BASE	10	3.3750
0004B2	00			550	DC	X'00'	BASE	10	0.0000

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000483 00                         551          DC    X'00'          BASE 10  0.0000
000484 00                         552          DC    X'00'          BASE 10  0.0000
000485 01                         553          DC    X'01'          BASE 10  0.0625
000486 01                         554          DC    X'01'          BASE 10  0.0625
000487 01                         555          DC    X'01'          BASE 10  0.0625
000488 04                         556          DC    X'04'          BASE 10  0.2500
000489 04                         557          DC    X'04'          BASE 10  0.2500
00048A 06                         558          DC    X'06'          BASE 10  0.5000
00048B 08                         559          DC    X'08'          BASE 10  0.5000
00048C 07                         560          DC    X'07'          BASE 10  0.4375
00048D 08                         561          DC    X'08'          BASE 10  0.6875
00048E 09                         562          DC    X'09'          BASE 10  0.5625
00048F 0C                         563          DC    X'0C'          BASE 10  0.7500
000490 0D                         564          DC    X'0D'          BASE 10  0.8125
000491 0E                         565          DC    X'0E'          BASE 10  0.8750
000492 10                         566          DC    X'10'          BASE 10  1.0000
000493 13                         567          DC    X'13'          BASE 10  1.1875
000494 02                         568          DC    X'02'          BASE 10  0.1250
000495 02                         569          DC    X'02'          BASE 10  0.1250
000496 02                         570          DC    X'02'          BASE 10  0.1250
000497 03                         571          DC    X'03'          BASE 10  0.1875
000498 03                         572          DC    X'03'          BASE 10  0.1875
000499 03                         573          DC    X'03'          BASE 10  0.1875
00049A 05                         574          DC    X'05'          BASE 10  0.3125
00049B 05                         575          DC    X'05'          BASE 10  0.3125
00049C 06                         576          DC    X'06'          BASE 10  0.3750
00049D 06                         577          DC    X'06'          BASE 10  0.3750
00049E 07                         578          DC    X'07'          BASE 10  0.4375
00049F 0A                         579          DC    X'0A'          BASE 10  0.6250
0004A0 0A                         580          DC    X'0A'          BASE 10  0.6250
0004A1 09                         581          DC    X'09'          BASE 10  0.5625
0004A2 0F                         582          DC    X'0F'          BASE 10  0.9375
0004A3 14                         583          DC    X'14'          BASE 10  1.2500
0004A4 15                         584          DC    X'15'          BASE 10  1.3125
0004A5 11                         585          DC    X'11'          BASE 10  1.0625
0004A6 12                         586          DC    X'12'          BASE 10  1.1250
                                  587          END
0004A8 00000025                   588               =F'37'
0004AC 00000108                   589               =F'456'
0004B0 00000F17                   590               =F'3863'
0004B4 00000E37                   591               =F'3639'
```

APPENDIX D

Assembler Listing of GEN3

NOT REPRODUCIBLE

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000000                                  GEN3     START 0
                                                 PRINT ON,NOGEN,NODATA
                                        ***********************************************************************
                                        * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., AUG 1970                  *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING NOR*
                                        *          MALLY DISTRIBUTED RANDOM NUMBERS.                          *
                                        * USAGE----X=RANORM(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER, IT    *
                                        *          PRIMES THE GENERATING SCHEME.                              *
                                        * EXAMPLE--X=RANORM(56865).                                           *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN  'COMMUNI-  *
                                        *          CATIONS OF ACM',VOL.7,NO.1,JAN 1964. THE PROGRAM DOCUMEN-   *
                                        *          TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.          *
                                        ***********************************************************************
                                                 ENTRY RANORM
000000 47F0 F00C          0000C          RANORM   B     12(0,15)
000004 C7                                        DC    XL1'07'
000005 40D9C1D5D6D9D4                            DC    CL7' RANORM'
00000C 90E8 D00C          0000C                  STM   14,8,12(13)
000010 0580                                      BALR  8,0
000012                                           USING *,8
000012 1850                                      LR    5,13
000014 41D0 8282          002C4                  LA    13,ARCA
000018 50D0 5008          00008                  ST    13,8(0,5)
00001C 5050 0004          00004                  ST    5,4(0,13)
000020 47F0 809E          00080          BRANCH   B     TEMP
000024 1866                                      SR    6,6
000026 5C40 82FA          0030C                  M     4,BPAT        GENERATE U(UNIFORM ON (0,1)).
00002A 5050 82FE          00310                  ST    5,LA
00002E 3800                                      SER   0,0           RESULT TO BE PLACED IN FPRG.
000030 1844                                      SR    4,4
000032 8D40 0006          00006                  SLDL  4,6           D1D2 IN R4.
000036 5540 86AF          006C0                  CL    4,=F'44'      44(BASE 10)=54(BASE 8).
00003A 4780 8038          0004A                  BC    11,L1         IF D1D2>=44 BRANCH TO L1.
00003C 8950 0006          00006                  SLL   5,6           R5=D5D6D7...
000042 4364 85A6          00588                  IC    6,A(4)        OBTAIN TABLE VALUE.
000046 47F0 8064          00076                  B     EX
00004A 8D40 0003          00003          L1      SLDL  4,3           R4=D1D2D3.
00004E 5540 86B2          006C4                  CL    4,=F'475'     475(BASE 10)=733(BASE 8).
000052 4780 8050          00062                  BC    11,L2         IF D1D2D3>=475 BRANCH TO L2.
000056 8950 0003          00003                  SLL   5,3           R5=D5D6D7...
00005A 4364 8460          00472                  IC    6,A-326(4)    OBTAIN TABLE VALUE.
00005E 47F0 8064          00076                  B     EX
000062 8D40 0003          00003          L2      SLDL  4,3           R4=D1D2D3D4.
000066 5540 86B6          006C8                  CL    4,=F'3961'    3961(BASE 10)=7571(BASE 8).
00006A 4780 8098          000CA                  BC    11,L3         IF D1D2D3D4>=3961 BRANCH TO L3.
00006E 5840 86BA          006CC                  S     4,=F'3703'    OBTAIN PROPER TABLE ADDRESS.
000072 4364 85A6          00588                  IC    6,A(4)        OBTAIN TABLE VALUE.
000076 1846                                      LR    4,6           R4=DISCRETE TABLE VALUE.
000078 8C40 0010          00010          FL      SRDL  4,16          R5=TABLE R.V. WITH D5D6.. TACKED ON.
00007C 5050 5302          00314                  ST    5,RES         CONVERT TO FLOATING POINT.
000080 5850 82FE          00310                  L     5,LA          ATTACH + OR -.
000084 5C40 82FA          0030C                  M     4,BPAT
000088 5050 82FE          00310                  ST    5,LA
00008C 1255                                      LTR   5,5
00008E 4720 8088          0009A                  BP    *+12
```

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000092 92C1 8302      C0314          56          MVI   RES,X'C1'
000096 47F0 808C            0009F    57          BC    15,*+8
00009A 9241 8302      00314          58          MVI   RES,X'41'
00009E 7A00 8302            00314    59          AE    0,RES        NORMALIZED RESULT IN FPRO.
0000A2 5E00 8286            002C8    60 OUT      L     13,A4FA+4
0000A6 98E9 000C            0000C    61          LM    14,9,12(13)
0000AA 92FF 000C      0000C          62          MVI   12(13),X'FF'
0000AE 07FE                          63          BCR   15,14
0000B0 5850 82FE            00310    64 TEMP     L     5,LA         REPLACES INSTR. AT BRANCH.
0000B4 D203 800E 809E C0020 00080    65          MVC   BRANCH,TEMP
0000BA 5850 1000            000C0    66          L     5,0(,1)
0000BE 5850 50C0            00000    67          L     5,0(,5)
0000C2 5050 82FE            00310    68          ST    5,LA
0000C6 47F0 8012            00024    69          B     BRANCH+4
0000CA 5850 82FE            00310    70 L3       L     5,LA         R5=U.
0000CE 4120 0004            00004    71          LA    2,4
0000D2 5550 868E            00600    72          CL    5,=X'FF4F10F0'
0000D6 4780 818E            00100    73          BC    11,L4        IF U>=FF4F10F0(BASE 16) GO TO L4.
0000DA 1162                          74          LNR   6,2
0000DC 1A62                          75          AR    6,2          INCREMENT I(R6).
0000DE 5556 83C6            00308    76          CL    5,B(6)       COMPARE U TO B(I).
0000E2 4780 80CA            000DC    77          BC    11,*-6       IF U>=B(I) CONTINUE LOOP.
0000E6 5555 8486            00498    78          CL    5,C(6)       COMPARE U TO C(I).
0000EA 4780 80EC            000FE    79          BC    11,L8        IF C(I)=<U<B(I) GO TO L8.
0000EE 5C40 82FA            0030C    80          M     4,BPAT       GENERATE A NEW U.
0000F2 5050 82FE            00310    81          ST    5,LA
0000F6 8860 0002            00002    82          SRL   6,2
0000FA 47F0 8184            001C6    83          B     L5
0000FE 5C40 82FA            0030C    84 L8       M     4,BPAT       GENERATE U1.
000102 1E75                          85          LR    7,5          SAVE U1.
000104 5C40 82FA            0030C    86          M     4,BPAT       GENERATE U2.
000108 5050 82FE            00310    87          ST    5,LA
00010C 1575                          88          CLR   7,5
00010E 4780 8106            00118    89          BC    11,L6        R5=MIN(U1,U2).
000112 1E57                          90          LR    5,7
000114 5870 82FE            00310    91          L     7,LA
000118 8860 0001            00001    92 L6       SRL   6,1
00011C 1844                          93          SR    4,4
00011E 4846 8546            00558    94          LH    4,D(6)
000122 1024                          95          LPR   2,4
000124 1837                          96          LR    3,7
000126 8830 0010            00010    97          SRL   3,16
00012A 1532                          98          CLR   3,2
00012C 4780 8126            00138    99          BC    11,L9        IF MAX(U1,U2)<D(I) GO TO L5.
000130 8860 0001            00001   100          SRL   6,1
000134 47F0 8184            001C6   101          B     L5
000138 8860 0001            00001   102 L9       SRL   6,1
00013C 5050 829A            002AC   103          ST    5,MIN
000140 1E75                         104          SLR   7,5          R7=MAX-MIN.
000142 8870 0008            00008   105          SRL   7,8
000146 5070 829E            00280   106          ST    7,DIF        CONVERT TO FLOATING POINT.
00014A 9240 829E      00280         107          MVI   DIF,X'40'
00014E 3822                         108          SER   2,2
000150 7420 829E            00280   109          AE    2,DIF
000154 8850 000C            0000C   110          SRL   5,12
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    FORAPR70   6/30/71
000159 5050 82A2          00284   111          ST    5,W          CONVERT TO FLOATING POINT.
00015C 9240 82A2    00284         112          MVI   W,X'40'
00016C 7840 86C2          00604   113          LE    4,=E'0.0625'
000164 3864                       114          LER   6,4
00016A 7840 82A2          00284   115          SE    4,W
00016A 7A60 82A2          00284   116          AE    6,W
00016F 1933                       117          SR    3,3
000170 4336 867E          00690   118          IC    3,A+216(6)   R3=TABLE RANDOM VARIABLE.
000174 8930 0001          00001   119          SLL   3,1          R3=R3*2.
000178 5030 82A2          00284   120          ST    3,W
00017C 9245 82A2    00284         121          MVI   W,X'45'
000180 7A60 82A2          00284   122          AE    6,W
000184 3C46                       123          MER   4,6
000186 7040 86C6          00608   124          ME    4,=E'0.5'    FPR4=-(MIN/16-1/16)*(2*A(I)+MIN/16+1/16)*.5
00018A 7040 82A2          00284   125          STE   4,W
                                  126          CALL EXP,(W)       CALCULATE EXP(W).
0001AA 7800 86CA          00600   138          SE    0,=E'1.0'    FPR0=EXP(W)-1.
0001AA 8560 0002          00002   139          SLL   6,2
0001AE 7C06 8306          00318   140          ME    0,E(6)       FPR0=(EXP(W)-1)*E(I).
000182 5850 82FE          00310   141          L     5,LA
000186 3902                       142          CER   0,2          COMPERE FPR0 TO DIF.
000188 4700 80EC          000FE   143          BC    13,L8        IF FPR0<=DIF GO TO L9.
0001BC 5850 829A          002AC   144          L     5,MIN
0001C0 3B00                       145          SER   0,0
0001C2 8860 0002          00002   146          SRL   6,2
0001C6 1844                       147 L5       SR    4,4
0001C8 4346 867E          00690   148          IC    4,A+216(6)   OBTAIN RANDOM VARIABLE.
0001CC 47F0 8066          00078   149          B     FL
0001D0 5850 82FE          00310   150 L4       L     5,LA
0001D4 1877                       151          SR    7,7
0001D6 5040 82FA          0030C   152          M     4,BPAT       OBTAIN U1.
0001DA 1844                       153          SR    4,4
0001DC 5050 82FF          00310   154          ST    5,LA
0001DE 8850 0008          00008   155          SRL   5,8          PREPARE FOR FLT. PT. CONVERSION.
0001E4 5050 82A6          00288   156          ST    5,U1
0001E8 5850 82FE          00310   157          L     5,LA
0001EC 1C45                       158          MR    4,5          R4=U1*U1.
0001EE 1E74                       159          ALR   7,4
0001F0 5850 82FE          00310   160          L     5,LA
0001F4 5040 82FA          0030C   161          M     4,BPAT       OBTAIN U2.
0001F8 5050 82FE          00310   162          ST    5,LA
0001FC 1844                       163          SR    4,4
0001FE 8850 0008          00008   164          SRL   5,8
000202 5050 82AA          0028C   165          ST    5,U2
000206 5850 82FE          00310   166          L     5,LA
00020A 1C45                       167          MR    4,5          R4=U2*U2.
00020C 1E74                       168          ALR   7,4          R7=U1**2+U2**2.
00020E 4710 81BE          00100   169          BO    L4           IF U1**2+U2**2>=1 BRANCH TO L4.
000212 8870 0009          00008   170          SRL   7,8
000216 5070 82AE          002C0   171          ST    7,S          CONVERT TO FLT. PT.
00021A 9240 82AE    002C0         172          MVI   S,X'40'
00021F 7A00 82AE          002C0   173          AE    0,S
000222 7000 82AE          002C0   174          STE   0,S
                                  175          CALL ALOG,(S)      CALCULATE LN(U1**2+U2**2).
00023E 7C00 86CE          006E0   187          ME    0,=E'-2.0'
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    FORAPR70   6/30/71

000242 7A00 86D2            006F4  188          AE    0,=E'9.0'
000246 7D00 82AE            002C0  189          DE    0,S
00024A 7000 82AE            002C0  190          STE   0,S
                                   191          CALL SQ[illegible],(S)
000266 9240 82A6      002B8        203          MVI   U1,X'40'     CONVERT TO FLT. PT.
00026A 3922                        204          SER   2,2
00026C 7A20 82A6            002B8  205          AE    2,U1
000270 3C20                        206          MER   2,0
000272 7920 86D6            006F8  207          CE    2,=E'3.0'
000276 4720 827E            00290  208          BC    2,L10        IF FPR2>3.0 BRANCH TO L10.
00027A 9240 82AA      002BC        209          MVI   U2,X'40'     CONVERT TO FLT. PT.
00027E 3922                        210          SER   2,2
000280 7A20 82AA            002BC  211          AE    2,U2
000284 3C20                        212          MER   2,0
000286 3800                        213          SER   0,0
000288 7920 86D6            006F8  214          CE    2,=E'3.0'
00028C 4700 818E            001D0  215          BC    13,L4        IF FPR2=<3.0 BRANCH TO L4.
000290 5850 82FE            00310  216 L10      L     5,LA
000294 5C40 82FA            0030C  217          M     4,BPAT
000298 5050 82FE            00310  218          ST    5,LA
00029C 1255                        219          LTR   5,5
00029E 4720 8292            002A4  220          BP    *+6
0002A2 3122                        221          LNER  2,2
0002A4 3802                        222          LER   0,2
0002A6 47F0 8090            007A2  223          B     OUT
0002AC                             224 MIN      DS    1F
0002B0                             225 DIF      DS    1F
0002B4                             226 W        DS    1F
0002B8                             227 U1       DS    1F
0002BC                             228 U2       DS    1F
0002C0                             229 S        DS    1F
0002C4                             230 AREA     DS    18F
00030C 48C27395                    231 BPAT     DC    XL4'48C27395'
000310                             232 LA       DS    1F
000314                             233 RES      DS    1F
000318 42100000                    234 E        DC    E'16.0'
00031C 41D58073                    235          DC    E'13.34385943'
000320 42138138                    236          DC    E'19.69230768'
000324 42111111                    237          DC    E'17.06666666'
000328 41F225F6                    238          DC    E'14.13426759'
00032C 41C42720                    239          DC    E'12.63455216'
000330 42124925                    240          DC    E'18.26571428'
000334 41F0540F                    241          DC    E'15.02052223'
000338 41A66396                    242          DC    E'10.40126568'
00033C 41869FA4                    243          DC    E'11.41397461'
000340 42155555                    244          DC    E'21.33333333'
000344 4198C29F                    245          DC    E'9.547515094'
000348 421C71C7                    246          DC    E'28.44444444'
00034C 419F4FAF                    247          DC    E'9.956553629'
000350 42174501                    248          DC    E'23.27272727'
000354 42249249                    249          DC    E'36.57142856'
00035B 42230000                    250          DC    E'32.0'
00035C 4192843C                    251          DC    E'9.169002116'
000360 41870E22                    252          DC    E'8.491731562'
000364 41801682                    253          DC    E'8.818041004'
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
000368	422AAAAB			254	DC	E'42.66666666'
00036C	417A321F			255	DC	E'7.637235440'
000370	41830046			256	DC	E'8.187566839'
000374	42333333			257	DC	E'51.19999999'
00037C	417F[illegible]			258	DC	E'7.903368655'
00037C	4172716E			259	DC	E'7.152692877'
000380	42400000			260	DC	E'64.0'
000384	4150598E			261	DC	E'5.834165752'
000388	416969C3			262	DC	E'6.525576115'
00038C	416FF77F			263	DC	E'6.931516923'
000390	42800000			264	DC	E'128.0'
000394	4162970F			265	DC	E'6.161878474'
000398	415444CF			266	DC	E'5.267287374'
00039C	42100000			267	DC	E'256.0'
0003A0	41656C02			268	DC	E'6.3[illegible]869363'
0003A4	41566328			269	DC	E'5.3[illegible]210855'
0003A8	415A2433			270	DC	E'5.635302894'
0003AC	41599904			271	DC	E'5.537357695'
0003B0	415053F0			272	DC	E'5.020504743'
0003B4	41524243			273	DC	E'5.141177028'
0003B8	414E7455			274	DC	E'4.994927053'
0003BC	418FE961			275	DC	E'11.[illegible]9447734'
0003C0	42555555			276	DC	E'85.33333333'
0003C4	4219999A			277	DC	E'25.6[illegible]'
0003C8	41AF2760			278	DC	E'10.[illegible]51C165'
0003CC	41763333			279	DC	E'7.397499660'
0003D0	41639048			280	DC	E'6.722816549'
0003D4	415FE6F0			281	DC	E'5.993865632'
0003D8	F70C2088			282 8	DC	X'F70C2088'
0003DC	F8272283			283	DC	X'F8272283'
0003E0	F86F9303			284	DC	X'F86F9303'
0003E4	F8870400			285	DC	X'F8870400'
0003E8	F8FBFEA0			286	DC	X'F8FBFEA0'
0003EC	F93F3937			287	DC	X'F93F3937'
0003F0	F9807426			288	DC	X'F9807426'
0003F4	F9C0C5F0			289	DC	X'F9C0C5F0'
0003F8	F9F850DE			290	DC	X'F9F850DE'
0003FC	FA3581DE			291	DC	X'FA3581DE'
000400	FA717F89			292	DC	X'FA717F89'
000404	FAA78087			293	DC	X'FAA78087'
000408	FAD03082			294	DC	X'FAD03082'
00040C	FB1156A5			295	DC	X'FB1156A5'
000410	FB4C180E			296	DC	X'FB4C180E'
000414	FB7CF968			297	DC	X'FB7CF968'
000418	FBAC6582			298	DC	X'FBAC6582'
00041C	FBBA168C			299	DC	X'FBBA168C'
000420	FC072AD8			300	DC	X'FC072AD8'
000424	FC31F07D			301	DC	X'FC31F07D'
000428	FC59C042			302	DC	X'FC59C042'
00042C	FC8C8F9A			303	DC	X'FC8C8F9A'
000430	FCA600E7			304	DC	X'FCA600E7'
000434	FCCAEEF0			305	DC	X'FCCAEEF0'
000438	FCEE547E			306	DC	X'FCEE547E'
00043C	FD08990F			307	DC	X'FD08990F'
000440	FD31F952			308	DC	X'FD31F952'

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000444 FD46D39F                  309          DC    X'FD46D39F'
000448 FD593D30                  310          DC    X'FD593D30'
00044C FD7915BF                  311          DC    X'FD7915BF'
000450 FD85D36                   312          DC    X'FD85D36'
000454 FDA339E6                  313          DC    X'FDA339E6'
000458 FD87A0AF                  314          DC    X'FD87A0AF'
00045C FDC9A2EA                  315          DC    X'FDC9A2EA'
000460 FDD96E0E                  316          DC    X'FDD96E0E'
000464 FDFCA777                  317          DC    X'FDFCA777'
000468 FDFD0A1D                  318          DC    X'FDFD0A1D'
00046C FE0DED49                  319          DC    X'FE0DED49'
000470 FE1F0F0C                  320          DC    X'FE1F0F0C'
000474 FE273FF6                  321          DC    X'FE273FF6'
000478 FE2F27A3                  322          DC    X'FE2F27A3'
00047C FE409811                  323          DC    X'FE409811'
000480 FE89339C                  324          DC    X'FE89339C'
000484 FE603425                  325          DC    X'FE603425'
000488 FEFD414D                  326          DC    X'FEFD414D'
00048C FF1CC2A3                  327          DC    X'FF1CC2A3'
000490 FF33C0E3                  328          DC    X'FF33C0E3'
000494 FF4F10F0                  329          DC    X'FF4F10F0'
000498 F7C0E758                  330 C        DC    X'F7C0E758'
00049C F8197206                  331          DC    X'F8197206'
0004A0 F8623897                  332          DC    X'F8623897'
0004A4 F8A210F0                  333          DC    X'F8A210F0'
0004A8 F8F44018                  334          DC    X'F8F44018'
0004AC F935F652                  335          DC    X'F935F652'
0004B0 F97AC777                  336          DC    X'F97AC777'
0004B4 F98E47B2                  337          DC    X'F98E47B2'
0004B8 F9F35BC8                  338          DC    X'F9F35BC8'
0004BC FA2F3596                  339          DC    X'FA2F3596'
0004C0 FA6F93C1                  340          DC    X'FA6F93C1'
0004C4 FA9F8DC6                  341          DC    X'FA9F8DC6'
0004C8 FAD762AF                  342          DC    X'FAD762AF'
0004CC FB0D2A1F                  343          DC    X'FB0D2A1F'
0004D0 FB47D73D                  344          DC    X'FB47D73D'
0004D4 FB731C08                  345          DC    X'FB731C08'
0004D8 FBA49731                  346          DC    X'FBA49731'
0004DC FBD726FA                  347          DC    X'FBD726FA'
0004E0 FBFED9BF                  348          DC    X'FBFED9BF'
0004E4 FC2FC4D6                  349          DC    X'FC2FC4D6'
0004E8 FC53839A                  350          DC    X'FC53839A'
0004EC FC764456                  351          DC    X'FC764456'
0004F0 FCA282A2                  352          DC    X'FCA282A2'
0004F4 FCC29AB5                  353          DC    X'FCC29AB5'
0004F8 FCDA1FBA                  354          DC    X'FCDA1FBA'
0004FC FD0531CD                  355          DC    X'FD0531CD'
000500 FD2263B2                  356          DC    X'FD2263B2'
000504 FD3C3FF6                  357          DC    X'FD3C3FF6'
000508 FD57C534                  358          DC    X'FD57C534'
00050C FD6F65A6                  359          DC    X'FD6F65A6'
000510 FD83396A                  360          DC    X'FD83396A'
000514 FD99A088                  361          DC    X'FD99A088'
000518 FDA904FA                  362          DC    X'FDA904FA'
00051C FDB8E0C9                  363          DC    X'FDB8E0C9'
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
000520	FDDA8064			364	DC	X'FDDA8064'
000524	FDF22AFA			365	DC	X'FDF22AFA'
000528	FDF59D92			366	DC	X'FDF59D92'
00052C	FF05A299			367	DC	X'FF05A299'
000530	FE1229E8			368	DC	X'FE1229E8'
000534	FE22D621			369	DC	X'FE22D621'
000538	FE2CD971			370	DC	X'FE2CD971'
00053C	FE6B6D88			371	DC	X'FE6B6D88'
000540	FE7E231D			372	DC	X'FE7E231D'
000544	FEBCE8B6			373	DC	X'FEBCE8B6'
000548	FEF23D46			374	DC	X'FEF23D46'
00054C	FF1732E6			375	DC	X'FF1732E6'
000550	FF2F8EC3			376	DC	X'FF2F8EC3'
000554	FF3F796D			377	DC	X'FF3F796D'
000558	FF7D			378 D	DC	X'FF7D'
00055A	FF38			379	DC	X'FF38'
00055C	FC29			380	DC	X'FC29'
00055E	FF76			381	DC	X'FF76'
000560	FF70			382	DC	X'FF70'
000562	FEF9			383	DC	X'FEF9'
000564	FD70			384	DC	X'FD70'
000566	FFBE			385	DC	X'FFBE'
000568	FE35			386	DC	X'FE35'
00056A	FEB8			387	DC	X'FEB8'
00056C	FAE1			388	DC	X'FAE1'
00056E	FDF3			389	DC	X'FDF3'
000570	F5C2			390	DC	X'F5C2'
000572	FE35			391	DC	X'FE35'
000574	F958			392	DC	X'F958'
000576	F0A3			393	DC	X'F0A3'
000578	F374			394	DC	X'F374'
00057A	FDB2			395	DC	X'FDB2'
00057C	FD70			396	DC	X'FD70'
00057E	FDB2			397	DC	X'FDB2'
000580	FD07			398	DC	X'FD07'
000582	FCE0			399	DC	X'FCE0'
000584	FD2F			400	DC	X'FD2F'
000586	F831			401	DC	X'F831'
000588	FD2F			402	DC	X'FD2F'
00058A	FCAC			403	DC	X'FCAC'
00058C	E189			404	DC	X'E189'
00058E	FBA5			405	DC	X'FBA5'
000590	FC29			406	DC	X'FC29'
000592	FC6A			407	DC	X'FC6A'
000594	C0C3			408	DC	X'C0C3'
000596	FBE7			409	DC	X'FBE7'
000598	FB22			410	DC	X'FB22'
00059A	B000			411	DC	X'B000'
00059C	FC29			412	DC	X'FC29'
00059E	FB22			413	DC	X'FB22'
000540	FB64			414	DC	X'FB64'
000542	FB64			415	DC	X'FB64'
000544	FA9F			416	DC	X'FA9F'
000546	FAE1			417	DC	X'FAE1'
000548	FA9F			418	DC	X'FA9F'

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F08APR70   6/30/71
0005AA FEF9                       419          DC    X'FEF9'
0005AC D645                       420          DC    X'D645'
0005AE F6BD                       421          DC    X'F6BD'
0005B0 FE76                       422          DC    X'FE76'
0005B2 FCAC                       423          DC    X'FCAC'
0005B4 FC6A                       424          DC    X'FC6A'
0005B6 FBA5                       425          DC    X'FBA5'
0005B8 00                         426 A        DC    X'00'
0005B9 C0                         427          DC    X'00'
0005BA C0                         428          DC    X'00'
0005BB 01                         429          DC    X'01'
0005BC C1                         430          DC    X'01'
0005BD C1                         431          DC    X'01'
0005BE 02                         432          DC    X'02'
0005BF 02                         433          DC    X'02'
0005C0 03                         434          DC    X'03'
0005C1 03                         435          DC    X'03'
0005C2 03                         436          DC    X'03'
0005C3 04                         437          DC    X'04'
0005C4 C4                         438          DC    X'04'
0005C5 C4                         439          DC    X'04'
0005C6 C8                         440          DC    X'08'
0005C7 C8                         441          DC    X'08'
0005C8 0C                         442          DC    X'0C'
0005C9 CD                         443          DC    X'0D'
0005CA CD                         444          DC    X'0D'
0005CB 0E                         445          DC    X'0E'
0005CC CF                         446          DC    X'CF'
0005CD 13                         447          DC    X'13'
0005CE 14                         448          DC    X'14'
0005CF 15                         449          DC    X'15'
0005D0 16                         450          DC    X'16'
0005D1 17                         451          DC    X'17'
0005D2 C2                         452          DC    X'02'
0005D3 C5                         453          DC    X'05'
0005D4 C5                         454          DC    X'05'
0005D5 06                         455          DC    X'06'
0005D6 C6                         456          DC    X'06'
0005D7 C7                         457          DC    X'07'
0005D8 07                         458          DC    X'07'
0005D9 09                         459          DC    X'09'
0005DA C9                         460          DC    X'09'
0005DB CA                         461          DC    X'0A'
0005DC CA                         462          DC    X'0A'
0005DD CB                         463          DC    X'0B'
0005DE 0B                         464          DC    X'0B'
0005DF CC                         465          DC    X'0C'
0005E0 0F                         466          DC    X'0F'
0005E1 10                         467          DC    X'10'
0005E2 11                         468          DC    X'11'
0005E3 12                         469          DC    X'12'
0005E4 C6                         470          DC    X'06'
0005E5 06                         471          DC    X'06'
0005E6 06                         472          DC    X'06'
0005E7 C6                         473          DC    X'06'
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000E8 C6                          474          DC    X'06'
000E9 C7                          475          DC    X'07'
000EA C7                          476          DC    X'07'
000EB 07                          477          DC    X'07'
000EC C9                          478          DC    X'09'
000ED C9                          479          DC    X'09'
000EE C9                          480          DC    X'09'
000EF CA                          481          DC    X'0A'
000F0 0A                          482          DC    X'0A'
000F1 0F                          483          DC    X'0F'
000F2 0F                          484          DC    X'0F'
000F3 0F                          485          DC    X'0F'
000F4 0F                          486          DC    X'0F'
000F5 0F                          487          DC    X'0F'
000F6 10                          488          DC    X'10'
000F7 10                          489          DC    X'10'
000F8 10                          490          DC    X'10'
000F9 10                          491          DC    X'10'
000FA 11                          492          DC    X'11'
000FB 11                          493          DC    X'11'
000FC 11                          494          DC    X'11'
000FD 16                          495          DC    X'16'
000FE 18                          496          DC    X'18'
000FF 18                          497          DC    X'18'
00100 18                          498          DC    X'18'
00101 18                          499          DC    X'18'
00102 18                          500          DC    X'18'
00103 18                          501          DC    X'18'
00104 19                          502          DC    X'19'
00105 1A                          503          DC    X'1A'
00106 1A                          504          DC    X'1A'
00107 1A                          505          DC    X'1A'
00108 1A                          506          DC    X'1A'
00109 1A                          507          DC    X'1A'
0010A 1A                          508          DC    X'1A'
0010B 18                          509          DC    X'18'
0010C 18                          510          DC    X'18'
0010D 18                          511          DC    X'18'
0010E 18                          512          DC    X'18'
0010F 1D                          513          DC    X'1D'
00110 1D                          514          DC    X'1D'
00111 1F                          515          DC    X'1F'
00112 20                          516          DC    X'20'
00113 20                          517          DC    X'20'
00114 20                          518          DC    X'20'
00115 22                          519          DC    X'22'
00116 23                          520          DC    X'23'
00117 23                          521          DC    X'23'
00118 27                          522          DC    X'27'
00119 00                          523          DC    X'00'
0011A 01                          524          DC    X'01'
0011B 05                          525          DC    X'05'
0011C 05                          526          DC    X'05'
0011D 05                          527          DC    X'05'
0011E 05                          528          DC    X'05'
```

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
000610	05			529	DC	X'05'
000620	07			530	DC	X'07'
000621	08			531	DC	X'08'
000622	08			532	DC	X'08'
000623	08			533	DC	X'08'
000624	08			534	DC	X'08'
000625	08			535	DC	X'08'
000626	08			536	DC	X'08'
000627	0C			537	DC	X'0C'
000628	0D			538	DC	X'0D'
000629	0F			539	DC	X'0F'
00062A	10			540	DC	X'10'
00062B	11			541	DC	X'11'
00062C	12			542	DC	X'12'
00062D	12			543	DC	X'12'
00062E	12			544	DC	X'12'
00062F	13			545	DC	X'13'
000630	13			546	DC	X'13'
000631	13			547	DC	X'13'
000632	14			548	DC	X'14'
000633	14			549	DC	X'14'
000634	15			550	DC	X'15'
000635	18			551	DC	X'18'
000636	19			552	DC	X'19'
000637	19			553	DC	X'19'
000638	19			554	DC	X'19'
000639	19			555	DC	X'19'
00063A	19			556	DC	X'19'
00063B	18			557	DC	X'18'
00063C	1C			558	DC	X'1C'
00063D	1C			559	DC	X'1C'
00063E	1C			560	DC	X'1C'
00063F	1C			561	DC	X'1C'
000640	1D			562	DC	X'1D'
000641	1D			563	DC	X'1D'
000642	1E			564	DC	X'1E'
000643	1F			565	DC	X'1F'
000644	1F			566	DC	X'1E'
000645	1F			567	DC	X'1F'
000646	1F			568	DC	X'1F'
000647	21			569	DC	X'21'
000648	21			570	DC	X'21'
000649	22			571	DC	X'22'
00064A	24			572	DC	X'24'
00064B	25			573	DC	X'25'
00064C	26			574	DC	X'26'
00064D	00			575	DC	X'00'
00064E	03			576	DC	X'03'
00064F	03			577	DC	X'03'
000650	03			578	DC	X'03'
000651	03			579	DC	X'03'
000652	04			580	DC	X'04'
000653	07			581	DC	X'07'
000654	07			582	DC	X'07'
000655	0D			583	DC	X'0D'

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT  SOURCE STATEMENT
000656 0E                        584        DC    X'0E'
000657 0E                        585        DC    X'0E'
000658 0F                        586        DC    X'0F'
000659 10                        587        DC    X'10'
00065A 10                        588        DC    X'10'
00065B 11                        589        DC    X'11'
00065C 11                        590        DC    X'11'
00065D 13                        591        DC    X'13'
00065E 14                        592        DC    X'14'
00065F 14                        593        DC    X'14'
000660 14                        594        DC    X'14'
000661 15                        595        DC    X'15'
000662 15                        596        DC    X'15'
000663 15                        597        DC    X'15'
000664 15                        598        DC    X'15'
000665 15                        599        DC    X'15'
000666 17                        600        DC    X'17'
000667 18                        601        DC    X'18'
000668 18                        602        DC    X'18'
000669 18                        603        DC    X'18'
00066A 1B                        604        DC    X'1B'
00066B 1C                        605        DC    X'1C'
00066C 1C                        606        DC    X'1C'
00066D 1E                        607        DC    X'1E'
00066E 1E                        608        DC    X'1E'
00066F 1E                        609        DC    X'1E'
000670 21                        610        DC    X'21'
000671 21                        611        DC    X'21'
000672 24                        612        DC    X'24'
000673 24                        613        DC    X'24'
000674 24                        614        DC    X'24'
000675 24                        615        DC    X'24'
000676 25                        616        DC    X'25'
000677 25                        617        DC    X'25'
000678 28                        618        DC    X'28'
000679 28                        619        DC    X'28'
00067A 28                        620        DC    X'28'
00067B 28                        621        DC    X'28'
00067C 28                        622        DC    X'28'
00067D 28                        623        DC    X'28'
00067E 29                        624        DC    X'29'
00067F 29                        625        DC    X'29'
000680 29                        626        DC    X'29'
000681 29                        627        DC    X'29'
000682 29                        628        DC    X'29'
000683 2A                        629        DC    X'2A'
000684 2A                        630        DC    X'2A'
000685 2A                        631        DC    X'2A'
000686 2A                        632        DC    X'2A'
000687 2B                        633        DC    X'2B'
000688 2B                        634        DC    X'2B'
000689 2B                        635        DC    X'2B'
00068A 2C                        636        DC    X'2C'
00068B 2C                        637        DC    X'2C'
00068C 2D                        638        DC    X'2D'
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000680 20                         639          DC    X'20'
000681 2E                         640          DC    X'2E'
000682 2F                         641          DC    X'2F'
000683 0F                         642          DC    X'0F'
000684 12                         643          DC    X'12'
000685 0C                         644          DC    X'0C'
000686 0E                         645          DC    X'0E'
000687 11                         646          DC    X'11'
000688 13                         647          DC    X'13'
000689 0D                         648          DC    X'0D'
00068A 10                         649          DC    X'10'
00068B 17                         650          DC    X'17'
00068C 15                         651          DC    X'15'
00068D 08                         652          DC    X'08'
00068E 18                         653          DC    X'18'
00068F 09                         654          DC    X'09'
000690 19                         655          DC    X'19'
000691 0A                         656          DC    X'0A'
000692 06                         657          DC    X'06'
000693 07                         658          DC    X'07'
000694 1A                         659          DC    X'1A'
000695 1E                         660          DC    X'1E'
000696 18                         661          DC    X'18'
000697 05                         662          DC    X'05'
000698 1F                         663          DC    X'1F'
000699 1D                         664          DC    X'1D'
00069A 04                         665          DC    X'04'
00069B 1E                         666          DC    X'1E'
00069C 21                         667          DC    X'21'
00069D 03                         668          DC    X'03'
00069E 28                         669          DC    X'28'
00069F 24                         670          DC    X'24'
0006A0 22                         671          DC    X'22'
0006A1 01                         672          DC    X'01'
0006A2 26                         673          DC    X'26'
0006A3 2C                         674          DC    X'2C'
0006A4 00                         675          DC    X'00'
0006A5 25                         676          DC    X'25'
0006A6 28                         677          DC    X'28'
0006A7 29                         678          DC    X'29'
0006A8 2A                         679          DC    X'2A'
0006A9 2E                         680          DC    X'2E'
0006AA 20                         681          DC    X'20'
0006AB 2F                         682          DC    X'2F'
0006AC 14                         683          DC    X'14'
0006AD 02                         684          DC    X'02'
0006AE 09                         685          DC    X'09'
0006AF 16                         686          DC    X'16'
0006B0 20                         687          DC    X'20'
0006B1 23                         688          DC    X'23'
0006B2 27                         689          DC    X'27'
                                  690          END
0006B0 0000002C                   691               =F'44'
0006B4 00000109                   692               =F'475'
0006B8 00000F79                   693               =F'3961'
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
0006CC 00000E77                  694              =F'3703'
0006D0 FF4F10F0                  695              =X'FF4F10F0'
0006D4 40100000                  696              =E'0.0625'
0006D8 40800000                  697              =E'0.5'
0006DC 41100000                  698              =E'1.0'
0006E0 C1200000                  699              =E'-2.0'
0006E4 41900000                  700              =E'9.0'
0006E8 41300000                  701              =E'3.0'
```

NOT REPRODUCIBLE

APPENDIX E

Assembler Listing of GEN4

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000000                                  GEN4     START 0
                                        ************************************************************************
                                        * SOURCE---L.E. CANNON,UGA,DEPT. OF STAT., JUN 1970                   *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING    *
                                        *          BINOMIALLY DISTRIBUTED RANDOM NUMBERS.                     *
                                        * USAGE----1. CALL BSETUP(P,IR). P IS THE PROBABILITY OF A SUCCESS.   *
                                        *          IR IS THE RANGE OF THE VARIABLE. BSETUP SETS UP THE TABLE  *
                                        *          FROM WHICH RANBI GENERATES A RANDOM NUMBER. BSETUP NEED BE *
                                        *          CALLED ONLY ONCE FOR A GIVEN SET OF PARAMETERS.            *
                                        *          2. X=RANBI(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER. IT *
                                        *          PRIMES THE GENERATING SCHEME. X IS THE RANDOM NUMBER GENER*
                                        *          ATED. IT IS RETURNED IN THE REAL MODE.                     *
                                        * EXAMPLE--CALL BSETUP(0.5,2)                                         *
                                        *          DO 1 I=1,100                                               *
                                        *          X=RANBI(55695)                                             *
                                        *             ...                                                     *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                        *          6',1(JAN. 1963),37.                                        *
                                        ************************************************************************
                                                 ENTRY BSETUP
000000 47F0 F00C          0000C          BSETUP   B     12(0,15)
000004 07                                         DC    XL1'07'
000005 40C2E2C5E3E4D7                             DC    CL7' BSETUP'
00000C 90EC D00C          0000C                   STM   14,12,12(13)
000010 05A0                                       BALR  10,0
000012                                            USING *,10
000012 18ED                                       LR    14,13
000014 4100 A25A          0026C                   LA    13,AREA
000018 5000 F008          00008                   ST    13,8(0,14)
00001C 50E0 D004          00004                   ST    14,4(0,13)
000020 5820 1000          00000                   L     2,0(,1)
000024 7820 2000          00000                   LE    2,0(,2)       FPR2=P.
000028 7840 AEAE          00EC0                   LE    4,=F'1.0'
00002C 3804                                       LER   0,4
00002E 3842                                       SER   4,2          FPR4=1-P=Q.
000030 7040 A2A2          00284                   STE   4,0
000034 5860 1004          00004                   L     6,4(,1)
000038 5860 6000          00000                   L     6,0(,6)       R6=N.
00003C 1856                                       LR    5,6
00003E 8950 0002          00002                   SLL   5,2           CONVERT N TO WORDS.
000042 3866                                       SER   6,6
000044 5060 A2A6          00288                   ST    6,NUM         CONVERTING TO FLOATING POINT.
000048 9246 A2A6     00288                        MVI   NUM,X'46'
00004C 7A60 A2A6          00288                   AE    6,NUM
000050 5870 AEB2          00EC4                   L     7,=F'-1'
000054 1A67                                       AR    6,7
000056 3C04                                       MER   0,4           FPR0=Q**N.
000058 8667 A044          00056                   RXH   6,7,*-2
00005C 4140 00C4          00004                   LA    4,4
000060 1833                                       SR    3,3
000062 3844                                       SER   4,4
000064 4100 0001          00001                   LA    12,1
000068 7900 A282          002C4                   CE    0,LLIM        IF Q**N<LLIM
00006C 4740 A08E          00A0                    BC    4,L0         BRANCH TO L0
000070 5030 A2AE          002C0                   ST    3,XST         OTHERWISE STORE R3=0 INTO XST.
```

NOT REPRODUCIBLE

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000074 7C03 A2B6          002C8    56          STE   0,P(3)      AND STORE Q**N INTO P(0).
000078 1A34                        57          AR    3,4
00007A 7A40 AEAE          00EC0    58 L1       AE    4,=E'1.0'   FPR4=X+1.
00007E 3C04                        59          DER   0,4         FPR0=P(X)/(X+1).
000080 3C06                        60          MER   0,6         FPR0=P(X)*(N-X)/(X+1).
000082 7C00 A2A2          002B4    61          DE    0,Q         FPR0=P(X)*(N-X)/((X+1)*Q).
000086 3C02                        62          MER   0,2         FPR0=P(X)*P*(N-X)/((X+1)*Q)
000088 7903 A2B2          002C4    63          CE    0,LLIM      IF FPR0=P(X+1)<LLIM
00008C 4740 A0B8          000CA    64          BC    4,CONVER    BRANCH TO CONVER
000090 7C03 A2B6          002C8    65          STE   0,P(3)      OTHERWISE STORE FPR0 INTO P(I).
000094 7B40 AEAE          00EC0    66          SE    6,=E'1.0'   FPR6=N-X.
000098 8734 A068          0007A    67          BXLE  3,4,L1      CONTINUE
00009C 47F0 A0B8          000CA    68          B     CONVER
0000A0 7A40 AEAE          00EC0    69 L0       AE    4,=E'1.0'   THIS SECTION OF CODE
0000A4 3C04                        70          DER   0,4         COMPUTES SUCCESSIVE
0000A6 3C06                        71          MER   0,6         PROBABILITIES UNTIL A
0000A8 7C00 A2A2          002B4    72          DE    0,Q         PROBABILITY>=LLIM IS
0000AC 3C02                        73          MER   0,2         ATTAINED, THE PROPER
0000AE 1B54                        74          SR    5,4
0000B0 7B60 AEAE          00EC0    75          SE    6,=E'1.0'
0000B4 5AC0 AEB6          00EC8    76          A     12,=F'1'
0000B8 7903 A2B2          002C4    77          CE    0,LLIM      STARTING POINT IS
0000BC 4740 A08E          000A0    78          BC    4,L0        THEN STORED
0000C0 5000 A2AE          002C0    79          ST    12,XST      AND A BRANCH IS
0000C4 1B54                        80          SR    5,4
0000C6 47F0 A068          0007A    81          BC    15,L1       MADE TO L1 FOR FURTHER CALCULATIONS.
0000CA 1B53                        82 CONVER   LR    5,3
0000CC 1B54                        83          SR    5,4
0000CE 1B33                        84          SR    3,3         R3=I.
0000D0 7823 A2B6          002C8    85 L3       LE    2,P(3)      CONVERTING TO FIXED POINT.
0000D4 7F20 A60A          006EC    86          AU    2,ZERO
0000D8 7023 A2B6          002C8    87          STE   2,P(3)
0000DC 5893 A2B6          002C8    88          L     9,P(3)      R9=P(I).
0000E0 8990 0008          00008    89          SLL   9,8
0000E4 5093 A2B6          002C8    90          ST    9,P(3)      STORE CONVERTED P(I).
0000E8 8734 A08E          000D0    91          BXLE  3,4,L3      CONTINUE
0000EC 18CC                        92          SP    12,12
0000EE 1888                        93          SR    11,11
0000F0 1877                        94          SR    7,7
0000F2 1866                        95          SR    6,6
0000F4 1833                        96          SR    3,3         R3=I.
0000F6 1888                        97 L4       SR    8,8         CLEAR R8.
0000F8 5893 A2B6          002C8    98          L     9,P(3)      R9=P(I).
0000FC 8D80 0004          00004    99          SLDL  8,4         R8=D1.
000100 1A69                       100          AR    6,8         R6=SUM(D1).
000102 8D80 0004          00004   101          SLDL  8,4         R8=D1D2.
000106 1A78                       102          AR    7,8         R7=SUM(D1D2).
000108 8D80 0004          00004   103          SLDL  8,4         R8=D1D2D3.
00010C 1A88                       104          AR    11,8        R11=SUM(D1D2D3).
00010E 8D80 0004          00004   105          SLDL  8,4         R8=D1D2D3D4.
000112 1AC8                       106          AR    12,8        R12=SUM(D1D2D3D4).
000114 8734 A0E4          000F6   107          BXLE  3,4,L4      CONTINUE.
000118 5060 A6B6          006C8   108          ST    6,S         STORE SUM(D1) INTO S(0).
00011C 5070 A68A          006CC   109          ST    7,S+4       STORE SUM(D1D2) INTO S(1).
000120 5080 A68E          006D0   110          ST    11,S+8      STORE SUM(D1D2D3) INTO S(2)
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NOT REPRODUCIBLE

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                  F08APR70   6/30/71

000124 5000 A6C2          00604   111          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
000128 8980 0004          00004   112          SLL   11,4         R11=SUM(D1D2D3)*16.
00012C 8970 0004          00004   113          SLL   7,4          R7=SUM(D1D2)*16.
000130 8960 0004          00004   114          SLL   6,4          R6=SUM(D1)*16.
000134 1833                       115          SR    3,3          R3=I.
000136 1888                       116 L5       SR    8,8          CLEAR R8.
000138 5853 A2B6          002C8   117          L     9,P(3)       R9=P(I).
00013C 8080 0004          00004   118          SLDL  8,4          R8=D1.
000140 1869                       119          SR    6,8          R6=SUM(D1)*16-SUM(D1).
000142 1878                       120          SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
000144 1888                       121          SR    11,8         R11=SUM(D1D2D3)*16-SUM(D1).
000146 1888                       122          SR    8,8          CLEAR R8.
000148 5080 0004          00004   123          SLDL  8,4          R8=D2.
00014C 1878                       124          SR    7,8          R7=R7-SUM(D2).
00014E 1888                       125          SR    11,8         R11=R11-SUM(D2).
000150 1888                       126          SR    8,8          CLEAR R8.
000152 8080 0004          00004   127          SLDL  8,4          R8=D3
000156 1888                       128          SR    11,8         R11=R11-SUM(D3).
000158 8734 A124          00136   129          BXLE  3,4,L5       CONTINUE.
00015C 18CC                       130          SR    12,12
00015E 5060 A6C6          00608   131          ST    6,N          STORE R6 INTO N(0).
000162 5070 A6CA          0060C   132          ST    7,N+4        STORE R7 INTO N(1).
000166 5080 A6CE          006E0   133          ST    11,N+8       STORE R11 INTO N(2).
00016A 1877                       134          SR    7,7
00016C 1800                       135          SR    0,0
00016E 4160 0001          00001   136          LA    6,1
000172 1833                       137 L6       SR    3,3          R3=I.
000174 5820 A2AE          002C0   138          L     2,XST        LOAD R2 WITH STARTING VALUE.
000178 1888                       139 L7       SR    8,8          CLEAR R8.
00017A 5893 A2B6          002C8   140          L     9,P(3)       R9=P(I).
00017E 8080 0004          00004   141          SLDL  8,4          R8=D1,R8=D2,R8=D3,R8=D4.
000182 5093 A2B6          002C8   142          ST    9,P(3)       RESTORE SHIFTED P(I).
000186 5980 AF8A          00ECC   143          C     8,=F'0'
00018A 4780 A192          001A4   144          BC    8,L8         BRANCH TO L8.
00018E 1A78                       145          AR    7,8          OTHERWISE STORE R2(DISCRETE R.V.)
000190 5970 AF8E          00ED0   146          C     7,=F'2000'   CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
000194 4720 A1A0          001B2   147          BC    2,L9
000198 1876                       148          SR    7,6          THE NUMBER OF TIMES CORRESPONDING
00019A 422C A6DF          006FC   149          STC   2,A(12)      TO THE DIGIT IN R8.
00019E 8706 A188          0019A   150          BXLE  12,6,*-4
0001A2 1A76                       151          AR    7,6
0001A4 1A26                       152 L8       AR    2,6          INCREMENT DISCRETE R.V.
0001A6 8734 A166          00178   153          BXLE  3,4,L7       CONTINUE.
0001A8 1A06                       154          AR    0,6          CONTINUE L6 LOOP
0001AC 1904                       155          CR    0,4          UNTIL FIRST 4 DIGITS OF
0001AE 4740 A160          00172   156          BC    4,L6         EACH P(I) HAVE BEEN SHIFTED.
0001B2 5830 A6C2          00604   157 L9       L     3,S+12
0001B6 5830 A6CE          006E0   158          S     3,N+8        THIS SECTION OF CODE
0001BA 193C                       159          CR    3,12         RESETS N(3)
0001BC 4780 A188          001CA   160          BC    8,OUT        IF TABLE LENGTH IS EXCEEDED.
0001C0 183C                       161          SR    3,12
0001C2 5A30 A6CE          006E0   162          A     3,N+8
0001C6 5030 A6CE          006E0   163          ST    3,N+8
0001CA                            164 OUT      DS    0H
0001CA 5800 A25E          00270   165          L     13,AREA+4
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NOT REPRODUCIBLE

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F38APR70   6/30/71
0001CE 98EC D00C            0000C  166          LM    14,12,12(13)
0001D2 92FF D00C      C000C        167          MVI   12(13),X'FF'
0001D6 07FE                        168          BCR   15,14        BRANCH TO CALLING ROUTINE.
                                   169          DROP  10
                                   170          ENTRY RANBI
0001E0                             171          USING RANBI,15
0001D8 07                          172          DC    XL1'07'
0001D9 404009C105C2C9              173          DC    CL7'  RANBI'
0001E0 9013 D018            00018  174 RANBI    STM   1,3,24(13)
0001E4 3800                        175          SER   0,0
0001E6 47F0 F070            00250  176 BRANCH   B     TEMP         BRANCH MADE ON FIRST CALL ONLY.
0001EA 5C20 F504            006E4  177          M     2,BPAT       GENERATE NEW U.
0001EE 5030 F508            006E8  178          ST    3,LA         STORE FOR NEXT CALL.
0001F2 1B22                        179          SR    2,2
0001F4 1B00                        180          SR    0,0
0001F6 8D20 0004            00004  181          SLDL  2,4          R2=D1.
0001FA 5920 F4E8            006C8  182          C     2,S          IF D1<S THEN
0001FE 4740 F05A            0023A  183          BC    4,EX         BRANCH TO EX
000202 8D20 0004            00004  184 N1       SLDL  2,4          R2=D1D2.
000206 5920 F4EC            006CC  185          C     2,S+4        IF D1D2>=S(1) THEN
00020A 4780 F036            00216  186          BC    11,N2        BRANCH TO N2
00020E 5820 F4E8            006D8  187          S     2,N          OTHERWISE OBTAIN PROPER TABLE ADDRESS
000212 47F0 F05A            0023A  188          B     EX           BRANCH TO EX.
000216 8D20 0004            00004  189 N2       SLDL  2,4          R2=D1D2D3.
00021A 5920 F4F0            006D0  190          C     2,S+8        IF D1D2D3>=S(2) THEN
00021E 4780 F04A            0022A  191          BC    11,N3        BRANCH TO N3
000222 5820 F4EC            006DC  192          S     2,N+4        OBTAIN PROPER TABLE ADDRESS
000226 47F0 F05A            0023A  193          B     EX           BRANCH TO EX.
00022A 8D20 0004            00004  194 N3       SLDL  2,4          R2=D1D2D3D4.
00022E 5920 F4F4            006D4  195          C     2,S+12       IF D1D2D3D4>=S(3) THEN
000232 4780 F006            001E6  196          BC    11,BRANCH    GO TO BRANCH(OBTAIN NEW U).
000236 5820 F500            006F0  197          S     2,N+8        OBTAIN PROPER TABLE ADDRESS.
00023A 4302 F510            006F0  198 EX       IC    0,A(2)       OBTAIN PROPER DISCRETE VALUE.
00023E 5000 F0DC            002BC  199          ST    0,RES        CONVERT TO FLOATING POINT.
000242 9246 F0DC      C02BC        200          MVI   RES,X'46'
000246 7A00 F0DC            002BC  201          AE    0,RES        NORMALIZED FLT. PT. RESULT IN FPRO.
00024A 9813 D018            00018  202          LM    1,3,24(13)
00024F 07FE                        203          BCR   15,14        RETURN.
000250 5830 F508            006E8  204 TEMP     L     3,LA         REPLACES THE INSTRUCTION AT BRANCH.
000254 0203 F006 F070 C01E6 00250  205          MVC   BRANCH,TEMP  REPLACE INSTRUCTION AT BRANCH.
00025A 5830 1000            00000  206          L     3,0(,1)      LOAD PRIMER(ONLY DONE ON FIRST CALL).
00025E 5830 3000            00000  207          L     3,0(,3)
000262 5030 F508            006E8  208          ST    3,LA
000266 47F0 F00A            001EA  209          B     BRANCH+4     GO TO INSTRUCTION FOLLOWING BRANCH.
00026C                             210 AREA     DS    18F
0002B4                             211 O        DS    1F
0002B8                             212 NUM      DS    1F
0002BC                             213 RES      DS    1F
0002C0                             214 XST      DS    1F
0002C4 3CFBA382                    215 LLIM     DC    E'0.0E0015'
0002C8                             216 P        DS    256F
0006C8                             217 S        DS    4F
0006D8                             218 N        DS    3F
0006F4 48C27395                    219 BPAT     DC    XL4'48C27395'
0006E8                             220 LA       DS    1F
```

NOT REPRODUCIBLE

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0006EC 40000000                  221 ZERO     DC    X'40000000'
0006F0                           222 A        DS    2000XL1
                                 223          END
000FCC 41100000                  224               =E'1.0'
000FC4 FFFFFFFF                  225               =F'-1'
000FC8 00000001                  226               =F'1'
000FCC 00000000                  227               =F'0'
000FD0 000007D0                  228               =F'2000'
```

NOT REPRODUCIBLE

APPENDIX F

Assembler Listing of GEN5

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F08APR70   6/30/71
000000                                  GENS     START 0
                                                 PRINT ON,NOGEN,NODATA
                                        ***********************************************************************
                                        * SOURCE---L.E. CANNON,UGA,DEPT. OF STAT., JUN 1970                  *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING   *
                                        *          POISSON DISTRIBUTED RANDOM NUMBERS.                       *
                                        * USAGE----1. CALL PSETUP(LAM). LAM IS THE MEAN OF THE POISSON DIS-  *
                                        *          TRIBUTION TO BE GENERATED. PSETUP SETS UP THE TABLE FROM  *
                                        *          RANPOI GENERATES A RANDOM NUMBER. PSETUP NEED BE CALLED   *
                                        *          ONLY ONCE FOR A GIVEN LAM.                                *
                                        *          2. X=RANPOI(IOD). 'IOD' MUST BE AN ODD INTEGER, IT PRIMES *
                                        *          THE GENERATING SCHEME. X REPRESENTS THE POISSON VARIABLE  *
                                        *          DO 1 I=1,100                                              *
                                        *          X=RANPOI(56951)                                           *
                                        *                ...                                                 *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                        *          6',1(JAN. 1963),37.                                       *
                                        ***********************************************************************
                                                 ENTRY PSETUP
000000 47F0 F00C          0000C          PSETUP   B     12(0,15)
000004 07                                        DC    XL1'07'
000005 40D7E2C5E3E4D7                            DC    CL7' PSETUP'
00000C 90EC D00C          0000C                  STM   14,12,12(13)
000010 05A0                                      BALR  10,0
000012                                           USING *,10
000012 18E0                                      LR    14,13
000014 4100 A232          00244                  LA    13,AREA
000018 5000 E008          00008                  ST    13,8(0,14)
00001C 50E0 D004          00004                  ST    14,4(0,13)
000020 5820 1000          00000                  L     2,0(,1)       LOAD ADDRESS OF LAM.
000024 7820 2000          00000                  LE    2,0(,2)       LOAD LAM INTO FPR2.
000028 3122                                      LNER  2,2           FPR2=-LAM.
00002A 7020 A27A          0028C                  STE   2,LAM
                                                 CALL EXP,(LAM)      COMPUTE EXP(-LAM).
000046 3022                                      LPER  2,2           FPR2=+LAM
000048 1855                                      SR    5,5           INDEX.
00004A 4150 0004          00004                  LA    6,4           INCREMENT.
00004E 4170 03FC          003FC                  LA    7,1020        DELIMITER.
000052 58C0 AE86          00E9B                  L     12,=E'1'
000056 3844                                      SER   4,4
000058 7900 A286          00298                  CE    0,LLIM        IF EXP(-LAM)<LLIM.
00005C 4740 A074          00086                  BC    4,LO          BRANCH TO LO.
000060 7005 A28A          0029C                  STE   0,P(5)        OTHERWISE STORE INTO P(0).
000064 5050 A282          00294                  ST    5,XST         STORE STARTING POINT.
000068 1A56                                      AR    5,6
00006A 7A40 AE8A          00E9C          L2      AE    4,=E'1'     FPR4=X.
00006E 3C02                                      MER   0,2           FPR0=EXP(-LAM)*LAM.
000070 3D04                                      DER   0,4           FPR0=EXP(-LAM)*LAM/X.
000072 7900 A286          00298                  CE    0,LLIM        IF FPR0<LLIM.
000076 4740 A090          000A2                  BC    4,CONVER      BRANCH TO CONVER.
00007A 7005 A28A          0029C                  STE   0,P(5)        OTHERWISE STORE FPR0 INTO P(1).
00007E 8756 A058          0006A                  BXLE  5,6,L2        CONTINUE
000082 47F0 A090          000A2                  B     CONVER
000086 7A40 AE8A          00E9C          LO      AE    4,=E'1'       THIS SECTION OF CODE
00008A 3C02                                      MER   0,2           COMPUTES SUCCESSIVE PROBILITIES
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    FOBAPRTO   6/30/71
00008C 3004                        67          DER   0,4          USING THE RECURSIVE RELATION UNTIL
00008E 5AC0 AE86          00E98    68          A     12,=F'1'     A PROBILITY>=LLIM IS ATTAINED.
000092 7900 A286          0029B    69          CE    0,LLIM       THE PROPER STARTING POINT IS STORED.
000096 4740 A074          00086    70          BC    4,L0         AND A BRANCH IS
00009A 50C0 A282          00294    71          ST    12,XST       MADE TO L2 FOR SUB-
00009E 47F0 A058          0006A    72          BC    15,L2        SEQUENT CALCULATIONS.
0000A2 1856                        73 CONVER   SR    5,6
0000A4 4140 0004          00004    74          LA    4,4
0000A8 1833                        75          SR    3,3          R3=I.
0000AA 7823 A28A          0029C    76 L3       LE    2,P(3)       CONVERTING TO FIXED POINT.
0000AE 7E20 A6AE          006C0    77          AU    2,ZERO
0000B2 7023 A28A          0029C    78          STE   2,P(3)
0000B6 5893 A28A          0029C    79          L     9,P(3)       R9=P(I).
0000BA 8990 0008          00008    80          SLL   9,8
0000BE 5093 A28A          0029C    81          ST    9,P(3)       STORE CONVERTED P(I).
0000C2 8734 A058          000AA    82          BXLE  3,4,L3       CONTINUE
0000C6 18CC                        83          SR    12,12
0000C8 1888                        84          SR    11,11
0000CA 1877                        85          SR    7,7
0000CC 1866                        86          SR    6,6
0000CE 1833                        87          SR    3,3          R3=I.
0000D0 1888                        88 L4       SR    8,8          CLEAR R8.
0000D2 5893 A28A          0029C    89          L     9,P(3)       R9=P(I).
0000D6 8D80 0004          00004    90          SLDL  8,4          R8=D1.
0000DA 1A68                        91          AR    6,8          R6=SUM(D1).
0000DC 8D80 0004          00004    92          SLDL  8,4          R8=D1D2.
0000E0 1A78                        93          AR    7,8          R7=SUM(D1D2).
0000E2 8D80 0004          00004    94          SLDL  8,4          R8=D1D2D3.
0000E6 1AB8                        95          AR    11,8         R11=SUM(D1D2D3).
0000E8 8D80 0004          00004    96          SLDL  8,4          R8=D1D2D3D4.
0000EC 1AC8                        97          AR    12,8         R12=SUM(D1D2D3D4).
0000EE 8734 A0BE          00000    98          BXLE  3,4,L4       CONTINUE.
0000F2 5060 A68A          0069C    99          ST    6,S          STORE SUM(D1) INTO S(0).
0000F6 5070 A68E          006A0   100          ST    7,S+4        STORE SUM(D1D2) INTO S(1).
0000FA 5080 A692          006A4   101          ST    11,S+8       STORE SUM(D1D2D3) INTO S(2)
0000FE 50C0 A696          006A8   102          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
000102 8990 0004          00004   103          SLL   11,4         R11=SUM(D1D2D3)*16.
000106 8970 0004          00004   104          SLL   7,4          R7=SUM(D1D2)*16.
00010A 8960 0004          00004   105          SLL   6,4          R6=SUM(D1)*16.
00010E 1833                       106          SR    3,3          R3=I.
000110 1888                       107 L5       SR    8,8          CLEAR R8.
000112 5893 A28A          0029C   108          L     9,P(3)       R9=P(I).
000116 8D80 0004          00004   109          SLDL  8,4          R8=D1.
00011A 1868                       110          SR    6,8          R6=SUM(D1)*16-SUM(D1).
00011C 1878                       111          SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
00011E 1888                       112          SR    11,8         R11=SUM(D1D2D3)*16-SUM(D1).
000120 1888                       113          SR    8,8          CLEAR R8.
000122 8D80 0004          00004   114          SLDL  8,4          R8=D2.
000126 1878                       115          SR    7,8          R7=R7-SUM(D2).
000128 1888                       116          SR    11,8         R11=R11-SUM(D2).
00012A 1888                       117          SR    8,8          CLEAR R8.
00012C 8D80 0004          00004   118          SLDL  8,4          R8=D3
000130 1888                       119          SR    11,8         R11=R11-SUM(D3).
000132 8734 A0FE          00110   120          BXLE  3,4,L5       CONTINUE.
000136 18CC                       121          SR    12,12
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71

000138 5060 A69A          006AC   122          ST    6,N          STORE R6 INTO N(0).
00013C 5070 A69F          006B0   123          ST    7,N+4        STORE R7 INTO N(1).
000140 5080 A6A2          006B4   124          ST    11,N+8       STORE R11 INTO N(2).
000144 1877                       125          SR    7,7
000146 1800                       126          SR    0,0
000148 4160 0001          00001   127          LA    6,1
00014C 1833                       128 L6       SR    3,3          R3=I.
00014E 5820 A282          00294   129          L     2,XST        LOAD R2 WITH STARTING VALUE.
000152 1888                       130 L7       SR    8,8          CLEAR R8.
000154 5953 A28A          0029C   131          L     9,P(3)       R9=P(I).
000158 8083 0004          00004   132          SLDL  8,4          R8=D1,R8=D2,R8=D3,R8=D4.
00015C 5093 A28A          0029C   133          ST    9,P(3)       RESTORE SHIFTED P(I).
000160 5980 A6BE          00EA0   134          C     8,=F'0'      IF R8=0.
000164 4780 A16C          0017E   135          BC    8,L8         BRANCH TO L8.
000168 1A79                       136          AR    7,8          OTHERWISE STORE R2(DISCRETE R.V.)
00016A 5970 AE92          00EA4   137          C     7,=F'2000'   CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
00016E 4720 A17A          0018C   138          BC    2,L9
000172 1876                       139          SR    7,6          THE NUMBER OF TIMES CORRESPONDING
000174 422C A6B2          006C4   140          STC   2,A(12)      TO THE DIGIT IN R8.
000178 87C6 A162          00174   141          BXLE  12,6,*-4
00017C 1A76                       142          AR    7,6
00017E 1A26                       143 L8       AR    2,6          INCREMENT DISCRETE R.V.
000180 8734 A140          00152   144          BXLE  3,4,L7       CONTINUE.
000184 1A06                       145          AR    0,6          CONTINUE L6 LOOP
000186 1904                       146          CR    0,4          UNTIL FIRST 4 DIGITS OF
000188 4740 A13A          0014C   147          BC    4,L6         EACH P(I) HAVE BEEN SHIFTED.
00018C 5830 A696          006A8   148 L9       L     3,S+12
000190 5830 A6A2          006B4   149          S     3,N+8        THIS SECTION OF CODE
000194 193C                       150          CR    3,12         RESETS N(3)
000196 4780 A192          001A4   151          BC    8,OUT        IF TABLE LENGTH IS EXCEEDED.
00019A 183C                       152          SR    3,12
00019C 5A30 A6A2          006B4   153          A     3,N+8
0001A0 5030 A6A2          006B4   154          ST    3,N+8
0001A4                            155 OUT      DS    0H
0001A4 5800 A236          00248   156          L     13,AREA+4
0001A8 98EC D00C          0000C   157          LM    14,12,12(13)
0001AC 92FF D00C     C000C        158          MVI   12(13),X'FF'
0001B0 07FE                       159          BCR   15,14        BRANCH TO CALLING ROUTINE.
                                  160          DROP  10
                                  161          ENTRY RANPOI
0001B4                            162          USING RANPOI,15
0001B7 07                         163          DC    XL1'07'
0001B8 40D9C1D5D7D6C9             164          DC    CL7' RANPOI'
0001BA 9013 D018          00018   165 RANPOI   STM   1,3,24(13)
0001BE 3800                       166          SER   0,0
0001C0 47F0 F070          0022A   167 BRANCH   B     TEMP         BRANCH MADE ON FIRST CALL ONLY.
0001C4 5C20 F4FF          006B8   168          M     2,BPAT       GENERATE NEW U.
0001C8 5030 F502          006BC   169          ST    3,LA         STORE FOR NEXT CALL.
0001CC 1822                       170          SR    2,2
0001CE 1800                       171          SR    0,0
0001D0 8020 0004          00004   172          SLDL  2,4          P2=D1.
0001D4 5920 F4E2          0069C   173          C     2,S          IF D1<S THEN
0001D8 4740 F05A          00214   174          BC    4,EX         BRANCH TO EX
0001DC 8C20 0004          00004   175 N1       SLDL  2,4          P2=D1D2.
0001E0 5920 F4E6          006A0   176          C     2,S+4        IF D1D2>=S(1) THEN
```

```
LOC  OBJECT CODE     ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F08APR70   6/30/71

0001E4 47B0 F036            001F0   177          BC    11,N2          BRANCH TO N2
0001E8 5B20 F4F2            006AC   178          S     2,N            OTHERWISE OBTAIN PROPER TABLE ADDRESS
0001EC 47F0 F05A            00214   179          B     EX             BRANCH TO EX.
0001F0 8D20 0004            00004   180 N2       SLDL  2,4            R2=D1D2D3.
0001F4 5920 F4EA            006A4   181          C     2,S+8          IF D1D2D3>=S(2) THEN
0001F8 47B0 F04A            00204   182          BC    11,N3          BRANCH TO N3
0001FC 5B20 F4F6            006B0   183          S     2,N+4          OBTAIN PROPER TABLE ADDRESS
000200 47F0 F05A            00214   184          B     EX             BRANCH TO EX.
000204 8D20 0004            00004   185 N3       SLDL  2,4            R2=D1D2D3D4.
000208 5920 F4EE            006A8   186          C     2,S+12         IF D1D2D3D4>=S(3) THEN
00020C 47B0 F006            001C0   187          BC    11,BRANCH      GO TO BRANCH(OBTAIN NEW U).
000210 5B20 F4FA            006B4   188          S     2,N+8          OBTAIN PROPER TABLE ADDRESS.
000214 4302 F50A            006C4   189 EX       IC    0,A(2)         OBTAIN PROPER DISCRETE VALUE.
000218 5000 F0D6            00290   190          ST    0,RES          CONVERT TO FLOATING POINT.
00021C 9246 F0D6      00290         191          MVI   RES,X'46'
000220 7A00 F0D6            00290   192          AE    0,RES          NORMALIZED FLT. PT. RESULT IN FPR0.
000224 9813 0018            00018   193          LM    1,3,24(13)
000228 07FE                         194          BCR   15,14          RETURN.
00022A 5830 F502            006BC   195 TEMP     L     3,LA           REPLACES THE INSTRUCTION AT BRANCH.
00022E D203 F006 F070 001C0 0022A   196          MVC   BRANCH,TEMP    REPLACE INSTRUCTION AT BRANCH.
000234 5830 1000            00000   197          L     3,0(,1)        LOAD PRIMER(ONLY DONE ON FIRST CALL).
000238 5830 3000            00000   198          L     3,0(,3)
00023C 5030 F502            006BC   199          ST    3,LA
000240 47F0 F00A            001C4   200          B     BRANCH+4       GO TO INSTRUCTION FOLLOWING BRANCH.
000244                              201 AREA     DS    18F
00028C                              202 LAM      DS    1F
000290                              203 RES      DS    1F
000294                              204 XST      DS    1F
000298 3CFBA882                     205 LLIM     DC    E'0.000015'
00029C                              206 P        DS    256F
00069C                              207 S        DS    4F
0006AC                              208 N        DS    3F
0006B8 48C27395                     209 BPAT     DC    XL4'48C27395'
0006BC                              210 LA       DS    1F
0006C0 40000000                     211 ZERO     DC    X'40000000'
0006C4                              212 A        DS    2000XL1
                                    213          END
000F98 00000001                     214                =F'1'
000F9C 41100000                     215                =E'1'
000FA0 00000000                     216                =F'0'
000FA4 00000700                     217                =F'2000'
```

APPENDIX G

Assembler Listing of GEN6

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000000                                  GEN6     START 0
                                        ************************************************************************
                                        * SOURCE---L.F. CANNON,UGA,DEPT. OF STAT., JUN 1970                    *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING NE-*
                                        *          GATIVE BINOMIAL DISTRIBUTED RANDOM NUMBERS.                *
                                        * USAGE----1. CALL NBSETU(P,IR). P=PROBABILITY OF A SUCCESS, IR=NUM-  *
                                        *          BER OF SUCCESSES. NBSETU SETS UP THE TABLE FROM WHICH RANE-*
                                        *          BI GENERATES A RANDOM NUMBER. NBSETU NEED BE CALLED ONLY   *
                                        *          ONCE FOR A GIVEN P AND IR.                                 *
                                        *          2. X=RANEBI(IOD). 'IOD' MUST BE AN ODD INTEGER, IT PRIMES  *
                                        *          THE GENERATING SCHEME. X REPRESENTS THE NUMBER OF FAILURES *
                                        *          BEFORE THE IR'TH SUCCESS.                                  *
                                        * EXAMPLE--CALL NBSETU(0.5,10)                                        *
                                        *          DO 1 I=1,100                                               *
                                        *          X=RANEBI(55695)                                            *
                                        *             ...                                                     *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                        *          6',1(JAN. 1963),37.                                        *
                                        ************************************************************************
                                                 ENTRY NBSETU
000000 47F0 F00C            0000C       NBSETU   B     12(0,15)
000004 07                                        DC    XL1'07'
000005 4005C7C2C5E3F4                            DC    CL7' NBSETU'
000000 90EC D00C            000CC                STM   14,12,12(13)
000010 05A0                                      BALR  10,0
000012                                           USING *,10
000012 18ED                                      LR    14,13
000014 4100 A24F            00260                LA    13,AREA
000018 5000 E008            00008                ST    13,8(0,14)
00001C 50E0 0004            00004                ST    14,4(0,13)
000020 5840 1000            00000                L     4,0(,1)       LOAD ADDRESS OF P.
000024 7840 4000            00000                LE    4,0(,4)       LOAD P INTO FPR4.
000028 7820 AE9E            00EB0                LE    2,=E'1'
00002C 3B24                                      SER   2,4           CALCULATE Q=1-P.
00002E 5860 1004            00004                L     6,4(,1)       LOAD ADDRESS OF R.
000032 5860 6000            00000                L     6,0(,6)       LOAD R.
000036 5050 A29A            002AC                ST    6,SUC         CONVERT R TO FLOATING POINT.
00003A 9246 A29A      002AC                      MVI   SUC,X'46'
00003E 7800 AE9E            00EB0                LE    0,=E'1'
000042 3B66                                      SER   6,6
000044 7A60 A29A            002AC                AE    6,SUC         FPR6=R (NORMALIZED FLT. PT.).
000048 4780 A046            00058                BC    8,SKIP        IF R=0 BRANCH TO SKIP
00004C 5870 AEA2            00EB4                L     7,=F'-1'
000050 1A67                                      AR    6,7
000052 3C04                                      MER   0,4           CALCULATE P**R.
000054 E667 A040            00052                BXH   6,7,*-2
000058 1B55                                 SKIP     SR    5,5
00005A 4160 0004            00004                LA    6,4
00005E 4170 03FC            003FC                LA    7,1020
000062 58C0 AEA6            00EB8                L     12,=F'1'
000066 3B44                                      SER   4,4
000068 7900 A2A2            002B4                CE    0,LLIM        IF P**R<LLIM.
00006C 4740 A08A            0009C                BC    4,LO          BRANCH TO LO.
000070 7005 A2A6            002B8                STE   0,P(5)        OTHERWISE STORE P**R INTO P(0).
000074 505D A29E            002B0                ST    5,XST         STORE R5=0 INTO XST.
```

```
LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
000079 1A56                        56          AR    5,6
00007A 7A40 AE9E          00EB0    57 L2      AE    4,=E'1'      FPR4=X+1.
00007E 3C06                        58          MER   0,6          FPR0=P(X)*(X+R).
000080 3D04                        59          DER   0,4          FPR0=P(X)*(X+R)/(X+1).
000082 3C02                        60          MER   0,2          FPR0=P(X+1)=P(X)*(X+R)*Q/(X+1).
000084 7900 A2A2          00284    61          CE    0,LLIM       IF P(X+1)<LLIM.
000088 4740 A0AC          0008E    62          BC    4,CONVER     BRANCH TO CONVER.
00008C 7005 A2A6          00288    63          STE   0,P(5)       OTHERWISE STORE FPR0=P(X+1) INTO P(R5).
000090 7A60 AE9E          00EB0    64          AE    6,=F'1'      INCREMENT X+R.
000094 8756 A06B          0007A    65          BXLE  5,6,L2       CONTINUE.
000098 47F0 A0AC          0008E    66          B     CONVER
00009C 7A40 AE9E          00EB0    67 L0      AE    4,=E'1'      THIS SECTION OF CODE
0000A0 3C06                        68          MER   0,6          CALCULATES SUCCESSIVE PROBA-
0000A2 3D04                        69          DER   0,4          BILITIES, USING THE RECURSIVE
0000A4 3C02                        70          MER   0,2          RELATION. UNTIL A PROBABILITY>=LLIM
0000A6 5A00 AEA6          00EB8    71          A     12,=F'1'     IS OBTAINED.
0000AA 7A60 AE9F          00EB0    72          AE    6,=F'1'      THE CORRESPONDING X-VALUE
0000AE 7900 A2A2          00284    73          CE    0,LLIM       IS STORED INTO XST
0000B2 4740 A08A          0009C    74          BC    4,L0         AND A BRANCH
0000B6 5000 A29E          00280    75          ST    12,XST       IS MADE TO L2
0000BA 47F0 A06B          0007A    76          BC    15,L2        FOR FUTHER CALCULATIONS.
0000BE 1856                        77 CONVER  SR    5,6
0000C0 4140 0004          00004    78          LA    4,4
0000C4 1833                        79          SR    3,3          R3=I.
0000C6 7823 A2A6          00288    80 L3      LE    2,P(3)       CONVERTING TO FIXED POINT.
0000CA 7E20 A6CA          006DC    81          AU    2,ZERO
0000CE 7023 A2A6          00288    82          STE   2,P(3)
000002 5893 A2A6          00288    83          L     9,P(3)
000006 8590 0008          00008    84          SLL   9,8
0000CA 5093 A2A6          00288    85          ST    9,P(3)
0000CE 8734 A084          000C6    86          BXLE  3,4,L3       CONTINUE
0000E2 18CC                        87          SR    12,12
0000E4 1888                        88          SR    11,11
0000E6 1877                        89          SR    7,7
0000E8 1866                        90          SR    6,6
0000EA 1833                        91          SR    3,3          R3=I.
0000EC 1888                        92 L4      SR    8,8          CLEAR R8.
0000EE 5893 A2A6          00288    93          L     9,P(3)       R9=P(I).
0000F2 8080 0004          00004    94          SLDL  8,4          R8=D1.
0000F6 1A69                        95          AR    6,8          R6=SUM(D1).
0000F8 8080 0004          00004    96          SLDL  8,4          R8=D1D2.
0000FC 1A78                        97          AR    7,8          R7=SUM(D1D2).
0000FE 8080 0004          00004    98          SLDL  8,4          R8=D1D2D3.
000102 1A88                        99          AR    11,8         R11=SUM(D1D2D3).
000104 8080 0004          00004   100          SLDL  8,4          R8=D1D2D3D4.
000108 1AC8                       101          AR    12,8         R12=SUM(D1D2D3D4).
00010A 8734 A0DA          000EC   102          BXLE  3,4,L4       CONTINUE.
00010E 5060 A6A6          006B8   103          ST    6,S          STORE SUM(D1) INTO S(0).
000112 5070 A6AA          006BC   104          ST    7,S+4        STORE SUM(D1D2) INTO S(1).
000116 5080 A6AE          006C0   105          ST    11,S+8       STORE SUM(D1D2D3) INTO S(2)
00011A 5000 A6B2          006C4   106          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
00011E 8980 0004          00004   107          SLL   11,4         R11=SUM(D1D2D3)*16.
000122 8970 0004          00004   108          SLL   7,4          R7=SUM(D1D2)*16.
000126 8960 0004          00004   109          SLL   6,4          R6=SUM(D1)*16.
00012A 1833                       110          SR    3,3          R3=I.
```

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    FORAPR70   6/30/71
00012C 1889                          111 L5      SR    8,8          CLEAR R8.
00012E 5893 A2A6          00288      112         L     9,P(3)       R9=P(I).
000132 8D80 0004          00004      113         SLDL  8,4          R8=D1.
000136 1869                          114         SR    6,8          R6=SUM(D1)*16-SUM(D1).
000138 1878                          115         SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
00013A 1889                          116         SR    11,8         R11=SUM(D1D2D3)*16-SUM(D1).
00013C 1889                          117         SR    8,8          CLEAR R8.
00013E 8D80 0004          00004      118         SLDL  8,4          R8=D2.
000142 1878                          119         SR    7,8          R7=R7-SUM(D2).
000144 1888                          120         SR    11,8         R11=R11-SUM(D2).
000146 1883                          121         SR    8,8          CLEAR R8.
000148 8D80 0004          00004      122         SLDL  8,4          R8=D3
00014C 1888                          123         SR    11,8         R11=R11-SUM(D3).
00014E 8734 A11A          0012C      124         BXLE  3,4,L5       CONTINUE.
000152 18CC                          125         SR    12,12
000154 5060 A6B6          006C8      126         ST    6,N          STORE R6 INTO N(0).
000158 5070 A6BA          006CC      127         ST    7,N+4        STORE R7 INTO N(1).
00015C 5080 A6BE          006D0      128         ST    11,N+8       STORE R11 INTO N(2).
000160 1877                          129         SR    7,7
000162 1800                          130         SR    0,0
000164 4160 0001          00001      131         LA    6,1
000168 1833                          132 L6      SR    3,3          R3=I.
00016A 5820 A29E          00280      133         L     2,XST        LOAD R2 WITH STARTING VALUE.
00016E 1889                          134 L7      SR    8,8          CLEAR R8.
000170 5893 A2A6          00288      135         L     9,P(3)       R9=P(I).
000174 8D80 0004          00004      136         SLDL  8,4          R8=D1,R8=D2,R8=D3,R8=D4.
000178 5093 A2A6          00288      137         ST    9,P(3)       RESTORE SHIFTED P(I).
00017C 5540 AEAA          00EBC      138         C     8,=F'0'      IF R8=0.
000180 4780 A188          0019A      139         BC    8,L8         BRANCH TO L8.
000184 1A78                          140         AR    7,8          OTHERWISE STORE R2(DISCRETE R.V.)
000186 5970 AFAE          00FC0      141         C     7,=F'2000'   CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
00018A 4720 A196          001A8      142         BC    2,L9
00018E 1876                          143         SR    7,6          THE NUMBER OF TIMES CORRESPONDING
000190 422C A6CE          006FC      144         STC   2,A(12)      TO THE DIGIT IN R8.
000194 87C6 A17E          00190      145         BXLE  12,6,=-4
000198 1A76                          146         AR    7,6
00019A 1A26                          147 L6      AR    2,6          INCREMENT DISCRETE R.V.
00019C 8734 A15C          0016E      148         BXLE  3,4,L7       CONTINUE.
0001A0 1A06                          149         AR    0,6          CONTINUE L6 LOOP
0001A2 1904                          150         CR    0,4          UNTIL FIRST 4 DIGITS OF
0001A4 4740 A156          00168      151         BC    4,L6         EACH P(I) HAVE BEEN SHIFTED.
0001A8 5830 A682          006C4      152 L9      L     3,S+12
0001AC 5830 A68E          006D0      153         S     3,N+8        THIS SECTION OF CODE
0001B0 193C                          154         CR    3,12         RESETS N(3)
0001B2 4780 A1AE          001C0      155         BC    8,OUT        IF TABLE LENGTH IS EXCEEDED.
0001B6 183C                          156         SR    3,12
0001B8 5A30 A6BE          006D0      157         A     3,N+8
0001BC 5030 A6BE          006D0      158         ST    3,N+8
0001C0                               159 OUT     DS    0H
0001C0 58D0 A252          00264      160         L     13,AREA+4
0001C4 98EC D00C          0000C      161         LM    14,12,12(13)
0001C8 92FF D00C     0000C           162         MVI   12(13),X'FF'
0001CC 07FE                          163         BCR   15,14        BRANCH TO CALLING ROUTINE.
                                     164         DROP  10
                                     165         ENTRY RANEBI
```

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 LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                    F08APR70   6/30/71
0001D6                              166          USING RANEBI,15
0001CE 07                           167          DC    XL1'07'
0001CF 40D9C1D5C5C2C9               168          DC    CL7' RANEBI'
0001D5 9013 D018            00018   169 RANEBI   STM   1,3,24(13)
0001DA 3800                         170          SER   0,0
0001DC 47F0 F070            00246   171 BRANCH   B     TEMP          BRANCH MADE ON FIRST CALL ONLY.
0001E0 5C20 F4FE            00604   172          M     2,BPAT        GENERATE NEW U.
0001E4 5C30 F507            00608   173          ST    3,LA          STORE FOR NEXT CALL.
0001E8 1822                         174          SR    2,2
0001EA 1800                         175          SR    0,0
0001EC 8020 0004            00004   176          SLDL  2,4           R2=D1.
0001F0 5920 F4E2            00588   177          C     2,S           IF D1<S THEN
0001F4 4740 F05A            00230   178          BC    4,EX          BRANCH TO EX
0001F8 8020 0004            00004   179 N1       SLDL  2,4           R2=D1D2.
0001FC 5920 F4F6            00688   180          C     2,S+4         IF D1D2>=S(1) THEN
000200 4780 F036            00200   181          BC    11,N2         BRANCH TO N2
000204 5920 F4E2            00588   182          S     2,N           OTHERWISE OBTAIN PROPER TABLE ADDRESS
000208 47F0 F05A            00230   183          B     EX            BRANCH TO EX.
00020C 8020 0004            00004   184 N2       SLDL  2,4           R2=D1D2D3.
000210 5920 F4FA            00600   185          C     2,S+8         IF D1D2D3>=S(2) THEN
000214 4780 F04A            00220   186          BC    11,N3         BRANCH TO N3
000218 5920 F4F6            00688   187          S     2,N+4         OBTAIN PROPER TABLE ADDRESS
00021C 47F0 F05A            00230   188          B     EX            BRANCH TO EX.
000220 8020 0004            00004   189 N3       SLDL  2,4           R2=D1D2D3D4.
000224 5920 F4FF            00604   190          C     2,S+12        IF D1D2D3D4>=S(3) THEN
000228 4780 F006            001DC   191          BC    11,BRANCH     GO TO BRANCH(OBTAIN NEW U).
00022C 5820 F4FA            00600   192          S     2,N+8         OBTAIN PROPER TABLE ADDRESS.
000230 4300 F50A            00608   193 EX       IC    0,4(2)        OBTAIN PROPER DISCRETE VALUE.
000234 5000 F002            00248   194          ST    0,RES         CONVERT TO FLOATING POINT.
000238 9246 F002      C02AB         195          MVI   RES,X'46'
00023C 7A00 F002            00248   196          AE    0,RES         NORMALIZED FLT. PT. RESULT IN FPRC.
000240 5813 0018            00018   197          LM    1,3,24(13)
000244 07FF                         198          BCR   15,14         RETURN.
000246 5830 F502            00608   199 TEMP     L     3,LA          REPLACES THE INSTRUCTION AT BRANCH.
00024A D203 F006 F070 C01DC 00246   200          MVC   BRANCH,TEMP   REPLACE INSTRUCTION AT BRANCH.
00024E 5830 1000            00000   201          L     3,0(,1)       LOAD PRIMER(ONLY DONE ON FIRST CALL).
000254 5830 3000            00000   202          L     3,0(,3)
000258 5030 F502            00608   203          ST    3,LA
00025C 47F0 F00A            001E0   204          B     BRANCH+4      GO TO INSTRUCTION FOLLOWING BRANCH.
000260                              205 AREA     DS    18F
00028B                              206 RES      DS    1F
00028C                              207 SUC      DS    1F
000280                              208 XST      DS    1F
000284 3CF8A982                     209 LLIN     DC    E'0.000015'
000239                              210 P        DS    256F
000688                              211 S        DS    4F
000608                              212 N        DS    3F
000604 4BC27395                     213 BPAT     DC    XL4'4BC27395'
00060A                              214 LA       DS    1F
00060C 40000000                     215 ZERO     DC    X'40000000'
000EC0                              216 A        DS    2000XL1
                                    217          END
000E80 41100000                     218                =E'1'
000E84 FFFFFFFF                     219                =F'-1'
000E88 00000001                     220                =F'1'
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000EBC 00000000                   221                =F'0'
000EC0 00000700                   222                =F'2000'
```

APPENDIX H

Assembler Listing Of GEN7

NOT REPRODUCIBLE

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                                                                                                                  PAGE    1

LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  FORAPR70   6/30/71

000000                                  GEN7     START 0
                                        ***********************************************************************
                                        * SOURCE---L.E. CANNON, UGA, DEPT. OF STAT., JUN 1970.                *
                                        * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING DIS*
                                        *          CRETE RANDOM VARIABLES WITH A SPECIFIED PROBABILITY.       *
                                        * USAGE----1. CALL DSETUP(P,IX,N). 'P' AND 'IX' ARE VECTORS OF PROBA- *
                                        *          BILITIES AND THE CORRESPONDING DISCRETE VARIABLES ( MUST   *
                                        *          BE INTEGERS<256). 'N' IS THE NUMBER OF ELEMENTS IN THE     *
                                        *          VECTORS. DSETUP SETS UP THE TABLE FROM WHICH RANDIS GENER- *
                                        *          ATES A RANDOM NUMBER. DSETUP NEED BE CALLED ONLY ONCE FOR  *
                                        *          A GIVEN 'P','IX', AND 'N'.                                 *
                                        *          2. X=RANDIS(IOD). 'IOD' MUST BE AN ODD INTEGER, IT PRIMES  *
                                        *          THE GENERATING SCHEME. X REPRESENTS THE DESIRED RANDOM     *
                                        *          VARIABLE, RETURNED IN FLOATING POINT.                      *
                                        * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                        *          6',1(JAN 1963),37.                                         *
                                        ***********************************************************************
                                                 ENTRY TSETUP
000000 47F0 F00C            0000C       TSETUP   B     12(0,15)
000004 07                                        DC    XL1'07'
000005 40E3E2C5E3E4D7                            DC    CL7' TSETUP'
00000C 90EC 000C            0000C                STM   14,12,12(13)
000010 05A0                                      BALR  10,0
000012                                           USING *,10
000012 18E0                                      LR    14,13
000014 4100 A18E            00100                LA    13,AREA
000018 5000 F008            00008                ST    13,8(0,14)
00001C 50E0 0004            00004                ST    14,4(0,13)
000020 5850 1008            00008                L     5,8(,1)
000024 5850 5000            00000                L     5,0(,5)       LOAD N INTO R5
000028 8950 0002            00002                SLL   5,2           CONVERT N TO WORDS (1 WORD= 4 BYTES).
00002C 5880 1000            00000                L     11,0(,1)      LOAD ADDRESS OF P VECTOR INTO R11.
000030 4140 0004            00004                LA    4,4
000034 1854                                      SR    5,4
000036 1833                                      SR    3,3           R3=I.
000038 7823 B000            00000       L3       LE    2,0(3,11)     LOAD LOAD FPR2 WITH P(I) FROM PARM. LIST.
00003C 7E20 A62E            00640                AU    2,ZERO        CONVERTING TO FIX POINT.
000040 7023 A20A            0021C                STE   2,P(3)
000044 5893 A20A            0021C                L     9,P(3)
000048 8590 0008            00008                SLL   9,8
00004C 5093 A20A            0021C                ST    9,P(3)
000050 8734 A026            00038                BXLE  3,4,L3        CONTINUE
000054 18CC                                      SR    12,12
000056 1888                                      SR    11,11
000058 1877                                      SR    7,7
00005A 1866                                      SR    6,6
00005C 1833                                      SR    3,3           R3=I.
00005E 1888                                 L4   SR    8,8           CLEAR R8.
000060 5893 A20A            0021C                L     9,P(3)        R9=P(I).
000064 8080 0004            00004                SLDL  8,4           R8=D1.
000068 1A68                                      AR    6,8           R6=SUM(D1).
00006A 8080 0004            00004                SLDL  8,4           R8=D1D2.
00006E 1A78                                      AR    7,8           R7=SUM(D1D2).
000070 8080 0004            00004                SLDL  8,4           R8=D1D2D3.
000074 1A88                                      AR    11,8          R11=SUM(D1D2D3).
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  FORAPRTO   6/30/71
000076 8080 0004          00004   56          SLDL  8,4          R8=D1D2D3D4.
00007A 1AC9                       57          AR    12,8         R12=SUM(D1D2D3D4).
00007C 8734 A04C          0005E   58          BXLE  3,4,L4       CONTINUE.
000080 5060 A60A          0061C   59          ST    6,S          STORE SUM(D1) INTO S(0).
000084 5070 A60E          00620   60          ST    7,S+4        STORE SUM(D1D2) INTO S(1).
000088 5080 A612          00624   61          ST    11,S+8       STORE SUM(D1D2D3) INTO S(2)
00008C 5000 A616          00628   62          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
000090 8990 0004          00004   63          SLL   11,4         R11=SUM(D1D2D3)*16.
000094 8970 0004          00004   64          SLL   7,4          R7=SUM(D1D2)*16.
000098 8960 0004          00004   65          SLL   6,4          R6=SUM(D1)*16.
00009C 1833                       66          SR    3,3          R3=I.
00009E 1888                       67 L5       SR    8,8          CLEAR R8.
0000A0 5893 A20A          0021C   68          L     9,P(3)       R9=P(I).
0000A4 8080 0004          00004   69          SLDL  8,4          R8=D1.
0000A8 1863                       70          SR    6,8          R6=SUM(D1)*16-SUM(D1).
0000AA 1879                       71          SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
0000AC 1889                       72          SR    11,8         R11=SUM(D1D2D3)*16-2UM(D1).
0000AE 1888                       73          SR    8,8          CLEAR R8.
0000B0 8080 0004          00004   74          SLDL  8,4          R8=D2.
0000B4 1879                       75          SR    7,8          R7=R7-SUM(D2).
0000B6 1889                       76          SR    11,8         R11=R11-SUM(D2).
0000B8 1888                       77          SR    8,8          CLEAR R8.
0000BA 8080 0004          00004   78          SLDL  8,4          R8=D3
0000BE 1889                       79          SR    11,8         R11=R11-SUM(D3).
0000C0 8734 A08C          0009E   80          BXLE  3,4,L5       CONTINUE.
0000C4 18CC                       81          SR    12,12
0000C6 5060 A61A          0062C   82          ST    6,N          STORE R6 INTO N(0).
0000CA 5070 A61E          00630   83          ST    7,N+4        STORE R7 INTO N(1).
0000CE 5080 A622          00634   84          ST    11,N+8       STORE R11 INTO N(2).
0000D2 1877                       85          SR    7,7
0000D4 1800                       86          SR    0,0
0000D6 5860 AE06          00E18   87          L     6,=F'1'
0000DA 5880 1004          00004   88          L     11,4(,1)     LOAD R11 WITH ADDRESS OF X VECTOR.
0000DE 1833                       89 L6       SR    3,3          R3=I.
0000E0 1888                       90 L7       SR    8,8          CLEAR R8.
0000E2 5893 A20A          0021C   91          L     9,P(3)       R9=P(I).
0000E6 8080 0004          00004   92          SLDL  8,4          R8=D1,R8=D2,R8=D3,R8=D4.
0000EA 5093 A20A          0021C   93          ST    9,P(3)       RESTORE SHIFTED P(I).
0000EE 5980 AE0A          00E1C   94          C     8,=F'0'      IF R8=0.
0000F2 4780 A0FE          00110   95          BC    8,L8         BRANCH TO L8.
0000F6 1A78                       96          AR    7,8          OTHERWISE STORE R2(DISCRETE R.V.)
0000F8 5970 AF0E          00E20   97          C     7,=F'2000'   CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
0000FC 4720 A10A          0011C   98          BC    2,L9
000100 5823 8000          00000   99          L     2,0(3,11)    LOAD R2 WITH DISCRETE VALUE.
000104 1876                      100          SR    7,6          THE NUMBER OF TIMES CORRESPONDING
000106 422C A632          00644   101          STC   2,A(12)      TO THE DIGIT IN R8.
00010A 87C6 A0F4          00106   102          BXLE  12,6,*-4
00010E 1A76                      103          AR    7,6
000110 8734 A0CE          000E0   104 L8       BXLE  3,4,L7       CONTINUE.
000114 1A06                      105          AR    0,6          CONTINUE L6 LOOP
000116 1904                      106          CR    0,4          UNTIL FIRST 4 DIGITS OF
000118 4740 A0CC          000DE   107          BC    4,L6         EACH P(I) HAVE BEEN SHIFTED.
00011C 5830 A616          00628   108 L9       L     3,S+12
000120 5830 A622          00634   109          S     3,N+8        THIS SECTION OF CODE
000124 193C                      110          CR    3,12         RESETS N(3)
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NOT REPRODUCIBLE

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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                    F08APR70   6/30/71

000126 4780 A122            00134  111          BC    9,OUT        IF TABLE LENGTH IS EXCEEDED.
00012A 183C                        112          SR    3,12
00012C 5830 A622            00634  113          A     3,N+8
000130 5030 A622            00634  114          ST    3,N+8
000134                             115 OUT      DS    0H
000134 5800 A1C2            00104  116          L     13,AREA+4
000138 98EC 000C            0000C  117          LM    14,12,12(13)
00013C 07FF                        118          BCR   15,14        BRANCH TO CALLING ROUTINE.
                                   119          DROP  10
                                   120          ENTRY RANDIS
000146                             121          USING RANDIS,15
00013E C7                          122          DC    XL1'07'
00013F 40D9C1D5C4C9E2              123          DC    CL7' RANDIS'
000144 9003 0014            00014  124 RANDIS   STM   0,3,20(13)
00014A 3800                        125          SFR   0,0
00014C 47F0 F070            00186  126 BRANCH   B     TEMP         BRANCH MADE ON FIRST CALL ONLY.
000150 5C20 F4F2            00638  127          M     2,BPAT       GENERATE NEW U.
000154 5030 F4F6            0063C  128          ST    3,LA         STORE FOR NEXT CALL.
000158 1B22                        129          SR    2,2
00015A 1B00                        130          SR    0,0
00015C 8C20 0004            00004  131          SLDL  2,4          R2=D1.
000160 5920 F406            0061C  132          C     2,S          IF D1<S THEN
000164 4740 F05A            001A0  133          BC    4,EX         BRANCH TO EX
000168 8020 0004            00004  134 N1       SLDL  2,4          R2=D1D2.
00016C 5920 F40A            00620  135          C     2,S+4        IF D1D2>=S(1) THEN
000170 4780 F036            0017C  136          BC    11,N2        BRANCH TO N2
000174 5820 F4F6            0062C  137          S     2,N          OTHERWISE OBTAIN PROPER TABLE ADDRESS
000178 47F0 F05A            001A0  138          B     EX           BRANCH TO EX.
00017C 8020 0004            00004  139 N2       SLDL  2,4          R2=D1D2D3.
000180 5920 F40F            00624  140          C     2,S+8        IF D1D2D3>=S(2) THEN
000184 4780 F04A            00190  141          BC    11,N3        BRANCH TO N3
000188 5820 F4FA            00630  142          S     2,N+4        OBTAIN PROPER TABLE ADDRESS
00018C 47F0 F05A            001A0  143          B     EX           BRANCH TO EX.
000190 8020 0004            00004  144 N3       SLDL  2,4          R2=D1D2D3D4.
000194 5920 F4F2            00628  145          C     2,S+12       IF D1D2D3D4>=S(3) THEN
000198 4780 F006            0014C  146          BC    11,BRANCH    GO TO BRANCH(OBTAIN NEW U).
00019C 5820 F4EF            00634  147          S     2,N+9        OBTAIN PROPER TABLE ADDRESS.
0001A0 4302 F4FE            00644  148 EX       IC    0,A(2)       OBTAIN PROPER DISCRETE VALUE.
0001A4 5000 F0D2            00218  149          ST    0,RES        CONVERT TO FLOATING POINT.
0001A8 9246 F002      C0218        150          MVI   RES,X'46'
0001AC 7A00 F0D2            00218  151          AE    0,RES        NORMALIZED FLT. PT. RESULT IN FPRO.
0001B0 9803 0014            00014  152          LM    0,3,20(13)
0001B4 07FF                        153          BCR   15,14        RETURN.
0001B6 5830 F4F6            0063C  154 TEMP     L     3,LA         REPLACES THE INSTRUCTION AT BRANCH.
0001BA 0203 F006 F070 C014C 00186  155          MVC   BRANCH,TEMP  REPLACE INSTRUCTION AT BRANCH.
0001C0 5830 1000            00000  156          L     3,0(,1)      LOAD PRIMER(ONLY DONE ON FIRST CALL).
0001C4 5830 3000            00000  157          L     3,0(,3)
0001C8 5030 F4F6            0063C  158          ST    3,LA
0001CC 47F0 F00A            00150  159          B     BRANCH+4     GO TO INSTRUCTION FOLLOWING BRANCH.
000100                             160 AREA     DS    19F
000218                             161 RES      DS    1F
00021C                             162 P        DS    256F
00061C                             163 S        DS    4F
00062C                             164 N        DS    3F
000638 48C27395                    165 BPAT     DC    XL4'48C27395'
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                                                                                                                  PAGE    4
  LCC  OBJECT CODE     ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                  F08APR70   6/30/71
00063C                                166 LA        DS     1F
000640 40000000                       167 ZERO      DC     X'40000000'
000644                                168 A         DS     2000XL1
                                      169           END
000E18 00000001                       170                  =F'1'
000E1C 00000000                       171                  =F'0'
000E20 00000700                       172                  =F'2000'
```

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