COMPARISON OF UNIAXIAL DEFORMATION IN SHOCK
AND STATIC LOADING OF THREE ROCKS

FINAL REPORT

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Deformation of a granite, tonalite and limestone in a plane shock wave was compared with triaxial deformation at $10^{-5}$ sec$^{-1}$ under the constraint of uniaxial strain. For granite and for limestone at moderate pressure, the stress-strain relation was nearly identical in the two tests; strain under these conditions was elastically recoverable. For the limestone at high pressure and for the tonalite at all pressures, volume decreased permanently. This permanent deformation, compared here for shock and slow strain-rate tests, had about the same strain rate dependence as observed typically in fracture of rocks under triaxial conditions.
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This work was supported by the Advanced Research Projects Agency under ARPA Order No. 862

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ABSTRACT

Deformation of a granite, tonalite and limestone in a plane shock wave was compared with triaxial deformation at $10^{-5}\text{sec}^{-1}$ under the constraint of uniaxial strain. For granite and for limestone at moderate pressure, the stress-strain relation was nearly identical in the two tests; strain under these conditions was elastically recoverable. For the limestone at high pressure and for the tonalite at all pressures, volume decreased permanently. This permanent deformation, compared here for shock and slow strain-rate tests, had about the same strain-rate dependence as observed typically in fracture of rocks under triaxial conditions.
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INTRODUCTION

The study of rock deformation under shock loading has broad application in geophysics. Apart from the obvious connection to design of engineering structures likely to experience shock loading, study of shocked specimens may provide information both on the high pressure equation of state and on the time-dependent mechanical properties of geologic materials. As with many new techniques, however, very little time has been devoted to careful calibration of the method, particularly by detailed comparison of results with those of overlapping techniques. In the present paper, we present the results of one such comparison, designed to throw light on the following questions. To what extent can elastic, recoverable behavior in shock loading be predicted from mechanical properties measured in conventional laboratory tests? How does the permanent deformation of shocked material compare with the deformation at much slower strain rates observed in conventional triaxial tests?
A comparison for three rocks, Westerly granite, Solenhofen limestone and Cedar City tonalite, is presented here. Behavior during loading by a plane shock wave from the work of Jones and Froula [1969] was compared with behavior during more or less conventional triaxial loading [Brace, 1970]; the first will be termed shock, the latter static loading. In our comparison, strain rate in shock loading was of the order of $10^5$ to $10^6$ sec$^{-1}$, whereas in static loading it was close to $10^{-3}$ sec$^{-1}$. Our static experiments differed from one previous comparison [Kumar, 1968] in that strain was uniaxial; that is, loading of the specimen was so controlled that two principal strains were zero. In a plane shock wave strain is also uniaxial, as material in the center of the target is unable to move laterally within the duration of the experiment. With strain uniaxial in both shock and static experiments, direct comparison of measured stresses and strains was simplified.

OBSERVATIONS

The rocks investigated

Although the three rocks studied were in some respects nonideal for a precise comparison, they represented, at the time the study was begun, materials which had been given the most attention in shock wave and in other static studies. For this reason we could compare not only shock with static but also
different shock and different static results. However, in future, more appropriate material should be selected. Because of alteration, the tonalite is particularly unsuited for careful experimental studies; the wide range of composition and porosity of this rock is particularly evident in La Mori's [1970] compilation. Even Solenhofen limestone has complicating features for the present purpose.

Pertinent physical properties of the granite, limestone and tonalite are given in Table 1. The granite and limestone were the same as described in Brace [1965] and Brace [1964], respectively, for the static experiments. The tonalite for both experiments came from the same general area of the Cedar City exposure, although not from the same block. Modal analyses of all three rocks are available in Brace [1970].

Table 1 contains bulk density and total porosity, obtained by immersion methods. Overall precision for the granite and limestone is about 0.5 percent for density. For tonalite, precision is probably not much better than 1 percent; the samples were friable and crumbly.

Static loading

As described in detail in Brace [1970], cylindrical samples were so loaded by axial stress and hydrostatic pressure that radial strain was zero. Strain gages were used to record strain; the only nonzero component, the axial strain $\varepsilon_1$, was equal to the volume change. A typical result for a rock of low
<table>
<thead>
<tr>
<th>Rock</th>
<th>Density g cm$^{-3}$</th>
<th>Porosity %</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td>Brace, 1965</td>
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<tr>
<td>Tonalite, Cedar City, Utah</td>
<td>shock: 2.57, static: 2.48</td>
<td>5, 7</td>
<td>Perkins, Green &amp; Friedman, 1970</td>
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<td></td>
<td></td>
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<td>La Mori, 1970</td>
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<td>Limestone, Solenhofen</td>
<td>shock: 2.57, static: 2.54</td>
<td>4.7, 4.8</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Brace, 1964</td>
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porosity, Westerly granite, is given in Figure 1, as a curve of axial stress, $\sigma_1$, vs radial stress, $\sigma_3$, and a curve of axial stress vs volume change. Strain rate was close to $10^{-5}\text{sec}^{-1}$, the rocks were nominally dry, at room temperature. Brown and Swanson's results [1970] for a different block of Westerly granite, to lower stresses, are also given in Figure 1. Error bars indicate the probable error of a single measurement, the pairs of points, the variation between identical experiments. Both compressive stress and strain are taken as positive in this paper.

The tonalite, a relatively porous material, had the $\sigma_1 - \epsilon_1$ curve given in Figure 2. Solenhofen limestone, of intermediate porosity, gave the result shown in Figure 3, which also gives data for a calcite marble. The marble had the same mineralogy as the limestone but less than 1 percent porosity, most of which was crack porosity [Brace, 1965].

As discussed more fully in Brace [1970], Westerly granite and the marble showed virtually no permanent volumetric strain as a result of one cycle of loading in uniaxial strain, whereas the tonalite and limestone experienced volumetric compaction of 4.2 and 2.3 percent, respectively. In spite of these large strains there were no faults or other features much larger than the scale of a grain. Deformation apparently occurred through cataclasis.
Shock loading

The shock results come from Jones and Froula [1969], where details of the procedure are given. For the present comparison, data have been put in the form of stress vs volumetric strain, equivalent in the static test to $\sigma_1$, vs $\epsilon_1$. The data, reduced to this form, are tabulated in Table 2 and plotted in Figures 1, 2 and 3. For Westerly granite, a plot of $\sigma_1$ vs $\sigma_3$ is also given; this was obtained from the shock results by comparing with a hydrostat obtained from Simmons and Brace [1965] at low pressure and La Mori [1970] at higher pressures. The highest pressures obtained by Jones and Froula [1969] exceeded the axial pressures attained in the static test. The highest pressures for which a comparison could be made was set by the confining pressures attainable in the static test (10 kb), so that only data from the shock study corresponding to this limit are given here.

DISCUSSION

The static and shock results for granite are seen (Figure 1) to be in close agreement, both for the measurements we have made and those reported elsewhere [Brown and Swanson, 1970; Grine, 1970] and also shown in Figure 1. This agreement
### TABLE 2. Shock Compressions

<table>
<thead>
<tr>
<th>$\sigma_1$, kb</th>
<th>granite</th>
<th>tonalite</th>
<th>limestone</th>
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<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.45</td>
<td>0.14</td>
</tr>
<tr>
<td>2</td>
<td>0.27</td>
<td>0.83</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>1.17</td>
<td>0.37</td>
</tr>
<tr>
<td>4</td>
<td>0.51</td>
<td>1.45</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>0.63</td>
<td>1.72</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>0.75</td>
<td>1.95</td>
<td>0.68</td>
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<tr>
<td>7</td>
<td>0.86</td>
<td>2.16</td>
<td>0.82</td>
</tr>
<tr>
<td>8</td>
<td>0.98</td>
<td>2.37</td>
<td>1.07</td>
</tr>
<tr>
<td>9</td>
<td>1.09</td>
<td>2.56</td>
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<td>10</td>
<td>1.21</td>
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<tr>
<td>12</td>
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<td>3.08</td>
<td>2.53</td>
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<td>13</td>
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<tr>
<td>22</td>
<td>2.59</td>
<td>4.39</td>
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is somewhat remarkable inasmuch as the specimens used all came from different blocks of Westerly granite and the shock results of Grine [1970] were obtained on material from a different quarry. By contrast, the shock and static results for the tonalite are very different (Figure 2). Throughout most of the loading path, the shock stress exceeded the static stress at any given strain. Between strains of about 2 to 7 percent, the ratio of these stresses remained fairly constant; the static stress was 0.60 to 0.76 of the shock stress.

For the limestone, Figure 3, the situation was apparently intermediate between that of granite and tonalite. Shock and static stresses were nearly identical at low stress, but differed at high stress. Above a stress of about 4.5 kb, the ratio of additional stress for shock loading to that for static loading ranged from 0.53 to 0.67 at a given strain. The stress-strain curve for the marble (Figure 3) is seen to have the same slope above 1 kb as the limestone does both in shock and static loading up to 4.5 kb.

Several of these observations suggest that granite, and limestone below 4.5 kb, behaved elastically during loading in uniaxial strain, whereas tonalite was permanently deformed starting at very low stress. For one thing, absence of any permanent strain in granite after a cycle of loading and unloading was direct evidence of a recoverable deformation. For another, agreement between different samples of the granite, and between limestone and marble is just what one would predict...
since it is bulk mineralogy, not details of texture and grain size, which determine elasticity of low porosity rocks at high pressure. Marble has a slight crack porosity, whereas the limestone does not; marble is seen to be more compliant than limestone below 1 kb.

The elasticity of these and other rocks in uniaxial strain is, in fact, not ideal, as stress-strain relations are not linear and abnormally high values of Poisson's ratio are evident [Brace, 1970]. These effects and their analysis are the subject of a separate study [Walsh and Brace, 1970].

Our observation that elasticity of granite and limestone is not strain-rate dependent, as implied by the agreement between shock and static stress-strain curves above, is not surprising if one assumes that the elasticity one observes has an atomic basis in the minerals in the rocks. It is also consistent with other observations of the effect of loading or strain rate on modulus. No effect has been observed for marble, sandstone, gabbro and limestone [Serdengecti and Boozer, 1961; Mogi, 1959; Green and Perkins, 1969] whereas granite, tuff and tonalite do exhibit a marked effect at atmospheric pressure [Green and Perkins, 1969; Perkins, Green and Friedman, 1970]. Handin [1970] found that the effect disappeared at a few kilobars confining pressure for Westerly granite. The reason for the increase in modulus with strain rate for tuff, tonalite, and granite at atmospheric pressure may be that frictional sliding on cracks, crack growth and
other strain-rate dependent effects are important during so-called elastic deformation of these rocks. This would seem to be borne out by Handin's [1970] observation, for in Westerly granite cracks are closed by pressures of a few kilobars. With tuff and tonalite, compaction (which involves local cracking) probably occurs at all pressures [Brace, 1970].

Although it might be argued that shock and static tonalite differ because of porosity differences (Table 1), it seems more likely that both this difference and the similar one for limestone are caused by the same effect. If porosity were the principal factor for the tonalite, it is hard to see why shock stress is such a constant factor of static stress even down to small strains. A more likely explanation is that these rocks have deformed by some mechanism that is strain-rate dependent. It is of interest to see if this dependence is of the right order, based on previous studies of time dependence.

Although some twinning of calcite in the limestone cannot be ruled out, microscopic study of the deformed limestone and tonalite suggest that cataclasis was the principal mechanism by which these rocks became permanently compacted. In detail, this probably involved small scale fracturing and grain rotation; rotation and sliding probably involved some fracturing on a still smaller scale.

A number of studies are available in which strain-rate dependence of fracture in rocks has been determined near room temperature. Early work was summarized by Handin [1966];
observations were typically made at strain rates which ranged from $10^{-7}$ to $10^{-1}\text{sec}^{-1}$. Newer studies [Kumar, 1968; Green and Perkins, 1969; Perkins, Green and Friedman, 1970], in which Hopkinson bar and other techniques are employed, have extended the range to $10^3\text{sec}^{-1}$. All of these results are collected in Figure 4. For convenience in comparing strain-rate effects, we give the percent increase in fracture strength per $10^3$ increase in strain rate. This quantity ranges from 10 to about 18 percent for silicate rocks and Solenhofen limestone; an average would be $15 \pm 5$ percent. This number is conspicuously different [Heard and Carter, 1968] in the case of calcite marble (5 percent) or carbonate and silicate rocks at 500 to 800° (40 to 250 percent).

If we take the values reported for tonalite (15 percent) and limestone (18 percent) from Green and Perkins [1969], we can calculate the ratio of shock to static stress to be expected here, based on a difference of strain rate of $10^{11}\text{sec}^{-1}$. We obtain a factor of 0.60 for the tonalite and 0.55 for the limestone. These are seen to be near the lower limit of the values observed and given above (0.60 and 0.53, respectively). Thus, the effect noted here does seem consistent with strain-rate dependence of fracture stress. This is somewhat surprising inasmuch as faulting plays a major role in the latter; as noted above, it is absent in uniaxial strain deformation. However, the explanation for this may be that faulting too depends on small scale fracturing; this seems consistent with modern studies of the faulting process [Wawersik and Fairhurst, 1970].
In several recent studies [Green and Perkins, 1969; Kumar, 1968; Perkins, Green and Friedman, 1970], the relation between strength and strain rate was of the form shown for Solenhofen limestone in Figure 5. That is, stress to fracture increased quite slowly up to a strain rate of about 10^3 sec^{-1} and then, beyond that, increased very rapidly. The slow increase corresponds to the strain-rate dependence discussed above, and to the data collected in Figure 4. The very rapid increase has been interpreted as the transition to a new more strongly strain-rate dependent mechanism of crack growth [Perkins, Green and Friedman, 1970] or to change in the stress state from uniaxial stress to uniaxial strain with increasing strain-rate [Green and Perkins, 1969]. We favor the second of these explanations. For one thing, a curve such as Figure 5 seems to be approaching a value which one might predict for uniaxial strain. We can estimate this value from Figure 3 by using a fracture strain of 0.7% percent for the limestone [Green and Perkins, 1969]. The maximum stress under these conditions is about 6 kb, as shown in Figure 5. The second reason for favoring transition to uniaxial strain is our own comparison of shock and static behavior. When strain is uniaxial in both, then, as shown in Figure 5, strain-rate dependency resembles that widely observed at strain rates below 10^2 sec^{-1}. 
Acknowledgments

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Shock wave data was generated under DASA Contract No. DA49-146-XZ-544, monitored by Clifton McFarland. The work was carried out with N. Froula, when one of the authors (A.H. Jones) was with the Materials and Structures Laboratory, General Motors Corporation, Warren, Michigan.
REFERENCES


La Mori, Phillip N., Static determination of the equation-of-state of Cedar City tonalite, DASA-2527, Final Report, 80 pp., May 1970.


**CAPTIONS**

**Figure 1** Uniaxial deformation of Westerly granite. The open circles and triangles are static, the closed squares shock loading. Grine's [1970] results are for Bradford granite. Typical error bars are shown; other data points have the same fractional errors.

**Figure 2** Uniaxial deformation of tonalite. Typical error bars are shown; other data points have the same fractional error.

**Figure 3** Comparison of shock and static deformation of Solenhofen with static deformation of calcite marble. Typical error bars only are shown for the static measurements; errors for the shock measurements were within the squares.

**Figure 4** Stress difference at fracture of various rocks as a function of strain rate. Circles give data from Handin [1966], diamonds from Perkins, Green and Friedman [1970], triangles from Green and Perkins [1969], squares from Heard and Carter [1968], crosses from Brace and Martin [1968], and x's from Kumar [1968]. For each rock the stress difference was taken at about the same total axial strain. Except as noted on the lines, all measurements were made at room pressure and temperature. The numbers at the left end of each
line through the data points reflect the slope of that line; the number is the percent increase in stress difference per $10^3$ increase.

Figure 5 Maximum stress, $\sigma_1$, as a function of strain rate for Solenhofen limestone [Green and Perkins, 1969] is given by the circles and solid curve. For comparison, static and shock values of $\sigma_1$ are given from Figure 3 assuming an axial strain, $\varepsilon_1$, of about 0.7 percent.
Fig. 1 Uniaxial deformation of Westerly granite
Fig. 2 Uniaxial deformation of tonalite
Fig. 3 Comparison of shock and static deformation of Solenhofen with static deformation of calcite marble.
Fig. 4 - Stress difference at fracture of various rocks as a function of strain rate.
Fig. 5 Maximum stress vs strain rate for Solenhofen Limestone from Green and Perkins, 1969, compared with static and Shock values from Figure 3.