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END-FIRE RADIATION FROM PLANAR AND LARGE CYLINDRICAL ARRAYS

Giorgio V. Borgiotti
Quirino Balzano

RAYTHEON CO. MISSILE SYSTEMS DIVISION
BEDFORD LABORATORIES
Hartwell Rd. Bedford, Massachusetts 01730

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13. ABSTRACT <p>The analysis and the design of the elements of a large array of circular apertures on a triangular grid is approached by modeling the antenna as an infinite structure rotationally symmetric and periodic along the cylinder axis. Because of this particular symmetry every possible excitation is the superposition, with suitable weights, of a set of fundamental excitations having uniform magnitude and linear phase progression in the azimuthal direction and in the direction of the cylinder axis ("eigenexcitations"). Thus, by invoking superposition the electromagnetic analysis of the array is reduced to the solutions of the simpler boundary value problems pertinent to the set of eigenexcitations. This is done by expanding the field in normal modes in the region exterior to the cylinder and in the waveguides feeding the apertures, followed by a field matching at the cylinder surface (obtained approximately through Galerkin's method). The realized gain pattern of the radiators can be modified to a considerable extent by using an "element pattern shaping network" (in the radiator waveguides), serving the purpose of matching the array for a selected eigenexcitation. Criteria for the network design are given. A series of numerical examples illustrates the technique and shows that a "flat" element pattern can be thus obtained with a gain fall off with respect to the peak of less than 6 db at 80 degrees.</p>			

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ABSTRACT

Endfire radiation in planar finite arrays of circular apertures in apertures in triangular arrangement is investigated by using a technique consisting of using the results obtained via the usual infinite array model as a zeroth order approximation of a perturbation procedure. It is shown that finite arrays can be scanned up to 90 degrees from broadside, still retaining substantial radiation. The endfire radiation can be enhanced through an appropriate design of the element feed network. With minor modifications the method can be simply applied to the approximate investigation of the radiation of an array on a cylinder (having a large radius in terms of wavelengths) in the axial direction.

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ENDFIRE RADIATION FROM PLANAR AND LARGE CYLINDRICAL ARRAYS

1. Background

The analysis of planar and cylindrical periodic arrays is usually based on the infinite array model [1-6]. In this way all the analytical problems related to "edge effects" are avoided by simply postulating that the environment is the same for each element. It is well known that according to this model a planar array cannot have any endfire gain. Also for cylindrical infinite periodic arrays the gain can be shown to go necessarily to zero in the direction of the cylinder axis. It is consequently clear that the infinite periodic array model becomes useless for those directions for which end-effect plays a fundamental role, and a formalism which better reflects the physics of the phenomena must be sought.

Considering the finiteness of the array makes the analysis very involved, and leads invariably to facing the problem of the inversion of large matrices with complex elements. The problem can however be circumvented through the use of a perturbation technique, based on the recognition that for a large array the "eigenexcitations" of the structure (eigenvectors of the element scattering matrix) [7] are "not too different" from those of an infinite structure (with the same element and spacing).

A technique based on this idea, which does not require any large matrix inversion, has been recently introduced and applied to the analysis of an array of uniform slits on a conducting ground plane, considering the radiators as "one mode" elements [7]. This report extends the results of [7] in three respects:

- The practical case of circular apertures in a triangular arrangement is considered.
- A technique for enhancing radiation in directions close to endfire is developed, based on the idea of modifying mutual coupling among elements, through the use of matching networks in the waveguides feeding the elements.

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- An approximate simple method for the analysis of the radiation of a finite cylindrical structure for axial scan has been introduced.

In Section 2 the formalism for planar arrays is very briefly discussed. Computed examples are presented in Section 3, focusing the attention on the influence of element matching networks on array endfire or quasi endfire radiation. In Section 4 it is shown how the planar array results can be useful in cylindrical array performance evaluation.

The development in this report is based on the algebraic technique developed in Reference [7], which for reasons of brevity, will be assumed familiar to the reader.

2. Edge Effects in Planar Periodic Arrays

The structure investigated consists of a periodic array having as elements circular waveguides terminated on a ground plane. The elements are arranged in a triangular grid and the array is infinite in y direction and has a finite length in x direction. The lattice geometry is indicated in Figure 1. The elements in each infinite column are assumed to be in phase: thus only scan in a plane orthogonal to the array edge will be considered. Free array excitation is considered, i. e., the elements are excited by a set of incident waves in the element waveguides.

The polarization is in the plane of scan and the waveguides are filled with a dielectric material having a dielectric constant $\epsilon = 2.5$. A matching network, located far enough from the aperture not to interfere with the evanescent waveguide mode, completes the idealized element.

In order to simplify the analysis, the functional form of the element electric transverse field distribution is assumed independent of scan condition and equal to that of the fundamental waveguide mode polarized in the x direction. Thus, the element interactions are assumed to effect only the element relative complex voltage levels. For small elements and polarization in the plane of scan this approximation has proved to be very good in infinite array analysis, and it can be safely conjectured that this will be true in the finite case also. The relative complex levels of the element voltage

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are obtained by using the technique of transforming the field problem into a network problem. The approach consists of considering each (infinite) column as a single element of an equivalent linear array, and determining the relative set of self and mutual admittances via the Fourier Transform method [8]. Denote by a_i the excitation of an element of the i th column (the excitation of all the other elements of the same column being the same). The set of the quantities a_i can be concisely indicated as a vector column

$$\underline{a} = \{a_i\} \quad (i = 1 \dots N-1) \quad (1)$$

in the input space of the array having a dimensionality N equal to the number of array columns. The element voltages similarly are denoted by the vector \underline{V} related to \underline{a} by:

$$\underline{V} = \sqrt{2} (Y_L \underline{U} + \underline{Y})^{-1} Y_L^{-1/2} \underline{a} \quad (2)$$

where standard normalization has been used (see for example [8]).

In Equation (2) \underline{U} is the unit matrix of order N and Y_L is the admittance looking into the element waveguides, assumed equal for each element. Thus, Y_L depends upon the internal admittance of the equivalent generator feeding the element and upon the element matching network. The elements of the N th order matrix \underline{Y} are the admittances between columns (as discussed in some detail in the appendix) given by:

$$Y_{it} = \frac{1}{k\eta} \frac{\pi}{h} \sum_{m=-\infty}^{+\infty} (-1)^{(i-t)} \int_{-\infty}^{+\infty} \left| \frac{k^2 \sigma_\rho^2 + w^2 \sigma_\psi^2}{w} \right| e^{ju(i-t)u} du \quad (3)$$

$$v = \frac{m\pi}{h}$$

where σ_ρ and σ_ψ are the polar components of the Fourier Transforms of the fundamental waveguide mode (nominally polarized in the x direction) whose expression is given in the appendix, w is the wavenumber in the z direction related to the two coordinates u, v of the wavenumber plane by:

$$w = \sqrt{k^2 - u^2 - v^2} \quad (4)$$

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(the appropriate branch of Equation (4) being chosen in order to satisfy radiation condition), k and η are free space propagation constant and characteristic impedance. The various integrals in Equation (3) must be calculated for $v = m\pi/h$, as the notation indicates.

To avoid the direct matrix inversion Equation (2), through a series of manipulations (described in detail in [7]) the following Neumann series is obtained:

$$\underline{\underline{V}} = \sum_{i=0}^{N-1} \underline{\underline{m}}(i) \frac{\underline{\underline{m}}^+(i) \underline{\underline{a}}}{\underline{\underline{y}}_g + \underline{\underline{Y}}(i)} + \underline{\underline{M}} (\underline{\underline{U}} \underline{\underline{Y}}_g + \underline{\underline{D}})^{-1} \left[\underline{\underline{Q}} (\underline{\underline{U}} \underline{\underline{Y}}_g + \underline{\underline{D}})^{-1} \right]^s \underline{\underline{M}}^+ \underline{\underline{a}} \quad (5)$$

where $\underline{\underline{m}}(i)$ are the set of vectors

$$\underline{\underline{m}}(i) = \left\{ N^{-1/2} \exp(-j2\pi ik/N) \right\} \quad (i, k=0, 1, \dots, N-1) \quad (6)$$

$\underline{\underline{M}}$ is a matrix whose columns are the vector $\underline{\underline{m}}(i)$. $\underline{\underline{Y}}(i)$ is the active admittance for a uniform excitation and a phase progression equal to $2\pi i/N$ for an infinite reference array with the same elements and spacing. $\underline{\underline{D}}$ is a diagonal matrix whose elements different from zero are given by $\underline{\underline{Y}}(i)$, (suitably ordered) and:

$$\underline{\underline{Q}} = \underline{\underline{M}}^+ \underline{\underline{Y}} \underline{\underline{M}} - \underline{\underline{D}} \quad (7)$$

Once the aperture voltages are found, the element active admittances and array patterns are determined through standard procedures (see for example [7]).

Since this investigation aims at determining the scan limitations of a finite array, the attention will be mainly focussed on endfire or quasi endfire scan. It is apparent that the realized gain pattern of the array in the direction of scan (gain referred to the power of the "free excitation" : i.e., including mismatch loss) depends essentially upon the nature of the network located in the element waveguide. In an infinite array it is possible to match the structure for radiation in any assigned direction (different from endfire). In a finite array instead the input admittance is different for each element,

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and a perfect match of the array can be obtained only by using different matching networks for different elements. However, it is physically clear that placing in each element transmission line equal networks which would yield perfect match at a direction close to grazing angle for an infinite array will improve endfire or quasi endfire radiation. This is obtained, of course, at the expense of broadside gain.

3. Endfire Radiation in a Planar Array

Consider an array with a number of columns $N = 26$. The element size and lattice are those of Figure 1. The "free" array excitation a consists of terms having equal magnitude for each column with a linear phase taper from one column to the other. Different choices of the element tuning network (equivalent to a shunt susceptance and a perfect transformer) are considered, yielding perfect impedance matches for different scan angles in an infinite reference array (i. e., with the same element and lattice¹). In Figure 2 the magnitudes of the aperture voltages of the elements versus element positions have been shown for broadside match. Three different scan directions have been considered. A long "spatial transient" is present for 80- and 90-degree scan conditions. The aperture voltages for endfire scan vary along the array tending to the short-circuit condition typical of an infinite array. The phase of the voltages is essentially linear with only little difference from that of the free excitation, and thus has not been indicated. Figure 3 shows the magnitude of the reflection coefficients for the same match condition and three scan directions. For endfire radiation the moduli of the reflection coefficients increase going toward the edge from which the radiation occurs. All the elements in this scan condition are badly mismatched as expected. Figures 4 and 5 show two array patterns (for this match condition) for broadside and endfire scan. Figures 6 through 9 show the effect of a different matching network. The reference infinite array is matched for scan at 80 degrees from broadside. It is apparent that now the

¹The evaluation of the active admittance in the finite reference array necessary for the calculation of the parameters of the tuning network is done through well known methods recently developed [5-6].

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array is matched in a much better way for extreme scan angle. The element active reflection coefficient now decreases when approaching the radiating edge (Figure 7). Figures 8 and 9 show the radiation patterns for broadside and endfire scan. For this match condition the difference of gain scanning from broadside to endfire is about 3.7 dB.

The difference of behavior between the two element match conditions is clearly illustrated by the curves of gain envelope versus scan angle in Figures 10 and 11.

4. Finite Arrays on Cylinders - An Approximate Analysis of the Axial Radiation

The "infinite array" method of analysis consists of considering a cylindrical array as the excited section of an infinite structure periodic in both the longitudinal and azimuthal directions [10]. This model neglects endfire effects and therefore is of no use in predicting the radiation pattern for directions close to the cylinder axis. Here below a very simple method is presented which is believed to yield reasonably accurate performance predictions for radiation patterns in directions belonging to the axial symmetry plane of the array.

Consider a structure consisting of N rings of elements on a cylindrical surface. The elements are the same and are arranged with the same lattice as in the planar array previously considered for the planar case. Suppose that only a region azimuthally confined between the two angles $-\phi_0$ and ϕ_0 is excited. The analysis of the radiative properties of this structure can be performed through a procedure requiring the inversion of matrices of N th order only. As discussed in detail elsewhere [9], this is obtained by decomposing the original problem into a set of simpler reduced problems concerning the analysis of linear arrays having as "elements" entire rings of radiators excited with constant amplitude and linear phase taper. These reduced problems can be in turn approached by using the technique described in Section 2, avoiding in this way even the direct inversion of the N th order matrices. The procedure will not be described in detail because in most cases it can be replaced by a useful simple approximate technique, based on the recognition that for a cylinder with a large radius both the realized gain

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pattern and the pattern of the isolated elements are, in the axial plane, practically identical to their counterparts for a planar array. Consequently, in the axial plane the cylindrical array pattern can be obtained from that of a finite planar array simply by multiplication by a factor α_p depending upon the azimuthal angular extension $2\phi_0$ of the array. This factor takes into account the polarization effects arising from the fact that axially polarized elements located in different azimuthal positions give differently polarized contributions to the radiation field on the cylinder axis.

Suppose that the array aperture has a rectangular shape, limited by the angular abscissa ϕ_0 and $-\phi_0$, symmetric with respect to the axial scan plane. Consider a planar array having the same elements and lattice, and a number of columns equal to the number of array rings. When all the elements are phased to contribute in phase in the scan direction the pattern of the conformal array in the axial plane is approximately obtained by multiplying the planar array pattern by the factor

$$\alpha_p(\theta_s) = \cos^2 \theta_s + \sin^2 \theta_s \frac{\sin \phi_0}{\phi_0} \quad (8)$$

θ_s being the angle from broadside in the plane of scan. Expression (8) is determined on the basis of simple geometrical considerations. In many cases for large cylinder radii and relatively small ϕ_0 's, the expression (Equation (8)) is very close to unity.

Figures 12 and 13 show the gain envelope versus scan angle for a 273-element cylindrical array with a rectangular aperture and an angular extension $2\phi_0 = 49$ degrees. The element size and lattice are those of Figure 1.

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APPENDIX

Denote by $\underline{E}_0(x, y)$ the transverse electric field distribution on the reference element, whose center is at the origin of the x, y coordinate system. Thus, the electric transverse field of the i th column can be written

$$\underline{E}^{(i)}(x, y) = \sum_{m=-\infty}^{+\infty} \underline{E}_0 [x - id, y - (2m+i)h] \quad (A1)$$

where d and h are the distances between columns and between rows, respectively. Introduce the Fourier Transform $\underline{G}(u, v)$ of $\underline{E}_0(x, y)$:

$$\underline{E}_0(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underline{G}(u, v) e^{-j(ux+vy)} du dv \quad (A2)$$

The transverse electric field of the i th column can be expressed as follows:

$$\underline{E}^{(i)}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underline{G}(u, v) e^{-j(ux+vy)} e^{j(uid+vih)} \frac{\pi}{h} \sum_{m=-\infty}^{+\infty} \delta(v - \frac{m\pi}{h}) du dv \quad (A3)$$

Use has been made in Equation (A3) of the well known Fourier representation of a periodic delta function.

The mutual admittance Y_{it} between two columns i and t is defined here as the short circuit aperture current of an element in the column i when all the elements of column t are excited with equal unitary voltages. By using the results of [5] it is easy to establish the expression (3) for Y_{it} .

The polar components of the fundamental circular waveguide mode polarized in x direction are given by:

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$$\xi_{\rho}(t, \mu) = \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{(x'_{11})^2 - 1}} \frac{\sin \mu}{t} J_1(at)$$

$$\xi_{\psi}(t, \mu) = \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{(x'_{11})^2 - 1}} \frac{a \cos \mu}{1 - \left(\frac{at}{x'_{11}}\right)^2} J'_1(at)$$

where a is the radius of the element aperture, x'_{11} is the first root of the equation $J'_1(x) = 0$ and

$$t = \sqrt{u^2 + v^2}$$

$$\cos \mu = \frac{u}{\sqrt{u^2 + v^2}}$$

$$\sin \mu = \frac{v}{\sqrt{u^2 + v^2}}$$

If in Equation (A3) $|i-t| \gg 1$ (practically > 3) the following asymptotic expression can be conveniently used:

$$Y_{it} = \frac{\pi^2}{k\eta h} \sum_{m=-\infty}^{+\infty} \left| k^2 \xi_{\rho}^2 + \xi_{\psi}^2 w^2 \right|_{u=u_m} H_0^{(2)}(u_m |i-t|d)$$

with:

$$u_m = \sqrt{k^2 - \left(\frac{m\pi}{h}\right)^2}$$

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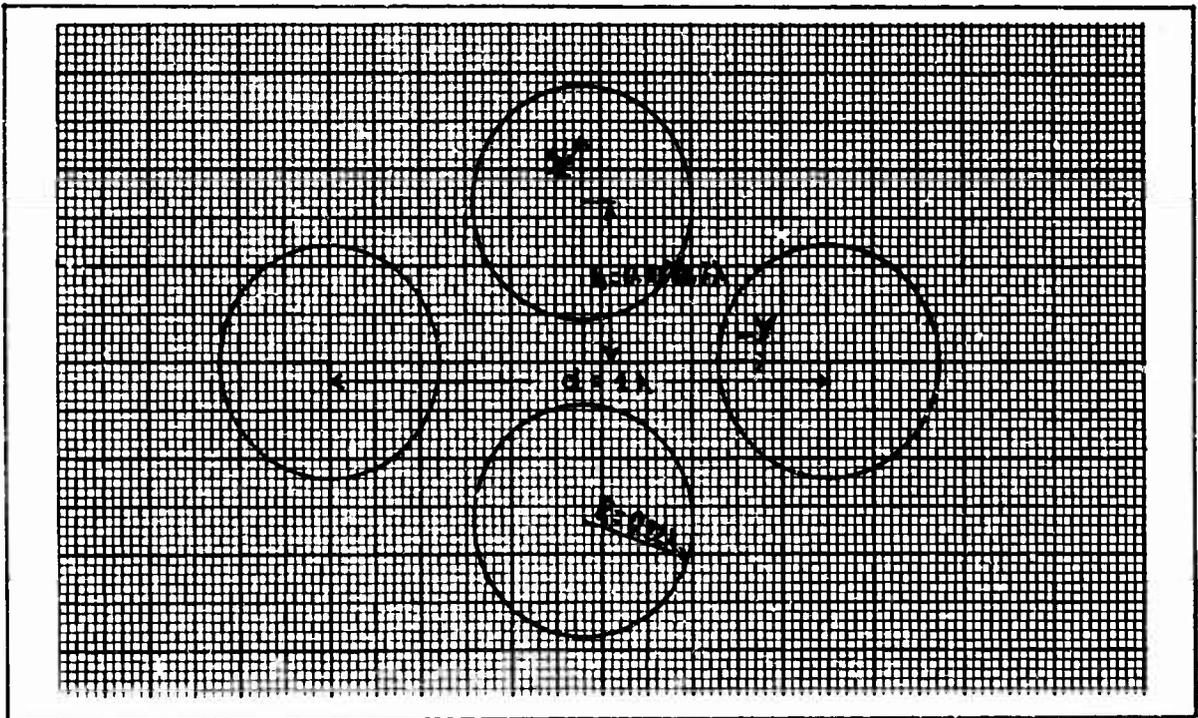


Figure 1 - Array Lattice

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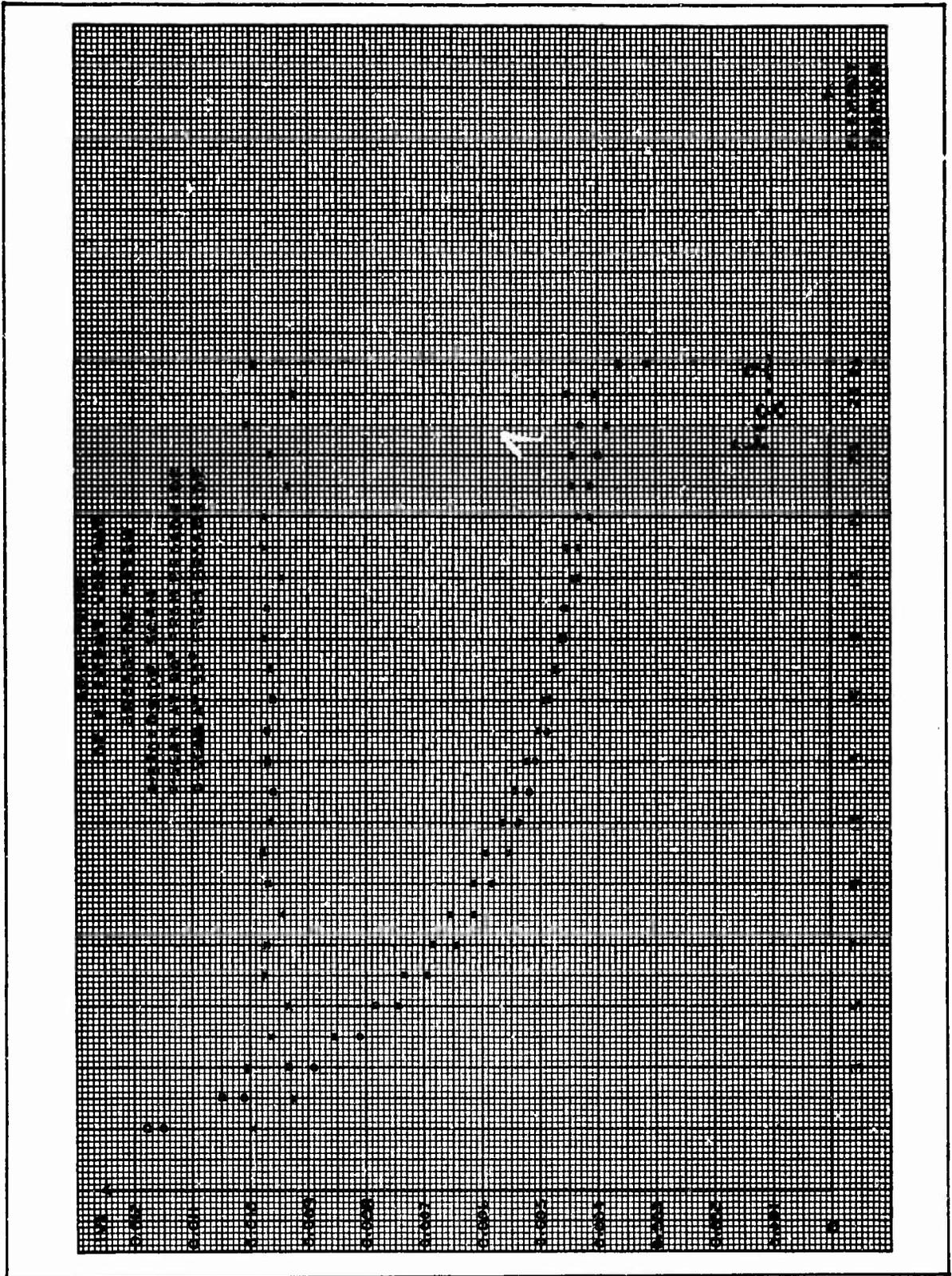


Figure 2 - Amplitude of Element Voltage

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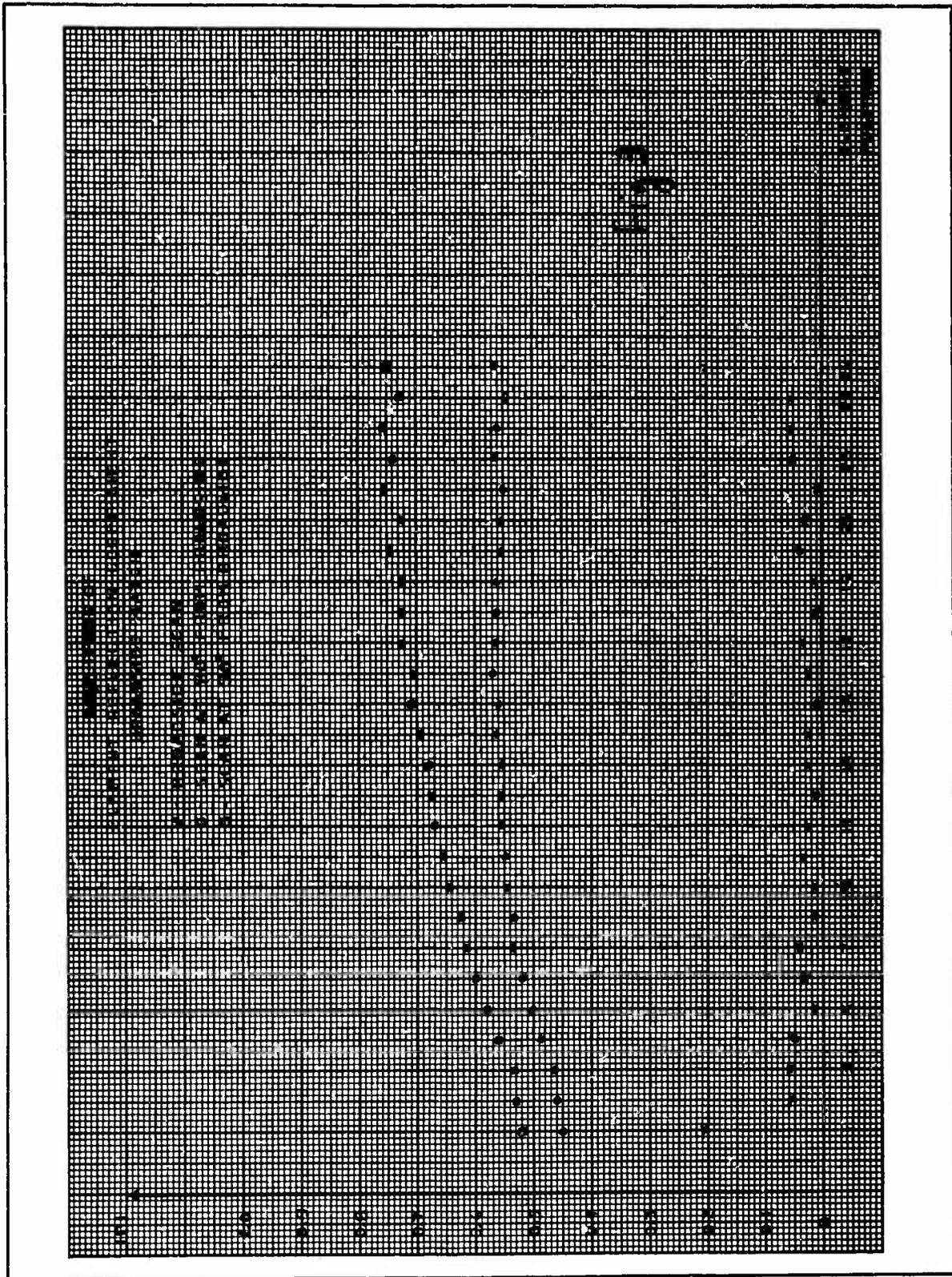


Figure 3 - Amplitude of Element Reflection Coefficient

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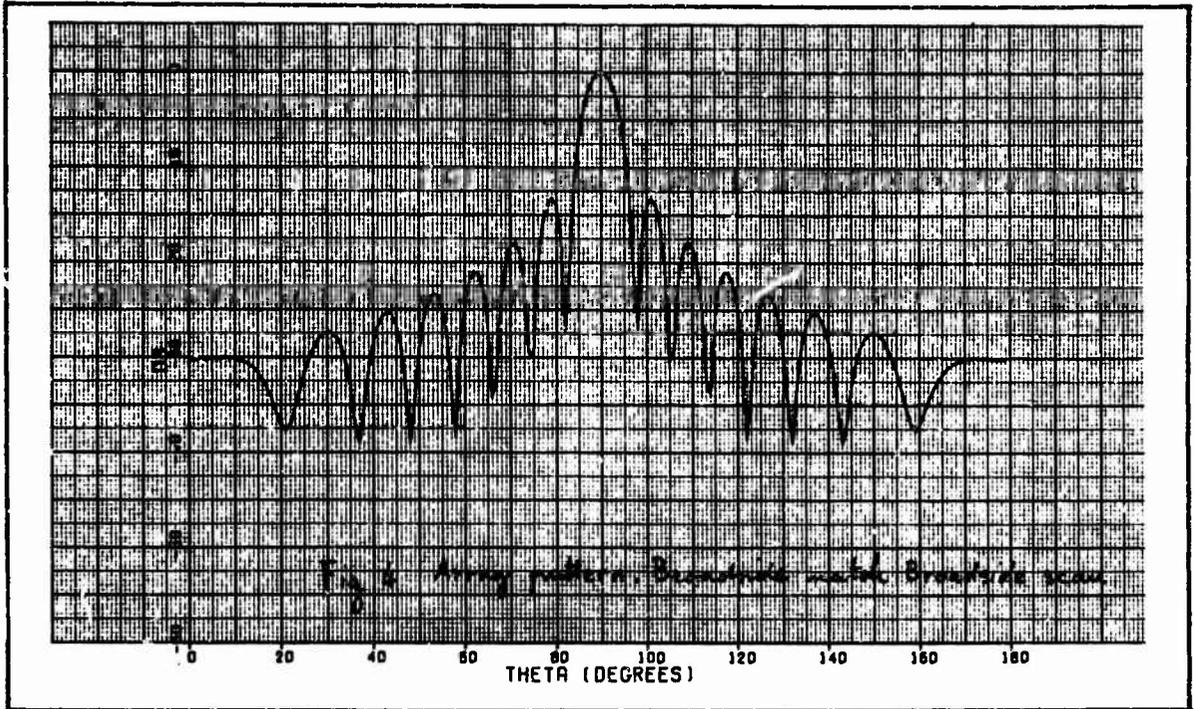


Figure 4 - Array Pattern, Broadside Match, Broadside Scan

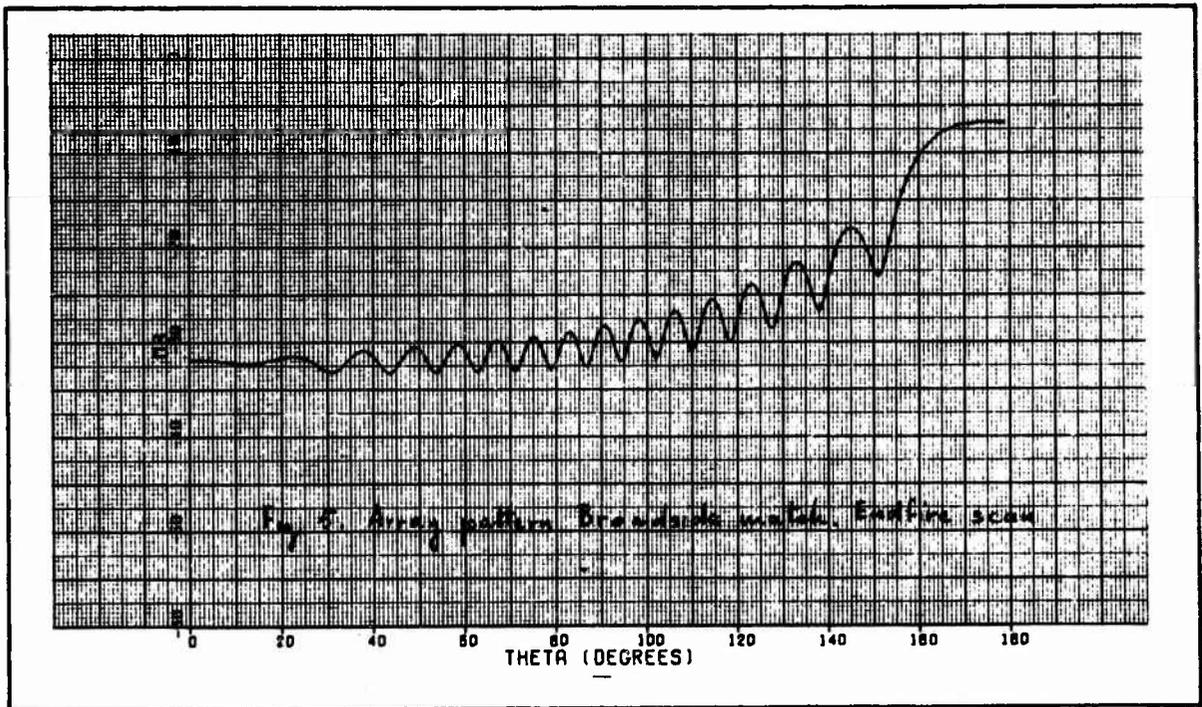


Figure 5 - Array Pattern, Broadside Match, Endfire Scan

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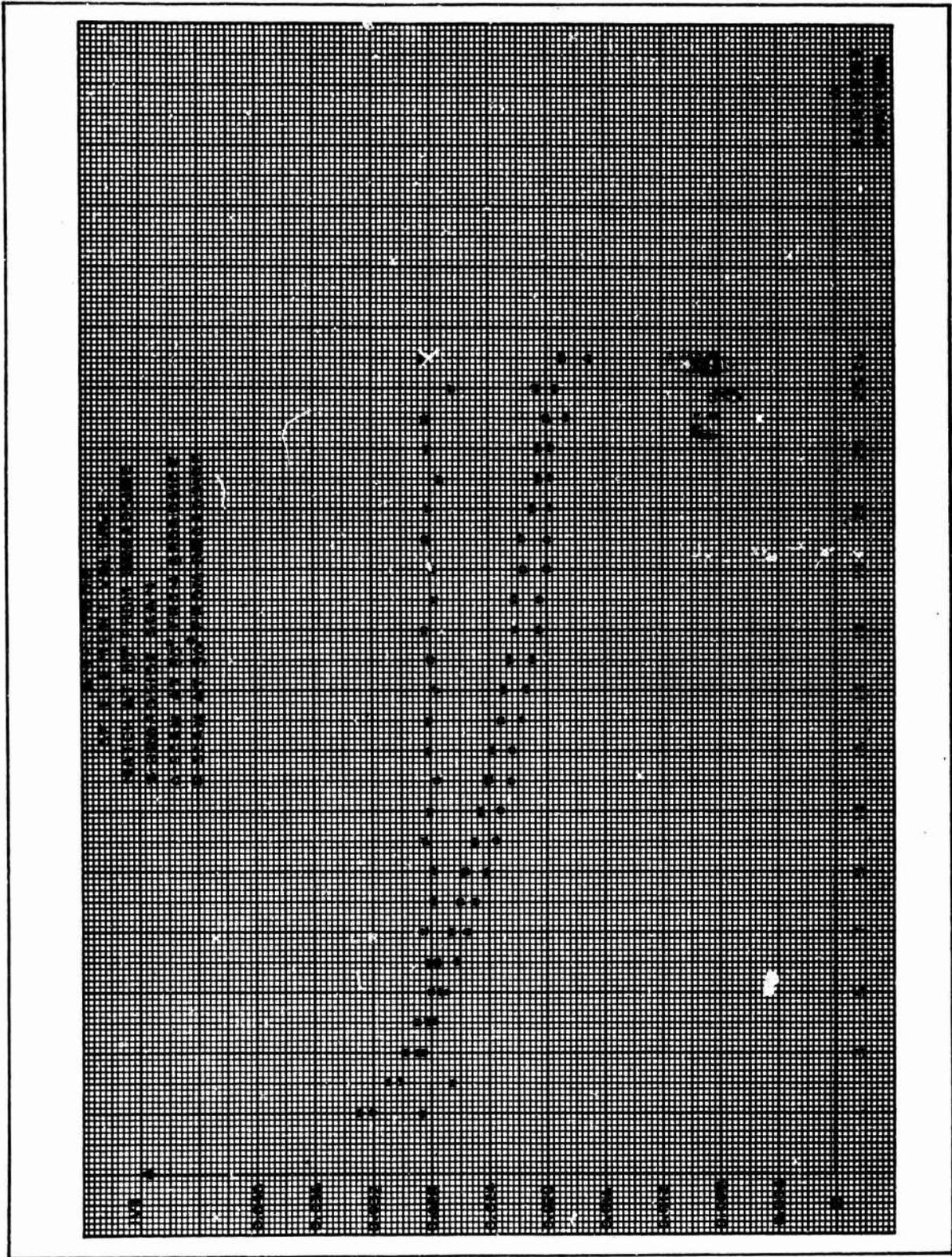


Figure 6 - Amplitude of Element Voltage

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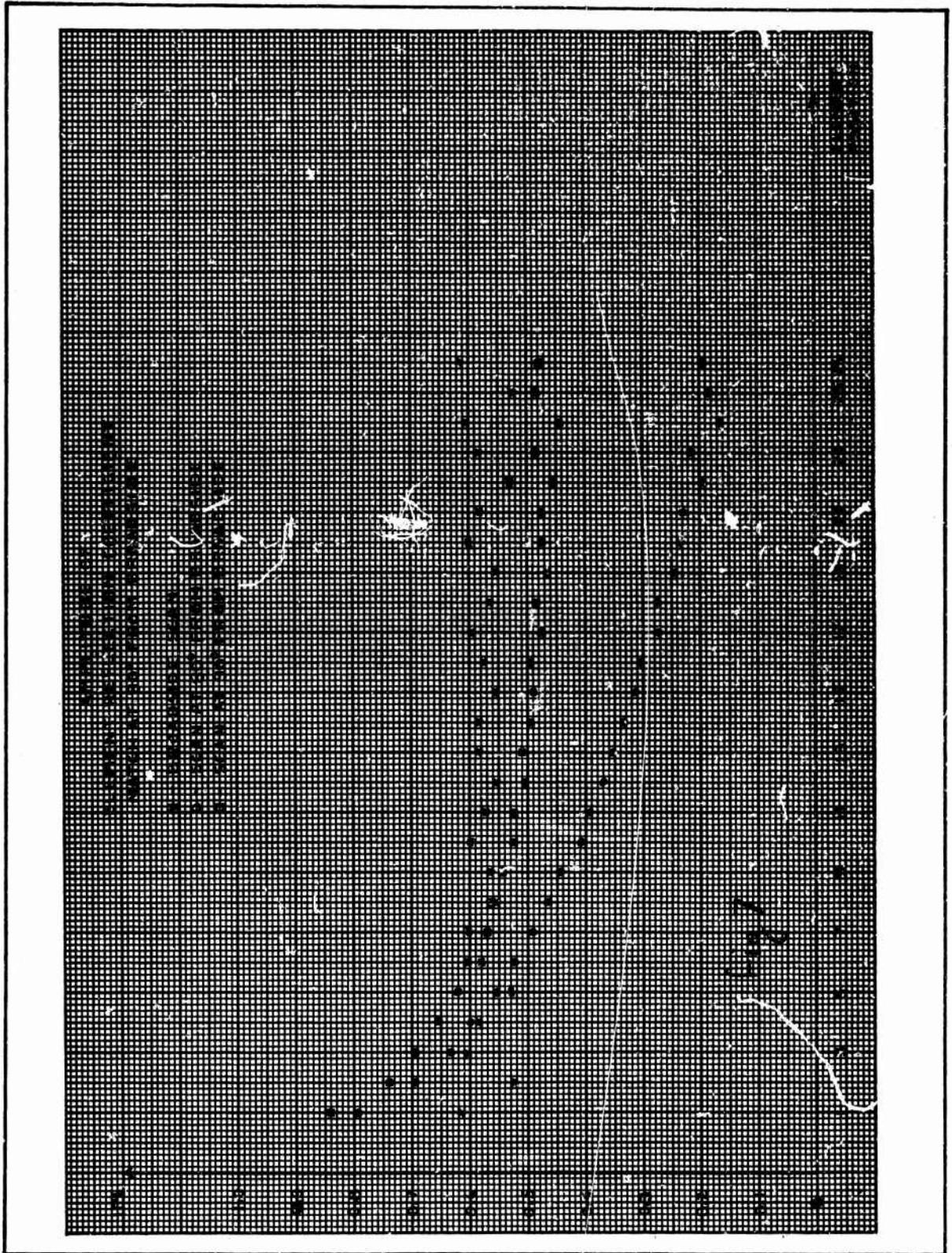


Figure 7 - Amplitude of Element Reflection Coefficient

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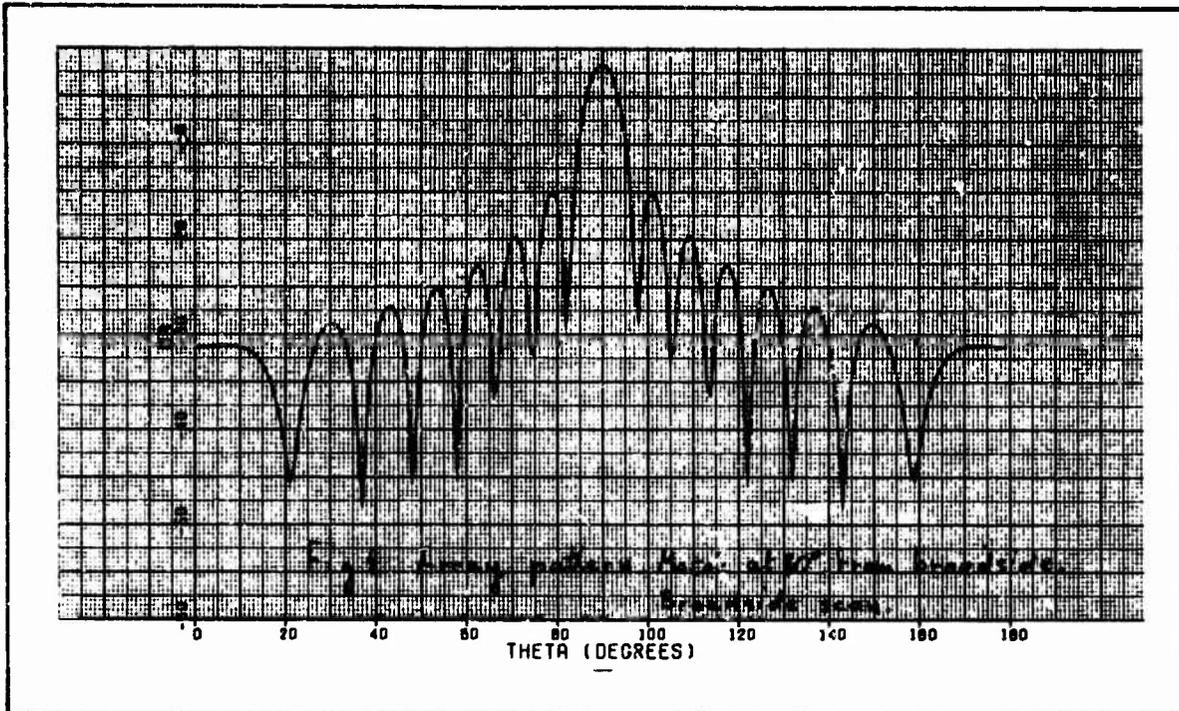


Figure 8 - Array Pattern, Match at 80 Degrees from Broadside, Broadside Scan

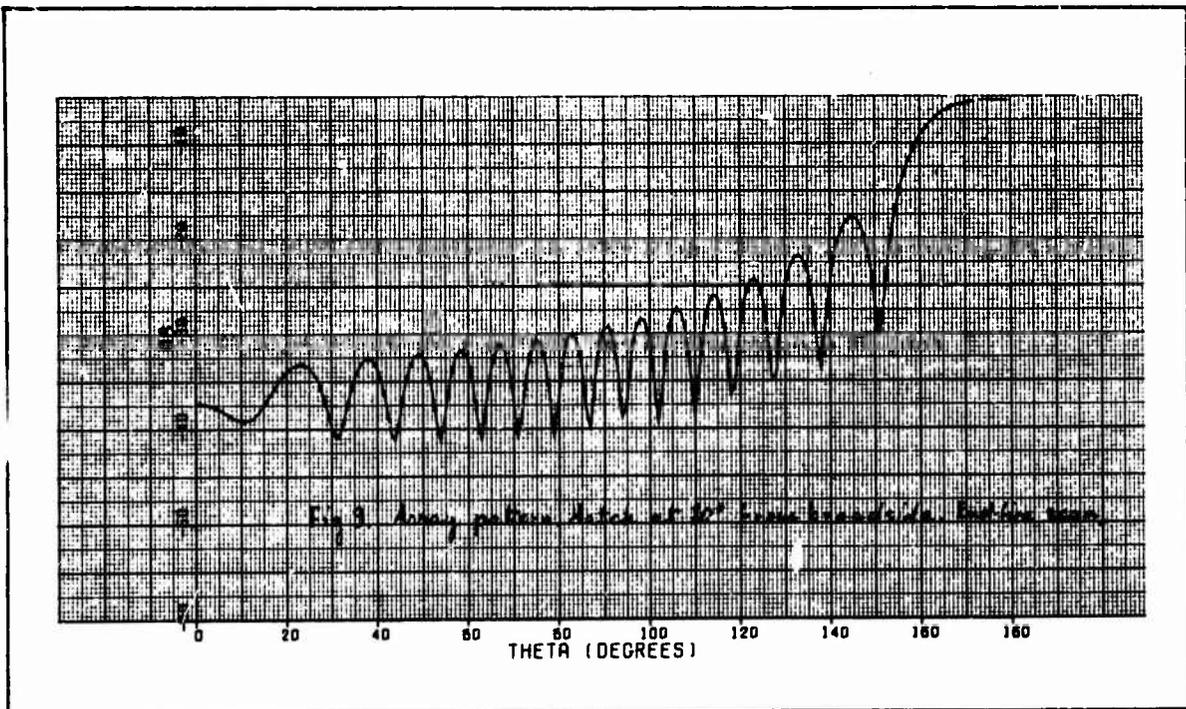


Figure 9 - Array Pattern, Match at 80 Degrees from Broadside, Endfire Scan

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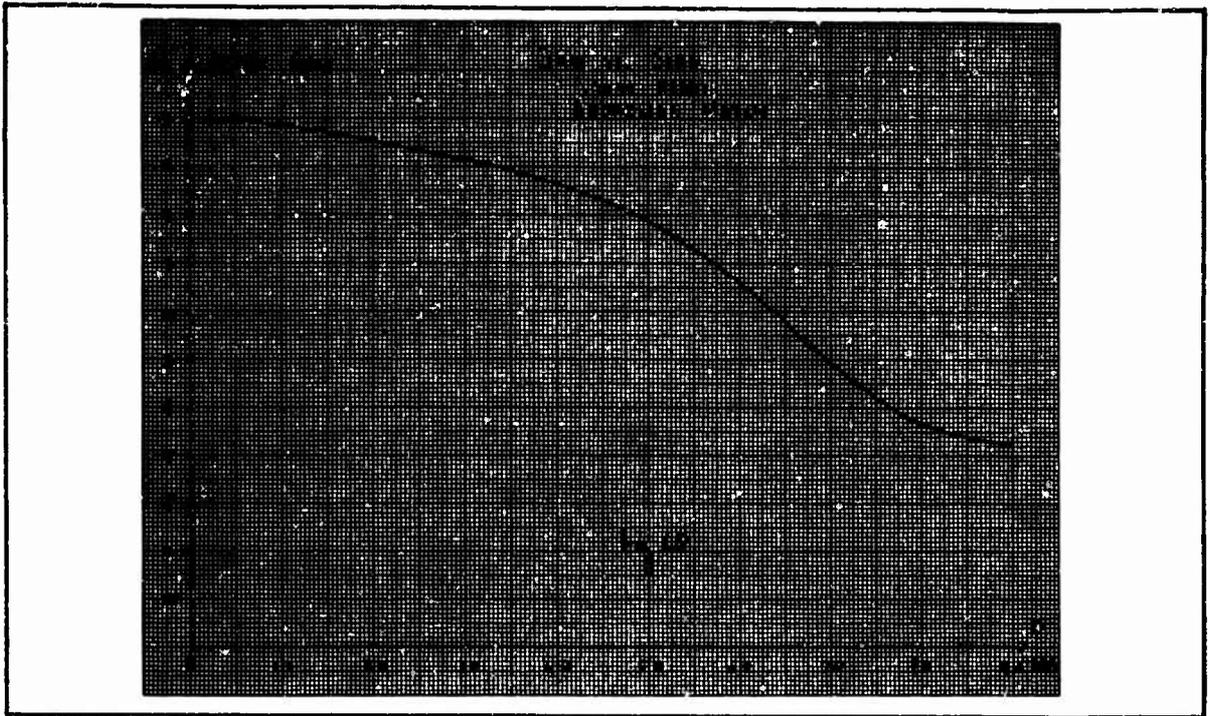


Figure 10 - Gain versus Scan, Planar Array, Broadside Match

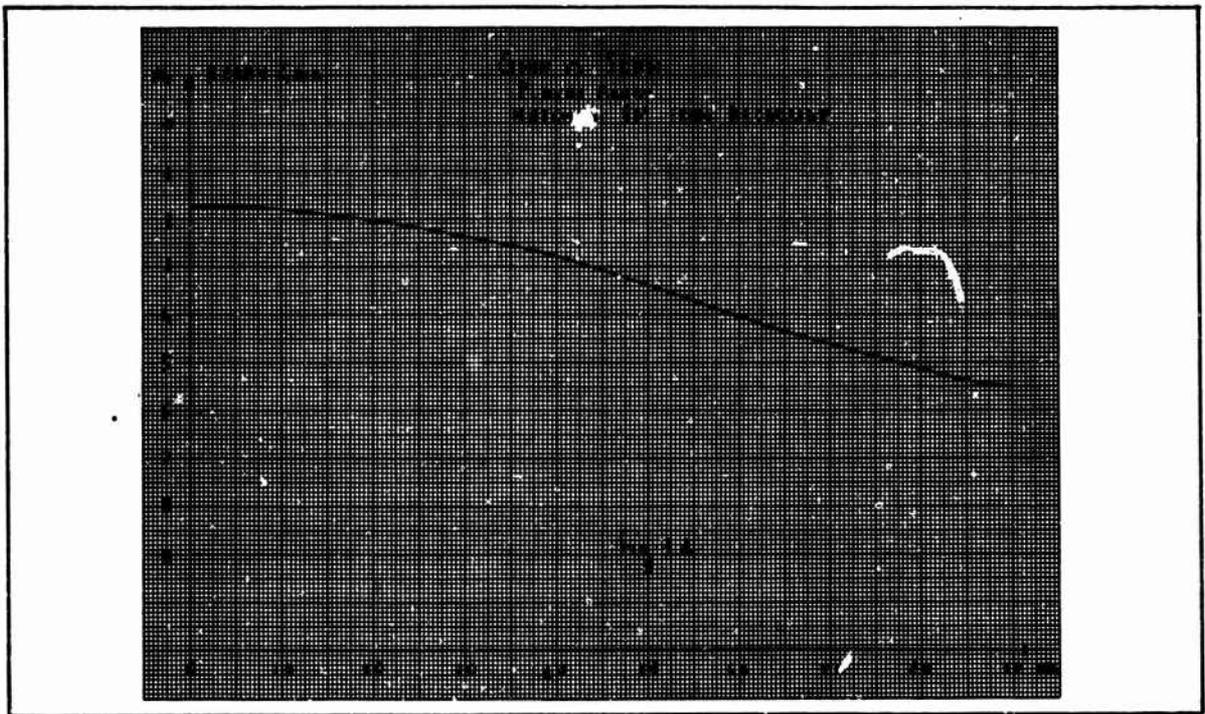


Figure 11 - Gain versus Scan, Planar Array, Match at 80 Degrees from Broadside

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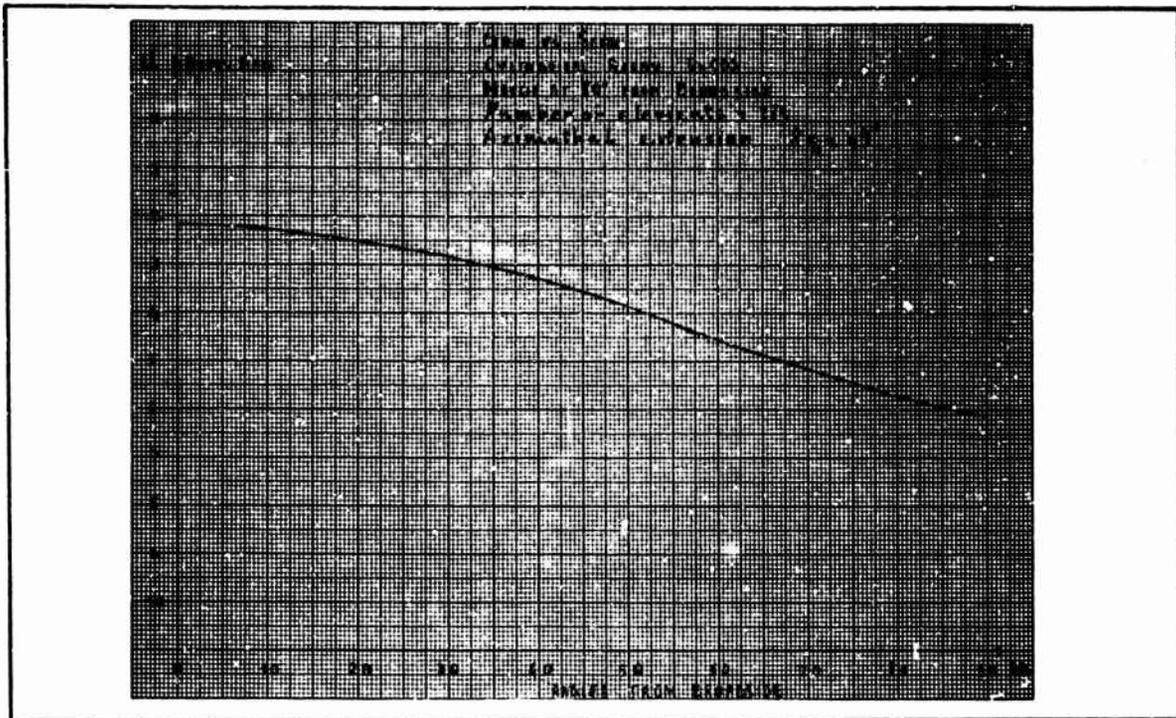


Figure 12 - Gain versus Scan, Cylindrical Array R = 50λ

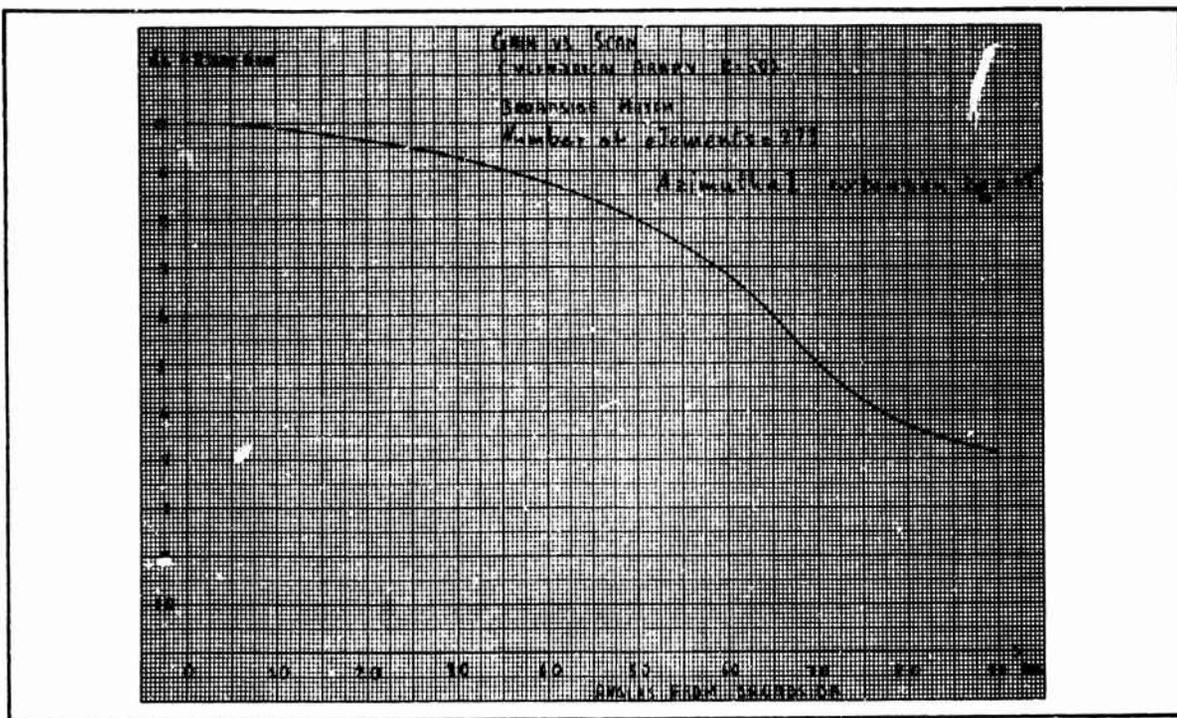


Figure 13 - Gain versus Scan, Cylindrical Array R = 50λ

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