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THE STABILITY DERIVATIVES OF THE NAVION AIRCRAFT ESTIMATED BY VARIOUS METHODS AND DERIVED FROM FLIGHT TEST DATA

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16. Abstract

For application to a new technique of predicting aerodynamic loads, the FAA sponsored a study with the Flight Mechanics section of the Princeton University Aerospace and Mechanical Sciences Department to determine the accuracy of dynamic stability derivatives as predicted for the North American Navion by several different sources. Using the results of flight tests as the norm, estimates from wind tunnel tests and several texts were compared and found to be accurate to within the order of 20% with a few exceptions. It is believed that the results of this study would be typical of single engine light aircraft with straight wing and tail, propeller propulsion, and low power effects.

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FOREWORD

The material reported herein is the result of a study by several members of the staff and students of the Flight Research Laboratory, Aerospace and Mechanical Sciences Department, Princeton University. The authors especially wish to recognize the substantial part in the project by the following undergraduate students who contributed their senior independent work:

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

C_L	Lift coefficient, $\frac{L}{\frac{1}{2}\rho V^2 S}$
C_m	Pitching-moment coefficient, $\frac{M}{\frac{1}{2}\rho V^2 S \bar{c}}$
C_y	Side-force coefficient, $\frac{Y}{\frac{1}{2}\rho V^2 S}$
C_ℓ	Rolling-moment coefficient, $\frac{L}{\frac{1}{2}\rho V^2 S b}$
C_n	Yawing-moment coefficient, $\frac{N}{\frac{1}{2}\rho V^2 S b}$
C_{y_β}	The side-force derivative; $\partial C_y / \partial\beta$; per radian
$C_{y_{\beta(vt)}}$	The component of C_{y_β} which is attributable to the vertical tail; per radian
C_{y_r}	$\frac{\partial C_y}{\partial(\frac{rb}{2V})}$; per radian
$C_{y_{r(vt)}}$	The component of C_{y_r} which is attributable to the vertical tail; per radian
C_{ℓ_β}	The dihedral effect; $\frac{\partial C_\ell}{\partial\beta}$; per radian
C_{ℓ_r}	Roll-due-to-yaw derivative; $\frac{\partial C_\ell}{\partial(\frac{rb}{2V})}$; per radian
C_{ℓ_p}	Roll damping; $\frac{\partial C_\ell}{\partial(\frac{pb}{2V})}$; per radian
$C_{\ell_{\delta a}}$	Aileron effectiveness; $\frac{\partial C_\ell}{\partial \delta a}$; per radian aileron deflection each side

LIST OF SYMBOLS (continued)

$C_{\ell_{\delta r}}$	Roll-due-to-rudder; $\frac{\partial C_\ell}{\partial \delta_r}$; per radian
C_{n_β}	Directional stability; $\frac{\partial C_n}{\partial \beta}$; per radian
$C_{n_{\beta(vt)}}$	The component of C_{n_β} which is attributable to the vertical tail
C_{n_r}	Yaw damping; $\frac{\partial C_n}{\partial (\frac{rb}{2V})}$; per radian
$C_{n_{r(vt)}}$	The component of C_{n_r} which is attributable to the vertical tail
C_{n_p}	The adverse yaw derivative; $\frac{\partial C_n}{\partial (\frac{pb}{2V})}$; per radian
$C_{n_{\delta a}}$	Aileron yaw; $\frac{\partial C_n}{\partial \delta_a}$; per radian
$C_{n_{\delta r}}$	Rudder effectiveness; $\frac{\partial C_n}{\partial \delta_r}$; per radian
C_{m_α}	Static stability derivative; $\frac{\partial C_m}{\partial \alpha}$; per radian
$C_{m_{\dot\alpha}}$	Angle of attack damping; $\frac{\partial C_m}{\partial (\frac{\dot\alpha \bar{c}}{2V})}$; per radian
C_{m_q}	Pitch rate damping; $\frac{\partial C_m}{\partial (\frac{\dot\theta \bar{c}}{2V})}$; per radian
$C_{m_{\delta_e}}$	Elevator effectiveness; $\frac{\partial C_m}{\partial \delta_e}$; per radian

LIST OF SYMBOLS (continued)

$C_{m_{i_t}}$	Tail effectiveness; $\frac{\partial C_m}{\partial i_t}$; per radian
C_{L_α}	Slope of the lift curve; $\frac{\partial C_L}{\partial \alpha}$; per radian
$C_{m_\alpha} + \frac{C_{L_\alpha} C_{m_q}}{4\mu}$	Static stability or frequency parameter; per radian
$C_{m_{\dot{\alpha}}} + C_{m_q}$	Pitch rate damping parameter; per radian
I_{xx}	Moment of inertia about the x-axis; slug-ft^2
I_{yy}	Moment of inertia about the y-axis; slug-ft^2
I_{zz}	Moment of inertia about the z-axis; slug-ft^2
T_2	The time constant of the spiral mode in Condition II
$(p/r)_d$	The ratio of the amplitude of the roll trace to the yaw trace in the Dutch roll mode
$d\sigma/d\beta$	The sidewash factor
b	Wingspan; ft
$\bar{c}$	Mean aerodynamic chord; (MAC); ft
c_f	Flap chord, ft
v	Velocity; ft/sec
C_L	Airplane lift coefficient
δ_f	Flap angle; positive down; deg
δ_a	Aileron deflection, radians each side

LIST OF SYMBOLS (continued)

The dimensional stability derivatives of the analog equations, contain mass and inertia parameters, like

$$L_{\alpha} \equiv \frac{1}{m}\,\frac{\partial L}{\partial \alpha}$$

$$M_{\dot{\theta}} \equiv \frac{1}{I_y}\,\frac{\partial M}{\partial \dot{\theta}}$$

$$N_{\beta} \equiv \frac{1}{I_z}\,\frac{\partial N}{\partial \beta}$$

$$N_{r} \equiv \frac{1}{I_z}\,\frac{\partial N}{\partial r}$$

The system is the same as in Reference 7.

INTRODUCTION

In connection with a new method (Reference 1) for estimating the structural loads applied to an airplane by flight in turbulence, the FAA has supported this brief study of the estimation of airplane stability derivatives. Numerical values of the latter are needed in formulas for the loads, and the accuracy of the loads estimate directly depends on the accuracy of the stability derivatives.

The new method for gust loads was a simplified one, intended for use by manufacturers of light general aviation aircraft, and so the stability derivatives of interest would be those of light planes with straight, high aspect-ratio wings and conventional tails, and propeller propulsion. The Navion airplane, which has been extensively operated and tested at Princeton University, was to be the subject of the stability derivative study.

The derivatives of the Navion were to be predicted by several different standard, or conventional, methods. They would be compared with full-scale wind-tunnel data recently available, and finally with values deduced from dynamic responses recorded in a special series of flight tests. The latter, of course, would be the norm against which the other estimates would be evaluated for accuracy.

The standard methods to be considered were selected more-or-less arbitrarily to be commonly available textbooks, NACA reports, and USAF DATCOM. Although all of those references call attention to the necessity for judgment and experience in applying their methods, formulas and charts; in the context of the present study it was deemed best to follow literally, without modification, the procedures presented. The estimates derived are what would be calculated by an inexperienced engineer - one who could follow instructions without mistakes, but one without the background or confidence to seek other data or to make arbitrary variations, as might seem desirable to an "old hand" at vehicle aerodynamics. The resulting accuracy is what might be expected from a new engineering graduate given the task to estimate the derivatives without guidance from a senior supervisor.

The numerical values of parameters predicted by the three textbooks were calculated first by a group of students and checked by members of the research staff at Princeton University. Methods and instructions were followed literally. Where alternate methods were given, the recommended ones were selected.

The USAF Datcom estimates were prepared by Douglas personnel, by request of the Air Force. The Datcom methods, of course, cover a wide range of airplane configurations with little emphasis, actually, on conventional light-plane types. Some of the methods are not directly applicable, so that good accuracy in this reference, for this problem, is not necessarily to be expected.

The lateral-directional parameters listed for NACA TR 1098 (Reference 2) do not include the important C_{y_β}, C_{n_β}, and C_{n_r} derivatives. The vertical tail contributes greatly to these, and although the reference describes methods for correcting empirical data, the basic data are not given in the reference. Consistent with the ground-rules of this investigation, the necessary reference to other material is disallowed, and no estimates are possible.

The wind-tunnel estimates of parameters are data taken in full-scale tests at Langley Research Center in 1969 (Reference 3). The airframe was originally an actual Navion, modified for variable tail incidence, special flap deflections, and reverse thrust propeller. For the wind-tunnel tests, an electric motor was installed. The two conditions involved correspond to a moderate cruise at 150 mph, and a landing pattern condition with 1/2 flap and power for level flight at 90 mph.

The flight values of parameters are derived from special flight tests of the Princeton variable-stability Navion, N91566. Techniques and special considerations are discussed in subsequent sections.

In the gust response calculations (Reference 1) both longitudinal and lateral-directional stability derivatives are involved. The following of the two groups are required:

Longitudinal

$$C_{L_\alpha},\ C_{m_\alpha},\ C_{m_{\dot\alpha}},\ C_{m_q}$$

Lateral-Directional

$$C_{y_\beta},\ C_{\ell_\beta},\ C_{n_\beta}$$

$$C_{y_r},\ C_{n_r}$$

and for the vertical tail alone

$$(C_{n_r})_{vt},\ (C_{y_\beta})_{vt},\ (C_{y_r})_{vt},\ (C_{n_\beta})_{vt}$$

Although they are not involved in the simplified calculations of Reference 1, two other derivatives are of general interest in stability calculations, and they are included in the following study, for completeness. They are

$$C_{\ell_r},\ C_{\ell_p}$$

The Navion Airplane

The physical characteristics of the Navion are presented by the three-view drawing of Figure 1 and the following tabulation of dimensions and inertial properties. Two flight conditions are considered, identified below as I and II. The first is a moderate level-flight cruise condition with flaps and gear up, and about 75% power; whereas the second is a landing pattern condition with half-flap deflection and power for level flight. The general characteristics are as follows.

TABLE I - NAVION AIRPLANE DIMENSIONS

Wing	
Area, S	184 ft^2
Sweep	2° 59'46"
Aspect Ratio, A	6.04
Taper Ratio, λ	.54
Mean-Aerodynamic Chord, $\bar{c}$	5.7 ft
Dihedral	7.5°
Incidence Root, i_{w_r}	$+2^\circ$
Incidence tip, i_{w_t}	-1°
Airfoil tip	NACA 6410 R
root	NACA 4415 R
Horizontal Tail	
Area	43 ft^2
Sweep	6°
Aspect Ratio	4.0
Taper Ratio	.67
Airfoil	NACA 0012
Incidence	-3°
Vertical Tail	
Area (above horizontal stabilizer)	12.5 ft^2
Airfoil	NACA 0013.2 Mod (root) NACA 0012.04 Mod (tip)
Fin Offset	2°

Propeller Characteristics

Diameter 84"
Number of blades 2
Side force factor 100

Power Plant

Continental engine; Model #10520B

HP Rating 285 at take-off at 2700 RPM

Control Surfaces

Surface	Area (ft^2)	Deflection (deg)	c_f/α
Flaps (plain)	83.6	40	.24
Stabilizer	30.0	-	-
Elevator	14.1	up 30° down 20°	.23
Aileron	5.4	20°	.18
Rudder	6.0	15°	.391 base .453 tip

Mass and Inertia Characteristics for Data of this Report

Gross Weight	2948 pounds
Center of Gravity	for the flight tests, 27.4% mac for the tabulated derivatives, 25.0% mac
I_x	1284 slug-ft^2
I_y	2773 slug-ft^2
I_z	3235 slug-ft^2

Flight Characteristics

	Condition I	Condition II
Density Altitude	5000 ft	5000 ft
Velocity	240 ft/sec TAS	144 ft/sec TAS
Flaps	0°	20°
Lift Coefficient	.28	.75
Thrust Coefficient, T_c' (est.)	.023	.057

6

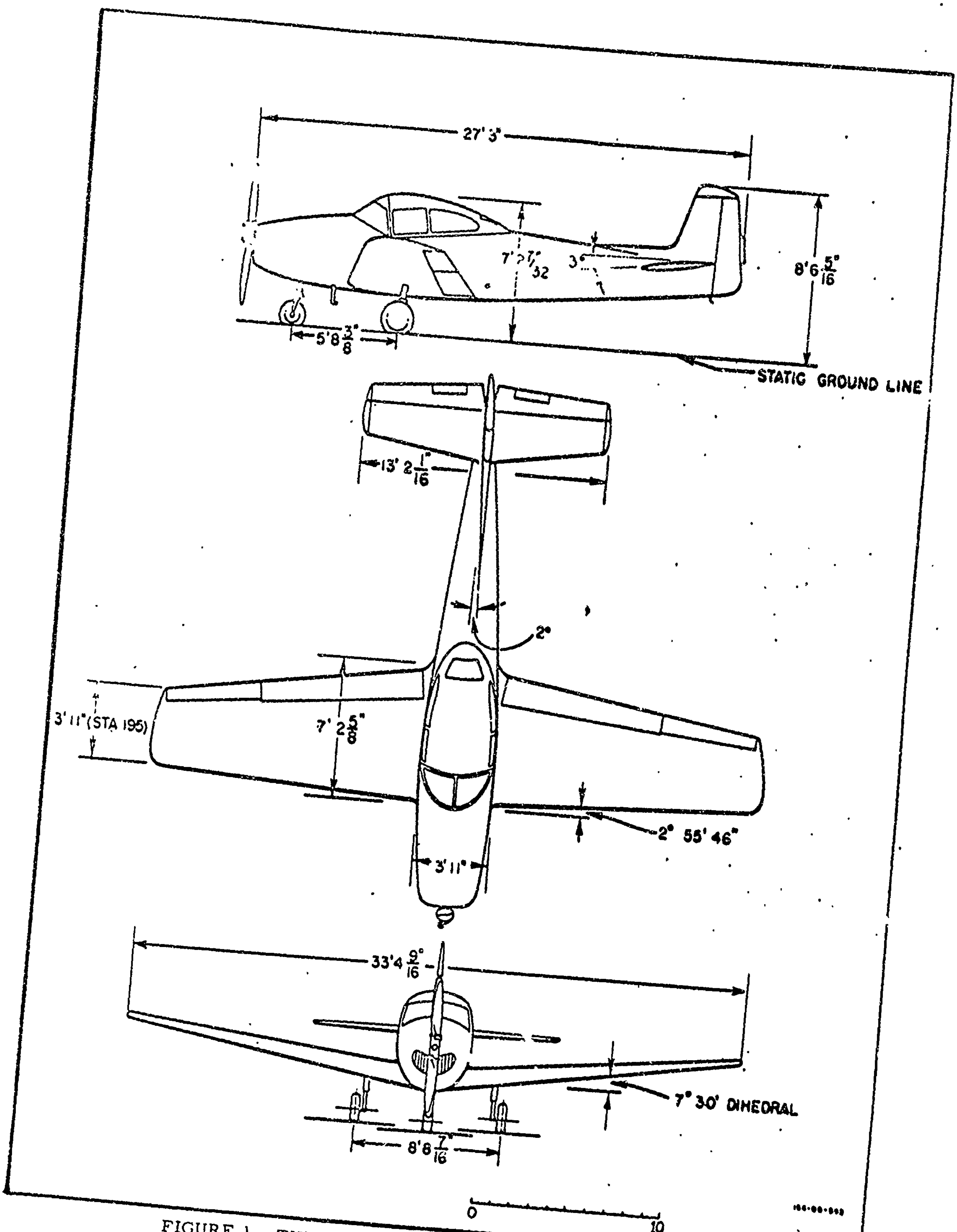

FIGURE 1. THREE-VIEW DRAWING OF NAVION AIRPLANE

Lateral-Directional Degrees of Freedom

1. Flight test derivatives

The matching, on an analog computer, of the flight-recorded transient responses has resulted in the derivatives tabulated in column 7 of Table 2.

The basis for the analog computer model was the set of more-or-less standard equations below:

$$(s - Y_v)\Delta\beta + \Delta r - \frac{g}{V}\Delta\varphi = \frac{1}{V} Y_{\delta_r} \Delta\delta_r$$

$$-L_\beta \Delta\beta - L_r \Delta r + s(s - L_p)\Delta\varphi = L_{\delta_r}\Delta\delta_r + L_{\delta_a}\Delta\delta_a$$

$$-N_\beta \Delta\beta + (s - N_r)\Delta r - N_p s\Delta\varphi = N_{\delta_r}\Delta\delta_r + N_{\delta_a}\Delta\delta_a$$

The various nondimensional derivatives are found, of course, from the parameters of these equations by applying the dimensionalizing factors: airplane mass, moment of inertia, wing area, span, dynamic pressure, etc. The moments of inertia have been separately determined by oscillating the airplane in different pendulum modes, with the following results

$$I_{xx} = 1284 \text{ slug-ft}^2$$

$$I_{zz} = 3235 \text{ slug-ft}^2$$

These are the corrected values for the weight and loading conditions of the flight tests. The product of inertia, I_{xz}, which is omitted from the above, is negligible for this airplane in these flight conditions where the X principal axis almost coincides with the flight path.

A few brief comments about the analog matches are in order (Figures 2 and 3). In the first place, two of the derivatives were determined from sources outside the flight test data. They are $C_{\ell_{\delta_a}}$ and $C_{n_{\delta_a}}$ evaluated from the wind-tunnel data of Reference 3. It was not feasible, because of the short roll time-constant of the airplane, to distinguish in the analog matching between C_{ℓ_p} and $C_{\ell_{\delta_a}}$ and between C_{n_p} and $C_{n_{\delta_a}}$. Because of this redundancy in the analog matches, over fairly wide ranges of values, the effects of changes in the aileron derivatives would be as follows

$$C_{\ell_p} \sim C_{\ell_{\delta a}}$$

$$C_{\ell_\beta} \sim C_{\ell_{\delta a}}$$

$$C_{\ell_r} \sim C_{\ell_{\delta a}}$$

$$\Delta C_{n_p} = \frac{C_{\ell_p}}{C_{\ell_{\delta a}}} \Delta C_{n_{\delta a}}$$

It is considered, however, that the aileron control derivatives would be the most reliable of any determined from the wind-tunnel data, and that the procedure followed is therefore the best that can be done.

The roll-due-to-yaw derivative, C_{ℓ_r}, has been determined from a requirement to match the spiral mode stability observed in special flight tests for the purpose. In Condition I, the spiral mode was exactly neutral; in Condition II, the mode was unstable, T_2 = 10 secs. These characteristics are matched by the derivatives tabulated. In the transients of Figures 2 and 3 the mode is not excited and not visible.

In addition to the transient responses and the special spiral stability runs, steady sideslips were performed to provide equilibrium equations involving static stability and control derivatives. Although in principle these data are capable of removing the redundancy in the determination of derivatives from flight transients; in practice, the necessary solutions were poorly conditioned, and the sideslip data could only be used to check for consistency the derivatives determined from the matching of transients. This check was made, and indeed the steady sideslip equilibrium equations are satisfied to within the accuracies of derivatives estimated below.

The Individual Derivatives (Table 2)

The stability and control derivatives tabulated are discussed individually below.

1) $C_{\ell_{\delta a}}$ The aileron effectiveness has been determined for Condition I directly from full-scale wind-tunnel data. In the table, the asterisk indicates that the flight value is not an independent result.

For Condition II, wind-tunnel data are not available and the flight value is arbitrary. Both $C_{\ell_{\delta a}}$ and C_{ℓ_p} should, however, be little affected by flap deflection; and it is considered to be a confirming factor that, starting with the same $C_{\ell_{\delta a}}$ in both Conditions I and II, essentially the same values of C_{ℓ_p} are found.

The control derivative is not involved in the estimation of loads, due to turbulence, and so its value has not been estimated by the various other references.

2) $C_{n_{\delta a}}$ The aileron yaw, for Condition I, has been evaluated from the wind-tunnel data. Again, the asterisk indicates that the flight value is not independent.

For Condition II, $C_{n_{\delta a}}$ should be different, since it would be affected by flap deflection and difference of lift coefficient. As discussed above, it can not, however, in matching transients, be distinguished from C_{n_p}, and what can be determined is only a linear combination of the two. The particular values tabulated for flight produce a proper match, but the asterisk indicates that they have not been separately identified.

3) $C_{n_{\delta r}}$ The rudder effectiveness is determined uniquely in the match of flight responses by the size of yaw rate response to the rudder pulse. The first three peaks in Figures 2 and 3 have been clipped by saturation of an instrumentation amplifier, and so they are ignored in making the match. The resulting accuracy is considered to be about 10 percent.

This derivative is not involved in the loads due to turbulence, and so estimates by the various references are omitted. The wind-tunnel value is available, however, for Condition I, and the agreement is excellent.

4) $C_{\ell_{\delta r}}$ The roll due to rudder, over a reasonable range of values, has been found to affect the transient responses very little on the Navion. Although a simple estimate was made and provided on the analog model, no real determination was feasible and no estimate of accuracy is possible. For these reasons the derivative is omitted entirely from the table.

5) C_{ℓ_p} Although the roll damping is not explicitly involved in the simplified method of loads estimation, its value is determined quite accurately (say 5 percent, relative to $C_{\ell_{\delta a}}$) by the match of "steady-state" roll rate response to the aileron doublet.

The values of C_{ℓ_p} are adequately predicted by the various references, except that Datcom (Reference 4) underestimates by about 30 percent.

6) C_{ℓ_r} Over the range of plausible values for the Navion, this roll-due-to-yaw derivative has no appreciable affect on the transient responses of Figures 2 and 3. The values tabulated under <u>flight</u> have been calculated so as to match the spiral stability observed in special flight tests for the purpose. The accuracy is perhaps 20 percent.

At least for Condition I, the various estimates appear satisfactory, except that Datcom (Reference 4) overestimates considerably.

7) C_{ℓ_β} The dihedral effect is evaluated, under <u>flight</u>, by matching the size of the roll-rate response to the rudder pulse. This effectively controls also the Dutch-Roll mode-shape, $(p/r)_d$. The accuracy is considered to be quite good, say, 15 percent.

The estimates of the various references are reasonable, with Datcom and Etkin (References 4 and 6) overestimating about 50 percent. Even the wind-tunnel value is too large!

8) C_{n_β} The directional stability parameter is adjusted to match the period of the Dutch-Roll mode. It is well and uniquely determined, with probable accuracy within 10 percent.

There is a wide variation of values from the different references. Seckel (Reference 7) seriously underestimates C_{n_β}, and even the wind-tunnel data yield a value too high by about 25 percent. Inconsistency of the reference estimates might, in fact, be expected from the warnings of Campbell and McKinney (Reference 2) that the important tail contribution can only be evaluated by reference to empirical data.

Although the details are by no means clear, it seems likely that the Seckel formulae underestimate the stable vertical tail contribution and overestimate the unstable fuselage part. Etkin's method (Reference 6) seems to overestimate seriously the tail contribution. A large part of the problem may be inability to evaluate the sidewash factor, $d\sigma/d\beta$, as indeed suggested by Campbell and McKinney.

The Etkin text (Reference 6), which appears to overestimate the tail contribution, leaves to judgment the evaluation of vertical tail area and aspect ratio, and so different users of this reference would find different values of the static stability.

9) C_{n_r} The yaw damping is adjusted to match the damping of the Dutch-Roll mode in the flight transients. Its effect is strong and direct, and the accuracy is considered to be about 15 percent.

Discrepancies among the references are almost as bad as for C_{n_β}. Datcom (Reference 4) apparently overestimates by about 50 percent. The statements of Campbell and McKinney (Reference 2) are again seemingly justified, and again the culprit may be sidewash effects. Etkin (Reference 6) again seems to overestimate the tail contribution.

10) C_{n_p} The adverse yaw derivative is determined by the size of the yaw rate response to the aileron doublet. For Condition I, where a wind-tunnel value of $C_{n_{\delta a}}$ was used, the C_{n_p} value is valid and should be accurate to within about 10 percent. For Condition II, as discussed earlier, the individual values of C_{n_p} and $C_{n_{\delta a}}$ are not really known, as indicated by the asterisk in the table, although a linear combination is determined and represented in the analog model.

None of the various references are very precise on a percentage basis, although in the range of values tabulated, the errors have little influence on the airplane behavior. Perkins and Hage (Reference 5) predicts the right value by neglecting a drag contribution!

11) $C_{y\beta}$ The flight value tabulated for the side-force derivative is derived from the bank-angle to sideslip ratio in steady sideslips. The reduction to $C_{y\beta}$ involves, of course, the side-force due to rudder, which is estimated from $C_{n\delta_r}$ and the tail length for rudder forces. The tabulated value is considered accurate within, say, 10 percent. For Condition II, sideslip data were not available, and the value was arbitrarily set the same as for Condition I. The fact that no valid determination was made is indicated by the asterisk for that case.

No certain explanation of the general discrepancy between the various references and the flight value is available, although it may be assumed that the difference is due to propeller, fuselage, and wing contributions not properly considered. The effects on the airplane would not be significant.

Longitudinal Degrees of Freedom

Longitudinal stability and control derivatives estimated by the various references, and derived from wind-tunnel data and flight test transients, are shown in Table 3. The wind-tunnel data would normally be the standard against which to compare the reference estimates, but in this case, the special flight test data, with certain qualifications, are considered to be preferable.

The flight test derivatives are derived from trial-and-error analog-matching of the transients shown in Figure 4.

The basis for the analog computer model was the following equations

$$[s + (D_v - T_v)]\,\Delta V + [D_\alpha - g]\,\Delta\alpha + g\,\Delta\theta = 0$$

$$\frac{L_v}{V_o}\,\Delta V + [s + \frac{L_\alpha}{V_o}]\,\Delta\alpha - s\,\Delta\theta = -\frac{1}{V_o}\,L_{\delta_e}\,\Delta\delta_e$$

$$-M_v\,\Delta V - [M_{\dot\alpha}\,s + M_\alpha]\,\Delta\alpha + s\,[s - M_{\dot\theta}]\,\Delta\theta = M_{\delta_e}\,\Delta\delta_e$$

$$\Delta n = \frac{V_o}{g}\,(s\,\Delta\theta - s\,\Delta\alpha)$$

The dimensional derivatives above are converted, of course, to their nondimensional counterparts by applying the factors of mass, inertia, area, etc.

For the kind of motion involved in the flight tests, where ΔV was practically negligible, it is well known that the parameters, which are in principle unique, are

C_{L_α}	- slope of lift-curve
$C_{m_{\delta_e}}$	- elevator effectiveness
$C_{m_\alpha} + \dfrac{C_{L_\alpha} C_{m_q}}{4\mu}$	- short-period frequency parameter
$C_{m_{\dot\alpha}} + C_{m_q}$	- short-period damping parameter

For the Navion airplane, besides the redundancy in the individual derivatives, there is the added difficulty that the short period mode is of high frequency and heavy damping, making it difficult to distinguish in practice between the last three parameters above. Ratios of any two to the third are easily found, but unique values of all three are quite difficult to resolve by simple matching. For this reason, C_{m_δ} was separately evaluated by a special flight test involving a difference in elevator position to trim at two different CG positions. This independent determination of C_{m_δ} resolved the above redundancy problem in a satisfactory way. As the Figure 4 shows, the tabulated values provide an excellent match to the flight test responses.

The analog model of the longitudinal dynamics, of course, contains the pitching moment-of-inertia, which has been separately determined from special oscillation tests. The value, for the flight condition and loading of the tests, was

$$I_{yy} = 2773 \text{ slug-ft}^2$$

Although only the "short-period" parameters above were varied in the trial and error matching, all three longitudinal degrees of freedom were provided in the model. The parameters of the drag equation and the coefficients of velocity change, ΔV, were pure estimates and no attempt was made to check their values. This detail is not significant, since it could be seen in the data that the velocity changes, during the transients shown, were very small and seemingly negligible in any case.

Discussion of the individual derivatives follows.

1) C_{m_δ}. The elevator effectiveness controls, in principle, the size of load-factor and pitch-rate responses. However, the static stability parameter

$$C_{m_\alpha} + \frac{C_{L_\alpha} C_{m_q}}{4\mu}$$

is also involved, and in the flight test transients, their effects can not really be separated, as previously noted. This was resolved by a separate flight test for $C_{m_{\delta_e}}$, with the result tabulated. The accuracy of this special result is perhaps 10 percent, and so the perfect agreement with the wind tunnel value is fortuitous.

The control derivative is not required in the estimation of loads due to turbulence, and so values from the various references are not tabulated.

2) C_{L_α} The important slope of the lift-curve is determined by matching the size of the load-factor responses in Figure 4. The accuracy, relative to the control effectiveness, should be excellent, perhaps 10 percent or better.

The estimates from the reference textbooks may be low because they only count wing lift. But of course for wing loads, one should only count the wing contribution, and so for that purpose the estimates may be satisfactory.

3) $C_{m_\alpha} + \frac{C_{L_\alpha} C_{m_q}}{4\mu}$ The static stability, or frequency, parameter could ideally be determined, in the match, by the timing of successive peaks in the pitch rate responses. Especially in Condition I, the natural frequency is so high that the method is insensitive; and so the parameter is used to control the size of pitch rate response instead, for the given $C_{m_{\delta_e}}$. Done in this way, the accuracy should be excellent - say 10 percent.

For Condition II, where the natural frequency was lower, a partly independent determination could be made, although not with good accuracy. In this case $C_{m_{\delta_e}}$ was set to the wind-tunnel value, which was validated in Condition I, and the resulting accuracy of the static stability parameter is estimated at 5 or 10 percent. A confirming feature of the matched responses is the perfect timing of the response peaks in both acceleration and pitch rate.

The various references exhibit an unpleasantly large variation in their estimates of this important stability parameter. The largest error, by Seckel (Reference 7), amounts to an equivalent CG shift of 12 percent mac; while the smallest error, by Datcom (Reference 4) represents about 2 percent mac. It seems probable that the large errors are due to problems of stimating the important stabilizer effectiveness, $C_{m_{i_t}}$. This would be significant in the estimation of horizontal tail loads due to turbulence, as for example in Reference 1.

Under wind-tunnel, the overall value is not directly available. It has been estimated using only the tail contribution to C_{m_q}, from the formula

$$\Delta_t C_{m_q} = 2 \frac{\ell_t}{\bar{c}} C_{m_{i_t}}$$

This qualification is indicated by asterisks.

4) $C_{m_{\dot{\alpha}}} + C_{m_q}$ The rate damping parameter is determined in matching by ratios between peak amplitudes in the pitch rate responses, Figure 4. They are quite sensitive to the parameter, and so the accuracy should be good, say, 10 percent.

Although the textbook formulas predict too small values, the Datcom estimates are a bit too large; but in fact all the estimates are probably satisfactory.

CONCLUSIONS

Based on the study of stability derivatives of the Navion, as estimated by various references and compared to full-scale wind-tunnel and flight test results, the following conclusions are drawn. They are presumed to apply generally to the estimation of those parameters for light single-engine propeller airplanes of conventional configuration with high aspect ratio straight wing and tail, to flight conditions at low angle of attack and where power and slipstream effects are small.

1) The important airplane slope of the lift-curve, C_{L_α}, is underestimated by the textbook references, probably because of neglecting body and tail lift. They may, however, be satisfactory for estimating wing loads.

2) The static stability in angle-of-attack, $C_{m_\alpha} + \frac{C_{L_\alpha} C_{m_q}}{4\mu}$ varies considerably as estimated by the different references. In terms of equivalent CG position, the errors vary from 2 percent to 12 percent chord. The difficulty appears to be with the important stabilizer effectiveness.

3) The rate damping term, $C_{m_{\dot\alpha}} + C_{m_q}$, appears to be underestimated by all the textbook references - by about 20 percent.

4) The directional stability, C_{n_β}, is predicted well by the references except one, where an overestimate of the fuselage contribution and a somewhat low tail term combine to produce a very low overall value.

5) The yaw damping, C_{n_r}, is overestimated badly ($\sim$ 50 percent) by two of the references. The difficulties with C_{n_r} and C_{n_β} are probably at least partly due to sidewash effects on the flow at the vertical tail. These are not adequately treated anywhere, and the statements of Campbell and McKinney, about the need for test data on a similar configuration, seem to be confirmed.

6) The roll damping, C_{ℓ_p}, is adequately predicted by the references save one, which comes in low by about 30 percent.

7) The dihedral effect, C_{ℓ_β}, is overestimated about 50 percent by two of the references. The others are satisfactory.

8) The roll due to yaw, C_{ℓ_r}, is adequately predicted by the references except one which overestimates it by some 90 percent.

9) The yaw due to roll, C_{n_p}, exhibits large fractional errors by the references; but in the flight condition considered, the level of the derivative is small, so that the errors are not very significant.

10) The side force derivative, C_{y_β}, is grossly underestimated by all the references; probably, however, without much affect on the airplane.

11) Particular values of all the derivatives cited above are listed in Tables 2 and 3. Estimates judged to be unacceptably erroneous are circled for easy identification.

12) It is amply clear that the accuracy of any of the cited references, followed literally, would be inferior to that of an "old hand" at airplane aerodynamics, who would be guided by experience and related test data to modify the predictions in "ad-hoc" fashion.

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TABLE II - LATERAL DIRECTIONAL DYNAMIC DERIVATIVES

Derivative	Textbooks: Perkins and Hage	Textbooks: Etkin	Textbooks: Seckel	NACA TR 1098	USAF Datcom	Wind Tunnel	Flight Test
C_{y_β}	-	-.28	-.20 -.20	-	-.35	-.77	-.61 -.61*
$(C_{y_\beta})_{vt}$	-	-	-.18 -.18	-	-	-	-
C_{ℓ_β}	-.062 -.062	$\boxed{-.103}$	-.077 -.077	-.072	$\boxed{-.100}$	-.109 -.097	-.067 -.051
C_{ℓ_r}	.070 .195	.084	.067 .141	.062	$\boxed{.130}$	-	.069 .27
C_{ℓ_p}	-.46 -.46	-.39	-.42 -.42	-.42	$\boxed{-.30}$	-	-.46 -.48
C_{n_β}	.073 .073	.094	$\boxed{.033}$ .049	-	.082	.109 .109	.086 .084
$(C_{n_\beta})_{vt}$	.109 .109	.139	.090 .090	-	-	-	-
C_{n_r}	-.087 -.096	$\boxed{-.141}$	-.065 -.076	-	$\boxed{-.120}$	-	-.088 -.163
$(C_{n_r})_{vt}$	-.084 -.084	-.139	-.061 -.061	-	-	-	-
C_{n_p}	-.035 -.098	-.017	-.010 -.023	-.010	-.030	-	-.038 -.141*
$C_{\ell_{\delta a}}$	-	-	-	-	-	.152	.152* .150*
$C_{n_{\delta a}}$	-	-	-	-	-	-.0047	-.0047* -.0013*
$C_{n_{\delta r}}$	-	-	-	-	-	-.077	-.075 -.093

NOTES

1. In each box, the number in the upper left corner is for Condition I and the number in the lower right corner is for Condition II.
2. The enboxed numbers are estimates which are considered to be unacceptably erroneous.
3. The asterisks signify numbers which were not found independently during the analog matching procedure (see Discussion).

TABLE III - LONGITUDINAL DYNAMIC DERIVATIVES

Derivative	Textbooks: Perkins and Hage	Textbooks: Etkin	Textbooks: Seckel	USAF Datcom	Wind Tunnel	Flight Test
C_{L_α}	4.36	4.25	4.54	5.50	4.52	6.04 6.40
C_{m_α}	-.83	-.715 -.872	-.545 -.545	-1.24	-.95 -1.03	-
$C_{m_{\dot\alpha}}$	-3.0	-4.91 -5.23	-4.91 -4.91	-6.58	-	-
C_{m_q}	-9.6	-9.75 -9.75	-9.50 -9.50	-13.29	-	-
$C_{m_{\delta_e}}$	-	-	-	-	-1.42 -1.55	-1.42 -1.55*
$C_{m_{i_t}}$	-1.82	-1.68 -1.68	-1.72 -1.72	-2.87	-2.03 -2.19	-
$C_{m_\alpha} + \frac{C_{L_\alpha} C_{M_q}}{4\mu}$	$\boxed{-1.08}$	$\boxed{-.97}$ -1.12	$\boxed{-.81}$ -.81	-1.68	-1.27* -1.35*	-1.55 -1.37
$C_{m_{\dot\alpha}} + C_{m_q}$	-12.6	-14.7	-14.4 -14.4	-19.8	-	-18.3 -15.5

NOTES

1. In each box, the number in the upper left corner is for Condition I and the number in the lower right corner is for Condition II.

2. The enboxed numbers are estimates which are considered to be unacceptably erroneous.

3. The asterisks signify numbers which were not found independently (see Discussion).

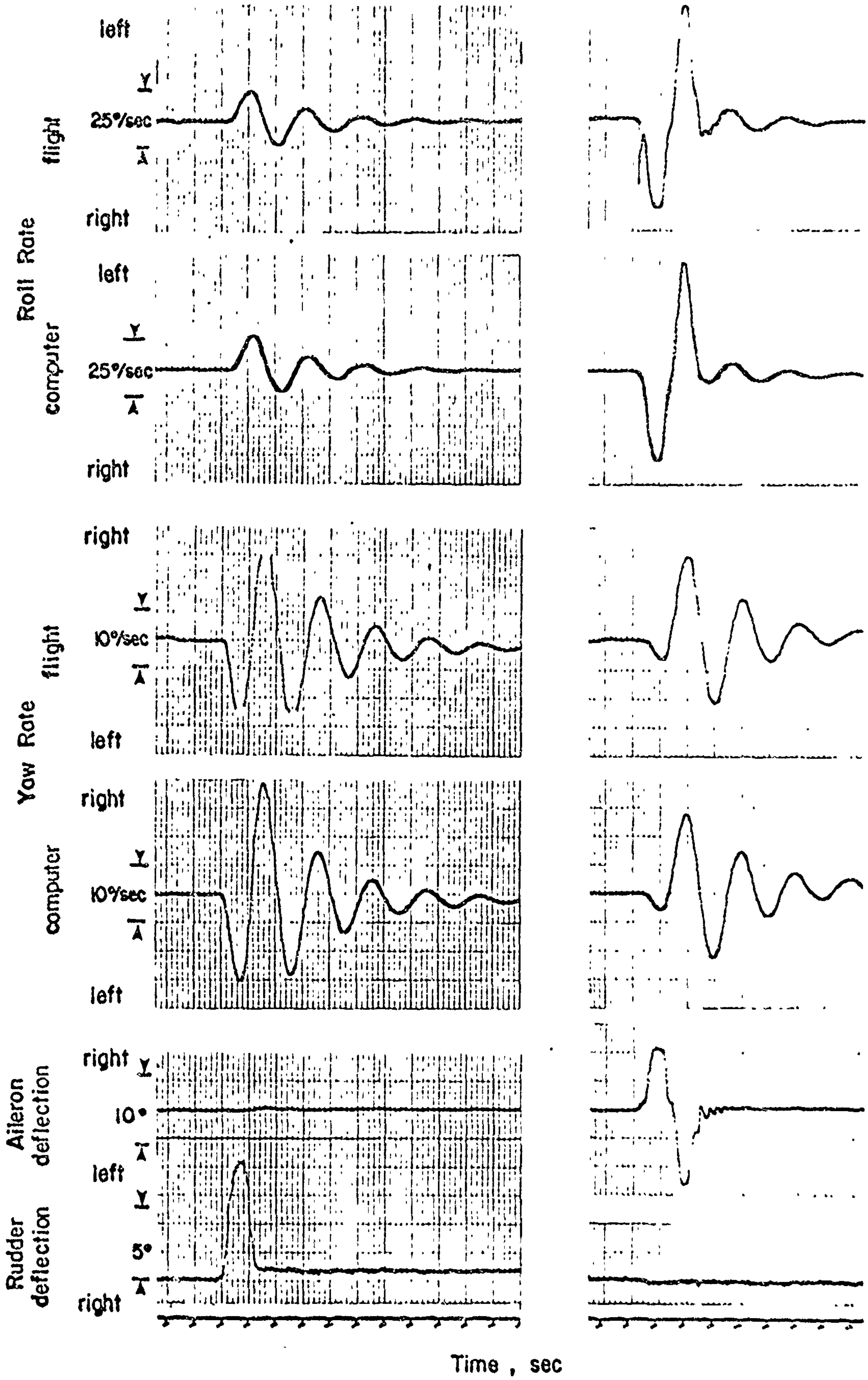

FIGURE 2 ANALOG COMPUTER MATCH OF CONTROL RESPONSES; CONDITION I
FLAPS UP , $C_L = .28$
LATERAL - DIRECTIONAL MOTION

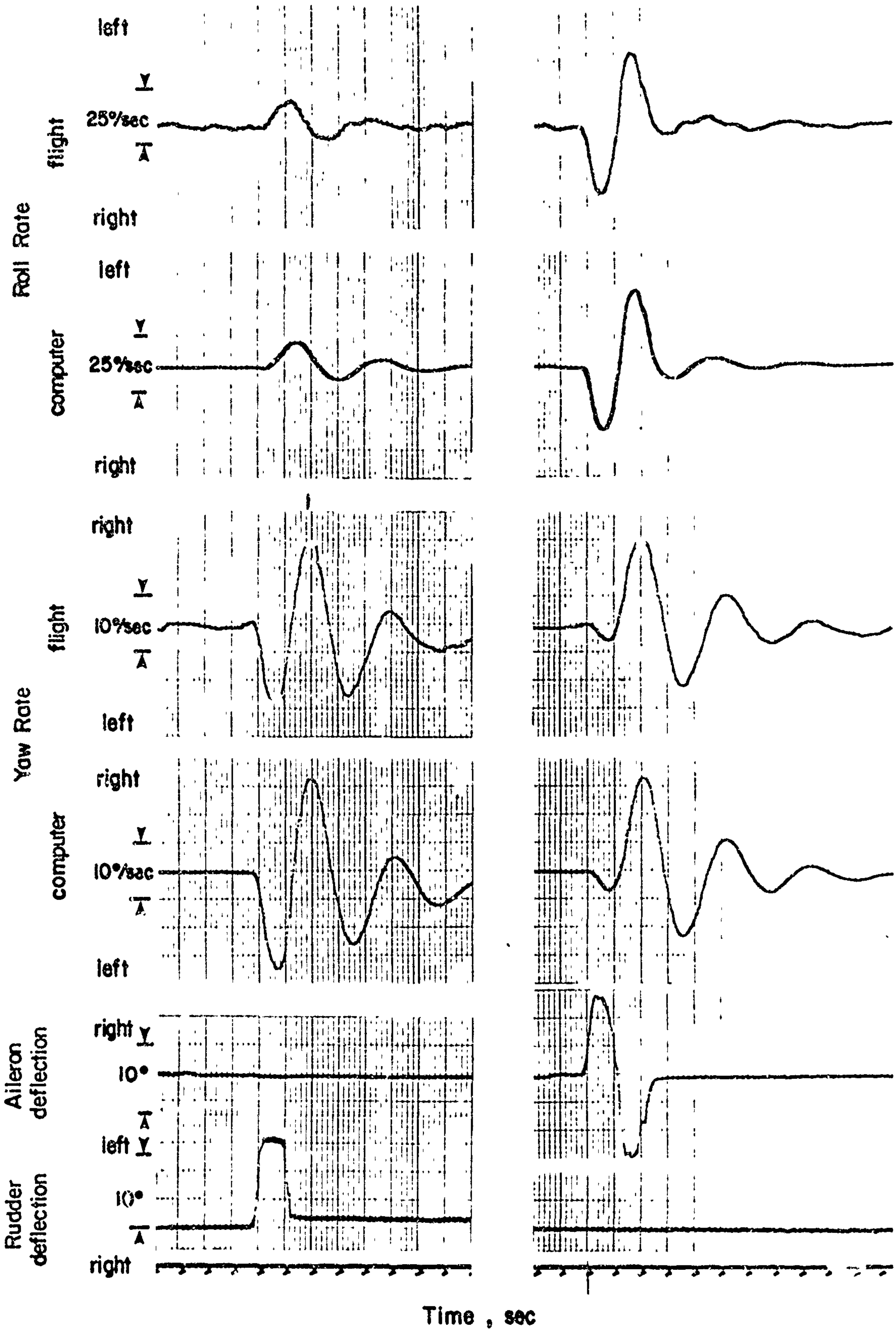

FIGURE 3 ANALOG COMPUTER MATCH OF CONTROL RESPONSES ; CONDITION II, $\delta_f = 20°$, $C_L = .75$
LATERAL - DIRECTIONAL MOTION

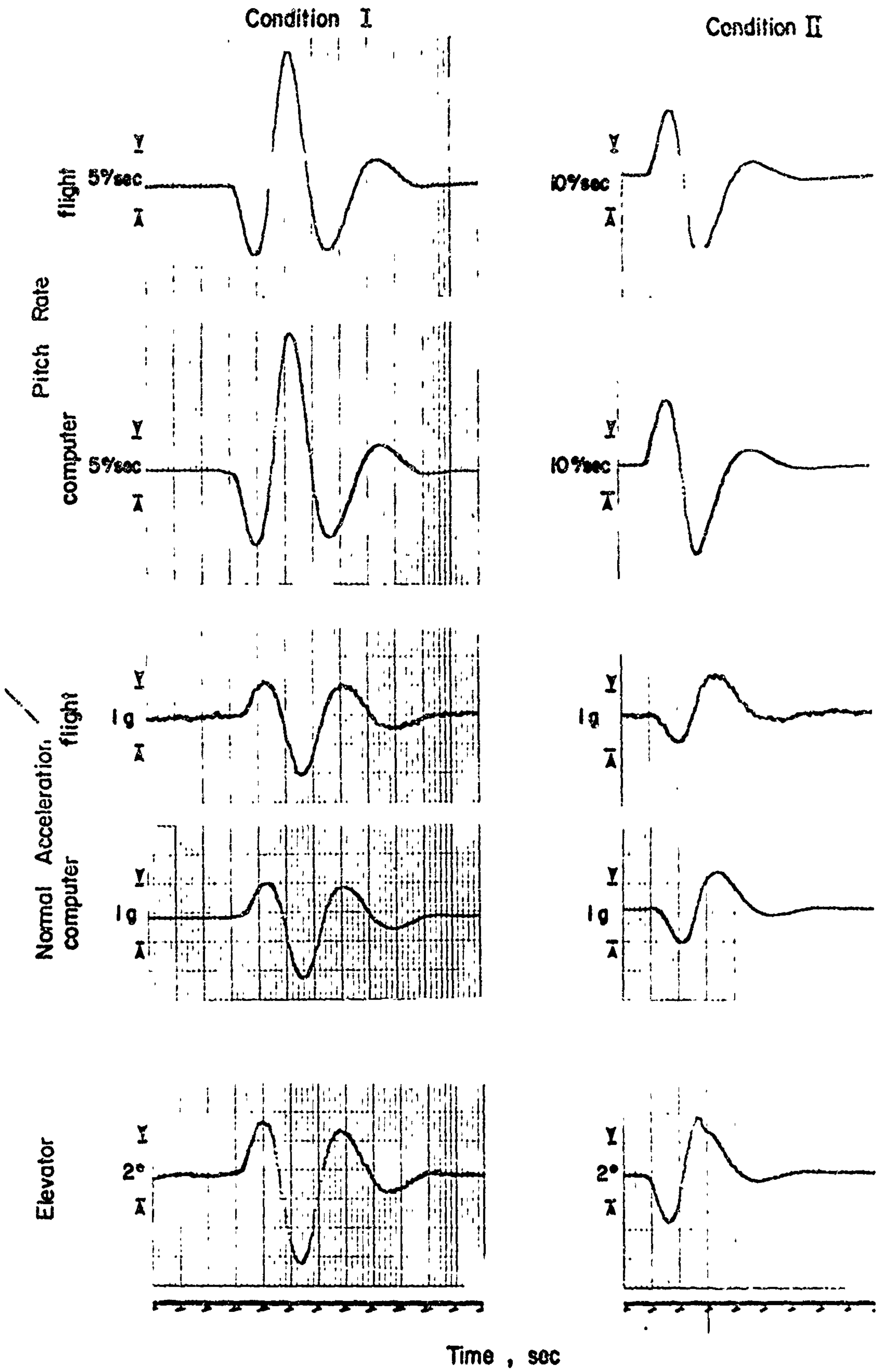

FIGURE 4 ANALOG COMPUTER MATCH OF CONTROL RESPONSE LONGITUDINAL MOTION

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