FLUSH ELECTROSTATIC PROBE IN A CONTINUUM REGIME

by

Samuel Lederman and Robert Bushman

POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT

of

AEROSPACE ENGINEERING

and

APPLIED MECHANICS

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The effects of area, geometry and applied bias upon the current collection of negatively biased flush electrostatic probes is experimentally investigated in a continuum regime. The effects of area, geometry and applied bias were found to be strongly interrelated and their influence on the collected ion current density very complex. It is found, in general, that the ion current density increases with increasing probe bias but decreases with increasing probe area.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrostatic Probe</td>
</tr>
<tr>
<td>Flush Probe</td>
</tr>
<tr>
<td>Ion Collection</td>
</tr>
<tr>
<td>Continuum Regime</td>
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</tbody>
</table>

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<tr>
<th>LINK A</th>
<th>LINK B</th>
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<tr>
<td>ROLE</td>
<td>WT</td>
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POLYTECHNIC INSTITUTE OF BROOKLYN
Department of Aerospace Engineering and Applied Mechanics

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FLUSH ELECTROSTATIC PROBE IN A CONTINUUM REGIME

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Farmingdale, New York

ABSTRACT

The effects of area, geometry and applied bias upon the current collection of negatively biased flush electrostatic probes is experimentally investigated in a continuum regime. The effects of area, geometry and applied bias were found to be strongly interrelated and their influence on the collected ion current density very complex. It is found, in general, that the ion current density increases with increasing probe bias but decreases with increasing probe area.

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** Associate Professor of Aerospace Engineering.

** Research Assistant.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I  Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II Review of the Chung and Bredfeldt-Scharfman Theories</td>
<td>2</td>
</tr>
<tr>
<td>III Experimental Facility</td>
<td>6</td>
</tr>
<tr>
<td>IV Experimental Results</td>
<td>7</td>
</tr>
<tr>
<td>V Discussion of Experimental Results and Conclusions</td>
<td>10</td>
</tr>
<tr>
<td>VI References</td>
<td>12</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Flush Probe Schematic Showing Probe Circuitry Ion Sheath and Boundary Layer Profiles on a Flat Plate</td>
</tr>
<tr>
<td>2</td>
<td>Schematic Diagram of the Shock Tube</td>
</tr>
<tr>
<td>3</td>
<td>Flat Plate and Test Model Insert</td>
</tr>
<tr>
<td>4</td>
<td>Photograph of Test Section Showing Flat Plate, Cylindrical Langmuir Probe and Test Model IV</td>
</tr>
<tr>
<td>5</td>
<td>Test Inserts I and II</td>
</tr>
<tr>
<td>6</td>
<td>Test Inserts III and IV</td>
</tr>
<tr>
<td>7</td>
<td>Typical Ion Current Traces</td>
</tr>
<tr>
<td>8</td>
<td>Current Voltage Characteristics</td>
</tr>
<tr>
<td>9</td>
<td>Current Area Characteristics</td>
</tr>
<tr>
<td>10</td>
<td>Effect of Area on Current Density for P₁=1.0 Torr</td>
</tr>
<tr>
<td>11</td>
<td>Effect of Area on Current Density for P₁=0.1 Torr</td>
</tr>
<tr>
<td>12</td>
<td>Nondimensional Current Density as a Function of Rₚ/dₛ</td>
</tr>
<tr>
<td>13</td>
<td>Axial Current Density Distribution for P₁=0.1 Torr</td>
</tr>
<tr>
<td>14</td>
<td>Axial Current Density Distribution for P₁=1.0 Torr</td>
</tr>
<tr>
<td>15</td>
<td>Comparison of Experimental and Theoretical Current Density Collection for P₁=0.1 Torr</td>
</tr>
<tr>
<td>16</td>
<td>Comparison of Experimental and Theoretical Current Density Collection for P₁=1.0 Torr</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

d  thickness

e  electron charge

J  current density

k  Boltzmann constant

L  probe length

m  mass

n  particle density

T  temperature

u  velocity

V_P  probe bias

V_{TH}  thermal velocity

X_2  streamwise distance

\delta  boundary layer thickness

\lambda  mean free path

\varepsilon_o  permittivity of free space

\omega  mobility

\nu  kinematic viscosity

Subscripts

e  electron

i  ion

n  neutral

0  free stream (behind incident shock)

s  sheath
I. INTRODUCTION

The plasma associated with a vehicle entering the earth's atmosphere at hypersonic velocity is the source of a number of observable phenomena. Among these phenomena is the change in radar cross-section of the entering vehicle, the communication difficulties with the vehicle, and recognition difficulties in the case of ICBMs, in terms of discrimination between actual, lethal missiles and accompanying penetration aids. One of the important observables dealt with in this context is the ionized particle number density. Since on entry, in addition to the air, ablating products are included, the chemistry and composition of the gases surrounding the entry vehicle and found in the wake become extremely complicated. Thus, the only recourse is to experimental research and the problem of diagnostic instruments therefore becomes one of major importance.

Among a variety of diagnostic instruments utilized to measure ionized particle density in a given plasma, the Langmuir or electrostatic probe has always been the most attractive to the experimentalist, notwithstanding the fact that interpretation of the measurements has not always been trivial. This attraction to the electrostatic probe can be attributed to its simplicity and wide dynamic range. There are basically three types of electrostatic probes in operation today; they are the spherical, cylindrical and flush mounted probes. Sufficient theoretical and experimental work has been done on spherical and cylindrical probes operated in different regimes to permit the experimentalist to relate the collected current density to the free stream ionized particle density.¹⁻⁴ In the case of the flush mounted electrostatic probe, this is not the case. The few theoretical or experimental
results available under certain conditions and specific cases.

The analysis of Chung of a one-dimensional geometry probe in a frozen flow, with no connection, is of special interest because the author reduced his results to a simple equation relating the current density to the free stream ionized particle number density.

Breitfeldt and Scharfman proposed another theory based on the planar space charge limited, mobility controlled, diode equation. The experiments conducted by Scharfman indicated that the measured current densities were bounded by the current densities predicted by the Breitfeldt-Scharfman and Chung equations. The above was based on the assumption that the collected current represents the saturation current.

In most of the work reported and known to the authors, the question of what constitutes ion saturation, the effect of increasing bias, the effect of geometry and orientation of the probes and the effect of size on the current collection has not been sufficiently explored.

In the present work an attempt is being made to answer some of these questions in a systematic way. To that end a 6" diameter pressure driven shock tube has been utilized. The tests were conducted in the ionized flow behind the incident shock where a continuum regime existed.

II. REVIEW OF THE CHUNG AND BREITFELDT-SCHARFMAN THEORIES

As indicated above, for theories have been advanced in recent years that deal with the operation of flush electrostatic probes in a continuum regime. In these treatments, one or more of the following assumptions have been made in order to simplify the fluid-electromagnetic equations. This resulted in solutions which are of limited applicability.
The primary assumption is, of course, that continuum flow holds throughout the flow field; i.e., all mean free paths are much smaller than the probe dimension. The next most common assumption made in these analyses is that the electron temperature is known at the edge of the velocity boundary layer. Third, the electron and ion temperatures are assumed constant or to form the same ratio within the flow field. Fourth, recombination effects are neglected on the probe surface; and fifth, charge separation does not occur to any high degree except near the probe surface.

One such theory which was developed, and which was not limited to any of the special non-typical flow fields arrived at by invoking many of the previous assumptions, was developed by Chung. This theory was applied to the stagnation point boundary layer flow of slightly ionized gases.

Later, Chung and Blankenship recast this analysis into a more useable form for determining free-stream charge density from measurements of net collected current, by considering the problem posed by parallel plate electrostatic probes. Further restrictions were made in order to obtain a simple form for the solution, namely, frozen boundary layer flow, constant ion mobility, low applied bias and knowledge of peak charge density. The result of this analysis was given by

\[ n_{e\infty} \sim \frac{2^{3/2} (X_s U_{\infty} V_{\infty})^{1/2} J}{1.00 + 0.94 \left( \frac{T_e}{T_{n\infty}} \right) e} \]  

\[ \delta = \left( \frac{V_{\infty} X_s}{U_{\infty}} \right)^{1/2} \]
and a characteristic mean free path defined by

$$\lambda = \frac{3\nu_{\text{th}}}{(v_{\text{th}}/v_{\text{th}})}$$

(3)

where the thermal velocity was given by

$$v_{\text{th}} = \frac{8kT_e/\mu_e}{\pi m_i}$$

(4)

Substituting equations (2) and (3) into equation (1), and expressing in terms of net current density, gives

$$J \sim \frac{1}{3} \left[ 1.00 + 0.94(T_e/T_n) \right] \cdot J[\tau]$$

(5)

where

$$J[\tau] = \frac{n_{e e}eV_{\text{th}}}{4} \cdot \left[ \frac{\lambda}{\delta} \right]$$

(6)

For the case of $T_e = T_n$, expression (5) reduces to

$$J \sim 0.92 \frac{n_{e e}eV_{\text{th}}}{4} \cdot \left[ \frac{\lambda}{\delta} \right]$$

(7)

Physically, this can be interpreted to mean that instead of a simple, collision-free current collection, the flux arriving at the probe is reduced by a factor $\left[ \frac{\lambda}{\delta} \right]$ in diffusing through a thickness $\delta$.

Equation (7), which gives the net current in terms of the reduced current flux, can be rewritten in a form familiar for electrostatic probes in a stationary plasma, namely,

$$J_s = \frac{n_{e e}eV_{\text{th}}}{4} \left[ \frac{\lambda}{\delta} \right]$$

(8)

or

$$J_s \sim n_{e e} \frac{U_{e e}v_{\text{th}}}{X_s}$$

(9)

Several features can be readily noted. First, for given flow conditions and distance from the leading edge, there is a saturation of current collected with applied bias; increases in applied bias once saturation current is reached would have no effect. As determined, experimentally in this work, this conclusion is incorrect. Second,
for given gas properties and charge density, the saturated current varies inversely with the square root of the streamwise distance. This conclusion is also not justified in general. Third, current collection is linearly related to charge density for given gas properties and distance from the leading edge of the flat plate.

Bredfeldt and Scharfman approached the problem posed by flush probe current collection and on the grounds of physical reasoning, supplemented by experimental evidence, arrived at a result similar to that obtained considering a stationary plasma.

Bredfeldt and Scharfman considered the case of current collection for which all mean free paths are less than the sheath thickness and assumed that the full, random thermal flux arriving at the sheath edge is collected by the probe. Furthermore, they assumed that the planar mobility controlled diode equations could be used to locate the sheath edge.

Solution of these equations gave expressions for the sheath thickness and current density in terms of parameters evaluated at the sheath edge; these expressions were

\[ d_s = \left[ \frac{9}{8} \varepsilon_0 \frac{u_i v_i^2}{j_s} \right]^{1/3} \]  

\[ j_s = \frac{n_e e v_i}{4} \]  

The simplicity of these equations is misleading; a velocity and charge density must first be calculated by a suitable boundary layer theory before they can be used, Fig. 1. For highly non-uniform boundary layers, this theory is not applicable, because the assumption that the mobility controlled diode equations can be used to locate the sheath edge is no longer valid.
Bradfeldt and Scharfman justify this result by theorizing that in a collision dominated flow, the charged particles are scattered deeper into the sheath by collisions occurring in the first mean free path after entering the sheath. Collection is thus more assured because the field is stronger near the probe than at the sheath edge.

While this theory does not explicitly reflect a dependence of ion current density upon applied bias, it does attempt to account for the variance between physical probe area and sheath area, which becomes significant at large values of applied bias. It is the flux collecting area of the probe sheath that is of importance.

This method assumes that the ion current is collected over a 90° segment of a cylinder bordering on the perimeter of the probe, with radius given by sheath thickness.

Thus, for probes whose streamwise and cross-stream dimensions are approximately equal,

$$C_D = \frac{A_{\text{sheath}}}{A_{\text{probe}}} = 0.500 + \pi ds \frac{(L+2ds)}{2A_p}$$

and for probes whose streamwise length is much greater than cross-stream width

$$C_D = \frac{A_{\text{sheath}}}{A_{\text{probe}}} = 0.500 + 0.250\pi ds (L+4ds).$$

III. EXPERIMENTAL FACILITY

A conventional double-diaphragm pressure driven shock tube was used to generate the required low density, high enthalpic flow. A schematic of the shock tube facility is presented in Fig. 2. The facility consists of a driver, driven, double diaphragm and test sections, a dump tank, instrumentation, vacuum and pressurization systems.
full description of this facility is given in Ref. 8.

The test section housed a flat plate, Figs. 3 and 4, to which removable model inserts were attached, Figs. 5 and 6. The leading edge of the flat plate was a sharp 20° wedge and the upper surface was mounted at zero angle of attack. All surfaces were machined smooth on the flat plate and the model inserts. A cylindrical Langmuir probe extended ahead of the leading edge of the flat plate, well into the undisturbed flow, as seen in Figs. 3 and 4.

The instrumentation consisted of the various pressure sensing devices, a shock speed timer and the data recording system. Test data were recorded on Tektronix 565 oscilloscopes, each equipped with a Polaroid camera.

The flush electrostatic probes used in these experiments were machined from brass to close tolerance in both size and geometry and imbedded in plastic, which served a dual purpose of support and insulation. A typical probe circuit is shown in Fig. 1. The negative bias furnished by a battery or regulated power supply was applied to the probes in series with a fixed resistor across which the collected ion current was measured. The specific geometries, areas and orientation of the flush probes with respect to the flow are shown in Figs. 5 and 6.

IV. EXPERIMENTAL RESULTS

With the shock tube and probes described previously, several series of tests were conducted. Due to the unreproducibility of the test, traceable to the inhomogeneity of the diaphragm material, the shock Mach number obtained with the 0.1 Torr driven pressure varied from 10.4 to 11.8 and for the 1.0 Torr driven pressure from 8.4 to 9.6.
Before attempting to explain the behavior of the probes under conditions of different bias, area, geometry and position on the plate, the uniformity of the flow field over the plate had to be ascertained. This was done by running a series of tests using a number of probes of the same size and bias, distributed over the plate. It was found that the collected current deviated no more than about 10% from the centerline probe current, on each line of probes. Some representative traces of the recorded raw data are shown in Fig. 7. Having established this, tests were conducted to determine the dependence of the collected ion current on the bias. Due to the Mach number variation between tests, the data obtained were, for the sake of comparison, reduced to a Mach number of 10.3 and 9.3 for the 0.1 and 1.0 Torr cases, respectively. The results of these tests are shown in Fig. 8, where the normalized collected ion current density is plotted vs. bias for the 0.1 and 1.0 Torr of driven pressure for probes of 1/4" diameter. The next series of tests was concerned with the dependence of the collected current density on the size of the probes. To achieve this the bias was maintained constant and the area was varied. The results of these tests are shown in Fig. 9, where the normalized ion current densities are plotted vs. the normalized area for the 0.1 and 1.0 Torr.

The current density as a function of bias with the area as a parameter and normalized to the current density of the smallest probe at the given bias is shown in Figs. 10 and 11 for the 0.1 Torr and 1.0 Torr initial pressure, respectively. The decrease in current density with the increase in area at constant bias is evident in both cases. There is also a slight decrease in current density with increasing bias. This behavior, however, is not monotonic as there is a minimum, then an increase in current density with the increase in bias in the 0.1 Torr initial pressure case.
In the 1.0 Torr case this effect could not be found because of breakdown at higher biases of the larger area probes. The combined effect of bias, sheath thickness and area is shown, for both cases of 0.1 and 1.0 Torr in Fig. 12. In this context, and due to the construction of the test inserts, the question of interference of probes with each other, particularly at high probe biases, arose. To answer this question, a series of tests was conducted with all probes at a given bias and a series with alternate probes grounded to the plate. From the measured current densities, no interference effect between adjacent probes could be detected.

The effect of position of the probe on the plate, on the collected current density is shown in Figs. 13 and 14, where the current density is plotted as a function of position of the probe from the leading edge of the plate with the applied bias as a parameter for the 0.1 and 1.0 Torr of driven air, respectively. It is evident that the prediction of behavior by Chung, based on boundary layer theory, is not completely satisfied. The deviation is quite strong and increases with the bias.

The current density data obtained in the above tests were compared to the current densities predicted by the theories of Chung, Bredfeldt and Scharfman, by relating the free stream ionized particle number densities to their corresponding ion current densities. The results are shown in Figs. 15 and 16 for the 0.1 and 1.0 Torr, respectively. It is evident in both cases that the theory of Chung is applicable at very low biases whereas the simple theory of Bredfeldt and Scharfman is more applicable at high biases. Neither is completely valid. The density calculated using either theory may deviate from the actual density by as much as a factor of 2 or more.
The effect of geometry was explored using circular and rectangular probes of the same area. No distinguishable effect could be detected. This is indicated in Figs. 15 and 16, where the geometry effects are compared for 1/8" diameter circular probes and 1/8" square probes.

V. DISCUSSION OF EXPERIMENTAL RESULTS AND CONCLUSIONS

As indicated above, the aim of this work was to obtain some experimental information on the behavior of flush electrostatic probes in a continuum flow. The effect of bias on current collection is of particular importance, since determination of the ion number density, which is the main objective of using electrostatic probes, can only be accomplished by relating the above.

An implication of Chung's analysis, from the linear relation between the collected ion current density and the free stream ion density, is a saturation ion current, or independence of the collected ion current from the applied bias. This, as seen from the experimental results as presented in Fig. 8, is clearly not the case. The current density increases with increasing bias. This increase is non-linear, depending on the density and degree of ionization as indicated in the figure and evident from Table I.

The effect of area of the collecting probes on the current density is shown in Fig. 9. It is evident that the collected current density decreases with the increase in area. The decrease in current density is also dependent on the degree of ionization. An explanation in terms of ion depletion is not possible, because the collected currents are very small, and the area effect increases with an increase in neutral density. This seems to be pointing towards the effects of mobility and collisions. A decrease of mobility with an increase in density results in a decrease in current density. The combined effect of area and
Sheath thickness is shown in Fig. 12. As can be seen here, the increase of mobility increases the sheath thickness and decreases the current density with an increase in area.

The current collection as a function of position on the plate is shown in Figs. 13 and 14. As is evident from the inspection of these figures, Chung's prediction of the collected current density, based on his boundary layer analysis, is not satisfied. The experimental data reveal a strong dependence on the bias and again on the neutral number density.

The next problem of this experimental investigation was the question of the applicability of the Chung and/or Bredfeldt-Scharfman theories to the reduction of the data. In Figs. 15 and 16, the results of the investigation are shown. Here ion current densities, as a function of the free stream charge densities, are computed using Chung's and Bredfeldt's relations. The actual measured current densities are then plotted vs. free stream ion number density as measured by cylindrical probe mounted in front of the plate. The ion number densities corresponding to the measured shock Mach numbers were thus confirmed. The data obtained with low biases seem to agree with the calculated densities according to Chung, whereas at a higher probe bias the data agreed more with the analysis of Bredfeldt and Scharfman. Neither is completely valid. From the data in Figs. 15 and 16, it is also evident that the geometry of the probes has a very minor effect on the current collection.

In conclusion, from the data presented above, it is evident that the flush mounted probe cannot be considered an absolute type of a diagnostic instrument for the following reasons:

1. The collected ion current density is bias dependent.
2. The collected ion current density is area dependent.

3. The collected ion current density is neutral density dependent (collision dependent).

4. The relationship between the collected current density and the free stream ion density is so complicated that no satisfactory analytical theory exists which is capable of relating the above. This does not mean that the flush electrostatic probe is useless. On the contrary, it is a simple rugged instrument which is capable of providing qualitative data in very harsh environment where the survival of other diagnostic instruments is nonexistent. In many cases, where the regime of operation and the flow field is known, it is also capable of providing valuable quantitative data.

VI. REFERENCES


<table>
<thead>
<tr>
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<th>$P_1 = 1.0 \text{ Torr}$</th>
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<tr>
<td>$T_\infty$</td>
<td>3000 K</td>
<td>3000 K</td>
</tr>
<tr>
<td>$P_\infty$</td>
<td>$1.73 \times 10^{-2} \text{ atm.}$</td>
<td>$1.41 \times 10^{-1} \text{ atm.}$</td>
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<tr>
<td>$M_\infty$</td>
<td>2.94</td>
<td>2.74</td>
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<tr>
<td>Reynolds Number/Unit Length</td>
<td>$7.0 \times 10^2 \text{ /cm}$</td>
<td>$5.0 \times 10^3 \text{ /cm}$</td>
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<tr>
<td>Plate Surface Temperature</td>
<td>300 K</td>
<td>300 K</td>
</tr>
<tr>
<td>Ion Temperature</td>
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<td>3000 K</td>
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<td>Electron Temperature</td>
<td>3000 K</td>
<td>3000 K</td>
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<tr>
<td>Charged Particle Density</td>
<td>$1.55 \times 10^{10} \text{ /cc.}$</td>
<td>$6.0 \times 10^{10} \text{ /cc.}$</td>
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<td>$\lambda_D$ Debye Shielding Distance</td>
<td>$3.0 \times 10^{-3} \text{ cm}$</td>
<td>$1.55 \times 10^{-3} \text{ cm}$</td>
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FIG. 1  FLUSH PROBE SCHEMATIC SHOWING PROBE CIRCUITRY
ION SHEATH AND BOUNDARY LAYER PROFILES ON A FLAT PLATE
FIG. 2

SCHEMATIC DIAGRAM OF THE SHOCK TUBE
FIG. 4 PHOTOGRAPH OF TEST SECTION SHOWING FLAT PLATE, CYLINDRICAL LANGMUIR PROBE AND TEST MODEL IV INSTALLED
FIG. 5  TEST INSERTS I AND II
FIG. 6  TEST INSERTS III AND IV
**FIG 7** TYPICAL ION CURRENT TRACES
FIG. 8 CURRENT-VOLTAGE CHARACTERISTICS
FIG. 9 CURRENT AREA CHARACTERISTICS
$P_i = 1.0 \text{ TORR}$

REFERENCE PROBE WAS
0.125 IN. MAINTAINED AT
CORRESPONDING BIAS

$0 \quad 0.250$
$0 \quad 0.3535$
$0 \quad 0.500$
$0 \quad 0.707$

**FIG. 10 EFFECT OF AREA ON CURRENT DENSITY**

$J/J_{\text{REF}}$ vs $V_p - \text{VOLTS}$
$P = 0.1 \text{ TORR}$

$J_{\text{REF}}$—CURRENT DENSITY
OF 0.125 IN. DIA PROBE
AT CORRESPONDING BIAS

<table>
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<th>SYMBOL</th>
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<tr>
<td>○</td>
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</tr>
<tr>
<td>□</td>
<td>0.3535</td>
</tr>
<tr>
<td>◆</td>
<td>0.500</td>
</tr>
<tr>
<td>△</td>
<td>0.707</td>
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FIG. II EFFECT OF AREA ON CURRENT DENSITY
FIG. 12 NONDIMENSIONAL CURRENT DENSITY AS A FUNCTION OF $R_p/D_s$
$P_1 = 0.1$ TORR \hspace{1cm} $M_S = 10.3$

![Graph showing normalized ion current density distribution](image)

**THEORY OF CHUNG** \hspace{1cm} $J \propto x^{-\frac{1}{2}}$

**BIAS SYMBOL**
- -3 V \hspace{1cm} ○
- -40 \hspace{1cm} □
- -80 \hspace{1cm} ◊
- -160 \hspace{1cm} △
- -240 \hspace{1cm} ▼

**REFERENCE PROBE WAS AT CORRESPONDING BIAS**

**FIG. 13** AXIAL CURRENT DENSITY DISTRIBUTION FOR $P_1 = 0.1$ TORR
$P_1 = 1.0 \text{ TORR}$
$M_S = 9.3$

**FIG. 14** AXIAL CURRENT DENSITY DISTRIBUTION FOR $P_1 = 1.0 \text{ TORR}$
FIG. 15 COMPARISON OF EXPERIMENTAL AND THEORETICAL CURRENT DENSITY COLLECTION FOR $P_i = 0.1$ TORR
FIG. 16 COMPARISON OF EXPERIMENTAL AND THEORETICAL CURRENT DENSITY COLLECTION FOR $P_i = 1.0$ TORR